

BLOOD FLOW IN AN ARTERY

Submitted in partial fulfillment of the requirements for the
award of Master of Science degree in Mathematics

By

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(DEEMED TO BE UNIVERSITY)

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **M.RAMYA (40630009)** who carried out the project entitled "**BLOOD FLOW IN AN ARTERY**" under the supervision from **November 2021 to May 2022**

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I **M. RAMYA** (**40630009**) hereby declare that the Project Report entitled "**BLOOD FLOW IN AN ARTERY**" done by me under the guidance of **Dr. H.GIRIJA BAI** (Internal) and ----- (External) at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** is submitted in partial fulfillment of the requirements for the award of Masters of Science Degree in Mathematics.

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ABSTRACT:

The aim of this study is a numerical simulation of hemodynamics in blood vessels with an aneurysm. Dilation of 0.25 is considered. Hemodynamic factors such as velocity and pressure are investigated using techniques in Computational fluid dynamics. The problem is solved by the finite volume method. Numerical simulation is prescribed using the CFD software Fluent and Gambit. High pressure and low velocity are observed in the interruption of the aneurysm. This is the indication of interruption of blood flow. These techniques based on computer flow are important for understanding the relationship between hemodynamic parameters and the risk of rupture.

KEYWORDS:

- Aneurysms
- Hemodynamics
- Computational fluid dynamics
- Blood flow
- Finite Volume Method

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CHAPTER-1

1.1 INTRODUCTION:

Hemodynamics is the study of physical factors involved in blood circulation. It refers to physical factors governing the blood in the circulatory system. An aneurysm is a balloon-like dilation found on the walls of a blood vessel or a sac formed by the localized dilation of the wall of an artery or a vein, or the heart. The Unprocessed aneurysm may rupture under insistent internal pressure, causing fatality or severe disability. Even an un ruptured aneurysm can lead to damage by interrupting the flow of blood or by impinging on the wall of the vessel, in some cases eroding nearby blood vessels, organs, or bone. Aneurysms are seen most often in large arteries such as the iliac, femoral, popliteal, carotid or renal arteries. Hemodynamic parameters are considered to be responsible for initiating the growth and rupture of aneurysms.

In arterial walls, if increased blood flow occurs, it leads to enlargement of vessel diameter and reduction of shear forces. To understand the aneurysm behavior, flow dynamics have been studied in multitude experimental models.

The major application of CFD is the Aerodynamic design of impeller blades of axial flow compressors and centrifugal compressors, blood flow analysis, Petroleum engineers to design optimum oil-recovery strategies.

CHAPTER-2

2.1 LITERATURE SURVEY:

The Pathlines of the flow particles were analyzed by Perktold K (1987)

David N. Ku (1997) discussed the study of arterial blood flow.

Malek, Alpher and Izumo (1999) were discussed the initiation, growth and rupture of aneurysms. It is extensively accepted that the wall of aneurysm is assumed to be rigid using (CFD)

Hassan et al. and J Neurosurg (2005) were discussed using Computational Fluid Dynamics analysis (CFDA) of simple models representing the three aneurysm categories. The authors analyzed geometry-related risk factors such as neck, width, parent artery curvature and angulation of the branching vessels.

H.Girija Bai et al (2014,2015) were discussed Hemodynamic factors such as velocity, pressure, wall shear stress and streamline patterns in blood vessels. High pressure and low velocity are observed in the region of aneurysms.

Mohammed Nasir Uddin, Md.Abdul Alim, Md.Mashud Karim and M.M.Alam (2021) discussed the important effects on the blood flow of the aneurysmatic artery. Blood velocity, pressure and wall shear stress profiles are presented graphically for Newtonian and non-Newtonian models.

CHAPTER – 3

3.1 AIM AND SCOPE:

The aim of this project aims to analyze numerical simulation of hemodynamics in arteries affected by aneurysms using Computational fluid dynamics (CFD).

In practice, widely used numerical methods are FDM, FVM, FEM etc. Compared to other numerical methods Finite Volume method is easy to implement and find the blood flow in arteries affected by aneurysms.

3.2 RESEARCH METHODOLOGY:

CFD Software Gambit is used to discretize the model. Governing equations were solved in FLUENT which uses the finite volume method. The file obtained in Gambit is imported to CFD software FLUENT and processed.

Computational fluid dynamics is used in many areas, such as engineering and the medical field. This new field provides very detailed information about fluid characteristics. Medical science lent this new technology to study hemodynamics within the body. An aneurysm, CFD help us to understand its formation, growth and rupture. CFD software Gambit is used to discretize the model.

3.3 FINITE VOLUME METHOD:

The Finite Volume Method (FVM) is a numerical technique that transforms the partial differential equations representing conservation laws over differential volumes into discrete algebraic equations over finite volumes (or elements or cells). In a similar fashion to the finite difference or finite element method, the first step in the solution process is the discretization of the geometric domain, which in the FVM, is discretized into non-overlapping elements or finite volumes. The partial differential equations are then discretized/transformed into algebraic equations by integrating them over each discrete element. The system of algebraic equations is then solved to compute the values of the dependent variable for each of the elements.

CHAPTER -4

4.1 GOVERNING EQUATIONS:

Equations of momentum and mass conservation for incompressible fluid can be written as $\nabla \cdot \bar{v} = 0$

$$\rho \left(\frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} \right) = -\nabla p + \mu \nabla^2 \bar{v}$$

Where: ρ - density of blood, $\bar{v}$ - velocity field, p – pressure, μ - co-efficient of viscosity

The Navier-Stoke System of equations: Incompressible Flow case:

The vector form of unsteady three-dimensional motion of a viscous, incompressible and Isothermal flow:

$$\frac{\partial \bar{Q}}{\partial t} + \frac{\partial \bar{E}_i}{\partial x} + \frac{\partial \bar{F}_i}{\partial y} + \frac{\partial \bar{G}_i}{\partial z} = \frac{\partial \bar{E}_v}{\partial x} + \frac{\partial \bar{F}_v}{\partial y} + \frac{\partial \bar{G}_v}{\partial z}$$

Where $\bar{Q}$ is the vector containing the primitive variables and is given by

$$\bar{Q} = \begin{bmatrix} 0 \\ u \\ v \\ w \end{bmatrix}$$

and E_i , F_i , G_i are the vectors containing the inviscid fluxes in the X, Y and Z directions are given by

$$E_i = \begin{pmatrix} u \\ u^2 + p \\ uv \\ uw \end{pmatrix}, F_i = \begin{pmatrix} v \\ vu \\ v^2 + p \\ vw \end{pmatrix}, G_i = \begin{pmatrix} w \\ wu \\ wv \\ w^2 + p \end{pmatrix}$$

The viscous fluxes in the x, y and z directions are defined as follows

$$E_v = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \end{pmatrix}, F_v = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \end{pmatrix}, G_v = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \end{pmatrix},$$

Since we made the assumptions of an incompressible flow, appropriate non dimensional terms and expressions for shear stress must be used these expressions are given as follows

$$\tau_{xx} = \frac{2}{Re_L} \frac{\partial u}{\partial x}, \quad \tau_{yy} = \frac{2}{Re_L} \frac{\partial v}{\partial y}, \quad , \tau_{zz} = \frac{2}{Re_L} \frac{\partial w}{\partial z},$$

$$\tau_{xy} = \frac{1}{Re_L} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) = \tau_{yx},$$

$$\tau_{xz} = \frac{1}{Re_L} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) = \tau_{zx},$$

$$\tau_{yz} = \frac{1}{Re_L} \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right) = \tau_{zy}$$

Transformation of the governing equations from Cartesian coordinates (x ,y, z, t) to computational space with generalized curvilinear coordinates (ξ, η, ζ, t)

$$\frac{\partial \widehat{Q}}{\partial t} + \frac{\partial \widehat{E}_t}{\partial \xi} + \frac{\partial \widehat{F}_t}{\partial \eta} + \frac{\partial \widehat{G}_t}{\partial \zeta} = \frac{\partial \widehat{E}_v}{\partial \xi} + \frac{\partial \widehat{F}_v}{\partial \eta} + \frac{\partial \widehat{G}_v}{\partial \zeta}$$

Where $\widehat{Q} = \frac{\bar{Q}}{J_x}$

$$\widehat{E}_t = \frac{1}{J_x} (\xi_x \vec{E}_t + \xi_y \vec{F}_t + \xi_z \vec{G}_t)$$

$$\widehat{F}_t = \frac{1}{J_x} (\eta_x \vec{E}_t + \eta_y \vec{F}_t + \eta_z \vec{G}_t)$$

$$\widehat{G}_t = \frac{1}{J_x} (\zeta_x \vec{E}_t + \zeta_y \vec{F}_t + \zeta_z \vec{G}_t)$$

$$\begin{aligned}\widehat{E}_v &= \frac{1}{J_x} (\xi_x \overrightarrow{E_v} + \xi_y \overrightarrow{F_v} + \xi_z \overrightarrow{G_v}) \\ \widehat{F}_v &= \frac{1}{J_x} (\eta_x \overrightarrow{E_v} + \eta_y \overrightarrow{F_v} + \eta_z \overrightarrow{G_v}) \\ \widehat{G}_v &= \frac{1}{J_x} (\varsigma_x \overrightarrow{E_v} + \varsigma_y \overrightarrow{F_v} + \varsigma_z \overrightarrow{G_v})\end{aligned}$$

$(\widehat{Q})$ is the vector containing the primitive variables.

$$\widehat{Q} = \frac{1}{J_x} \begin{pmatrix} 0 \\ u \\ v \\ w \end{pmatrix}$$

$$\widehat{E}_i = \frac{1}{J_x} \begin{bmatrix} U \\ uU + p\xi_x \\ vU + p\xi_y \\ wU + p\xi_z \end{bmatrix}; \quad \widehat{F}_i = \frac{1}{J_x} \begin{bmatrix} V \\ uV + p\eta_x \\ vV + p\eta_y \\ wV + p\eta_z \end{bmatrix}, \quad \widehat{G}_i = \frac{1}{J_x} \begin{bmatrix} W \\ uW + p\varsigma_x \\ vW + p\varsigma_y \\ wW + p\varsigma_z \end{bmatrix}$$

BOUNDARY CONDITION:

$U = 0, V = 0$, on the dilated vessel CD

$f_x = 0, v = 0$ on the central axis AB

$f_x = 0, f_y = 0$ on the outflow segment BC

The file obtained in Gambit is imported into the software program FLUENT and processed. We take density $\rho = 1060$ [Kg/ms] and dynamic viscosity $\eta = 0.003$ [Kg/ms], input velocity 0.3[Kg/ms]. Although blood has non-Newtonian behavior, in the simulation it is considered Newtonian because there were no significant differences in the distribution of wall shear stress.

Average blood flow speed in the artery is 0.344 [m/s] and peak systolic drop occurs at $t = 0.13$ [s].

CHAPTER – 5

5.1 RESULT AND DISCUSSION:

Governing equations were solved in Computational fluid dynamics which uses finite volume method for discretization. Detailed study of blood flow analysis in arteries is made.

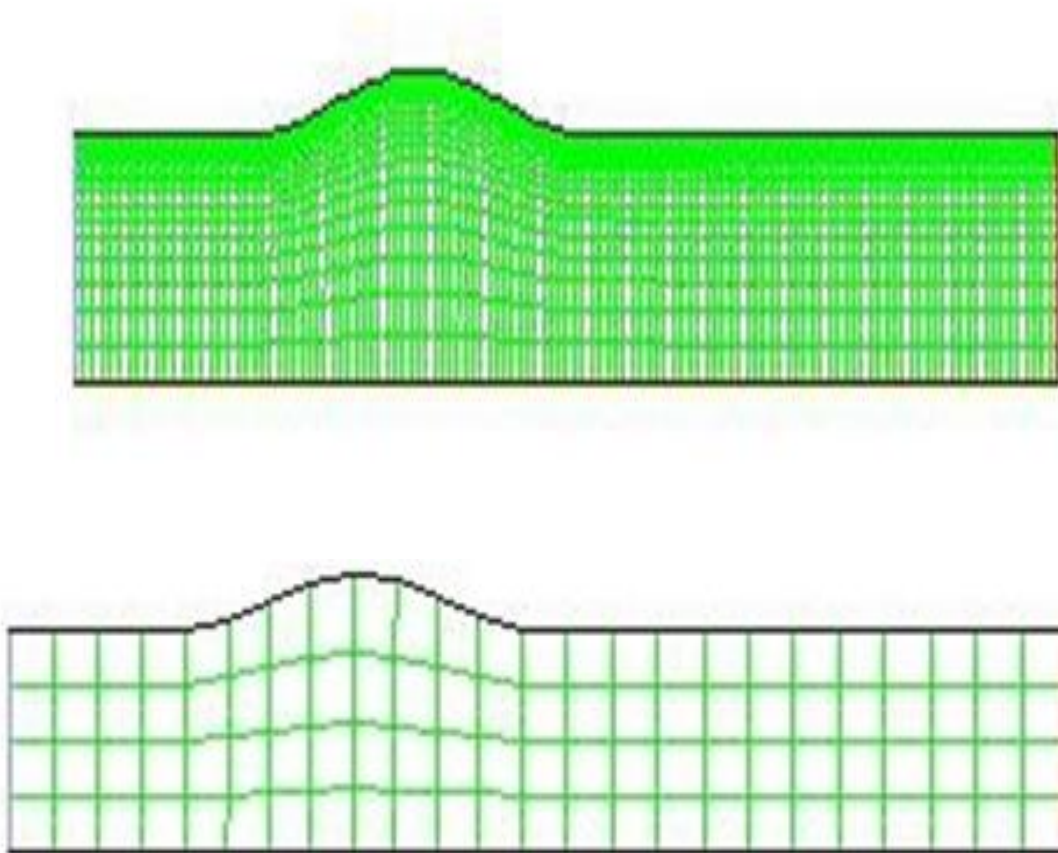


Figure 1 Grid of Single Aneurysm

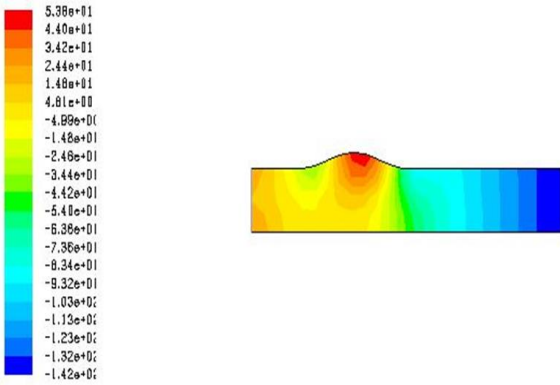
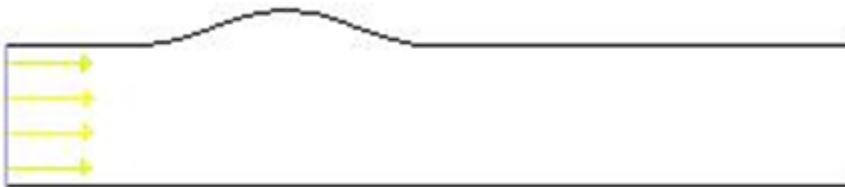
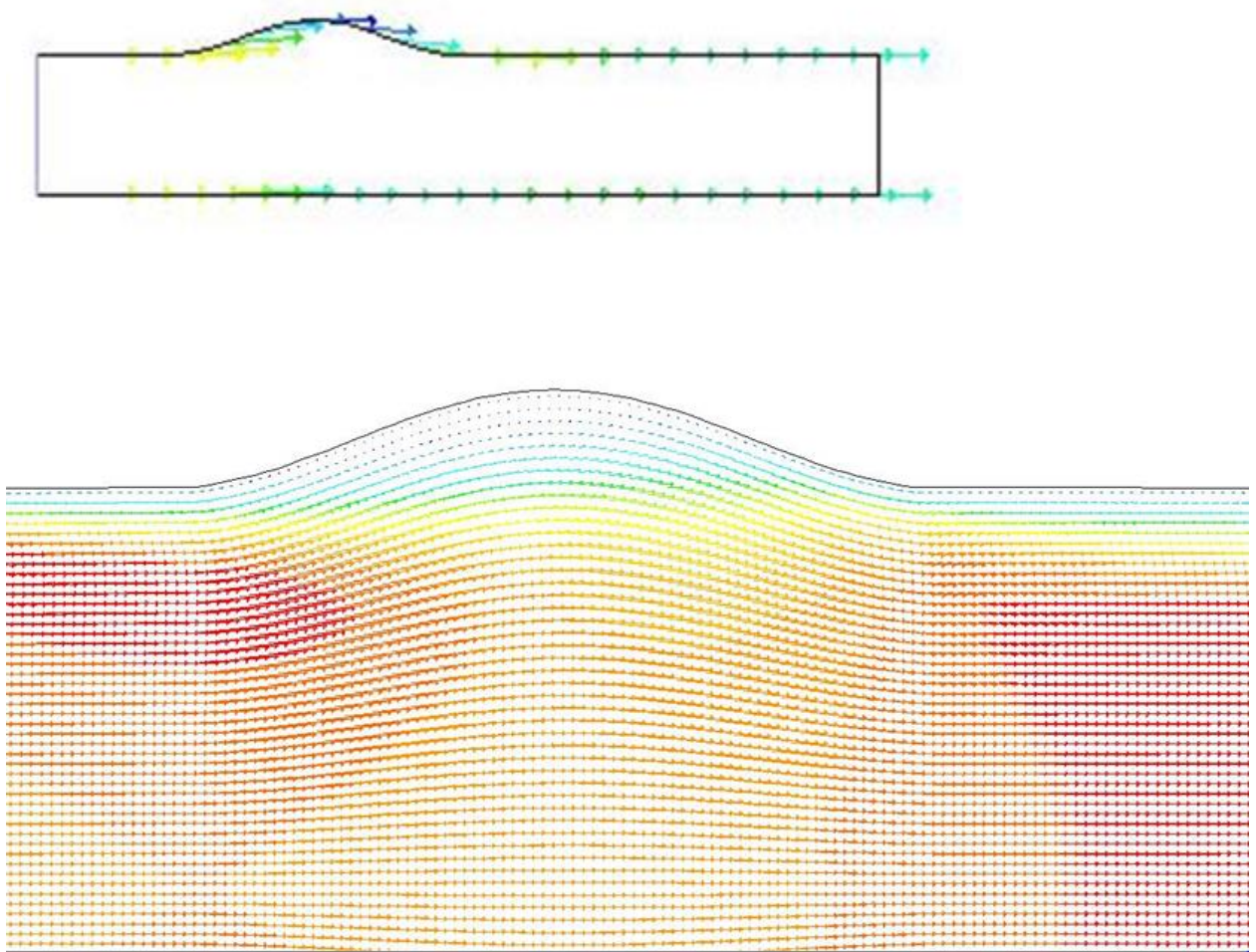


Figure 2 Contours of Static Pressure

Static Pressure means the force per unit area that is exerted by a fluid upon a surface at rest relative to the fluid.

Static pressure varies between -1.42×10^2 Pa to 5.38×10^1 Pa as shown in figure 2





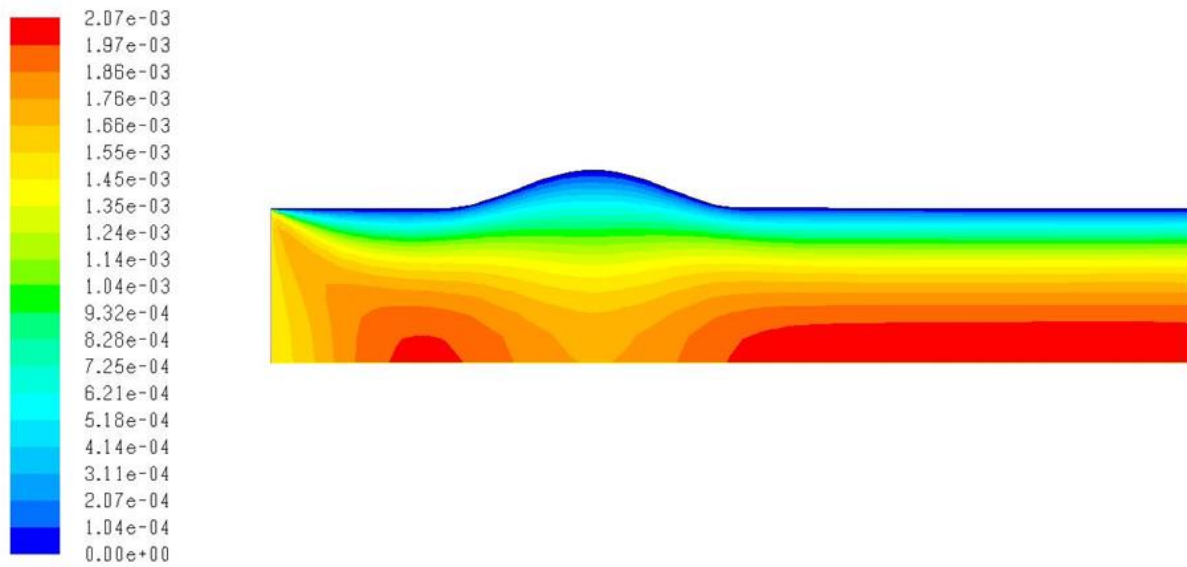


Figure 3. Contours of Velocity Magnitude (m/s)

Velocity varies between 0 to 2.07×10^{-3}

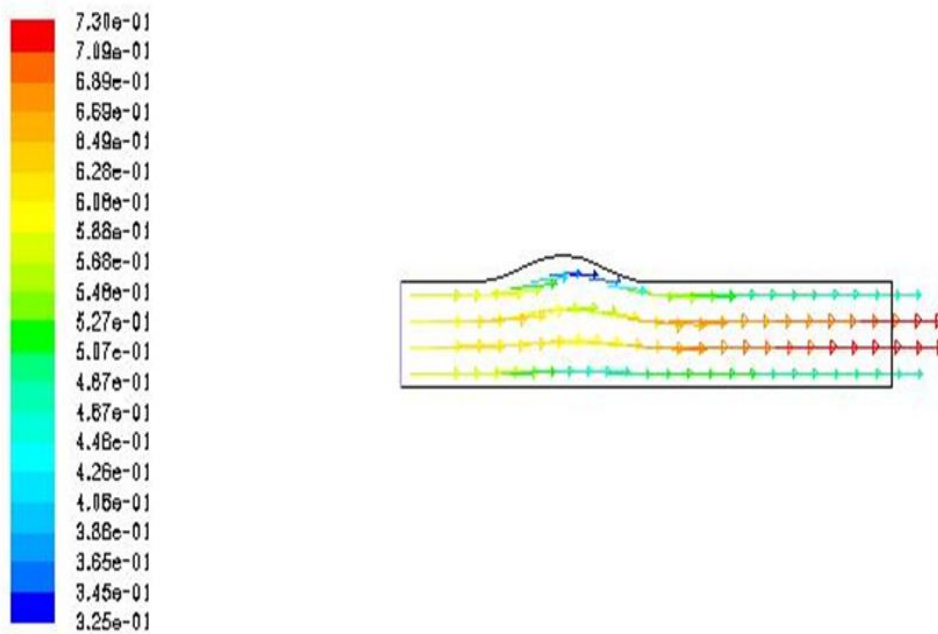
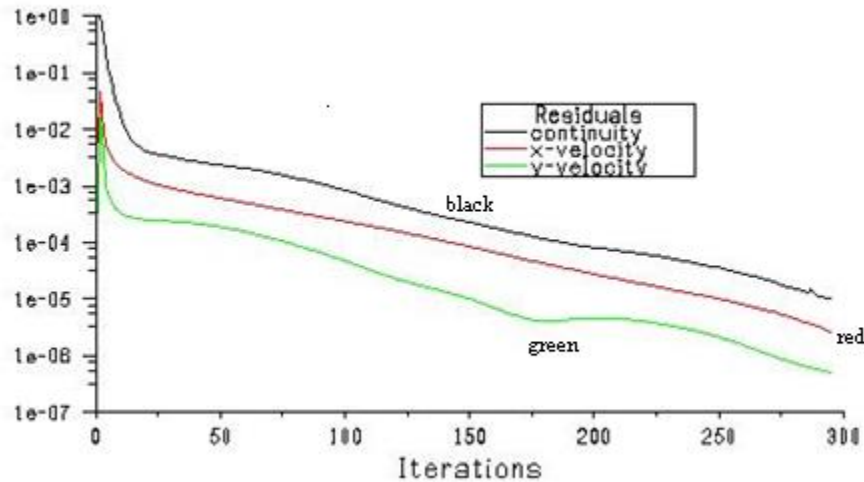


Figure 4. Stream lines – velocity vectors for single Aneurysm

Streamlines are a family of curves that are instantaneously tangent to the velocity vector of the flow. They show the direction of a fluid element that travel at any point at any time. The streamlines are representative of the speed in a given time as shown in the figure 4.



Navier stroke equations was solved by FVM and solution were converged as shown in figure 5

By considering equation of continuity - ----- (black)

Momentum equation x velocity - ---- (red)

Momentum equation Y velocity - ---- (green)

Figure 5. Iteration

RESULTS:

Governing equations were solved in FLUENT which uses finite volume method for discretization conducted in Gambit. Detailed study of velocity, pressure and streamlines are made.

The velocity and pressure of the fluid vary, through an artery that narrows or widens. The artery is approximately circular, velocity distribution is considered parabolic in the artery entry, with a maximum into the centre of the vessel and a minimum close to the vessel wall. Each aneurysm has its unique hemodynamic profile, but many aneurysms have the same characteristics. The most common jet inflow had a maximum collision in the aneurysm neck. After impact on the wall of the aneurysms, the inflow jet disintegrates into one or more “whirlpools”, depending on the aneurysm geometry.

CHAPTER - 6

6.1 SUMMARY AND CONCLUSION:

- Significant changes in blood flow are found in this investigation. The blood velocity has been reduced due to the presence of permeability aneurysm is another important finding.
- The effect of the blood flow on the wall shear stress is an important factor to determine fatal arterial diseases.

Flow analysis shows that the flow model is not similar for all aneurysms. Flow characteristics are highly dependent on the geometry of the vessels and aneurysms. High pressure and low velocity are obtained in the region of aneurysm. These techniques based on computer flow study are important for understanding the relationship between hemodynamic parameters and risk of rupture and will be very useful since they can be done in a routine clinical environment.

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THANK YOU