

# **UNSTEADY MHD FLOW THROUGH POROUS MOVING CHANNEL**

Submitted in partial fulfillment of the requirements for the award of  
Bachelor of Science degree in Mathematics (B.sc Mathematics)

BY

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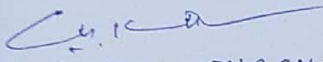
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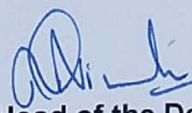
## DEPARTMENT OF MATHEMATICS

### BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **JEYASUDHA.V** (39060005) and **PRIYADHARSHINI.D** (39060008) who carried out the project entitled "**UNSTEADY MHD FLOW THROUGH POROUS MOVING CHANNEL**" under our supervision from **AUGUST 2021** to **APRIL 2022**.

  
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## DECLARATION

We JEYASUDHA.V(39060005), PRIYADHARSHINI.D(39060008) hereby declare that the Project Report entitled "UNSTEADY MHD FLOW THROUGH POROUS MOVING CHANNEL" done by me under the guidance of Dr.C.K.KIRUBHASHANKAR (Internal) at Sathyabama Institute of Science and Technology submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in Mathematics.

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## **ABSTRACT**

The unsteady two dimensional laminar stream of an incompressible viscous liquid between two parallel plates in the presence of a transverse magnetic field is considered by accepting the normal wall velocities to be equal. We study the steady MHD flow between moving plates with small uniform suction at the lower stationary plate. The velocity profile for the stream of the liquid is examined for various qualities. Also, the impacts of velocity potential are researched and discussed graphically.

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# CHAPTER 1

# **INTRODUCTION**

## **1.1 FLUID DYNAMICS**

Fluid Mechanics is a branch of science that deals with the behavior of fluids (liquids or gasses) at rest as well as in motion. Thus this branch of science deals with the static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure forces are not considered, is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of Science is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; the influence of wind on building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid

## **1.2 TYPES OF FLOW**

- IMPORTANCE OF FLUID FLOW
- MAGNETOHYDRODYNAMIC FLOWS
- FLOW THROUGH POROUS MEDIUM
- NEWTONIAN FLOWS

### **1.2.1 IMPORTANCE OF FLUID FLOW**

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of fluid under the effect of external forces.

### **1.2.2 MAGNETOHYDRODYNAMIC FLOWS**

MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Magnetohydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil.

### **1.2.3 FLOW THROUGH POROUS MEDIUM**

A medium that is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity ( $\epsilon$ ) and permeability ( $k_i$ ). The porosity of a porous medium describes how densely the material is packed. The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by the void space. Thus  $1 - \epsilon$  is the fraction that is occupied by a solid. Porosity is a fraction between 0 and 1, typically ranging from less than 0.01 for solid granite to more value 1. For natural media,  $\epsilon$  does not normally exceed 0.6.

The permeability is related to pore-size distribution since the distribution of the sizes of entrances, exits and lengths of the pore walls make up the major resistance to flow. The permeability is the single parameter that reflects the conductance of a given pore structure. The dimensions of the permeability are length squared. In the oil industry, it is measured in 'Darcy's with  $1 \text{ darcy} = 9.87 \times 10^{-9} \text{ cm}^2$ . The permeability and porosity are related since if the porosity is zero the permeability is zero.

#### **1.2.4 NEWTONIAN FLOWS**

Fluids can be defined as substances that flow or deform under the application of shear stress and these include liquids and gasses. They are part of engineering studies in many tertiary institutions. Basically, in the study of science, fluids are divided into two broad groups. These divisions in this write-up are known as types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids that obey Newton's Law of viscosity. The Newtonian viscosity is a physical property that is only a function of pressure and temperature. The flow properties of an incompressible Newtonian fluid can be characterized solely in terms of density and viscosity.

# CHAPTER 2

## LITERATURE SURVEY

Flow through porous media has many geophysical and engineering applications. Berman AS [1953] analyzed the Navier-Stokes equations to obtain a complete description of the fluid flow in a channel having a rectangular cross-section and two equally porous walls. The scope has been limited to two-dimensional incompressible steady-state laminar flow. The solution of the flow equations leads to detailed expressions for the dependence of the velocity components and the pressure on position coordinates, channel dimensions, and fluid properties.

Hartmann and Lazarus [1937] investigated the effect of a transversely uniform MHD on the flow of conductive fluid between two infinitely parallel and fixed insulating plates. The equation describing MHD flow is a synthesis of the continuous equation and the Navier-Stokes equation. Governing equations are differential equations that require analytical or numerical solutions.

Hartmann J. [1937] considered the magnetic field in the laminar flow of conductive liquids.

Hassaninen I. A. and Mansour M.A. [1990] Analyzed the two-dimensional unsteady flow of a viscous, incompressible electrically- conducting fluid through a porous medium bounded by two infinite parallel plates under the the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin friction are derived. The effects of the magnetic parameters, the permeability of the porous medium and the frequency parameter are discussed.

Hamza E. A [1999], A similar solution has been obtained for a flow between two parallel plates (rectangular or circular) approaching or receding from each other with suction or injection at the porous plate. Approximate analytic solutions based on the regular perturbation technique are given and a numerical solution of the resulting nonlinear differential equations is presented. The effects of injection and suction on the velocity profiles, pressure distribution and shearing stress on the

wall are studied. In the case of the flow between circular plates, the effects on load capacity due to mass transfer are investigated.

Soundalgekar V. and Uplekar [1986] investigated an exact analysis of a generalized MHD Couette flow, taking into the Hall current is presented. Expressions for the axial and transverse components of velocity, the skin friction, are derived. These are shown in graphs. The exact solution to the energy equation, assuming the linear variation of the temperature along with the plate, is also derived and this is shown graphically. The effects of different parameters are discussed Sing & Ram [1978] solved the differential equation by Laplace transform method, considering the laminar flow of conductive fluid laminar flow through a channel with a transverse magnetic field under a periodic pressure gradient.

Cox.S.M [1991] considered the flow of a viscous incompressible fluid in a parallel-walled channel, driven by steady uniform suction through the porous channel walls. A similarity transformation reduces the Navier-Stokes equations to a single partial differential equation (PDE) for the stream function, with two-point boundary conditions. We discuss the bifurcations of the steady solutions first and show how a pitchfork bifurcation is unfolded when the symmetry of the problem is broken.

Attia, H. A. and Kotb, N. A., (1996) studied the steady flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel infinite insulated horizontal plates and the heat transfer through it are studied. The upper plate is given a constant velocity while the lower plate is kept stationary. The viscosity of the fluid is assumed to vary with temperature. The effect of an external uniform magnetic field, as well as the action of an inflow perpendicular to the plates together with the influence of the pressure gradient on the flow and temperature distributions, are reported. A numerical solution for the governing non-linear ordinary differential equations is developed.

Chiam T.C. (1993) investigated the boundary layer equations arising from the effects of a continuously moving flat plate under the action of a transverse applied magnetic field is converted into a system of ordinary differential equations subject to two-point boundary conditions by using the Local Non Similarity Method. An

iterative method incorporating the shooting method is presented for solving both the two-equation and three-equation models.

Verma, P.D., and Bansal, J. L., (1966) analyzed an exact solution of the Navier-Stokes equations reduced to third and second order nonlinear differential equations with appropriate boundary conditions. The longitudinal and transverse velocity profiles for  $\lambda=0, 1$  and  $R=100$  are drawn. It is noted that for large values of  $x^+x^+$ , an adverse pressure gradient is developed which causes a back flow. There is an increase in pressure even for very small fluid suction along with the stationary plate. The skin friction and the flow coefficient decrease concerning the suction velocity and the distance along with the stationary plate. For  $\lambda=0$ , the results transform to the known results for plane Couette flow without suction.

R. Delhi Babu, S. Ganesh & C.K.Kirubhashankar (2020) studied the impact of the unsteady magnetohydrodynamic flow of dust particles on the velocity of electrically directed viscous fluid between parallel porous plates with an angular velocity at the sight of a transverse magnetic field. The differential equations managing the flow are explained by similarity transformation. The velocity of the fluid, the velocity of the dust particles, and pressure distribution are obtained. The outcomes are the velocity of the dust particle is higher than that velocity of the fluid particles for each of the parameters examined.

C.K.Kirubhashankar, Y. Immanuel and S. Vaidya Subramanian (2020) discussed an exact analytic solution for the unsteady MHD fluid flow between the parallel plate's dependent on the Homotopy technique. In this study, the unsteady magnetohydrodynamic (MHD), viscid, condensed and conducting fluid stream problem under a transverse magnetic field is discussed. Solutions are proposed for a variety of Hartmann number values and time.

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C.K.Kirubhashankar (2018), proposed the steady two-dimensional laminar stream of an incompressible viscous liquid between two parallel plates in the presence of a transverse magnetic field is considered by accepting the normal wall velocities to be equal. We study the steady MHD flow between moving parallel plates with small uniform suction at the lower stationary plate. The velocity profile for the stream of the liquid and pressure gradient are examined for various qualities. Also, the impacts of velocity potential are researched and discussed graphically

# CHAPTER 3

## RESEARCH METHODOLOGY

### 3.1 AIM AND SCOPE

#### 3.1.1 *The main objectives of the work are:*

- To study the effect of individual factors like Magnetic field and Reynolds number.
- To study the solution for the unsteady MHD fluid flow through the parallel plate channel with heat source and the external transverse magnetic field is presented.
- The effects of velocity are studied for different parameters like magnetic field parameters.

- **3.1.2 *The scope of the work is:***

It not only helps the people working in the field of Physiological fluid dynamics but also the medical practitioners.

### 3.2 DESCRIPTION OF RESEARCH WORK

This research work focuses on the MATLAB implementation of the interpretation of analytical solutions obtained for various fluid flow problems.

#### 3.2.1 *Research Problem*

The main objective of the research work is to analyze the flow of fluids through parallel porous plates in the presence/absence of magnetic effects

### **3.2.2 Research Methodologies**

Analytical solutions are obtained by transforming the partial differential equation concerning the flow into an ordinary linear differential equation with constant coefficients by using similarity transformation. The arbitrary constant is then calculated by using the boundary condition of the respective problem and the results are interpreted by using MATLAB applications

# CHAPTER 4

## MATHEMATICAL FORMULATION OF THE MODEL

Unsteady Magnetohydrodynamic Stokes Flow between Parallel Plates with One Plate Moving Uniformly and the Other Plate at Rest with Uniform Suction

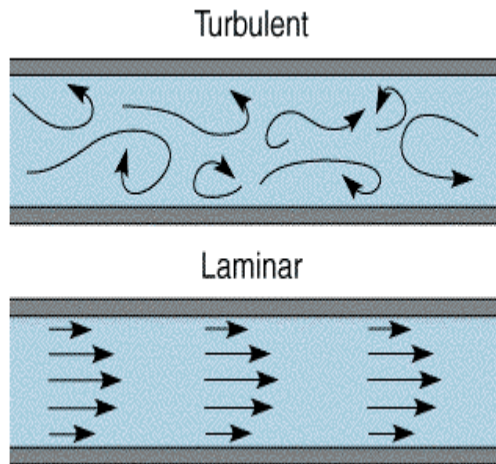


Fig: 4.1 Laminar and turbulent flows

The flow investigation has been carried out by studying the effect of magnetic fields. The equations which describe the MHD flow are a combination of continuity equations and Navier-Stokes equations of fluid dynamics. The governing equations are differential equations that have to be solved either analytically or numerically and the solution is expressed in terms of Hartmann number.

### Assumptions

- The flow of an incompressible viscous fluid between two parallel porous plates  $y = -h$  and  $y = h$  is considered in the presence of a transverse magnetic field which is applied perpendicular to the walls.
- The MHD Stokes flow through two parallel plates with one in uniform motion and the other plate at rest and uniform suction at the stationary plate.
- Let  $u$  and  $v$  be the velocity components in the  $x$  and  $y$  directions respectively in the flow field at time  $t$

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Equations of momentum are:

$$\frac{\partial u}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma B_0^2 u}{\rho} \quad (2)$$

$$\frac{\partial v}{\partial t} + \frac{1}{\rho} \frac{\partial p}{\partial y} = \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \quad (3)$$

Let us choose the solutions of the equations (1)-(3) respectively as

$$\left. \begin{aligned} u &= u(x, y) e^{i\omega t} \\ v &= v(x, y) e^{i\omega t} \\ p &= p(x, y) e^{i\omega t} \end{aligned} \right\} \quad (4)$$

With the boundary conditions:

$$\left. \begin{aligned} u(x, h) &= U, & u(x, -h) &= 0 \\ v(x, h) &= 0, & v(x, -h) &= -v_0 \end{aligned} \right\} \quad (5)$$

Let  $\eta = \frac{y}{h}$  be the dimensionless distance and let  $\nu = \frac{\mu}{\rho}$  be the kinematic viscosity and the equations (1), (2) and (3) become

$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial u}{\partial \eta} = 0 \quad (6)$$

$$i\omega u = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \eta^2} \right) - \frac{\sigma B_0^2 u}{\rho} \quad (7)$$

$$i\omega v = -\frac{1}{\rho h} \frac{\partial p}{\partial \eta} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \eta^2} \right) \quad (8)$$

The boundary conditions are converted into

$$\left. \begin{aligned} u(x,1) &= U, & u(x,-1) &= 0 \\ v(x,1) &= 0, & v(x,-1) &= -v_0 \end{aligned} \right\} \quad (9)$$

the stream function is defined as

$$\psi(x,\eta) = (hu(0) - v_0 x) f(\eta) \quad (10)$$

$$u(x,y) = \frac{\partial \psi}{\partial y} \quad \& \quad v(x,y) = -\frac{\partial \psi}{\partial x} \quad (11)$$

From equations (2), (3), (10) and (11), we have

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = (hu(0) - v_0 x) \left[ i\omega f' - \frac{v}{h^2} f'''(\eta) + \frac{\sigma B_0^2}{\rho} f' \right] \quad (12)$$

$$-\frac{1}{\rho h} \frac{\partial p}{\partial \eta} = i\omega v_0 f(\eta) - \frac{v v_0}{h^2} f''(\eta) \quad (13)$$

Partially differentiation of equations (12) & (13) with respect to 'η' & 'x' respectively

$$\frac{\partial^2 p}{\partial \eta \partial x} = (hu(0) - v_0 x) \frac{\partial}{\partial \eta} \left( i\omega f' - \frac{v}{h^2} f'''(\eta) + \frac{\sigma B_0^2}{\rho} f' \right) \quad (14)$$

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \quad (15)$$

From equations (14) & (15), we get

$$\frac{d}{d\eta} \left( i\omega f' - \frac{\nu}{h^2} f'''(\eta) + \frac{\sigma B_0^2}{\rho} f' \right) = 0 \quad (16)$$

where M is the Hartmann number and  $\alpha^2 = \frac{i\omega}{\nu}$

Analytical Solution

Equation (16) reduces to the form

$$(D^3 - (\alpha^2 h^2 + M^2)D)f(\eta) = K \quad (17)$$

with the boundary conditions

$$\left. \begin{aligned} f(1) &= 0, & f(-1) &= -1 \\ f'(1) &= 0, & f'(-1) &= 0 \end{aligned} \right\} \quad (18)$$

Hence the solution of (17) subject to the boundary condition (18) is

$$f(\eta) = \frac{(1-\eta)\sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2} + \sinh(\sqrt{\alpha^2 h^2 + M^2} \eta) - \sinh \sqrt{\alpha^2 h^2 + M^2}}{2(\sinh \sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2})} \quad (19)$$

Substituting the value of  $f(\eta)$  in the stream function

$$\psi(x, \eta) = (hu(0) - v_0 x) f(\eta)$$

Using the stream function  $\psi$  and equation (11), the axial and radial velocity of the flow are

$$u(x, y) = \left( u(0) - \frac{v_0 x}{h} \right) e^{i\omega t} \left( \frac{\cosh(\sqrt{\alpha^2 h^2 + M^2} y/h) - \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2}}{2(\sinh \sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2})} \right) \quad (20)$$

$$v(x, y) = v_0 e^{i\omega t} \left[ \frac{(1 - (y/h))\sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2} + \sinh(\sqrt{\alpha^2 h^2 + M^2} y/h) - \sinh \sqrt{\alpha^2 h^2 + M^2}}{2(\sinh \sqrt{\alpha^2 h^2 + M^2} - \sqrt{\alpha^2 h^2 + M^2} \cosh \sqrt{\alpha^2 h^2 + M^2})} \right] \quad (21)$$

# CHAPTER 5



## SENSITIVE ANALYSIS

From the above discussions, we have analyzed the fluid flow between two parallel porous plates, where the upper plate is moving uniformly. Fluid suction is through the lower fixed plate. Here the effects of velocity profiles and pressure distribution are studied.

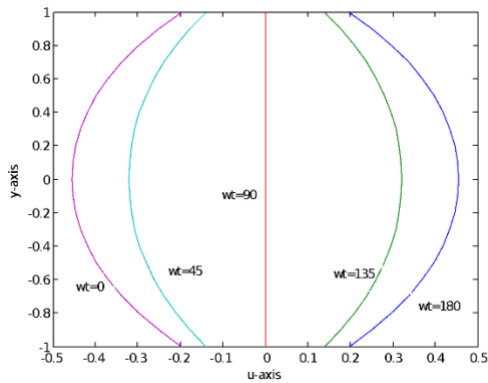


Fig 5.1 Axial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 0$ ,  $M = 1$  and for different values of  $\omega t$ .

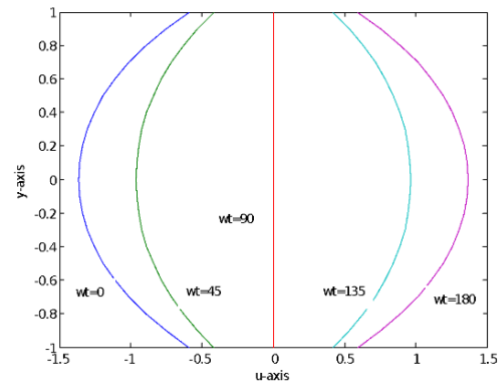


Fig 5.2 Axial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 4$ ,  $M = 1$  and for different values of  $\omega t$ .

Fig 5.1 and Fig 5.2 describes that the magnitude of the axial velocity increases from  $x = 0$  to  $x = 4$  for different values of  $\omega t$ .

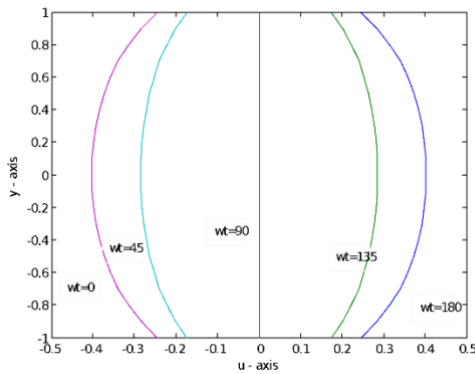


Fig 5.3 Axial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 0$ ,  $M = 2$  and for different values of  $\omega t$

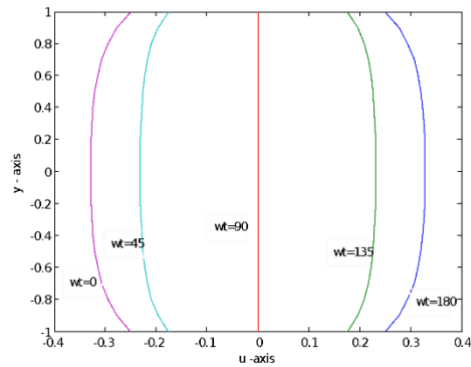


Fig 5.4 Axial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 0$ ,  $M = 4$  and for different value of  $\omega t$

Fig 5.3 and Fig 5.4 illustrates that the magnitude of the axial velocity decreases from  $M = 2$  to  $M = 4$  for different values of  $\omega t$ .

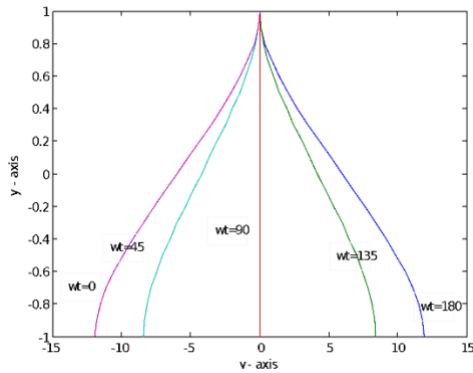


Fig 5.5 Radial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 0$ ,  **$M = 2$**  and for different values of  $\omega t$

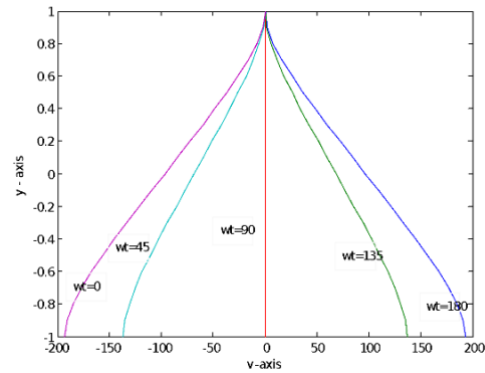


Fig 5.6 Radial velocity profiles when  $u_0 = 0.5$ ,  $v_0 = 0.5$ ,  $h=1$ ,  $\alpha = 1$ ,  $x = 0$ ,  **$M = 4$**  and for different values of  $\omega t$

Fig 5.5 and Fig 5.6 shows that the magnitude of the radial velocity increases from  $M = 2$  to  $M = 4$  for different values of  $\omega t$ .

# CHAPTER 6

## CONCLUSION

Analytical solutions are obtained for velocity profiles for the flow of the fluid and are discussed for different values of  $\omega t$ . From the discussions above, we come to the conclusions:

- The axial velocity experience increases in magnitude for different values of  $\omega t$  from  $x = 0$  to  $x = 4$ .
- The magnitude of the axial velocity decreases from  $M = 2$  to  $M = 4$  for different values of  $\omega t$ .
- The magnitude of the radial velocity decreases from  $x = 0$  to  $x = 4$  for different values of  $\omega t$ .
- The magnitude of the radial velocity increases from  $M = 2$  to  $M = 4$  for different values of  $\omega t$ .
- In the case of radial velocity, the magnitude of these stresses at the lower plate is very high compared to the respective magnitudes at the upper plate.
- The effect of increasing Hartmann number  $M$  is to decrease the transverse component of velocity, while it increases the radial component of velocity.
- The magnitude of the velocity component  $u$  reduces and  $v$  increases with increasing the intensity of the magnetic field  $M$ , also the resultant velocity experiences retardation with increasing  $M$ .

The transverse magnetic field decelerates the fluid motion. When the strength of the magnetic field increases, the flow velocity decreases

## REFERENCES

- [1] Berman A.S., "Laminar flow in channels with porous walls", J. Appl. Phys., Vol.24, (1953), pp.1232-1235.
- [2] Hartmann J. and Lazarus F., Kgl. Danske VidenskabSelskab, Mat-Fys. 15(6, 7), (1937).
- [3] Hartmann .J , Hf-Dynamics-I, "Theory of the laminar flow of an electrically conducting liquid in a homogeneous magnetic field", Kgl. Danske Videnskab. Selskab Mat. - Fys. Medd Vol15,(1937), p.6.
- [4] Hassaninen I. A. and Mansour M.A, "Unsteady Magnetic flow through a Porous Medium between two infinite parallel" Astrophysics and Space Science Springer Netherlands,(1990),Vol.163, No.2, pp.241-246.
- [5] Hamza E.A., "Suction and injection effects on a similar flow between parallel plates", J Phys 32, 656.(1999)
- [6] Cox S.M , 'Two-dimensional flow of viscous fluid in a channel with porous walls', J.Fluid Mech., Vol. 227,(1991),pp.1-33.
- [7] Soundalgekar V. and Uplekar A, 'Steady MHD Couette flow with heat transfer considering varying temperature', IEEE Trans Plasma Sci, Vol.14, (1968),p.579.
- [8] Sing C. B & P.C. Ram, 'Unsteady Magnetohydrodynamic Fluid flow through a channel: Journal of Scientific Research, XXVIII, No.2 (1978).
- [9] Attia, H. A. and Kotb, N. A. 'MHD Flow between Two Parallel Plates with Heat Transfer,' ACTA Mechanic, Vol. 117,(1996), p. 215.
- [10] Chiam T.C, 'Magnetohydrodynamic boundary layer flow due to a continuously moving flat plate', Comp. Math. Appl. 26, 1–7. International Conference on Mathematical Sciences (ICMS 2020) Journal of Physics: Conference Series 1770 (2021) 012047 IOP Publishing doi:10.1088/1742-6596/1770/1/0120478 (1993)
- [11] Verma, P.D., and Bansal, J. L., ' Flow of a viscous incompressible fluid between two parallel .plates, one in uniform motion and the other at rest with uniform suction at the stationary plate', Proc. Indian Acad. Sci., A 64, 385-396.(1966).
- [12] KirubhaShankar C.K, Ganesh.S and Mohamed Ismail.A9 , "Unsteady Magnetohydrodynamic Stokes Flow Between Parallel Plates with One Plate

- [13] Moving Uniformly and the Other Plate at Rest with Uniform Suction”, National Conference on Recent Trends in Mathematical Computing, 23, August 2013 - 24, August 2013,(2013), 404-409.
- [14] C.K.KirubhaShankar C.K, Dr.S.Ganesh, Mohamed Ismail.A , “Analysis of Electrically Conducting Magneto Hydrodynamic Viscous Flow between Two Moving Parallel Permeable Plates”, International Journal of Science, Engineering and Technology Research (IJSETR), Volume 3, Issue 1,(2014),23 – 26.
- [15] KirubhaShankar.C.K , Dr.S.Ganesh, and Mohamed Ismail.A(2013), “An Exact Solution of the Problem of Unsteady MHD Flow Through Parallel Porous Plates”, Int'l Journal of Advances in Mechanical & Automobile Eng. (IJAMAE) Vol. 1, Issue 1(2014) ISSN 2349-1485 EISSN 2349-1493,(2013),39 – 42.
- [16] Delhi Babu, R., Ganesh, S., & Anish, M. . Steady two dimensional MHD stokes flow between two parallel plates under angular velocity with one plate moving uniformly and the other plates at rest and uniform suction at the stationary plate. International Journal of Ambient Energy,(2019) 1-4.
- [17] Ganesh, S., Anand, V. W. J., Chandrasekar, P., & Anish, M. . Effects of Magnetohydrodynamic Unsteady fluid flow over a Narrow channel in the axial and radial directions. International Journal of Ambient Energy,(2019) 1-5.
- [18] Delhi Babu.R, Ganesh.S& .Kirubhashankar.C.K , “An exact solution of unsteady magnetohydrodynamic flow of dusty fluid between parallel porous plates with an angular velocity”, International Journal of Ambient Energy, DOI: 10.1080/01430750.2020.1725636.(2020)
- [19] Delhi Babu, R., & Ganesh, S. Unsteady MHD stokes flow of a viscous fluid between parallel porous plates under angular velocity. International Journal of Ambient Energy,(2020) 1-17.
- [20] Ganesh, S., Delhi Babu, R., & Culathur Karunakaran, K. . Analysis of Unsteady two dimensional MHD flow between two parallel plates under angular velocity with one plate moving uniformly and the other plate at rest and uniform suction at the stationary plate. International Journal of Ambient Energy,(2020) 1-7.
- [21] Chandrasekar, P., & Ganesh, S. . Investigation of Inclined Magnetic Unsteady MHD Non-Newtonian Fluid Flow between uniform progressing and upholding Parallel Plates. In 2020 International Conference on Emerging Trends

in Information Technology and Engineering (ic-PETITE) (pp. 1-5). IEEE.(2020 February)

- [22] KirubhaShankar.C.K, Immanuel.Y and Vaidya Subramanian .S, “Homotopy method for solving unsteady MHD fluid flow between two parallel plates”, Journal of Advanced Research in Dynamical and Control Systems, Vol. 12,01-Special Issue,741-745,DOI:10.5373/JARDCS/V12SP1/20201124,(2020).
- [23] KirubhaShankar C.K , “Steady Magneto Hydrodynamic Laminar Stream of Viscous Liquid with Constantly Moving Parallel Plates”, Journal of Advanced Research in Dynamical and Control Systems, 03-Special Issue, (2018),1348-1353.
- [24] KirubhaShankar,C,K, Vaidya Subramanian.S, Immanuel.Y, Muniappan.P , “Thermal Effects on Magneto Hydrodynamic Casson Liquid Stream Between Electrically Conducting Plates” International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN:2278-3075, Volume-9 Issue-4, (2020),213–217.