

THE INVENTORY MODEL FOR DETERIORATING PRODUCTS WITH CONSTANT DEMAND

Submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in
Mathematics

by

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BONAFIDECERTIFICATE

This is to certify that this Project Report is the Bonafide work of RITHIKA. M (39060009) , SUSEELA.V (39060013) who carried out the project entitled "**THE INVENTORY MODEL FOR DETERIORATING PRODUCTS WITH CONSTANT DEMAND** " under our supervision from November 2021 to April 2022.

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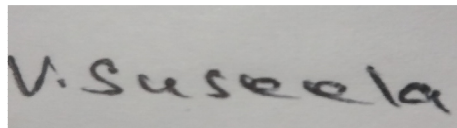
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DECLARATION

We **Rithika. M (39060009)**, **Suseela. V (39060013)** hereby declare that the Project Report entitled “**The Inventory Model For Deteriorating Products With Constant Demand**” done by us under the guidance of **Asst. Prof. P. Muniappan** (Internal) and _____(External) at Sathyabama Institute of Science and Technology is submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in Mathematics.

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ABSTRACT

In this paper, an optimal inventory cost for a deteriorating products and constant demand is consider. The inventory strategy in turn depends on the cost and the situation that exist between vendor and buyer, in general inventory affects the production process and supply chain activities. Integrated decision inventory improves performance and optimizes supply chain activities. However, its disadvantage is the Integrated Procurement Production inventory model, which manages the raw material inventory in procurement activities, works in process, and finished products. The objective is to find optimal values of replenishment cycle length (T), which minimize the total inventory cost.

CONTENT

1. INVENTORY
 - 1.1. INVENTORY
 - 1.1.1. WHAT IS INVENTORY
 - 1.1.2. UNDERSTANDING INVENTORY
 - 1.2. INVENTORY MANAGEMENT
 - 1.3. INVENTORY CONTROL
 - 1.4. TYPES OF INVENTORY
 - 1.4.1.
 - 1.5. VARIOUS COST IN INVENTORY CONTROL
 - 1.6. METHODS
2. INTRODUCTION
 - 2.1. AIM
 - 2.2. OBJECTIVE
 - 2.3. ASSUMPTIONS AND NOTATIONS
 - 2.3.1. ASSUMPTIONS
 - 2.3.2. NOTATIONS
3. FORMULATION
 - 3.1. MATHEMATICAL MODEL FORMULATION
 - 3.1.1. HOLDING COST
 - 3.1.2. DETERIORATING COST
 - 3.1.3. SETUP COST
 - 3.1.4. AVERAGE TOTAL COST
 - 3.1.5. FINDING OPTIMUM TIME PERIOD
 - 3.1.6. ALGORITHMS
4. RESULTS
5. CONCLUSION

1. INVENTORY

Inventories are basically stocks of goods being held for future use or sale. Maintaining inventories is necessary for any company dealing with physical products, including manufacturers, wholesalers, and retailers. Storing and carrying inventory contain some costs, but facing problem of out of stock and costs associated to it make huge negative impact on the growth of the company.

“Inventory refers to all the items, goods, merchandise, and materials held by a business for selling in the market to earn a profit.”

What Is Inventory?

The term inventory refers to the raw materials used in production as well as the goods produced that are available for sale. A company's inventory represents one of the most important [assets](#) it has because the [turnover of inventory](#) represents one of the primary sources of revenue generation and subsequent earnings for the company's shareholders. There are three types of inventory, including raw materials, work-in-progress, and finished goods. It is categorized as a current asset on a company's [balance sheet](#).

Understanding Inventory:

Inventory is a very important asset for any [company](#). It is defined as the array of goods used in production or finished goods held by a company during its normal course of business. There are three general categories of inventory, including raw materials (any supplies that are used to produce finished goods), work-in-progress (WIP), and finished goods or those that are ready for sale.

As noted above, inventory is classified as a current asset on a company's balance sheet, and it serves as a buffer between [manufacturing](#) and order fulfillment. When an inventory item is sold, its carrying cost transfers to the [cost of goods sold](#) (COGS) category on the income statement.

Company management, analysts, and investors can use a company's inventory turnover to determine how many times it sells its products over a certain period of time. Inventory turnover can indicate whether a company has too much or too little inventory on hand.

INVENTORY MANAGEMENT

“Inventory management is a discipline primarily about specifying the shape and placement of stocked goods”

Possessing a high amount of inventory for a long time is usually not a good idea for a business. That's because of the challenges it presents, including storage costs, spoilage costs, and the threat of obsolescence.

Possessing too little inventory also has its disadvantages. For instance, a company runs the risk of [market share](#) erosion and losing profit from potential sales.

[Inventory management](#) forecasts and strategies, such as a [just-in-time](#) (JIT) inventory system (with [back flush costing](#)), can help companies minimize inventory costs because goods are created or received only when needed.

What Is Inventory Management?

Inventory management manages the process of ordering, storing and using inventory, both at the level of the raw materials used as well as finished goods. Inventory management helps businesses identify which stock to order, how much to order and when.

Sometimes known as stock management, the practice involves tracking the lifecycle of raw and finished goods from the moment of ordering a product from suppliers to delivering it to customers and everything in between. Inventory management is a key part of the supply chain and ensures there is sufficient stock on hand to fulfill customer orders, but not more than a business can sell. For example, a taco truck can use inventory data to correctly predict it needs five pounds of tomatoes for a week. The business won't waste money by throwing out overripe tomatoes that it can't use. Similarly, an event planner won't spend money renting extra space to store paper cups that she won't use. Learn more about inventory management techniques.

Implementing an effective inventory management practice can help your business hold less inventory on hand, increase order fulfillment accuracy and decrease costs.

Some of the fundamental elements of inventory management include:

- Managing stock end-to-end, from ordering to selling.

- Forecasting demand using past sales data to estimate future customer product demand. With a better understanding of possible demand, companies can make better decisions about inventory. Management can use this data to understand market potential, adjust pricing and identify ways to grow.
- Determining when to replenish stock.
- Tracking inventory turns, or how often an item sells or moves within a certain time.
- Performing cycle counts, or regular checks of a small set of goods to confirm stock levels..
- Conducting audits to verify that inventory counts are correct as the stock moves through each stage.

Inventory control starts from the moment stock enters your warehouse till it leaves for shipments. It has to do with all the operational activities done to manage stock in the warehouses and different tactics employed to achieve continuous physical availability of inventory.

With the wide range of available software, inventory control also means quality control, tracking locations and bins, tracking items with barcodes, SKUs, and so on. By implementing an efficient inventory control system, you can optimize your stock to cut on what doesn't sell and procure more of what sells fast.

INVENTORY CONTROL

What Is Inventory Control?

Inventory control is a part of the inventory management process. It is the daily activity of managing stock within the warehouse. Inventory control activities include receiving, storing and transferring stock, as well as tracking and fulfilling orders and returns.

Taking control over your stock rotation is an important part of inventory control and an overall efficient inventory management process. Defining the flow of stock ensures you have control over what items are used to fulfill customer orders, and when. Depending on your product portfolio you can dictate how product is deployed to fulfill customer orders. Some common inventory control methods are:

- FIFO (First in, First out) – the oldest inventory is used first to fulfill customer orders.
- LIFO (Last in, First out) – the inventory received most recently is used to fulfill customer orders.
- FEFO (First expiring, First out) – the inventory closest to its expiration date is used to fulfill customer orders.

Warehouse organization also falls under inventory control. When a new product arrives, staff scan the barcode or RFID tag using a mobile scanner. Each product type received has a unique code. Companies can also assign high-value products unique codes. For example, an expensive device such as a high-end sound system can be tagged based on its unique serial number.

The team checks inbound shipments and notes delivery details. When a product is sold, fulfillment records and packs orders to go out.

Vendor-managed inventory (VMI) is the business model that removes inventory management from the retailer. The customer deals directly with the vendor supplying the goods (although they are not always aware of this arrangement). The vendor decides when to refill orders and how much to send. Often these choices are based on data it receives from point-of-sale (POS) systems.

Some of the fundamental elements of inventory control include:

- Putting stock away and tracking its exact location within a warehouse.
- Ensuring stock stays in good condition, with no spoilage, and is used before its expiration date.
- Locating popular items near packing areas to speed order fulfillment.

The benefits of inventory control can include higher revenue and happier customers. When you track stock accurately, you reduce stockouts and backorders. Reducing excess stock and obsolete inventory can help eliminate spoilage and remove added storage costs.

Inventory Management vs Inventory Control

Inventory management and inventory control are similar but have different focuses. Inventory management handles forecasting and ordering stock. Inventory control, also known as stock control, is a part of inventory management that handles the stock on-hand.

Differences Between Inventory Control and Inventory Management

Inventory control is responsible for the movement of inventory within the warehouse. With stock control, you track which goods or materials you have and in

which quantities. You also track the condition and status of items. By contrast, inventory management encompasses the entire process of forecasting demand, ordering and managing stock on hand. This practice looks to the future to see what customers will want to purchase and places orders accordingly.

Similarities of Inventory Control and Inventory Management

Both inventory control and inventory management track and manage stock. Inventory control is a part of the overall inventory management process and tracks daily trends for each item. Inventory management follows broader trends over longer periods. Both practices use mobile devices to for barcodes and radio frequency identification (RFID) scans for precise, real-time updates.

Inventory control objectives

- Ensure stock availability
- Prevent wastage
- Maximize profitability
- Save storage space

Ensure stock availability

Having the right inventory levels is only a part of the problem to ensure smooth flow of goods in warehouses. For a manufacturing company, there are bottlenecks caused by disorganized warehouse structures or related items being placed too far away.

Inventory control's objective here is to ensure a steady supply of raw materials and subassemblies (manufactured or outsourced) from the warehouses without interruptions for fulfilling manufacturing orders on time. This also applies to distribution businesses where inventory has to be picked and packed for shipping orders.

Prevent wastage

Inventory waste happens in two cases—either the stock is defective or the stock has expired. It also happens when stock is idle and there's no market demand. Here, inventory control aims to prevent defective stock from entering your warehouses with quality control, and to prevent dead stock or accumulation of expired items.

Maximize profitability

Poor inventory control impacts profits negatively. This is not surprising once you realize that one of the other objectives is avoiding stock wastage. Not only that, but poor inventory control also means difficulty in picking items from warehouses where required, be it manufacturing orders or distribution activities. If the orders are not fulfilled on time, customers won't be happy. Hence, good inventory control positively impacts revenue in two ways—by avoiding money lost in expired stock and eliminating any inefficiencies during stock movements between storage areas.

Save storage space

Stocking inventory requires physical space. Actively managing your inventory, warehouse layouts, locations, saves on warehouse space and hence rent costs. It also involves doing away with excess stock, dead stock, and expired stock. If your storage areas are planned well and aren't occupied by dead stock, you'll be saving on storage space for the products that sell better.

TYPES OF INVENTORY

The three types of inventories are direct material inventory, work in progress inventory and the finished goods inventory where the direct material inventory includes the stock of raw material which the company has purchased for its use in production; work in progress inventory is the cost accumulated to the goods that are partially completed and the finished goods inventory is the stock that has finished all stages of production and is now available for sale.

Inventory means those current assets, which have been or will be converted into the final products of a company for sale in the near future. In other words, inventory represents finished goods or goods in different stages of production that a company keeps at its premises or at third-party locations with ownership interest retained until goods are sold.

The three types of inventory most commonly used are

- Raw Materials (raw material for making finished goods)
- Work-in-Progress (items in the process of making finished goods for sale).
- Finished Goods (available for selling to customers)

It is essential to understand the type of inventory you have, so you can make better decisions while choosing the best inventory system to calculate the cost of goods sold (COGS).

RAW MATERIAL

Raw materials are any items used to manufacture finished goods. Raw materials are mainly purchased or produced by your company to make your finished product. Raw materials are the basic materials that a manufacturing company buys from its suppliers, and that is used by the former to convert them into the final products by applying a set of manufacturing processes. For example, aluminum scrap is the raw material for a company that produces aluminum ingots. Flour is the raw material for a company that produces bread or pizza. Similarly, metal parts and ingots are the raw materials bought by a company that manufactures cars, and crude oil is the raw material for an oil refinery.

It is prevalent and easy to observe that the final products of one company are bought as raw materials for some other company. For instance, many oil drilling companies produce crude oil as their final product. On the other hand, the same crude oil is bought by oil refining companies as raw materials to manufacture their final products, i.e., gasoline, kerosene, paraffin, etc. It is essential to optimize raw material inventory. It is because if a company keeps too much raw material inventory in stock, it will incur higher carrying costs, and there is also the undesirable possibility of the inventory getting obsolete. For example, in the pharmaceutical or food industry, the raw materials may be perishable. If not used within a stipulated time limit, they can get expired and can't be used in production. On the other hand, a company must have a certain minimum level of inventory at all times to cater to the production volumes, which mostly follow the trend of the market demand. Thus, the optimization of raw material inventory is essential

Materials that are needed to turn your inventory into a finished product are raw materials. These inventory items are bits and pieces of component parts that are currently in stock but have not yet been used in either work-in-process or finished goods inventory.

There are two types of raw materials: direct materials—which are used *directly* in finished goods, and indirect materials—which are part of overhead or factory costs.

Example,

direct raw materials might be leather to make belts for your company would fall under this category. Or, if you sell artificial flowers for your interior design business, the cotton used would be considered direct raw materials, too.

Indirect raw materials might be lightbulbs, batteries, or anything else that indirectly contributes to keeping your shop running.

WORK-IN-PROCESS

Work-in-progress inventory refers to unfinished items in the process of making finished goods for sales. Work-in-progress inventory is unfinished items or components currently in production, but not yet ready for sale to the customers.

Work in progress inventory can also be called semi-finished goods. They are the raw materials that have been taken out of the raw materials store and are now undergoing the process of their conversion into the final products. These are the partly processed raw materials lying on the production floor. And they have also not reached the stage where they have been converted into the final product.

The extent of inventory locked-up as work in progress is lower the better. It is understandable as the inventory under process is of no use till it gets converted into the final product. It may be saleable at some price, but it cannot be sold to generate any revenue for the company's core business. In fact, in lean manufacturing systems, the work in progress inventory is considered as waste.

So it is most desirable that the volume of inventory that is lying in the form of work in progress be minimized, and the time is taken to convert it into the final, also be minimized so that the locked-up value can be released as quickly as possible. The idea is that this capital, which is locked-up in the form of work in progress inventory, can otherwise be invested somewhere else to achieve much better returns.

Inventory that is being worked on is Work-In-Process (WIP), just like the name sounds. From a cost perspective, WIP includes raw materials (plus, sometimes labor costs) that are still "in production" when the accounting period ends.

In other words, whatever direct and indirect raw materials your business is using to create finished goods is WIP inventory.

Example, If you sell medical equipment, the packaging would be considered WIP. That's because the medicine cannot be sold to the consumer until it is stored in proper packaging. It's literally a work-in-process.

Another example would be a custom wedding dress that's not quite finished when the end of the fiscal year rolls around. That lace, silk, and taffeta are no longer raw materials, but they're not quite a "finished goods" wedding dress, either.

Maintenance, Repair, and Operating (MRO) inventory

MRO refers to Maintenance, repair and operation supplies. These are materials, equipment and supplies used in the production process at a manufacturing plant but are not part of the finished goods being produced.

Inventory management is an attempt to have the right stock, in the right place, at the right time, and at the right cost. The goal is to minimize cost by helping facilities know when to purchase more inventory based on normal usage rates.

The way you define inventory depends on your industry. For a retail store, inventory is the products the store is trying to sell customers, like suits or dresses. When it comes to maintenance, inventory is the parts used to make assets function properly, like motors, bearings, fans or filters. The aim for maintenance teams is to have the right inventory on hand in the right amounts to repair or improve assets, while also considering the space available in their budgets and storerooms.

MRO inventory management, or maintenance inventory management, is the process of procuring, storing, using, and replenishing the materials and supplies used for maintaining assets at the lowest possible cost. This process involves ensuring you have stock on hand while factoring in available storage space and budget.

Examples: include janitorial supplies, safety equipment, maintenance and repair supplies, technology, office supplies and laboratory equipment

Decoupling inventory

Decoupling inventory is the term used when product manufacturers set aside extra raw materials or work in progress items for all or some stages in a production line, so that a low-stock situation or breakdown at one stage doesn't slow or stop operations.

The process of separating inventory within a manufacturing procedure in order to mitigate one stage of manufacturing from slowing down another stage of manufacturing. What is the purpose of decoupling in inventory management?

Decoupling inventory is a valuable tool in inventory management. It acts as a buffer that allows you to continue sales and order fulfillment processes even as production issues are happening. This lets you avoid lost sales and defends your supply chain from the bullwhip effect. What is the purpose of decoupling in inventory management?

Example: Decoupling inventory is a valuable tool in inventory management. It acts as a buffer that allows you to continue sales and order fulfillment processes even as production issues are happening. This lets you avoid lost sales and defends your supply chain from the bullwhip effect.

FINISHED GOODS

Finished goods are your completed products available for selling to customers. Finished goods can be manufactured or purchased from a supplier. Any product that is sold to your customers is your finished good.

Finished goods are indeed the final products obtained after the application of the manufacturing processes on the raw materials and the semi-finished goods discussed above in the article. They are saleable, and their sale contributes fully to the revenue from the core operation of the company.

Regarding the level of finished goods inventory, there are two types of industries that we need to look at. First, we would take the industries in which the finished goods are mass-produced, and the sale happens after the production. Examples of such industries are the FMCG industry and the oil industry. For a company in such an industry, the correct approach is to maintain the finished goods inventory similarly as the raw material inventory is maintained, i.e., at an optimized level as per the demand in the market.

The other type of industry is one in which the goods are manufactured on demand, i.e., the order is first received, and then the production starts. An example of such industries is the capital goods industry and the customized goods industry. For a company in such an industry, it is neither necessary nor advisable to keep any inventory of finished goods because their finished goods kept ready in stock might never get sold even if they have the slightest deviation from the specifications of the new orders coming from the customers. So they may never get a return on their investment gone in making the finished goods ready.

Maybe the most straightforward of all inventory types is finished goods inventory. That inventory you have listed for sale on your website? Those are finished goods. Any product that is ready to be sold to your customers falls under this category.

Example, Finished goods could be a pre-packaged fruit salad, a monogrammed bathrobe, or a custom-built laptop ready for an employee to use

VARIOUS COST IN INVENTORY CONTROL

.Ordering, holding, carrying, shortage and spoilage costs make up some of the main categories of inventory-related costs. These groupings broadly separate the many different inventory costs that exist, and below we will identify and describe some examples of the different types of cost in each category.

The other requires a certain amount of calculation to understand the impact it has on your Gross Profit. Let's look at types of costs :

1. Ordering Costs

Ordering costs include payroll taxes, benefits and the wages of the procurement department, labor costs etc. These costs are typically included in an overhead cost pool and allocated to the number of units produced in each period.

$$C_3 = (\text{Cost per order}) \times (\text{Number of order in unit time})$$

- Transportation costs
- Cost of finding suppliers and expediting orders
- Receiving costs
- Clerical costs of preparing purchase orders
- Cost of electronic data interchange.

2. Inventory Holding Costs

This is simply the amount of rent a business pays for the storage area where they hold the inventory. This can be either the direct rent the company pays for all the warehouses put together or a percentage of the total rent of the office area utilized for storing inventory.

- Inventory services costs
- Inventory risk costs
- Opportunity cost - money invested in inventory
- Storage space costs
- Inventory financing costs

3. Shortage Costs

Shortage costs, also known as stock-out costs, occurs when businesses become out of stock for various reasons. This is denoted by ' C_2 '. shortage cost is the cost incurred when the goods cannot be supplied on demand. The shortage cost can be calculated by the following expression.

$$C_2 = (\text{Shortage cost}) \times (\text{number of the units short in the inventory}).$$

Shortage costs some of the reason might be as below:

- Emergency shipments costs
- Disrupted production costs
- Customer loyalty and reputation

4. Spoilage Costs

Perishable inventory stock can rot or spoil if not sold in time, so controlling inventory to prevent spoilage is essential. Products that expire are a concern for many industries. Industries such as the food and beverage, pharmaceutical, healthcare and cosmetic industries, are affected by the expiration and use-by dates of their products.

5. Inventory Carrying Costs

This is the lesser-known aspect of inventory cost. This cost requires a certain amount of calculation to understand the extent of its impact on your P&L statement. Inventory

carrying costs refers to the amount of interest a business loses out on the unsold stock value lying in the warehouses. This includes the expenses like provision of warehouse, payment of salaries to employees deterioration cost and depreciation, etc.

Carrying not can be calculated by the relation

$C_1 = (\text{Cost of carrying one unit of an item}) * (\text{Average number of units of the item carried in the inventory})$.

2. INTRODUCTION

Inventory is the major strategy that needs careful decision when it comes to production. The decision becomes more critical when the product has limited shelf life such as chemicals, drugs, food products etc., The inventory strategy in turn depends on the cost and the situation that exist between vendor and buyer. The vendor or the buyer sometimes may face the situation of shortages. Shortages refer to the inability to meet the demand at the required time schedule as preferred by the customer. There are three types of shortages. They are completely backlogged shortages, partially backlogged shortages or lost sales. The term completely backlogged shortages means the shortages which are duly fulfilled by the vendor during the shortage period. Sometimes the shortages may not be completely backlogged and only a part of demand can be met by the vendor during the shortage period, which is termed as partially backlogged. Lost sales means the demand is totally not fulfilled during shortage period. Hence an attempt has been made to use the completely backlogged shortage for fixed life time products. In a co-ordination strategy quantity discount mechanisms are necessary to maximize the profit of buyer and the vendor. In this situation vendor provides a discount to the buyer to increasing his saving percentage and the buyer alter his order quantity with quantity discount. So quantity discount plays an important role in co-ordination strategy.

Past researchers concentrates various types of demands like stock dependent, time dependent, price dependent and inventory level dependent demands. Chung – Yuan Dye [6] developed inventory model with stock dependent demand and partial backlogging shortages with permissible delay in payments. Gupta and Vrat [8] framed inventory model for stock dependent consumption rate. Mandal and Phaujdar [12] developed an inventory model for deteriorating items and stock dependent consumption rate. Padmanabhan and Virat [14] considered EOQ models for perishable items under stock dependent selling rate. Chang and Dye [4] assumed time varying demand and partially backlogging shortages. Backer and Urban [2] developed deterministic inventory system with an inventory level dependent demand. Sarker et al. [16] assumed an order level lot size model with inventory level dependent demand. BiswajitSarkar [3] reviewed an EOQ model with delay in payments and time varying deterioration rate. ShibSankar Sana [17] studied an EOQ model with perishable items for price sensitive demand. A.K. Maiti et al. [13] developed inventory model for stochastic lead time and price dependent demand for advanced payments. Jing Chen and Peter C. Bell [11] had researched on coordinating supply chain with customer returns and price dependent stochastic demand. The optimal inventory policy with price and credit demand was analyzed by Chia – Huei Ho [5].

Ordering policies of deteriorating items under permissible delay in payments was considered by Aggarwal and Jaggi [1]. EOQ model for permissible delay in payments

was performed by Goyal [7]. Exponential deteriorating product under condition of permissible delay in payments was assumed by Hwang and Shrinin [9]. Shinn et al. [18] developed permissible delay in payments and quantity discount for freight cost. Jamal et al. [10] considered ordering policy for deteriorating items with shortages and delay payments. SaoussenKrichen et al. [15] analyzed single supplier multiple cooperative retailers inventory model with quantity discount and permissible delay in payments. Yanlai Liang and Fangming Zhou [20] developed inventory model for deteriorating items under conditionally permissible delay in payments. R. Uthayakumar and P. Parvathi [19] considered a deterministic inventory model for deteriorating items with time dependent demand and partially backlogged shortages with permissible delay in payments. Aggarwal, Jaggi [1] and Hwang, Shinn [9] extended Goyal's [7] model to consider the deterministic inventory model with a constant deterioration rate. Jamal et al. [10] extend Aggarwal and Jaggi's model to allow for shortages. We extend R. Uthayakumar and P. Parvathi's [19] model for time dependent quadratic demand with shortages and permissible delay in payments.

2.1. AIM:

This project is to develop a mathematical model of an efficient inventory strategies for a distribution company such that using this system they optimize the inventory and minimize the inventory cost.

2.2. OBJECTIVE:

To find the optimal value of replenishment cycle length (T) which minimize the total inventory cost.

2.3. Assumptions and Notations

The following assumptions and notations are used to develop the model.

2.3.1. Assumptions

- (1) Demand is constant.
- (2) Lead time is zero.
- (3) All items order by the vendor arrive fresh and new. i.e., their age equals zero.

2.3.2. Notations

$I(t)$	- The inventory level at time t .
θ	- Deterioration rate a fraction of the on-hand inventory.
h	- Holding cost excluding interest charges, \$ per unit per year.
D	- Demand per unit per year.
T	- The length of replenishment cycle.
P	- Deteriorating cost per unit per year.
A	- Set cost per cycle per year
TC	- The average total inventory cost per unit time.

3.FORMULATION

3.1.MATHEMATICAL MODEL FORMULATION

The model can be formulated as follows, at $t=0$, the inventory level in $I(t)$. And inventory level is decreased due to demand as well as deteriorating items. Hence, by using Differential equation, we have,

$$\frac{dI(t)}{dt} = D - \theta(t)I(t); 0 \leq t \leq T$$

$$\frac{dI(t)}{dt} + \theta(t)I(t) = D$$

Now, let $P = \theta$; $Q = D$ here

$$\therefore \int P dt = \int \theta dt = \theta t$$

$$I(t)e^{\int P dt} = \int Q e^{\int P dt} \cdot dt + c$$

$$I(t)e^{\theta t} = \int D e^{\theta t} \cdot dt + c$$

$$I(t)e^{\theta t} = D \int e^{\theta t} \cdot dt + c$$

$$I(t)e^{\theta t} = D \frac{e^{\theta t}}{\theta} + c \dots (1)$$

$$I(T)e^{\theta T} = \frac{De^{\theta T}}{\theta} + c = 0$$

$$\text{Therefore, } c = \frac{-De^{\theta T}}{\theta}$$

Now substituting c in **equation (1)**,

$$I(t)e^{\theta t} = \frac{De^{\theta t}}{\theta} - \frac{De^{\theta T}}{\theta}$$

$$I(t)e^{\theta t} = \frac{D}{\theta} [e^{\theta t} - e^{\theta T}]$$

$$I(t) = \frac{D}{\theta} \left[\frac{e^{\theta t}}{e^{\theta t}} - \frac{e^{\theta T}}{e^{\theta t}} \right]$$

$$I(t) = \frac{D}{\theta} [1 - e^{\theta t - \theta T}]$$

$$I(t) = \frac{D}{\theta} [1 - e^{\theta(T-t)}]; \quad 0 \leq t \leq T \quad \dots \dots (2)$$

$$3.1.1. \text{ Holding cost} = \frac{h}{T} \int_0^T I(t) dt$$

$$= \frac{h}{T} \int_0^T \frac{D}{\theta} [1 - e^{\theta(T-t)}] dt$$

$$= \frac{Dh}{T\theta} \int_0^T [1 - e^{\theta(T-t)}] dt$$

$$= \frac{Dh}{T\theta} \left[t - \frac{e^{\theta(T-t)}}{-\theta} \right]_0^T$$

$$= \frac{Dh}{T\theta} \left[T + \frac{1}{\theta} - \frac{e^{\theta T}}{\theta} \right]$$

$$= \frac{Dh}{T\theta} \left[T + \frac{1 - e^{\theta T}}{\theta} \right] \quad \dots \dots (3)$$

$$3.1.2. \text{ Deteriorating cost} = \frac{p\theta}{T} \int_0^T I(t) dt$$

$$= \frac{p\theta}{T} \int_0^T \frac{D}{\theta} [1 - e^{\theta(T-t)}] dt$$

$$= \frac{Dp}{T} \left[t - \frac{e^{\theta(T-t)}}{-\theta} \right]_0^T$$

$$= \frac{Dp}{T} \left[T + \frac{1}{\theta} - \frac{e^{\theta T}}{\theta} \right]$$

$$= \frac{Dp}{T} \left[T + \frac{1 - e^{\theta T}}{\theta} \right] \quad \dots \dots (4)$$

$$3.1.3. \text{ Setup cost} = \frac{A}{T} \quad \dots \dots (5)$$

3.1.4. THE TOTAL AVERAGE COST

The total average inventory cost formulated as follows

TOTAL AVERAGE COST = HOLDING COST + DETERIORATING COST + SETUP COST.

$$\begin{aligned}
 \therefore Tc &= \frac{hD}{T\theta} \left(T + \frac{1 - e^{\theta T}}{\theta} \right) + \frac{Dp}{T} \left(T + \frac{1 - e^{\theta T}}{\theta} \right) + \frac{A}{T} \\
 &= \frac{1}{T} \left[A + \frac{hD}{\theta} \left(T + \frac{1 - e^{\theta T}}{\theta} \right) + Dp \left(T + \frac{1 - e^{\theta T}}{\theta} \right) \right] \\
 &= \frac{1}{T} \left[D \left(T + \frac{1 - e^{\theta T}}{\theta} \right) \left(\frac{h}{\theta} + p \right) + A \right] \theta \\
 &= \frac{1}{T} \left[A + D \left(\frac{h + p\theta}{\theta} \right) \left(\frac{T\theta + (1 - e^{\theta T})}{\theta} \right) \right] \\
 &= \frac{1}{T} \left[A + \frac{D}{\theta^2} (h + p\theta) (T\theta + (1 - e^{\theta T})) \right] \quad \text{..... (6)}
 \end{aligned}$$

To find the optimum time period,

Differentiate the total average cost with respect to T,

$$\begin{aligned}
 \frac{dTc}{dT} &= \left[\frac{1}{T} \left[A + \frac{D}{\theta^2} (h + p\theta) (T\theta + 1 - e^{\theta T}) \right] \right] \frac{d}{dT} \\
 \frac{dTc}{dT} &= \left[A + \frac{D}{\theta^2} (h + p\theta) (T\theta + 1 - e^{\theta T}) \right] \left(-\frac{1}{T^2} \right) + \frac{1}{T} \left[\left[\frac{D}{\theta^2} (h + p\theta) \right] (\theta - e^{\theta T}(\theta)) \right] \\
 &\quad \text{..... (7)}
 \end{aligned}$$

$$\begin{aligned}
 \frac{d^2Tc}{dT^2} &= \left[A + \frac{D}{\theta^2} (h + p\theta) (T\theta + 1 - e^{\theta T}) \right] \left(-\frac{1}{T^2} \right) \frac{d}{dT} + \frac{1}{T} \left[\left[\frac{D}{\theta^2} (h + p\theta) \right] (\theta - e^{\theta T}(\theta)) \right] \frac{d}{dT} \\
 \frac{d^2Tc}{dT^2} &= \left[A + \frac{D}{\theta^2} (h + p\theta) (T\theta + 1 - e^{\theta T}) \right] \left(\frac{1}{T^3} \right) + \left[\frac{D}{\theta^2} (h + p\theta) (\theta - e^{\theta T}(\theta)) \right] \left(-\frac{1}{T^2} \right) + \\
 &\quad \left[\frac{D}{\theta^2} (h + p\theta) (\theta - e^{\theta T}(\theta)) \right] \left(-\frac{1}{T^2} \right) + \left[\frac{D}{\theta^2} (h + p\theta) (-e^{\theta T}(\theta^2)) \right] \left(\frac{1}{T} \right) \quad \text{.....(8)}
 \end{aligned}$$

Algorithm

1. Set a trial value of T.
2. Using Equation by using (7) find the value of t.
3. Using Equation (8), find T.

4. Repeat steps 7 – 8 till $T^* = T$
5. Compute the corresponding $TC(T^*)$

For optimality, $\frac{dTC}{dT} = 0$, and $\frac{d^2TC}{dT^2} > 0$, we get the optimum cycle period