

MAGNETO HYDRODYNAMIC CASSON FLUID FLOW THROUGH A POROUS CHANNEL

Submitted in partial fulfillment of the requirements for the award of

Master of Science Degree in Mathematics

By

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SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **M. ANBU HANISTA (39630003)**. Who carried out the project entitled "**MAGNETO HYDRODYNAMIC CASSON FLUID FLOW THROUGH a POROUS CHANNEL**" under our supervision from January 2021 to April 2021.

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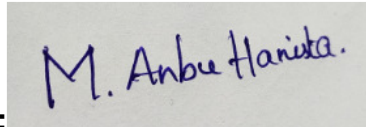
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DECLARATION

IM.ANBU HANISTA(39630003) hereby declare that the Project Report entitled "**MAGNETOHYDRODYNAMIC CASSON FLUID FLOW THROUGH POROUS CHANNEL**" done by me under the guidance of **Dr .Y.IMMANUEL** (Internal) and __ (External) at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Master of Science Degree in Mathematics

DATE:



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ABSTRACT

The aim of the present paper is to study the Casson flow of viscous fluid moving steadily through a porous channel with constant velocity under the influence of uniform magnetic field applied perpendicularly to the flow of viscous liquid. With the help of similarity transformations, the governing partial differential equations corresponding to the momentum are reduced to linear ordinary differential equations. Analytical solution of the equation is obtained.

KEYWORDS

- Flow
- viscous liquid
- flat plates
- magnetic field

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CHAPTER-1

1. INTRODUCTION

FLUID DYNAMICS

Fluid dynamics is “the branch of applied Science that is concerned with the movement of fluids (liquids or gases) at rest as well as in motion”. Fluids are the principle transport media and hence play a central role in nature (wind, rivers, ocean currents, blood etc). Fluids are a source of energy generation (power dams). Fluid dynamics is one of two branches of fluid mechanics, which is study of fluid and how forces affect them. Thus, this branch of Science deals with the static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of Science is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; influence of wind upon building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid. They have several engineering applications such as Mechanical engineering (hydraulic brakes, hydraulic press etc.), Electrical engineering (semi conductor industries), chemical engineering (centrifuges), Aerospace engineering (aerodynamics).

1.1 APPLICATION OF FLUID FLOW

Fluid flow is a part of fluid mechanics and deals with fluid dynamics. It involves the motion of the fluid subjected to unbalanced forces. Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces. Important identification of type of flow is, the flow of the fluid through a pipe can be viscous or turbulent and it can be determined by Reynolds number. Reynolds number have no limit. In engineering, the application of peristaltic flow models and of direct simulation of turbulence method are analyzed.

1.2 NON-NEWTONIAN FLUIDS

A Newtonian fluid is one where there is a linear relationship between stress and strain-rate. If the properties are predominantly solid like, the materials are called non-Hookean and it is described as viscoelastic. If they are predominantly fluid like, they are called non-Newtonian fluid and its materials are described as elastoviscous. They are part of engineering study in many tertiary institutions. Basically, in the study of science, fluids are divided into two broad groups. These divisions in this write-up are known as types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids that obey Newton Law of viscosity. A non-Newtonian fluid does not follow Newton's law of viscosity that is constant viscosity independent of stress. The Newtonian viscosity is a physical property which is only a function of pressure and temperature. The flow properties of an incompressible Newtonian fluid can be characterized solely in terms of the density and viscosity. These include non-Newtonian fluid friction reduction, surfactant applications to large scale heating and cooling systems, scale-up, and flow tracers.

1.3 MAGNETOHYDRODYNAMIC FLOWS

In hydro magnetic flow, we study of the flow of electrically conducting fluid in presence of Maxwell electromagnetic field. The flow of conducting fluid is effectively changed by the presence of the magnetic field where as the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomena is called Magneto Hydrodynamics and in short written as MHD. There are wider applications in engineering technology, cosmology, astrophysics and other applied sciences.

1.4 FLOW THROUGH POROUS CHANNEL

The instance of laminar flow through porous channel is the idealisation of the flow behaviour that occurs in the real world in corresponding geometries. Flow through porous channel is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Porosity is a fraction between 0 and 1, typically ranging from less than 0.01 for solid granite to more than 0.5 for peat and

clay. For manmade materials such as metallic frames, ε can approach to the value 1. For natural media, ε does not normally exceed 0.6.

The permeability is related to pore-size distribution since the distribution of the sizes of entrances, exits and lengths of the pore walls make up the major resistance to flow. The permeability is the single parameter that reflects the conductance of a given pore structure. The dimensions of the permeability are length squared. In oil industry it is measured in 'Darcy' with $1\text{darcy} = 9.87 \times 10^{-9} \text{ cm}^2$. The permeability and porosity are related since if the porosity is zero the permeability is zero.

CHAPTER-2

2. LITERATURE SURVEY

Unsteady magnetohydrodynamic flow through a porous medium between two infinite parallel plates. I. A. Hassanien & M. A. Mansour (1990) has analyzed the analysis of a two-dimensional unsteady flow of a viscous, incompressible electrically-conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter are discussed.

The subject of MHD is largely perceived to have been initiated by Swedish electrical engineer Hannes Alfvén [1942]. Davidson [2001] has analyzed the Magneto Hydrodynamics flows. If an electrically conducting fluid is placed in a constant magnetic field, the motion of the fluid induces currents which create forces on the fluid. The production of these currents has led to the design of among other devices the MHD generators for electricity production. The equations which describe MHD flow are a combination of continuity equation and Navier-Stokes's equations of fluid dynamics and Maxwell's equations of electromagnetism.

Shercliff [1956] studied the steady motion of an electrically conducting fluid in pipes under transverse magnetic fields. Drake [1965] considered flow in a channel due to periodic pressure gradient and solved the resulting equation by separation of variables method.

Unsteady MHD Flow of a Casson Fluid in a Parallel Plate Channel Heat And Mass Transfer of Chemical Reaction. C.K. Kirubhashankar, S. Ganesh (2012). presents the study of unsteady magneto hydrodynamic flow of Casson fluid accompanied by heat transfer through parallel plate in the presence of chemical reaction. Casson fluid model is used to characterize the non-Newtonian fluid behavior. The present fluid model is working to suggest rheological liquids encountered in biotechnology (medical creams) and chemical engineering. This rheological model introduces additional terms into the momentum equation. The

basic equations governing the flow, heat transfer, and concentration are reduced to a set of ordinary differential equations by using appropriate transformation for variables. It is recognized that the Schmidt number (Sc) and Chemical reaction parameter (Cr) have similar effects on the velocity profile.

Hall effects on heat and mass transfer flow through porous medium P. C. Ram, C. B. Singh & U. S. Singh (1984) proposed Effects of Hall current on free convection and mass transfer flow through a porous medium bounded by a vertical surface has been analyzed. The problem is solved analytically. The velocity profiles are shown on graphs. Effects of (Hall parameter). K^* (permeability parameter), and Sc (Schmidt number) on velocity are discussed

Unsteady flow of viscous fluid between two parallel porous plates.S.Ganesh,S.Krishnambal (2007), discussed the Unsteady stokes flow of an electrically conducting viscous, incompressible fluid between two parallel porous plates of a channel in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate is discussed. An exact solution is obtained for all values of R (Suction Reynolds number) and M (Hartmann number). Expressions for the velocity components and the pressure are obtained. The graphs of axial and radial velocity profiles have been drawn for different values of M .

C.K.Kirubhashankar (2018), proposed the steady two-dimensional laminar stream of an incompressible viscous liquid between two parallel plates in the presence of a transverse magnetic field is considered by accepting the normal wall velocities to be equal. We study the steady MHD flow between moving parallel plates with small uniform suction at the lower stationary plate. The velocity profile for the stream of the liquid and pressure gradient are examined for various qualities. Also, the impacts of velocity potential are researched and discussed graphically.Hassaninen I. A. and Mansour M.A. [1990] Analyzed the two-dimensional unsteady flow of a viscous, incompressible electrically-conducting fluid through a porous medium bounded by two infinite parallel plates under the

action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter is discussed.

CHAPTER-3

3. RESEARCH METHODOLOGY

3.1 AIM AND SCOPE

The main objectives of the work reported are:

- To study the Casson flow of viscous fluid moving steadily through porous channel with constant velocity.
- To study the effect of the magnetic field and the Reynolds number.
- The effects of velocity are studied for the increases with the decrease of magnetic field parameter.

The scope of the work reported is:

- It not only helps the people working in the field of Physiological fluid dynamics but also to the medical practitioners.

3.2 DESCRIPTION OF RESEARCH WORK

This research work focuses on the MATLAB implementation of interpretation of analytical solutions obtained for various fluid flow problems.

Research Problem

The main objective of the research work is to analyze the flow of fluids through parallel porous channel in the presence/absence of magnetic effects.

Research Methodologies

Analytical solutions are obtained by transforming the partial differential equation concerning the flow into an ordinary linear differential equation with constant coefficients by using similarity transformation. The arbitrary constant are

then calculated by using boundary condition of the respective problem and the results are interpreted by using MATLAB applications

CHAPTER-4

4. MATHEMATICAL FORMULATION

Consider the two-dimensional flow of an incompressible Casson fluid over an exponentially shrinking/stretching porous sheet at $y = 0$, with the flow being confined in $y > 0$. The rheological equation of state for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} \left(\mu_B + \frac{P_y}{\sqrt{2\pi}} \right) 2e_{ij} & \pi > \pi_c \\ \left(\mu_B + \frac{P_y}{\sqrt{\pi_c}} \right) 2e_{ij} & \pi < \pi_c \end{cases}$$

where π_{ij} is the (i, j)-th component of the stress tensor, $\pi = e_{ij}e_{ij}$ and e_{ij} are the (i, j)-th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is a critical value of this product based on the non-Newtonian model, μ_B is plastic dynamic viscosity of the non-Newtonian fluid, and p_y is the yield stress of the fluid. The equations governing the flow are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\nu \rho}{k} u \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(1 + \frac{1}{\beta} \right) \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

4.1 ASSUMPTIONS

1. Flow between non conducting two parallel plates.
2. Viscosity of the fluid is considered as constant.
3. One plate is in uniform motion and the other plate (stationary plate) at rest with uniform suction.

4.2 NOTATIONS

ρ - Density of the fluid

μ - Coefficient of viscosity

σ - Electrical conductivity of the fluid

ν - porosity ($\nu = \mu/\rho$)

B_0 - Electromagnetic induction

H_0 - Transverse magnetic field

$\beta = \mu B_0 \sqrt{2\pi\sigma/\rho\nu}$ is the Casson parameter,

M - Hartmann number $\left(M = \sqrt{\frac{\sigma B_0^2 h^2}{\rho\nu}} = B_0 h \sqrt{\frac{\sigma}{\mu}} \right)$

4.3 GENERAL SOLUTION OF ANALYTICAL PROBLEM

The continuity equation yields

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \text{ but } \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

i.e., $\frac{\partial u}{\partial x} = 0$, from which infer that u is independent of x .

This makes the non-linear term $(q \cdot \nabla)q$ in the Navier-Stokes equation zero. We neglect body forces f_B which are mainly due to gravity in the Navier-Stokes equations. Assuming the flow variables are independent of z – direction, It means that the governing equations for this flow are

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\beta} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k} u \quad (4)$$

$$0 = -\frac{1}{\rho} \frac{\partial p}{\partial y} \quad (5)$$

Where, $\nu = \frac{\mu}{\rho}$

i.e., p is the function of x only.

Assume the characteristic velocity v_0 moving perpendicular to the fluid at a constant pressure gradient. Now the governing equations are

$$v_0 \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\mu}{k} u \quad (6)$$

If the flow is influenced by transverse magnetic field, then the governing equation will be

$$v_0 \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu}{k} u \quad (7)$$

The boundary conditions for generalized flow are $\begin{cases} u = 0, y = 0 \\ u = 0, y = 0 \end{cases}$

Introducing the following non-dimensional quantities

$$x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad u' = \frac{u}{v_0}, \quad p' = \frac{p}{\rho v_0^2} \quad (7a)$$

Equation (7) becomes

$$\frac{v_0^2}{h} \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\rho v_0^2}{h} \frac{\partial p}{\partial x} + \nu \frac{v_0}{h^2} \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu v_0}{k} u$$

multiply by $\frac{h^2}{\nu v_0}$,

$$\frac{h v_0}{\nu} \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\rho v_0}{h} \frac{\partial p}{\partial x} + \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 h^2}{\nu \rho} u - \frac{h^2 v_0}{\nu k} u \quad (7b)$$

integrate,

$$\frac{d^2 p}{dx^2} = 0 \quad \text{i.e., } \frac{dp}{dx} = -P$$

where 'P' is a positive constant.

Now the governing equation will be

$$\left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - R \frac{\partial u}{\partial y} - (M^2 h + A^2 h) u = -RP \quad (8)$$

$$\text{where, } R = \frac{h v_0}{v}, \quad A^2 = \frac{\mu h}{v k}, \quad M = \sqrt{\frac{\sigma B_0^2 h}{\rho v}}$$

$$\text{with boundary conditions } \begin{cases} u = 0, y = 0 \\ u = 0, y = 1 \end{cases}$$

4.4 General Solution of Formation of the Problem

Equation (8) reduces to the form

$$(D^2 - \gamma R D - \gamma(M^2 + A^2)h)u(y) = -RP \quad (9)$$

With boundary conditions

$$\begin{aligned} u &= 0, y = 0 \\ u &= 0, y = 1 \end{aligned} \quad (10)$$

Auxiliary equation of the above equation is

$$m^2 - \gamma R m - \gamma(M^2 + A^2)h = 0$$

$$m = \frac{1}{2}(\gamma R \pm \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})$$

Complementary function is $c_1 e^{\frac{1}{2}(\gamma R + \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} + c_2 e^{\frac{1}{2}(\gamma R - \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})}$

Particular integral is $\frac{-\gamma R P e^{(0)y}}{(D^2 - \gamma R D - \gamma(M^2 + A^2)h)} = \frac{R P}{(M^2 + A^2)h}$

Hence

$$u(y) = c_1 e^{\frac{1}{2}(\gamma R + \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})y} + c_2 e^{\frac{1}{2}(\gamma R - \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})y} + \frac{R P}{(M^2 + A^2)h} \quad (11)$$

Applying the boundary conditions (9) in (11)

$$0 = u(0) = c_1 + c_2 + \frac{R p}{(M^2 + A^2)h} \quad (12)$$

$$0 = u(1) = c_1 e^{\frac{1}{2}(\gamma R + \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} + c_2 e^{\frac{1}{2}(\gamma R - \sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} + \frac{R p}{(M^2 + A^2)h} \quad (13)$$

(13) $\times e^{-\gamma R/2}$, we get

$$c_1 e^{\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} + c_2 e^{-\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} = -\frac{R p}{(M^2 + A^2)h} e^{-\gamma R/2} \quad (14)$$

Using (12) in (14),

$$c_1 e^{\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} + \left(-\frac{R p}{(M^2 + A^2)h} - c_1\right) e^{-\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} = -\frac{R p}{(M^2 + A^2)h} e^{-\gamma R/2}$$

$$c_1 \left(e^{\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} - e^{-\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} \right) - \frac{R p}{M^2 h} e^{-\frac{1}{2}(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)})} = -\frac{R p}{(M^2 + A^2)h} e^{-\gamma R/2}$$

$$\begin{aligned}
& 2c_1 \sinh\left(\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)\right) \\
& = -\frac{Rp}{(M^2 + A^2)h} \left(e^{-\gamma R/2} - e^{-\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)}\right)
\end{aligned}$$

$$\begin{aligned}
& c_1 \\
& = -\frac{Rp}{(M^2 + A^2)h} \left(\left(e^{-\gamma R/2} - e^{-\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)} \right) / 2 \sinh\left(\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)\right) \right)
\end{aligned}$$

(15)

Substituting (15) in (12),

$$\begin{aligned}
c_2 & = -c_1 - \frac{Rp}{(M^2 + A^2)h} \\
& = -\frac{Rp}{(M^2 + A^2)h} \left(\left(e^{-R/2} - e^{-\frac{1}{2}\left(\sqrt{R^2 + 4h(M^2 + A^2)}\right)} \right) / 2 \sinh\left(\frac{1}{2}\left(\sqrt{R^2 + 4h(M^2 + A^2)}\right)\right) \right) \\
& \quad - \frac{Rp}{(M^2 + A^2)h} \\
c_2 & = -\frac{Rp}{2 \sinh\left(\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)\right) (M^2 + A^2)h} \\
& \times \left(\left(e^{-\gamma R/2} - e^{-\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)} \right) - \left(e^{\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)} - e^{-\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)} \right) \right) \\
& = -\frac{Rp}{2 \sinh\left(\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)\right) (M^2 + A^2)h} \left(\left(e^{-\gamma R/2} \right. \right. \\
& \quad \left. \left. - e^{\frac{1}{2}\left(\sqrt{\gamma^2 R^2 + \gamma 4h(M^2 + A^2)}\right)} \right) \right)
\end{aligned} \tag{161}$$

Substituting (7) and (16) in (11), Solution of equation (11) is Solution of the equation (9) is

$$U(y) = \frac{RP}{(M^2 + A^2)h \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right)} \left\{ \begin{aligned} &e^{\gamma R y/2} \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) (y - 1) \\ &- e^{\gamma R (y-1)/2} \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) y \\ &+ \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) \end{aligned} \right.$$

4.5 VOLUMETRIC FLOW PER UNIT TIME

The volumetric flow per unit time

$$Q = \int_0^1 u(y) dy$$

$$= \frac{RP}{2(M^2 + A^2)h \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right)} \left\{ \begin{aligned} &e^{\gamma R y/2} \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) (y - 1) \\ &- e^{\gamma R (y-1)/2} \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) y \\ &+ \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right) \end{aligned} \right.$$

$$= \frac{RP}{2(M^2 + A^2)h \sinh \left(\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2} \right)} \left\{ \begin{aligned} &e^{\frac{\gamma R y}{2}} \left(e^{\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} \right. \\ &\quad \left. - e^{\frac{-\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} \right) (y - 1) \\ &- e^{\frac{\gamma R (y-1)}{2}} \left(e^{\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} \right. \\ &\quad \left. - e^{\frac{-\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} \right) y \\ &+ \left(e^{\frac{\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} - e^{\frac{-\sqrt{(\gamma R)^2 + 4h\gamma(M^2 + A^2)}}{2}} \right) \end{aligned} \right.$$

$$Q = \int_0^1 u(y) dy$$

$$= \frac{RP}{(M^2 + A^2) h e^{\frac{\gamma R}{2} \sinh \left(\left(\sqrt{\gamma} R^2 \right) + 4 h \gamma (M^2 + A^2) \right) / 2}}$$

CHAPTER-5

5. PERFORMANCE ANALYSIS

5.1 SENSITIVE ANALYSIS AND DISCUSSION

A similarity solution of the Navier Stokes equation for the Casson fluid flow between two parallel plates has been presented. Employing perturbation method, we have obtained approximate analytical solutions for the velocity of the fluid flow of the two-dimensional Casson fluid between two parallel plates.

Figure 5.1 show that effects of Casson parameter β on velocity profiles. Both axial velocity and magnitude of axial velocity is found to increase with increasing values of Casson parameter β .

From figure 5.2 it is observed that the axial velocity increases and magnitude of axial velocity decreases with the increasing magnetic field.

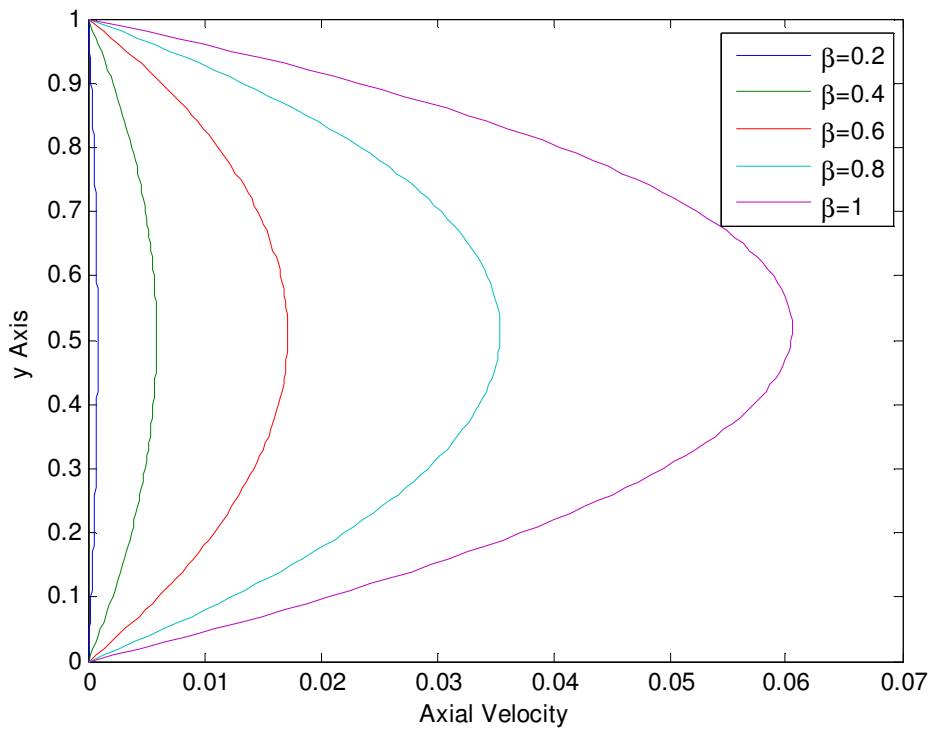
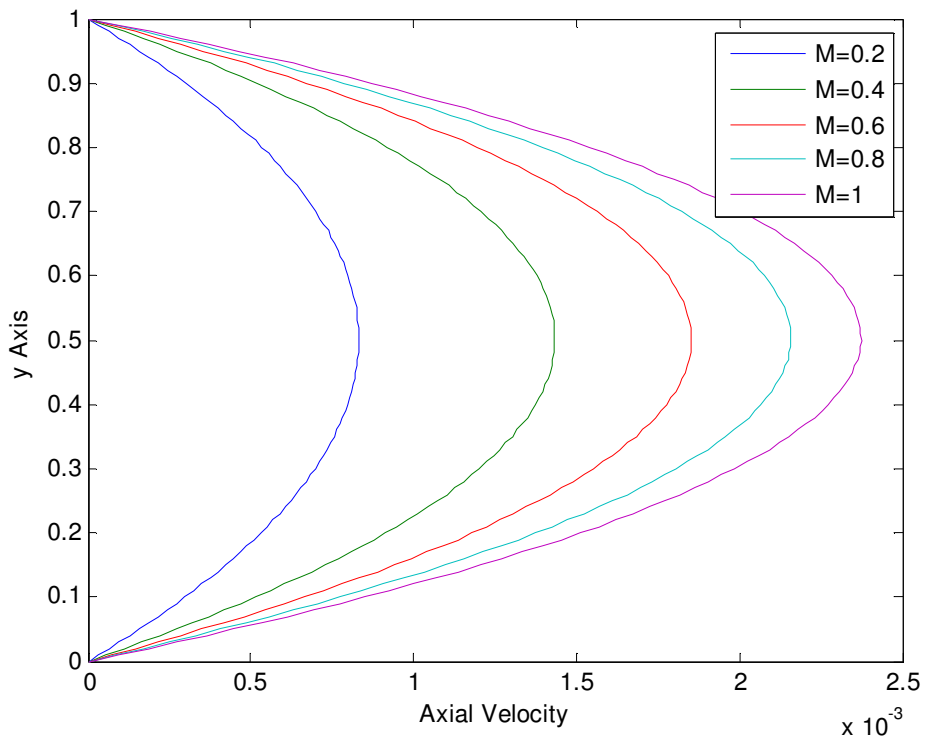


Fig 5.1 velocity profiles for $h=1; y=0:0.01:1; p=0.2; R=0.2; M=0.2; c=0.2; A=0.2$; Different values for β and M .



**Fig 5.2 velocity profiles for $h=1; y=0:0.01:1; p=0.2; R=0.2; M=0.2; c=0.2; A=0.2;$
Different value of M**

CHAPTER-6

6. SUMMARY AND CONCLUSIONS

6.1 CONCLUSION

In this paper the effect of Magnetic field and Casson parameter are studied on viscous fluid moving steadily through a porous channel with constant velocity under the influence of electrically conducting viscous incompressible fluid between two parallel plates through porous channel.

- Both axial velocity and magnitude of axial velocity is found to increases with increasing values of Casson parameter β .
- The axial velocity increases and magnitude of axial velocity decreases with the increasing magnetic field.

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