

OPTIMIZATION OF BANK LOAN MODEL BY FOURIER MOTZKIN ELIMINATION ALGORITHM FOR SOLVING LINEAR PROGRAMMING PROBLEMS

Submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in Mathematics (B.Sc Mathematics)

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INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited with Grade “A” by NAAC**

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This is to certify that this Project Report is the bonafide work of **Ruby Shalini.T (39060010), Bala Monica.N (39060004)** who carried out the project entitled **"Optimization of bank loan model by Fourier motzkin elimination algorithm for solving linear programming problem"** under our supervision from **November 2021 to May 2022.**

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DECLARATION

I **Ruby Shalini.T (39060010), Bala Monica.N (39060004)** hereby declare that the Project Report entitled “**Optimization of bank loan model by Fourier motzkin elimination algorithm for solving linear programming problems**” done by us under the guidance of **Dr. C. Kavitha** (Internal) and _____ (External) at **Sathyabama Institute of Science and Technology** is submitted in partial fulfillment of the requirements for the award of Bachelor of Science degree in Mathematics.

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ABSTRACT:

This study uses the Fourier Motzkin Elimination approach to tackle linear programming (LP) problems. This model is used to determine the best net return in a bank for loans such as personal loans, automobile loans, house loans, agricultural loans, and commercial loans. We usually utilize simplex, dual, and graphical methods to handle linear programming problems, but we employ the Fourier Motzkin Elimination Algorithm to solve linear programming problems. The advantage of this approach is that it is quite easy to achieve convergence when compared to other ways. The challenge is solved by describing the bank profit optimization problem as a linear programming problem and utilizing the Fourier Motzkin Elimination Algorithm to compute the optimal.

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CHAPTER-1

1. INTRODUCTION:

Operation Research is an analytical method for problem-solving and decision-making which is very useful in the management of an organization. K. Arumugam et.al finds the fuzzy linear programming problem into fuzzy linear system of equations, then a fourier motzkin elimination algorithm is used to solve the above converted fuzzy linear system of equations [1]. S.I. Bastrakov et.al concerns the elimination of a set of variables from a system of linear inequalities [2]. Beyza Ahlatcioglu ozkok et.al finds the fuzzy optimal and approximate for finding fuzzy optimal solution for fully fuzzy linear programming problems with mixed constraints [3]. H.P.Williams et.al concerns the fourier motzkin elimination method, which can be used for solving linear programming problems, can be extended to deal with integer programming problems [4]. Rui Juan Jing et.al finds the method for removing all redundant inequalities generated by fourier motzkin elimination [5]. Ivan Simecek et.al finds the analysis, design, and implementation of a parallel solver for distributed memory for large system of linear inequalities using the fourier motzkin elimination algorithm [6]. Geir Dahl et.al finds the fourier motzkin elimination is a classical method for solving linear inequalities in which one variable is elimination in each iteration [7]. N.Yu Zolotykh et.al finds a new fast technique for verifying chernikov rules in fourier motzkin elimination [8]. Sempavazhaam et.al finds the application of LPP is cost reduction in various business fields [9]. Mayank Shekhar Jha et.al finds the fault diagnosis problem of uncertain systems in the context of bond graph modelling technique [10].

CHAPTER-2

2. LITERATURE SURVEY:

IN 2017, K.Arumugam, R.Abdul Saleem and K.Sugantha began to discuss the fourier motzkin elimination algorithm of hexagonal fuzzy numbers for solving fuzzy linear programming problems [1]. S.I. Bastrakov, A.V. Churkin and N.Yu.Zolotykh discussed about accelerating fourier-motzkin elimination using Bit Pattern tress in 2020 [2]. Beyza Ahlatcioglu ozkok discussed in finding the fuzzy optimal solution to fully fuzzy linear programming problems with trapezoidal fuzzy numbers [3]. H.P.Williams discussed about fourier motzkin elimination extension to integer programming problems in 1976 [4]. Rui Juan Jing, Marc Moreno-Maza and Delaram Talaashrafi proceeded with a discussion on complexity estimates for fourier motzkin elimination in 2020 [5]. Ivan Simecek, Richard Fritish and Robert Lorencz discussed parallel solvers of larger systems of linear inequalities using fourier motzkin elimination [6].

CHAPTER-3

3.1 MATHEMATICAL FORMULATION OF LINEAR PROGRAMMING

The mathematical expression of a general linear programming problem is as follows:

Maximize or Minimize $Z = \sum_{j=1}^n c_j x_j$

Subject to $\sum_{j=1}^n a_{ij} x_j \{ \leq, =, \geq \} b_i; i=1,2,\dots,m$

$$x_j \geq 0; b_i \geq 0$$

where one and only one of the signs ($\leq$, $=$, $\geq$) holds for each constraint and the sign may vary from one constraint to another. Here c_j ($j=1, 2, \dots, n$) are called profit or cost coefficients, x_j ($j=1, 2, \dots, n$) are called decision variables. The set of feasible solution to the linear programming problem is

$$S = \{(x_1, x_2, \dots, x_n)^T : (x_1, x_2, \dots, x_n)^T \in R^n\}$$

The set S is called the constraints set, feasible set or feasible region of linear programming.

In matrix vector notation the above problem can be expressed as :

Maximize or Minimize $Z = cx$

Subject to $Ax (\leq, =, \geq) b$

$$x \geq 0; b \geq 0$$

where A is an $m \times n$ matrix, x is an $n \times 1$ column vector, b is an $m \times 1$ column vector and c is a $1 \times n$ row vector, $b \in R^m$; $x, c \in R^n$.

3.2 MATHEMATICAL FORMULATION OF FOURIER MOTZKIN ELIMINATION

We briefly review the Fourier-Motzkin elimination method. Consider again a linear system $Ax \leq b$ where $A \in R^{m,n}$, $b \in R^m$ and let $I := \{1, 2, \dots, m\}$. We write the system in component form

$$\begin{aligned}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\
 &\vdots \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m
 \end{aligned} \tag{1}$$

Say that we want to eliminate x_1 from the system (1). For each i where $a_{i1} \neq 0$ we multiply the i 'th inequality $\sum_{j=1}^n a_{ij} x_j \leq b_i$ by $1/|a_{i1}|$. This gives an equivalent system

$$\begin{aligned}
 x_1 + a'_{i2}x_2 + \dots + a'_{in}x_n &\leq b'_i \quad (i \in I^+) \\
 a_{i2}x_2 + \dots + a_{in}x_n &\leq b_i \quad (i \in I^0) \\
 -x_1 + a'_{i2}x_2 + \dots + a'_{in}x_n &\leq b'_i \quad (i \in I^-)
 \end{aligned} \tag{2}$$

where $I^+ = \{i : a_{i1} > 0\}$, $I^0 = \{i : a_{i1} = 0\}$, $I^- = \{i : a_{i1} < 0\}$, $a'_{ij} = a_{ij}/|a_{i1}|$ and $b'_i = b_i/|a_{i1}|$. Thus, the row index set $I = \{1, 2, \dots, m\}$ is partitioned into subsets I^+ , I^0 and I^- , some of which may be empty. It follows that $x_1, x_2, \dots, x_n$ is a solution of the original system (1) if and only if $x_2, x_3, \dots, x_n$ satisfy

$$\begin{aligned}
 \sum_{j=2}^n a'_{kj}x_j - b'_k &\leq b'_i - \sum_{j=2}^n a'_{ij}x_j \quad (i \in I^+, k \in I^-) \\
 \sum_{j=2}^n a_{ij}x_j &\leq b_i \quad (i \in I^0)
 \end{aligned} \tag{3}$$

and x_1 satisfies

$$\max_{k \in I^-} (\sum_{j=2}^n a'_{kj} x_j - b'_k) \leq x_1 \leq \min_{i \in I^+} (b'_i - \sum_{j=2}^n a'_{ij} x_j) \quad (4)$$

If I^- (resp. I^+) is empty, the first set of constraints in (3) vanishes and the maximum (resp. minimum) in (4) is interpreted as ∞ (resp. $-\infty$). If I^0 is empty and either I^- or I^+ is empty too, then we terminate: the general solution of $Ax \leq b$ is obtained by choosing $x_2, x_3, \dots, x_n$ arbitrarily and choosing x_1 according to (4). The constraint in (4) says that x_1 lies in a certain interval which is determined by $x_2, x_3, \dots, x_n$. The polyhedron defined by (3) is the projection of P along the x_1 -axis, i.e., into the space of the variables $x_2, x_3, \dots, x_n$. then proceed and similarly eliminate x_2, x_3 etc. Eventually one obtains a system $l \leq x_n \leq u$.

If $l > u$ one concludes that $Ax \leq b$ has no solution, otherwise one may choose $x_n \in [l, u]$, and then choose x_{n-1} in an interval which depends on x_n etc. This back Substitution produces a solution $x = (x_1, x_2, \dots, x_n)$ to $Ax \leq b$. Moreover, every solution of $Ax \leq b$ may be produced in this way. (If the system is inconsistent, this might possibly be discovered at an early stage and one terminates).

The number of constraints may grow exponentially fast as variables are eliminated using fourier motzkin elimination. Actually, a main problem in practice is that the number of inequalities becomes “too large” during the elimination process, even when redundant inequalities are removed. It is therefore of interest to know situation where the project linear system is not very large or, at least, have some interesting structure. These questions are discussed in the remaining part of the page. We refer to [7] for a further discussion of fourier motzkin elimination algorithm and a collection of references on this method.

3.3 METHODOLOGY:

FOURIER MOTZKIN ELIMINATION ALGORITHM

STEP 1: Formulate the linear programming problem from the given problem

STEP 2: Change the objective function as inequality and join it with the constraints. Now, we get the linear system of the equation. For max use $\leq$ and for min use $\geq$ inequality.

STEP 3: Change all the inequalities of the fuzzy system into $\geq$ for minimization problem and $\leq$ for maximization problem.

STEP 4: Eliminate one by one in $x_1, x_2, x_3, x_4, \dots, x_n$

- ❖ Divide each equation by its modulus value of x_1 coefficient for all the equations.
- ❖ Now, we have three classes of x_1 coefficient i.e, “-1”, or “+1” or “0” in the system of linear equation.
- ❖ Adding or subtracting any two classes of equations to eliminate x_1 .

STEP 5: Repeat step 4 until all the ‘n’ variables are eliminated.

STEP 6: After eliminating all the ‘n’ variables, we get the z values and substitute the z in above,

we get the values of variables in back substitution.

3.4 NUMERICAL EXAMPLE;

- These data are taken from the Taha book written by A.Hamdy
- There are many types of loans such as Personal loans, Car loans, Home loans, Farm loans, and Commercial loans. The Interest rate and Bad-debt ratio of the loans are taken as data.
- Total funds - \$12 million.
- At least 40% funds to Farm and Commercial loans.
- At least 50% funds to Personal, Car, and Home loans.
- Bad debt should not exceed 4% of all loans.

TYPE OF LOAN	INTEREST RATE	BAD-DEBT RATIO
PERSONAL	0.140	0.10
CAR	0.130	0.07
HOME	0.120	0.03
FARM	0.125	0.05
COMMERCIAL	0.100	0.02
TOTAL FUNDS - \$12 million		
At least 40% funds to farm & commercial loans		
At least 50% funds to personal, car & home loans		
Bad debt should not exceed 4 % of all loans		

FORMATION OF MATHEMATICAL MODEL

Using these data;

x_1 = personal loans

x_2 = car loans

x_3 = home loans

x_4 = farm loans

x_5 = commercial loans

$$\text{Total interest} = 0.14(0.9x_1) + 0.13(0.93x_2) + 0.12(0.97x_3) + 0.125(0.95x_4) + 0.1(0.98x_5)$$

$$= 0.126x_1 + 0.1209x_2 + 0.1164x_3 + 0.11875x_4 + 0.098x_5$$

Also we have,

$$\text{Bad debt} = 0.1x_1 + 0.07x_2 + 0.03x_3 + 0.05x_4 + 0.02x_5$$

$$\text{Maximize } Z = \text{Total interest} - \text{Bad debt}$$

$$\begin{aligned} &= (0.126x_1 + 0.1209x_2 + 0.1164x_3 + 0.11875x_4 + 0.098x_5) - \\ &\quad (0.1x_1 + 0.07x_2 + 0.03x_3 + 0.05x_4 + 0.02x_5) \\ &= 0.026x_1 + 0.0509x_2 + 0.0864x_3 + 0.06875x_4 + 0.078x_5 \end{aligned}$$

The problem has five constraints:

1. Total funds should not exceed \$12(million):

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 12$$

2. Farm and Commercial loans equal at least 40% of all loans:

$$0.4x_1 + 0.4x_2 + 0.4x_3 - 0.6x_4 - 0.6x_5 \leq 0$$

3. Home loans should equal at least 50% of personal, car and home loans:

$$0.5x_1 + 0.5x_2 - 0.5x_3 \leq 0$$

4. Bad debts should not exceed 4% of all loans:

$$0.06x_1 + 0.03x_2 + 0.01x_3 + 0.01x_4 - 0.02x_5 \leq 0$$

5. Nonnegativity:

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

FORMATION

$$\text{Maximize } Z = 0.026x_1 + 0.0509x_2 + 0.0864x_3 + 0.06875x_4 + 0.078x_5$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 12$$

$$0.4x_1 + 0.4x_2 + 0.4x_3 - 0.6x_4 - 0.6x_5 \leq 0$$

$$0.5x_1 + 0.5x_2 - 0.5x_3 \leq 0$$

$$0.06x_1 + 0.03x_2 + 0.01x_3 + 0.01x_4 - 0.02x_5 \leq 0$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

STEP 1: Change objective as $\leq$

$$Z \leq 0.026x_1 + 0.0509x_2 + 0.0864x_3 + 0.06875x_4 + 0.078x_5$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \leq 12$$

$$0.4x_1 + 0.4x_2 + 0.4x_3 - 0.6x_4 - 0.6x_5 \leq 0$$

$$0.5x_1 + 0.5x_2 - 0.5x_3 \leq 0$$

$$0.06x_1 + 0.03x_2 + 0.01x_3 + 0.01x_4 - 0.02x_5 \leq 0$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 0$$

STEP 2: In objective & non-negative constraints bring to the form of linear inequality ($\leq$)

$$-0.026x_1 - 0.0509x_2 - 0.0864x_3 - 0.06875x_4 - 0.078x_5 + Z \leq 0$$

$$x_1+x_2+x_3+x_4+x_5 \leq 12$$

$$0.4x_1+0.4x_2+0.4x_3-0.6x_4-0.6x_5 \leq 0$$

$$0.5x_1+0.5x_2-0.5x_3 \leq 0$$

$$0.06x_1+0.03x_2+0.01x_3+0.01x_4-0.02x_5 \leq 0$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0$$

$$-x_3 \leq 0$$

$$-x_4 \leq 0$$

$$-x_5 \leq 0$$

STEP 3 : Divide the coefficient of x_1

$$-x_1-1.95769x_2-3.32308x_3-2.64423x_4-3x_5+38.46154z \leq 0$$

$$x_1+x_2+x_3+x_4+x_5 \leq 12$$

$$x_1+x_2+x_3-1.5x_4-1.5x_5 \leq 0$$

$$x_1+x_2-x_3 \leq 0$$

$$x_1+0.5x_2-0.16667x_3+0.16667x_4-0.33333x_5 \leq 0$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0$$

$$-x_3 \leq 0$$

$$-x_4 \leq 0$$

$$-x_5 \leq 0$$

STEP 4: Now, we have first class, second class , third class equations. If the coefficient of x_1 is -1 it is first class, the coefficient of x_1 is +1 it is second class, then the coefficient of x_1 is 0 it is third class.

STEP 5: To eliminate x_1 , we have 2 equations in the first class and 4 equations in the second class. Adding first class and second class equations we get 8 equations without involving x_1 .

STEP 6: To eliminate x_2 , divide the coefficient of x_2 in the obtained equations. No ,we have 5 first class equations and 4 second class equations. Adding first class and second class equations we get 20 equations without involving x_2 .

STEP 7: To eliminate x_3 , divide the coefficient of x_3 in the obtained equations. Now ,we have 19 first class equations and 2 second class equations. Adding first class and second class equations we get 38 equations without involving x_3 .

STEP 8:To eliminate x_4 , divide the coefficient of x_4 in the obtained equations. No ,we have 27 first class equations and 12 second class equations. Adding first class and second class equations we get 324 equations without involving x_4 .

STEP 9: To eliminate x_5 , divide the coefficient of x_5 in the obtained equations. No ,we have 299 first class equations and 10 second class equations. Adding first class and second class equations we get 2990 equations without involving x_5 .

STEP 10:After eliminating x_5 , we get the system of linear inequalities in terms of z. In that z equation, make the coefficient of z as 1. Then select the minimum value of z.

STEP 11:Now , substitute the value of z in the equation which contains only x_5 . After finding x_5 using back substitution we get the values of x_1, x_2, x_3, x_4 .

OUTPUT

The optimal solution is $Z=0.99648$, $x_1=0$, $x_2=0$, $x_3=7.2$, $x_4=0$, $x_5=4.8$

CHAPTER-4

4.1 RESULTS AND DISCUSSION

In this section, we compare our method with all other available methods and we find that our method is better than any other available method. The reasons are as follows;

- Usually we use Simplex, Dual and Graphical methods to solve linear programming problems. But now we used Fourier motzkin elimination algorithm to solve linear programming problems.
- We can solve any type of linear programming problem by this methodology.
- Its computational steps are so easy from other methods.
- The final result converges quickly in this method.
- Using a computer program, we get the optimal solution of the LPP problem very quickly.

CHAPTER-5

5.1 CONCLUSION

In this project, we describe the bank profit optimization problem as a linear programming problem using Fourier motzkin elimination algorithm. The model is to find the optimum profit of net return in a bank, in regions of interest from loans such as personal loans, vehicle loans, home loans, agricultural loans and commercial loans. The solution of the problem is to describe the bank's profit optimization problem and calculate the optimal profit using the Fourier motzkin elimination algorithm. The linear programming problem has been solved using the Fourier motzkin elimination algorithm. We obtained the optimal profit of net return in a bank.

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