

TOTAL MAGIC CORDIAL LABELING OF SOME GRAPHS

Submitted in partial fulfillment of the requirements for the
award of Master of Science degree in Mathematics

By

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(DEEMED TO BE UNIVERSITY)

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **C. JAYALAKSHMI (40630004)** who carried out the project entitled "**TOTAL MAGIC CORDIAL LABELING OF SOME GRAPHS**" under our supervision from **November 2021 to May 2022**.

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ABSTRACT:

A graph $G (V, E)$ is said to have a total magic cordial labeling if there exists a mapping $f: V(G) \cup E(G) \rightarrow \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C(\text{mod } 2)$ for all edges $ab \in E(G)$ provided the condition $|f_0 - f_1| \leq 1$ is hold, where $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively. This project found that Octopus graph, Vanessa graph, Bow graph, Lilly graph, Tortoise graph and Lotus graph have total magic cordial labeling for all values of n .

KEYWORDS:

Graph labeling, Cordial labeling, Total magic cordial labeling, Octopus graph, Vanessa graph, Bow graph, Lilly graph, Tortoise graph and Lotus graph.

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CHAPTER – 1

INTRODUCTION

1.1 GRAPH THEORY

Many structures involving real-world situations can be conveniently represented on paper by means of a diagram consisting of a set of points (usually drawn as small circles or dots) together with lines (or curves) joining some or all pairs of these points. For example, the points in a diagram could represent different cities in a country, and a line joining two points that does not pass through a third point may indicate that there is direct air services between the two cities represented by those two points. A mathematical abstraction of such structures involving points and lines leads us to the concept of graphs and digraphs. **Graph theory** is the study of graphs, which are Mathematical structures used to model pairwise relations between objects.

A **graph** G consists of a set V of vertices and a collection E of unordered pairs of vertices called edges. A graph is symbolically represented as $G = (V, E)$. When there is no concern about the direction of an edge the graph is called undirected. The unordered pair of vertices are joined by a line for the undirected graph but ordered pair of vertices are joined by a line for the directed graph.

1.2 APPLICATION OF GRAPH THEORY

Graph theoretical concepts are widely used to study and model various applications in different areas. In Google Maps, various locations are represented as vertices or nodes and the roads are represented as edges and graph theory is used to find the shortest path between two nodes. Graph is also helpful in constructing the molecular structure as well as lattice of the molecule. It also helps us to show the bond relation in between atoms and molecules, also help in comparing structure of one molecule to other. Graph theory provides many useful applications in operational research which is an important field in Mathematics. Graphs are used to represent the routes between the cities. With the help of tree that is a type of graph, we can create hierarchical ordered information such as family tree.

1.3 GRAPH LABELING

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa (1967) who is the father of graph labeling. Rosa introduced a function f from a set of vertices in a graph G to the set of integers $\{0, 1, 2, \dots, q\}$ so that each edge xy is assigned the label $|f(x) - f(y)|$, all labels are distinct. Rosa called this labeling as β -valuation. Independently, Golomb (1972) studied the same type of labeling and called as graceful labeling. After the introduction of graceful labeling, various labeling of graphs such as cordial labeling, magic labeling, anti-magic labeling, prime labeling, total magic cordial labeling, odd and even graceful labeling etc., More than 2000 and odd papers have been published and was discussed in dynamic survey by Professor J.A. Gallian (2021).

1.4 APPLICATION OF GRAPH LABELING

Graph labeling is currently an emerging area in the research of graph theory. Labeled graphs serve as useful mathematical models for a broad range of applications such as X-ray Crystallography, data security, radar, coding theory, bioinformatics, astronomy, circuit design and communication networks. Also, it is used to derive a complete complexity in computational techniques, cloud computing, cryptography, software testing, data mining. It is also used to represent the protein structure and pattern discovery using graph mining.

1.5 PRELIMINARIES AND BASIC TERMINOLOGY

This section comprises the basic definitions which are used in this study.

A **graph labeling** is an assignment of integers to the vertices or edges, or both, subject to certain conditions. If the domain of the mapping is the set of vertices (or edges) then the labeling is called vertex labeling (or edge labeling). If the domain of the mapping is the set of vertices and edges then the labeling is called total labeling.

Let $G(V, E)$ be a finite undirected graph. Let $f : V \rightarrow \{0,1\}$ and for each edge

xy assign the label $|f(x) - f(y)|$. We call f a **cordial labeling** of G if the following condition is satisfied: $|v_f(1) - v_f(0)| \leq 1$, $|e_f(1) - e_f(0)| \leq 1$ where $v_f(i)$ and $e_f(i)$, $i = \{0,1\}$ is the number of vertices and edges of G , respectively, with label i . A graph G is cordial if it admits cordial labeling.

A graph $G(V, E)$ is said to have a **total magic cordial (TMC)** labeling if there exists a mapping $f: V(G) \cup E(G) \rightarrow \{0,1\}$ such that $f(a) + f(b) + f(ab) \equiv C(\text{mod } 2)$ for all edges $ab \in E(G)$ provided the condition $|f_0 - f_1| \leq 1$ is hold, where $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where u_i, e_i , $i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

A **Path** of a graph G is a finite sequence of edges which joins a sequence of vertices which are all distinct. The Path graph P_n has n vertices and $n - 1$ edges.

The **Fan graph** F_n , ($n \geq 2$) is obtained by joining all vertices of the Path P_n to a further vertex called the center and contains $n + 1$ vertices and $2n - 1$ edges.

A **Star graph** is a complete bipartite graph $K_{1,n}$. It has $n + 1$ vertices and n edges.

The **Octopus graph** O_n , ($n \geq 2$) is constructed by a fan graph F_n , ($n \geq 2$) joining a star graph $K_{1,n}$ with sharing a common vertex which is the center of the star. In other words, $O_n = F_n + K_{1,n}$, where n is any positive integer. It has $2n + 1$ vertices and $3n - 1$ edges.

The **Vanessa graph** V_n , ($n \geq 2$) is constructed by two fan graphs F_n , ($n \geq 2$) of the same order joining a star graph $K_{1,n}$ with sharing a common vertex which is the center of the star. In other words, $V_n = 2F_n + K_{1,n}$, where n is any positive integer. It has $3n + 1$ vertices and $5n - 2$ edges.

A **Bow graph** is defined to be a double shell in which each shell has any order.

The **Lilly graph** I_n , ($n \geq 2$) is a graph obtained by joining two star graphs

$2K_{1,n}$, ($n \geq 2$) and two path graphs $2P_n$, ($n \geq 2$) with sharing a common vertex. In other words, $I_n = 2K_{1,n} + 2P_n$, where n is any positive integer. It has $4n - 1$ vertices and $4n - 2$ edges.

The **Tortoise graph T_n** , ($n \geq 4$) is the graph obtained from a path graph P_n by attaching an edge between v_i and u_i for $1 \leq i \leq n$.

The graph obtained from shell graph by adding a vertex in between each pair of adjacent vertices on the cycle and adding an edge in apex and two or more chords is known as **Lotus graph Lo_n** . It has $4n + 9$ vertices and $6n + 11$ edges.

CHAPTER – 2

2.1 LITERATURE SURVEY

Cahit I. (1987) introduced the concept of cordial labeling which is a weaker version of graceful and harmonious labeling. The notion of totally magic cordial (TMC) labeling was introduced by Cahit I. (2002). He proved that the following graphs have a TMC labeling: $K_{m,n}$ ($m, n > 1$), trees, cordial graphs and K_n if and only if $n = 2, 3, 5$ or 6 . Jeyanthi P., Angel Benseera N. and Immaculate Mary M. (2013) proved that the following graphs have total magic cordial labelings: (p, q) graphs with $|p - q| \leq 1$, Flower graph Fl_n for $(n \geq 3)$, Ladder graph. Parameswari R. and Rajeswari R. (2013), (2015) extended total magic cordial labeling for directed graphs and proved the same for Paley digraphs, also proved that (P_n, K_1) , Gear and shadow graphs admits total magic cordial labeling. Jeyanthi P. and Angel Benseera N. (2015) proved total magic cordial labeling of some graphs. Parameswari R. (2015) presented the total magic cordial labeling of square and shadow of Bistar graph. Selvam B., Thirusangu K., Indira P. (2016) investigated the total magic cordial labeling of extended duplicate graph of triangular snake. Selvam B., Avudainayaki R. and Ulaganathan P. P. (2016) investigated the total magic cordial labeling of extended duplicate graph of arrow graph. Jeyanthi P., Angel Benseera and Cahit I. (2016) investigated the total magic cordial labeling of mP_n and mK_n . Parameswari R. and Rajeswari R. (2018) presented the total magic cordial labeling of split graph of cycle, wheel and fan graph.

CHAPTER – 3

3.1 AIM AND SCOPE

In real world, there are many problems that can be defined with the help of graph. In this project, the total magic cordial labeling has been applied to Octopus, Vanessa, Bow, Lilly, Tortoise and Lotus graphs such that it has given various ideas to implement in other graphs to develop this area of graph theory.

CHAPTER – 4

MAIN RESULTS

4.1 TOTAL MAGIC CORDIAL LABELING OF OCTOPUS GRAPH

THEOREM:

The Octopus graph O_n ($n \geq 2$) for every n admits Total magic cordial labeling.

PROOF:

Let $G = O_n$ ($n \geq 2$) be the Octopus graph with $|V| = 2n + 1$ vertices and $|E| = 3n - 1$ edges. The vertex set is defined as $V = V_1 \cup V_2 \cup V_3$ where $V_1 = \{u_0\}$, $V_2 = \{u_i \text{ for } 1 \leq i \leq n\}$ and $V_3 = \{v_i \text{ for } 1 \leq i \leq n\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3$ where $E_1 = \{u_0 u_i \text{ for } 1 \leq i \leq n\}$, $E_2 = \{u_0 v_i \text{ for } 1 \leq i \leq n\}$ and $E_3 = \{v_i v_{i+1} \text{ for } 1 \leq i \leq n - 1\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_0 u_i) = 0 \text{ for all } i$$

$$f(u_0 v_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i v_{i+1}) = 1 \text{ for } 1 \leq i \leq n - 1$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that $G = O_n$ ($n \geq 2$) admits total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

CASE (I): When n is odd

$$f_0 = \binom{n-1}{2} + \binom{n+1}{2} + n + \binom{n+1}{2} = \binom{5n+1}{2}$$

$$f_1 = 1 + \binom{n+1}{2} + \binom{n-1}{2} + \binom{n-1}{2} + (n-1) = \binom{5n-1}{2}$$

$$\therefore |f_0 - f_1| = \left| \binom{5n+1}{2} - \binom{5n-1}{2} \right| = 1$$

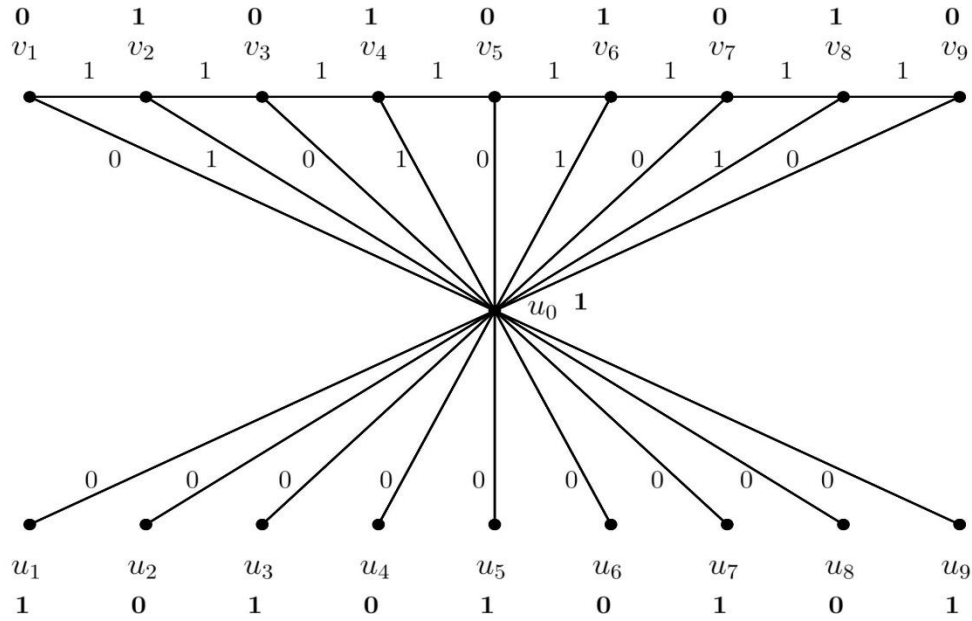


Fig: 4.1.1: Illustration of TMC labeling of O_9

CASE (II): When n is even

$$f_0 = \binom{n}{2} + \binom{n}{2} + n + \binom{n}{2} = \binom{5n}{2}$$

$$f_1 = 1 + \binom{n}{2} + \binom{n}{2} + \binom{n}{2} + (n-1) = \binom{5n}{2}$$

$$\therefore |f_0 - f_1| = \left| \binom{5n}{2} - \binom{5n}{2} \right| = 0$$

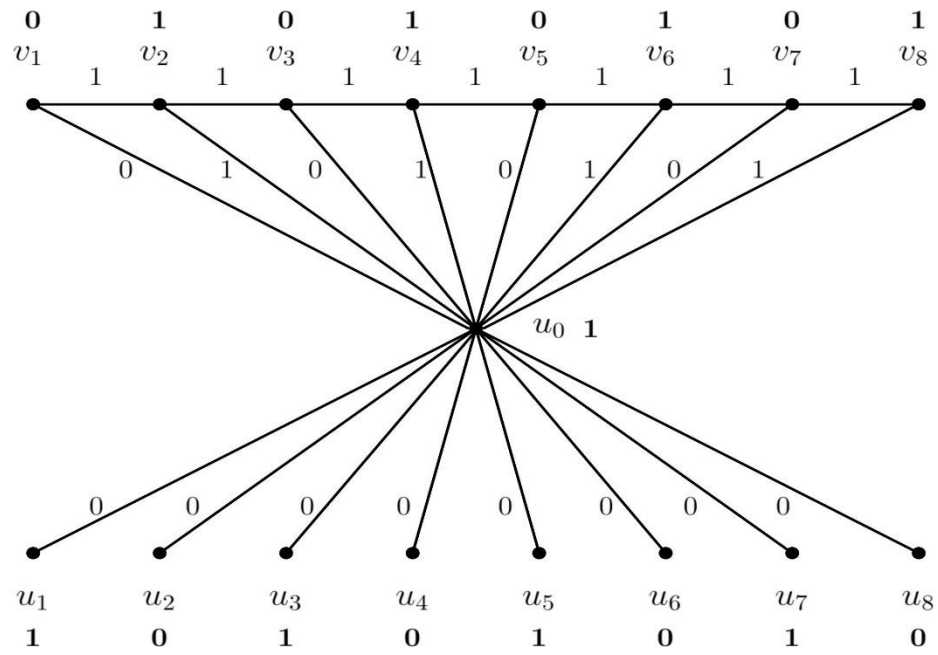


Fig: 4.1.2: Illustration of TMC labeling of O_8

In both cases, it is verified that $|f_0 - f_1| \leq 1$.

Hence, Octopus graph O_n ($n \geq 2$) admits total magic cordial labeling for every n .

4.2 TOTAL MAGIC CORDIAL LABELING OF VANESSA GRAPH

THEOREM:

The Vanessa graph V_n ($n \geq 2$) for every n admits Total magic cordial labeling.

PROOF:

Let $G = V_n$ ($n \geq 2$) be the Vanessa graph with $|V| = 3n + 1$ vertices and $|E| = 5n - 2$ edges. The vertex set is defined as $V = V_1 \cup V_2 \cup V_3 \cup V_4$ where $V_1 = \{u_0\}$, $V_2 = \{u_i \text{ for } 1 \leq i \leq n\}$, $V_3 = \{v_i \text{ for } 1 \leq i \leq n\}$ and $V_4 = \{v_i' \text{ for } 1 \leq i \leq n\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_0 u_i \text{ for } 1 \leq i \leq n\}$, $E_2 = \{u_0 v_i \text{ for } 1 \leq i \leq n\}$, $E_3 = \{u_0 v_i' \text{ for } 1 \leq i \leq n\}$, $E_4 = \{v_i v_{i+1} \text{ for } 1 \leq i \leq n - 1\}$ and $E_5 = \{v_i' v_{i+1}' \text{ for } 1 \leq i \leq n - 1\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i') = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_0 u_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_0 v_i) = 0 \quad \text{for } 1 \leq i \leq n$$

$$f(u_0 v_i') = 1 \quad \text{for } 1 \leq i \leq n$$

$$f(v_i v_{i+1}) = 0 \quad \text{for } 1 \leq i \leq n - 1$$

$$f(v_i' v_{i+1}') = 1 \quad \text{for } 1 \leq i \leq n - 1$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that $G = V_n (n \geq 2)$ admits total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

CASE (I): When n is odd

$$f_0 = \left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right) + \left(\frac{n-1}{2}\right) + \left(\frac{n+1}{2}\right) + n + (n-1) = 4n$$

$$f_1 = 1 + \left(\frac{n-1}{2}\right) + \left(\frac{n-1}{2}\right) + \left(\frac{n+1}{2}\right) + \left(\frac{n-1}{2}\right) + n + (n-1) = 4n - 1$$

$$\therefore |f_0 - f_1| = |4n - (4n - 1)| = 1$$

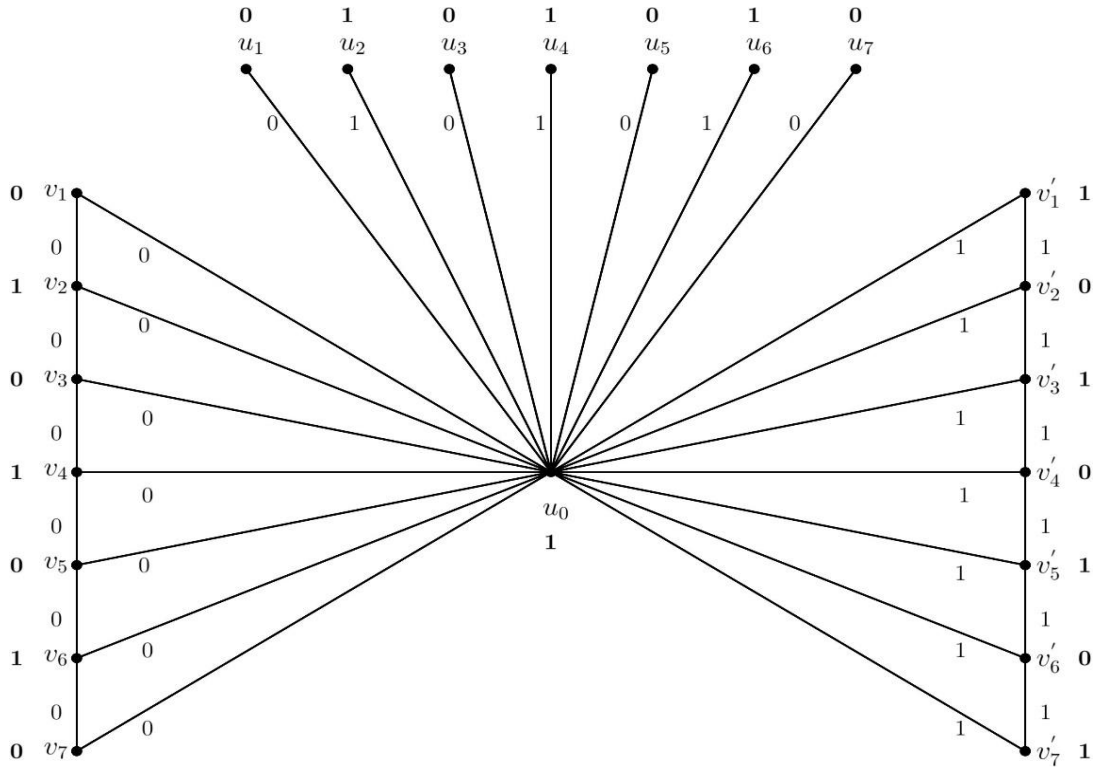


Fig: 4.2.1: Illustration of TMC labeling of V_7

CASE (II): When n is even

$$f_0 = \binom{n}{2} + \binom{n}{2} + \binom{n}{2} + \binom{n}{2} + n + (n - 1) = 4n - 1$$

$$f_1 = 1 + \binom{n}{2} + \binom{n}{2} + \binom{n}{2} + \binom{n}{2} + n + (n - 1) = 4n$$

$$\therefore |f_0 - f_1| = |(4n - 1) - 4n| = 1$$

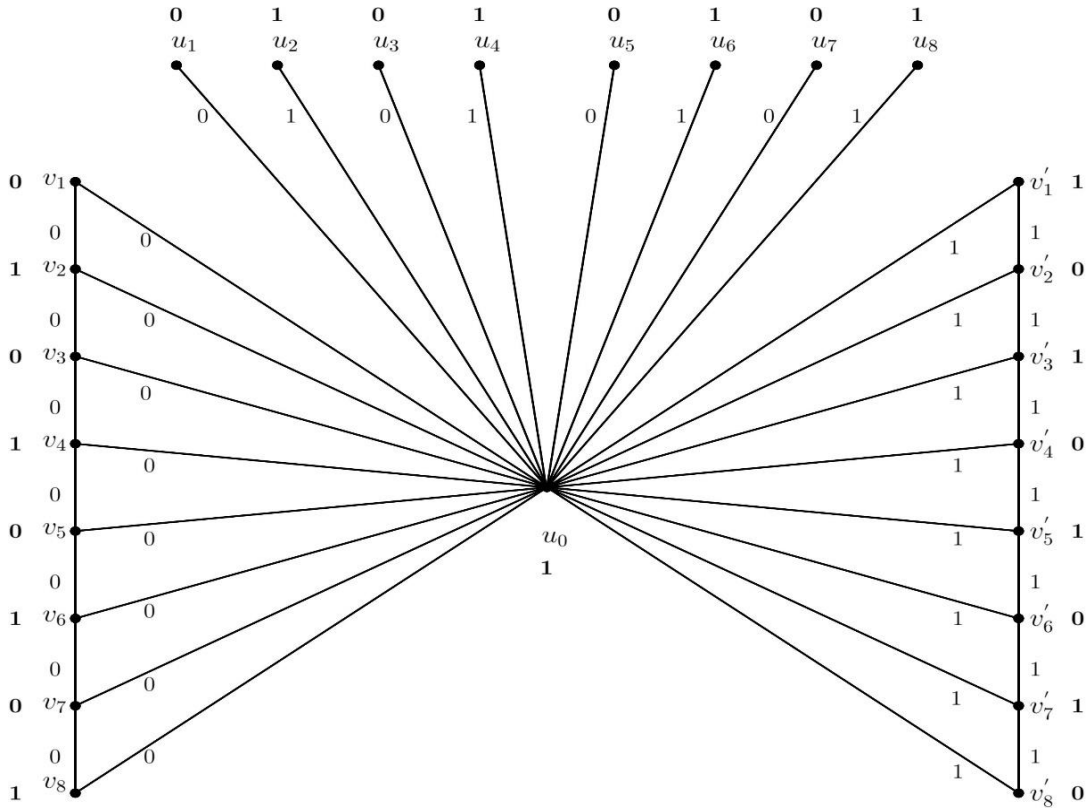


Fig: 4.2.2: Illustration of TMC labeling of V_8

In both cases, it is verified that $|f_0 - f_1| \leq 1$.

Hence, Vanessa graph V_n ($n \geq 2$) admits total magic cordial labeling for every n .

4.3 TOTAL MAGIC CORDIAL LABELING OF BOW GRAPH

THEOREM:

Every Bow graph obtained from shell graph of order 'm' admits total magic cordial labeling.

PROOF:

Let G be a Bow graph obtained from a shell graph whose order is same on both sides say 'm' excluding the apex.

Here G has $2n + 1$ vertices and $4n - 1$ edges.

The vertex set is defined as $V = V_1 \cup V_2 \cup V_3$ where $V_1 = \{u_0\}$, $V_2 = \{u_i \text{ for } 1 \leq i \leq n\}$ and $V_3 = \{v_i \text{ for } 1 \leq i \leq n\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_0 u_i \text{ for } 1 \leq i \leq n\}$, $E_2 = \{u_0 v_i \text{ for } 1 \leq i \leq n\}$, $E_3 = \{u_i u_{i+1} \text{ for } 1 \leq i \leq n - 1\}$ and $E_4 = \{v_i v_{i+1} \text{ for } 1 \leq i \leq n - 1\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = \begin{cases} 1 & i \text{ is odd} \\ 0 & i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 0 & i \text{ is odd} \\ 1 & i \text{ is even} \end{cases}$$

$$f(u_0 u_i) = 0 \quad \text{for } 1 \leq i \leq n$$

$$f(u_0 v_i) = 0 \quad \text{for } 1 \leq i \leq n$$

$$f(u_i u_{i+1}) = 1 \quad \text{for } 1 \leq i \leq n - 1$$

$$f(v_i v_{i+1}) = 1 \quad \text{for } 1 \leq i \leq n - 1$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that Bow graph with same order 'm' admits Total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

$$f_0 = \left(\frac{n-1}{2}\right) + \left(\frac{n+1}{2}\right) + n + n = 3n$$

$$f_1 = 1 + \left(\frac{n+1}{2}\right) + \left(\frac{n-1}{2}\right) + n - 1 + n - 1 = 3n - 1$$

$$\therefore |f_0 - f_1| = |3n - (3n - 1)| = 1$$

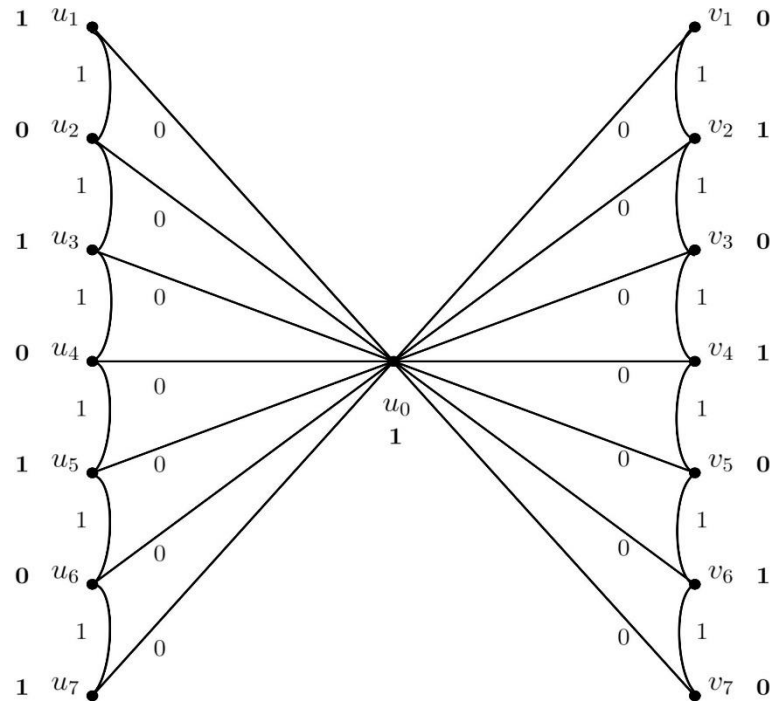


Fig: 4.3.1: Illustration of TMC labeling of Bow graph when $n = 7$

It is verified that $|f_0 - f_1| \leq 1$.

Hence, every bow graph obtained from shell graph of order 'm' admits total magic cordial labeling for every n .

4.4 TOTAL MAGIC CORDIAL LABELING OF LILLY GRAPH

THEOREM:

The Lilly graph I_n ($n \geq 2$) for every n admits total magic cordial labeling.

PROOF:

Let $G = I_n$ ($n \geq 2$) be the Lilly graph with $|V| = 4n - 1$ vertices and $|E| = 4n - 2$ edges.

The vertex set is defined as $V = V_1 \cup V_2 \cup V_3 \cup V_4$ where $V_1 = \{u_0\}$, $V_2 = \{u_i \text{ for } 1 \leq i \leq 2n\}$, $V_3 = \{v_i \text{ for } 1 \leq i \leq n - 1\}$ and $V_4 = \{v_i' \text{ for } 1 \leq i \leq n - 1\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_0 u_i \text{ for } 1 \leq i \leq 2n\}$, $E_2 = \{u_0 v_1\}$, $E_3 = \{u_0 v_1'\}$, $E_4 = \{v_i v_{i+1} \text{ for } 1 \leq i \leq n - 2\}$ and $E_5 = \{v_i' v_{i+1}' \text{ for } 1 \leq i \leq n - 2\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_0) = 0$$

$$f(u_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = 1 \text{ for } 1 \leq i \leq n - 1$$

$$f(v_i') = 0 \text{ for } 1 \leq i \leq n - 1$$

$$f(u_0 u_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_0 v_1) = 1$$

$$f(u_0 v_1') = 0$$

$$f(v_i v_{i+1}) = 1 \text{ for } 1 \leq i \leq n - 2$$

$$f(v_i' v_{i+1}') = 0 \text{ for } 1 \leq i \leq n - 2$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that $G = I_n (n \geq 2)$ admits total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

$$f_0 = 1 + n + (n - 1) + n + 1 + (n - 2) = 4n - 1$$

$$f_1 = n + (n - 1) + n + 1 + (n - 2) = 4n - 2$$

$$\therefore |f_0 - f_1| = |(4n - 1) - (4n - 2)| = 1$$

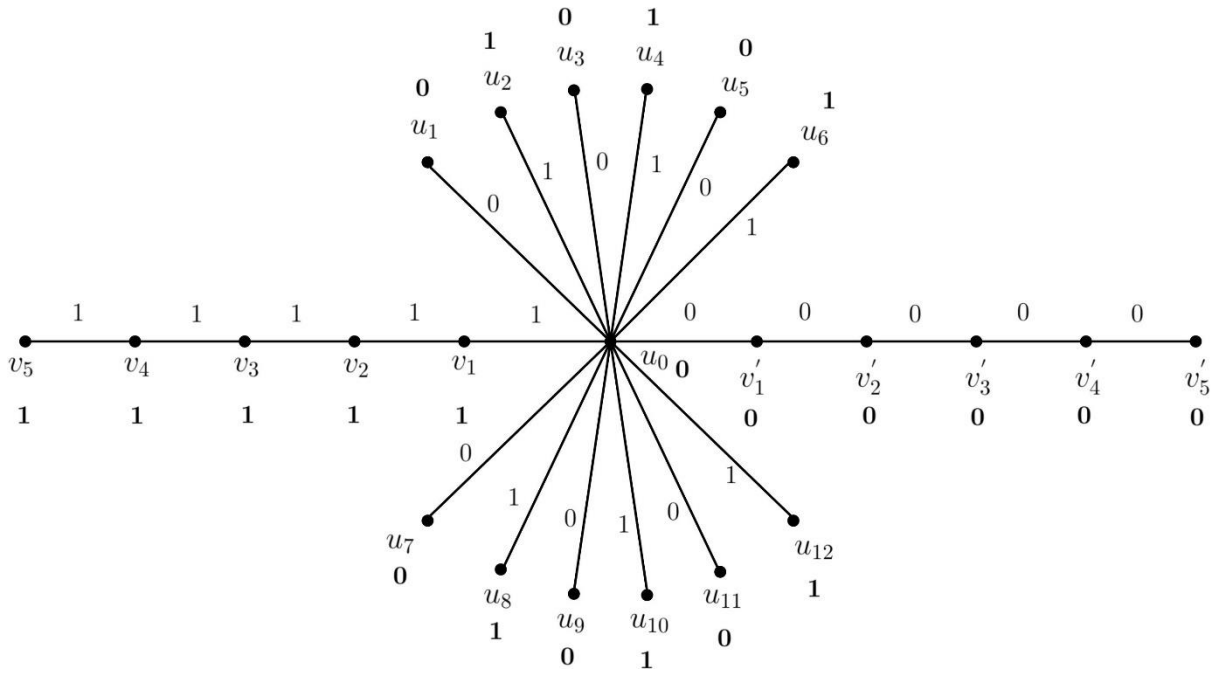


Fig: 4.4.1: Illustration of TMC labeling of I_6

It is verified that $|f_0 - f_1| \leq 1$.

Hence, Lilly graph $I_n (n \geq 2)$ admits total magic cordial labeling for every n .

4.5 TOTAL MAGIC CORDIAL LABELING OF TORTOISE GRAPH

THEOREM:

The Tortoise graph T_n ($n \geq 4$) for every n admits Total magic cordial labeling.

PROOF:

Let $G = T_n$ ($n \geq 4$) be the Tortoise graph with $|V| = 2n + 1$ vertices and $|E| = 3n$ edges.

The vertex set is defined as $V = V_1 \cup V_2 \cup V_3$ where $V_1 = \{u_0\}$, $V_2 = \{u_i \text{ for } 1 \leq i \leq n\}$ and $V_3 = \{v_i \text{ for } 1 \leq i \leq n\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4 \cup E_5$ where $E_1 = \{u_0u_1\}$, $E_2 = \{u_0v_1\}$, $E_3 = \{v_i v_{i+1} \text{ for } 1 \leq i \leq n-1\}$, $E_4 = \{u_i u_{i+1} \text{ for } 1 \leq i \leq n-1\}$ and $E_5 = \{u_i v_i \text{ for } 1 \leq i \leq n\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_0) = 1$$

$$f(u_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} 0 & \text{if } i \text{ is odd} \\ 1 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_0u_1) = 0$$

$$f(u_0v_1) = 1$$

$$f(v_i v_{i+1}) = 1 \quad \text{for } 1 \leq i \leq n-1$$

$$f(u_i u_{i+1}) = 0 \quad \text{for } 1 \leq i \leq n-1$$

$$f(u_i v_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that $G = T_n (n \geq 4)$ admits total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

CASE (I): When n is odd

$$f_0 = \left(\frac{n+1}{2}\right) + \left(\frac{n+1}{2}\right) + 1 + (n-1) + \left(\frac{n-1}{2}\right) = \left(\frac{5n+1}{2}\right)$$

$$f_1 = 1 + \left(\frac{n-1}{2}\right) + \left(\frac{n-1}{2}\right) + 1 + (n-1) + \left(\frac{n+1}{2}\right) = \left(\frac{5n+1}{2}\right)$$

$$\therefore |f_0 - f_1| = \left| \left(\frac{5n+1}{2}\right) - \left(\frac{5n+1}{2}\right) \right| = 0$$

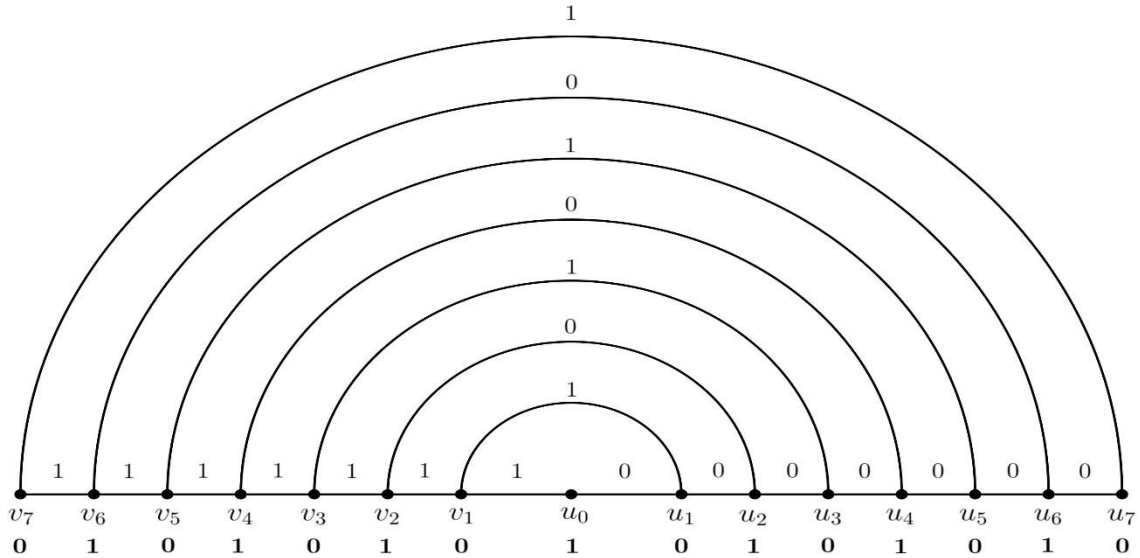


Fig: 4.5.1: Illustration of TMC labeling of T_{15}

CASE (II): When n is even

$$f_0 = \binom{n}{2} + \binom{n}{2} + 1 + (n-1) + \binom{n}{2} = \left(\frac{5n}{2}\right)$$

$$f_1 = 1 + \binom{n}{2} + \binom{n}{2} + 1 + (n-1) + \binom{n}{2} = \left(\frac{5n}{2} + 1\right)$$

$$\therefore |f_0 - f_1| = \left| \left(\frac{5n}{2}\right) - \left(\frac{5n}{2} + 1\right) \right| = 1$$

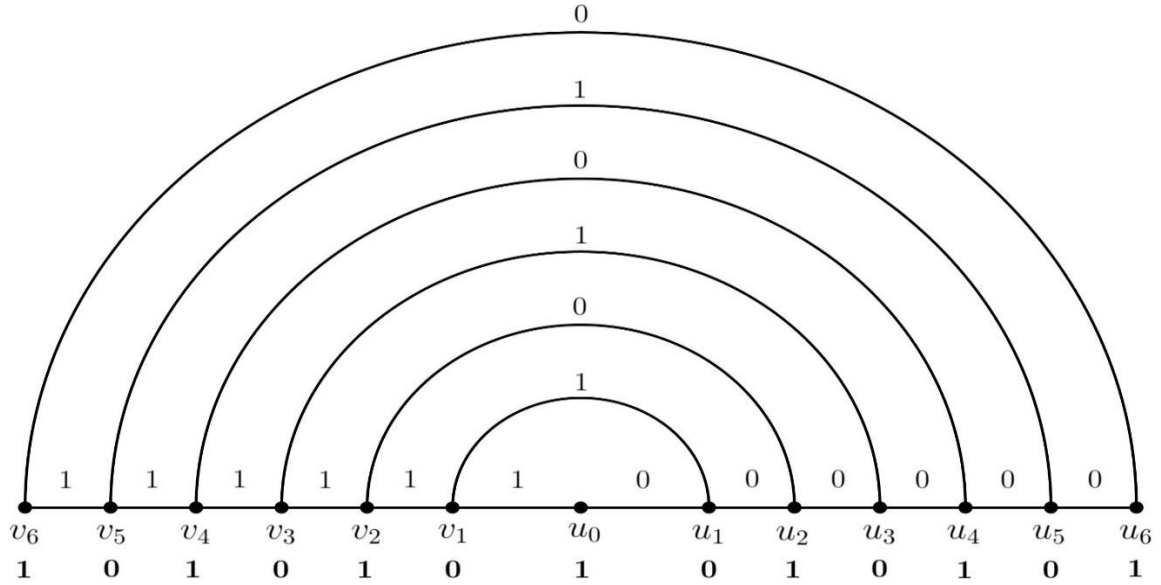


Fig: 4.5.2: Illustration of TMC labeling of T_{13}

In both cases, it is verified that $|f_0 - f_1| \leq 1$.

Hence, Tortoise graph T_n ($n \geq 4$) admits total magic cordial labeling for every n .

4.6 TOTAL MAGIC CORDIAL LABELING OF LOTUS GRAPH

THEOREM:

The Lotus graph Lo_n for every n admits Total magic cordial labeling.

PROOF:

Let $G = Lo_n$ be the Lotus graph with $|V| = 4n + 9$ vertices and $|E| = 6n + 11$ edges.

The vertex set is defined as $V = V_1$ where $V_1 = \{u_i \text{ for } 1 \leq i \leq n\}$.

The edge set is defined as $E = E_1 \cup E_2 \cup E_3 \cup E_4$ where $E_1 = \{u_1 u_2\}$, $E_2 = \{u_2 u_{2i+1} \text{ for } 1 \leq i \leq \left(\frac{n-1}{2}\right)\}$, $E_3 = \{u_{i+1} u_{i+2} \text{ for } 2 \leq i \leq \left(\frac{n-1}{2}\right)\}$ and $E_4 = \{u_i u_{i+1} \text{ for } \left(\frac{n-1}{2}\right) + 2 \leq i \leq n - 1\}$.

We define the mapping $f: V \cup E \rightarrow \{0,1\}$ as follows:

$$f(u_i) = \begin{cases} 1 & \text{if } i \text{ is odd} \\ 0 & \text{if } i \text{ is even} \end{cases}$$

$$f(u_1 u_2) = 0$$

$$f(u_2 u_{2i+1}) = \begin{cases} 0 & \text{for } 1 \leq i \leq \left(\frac{n-1}{4}\right) \\ 1 & \text{for } \left(\frac{n-1}{4}\right) + 1 \leq i \leq \left(\frac{n-1}{2}\right) \end{cases}$$

$$f(u_{i+1} u_{i+2}) = 1 \quad \text{for } 2 \leq i \leq \left(\frac{n-1}{2}\right)$$

$$f(u_i u_{i+1}) = 0 \quad \text{for } \left(\frac{n-1}{2}\right) + 2 \leq i \leq n - 1$$

Let f_0 denote the number of vertices and edges labeled zero and f_1 denote the number of vertices and edges labeled one.

That is, $f_0 = u_0 + e_0$ and $f_1 = u_1 + e_1$ where $u_i, e_i, i \in \{0,1\}$ are the number of vertices and edges labeled with i respectively.

To prove that $G = Lo_n$ admits total magic cordial labeling, we need to prove that number of vertices and edges with label 0 and number of vertices and edges with label 1 differ at most by 1.

That is, to prove that $|f_0 - f_1| \leq 1$.

$$f_0 = \left(\frac{n-1}{2}\right) + 1 + \left(\frac{n-1}{4}\right) + \left[\left(\frac{n-1}{2}\right) - 1\right] = \left(\frac{5n-5}{4}\right)$$

$$f_1 = \left(\frac{n+1}{2}\right) + \left(\frac{n-1}{4}\right) + \left[\left(\frac{n-1}{2}\right) - 1\right] = \left(\frac{5n-5}{4}\right)$$

$$\therefore |f_0 - f_1| = \left|\left(\frac{5n-5}{4}\right) - \left(\frac{5n-5}{4}\right)\right| = 0$$

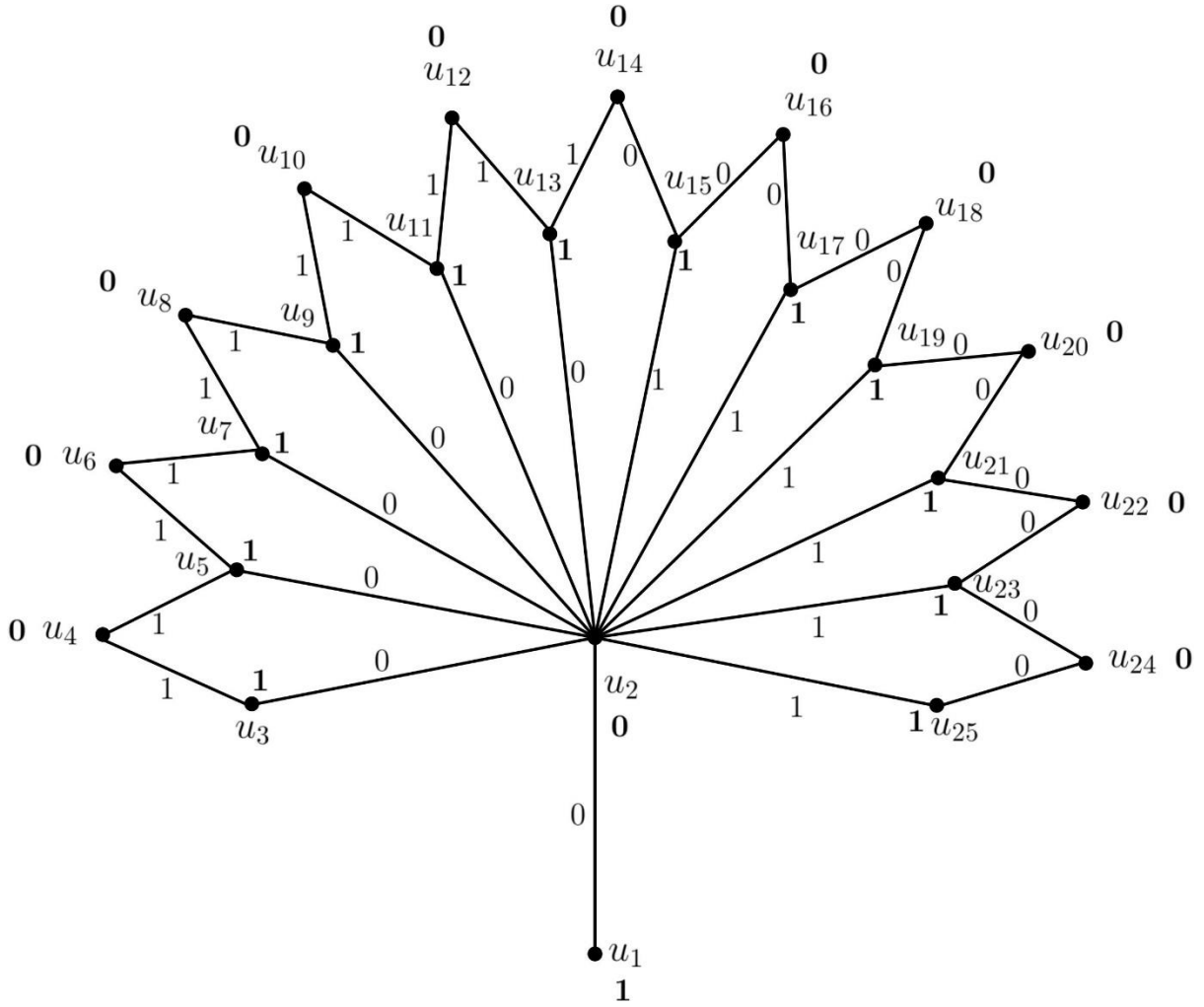


Fig: 4.6.1: Illustration of TMC labeling of Lo_{25}

It is verified that $|f_0 - f_1| \leq 1$.

Hence, Lotus graph Lo_n admits total magic cordial labeling for every n .

CHAPTER – 5

5.1 CONCLUSION

In this paper we showed the existence of total magic cordial labeling for flower graphs namely Octopus graph, Vanessa graph, Bow graph, Lilly graph, Tortoise graph and Lotus graph. It was inspiring to investigate total magic cordial labeling on flower graphs. This labeling can be extended to various types of graphs and examined in future.

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