

**MODULAR MULTIPLICATIVE DIVISOR LABELING
OF COMPLETE BIPARTITE GRAPH $K_{m,n}$**

Submitted in partial fulfillment of the requirements for the award of
Master of Science degree in Mathematics (M.Sc. Mathematics)

By

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**DEPARTMENT OF MATHEMATICS
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**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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DEPARTMENT OF MATHEMATICS BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **R.Radhika (39630009)** who carried out the project entitled **MODULAR MULTIPLICATIVE DIVISOR LABELING OF COMPLETE BIPARTITE GRAPH $K_{m,n}$** under my supervision from August 2020 to April 2021.

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DECLARATION

I **R.Radhika (39630009)** hereby declare that the project report entitled **MODULAR MULTIPLICATIVE DIVISOR LABELING OF COMPLETE BIPARTITE GRAPH** done by me under the guidance of **DR. R.REVATHI** (Internal) and__ (External) at **Sathyabama Institute of Science and Technology** is submitted in partial fulfillment of the requirements for the award of master of science degree in Mathematics.

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ABSTRACT:

A graph $G(V, E)$ with vertex set V is said to have modular multiplicative divisor labeling if there exist a bijection $f: V(G) \rightarrow \{1, 2, \dots, |V|\}$ and the induced function $f^*: E(G) \rightarrow \{0, 1, 2, \dots, |V|-1\}$ where $f^*(uv) = f(u)f(v) \bmod |V|$ such that $|V|$ divides the sum of all edge labels of G . In this paper we study MMD labeling of complete bipartite graph and some open problems.

KEYWORDS:

MMD labeling, graph labeling, arbitrary super subdivision graph.

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LIST OF SYMBOLS

$K_{m,n}$ - Complete Bipartite Graph

$|V|$ - Number of vertices of a graph G

$|E|$ - Number of edges of a graph G

C_n - Cycle graph with n vertices

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CHAPTER-1

1 INTRODUCTION

1.1 Graph Theory

In mathematics, graph theory is the study of graphs, which are mathematical structures used to model pairwise relations between objects. A graph in this context is made up of vertices which are connected by edges. A distinction is made between undirected graphs, where edges link two vertices symmetrically and directed graphs, where edges link two vertices asymmetrically. Other variations in the types of graph that are commonly considered can be found in graphs [23,25] in discrete mathematics. Graphs are one of the prime objects of study in discrete mathematics.

V and E are usually taken to be finite, and many of the well-known results are not true (or are rather different) for infinite graphs because many of the arguments fail in the infinite case. Moreover, V is often assumed to be non-empty, but E is allowed to be the empty set. The order of a graph is $|V|$, its number of vertices. The size of a graph is $|E|$, its number of edges. The degree or valency of a vertex is the number of edges that are incident to it, where a loop is counted twice.

In the edge xy directed from x to y , the vertices x and y are called the endpoints of the edge. The edge yx is called the inverted edge of xy . The edge is said to join x and y and to be incident on x and on y . A vertex may exist in a graph and not belong to an edge. A loop is an edge that joins a vertex to itself. Multiple edges are two or more edges that join the same two vertices.

1.2 Graph Labeling

Graph labeling is one of the fascinating areas of graph theory with wide ranging applications. Graph labeling was first introduced in the 1960's. A graph labeling is an assignment of integers to the vertices or the edges, or both, subject to certain conditions. If the domain is the set of vertices, then the labeling is called the vertex labeling. If the domain is the set of edges, then the labeling is called the edge labeling. If the labels are assigned to the vertices and also to the edges of a graph, such a labeling is called total labeling. An enormous body of literature has grown around graph labeling in the last four decades.

when the edge labels are members of an ordered set (e.g., the real numbers), it may be called a weighted graph.

when used without qualification, the term labeled graph generally refers to a vertex-labeled graph with all labels distinct. Such a graph may equivalently be labeled by the consecutive integers $\{1, 2, \dots, |V|\}$ where $|V|$ is the number of vertices in the graph. For many applications, the edges or vertices are given labels that are meaningful in the associated domain. For example, the edges may be assigned weights representing the "cost" of traversing between the incident vertices.

In the above definition a graph is understood to be a finite undirected simple graph. However, the notion of labeling may be applied to all extensions and generalizations of graphs. For example, in automata theory and formal language theory it is convenient to consider labeled multigraphs, i.e., a pair of vertices may be connected by several labeled edges.

1.3 Applications of Graph Labeling

Labeled graphs provide mathematical models for a broad range of applications. The qualitative labeling of a graph elements have been used in diverse fields[15,19] such as conflict resolutions in social psychology, energy crises etc. Quantitative labeling of graph elements[1,2,3] have been used in missile guidance codes, radar location codes, coding theory, x-ray crystallography, astronomy, circuit design, communication network addressing, data base management, secret sharing schemes, models for constraint programming over finite domains, [10], [6], [17] and network passwords [8] and [18]. Wang, B. Yao, and M. Yao [16], graph labelings are used for incorporating redundancy in disks, designing drilling machines, creating layouts for circuit boards, and configuring resistor networks.

CHAPTER-2

2.1 LITERATURE SURVEY

In 2000 G.Sethuraman and P.Selvaraju [13] have shown that Arbitrary supersubdivision of paths are graceful and for every cycle $C_n, n \geq 3$ there exist Super subdivision of C_n which are graceful.

In 2005 Sundaram, Ponraj, and Somasundaram introduced [20,21,22] the notion of product cordial labelings. A product cordial labeling of a graph G with vertex set V is a function f from V to $\{0, 1\}$ such that if each edge uv is assigned the label $f(u)f(v)$, the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1

In 2014 R Revathi and Rajeswari have shown that some path related graphs admits modular multiplicative divisor labeling and also studied the upper bound for the number of edges in a modular multiplicative divisor graphs of order $n > 3$, $n \equiv 1 \pmod{2}$ and $n \equiv 1, 2 \pmod{3}$. We investigated the upper bound for number of edges $\lambda(n)$ in a modular multiplicative divisor graph of order $n > 3$, for $n \equiv 0 \pmod{2}$ and $n \equiv 3 \pmod{6}$.

In 2017 Kanani and Chhaya [29] and [5,7,9] prove the following graphs are strongly multiplicative: the total graph, splitting graph, shadow graph of paths, triangular snakes, splitting graphs of stars and bistars.

In 2018 R Revathi and S Ganesh [4] have shown that MMD labeling for bistar related graphs and also studied the structural property of shadow graph of bistar and [11,12,14] investigated the maximum number of edges $\lambda(n)$ in a MMD graph of order $n > 3$, except $n \equiv 1 \pmod{2}$ and $n \equiv 1, 2 \pmod{3}$. Also to find for $r \neq s$, MMD labeling of bistar $B_{r,s}$.

In 2019 R.Revathi and R Mary Jeya Jothi have shown that the existence of MMD labeling of corona of star S_n with disjoint union of $2m$ copies of K_1 and MMD labeling of the join of C_n^+ (adding pendant edge for each vertex of the cycle C_n) with disjoint union of $2m$ copies of K_1 .

Acharya [24,26] proved that every graph can be embedded as an induced subgraph of a graceful graph and a connected graph can be embedded as an induced subgraph of a graceful connected graph, Graham and Sloane [27,28,29] defined a graph G with q edges to be harmonious if there is an injection f from the vertices of G to the group of integers modulo q such that when each edge xy is assigned the label $(f(x) + f(y)) \pmod{q}$, the resulting edge labels are distinct

CHAPTER 3

3.1 DEFINITIONS

3.1.1 Bipartite Graph

A Bipartite graph is a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent.

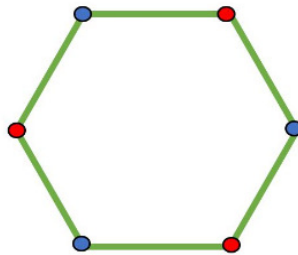


Fig 3.1.1 Bipartite graph

3.1.2 MMD Labeling

A graph $G (V,E)$ with vertex set V is said to have modular multiplicative divisor labeling if there exist a bijection $f:V(G) \rightarrow \{1,2,\dots,|V|\}$ and the induced function $f^*:E(G) \rightarrow \{0,1,2,\dots,|V|-1\}$ where $f^*(uv) = f(u) f(v) \bmod |V|$ such that $|V|$ divides the sum of all edge labels of G .

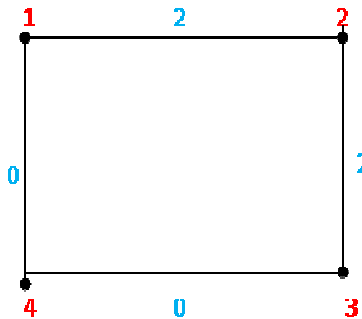


Fig 3.1.2 MMD Graph

sum of all edge labels = $0+2+2+0 = 4$ which is multiple of 4(=p)

3.1.3 Arbitrary super subdivision graph

A super subdivision H of a graph G is said to be an arbitrary super subdivision of a graph of G if every edge of G is replaced by an arbitrary $K_{2,m}$ (m may vary for each edge arbitrarily)

Example 3.1.1

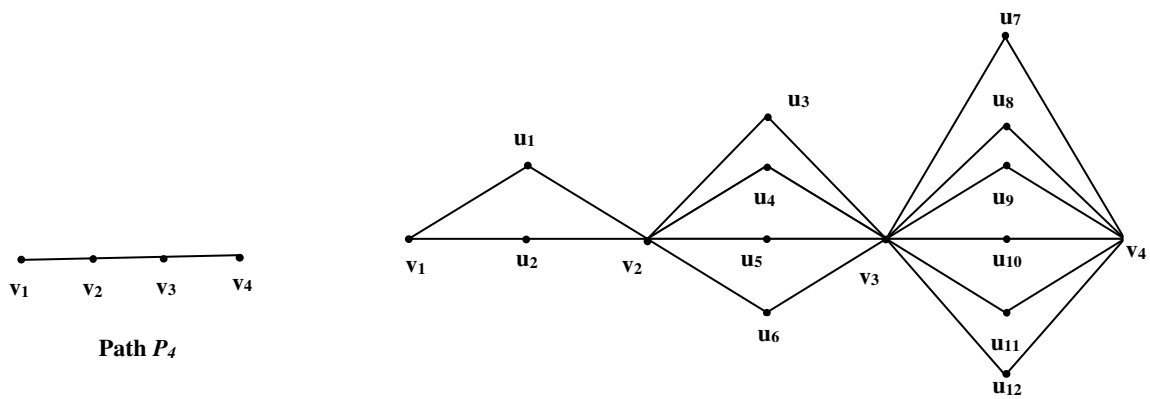


Fig 3.1.3 Arbitrary super subdivision Graph

3.2 MAIN RESULT

3.2.1 THEOREM: Arbitrary super subdivision of a bipartite graph admits MMD labeling

PROOF:

Let $K_{m,n}$ be a bipartite graph with $a_1, a_2, \dots, a_m$ vertices in m-vertices part and $b_1, b_2, \dots, b_n$ vertices in n-vertices part of $K_{m,n}$. As the graph is simple we have at most mn edges $u_1v_1, u_1v_2, \dots, u_nv_m$. Let w_k^{11} be the subdividing vertices of u_1v_1 $1 \leq k \leq x_1$ and let $w_k^{12}, 1 \leq k \leq x_2$ be the subdividing vertices of u_1v_2 . Assuming like this we get,

EDGE	SUBDIVIDING VERTICES OF EDGES	NO.OF SUBDIVIDING VERTICES OF EDGE
------	----------------------------------	---------------------------------------

$u_1 v_1$	$w_k^{11} \quad 1 \leq k \leq x_1$	x_1
$u_1^n v_2$	$w_k^{12} \quad 1 \leq k \leq x_{(n-1)m+1}$	$x_{(n-1)m+1}$
$u_1 v_2$	$w_k^{n2} \quad 1 \leq k \leq x_{(n-1)m+2}$	$\cdot$
$\cdot$	$\cdot$	$\cdot$
$u_1^n v_m$	$w_k^{1m} \quad 1 \leq k \leq x_{nm}$	x_{nm}
$u_2 v_1$	$w_k^{21} \quad 1 \leq k \leq x_{m+1}$	x_{m+1}
$u_2 v_2$	$w_k^{22} \quad 1 \leq k \leq x_{m+2}$	$\cdot$
$\cdot$	$\cdot$	$\cdot$
$\cdot$	$\cdot$	$\cdot$
$u_2 v_m$	$w_k^{2m} \quad 1 \leq k \leq x_{2m}$	x_{2m}
$\cdot$	$\cdot$	$\cdot$
$\cdot$	$\cdot$	$\cdot$
$u_{n-1} v_m$	$w_k^{(n-1)m} \quad 1 \leq k \leq x_{(n-1)m}$	$x_{(n-1)m}$

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Let $S_1, S_2, \dots, S_m, S_{m+1}, S_{m+2}, \dots, S_{2m}, \dots, S_{(n-1)m}, S_{(n-1)m+1}, \dots, S_{nm}$ be the sets consisting of subdividing vertices of the edges $u_1 v_1, u_1 v_2, \dots, u_1 v_m, u_2 v_1, u_2 v_2, \dots, u_2 v_m, \dots, u_{n-1} v_m,$

$u_{n-1} v_1, u_{n-1} v_2, \dots, u_{n-1} v_m.$

That is

$$S_1 = \{w_1^{11}, w_2^{11}, w_3^{11}, \dots, w_{x_1}^{11}\}$$

$$S_2 = \{w_1^{12}, w_2^{12}, w_3^{12}, \dots, w_{x_2}^{12}\}$$

$\cdot$

$\cdot$

$$S_m = \{w_1^{1m}, w_2^{1m}, w_3^{1m}, \dots, w_{x_m}^{1m}\}$$

$$S_{m+1} = \{w_1^{21}, w_2^{21}, w_3^{21}, \dots, w_{x_{m+1}}^{21}\}$$

$$S_{m+2} = \{w_1^{22}, w_2^{22}, w_3^{22}, \dots, w_{x_{m+2}}^{22}\}$$

.

.

$$S_{2m} = \{w_1^{2m}, w_2^{2m}, \dots, w_{x_{2m}}^{2m}\}$$

$$S_{2m+1} = \{w_1^{31}, w_2^{31}, \dots, w_{x_{2m+1}}^{31}\}$$

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.

$$S_{(n-1)m} = \{w_1^{(n-1)m}, w_2^{(n-1)m}, \dots, w_{x_{(n-1)m}}^{(n-1)m}\}$$

$$S_{(n-1)m+1} = \{w_1^{n1}, w_2^{n1}, w_3^{n1}, \dots, w_{x_{(n-1)m+1}}^{n1}\}$$

$$S_{(n-1)m+2} = \{w_1^{n2}, w_2^{n2}, w_3^{n2}, \dots, w_{x_{(n-1)m+2}}^{n2}\}$$

.

.

.

$$S_{(n-1)m+m} = S_{nm} = \{w_1^{nm}, w_2^{nm}, \dots, w_{x_{nm}}^{nm}\}$$

Such that $|S_i| = x_i \quad 1 \leq i \leq nm$. Therefore the cardinality of all subdividing vertices

$$\text{is } \sum_{i=1}^{nm} x_i.$$

Let it be M . Hence the total number of vertices in an arbitrary supersubdivision of a bipartite graph is at most the sum of subdividing vertices and vertices of a bipartite graph.

(ie) $n[V(\text{ASS}(k_{n,m}))] \leq M+n[V(K_{n,m})] = R(\text{say})$. Let each x_i is congruent to 0 (mod 2).
Label the subdividing vertices as follows.

Partition each set S_i into two subsets S_{ia} and S_{ib} with $n[S_{ia}] = n[S_{ib}] = \frac{x_i}{2}$

Label the vertices of S_{1a} from 1 to $\frac{x_1}{2}$, Label the vertices of S_{2a} from $\frac{x_1}{2} + 1$ to $\frac{x_1 + x_2}{2}$

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Proceeding like this, label the vertices of S_{nma} from $\frac{x_1 + x_2 + \dots + x_{nm-1}}{2} + 1$ to $\frac{x_1 + x_2 + \dots + x_{nm}}{2}$.

Label the vertices of S_{1b} from $R - \frac{x_1}{2}$ to $R-1$.

That is $\{R-1, R-2, \dots, R - \frac{x_1}{2}\}$.

Label the vertices of S_{2b} from the set

$$\left\{ R - \left(\frac{x_1}{2} + 1 \right), R - \left(\frac{x_1}{2} + 2 \right), \dots, R - \left(\frac{x_1 + x_2}{2} \right) \right\}$$

proceeding like this, label the vertices of S_{nmb} from the set

$$\left\{ R - \left(\frac{x_1 + x_2 + \dots + x_{nm-1}}{2} + 1 \right), R - \left(\frac{x_1 + x_2 + \dots + x_{nm-1}}{2} + 2 \right), \dots, R - \left(\frac{x_1 + x_2 + \dots + x_{nm}}{2} \right) \right\}$$

Remaining $R-M$ vertices are labeled with

$$\frac{x_1 + x_2 + \dots + x_{nm}}{2} + 1, \frac{x_1 + x_2 + \dots + x_{nm}}{2} + 2, \dots,$$

$$R - \left(\frac{x_1 + x_2 + \dots + x_{nm}}{2} + 2 \right), R - \left(\frac{x_1 + x_2 + \dots + x_{nm}}{2} + 1 \right), R.$$

Let u_1v_1 be an arbitrary edge of $K_{n,m}$. In the ASS of the edge u_1v_1 , edges incident with the vertex u_1 bears the labels

$$f(u_1), 2f(u_1), 3f(u_1), \dots, \frac{x_1}{2} f(u_1), (R-1)f(u_1), (R-2)f(u_1), \dots, (R - \frac{x_1}{2})f(u_1)$$

summing up the labels of the edges incident with the vertex u_1 we get,

$$f(u_1)[(1+R-1)+(2+R-2)+\dots+(\frac{x_1}{2} + R - \frac{x_1}{2})]$$

$$= f(u_1)[R+R+\dots\frac{x_1}{2} \text{ times}]$$

$$= \frac{x_1}{2} f(u_1)R.$$

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Similarly summing up the labels of the edges incident with the vertex v_1

results $\frac{x_1}{2} f(v_1)R$. Adding the weight of the edges incident with the vertices u_1 and v_1

gives the result $\frac{x_1}{2} [f(u_1) + f(v_1)]R$ which is congruent to zero mod R . it is explained in figure 3.1. since the edge u_1v_1 is arbitrary, it is true for any edge of $K_{n,m}$

Hence the Result.

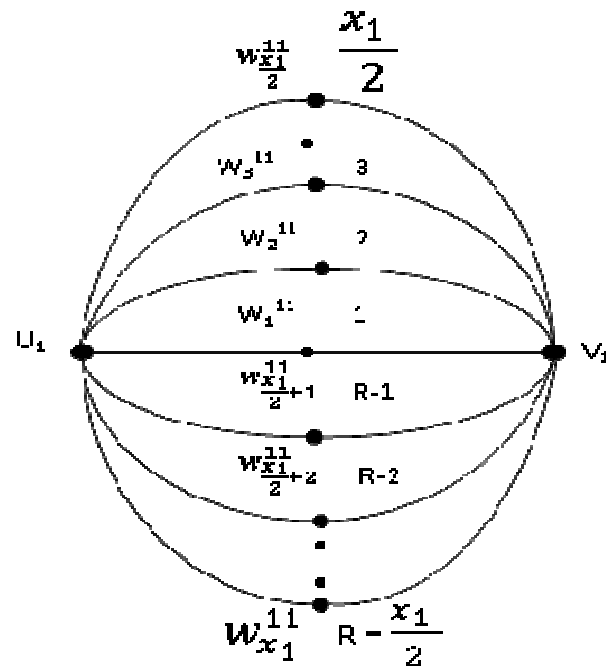


Fig 3.2.1 Arbitrary super subdivision of the edge u_1v_1

REMARK:

If the number of vertices of $ASS(K_{n,m})=R$ then it is a complete bipartite graph.

Example:3.2.1

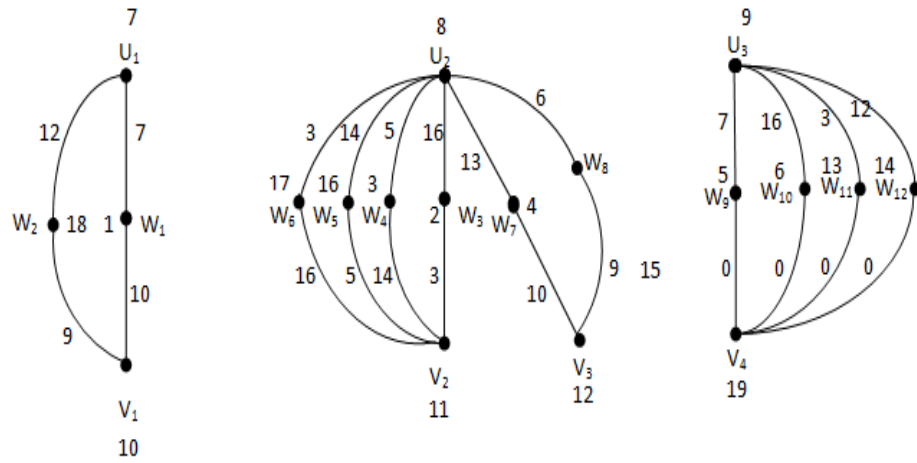


Fig 3.2.2 MMD Labeling of Arbitrary Super Subdivision of $K_{3,4}$

CHAPTER-4

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SUMMARY

4.1 CONCLUSION:

Discussed and found MMD Labeling for a larger family of arbitrary super subdivision of bipartite graphs

OPEN PROBLEM

Does there exist an ASS of $K_{n,m}$ with MMD labeling for all x_i (except the case discussed above)?

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