

SOME ASPECTS OF NEAR-RING GROUPS

Submitted in partial fulfillment of the requirements for the award of
Bachelor of Science Degree in Mathematics

By

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**DEPARTMENT OF MATHEMATICS
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SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

Accredited with Grade "A" by NAAC

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **Eluri Sri Ram (38060003)** Who carried out the project entitled "SOME ASPECTS OF NEAR-RING GROUPS" under my supervision from **January 2021** to **April 2021**.

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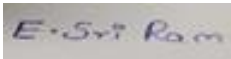
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DECLARATION

I **Eluri Sri Ram(38060003)** hereby declare that the Project Report entitled "**SOME ASPECTS OF NEAR-RING GROUPS**" done by me under the guidance of **Dr.R.Delhi Babu**(Internal) and **_**(External) at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Bachelor of Science Degree in Mathematics

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ABSTRACT

Basic definitions and results of algebraic structures in particular rings, Modules, near-rings, near-ring groups, lattices are covered. Further it deals Fundamental definitions and results of near-rings, near-ring groups and them Substructures, Posets, connection with rings and modules.

KEYWORDS

Near-ring, near-ring groups

POSETs and Lattices

Rings and Modules

CONTENT

CHAPTER-1 PAGENO

1.INTRODUCTON

1.1.Near-ring.	5
1.2. Near-ring Groups.	5
1.3. Goldie Ring.	6

CHAPTER-2

2.LITERATURE SURVEY.	9
----------------------	---

CHAPTER-3

3.RESEARCH AND METHODOLOGY

3.1. AIM and scope.	12
3.2. Description and research work.	12
3.3. Research problem.	12

CHAPTER-4

4.PERFORMANCE ANALYSIS

4.1. Near-ring and near-ring groups and their substructures.	14
4.2. POSETs and Lattices.	20
4.3. Rings and Modules.	21

CHAPTER-5

5.SUMMARY AND CONCLUSION

5.1 Conclusion.	26
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REFERENCE 27

LIST OF TABLES

TABLE NO.	TITLE.	PAGE NO.
Tab. 4.1.	Let $(G = Z_3 = \{0,1,2\})$ and N be the 15 of all mappings in G Defined in table 4.1.	15
Tab. 4.2.	$N = \{0, a, b, c\}$ is a near-ring under the "+".	16
Tab. 4.3.	$N = \{0, a, b, c\}$ is a near-ring under the "x".	16
Tab. 4.4.	$N = \{0, a, b, c, x, y\}$ is a near-ring under the "+".	17
Tab. 4.5.	$N = \{0, a, b, c, x, y\}$ is a near-ring under the "x".	17

CHAPTER 1

1. INTRODUCTION

1.1 Near-ring

A near-ring is an algebraic system with binary operation of addition and multiplication satisfying all the ring axioms except, possibly, one of the distributive laws and commutativity of addition. Historically, the first step towards near-rings was an axiomatic research done by Dickson in 1905. The beginning 30's saw the first proper near-ring considerations. Since then, the theory of near-rings has been developed much of the same way as rings. Near-rings are now a sophisticated theory which has found numerous applications in various areas viz. interpolation theory, group theory, polynomials and matrices. Beautiful and efficient block designs can be constructed from finite near-rings. In recent years Pilz, Meldrum and others have obtained elegantly the relations between near-rings and automata, near-rings and dynamical system, seminear-rings and rooted trees. The development of near-rings and near-fields has matured to the point where the theory is substantial, the applications are numerous and both are amply documented. In 1938, H. Wieland initiated a structure theory of near-rings. Since then, extensive research works are being carried out on near-rings and near-ring groups in which structure theory is one area of importance, Beidleman, Oswald, Ligh and others have done considerable work on various aspects of near-rings, near-ring groups with chain conditions. In 70's, Oswald has obtained the structure theory of near-rings in which each near-ring subgroup is principal. As Meldrum comments on our paper- On near-rings with ACC on annihilators- this is a difficult area in which there is not a great deal of work being done.

1.2 Near-ring Groups

Near-ring theory still awaits useful contribution to this subject for its development. Here we devote ourselves to obtain some special 2 types of structure theorems on near-rings and near-ring groups satisfying different chain conditions. In this dissertation important and interesting investigations are being carried out on near-rings with chain conditions on annihilators

having no infinite independent family of near-ring subgroups with parts having minimum conditions. This investigation is an attempt to study one subalgebraic structure of above type of near-rings and one may expect to get more elegant structure theorems in case of above type of near-rings. W.Blackett studied simple and semisimple near-rings around 1950. S.C.Choudhary, Mason, Mason and others have generalized that concept to strictly semisimple near-rings. Here we further extend the above concept to strictly-1 semisimple near-ring groups. Near-rings are generalizations of rings. It is natural to generalize various concepts of rings to near-rings. Betsh, Beidieman, Ramakotaiah, Ligh, Clay, Choudhary and others had generalized various concepts to near-rings. Due to non-ring character of a near-ring the results have their own beauty. We extend the notion of finite spanning dimension in modules due to Fleury to near-ring groups. We also deal with near-rings and near-ring groups with various types of chain conditions on annihilators together with strictly finite Goldie dimension which is an extension of finite Goldie dimension due to Satyanarayana, Cohn, Jensen, Brungs, Stephenson have studied algebraic structures like modules, rings with distributive property. Extending this concept to a near-rings and near-ring groups, termed as d near-rings and d near-ring groups, we investigate some characteristics of such type of near-rings and near-ring groups. Work on semiprime near-ring goes back to Blackett. Recently different types of semiprime near-rings were studied by Groenwald, Rao, Stefano discussed a generalized semiprime character, the strongly semiprime near-rings play an important role.

1.3 Goldie ring

There are various ways to generalize prime ideals of rings to near-rings-each with its own advantages. Together with Groenwald, Veldsman and others have generalized prime ideals. We also present another generalization of prime ideals. The quest to characterize minimal ideals goes back to Blackett. But the first paper devoted entirely to characterizing them is due to Scott, J. Birkenmeir and Heatherly, Groenwald and others

have investigated many characteristics of minimal ideals. In some of our results we also deal with minimal ideals. Objective of the work: Our objective is to study near-rings with acc on annihilators having no infinite direct sum of ideals(subgroups) with parts satisfying the acc or the dec on its substructures. It motivates us to study the strictly Artinian radical ((s-A) radical) of such a near-ring. One of the motivations for studying this radical is that, after slicing the radical out, the process yields a near-ring which is simpler in some sense and hence is more amenable. Also, one expects to get information about the original near-ring from the structures of near-rings obtained by slicing out such a radical. A Our aim to study N-groups is to obtain some elegance structure theorems, to develop some ways to see the effect of projectivity, strictly-1 semisimple character and chain conditions. It A is worthwhile to mention that chain near-rings with the help of d character give rise to some important features of near-rings and possibly, this is another interesting aspect of such near-rings and near-ring groups with which a lot of work may be carried out. The decomposition of an ideal into primary ideals is a traditional pillar of ideal theory. It provides the algebraic foundation for decomposing an algebraic variety into its irreducible components. From another point of view primary decomposition provides a generalization of the 4 factorizations of an integer as a product of prime powers. In a Noetherian ring, every ideal has a primary decomposition. In case of a Noetherian module (a particular case of module with finite Goldie dimension) such a decomposition is known. It is our motivation to find out a decomposition of zero of an FSD-1 N-group (which is a dual concept of finite Goldie dimension). We aim to see the differences of such decomposition of the zero with that of a strictly finite Goldie dimension N-group. One may expect to bring into light many aspects of this decomposition theory of finite spanning dimension as well as that of finite Goldie dimension N-group.

CHAPTER 2

2. LITERATURE SURVEY

A structure theorem in near-rings is seen in 1938 initiated by H.Wielandt. Since then, extensive research work is being carried out on near-rings and near-ring groups in which structure theory is one area of importance. Beidleman, Oswald, Ligh and others have done considerable work on various aspects of near-rings, near-ring groups with chain conditions. In 70's, Oswald [1974] has obtained the structure theory of near-rings in which each near-ring subgroup is principal. Near-ring theory still awaits useful contribution to this subject for its development. Here we devote ourselves to obtain some special types of structure theorems on near-rings and near-ring groups satisfying different chain conditions. As near-rings are generalizations of rings, it is natural to generalize various concepts of rings to near-rings. Betsh, Beidleman, Ramakotaiah, Ligh, Clay, Chowdhury and others have generalized various concepts to near-rings. Due to non-ring character of a near-ring the results have their own beauty. Oswald's consideration of near-rings [1974] in which every N-subgroup is principal (prs near-rings), gives rise to many interesting results with chain conditions on annihilators. Baruah's work [1984] on the above type of near-rings is directed towards the idea of l.c.m and g.c.d of near-rings. < I in this dissertation important and interesting investigations are being carried out on near-rings with chain conditions on annihilators and in some cases having no infinite independent family of near-ring subgroups. Linearly ordered near-rings and near-ring groups are another important aspects to1 the contribution with a number of extended ' 1 ideas namely left serial near-rings, principal sequence N-groups, restricted principal sequence N-groups, D-regularity of Divinsld and above all, the interesting one-sided integral power of radical of a near-ring which arises on a near-ring with its duo or sub commutative character in the sense of Dheena [1987]. It is another noticeable achievement on what the author claims. The non-associative character of the product of two near-ring subgroups in unusual sense is playing key-role to this one-sided power of the radical. Some results on another difficult area namely on near-rings with ascending chain condition on

annihilator in which there is not a great deal of work been done is another interesting area that the author has chosen for carrying out at least some work. Near-rings and near-ring groups or near-ring modules with finite Goldie dimension are being developed with many branches and matured healthy symptoms for carrying out fruitful research work since Oswald, Satyanarayana, Chowdhury, Masum, Saikia at all. The projectivity and injectivity of near-ring groups have been viewed from some deviated points giving rise to some analogous concepts viz. normal injective hull throwing a new dimension to the injectivity theory of near-ring groups. Semi-simple notions of an N-group have been devised so as to extend the existing idea of the socle of an N-group. Many results on multifaced simple character of near-ring groups have been successfully co-ordinated with these notions.

CHAPTER 3

3. RESEARCH METHODOLOGY

3.1 AIM AND SCOPE

The main objectives of the work reported are:

- To study near-ring and near-ring groups.
- Basic definitions and results of algebraic structures in particular rings.
- Modules, near-rings, near-ring groups, lattices.
- Further it deals Fundamental definitions and results of near-rings, near-ring groups.
- Substructures, Posets, connection with rings and modules.

The scope of the work reported is:

- Many applications involve the subclass of near-rings known as near-fields; for these see the article on near-fields.
- There are various applications of proper near-rings, i.e., those that are neither rings nor near-fields.

3.2 DESCRIPTION OF RESEARCH WORK

It deals with various applications of proper near-rings, i.e., those that are neither rings nor near-fields.

3.3 Research Problem

It deals with balanced incomplete block designs using planar near-rings. These are a way to obtain difference families using the orbits of a fixed point free automorphism group of a group.

CHAPTER 4

Some basic definitions and results of some algebraic structures in Particular rings, modules, near-rings, near-ring groups, lattices etc., are covered in this introductory chapter. There are three subsections which comprises the Chapter. The first section is about fundamental definitions and results of near-Rings, near-ring groups and their substructures. The second section contains Some relevant information of posets etc. and the remaining section contain all That is necessary in connection with rings and modules.

4.1 Near-ring and near-ring groups and their substructures:

4.1.1 Definition: A triple $(N, +, \cdot)$ where N is a non-empty set '+' and ' $\cdot$ ' are two operations in N , if

- i) $(N, +)$ is a group (not necessarily abelian)
- ii) $(N, \cdot)$ is a semi group.
- iii) $(a + b) \cdot c = a \cdot c + b \cdot c$ for all $a, b, c \in N$

$a \cdot (b + c) = ab + a \cdot c$ then the corresponding triple is a left near-ring.

Unless otherwise specified ab will mean $a \cdot b$ for all $a, b \in N$ and N will mean a near-ring.

In contrast to ring constructed naturally in the set say $E(H)$ of endomorphism of an additive abelian group, say w.r.t the operations

- i) $(f+g)x = f(x) + g(x)$
- ii) $(fg)x = f(g(x))$

for all $f, g \in E(H)$ and $x \in H$, the set say $M(G)$ of all mapping of an additive group say G not necessarily abelian w.r.t operations defined as (i), (ii) gives rise naturally a near-ring.

4.1.2 Definition: The additive identity of the group $(N, +)$ of a near-ring N is the zero element and it is denoted by 0 .

4.1.3 Property: In a near-ring N , $0 \cdot a = 0$ for all $a \in N$.

4.1.4 Example: Let $(G = Z_3 = \{0, 1, 2\})$ and N be the set of all mappings in G Defined in table 4.1. Here N is a near-ring w.r.t. the operations

i) $(f+g)x = f(x)+g(x)$

ii) $(fg)x = f(g(x))$ for all $f, g \in E(H)$ and $x \in H$ with the zero element f_0 .

Table 4.1

$G \rightarrow G$	0	1	2
f_0	0	0	0
f_1	0	1	2
f_2	0	0	1
f_3	0	0	2
f_4	0	1	0
f_5	0	1	1
f_6	0	2	0
f_7	0	2	1

$G \rightarrow G$	0	1	2
f_8	0	2	2
f_9	1	0	0
f_{10}	1	0	1
f_{11}	1	0	2
f_{12}	1	1	0
f_{13}	1	1	1
f_{14}	1	1	2
f_{15}	1	2	0

$G \rightarrow G$	0	1	2
f_{16}	1	2	1
f_{17}	1	2	2
f_{18}	2	0	0
f_{19}	2	0	1
f_{20}	2	0	2
f_{21}	2	1	0
f_{22}	2	1	1
f_{23}	2	1	2
f_{24}	2	2	0
f_{25}	2	2	1
f_{26}	2	2	2

4.1.5 Definition: A near-ring N is a zero-symmetric near-ring if $a.0 = 0$, for all $a \in N$.

4.1.6 Definition: If the semi-group $(N,0)$ of a near-ring possesses an element 1 such that $a.1=a=1.a$ for all $a \in N$ then 1 is the identity or the unity

element of N . Throughout our discussion, unless otherwise specified, N stands for zero-symmetric near-ring with unity.

4.1.7 Definition: An element $x \in N$ is a distributive element if $x(a+b) = xa+xb$, for all $a, b \in N$.

4.1.8 Properties: If x is a distributive element of N , then

- i) $x \cdot 0 = 0$
- ii) $x(-a) = -(xa)$ for all $a \in N$.
- iii) $(-x)(-a) = xa$ for all $a \in N$.

4.1.9 Definition: A group $(E, +)$ is an N -group (near-ring group), denoted by N^E when there exists a mapping $N \times E$ into E such that

- i) $(n_1 + n_2)e = n_1e + n_2e$
- ii) $(n_1 \cdot n_2)e = n_1(n_2e)$
- iii) $1 \cdot e = e$ for all $n_1, n_2 \in N, e \in E$. In what follows, E will mean the N -group N^E .

4.1.10 Properties:

- i) $0 \cdot e = 0$ (the first 0 is the zero element of N and the second zero is the zero element of E)
- ii) $(-n)e = -ne$ and
- iii) $(n_1 - n_2)e = n_1e - n_2e$ for all $e \in E, n_1, n_2 \in N$.

4.1.11 Definition: For two non-empty sets B and C of N we have the set product $BC = \{ab \mid a \in B, b \in C\}$. Following substructures in case of N and E have considerable importance in further discussion.

4.1.12 Examples: $N = \{0, a, b, c\}$ is a near-ring under the operations defined

Table 4.2

Table 4.3

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

0	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	b	b
c	0	a	c	c

by the following two tables of the operations '+' and '.' respectively. Here for $B = \{0, a\}$, $C = \{0, a, b\}$ and $D = \{0, a, c\}$, we have $CN=C$, $DN=D$, whereas $BN=B$ and $NB=B$. This enables us to define the following

4.1.13 Definitions: A non-empty subset B of N is a

- i) a right N -subset of N if $BN \subseteq B$.
- ii) a left N -subset of N if $NB \subseteq B$.
- iii) an invariant subset of N if $BN \subseteq B$ and $NB \subseteq B$

It is noticeable that an invariant subset of N is a left as well as right N -subset of N . Again, every left (right) N -subset contains the zero element of N .

4.1.14 Lemma: Intersection of any numbers of left (right) N -subsets of N is again a left (right) N -subset of N .

4.1.15 Lemma: Intersection of any numbers of invariant subsets of N is again an invariant subset of N .

4.1.16 Lemma: If B is a left N -subset and C is a right N -subset of N , then BC is an invariant subset of N .

4.1.17 Lemma: If I is a right N -subset of N and $x \in A$, then xI is a right N -subset of N .

4.1.18 Corollary: For $x \in N$, xN is a right N -subset of N .

4.1.19 Lemma: For two right N -subsets B and C of N , the sum $B + C = \{b+c \mid b \in B, c \in C\}$ is also a right N -subset of N . Now we observe some other substructures of N as given below.

4.1.20 Example: $N = \{0, a, b, c, x, y\}$ is a near-ring under the operation defined by the following tables.

Table 4.4

+	0	a	b	c	x	y
0	0	a	b	c	x	y
a	a	0	y	x	c	b
b	b	x	0	y	a	c
c	c	y	x	0	b	a
x	x	b	c	a	y	0
y	y	c	b	a	0	x

Table 4.5

0	0	a	b	c	x	y
0	0	0	0	0	0	0
a	0	0	a	a	a	b
b	0	0	b	c	c	c
c	0	0	c	b	b	a
x	0	0	x	y	y	0
y	0	0	y	x	x	x

Here we get $X=\{0,a\}$ as a subgroup of $(N,+)$ such that

- (i) $XN=X$
- (ii) $NX\subset X$
- (iii) $XX\subset X$

4.1.21 Definition: If H is a subgroup of $(N,+)$, then H is

- i) a left A -subgroup of A if $NH \subseteq H$
- ii) a right N -subgroup of A if $HN \subseteq H$
- iii) sub near-ring of A if $HH \subseteq H$
- iv) An invariant sub near-ring of N if $NH \subseteq H, HN \subseteq H$.

obviously, a right (left) N -subgroup of N is a sub near-ring of N .

4.1.22 Definition: An element $x \in N$ is a distributive element if

$$X(a+b) = xa + xb, \text{ for all } a,b \in N.$$

4.1.23 Definition: A sub near-ring B of N is distributively generated(dg) if

There exists a multiplicative sub semi-group D of distributive elements of N such that the additive group $(B,+)$ is generated by D . The near-ring N is Said to be distributively generated near-ring (dgnr) when $B = N$ denoted $N = (D)$. Clearly any element of $N(=N(D))$ is of the form $\sum \pm d_1, d_2 \in D$.

4.1.24 Lemma: If I is a left N -subgroup of N and $x \in N$, then Ix is a left N -subgroup of N .

4.1.25 Corollary: For any element $x \in N$, Nx is a left N -subgroup of N .

4.1.26 Lemma: Intersection of any number of left(right) N -subgroup of N is a left(right) N -subgroup of N .

4.1.27 Definition: In 1.1.21(i), if $N = E$, then H is an N -subgroup of E . In

other words, subgroup H of $(E, +)$ is an N -subgroup of E if $NH \subseteq H$.

Clearly, a left N -subgroup of N is an N -subgroup of N^N and vice versa.

4.1.28 Lemma: Intersection of any number of N -subgroup of E is a left (right) N -subgroup of E .

4.1.29 Lemma: If M is an N -subgroup of E and S is an N -subgroup of N^M then S is an N -subgroup of E .

4.1.30 Lemma: If C is an N -subgroup of E and I is a left N -subgroup of N , then Ix is an N -subgroup of C , for $x \in C$.

4.1.31 Definitions: A normal subgroup I of $(N, +)$ is

- i) a right ideal of N if $xn \in I$ for all $x \in I, n \in N$.
- ii) a left ideal of N if $n_1(x+n_2)-n_1n_2 \in I$ for all $x \in I, n_1, n_2 \in N$.
- iii) an ideal of N if I is a right as well as left ideal of N .

As I is a normal subgroup of $(N, +)$, the condition for $n_1(n_2+x)-n_1n_2 \in I$ for all $x \in I, n_1, n_2 \in N$ is equivalent to that of a left ideal of N .

Obviously an ideal is itself a near-ring and so every ideal of N is a sub near-ring of N .

4.1.32 Lemma: If B is a left N -subgroup of N and C is a left ideal of N , then $B+C$ is a left N -subgroup of N .

4.1.33 Definition: A normal subgroup M of $(E, +)$ is an ideal of E if $n(e+m)-ne \in M$ for all $n \in N$ and $e \in E$.

Normal character of M facilitates us to replace the above condition by $n(m+e)-ne \in M$.

There is a coincidence between the left ideal of a near-ring N and the ideal of N^N .

4.1.34 Lemma: If H is an N -subgroup of $B + C$ and M is an ideal of E , then $H+M$ is an N -subgroup of E .

4.1.35 Lemma: Intersection of any number of ideals of E is again a deal of E .

4.1.36 Lemma: If M is an ideal of E and S is an N -subgroup of N such that $M \subseteq S$, then M is an ideal of N^S .

4.1.37 Lemma: If H is an N -subgroup of E and M is an ideal of E , then

$H \cap M$ is an ideal of N^H .

4.1.38 Corollary: If I and B are two ideals of E , then $I+B$ is also an ideal of E .

4.1.39 Definition: A near-ring N is a duo near-ring if for every $a, b \in N$, $ab = ba = da$, for some $c, d \in N$.

4.1.40 Definition: A near-ring K is left Goldie if It has no infinite direct sum of its left ideals and satisfies the ascending chain condition (acc) on its left annihilators. On the contrary, a near-ring K is right Goldie If It has no infinite independent family of right K subsets and K satisfies the acc on right annihilators [162].

4.2. POSETs and Lattices:

4.2.1 Definition: A binary relation R on a non-empty set A is a partial order (ordering) on X if it satisfies the following.

1. R is reflexive on X ; that is, $(x, x) \in R$ for all $x \in X$.
2. R is transitive; that is, $(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$ and
3. R is anti-symmetric; that is for any x and $y \in X$, $(x, y) \in R$ and $(y, x) \in R \Rightarrow x = y$

If R is a binary operation on a set X and $x, y \in X$, then we usually write xRy for $(x, y) \in R$. The partial orders are commonly denoted by the symbols $\leq, \geq, \subseteq, \supseteq$, etc. In this terminology, a binary relation ' $\leq$ ' on a non-empty set X is a partial order on X if, for x, y and $z \in X$.

1. $X \leq x$ (reflexive)
2. $X \leq y$ and $y \leq z \Rightarrow x \leq z$ (transitive)
3. $X \leq y$ and $y \leq x \Rightarrow x = y$ (anti-symmetric)

4.2.2 Example:

(i) The usual order on the real number system R , defined by $a \leq b$ if and only if $b-a$ is non-negative, is a partial order on R . If $\leq$ is a partial order on X and Y is a non-empty subset of X , then $\leq$ (on $X \times Y$) is a partial order on Y and is called the 'ordering $\leq$ restricted to Y ' and is usually denoted by $\leq_Y$.

(ii) Consider the set Z^+ of positive integers. For any a and b in Z^+ , define a/b

if and only if a divides b , that is, $a \mid b$ for some $c \in \mathbb{Z}^+$. Then $\mid$ is a partial order on $\mathbb{Z}^+$.

(iii) Let P be a non-empty class of sets and, for any A and B in P , define $A \subseteq B$ if and only if A is a subset of B . Then ' $\subseteq$ ' is a partial order on P and is called the 'inclusion ordering'.

4.2.3 Definition: If X is a non-empty set and $\leq$ is a partial order on X , then the pair $(X, \leq)$ is called a partially ordered set (POSET).

4.2.4 Definition: Let $(X, \leq)$ be a partially ordered set and $a \in X$. Then a is a maximal element in X if, for any $x \in X$, $a \leq x \Rightarrow a = x$.

Dually, a is a minimal element if, for any $x \in X$, $x \leq a \Rightarrow x = a$.

We often write $x < y$ if $x \leq y$ and $x \neq y$. In this terminology, an element a is maximal.

If $\geq$ is the dual of $\leq$ then clearly an element a is maximal in $(X, \leq)$ if and only if it is minimal in $(X, \geq)$.

4.2.5 Definition: A partially ordered set $(X, \leq)$ satisfies the ascending chain condition if, any ascending sequence $x_1 \leq x_2 \leq x_3 \dots$ in X terminating at a finite stage, in the sense that, there exists a positive integer n such that $x_n = x_{n+1} = x_{n+2} = \dots$.

Similarly, $(X, \leq)$ satisfies the descending chain condition if any descending sequence $x_1 \geq x_2 \geq \dots$ terminates at a finite stage, clearly $(X, \leq)$ satisfies the ascending chain condition if and only if its dual $(X, \geq)$ satisfies the descending chain condition.

4.2.6 Examples:

(i) The set $\mathbb{Z}^+$ of all integers together with the usual ordering satisfies neither the ascending chain condition nor the descending chain condition.

(ii) The set $\mathbb{Z}^+$ of positive integers satisfies the descending chain condition and does not satisfy the ascending chain condition.

(iii) Any finite partially ordered set satisfies both the ascending and descending chain conditions.

4.2.7 Theorem: A partially ordered set $(X, \leq)$ satisfies the ascending (descending) chain condition if and only if every non-empty subset of X has a maximal (respectively minimal) element. In particular, any finite partially ordered set possesses maximal elements and minimal elements. In the next section we present some preliminary definitions and results of rings and modules.

4.3 Rings and Modules:

4.3.1 Definition: If $(R, +, \cdot)$ is a ring, then the group $(R, +)$ is the additive group of the ring R . A non-empty subset S of a ring R is a subring of R if S is itself a ring with respect to the operation $+$ and $\cdot$ in R . If there exists an element 1 in the ring R such that for every element $a \in R$, $a \cdot 1 = a$ then 1 is the unity of R .

4.3.2 Definition: An element of a ring R is a left (right) zero divisor in R if There exists some $b \neq 0$ in R such that $ab = 0$ ($ba = 0$).

4.3.3 Definition: An element of a ring R is a regular element of R if it is Neither a left nor a right zero divisor in R .

4.3.4 Definition: An element e of a ring R is an idempotent if $e^2 = e$.

4.3.5 Definition: An element a of a ring R is nilpotent if we have $a^n = 0$ For some positive integer n . It is to be noted that every nilpotent element is a divisor of zero. A Regular element n is non-nilpotent.

4.3.6 Definition: A map f from a ring R to a ring R' is a homomorphism if $f(a + b) = f(a) + f(b)$ and $f(ab) = f(a)f(b)$ for all $a, b \in R$. In particular, it is a monomorphism (or an embedding) if it is Injective, an epimorphism if it is surjective and isomorphism if it is bijective.

The set $\{x \in R \mid f(x) = 0\}$, the additive identity of S is the kernel of f denoted $\text{Ker}(f)$. It is to be noted that $\text{Ker}(f)$ is an ideal of the ring R . Now we note here some preliminary information about modules.

4.3.7 Definition: A right R -module M is Artinian (Noetherian) and has the d.c.c. (a.c.c.) for sub modules if every non empty set of sub modules of M has a minimal (maximal) element or equivalently, if every descending (ascending) chain of sub modules of M becomes stationary after a finite number of steps.

4.3.8 Proposition: A module M is Noetherian if and only if every sub modules of M is finitely generated.

4.3.9 Proposition: Let N be a sub module of a module M . Then M is Artinian (Noetherian) if and only if N and M/N are both Artinian (Noetherian).

4.3.10 Definition: If in case of the module $^M R$, M has no infinite direct sum of its submodule, then M is finite dimensional.

4.3.11 Definition: A Goldie module [38] is a module M over a ring R if,

(i) M is finite dimensional

(ii) R satisfies the ascending chain condition (acc.) for annihilators of subsets of M in R i.e.) any collection of annihilators of subsets of M in R satisfies the maximal condition.

4.3.12 Definition: The intersection of all prime ideals of the ring R is the nil-radical of R . A ring R is prime sub-commutative in the sense that for any subset T and prime sub-module S of R , $TS = ST$.

4.3.13 Definition: Let R be a ring. Then, by a right R -module we mean an Abelian group $(M, +)$ and a ring R such that there is a map $M \times R \rightarrow M$ such that $(m, r) \rightarrow mr$ with the conditions

$$i) (m_1, m_2)r = m_1r + m_2r$$

$$ii) M(r_1 + r_2) = mr_1 + mr_2$$

$$iii) M(r_1 r_2) = (mr_1)r_2 \text{ for any } r_1, r_2 \in R$$

4.3.14 Definition: Let M be a right (left) R -module. Then an abelian subgroup N of M is a sub module of M in case N is a right (left) R module on its own right. We note that if M be a right R -module and X be a non empty subset of M then XR is a sub module of M .

4.3.15 Proposition: If M be a right R -module and ζ be any arbitrary collection of sub modules of M , then the intersection of members of ζ is a sub module of M .

4.3.16 Proposition: If M be a right R -module and $M_1, M_2, \dots, M_n$ are sub modules of M , then the set

$M_1 + M_2 + \dots + M_n \sim \{m_1 + \dots + m_n \mid m_i \in M_i, (i = 1, 2, \dots, n)\}$ is a sub module of M .

4.3.17 Definition: A ring R is reduced if its nil-radical $J = 0$.

4.3.18 Definition: If the module ${}^R R$ is Goldie, then R is a right Goldie ring (i.e.) if

i) R has no infinite direct sum of its right ideals

ii) R satisfies the acc on the right annihilators of subsets of R in R .

Similarly, a Goldie module ${}_R M$ is such that it is a left module over the ring R the two left analogous conditions hold and if in case of the ring R , if ${}^R R$ is Goldie, then R is left Goldie ring.

4.3.19 Note: Similarly, in case of a module M , the acc on annihilators of subsets of M in R is equivalent to the acc on the annihilators of subsets of R in M . Same is the case for a module R^M too. It is to be noted that homomorphic image of a Goldie module need not be Goldie again.

4.3.20 Definition: A Goldie module is fully Goldie if any homomorphic image of it is again Goldie.

4.3.21 Definition: When a ring R is such that ${}^R R$ is torsion-free as a module, then R is a torsion free ring.

4.3.22 Definition: A ring R is nilpotent if we have some positive integer n such that $R^n = 0$.

4.3.23 Definition: Let R and S be two rings. An Abelian group M is a left R -right S -bimodule in case M is both a left R -module and a right S -module for which the two scalar multiplications jointly satisfy $(xs) \cdot (rx) = x(sr)$

Usually stick to the following notations:

M_R means M is a right R -module

${}_R M$ means M is a left R -module

${}_R M_S$ means M is a left R -right- S bimodule

${}_S M$ means M is a left R -left S -bimodule

CHAPTER 5

5. SUMMARY AND CONCLUSIONS

5.1 CONCLUSION

Throughout this paper, we have dealt with Near-ring groups and their substructures, Posets and Lattices, Rings and Modules. The following results we conclude it,

- (i) A Partially ordered set $(X, \leq)$ satisfies the ascending or descending chain condition if and only if every non-empty subset of X has a maximal (respectively minimal) element.
- (ii) Every quotient ring of a right Artinian ring is right Artinian. (Right Artinian: A ring is right Artinian if the right module R_R is right Artinian)

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