

NUMERICAL SOLUTIONS OF FUZZY DIFFERENTIAL EQUATIONS BY RUNGE-KUTTA METHOD

Submitted in partial fulfillment of the requirements for the award of

Master of Science Degree in Mathematics

By

ALICE JENIFER .C (39630001)



**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

**Accredited with Grade “A” by NAAC I 12B Status
by UGC I Approved by AICTE**

**JEPPIAAR NAGAR, RAJIV GANDHI SALAI,
CHENNAI - 600 119**

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Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai – 600 119

www.sathyabama.ac.in

DEPARTMENT OF MATHEMATICS

BONAFIDE CERTIFICATE

This is to certify that this Project report is the bonafide work of **ALICE JENIFER. C (39630001)** who carried out the project entitled “**NUMERICAL SOLUTION OF FUZZY DIFFERENTIAL EQUATIONS BY RUNGE-KUTTA METHOD**” under our supervision from **JANUARY 2021** to **April 2021**.

Internal Guide

Dr. C. KAVITHA

Head of the Department (Name in Capital letters with Seal & Signature)

(Dr. M. NIRMALA)

Submitted for Viva voce Examination held on _____

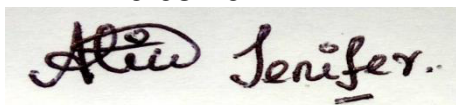
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External Examiner

DECLARATION

I **ALICE JENIFER .C (39630001)** hereby declare that the Project Report entitled "**NUMERICAL SOLUTION OF FUZZY DIFFERENTIAL EQUATIONS BY RUNGE-KUTTA METHOD**" done by me under the guidance of **Dr C. KAVITHA** (Internal) and at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Master of Science Degree in Mathematics

DATE: 26-03-2021

A handwritten signature in dark ink, appearing to read "Alice Jenifer", is written on a light-colored rectangular background.

PLACE: CHENNAI SIGNATURE OF THE CANDIDATE

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ABSTRACT:

We present a numerical algorithm for solving fuzzy initial value problems in this project. We consider an ordinary first order differential equation that's been converted to a fuzzy differential equation by using a membership function .The Runge-Kutta method is used to solve the linear fuzzy differential equation. As a result, the Runge-Kutta second order method, Runge-Kutta third order method, and Runge-Kutta fourth order method are used to solve the initial valued fuzzy differential equation. The numerical values are computed by varying the step size. This method provides a thorough error analysis

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CHAPTER 1

1. INTRODUCTION:

Fuzzy set theory and fuzzy logic are part of a branch of mathematics. It began in 1965, following the publication of Lotfi Asker Zadeh seminal work Fuzzy sets. Typically, mathematical concepts are fuzzified by generalising the characteristic function to membership functions. A differential equation is a mathematical equation that connects one or more functions and their derivatives. When there is a lot of uncertainty or ambiguity, we use a fuzzy differential equation (FDE) which has expanded rapidly in recent years. Such uncertainties or lack of precision can be overcome using a fuzzy differential equation. Complexities in Fuzzy arithmetic may make exact solutions to Fuzzy differential equations difficult to obtain in practise. Chang and Zadeh defined the concept of a fuzzy derivative in 1972. Kandel and Byatt coined the term “Fuzzy Differential Equation” in 1987. The most common approach is to use the Hukuhara differentiability for the fuzzy value functions, but this has a disadvantage. As a result, the fuzzy solution behaves very differently than the crisp solutions. Later, as an extension of the Hukuhara derivative, Seikkala introduces fuzzy derivative notations. The numerical solution of an FDE is now obtained naturally by extending the existing classical methods to the fuzzy case. To solve the fuzzy linear differential equation, we used the Runge-Kutta second order, third order, and fourth order methods.

1.1 FUZZY LOGIC:

Fuzzy logic is an approach to computing based on “degrees of truth” rather than the usual “true or false “(1 or 0) Boolean logic on which the modern computer is based.

1.2 FUZZY SET:

In fuzzy sets, each element is mapped to $[0,1]$ by membership function, $\mu_A : X \rightarrow [0,1]$, Where $[0,1]$ means real numbers between 0 and 1 (including 0 and 1).

1.3 MEMBERSHIP FUNCTION:

A membership function for a fuzzy set A on the universe X is defined as $\mu_A : X \rightarrow [0,1]$, where each element of X is mapped to a value between 0 and 1. This value, called membership value or degree of membership, quantifies the grade of membership of the element in X to the fuzzy set A . Fuzzy differential equation:

1.4 α - CUT:

Let $\mu \in A$ and $\alpha \in [0,1]$. Then $[\mu_A]\alpha = \{x \in X \mid \mu(x) > \alpha\}$ are called the α -cut or α level cut.

1.5 FUZZY DIFFERENTIAL EQUATION:

Fuzzy differential equations (FDEs) appear as a natural way to model the propagation of epistemic uncertainty in a dynamical environment. There are several interpretations of a fuzzy differential equation. This interpretation has the disadvantage that solutions of a fuzzy differential equation have always an increasing length of the support. This fact implies that the future behavior of a fuzzy dynamical system is more and more uncertain in time. This phenomenon does not allow the existence of periodic solutions or asymptotic phenomena. That is why different ideas and methods to solve fuzzy differential equations have been developed.

1.6 FUZZY NUMBER:

A fuzzy set A is defined on the set of real numbers R is said to be a fuzzy number if its membership function $A: R \rightarrow [0,1]$ has the following

- A is convex and normal
- Support and α cut of A must be bounded
- A is piecewise continuous

1.7 TRIANGULAR FUZZY NUMBER:

The triplet $\{a,b,c\}$ is known as triangular fuzzy number, where a and c are the upper and lower boundary whose membership degree '0' and b the upper boundary whose membership degree is 1.

$$\mu_A(x) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{b-a}, & a < x \leq b \\ 1, & x = b \\ \frac{c-x}{c-b}, & b < x \leq c \\ 0, & x \geq c. \end{cases}$$

1.8 EXTENSION PRINCIPLE:

Suppose that f is a function from X to Y and A is a fuzzy set on X defined as

$$A = \frac{\mu_A(x_1)}{x_1} + \frac{\mu_A(x_2)}{x_2} + \dots + \frac{\mu_A(x_n)}{x_n}$$

the extension principle states that the image of fuzzy set A under the mapping f(.) can be expressed as a fuzzy set B. where $y_i=f(x_i)$

$$B = f(A) = \frac{\mu_A(x_1)}{y_1} + \frac{\mu_A(x_2)}{y_2} + \dots + \frac{\mu_A(x_n)}{y_n}$$

CHAPTER 2

2.1 LITERATURE SURVEY:

S.Narayanamoorthy et.al (2015) introduced a new method to solve the numerical algorithm for fuzzy initial value problem and the fuzzy differential equation. They are solved by Runge -Kutta third order method.

T. Jayakumar et.al (n.d.) studied the differential equation with fuzzy initial value. A numerical method is used to approximate the fuzzy solution by using partition of fuzzy interval.

R. Ravichanthiran et.al (2017) worked on some boundary value problem for fuzzy differential equations with functional dependence. Impulsive differential equation with various boundary conditions.

S. Siah Mansouri et.al(2012) introduce a method to solve Nth order fuzzy differential equation is derived. The theorems which shows that Nth order fuzzy differential equation is equivalent to system of ordinary differential equations. So, the numerical method for ordinary differential equation can be used for Nth order fuzzy differential equation.

T.Jeyaraj et.al (2019) introduced numerical solution of second order fuzzy differential equation by Runge–Kutta method with an alternation in parameters in order to increase the order of accuracy of the solution is presented.

K. Kanagarajan et.al (2015) studied the numerical solution of hybrid fuzzy differential equations by using Runge-Kutta method of order five. This method is adopted to solve the dependency problem in fuzzy computation. Numerical examples are presented to illustrate the theory.

S. Rubanraj et.al (2015) proposed a method of computing approximate solution of the fuzzy differential equation with initial conditions. The sixth order Runge-Kutta method is discussed in detail followed by a complete error analysis.

Amirah Ramli et.al (n.d.) studied on numerical algorithm which is applied for solving fuzzy differential equations based on generalized

Hukuhara differentiability. In order to enhance the order of accuracy of the solutions, a developed fourth order Runge-Kutta method is apply for solving fuzzy differential equations. The purpose of this study is to explore the explicit methods which we believe that most of the explicit methods can be improved and modified to cater and solve fuzzy differential equations. This paper is divided into six sections. In the first section, some basic definitions and theorem are reviewed. The numerical examples are given to illustrate the efficiency of the method and the comparison with the existing method is discussed.

Ali Ahmadian et.al (2019) extended the fourth-order Runge–Kutta method is studied to approximate the solutions of first-order fuzzy differential equations using a generalized characterization theorem. In this method, new parameters are utilized in order to enhance the order of accuracy of the solutions using evaluations of both f and f' , instead of using the evaluations of f only. The proposed extended Runge–Kutta method and its error analysis, which guarantees pointwise convergence, are given in detail. Furthermore, the accuracy and efficiency of the proposed method are demonstrated in a series of numerical experiments.

P. Rajkumar et.al (2015) proposed a method of computing approximation of the solution for fuzzy differential equation with initial conditions using Runge-Kutta of order seven in order to increase the order of the accuracy of the solution. This method is discussed in details followed by a complete error analysis.

S. Sindu Devi et.al (2018) proposed a fuzzy version of Runge- Kutta method of order four, for the solution of first order linear fuzzy differential equations without converting them to crisp form. The accuracy and efficiency of the proposed method is illustrated by solving a fuzzy initial value problem with trapezoidal fuzzy number.

V. Parimala et.al (2019) they presented the solution for first order fuzzy differential equation by Runge –Kutta method of order two with new parameters that increase the order of accuracy of the solution. This method is discussed in detail and followed by a complete error analysis. The accuracy and

efficiency of the proposed method is illustrated by solving a fuzzy initial value problem with trapezoidal fuzzy number.

N. Parandin et.al (2014) solved fuzzy ordinary differential equations of the n th order by Runge-Kutta method have been done, and the convergence of the proposed method is proved. This method is illustrated by some numerical examples.

CHAPTER 3

3.1 AIM AND SCOPE:

In differential equation, there exist a vagueness and uncertainty, therefore fuzziness occurs. As a result we present fuzzy differential equation. We know that, in a fuzzy set, each element is mapped to $[0, 1]$ by membership function $\mu_A : x \rightarrow [0, 1]$. We

assign membership function to the initial valued problem and we derive an approximation solution .

AIM:

- Our aim is to solve initial valued problem, numerically.
- The fuzzy differential equation is solved using the Runge – Kutta second order method, the Runge – Kutta third order method and the Runge – Kutta fourth order method, which is discussed in detail.

SCOPE:

- The temperature of several objects can be modelled using this Fuzzy Differential Equation, which is used to calculate the time required for a beverage to cool down.
- Many social systems can be modeled using variable separable types, such as the Lotka - Volterra prey predator problem.

CHAPTER 4

4.METHODOLOGY:

4.1 ARITHMETIC OPERATION ON FUZZY NUMBERS:

The fuzzy number $A \in [0,1]$ can also be represented by an ordered pair of $A = (\underline{a}(r), \overline{a}(r))$ for $0 \leq r \leq 1$ which satisfies

- $\underline{a}(r)$ is a bounded monotonic increasing right continuous
- $\bar{a}(r)$ is a bounded monotonic decreasing left continuous function
- $\underline{a}(r) \leq \bar{a}(r)$, $0 \leq r \leq 1$

4.2 FUZZY INITIAL VALUE PROBLEM:

Consider the first order fuzzy initial value differential equation is given by

$$y'(x) = f(x, y(x)), t \in [x_0, X] \text{ and the initial condition } y(x_0) = y_0,$$

where y is a fuzzy function of x , $f(x, y)$ is a fuzzy function of the crisp variable x and the fuzzy derivative of y and $y(x_0) = y_0$ is a triangular shaped fuzzy number.

We denote the fuzzy function y by $y = [\underline{y}, \bar{y}]$.

The α -level set of $y(x)$ for $x \in [x_0, X]$ is defined as

$$[y(x)]_\alpha = [\underline{y}(x; \alpha), \bar{y}(x; \alpha)],$$

$$[y(x_0)]_\alpha = [\underline{y}(x_0; \alpha), \bar{y}(x_0; \alpha)], \alpha \in (0, 1]$$

We write $[f(x, y)] = [\underline{f}(x, y), \bar{f}(x, y)]$ and

$$f(x, y) = F[x, \underline{y}, \bar{y}],$$

$$f(x, y) = G[x, \underline{y}, \bar{y}], \text{ because of } y' = f(x, y)$$

we have $f(x, y(x); \alpha) = F[x, \underline{y}(x; \alpha), \bar{y}(x; \alpha)],$

$$f(x, y(x); \alpha) = G[x, \underline{y}(x; \alpha), \bar{y}(x; \alpha)].$$

By using the extension principle, we have the membership function

$$\mu_{f(t, y(x))}(s) = \sup\{\mu_{y(x)}(\tau) / s = f(x, \tau)\}$$

so fuzzy number $f(x, y(x))$, it follows that

$$[f(t, y(x))]_\alpha = [\underline{f}(x, y(x); \alpha), \bar{f}(x, y(x); \alpha)], \alpha \in (0, 1].$$

Here, $f(x, y(x); \alpha) = \min\{f(x, u) \mid u \in [y(x)]_\alpha\}$ and

$$\bar{f}(x, y(x); \alpha) = \max \{f(x, u) \mid u \in [y(x)]^r\}$$

4.3 SECOND ORDER FUZZY RUNGE- KUTTA METHOD:

The general second order Runge Kutta method formula is

$$y_{n+1} = y_n + k_2$$

Where, $k_1 = h f(x_n, y_n)$,

$$k_2 = h f(x_n + h/2, y_n + k_1/2)$$

Then, let $Y = [Y_1, Y_2]$ be exact solution

And $y = [y_1, y_2]$ be the approximated solution of the Fuzzy initial value problem

$$[Y(t)]^r = [Y_1(t, r), Y_2(t, r)]^r = [y_1(t, r), y_2(t, r)]^r.$$

FOR Y [Y₁, Y₂], the exact solution of fuzzy differential equation is given by,

$$Y_1(t_{n+1}; r) = Y_1(t_n; r) + k_2,$$

Where, $k_1 = h f(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h f(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$Y_2(t_{n+1}; r) = Y_2(t_n; r) + k_2,$$

Where, $k_1 = h g(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h g(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

FOR y [y₁, y₂], the approximation solution of fuzzy differential equation is given by,

$$Y_1(t_{n+1}; r) = Y_1(t_n; r) + k_2,$$

Where, $k_1 = h f(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h f(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$Y_2(t_{n+1}; r) = Y_2(t_n; r) + k_2,$$

Where, $k_1 = h g(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h g(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

4.4 THIRD ORDER FUZZY RUNGE KUTTA METHOD:

The general Third Order Runge-Kutta Method formula is

$$y_{n+1} = y_n + h/6 [k_1 + 4k_2 + k_3],$$

Where, $k_1 = f(x_n, y_n)$

$$k_2 = f(x_n + h/2, y_n + h/2 k_1)$$

$$k_3 = f(x_n + h, y_n - hk_1 + 2hk_2).$$

Then, let $Y = [Y_1, Y_2]$ be exact solution

And $y = [y_1, y_2]$ be the approximated solution of the Fuzzy initial value problem

$$[Y(t)]_r = [Y_1(t, r), Y_2(t, r)] \quad r = [y_1(t, r), y_2(t, r)] \quad r.$$

FOR $Y [Y_1, Y_2]$, the exact solution of fuzzy differential equation is given by,

$$Y_1(t_{n+1}; r) = Y_1(t_n; r) + h/6 [k_1 + 4k_2 + k_3],$$

Where, $k_1 = f(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = f(t_n + h/2, Y_1(t_n; r) + h/2 k_1, Y_2(t_n; r) + h/2 k_1)$$

$$k_3 = f(t_n + h, Y_1(t_n; r) - hk_1 + 2hk_2, Y_2(t_n; r) - hk_1 + 2hk_2)$$

$$Y_2(t_{n+1}; r) = Y_2(t_n; r) + h/6 [k_1 + 4k_2 + k_3],$$

Where, $k_1 = g(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = g(t_n + h/2, Y_1(t_n; r) + h/2 k_1, Y_2(t_n; r) + h/2 k_1)$$

$$k_3 = g(t_n + h, Y_1(t_n; r) - hk_1 + 2hk_2, Y_2(t_n; r) - hk_1 + 2hk_2)$$

FOR $y [y_1, y_2]$, the approximation solution of fuzzy differential equation is given by,

$$Y_1(t_{n+1}; r) = Y_1(t_n; r) + h/6 [k_1 + 4k_2 + k_3],$$

Where, $k_1 = f(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = f(t_n + h/2, Y_1(t_n; r) + h/2 k_1, Y_2(t_n; r) + h/2 k_1)$$

$$k_3 = f(t_n + h, Y_1(t_n; r) - hk_1 + 2hk_2, Y_2(t_n; r) - hk_1 + 2hk_2)$$

$$Y_2(t_{n+1}; r) = Y_1(t_n; r) + h/6 [k_1 + 4k_2 + k_3],$$

Where, $k_1 = g(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = g(t_n + h/2, Y_1(t_n; r) + h/2 k_1, Y_2(t_n; r) + h/2 k_1)$$

$$k_3 = g(t_n + h, Y_1(t_n; r) - hk_1 + 2hk_2, Y_2(t_n; r) - hk_1 + 2hk_2)$$

4.5 FOURTH ORDER FUZZY RUNGE KUTTA METHOD:

The General Fourth Order Runge-Kutta Method formula is

$$y_{n+1} = y_n + 1/6 [k_1 + 2k_2 + 2k_3 + k_4],$$

Where, $k_1 = h f(x_n, y_n)$,

$$k_2 = h f(x_n + h/2, y_n + k_1/2)$$

$$k_3 = h f(x_n + h/2, y_n + k_2/2)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

Then, let $Y = [Y_1, Y_2]$ be exact solution

And $y = [y_1, y_2]$ be the approximated solution of the Fuzzy initial value problem

$$[Y(t)] r = [Y_1(t, r), Y_2(t, r)] r = [y_1(t, r), y_2(t, r)] r.$$

FOR $Y[Y_1, Y_2]$, the exact solution of fuzzy differential equation is given by,

$$y_1(t_{n+1}; r) = Y_1(t_n; r) + 1/6 [k_1 + 2k_2 + 2k_3 + k_4],$$

Where, $k_1 = h f(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h f(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$k_3 = h f(t_n + h/2, Y_1(t_n; r) + k_2/2, Y_2(t_n; r) + k_2/2)$$

$$k_4 = h f(t_n + h, y_1(t_n; r) + k_3, y_2(t_n; r) + k_3)$$

$$y_2(t_{n+1}; r) = Y_2(t_n; r) + 1/6 [k_1 + 2k_2 + 2k_3 + k_4],$$

Where, $k_1 = h g(t_n, [Y_1(t_n; r), Y_2(t_n; r)])$,

$$k_2 = h g(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$k_3 = h g(t_n + h/2, Y_1(t_n; r) + k_2/2, Y_2(t_n; r) + k_2/2)$$

$$K_4 = h g(t_n + h, y_1(t_n; r) + k_3, y_2(t_n; r) + k_3)$$

FOR $y[y_1, y_2]$, the approximation solution of fuzzy differential equation is given by,

$$y_1(t_{n+1}; r) = Y_1(t_n; r) + 1/6 [k_1 + 2k_2 + 2k_3 + k_4],$$

$$\text{Were, } k_1 = h f(t_n, [Y_1(t_n; r), Y_2(t_n; r)]),$$

$$k_2 = h f(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$k_3 = h f(t_n + h/2, Y_1(t_n; r) + k_2/2, Y_2(t_n; r) + k_2/2)$$

$$k_4 = h f(t_n + h, y_1(t_n; r) + k_3, y_2(t_n; r) + k_3)$$

$$y_2(t_{n+1}; r) = Y_2(t_n; r) + 1/6 [k_1 + 2k_2 + 2k_3 + k_4],$$

$$\text{Were, } k_1 = h g(t_n, [Y_1(t_n; r), Y_2(t_n; r)]),$$

$$k_2 = h g(t_n + h/2, Y_1(t_n; r) + k_1/2, Y_2(t_n; r) + k_1/2)$$

$$k_3 = h g(t_n + h/2, Y_1(t_n; r) + k_2/2, Y_2(t_n; r) + k_2/2)$$

$$k_4 = h g(t_n + h, y_1(t_n; r) + k_3, y_2(t_n; r) + k_3)$$

Clearly $y_1(t_n; r)$ and $y_2(t_n; r)$ converges to $Y_1(t_n; r)$ and $Y_2(t_n; r)$

CHAPTER 5

5.1 RESULTS AND DISCUSSION:

Numerical solution:

Consider the first order linear differential equation, $Y'(t) = x^2y$, $t > 0$,
 $y(0) = (0.75 + 0.25r, 1.25 - 0.25r)$

Solution:

The exact solution is given by,

$$y(t, r) = [(0.75 + 0.25r) e^{t^3/3}, (1.25 - 0.25r) e^{t^3/3}]$$

at $t = 1$ and $h=0.2$

$$y(1, r) = [(0.75 + 0.25r) e^{1/3}, (1.25 - 0.25r) e^{1/3}], 0 \leq r \leq 1$$

Comparison between the exact solution and the approximate solution of second order, third order and fourth order Runge - Kutta method , is given by complete error analysis.

TABLE 1: The exact solution for the initial value problem is given by,

r	t	$\underline{y}$	$\bar{y}$
0	1	1.04671	1.74452
0.2	1	1.11649	1.67473
0.4	1	1.18627	1.60495
0.6	1	1.25605	1.53517
0.8	1	1.32583	1.46539
1	1	1.39561	1.39561

TABLE 2: Approximate solution of initial value problem of Runge Kutta second order, third order and fourth order are given by,

r	y_1	y_2	y_1	y_2	y_1	y_2
0	0.94964	1.58275	0.9560664	1.593444	0.95597	1.59329
0.2	1.01296	1.51944	1.01980416	1.529706	1.0197056	1.529558
0.4	1.07627	1.45613	1.0835416	1.465968	1.083437	1.4658269
0.6	1.13958	1.39282	1.147279	1.4022306	1.147168	1.402095
0.8	1.20289	1.32951	1.211017	1.3384928	1.210897	1.33836376
1	1.2662	1.2662	1.2747552	1.2747552	1.27463215	1.27463215

CHAPTER 6

6.1 CONCLUSION:

The numerical solution to the fuzzy differential equation has been found. A new technique for solving the fuzzy initial valued problem by fuzzy differential equation by using membership function has been developed for the Runge - Kutta

second order method, Runge - Kutta third order method, and Runge -Kutta fourth order method, as discussed in the methodology.

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