

DIGITAL IMAGE PROCESSING ON MEDICAL CONDITIONS USING GRAPH THEORY

Submitted in partial fulfillment of the requirements for the award of Masters of Science degree in Mathematics

By

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**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited with Grade "A" by NAAC
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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of DEEPA R (40630001) who carried out the project entitled "DIGITAL IMAGE PROCESSING ON MEDICAL CONDITIONS USING GRAPH THEORY" under our supervision from January 2022 to March 2022.

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ABSTRACT:

Image processing techniques have been used in a wide variety of applications. The image is taken as input and after noise removal; the image is segmented using simple thresholding, multiple thresholding, and optimal thresholding in the threshold value method. In the spanning tree, the resultant image is segmented using the image segmentation algorithm (prim algorithm). In graph theory using color spatial clustering with consensus region merging, various regions merged. In this paper, we make a comparative study using image processing.

KEYWORDS:

Bipartite graph,

Minimal spanning tree,

Cut vertex,

Normalized cut,

Threshold graph,

Euler graph,

Segmentation, etc.

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CHAPTER – 1

1. INTRODUCTION:

1.1 GRAPH THEORY:

Graph Theory is the study of lines and points. It is a sub-field of mathematics that deals with graphs: diagrams that involve points and lines and which often pictorially represent mathematical truths. Graph theory is the study of the relationship between edges and vertices. Graph theory is used to find the shortest path in a road or a network. In Google Maps, various locations are represented as vertices or nodes, the roads are represented as edges, and graph theory is used to find the shortest path between two nodes.

1.2 APPLICATION OF GRAPH THEORY:

1] Computer science

In computer science graph theory is used like:

- Graphs are used to represent networks of communication.
- Graph theory is used to find the shortest path on a road or a network.
- In Google Maps, various locations are represented as vertices or nodes, the roads are represented as edges, and graph theory is used to find the shortest path between two nodes.

2] Electrical engineering

In Electrical Engineering, graph theory is used in designing circuit connections. These circuit connections are named topologies. Some topologies are series, bridge, star, and parallel topologies.

3] Mathematics

In mathematics, operational research is an important field. Graph theory provides many useful applications in operational research. Like:

- Minimum cost path.
- A scheduling problem.

4] Other branches of science

- In physics and chemistry, graph theory is used to study molecules.
- The 3D structure of complicated simulated atomic structures can be studied quantitatively by gathering statistics on graph-theoretic properties related to the topology of the atoms.
- Nodes in biological networks represent biomolecules such as genes, proteins, or metabolites and edges connecting these nodes indicate functional, physical, or chemical interactions between the corresponding biomolecules.
- Graph theory is used in transcriptional regulation networks.

1.3 IMAGE PROCESSING:

Image processing is a method to perform some operations on an image, to get an enhanced image, or to extract some useful information from it. It is a type of signal processing in which input is an image and output may be an image or characteristics/features associated with that image. Nowadays, image processing is among rapidly growing technologies. It forms a core research area within engineering and computer science disciplines too.

Image processing includes the following three steps:

- Importing the image via image acquisition tools;
- Analyzing and manipulating the image;
- Output in which result can be altered image or report that is based on image analysis.

There are two types of methods used for image processing namely, analog and digital image processing. Analog image processing can be used for hard copies like printouts and photographs. Image analysts use various fundamentals of interpretation while using these visual techniques. Digital image processing techniques help in manipulation of the digital images by using computers. The three general phases that all types of data have to undergo while using digital techniques are pre-processing, enhancement, and display, information extraction.

1.4 APPLICATION OF IMAGE PROCESSING:

1]. Image Correction, Sharpening, and Resolution Correction:

Often, we wish we could make old images better. And that is possible nowadays. Zooming, sharpening, edge detection and high dynamic range editing all fall under this category. All these steps help in enhancing the image. Most editing software and Image correction code can do these things easily.

2] Medical Technology:

In the medical field, Image Processing is used for various tasks like PET scans, X-Ray Imaging, Medical CT, UV imaging, Cancer Cell Image processing, and much more. The introduction of Image Processing to the medical technology field has greatly improved the diagnostics process.

3]. Computer / Machine Vision:

One of the most interesting and useful applications of Image Processing is in Computer Vision. Computer Vision is used to make the computer see, identify things, and process the whole environment as a whole. An important use of Computer Vision is Self Driving cars, Drones, etc. CV helps in obstacle detection, path recognition, and understanding the environment.

4] Pattern recognition:

Pattern recognition is a part of Image Processing that involves AI and Machine Learning. Image processing is used to find out various patterns and aspects in images. Pattern Recognition is used for Handwriting analysis, Image recognition, Computer-aided medical diagnosis, and much more.

5]. Video Processing:

Video is a fast movement of images. Various image processing techniques are used in Video Processing. Some methods of Video Processing are noise removal, image stabilization, frame rate conversion, detail enhancement, and much more.

1.5 BASIC DEFINITION

GRAPH:

A graph, G , comprises a set V of vertices and a set E of edges. The vertex set can be anything but is most commonly a collection of letters or numbers. The set of edges is a set of doubleton subsets of V . That is $E \subseteq \{\{a, b\}: a, b \in V \text{ and } a \neq b\}$. We denote the graph by $G(V, E)$.

SUBGRAPH:

A graph whose vertices and edges are subsets of another graph.

CUT VERTEX:

It is an edge of a graph in which removal of it increases the number of components of the graph.

BIPARTILE GRAPH:

It is a resultant of a given graph such that the given vertex set is divided into two subsets so that if we consider any edge of the resultant graph, one vertex is from the set and another is from the set.

MINIMAL SPANNING TREE (MST):

The subgraph which contains the total weight of the edges should be minimum.

- A spanning subgraph (n-1)
- A tree (not form cycle)
- The Total sum of the weight of edges must be minimum.

DEFINITION OF NORMALIZED CUT:

Let v be the vertex, which is divided into subsets v_1 and v_2 , if you consider every edge of the bipartite graph, it is joined by the two vertices such that $v_1 \in V_1$ and $v_2 \in V_2$ i.e. one vertex from the set V_1 and another from the set V_2 where both V_1 and V_2 are the subsets of the set V .

$$Ncut(A, B) = \left[\frac{cut(A, B)}{vol A} + \frac{cut(A, B)}{vol B} \right]$$

Where,

$$Vol(A) = \sum v_i \in A, v_j \in V \quad (V_i, V_j)$$

$$Vol(B) = \sum v_i \in B, v_j \in V \quad (V_i, V_j)$$

THRESHOLD GRAPH:

A graph is a threshold graph if it can be constructed from the empty graph by repeatedly adding either an isolated vertex or a dominating vertex. A graph is a threshold graph if there is a real number S (the threshold) and for every vertex v , there is a real weight a_v such that: VW is an edge iff

$$a_v + a_w \geq S.$$

ADJACENT:

Two nodes v_i and v_j are called adjacent if $\exists e_{ij} \in E$ or $\exists e_{ji} \in E$, which is denoted by $v_i \sim v_j$. Two edges e_{ij} and e_{sk} are adjacent if they share a common vertex, that is, if

$$i = s, i = k, j = s, \text{ or } j = k.$$

EULER'S GRAPH:

A connected graph G is called an Euler graph, if there is a closed trail that includes every edge of the graph G . Euler Path - An Euler path is a path that uses every edge of a graph exactly once. An Euler path starts and ends at different vertices.

CHAPTER – 2

2. LITERATURE SURVEY:

[1] Image segmentation is a key study area of image processing. Image segmentation is done through the various methods. According to Salem Saleh Al-Amri. N. V, Kalyankar, Khamitkar. S. D [31] threshold methods are classified into three processes simple thresholding, multiple thresholding, and optimal thresholding.

[2] According to Nikhil. R Pal and Sankar. K Pal [23], image segmentation is done in several ways and those methods are mainly divided into two ways, which are the layer-based segmentation method and the block-based segmentation method.

[3] Anjna E and Rajandeep Kaur E [8] also describe a few segmentation techniques. Image segmentation is very practical for image compression or image recognition applications and they have discussed various image segmentation techniques in their paper.

[4] According to Andrew. M. C and Alasdair [9], another method of doing image segmentation is global thresholding. This method is suitable for the image which has uneven lightening.

[5] According to Peng. B, Zhang. L and Zhang. D [28], among many different segmentation techniques, graph theoretical approaches are helpful in practical applications, especially in medical imaging like cancer and tumor cells. Graphs are used to model many problems in the real world in various fields.

CHAPTER – 3

3. PHASES OF A DIGITAL IMAGE PROCESSING (DIP):

1. Image Acquisition
2. Image Enhancement
3. Image Restoration
4. Colour Image Processing
5. Wavelets and Multiresolution Processing
6. Compression
7. Morphological Processing
8. Segmentation
9. Representation and Description
10. Recognition

IMAGE ACQUISITION:

Image acquisition involves using an image sensor and hardware capable of changing the signal data from the sensors to a digitized or analog form. The use of image acquisition and analysis techniques is proposed here as a potential method of obtaining a better understanding of the different phenomena that can happen during drying. Such techniques can quantify the morphological, textural, and color features of materials and the changes that occur in them.

IMAGE ENHANCEMENT:

Image enhancement is the procedure of improving the quality and information content of original data before processing. Common practices include contrast enhancement, spatial filtering, density, and slicing. Spatial filtering improves the naturally occurring linear features like fault, shear zones, and lineaments.

IMAGE RESTORATION:

Image restoration is performed by reversing the process that blurred the image and such is performed by imaging a point source and using the point source image, which is called the Point Spread Function (PSF) to restore the image information lost to the blurring process. The purpose of image restoration is to estimate the original image from the degraded data. Applications range from medical imaging, and astronomical imaging, to forensic science, etc.

COLOR OF IMAGE PROCESSING:

In color image processing, an abstract mathematical model known as color space is used to characterize the colors in terms of intensity values. This color space uses a three-dimensional coordinate system. For different types of applications, several different color spaces exist.

WAVELETS AND RESOLUTION PROCESSING:

Wavelet transform is used to analyze a signal (image) into different frequency components at different resolution scales which is multi-resolution). This allows revealing the images spatial and frequency attributes simultaneously. The multi-resolution theory incorporates image pyramid and sub and coding techniques.

IMAGE COMPRESSION:

Image compression is minimizing the size in bytes of a graphics file without degrading the quality of the image to an unacceptable level. The reduction in file size allows more images to be stored in a given amount of disk or memory space.

MORPHOLOGICAL PROCESSING:

In the morphological processing of images, pixels are added or removed from the images. The structure and shape of the objects are analyzed so that they can be identified. The basic operations in this processing are binary convolution and correlation that is based on logical operations rather than arithmetic operations.

SEGMENTATION:

Image segmentation is a branch of digital image processing that focuses on partitioning an image into different parts according to its features and properties. The primary goal of image segmentation is to simplify the image for easier analysis. The parts in which you divide the image are called Image Objects.

REPRESENTATION AND DESCRIPTION:

After an image has been segmented into object and background regions, one intends to represent and describe them in characteristic features for computer processing during pattern recognition or in quantitative codes for efficient storage during image compression.

RECOGNITION:

Image recognition is used to perform many machine-based visual tasks, such as labeling the content of images with meta-tags, performing image content search, and guiding autonomous robots, self-driving cars, and accident-avoidance systems.

CHAPTER – 4

4. METHODOLOGY:

4.1 IMAGE SEGMENTATION USING MINIMAL SPANNING TREE:

4.1.1 Convert Image to Graph Shape:

The gray level of each pixel in the image is represented in matrix form, where the value of the matrix elements expresses the level of the image, while the position of the matrix element represents the pixel coordinates of the image. Each pixel in the image is identified as the neighboring dots and pixels with the right, left, up where $I(v)$ is the gray level at the point (pixels) v ., down, right top right, lower right corner, upper left corner of the lower-left corner connected by aside. The weight of each side is obtained by using the formula:

$$W(V_i, V_j) = |V_i - V_j| + 1$$

So we will get the weighted graph represented in the form of an adjacency matrix

4.1.2 Minimum Range Tree Process:

In this process, the algorithm used is the Prim algorithm by adding the criteria of similarity. That is, the selection of sides is not only based on the minimum weight but also on considering the criteria of similarity. A side is said to meet the criteria of similarity if the weights are less than the value of the likeness criterion in which this value is any value. As a result, there will be some minimum range trees.

4.1.3 Shaping Image Segmentation Results:

Each point (pixel) in a minimum range tree has the same color. The color of each pixel on a minimum range tree is determined by looking for the average pixel gray level in the minimum range tree. Then, the average obtained will determine the color of pixels on the condition,

{Black, if average $1 > T$

{White, if average $1 \leq T$

Where,

$$T = \frac{maxv+minv}{2}$$

4.2 IMAGE SEGMENTATION USING THRESHOLDING METHODS:

To analyze medical X-ray images through thresholding methods for medical imaging applications. For analysis of X-ray bone images, different thresholding methods are being used and the results are compared with each other. The general thresholding techniques used in our research are simple thresholding with different thresholding values, multiple thresholding, and optimal thresholding. The segmentation generates an enhanced image using intensity-based segmentation on the x-ray image.

4.2.1 SIMPLE THRESHOLDING:

Generally, digital images are viewed as a 2D matrix or 2 variables function consisting of discrete points called pixels. There is a measurable difference between a gray level image and color images in the range of pixel counts. In image processing, it is simple to take a grey-level image in comparison to a color image. In the clustering method of thresholding the gray level samples are clustered into two parts such as foreground and background. For simplicity, the gray level image is converted to a binary image. By selecting an adequate threshold value T , the binary image can be created by converting a gray-level image.

The advantage of obtaining first a binary image is that it reduces the complexity of the data and simplifies the process of recognition and classification. The most widely used way to convert a gray-level image to a binary image is to select a single threshold value (T). Then all the gray level values below T are treated as background and above T are considered to be part of the object. The color code of the background is black (1), and the foreground color is white (0). The segmentation problem becomes one of selecting the proper value for the threshold T . Threshold technique can be expressed in the equation form given below.

$$T = T[x, y, m(x, y), n(x, y)]$$

T is the threshold value where x and y are the coordinates of the threshold value point $m(x, y)$, $n(x, y)$ are points of the gray level image pixels. Threshold image $f(x, y)$ can be defined as:

$$f(x, y) = \begin{cases} 1, & \text{if } m(x, y) > T \\ 0, & \text{if } n(x, y) \leq T \end{cases}$$



Fig 4.1

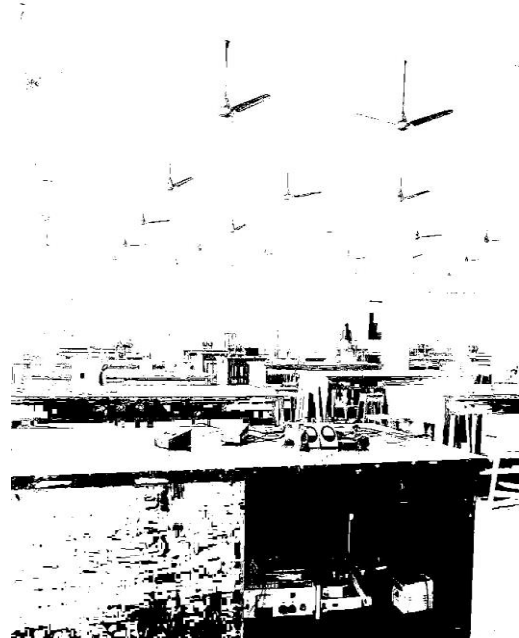


Fig 4.2

4.2.2 MULTIPLE THRESHOLDING:

Threshold-based algorithms are divided into single and multiple thresholding categories. The multiple thresholding approach focuses on finding multiple thresholds which aim to separate multiple objects. When several desired segments in an image are introduced, then threshold segmentation can be extended to use multiple thresholds to segment an image into more than two segments: all pixels with a value lesser than the first threshold are assigned to segment number 0, all pixels with values between the first and second threshold are assigned to segment number 1, all pixels with values between

the second and third threshold are assigned to segment 2, etc. If n thresholds ($t_1, t_2, \dots, t_n$) are used:

$$F(V) = \begin{cases} 0, & \text{if } f(x, y) < t_1 \\ 1, & \text{if } t_1 \leq V < t_2 \\ 2, & \text{if } t_2 \leq V < t_3 \\ \dots & \dots \\ n, & \text{if } t_n < V \end{cases}$$

After thresholding, the existing image has been segmented into $n+1$ distinct segments identified by the grey values 0 to n respectively.



Fig 4.3



Fig 4.4

4.2.3 OPTIMAL THRESHOLDING:

Optimal thresholding is applied where the histogram is the sum of two overlapping distributions. It is a technique that approximates the histogram using a weighted sum of distribution functions. The distribution function sets a threshold in such a way that the number of incorrectly segmented pixels (as predicted from the approximation) is minimal. In optimal thresholding, a criterion function is devised that yields some measure of separation between regions. A criterion function is applied for the intensity and that which maximizes this function is chosen as the threshold.

4.3 GRAPH THEORY USING COLOR SPATIAL CLUSTERING WITH CONSENSUS REGION MERGING:

1. Graph Region Merging (GRM).
2. Random Region Merging (RRM).
3. Consensus Region Merging (CRM).

4.3.1 GRAPH REGION MERGING (GRM):

The graph Region Merging (GRM) algorithm using the broader setting of weighted graphs GRM starts by considering each pixel v as its region (a singleton $\{v\}$) and eventually merges following a predefined order of the regions relying on a statistical merging predicate To describe GRM using the framework of graphs, we first convert an image I into a weighted graph $G = (V, E, w)$ using the 4-connectivity of pixels. The nodes $v \in V$ correspond to pixels, and edges $e = (a, b) \in E$ iff $\|a - b\|_1 = 1$ where $\|\cdot\|$ denote the L_1 norm (C4 South/North/East/West neighboring pixels). This yields $2(w-1)(h-1) + w+h-2$ unoriented edges for an image of size $w \times h$. The weight $w(e) = |I_a - I_b|$ of an edge $e = (a, b)$ is set to the difference of the intensity channel, where I_v denotes the intensity at pixel v .

Region merging is efficiently implemented inequalities. For a region R_x , let I_x denote the average channel value, and n_x the number of pixels contained inside that region. For our purpose, we shall use the following simpler predicate:

$$Merge\ predicate(R_a, R_b) = \begin{cases} true, if |I_a - I_b| < \frac{255}{2 \log \max(n_a, n_b)} \\ false, otherwise \end{cases}$$

4.3.2 RANDOM REGION MERGING (RRM):

To get a “population of segmentations,” we add random behavior in GRM. First, we apply an arbitrary random permutation on the edge pairs (we purely discard the former $w(e)$ values). Second, we consider the following non-deterministic random predicate:

$$Random\ predicate(R_a, R_b) = \begin{cases} true, if |I_a - I_b| < \frac{255U}{2 \log \max(n_a, n_b)} \\ false, otherwise \end{cases}$$

Where,

U denotes a uniform random variate in $[0, 1)$.

4.3.3 CONSENSUS REGION MERGING (CRM):

Consensus Region Merging (CRM) outputs segmentation and a soft contour map. The CRM algorithm consists of computing I independent random segmentations using RRM and then combining the sample outcome into one overall segmentation.

The key idea consists in adding one vote to an edge e whenever its merging predicate was evaluated to true. We then set us to the number of votes, and run GRM to export a hard segmentation. In addition, we also compute a soft contour detector as follows: Each edge adds to the weights of its two extremity pixels the number of times it was selected for merging.

5. CONCLUSION:

The image segmentation process by applying graph theory using color spatial clustering with consensus region merging is useful in medical imaging for better analysis during diagnosis. This method shows a better quality of the image parameter as it has lower MSE, and NAE values. MSE value is comparatively small for both MRI scan and X-ray images since lower MSE gives higher images quality. Normalized absolute error value of proposed method relatively less as the small value of NAE gives the good quality image.

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