

# **UNSTEADY OSCILLATORY FLOW OF DUSTY CASSON FLUIDS BETWEEN RADIATED PARALLEL PLATES**

Submitted in partial fulfillment of the requirements for the award of  
Master of Science Degree in Mathematics

By

**T. ANANDHI (39630002)**



**DEPARTMENT OF MATHEMATICS  
SCHOOL OF SCIENCE AND HUMANITIES**

**SATHYABAMA**  
INSTITUTE OF SCIENCE AND TECHNOLOGY  
(DEEMED TO BE UNIVERSITY)  
Accredited with Grade "A" by NAAC  
JEPPIAAR NAGAR, RAJIV GANDHI SALAI,  
CHENNAI - 600 119

**April 2021**



# SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY  
(DEEMED TO BE UNIVERSITY)

Accredited with "A" grade by NAAC

Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai – 600 119

[www.sathyabama.ac.in](http://www.sathyabama.ac.in)

## DEPARTMENT OF MATHEMATICS

### BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **T. ANANDHI (39630002)**. Who carried out the project entitled "**UNSTEADY OSCILLATORY FLOW OF DUSTY CASSON FLUIDS BETWEEN RADIATED PARALLEL PLATES**" under our supervision from **January 2021** to **April 2021**.

Internal Guide

External Guide

(Dr. M. NIRMALA)

Head of the Department

---

Submitted for Viva voce Examination held on \_\_\_\_\_

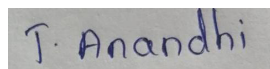
Internal Examiner

External Examiner

## DECLARATION

I **T. ANANDHI (39630002)** hereby declare that the Project Report entitled **“UNSTEADY OSCILLATORY FLOW OF DUSTY CASSON FLUIDS BETWEEN RADIATED PARALLEL PLATES”** done by me under the guidance of **Dr.C. K KIRUBHASHANKAR** (Internal) and \_\_\_\_\_(External) **at SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Master of Science Degree in Mathematics.

**DATE:**

A rectangular box containing a handwritten signature in blue ink that reads "T. Anandhi".

**PLACE: CHENNAI**

**SIGNATURE OF THE CANDIDATE**

## ACKNOWLEDGEMENT

I am pleased to acknowledge my sincere thanks to Board of Management of **SATHYABAMA**, for their kind encouragement in doing this project and for completing it successfully. I am grateful to them.

I convey my thanks to **DrM. Nirmala**, Head of the Department, Dept. of Mathematics for providing me necessary support and details at the right time during the progressive reviews.

I would like to express my sincere and deep sense of gratitude to my Project Guide **Dr .C. K. Kirubhashankar** for his valuable guidance, suggestions and constant encouragement paved way for the successful completion of my Project work.

I wish to express my thanks to all Teaching and Non-teaching staff members of the Department of Mathematics who were helpful in many ways for the completion of the project.

## **ABSTRACT**

The unsteady oscillatory flow of dusty Casson fluids over a stretching porous surface having a prescribed surface temperature is investigated. Similarity transformations are employed to transform the governing partial differential equations into ordinary differential equations. The velocity and the temperature have been analyzed for different variations of the governing parameters Casson parameter, heat source parameter, and Magnetic field are analyzed and discussed in detail. It was found that heat source and magnetic field modify the flow patterns and increase the temperature of the fluid.

## **KEY WORDS**

Dusty flow

Parallel Porous Plate

Boundary Layer

Heat Source

Magnetic Field

Casson fluid.

## **CONTENTS**

| <b>Chapter 1</b>                          | <b>Page No</b> |
|-------------------------------------------|----------------|
| <b>1. Introduction</b>                    |                |
| 1.1 IMPORTANCE OF FLUID DYNAMICS          | 2              |
| 1.2 MAGNETOHYDRODYNAMIC FLOW              | 2              |
| 1.3 DUSTY FLUID                           | 3              |
| 1.4 NON-NEWTONIAN FLOWS                   | 3              |
| 1.5 BOUNDARY LAYER FLOW OF A CASSON FLUID | 3              |
| <b>Chapter 2</b>                          |                |
| 2 LITERATURE SURVEY                       | 5              |
| <b>Chapter 3</b>                          |                |
| <b>3. RESEARCH METHODOLOGY</b>            |                |
| 3.1 AIM AND SCOPE                         | 9              |
| 3.2 DESCRIPTION OF RESEARCH WORK          | 9              |
| <b>Chapter 4</b>                          |                |
| <b>4. MODEL FORMULATION</b>               |                |
| 4.1 MATHEMATICAL FORMULATION              | 12             |
| 4.2 ANALYTICAL SOLUTION OF THE PROBLEM    | 14             |
| <b>Chapter 5</b>                          |                |
| <b>5. PERFORMANCE ANALYSIS</b>            |                |

|                                                                    |    |
|--------------------------------------------------------------------|----|
| 5.1 SENSITIVE ANALYSIS AND DISCUSSION                              | 20 |
| 5.2 EFFECTS OF DIFFERENT PHYSICAL PARAMETERS<br>ON VELOCITY FIELDS | 20 |

## **Chapter 6**

|                  |    |
|------------------|----|
| 6.1 CONCLUSION   | 25 |
| <b>REFERENCE</b> | 26 |

## LIST OF FIGURES

| FIGURE NO. | TITLE                                      | PAGE NO |
|------------|--------------------------------------------|---------|
| Fig 5.1    | Axial velocity of fluid when $y$ increases | 21      |
| Fig 5.2    | Axial velocity of fluid when $N$ increases | 22      |
| Fig 5.3    | Axial velocity of fluid when $S$ increases | 23      |



# **CHAPTER-1**

# **1. INTRODUCTION**

## **FLUID DYNAMICS**

Fluid Mechanics is that branch of Science which deals with the behaviour of fluids (liquids or gases) at rest as well as in motion. Thus this branch of Science deals with the static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of Science is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; influence of wind upon building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid.

### **1.1 IMPORTANCE OF FLUID FLOW**

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces.

### **1.2 MAGNETOHYDRODYNAMIC FLOWS**

MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Magnetohydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil.

### **1.3 DUSTY FLUID**

A dusty fluid is the mixture of fluid and fine dust particles. Its study is important in areas like environmental pollution, smoke emission from vehicles, emission of effluents from industries, cooling effects of air conditioners, flying ash produced from thermal reactors and formation of raindrops, etc. Also it is useful in the study of lunar ash flow which explains many features of lunar soil

### **1.4 NON-NEWTONIAN FLOWS**

Fluids can be defined as substances that flow or deform under the application of shear stress and these include liquids and gases. They are part of engineering study in many tertiary institutions. Basically, in the study of science, fluids are divided into two broad groups. These divisions in this write-up are known as types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids that obey Newton Law of viscosity. The Newtonian viscosity is a physical property which is only a function of pressure and temperature. The flow properties of an incompressible Newtonian fluid can be characterized solely in terms of the density and viscosity.

### **1.5 BOUNDARY LAYER FLOW OF A CASSON FLUID**

The radiations due to heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends, to some extent to the heat controlling factors. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radioactive heat transfer from a vertical wall to conductive gray fluids.

The purpose of this present work is to extend the flow and heat transfer analysis in boundary layer flow of a Casson fluid over an exponentially stretching sheet. Combined effects of suction/blowing and thermal radiation are investigated.

# **CHAPTER-2**

## 2. LITERATURE SURVEY

Flow through porous media has many geophysical and engineering applications. Berman AS [1953] analyzed the Navier-Stokes's equations have been solved to obtain a complete description of the fluid flow in a channel having a rectangular cross section and two equally porous walls. The scope has been limited to two-dimensional incompressible steady-state laminar flow. The solution of the flow equations leads to detailed expressions for the dependence of the velocity components and the pressure on position coordinates, channel dimensions, and fluid properties.

Hartmann and Lazarus [1937] investigated the effect of a transversely uniform MHD on the flow of conductive fluid between two infinitely parallel and fixed insulating plates. The equation describing MHD flow is a synthesis of the continuous equation and the Navier-Stokes's equation. Governing equations are differential equations that require analytical or numerical solutions.

Hartmann J. [1937] considered the magnetic field in the laminar flow of conductive liquids.

Hassaninen I. A. and Mansour M.A. [1990] Analyzed the two-dimensional unsteady flow of a viscous, incompressible electrically- conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter are discussed.

Hamza E.A [1999], A similarity solution has been obtained for a flow between two parallel plates (rectangular or circular) approaching or receding from each other with suction or injection at the porous plate. Approximate analytic solutions based on the regular perturbation technique are given and a numerical solution of the resulting nonlinear differential equations is presented. The effects of injection and suction on the velocity profiles, pressure distribution and shearing stress on the wall are studied. In the case of the flow between circular plates the effects on load capacity due to mass transfer are investigated.

Soundalgekar V. and Uplekar [1986] investigated an exact analysis of a generalized MHD Couette flow, on taking into account the Hall current, is presented. Expressions for the axial and transverse components of velocity, the skin friction, are derived. These are shown in graphs. The exact solution to the energy equation, assuming the linear variation of the temperature along the plate, is also derived and this is shown graphically. The effects of different parameters are discussed.

Sing & Ram [1978] solved the differential equation by Laplace transform method, considering the laminar flow of conductive fluid laminar flow through a channel with a transverse magnetic field under a periodic pressure gradient.

Cox.S.M [1991] considered the flow of a viscous incompressible fluid in a parallel-walled channel, driven by steady uniform suction through the porous channel walls. A similarity transformation reduces the Navier-Stokes equations to a single partial differential equation (PDE) for the stream function, with two-point boundary conditions. We discuss the bifurcations of the steady solutions first, and show how a pitchfork bifurcation is unfolded when a symmetry of the problem is broken.

Attia, H. A. and Kotb, N. A., (1996) studied the steady flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel infinite insulated horizontal plates and the heat transfer through it are studied. The upper plate is given a constant velocity while the lower plate is kept stationary. The viscosity of the fluid is assumed to vary with temperature. The effect of an external uniform magnetic field as well as the action of an inflow perpendicular to the plates together with the influence of the pressure gradient on the flow and temperature distributions are reported. A numerical solution for the governing non-linear ordinary differential equations is developed.

Chiam T.C. (1993) investigated the boundary layer equations arising from the effects of a continuously moving flat plate under the action of a transverse applied magnetic field are converted into a system of ordinary differential equations subject to two-point boundary conditions by using the Local Nonsimilarity Method. An iterative method incorporating the shooting method is presented for solving both the two-equation and three-equation models.

Verma, P.D., and Bansal, J. L., (1966) analyzed an exact solution of the Navier-Stoke's equations reduced to third and second order non-linear differential

equations with appropriate boundary conditions is obtained. The longitudinal and transverse velocity profiles for  $\lambda=0.1$ , 1 and  $R=100$  is drawn. It is noted that for large values of  $x^*$ , an adverse pressure gradient is developed which causes a back flow. There is increase in pressure even for very small fluid suction along the stationary plate. The skin friction and the flow coefficient decrease with reference to the suction velocity and the distance along the stationary plate. For  $\lambda=0$ , the results transform to the known results for plane couette flow without suction.

R. Delhi Babu, S. Ganesh & C. K. Kirubhashankar (2020) studied the impact of unsteady magnetohydrodynamic flow of dust particles on the velocity of electrically directed viscous fluid between parallel porous plates with an angular velocity at the sight of a transverse magnetic field. The differential equations managing the flow are explained by similarity transformation. The velocity of the fluid, velocity of the dust particles and pressure distribution are obtained. The outcomes are the velocity of the dust particle is higher than that velocity of the fluid particles for each of the parameters examined.

C.K. Kirubhashankar, Y. Immanuel and S. Vaithyasubramanian (2020) discussed an exact analytic solution for the unsteady MHD fluid flow between the parallel plate's dependent on the Homotopy technique. In this study, the unsteady magneto hydrodynamic (MHD), viscid, condensed and conducting fluid stream problem under transverse magnetic field is discussed. Solutions are proposed for variety of Hartmann number values and time.

C.K. Kirubhashankar (2018), proposed the steady two-dimensional laminar stream of an incompressible viscous liquid between two parallel plates in the presence of a transverse magnetic field is considered by accepting the normal wall velocities to be equal. We study the steady MHD flow between moving parallel plates with small uniform suction at the lower stationary plate. The velocity profile for the stream of the liquid and pressure gradient are examined for various qualities. Also, the impacts of velocity potential are researched and discussed graphically.

# **CHAPTER-3**

### **3. RESEARCH METHODOLOGY**

#### **3.1 AIM AND SCOPE**

**The main objectives of the work reported are:**

- To study the effect of individual factors like Magnetic field and Reynolds number.
- To study the solution for the unsteady oscillatory flow of dusty Casson fluids over a stretching porous surface having a prescribed surface temperature through the parallel plate channel with heat source and external transverse magnetic field is presented.
- The effects of velocity are studied for the parameters like Casson parameter, magnetic field parameter and Heat source parameter.

**The scope of the work reported is:**

- It not only helps the people working in the field of Physiological fluid dynamics but also to the medical practitioners.

#### **3.2 DESCRIPTION OF RESEARCH WORK**

This research work focuses on the MATLAB implementation of interpretation of analytical solutions obtained for various fluid flows.

##### **Research Problem**

The main objective of the research work is to analyze the flow of fluids through parallel porous plates in the presence/absence of magnetic effects.

## **Research Methodologies**

Analytical solutions are obtained by transforming the partial differential equation concerning the flow into an ordinary linear differential equation with constant coefficients by using similarity transformation. The arbitrary constant is then calculated by using boundary condition of the respective problem and the results are interpreted by using MATLAB applications

# **CHAPTER-4**

#### 4. MODEL FORMULATION

Consider the flow of an incompressible Casson fluid over an exponentially shrinking/stretching porous sheet through the length of the channel and the distance between the two plates is assumed to be  $h$ . The medium inside the channel is assumed to be porous. Let  $u$  and  $v$  as velocity components in the directions of  $x$  and  $y$  respectively (axial and normal respectively) at time  $t$  in the flow field.

The rheological equation of state for an isotropic and incompressible flow of a Casson fluid as follows:

$$\tau_{ij} = \begin{cases} \left( \mu_B + \frac{p_y}{\sqrt{2\pi}} \right) 2e_{ij}, & \pi > \pi_c, \\ \left( \mu_B + \frac{p_y}{\sqrt{2\pi_c}} \right) 2e_{ij}, & \pi < \pi_c \end{cases}$$

where  $\tau_{ij}$  is the  $(i, j)$  th component of the stress tensor,  $\pi = e_{ij} e_{ij}$  and  $e_{ij}$  are the  $(i, j)$  th component of the deformation rate,  $\pi$  is the product of the component of deformation rate with itself,  $\pi_c$  is a critical value of this product based on the non-Newtonian model,  $\mu_B$  is plastic dynamic viscosity of the non-Newtonian fluid, and  $p_y$  is the yield stress of the fluid.

##### 4.1 MATHEMATICAL FORMULATION

The governing equations of such type of flow are, in the usual notations,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} &= v \left( 1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{v}{K} u \\ &- k^* N(u - u_p) - \frac{\sigma B_0^2 u}{\rho} \alpha + g\beta(T - T_0) \end{aligned} \quad (2)$$

$$\frac{\partial T}{\partial t} = \frac{K'}{\rho C_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho C_p} (T - T_0) \quad (3)$$

where  $u$  and  $v$  as velocity components in the directions of  $x$  and  $y$  respectively (axial and radial respectively) at time  $t$  in the flow field.  $\nu$  is the kinematic fluid viscosity,  $\rho$  is the fluid density,  $\gamma = \mu_B \sqrt{2\pi c/p_y}$  is the Casson parameter,  $\sigma$  is the electrical conductivity of the fluid, and  $B_0$  is the strength of magnetic field applied in the  $y$  direction.  $\rho$  and  $\mu$  is the density and viscosity of the blood while  $p^*$  stands for pressure.  $K'$  is thermal conductivity,  $K$  is the permeability of the porous medium;  $C_P$  is the specific heat at constant pressure.  $Q$  is the quantity of heat,  $T$  is the temperature of the fluid and  $T_0$  is the temperature of the wall and  $\beta$  is the volumetric expansion parameter while  $\theta$  is the temperature distribution.

The boundary conditions are taken as:

$$\theta = e^{-\lambda^2 t}, u = e^{-\lambda^2 t} \text{ at } y = h$$

$$\theta = 0, u = 0 \text{ at } y = 0$$

Let us introduce the non-dimensional variables,

$$\begin{aligned} x^* &= \frac{x}{h}, y^* = \frac{y}{h}, u^* = \frac{u}{m/2\rho h}, v^* = \frac{v}{m/2\rho h}, t^* = \frac{t}{\rho h^2/\mu}, \\ p^*(x, t) &= \frac{\frac{dp}{dx}}{\mu m/2\rho^2 h^3}, \theta^* = \frac{\theta}{\mu m/2\rho^2 h^3} \end{aligned} \quad (4)$$

Substituting equation (4) into equations (1) – (3), we get

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (5)$$

$$\begin{aligned} \frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} &= \nu \left( 1 + \frac{1}{\gamma} \right) \frac{\partial^2 u}{\partial y^2} - \frac{\nu}{K} u - k^* N(u - u_p) - \frac{\sigma B_0^2 u}{\rho} \\ &\quad + g\beta(T - T_0) \end{aligned} \quad (6)$$

$$\frac{\partial \theta}{\partial t} - \frac{1}{vP_r} \frac{\partial^2 \theta}{\partial y^2} + \frac{S}{vP_r} \theta \quad (7)$$

where the heat source parameter,  $S = \frac{Qh^2}{K'}$ . Prandtl number,  $P_r = \frac{\mu c_p}{K'}$

#### 4.2 ANALYTICAL SOLUTION OF THE PROBLEM

With the discussions in the section 3.2, let us choose the solutions of the equations (5) – (7) respectively as

$$u(y, t) = F(y)e^{-\lambda^2 t} \quad (8)$$

$$v(y, t) = G(y)e^{-\lambda^2 t} \quad (9)$$

$$\theta(y, t) = H(y)e^{-\lambda^2 t} \quad (10)$$

$$u_p(y, t) = u_p(y)e^{-\lambda^2 t}$$

Differentiate with respect to 't'

$$\frac{\partial u_p}{\partial t} = -\lambda^2 u_p(y) e^{-\lambda^2 t}$$

$$m_p(-\lambda^2 u_p(y) e^{-\lambda^2 t}) = k^* N(u - u_p) e^{-\lambda^2 t}$$

$$u_p = \frac{k^* u}{k^* - m_p \lambda^2}$$

$$u_p = c^2 u$$

Where  $c^2 = \frac{k^*}{k^* - m_p \lambda^2}$

$$\frac{\partial u}{\partial t} + p = \left(1 + \frac{1}{\gamma}\right) \frac{\partial^2 u}{\partial y^2} - \left(Ha^2 \sin^2 \alpha + \frac{1}{\kappa} + k^* N - k^* N c^2\right) u + g\beta$$

$$\begin{aligned} & \lambda^2 F(y) e^{-\lambda^2 t} + p e^{-\lambda^2 t} \\ & - \left(1 + \frac{1}{\gamma}\right) F''(y) e^{-\lambda^2 t} - \left(\lambda^2 - Ha^2 \sin^2 \alpha - \frac{1}{\kappa} + k^* N \right. \\ & \left. - k^* N c^2\right) F(y) e^{-\lambda^2 t} + g\beta H(y) e^{-\lambda^2 t} \end{aligned}$$

$$F'' + \left(\frac{\gamma}{1+\gamma}\right)(\lambda^2 - H a^2 \sin^2 \alpha - \frac{1}{k} + k^* N(c^2 - 1))F(y) = \frac{\gamma}{1+\gamma} \left(p - g\beta \frac{\sin(\delta y)}{\sin \delta}\right)$$

$$F''(y) + \zeta^2 F(y) = \frac{\gamma}{1+\gamma} \left(p - g\beta \frac{\sin(\delta y)}{\sin \delta}\right)$$

where,

$$\zeta^2 = \left(\frac{\gamma}{1+\gamma}\right)(\lambda^2 - H a^2 \sin^2 \alpha - \frac{1}{k} + k^* N(c^2 - 1))$$

The boundary conditions are transformed to

$$\begin{aligned} H &= 0, F = 0 \text{ at } y = 0 \\ H &= 1, F = 1 \text{ at } y = 1 \end{aligned} \quad (11)$$

By (5) and (9)

$$G = A \quad (13)$$

where A is an arbitrary constant.

By (10), equation (4.7) becomes

$$\begin{aligned} -\lambda^2 H(y) e^{-\lambda^2 t} &= \frac{1}{v P_r} H''(y) e^{-\lambda^2 t} + \frac{S}{v P_r} H(y) e^{-\lambda^2 t} \\ \frac{1}{v P_r} H''(y) e^{-\lambda^2 t} + \frac{S}{v P_r} H(y) e^{-\lambda^2 t} + \lambda^2 H(y) e^{-\lambda^2 t} &= 0 \\ H''(y) + \delta^2 H(y) &= 0 \end{aligned} \quad (14)$$

where  $\delta^2 = S + \lambda^2 P_r v$

Solution of equation (14) using the boundary condition (11) is as follows:

Auxiliary equation of (14)

$$m^2 + \delta^2 = 0 \quad m = \pm i\delta$$

Hence,

$$H(y) = c_1 \cos(\delta y) + c_2 \sin(\delta y) \quad (15)$$

$$\text{At } y = 0, H = 0: 0 = c_1 \cos \delta \quad (15a)$$

$$\text{At } y = 1, H = 1: 1 = c_1 \cos \delta + c_2 \sin \delta \quad (15b)$$

From (15a) and (15b), we have

$$c_1 = 0 \text{ and } c_2 = \frac{1}{\sin \delta}$$

Hence (15) becomes

$$H(y) = \frac{\sin(\delta y)}{\sin \delta} \quad (16)$$

From (10) and (16) the temperature distribution is given by

$$\theta(y, t) = \left( \frac{\sin(\delta y)}{\sin \delta} \right) e^{-\lambda^2 t} \quad (17)$$

Using the equation (16) into equation (12) we get

$$F''(y) + \zeta^2 F(y) = \frac{\gamma}{1+\gamma} \left( p - g\beta \frac{\sin(\delta y)}{\sin \delta} \right) \quad (18)$$

Solution of equation (18) using the boundary condition (11) is as follows:

Auxiliary equation of (18)

$$m^2 + \zeta^2 = 0, \text{ implies } m = \pm i\zeta$$

$$\text{Hence, C.F.} = c_3 \cos(\zeta y) + c_4 \sin(\zeta y)$$

$$\text{P.I.}_1 = \frac{\gamma}{1+\gamma} p \left( \frac{e^{(0)y}}{0^2 + \zeta^2} \right) = \frac{\gamma}{1+\gamma} \frac{p}{\zeta^2}$$

$$\text{P.I.}_2 = \frac{\gamma}{1+\gamma} \frac{g\beta}{\sin \delta} \left( \frac{\sin \delta y}{-\delta^2 + \zeta^2} \right) = \frac{\gamma}{1+\gamma} \frac{g\beta}{\sin \delta} \left( \frac{\sin \delta y}{\zeta^2 - \delta^2} \right)$$

Hence

$$F(y) = c_3 \cos \zeta y + c_4 \sin \zeta y + \frac{\gamma}{1+\gamma} \frac{p}{\zeta^2} + \frac{\gamma}{1+\gamma} \frac{g\beta}{\sin \delta} \left( \frac{\sin \delta y}{\zeta^2 - \delta^2} \right) \quad (19)$$

At  $y = 1, F = 1$

$$1 = c_3 \cos \zeta + c_4 \sin \zeta + \frac{\gamma}{1 + \gamma \zeta^2} + \frac{\gamma}{1 + \gamma \sin \delta} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right) \quad (19a)$$

At  $y = 0, F = 0$

$$0 = c_3 + \frac{\gamma}{1 + \gamma \zeta^2} \quad (19b)$$

From (19b), we have

$$c_3 = -\frac{\gamma}{1 + \gamma \zeta^2}$$

From (19a), we have

$$1 = -\frac{\gamma}{1 + \gamma \zeta^2} \cos \zeta + c_4 \sin \zeta + \frac{\gamma}{1 + \gamma \zeta^2} + \frac{\gamma}{1 + \gamma} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right)$$

$$1 = \frac{\gamma}{1 + \gamma \zeta^2} (1 - \cos \zeta) + c_4 \sin \zeta + \frac{\gamma}{1 + \gamma} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right)$$

$$c_4 \sin \zeta = 1 - \frac{\gamma}{1 + \gamma} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right) + \frac{\gamma}{1 + \gamma \zeta^2} (\cos \zeta - 1)$$

$$c_4 = \frac{1}{\sin \zeta} \left\{ 1 - \frac{\gamma}{1 + \gamma} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right) + \frac{\gamma}{1 + \gamma \zeta^2} (\cos \zeta - 1) \right\}$$

Hence

$$F(y) = c_3 \cos \zeta y + c_4 \sin \zeta y + \frac{\gamma}{1 + \gamma \zeta^2} + \frac{\gamma}{1 + \gamma \sin \delta} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right) \quad (20)$$

From equation (8) and (20) the velocity of the flow of the fluid parallel to the direction of the channel is obtained as,

$$u(y, t) = \left( c_3 \cos \zeta y + c_4 \sin \zeta y + \frac{\gamma}{1 + \gamma \zeta^2} + \frac{\gamma}{1 + \gamma \sin \delta} \left( \frac{g\beta}{\zeta^2 - \delta^2} \right) \right) e^{-\lambda^2 t} \quad (21)$$

From equations (9) and (13), the velocity of the fluid flow perpendicular to the direction of the channel is given by

$$v(y,t) = Ae^{-\lambda^2 t} \quad (22)$$

Equations (17), (21) and (22) show the temperature distribution, the axial velocity and normal velocity respectively.

# **CHAPTER-5**

## 5. PERFORMANCE ANALYSIS

### 5.1 SENSITIVE ANALYSIS AND DISCUSSION

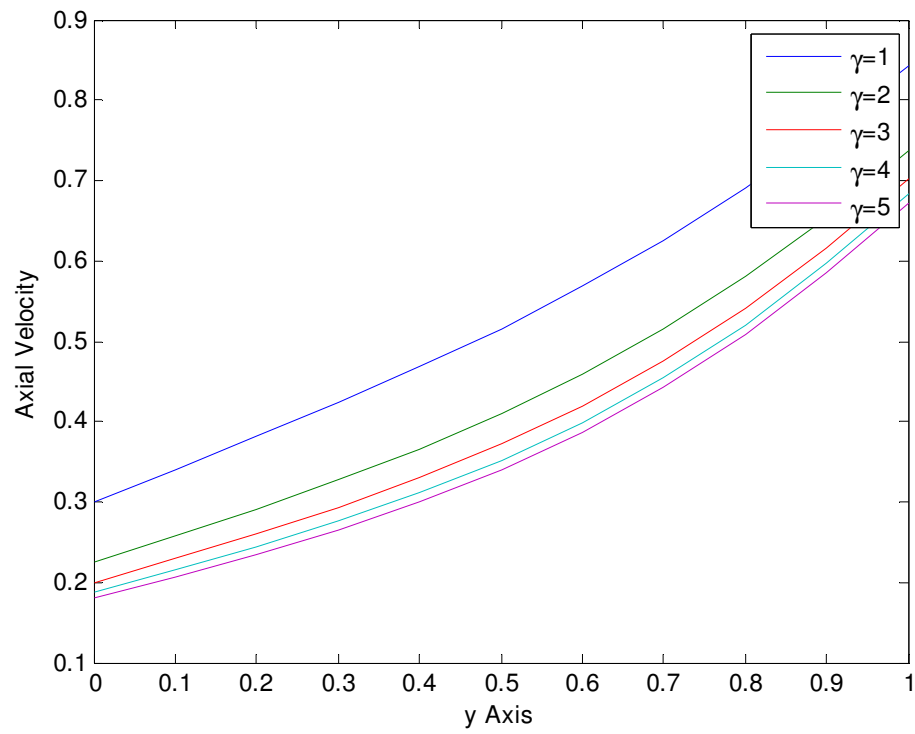
From what has been discussed above, we have performed fluid flow between two parallel plates, causing them to move in parallel with dusty particles. Fluid suction is through the lower fixed plate. Here the effects of velocity profiles and pressure distribution are studied. To study the behavior of the velocity profiles for Casson parameter, dusty particles and heat source curves are drawn for various values of the parameters that describe the flow. The results obtained for the steady flow are displayed in Figures 5.1, 5.2 and 5.3.

### 5.2 EFFECTS OF DIFFERENT PHYSICAL PARAMETERS ON VELOCITY FIELDS

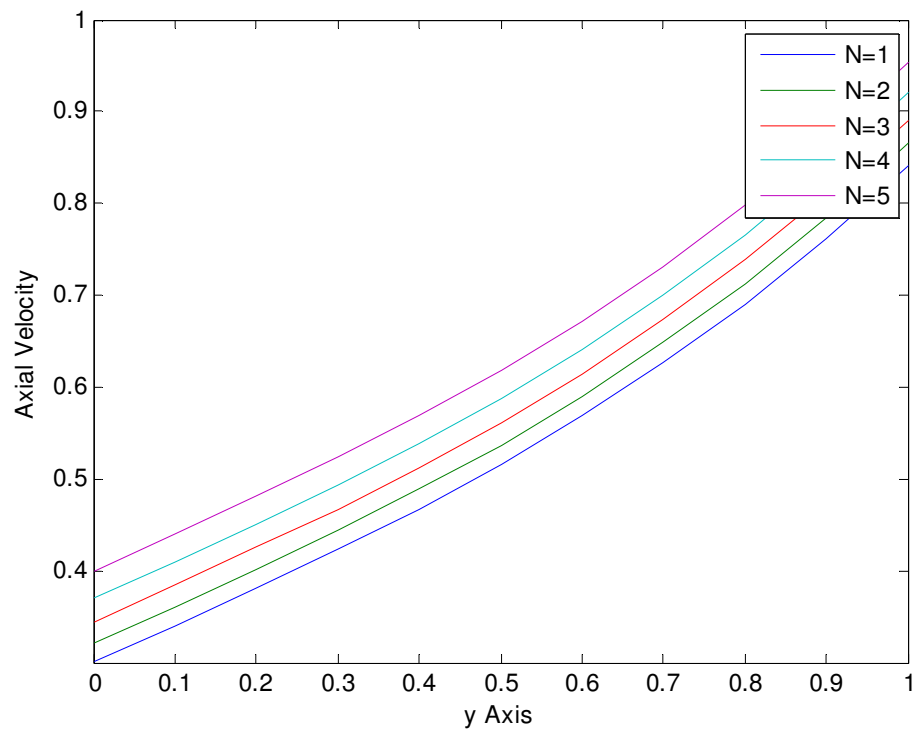
Effects of Casson parameter  $\gamma$  on velocity profiles for unsteady motion are clearly exhibited in Figure 5.1 the behavior of velocity with increasing  $\gamma$  is noted at  $\gamma = 1, 2, 3, 4, 5$ ,  $\lambda = 0.2$ ,  $y = 0:0.1:1$ ,  $S = 0.5$ ,  $v = 0.9$ ,  $K = 0.8$ ,  $t = 0.6$ ,  $g = 9.8$ ,  $\beta = 0.4$ ,  $p = 0.2$ ,  $K' = 2$ ,  $N = 1$ ,  $M = 1$ . It is observed that the axial velocity is found to decreasing with increasing Casson parameter  $\gamma$ .

Figure 5.2 exhibits the axial velocity profiles for several values of number of elements in dusty particles  $N = 1, 2, 3, 4, 5$  at  $\lambda = 0.2$ ,  $y = 0:0.1:1$ ,  $S = 0.5$ ,  $v = 0.9$ ,  $K = 0.8$ ,  $t = 0.6$ ,  $\gamma = 1$ ,  $g = 9.8$ ,  $\beta = 0.4$ ,  $p = 0.2$ ,  $K' = 2$ ,  $N = 1$ ,  $M = 1$ . It is observed that axial velocity increases with increasing the volume of dusty particles.

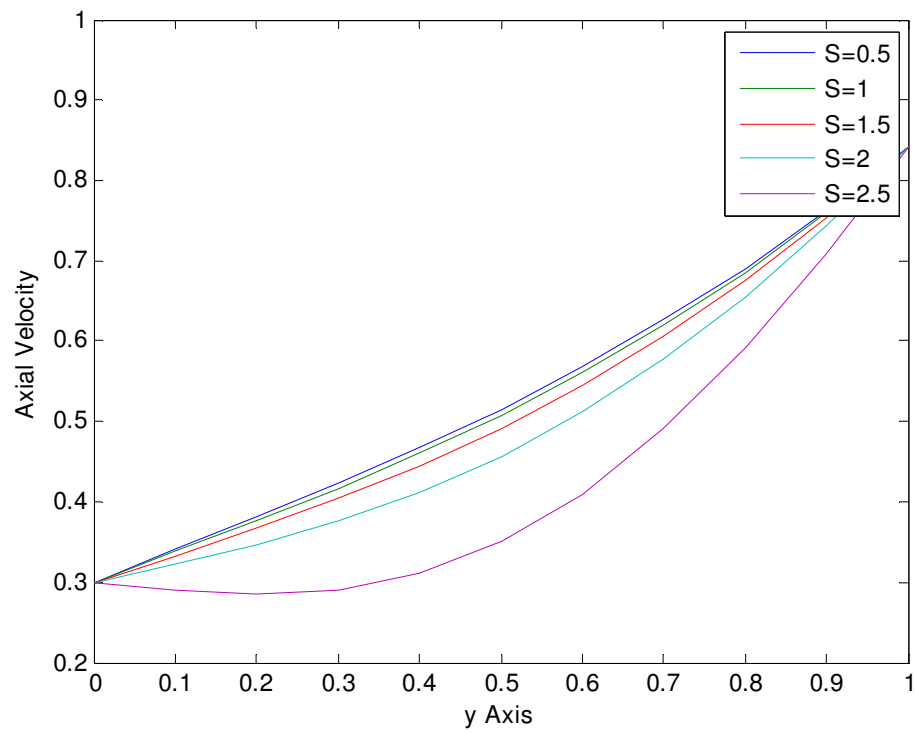
Figure 5.3 exhibits the axial velocity profiles for several values of heat source parameter  $S = 0.5, 1, 1.5, 2, 2.5$  at  $\lambda = 0.2$ ,  $y = 0:0.1:1$ ,  $S = 0.5$ ,  $v = 0.9$ ,  $K = 0.8$ ,  $t = 0.6$ ,  $\gamma = 1$ ,  $g = 9.8$ ,  $\beta = 0.4$ ,  $p = 0.2$ ,  $K' = 2$ ,  $N = 1$ ,  $M = 1$ . It is observed that axial velocity decreases with increasing the heart source parameter  $S$ .



**Figure 5.1 Axial velocity of fluid when  $\gamma$  increases**



**Figure 5.2 Axial velocity of fluid when N increases**



**Figure 5.3 Axial velocity of fluid when S increases**

# CHAPTER-6

## 6.1 CONCLUSIONS

The MHD flow of dusty Casson fluid and heat transfer over a stretching sheet are investigated taking into consideration the thermal radiation effect. From the study, the following remarks can be summarized.

- ▶ The axial velocity decreases with increasing the heat source parameter ( $S$ ).
- ▶ The axial velocity decreases with increasing the Casson fluid parameter ( $\gamma$ ).
- ▶ The axial velocity increases with increasing the dusty particles ( $N$ ).

Finally, the solution is presented graphically.

## REFERENCES

- [1] Attia, H. A. and Kotb, N. A., (1996): 'MHD Flow between Two Parallel Plates with Heat Transfer,' ACTA Mechanica, Vol. 117, p. 215.
- [2] Berman A.S. (1953), "Laminar flow in channels with porous wall", J. Appl. Phys., Vol.24, pp.1232-1235.
- [3] C.K.Kirubhashankar (2018), "Steady Magneto Hydrodynamic Laminar Stream of Viscous Liquid with Constantly Moving Parallel Plates", Journal of Advanced Research in Dynamical and Control Systems, 03-Special Issue, 1348-1353.
- [4] C.K.Kirubhashankar, Y. Immanuel and S. Vaithyasubramanian (2020), "Homotopy method for solving unsteady MHD fluid flow between two parallel plates", Journal of Advanced Research in Dynamical and Control Systems, Vol. 12, 01-Special Issue, 741-745, DOI: 10.5373/JARDCS/V12SP1/20201124
- [5] Chiam T.C. (1993), 'Magnetohydrodynamic boundary layer flow due to a continuously moving flate plate', Comp. Math. Appl. 26, 1–7.
- [6] Cox S.M. (1991), 'Two-dimensional flow of viscous fluid in a channel with porous walls', J.Fluid Mech., Vol. 227, pp.1-33.
- [7] Hamza E.A., (1999), "Suction and injection effects on a similar flow between parallel plates", J Phys 32, 656.
- [8] Hartmann J. (1937), Hf-Dynamics-I, "Theory of the laminar flow of an electrically conducting liquid in a homogeneous magnetic field", Kgl. Danske Videnskab. Selskab Mat. - Fys. Medd Vol15, p.6.
- [9] Hartmann J. and Lazarus F. (1937), Kgl. Danske Videnskab Selskab, Mat-Fys. 15(6, 7).
- [10] Hassaninen I. A. and Mansour M.A.(1990), "Unsteady Magnetic flow through a Porous Medium between two infinite parallel" Astrophysics and Space Science Springer Netherlands, Vol.163, No.2,pp.241-246.
- [11] R. Delhi Babu, S. Ganesh & C. K. Kirubhashankar (2020), "An exact solution of unsteady magnetohydrodynamic flow of dusty fluid between parallel porous plates with an angular velocity", International Journal of Ambient Energy, DOI: 10.1080/01430750.2020.1725636.
- [12] Sing C.B & P.C.Ram, 'Unsteady Magnetohydrodynamic Fluid flow through a channel: Journal of Scientific Research, XXVIII, No.2 (1978).

- [13] Soundalgekar V. and Uplekar A. (1986), 'Steady MHD Couette flow with heat transfer considering varying temperature', IEEE Trans Plasma Sci, Vol.14, p.579.
- [14] Verma, P.D., and Bansal, J. L., (1966), 'Flow of a viscous incompressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate', Proc. Indian Acad. Sci., A 64, 385-396.