

UNSTEADY TWO-DIMENSIONAL LAMINAR FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES IN PRESENCE OF TRANSVERSE MAGNETIC FIELD

Submitted in partial fulfilment of the requirements for the award of
Master of Mathematics

by

M.DIVYA (Reg.No.39630004)



**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited with Grade "A" by NAAC I 12B

Status by UGC I Approved by AICTE

**JEPPIAAR NAGAR, RAJIV GANDHI SALAI,
CHENNAI - 600 119**

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Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai – 600 119

www.sathyabama.ac.in

BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **M.Divya(39630004)** who carried out the project entitled "**UNSTEADY TWO-DIMENSIONAL LAMINAR FLOW OF A VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES IN PRESENCE OF A TRANSVERSE MAGNETIC FIELD**" under my supervision from **January 2021 to April 2021**

Internal Guide

(Name in Capital letters
with Signature) Signature)

External Guide(if Applicable)

(Name in capital letters with

Head of the Department

(Name in Capital letters with Seal & Signature)

(M. NIRMALA)

Submitted for Viva voce Examination held on _____

Internal Examiner
(Name in Capital letters
with Signature)

External Examiner
(Name in capital letters with
Signature)

DECLARATION

I **M.Divya (39630004)** hereby declare that project Report entitled “**UNSTEADY TWO-DIMENSIONAL LAMINAR FLOW OF A VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES IN PRESENCE OF A TRANSVERSE MAGNETIC FIELD**” done by me under the guidance of **Dr.S.GANESH**(Internal) and _____(External) at **Sathyabama Institute of Science and Technology** is submitted in partial fulfillment of the requirements for the award of master of science degree in Mathematics.



DATE : 31.03.2021

PLACE : CHENNAI

SIGNATURE OF THE CANDIDATE

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ABSTRACT:

The unsteady Magnetohydrodynamic flow between two parallel porous plates is a classical problem whose solution has many applications in magnetohydrodynamic (MHD) power generators, cooling system, aerodynamics heating, polymer technology, petroleum industry, centrifugal separation of matter from fluid, purification of crude oil and fluid droplets and sprays.

KEYWORDS:

Unsteady laminar flow, magnetohydrodynamic flow, Suction Reynolds number, Hartmann number, Pressure gradient

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LIST OF SYMBOLS AND ABBREVIATIONS:

σ -electrical conductivity

$B_0 = \mu_e H_0$ - electromagnetic induction

μ_e -magnetic permeability

H_0 -transverse magnetic field

ω - frequency.

$\nu = \frac{\mu}{\rho}$ - Kinematic Viscosity

ρ - density of the fluid

μ - coefficient of viscosity

p - pressure

R - Suction Reynolds number

M - Hartmann number

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CHAPTER-1

1 INTRODUCTION:

1.1 FLUID DYNAMICS:

Fluid dynamics is "the branch of applied science that is concerned with the movement of liquids and gases," according to the American Heritage Dictionary. Fluid dynamics is one of two branches of fluid mechanics, which is the study of fluids and how forces affect them. (The other branch is fluid statics, which deals with fluids at rest.)

Scientists across several fields study fluid dynamics. Fluid dynamics provides methods for studying the evolution of stars, ocean currents, weather patterns, plate tectonics and even blood circulation. Some important technological applications of fluid dynamics include rocket engines, wind turbines, oil pipelines and air conditioning systems

1.2 VISCIOUS PROBLEM:

Viscous problems are those problems in which the fluid friction has a significant effect on the fluid motion

1.3 REYNOLDS NUMBER:

Reynolds number is the ratio between the inertial forces and viscous forces.

$$\text{Reynolds number} = \frac{\text{Inertial force}}{\text{Viscous force}}$$

Viscous force

$$Re = \frac{\rho V D}{\mu}$$

Where,

Re is the Reynolds number

ρ is the density of the fluid

V is the velocity of flow

D is the pipe diameter

μ is the viscosity of the fluid

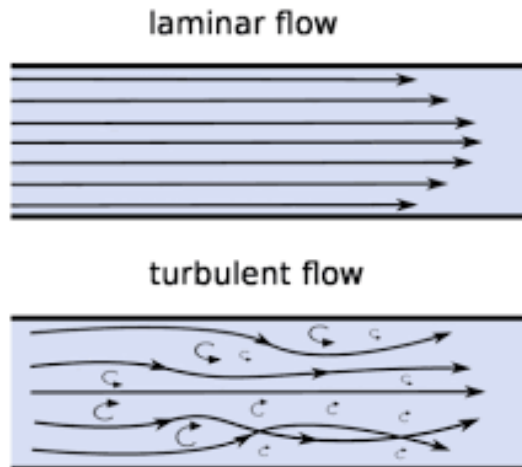
Reynolds number is dimensionless quantity to determine the type of flow pattern as laminar or turbulent while flowing through a pipe.

1.4 LAMINAR FLOW:

Laminar flow is a smooth free flow of fluid in parallel layers with no disruption.

1.5 TURBULENT FLOW:

Turbulent flow is a flow regime characterized by chaotic property changes.

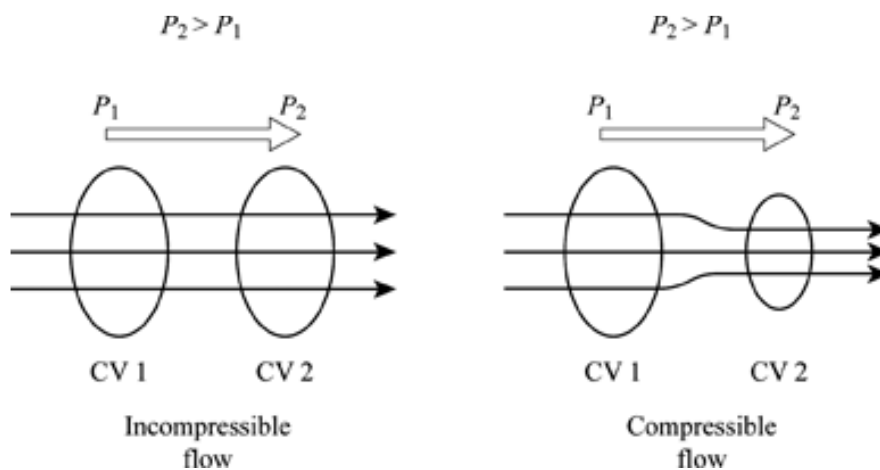


1.6 INCOMPRESSIBLE FLOW:

The changes in pressure and temperature are very small and that results in change of density which are negligible is called Incompressible flow.

1.7 COMPRESSIBLE FLOW:

Compressible flow is a flow that deals with fluids in which the fluid density varies significantly in response to a change in pressure



CHAPTER-2

2. LITERATURE SURVEY:

Hassanien and Mansour (1990) discussed the Unsteady magnetic flow through a porous medium between two infinite parallel plates. Bagchi (1996) studied the problem of Unsteady flow of viscoelastic Maxwell fluid with transient pressure gradient through a rectangular channel. Attia and Kotb (1996) studied the Steady, fully developed MHD flow and heat transfer between two parallel plates with temperature dependant viscosity.

Attia (1999) extended the problem to the transient state. Ezzat, Othman and Helmy (1999) studied the problem of micropolar magnetohydrodynamic boundary layer flow. Aboul-Hassan and Attia (2002) discussed the flow of a conducting Visco elastic fluid between two horizontal porous plates in the presence of a transverse-magnetic field. Soundalgekar, Vighnesam and Takhar

(2003) discussed the effects of suction and injection on MHD heat transfer in stagnation point flow with variable body temperature.

Nabil, Eldabe, Galal, Moatimid and HodaSm Ali (2003) studied the MHD flow of Non-Newtonian visco-elastic fluid through a porous medium near an accelerated plate. Attia (2004) has considered the Unsteady Hartmann flow with heat transfer of a viscoelastic fluid considering the Hall effect. Hayat, Wang and Hutter (2004) studied the Hall effects on the Unsteady hydromagnetic oscillatory flow of a second-grade fluid.

Krishnambal and Ganesh (2004) discussed the Unsteady stokes flow of viscous fluid between two parallel porous plates. Hazeem Ali Attia (2005) studied the Unsteady laminar flow of an incompressible viscous fluid and heat transfer between two parallel plates in the presence of a uniform suction and injection considering variable properties. Hazeem Ali Attia (2005) studied the Unsteady flow of a dusty conducting fluid between parallel porous plates with temperature dependent viscosity.

CHAPTER-3

AIM AND SCOPE:

The Unsteady Magnetohydrodynamic flow between two parallel porous plates is a classical problem whose solution has many applications in magnetohydrodynamic (MHD) power generators, cooling system, aerodynamics heating, polymer technology, petroleum industry, and centrifugal separation of matter from fluid, purification of crude oil and fluid droplets and sprays.

The objective of this chapter is to analyse the Unsteady Magnetohydrodynamic Stokes flow of viscous fluid between two parallel porous plates when the fluid is being withdrawn through both the walls of the channel at the same rate. The problem is reduced to a third order nonlinear differential equation which depends on a Suction Reynolds number R and a Hartmann number M for which an exact solution is obtained.

CHAPTER-4

MATHEMATICAL FORMULATION:

The unsteady laminar flow of an incompressible viscous fluid between two parallel porous plates is considered in the presence of a transverse magnetic field H_0 applied perpendicular to the walls. The origin is taken at the centre of the channel and let x and y be the coordinate axes parallel and perpendicular to the channel walls.

The length of the channel is assumed to be L and $2h$ is the distance between the two plates. Let u and v be the velocity components in the x and y directions respectively.

$$\text{The equation of continuity is } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Equations of momentum are

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \sigma_e B_0^2 u \quad (2)$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

where σ is the electrical conductivity and $B_0 = \mu_e H_0$, B_0 is the electromagnetic induction, μ_e being the magnetic permeability and H_0 is the transverse magnetic field. The boundary conditions are

$u = 0$ on $y = h$ and $y = -h$; $v = v_0 e^{i\omega t}$ on $y = h$ and $v = -v_0 e^{i\omega t}$ on $y = -h$ where v_0 is the velocity of suction at the walls of the channel.

Let $\eta = \frac{y}{h}$, $u = u(x, \eta) e^{i\omega t}$, $v = v(x, \eta) e^{i\omega t}$, $p = p(x, \eta) e^{i\omega t}$ where ω is the frequency.

The equations (1), (2) and (3) become

$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \eta} = 0 \quad (4)$$

$$\rho i \omega u e^{i\omega t} = -\frac{\partial p}{\partial x} e^{i\omega t} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial \eta^2} \right) e^{i\omega t} - \sigma_e B_0^2 u e^{i\omega t}$$

i.e.
$$\rho i \omega u = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \eta^2} \right) - \sigma_e B_0^2 u \quad (5)$$

$$\rho i \omega v e^{i\omega t} = -\frac{\partial p}{\partial y} e^{i\omega t} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial \eta^2} \right) e^{i\omega t}$$

i.e.
$$\rho i w v = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \eta^2} \right) \quad (6)$$

Let $\nu = \frac{\mu}{\rho}$ = Kinematic Viscosity, ρ the density of the fluid, μ the coefficient of viscosity and p the pressure.

The boundary conditions are converted into

$$u(x,1) = 0, \quad u(x,-1) = 0 \quad (7)$$

and
$$v(x,1)=v_0, \quad v(x,-1)=-v_0 \quad (8)$$

Let ψ be the stream function such that

$$u = \frac{1}{h} \frac{\partial \psi}{\partial \eta} \quad (9)$$

$$v = -\frac{\partial \psi}{\partial x} \quad (10)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\eta) = (hU(0) - v_0 x) f(\eta) \quad (11)$$

where $U(0)$ is the average entrance velocity at $x = 0$. From equation (11), the velocity components (9) and (10) are given by

$$u = \frac{1}{h} (hU(0) - v_0 x) f'(\eta) \quad (12)$$

$$v = v_0 f(\eta) \quad (13)$$

where the prime denotes the differentiation with respect to the dimensionless variable $\eta = \frac{y}{h}$. Since the fluid is being withdrawn at constant rate from both the walls, v_0 is independent of x .

Using (12) and (13) in (5) and (6), the equations of momentum reduce to

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(i\omega f'(\eta) - \frac{\nu}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right) \quad (14)$$

$$-\frac{1}{h\rho} \frac{\partial p}{\partial \eta} = i\omega v_0 f(\eta) - \frac{\nu \cdot v_0 f''(\eta)}{h^2} \quad (15)$$

Now differentiating (15) w.r.t. 'x', the following equation is obtained.

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \quad (16)$$

Differentiating (14) w.r.t. 'η', it follows that

$$-\frac{1}{\rho} \frac{\partial^2 p}{\partial x \partial \eta} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \frac{d}{d\eta} \left(i\omega f'(\eta) - \frac{\nu}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right) \quad (17)$$

From (16), equation (17) can be written as

$$\frac{d}{d\eta} \left(i\omega f'(\eta) - \frac{\nu}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right) = 0 \quad (18)$$

which is true for all x .

Let $R = \text{Suction Reynolds number} = \frac{h v_0}{\nu}$

$M = \text{Hartmann number} = B_0 h \left(\frac{\sigma_e}{\nu \rho} \right)^{\frac{1}{2}}$

Integrating (18) w.r.t. η and substituting the above expressions, it follows that

$$\begin{aligned}
 f'''(\eta) - \frac{i\omega h^2}{\nu} f'(\eta) - \frac{\sigma_e B_0^2 h^2}{\rho \nu} f'(\eta) &= K \\
 \Rightarrow f'''(\eta) - \left(\frac{i\omega h^2}{\nu} + \frac{\sigma_e B_0^2 h^2}{\rho \nu} \right) f'(\eta) &= K \\
 \Rightarrow f'''(\eta) - \alpha^2 h^2 f'(\eta) - a_1 R f'(\eta) &= K \quad (19)
 \end{aligned}$$

where $R = \frac{h\nu_0}{\nu}$ and $a_1 = \frac{\sigma_e h B_0^2}{\rho \nu_0}$ and K is an arbitrary constant.

Boundary conditions on $f(\eta)$ are

$$f(1) = 1, \quad f(-1) = -1, \quad f'(1) = 0 \quad \text{and} \quad f'(-1) = 0 \quad (20)$$

Hence the solution of the equations of motion and continuity is given by a linear third order differential equation (19) subject to the boundary conditions (20).

$\therefore$ Equation (19) can be rewritten as

$$f'''(\eta) - (\alpha^2 h^2 + M^2) f'(\eta) = K$$

where $a_1 R = M^2$ and $\alpha^2 = \frac{i\rho\omega}{\mu}$

$$\text{i.e. } (D^3 - (\alpha^2 h^2 + M^2) D) f(\eta) = K$$

$$\text{i.e. } D(D^2 - (\alpha^2 h^2 + M^2)) f(\eta) = K$$

where

$$D = \frac{d}{d\eta} \quad \text{and} \quad D^2 = \frac{d^2}{d\eta^2}$$

The roots of the auxiliary equation for the above equation are $m=0$,

$$m = \pm \sqrt{\alpha^2 h^2 + M^2}$$

Hence the complimentary function is $Ae^{0\eta} + Be^{\sqrt{\alpha^2 h^2 + M^2} \cdot \eta} + C e^{-\sqrt{\alpha^2 h^2 + M^2} \cdot \eta}$

$$P.I = \frac{1}{(D^3 - (\alpha^2 h^2 + M^2)D)} \cdot K e^{0\eta}$$

$$\Rightarrow P.I = K\eta \cdot \frac{1}{3D^2 - (\alpha^2 h^2 + M^2)} \cdot e^{0\eta}$$

$$= \frac{-K\eta}{\alpha^2 h^2 + M^2}$$

$$\therefore f(\eta) = A + B e^{\sqrt{\alpha^2 h^2 + M^2} \eta} + C e^{-\sqrt{\alpha^2 h^2 + M^2} \eta} - \frac{K\eta}{\alpha^2 h^2 + M^2} \quad (21)$$

In order to obtain the values of the arbitrary constants, the boundary conditions are applied on $f(\eta)$.

Since $f(1) = 1$,

$$\Rightarrow 1 = A + B e^{\sqrt{\alpha^2 h^2 + M^2}} + C e^{-\sqrt{\alpha^2 h^2 + M^2}} - \frac{K}{\alpha^2 h^2 + M^2} \quad (22)$$

Since $f(-1) = -1$

$$\Rightarrow -1 = A + B e^{-\sqrt{\alpha^2 h^2 + M^2}} + C e^{\sqrt{\alpha^2 h^2 + M^2}} + \frac{K}{\alpha^2 h^2 + M^2} \quad (23)$$

Adding (22) and (23), it is seen that

$$0 = 2A + B \left(e^{\sqrt{\alpha^2 h^2 + M^2}} + e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) + C \left(e^{\sqrt{\alpha^2 h^2 + M^2}} + e^{-\sqrt{\alpha^2 h^2 + M^2}} \right)$$

$$\Rightarrow 0 = A + B \cosh \sqrt{\alpha^2 h^2 + M^2} + C \cosh \sqrt{\alpha^2 h^2 + M^2} \quad (24)$$

(22) - (23), it follows that

$$2 = B \left(e^{\sqrt{\alpha^2 h^2 + M^2}} - e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) - C \left(e^{\sqrt{\alpha^2 h^2 + M^2}} - e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) - \frac{2k}{(\alpha^2 h^2 + M^2)}$$

$$\Rightarrow 1 = B \sinh \sqrt{\alpha^2 h^2 + M^2} - C \sinh \sqrt{\alpha^2 h^2 + M^2} - \frac{K}{(\alpha^2 h^2 + M^2)} \quad (25)$$

$$f'(\eta) = \sqrt{\alpha^2 h^2 + M^2} \cdot B \cdot \left(e^{\eta \sqrt{\alpha^2 h^2 + M^2}} \right) - \sqrt{\alpha^2 h^2 + M^2} \cdot C \cdot e^{-\eta \sqrt{\alpha^2 h^2 + M^2}} - \frac{K}{(\alpha^2 h^2 + M^2)}$$

Since $f'(1)=0$

$$\Rightarrow 0 = \sqrt{\alpha^2 h^2 + M^2} \cdot B e^{\sqrt{\alpha^2 h^2 + M^2}} - \sqrt{\alpha^2 h^2 + M^2} \cdot C e^{-\sqrt{\alpha^2 h^2 + M^2}} - \frac{K}{(\alpha^2 h^2 + M^2)} \quad (26)$$

Since $f'(-1)=0$

$$\Rightarrow 0 = \sqrt{\alpha^2 h^2 + M^2} \cdot B e^{-\sqrt{\alpha^2 h^2 + M^2}} - \sqrt{\alpha^2 h^2 + M^2} \cdot C e^{\sqrt{\alpha^2 h^2 + M^2}} - \frac{K}{(\alpha^2 h^2 + M^2)} \quad (27)$$

Adding (26) and (27), it is seen that

$$0 = \sqrt{\alpha^2 h^2 + M^2} \cdot B \left(e^{\sqrt{\alpha^2 h^2 + M^2}} + e^{-\sqrt{\alpha^2 h^2 + M^2}} \right)$$

$$- \sqrt{\alpha^2 h^2 + M^2} \cdot C \cdot \left(e^{\sqrt{\alpha^2 h^2 + M^2}} + e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) - \frac{2K}{\alpha^2 h^2 + M^2}$$

$$\Rightarrow B \cdot \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2} - C \cdot \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2}$$

$$\frac{-K}{\alpha^2 h^2 + M^2} = 0 \quad (28)$$

(26) - (27), it follows that

$$\begin{aligned}
0 &= \sqrt{\alpha^2 h^2 + M^2} \cdot B \left(e^{\sqrt{\alpha^2 h^2 + M^2}} - e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) \\
&\quad + \sqrt{\alpha^2 h^2 + M^2} \cdot C \left(e^{\sqrt{\alpha^2 h^2 + M^2}} - e^{-\sqrt{\alpha^2 h^2 + M^2}} \right) \\
&\Rightarrow \sqrt{\alpha^2 h^2 + M^2} \cdot B \cdot \sinh \sqrt{\alpha^2 h^2 + M^2} + C \cdot \sqrt{\alpha^2 h^2 + M^2} \cdot \sinh \sqrt{\alpha^2 h^2 + M^2} = 0 \\
&\Rightarrow B = -C \tag{29}
\end{aligned}$$

$$(24) \Rightarrow A - C \cosh \sqrt{\alpha^2 h^2 + M^2} + C \cosh \sqrt{\alpha^2 h^2 + M^2} = 0.$$

$$\Rightarrow A = 0 \tag{30}$$

$$(25) \Rightarrow -C \sinh \sqrt{\alpha^2 h^2 + M^2} - C \sinh \sqrt{\alpha^2 h^2 + M^2} = \frac{K}{\alpha^2 h^2 + M^2} + 1$$

$$\Rightarrow C = -\frac{(K + (\alpha^2 h^2 + M^2))}{2(\alpha^2 h^2 + M^2) \sinh \sqrt{\alpha^2 h^2 + M^2}} \tag{31}$$

$$(28) \Rightarrow \sqrt{\alpha^2 h^2 + M^2} (-C) \cosh \sqrt{\alpha^2 h^2 + M^2}$$

$$- \sqrt{\alpha^2 h^2 + M^2} \cdot (C) \cdot \cosh \sqrt{\alpha^2 h^2 + M^2} = \frac{K}{(\alpha^2 h^2 + M^2)}$$

$$\Rightarrow -2C \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2} = \frac{K}{(\alpha^2 h^2 + M^2)}$$

$$\Rightarrow \sqrt{\alpha^2 h^2 + M^2} \cdot \frac{(K + \alpha^2 h^2 + M^2) \cosh \sqrt{\alpha^2 h^2 + M^2}}{(\alpha^2 h^2 + M^2) \sinh \sqrt{\alpha^2 h^2 + M^2}} = \frac{K}{\alpha^2 h^2 + M^2}$$

$$\Rightarrow K = K \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2} + (\alpha^2 h^2 + M^2)^{3/2} \coth \sqrt{\alpha^2 h^2 + M^2}$$

$$\Rightarrow K \left(1 - \sqrt{\alpha^2 h^2 + M^2} \cot h \sqrt{\alpha^2 h^2 + M^2} \right) = (\alpha^2 h^2 + M^2)^{3/2} \cot h \sqrt{\alpha^2 h^2 + M^2}$$

$$\text{i.e.,} \quad K = \frac{(\alpha^2 h^2 + M^2)^{3/2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2}}{\left(1 - \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2} \right)} \quad (32)$$

$$\therefore C = - \left(\frac{\frac{(\alpha^2 h^2 + M^2)^{3/2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2}}{(1 - \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2})} + \alpha^2 h^2 + M^2}{2(\alpha^2 h^2 + M^2) \text{Sinh} \sqrt{\alpha^2 h^2 + M^2}} \right)$$

(from 31)

$$\Rightarrow C = \frac{-1}{\left(2 \text{Sinh} \sqrt{\alpha^2 h^2 + M^2} \right) \left(1 - \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2} \right)} \quad (33)$$

$$\Rightarrow B = \frac{1}{\left(2 \text{Sinh} \sqrt{\alpha^2 h^2 + M^2} \right) \left(1 - \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2} \right)} \quad (34)$$

(Since B=-C)

Substituting the values of the arbitrary constants in f(η), the expression is

$$f(\eta) = \frac{1}{\left(1 - \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2} \right)} \cdot \left(\frac{\text{Sinh} \sqrt{\alpha^2 h^2 + M^2} \eta}{\text{Sinh} \sqrt{\alpha^2 h^2 + M^2}} - \eta \sqrt{\alpha^2 h^2 + M^2} \text{Coth} \sqrt{\alpha^2 h^2 + M^2} \right) \quad (35)$$

Hence the expressions for the velocity components are

$$u = \frac{1}{h} [hU(0) - v_0 x] f'(\eta) e^{i\alpha x}$$

$$\begin{aligned}
&= \left(U(0) - \frac{v_0 x}{h} \right) \left(\frac{1}{1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2}} \right) \\
&\quad \left(\frac{\sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2} \right) e^{i\alpha x} \\
&= \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(\frac{\sqrt{\alpha^2 h^2 + M^2}}{1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2}} \right) \\
&\quad \left(\frac{\cosh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \coth \sqrt{\alpha^2 h^2 + M^2} \right) e^{i\alpha x} \tag{36}
\end{aligned}$$

$$v = v_0 f(\eta) e^{i\alpha x}$$

$$\begin{aligned}
&= v_0 \left(\frac{1}{1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2}} \right) \\
&\quad \cdot \left(\frac{\sinh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \eta \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2} \right) e^{i\alpha x} \tag{37}
\end{aligned}$$

4.1 PRESSURE DISTRIBUTION

From equation (14), it is seen that

$$\frac{-1}{\rho} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(i\omega f'(\eta) - \frac{v}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right)$$

Dividing by $-\frac{v}{h^2}$, it follows that

$$\frac{h^2}{\rho v} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(\frac{i\omega h^2}{v} f'(\eta) + f'''(\eta) - \frac{\sigma_e B_0^2}{\rho} \cdot \frac{h^2}{v} f'(\eta) \right)$$

$$\frac{h^2}{\rho\nu} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \left(f'''(\eta) - \frac{h^2}{\nu} \left(i\omega + \frac{\sigma_e B_0^2}{\rho} \right) f'(\eta) \right)$$

and since $f'''(\eta) - \alpha^2 h^2 f'(\eta) - a_1 R f'(\eta) = K$ (from 19)

$$\text{i.e. } f'''(\eta) - \frac{h^2 i\omega}{\nu} f'(\eta) - M^2 f'(\eta) = K$$

$$\frac{h^2}{\rho\nu} \frac{\partial p}{\partial x} = K \left(U(0) - \frac{v_0 x}{h} \right) (\text{from (19)})$$

$$\therefore \frac{\partial p}{\partial x} = \frac{K\rho\nu}{h^2} \left(U(0) - \frac{v_0 x}{h} \right)$$

$$= \frac{K\mu}{h^2} \left(U(0) - \frac{v_0 x}{h} \right) \because \left(\nu = \frac{\mu}{\rho} \right) \quad (38)$$

Now, from equation (15) it is seen that

$$-\frac{1}{h\rho} \frac{\partial p}{\partial \eta} = i\omega v_0 f(\eta) - \frac{v_0}{h^2} f''(\eta)$$

$$\frac{\partial p}{\partial \eta} = -i\omega v_0 \rho h f(\eta) + \frac{v_0 \rho}{h} f''(\eta) (\because \mu = \rho\nu)$$

$$\frac{\partial p}{\partial \eta} = \frac{\mu v_0}{h} f''(\eta) - i\omega v_0 h \rho f(\eta) \quad (39)$$

Since $dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy$

$$= \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial \eta} d\eta \because \left(\eta = \frac{y}{h} \right)$$

$$\Rightarrow dp = \frac{K\mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{h} \right) dx + \left(\frac{\mu v_0}{h} f''(\eta) - i\omega v_0 h \rho f(\eta) \right) d\eta \quad (40)$$

Integrating (40), it is clearly seen that

$$\begin{aligned} p(x, \eta) &= \frac{K\mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \left(\frac{\mu v_0}{h} \int f''(\eta) d\eta - i\omega v_0 h \rho \int_0^n f(\eta) d\eta \right) + K_1 \\ p(x, \eta) &= \frac{K\mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \left(\frac{\mu v_0}{h} f'(\eta) - i\omega v_0 h \rho \int_0^n f(\eta) d\eta \right) + K_1 \end{aligned} \quad (41)$$

$\therefore$ The pressure drop is given by

$$\Rightarrow p(x, \eta) - p(0, 0) = \frac{K\mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \frac{\mu v_0}{h} (f'(\eta) - f(0)) - i\omega \rho v_0 h \int_0^n f(\eta) d\eta \quad (42)$$

CHAPTER-5

5. DISCUSSION

The graphs of the axial velocity and radial velocity profiles have been drawn for different values of M . Figures 1, 2 and 3 represent the axial velocity profiles at different cross sections of the channel namely at $x=0, x=3$ and $x=5$ when the average entrance velocity is $u_0=0.5$ and $h = 1.0$. The magnitude of the axial velocity increases as x increases from $x=0$ to $x=5$ for different values of ωt , namely $\omega t = 0, \pi/4, \pi/2, 3\pi/4$, and π respectively.

The Figure 4 represents the axial velocity profiles of u at $x=0, h=1.0$ when the inlet velocity is increased to $u_0=1.0$ from $u_0 = 0.5$. It is clearly seen that the magnitudes of the axial velocity u are more when the values of x are increased and also the

magnitudes of the axial velocity u are more when the inlet velocity is increased. When the Hartmann number M is increased from $M = 1$ to $M = 5$, we see that the magnitude of the axial velocity profiles decrease as seen in Figure 5.

The Figures 6 and 7 represent the radial velocity profiles of v at $v_0 = 0.5$, $h = 1.0$, $\alpha = 1.0$ and for different values of M . As M increases from $M = 1.0$ to $M = 5.0$, it is seen that there is a marginal increase in the magnitude of the radial velocity. It is also seen from the Figures 6 and 7 that the radial velocity vanishes for $\omega t = \pi/2$ and the radial velocity profiles are non linear for other values of ωt .

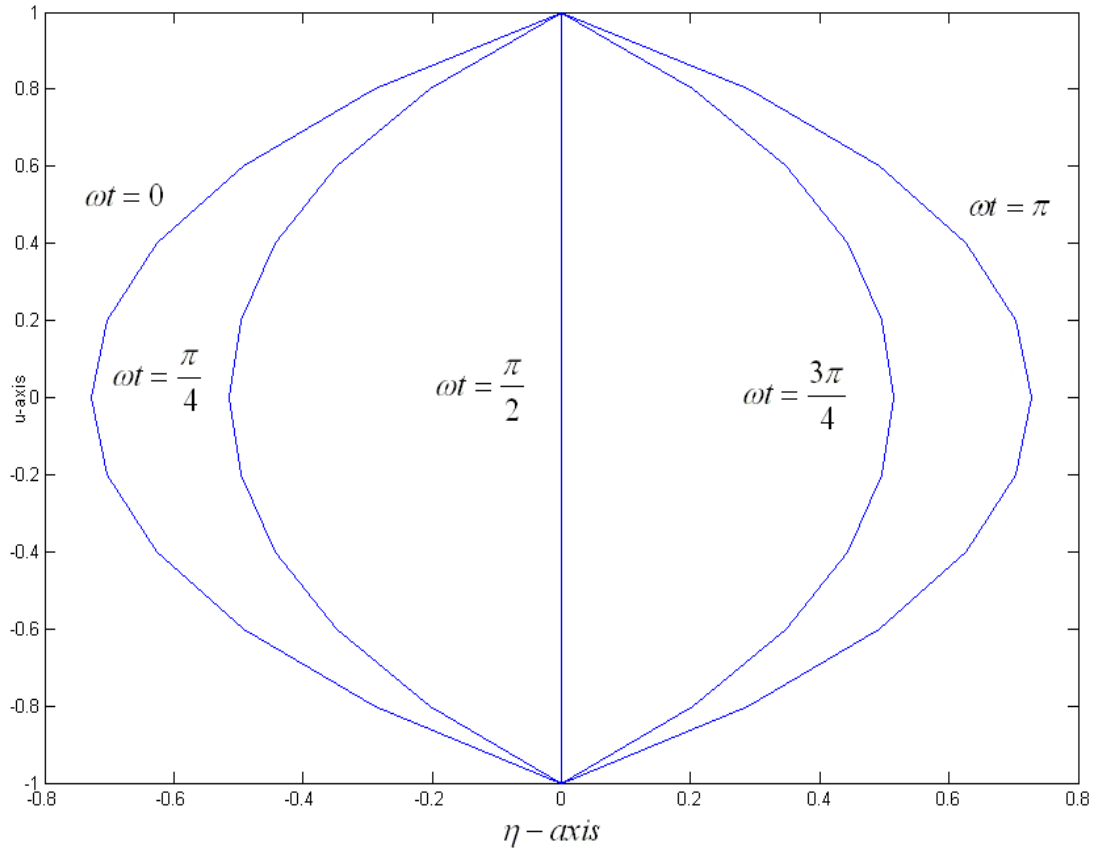


Figure 5.1: Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

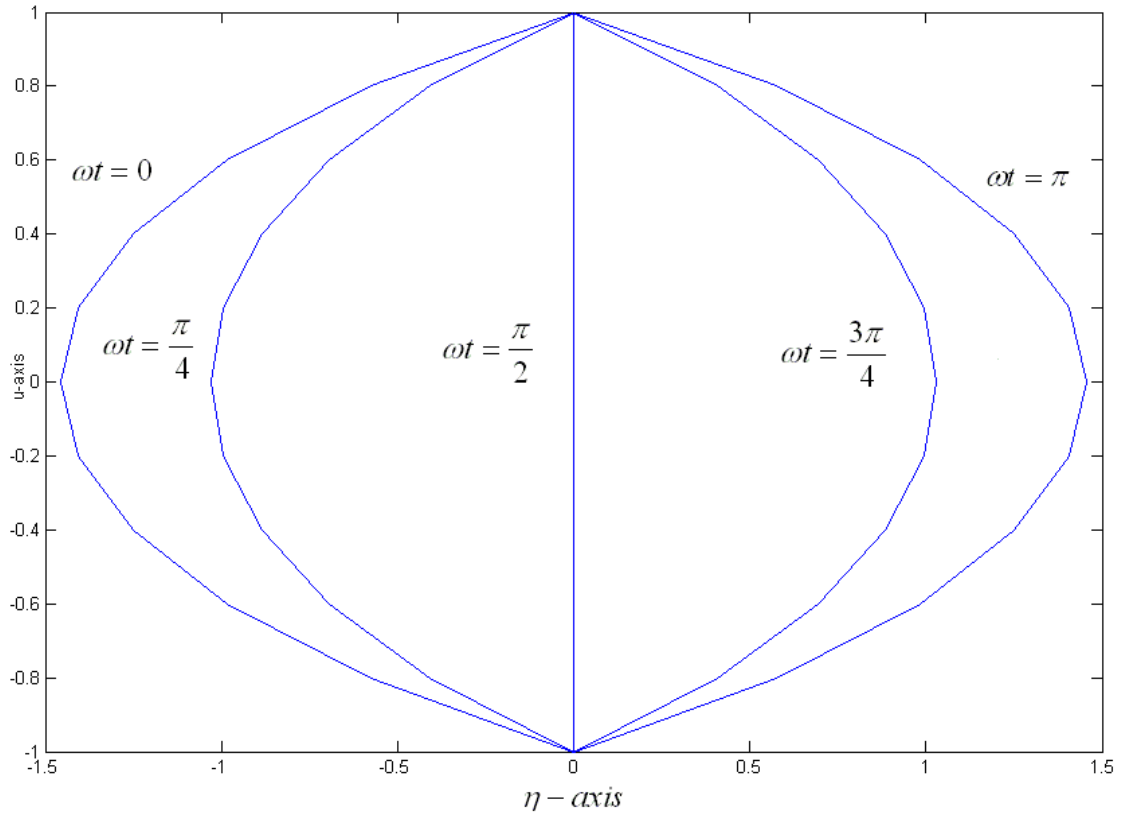


Figure 5.2: Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=3$ and for different values of ωt

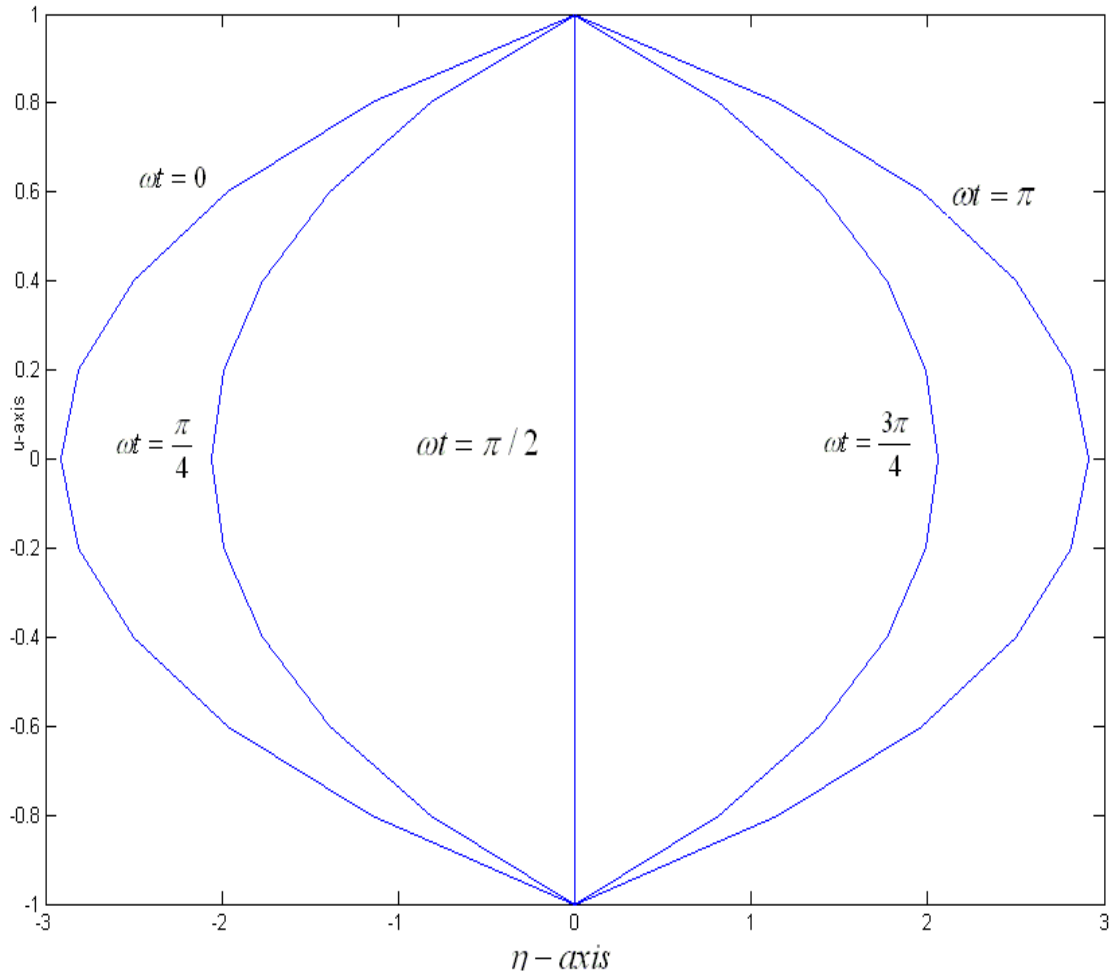


Figure 5.3: Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=5$ and for different values of ωt

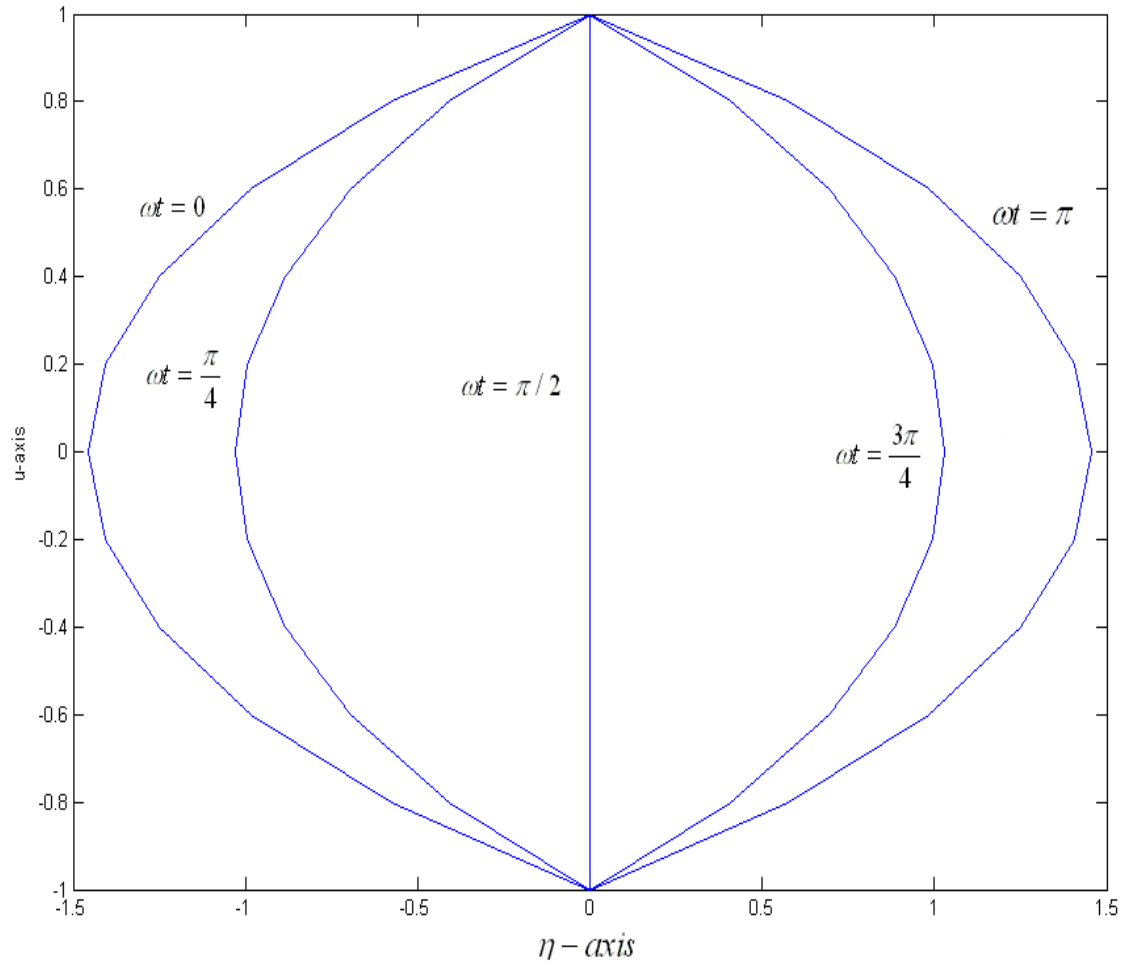


Figure 5.4: Axial velocity profiles when $u_0=1.0$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

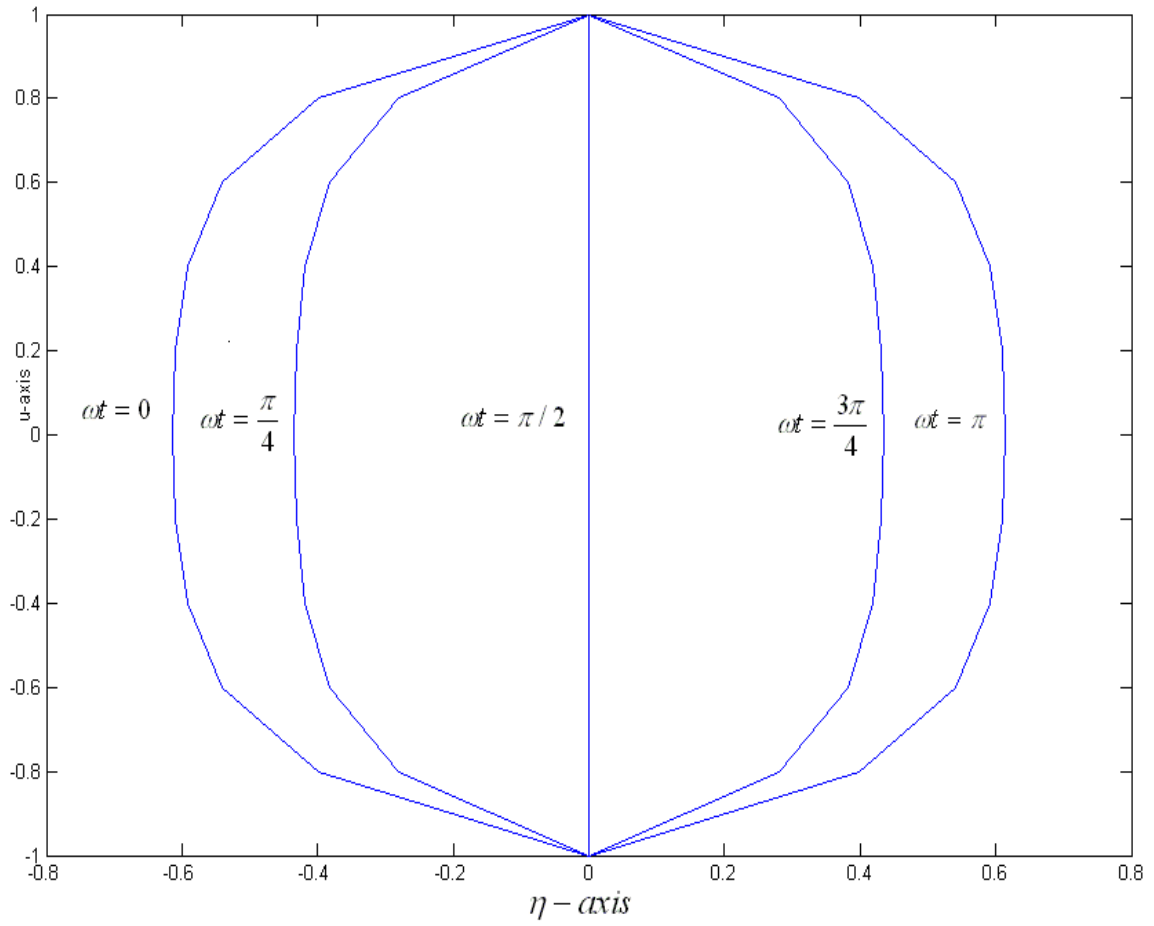


Figure 5.5: Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=5.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

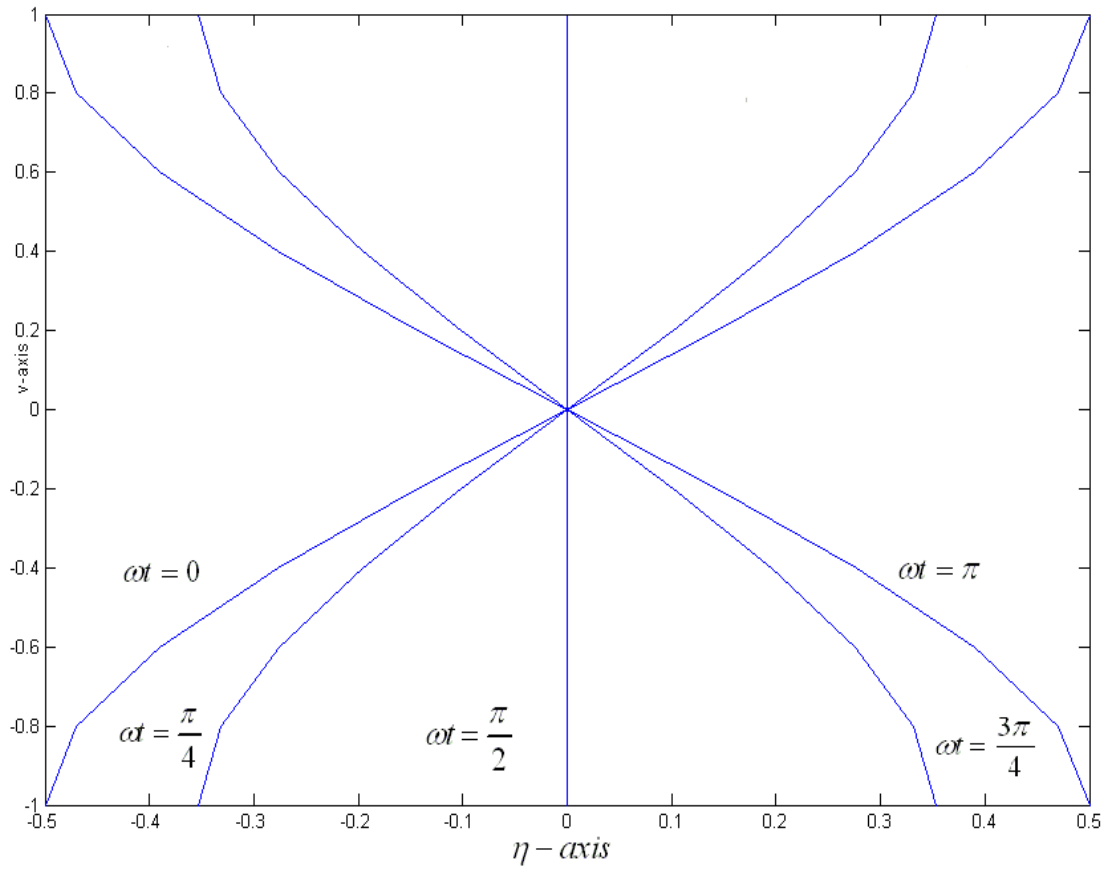


Figure 5.6: Radial velocity profiles when $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

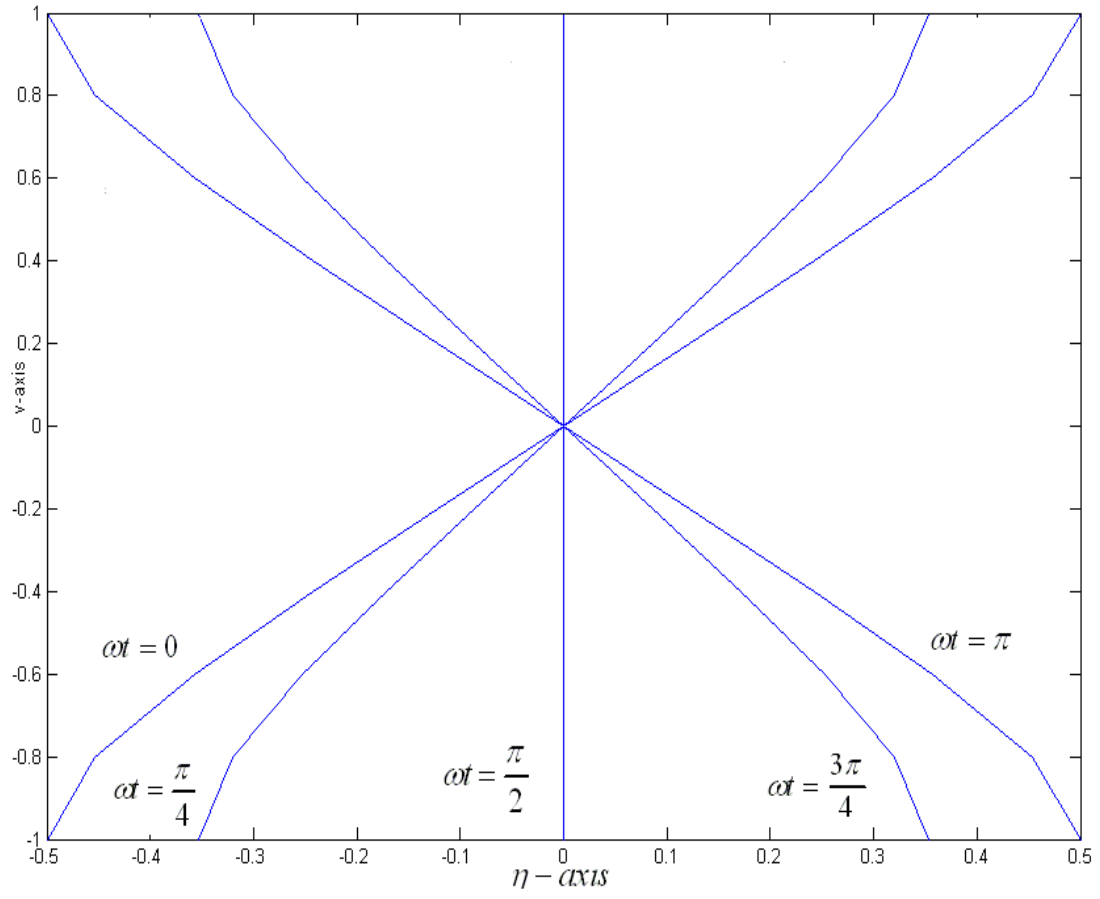


Figure 5.7: Radial velocity profiles when $v_0=0.5$, $h=1.0$, $M=5.0$, $\alpha=1.0$ and for different values of ωt

CHAPTER-6

6. CONCLUSION

In the above analysis a class of solutions of unsteady magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates is presented, in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate.

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