

ANALYSING THE EXISTENCE OF MEAN LABELING FOR SOME SPECIAL GRAPHS

Submitted in partial fulfillment of the requirements for the award of
Bachelor of Science Degree in Mathematics

By

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **ANGELIN RESHMI T (38060001)**. Who carried out the project entitled "**ANALYSING THE EXISTENCE OF MEAN LABELING FOR SOME SPECIAL GRAPHS**" under our supervision from **January 2021 to March 2021**.

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DECLARATION

I **ANGELIN RESHMI T (38060001)** hereby declare that the Project Report entitled "**ANALYSING THE EXISTENCE OF MEAN LABELING FOR SOME SPECIAL GRAPHS**" done by me under the guidance of **Dr. R. MANOHARAN** (Internal) and **_** (External) at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Bachelor of Science Degree in Mathematics

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ABSTRACT

A graph G with p vertices and q edges is a mean graph if there is an injective function f from the vertices of G to $\{0,1,2,\dots,q\}$ such that when each edge uv is labeled with $\lfloor \frac{f(u)+f(v)}{2} \rfloor$ if $\lfloor \frac{f(u)+f(v)}{2} \rfloor$ is even and $\lfloor \frac{f(u)+f(v)+1}{2} \rfloor$ if $\lfloor \frac{f(u)+f(v)}{2} \rfloor$ is odd then the resulting edges are distinct. In this paper we investigate the existing of mean labeling.

KEYWORDS

Mean labeling

Double fan graph

Middle graph

Ladder graph

Banana tree graph

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CHAPTER – 1

INTRODUCTION

1.1 Graph Theory

It is a sub-field of mathematics which deals with graphs: diagrams that involve points and lines and which often pictorially represent mathematical truths. Graph theory is the study of the relationship between edges and vertices. Formally, a graph is a pair (V, E) , where V is a finite set of vertices and E a finite set of edges.

Graphs are therefore mathematical structures used to model pairwise relations between objects. They are found on road maps, constellations, when constructing schemes and drawings. Graphs underlie many computer programs that make modern communication and technological processes possible. They contribute to the development of thinking, both logical and abstract.

Swiss Mathematician Leonhard Euler transferred the Königsberg bridge problem into a graph and wrote a paper and issued it in 1736. He is well thought-out as the originator of graph theory. In detail, the word “graph” was introduced in a paper by Sylvester which was issued in 1878. The first book on graph theory was written by Denes König and was issued in 1936.

1.2 APPLICATION OF GRAPH THEORY

1 Computer science

In computer science graph theory is used like:

- Graphs are used to represent networks of communication.
- Graph theory is used to find shortest path in road or a network.
- In Google Maps, various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find the shortest path between two nodes.

2Electrical engineering

In Electrical Engineering, graph theory is used in designing of circuit connections. These circuit connections are named as topologies. Some topologies are series, bridge, star and parallel topologies.

3 Mathematics

In mathematics, operational research is the important field. Graph theory provides many useful applications in operational research. Like:

- Minimum cost path.
- A scheduling problem.

4 Other branches of science

- In physics and chemistry, graph theory is used to study molecules.
- The 3D structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms.
- Nodes in biological networks represent biomolecules such as genes, proteins or metabolites, and edges connecting these nodes indicate functional, physical or chemical interactions between the corresponding biomolecules.
- Graph theory is used in transcriptional regulation networks.

5 General

Graphs are used to represent the routes between the cities. With the help of tree that is a type of graph, we can create hierarchical ordered information such as **family tree**.

1.3 GRAPH LABELING

In the mathematical discipline of graph theory, a graph labelling is the assignment of labels, traditionally represented by integers, to edges and / or vertices of a graph. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Somasundaram and Ponraj have introduced the notion of mean labeling of graphs. When used without qualification, the term labeled graph generally refers to a vertex-labeled graph with all labels distinct. Such a graph may equivalently be labeled by the consecutive integers where n is the number of vertices in the graph. For many applications, the edges or vertices are given labels that are meaningful in the associated domain. For example, the edges may be assigned weights representing the "cost" of traversing between the incident vertice.

Most graph labeling trace their origins to labelings presented by Alexander Rosa in his 1967 paper. Rosa identified three types of labelings, which he called α , β -, and ρ -labelings. β -labelings were later renamed as "graceful" by Solomon Golomb, and the name has been popular since.

Types:

Graceful labelling, Edge-graceful labelling, Harmonious labelling, Lucky labelling.

1.4 APPLICATION OF GRAPH LABELING

Graph Labeling plays a vital role in many other field. It has various applications within mathematics as well as to several areas of computer science and communication networks. It is used in many applications like coding theory, x - ray crystallography, radar, astronomy, circuit design, communication network addressing, database management.

1.5 PRELIMINARIES AND BASIC TERMINOLOGY

This section comprises the basic definitions which are used in this study.

The double fan D_{fn} consists of two fan graph that have a common path

the double fan D_{fn} is obtained by $P_n + 2K_1$.

The middle graph $M(G)$ of a graph G is the graph whose vertex set is $V(G) \cup E(G)$ and in which two vertices are adjacent if and only if either they are adjacent edges of G or one is a vertex of G and the other is an edge incident on it.

A graph is called **regular graph** if degree of each vertex is equal. A graph is called **K regular** if degree of each vertex in the graph is K

A caterpillar or **caterpillar tree** is a tree in which all the vertices are within distance 1 of a central path.

The ladder graph L_n ($n \geq 2$) is the product graph $P_2 \times P_n$ Which contains **$2n$** vertices and **$3n - 2$** edges.

Banana tree (n, k) as defined is graph obtained by connecting one leaf of each of n copies of an **k – star** graph with single root vertex that is distinct from all the stars.

CHAPTER–2

2.1 LITERATURE SURVEY

The different types of graph labeling methods are presented here and the literature survey pertaining to them is also showcased in this section.

The origin of graph labeling, traces back to the introduction of graceful labeling by Rosa [88] in 1967. One of the valuations termed as the β -labeling, introduced by Rosa in 1967, served as a tool to attack the Ringel-Kotzig's Conjecture.

Golomb later renamed this β -valuation as graceful labeling and this is the most widely used term in recent years. A graceful labeling of a graph G with q edges is an injection $f: V(G) \rightarrow \{0, 1, 2, \dots, q\}$ with the property that the resulting edge labels are also distinct, where an edge incident with vertices u and v is assigned the label $f(u) - f(v)$.

S. Somasundaram and R. Ponraj introduced mean labeling of graphs in (2003a) and investigate mean labeling for some standard graphs in (2003b) and in (2004). They introduced Harmonic mean labeling of graphs in (Somasundaram, Ponraj & Sandhya) and proved Harmonic mean labeling of some standard graphs in (Sandhya, Somasundaram & Ponraj, 2012).

According to Ramya, Ponraj and Jeyanthi, a super mean graph is a mean graph in which vertex labels and the edge labels are $\{1, 2, \dots, p+q\}$. It has been also proved that P_n , odd cycles, P_n^2 , L_n , K_1 , $C_m \cup P_n$ ($n \geq 2$), $B_{n,n}$ and $B_{n+1,n}$ admit super mean labeling while K_n and $K_{1,n}$ are not super mean graphs for $n > 3$.

Christian Barrientos and Elliot Krop presented several results on mean graphs. They studied the mean labelings of some regular graphs; including the disjoint union of n triangles as well as the non-existence of a mean labeling when G is a r -regular

graph of order n and $r \in \{n-1, n-2\}$. The main result of shows how to transform an α -labeling into a mean labeling. They introduced the concept of α -mean labelings, mainly use α -mean graphs to construct new α -mean graphs using two distinct methods. They completely solved the problem of α -mean labelings of quadrilateral snakes.

CHAPTER - 3

3.1 AIM AND SCOPE

In this paper I am going to analysis some results for Double fan graph and Middle graph whether Mean labeling is applicable or not. Also, I am going to find more new graphs for Ladder graph and Banana tree graph using my references.

The main objectives of the work reported is:

To analysis the existence of Mean labeling for the following four graphs

- Double fan graph
- Middle graph
- Ladder graph
- Banana tree graph

CHAPTER – 4

4. MEAN LABELING OF SOME SPECIAL GRAPHS

In this section we have discussed the existence of mean labeling for some special graphs like double fan graph, Middle graph, Ladder graph and Banana tree graph.

4.1 MEAN LABELING FOR DOUBLE FAN GRAPH $C(r \cdot Df_n)$

THEOREM: $C(r \cdot Df_n)$ is a mean graph for n even, where $r \equiv 0 \pmod{2}$.

PROOF: Let $G = C(r \cdot Df_n)$ be a cycle of double fan Df_n with r copies.

Let f be the mean labeling function for Df_n defined as

$$f(u) = 0; f(v) = 3n - 1;$$

$$f(v_j) = 3j - 1, j \equiv 1 \pmod{2},$$

$$f(v_j) = 3j - 2, \text{ otherwise.}$$

In graph G we see that the vertices $|V(G)| = r(n+2)$ and the edges $|E(G)| = 3rn$. Let $u_i, v_{i,j}$ ($0 \leq j \leq n$) be vertices of i -th copy of Df_n , $\forall i = 1, 2, \dots, r$, where the vertices of i -th copy of Df_n is $p = n+2$ and edges of i -th copy of Df_n is $q = 3n-1$.

Join the vertices u_i to $v_{i+1,0}$, $\forall i = 1, 2, \dots, r-1$ and $v_{r,1}$ to $v_{1,0}$ by an edge to form the cycle of r copies of double fan.

We define the labeling function $g : V(G) \rightarrow \{0, 1, \dots, Q\}$ as follows,

$$g(u_1) = f(u) + (Q - q); g(v_1, 0) = f(v) + (Q - q),$$

$$g(v_1, j) = f(v_j) + (Q - q), \forall j = 1, 2, \dots, n;$$

$$g(u_i) = g(u_{i-1}) - (q + 1), \forall i = 2, 3, \dots, r/2,$$

$$g(v_{i,j}) = g(v_{i-1,j}) - (q + 1), \forall j = 0, 1, 2, \dots, n, \forall i = 2, 3, \dots, r/2;$$

$$g(u_{r/2+1}) = g(u_{r/2}) - (q + 2), g(v_{r/2+1,j}) = g(v_{r/2,j}) - (q + 2), \forall j = 0, 1, 2, \dots,$$

n ;

$$g(u_i) = g(u_{i-1}) - (q + 1), \forall i = r^2 + 2, r^2 + 3, \dots, r,$$

$$g(v_{i,j}) = g(v_{i-1,j}) - (q + 1), \forall j = 0, 1, 2, \dots, n, \forall i = r^2 + 2, r^2 + 3, \dots, r.$$

Above labeling pattern give rise mean labeling to the given cycle of double fan.

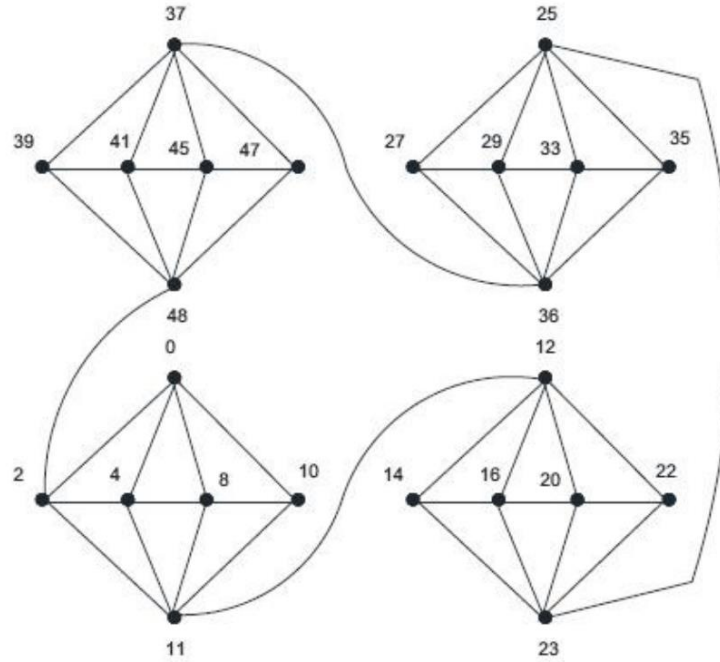


Fig 4.1 $C(4 \cdot Df4)$ and its mean labeling

4.2 MEAN LABELING FOR MIDDLE GRAPH $C(r \cdot M(C_n))$

THEOREM: $C(r \cdot M(C_n))$ is a mean graph, where $r \equiv 0 \pmod{2}$.

PROOF : Let $G = C(r \cdot M(C_n))$ be a cycle of middle graph $M(C_n)$ with r copies. Let f be the mean labeling function for $M(C_n)$ defined as

$C(r \cdot M(C_n))$ is a mean graph, where $r \equiv 0 \pmod{2}$

Case I : $n \equiv 1 \pmod{2}$

$$f(v_j) = 3(j - 1), \forall j = 1, 2, \dots, (n - 1)/2$$

$$f(v'_j) = 3j - 1, \forall j = 1, 2, \dots, (n - 1)/2 ;$$

$$f(v_j) = (3n - 1)/2, \text{ for } j = (n + 1)/2,$$

$$f(v'_{j-1}) = (3n + 5)/2, \text{ for } j = (n + 1)/2;$$

$$f(v_j) = 3j, \forall j = (n + 3)/2, (n + 5)/2, \dots, n,$$

$$f(v'_{j-1}) = 3j - 1, \forall j = (n + 3)/2, (n + 5)/2, \dots, n$$

Case 2 : $n \equiv 0 \pmod{2}$

$$f(v_j) = 3(j - 1), \forall j = 1, 2, \dots, n/2,$$

$$f(v'_{j-1}) = 3j - 1, \forall j = 1, 2, \dots, n/2;$$

$$f(v_n) = 3n; f(v'_{n-1}) = 3n - 2; f(v'_{n/2+1}) = 3n/2; f(v_j) = 3j + 2, \forall j = n/2 + 1, n/2 + 2, \dots, n - 1;$$

$$f(v'_{j-1}) = 3j - 5, \forall j = n/2 + 2, n/2 + 3, \dots, n - 1.$$

In graph G we see that the vertices $|V(G)| = P = 2rn$ and the edges $|E(G)| = Q = 3rn + r$. Let $v_{i,j}$ ($1 \leq j \leq n$), $v_{0,i,j}$ ($1 \leq j \leq n$) be vertices of i -th copy of $M(C_n)$, $\forall i = 1, 2, \dots, r$, where the vertices of i -th copy of $M(C_n)$ is $p = 2n$ and edges of i -th copy of $M(C_n)$ is $q = 3n$. Join the vertices $v_{i,1}$ to $v_{i+1,n}$, $\forall i = 1, 2, \dots, r - 1$ and $v_{0,r,n-1}$ to $v_{1,n}$ by an edge to form the cycle of r copies of middle graph.

We define the labeling function $g : V(G) \rightarrow \{0, 1, \dots, Q\}$ as follows,

$$g(v_{1,j}) = f(v_j) + (Q - q), \forall j = 1, 2, \dots, n,$$

$$g(v'_{1,j}) = f(v'_{j-1}) + (Q - q), \forall j = 1, 2, \dots, n;$$

$$g(v_{i,j}) = g(v_{i-1,j}) - (q + 1), \forall j = 1, 2, \dots, n, \forall i = 2, 3, \dots, r/2,$$

$$g(v'_{i,j}) = g(v'_{i-1,j}) - (q + 1), \forall j = 1, 2, \dots, n, \forall i = 2, 3, \dots, r/2;$$

$$g(v_{r/2+1,j}) = g(v_{r/2,j}) - (q + 2), \forall j = 1, 2, \dots, n,$$

$$g(v'_{r/2+1,j}) = g(v'_{r/2,j}) - (q + 2), \forall j = 1, 2, \dots, n;$$

$$g(v_{i,j}) = g(v_{i-1,j}) - (q + 1), \forall j = 1, 2, \dots, n, \forall i = r/2 + 2, r/2 + 3, \dots, r,$$

$$g(v_i', j) = g(v_{i-1}, j) - (q + 1), \forall j = 1, 2, \dots, n, \forall i = r/2 + 2, r/2 + 3, \dots, r.$$

Above labeling pattern give rise mean labeling to the given cycle of middle graph.

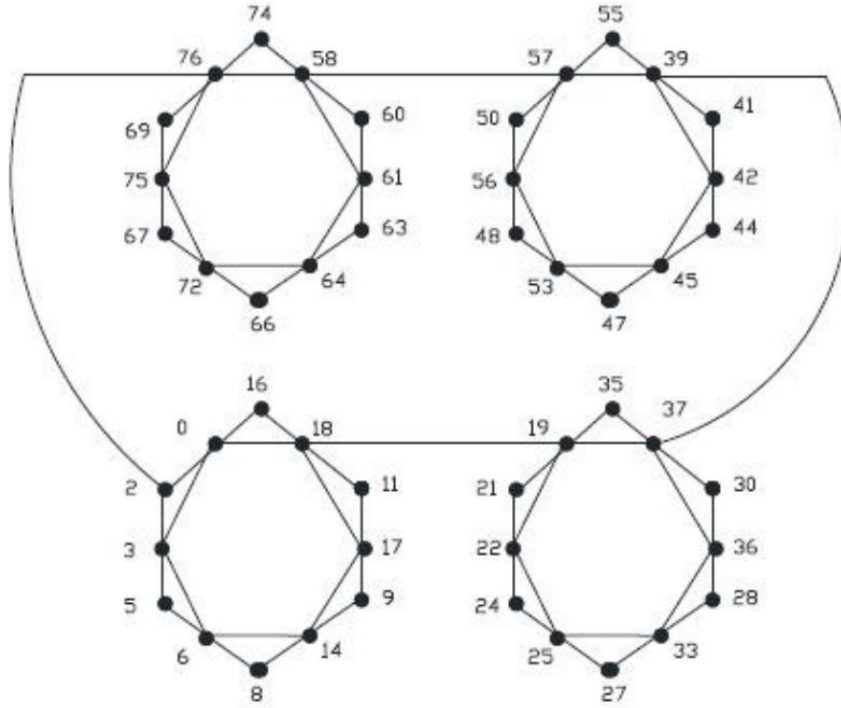


Fig 4.2 $C(4 \cdot M(C_6))$ and its mean labeling

4.3 SOME MORE RESULTS OF LADDER GRAPH

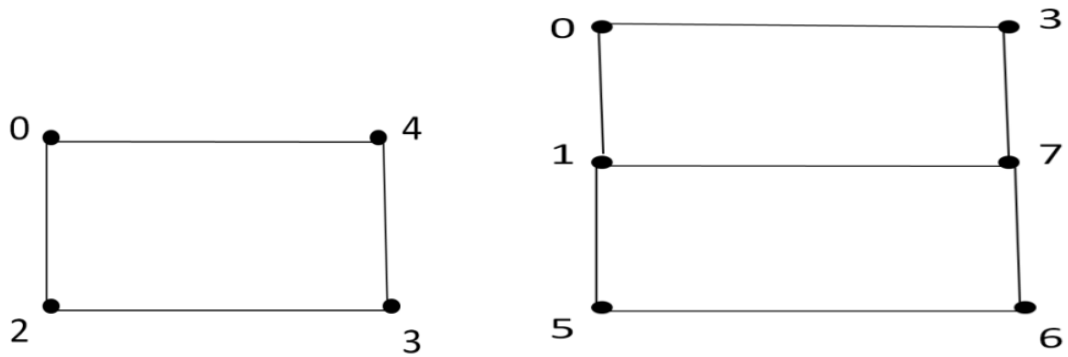
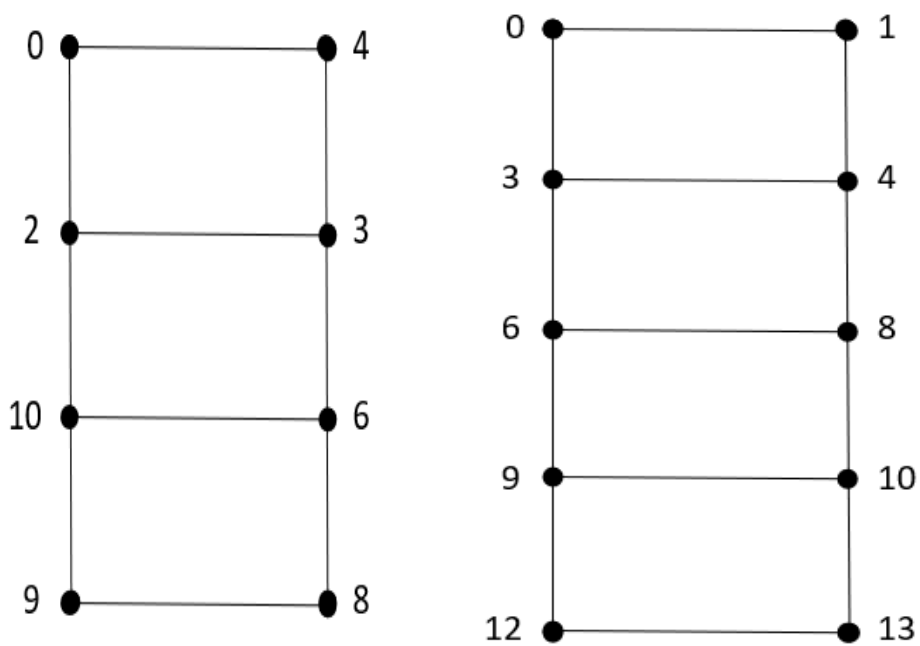


Fig 4.3(a)(i) Ladder graph for $n=2$ Fig 4.3(b)(ii) Ladder graph for $n=3$



Fig

4.3(c)(iii) Ladder graph for $n=4$ Fig 4.3(d)(iv) Ladder graph for $n=5$

4.4 SOME MORE RESULTS OF BANANA TREE GRAPH

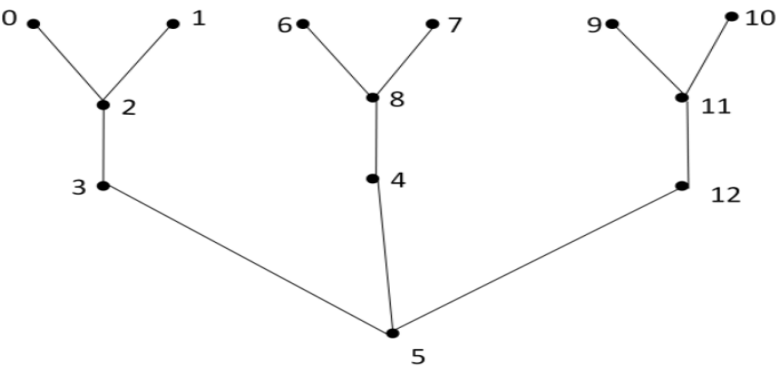


Fig 4.4(a)Mean labeling for $B_{3,4}$

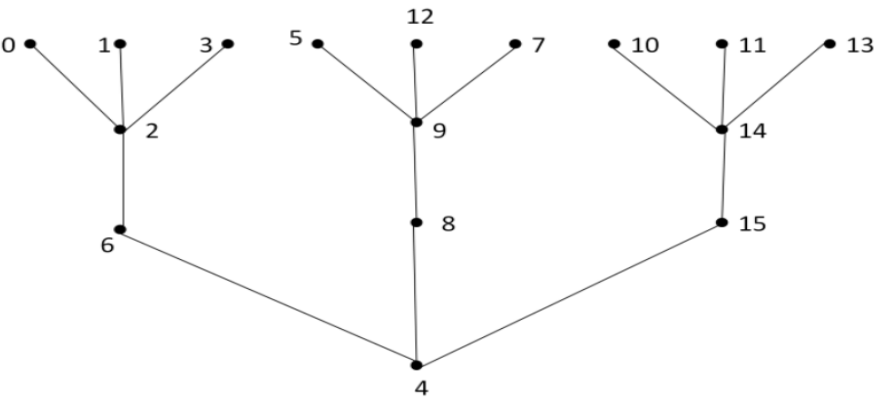


Fig 4.4(b)Mean labeling for $B_{3,5}$

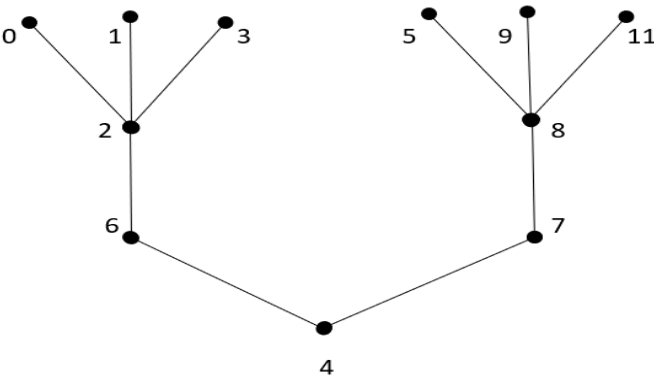


Fig 4.4(c)Mean labeling for $B_{3,5}$

CHAPTER – 5

5.1 CONCLUSION

It is inspiring to analyze Mean labeling for some graphs. In this paper we have computed Mean labeling for Double fan graph and Middle graph. Also found mean labeling some new Ladder graphs and Banana tree graphs. This can be further extended to fuzzy labeling.

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