

STUDY ON SQUARE SUM LABELING OF SOME GRAPHS

Submitted in partial fulfillment of the requirements for the award of
Bachelor of science degree in mathematics (B.sc Mathematics)

BY

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**(DEEMED TO BE UNIVERSITY)
Accredited with Grade "A" by NAAC**

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **R. ANISHA MARGRET (38060002)** who carried out the project entitled "**STUDY ON SQUARE SUM LABELING OF GRAPHS**" under our supervision from **JANUARY 2021 to APRIL 2021.**

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DECLARATION

I **R.ANISHA MARGRET (38060002)** hereby declare that the Project Report entitled "**STUDY ON SQUARE SUM LABELING OF SOME GRAPHS**" done by me under the guidance of **Dr.R.PARAMESWARI** (Internal) at **Sathyabama Institute of Science and Technology** submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in Mathematics.

DATE: 29.3.2021

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SIGNATURE OF THE CANDIDATE

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ABSTRACT

Let $G(V, E)$ be a graph where V denotes the set of vertices with $|V| = p$ and E denotes the set of edges with $|E| = q$. A Graph $G = G(V, E)$ is said to be square sum graph if there exist a mapping $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ which is bijection such that the induced function $f^*: E(G) \rightarrow N$ defined by $f^*(uv) = [f(u)]^2 + [f(v)]^2$, for every $uv \in E(G)$ is injective. This article addresses the square sum labeling of Octopus graph O_n and Vanessa graph V_n which admits square sum labeling.

KEYWORDS

Graph labeling, square sum labeling, Fan graph, Vanessa graph, Octopus graph.

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CHAPTER – 1

INTRODUCTION

1.1 Graph Theory

Graph theory is a promising division of Mathematics; any real life problem can be converted as pictorial representation, simply saying as a graph. It is a diagram which encloses set of dots jointly with lines connecting certain pair of dots. Swiss Mathematician Leonhard Euler transfer the Königsberg bridge problem in to a graph and wrote a paper and issued in 1736. He is well thought-out as the originator of graph theory. In detailed, the word “graph” was introduced in a paper by Sylvester which was issued in 1878. The first book on graph theory was written by Denes König and was issued in 1936.

A graph G consists of a set of objects $V = \{v_1, v_2, v_3, \dots\}$ called vertices (also called points or nodes) and other set $E = \{e_1, e_2, e_3, \dots\}$ whose elements are called edges (also called lines or arcs). The set $V(G)$ is called the vertex set of G and $E(G)$ is the edge set. Usually the graph is denoted as $G = (V, E)$. When a graph has an ordered pair of vertices, it is called a directed graph. The edges of the graph represent a specific direction from one vertex to another, were in undirected graph it has unordered pair of vertices with no specific direction.

1.2 APPLICATION OF GRAPH THEORY

Graph theoretical concepts are widely used to study and model various applications, in different areas. They include, study of molecules, construction of bonds in chemistry and the study of atoms. Graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist and the edges represent migration path or movement between the regions. Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest

spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph.

1.3 GRAPH LABELING

By assigning the vertices or edges or both by integers upon certain conditions is called graph labeling. Alexander Rosa introduced the concept of Graph Labeling in 1967. He identified labelings α , β -, and ρ -labelings. β -labelings renamed as "graceful" by Solomon Golomb. Later various labelings were developed. A dynamic survey on graph labeling is frequently updated by J. A. Gallian. More than 2500 papers were published in this field. Many labeling namely magic, cordial, sequential, prime, square sum, square difference, anti magic, Z_3 magic were introduced by different authors.

1.4 APPLICATION OF GRAPH LABELING

Graph Labeling plays a vital role in many other field. It has various applications within mathematics as well as to several areas of computer science and communication networks. It is used in many applications like coding theory, x - ray crystallography, radar, astronomy, circuit design, communication network addressing, database management, sociology etc.

1.5 PRELIMINARIES AND BASIC TERMINOLOGY

This section comprises the basic definitions which are used in this study.

An Open Walk in which no vertex appears more than once is called a **Path P_n** . It is also called as simple path or elementary path.

A **Star S_n** is a complete bipartite graph $K_{m,n}$ for $m = 1$.

A **Fan graph F_n** is defined as the graph $K_1 + P_n$, where K_1 is the singleton graph and P_n is the path on n vertices.

An **Octopus Graph** O_n , ($n \geq 2$) can be constructed by a fan graph F_n , ($n \geq 2$) joining a star graph $K_{1, n}$ with sharing a common vertex, where n is any positive integer. ie., $O_n = F_n + K_{1, n}$.

A **Vanessa Graph** V_n , ($n \geq 2$) can be constructed by two fan graphs F_n , ($n \geq 2$) joining a star graph $K_{1, n}$ with sharing a common vertex, where n is any positive integer. ie., $V_n = 2F_n + K_{1, n}$.

Graph Labeling is assigning the vertices or edges or both by integers upon certain conditions.

Let $G = (V, E)$ be a (p, q) - graph and let $f: V(G) \rightarrow \{0, 1, 2, \dots, p-1\}$ be a bijection. We define f^* on $E(G)$ by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. If f^* is injective on $E(G)$, where $f^*: E(G) \rightarrow N$, then f is called a **Square sum labeling**. The graph G is said to be a square sum graph if G admits a square sum labeling.

CHAPTER– 2

2.1 LITERATURE SURVEY

V. Ajitha, S. Arumugam and K. A. Germina introduced Square sum Labeling.

V. Ajitha, S. Arumugam and K. A. Germina [2009] proved that the Complete graph K_n is Square sum if and only if n less than or equal to 5. And proved that trees, cycles, friendship graph $C_3^{(n)}$, the complete lattice grids $L_{m,n} = P_m \times P_n$ are square sum graphs.

J. Shiama [2012] proved square sum labeling for some middle and Total graphs.

G. V. Ghodasara, Mitesh J. Patel [2017] proved square sum labeling for bistar related graphs.

A. Edward Samuel and S. Kalaivani [2017] proved the square sum labeling for some lilly related graphs. Lilly graph I_n is constructed by joining two star graphs $K_{1,n}$ and two path graphs P_n with sharing a common vertex.

In this chapter we have chosen some graphs based on Path graph and star graph namely lilly graph, square graph of comb and jelly fish graph discussed in detailed.

THEOREM

The lilly graph I_n , ($n \geq 2$) admits square sum labeling, where n is any positive integer.

PROOF

Let G be the lilly graph I_n , ($n \geq 2$). Let $\{u_1, u_2, \dots, u_{4n-1}\}$ be the vertices of I_n . Here $|V(I_n)| = 4n - 1$. Define a labeling $f: V(I_n) \rightarrow \{0, 1, 2, \dots, 4n-2\}$ as follows

$f(u_i) = i - 1$ for $1 \leq i \leq 4n - 1$. Clearly vertex labels are distinct. And f induces a function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = [f(u)^2] + [f(v)^2]$. Then for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Thus f admits square sum labeling. Therefore I_n is a square sum graph.

G. Subashini, K. Ramanathan and K. Manimekalai [2018] proved for pyramid graph and Hanging Pyramid graph admits square sum labeling.

E. Esakkiammal, B. Deepa, K. Thirusangu [2018] proved that square graph of comb is the square sum graph. The comb graph is defined as $P_n \odot K_1$ where it

has $2n$ number of vertices and $2n-1$ number of edges. For a given graph G , the square graph G^2 is a graph on the same vertex set but in which two vertices are

adjacent if and only if they are at distance at most 2 in G . The square graph of comb graph is denoted by $(P_n \odot K_1)^2$ and has vertex set $V = \{v_i \cup u_i / 1 \leq i \leq n\}$ and the edge set is $E = \{v_i u_i / 1 \leq i \leq n\} \cup \{u_i u_{i+2} / 1 \leq i \leq n-2\} \cup \{u_i u_{i+1} / 1 \leq i \leq n-1\} \cup \{u_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\}$. This set has $2n$ vertices and $5(n-1)$ edges.

THEOREM The square graph of comb admits square sum labeling.

PROOF

Let G be a square graph of comb $(P_n \odot K_1)^2$. Let $\{u_1, u_2, \dots, u_n\} \cup \{v_1, v_2, \dots, v_n\}$ be the vertices of $(P_n \odot K_1)^2$. Here, it has $2n$ vertices. Define a Labeling $f: V(G) \rightarrow \{0, 1, \dots, 2n-1\}$ as follows

$$f(u_i) = 2i - 2 \quad 1 \leq i \leq n$$

$$f(v_i) = 2i - 1 \quad 1 \leq i \leq n. \text{ Clearly vertex labels are distinct.}$$

And f induces a function $f^*: E(G) \rightarrow \mathbb{N}$ given by $f^*(uv) = [f(u)^2] + [f(v)^2]$. Then for any edge $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Thus f admits square sum labeling. Therefore square graph of comb is a square sum graph.

K. A. Germina and Reena Sebastian [2017] proved square sum labeling for Class of Planar graphs. They prove square sum labeling for Jelly Fish graph.

THEOREM The Jellyfish graph $J(m, n)$ is square sum for all $m, n \geq 0$.

PROOF

Let G be a Jellyfish graph $J(m, n)$, $m, n \geq 0$. Let $\{u, v, x, y\} \cup \{x_1, x_2, \dots, x_n\} \cup \{y_1, y_2, \dots, y_n\}$ be the vertices of $J(m, n)$. Here it has $m+n+4$ vertices. Define a labeling $f: V(J(m, n)) \rightarrow \{0, 1, \dots, m+n+3\}$ as follows.

$$f(x_i) = i; 1 \leq i \leq m,$$

$$f(u) = m + 1, f(x) = 0,$$

$$f(v) = m + 2, f(y) = m + 3,$$

$f(y_i) = m + 3 + i; 1 \leq i \leq n$. Clearly vertex labels are distinct.

And f induces a function $f^*: E(G) \rightarrow \mathbb{N}$ given by $f^*(uv) = [f(u)^2] + [f(v)^2]$. Then for any edge $f^*(e_i) \neq f^*(e_j), i \neq j$. Thus f admits square sum labeling. Therefore Jellyfish graph is a square sum graph.

CHAPTER - 3

3.1 AIM AND SCOPE

Graph is applied in real life to solve the real-life applications. Inspired by various Square sum Graphs, in this paper Square sum labeling introduced to Octopus graph and Vanessa graph.

CHAPTER – 4

4. RESULT

4.1 SQUARE SUM LABELING OF OCTOPUS GRAPH

THEOREM

The Octopus Graph (O_n), ($n \geq 2$) admits square sum labeling, where n is any positive integer.

PROOF

Let G be the Octopus Graph O_n , ($n \geq 2$). Let $\{u_1, u_2, \dots, u_n\} \cup \{x\} \cup \{v_1, v_2, \dots, v_n\}$ be the vertices of O_n . Here $|V(O_n)| = 2n+1$. Define a labeling $f: V(O_n) \rightarrow \{0, 1, 2, \dots, 2n\}$ as follows:

$$f(u_i) = i-1; 1 \leq i \leq n,$$

$$f(x) = n,$$

$$f(v_i) = n+i; 1 \leq i \leq n. \text{ Clearly vertex labels are distinct.}$$

Let $E = \{(u_i, x) / 1 \leq i \leq n\} \cup \{(x, v_i) / 1 \leq i \leq n\} \cup \{(v_i, v_{i+1}) / 1 \leq i \leq n\}$ be the edges of O_n . Here $|E(O_n)| = 3n-1$.

And f induces a function $f^*: E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. For any edge e_i, e_j $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Thus f admits square sum labeling. This proves that Octopus graph O_n is a square sum graph.

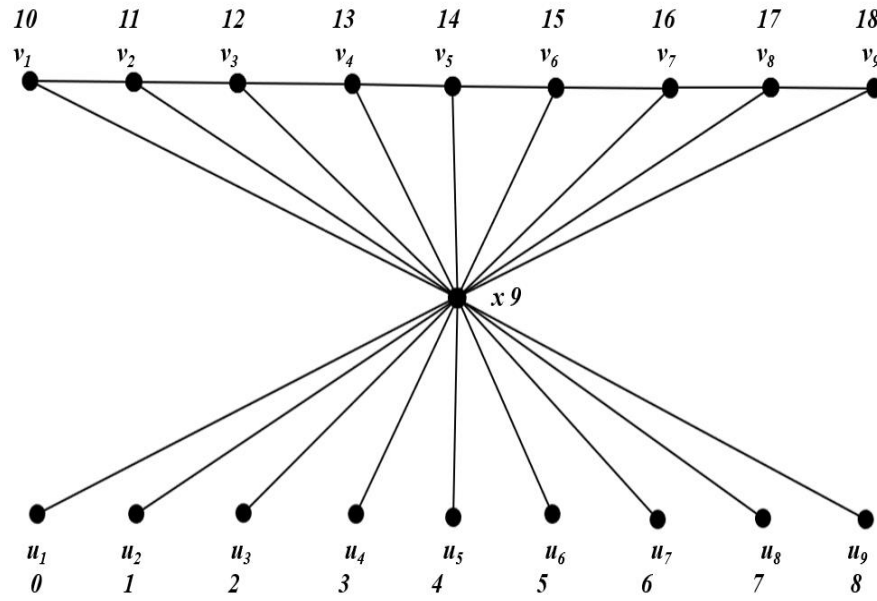


Fig 4.1.1 Octopus Graph $O_9 = F_9 + K_{1,9}$

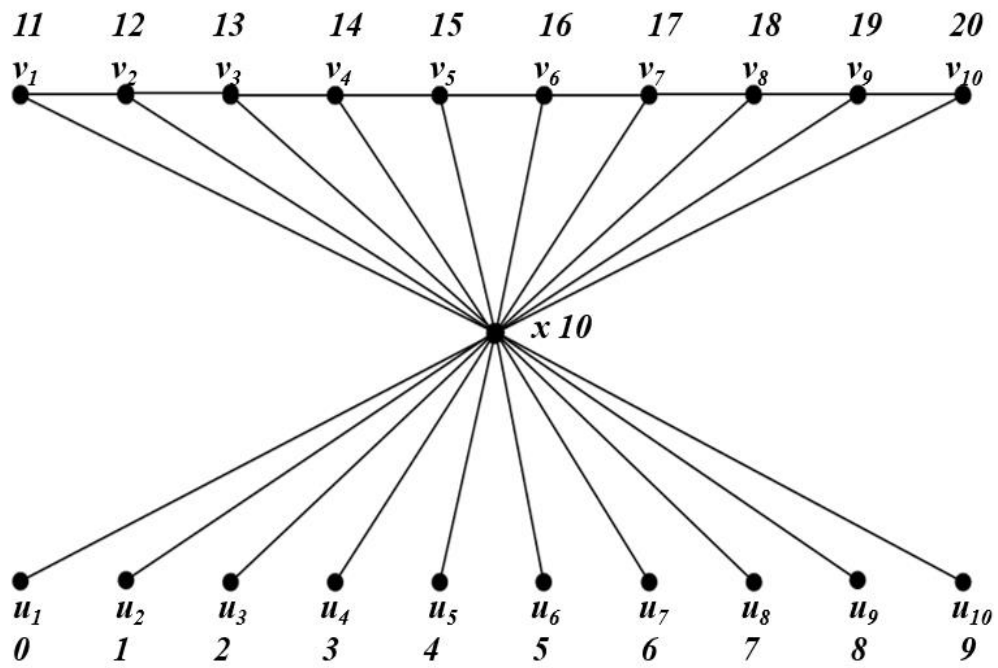


Fig 4.1.2 Octopus Graph $O_{10} = F_{10} + K_{1,10}$

4.2 SQUARE SUM LABELING OF VANESSA GRAPH

THEOREM

The Vanessa Graph (V_n) , $(n \geq 2)$ admits square sum labeling, where n is any positive integer.

PROOF

Let G be the Vanessa Graph V_n , $(n \geq 2)$. Let $\{u_1, u_2, \dots, u_n\} \cup \{u\} \cup \{x\} \cup \{v\} \cup \{v_1, v_2, \dots, v_{2n-2}\}$ be the vertices of V_n . Here $|V(V_n)| = 3n+1$.

Define a labeling $f : V(V_n) \rightarrow \{0, 1, 2, \dots, 3n\}$ as follows:

$$f(u_i) = i-1 ; 1 \leq i \leq n,$$

$$f(u) = n; f(x) = n+1;$$

$$f(v) = n+2,$$

$$f(v_i) = n+2+i ; 1 \leq i \leq 2n-2.$$

Clearly vertex labels are distinct.

Let $E = \{(u_i, x) / 1 \leq i \leq n\} \cup \{u, x\} \cup \{x, v\} \cup \{(v_i, x) / 1 \leq i \leq n\} \cup \{(v_i, v_{i+1}) / 1 \leq i \leq 2n-2\}$ be the edges of V_n . Here $|E(V_n)| = 5n-2$.

And f induces a function $f^* : E(G) \rightarrow N$ given by $f^*(uv) = [f(u)]^2 + [f(v)]^2$. For any edge e_i, e_j $f^*(e_i) \neq f^*(e_j)$, $i \neq j$. Thus f admits square sum labeling. This proves that Vanessa graph V_n is a square sum graph.

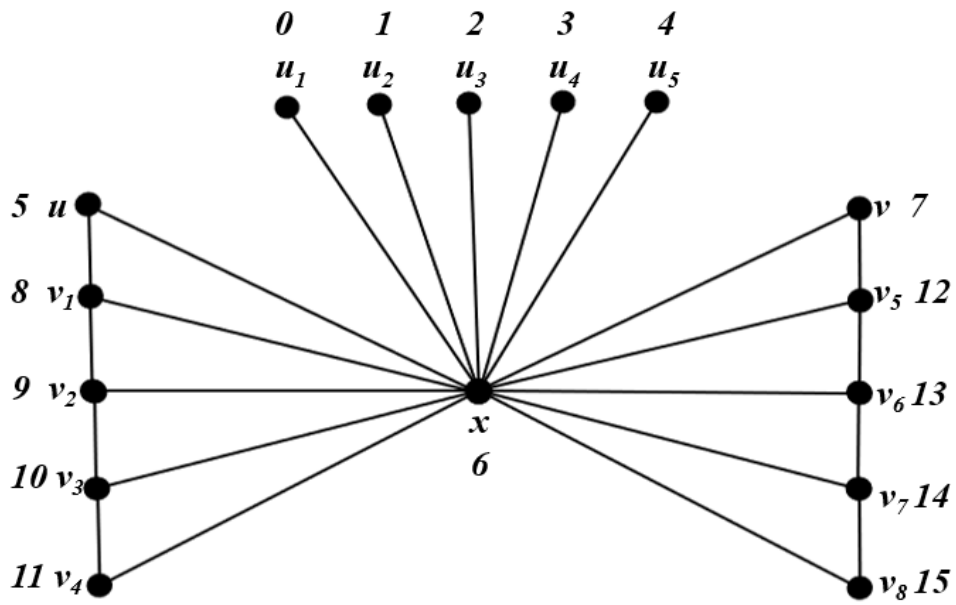


Fig 4.2.1 Vanessa graph $V_5 = 2F_5 + K_{1,5}$

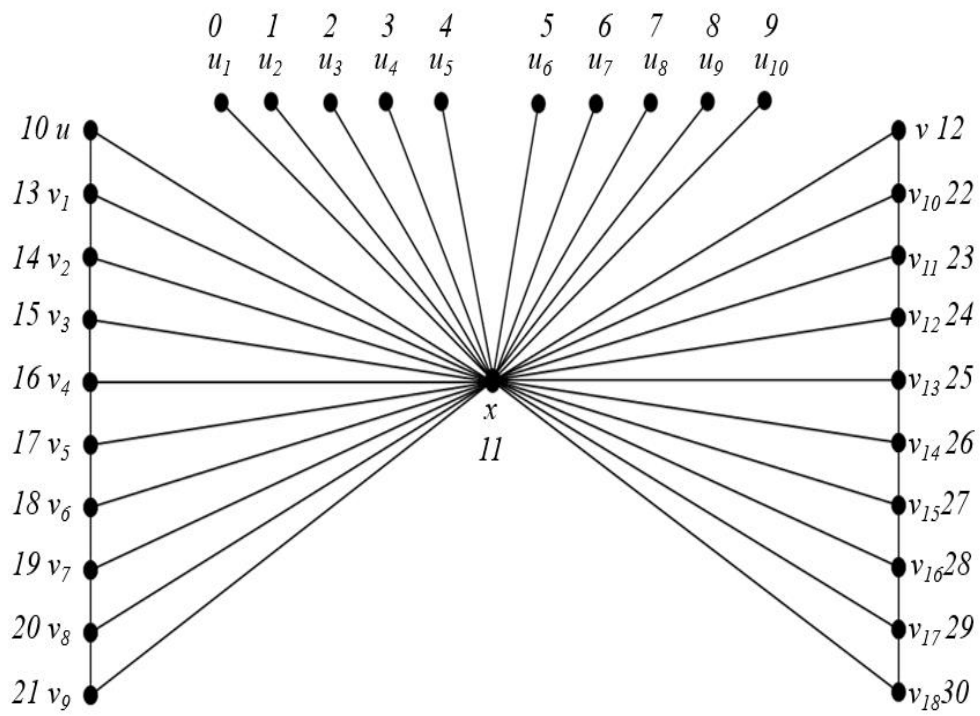


Fig 4.2.2 Vanessa graph $V_{10} = 2F_{10} + K_{1,10}$

CHAPTER – 5

5.1 CONCLUSION

It is inspiring to analyze Square sum labeling for some graphs. In this paper, we proved that Octopus graph and Vanessa graph are Square sum graphs. This can be extended for many challenging graphs in future.

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