

# CHROMATIC NUMBER OF SOME PERFECT GRAPHS

Submitted in partial fulfillment of the requirements  
for the award of Master of Science degree in Mathematics

by

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**DEPARTMENT OF MATHEMATICS  
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## **SATHYABAMA**

**INSTITUTE OF SCIENCE AND TECHNOLOGY  
(DEEMED TO BE UNIVERSITY)**

**Accredited with Grade "A" by NAAC | 12B Status by UGC Approved by AICTE  
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**APRIL – 2021**



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**BONAFIDE CERTIFICATE**

This is to certify that this Project Report is the bonafide work of **I. PAVITHRA(39630008)** who carried out the project entitled “**CHROMATIC NUMBER OF SOME PERFECT GRAPH**” under my supervision from **JANUARY 2021** to **April 2021**.

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## DECLARATION

I **PAVITHRA I (39630008)** hereby declare that the Project Report entitled  
“**CHROMATIC NUMBER OF SOME PERFECT GRAPHS**” done by me under the guidance of **Dr.**  
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**TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Master of  
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**PLACE: CHENNAI**



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## ACKNOWLEDGEMENT

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## ABSTRACT

A proper coloring is an assignment of labels, called colors to the vertices of a graph such that no two adjacent vertices share the same color. Let  $\mathbb{T}(V,E)$  be a graph with  $V(\mathbb{T})=\{q_1,q_2,\dots, q_n\}$ . The double graph  $D(\mathbb{T})$  of  $\mathbb{T}$  is the graph with two copies of  $\mathbb{T}$  having vertex sets  $\mathbb{T}_1=\{x_1,x_2,\dots, x_n\}$  and  $\mathbb{T}_2=\{y_1,y_2,\dots,y_n\}$  and the edge set consists of the original edge set of every copy of  $\mathbb{T}$  together with two edges  $x_i,y_j$  and  $x_j,y_i$ , added for every  $q_iq_j \in E(\mathbb{T})$ ,  $1 \leq i \leq n$ ,  $1 \leq j \leq n$ ,  $i \neq j$ . The Strong double graph  $SD(\mathbb{T})$  is a graph constructed from the double graph with  $V(SD(\mathbb{T})) = V(D(\mathbb{T}))$  and  $E(SD(\mathbb{T})) = E(D(\mathbb{T})) \cup \bigcup_{i=1}^n (x_i y_i)$ . This paper discusses proper coloring of Double graph and Strong Double graph of some standard perfect graphs.

## KEYWORDS

- Proper coloring
- chromatic number
- Double graph
- strong double graph
- Perfect graphs

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## LIST OF SYMBOLS

(i)  $P_{(n,1)}$ - Pan graph

(ii)  $T_{(m,n)}$ - Tadpole graph

(iii)  $B_{(m,n)}$  - Barbell graph

(iv)  $K_n$ -Complete graph

(v)  $C_n$  - Cycle graph

(vi)  $D(\mathbb{I})$ -Double graph

(vii)  $SD(\mathbb{I})$  -Strong Double graph

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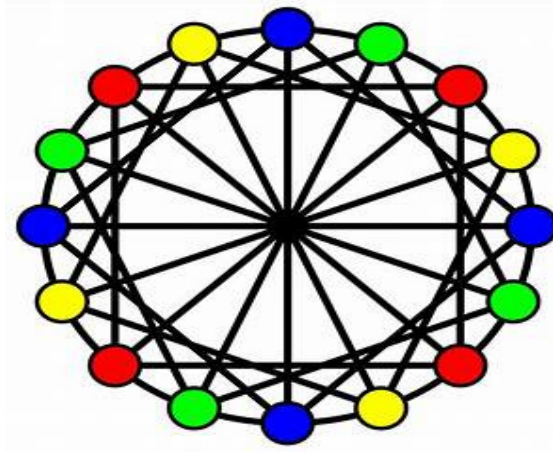
# **CHAPTER-I**

## 1. INTRODUCTION

In graph theory, graph coloring is a special case of graph label is an assignment of labels called “colors” to the vertices and edges of a graph subject to certain constraints. Eulerian refers to the Swiss mathematician Leonhard Euler, who invented **graph theory** in the 18th century. vertex coloring is the process of coloring the vertices of a graph such that no two adjacent vertices share the same color which is also known as proper coloring. The chromatic number of a graph is the smallest number of colors needed to color the vertices so that no two adjacent vertices share the same color. A perfect graph is a graph in which the chromatic number of every induced subgraph equals the order of the largest clique of that subgraph. The graph coloring is applied in Scheduling, transportation, assignment problems and also in biological field like drug targets and determine the role of Proteins and genes .M.Chodnovsky et.al discussed coloring perfect graphs with no balanced skew-partitions[4]. Graph coloring problems is also discussed by Jensen Tommy R[5]. H. Ait Haddadene et.al. gave an algorithm for coloring some perfect graphs[6]. Lovasz ,Loaszlo gave a characterization of perfect graphs.[3][8][10]. Motivated by the above results in this paper proper coloring of Double graph and Strong Double graph of Pan graph, Tadpole graph and Barbell graph were discussed and established relationship between their chromatic number.

## 1.1 GRAPH COLORING

In **graph** theory, **graph coloring** is a special case of **graph** labeling; it is an assignment of labels traditionally called "colors" to elements of a **graph** subject to certain constraints.



**Fig 1.1** Graph Coloring

## 1.2 TYPES OF COLORING

### 1.2.1 FACE AND MAP COLORING

### 1.2.2 LIST COLORING

### 1.2.3 PATH COLORING

### 1.2.4 TOTALCOLORING

### 1.2.5 PROPER COLORING

### 1.2.1 FACE AND MAP COLORING

The face is the method used to color area on a political map, and so it is the coloring associated with the Four Color theorem



Fig 1.1.1 Face and map coloring

### 1.2.2 LIST COLORING

In list coloring every vertex and edges could have different list . In terms of graph theory, a vertex list coloring is a mapping, which assign every vertex a color from the vertex's list such that no two adjacent vertices have the same color.

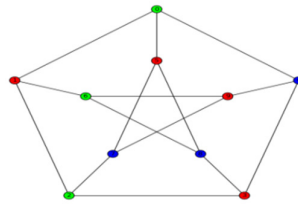


Fig 1.1.2 List coloring

### 1.2.3 PATH COLORING

Path coloring of a graph can be used as a mathematical model of call scheduling in a network, where a set of requests to transfer data between certain nodes is to be handled, and path which have common edges, or intersecting vertices, must be given different color. Another possible application of path coloring is the coloring of tourist

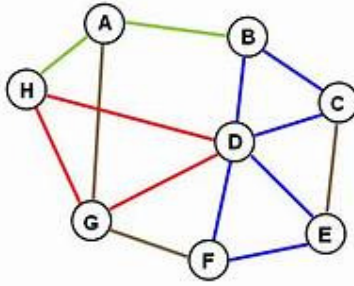


Fig 1.1.3 Path coloring

### 1.2.4 TOTAL COLORING

In the total coloring of a graph, vertices and edges are colored simultaneously. Every two adjacent vertices, any two edges having a common vertex, as well as every incident pair (vertex, edges) must be assigned different colors

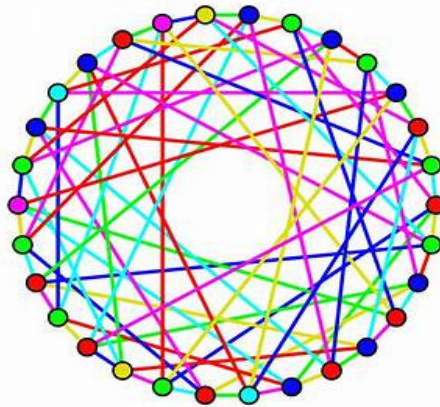


Fig1.1.4 Total coloring

### 1.2.5 PROPER COLORING

In graph theory, graph coloring is a special case of graph label it is an assignment of labels called “colors” to the vertices and edges of a graph subject to certain constraints vertex coloring is the process of coloring the vertices of a graph such that no two adjacent vertices share the same color which is also known as proper coloring

# **CHAPTER-II**

## 2. LITERATURE SURVEY

Berge, Claude has proposed a project called "Perfect graphs" in 1963 by using the concepts of graph theory and found the perfect graph[1].

Golumbic, Martin Charles has proposed a project called "Algorithm Graph Theory and Perfect Graphs" in 2004 by using graph theory concepts and found the perfect graph[2]. Lovasz, Laszlo; Beineke, Lowell W.; Wilson, Robin J.

has proposed a project called "perfect Graphs" in 1983 by using the selected topics in Graph theory and found the perfect graph[3]. M. Chodnovsky, N. Trotignon, T. Trunck and K. Vuskovic

has proposed a project called "coloring perfect graphs with no balanced skew-partitions" in 2015 by using the skew partitions in Graph Theory and found the coloring perfect Graph[4]. Jensen Tommy R., Wiley

undergo "Graph coloring problems" in 1995 and found Graph Coloring[5]. H. Ait Haddadene,

S. Gravier, F. Maffray has proposed a project called "An Algorithm for coloring some perfect graphs" in 1998 and found perfect graphs by using the concept of Graph Theory[6]. Lovasz, Laszlo

has proposed a project called "A characterization of perfect graphs" in 1972 by using the concept of Graph Theory and found the perfect Graph[7]. Chudnovsky, Maria;

Robertson, Neil; Seymour, Paul; Thomas, Robin has proposed a project called "The strong perfect graph theorem" in 2006 by using the concept of Graph Theory and found the strong perfect Graph[8]. Lovasz, Laszlo

has proposed a system called "Normal hypergraphs and the perfect graph conjecture" in 1972 by using the concept of Graph Theory and found the perfect graph[9].

# CHAPTER III

### 3. OUR RESEARCH

#### 3.1 PRELIMINARIES

##### 3.1.1 Perfect graph

In a graph theory, a perfect graph is a graph which the chromatic number of every induced subgraph equals to the order of the largest clique of that subgraph .

$$\chi(G) = \omega(G).$$

##### 3.1.2 Double graph

Let  $\Gamma$  be a graph with  $V(\Gamma) = \{q_1, q_2, \dots, q_n\}$  and the vertex set of double graph  $D(\Gamma)$  are given by the two set  $\Gamma_1 = \{x_1, x_2, \dots, x_n\}$  and  $\Gamma_2 = \{y_1, y_2, \dots, y_n\}$ . For  $q_i \in V(D(\Gamma))$  there are two vertices  $x_i$  and  $y_i$  in  $V(D(\Gamma))$ . The Double graph  $D(\Gamma)$  consist of the original edge set of every Copy of  $\Gamma$  , and for  $q_i q_j \in E(\Gamma)$ , two more edges  $x_i y_j$  and  $x_j y_i$  are added.

##### 3.1.3 Strong Double graph

Let  $\Gamma$  be a graph with  $V(\Gamma) = \{q_1, q_2, \dots, q_n\}$  and the vertex set of double graph  $D(\Gamma)$  are given by the two set  $\Gamma_1 = \{x_1, x_2, \dots, x_n\}$  and  $\Gamma_2 = \{y_1, y_2, \dots, y_n\}$ . The Strong double graph  $SD(\Gamma)$  consist of the original edges and the every copy of  $\Gamma$ , and for  $q_i q_j \in E(\Gamma)$ , more edges  $x_i y_j$ ,  $x_j y_i$  and  $x_i y_i$  are added

##### 3.1.4 Complete graph

A complete graph is a graph in which each pair of graph vertices is connected by a edge. The complete graph is denoted by  $K_n$  and vertices is denoted by "n" and edges is denoted by  $\frac{n(n-1)}{2}$ .

### 3.1.52-Coloring

A graph is 2-colorable if we can color each of its vertices with one of two colors, say red and blue, in such a way that no two red vertices are connected by an edge, and no two blue vertices are connected by an edge. Then every vertex adjacent to its gets colored red

### 3.1.63-Coloring

Let  $x$  be a vertex in any proper 3-coloring of  $G$ , if it exists, the vertex  $x$  either gets the same color as  $v$  or  $x$  receives a different color than  $v$ . therefore it is enough to determine if any of the graphs  $G \cup xv$  are 3-colorable

### 3.1.7. Chromatic number

The smallest number of colors needed to color a graph  $G$  is called its chromatic number. For example, the following can be colored minimum 2 colors.

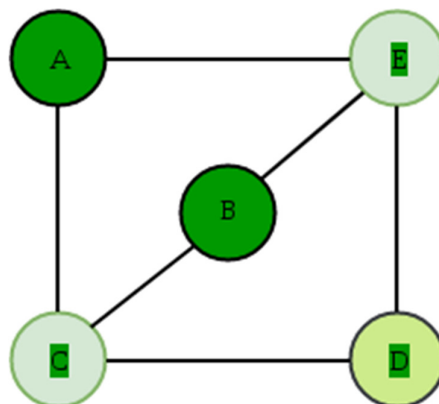


Fig3.1.7 Chromatic number

## 3.2 APPLICATIONS OF GRAPH COLORING

### 3.2.1 AIRCRAFT

Assuming that there are  $k$  aircrafts and they have to be assigned  $n$  flights. The  $i^{\text{th}}$  flight should be during the time interval  $(a_i, b_i)$ . If two flights overlap, then the same aircraft cannot be assigned to both the flights. This problem is modelled as a graph as follows. The vertices of the graph correspond to the flights. Two vertices will be connected, if the corresponding time intervals overlap. Therefore, the graph is an interval graph that can be coloured optimally in polynomial time.

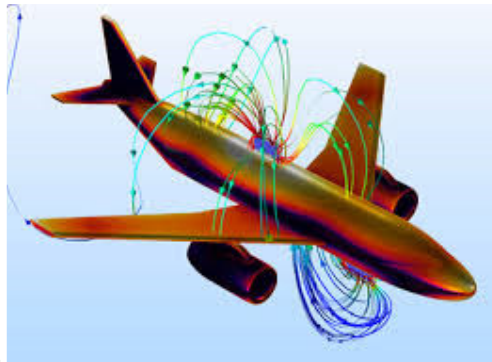


Fig 3.2.1 Aircraft

### 3.2.2 Job Scheduling

Job scheduling is the process of allocating system resources to many different tasks by an operating system. The system handles prioritized job queues that are awaiting CPU time and it should determine which job to be taken from which queue and the amount of time to be allocated for the job.

B. Time table scheduling

Consider a college with  $M$  teachers  $T_1, T_2, T_3, \dots, T_n$  and  $n$  classes  $C_1, C_2, C_3, \dots, C_n$  given that teacher  $T_i$  is required to teach class  $C_j$  for  $P_{ij}$  periods. Schedule a complete time table in the minimum possible numbers of periods. The problem can be formulated as graph theory model in which a bipartite graph  $G = (V, E)$  is devised with  $V = V_1 \cup V_2$  where  $V_1 = T_1, T_2, \dots, T_n$  and  $V_2 = C_1, C_2, \dots, C_n$  and points  $T_i$  and  $C_j$  are joined by  $P_{ij}$  lines. The timetabling problem is equivalent to coloring the points in  $V_1$  so that no two adjacent points have the same colour with as few colours as possible –each Colour representing a distinct period.

### 3.2.3 MAP COLORING

Map coloring is the act of assigning different colors to different features on a map. There are two very different uses of this term. ... The second is in mathematics, where the problem is to determine the minimum number of colors needed to color a map so that no two adjacent features have the same color.



Fig 3.2.3 Map coloring

# **CHAPTER-IV**

## 4. PROPER COLORING OF PAN GRAPH $P_{(n,1)}$

### 4.1. Pan Graph

The pan graph is the graph obtained by joining a cycle graph  $C_n$  to a singleton graph with a bridge. It is denoted by  $P_{(n,1)}$

#### 4.1.1. Proper coloring of Double graph of Pan graph $P_{(n,1)}$

The double graph of n-pan graph with  $2(n+1)$  vertices and  $5(n+1)$  edges admits proper coloring and the chromatic number is

$$\chi(P_{n,1}) = \begin{cases} 2, n \equiv 0 \pmod{2} \\ 3, n \equiv 1 \pmod{2} \end{cases}$$

#### Proof

The n-pan graph ' $P_{n,1}$ ' is the graph obtained by joining a cycle graph  $C_n$  to a singleton graph  $K_1$  with a bridge. It has  $(n+1)$  vertices and  $(n+1)$  edges. Let  $V(P_{n,1}) = \{x_1, x_2, \dots, x_n\}$  and  $E(P_{n,1}) = \{x_i, x_{i+1}, 1 \leq i \leq n-1\} \cup (x_n, x_1) \cup (x_1, x_{n+1})$ . The double graph of  $P_{n,1}$  denoted by  $D(P_{n,1})$  has vertex set given by  $V_1 \cup V_2$  where  $V_1 = \{u_1, u_2, \dots, u_{n+1}\}$  and  $V_2 = \{v_1, v_2, \dots, v_{n+1}\}$ . For every  $x_i \in V(P_{n,1})$  there are two vertices  $u_i$  and  $v_i$  in  $V(D(P_{n,1}))$ . The double graph  $D(P_{n,1})$  consists of the original edge set for every copy of  $P_{n,1}$  and for every  $(x_i, y_j) \in E(P_{n,1})$ , two more edges  $u_i v_j$  and  $v_j u_i$  are added. The proper coloring of  $D(P_{n,1})$  is as follows.

#### Case 1: $n \equiv 0 \pmod{2}$

Starting with the vertex  $u_1$  of the first copy alternatively assign color 1 and 2 around the circular vertices of  $C_n$  and assign color 2 to the vertex of  $K_1$ . Since  $u_i$  and  $v_i, 1 \leq i \leq n+1$  are not adjacent, assign the same color of  $u_i$  for  $v_i$  which gives a proper coloring of double graph of n-pan graph.  $\chi(D(P_{n,1})) = 2$

#### Case 2: $n \equiv 1 \pmod{2}$

If we assign label 1 and 2 alternatively to the vertices of  $C_n$ ,  $V_1$  and  $V_n$  will get same label as  $n$  is odd. Therefore assign color 3 to the vertex  $V_n$ , and assign color 2 to the vertex of  $K_1$ . As  $u_i$  and  $v_i$  are not adjacent, assign same colors to the second copy gives a proper coloring of double graph of pan graph. Therefore  $\chi(D(P_{n,1})) = 3$ ,  $n$  is odd.

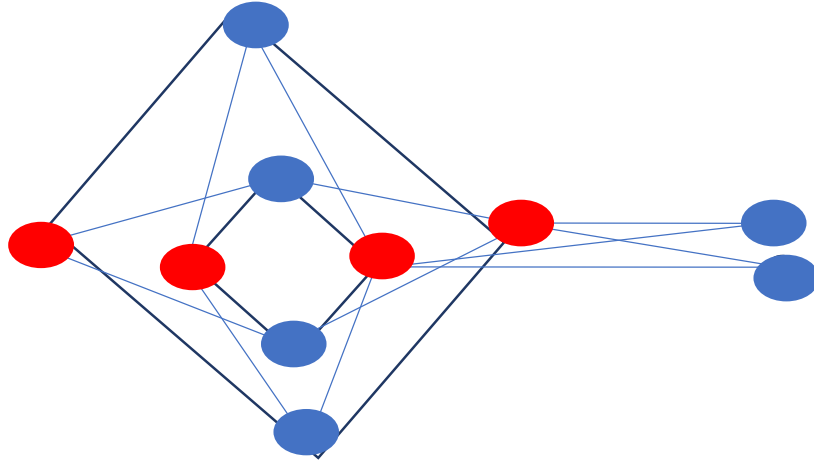


Fig 4.1.1 Double Pan graph  $D(P_{(n,1)})$ ,  $\chi(D(P_{(n,1)})) = 2$

#### 4.1.2 .Proper coloring of Strong Double graph of Pan graph $P_{(n,1)}$

The strong double graph of  $n$ -pan graph with  $2(n+1)$  vertices and  $5(n+1)$  edges admits proper coloring and the chromatic number is

$$\chi(SD(P_{(n,1)})) = \begin{cases} 4, & n \equiv 0 \pmod{2} \\ 6, & n \equiv 1 \pmod{2} \end{cases}$$

##### Proof

The  $n$ -pan graph 'G' is the graph obtained by joining a cycle graph  $C_n$  to a singleton graph  $K_1$  with a bridge. It has  $n+1$  vertices and  $n+1$  edges. Let  $V(G) = \{x_1, x_2, \dots, x_n\}$  and  $E(G) = \{(x_i, x_{i+1}), 1 \leq i \leq n-1\} \cup (x_n, x_1) \cup (x_1, x_{n+1})$ . The Strong double graph of  $G$  denoted by  $SD(G)$  has vertex set given by  $V_1 \cup V_2$  where  $V_1 = \{u_1, u_2, \dots, u_{n+1}\}$  and  $V_2 = \{v_1, v_2, \dots, v_{n+1}\}$ . For every  $x_i \in V(G)$  there are two vertices  $u_i$  and  $v_i$  in  $V(SD(G))$ . The Strong double graph  $SD(G)$  consists of the original edge set for every copy of  $G$  and for every edge  $(x_i, x_j) \in E(G)$ , more edges  $(u_i v_j), (v_j u_i)$  and  $(u_i v_i)$  are added. The proper coloring of  $SD(G)$  is as follows.

##### Case 1: $n \equiv 0 \pmod{2}$

Starting with the vertex  $u_1$  of the first copy alternatively assign colors 1 and 2 around the circular vertices of  $C_n$  and assign color 2 to the vertex of  $K_1$ . Since  $(v_i u_j), (v_j u_i)$  and  $(v_i u_i)$  are the edges of each  $v_i$  is adjacent to vertices assign color 3 and 4 to the vertices of second copy  $C_n$  starting with  $V_1$  and assign color 4 to the vertices of  $K_1$  of the second copy gives a proper coloring of  $SD(G)$ . Therefore

$$\chi(\text{SD}(G)) = 4, n \text{ is even}$$

**Case 2:**  $n \equiv 1 \pmod{2}$

If we assign label 1 and 2 alternatively to the vertices of  $C_n$ ,  $V_1$  &  $V_n$  will get same label as  $n$  is odd. Therefore assign color 3 to the vertex  $V_n$ , and assign color 2 to the vertex of  $K_1$ . Since  $(v_i u_j), (v_j u_i)$  and  $(v_i u_i)$  are the edges, assign colors 4 and 5 alternatively starting with  $u_1$  to the second copy of  $C_n$  and assign color 6 to the last vertex as it is adjacent to all the 5 colors and assign color 4 to the vertex of  $K_1$  gives proper coloring of  $\text{SD}(G)$ . Therefore  $\chi(\text{SD}(G)) = 6, n$  is odd

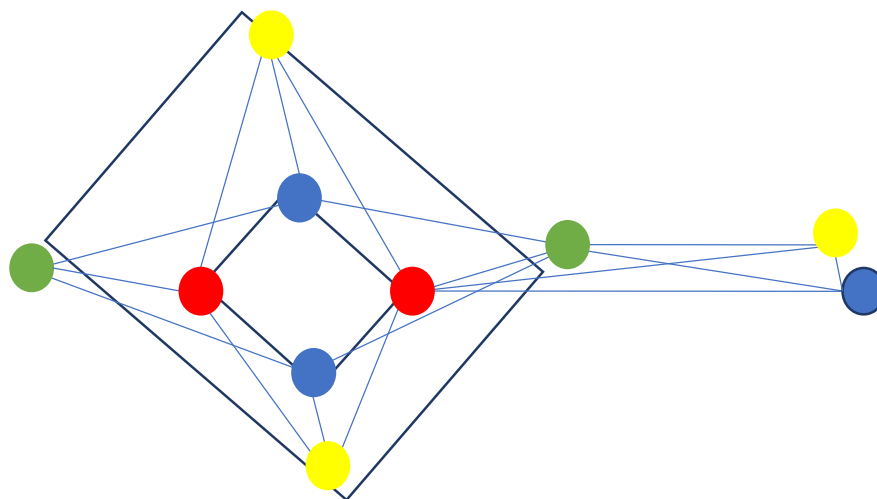


Fig 4.1.2 Strong Double Pan Graph  $\text{SD}(P_{(n,1)})$ ,  $\chi(\text{SD}(G)) = 4$

**4.1.3. Corollary**

For a Pan graph  $P_{(n,1)}$ ,  $\chi(\text{SD}(P_{(n,1)})) = \chi(P_{(n,1)}) + \chi(D(P_{(n,1)}))$

# **CHAPTER-V**



## 5. PROPER COLORING OF TADPOLE GRAPH $T_{(m,n)}$

### 5.1 Tadpole Graph

Tadpole graph  $T_{(m,n)}$  is obtained by joining a cycle graph  $C_m$  to a path graph  $P_n$  with a bridge.

#### 5.1.1. Proper coloring of Double graph Tadpole graph $T_{(m,n)}$

The Double graph of Tadpole graph  $T_{m,n}$  admits proper coloring and its Chromatic

$$\text{number } \chi(T_{m,n}) = \begin{cases} 2, n \equiv 0 \pmod{2} \\ 3, n \equiv 1 \pmod{2} \end{cases}$$

#### Proof

Consider the Tadpole graph  $T_{m,n}$ ,  $m \geq 3, n \geq 1$  is obtained by joining a cycle  $C_m$ , to a path  $P_n$  with an edge. Let  $\{v_1, v_2, \dots, v_m\}$  be the vertices of  $C_m$  and  $v_{m+1}, \dots, v_{m+n}$  be the vertices of  $P_n$  and  $v_m$  is adjacent to  $v_{m+1}$ . Double graph of  $T_{m,n}$  is constructed by making two copies of  $T_{m,n}$  with Vertex set  $V = \{v_1, v_2, \dots, v_{m+n}, u_1, u_2, \dots, u_{m+n}\}$  and Edge set  $E = \{v_i v_{i+1}, 1 \leq i \leq m+n-1\} \cup \{u_i u_{i+1}, 1 \leq i \leq m+n-1\} \cup \{u_i v_{i+1}, 1 \leq i \leq m+n-1\} \cup \{v_i u_{i+1}, 1 \leq i \leq m+n-1\} \cup \{v_m v_1\} \cup \{u_m u_1\}$ .

#### Case 1: $m \equiv 0 \pmod{2}$

Starting with the vertex  $v_1$  of the first copy alternatively assign color 1 and 2 around the circular vertices of  $C_m$ . As  $m$  is even,  $v_m$  receives color 2 and  $v_{m+1}$  is adjacent to  $v_m$ , assign color 1 and 2 to the vertices of the path  $P_n$  alternatively. Since  $u_i$  &  $v_i, 1 \leq i \leq n+1$  are not adjacent, assign the same color of  $v_i$  to  $u_i$  resulting a proper coloring of double graph of tadpole graph.  $\chi(D(T_{m,n})) = 2$

#### Case 2: $m \equiv 1 \pmod{2}$

If we assign label 1 and 2 alternatively to the vertices of  $C_n$ ,  $v_1$  &  $v_m$  will get same label as  $m$  is odd. Therefore assign color 3 to the vertex  $v_m$ , and assign color 1 and 2 to the vertices of  $P_n$ . As  $u_i$  &  $v_i$  are not adjacent, assign same colors to the second copy gives a proper coloring of double graph of Tadpole graph. Therefore  $\chi(D(T_{m,n})) = 3$ ,  $m$  is odd.

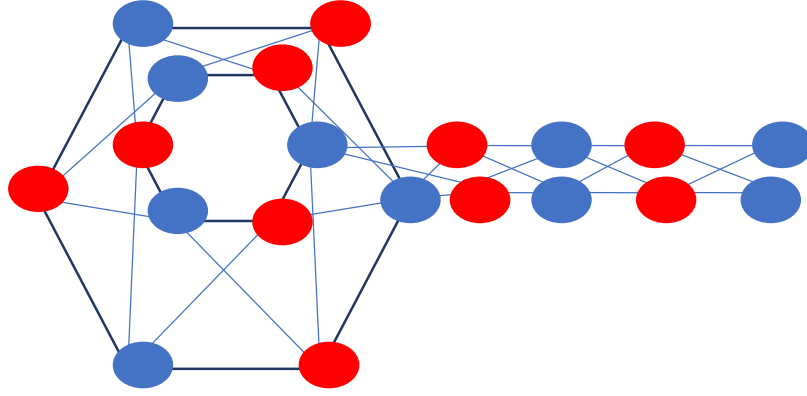


Fig 5.1.1 Double Tadpole graph  $D(T_{(m,n)})$ ,  $\chi(T_{m,n}) = 2$ ,  $m \equiv 0 \pmod{2}$

### 5.1.2. Proper coloring of Strong Double graph Tadpole graph $T_{(m,n)}$

Proper vertex coloring of Strong Double graph of Tadpole graph  $T_{(m,n)}$  is

$$\chi(SD(T_{(m,n)})) = \begin{cases} 4, m \equiv 0 \pmod{2} \\ 6, m \equiv 1 \pmod{2} \end{cases}$$

#### Proof

Consider the Tadpole graph  $T_{(m,n)}$ ,  $m \geq 3$ ,  $n \geq 1$  is obtained by joining a cycle  $C_m$  to a path  $P_n$  with an edge. Let  $\{v_1, v_2, \dots, v_m\}$  be the vertices of  $C_m$  and  $v_{m+1}, \dots, v_{m+n}$  be the vertices of  $P_n$  and  $v_m$  is adjacent to  $v_{m+1}$ . Strong Double graph

$SD(T_{(m,n)})$  is constructed by making two copies of  $T_{(m,n)}$  with Vertex set  $V = \{v_1, v_2, \dots, v_{m+n}, u_1, u_2, \dots, u_{m+n}\}$  and Edge set  $E = \{v_i v_{i+1}, 1 \leq i \leq m+n-1\} \cup \{u_i u_{i+1}, 1 \leq i \leq m+n-1\} \cup \{v_i u_{i+1}, 1 \leq i \leq m+n-1\} \cup \{v_i u_i, 1 \leq i \leq m+n-1\} \cup \{v_m v_1\} \cup \{u_m u_1\} \cup \{v_m u_1\} \cup \{v_i u_i, 1 \leq i \leq m+n\}$ .

#### Case 1: $n \equiv 0 \pmod{2}$

Starting with the vertex  $v_1$  of the first copy alternatively assign colors 1 and 2 around the circular vertices of  $C_m$ . As  $m$  is even,  $v_m$  receives color 2 and  $v_{m+1}$  is adjacent to  $v_m$ , assign color 1 and 2 to the vertices of the path  $P_n$  alternatively. Since  $(v_i u_i), (v_j u_j)$  and  $(v_i u_i)$  are the edges of each  $v_i$  is adjacent to  $u_i$ , assign color 3 and 4 to the vertices of second copy of  $C_m$  and  $P_n$  resulting a proper coloring of  $SD(T_{(m,n)})$ . Therefore  $\chi(SD(T_{(m,n)})) = 4$ ,  $n$  is even

**Case 2:**  $n \equiv 1 \pmod{2}$

If we assign color 1 and 2 alternatively to the vertices of  $C_m$ ,  $V_1$  &  $V_m$  will get same label as  $m$  is odd. Therefore assign color 3 to the vertex  $V_m$ , and assign colors 1 and 2 alternatively to the vertices of  $P_n$ . Since  $(v_i u_j), (v_j u_i)$  and  $(v_i u_i)$  are the edges, assign colors 4, 5 and 6 to the second copy of  $T_{m,n}$ , gives proper coloring of  $SD(G)$ . Therefore  $\chi(SD(G)) = 6$ ,  $m$  is odd

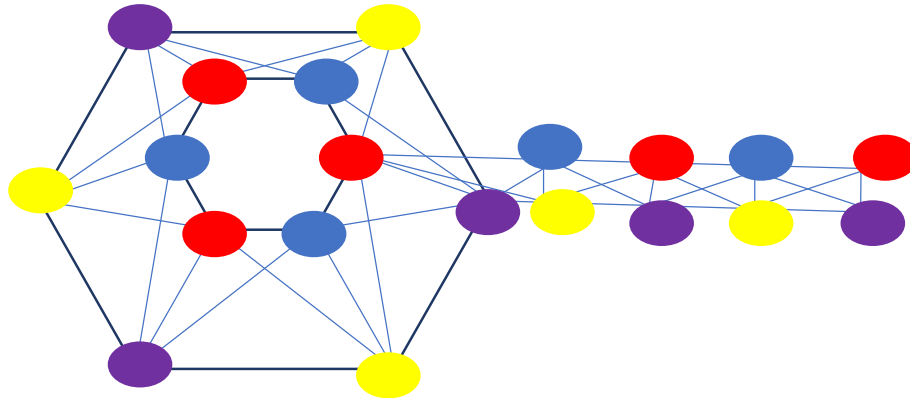


Fig 5.1.2 Strong Double Tadpole graph  $T_{(m,n)}$ ,  $\chi(SD(G)) = 4$

**5.1.3. Corollary**

For a Tadpole graph  $T_{(m,n)}$  :  $\chi(SD(T_{m,n})) = \chi(T_{m,n}) + \chi(D(T_{m,n}))$

# CHAPTER-VI

## 6. PROPER COLORING OF BARBELL GRAPH $B_{(m,n)}$ .

### 6.1.Barbell Graph

The Barbell graph is the simple graph obtained by connecting two copies of a complete graph  $K_n$  by a bridge. A barbell graph is denoted by  $B_{(m,n)}$ .

#### 6.1.1. Proper coloring of Double graph Barbell graph $B_{(m,n)}$

The chromatic number of Double Barbell graph is given by  $\chi(D(B_{m,n})) = m$

##### Proof

The Barbell graph is obtained by connecting  $n$ -copies of a complete graph  $K_m$  by a bridge  $v_m^j v_1^{j+1}$ ,  $1 \leq j \leq n-1$ . The double barbell graph is obtained by two copies of barbell graph  $B_{(m,n)}$  with vertex set  $V = \{v_i^j / 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{u_i^j / 1 \leq i \leq m, 1 \leq j \leq n\}$  and edge set

$E = U_{j=1}^n \{(v_i^j v_k^j) / 1 \leq j \leq m-1, i+1 \leq k \leq m\} \cup \{(v_m^j v_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_i^j v_k^j) / 1 \leq j \leq m-1, i+1 \leq k \leq m\} \cup \{(u_m^j u_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_i^j v_k^j) / 1 \leq j \leq m, i+1 \leq k \leq m\} \cup \{(u_i^j v_k^j) / 1 \leq j \leq m-1, i+1 \leq k \leq m\} \cup \{(v_m^j u_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_m^j v_1^{j+1}) / 1 \leq j \leq n-1\}$ . Let the vertices  $v_i^j$ ,  $1 \leq i \leq m-1$ ,  $1 \leq j \leq n$  be assigned color 'i' respectively. Since  $v_i^j$ ,  $1 \leq i \leq m-1$ ,  $1 \leq j \leq n$  and  $u_i^j$ ,  $1 \leq i \leq m-1$ ,  $1 \leq j \leq n$  are not adjacent, we can assign color i to the vertices  $u_i^j$ ,  $1 \leq i \leq m-1$ ,  $1 \leq j \leq n$  respectively. This gives proper coloring of  $D(B_{m,n})$ . Therefore  $\chi(D(B_{m,n})) = m$

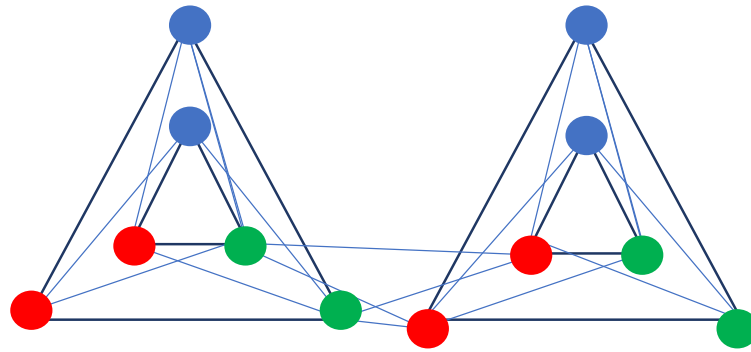


Fig 6.1.1 Double Barbell graph  $B_{(m,n)}$

### 6.1.2. Proper coloring of Strong Double graph Barbell graph $B_{(m,n)}$

The chromatic number of Strong Double Barbell graph is given by  $\chi(SD(B_{(m,n)})) = m$

#### Proof

The Barbell graph is obtained by connecting  $n$ -copies of a complete graph  $K_m$  by a bridge  $v_m^j v_1^{j+1}, 1 \leq j \leq n-1$ . The Strong double barbell graph is obtained by two copies of barbell graph  $B_{(m,n)}$  with vertex set  $V = \{v_i^j / 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{u_i^j / 1 \leq i \leq m, 1 \leq j \leq n\}$  and edge set  $E = \bigcup_{j=1}^n \{(v_i^j v_k^j) / 1 \leq i < k \leq m\} \cup \{(v_m^j v_1^{j+1}) / 1 \leq j \leq n-1\} \cup \bigcup_{j=1}^n \{(u_i^j v_k^j) / 1 \leq j \leq m-1, i+1 \leq k \leq m\} \cup \{(u_m^j u_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_i^j v_k^j) / 1 \leq j \leq m, i+1 \leq k \leq m\} \cup \{(u_i^j v_k^j) / 1 \leq j \leq m-1, i+1 \leq k \leq m\} \cup \{(v_m^j u_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_m^j v_1^{j+1}) / 1 \leq j \leq n-1\} \cup \{(u_i^j v_k^j) / 1 \leq i \leq m, i+1 \leq j \leq n\}$ . Let the vertices  $v_i^j, 1 \leq i \leq m, 1 \leq j \leq n$  be assigned color 'i' respectively. Since  $v_i^j, 1 \leq i \leq m, 1 \leq j \leq n$  and  $u_i^j, 1 \leq i \leq m, 1 \leq j \leq n$  are adjacent, we can assign color  $i+m$  to the vertices  $u_i^j, 1 \leq i \leq m, 1 \leq j \leq n$ . Since no two vertices receive the same color, all the vertices of  $SD(B_{(m,n)})$  have a proper coloring. Therefore  $\chi(SD(B_{(m,n)})) = 2m$

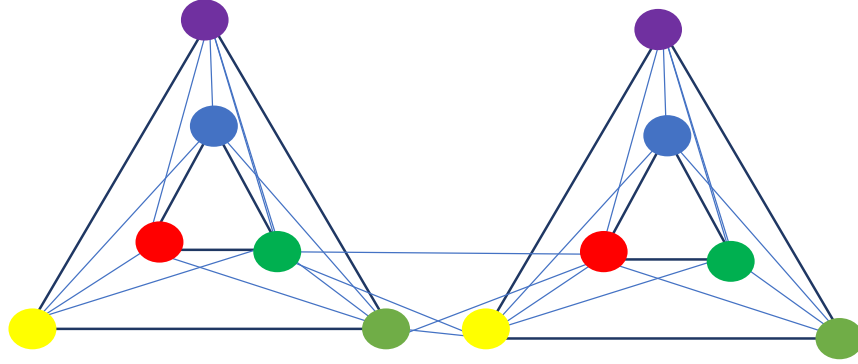


Fig 6.1.2 Strong Double Barbell graph  $B_{(m,n)}$

### 6.1.3. Corollary

For a barbell graph  $B_{(m,n)}$ ,  $\chi(SD(B_{(m,n)})) = \chi(B_{(m,n)}) + \chi(D(B_{(m,n)}))$

# CHAPTER-VII

## **7. CONCLUSION**

In practice, various problems of interest but in general in flexible in nature can be solved competently when constrained to the class of perfect graphs. Coloring of perfect graphs has applications in frequency assignments for telecommunications systems, integer programming and optimization etc. This paper deals with chromatic number of double and strong double graph of some classes of perfect graphs. In future, this can be extended to other classes of perfect graphs.

## **PAPER PRESENTED**

**R.Rajeswari and I.Pavithra, Proper coloring of double and strong double graph of perfect graph**, 2nd International Conference on mathematical techniques and applications, the Department of Mathematics, SRM Institute of Science and technology, Tamilnadu

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