

LOCATION DOMINATION AND ITS APPLICATIONS

Submitted in partial fulfilment of the requirements for the award of
Master of Science degree in Mathematics (M.Sc. Mathematics)

By

K.Ezhilarasi (39630005)



**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

**Accredited with Grade "A" by NAAC I 12B Status by UGC I Approved by AICTE
JEPPIAAR NAGAR, RAJIV GANDHI SALAI, CHENNAI - 600 119**

APRIL - 2021



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Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai-600 119

www.sathyabama.ac.in

DEPARTMENT OF MATHEMATICS

BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **K.Ezhilarasi (39630005)**, who carried out the project entitled "**Location domination and its applications**" under our supervision from **January 2021 to April 2021**.


26/03/21

(Dr. D. ANGEL)

Internal Guide
(Name in Capital letters
with Signature)

External Guide (if Applicable)
(Name in Capital letters
with signature)



(Dr. M. NIRMALA)

Head of the Department
(Name in Capital letters with Seal & Signature)

Submitted for Viva voce Examination held on _____

Internal Examiner

(Name in Capital letters
with Signature)

External Examiner

(Name in Capital letters
with signature)

DECLARATION

IK.Ezhilarasi(39630005) hereby declare that the Project Report entitled
“**Location Domination and its Applications**” done by me under the guidance
of **Dr.D.Angel** (Internal) and _____ (External) at **Sathyabama**
Institute of Science and Technology is submitted in partial fulfilment of the
requirements for the award of Master of Science degree in Mathematics.

DATE: 26-03-2021

PLACE: CHENNAI

Ezhilarasi. K

SIGNATURE OF THE CANDIDATE

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ABSTRACT

Safeguard analysis of a facility is an important concern for decades in any facility or network and still face enormous challenges. There are various problems in the literature related to domination. This work deals with location domination problem.

A location dominating set is a dominating set S that locates all the vertices in the sense that every vertex not in S is uniquely determined by its neighborhood in S . The location domination number of G , denoted by $\gamma_L(G)$, is the minimum cardinality of a locating dominating set in G .

Location domination are significant in dealing with security of networks. The protection analysis for these networks can be modelled as a graph $G = (V, E)$ where each vertex can represent a room, staircase, etc., and if two rooms are adjacent then they are connected by an edge.

The problem of solving location domination for graphs is placing detection devices at a set S of vertex location, so that one can accurately locate an intruder (fire, thief, etc.), that is, a malfunctioned node in that network.

In this project, the location dominating set problem for generalized friendship graphs is solved. Generalized friendship graphs are used to analyze social networks. Domination in social networks is used to find the most powerful node in such networks and location domination is significant in locating the faulty node in that network.

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CHAPTER-1

INTRODUCTION

Graph theory is a dynamic field in theory, application and practice which is useful in establishing important connections with mainstream areas of pure and applied Mathematics. Graph theory has now burgeoned into one of the fastest growing branches of Mathematics. Graphs are among the most ubiquitous models of both natural and human-made structures. Graph theory research from the middle of the twentieth century has been motivated by problems in combinatorial optimization, computer science, operations research, game theory, decision theory, urban planning, molecular biology, social science, geometry and so on. Graph theory has now grown to touch almost every mathematical discipline, in one form or another, and it likewise borrows from elsewhere tools for its own problems.

Graphs are used in the field of chemistry to model chemical compounds which include the study of molecules and atoms and construction of bonds. In computational biochemistry some sequences of cell samples have to be excluded to resolve the conflicts between two sequences. This is modelled in the form of graph where the vertices represent the sequences in the sample. An edge will be drawn between two vertices if and only if there is a conflict between the corresponding sequences. The aim is to remove possible vertices and to eliminate all conflicts. In statistical physics, graphs can represent local connections between interacting parts of a system, as well as the dynamics of a physical process on such systems.

Graph theory is used in sociology to measure actor's prestige or to explore diffusion mechanisms notably through the use of social network analysis software. Graph theory is used in biology and conservation efforts where, a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affect other species.

Graph theoretical concepts are widely used in Operations Research where graphs are used in planning and scheduling large complicated projects. There is a lot more abstract Mathematics in electrical and electronic engineering. For example, in communication theory, signal processing, networking etc. where networks involve nodes which communicate with each other. Therefore, in recent years, information security has devoted attention to the use of graph theoretic techniques to study the prevention and mitigation of exploits of networks.

Graph theory serves as a tool to solve a number of problems in other disciplines particularly in distributed computing. The digital image processing techniques used in image analysis by which information from images is extracted can be improved using a graph theoretic approach. Moreover, researchers in graph theory and computation are making efforts to discover efficient algorithms for solving these problems. In brief, graph theory has its unique impact in diverse fields and is growing large now a days. Anyone who delves into the topic will quickly see that the lifeblood of graph theory is the abundance of tricky questions and clever answers.

1.1 PRELIMINARIES

In this section we present many of the graph theoretic terms used in this project. We use standard graph theory terminology (Frank Harary, 1993).

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G , called edges. The vertex set and the edge set of G are denoted by $V(G)$ and $E(G)$ respectively. If $e = (u, v)$ is an edge of G , we say that u and v are adjacent and that each vertex is incident with e . The degree of a vertex v in G , denoted by $\deg v$, is the number of edges incident with v . We will denote by $\delta(G)$ and $\Delta(G)$ the smallest and the largest degree of a vertex in G , respectively.

A graph G is called r -regular if $\deg v = r$, for each $v \in V(G)$. A vertex v is called a pendant vertex if $\deg v = 1$ and an isolated vertex if $\deg v = 0$. An edge e in a graph G is called a pendant edge if it is incident with a pendant vertex. An edge with identical ends is called a loop and an edge with distinct end is a link. A graph is simple if it has no loops and no two of its links join the same pair of vertices.

A directed graph is a graph, in which the vertices are connected together by edges where all the edges are directed from one vertex to another. An undirected graph is a graph, in which the vertices are connected together by edges, where all the edges are bidirectional.

A complete graph is a graph in which each pair of vertices is connected by an edge. The complete graph with n vertices is denoted by K_n . Figure 1.1 is an example of a complete graph on 4 vertices. The complement $\bar{G}$ of a graph G also has $V(G)$ as its vertex set, and two vertices are adjacent in $\bar{G}$ if and only if they are not adjacent in G . An empty graph is a graph with no edges.

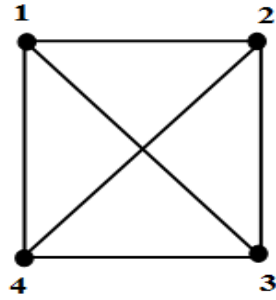


Figure 1.1: A complete graph K_4

The neighbourhood of a vertex i denoted by $N(i)$ is the set of vertices adjacent to i given by $N(i) = \{j \in V(G) : (i, j) \in E(G)\}$. The closed neighborhood of a vertex v , denoted by $N[v]$ is given by $N[v] = N(v) \cup \{v\}$. Vertices x and y are neighbors if (x, y) is an edge in G . In Figure. 1.2, $N(3) = \{2, 4, 5\}$ and $N[3] = \{2, 3, 4, 5\}$.

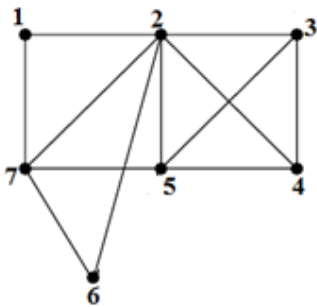


Figure 1.2: Neighbourhood of a graph

1.2 MOTIVATION

Determining a malfunctioning component in a processor network gives the motivation for location dominating sets. The network is modelled by a graph $G = (V, E)$ where V is the set of processors and E is the set of bidirectional communication links between them. We choose a subset S of processors. The problem of solving location domination for graphs is placing detection devices at a set S of vertex location, so that one can accurately locate an intruder (fire, thief, etc.), that is, a malfunctioned node in that network.

This work focuses on solving the location dominating set problem for generalized friendship graphs. Generalized friendship graphs are used to analyze social networks. Domination in social networks is used to find the most powerful node in such networks and location domination is significant in locating the faulty node in that network.

CHAPTER-2

REVIEW OF LITERATURE

In this Chapter a brief survey of work on location domination is presented.

2.1 A SURVEY

Slater introduced location domination and he has applied his location domination concept in acyclic graphs (Slater et al., 1987). Since then, there has been some progress in finding the location domination number of different graphs. Well-covered graph is one where every maximal independent set of vertices is maximum. The concept of location dominating set for well-covered graph was found by A. Finbow and A. Finbow et al., 1988).

T.W. Haynes presented the NP-completeness, frame works, approach and generalized graph domination from mathematical properties (T.W. Haynes et al., 1988). The single-fault tolerant location dominating set was analyzed by P.J. Slater (P.J. Slater 2002). Liro Honkala studied the smallest possible density of a location dominating set in a grid graph in 2006 (Liro Honkala et al., 2006). Muhammad Fazil extended the location dominating sets to hypergraphs (Muhammad Fazil et al., 2007).

In the year of 2012, Michael A. Henning showed that the location total domination number of a claw-free cubic graph is at most one-half its order (Michael A. Henning et al., 2012). S.R. Canoy characterized the location dominating set in corona of composition of graphs in 2013 (S.R. Canoy et al., 2013). C. Hernando determined some Nordhaus-Gaddum bounds for the location domination number of β , η and λ (C. Hernando et al., 2014). F. Foucaud studied location domination for the class of line graphs (F. Foucaud et al., 2017, G. Rajasekar et al., 2019).

The m -shadowing graph of a connected graph G is constructed by taking m -copies of G . Location dominating sets for these graphs were discussed by Dafik (Dafik et al., 2018). R. Adawiyah analyzed the location edge domination number of

wheel graphs and upper bound of location edge domination number (R. Adawiyah et al., 2018). Indra Rajasingh determined the location domination and location total domination numbers for hyper trees and sibling trees (Indra Rajasingh et al., 2020). Jia Guo discussed the bounds of location total-domination number of grid graphs (Jia Guo et al., 2020).

In the next chapter, location domination number is obtained for generalized friendship graphs. The location dominating sets are helpful to identify the location of the problem by a set of sensors placed at these positions.

CHAPTER -3

LOCATION DOMINATION IN GRAPHS

3.1 DOMINATION

In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D . The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G .

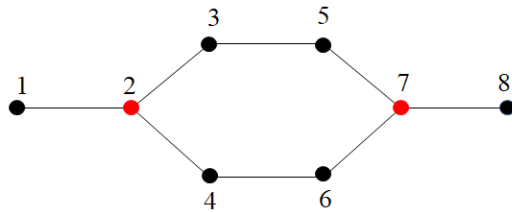


Fig. 3.1: *Dominating set of $G = \{2, 7\}$, $\gamma(G) = 2$*

3.2 LOCATION DOMINATION

The neighborhood of a vertex in a graph G is defined on the same vertex set as G in which two vertices are adjacent, if they have a common neighbor in G (See Fig.3.2). A locating dominating set is a dominating set S that locates all the vertices in the sense that every vertex not in S is uniquely determined by its neighborhood in S . $\gamma_L(G)$ is the minimum cardinality of a location dominating set (C.J. Colbourn et al., 1987, J. Gimbel et al., 1988) in a graph G (See Fig.3.3).

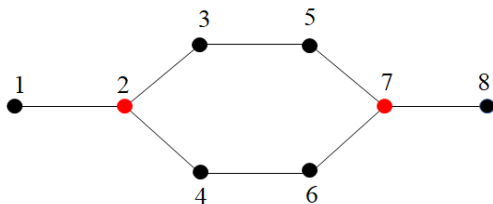


Fig. 3.2: $N(2) = \{3, 4, 1\}$ and $N(7) = \{5, 6, 8\}$

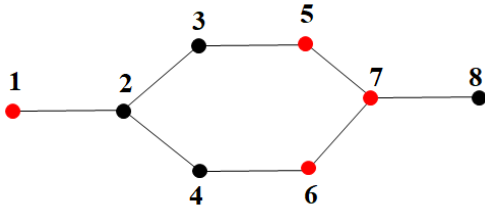


Fig. 3.3: Location dominating set $S = \{1, 5, 6, 7\}$ and $\gamma_L(G) = 4$

3.3APPLICATIONS

For safeguard analysis of a facility, such as a fire protection study of nuclear power plants, the facility can be modelled by a graph $G = (V, E)$ where each vertex can represent a room, staircase, etc., and if two rooms are adjacent they are connected by an edge. Location domination in graph $G = (V, E)$ is beneficial for placing detection devices, such as surveillance cameras or fire alarms in a system to monitor this facility. On placing the detection devices at a set $S = \{1, 5\}$ of vertex location we can precisely locate an intruder (fire, thief, etc.,) in the facility (See Fig. 3.4).

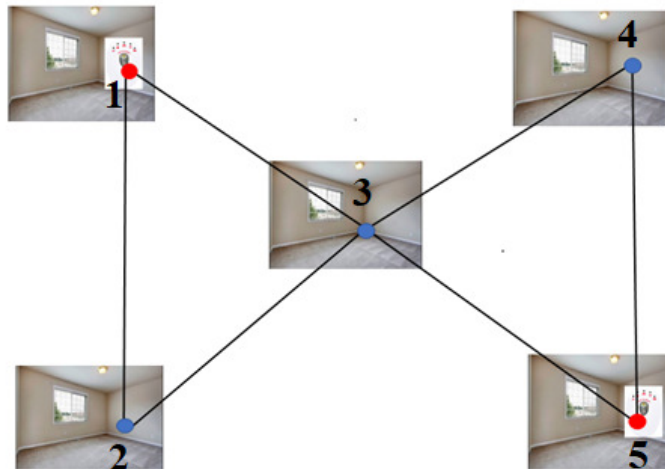


Fig. 3.4: Location dominating set

CHAPTER - 4

LOCATION DOMINATION OF GENERALIZED FRIENDSHIP GRAPHS

4.1 GENERALIZED FRIENDSHIP GRAPHS $F_{n, k}$

The generalized friendship graph $F_{n, k}$ is a graph with $n(k - 1) + 1$ vertices and nk edges. These graphs are constructed by joining n number of copies of the cycle graph C_k with a common vertex, where the value of n is greater than one (See Fig.4.1). Generalized Friendship graphs are used to analyze social networks. Domination in social networks is used to find the most powerful node in such networks and location domination is significant in locating the unusable node in that network (P.J. Slater 1988, T.W. Haynes et al., 1988).

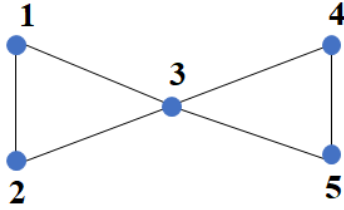


Fig.4.1: Generalized Friendship graph $F_{2, 3}$

4.2 LOCATION DOMINATION NUMBER OF $F_{n, k}$

Theorem 4.2.1

If G is a generalized friendship graph $F_{n, k}$, then $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor$ is valid for all $n > 1$ and $k \not\equiv 0 \pmod{6}$.

Proof:

We prove the result by mathematical induction method.

Here $n > 1$.

Case 1: Let $k = 3$

let $n = 2$

We prove that $\gamma_L(F_{2,3}) = 2$.

By definition of location domination, the red-colored vertices from Fig.4.2, gives the minimum location dominating set, $S = \{1,5\}$.

Therefore, $\gamma_L(F_{2,3}) = 2$.

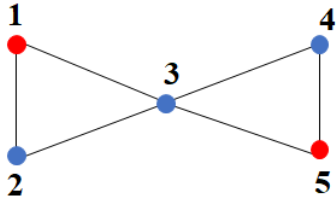


Fig. 4.2: Location Domination number for $F_{2,3}$ is 2

Let $n = 3$.

We shall prove that $\gamma_L(F_{3,3}) = 3$.

Since $F_{3,3}$ is constructed from $F_{2,3}$ the minimum location dominating set S for $F_{3,3} = \{1,5\} \cup \{7\}$.

Therefore, $\gamma_L(F_{3,3}) = \gamma_L(F_{2,3}) + 1 = 2 + 1 = 3$ (See Fig.4.3).

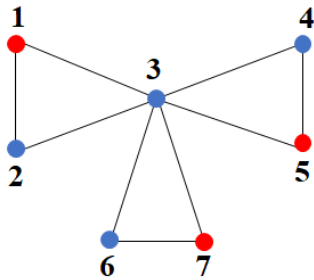


Fig. 4.3: Location Domination number for $F_{3,3}$ is 3

Similarly, for $n = 4$,

We know that, $F_{4, 3}$ is constructed from $F_{3, 3}$.

From Fig. 4.4, the red-colored vertices are the minimum location dominating set S for $F_{4, 3} = \{1,5,7\} \cup \{9\}$.

Thus, $\gamma_L(F_{4, 3}) = \gamma_L(F_{3, 3}) + 1 = 3 + 1 = 4$.

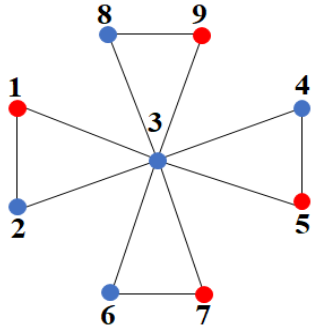


Fig. 4.4: Location Domination number for $F_{4, 3}$ is 4

Now we shall prove the result for a general n .

Assuming that $\gamma_L(F_n) = n$, we prove that $\gamma_L(F_{n+1}) = n + 1$.

Since $F_{n+1, 3}$ is constructed from $F_{n, 3}$ by adding a triangle to the common vertex, only one vertex is chosen for S from that triangle.

Therefore, $\gamma_L(F_{n+1, 3}) = \gamma_L(F_{n, 3}) + 1 = n + 1$

Hence for any value of n , $\gamma_L(F_{n, 3}) = n$.

Case 2: Let $k = 4$.

We prove the result for any $n > 1$ by mathematical induction method on n for different values of $k > 3$ and finally derive the result for a general k . That is, for $n > 1$, we will prove that $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor$.

Assuming that $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 2n$,

we shall prove that $\gamma_L(F_{n+1, k}) = 2(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1,k}$ is constructed from $F_{n,k}$ and $F_{n+1,k}$ differs from $F_{n,k}$ by one cycle C_4 , we require to choose 2 more from vertices C_4 . Therefore, $\gamma_L(F_{n+1,k}) = \gamma_L(F_{n,k}) + 2 = 2n + 2 = 2(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

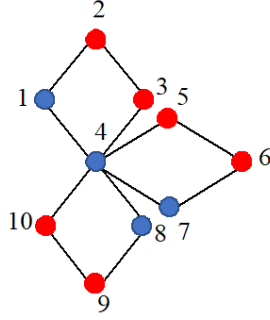


Fig. 4.5: Location Domination number for $F_{3,4}$ is 6

With a similar argument, we can prove for $k = 5$,

By assuming, $\gamma_L(F_{n,k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 2n$,

we shall prove that $\gamma_L(F_{n+1,k}) = 2(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1,k}$ is constructed from $F_{n,k}$,

$$\gamma_L(F_{n+1,k}) = \gamma_L(F_{n,k}) + 2 = 2n + 2 = 2(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor.$$

Case 3: Let $k = 7$.

Assume that $\gamma_L(F_{n,k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 3n$,

we shall prove that $\gamma_L(F_{n+1,k}) = 3(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1,k}$ is constructed from $F_{n,k}$ and $F_{n+1,k}$ differs from $F_{n,k}$ by one cycle C_7 , we require to choose 3 more from vertex C_7 . Therefore, $\gamma_L(F_{n+1,k}) = \gamma_L(F_{n,k}) + 3 = 3n + 3 = 3(n + 1) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

Case 4: Let $k = 8$

Assume that, $\gamma_L(F_{n,k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 4n$,

we shall prove that $\gamma_L(F_{n+1, k}) = 4(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1, k}$ is constructed from $F_{n, k}$, and $F_{n+1, k}$ differs from $F_{n, k}$ by one cycle C_8 , we require to choose 4 more from vertices C_8 to be present in the location domination set.

Therefore, $\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 4 = 4n + 4 = 4(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Also, we can prove for $k = 9$,

Assume that, $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 4n$,

we shall prove that $\gamma_L(F_{n+1, k}) = 4(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1, k}$ is constructed from $F_{n, k}$,

$\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 4 = 4n + 4 = 4(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Case 5: Let $k = 10$

Assume that $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 5n$,

we shall prove that $\gamma_L(F_{n+1, k}) = 5(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1, k}$ is constructed from $F_{n, k}$ and $F_{n+1, k}$ differs from $F_{n, k}$ by one cycle 10, we require to choose 5 more from vertices C_{10} . Therefore,

$\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 5 = 5n + 5 = 5(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

In a similar way, we can prove for $k = 11$,

By assuming, $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor = 5n$,

we shall prove that $\gamma_L(F_{n+1, k}) = 5(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1, k}$ is constructed from $F_{n, k}$,

$\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 5 = 5n + 5 = 5(n+1) = (n+1) \left\lfloor \frac{k}{2} \right\rfloor$.

Considering, the above cases, for a general n , in each cycle C_k there are $\left\lfloor \frac{k}{2} \right\rfloor$ number of vertices which are to be chosen in the minimum location dominating set.

Since $F_{n,k}$ is constructed by joining n copies of the cycle C_k with the common vertex, the cardinality of minimum location dominating set is $n \left\lfloor \frac{k}{2} \right\rfloor$.

Assuming that $\gamma_L(F_{n, k}) = n \left\lfloor \frac{k}{2} \right\rfloor$

we shall provethat $\gamma_L(F_{n+1, k}) = (n + 1) \left\lfloor \frac{k}{2} \right\rfloor$.

Since $F_{n+1, k}$ is constructed from $F_{n, k}$, $F_{n+1, k}$ differs from $F_{n, k}$ by one cycle C_k , we require to choose $\left\lfloor \frac{k}{2} \right\rfloor$ from vertices C_k .

Therefore, we have, $\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + \left\lfloor \frac{k}{2} \right\rfloor$

$$= n \left\lfloor \frac{k}{2} \right\rfloor + \left\lfloor \frac{k}{2} \right\rfloor$$

$$= (n + 1) \left\lfloor \frac{k}{2} \right\rfloor.$$

Theorem 4.2.2:

If $G = F_{n, k}$ is a generalized friendshipgraph, then $\gamma_L(F_{n, k}) = \left(\frac{n(k-2)}{2} \right) + 1$ when $k \equiv 0(mod 6)$.

Proof:

We prove the result by mathematical induction method on n for different values of k , where $k \equiv 0(mod 6)$ and show that $\gamma_L(F_{n, k}) = \left(\frac{n(k-2)}{2} \right) + 1$.

Case 1: Let $k = 6$.

Assuming that the result is true for n , that is,

$$\gamma_L(F_{n, k}) = \left(\frac{n(k-2)}{2} \right) + 1 = \left(\frac{n(6-2)}{2} \right) + 1 = 2n + 1,$$

we shall prove that $\gamma_L(F_{n+1, k}) = \left(\frac{(n+1)(k-2)}{2} \right) + 1$.

In other words, we shall prove that $\gamma_L(F_{n+1, k}) = 2(n + 1) + 1$.

We observe that for every $F_{n, k}$, there are n copies of C_6 attached to the common vertex, we choose 3 vertices from the first cycle C_6 and 2 vertices from the remaining $(n-1)$ cycles C_6 (See Fig. 4.6).

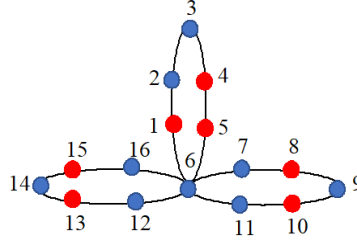


Fig. 4.6: Location Domination number for $F_{3, 6}$ is 7

Since $F_{n+1, k}$ is constructed from $F_{n, k}$ and $F_{n+1, k}$ differs from $F_{n, k}$ by one cycle C_6 , we require to choose 2 more from vertices C_6 .

$$\text{Therefore, } \gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 2 = 3 + 2(n - 1) + 2.$$

$$\text{This implies } \gamma_L(F_{n+1, k}) = 3 + 2n - 2 + 2.$$

$$\text{Thus, } \gamma_L(F_{n+1, k}) = 3 + 2n = 1 + 2 + 2n = 2(n + 1) + 1.$$

Case 2: Let $k = 12$.

Assuming that the result is true for n , that is,

$$\gamma_L(F_{n, k}) = \left(\frac{n(k-2)}{2} \right) + 1 = \left(\frac{n(12-2)}{2} \right) + 1 = 5n + 1,$$

$$\text{we shall prove that } \gamma_L(F_{n+1, k}) = \left(\frac{(n+1)(k-2)}{2} \right) + 1.$$

$$\text{In other words, we shall prove that } \gamma_L(F_{n+1, k}) = 5(n + 1) + 1.$$

We observe that for every $F_{n, k}$, there are n copies of C_{12} attached to the common vertex, we choose 6 vertices from the first cycle C_{12} and 5 vertices from the remaining $(n-1)$ cycles C_{12} .

Since $F_{n+1, k}$ is constructed from $F_{n, k}$ and $F_{n+1, k}$ differs from $F_{n, k}$ by one cycle C_{12} , we require to choose 5 more from vertices C_{12} . Therefore, $\gamma_L(F_{n+1, k}) = \gamma_L(F_{n, k}) + 5 = 6 + 5(n - 1) + 5.$

$$\text{This implies } \gamma_L(F_{n+1, k}) = 6 + 5n - 5 + 5.$$

$$\text{Thus, } \gamma_L(F_{n+1, k}) = 6 + 5n = 1 + 5 + 5n = 5(n + 1) + 1.$$

Considering the above cases, for a general n , in each cycle C_k , we require to choose $\frac{k}{2}$ vertices from the first cycle C_k and $\left(\frac{k}{2} - 1\right)$ vertices from the remaining $(n-1)$ cycles C_k .

Location dominating sets are equivalent to identifying codes but have an interesting variation in that only the nodes which do not belong to the location dominating set are required to have unique identifying sets (F. Foucaud 2013, M. Blidia et al., 2007).

Previously, location domination sets and identifying codes were adapted in the fault diagnosis in the sustainment of multiprocessor systems. Contemporaneously, identifying codes and locating dominating sets are used in joint monitoring and routing in wireless sensor networks and contamination detection in environmental monitoring (Müller et al., 2009).

CHAPTER - 5

CONCLUSION

5.1 SUMMARY

In Chapter 1 we started with the introduction and in Chapter 2, we presented a brief survey of work on location domination number. Chapter 3 includes the definitions of domination, location domination and its applications. We then moved on, in Chapter 4 where the location domination of generalized friendship graphs is studied. Finally, we conclude this Chapter 5 by summarizing the significance of our project work and provided open problems for future research.

5.2 CONCLUSION

Computer networks can be modelled as graphs and location domination helps in locating the malfunctioned nodes. The location dominating sets are helpful to identify the location of the problem by a set of sensors placed at these positions in the facility. Calculating location dominating sets are of interest in safeguard applications of graphical models of facilities $G = (V, E)$ where each vertex can represent a room, staircase, etc., In this project, location domination number of generalized friendship graphs are obtained. There is still much more research to be done on this topic of location dominating sets. Further research can be considered on various other computer architectures like butterfly, Benes, cube connected cycles network, etc.,

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