

GRACEFUL LABELINGS OF REGULAR DIGRAPHS

Submitted in partial fulfilment of the requirements for the award of Bachelor of
Science degree in Mathematics (B.sc Mathematics)

by

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**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE & HUMANITIES**

SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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This is to certify that this Project Report is the bonafide work of **R. SANDHYA (38060008)** who carried out the project entitled "**GRACEFUL LABELINGS OF REGULAR DIGRAPHS**" under my supervision from **JANUARY 2021** to **APRIL 2021**.

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I **SANDHYA R(38060008)** hereby declare that the Project Report entitled
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partial fulfilment of the requirements for the award of Bachelor of Science degree
in Mathematics.

DATE: 26.03.2021
PLACE: Chennai



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ABSTRACT

A digraph $G(V, E)$ with p vertices is said to be line graceful if there exists an injection $f : E \rightarrow \{1, 2, \dots, |V|-1\}$ such that the induced mapping $f^* : V \rightarrow \{0, 1, \dots, |V|-1\}$ is given by $f^*(v_i) = (\sum f(e_{ij})) \pmod{|V|}$ is bijective. If f is an edge odd graceful labeling of a digraph $G(V, E)$ with q edges, then the labeling ϕ defined by $\phi(e) = 2q - f(e)$ for all $e \in E(G)$ is again an edge odd graceful labeling of G and is called complementary edge graceful labeling. This project discusses the existence of line graceful labeling and complementary edge graceful for the directed graph with vertices as the field elements called Paley digraphs.

Keywords: Graph Labeling, Paley digraph, edge graceful labelling, line graceful labeling, edge odd graceful labeling, complementary edge graceful labeling.

CONTENTS

ABSTRACT	(i)
LIST OF FIGURES	(iv)
CHAPTERS	
1. INTRODUCTION	2
1.1 Graph Theory	2
1.2 Digraph	2
1.3 Cayley Digraph	2
1.4 Paley Digraph	3
1.5 Graph Labeling	4
1.6 Graceful Labeling	5
2. LITERATURE SURVEY	7
3. AIM AND SCOPE OF THE PRESENTATION	10
3.1 Applications	10
3.1.1 Graph theory	10
3.1.2 Graph Labeling	11
3.1.3 Graceful Labeling	11
3.1.4 Paley graph	11

4. PRELIMINARIES	14
4.1 A Graph	14
4.2 Walk, Paths and Cycle	14
4.3 Digraph	15
4.4 Cayley Digraph	15
4.5 Paley Digraph	15
4.6 Graph Labeling	16
4.7 Graceful Labeling	16
4.8 Edge Graceful Labeling	17
4.9 Line Graceful Labeling	17
4.10 Edge Odd Graceful Labeling	17
4.11 Complementary Edge Graceful Labeling	17
5. MAIN RESULTS	19
Theorems	19
5.1 Line graceful labeling of Paley diagram	19
5.2 Corollary	20
5.3 Complementary Edge Graceful labeling of Paley diagram	20
5.4 Corollary	21
6. SUMMARY AND CONCLUSION	23
REFERENCES	23

LIST OF FIGURES

1.1 Konigsberg bridge Problem	2
1.2 Cayley digraph	3
1.3 Paley digraph	4
1.4 Graph Labeling	5
1.5 Graceful Labeling	5
3.1 Computer Network	10
3.2 Graceful Labeling in Crystallography	11
3.3 Paley graph of locally linear graph	12
4.1 A Graph in graph theory	14
4.2 Walk, Paths and Cycle in graph	14
4.3 Digraph	15
4.4 Cayley digraph	15
4.5 Paley digraph	16
4.6 Graph Labeling	16
4.7 Graceful Labeling	17
5.1 Line graceful labeling of Paley digraph $P(7)$	20
5.2 Complementary Edge Graceful Labeling of Paley digraph $P(7)$	21

CHAPTER – 1

1. INTRODUCTION

1.1. GRAPH THEORY

Graph theory, branch of mathematics concerned with networks of points connected by lines. The subject of graph theory had its beginnings in recreational math problems, but it has grown into a significant area of mathematical research, with applications in chemistry, operations research, social sciences, and computer science.

The history of graph theory may be specifically traced to 1735, when the Swiss mathematician Leonhard Euler solved the Königsberg bridge problem. The Königsberg bridge problem was an old puzzle concerning the possibility of finding a path over every one of seven bridges that span a forked river flowing past an island but without crossing any bridge twice. Euler argued that no such path exists. His proof involved only references to the physical arrangement of the bridges, but essentially, he proved the first theorem in graph theory.

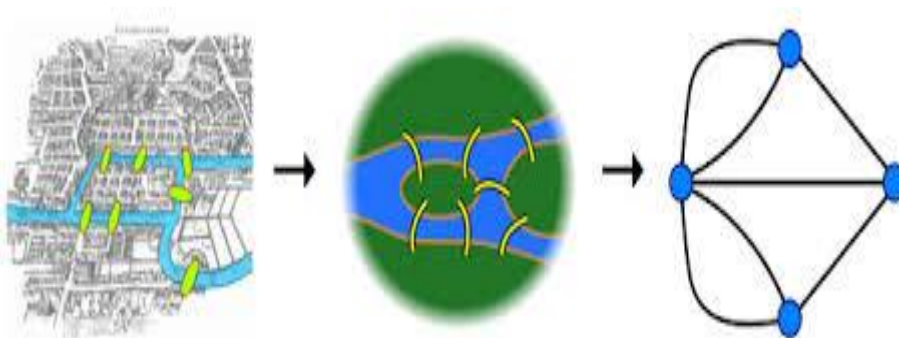


Fig.1.1 Königsberg bridge problem

1.2. DIGRAPH

A directed graph in which the path begins and ends on the same vertex (a closed loop) such that each vertex is visited exactly once is known as a Hamiltonian circuit. The 19th-century Irish mathematician William Rowan Hamilton began the systematic mathematical study of such graphs.

1.3. CAYLEY DIGRAPH

The concept of Cayley graph plays a vital role in dealing with certain optimization problems, especially routing problems in interconnection networks of

a parallel computer. Parallel processing and supercomputing continue to exert great influence in the development of modern science and engineering. The network comprising of processors plays a vital role in facilitating the communication between processors in a computer system. Some of the popular interconnection schemes are rings, torus and hypercubes. Their popularity stems from the commercial availability of machines with these architectures. These three families of graphs – rings, torus and hypercube – share a common property of being a Cayley graph. Many important problems in networks have been modelled by Cayley graphs. Since Cayley graphs have the property of vertex transitivity, it is possible to implement routing and communication schemes at each node of the network. One of the principal issues concerning routing problems is identification of perfect dominating sets in Cayley graphs. This will help in identifying optimal substructures in order to facilitate the communication between processors.

Cayley graph is a discrete structure created out of groups, more specifically from a finite group Γ and its generating set Ω . A non-empty subset $\Omega \subset \Gamma$ is called a generating set for Γ , denoted by $\Gamma = \langle \Omega \rangle$, if every element of Γ can be expressed as a product of elements in Ω . For a generating set Ω of Γ , we assume that C_1 : The identity element $e \notin \Omega$ and C_2 : If $a \in \Omega$, then $a^{-1} \in \Omega$.

Circulant graphs are Cayley graphs on the simplest of groups and thus may provide direction into Cayley graphs on other groups, particularly finite groups. Any finite vertex-transitive graph of prime order is a circulant graph and hence the study of vertex transitive graphs can gain from the study of circulant graphs.

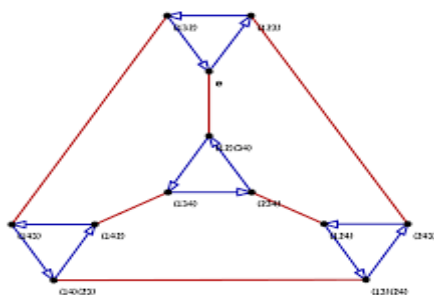


Fig. 1.2 Cayley digraph

1.4. PALEY DIGRAPH

In mathematics, Paley graphs are dense undirected graphs constructed from the members of a suitable finite field by connecting pairs of elements that

differ by a quadratic residue. The Paley graphs form an infinite family of conference graphs, which yield an infinite family of symmetric conference matrices. Paley graphs allow graph-theoretic tools to be applied to the number theory of quadratic residues, and have interesting properties that make them useful in graph theory more generally.

Paley graphs are named after Raymond Paley. They are closely related to the Paley construction for constructing Hadamard matrices from quadratic residues (Paley 1933). They were introduced as graphs independently by Sachs (1962) and Erdős & Rényi (1963). Sachs was interested in them for their self-complementarity properties, while Erdős and Rényi studied their symmetries.

Paley digraphs are directed analogs of Paley graphs that yield antisymmetric conference matrices. They were introduced by Graham & Spencer (1971) (independently of Sachs, Erdős, and Rényi) as a way of constructing tournaments with a property previously known to be held only by random tournaments: in a Paley digraph, every small subset of vertices is dominated by some other vertex.

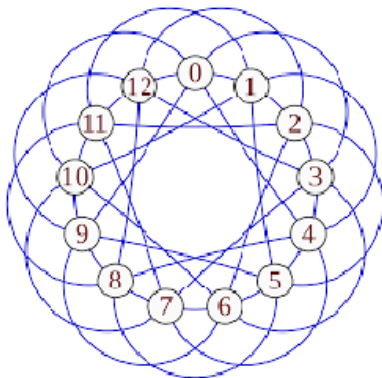


Fig. 1.3 Paley digraph

1.5. GRAPH LABELING

A graph labeling is an assignment of integers to edges or vertices or both. Graph labellings trace their origins to labellings presented by Alexander Rosa in his 1967 paper. Rosa identified three types of labellings, which he called α , β -, and ρ -labellings. β -labellings were later renamed as "graceful" by Solomon Golomb, and the name has been popular since. Some labelings are

- Graceful Labeling

- Edge-Graceful Labeling
- Harmonious Labeling
- Magic Labeling
- Cordial Labeling etc.

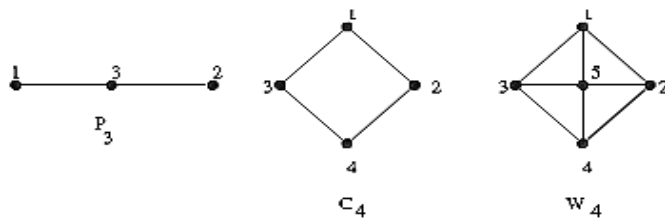


Fig. 1.4 Graph Labeling

1.6. GRACEFUL LABELING

The name "graceful labeling" is due to Solomon W. Golomb; this type of labeling was originally given the name β -labeling by Alexander Rosa in a 1967 paper on graph labelings.

A major conjecture in graph theory is the graceful tree conjecture or Ringel–Kotzig conjecture, named after Gerhard Ringel and Anton Kotzig, and sometimes abbreviated GTC. It hypothesizes that all trees are graceful. It is still an open conjecture, although a related but slightly weaker conjecture known as Ringel's conjecture was proven true in 2020. The Ringel–Kotzig conjecture is also known as the "graceful labeling conjecture". Kotzig once called the effort to prove the conjecture a "disease". Another weaker version of graceful labelling is the near graceful labeling, in which the vertices can be labeled using some subset of the integers between 0 and $m+1$ inclusive, such that no two vertices share a label, and each edge is uniquely identified by the absolute difference between its endpoints, such that this magnitude lies between 1 and $m+1$ inclusive. Another conjecture in graph theory is the Rosa's Conjecture, named after Alexander Rosa, which says that all triangular cacti are graceful or nearly-graceful.

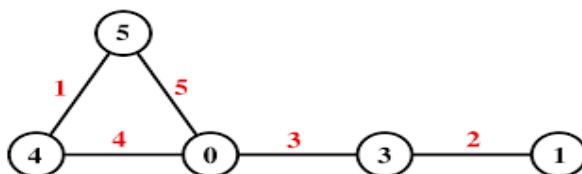


Fig. 1.5 Graceful labeling

CHAPTER– 2

2. LITERATURE SURVEY

An dynamic survey of graph labeling has presented by Gallian . J. A and approach the electronic journal of combinations[1]. One of the main topics in graph theory is labeling the graph which was introduced by Rosa in 1966 [2]. A graph labeling is an assignment of integers to edges or vertices or both. The labeling technology has many applications in real life such as Radar, Astronomy, X-Ray, and communication system. Sin Min Lee has introduced super vertex graceful labelling. He has analysed the super vertex gracefulfulness of complete bipartite graphs. Also he has discussed the amalgamation of graphs under graceful mapping and Fibonacci graceful labelling. Also, he has analysed the role of order and size of cycles, crowns, wheels under super vertex graceful map. In further, A. Rosa has described on certain valuations of the vertices of the graph and they also involves an international symposium on theory of graphs [2].

G.S. Bloom and D.F.Hsu have presented an approach for graceful digraphs and obtained a problem of network addressing[3]. They also identifies on graceful directed graphs that are computational methods of some algebraic systems and explained graph theory with applications to algorithms and computers. Further, they described the graceful directed graphs of discrete mathematics . In [6], S.Lo, identifies an edge graceful labeling of graphs . S.M. Lee et al. presented the k-edge graceful trees, preprint . And also, they designed the super edge graceful Eulerian graphs[8]. B. Gayathri et al. introduced the even edge graceful labeling of some cycle related graphs[9].

Akers S.B and Krishnamurthy.B have presents a group theoretic model for symmetric interconnection networks IEEE trans [10]. Annexstein, F., Baumslag, M. and Rosenberg, A.L. described a group action graphs and Parallel architectures [11]. Joseph A. Murugesan. N and Uma. R [13-18] have obtained a Conjecture on Amalgamation of graceful graphs with star graphs . Further, they proposed a Super vertex gracefulfulness of complete bipartite graph. They also interfaced the graceful labeling of some graphs and their subgraphs and identifies the Fibonacci graceful labeling of graphs. They also presented a study on vertex graceful graphs and the Super Vertex gracefulfulness of some special graphs. J.A. Bondy and U.S.R. Murty explained the graph theory with applications [19]. M.A. Perumal et al. described Super graceful labeling for some special graphs . Harary

defines graph theory in Narosa Publishing house. Babai and Laszlo considered the automorphism groups, isomorphism and reconstruction [20-22].

CHAPTER– 3

3. AIM AND SCOPE OF THE PRESENTATION

The concept of graceful labeling is one of the best known labeling methods of graphs, mainly because of their applications to physical problems. It is to determine the existence of line graceful and complementary graceful labeling for vertex symmetric directed graphs namely Paley digraphs.

3.1 APPLICATIONS

3.1.1 Graph theory

COMPUTER NETWORK

- In computer network, the **relationships among interconnected computers** within the network, follow the principles of graph theory.
- Graph theory is also used in **network security**.
- We can use the vertex coloring algorithm to find a proper **coloring of the map** with four colors.
- Vertex coloring algorithm may be used for assigning at most four different frequencies for any **GSM (Grouped Special Mobile) mobile phone networks**.

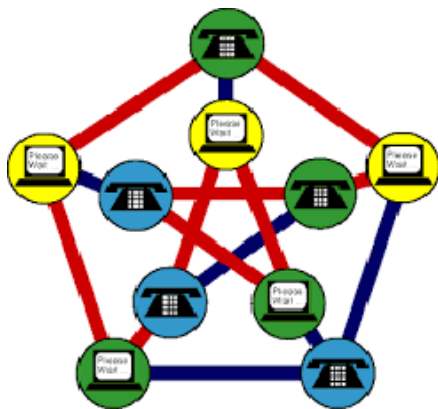


Fig. 3.1 Computer Network

3.1.2 Graph Labeling

- Graph Labeling used in many applications like coding theory, x-ray crystallography, radar, astronomy, circuit design, communication network addressing, data base management.
- Labeling of graphs in heterogeneous fields to some extent but mainly focuses on the communication networks.
- X-ray diffraction is one of the most powerful techniques for characterizing the structural properties of crystalline solids, in which a beam of X-rays strikes a crystal and diffracts into many specific directions.
- A communication network is composed of nodes, each of which has computing power and can transmit and receive messages over communication links, wireless or cabled. The basic network topologies are include fully connected, mesh, star, ring, tree, bus.

3.1.3. Graceful Labeling

- It can be effectively used in ATM network infrastructure, as the labeled flows can be mapped to ATM virtual circuit identifiers and vice-versa.
- Mathematical programming technique is presented to model the graceful labeling problem.
- A “branching method” is developed to solve the problem for special classes of graphs.

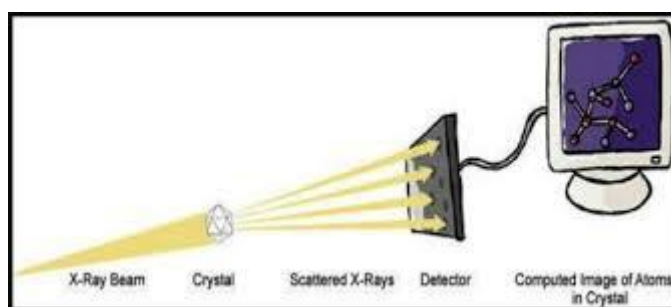


Fig. 3.2 Graceful Labeling in Crystallography

3.1.4. Paley graph

- The Paley graph of order 9 is a locally linear graph, a rook's graph, and the graph of the 3-3 duoprism.

- The Paley graph of order 13 has book thickness 4 and queue number 3 (Wolz 2018).
- The Paley graph of order 17 is the unique largest graph G such that neither G nor its complement contains a complete 4-vertex subgraph (Evans et al. 1981). It follows that the Ramsey number $R(4, 4) = 18$.
- The Paley graph of order 101 is currently the largest known graph G such that neither G nor its complement contains a complete 6-vertex subgraph.

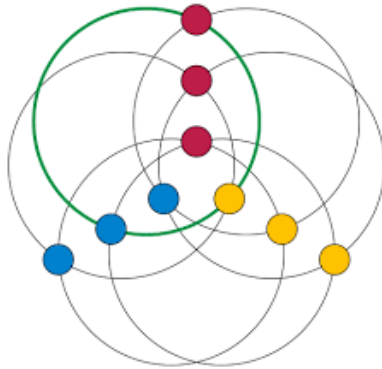


Fig. 3.3 Paley graph of locally linear graph

CHAPTER– 4

4. PRELIMINARIES

In this division we have some basic definitions

4.1. A Graph

Graph is a mathematical representation of a network and it describes the relationship between lines and points. A graph consists of some points and lines between them. The length of the lines and position of the points do not matter. Each object in a graph is called a node.

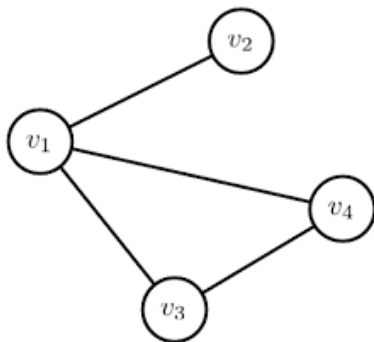


Fig. 4.1 A Graph in graph theory

4.2.Walk, Paths and Cycle

- A walk in a graph is a sequence of vertices each linked by some edge belonging to the graph.
- A Path is a walk that uses no edge more than once.
- A Circuit is a path that begins and ends at the same vertex.

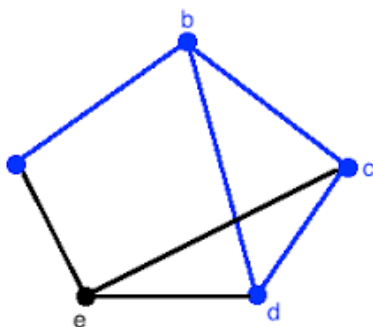


Fig. 4.2 Walk, Paths and Cycle in graph

4.3. Digraph

A digraph D is a Pair (V, A) , where V is a nonempty set whose elements are called the vertices and A is the subset of the set of ordered pairs of distinct elements of V . the elements of A are called the arcs of D .

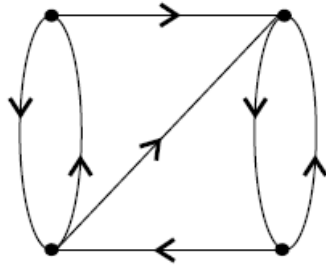


Fig. 4.3 Digraph

4.4. Cayley digraph

Cayley digraphs are defined with a group G and a subset S of G : the vertices of the Cayley digraph $\text{Cay}(G, S)$ are the elements of the group, and its arcs are all the couples (a, as) with $a \in G$ and $s \in S$.

The digraph $\text{Cay}(G, S)$ is strongly connected if and only if S generates G .

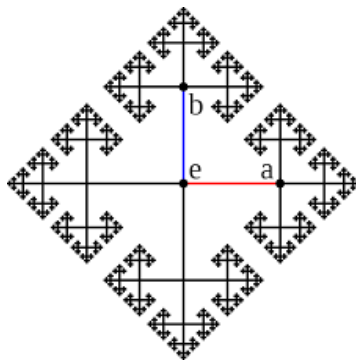


Fig.4.4 Cayley digraph

4.5. Paley digraph

Paley digraph $P = (V, E)$ is a directed graph with vertices as finite field elements F_q with $q \equiv 3 \pmod{4}$ and edges as $E(P) = \{(u, v) : u, v \in F_q, u - v \in (F_q^*)^2\}$.

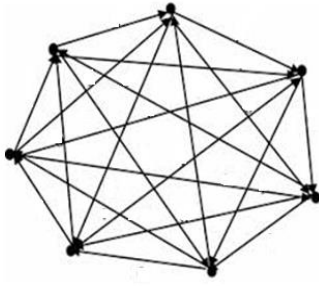


Fig.4.5 Paley digraph

4.6. Graph Labeling

In the mathematical discipline of graph theory, a graph labelling is the assignment of labels, traditionally represented by integers, to edges and/or vertices of a graph.

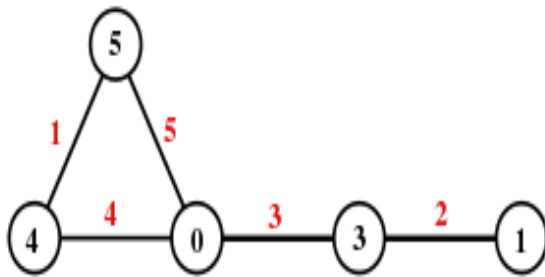


Fig.4.6 Graph Labeling

4.7. Graceful Labeling

Let $G = (V, E)$ be a simple graph with p vertices and q edges. Let $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$ is a one to one mapping. f is called graceful mapping if the induced mapping $f^+: E(G) \rightarrow \{1, 2, \dots, q\}$ defined by $f^+(uv) = |f(u) - f(v)|$ is also a one to one, onto mapping.

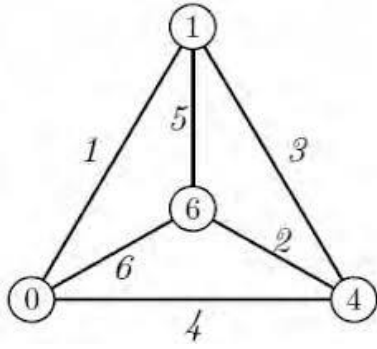


Fig.4.7 Graceful Labeling

4.8. Edge Graceful Labeling

A digraph is edge graceful if there occurs a bijection $f : E \rightarrow \{1, 2, \dots, |E|\}$ in a manner that the induced mapping $f^* : V \rightarrow \{0, 1, \dots, |V|-1\}$ is given by $f^*(v_i) = (\sum f(e_{ij})) \pmod{|V|}$ where the summation is taken over all the outgoing arcs of v_i , e_{ij} is the j^{th} outgoing edge of the node v_i is a bijection.

4.9. Line Graceful Labeling

A digraph $G(V, E)$ with p vertices is said to be line graceful if there exists an injection $f : E \rightarrow \{1, 2, \dots, |V|-1\}$ such that the induced mapping $f^* : V \rightarrow \{0, 1, \dots, |V|-1\}$ is given by $f^*(v_i) = (\sum f(e_{ij})) \pmod{|V|}$ is bijective.

4.10. Edge Odd Graceful Labeling

A digraph $G(p, q)$ is edge odd graceful if there is a bijection $f : E(G) \rightarrow \{1, 3, 5, \dots, 2q-1\}$ such that when each vertex is assigned the sum of the labels of its outgoing arcs under mod $2q$, the resulting vertex labels are distinct.

4.11. Complementary Edge Graceful Labeling

If f is an edge odd graceful labeling of a digraph $G(V, E)$ with q edges, then the labeling ϕ defined by $\phi(e) = 2q - f(e)$ for all $e \in E(G)$ is again an edge odd graceful labeling of G and is called complementary edge graceful labeling.

CHAPTER – 5

5. MAIN RESULTS

In this chapter Line graceful labelling and Complementary edge graceful labelling of a special class of vertex symmetric digraph namely Paley digraph is discussed

Theorem 5.1

Let G be a quadratic residue digraph with finite field elements as vertices $v_1, v_2, \dots, v_q$, q is prime and $q \equiv 3 \pmod{4}$. If each vertex of G has p incoming and p outgoing arcs with $|E| = pq$, where $p = \frac{q-1}{2}$ then G is line graceful graph provided , $\text{g.c.d of } (p,q) = 1$.

Proof

Consider any prime number q which is congruent to 3 (mod 4). Let us construct a Paley directed graph by using the definition. We denote the vertex set $V(P)$ as $\{v_1, v_2, \dots, v_q\}$ and the Edge set as $E = \{E_1 \cup E_2 \cup \dots \cup E_p\}$, $p = \frac{q-1}{2}$ where $E_1 = \{e_{1j} \text{ for } 1 \leq j \leq q\}$, $E_2 = \{e_{2j} \text{ for } 1 \leq j \leq q\}$, ..., $E_p = \{e_{pj} \text{ for } 1 \leq j \leq q\}$. To verify every Paley digraph admits line Graceful labeling, let us define $f : E(G) \rightarrow \{0, 1, 2, \dots, q-1\}$ as $f(e_{ij}) = i+j-2 \pmod{q}$ $1 \leq j \leq q, 1 \leq i \leq p$, and define the induced map $f^*: V \rightarrow \{0, 1, 2, \dots, |q| - 1\}$ as $f^*(v_j) = \sum_{i=1}^p f(e_{ij}) \pmod{q}$ where e_{ij} is an outgoing arc of j^{th} vertex.

Consider a random vertex $v_j \in V$ of the directed quadratic residue graph $P(q)$. In quadratic residue digraph, the incoming and outgoing arcs are the same

$$\begin{aligned} f^*(v_j) &= \sum_{i=1}^p f(e_{ij}) \pmod{q} \\ &= pi - 2p + 1 + 2 + \dots + p \pmod{p} \\ &= p/2(2i - 3 + p) \pmod{p} \end{aligned}$$

Hence all v_j are distinct for $1 \leq j \leq q$ and $\text{g.c.d}(m,p) = 1$

This shows every directed Paley graph is line Graceful.

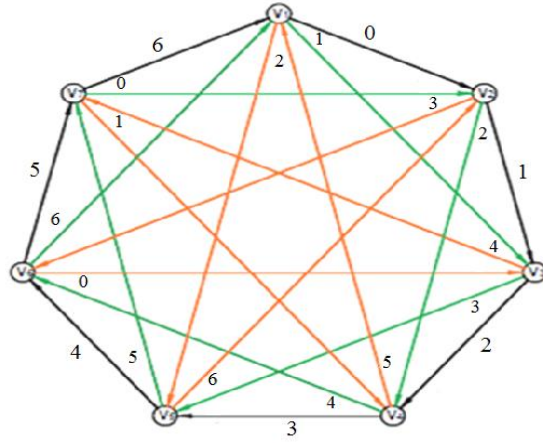


Fig. 5.1 Line graceful labeling of Paley digraph $P(7)$

Corollary 5.2.

In Paley digraph since p is prime $\text{g.c.d}(p,q)=1$ always. Therefore it admits line graceful labeling whereas the line digraph of paley digraph never admits line graceful labeling.

Theorem 5.3.

Let G be a quadratic residue digraph with finite field elements as vertices $v_1, v_2, \dots, v_q$, q is prime and $q \equiv 3 \pmod{4}$. If each vertex of G has p incoming and p outgoing arcs with $|E| = pq$, where $p = \frac{q-1}{2}$ then G admits complementary edge graceful labeling.

Proof

Define $f : E(G) \rightarrow \{1, 3, 5, \dots, 2q-1\}$ as

$$f(e_{ij}) = 2(j-1)q + 2i - 1, \quad 1 \leq i \leq q, \quad 1 \leq j \leq p$$

Now define $\phi(e_{ij}) = 2p - f(e_{ij})$

$$= 2p - [2(j-1)q + 2i - 1]$$

$$f^*(v_j) = \sum_{i=1}^p \phi(e_{ij})$$

$$= 2pq - 2ip + 1(p) - 2q - 4q - \dots - 2(p-1)q$$

$$= pq + p + q - 2pi \pmod{2q}$$

This gives distinct values for every i under mod $2q$. Therefore the paley digraph admits complementary edge graceful lableing.

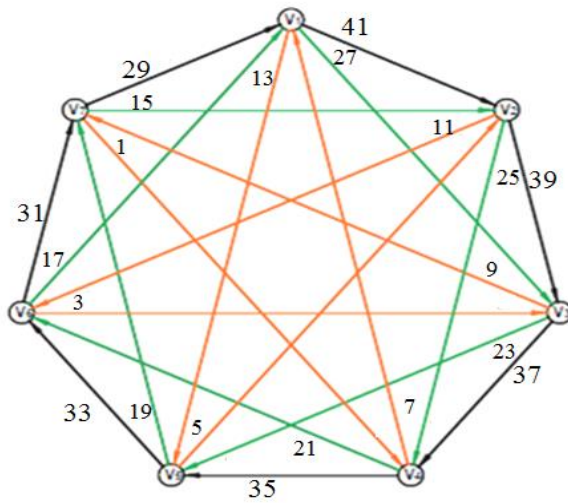


Fig. 5.2 Complementary Edge Graceful Labeling

Of Paley diagram $P(7)$

Corollary 5.4.

In Paley digraph , edge odd graceful labeling exist iff it is Complementary edge graceful.

CHAPTER – 6

6. SUMMARY AND CONCLUSION

This project discussed about various types of graceful labelings and shown the existence of line graceful and complementary edge graceful labeling for vertex symmetric directed graphs namely paley digraphs and derived the relationship between them. In Future study, Various other labelings can be discussed for the same family of graphs.

PAPER PRESENTATION

R.Rajeswari, R.Sandhya and R. Parameswari, “Graceful Labelings of Paley Digraphs”, in 2nd International Conference on mathematical techniques and applications ,the Department of Mathematics, SRM Institute of Science and technology, Tamilnadu

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