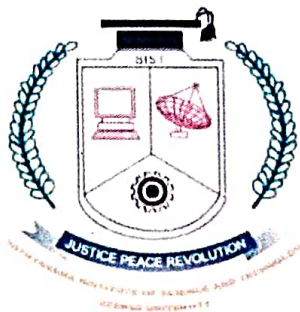


THE MATHEMATICAL EFFECT OF STEADY MAGNETOHYDRODYNAMIC STOKES FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES WITH AN ANGULAR VELOCITY

Submitted in partial fulfillment of the requirements for the award of
Master of Science Degree in Mathematics (M.Sc Mathematics)

By

N.SARAL (40630010)



**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

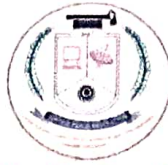
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**INSTITUTE OF SCIENCE AND TECHNOLOGY
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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **N.SARAL (40630010)**. Who carried out the project entitled **"THE MATHEMATICAL EFFECT OF STEADY MAGNETOHYDRODYNAMIC STOKES FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES WITH AN ANGULAR VELOCITY"** under our supervision from **November 2021 to May 2022**.

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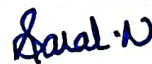
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DECLARATION

I N. SARAL (40630010) hereby declare that the Project Report entitled "THE MATHEMATICAL EFFECT OF STEADY MAGNETOHYDRODYNAMIC STOKES FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL POROUS PLATES WITH AN ANGULAR VELOCITY" done by me under the guidance of Dr. R. DELHI BABU (Internal) and _____ (External) at SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY submitted in partial fulfillment of the requirements for the award of Master of Science Degree in Mathematics.

DATE: 12.05.2022

PLACE: CHENNAI



SIGNATURE OF THE CANDIDATE

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ABSTRACT:

In this paper we discussed Steady Magnetohydrodynamic stokes flow of an electrically conducting viscous, incompressible fluid between two parallel porous plates of a channel with an angular velocity in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate. An exact solution is obtained for all values of R (Suction Reynolds number) and M_1 (Hartmann number with angular velocity). Expressions for the velocity components and the pressure are obtained. The graphs of axial and radial velocity profiles have been drawn for different values of M_1 . If the distance between the two plates is assumed to be h and the result obtained here reduces to the results of Ganesh and Krishnambal (2007) when the angular velocity Ω is zero.

KEYWORDS:

Magnetohydrodynamic (MHD) flow;
Viscous fluid flow;
Porous parallel plates;
Angular velocity (Ω).

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CHAPTER-1

1. INTRODUCTION

FLUID DYNAMICS

Fluid Mechanics is that branch of Science which deals with the behavior of fluids (liquids or gases) at rest as well as in motion. Thus this branch of Science deals with the static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of Science is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; influence of wind upon building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid.

1.1 IMPORTANCE OF FLUID FLOW

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces.

1.2 MAGNETOHYDRODYNAMIC FLOW

MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Magnetohydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil.

1.3 FLOW THROUGH POROUS MEDIUM

A medium which is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity (ϵ) and permeability (k_1). The porosity of a porous medium describes how densely the material is packed. The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by the void space. Thus $1-\epsilon$ is the fraction that is occupied by solid. Porosity is a fraction between 0 and 1, typically ranging from less than 0.01 for solid granite to more than 0.5 for peat and clay. For manmade materials such as metallic frames, ϵ can approach to the value 1. For natural media, ϵ does not normally exceed 0.6.

The permeability is related to pore-size distribution since the distribution of the sizes of entrances, exits and lengths of the pore walls make up the major resistance to flow. The permeability is the single parameter that reflects the conductance of a given pore structure. The dimensions of the permeability are length squared.

1.4 VISCOUS FLOW

Flow of a fluid in which its velocity at any point is constant or varies in a regular manner. For viscous material, there is **laminar (slow, smooth, parallel) flow**; one must exert a force to maintain motion because of internal frictional resistance to flow, called the viscosity. Viscosity varies with the applied stress, strain rate, and temperature.

1.5 VISCOUS FLUID

Fluids that have higher viscosity are known as viscous fluid. Examples of viscous fluids are honey and ketchup. The fluid that has more flow resistance is referred to as viscous fluid. Viscosity is a fluid factor, which is the degree of the amount of resistance between the fluid layers.

1.6 ANGULAR VELOCITY

Angular velocity is the vector measure of the rotation rate, which refers to how fast an object rotates or revolves relative to another point.

1.7 NON- NEWTONIAN FLOWS

Fluids can be defined as substances that flow or deform under the application of shear stress and these include liquids and gases. They are part of engineering study in many tertiary institutions. Basically, in the study of science, fluids are divided into two broad groups. These divisions in this write-up are known as types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids that obey Newton Law of viscosity. The Newtonian viscosity is a physical property which is only a function of pressure and temperature. The flow properties of an incompressible Newtonian fluid can be characterized solely in terms of the density and viscosity.

CHAPTER-2

LITERATURE SURVEY

Flow through porous media has many geophysical and engineering applications. Berman AS [1953] analyzed the Navier-Stokes equations have been solved to obtain a complete description of the fluid flow in a channel having a rectangular cross section and two equally porous walls. The solution of the flow equations leads to detailed expressions for the dependence of the velocity components and the pressure on position coordinates, channel dimensions, and fluid properties.

Hartmann and Lazarus [1937] investigated the effect of a transversely uniform MHD on the flow of conductive fluid between two infinitely parallel and fixed insulating plates. The equation describing MHD flow is a synthesis of the continuous equation and the Navier-Stokes equation. Governing equations are differential equations that require analytical or numerical solutions.

Hartmann J. [1937] considered the magnetic field in the laminar flow of conductive liquids.

Hassaninen I. A. and Mansour M.A. [1990] Analyzed the two-dimensional unsteady flow of a viscous, incompressible electrically- conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter are discussed.

Hamza E.A [1999], A similarity solution has been obtained for a flow between two parallel plates (rectangular or circular) approaching or receding from each other with suction or injection at the porous plate. Approximate analytic solutions based on the regular perturbation technique are given and a numerical solution of the resulting nonlinear differential equations is presented. The effects of injection and suction on the velocity profiles, pressure distribution and shearing stress on the wall are studied. In the case of the flow between circular plates the effects on load capacity due to mass transfer are investigated.

Soundalgekar V. and Uplekar [1986] investigated an exact analysis of a generalized MHD Couette flow, on taking into account the Hall current, is presented. Expressions for the axial and transverse components of velocity, the skin friction, are derived. These are shown in graphs. The exact solution to the energy equation, assuming the linear variation of the temperature along the plate, is also derived and this is shown graphically. The effects of different parameters are discussed.

Sing & Ram [1978] solved the differential equation by Laplace transform method, considering the laminar flow of conductive fluid laminar flow through a channel with a transverse magnetic field under a periodic pressure gradient.

Attia, H. A. and Kotb, N. A., (1996) studied the steady flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel infinite insulated horizontal plates and the heat transfer through it are studied. The effect of an external uniform magnetic field as well as the action of an inflow perpendicular to the plates together with the influence of the pressure gradient on the flow and temperature distributions are reported. A numerical solution for the governing non-linear ordinary differential equations is developed.

Chiam T.C. (1993) investigated the boundary layer equations arising from the effects of a continuously moving flat plate under the action of a transverse applied magnetic field are converted into a system of ordinary differential equations subject to two-point boundary conditions by using the Local No similarity Method. An iterative method incorporating the shooting method is presented for solving both the two-equation and three-equation models.

R. Delhi Babu, S. Ganesh & C. K. Kirubhashankar (2020) studied the impact of unsteady Magnetohydrodynamic flow of dust particles on the velocity of electrically directed viscous fluid between parallel porous plates with an angular velocity at the sight of a transverse magnetic field. The differential equations managing the flow are explained by similarity transformation. The velocity of the fluid, velocity of the dust particles and pressure distribution are obtained. The outcomes are the velocity of the dust particle is higher than that velocity of the fluid particles for each of the parameters examined.

CHAPTER-3

3. RESEARCH METHODOLOGY

3.1 AIM AND SCOPE

The main objectives of the work reported are:

- To study the effect of individual factors like Magnetic field, Hartmann number and Reynolds number.
- To study the solution for the steady MHD fluid flow through the parallel plate channel with heat source and external transverse magnetic field is presented.
- The effects of velocity are studied for the parameter magnetic field parameters.

The scope of the work reported is:

It not only helps the people working in the field of Physiological fluid dynamics but also to the medical practitioners.

3.2 DESCRIPTION OF RESEARCH WORK

This research work focuses on the MATLAB implementation of interpretation of analytical solutions obtained for various fluid flow problems.

Research Problem

The main objective of the research work is to analyze the flow of fluids through parallel porous plates in the presence/absence of magnetic effects.

Research Methodologies

Analytical solutions are obtained by transforming the partial differential equation concerning the flow into an ordinary linear differential equation with constant coefficients by using similarity transformation. The arbitrary constant is then calculated by using boundary condition of the respective problem and the results are interpreted by using MATLAB applications.

CHAPTER- 4

4. MATHEMATICAL FORMULATION

The steady laminar flow of an incompressible viscous fluid between two parallel porous plates with an angular velocity is considered in the presence of a transverse magnetic field H_0 applied perpendicular to the walls. The origin is taken at the centre of the channel and let x and y be the coordinate axes parallel and perpendicular to the channel walls.

Consider L is the length of the channel and $2h$ is the distance between the two parallel plates. Let u be the velocity component in x direction and v be the velocity component in y direction respectively and Ω to be an angular velocity.

The equation of continuity is
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Equations of momentum are

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2\Omega u - \sigma_e B_0^2 u \quad (2)$$

$$\rho \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - 2\Omega v \quad (3)$$

Where Ω is the angular velocity, σ_e is the electrical conductivity and B_0 is the Electromagnetic induction = $\mu_e H_0$, μ_e being the magnetic permeability and H_0 is the transverse magnetic field.

In Steady State, $\frac{\partial u}{\partial t} = 0$ and $\frac{\partial v}{\partial t} = 0$ and the equations (2) and (3) becomes,

$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + 2\Omega u - \sigma_e B_0^2 u \quad (4)$$

$$0 = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - 2\Omega v \quad (5)$$

4.1 NOTATIONS

σ_e - Electrical conductivity

$B_0 = \mu_e H_0$

B_0 - electromagnetic induction,

μ - Coefficient of viscosity

μ_e - Magnetic permeability

ψ - Stream function

η - Dimensionless Variable

H_0 - Transverse magnetic field

4.2 ANALYTICAL SOLUTION OF THE PROBLEM

The associated boundary conditions are

$u = 0$ on $y = h$ and $y = -h$; $v = v_0$ on $y = h$ and $v = -v_0$ on $y = -h$

Where v_0 is the velocity of suction at the walls of the channel.

Let the dimensionless variable be $\eta = \frac{y}{h}$, $u = u(x, y)$, $v = v(x, y)$, $p = p(x, y)$.

The equations (1), (4) and (5) becomes

$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \eta} = 0 \quad (6)$$

$$0 = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \eta^2} \right) + 2\Omega u - \sigma_e B_0^2 u \quad (7)$$

$$0 = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \eta^2} \right) - 2\Omega v \quad (8)$$

Let $\nu = \frac{\mu}{\rho}$ = Kinematic Viscosity, where ρ is the density of the fluid, μ is

the coefficient of viscosity and p the pressure.

The associated boundary conditions are converted into

$$u(x,1) = 0, u(x,-1) = 0 \quad (9)$$

$$\text{and } v(x,1) = v_0, v(x,-1) = -v_0 \quad (10)$$

Let ψ be the stream function such that

$$u = \frac{1}{h} \frac{\partial \psi}{\partial \eta} \quad (11)$$

$$v = -\frac{\partial \psi}{\partial x} \quad (12)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x,\eta) = (hU(0) - v_0 x) f(\eta) \quad (13)$$

Where $U(0)$ is the average entrance velocity at $x = 0$.

By using equation (13), the velocity components (11) and (12) reduces to

$$u = \frac{1}{h} (hU(0) - v_0 x) f'(\eta) \quad (14)$$

$$v = v_0 f(\eta) \quad (15)$$

Where the prime denotes the differentiation with respect to the dimensionless

variable $\eta = \frac{y}{h}$. Since the fluid is being withdrawn at constant rate from both the

walls, v_0 is independent of x .

Substituting equations (14) and (15) in equations (7) and (8), then the equations of momentum reduce to

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(-\frac{\nu}{h^2} f''''(\eta) + \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) \right) \quad (16)$$

$$-\frac{1}{h\rho} \frac{\partial p}{\partial \eta} = -\frac{\nu \cdot v_0}{h^2} f''(\eta) + \frac{2\Omega v_0}{\rho} f(\eta) \quad (17)$$

Differentiating equation (17) w.r.to ' x ' partially we get,

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \quad (18)$$

Differentiating equation (16) w.r.to 'η' we get,

$$\frac{-1}{\rho} \frac{\partial^2 p}{\partial x \partial \eta} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \frac{d}{d\eta} \left(-\frac{\nu}{h^2} f'''(\eta) + \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) \right) \quad (19)$$

From equations (18) and (19) we get,

$$\frac{d}{d\eta} \left(-\frac{\nu}{h^2} f'''(\eta) + \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) \right) = 0 \quad (20)$$

Let $R = \text{Suction Reynolds number} = \frac{h v_0}{\nu}$

$$M_1 = \text{Hartmann number with angular velocity} = B_0 h \left(\frac{\sigma_e}{\nu \rho} - \frac{2\Omega}{B_0^2 \nu \rho} \right)^{\frac{1}{2}}$$

Integrating equation (20) w.r.to 'η' and using the above expressions we get,

$$f'''(\eta) - \frac{h^2}{\nu} \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) = K$$

$$f'''(\eta) - a R f'(\eta) = K \quad (21)$$

Where $R = \frac{h v_0}{\nu}$, $a = \frac{\sigma_e h B_0^2}{\rho v_0} - \frac{2\Omega h}{\rho v_0}$ and K is an arbitrary constant.

The associated boundary conditions of f (η) are

$$f(1) = 1, f(-1) = -1, f'(1) = 0 \text{ and } f'(-1) = 0 \quad (22)$$

Hence the solution of the equations of motion and continuity is given by a Nonlinear third order differential equation (21) subject to the boundary conditions (22).

∴ Equation (21) can be rewritten as

$$f'''(\eta) - M_1^2 f'(\eta) = K \quad \text{Where } aR = M_1^2$$

$$(D^3 - M_1^2 D)f(\eta) = K$$

$$D(D^2 - M_1^2)f(\eta) = K$$

$$\text{Where } D = \frac{d}{d\eta} \text{ and } D^2 = \frac{d^2}{d\eta^2}$$

The roots of the auxiliary equation for the above equation is $m=0$, $m = \pm M_1$

Hence the complimentary function is $Ae^{0\eta} + Be^{M_1\eta} + C e^{-M_1\eta}$

$$\text{C.F} = A + Be^{M_1\eta} + C e^{-M_1\eta}$$

$$\text{P.I} = \frac{1}{(D^3 - M_1^2 D)} \cdot K e^{0\eta}$$

$$\text{P.I} = K\eta \cdot \frac{1}{3D^2 - M_1^2} \cdot e^{0\eta}$$

$$\text{P.I} = \frac{-K\eta}{M_1^2}$$

$$f(\eta) = \text{C.F} + \text{P.I}$$

$$\therefore f(\eta) = A + Be^{M_1\eta} + Ce^{-M_1\eta} - \frac{K\eta}{M_1^2} \quad (23)$$

In order to find the values of the arbitrary constants, the associated boundary conditions are applied on $f(\eta)$.

Since $f(1) = 1$,

$$1 = A + Be^{M_1} + C e^{-M_1} - \frac{K}{M_1^2} \quad (24)$$

Since $f(-1) = -1$

$$-1 = A + Be^{-M_1} + C e^{M_1} + \frac{K}{M_1^2} \quad (25)$$

Adding equations (24) and (25) we get,

$$0 = 2A + B(e^{M_1} + e^{-M_1}) + C(e^{M_1} + e^{-M_1}) - \frac{2K}{M_1^2}$$

$$0 = A + B \cosh M_1 + C \cosh M_1 \quad (26)$$

Subtracting from equation (24) to equation (25) we get,

$$2 = B(e^{M_1} - e^{-M_1}) - C(e^{M_1} - e^{-M_1}) - \frac{2K}{M_1^2}$$

$$1 = B \sinh M_1 - C \sinh M_1 - \frac{K}{M_1^2} \quad (27)$$

Differentiate equation (23) w.r.to ' η ' we get,

$$f'(\eta) = M_1 B e^{M_1 \eta} - M_1 C e^{-M_1 \eta} - \frac{K}{M_1^2}$$

Since $f'(1) = 0$

$$0 = M_1 B e^{M_1} - M_1 C e^{-M_1} - \frac{K}{M_1^2} \quad (28)$$

Since $f'(-1) = 0$

$$0 = M_1 B e^{-M_1} - M_1 C e^{M_1} - \frac{K}{M_1^2} \quad (29)$$

Adding equations (28) and (29) we get,

$$0 = M_1 B (e^{M_1} + e^{-M_1}) - M_1 C (e^{M_1} + e^{-M_1}) - \frac{2K}{M_1^2}$$

$$B M_1 \cosh M_1 - C M_1 \cosh M_1 - \frac{K}{M_1^2} = 0 \quad (30)$$

Subtracting from equation (28) to equation (29) we get,

$$0 = M_1 B (e^{M_1} - e^{-M_1}) + M_1 C (e^{M_1} - e^{-M_1})$$

$$B M_1 \sinh M_1 + C M_1 \sinh M_1 = 0$$

$$\Rightarrow B = -C \quad (31)$$

$$\text{Equation (26)} \Rightarrow A - C \cosh M_1 + C \cosh M_1 = 0$$

$$\Rightarrow A = 0 \quad (32)$$

Substitute equation (31) in equation (27) we get,

$$\begin{aligned} -C \sinh M_1 - C \sinh M_1 &= \frac{K}{M_1^2} + 1 \\ \Rightarrow C &= -\frac{(K + M_1^2)}{2M_1^2 \sinh M_1} \end{aligned} \quad (33)$$

Substitute equation (31) in equation (30) we get,

$$M_1 (-C) \cosh M_1 - M_1 (C) \cosh M_1 = \frac{K}{M_1^2}$$

$$-2M_1 C \cosh M_1 = \frac{K}{M_1^2}$$

$$M_1 \frac{(K + M_1^2) \cosh M_1}{(M_1^2) \sinh M_1} = \frac{K}{M_1^2} \quad (\text{by equation (33)})$$

$$K = M_1 K \coth M_1 + M_1^3 \coth M_1$$

$$K(1 - M_1 \coth M_1) = M_1^3 \coth M_1$$

$$K = \frac{M_1^3 \coth M_1}{(1 - M_1 \coth M_1)} \quad (34)$$

$$\therefore C = - \frac{\left(\frac{M_1^3 \coth M_1}{1 - M_1 \coth M_1} + M_1^2 \right)}{2 M_1^2 \sinh M_1} \quad (\text{from equation (33)})$$

$$C = \frac{-M_1^2}{2 M_1^2 \sinh M_1 (1 - M_1 \coth M_1)}$$

$$C = \frac{-1}{(2 \sinh M_1)(1 - M_1 \coth M_1)} \quad (35)$$

Since $B = -C$

$$B = \frac{1}{(2 \sinh M_1)(1 - M_1 \coth M_1)} \quad (36)$$

Substituting the values of A, B and C in $f(\eta)$ we get,

$$f(\eta) = \frac{1}{(1 - M_1 \coth M_1)} \left(\frac{\sinh M_1 \eta}{\sinh M_1} - \eta M_1 \coth M_1 \right) \quad (37)$$

Hence the expressions for the velocity components u and v are

$$\begin{aligned}
 u &= \frac{1}{h} [hU(0) - v_0 x] f'(\eta) \\
 &= \left(U(0) - \frac{v_0 x}{h} \right) \left(\frac{1}{(1 - M_1 \text{Coth} M_1)} \right) \left(\frac{M_1 \text{Cosh} M_1 \eta}{\text{Sinh} M_1} - M_1 \text{Coth} M_1 \right) \\
 u &= \left(U(0) - \frac{v_0 x}{h} \right) \left(\frac{M_1}{(1 - M_1 \text{Coth} M_1)} \right) \left(\frac{\text{Cosh} M_1 \eta}{\text{Sinh} M_1} - \text{Coth} M_1 \right) \quad (38)
 \end{aligned}$$

$$\begin{aligned}
 v &= v_0 f(\eta) \\
 v &= v_0 \left(\frac{1}{1 - M_1 \text{Coth} M_1} \right) \left(\frac{\text{Sinh} M_1 \eta}{\text{Sinh} M_1} - \eta M_1 \text{Coth} M_1 \right) \quad (39)
 \end{aligned}$$

PRESSURE DISTRIBUTION

From equation (16),

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \left(-\frac{v}{h^2} f''''(\eta) + \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) \right) \quad (40)$$

Dividing by $-\frac{v}{h^2}$, it follows that

$$\frac{h^2}{\rho v} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \left(f'''(\eta) - \frac{h^2}{v} \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) \right)$$

By equation (21),

$$f'''(\eta) - a R f'(\eta) = K$$

Since $R = \frac{h v_0}{v}$, $a = \frac{\sigma_e h B_0^2}{\rho v_0} - \frac{2\Omega h}{\rho v_0}$

$$f'''(\eta) - \frac{h v_0}{v} \left(\frac{\sigma_e h B_0^2}{\rho v_0} - \frac{2\Omega h}{\rho v_0} \right) f'(\eta) = K$$

$$f'''(\eta) - \frac{h^2}{v} \left(\frac{\sigma_e B_0^2}{\rho} - \frac{2\Omega}{\rho} \right) f'(\eta) = K \quad (41)$$

From equations (40) and (41) we get,

$$\begin{aligned}
 \frac{h^2}{\rho v} \frac{\partial p}{\partial x} &= K \left(U(0) - \frac{v_0 x}{h} \right) \\
 \therefore \frac{\partial p}{\partial x} &= \frac{K \rho v}{h^2} \left(U(0) - \frac{v_0 x}{h} \right) \\
 &= \frac{K \mu}{h^2} \left(U(0) - \frac{v_0 x}{h} \right) \quad (\because v = \frac{\mu}{\rho} \Rightarrow \mu = \rho v)
 \end{aligned} \tag{42}$$

From equation (17) we get,

$$\begin{aligned}
 -\frac{1}{h \rho} \frac{\partial p}{\partial \eta} &= -\frac{w_0}{h^2} f''(\eta) + \frac{2 \Omega v_0}{\rho} f(\eta) \\
 \frac{\partial p}{\partial \eta} &= \frac{w_0 \rho}{h} f''(\eta) - 2 h \Omega v_0 f(\eta) \quad (\because \mu = \rho v) \\
 \frac{\partial p}{\partial \eta} &= \frac{\mu w_0}{h} f''(\eta) - 2 h \Omega v_0 f(\eta)
 \end{aligned} \tag{43}$$

Since $dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial y} dy$

$$= \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial \eta} d\eta \quad \because \left(\eta = \frac{y}{h} \right)$$

$$dp = \frac{K \mu}{h^2} \left(U(0) - \frac{v_0 x}{h} \right) dx + \left(\frac{\mu w_0}{h} f''(\eta) - 2 h \Omega v_0 f(\eta) \right) d\eta \tag{44}$$

Integrating on both sides we get,

$$p(x, \eta) = \frac{K \mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \left(\frac{\mu w_0}{h} \int f''(\eta) d\eta - 2 h \Omega v_0 \int_0^\eta f(\eta) d\eta \right) + K_1$$

$$p(x, \eta) = \frac{K \mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \left(\frac{\mu w_0}{h} f'(\eta) - 2 h \Omega v_0 \int_0^\eta f(\eta) d\eta \right) + K_1$$

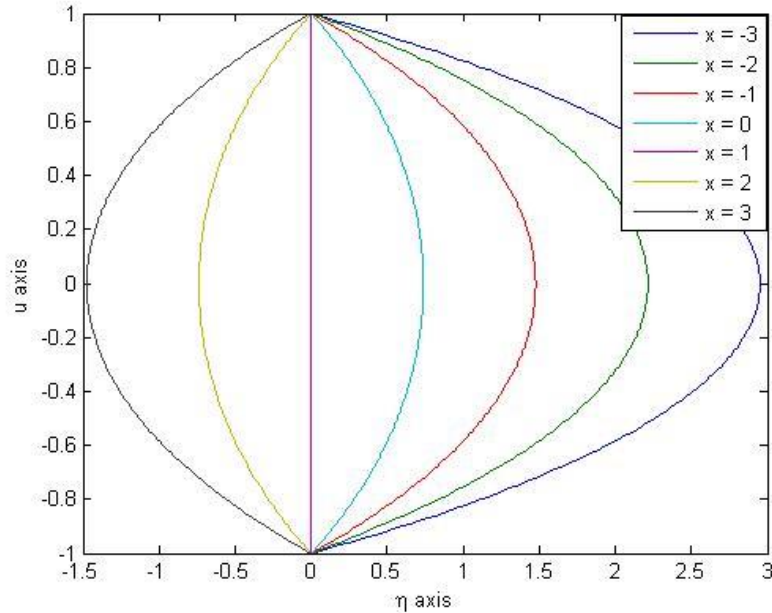
∴ The pressure drop is given by

$$p(x, \eta) - p(0,0) = \frac{K\mu}{h^2} \left(U(0)x - \frac{v_0 x^2}{2h} \right) + \frac{\mu v_0}{h} (f'(\eta) - f'(0)) - 2h\Omega v_0 \int_0^\eta f(\eta) d\eta \quad (45)$$

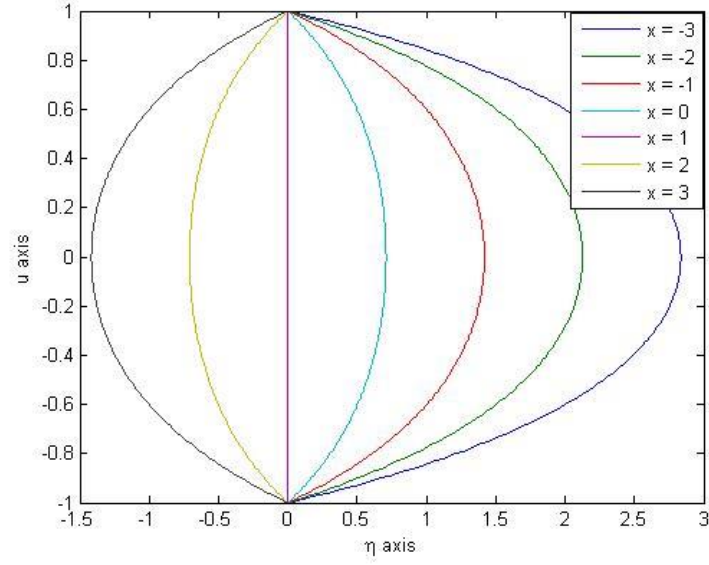
CHAPTER-5

RESULT AND DISCUSSION

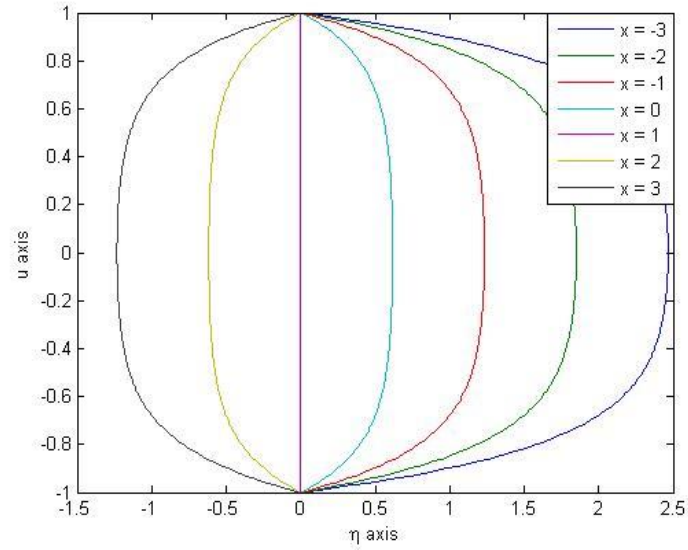
The graphs of the axial velocity and radial velocity profiles have been drawn for different values of M_1 . Figures 2.1, 2.2, 2.3 represents the axial velocity profiles at different values of M_1 namely $M_1=1$, $M_1=2$, and $M_1=5$ when the average entrance velocity is $u_0 = 0.5$ and $h = 1.0$, $x = -3$ to 3 . Figures 2.4, 2.5, 2.6 represents the axial velocity profiles at different values of M_1 namely $M_1=1$, $M_1=2$, and $M_1=5$ when the average entrance velocity u_0 is increased from $u_0 = 0.5$ to $u_0 = 1.0$ and $h = 1.0$, $x = -3$ to 3 . From the figures 2.1 to 2.6, it is observed that the magnitudes of the axial velocity u increase as x varies from -3 to 0 and decreases for x varying from 2 to 3 . For $x = 1$, the axial velocity profile is found to be linear. The Figure 2.7 represent the radial velocity profiles of v at $v_0=0.5$, $h=1.0$, $\alpha=1.0$ and for different values of M_1 namely $M_1=1$, $M_1=2$, and $M_1=5$. As M_1 increases from $M_1=1$ to $M_1=5$, it is seen that there is a marginal increase in the magnitude of the radial velocity.



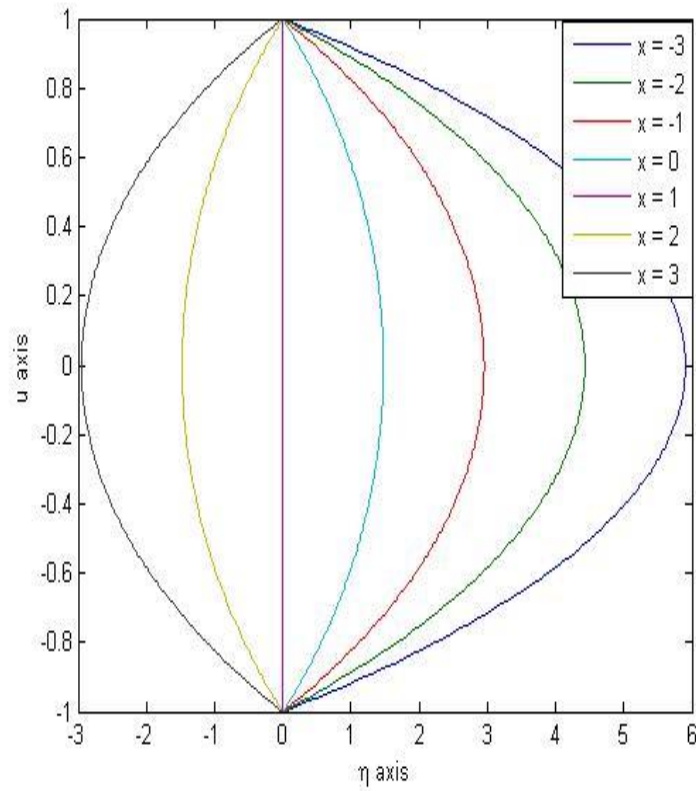
**Figure 5.1: Axial velocity profiles for $x=-3$ to 3 increases by 1
as $u_0 = 0.5$, $v_0 = 0.5$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 1$**



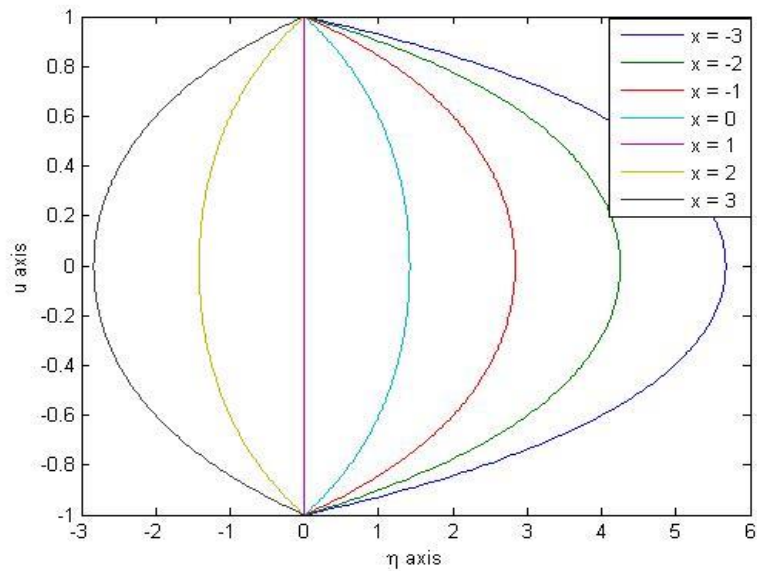
**Figure 5.2: Axial velocity profiles for $x = -3$ to 3 increases by 1
as $u_0 = 0.5$, $v_0 = 0.5$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 2$**



**Figure 5.3: Axial velocity profiles for $x = -3$ to 3 increases by 1
as $u_0 = 0.5$, $v_0 = 0.5$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 5$**



**Figure 5.4: Axial velocity profiles for $x = -3$ to 3 increases by 1
as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 1$**



**Figure 5.5: Axial velocity profiles for $x = -3$ to 3 increases by 1
as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 1$**

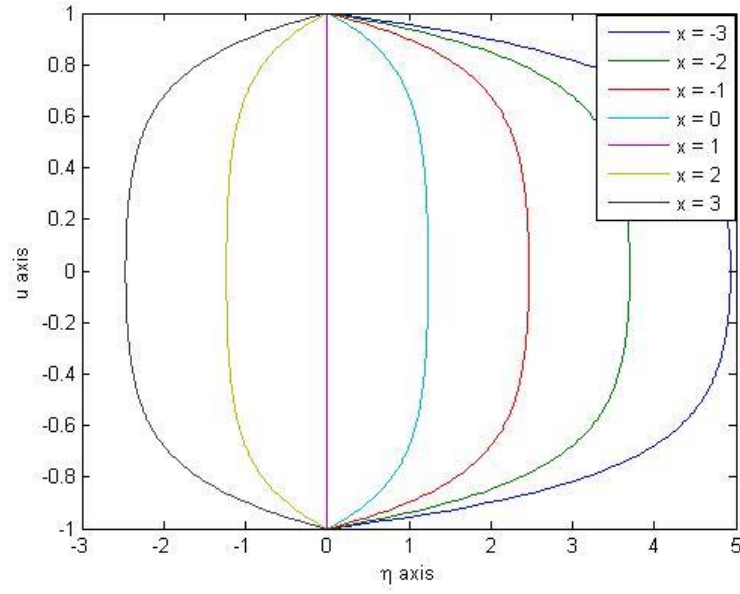


Figure 5.6: Axial velocity profiles for $x = -3$ to 3 increases by 1 as $u_0 = 1$, $v_0 = 1$, $\alpha = 1.0$, $h = 1.0$, $M_1 = 5$

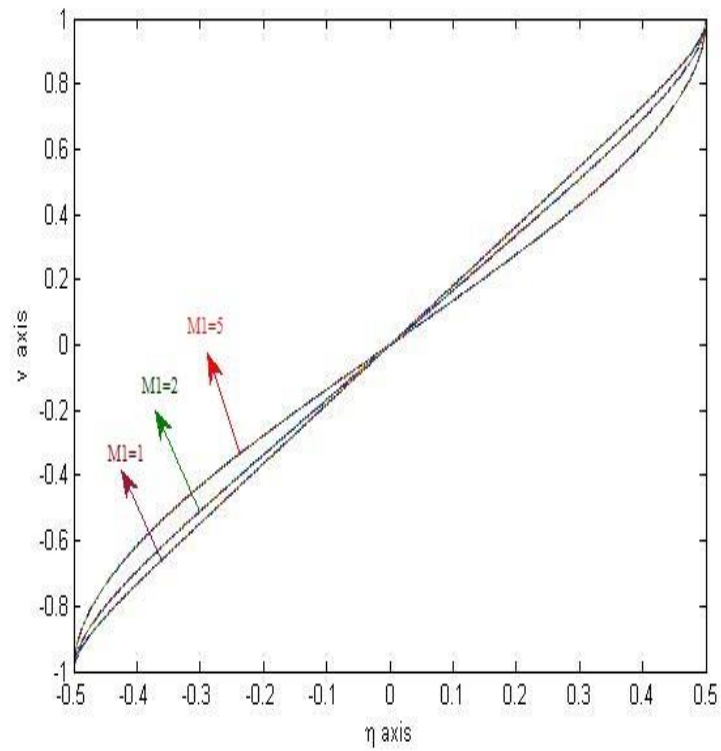


Figure 5.7: Radial velocity profiles for $M_1 = 1$, $M_1 = 2$, $M_1 = 5$,

CHAPTER-6

6.1 CONCLUSION

In the above analysis a class of solutions of steady Magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates with an angular velocity is presented, in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate.

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