

INVESTIGATION OF SWIRL CHARACTERISTICS OF CONVERGENT DIVERGENT NOZZLE

Submitted in partial fulfillment of the requirements for the award of

Bachelor of Engineering Degree in
Aeronautical Engineering

By

Sanapala Komesh₁(37260019)
Gajula Veerabhadra Rao(37260048)



DEPARTMENT OF MECHANICAL ENGINEERING

SCHOOL OF MECHANICAL ENGINEERING

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited with grade "A" by NAAC

JEPPIAAR NAGAR, RAJIV GANDHI SALAI, CHENNAI – 600119

APRIL 2021



SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited with "A" grade by NAAC
Jeppiaar Nagar, Rajiv Gandhi Salai, Chennai - 600 119
www.sathyabama.ac.in



DEPARTMENT OF MECHANICAL ENGINEERING

BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of SANAPALA KOMESH(Reg. No. 37260019) and GAJULA VEERABHADRA RAO (Reg. No. 37260048) who carried out the project entitled **"INVESTIGATION OF SWIRL CHARACTERISTICS OF CONVERGENT DIVERGENT NOZZLE"** under our supervision from January 2020 to April 2021.

Internal Guide
P Gunasekar

Head of the Department
Dr. G. ARUNKUMAR, ME., PhD.,

Submitted for Viva voce Examination held on _____

Internal Examiner

External Examiner

DECLARATION

I SANAPALA KOMESH(Reg. No. 37260019) and GAJULA VEERABHADRA RAO (Reg. No. 37260048) hereby declare that the Project Report entitled “**INVESTIGATION OF SWIRL CHARACTERISTICS OF CONVERGENT DIVERGENT NOZZLE**” done by me under the guidance of **P Gunasekar** is submitted in partial fulfillment of the requirements for the award of Bachelor of Engineering degree in Aeronautical Engineering.

DATE: 07.04.2021

SIGNATURE OF THE CANDIDATE

PLACE: CHENNAI

ACKNOWLEDGEMENT

I am pleased to acknowledge my sincere thanks to **Board of management of SATHYABAMA** for their kind encouragement in doing this project and for completing it successfully. We are grateful to them.

I convey my thanks to **Mr. Venkatesh, Mr. Madhan and Dr. Boomadevi, Ph.D., Head of the Department, Department of Aeronautical Engineering** for providing the necessary support and details at the right time during the progressive reviews.

I would like to express my sincere and deep sense of gratitude to our Project Guide **Mr. P .GUNASEKAR**, for his valuable guidance, suggestions and constant encouragement paved way for the successful completion of my project work.

I wish to express my thanks to all Teaching and Non-teaching staff members of Department of **AERONAUTICAL ENGINEERING** who were helpful in many ways for the completion of the project.

ABBREVIATIONS

CD – CONVERGENT DIVERGENT

L-LENGTH OF NOZZLE (in meters)

M-MACH NUMBER

X-LOCAL LENGTH OF NOZZLE (in meters)

P-STATIC PRESSURE

NPR- NOZZLE PRESSURE RATIO

V- VELOCITY

ABSTRACT

In this work the swirl effect is analyzed in a 3-Dimensional Convergent Divergent Nozzle (Supersonic) under various back pressure conditions.

Design and model of the 3D CD nozzle is done by using Fusion 360. Solving the Governing equations like continuity, momentum and energy equations gave us the flow field through the CD Nozzle.

We found that the effects of swirl and increase in back pressure helps in weakening the strength of shock. Reason being, when we increase the back pressure it makes the shock to move upstream towards throat in the divergent portion and whereas the swirl stretches the shock axially downstream to throat and weakens the shock ,so shock strength also weakens.

Keywords : Supersonic, compressible flows, shock wave, mach number, turbulence, Swirl flows.

TABLE OF CONTENTS

ABSTRACT

LIST OF SYMBOLS AND ABBREVIATIONS

CHAPTER NO	TITLE	PAGE NO.
1	INTRODUCTION	1
2	LITERATURE REVIEW	3
3	AIM AND SCOPE OF PRESENT INVESTIGATION	6
3.1	AIM	6
3.2	SCOPE	6
4	METHODS AND ALOGORITHIMS USED	9
4.1	GEOMETRICAL MODELING	9

4.2	BOUNDARY CONDITIONS	9
4.3	EXPERIMENTAL SETUP	10
4.4	ALGORITHMS	14
4.5	FORMULA USED	14
4.6	VALIDATION	15
5	RESULTS AND DISCUSSION	17
5.1	Result	17
5.1.1	EFFECT OF BACK PRESSURE ON SHOCK WAVE	17
5.1.2	EFFECT OF SWIRL AND SHOCK WAVE	18
5.2	PROTOTYPE USED AND EXPERIMENTAL ANALYSIS	21
6	CONCLUSION	32

CHAPTER 1

INTRODUCTION

The objective is to understand and analyze the effect of swirl on shock in a supersonic air flow through CD Nozzle.

The swirl flow in supersonic flow has many applications

- Improved mixing

- Noise reduction in Convergent Divergent Nozzle (CD Nozzle).

When the tangential component of velocity is introduced to the flow the flow becomes more complex as compared to one dimensional flow. From the swirling flow models by Mager, King, Lewellen, Mager gave some conclusions, one of which is that at each axial position there exists a core zone between the axis and a certain Radius where no flow exists and hence it could be represented by zero density.

A simple analytical approach similar to that of Mager treating a swirl type of similar characteristics has been introduced by Cutler and Barnwell who also conducted experiments of swirling flow through a CD Nozzle. King solved the specific case of a tangential velocity distribution as a solid body rotation. His solution assumed low swirl intensity through the nozzle and given values of Mach number along the axis. It came out to be that the specific swirl Lewellen made significant analytical as well as experimental contributions. Their analytical model assumed isentropic flow of a perfect gas, predicting the effects of swirl by expanding the one dimensional flow theory.

The highly underexpanded nozzle jet was investigated experimentally and analytically by Adamson and Nicholls. Who presented a method for calculating the position of the first Mach disk.

In their calculation the axial pressure distribution on the flow centerline downstream of the nozzle exit was used to define a fictitious nozzle extension.

supersonic flow of swirling jets were conducted in a stagnant atmosphere by Cutler there the swirling jets were created by tangential injection into a swirl chamber and accelerated through a CD Nozzle but in our case we cannot afford it so what we did is we twisted the Square CD nozzle and Hexagonal CD nozzle from the throat part till the exit of CD nozzle means we twisted the Divergent part of CD nozzle, that means initial twist near the throat is 30 degrees but the twist gradually increases as we go downstream from the throat the increase in twist is small. The results which we got was not accurate which we could have got by injection of swirl, but by twisting we got a result close to our desired result.

Gostintsev studied an underexpanded supersonic swirling gas jet issuing from a convergent nozzle. They showed that the effect of rotation on the wave structure of an axis symmetric jet is qualitatively showing some resemblance to the effect of reduction in overpressure ratio. Batson and Sforzini studied the swirling flow through a convergent nozzle. It was found that the tangential component of velocity decreases and the axial component of velocity increases as the flow passes through the nozzle

SWIRL NUMBER

Swirl number is the ratio of tangential momentum by axial momentum means swirl number is based on the axial and tangential component of velocity magnitude.

In our study we will study the pressure characteristics and variation of axial velocity when swirl is given to the CD Nozzle and flow is passed through that CD Nozzle.

CHAPTER 2

LITERATURE REVIEW

Durst et al., (1974) incontestable that in two-dimensional plane, bilateral flows will exist in sudden-expansion ducts for less than a restricted vary of Sir Joshua Reynolds numbers. At higher Sir Joshua Reynolds variety, at the tip of the unforeseen enlargement tiny disturbance was generated and amplified within the shear layers that square measure fashioned between the most flow and also the recirculation flow within the corners. Rathakrishnan & Sreekanth, (1984) studied flows in pipes with abrupt enlargement. They complete that the non-dimensional base pressure depends and varies by parameters like enlargement space ratios, the general pressure ratios and also the duct length-to-diameter ratios. They found that at a given overall pressure quantitative relation and a given space quantitative relation, we are able to realize length-to-diameter quantitative relation of the enlargement that shows minimum pressure loss within the nozzle at the nozzle exit on the symmetry axis. For minimum base pressure at given nozzle and enlargement space quantitative relation, the duct length ought to exceed a hard and fast minimum worth. In flow through pipes with unforeseen enlargement if the bottom pressure minimisation isn't adequate, therefore the full pressure loss should even be thought-about to attain most favourable performance. Khan et al.,(2002-2012) through an experiment explained the quality of micro-jets to manage base pressure during a suddenly swollen axisymmetric duct by considering parameters like completely different ratio, area ratio, NPR, and length-to-diameter (L/D) quantitative relation in enlargement of a CD nozzle with and while not

management of fluid flows. However, it absolutely was found that no numerical and analytical study was explained to manage the bottom pressure at CD nozzle from micro-jets. The numerical and experimental studies administered by varied students have evaluated the accuracy of the turbulence model for expecting the flow field and also the nozzle performance exactly. Two-dimensional axisymmetric compressible flow analysis through a CD nozzle has been studied with the assistance of ANSYS FLUENT by mistreatment mistreatment turbulence model and Spalart-Allmaras turbulence model, a comparative investigation between the models on the idea pressure, velocity, temperature contours and vectors to establish the economical style conditions for CD nozzles conjointly been done by Najjar et al., (2013). The variation of flow parameters like Pressure, Temperature, rate and Density is pictured mistreatment CFD and conjointly determined the simulation of shockwave (Patel et al., 2016

Zhang & Kim in theory and numerically studied the particle–gas flows through a CD nozzle.

Homogenized and equilibrium model for no slip condition in rate and temperature happens between particle and gas section thought of to derive mass flow and speed of sound for point in time flows. Particle mass loading to analyze its impact on choking phenomena for particle–gas flows at completely different nozzle pressure ratios has been carried. CFD simulation code and Solid works CAD tool for modelling used and analysed the results. during this performance discovered that the fluid leaves the throat and issue to the divergent region of the CD nozzle.

CHAPTER 3

AIM AND SCOPE OF PRESENT INVESTIGATION

(3.1) AIM –To understand the effect of swirl on shock wave during the flow through 3 Dimensional Convergent Divergent Nozzle.

(3.2) Scope of Present investigation

As we know that CD Nozzle is used to produce supersonic flow at outlet and we know that to get a supersonic flow at exit then the throat must be choked. So the back pressure will be sufficient that the flow after getting choked at throat accelerate to a supersonic value and the supersonic flow in a divergent portion means the flow will further gets accelerated to give a supersonic value at exit. We will get the supersonic value at the exit only when the Nozzle Pressure Ratio (NPR) is more than a certain value that value is 2.863, and if the NPR is lower than a different certain value that value is 1.974, then there will be no flow because of instabilities. So when the NPR value is in between 1.974 and 2.863 then there will be a Normal shock wave formation in the divergent portion of CD Nozzle because of which the flow becomes subsonic, but any way we considered the flow to be isentropic through the CD Nozzle.

So let's see what happens if we introduce the swirl at the divergent portion by not considering isentropic flow and decrease the back pressure at the same time. So when we introduce the swirl at the divergent portion of the CD nozzle keeping the NPR in between 1.974 and 2.863 and decrease the back pressure then the normal shock wave which is formed is stretched towards the downstream direction due to swirl and due to the decrease in back pressure the shock wave moves upstream towards the throat which weakens the shock strength which might help preventing the flow from becoming fully subsonic at exit of CD Nozzle and instead it can help to reach to a closer value of supersonic flow at exit.

Secondly due to swirl the total mass flow rate at exit is more as compared to when there is no swirl, because when we introduce swirl there is an additional tangential component of velocity added to the initial axial component of velocity, so now we will have a resultant mass flow which will be more.

This increased mass flow rate will help in producing greater thrust.

CHAPTER 4

METHODS AND ALGORITHMS USED NUMERICAL MODELLING

The geometry of the Nozzle which is used in this analysis is a CD Nozzle. This CD nozzle has cylindrical inlet and conical convergent and divergent section and circular arc design between these sections.

Numerical modeling consists of three processes namely

- Pre processing (geometric modelling, grid generation and applying boundary conditions is involved)
- Processing
- Post processing

We will conduct our analysis for three kind of CD Nozzle

- 1) Circular CD Nozzle
- 2) Square CD Nozzle
- 3) Hexagonal CD Nozzle

For Inlet mach 0.3

-For Circular CD Nozzle

Inlet to throat area ratio is 2.074

-For Square CD Nozzle

Inlet to throat area ratio is 1.438

-For Hexagonal CD Nozzle

Inlet to throat area ratio is 1.440

(4.1) GEOMETRICAL MODELING

The design of 3D convergent divergent nozzle is done in ANSYS (design software). The complex physics part is taken care by considering 360 degree of CD nozzle. Reason for complexity is due to supersonic flow and turbulence. For CD nozzle CFD is used to generate grid with cell count of hexahedral as 398735.

The tangential component of velocity is varied for various back pressure. The radial velocity is negligible.

(4.2) Boundary Conditions

Boundary Conditions used for this work are;

1) Inlet - For the boundary conditions at inlet we can say, inlet pressure of 101325 pascal with axial and tangential component of velocity. 0% tangential velocity component means it is no swirl condition.

2) Outlet - For the outlet pressures, the pressure here means at outlet is changed means pressure is variable to capture the shock inside the nozzle. From our referred base papers it is found that outlet pressure of pa results with the shock formation the downstream of throat.

3) wall - Adiabatic, no slip boundary conditions at wall. This Wall boundary conditions are applicable to all the faces in contact with the flow.

4) Fluid Properties - Fluid is air. And here air means Ideal gas. It is considered as ideal gas because there is change in density of the fluid at the divergent part of CD nozzle means it is compressible.

5) Pressure conditions - 0 Pascal operating pressure is considered for all the cases.

(4.3) EXPERIMENTAL SETUP

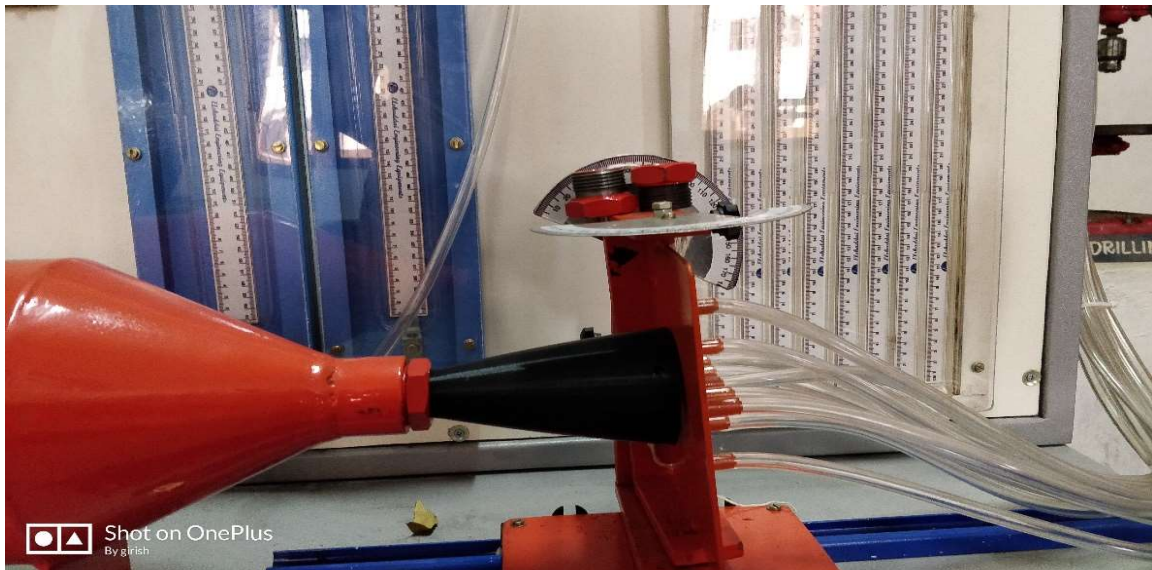


Fig No.4.0 Testing of Hexagonal CD Nozzle

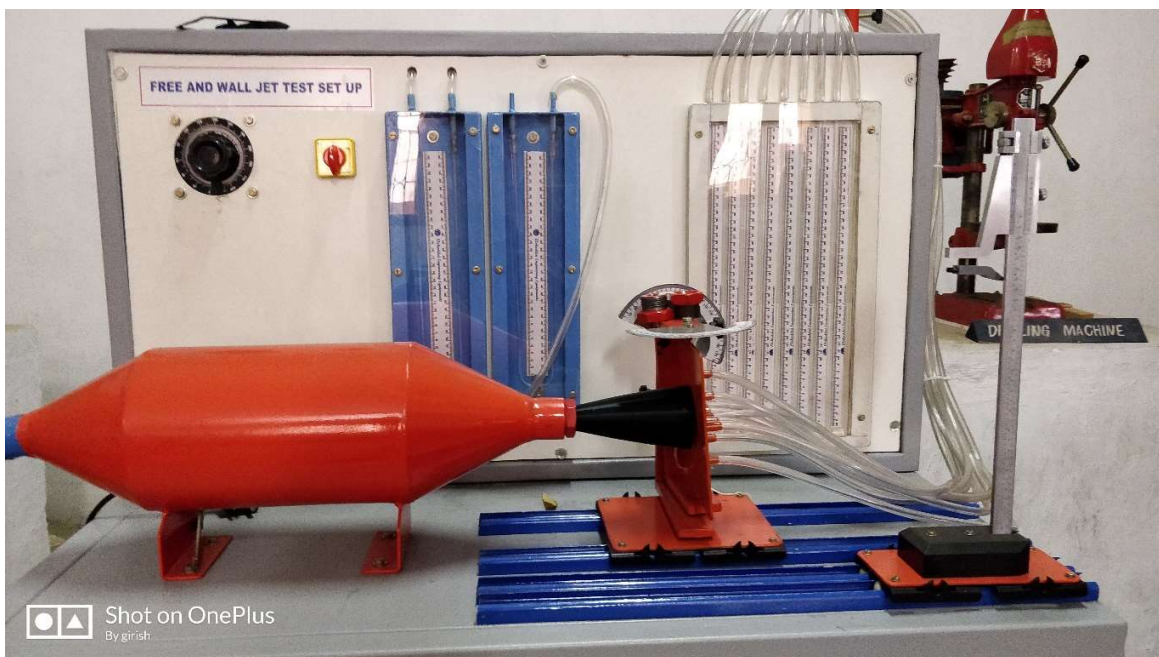


Fig No.4.1 Testing of Hexagonal CD Nozzle



Fig No.4.2 Testing of Square CD Nozzle



Fig No 4.3 Testing of Square CD Nozzle

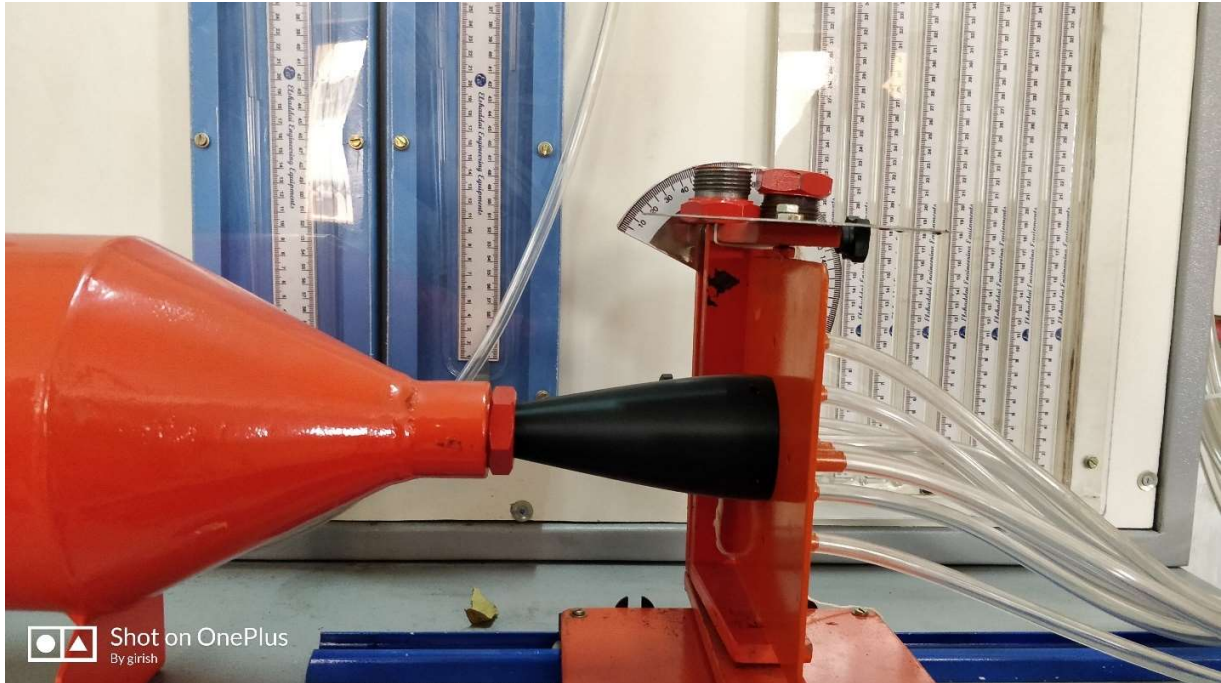


Fig no 4.4 Testing of Plane Circular CD Nozzle.

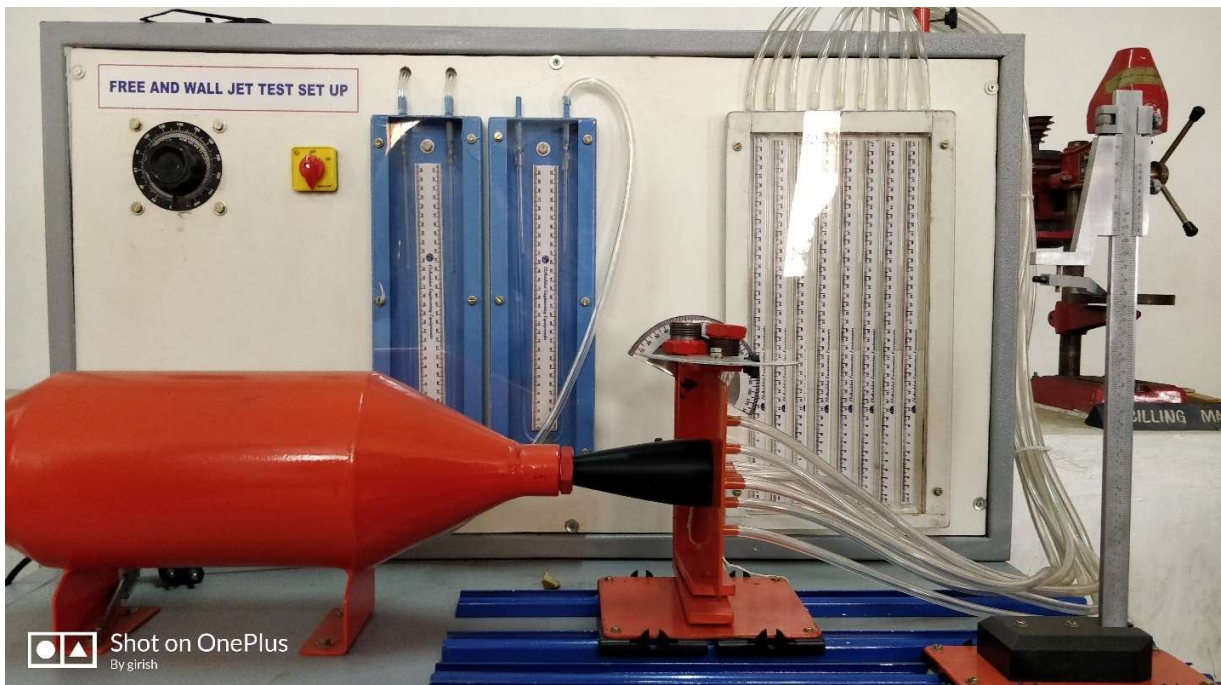


Fig no 4.5 Testing of Plane Circular CD Nozzle.



Fig No4.6 FREE AND WALL JET SET UP

(4.4)ALGORITHIMS

Conservation of Mass:

$$\nabla \cdot (\rho \vec{V}) = 0$$

Conservation of Momentum:

$$\text{x-momentum : } \nabla \cdot (\rho u \vec{V}) = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$\text{y-momentum : } \nabla \cdot (\rho u \vec{V}) = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$\text{z-momentum : } \nabla \cdot (\rho u \vec{V}) = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}$$

Conservation of Energy:

$$\frac{\partial}{\partial x_i} (\rho \mu_i T) = \frac{\partial}{\partial x_i} \left[\left(\frac{\mu}{Pr} + \frac{\mu_t}{\sigma_T} \right) \frac{\partial T}{\partial x_i} \right] + S_R$$

Ideal gas equation:

$$pV = mRT$$

Where, R is the characteristic gas constant.

(4.5)Formula used

$$\left(\frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[\frac{2}{\gamma + 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma + 1)/(\gamma - 1)}$$

A - local area

A*-Throat area

M - mach number

VALIDATION(4.6)

We will compare the flow which is predicted through the Convergent Divergent Nozzle.

The pressure at inlet is atmospheric pressure and outlet pressure will correspond to the value so that the inlet mach number is 0.3. First we will do this for no swirl condition means the flow will only have axial component, and the mach number predicted at outlet is supersonic so then we compared the predicted and experimental value the variation which we got was less than 2%. SO we can further proceed for analysis of flow through CD Nozzle of different shapes means square and hexagonal and we can involve swirl condition.

CHAPTER 5

RESULT, DISCUSSION AND PERFORMANCE ANALYSIS

(5.1)RESULT

(5.1.1)EFFECT OF BACK PRESSURE ON SHOCK WAVE

We will predict the formation of shockwave in the divergent section by using CFD technique. The ratio of inlet pressure to the exit pressure is nozzle pressure ratio (NPR) and there is a certain NPR above which no shockwave is formed and flow expands normally that value of NPR is 2.863 and there is a certain NPR below which there is no flow produced means there will be some instability and that value of NPR is 1.974. The flow is normal to boundary in all the cases. From the result we get that by decreasing NPR value shockwave propagate or moves towards the throat that is upstream towards the throat. If we increase the back pressure then the NPR value will decrease.

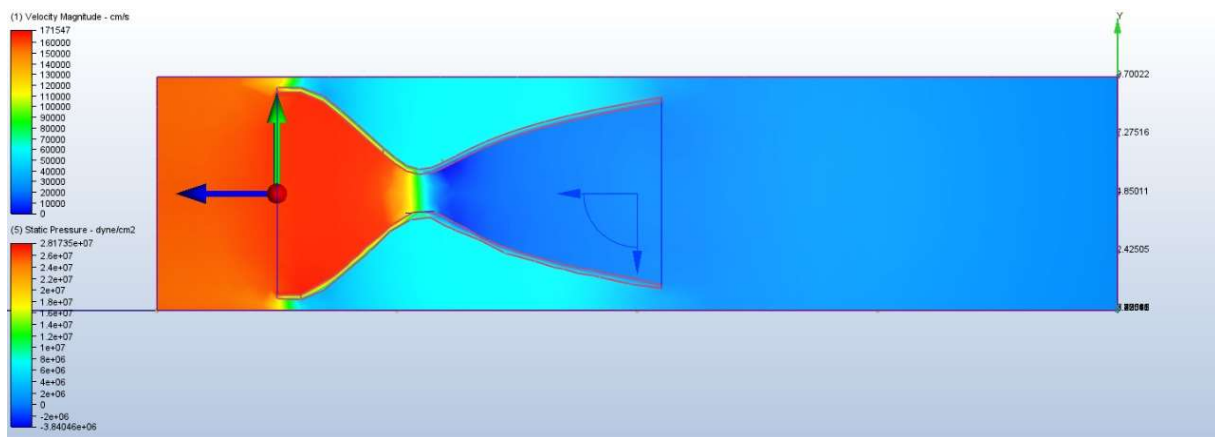


Fig No 5.0HEXAGONAL CONVERGENT DIVERGENT NOZZLE

As per the analysis we can see that the flow is supersonic at outlet because the NPR is more than 2.863 so there will be no shock formation in the divergent part of CD Nozzle and the flow will be supersonic at outlet . Here rightmost side of CD Nozzle where light blue region is shown is inlet, and the leftmost side where light orange region is shown is

outlet.

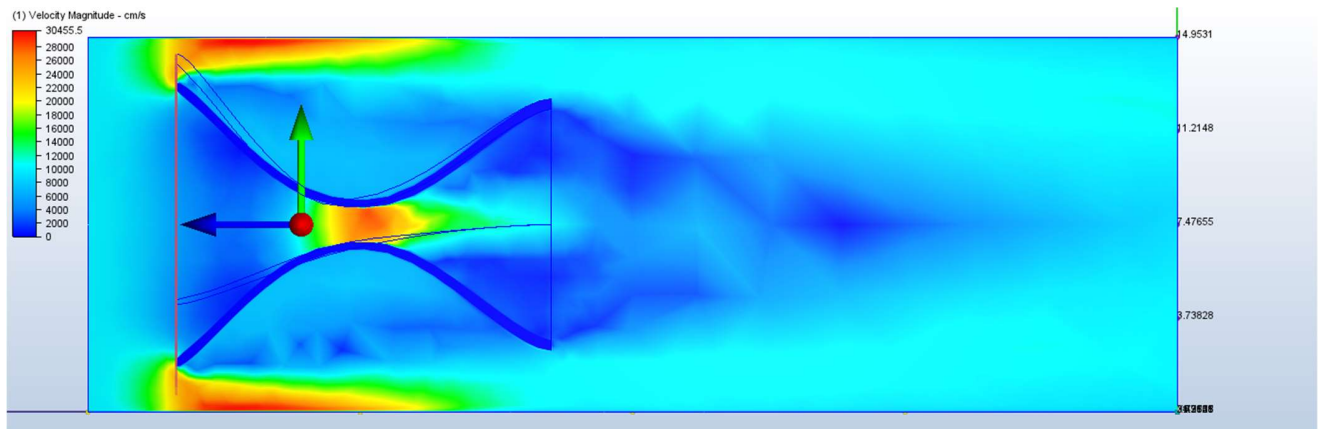


Fig No 5.1 SQUARE CONVERGENT DIVERGENT NOZZLE

As we can see above that the Square CD Nozzle is not as efficient as Hexagonal CD Nozzle .As a matter of fact there is no supersonic flow at outlet.

(5.1.2) EFFECT OF SWIRL ON SHOCK WAVE

Our objective is to study and analyze the swirl effects during supersonic flow field so what we did is we twisted or swirled the divergent part of CD nozzle by a certain angle and twist angle is varying along the downstream direction the variation is small, this twist is given so that we can achieve a swirl .This is done for square and hexagonal nozzle. As a twist in circular (plane) CD nozzle would not make a difference so it observed that by giving a swirl for supersonic flow condition it results in increase of mass flow near the axis which increase the axial velocity and also the thrust at exit, mass flow rate is increased because with addition of swirl there is tangential component of velocity also along with axial component of velocity. The analysis tells us that the supersonic swirling flow has some effect on the shock wave. It is analyzed that the shock wave is stretched towards outlet when the tangential component of velocity is increased means swirl is increased. In the graphs the flow properties are

compared by giving 30 degree and 60degree twist at the divergent part of the CD nozzle , as shown in figure below.

HEXAGONAL CD NOZZLE

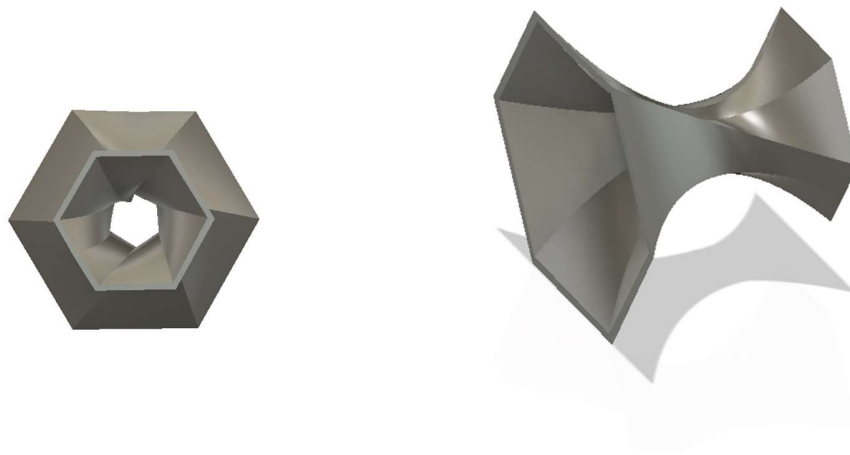


Fig no 5.3 HEXAGONAL CD NOZZLE

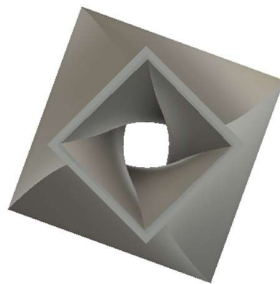
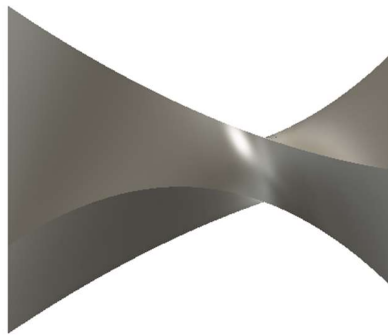


Fig No 5.4 SQUARE CD NOZZLE

(5.2) EXPERIMENTAL SETUP AND PROTOTYPE USED

FABRICATED MODEL OF HEXAGONAL CD NOZZLE fig(5.5,5.6 ,5.7)



Fig No 5.5



Fig No 5.6

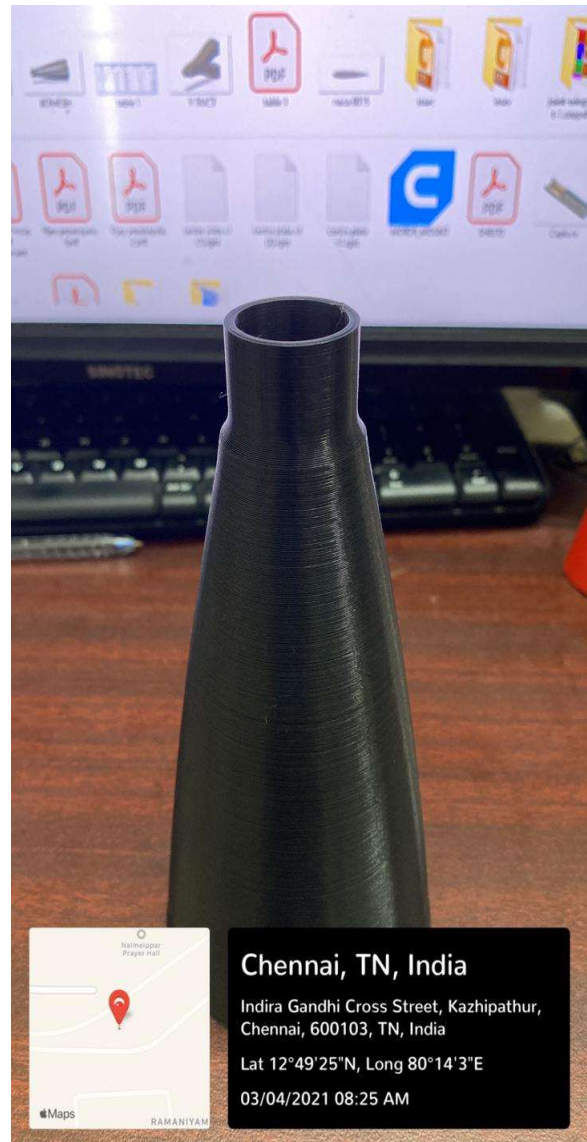


Fig No 5.7



Fig No 5.8

Fabricated model of Plane Circular CD Nozzle

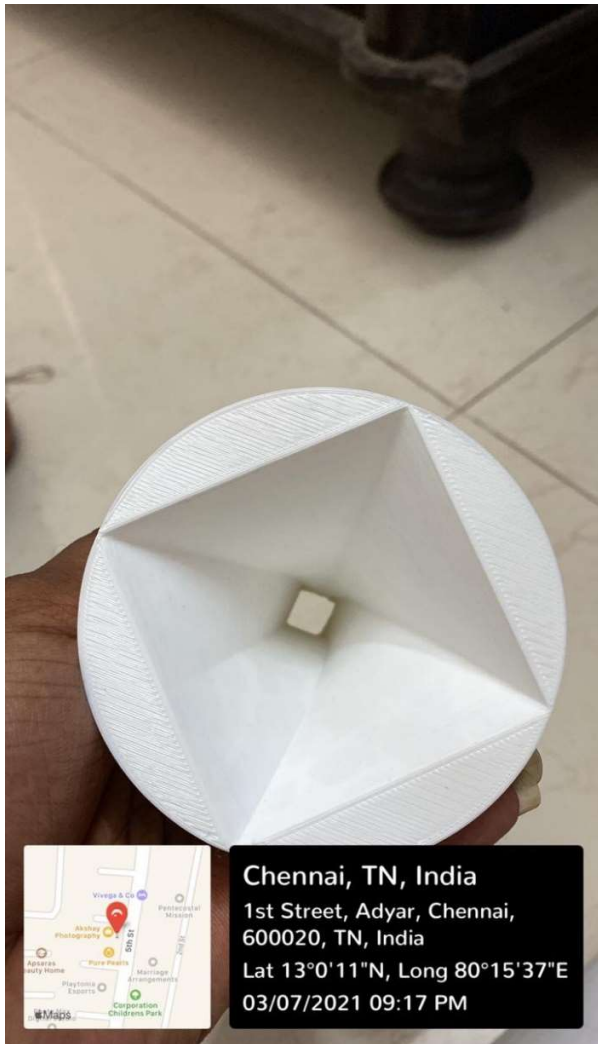


Fig No 5.9

Fabricated model of Square CD Nozzle



Fig No 5.10

Fabricated Model of Square CD Nozzle

(5.3)VARIATION OF PRESSURE RATIO AND MACH NUMBER ALONG THE AXIAL LENGTH OF CD NOZZLE

The Non dimensional parameters of pressure ratio and mach number are compared along the nozzle length. The shock is formed at the divergent portion of the CD Nozzle so when the the swirl is increased the shoch is stretched towards the outlet section

that's why swirl helps in the increase in the axial velocity at the outlet section of the Nozzle

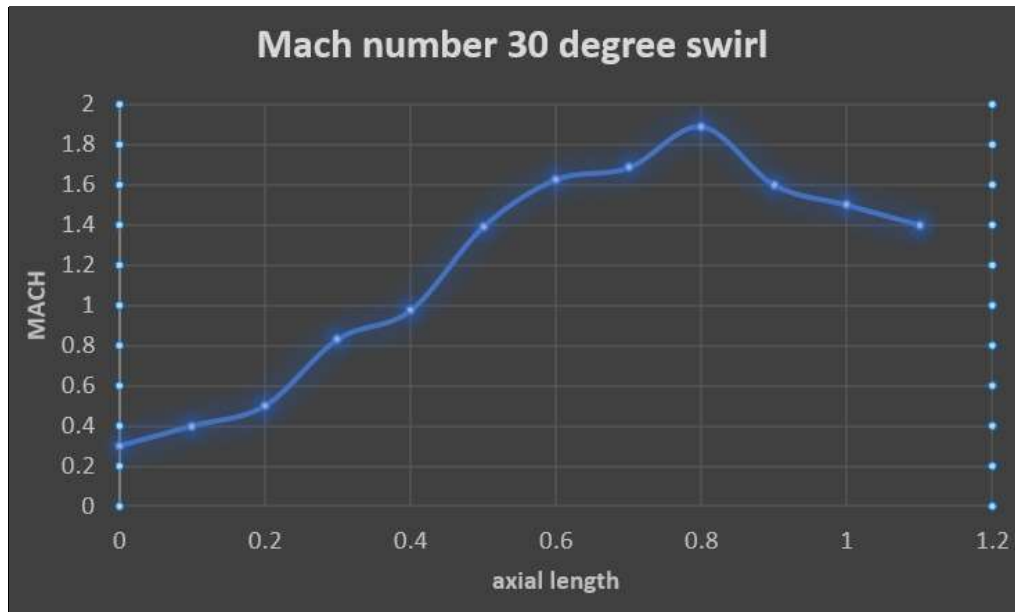


Fig 5.11

30° twist

Mach	X/L(axial distance)
0.3	0.0
0.4	0.1
0.510	0.2
0.622	0.3
0.823	0.4
0.974	0.5
1.391	0.6
1.627	0.7
1.686	0.8
1.891	0.9
0.503	1.0
0.728	1.1
0.749	1.2

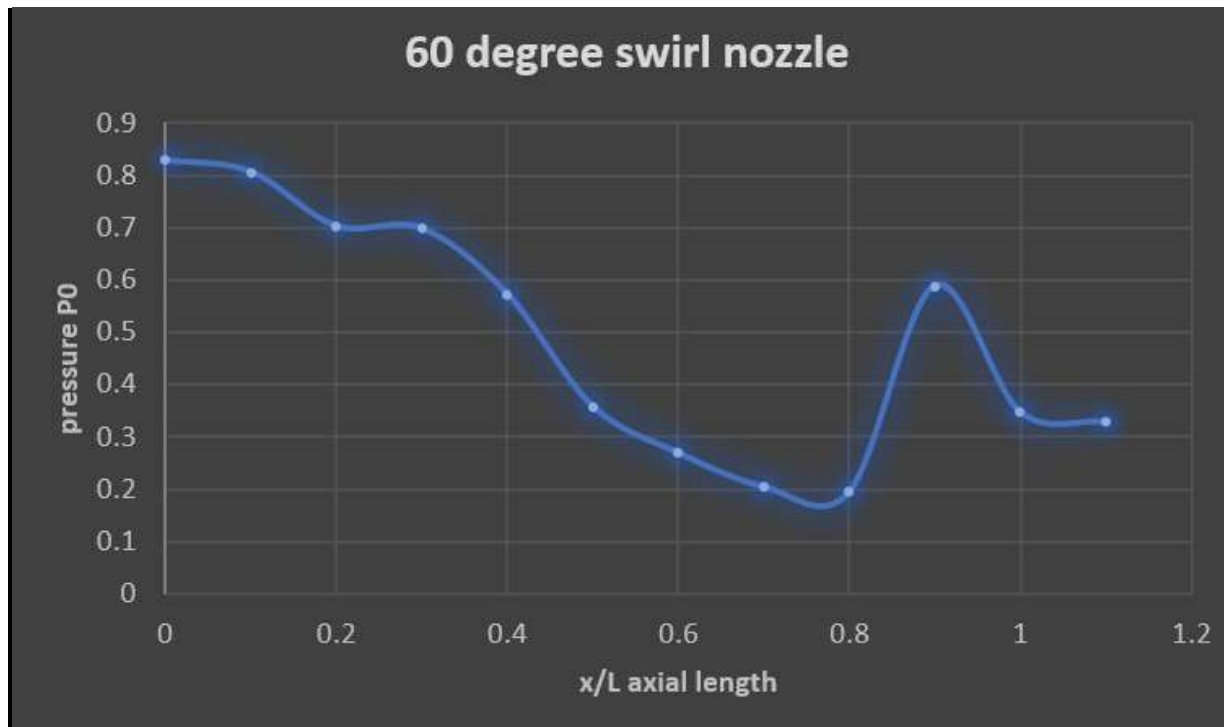


Fig 5.12

60° twist

Pressure(p/p_0)	X/L(axial distance)
0.828	0.0
0.806	0.1
0.702	0.2
0.698	0.3
0.573	0.4
0.358	0.5
0.269	0.6
0.209	0.7
0.194	0.8
0.588	0.9
0.352	1.0
0.328	1.1

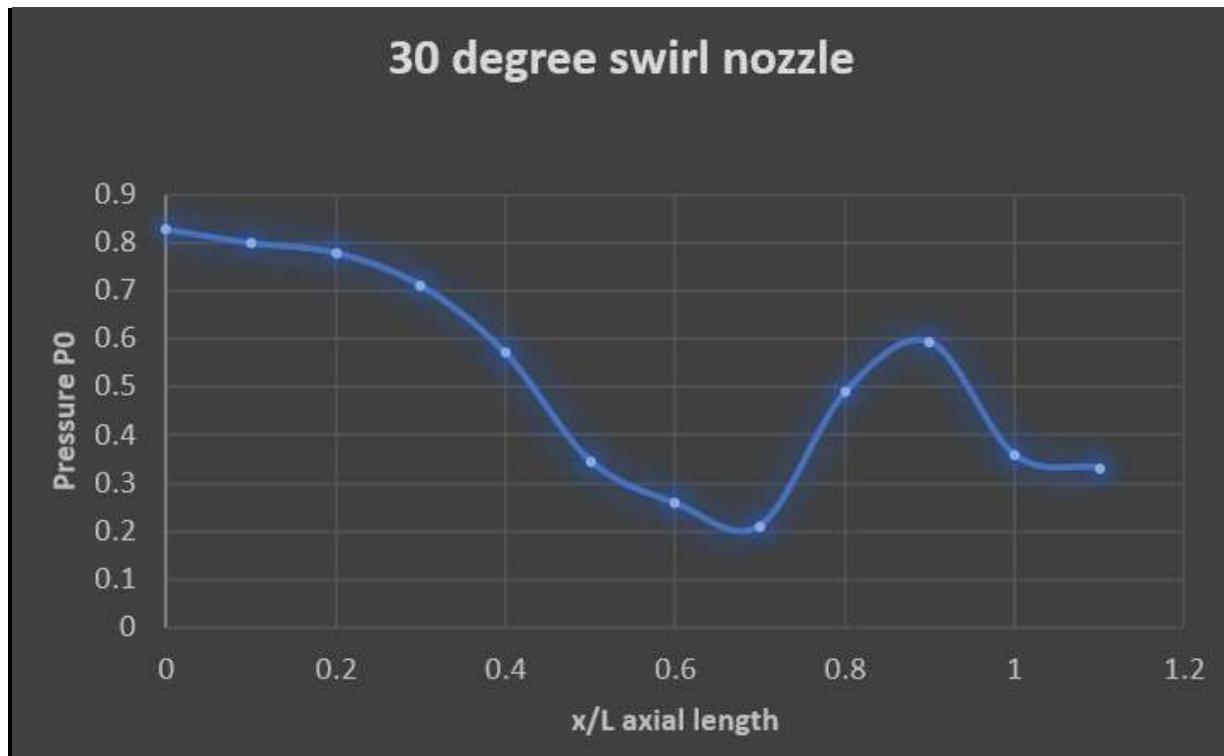


Fig 5.13

30° twist

Pressure(p/p_0)	X/L(axial distance)
0.829	0.0
808	0.1
0.78	0.2
0.712	0.3
0.571	0.4
0.346	0.5
0.258	0.6
0.211	0.7
0.148	0.8
0.594	0.9
0.358	1.0
0.332	1.1

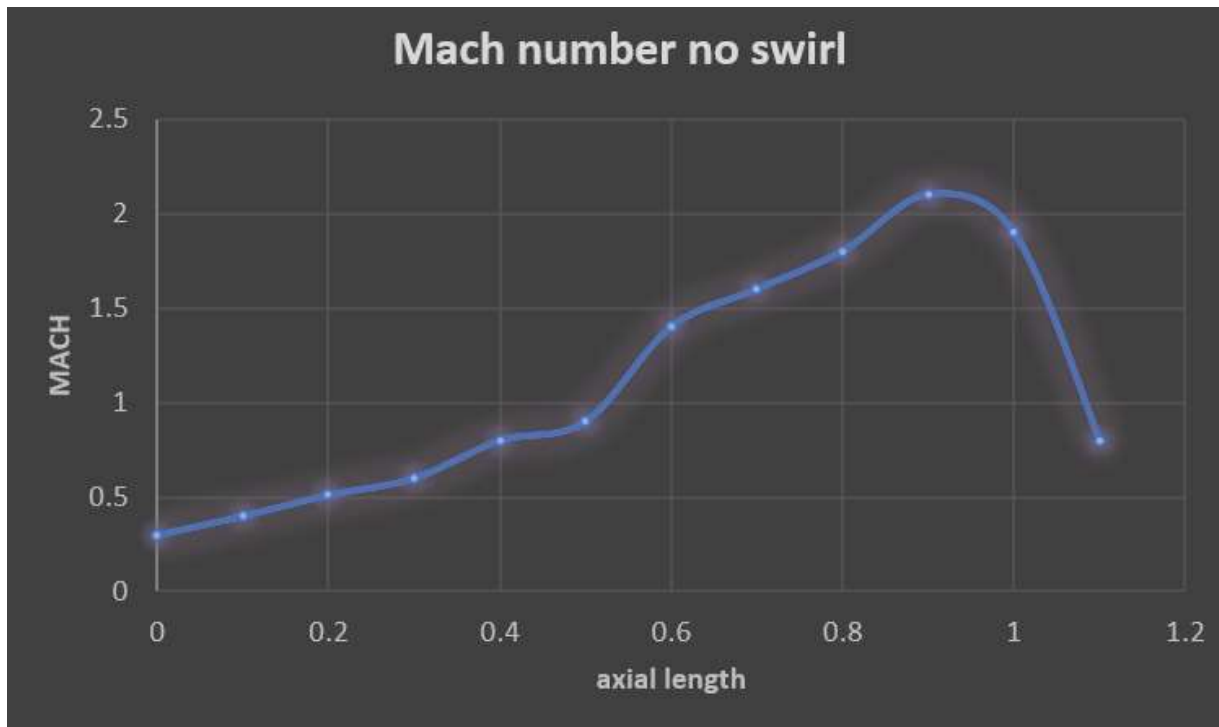


Fig 5.14

No Swirl

Mach	X/L(axial distance)
0.3	0.0
0.4	0.1
0.512	0.2
0.623	0.3
0.823	0.4
0.982	0.5
1.481	0.6
1.628	0.7
1.892	0.8
2.112	0.9
0.509	1.0
0.512	1.1
0.632	1.2

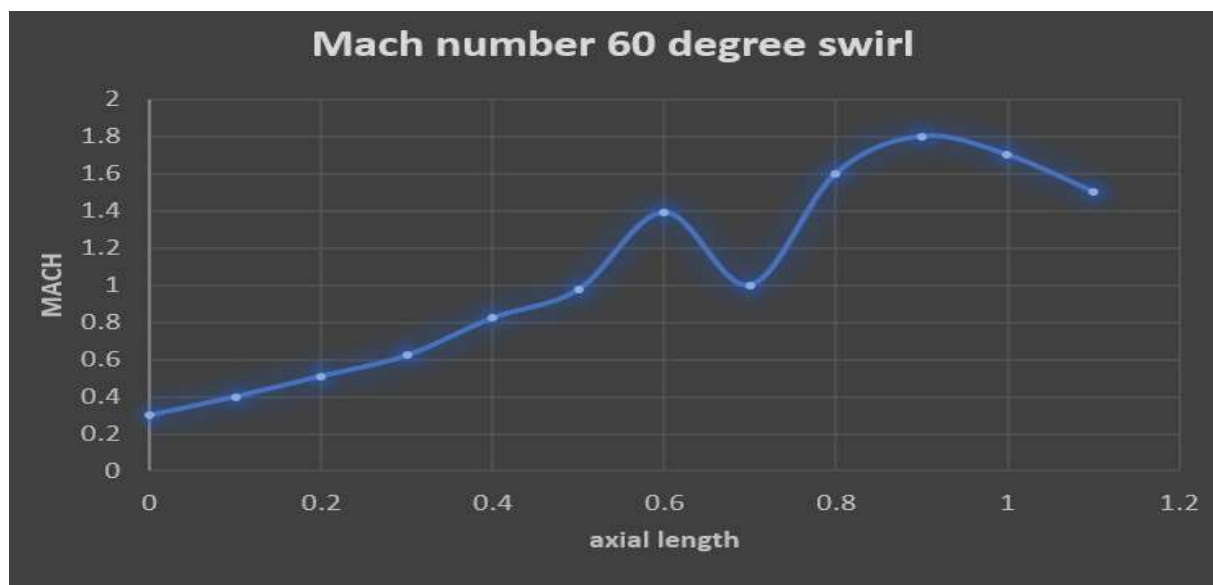


Fig 5.15

60° twist

Mach	X/L(axial distance)
0.3	0.0
0.4	0.1
0.510	0.2
0.622	0.3
0.823	0.4
0.974	0.5
1.391	0.6
1.027	0.7
1.662	0.8
1.811	0.9
0.500	1.0
0.592	1.1
0.621	1.2

CHAPTER 6

SUMMARY AND CONCLUSION

(6.1)SUMMARY

In this work the swirl effect is analysed in a 3D CD nozzle (supersonic) under various back pressure conditions. we found that the effects of swirl and increase in back pressure helps in weakening the strength of shock .Reason being when we increase the back pressure it makes the shock to move upstream; towards throat in the divergent portion and whereas the swirl stretches the shock axially downstream to the throat and weakens the shock (Shock strength).

(6.2) CONCLUSION

Increasing the swirl component from 0 to 15% decreases the shock strength.

There is a stretching of shock wave towards downstream at exit , when we increase the swirl component.

Due to swirl component there is increase of mass flow rate along the axial plane at exit.

Shock wave is formed when the NPR is between 1.974 and 2.863.

Beyond 2.863 there is no shock wave formed in the divergent portion of the CD Nozzle means the flow is expanded normally through the CD Nozzle.

The analysis result of Hexagonal CD Nozzle showed better and close to our desired results as compared to the results obtained from the Square CD Nozzle.

So we conclude that the Hexagonal CD Nozzle shows better flow characteristics as compared to Square CD Nozzle when Swirl is introduced.

REFERENCE

- [1] Mager, A., "Approximate Solution of Isentropic Swirling Flow Through a Nozzle," ARS Journal, Vol. 31, No. 8, 1961, pp. 1140–1148.
- [2] King, W. S., "On Swirling Nozzle Flows," Journal of Spacecraft and Rockets, Vol. 4, No. 10, 1967, pp. 1404, 1405.
- [3] Lewellen, W. S., Burns, W. J., and Strickland, H. J., "Transonic Swirling Flows," AIAA Journal, Vol. 7, No. 7, 1969, pp. 1290–1297.
- [4] Gillespie, T. D., and Shearer, J. L., "The Control of Thrust and Flow Rate in Choked Nozzles by Vortex Generation," Fluidics Quarterly, 1972, pp. 50– 55.
- [5] Cutler, A. D., and Barnwell, R. W., "Vortex Flow in a Convergent– divergent Nozzle," AIAA Journal, Vol. 37, No. 10, 1999, pp. 1329–1331.
- [6] Adamson, T. C., Jr., and Nicholls, J. A., "On the Structure of Jets from Highly Underexpanded Nozzles into Still Air," Journal of the Aero/ Space Sciences , Vol. 26, No. 1, Jan. 1959, pp. 16– 24.
- [7] Cutler, A. D., Levey, B. S., and Kraus, D. K., "An Experimental Investigation of Supersonic Swirling Jets," 24th AIAA Fluid Dynamics Conference, Orlando, FL, AIAA Paper 93-2922, July 1993.
- [8] Gostintsev, Y. A., Zelentsov, V. V., Ilyukhin, V. S., and Pokhil, P. F., "Structure of Underexpanded Supersonic Swirling Gas Jet," Fluid Dynamics , Vol. 4, No. 5, Sept. 1969, pp. 158 – 162.
- [9] Batson, J. L., and Sforzini, R. H., "Swirling Flow Through a Nozzle," Journal of Spacecraft and Rockets, Vol. 7, No. 2, 1970, pp. 159 – 163.
- [10] Dutton, J . C., "Swirling Supersonic nozzle f low," Journal of propulsion and power, vol.3, July 1987, pp. 342-349.
- [11] Guderley K G and Tabak D, "On the determination of optimum supersonic throat nozzles of given length for a flow with swirl" Theoretical report: Aerospace Research Laboratories Report ARL 66-0013, 1967.
- [12] Ekanayake E K S, Gear J A, Ding Y, "Numerical simulation of a supersonic convergent divergent nozzle with divergent angle variations for underexpanded condition" 17th Australasian Fluid Mechanics Conference, December 2010, pp 5 -9.

[13] Ekanayake E K S, Gear J A, Ding Y, “Flow simulation of a 2-dimensional rectangular supersonic convergent divergent nozzle” ANJIAM Journal 51 (EMAC 2009) 2010, pp C377 – C392.

[14] Xiao Q., Tsai H.M., and Papamoschou D, “Numerical Investigation of supersonic nozzle flow separation”. AIAA Journal, Vol 45, No 3, March 2007.