

MATHEMATICAL INVENTORY MODEL BY USING DIFFERENTIAL EQUATION

Submitted in partial fulfillment of the requirements for the award of
Master of Science Degree in Mathematics

By

JAYALAKSHMI K (Reg. No.39630006)



**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited with Grade "A" by NAAC

**JEPPIAAR NAGAR, RAJIV GANDHI SALAI,
CHENNAI - 600 119**

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Accredited with "A" grade by NAAC
Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai – 600 119
www.sathyabama.ac.in

DEPARTMENT OF MATHEMATICS

BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **Ms.JAYALAKSHMI(39630006)** who carried out the project entitled "**MATHEMATICAL INVENTORY MODEL BY USING DIFFERENTIAL EQUATION**" under my supervision from August 2020 to April 2021.

Internal Guide
Guide

External

(M. NIRMALA)
Head of the Department

Submitted for Viva voce Examination held on _____

Internal Examiner

External Examiner

DECLARATION

I **JAYALAKSHMI K(39630006)** hereby declare that the project report entitled “**MATHEMATICAL MODELBY USING DIFFERENTIAL EQUATION**” done by me completed under the guidance of **DR.P.MUNIAPPAN** is submitted in partial fulfillment of the requirements for the award of **Master Degree in Mathematics**.



DATE:

PLACE:

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ABSTRACT

As the benefits of adopting optimal inventory system is realized as impressive in supply chain, the project on developing inventory models for supply chain system has achieved significance. In this project deal with the Mathematical inventory model by using differential equation. And also, we develop an optimal inventory replenished system for deteriorating product with constant demand. Shortages are allowed in the inventory system. Our objective is system to find the optimal inventory level for minimizing the total inventory system. Finally, numerical results are presented to analyze the sensitivity of the optimal policies with respect to changes in some parameters of the system.

Keywords: Production, Inventory, Deteriorating Products, Partially Backlogged Shortages.

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CHAPTER 1

1.0 INTRODUCTION

Stock-out situation plays an important role in business environment. The stock-out situation occurs under many circumstances in any business. Most of the inventory models were developed under the assumption of inventory control theory, which states that “shortages are allowed completely backlogged”. But in day-to-day practice, this assumption is not realistic, because the majority of customers won’t wait for long time in purchasing goods from a particular shop, with an exception some people wait and purchase in the same shop because of retailer behavior, price, quality of products and also the locality of the shop. As a result, shortages are considered as partially backlogged.

1.1 INVENTORY

The word inventory refers to any kind of resource that has economic value and is maintained to fulfil the present and future needs of an organization. Such resources may be classified into three categories:

- physical resources such as raw materials, semi-finished goods, finished goods, spare parts, lubricants, etc.,
- human resources as unused labour (manpower), and
- financial resources such as working capital, etc.
- INVENTORY

1.2 INVENTORY CONTROL

Inventory control is an important part of any business that must have a stock of products or items on hand. It is a planned approach to determining what to order; when to order, how much to order and how much items are to be kept in stock so that costs associated with purchasing and stocking up are optimal without interrupting manufacture and sales. Basically inventory control deals with two problems.

- How much should be ordered? (Order quantity)
- When should an order be placed? (Order level)

Inventory control is a major element of an efficient operation for any business that buys and resells goods. Typical goals of inventory management include maintaining

low inventory holding costs while also meeting all customer demand for goods. Many companies rely on computer software programs for inventory management. An inventory may be defined as an idle resource that possesses economic value. It is an item stored or reserved for meeting future demand. Such items may be materials, machines, money or even human resource.

1.3 INVENTORY FUNCTION

Since investment in inventory represents a substantial portion of the total capital in any business.

TRANSIT

Since the movement of item cannot be instantaneous, optimal inventory level is required for shipment of inventory items to distribution centers and customers from production centers. such an inventory is called a process inventory. In transit inventory is defined as the inventory of products that is held due to transportation lead time. Transit inventory is also called transportation inventory or goods in transit or pipeline. The amount of pipeline inventory depends on the time required for shipment and the nature of the demand.

CYCLE

It is the inventory necessary to meet the average demand during the successive replenishments. The amount of such inventory depends on the production lot size, economical shipment quantities, warehouse space available, replenishment lead time, price-quantity schedules, and inventory carrying cost, etc.

BUFFER

Buffer inventory is the extra stock of either raw material raw material or final product a company maintains as a protection against unforeseen circumstances. In simple words, we can see it is the excess inventory that a company keeps in reserves to protect itself against an uncertain future.

ANTICIPATION

It is also called Seasonal inventory. For manufactures, anticipation inventory allows them to build up inventory when demand is low so that when demand picks up the increased inventory will be slowly depleted and the firm does not have to react by

increasing time. Therefore, the firm has avoided both excessive overtime due to increased demand and hiring costs due to increased demand. It also has avoided layoff costs associated with production cut-backs, or worse, the idling or shutting down of facilities. This process is sometimes called "smoothing" because it smooths the peaks and valley in demand, allowing the firm to maintain a constant level of output and a stable workforce.

1.4 INVENTORY MODELS

Inventory models can be classified as

- Deterministic inventory models
- Probabilistic inventory models

DETERMINISTIC INVENTORY MODELS

Demand size is known and well defined will be referred to a deterministic inventory model. In this case, the demand size may be static or dynamic from time to time. The deterministic method concedes a single best estimation of inventory reserves grounded on recognized engineering, geological, and economic information. Deterministic models of inventory control are used to determine the optimal inventory of a single item when demand is mostly largely obscure. Under this model inventory is built up at a constant rate to meet a determined, or accepted, demand. A deterministic circumstance is one in which the system parameters can be ascertained precisely. This is also known as a situation of sureness since it is realized that whatever are ascertained, things are sure to occur the same way. Also the information about the system under thought should be whole so that the parameters can be determined with confidence. But this kind of system rarely exists, and it is for sure that some uncertainty is always associated with the system. Deterministic models presume the state of affairs to be deterministic and consequently render the numerical model to optimize on system arguments. Since it conceives the system to be deterministic, it automatically means that one has full information about the system.

PROBABILISTIC OR STOCHASTIC INVENTORY MODELS

Demand or lead time or both are not known with uncertainty, but it is possible to ascertain its probability distribution is called probabilistic or stochastic inventory models.

The probabilistic method employs the known economic, geological and engineering data to produce a collection of approximate stock reserve quantities and their related probabilities. Each inventory reserve categorization gives a signal of the prospect of revival. The advantage of a probabilistic approach lies in the fact that by using values lying within a bandwidth and modeled by a defined distribution density, the reality can be modeled better than by using deterministic figures. Probabilistic situation is also known as a situation of uncertainty. Although this is present everywhere, the vagueness always makes us comfortless. So people keep attempting to lessen uncertainty. Probabilistic inventory prototypes consisting of probabilistic demand and supply are more suitable in many real circumstances. But, such models also create larger trouble in analysis and often become uncontrollable.

CHAPTER 2

2. LITERATURE SURVEY

C.T.Chang and L.Y.Ouyanget al.[1] analyzed an Optimal ordering policies for deteriorating items using a cash-flow analysis when a trade credit is linked to order quantity.

S.P.Aggarwal and C.K.Jaggi[2] developed an Ordering policies of deteriorating items under permissible delay in payments. Z.T.Balkhi[3] analyzed an optimal solution of a general lot size inventory model with deteriorated and imperfect products, taking into account inflation and time value of money. A.Guria and B.Daset al[4] analyzed an Inventory policy for an item with inflation induced purchasing price, selling price and demand with immediate part payment. Y.Liang and F.Zhou[5] developed a two-warehouse inventory model for deteriorating items

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dependent demand and two-level trade credit.

CHAPTER 3

DETERMINISTIC INVENTORY CONTROL MODEL WITH VARIOUS FACTORS INVOLVED IN INVENTORY PROBLEMS ANALYSIS

3.1 TYPES OF INVENTORY

- Raw materials
- Work-in-progress (WIP)
- Finished goods

RAW MATERIALS

Raw materials are inventory items that are used in the manufacturer. The materials, components, fuels etc. used in the manufacturing of products.

WORK IN PROGRESS

Work-in-process is made up of all the materials, parts (components), assemblies, and subassemblies, that are being processed or are waiting to be processed within the system. This is generally include all material-from raw material that has been released for initial processing up to material that has been completely processed and is awaiting final inspection and acceptance before inclusion in finished goods.

FINISHED GOODS

A finished good is a completely part that is ready for a customer order. Therefore, finished goods inventory is the stock of completed products. These goods have been inspected and have passed final inspection requirements so that they can be transferred out of work-in-process and into finished goods inventory. From this point, finished goods can be sold directly to their final user, sold to retailers, sold to wholesalers, sent to distribution centers, or held in anticipation of a customer order.

3.2 INVENTORY COSTS

In making any decision that affects inventory size, the following costs must be considered:

- Setup cost
- Purchase or Production cost
- Carrying or Holding cost
- Handling cost

- Storage cost
- Deterioration, Obsolescence cost
- Shortage cost

SETUP COST

This includes the fixed cost associated with obtaining goods through the placing of orders or purchasing or manufacturing or setting up machinery before starting production. The costs include ordering of raw materials for production purposes, advertisements, postage, telephone charges, and travel expenditure and so on. These are also called order costs or replenishment costs per production run.

PURCHASE OR PRODUCTION COST

The cost of purchasing (or producing) a unit of an item is known as purchase (or production) cost.

CARRYING OR HOLDING COST

The cost associated with carrying or holding goods in stock is called holding or carrying cost per unit of the item for a unit of time. It's assumed to vary directly with the size of the inventory as well as the time the item is held in stock.

HANDLING COST

These include all costs associated with movement of stock, cost of labor and so on.

STORAGE COST

This involves the rent for storage space or depreciation and interest even if own space is used.

DETERIORATION, OBSOLESCENCE COST

The cost associated with the items in stock being out of fashion or the items undergoing chemical changes during storage, date expiring and so on.

SHORTAGE COST

The shortage of items occurs when actual demand can't be fulfilled from the existing stock. This cost arises due to a shortage of goods and may cause loss of sales. Goodwill may be lost either by a delay in meeting the demand or being unable to meet the demand. The shortage can be looked in three different ways.

- Completely backlogged
- Partially backlogged
- Lost Sale

COMPLETELYBACKLOGGED

During stock out period the customers will wait upto the next replenishment is known as completely backlogged shortages.

PARTIALLYBACKLOGGED

In this type, during the out of stock period some of the customers wait and others do not wait till the end of the shortage period .

LOSTSALES

In this type, customers are not ready to wait and therefore there's a loss of sale.

3.3 REASONS FOR HOLDING INVENTORY

The main reasons for holding inventories are

- To avoid inventory shortages
- To take advantage of quantity discounts
- To protect against price increases
- To avoid uncertainty associated with market fluctuations
- To carry buffer stock in case of delayed deliveries by the supplier

EXAMPLES OF COST

INVENTORY FINANCING COSTS

This includes everything related to the investment made in inventory, including costs like interest on working capital Financing costs can be complex depending on the business.

OPPORTUNITY COST OF THE MONEY INVESTED IN INVENTORY

This is found by factoring in the lost alternatives of tying money up in inventory, such as investing in term deposits or mutual funds.

STORAGE SPACE COSTS

These are costs related to the place where the inventory is stored and will vary by location. There will be the cost of the storage facility itself, or lease payments if it is

not owned. Then there are facility maintenance costs like lighting, heating, and ventilation. Depreciation and property taxes are also included in this.

INVENTORY SERVICES COSTS

This includes the cost of the physical handling of the goods, as well as insurance, security, and IT hardware, and application if these are used. Expenses related to inventory control and cycle counting are further examples.

INVENTORY COSTS

A major cost is shrinkage, which is the loss of products between purchasing from the supplier and final sale due to any number of reasons: theft, vendor fraud, shipping errors, damage in transit or storage. The other main example is obsolescence, which is the cost of goods going past their use-by dates, or otherwise becoming outdated.

SHORTAGE COST

Shortage or stock out cost & cost of replenishment

- Cost of Loss, Pilferage, shrinkage and obsolescence etc.
- Cost of Logistics

Sales discounts, volume discounts and other related costs

3.4 FACTORS OF INVENTORY MANAGEMENT

The factors which play an important role in the study of inventory analysis are

- Demand
- Lead time
- Order cycle
- Stock replenishment
- Time horizon
- Reorder level

DEMAND

Demand is the number of units required per period and may be known exactly or in terms of probabilities or be completely unknown. If the demand is known, it may be either fixed or variable per unit time. Problems in which the demand is known and

fixed are called deterministic problems. If the demand is assumed to be a random variable, the problems are called stochastic or probabilistic problems.

LEAD TIME

The time gap between placing of an order and its actual arrival in the inventory is known as lead time. The level of inventory of an item depends upon the length of its lead time.

The longer the lead time, the higher in the average inventory. Lead time has two components, namely

- Administrative lead time
- Delivery lead time

ADMINISTRATIVE LEAD TIME

Time from initiation of procurement action to the placing of an order.

DELIVERY LEAD TIME

Time from placing an order to the delivery of the ordered material.

ORDER CYCLE

The time period between placements of two successive orders is referred to as an order cycle. The order may be placed on the basis of following two types of inventory review systems

- Continuous review system
- Periodic review system

CONTINUOUS REVIEW SYSTEM

The record of the inventory level is checked continuously until a specified point is reached where a new order is placed.

PERIODIC REVIEW SYSTEM

The inventory levels are viewed at equal time intervals and orders are placed at such intervals. The quantity ordered at each time depends on the available inventory level at the time of review.

STOCK REPLENISHMENT

The replacement of stock may occur instantaneously or uniformly. Instantaneous replenishment occurs in case the stock is purchased from the outside sources where uniform replenishment occurs when the product is manufactured by the company.

TIME HORIZON

The time period over which the inventory level will be controlled is called the time horizon.

REORDER LEVEL

The level between the maximum and minimum stock, at which purchasing (or production) activities start for replenishment is called reorder level.

3.5 FACTORS AFFECTING INVENTORY

FINANCIAL FACTORS

Factors such as the cost of borrowing money to stock enough. Inventory can greatly influence inventory management. In this case, your finances may fluctuate according to the economy, and it is wise to keep an eye on changing interest rates to help plan your spending. Then costs associated with stocking inventory is another factor that can influence inventory management. This is especially salient when preparing for the end of year tax returns. Other financial factors include the expenses associated with warehouse operations and transportation costs changes in these factors may require you to alter your inventory management processes accordingly. Fluctuations in the cost of fuel, for example, may require you to rethink your transportation methods to reduce costs. You may choose to purchase your own trucks or use outside contractors for transportation, which again will change the way you manage inventory.

SUPPLIERS

Suppliers can have a huge influence on inventory control. Successful businesses require reliable suppliers in order to plan spending and arrange production. An unreliable or unpredictable supplier can have huge knock-on effects for inventory control. It can be a good idea to ensure you have a reliable back up supplier to prevent product shortages or delays in the manufacturing process.

LEAD TIME

Lead time is the time it takes from the moment an item is ordered to the moment it arrives. Lead time will vary widely depending on the product type and the various manufacturing processes involved, and therefore changes in these factors can require changes to inventory management.

Outsourcing manufacturing processes to other countries due to lower production costs may result in longer waiting times. Producing the same goods locally may cost more but take less time, and therefore you may need to adjust your stock levels accordingly.

PRODUCT TYPE

Inventory management must take into consideration the different types of products in stock. For example, some products may be perishable and therefore have a shorter shelf life than others. In this case inventory must be managed to ensure that these items are rotated in line with expiration dates.

MANAGEMENT

Ultimately, responsibility for managing your business' inventory sits with you and any co-owners. While you may have multiple employees acting as managers to oversee inventory processes, they typically will not have the same stake in the business as you do.

EXTERNAL FACTORS

There are multiple external factors that may affect inventory control. For example, economic downturns may occur and this is something that you will generally have very little control over. Assessing the economy is a must in order to guard against stock outs or a buildup of excess inventory. Other factors may include the real estate markets or the extent of local competition. These factors are also largely out of your control, so it is a good idea to assess the external climate regularly in order to stay prepared.

CHAPTER 4

ANALYSIS ON DETERMINISTIC INVENTORY MODEL WITH CONSTANT DEMAND AND RE ORDER LEVEL

4.1 INTRODUCTION

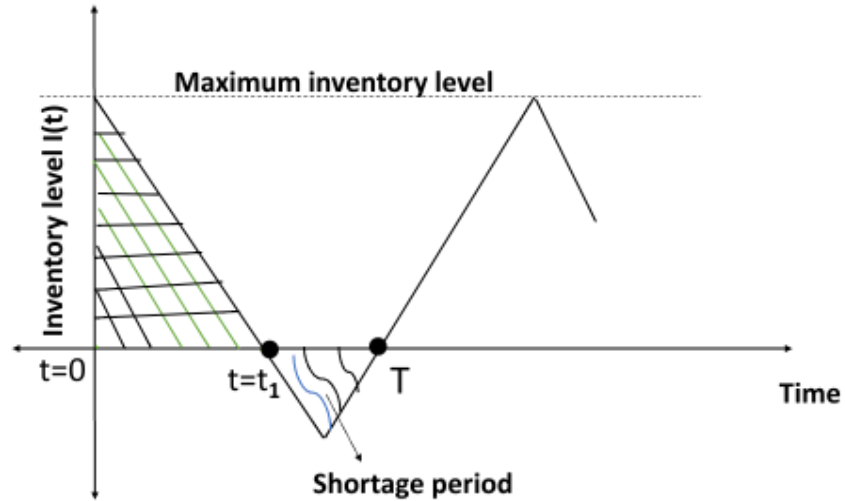
As the benefits of adopting optimal inventory system is realized as impressive in supply chain, the project on developing inventory models for supply chain system has achieved significance. In this project deal with the Mathematical inventory model by using differential equation. And also, we develop an optimal inventory replenished system for deteriorating product with constant demand. Shortage are allowed in the inventory system. Our objective is system to find the optimal inventory level for minimizing the total inventory system. Finally, numerical results are presented to analyze the sensitivity of the optimal policies with respect to changes in some parameters of the system.

4.2 NOTATIONS AND ASSUMPTIONS

The following notations and assumptions are used in the Mathematics model.

- D is the constant demand rate per unit time
- r denotes the setup cost /unit/unit time
- h denotes the holding cost/unit/unit time
- s denotes shortage cost/unit/unit time
- t_1 denotes average shortage time
- T denotes total time
- $I(t)$ is the total inventory level at time origin T
- $I_1(t)$ is the inventory level at time $0 \leq t \leq t_1$
- $I_2(t)$ is the inventory level at time $t_1 \leq t$
- θ denotes deteriorating rate a fraction of the on-hand inventory

4.3 Mathematical Formulation



As in the above inventory system, the reduction of the inventory is due to the combined effect of the demand as well as the deterioration in the $[0, t_1]$ and the shortages in the interval $[t_1, T]$. Therefore, we use

Differential equations as follows

$$\frac{dI_1(t)}{dt} = -\theta I(t) - D, 0 \leq t \leq t_1 \quad \dots (1)$$

$$\frac{dI_2(t)}{dt} = -D, t_1 \leq t \leq T \quad \dots (2)$$

With boundary conditions $I_1(t) = 0$ and $I(T) = 0$

$$\text{Setup cost} = A \quad \dots (3)$$

$$\text{Holding cost} = h \int_0^{t_1} I_1(t) dt \dots (4)$$

$$\text{Shortage cost} = s \int_{t_1}^T I_2(t) dt \dots (5)$$

Now the total cost of the system is formulated as follows

$$\text{Total inventory of average cost} = \frac{1}{T} \{ \text{setup cost} + \text{Holding cost} + \text{shortage cost} \} \quad \dots (6)$$

$$Z(t) = \frac{1}{T} \{ A + h \int_0^{t_1} I_1(t) dt + s \int_{t_1}^T I_2(t) dt \} \dots (7)$$

$$I_1(t) = \frac{D}{\theta} [e^{\theta(t_1-t)} - 1] \text{ and } I_2(t) = -D(t - t_1) \dots (8)$$

Substitute equation (8) in (7)

$$Z(t) = \frac{1}{T} \{ A + h \int_0^{t_1} \frac{D}{\theta} [e^{\theta(t_1-t)} - 1] dt + s \int_{t_1}^T -D(t - t_1) dt \} \quad \dots (9)$$

$$= \frac{1}{T} \{ A + \frac{hD}{\theta} [-\frac{1}{\theta} - t_1 + \frac{e^{\theta t_1}}{\theta}] - sD [\frac{T^2}{2} - t_1 T - \frac{t_1^2}{2} + t_1^2] \} = \frac{1}{T} \{ A - \frac{hD}{\theta^2} [e^{\theta t_1} - \theta t_1 - 1] - \frac{sD}{2} [T^2 - 2t_1 T - t_1^2 + 2t_1^2] \}$$

$$= \frac{1}{T} A - \frac{hD}{\theta^2} [e^{\theta t_1} - \theta t_1 - 1] - \frac{sD}{2} [T^2 - 2t_1 T + t_1^2]$$

$$Z(t) = \frac{1}{T} \{ A - \frac{hD}{\theta^2} [e^{\theta t_1} - \theta t_1 - 1] - \frac{sD}{2} (T - t_1)^2 \} \dots (10)$$

differential equation with respect to t_1 and T

$$\frac{\partial Z(t)}{\partial t_1} = \frac{1}{T} \{ -\frac{hD}{\theta^2} [\theta e^{\theta t_1} - \theta] + \frac{sD}{2} 2(T - t_1) \}$$

$$= \frac{D}{T} \{ \frac{h}{\theta} [1 - e^{\theta t_1}] + s(T - t_1) \} \dots (11)$$

$$\frac{\partial Z(t)}{\partial t_1} = 0$$

$$0 = \frac{D}{T} \{ \frac{h}{\theta} [1 - e^{\theta t_1}] + s(T - t_1) \}$$

$$\Rightarrow \frac{h}{\theta} [1 - e^{\theta t_1}] + s(T - t_1) = 0$$

$$\frac{h}{\theta} - \frac{he^{\theta t_1}}{\theta} + sT = st_1$$

$$\Rightarrow t_1 = \frac{1}{s} \left[\frac{h}{\theta} - \frac{he^{\theta t_1}}{\theta} + sT \right] \dots (12)$$

$$\frac{\partial Z(t)}{\partial T} = 0 + \frac{hD}{T^2 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) - \frac{sD}{2T} (T^2 + t_1^2 - 2Tt_1)$$

$$\frac{\partial Z(t)}{\partial T} = \frac{hD}{T^2 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) - \frac{sD}{2} \left(1 + \frac{t_1^2}{T^2} \right)$$

$$\frac{\partial Z(t)}{\partial T} = 0$$

$$0 = \frac{hD}{T^2 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) - \frac{sD}{2} \left(1 + \frac{t_1^2}{T^2} \right)$$

$$\Rightarrow \frac{hD}{T^2 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) = \frac{sD}{2} \left(1 + \frac{t_1^2}{T^2} \right)$$

$$\frac{h}{sT^2 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) = \frac{T^2 + t_1^2}{T^2}$$

$$\frac{h}{s\theta^2} (e^{\theta t_1} - \theta t_1 - 1) = T^2 + t_1^2$$

$$\frac{h}{s\theta^2} (e^{\theta t_1} - \theta t_1 - 1) - t_1^2 = T^2$$

$$\Rightarrow T^2 = \frac{h}{s\theta^2} (e^{\theta t_1} - \theta t_1 - 1) - t_1^2 \dots (13)$$

$$r = \frac{\partial^2 Z(t)}{\partial t_1^2} = \frac{D}{T} \left\{ \frac{h}{\theta} (-\theta e^{\theta t_1}) + s(-1) \right\}$$

$$r = \frac{\partial^2 Z(t)}{\partial t_1^2} = -\left(\frac{D}{T} \left\{ \frac{h}{\theta} (\theta e^{\theta t_1}) + s \right\} \right)$$

$$r = -\frac{D}{T} \{ \theta e^{\theta t_1} + s \}$$

$$t = \frac{\partial^2 Z(t)}{\partial T^2} = -2 \frac{hD}{T^3 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) - 2 \left(\frac{t_1^2}{T^3} \right)$$

$$t = -2 \left\{ \frac{hD}{T^3 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) + \left(\frac{t_1^2}{T^3} \right) \right\}$$

$$s = \frac{\partial^2 Z(t)}{\partial t_1 \partial T} = \frac{\partial}{\partial t_1} \left\{ -2 \frac{hD}{T^3 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) - 2 \left(\frac{t_1^2}{T^3} \right) \right\}$$

$$s = -2 \frac{hD}{T^3 \theta^2} (\theta e^{\theta t_1} - \theta) - 4 \left(\frac{t_1}{T^3} \right)$$

$$\frac{\partial^2 Z(t)}{\partial t_1^2} \times \frac{\partial^2 Z(t)}{\partial T^2} - \left(\frac{\partial^2 Z(t)}{\partial t_1 \partial T} \right)^2 > 0$$

$$\begin{aligned} -\left\{ \frac{D}{T} \{ \theta e^{\theta t_1} + s \} \times 2 \left\{ \frac{hD}{T^3 \theta^2} (e^{\theta t_1} - \theta t_1 - 1) + \left(\frac{t_1^2}{T^3} \right) \right\} \right\} \\ + \left(2 \frac{hD}{T^3 \theta^2} (\theta e^{\theta t_1} - \theta) + 4 \left(\frac{t_1}{T^3} \right) \right) > 0 \end{aligned}$$

...(14)

By solving the equation (12), (13) and (14) simultaneously the optimal values can be obtained.

Also we develop the following algorithm to find the optimal values of $t_1^* T^*$.

4.4 ALGORITHM:

STEP 1

Perform (i)-(iv)

- i. Start with $t_1 = T$
- ii. Substitution t_1 into equation (12) evaluate T_1
- iii. Using T_1 determine t_1 from equation (13)
- iv. Repeat (ii) and (iii) until no change occurs in the values of t_1 and T

STEP 2

Compare t_1 and T

- i. If $T \leq t_1$, t_1 is feasible. The values of t_1 and T obtained value, then go to next.
- ii. If $T > t_1$, t_1 is not feasible.
- iii. Set $t_1^* = t_1$ and evaluate the corresponding values of T^* from equation (39), then go to Step 3.

STEP 3

Compute the corresponding $TC^*(t_1^* = T^*)$ and Q^* .

4.5 NUMERICAL ANALYSIS

EXAMPLE 1

We consider an inventory system with the following data:

$A = 250, D = 50, h = 0.5, s = 2.5, \theta = 0.08, r = 0.1, t = 0.0833$ in appropriate units.

Applying the above said algorithm the computational result.

The following optimal values: $t_1 = 0.2716; T = 1.4511; TC = 2339.00, Q = 567.41$

EXAMPLE 2

We consider an inventory system with the following data:

$A=250, D=100, h=1.2, s=2.2, \theta=0.08, r=0.1, t=0.0833$ in appropriate units. Applying the above said algorithm the computational result.

The following optimal values: $t_1 = 0.3854; T = 1.0401; TC = 2542.60, Q = 771.20$

5. RESULT AND CONCLUSION

In this project, a deterministic inventory model has been framed for deteriorating items with constant demand consumption rate over a finite planning horizon. Shortages are allowed. Also, the effects of the time value of money in formulating the inventory policy are considered. Also given an analytic formulation of the problem on the framework described above and presented an optimal solution procedure to find the optimal economic order level. The results show that, the effect of inflation and time value of money on present value of total cost is more significant and increasing the fresh product time increases the order quantity and decreases the total cost. Hence, this model incorporates some realistic features that are likely to be associated with some kinds of inventory. The model is very useful in their retail business.

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