

A STUDY ON ISO-TRIANGULAR ARRAY GRAMMARS AND ISO-TRIANGULAR ARRAY LANGUAGES

Submitted in partial fulfillment of the requirements for the award of
BACHELOR OF SCIENCE degree in MATHEMATICS (B. Sc. Mathematics)

By

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SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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APRIL - 2024



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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **JOTHIKA.A (41060006)**, **KARISHMA. K (41060007)** who carried out the project entitled "**A STUDY ON ISO-TRIANGULAR ARRAY GRAMMARS AND ISO-TRIANGULAR ARRAY LANGUAGES**" under our supervision from **JANUARY 2024** to **APRIL 2024**.

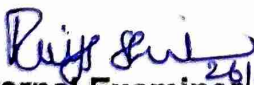

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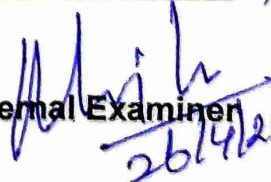
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DECLARATION

We JOTHIKA. A (41060006), KARISHMA. K (41060007) hereby declare that the Project Report entitled "A STUDY ON ISO- TRIANGULAR ARRAY GRAMMARS AND ISO-TRIANGULAR ARRAY LANGUAGES" done by us under the guidance of Dr. K. BHUVANESWARI (Internal) at SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY is submitted in partial fulfillment of the requirements for the award of BACHELOR OF SCIENCE degree in MATHEMATICS.

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ACKNOWLEDGEMENT

We are pleased to acknowledge our sincere thanks to The Board of Management of **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** for their encouragement in doing this project and for completing it successfully. We are grateful to them.

we convey our gratitude to **Dr. REKHA CHAKRAVARTHI** Dean, School of Science and Humanities and, **Dr. M. NIRMALA** Head of the Department, Dept. of Mathematics for providing us necessary support during the progressive reviews.

we would like to express our sincere and deep sense of gratitude to our Project Guide **Dr. K. BHUVANESWARI** for her valuable guidance, suggestions and constant encouragement paved way for the successful completion of our project work.

we wish to express our thanks to all Teaching and Non-teaching staff members of the Department of Mathematics who were helpful in many ways for the completion of the project.

ABSTRACT

In the domain of computer science, the theory of formal language and automata is efficient in solving a problem by a computational model. Generating two-dimensional picture languages is a study in formal language theory. To generate a picture language, many computing models are proposed in the literature. The generative grammars and the corresponding classes of automata are well known in the literature to compute the string languages and two-dimensional languages. A string language is defined as a set of strings over some finite alphabets, which are characterized by a grammar, a logical system.

Among the models of computation for the two-dimensional picture languages, the tile pasting system is a model. Adopting the notion of the tile pasting system, the triangular tile pasting system is introduced by Kalyani et al., (2006) which pastes edge-to-edge isosceles right-angled triangles in the two-dimensional plane, in which the pasting rules have been defined. Some classes of two-dimensional picture languages are generated and examined for their generative power.

Iso-array rewriting system models with the rules of context-free iso-array grammar generates some classes of 2D language. This system allows two isosceles right-angled triangular tiles to get glued under specified rules to form 2D pictures and Kolam patterns etc. By connecting the two areas tile pasting system and P system, a new model, the triangular tile pasting P system for triangular picture languages is proposed in the literature by K. Bhuvaneswari et al (2014).

In this project, some classes of two-dimensional picture languages using the iso-triangular array grammars. A comparative study on the generative power of the iso-triangular array grammars is examined.

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LIST OF ABBREVIATIONS

CFIAG – Context-Free Iso-Array Grammar

CFBPIAG – Context-Free Basic Puzzle Iso-Array Grammar

CFBPIAL – Context-Free Basic Puzzle Iso-Array Language

CITAG – Contextual Iso-Triangular Array Grammar

RITAG - Regular Iso-Triangular Array Grammar

RITAL – Regular Iso-Triangular Array Language

TTPS - Triangular Tile Pasting System

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CHAPTER 1

1. INTRODUCTION

This chapter presents extensively the introduction to formal language theory, two-dimensional picture languages, and iso-triangular array grammars.

1.1 FORMAL LANGUAGE THEORY

It is a branch in the theory of computation and it deals with strings or words over by finite sets or infinite sets of alphabets processed by the Chomsky hierarchy. With the interest in pattern recognition and image processing in the theory of computer science, several models are proposed (Siromoney et al. 1972, 1973) to recognize and generate two-dimensional objects. Handbook of Formal Languages" (Rozenberg 1997), which gives extensive coverage of the theoretical studies as well as applications of formal language theory. The models made an exclusive development in formal language theory and it gives an enriched handbook of Formal language theory. Many array grammar models have been introduced in formal language theory, by extending the rewriting feature of strings in Chomsky grammars.

1.2 TWO-DIMENSIONAL PICTURE LANGUAGES

A two-dimensional picture language is a rectangular array of symbols over alphabets. Many computing models have been proposed in the literature to generate two-dimensional picture languages. Triangular patterns and hexagonal patterns are known to occur in picture processing and image analysis (Middleton 2005). This study aimed to introduce iso-triangular arrays and hexagonal array grammars. Kalyani et al., (2006) introduced iso-array grammars for triangular pictures. Siromoney et al., (1976) introduced hexagonal array grammar to generate hexagonal patterns. Subramanian et al., (1979) presented the hierarchy of hexagonal array grammar and some of its properties. Dersanambika et al. (2005, 2004) introduced local and recognizable hexagonal picture languages.

1.3 ISO-TRIANGULAR ARRAY GRAMMARS

This section defines context-free iso-array grammar, context-free basic puzzle iso-array grammar, and contextual iso-triangular array grammar. The language generated by iso-triangular array grammar is called iso-triangular array language. The tile pasting system allows two isosceles right-angled triangular tiles to get glued under specified rules to form labeled or colored patterns. The sequence of isosceles right-angled triangles are generated using this grammar.

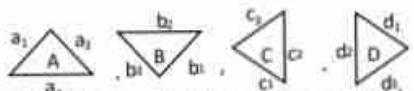
1.4 BASIC DEFINITIONS

Definition 1.4.1. ARRAY GRAMMAR

An array grammar is defined as a 5-tuple $G = (V, V_T, P, \#, S)$, where V is a finite set of total alphabets, $V_T \subset V$ is the set of terminal alphabets, $\# \in V - V_T$ is called the blank symbol, $S \in V - V_T$ is called the initial symbol, P is a finite set of rewriting rules, for any production $\alpha \rightarrow \beta$,

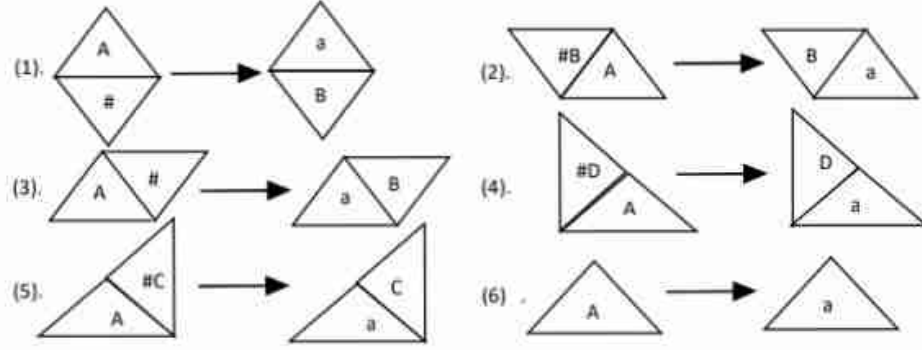
1. α contains non- $\#$'s.
2. If the non- $\#$'s of α do not touch the border of α , then the non- $\#$'s of β must be connected and non-empty.
3. Otherwise,
 - (i) Every connected component of non- $\#$'s in β must contain the intersection of some component of non- $\#$'s in α with border of α .
 - (ii) Conversely, every such intersection must be contained in some component of non- $\#$'s in β .

Definition 1.4.2. ISO-TRIANGULAR TILES

Let  be a finite set of labeled isosceles right-angled triangular tiles of dimensions $1/\sqrt{2}$, $1/\sqrt{2}$, and 1 unit are obtained by intersecting a unit square by its diagonals.

Definition 1.4.3. REGULAR ISO-ARRAY GRAMMAR

A regular iso-array grammar (RIAG) is a structure $G = (N, T, P, S)$ where $N = \{A, B, C, D\}$, $T = \{a, b, c, d\}$ are finite sets of symbols (isosceles right angled triangular tiles) and $N \cap T = \emptyset$. Elements of N and T are called non-terminals and terminals respectively. $S \in N$, S is the start symbol of the axiom, and P consists of rules of the following forms:



Similar rules can be given for the other tiles B , C and D . The set of all languages generated by RIAG is RIAL.

Definition 1.4.4. CONTEXTUAL ISO-TRIANGULAR ARRAY GRAMMAR

A contextual iso-triangular array grammar (CITAG) is a structure $G = (V, \#, P, A)$ where V is a set of all terminal symbols and $\#$ is a set of all empty symbols. A is a finite set of axioms in V^{+2} (The set of all finite connected arrays over V) and P is a finite set of rules of the form $(U_\alpha, \alpha, U_\beta, \beta)$ with the following conditions.

(i) $U_\alpha, U_\beta \subseteq \mathbb{Z}^2$, $U_\alpha \cap U_\beta = \emptyset$ and U_α, U_β are finite, and at least U_α is non-empty.

(ii) $\alpha: U_\alpha \rightarrow V \cup \left\{ \begin{array}{c} \triangle \\ \# \end{array}, \begin{array}{c} \nabla \\ \# \end{array}, \begin{array}{c} \triangleleft \\ \# \end{array}, \begin{array}{c} \triangleright \\ \# \end{array} \right\}$, $\beta: U_\beta \rightarrow V$ such that $\text{shape}(\alpha) = \{w \in U_\alpha \mid \alpha(w) \in V\}$. (U_α, α) is called the selector and (U_β, β) the context of the production (α, β) , U_α is called the selector area and U_β is the context area.

For arrays $C_1, C_2 \in V^{+2}$, we can find a sub-array that corresponds to the selector (U_α, α) and if the place corresponding to (U_β, β) are labeled only by the blank symbol, then we can add the context (U_β, β) and thereafter we get the iso-triangular array C_2 . Formally, C_2 is derivable from C_1 by the contextual iso-triangular array production $p \in P$, $p = (U_\alpha, \alpha, U_\beta, \beta)$ can be written as $C_1 \Rightarrow_p C_2$.

CHAPTER 2

LITERATURE SURVEY

Many array grammar models have been introduced in formal language theory, by extending the rewriting feature of strings in Chomsky grammars. Moreover, in the field of image analysis and picture generation, two-dimensional (array) grammar models have been used and many problems are investigated. Here arrays are rewritten only by the array rewriting rules of Chomsky grammars. Parallel rewriting has been considered in [11] and [12] to generate the two-dimensional picture languages. The generative power of the model is investigated.

The contextual string grammar model is one among the grammar models, introduced by Markus and it is extended to arrays to generate two-dimensional picture languages in a two-dimensional plane. Motivated by different problems in the framework of image analysis and picture generation, Freund R. [5] (2000&2006) introduced array grammar system and contextual d-dimensional array grammar. The fundamental difference between contextual and Chomsky grammars or Lindenmayer systems is that in contextual grammars we do not rewrite symbols, but we adjoin symbols to the current string and the symbols once introduced which cannot be modified, remain in the final generated string. Also erasing the symbol is considered. In [3] Contextual array rewriting p system is proposed and examined the computational power of the array rewriting p system.

The generative grammars and the corresponding classes of automata are well known [6] (Hopcroft 1979) in the literature. An automaton is a machine, which recognizes two-dimensional picture languages. [4] Ceterchi et. al. (2003) proposed an array of rewriting grammar for two-dimensional array languages. Two-dimensional languages or array languages also play important roles in character recognition and pictorial information system design. Handbook of Formal Languages" has a chapter on two-dimensional picture languages that provides a comprising discussion of the topic of grammar and automata. Context-Free iso-array rewriting p system is introduced and generated some 2D picture languages by Bhuvaneswari et al. [1] and [2] (2014).

In general, tiling is a branch concerned with shapes. An extended tile pasting system (Robinson et al. 2007) is proposed for Kolam patterns. Generating two-dimensional pictures by tile pasting systems motivates the introduction of new tiles and tile pasting systems [8] and [9] (Kalyani et. al. 20006, 2006a) to generate triangular pictures. Many computing models have been proposed in the literature to generate two-dimensional picture language. Local and recognizable iso picture languages investigated in [7]. Triangular tile rewriting grammars and triangular picture languages have been investigated in [10].

CHAPTER 3

3. AIM

- Generating 2-D images (iso-triangular pictures) by regular iso-triangular array grammar.
- Examination of computational powers on iso-triangular arrays by iso-array grammar rule.
- Generating 2-D images (Iso-triangular pictures) by context-free iso-array grammar.
- Generating 2-D images (iso-triangular tiles) by context-free basic puzzle iso-array grammar.
- Comparison study between the class of languages generated by context-free iso-array grammar, basic puzzle iso-array grammar, and contextual iso-array grammar.

SCOPE

- Medical image retrieval
- Traffic sensing technology
- Image reconstruction
- Face detection

CHAPTER 4

ISO-TRIANGULAR ARRAY GRAMMAR AND 2D LANGUAGES

Definition 4.1. INTRODUCTION TO 2D ARRAY GRAMMAR

A 2D array grammar defines the syntax and rules for declaring, accessing, and manipulating two-dimensional arrays in a programming language. It includes specifications for how to declare the array, how to access elements using row and column indices, and how to perform operations such as iteration and manipulation on the array.

Definition 4.1. CONTEXT-FREE ISO-ARRAY GRAMMAR

A context-free iso-array grammar (CFIAG) is a structure $G = (N, T, P, S)$ where N and T are finite nonempty sets of symbols (isosceles right-angled triangular tiles), $N \cap T = \emptyset$. Elements of N and T are called non-terminals and terminals respectively. $S \in N$ is the start symbol and P consists of rules of the form $\alpha \rightarrow \beta$, where α and β are finite connected arrays of one or more iso triangular arrays over

$N \cup T \cup \{ \triangle_{\#A}, \triangle_{\#B}, \triangle_{\#C}, \triangle_{\#D} \}$ and satisfy the following conditions:

1. The shapes of α and β are identical.
2. α contains exactly one non terminal and possibly one or more empty symbols and Terminals in α are not rewritten.

Application of the rule $\alpha \rightarrow \beta$ preserves the contentedness of the host array (that is, the application of the rule to a connected array results in a connected array).

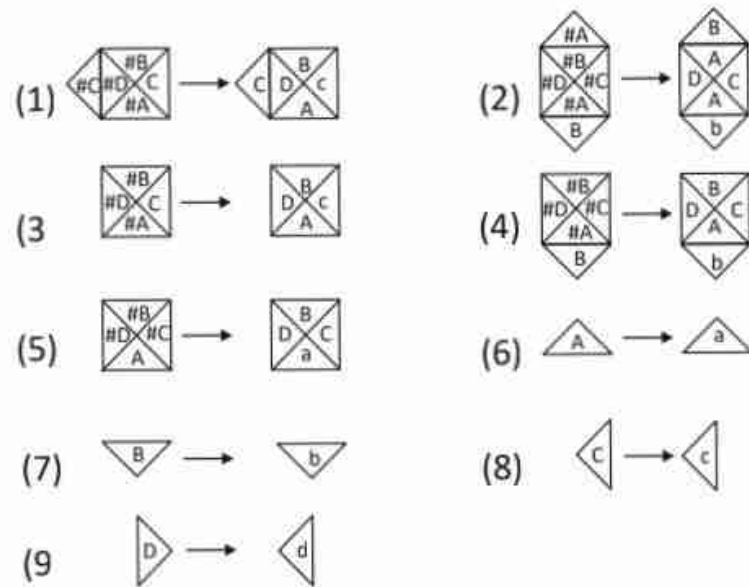
EXAMPLE 4.1.1.

A class of iso-triangular array language is generated by context-free iso-array grammar rules.

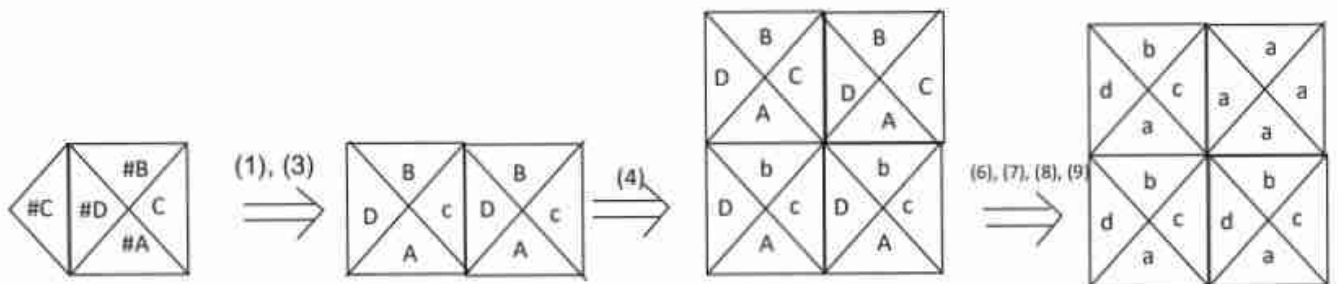
Consider the context-free iso-array grammar $G = (N, T, P, S)$, where

$$N = \{ \triangle_A, \triangle_B, \triangle_C, \triangle_D \}, T = \{ \triangle_a, \triangle_b, \triangle_c, \triangle_d \}, S = \triangle_C$$

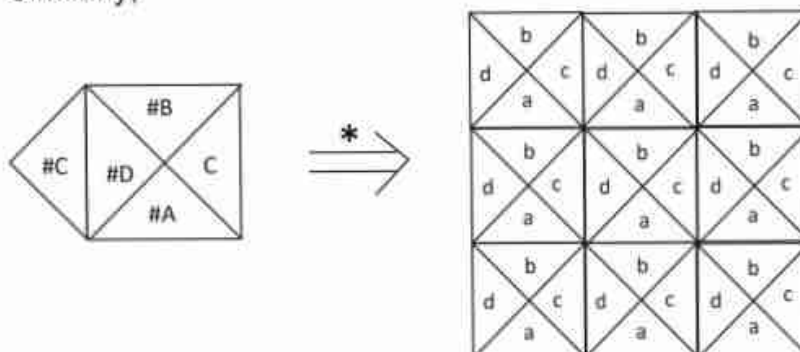
and P consisting of the following rules



The sample derivations of the iso-triangular picture patterns of $L(G)$ are shown below;



Similarly,



The iso-triangular picture language consists of square is shown below:

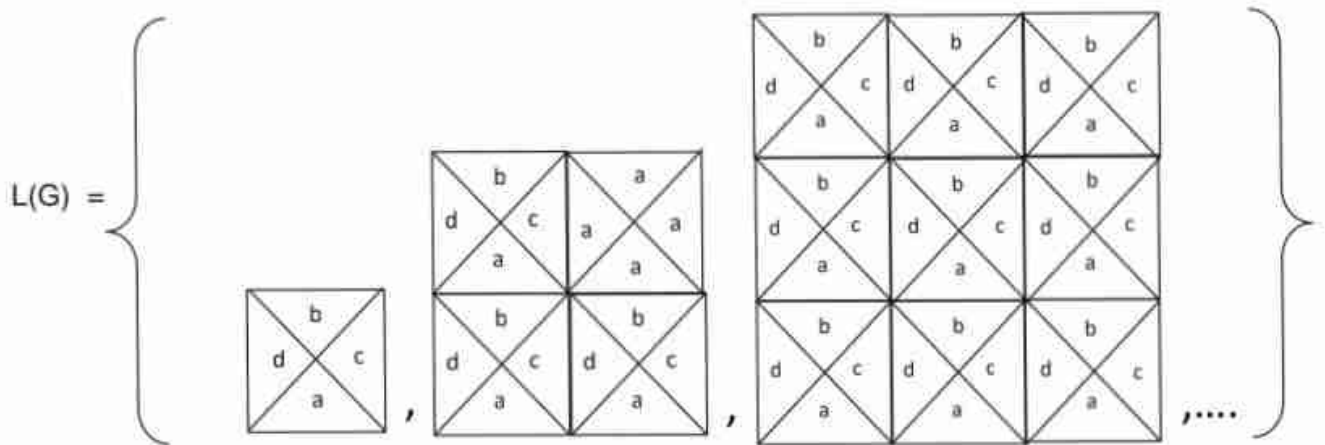


Fig.4.1. A class of square picture language

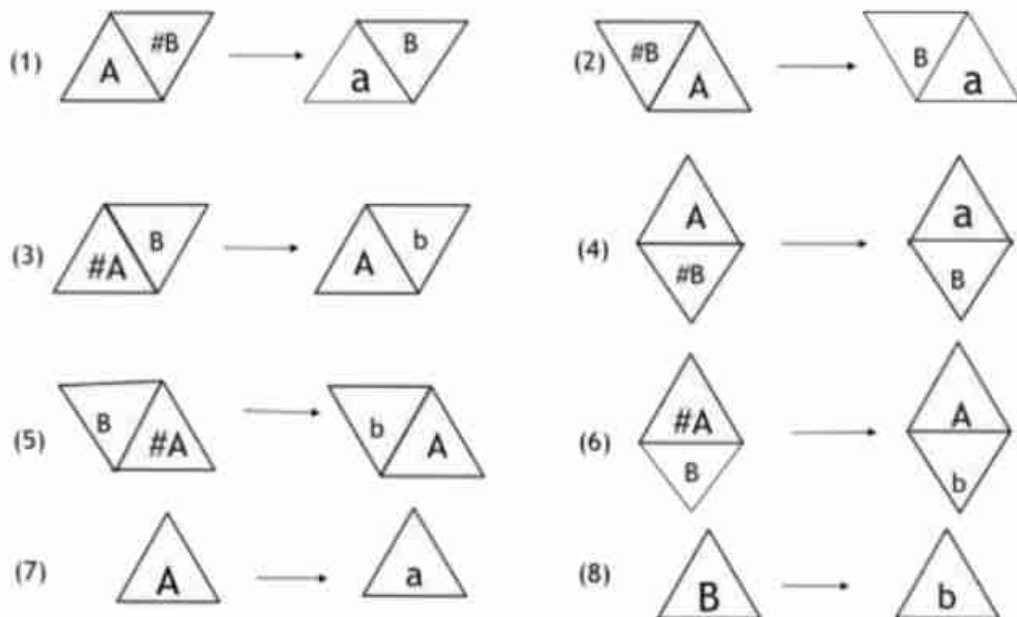
EXAMPLE: 4.1.2.

A class of iso-triangular array language is generated by context-free-iso-array grammar rules.

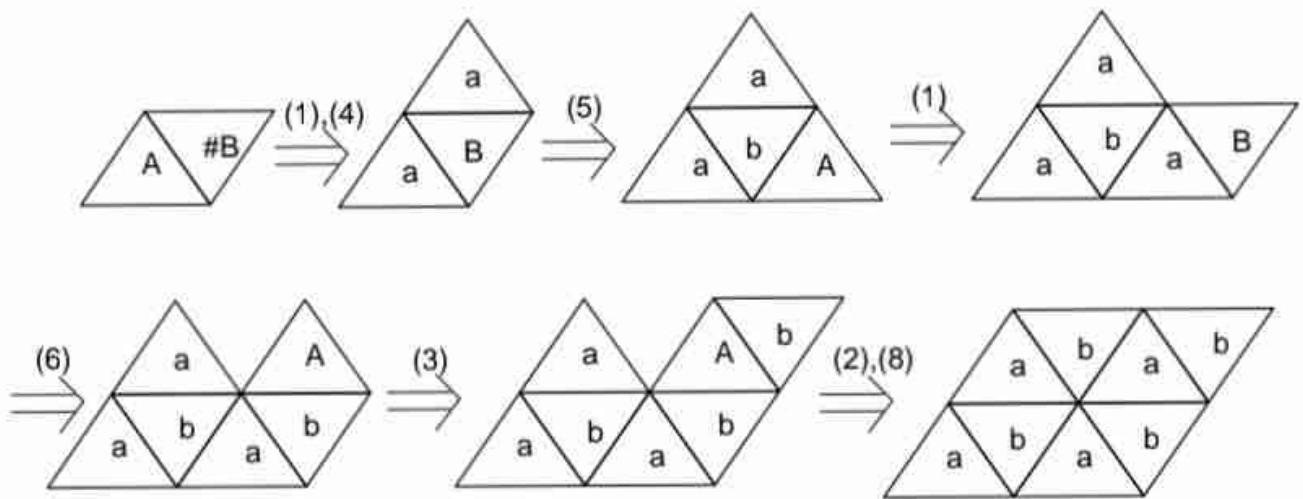
Consider the context-free iso-array grammar $G = (N, T, P, S)$, where

$$N = \{ \triangle A, \nabla B, \triangleleft C, \triangleright D \}, T = \{ \triangle a, \nabla b, \triangleleft c, \triangleright d \}, S = \triangle A$$

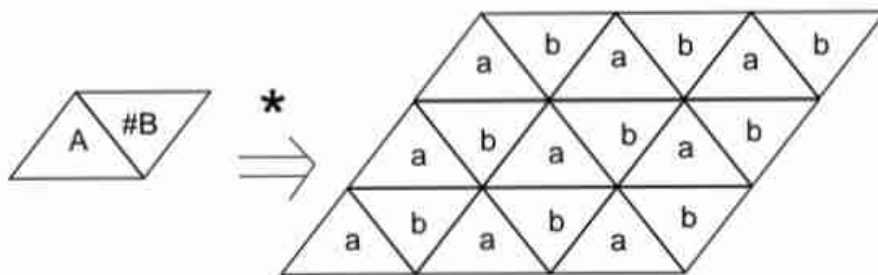
and P consisting of the following rules:



The sample derivations of the iso-triangular picture patterns of $L(G)$ are shown below:



Similarly,



The iso-triangular picture language consisting of Rhombus is shown below:

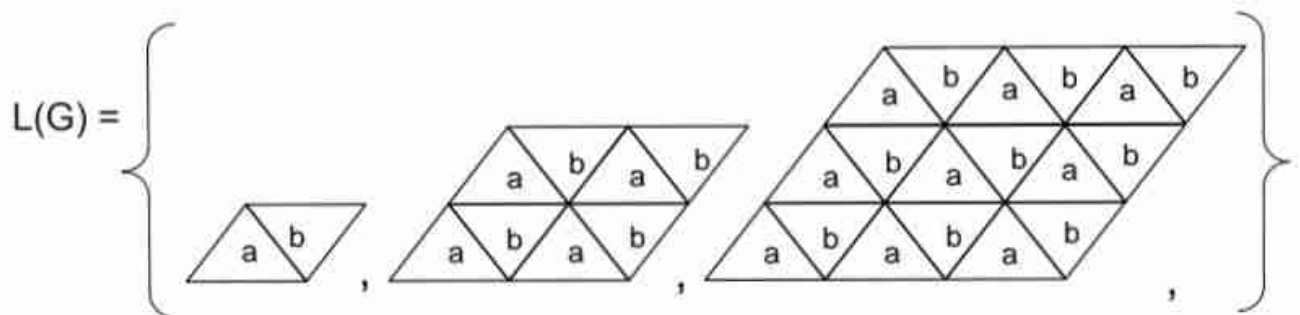
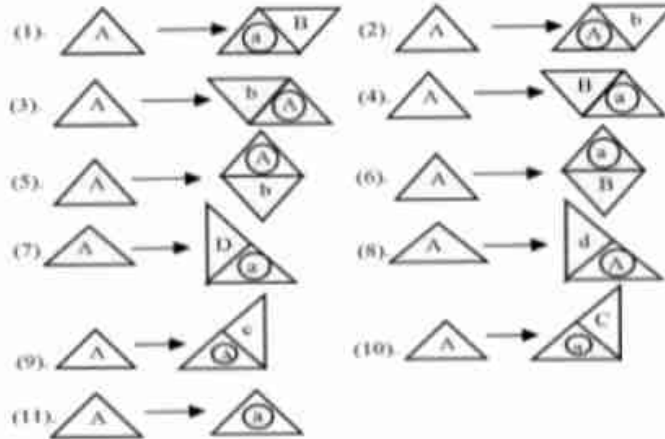


Fig.4.2. A class of 2D language consisting of Rhombuses

Definition 4.2. CONTEXT-FREE BASIC PUZZLE ISO-ARRAY GRAMMAR

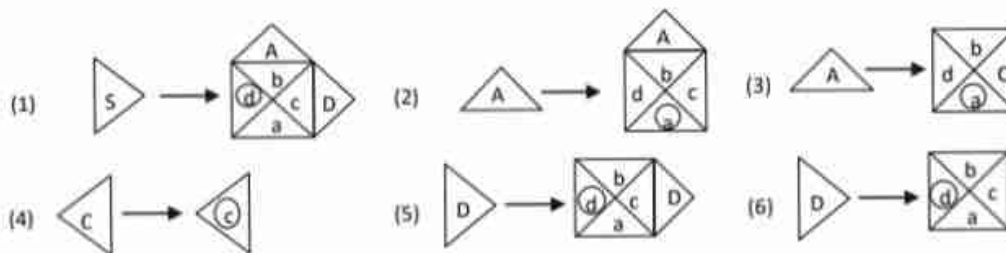
A Context-free basic puzzle Iso-Array Grammar (CFBPIAG) is a structure $G = (N, T, P, S)$, where N , T , and S are defined as in Definition 1.4.3. and P is the set of rewriting rules of the form $X \rightarrow \alpha$, where X can be any of the tiles and α is a finitely connected array of one or more iso-triangular arrays, each cell containing either a non-terminal or a terminal symbol, with the symbol in one of the cells of α being circled.



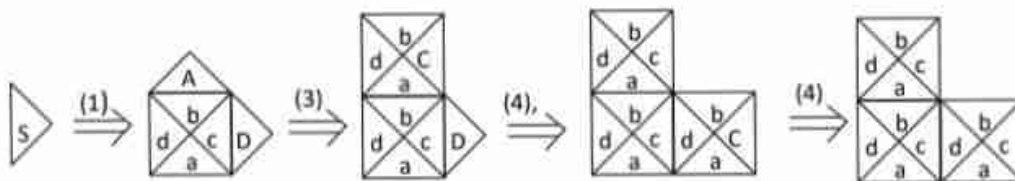
Similar rules can be given for the other tiles B , C , D . The set of all languages generated by CBFPIAG is CBFPIAL.

Example 4.2.1. The context-free puzzle iso-array grammar generates digitized staircase Patterns.

Consider $G = (N, T, P, S)$ where $N = \{S, A, B, D\}$, $T = \{a, b, c, d\}$, $S = \begin{array}{|c|} \hline S \\ \hline \end{array}$ and P consists of the following rules;



The first member of CFBPIAG is shown below in the derivation steps.



CHAPTER 5

5. COMPARISON RESULT-1

A class of picture language consisting of Rhombuses can be generated by context-free iso-triangular array grammar and also if can be generated by the Basic puzzle iso-array grammar.

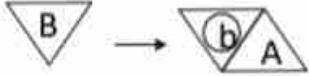
Proof:

A class of picture language consisting of rhombuses can be generated by context-free iso-triangular array grammar refer (4.1.2).

Now, consider the context-free basic puzzle iso-array grammar $G = (N, T, P, S)$, where

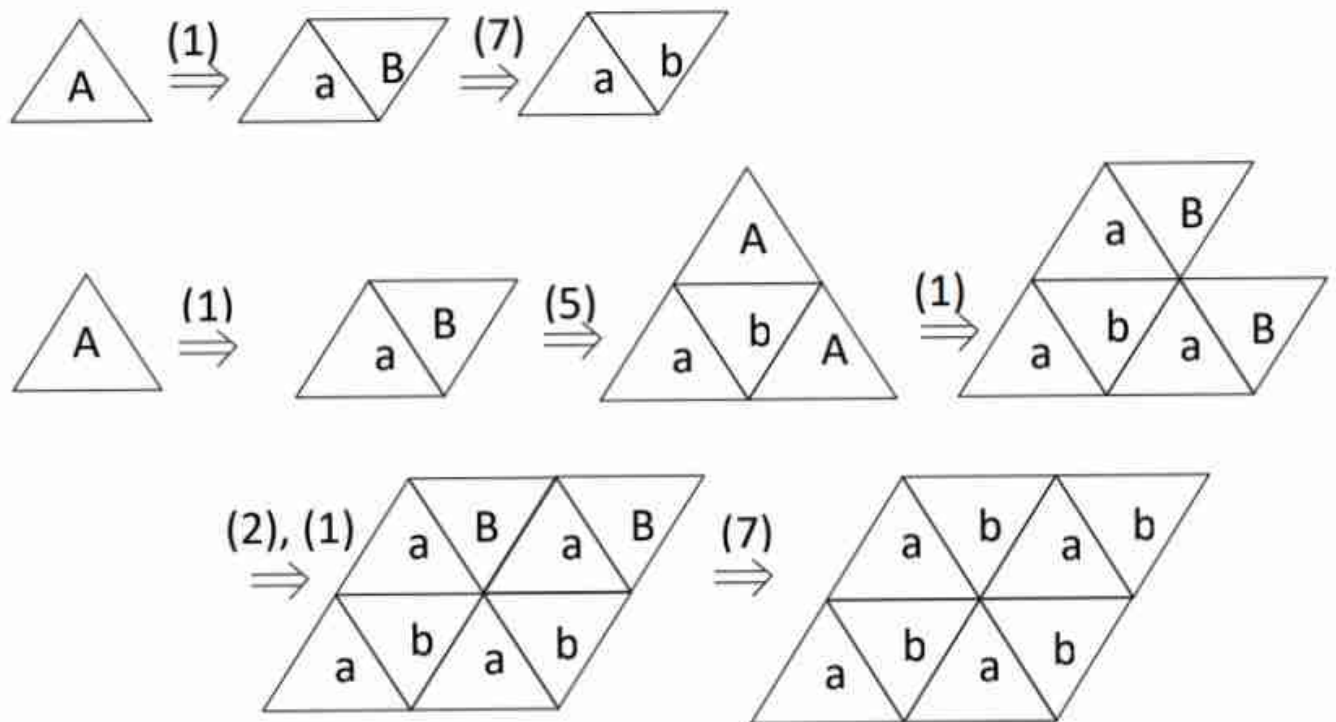
$$N = \{ \triangle A, \nabla B, \diamond C, \diamond D \}, T = \{ \triangle a, \nabla b, \diamond c, \diamond d \}, S = \triangle A$$

and P consisting of the following rules:

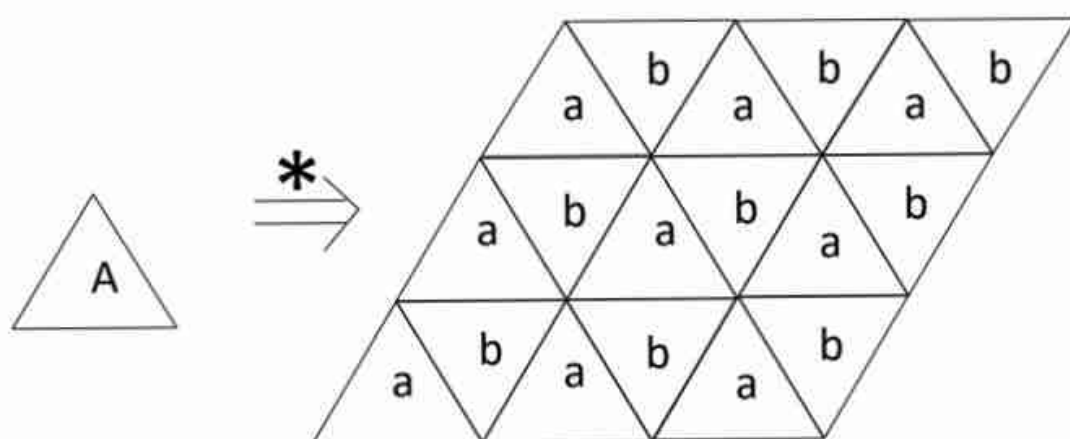
- | | |
|--|--|
| <p>(1) </p> | <p>(2) </p> |
| <p>(3) </p> | <p>(4) </p> |
| <p>(5) </p> | <p>(6) </p> |
| <p>(7) </p> | <p>(8) </p> |

The evaluation starts with the initial iso-triangular tile A which is rewritten to the tile B by rule (1) after that rule (9) is applied to generate a rhombus pattern. After that, the resultant picture is generated. the picture pattern rhombus is generated in this step. To generate the 2^{nd} member of the language, a copy of the first member is considered. In the next step the left-hand side of the triangle, the tile A rewritten to the resultant pattern by the rule (5) and then by the rule (1), rule (9), and rule (7) we have obtained 2^{nd} member of. the member of 3^{rd} picture language is shown below. the language consists of iso-array that is generated by context-free basic puzzle iso-array grammar. But we can also generate this pattern using context-free iso-triangular array grammar.

The sample derivations of the iso-triangular picture patterns of $L(G)$ are shown below



Similarly,



The iso-triangular picture language is shown below:

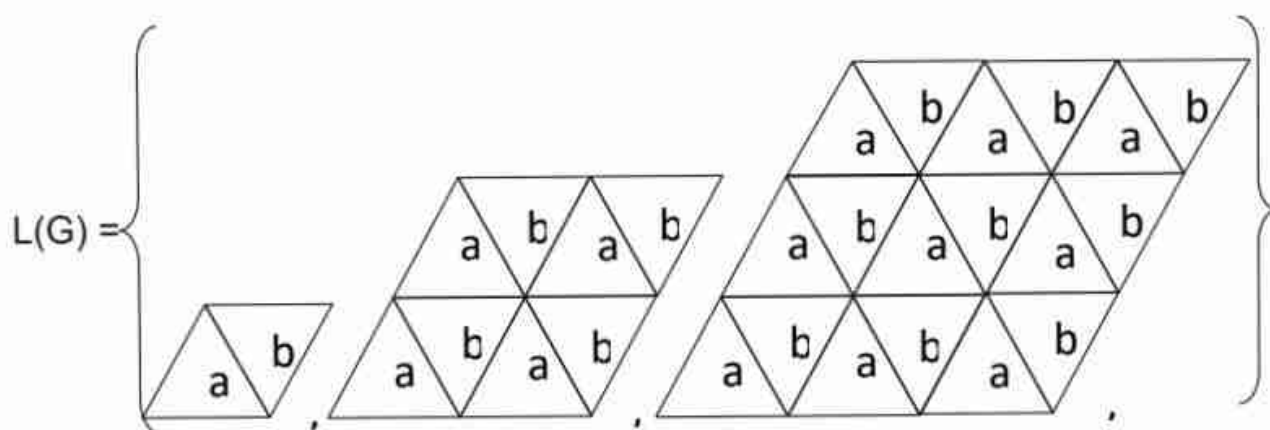


Fig.5.1. 2D language consisting of Rhombus

A class of picture language consisting of Rhombuses can be generated by context-free iso-triangular array grammar and Basic puzzle iso-array grammar.

COMPARISON RESULT -2

The parallel array rewriting is considered in the generation of two-dimensional picture languages.

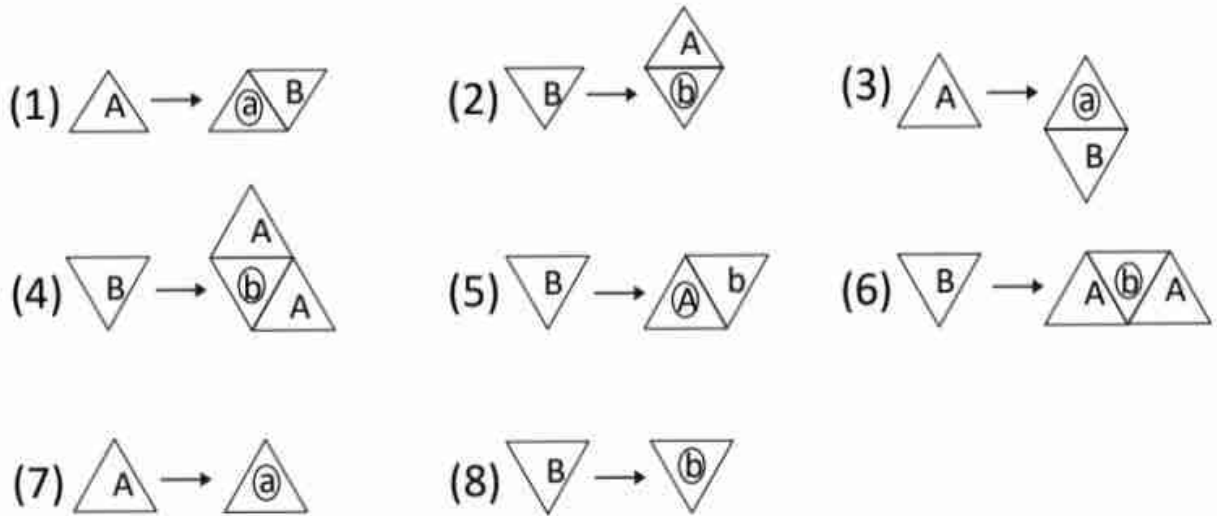
Proof:

Using the parallel iso-array rewriting, the class of picture language consisting of triangular arrays can be generated by Context-Free basic puzzle iso-array Grammar

Now, consider the context-free basic puzzle iso-array grammar $G = (N, T, P, S)$, where

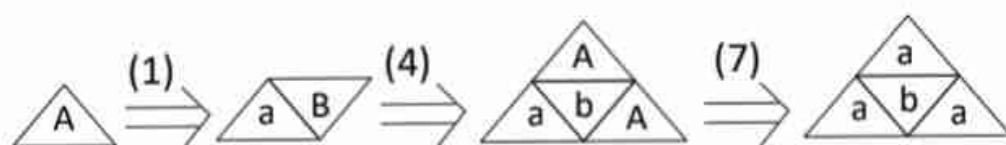
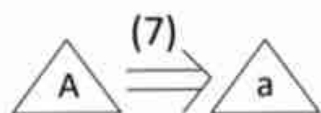
$$N = \{ \triangle A, \nabla B, \triangleleft C, \triangleright D \}, T = \{ \triangle a, \nabla b, \triangleleft c, \triangleright d \}, S = \triangle A$$

and P consisting of the following rules:

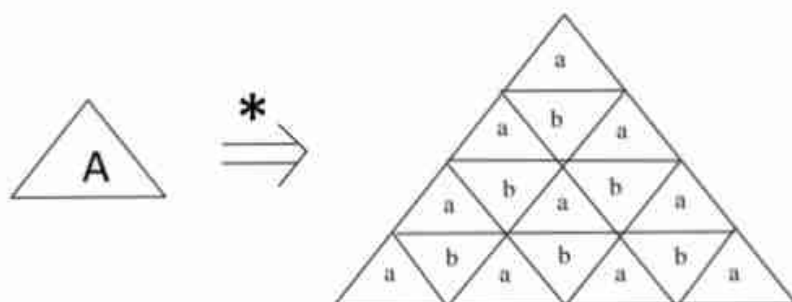


The evaluation starts with the initial iso-triangular array A which is rewritten by the rule (7). After that, the resultant picture is generated. the picture pattern of the iso-triangular array is generated in this step. To generate the 2nd member of the language, a copy of the first member is considered. In the next step the left-hand side of the array B is rewritten to the resultant pattern by rule (1) and then by rule (4) and rule (7), the 2nd member of iso-triangular array is generated. the member of

the 3rd picture language is shown below. the language consists of triangles that can be generated by context-free iso-triangular array grammar. Using parallel iso-array rewriting the number of derivation steps is reduced compare to basic puzzle iso-array rewriting grammar model [1]



Similarly,



The iso-triangular picture language is shown below:

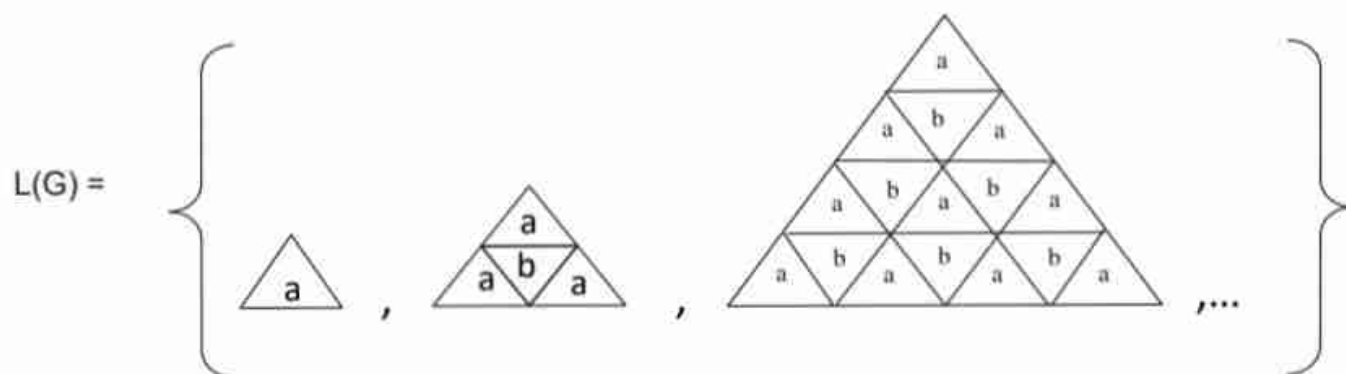


Fig.5.2. Language consisting of triangular arrays

COMPARISON RESULT - 3

The computational power of Basic puzzle iso array grammar is examined by generating the two-dimensional language consisting of Rhombuses.

Proof:

By (Result 1)

Generating 2D picture language consisting of rhombuses can offer convenience in representing certain symbols. Especially if the circle symbol is good for certain contexts. (Refer to 4.1.2 and result 1). That is the circled symbol may be terminal or non-terminal. The symbol in the circle is rewritten.

COMPARISON RESULT – 4

HEXAGONAL PICTURE LANGUAGE

Let I be a finite non-empty set of symbol. The set of all hexagonal arrays over I (including the empty array $\#$) is defined by I^{*+H} and $I^{*+H} = I^{*+H} - \{\#\}$. The size of the hexagonal array is defined by parameters LU(Left up), LD(Left down), RU(Right up), RD (Right down), U(Up) and D(Down) as shown in Fig.5.3.

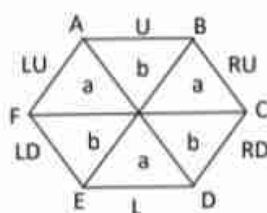


Fig.5.3. A Hexagonal Array

A class of picture language consisting of Hexagon can be generated by CITAP but cannot be generated by the CFIAG.

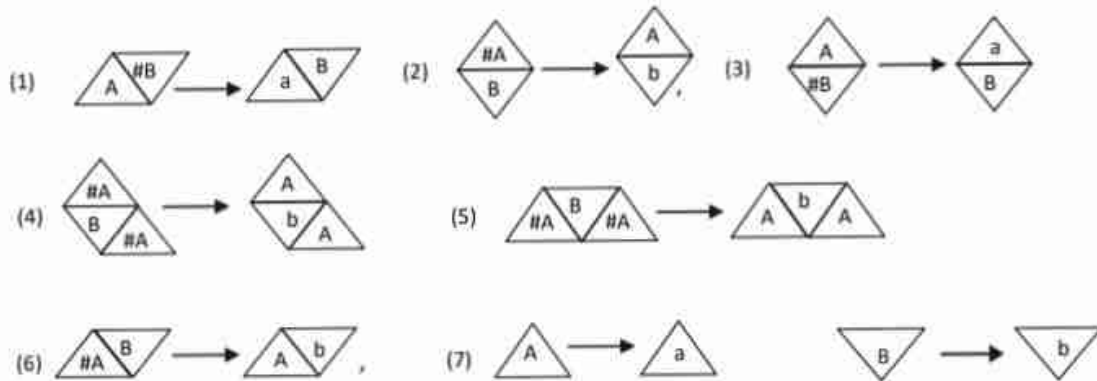
Proof:

From [2] we know that CITAP₃ (Π H) with adjoining rules generates a class of language consisting of hexagons.

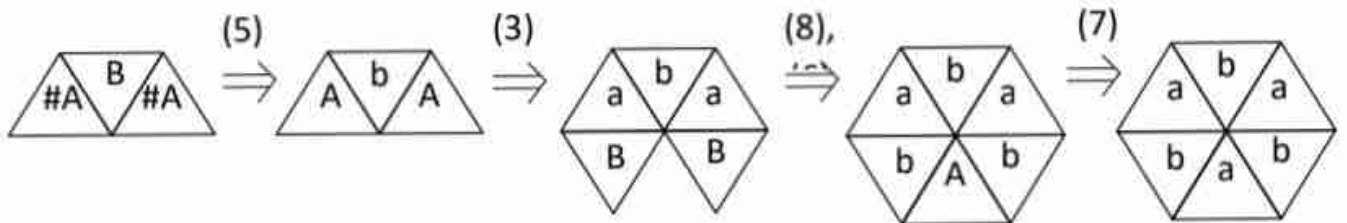
Now, consider the context-free iso-array grammar $G = (N, T, P, S)$, where

$$N = \{ \triangle A, \nabla B, \triangleleft C, \triangleright D \}, T = \{ \triangle a, \nabla b, \triangleleft c, \triangleright d \}, S = \nabla B$$

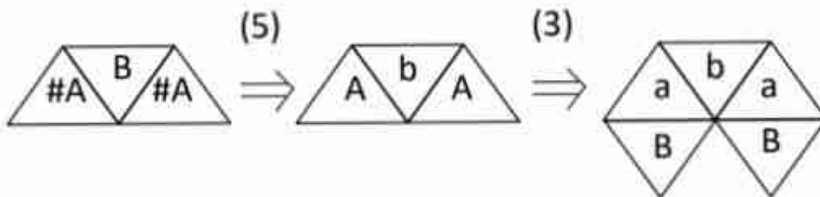
and P consisting the following rules:



The first member of CFIAG is shown below in the derivation steps.



Similarly, the second member of CFIAG is



By the above rules we cannot generate the second member of CFIAG. So, we cannot proceed the result. From the Literature, we have noticed that contextual iso-array rewriting grammar model with adjoining rules generate a class of language consisting of hexagons. But the hexagonal picture language cannot be generated by the CFIAG.

CHAPTER-6

CONCLUSION

In this project, iso-triangular array grammars are considered and generated in some classes of 2D picture languages. The generating powers of iso-triangular array grammars are examined with the existing model called iso-array rewriting models with RIAG and contextual iso-array rewriting grammar. The application of rewriting in CFBPIAG enhances the feature of the computation of 2D picture languages.

FUTURE WORK

Iso-array rewriting P system model will be considered for the 2D picture languages using iso-array rewriting grammars. The computational power of the existing models with Iso-array rewriting P system model will be examined.

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