

WARRANTY MODELLING FOR REMANUFACTURED PRODUCTS IN REVERSE SUPPLY CHAIN

Submitted in partial fulfilment of the requirements for the award of
BACHELOR OF SCIENCE in MATHEMATICS

By

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**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of Jayapriya. R (41060005), Jasmin Jenifer. R (41060004) who carried out the project entitled "Warranty modelling for remanufactured products in reverse supply chain" under my supervision from November 2023 to April 2024.

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DECLARATION

We Jayapriya. R (41060005) and Jasmin Jenifer. R (41060004) hereby declare that the Project Report entitled **Warranty modelling for remanufactured products in reverse supply chain** done by us under the guidance of Dr. S. Sangeetha, Department of Mathematics, Sathyabama Institute of Science and Technology is submitted in partial fulfilment of the requirements for the award of **Bachelor of Science** degree in **Mathematics**.

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ABSTRACT

In recent decades, supply chain management has emerged as one of the most important operational strategies for improving the competitiveness of organizations. In reality, consumers are often unable to determine or judge the quality of the product before purchase. To alleviate this uncertainty in the minds of consumers, manufacturers often insure their customers against this risk of product breakdown by bundling a manufacturer's warranty at the time of sale. Consumers often view remanufactured products as inferior and are less willing to pay for them. The warranty policy and its effect on consumer behaviour from the perspective of consumers is analysed in this study. Three marketing settings are developed to examine the impact of warranty for the manufacturer's profit. This report analyses the warranty associated with the products can command higher prices in the marketplace and accordingly can enhance the manufacturer's profit. The numerical analysis helps provide managers with insight on the base warranty and its implication.

Keywords: Warranty; Consumer utility; Remanufactured products; Profit function

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LIST OF SYMBOLS

c_n	Unit manufacturing cost of a new product
c_r	Unit manufacturing cost of a remanufactured product
p_{ni}	Retail price charged by OEM in three settings ($i=1,2,3$)
p_{ri}	Retail price charged by the manufacturer in three settings ($i=1,2,3$)
α	Retail price charged by the manufacturer in three settings ($i=1,2,3$)
β	Discount factor for remanufactured products
q_{ni}	Demand function for new products in three settings ($i=1,2,3$)
q_{ri}	Demand function for remanufactured products in three settings ($i=1,2,3$)

CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The profitability of remanufacturing systems has been extensively studied in the closed loop supply chain literatures. Successful examples, such as IBM, Kodak, Xerox show that remanufacturing can be profitable. To deal with problems like technological advancements, waste generation, management and pollution, remanufacturing has been widely adopted as an effective method [7,8]. The activity of taking back and remanufacturing leads to sufficiently high cost savings [6] and results in higher profit for the manufacturer. But remanufacturing is difficult due to the little guidance for the managers. Considering the above characteristics of remanufactured products effective measures are to be taken to promote the development of remanufacturing. In an intensely competitive marketplace, pricing is a kind of important business behavior. The competing manufacturers are often play a price war to attract consumers. As the consumer products becoming more and more complex, consumers are often unable to judge the quality of product before purchase. To reduce consumer's uncertainty of the product, manufacturers usually provide some kind of warranty with their products, e.g., Hyundai and Jaguar land rover in the automobile market, Panasonic, Samsung, and Sears in the home electronics market. This report studies the warranty strategy choice of the manufacturers and mathematically derives the optimal values of warranty level, demand, and price on the goods. The primary focus of this study is to analyze the warranty policy using a supply chain framework.

1.2 BASIC DEFINITIONS

- **SUPPLY CHAIN MANAGEMENT**

Supply chain management (SCM) is the centralized management of the flow of goods and services to and from a company and includes all of the processes involved in transforming raw materials and components into finished products.

- **REVERSE SUPPLY CHAIN**

It is the series of activities required to retrieve a used product from a customer in order to either dispose of it or reuse it.

- **REMANUFACTURING**

Remanufacturing is the process of recovering value from the used products by replacing and reassembling the components of a part or product to a like new condition. [11]

- **WARRANTY**

Warranty is defined as a written assurance that the manufacturer of a product will guarantee the quality and reliability of a product in terms of correcting any legitimate problems with the product at no additional cost, for some expressed or implied period of time or use. [9]

CHAPTER 2

LITERATURE SURVEY

2.1 LITERATURE SURVEY

Blischke and Murthy [1] developed a comprehensive review in the warranty policy of consumer products. Decroix [5] studied an oligopoly that the durable goods manufactures seeking optimal values for three decision variables: product warranty, reliability and price. Chattopadhyay and Murthy [2] addressed the problem that when the items are sold with free replacement or pro-rata warranties and proposed probabilistic models to compute the expected warranty cost to the manufacturer. Peggo and Lam [10] investigated an extended warranty model that includes a free repair period and an extended warranty period. Chen et al. [3] considered manufacture's pricing strategies in a two echelon supply chain that comprise one manufacturer and two retailers who face warranty period dependent demand. Dai et al. [4] examined the relationship of product quality and product warranty and their influence on supply chain performance.

The current study analyzes the warranty strategy choice of the manufactures and derives the optimal setting of warranty level, demand, and price on their goods in the market using a supply chain framework.

CHAPTER 3

AIMS AND SCOPE

3.1 AIMS AND SCOPE

The scope of this study is to derive managerial insights for manufacturers in reverse supply chain with regard to the influence of manufacturers strategy on decision variables and performances. The main aim is to identify the demand conditions under which policy is comfortable for the remanufacturers. The primary focus of this work is to analyse the warranty policy under a supply chain setting.

CHAPTER 4

MATHEMATICAL FORMULATION

4.1 MODEL DESCRIPTION

In this study a simple situation is investigated that involves two competing manufacturers. The Original Equipment Manufacturer produces new products and the competitive remanufacturer produces remanufactured products, i.e., a natural alternative to the new products. Both the two types of products sold directly to the customers. The consumers are homogeneous in all respects except their valuation of the product. Remanufactured products are often sold side by side with new products and provided for lower cost alternative to customers, so that consumers discount the valuation of new product by α and the remanufactured product by β . That is, a customer who has valuation αv for a new product and βv for remanufactured product, where $0 \leq \beta \leq \alpha \leq 1$ is the valuation discount factor.

4.2 MATHEMATICAL FORMULATION OF THE MODEL

Two manufacturers are independent decision makers and each looks at its own profit when setting retail prices and warranty strategy. The goal in this section is to identify the demand conditions under which policy is profitable.

4.2.1 Case 1: Both OEM and Remanufacturer doesn't provide warranty

The preferences of the consumers are described by a utility functions $u_n = \alpha v - p_n$ if the consumer buys one unit of new products at a price , and $u_r = \beta v - p_r$ if the consumer buys one unit of remanufactured products at a price. The value equals to 0 if the consumer does not buys at all. The parameter v is uniformly distributed with unit density across the population of consumers between 0 to 1.

A customer decides to buy a new product if $u_n \geq u_r$ and $u_n > 0$ and a remanufactured product if $u_r \geq u_n$ and $u_r > 0$. Then the demand functions of the varieties are

$$q_{n_1} = 1 - v_{12} = 1 - \frac{p_n - p_r}{\alpha - \beta} \quad \text{and} \quad q_{r_1} = v_{12} - v_{11} = \frac{p_n - p_r}{\alpha - \beta} - \frac{p_r}{\beta}$$

Correspondingly, the profit functions are

$$\pi_{n_1} = (p_n - c_n)q_n$$

$$\pi_{r_1} = (p_r - c_r)q_r$$

Substituting the demand functions in equations and maximizing the profit functions, the optimal prices and profits without warranty are obtained as follows:

$$p_{n_1}^* = \frac{2\alpha^2 - 2\alpha\beta + 2\alpha c_n + \alpha c_r}{4\alpha - \beta}$$

$$p_{r_1}^* = \frac{\alpha\beta - \beta^2 + 2\alpha c_r + \beta c_n}{4\alpha - \beta}$$

$$\pi_{n_1}^* = \frac{[2\alpha^2 - 2\alpha\beta - (2\alpha - \beta)c_n + \alpha c_r]^2}{(\alpha - \beta)(4\alpha - \beta)}$$

$$\pi_{r_1}^* = \frac{\alpha[\alpha\beta - \beta^2 + \beta c_n - (2\alpha - \beta)c_r]^2}{\beta(\alpha - \beta)(4\alpha - \beta)^2}$$

4.2.2 Case 2: Remanufacturer provide warranty, whereas OEM does not

The warranty level of the supply chain is measured by a variable y . To achieve this warranty performance level, certain investment has to be committed. The cost incurred by the remanufacturer is assumed to take a quadratic form: $c(y) = \frac{1}{2}cy^2$, which is independent of the production quantity.

To characterize the difference in service preference, it is assumed that a consumer buying a product with a warranty obtains an extra benefit by (relative to one unit of non-service product) when he or she buys one unit of goods with a given service activity. This implies that the extra benefit will be zero if the supply chain system does not provide a differentiated product with service commitment. This extra benefit formulation captures consumers' different Willingness to pay for a product with a given service activity.

The utility functions are,

$$u_n = \alpha v - p_{n_2}$$

$$u_r = \beta v - p_{r_2} + by_{r_2}$$

The demand functions are,

$$q_{n_2} = 1 - \frac{p_{n_2} - p_{r_2} + by_{r_2}}{\alpha - \beta}$$

$$q_{r_2} = \frac{\beta p_{n_2} - \alpha p_{r_2} + b\alpha y_{r_2}}{\beta(\alpha - \beta)}$$

The two profit functions respectively, are

$$\begin{aligned}\pi_{n_2} &= (p_{n_2} - c_n)q_{n_2} \\ &= (p_{n_2} - c_n) \left(1 - \frac{p_{n_2} - p_{r_2} + by_{r_2}}{\alpha - \beta} \right) \\ \pi_{r_2} &= (p_{r_2} - c_r)q_{r_2} - \frac{1}{2}cy_{r_2}^2 \\ &= (p_{r_2} - c_r) \left(\frac{\beta p_{n_2} - \alpha p_{r_2} + b\alpha y_{r_2}}{\beta(\alpha - \beta)} \right) - \frac{1}{2}cy_{r_2}^2\end{aligned}$$

Since $\frac{\partial^2 \pi_{n_2}}{\partial p_{n_2}^2} = \frac{-2}{\alpha - \beta} < 0$, the profit function π_{n_2} is strictly concave in prices. From the above equation π_{r_2} , the Hessian matrix H of $\pi_{r_2}(p_{r_2}, y)$ is obtained as

$$\pi_{r_2}(p_{r_2}, y) = \begin{bmatrix} \frac{-2\alpha}{\beta(\alpha - \beta)} & \frac{b\alpha}{\beta(\alpha - \beta)} \\ \frac{b\alpha}{\beta(\alpha - \beta)} & -c \end{bmatrix}$$

For π_{r_2} to be maximum in $(p_{r_2}^*, y_{r_2}^*)$, the Hessian matrix H should be negative definite.

Define $H' = 2\beta(\alpha - \beta) - \frac{b^2\alpha}{c}$. It means that the warranty investment should not be too inexpensive, which is consistent.

The first order condition $\frac{\partial \pi_{n_2}}{\partial p_{n_2}} = 0$ and $\frac{\partial \pi_{r_2}}{\partial p_{r_2}} = 0$ imply

$$1 - \frac{p_{n_2} - p_{r_2} + by_{r_2}}{\alpha - \beta} + (p_{n_2} - c_n) \frac{-1}{\alpha - \beta} = 0$$

$$\frac{\beta p_{n_2} - \alpha p_{r_2} + b\alpha y_{r_2}}{\beta(\alpha - \beta)} + (p_{r_2} - c_r) \frac{-\alpha}{\beta(\alpha - \beta)} = 0$$

The following are the optimal prices:

$$p_{n_2}^* = \frac{2\alpha^2 - 2\alpha\beta + 2\alpha c_n + \alpha c_r - b\alpha y_{r_2}}{4\alpha - \beta}$$

$$p_{r_2}^* = \frac{\alpha\beta - \beta^2 + 2\alpha c_r + \beta c_n + 2b\alpha y_{r_2} - b\beta y_{r_2}}{4\alpha - \beta}$$

The optimal level of service obtains from the first order condition:

$$y_{r_2}^* = \frac{2b\alpha(2\alpha - \beta)(\alpha\beta - \beta^2 + \beta c_n - 2\alpha c_r + \beta c_r)}{c\beta(\alpha - \beta)(4\alpha - \beta)^2 - 2b^2\alpha(2\alpha - \beta)^2}$$

$$\pi_{n_2}^* = \frac{(2\alpha^2 - 2\alpha\beta - 2\alpha c_n + \beta c_n + \alpha c_r - b\alpha y_{r_2}^*)^2}{(\alpha - \beta)(4\alpha - \beta)^2}$$

$$\pi_{r_2}^* = \frac{\alpha}{(\alpha - \beta)(4\alpha - \beta)^2} (\alpha\beta - \beta^2 + \beta c_n + \beta c_r - 2\alpha c_r + 2b\alpha y_{r_2}^* - b\beta y_{r_2}^*)^2 - \frac{1}{2} c y_{r_2}^{*2}$$

Recalling the above two cases, we observe that the remanufacturer gains higher profit in case 2 than in case 1, while the OEM suffers benefit injury. For this reason, they developed another case.

4.2.3 Case 3: Both OEM and Remanufacturer provides warranty

Similar to case 2, from the first order condition the following are the optimal prices:

$$P_{n_3}^* = \frac{1}{4\alpha - \beta} (2\alpha^2 - 2\alpha\beta + 2\alpha c_n + \alpha c_r - b\alpha y_{r_3} + e(2\alpha - \beta)y_{n_3})$$

$$P_{r_3}^* = \frac{1}{4\alpha - \beta} (\alpha\beta - \beta^2 + 2\alpha c_r + \beta c_n - \beta e y_{n_3} + b(2\alpha - \beta)y_{r_3})$$

Substitution these equations into the profit expressions, the equilibrium conditions are obtained as follows:

$$\pi_{n_3}^* = \frac{2\alpha^2 - \alpha\beta + 2\alpha c_n + \alpha c_r + e(2\alpha - \beta)y_{n_3} - c_n(4\alpha - \beta)}{(\alpha - \beta)(4\alpha - \beta)}$$

$$\pi_{r_3}^* = \frac{\alpha}{(\alpha - \beta)(4\alpha - \beta)^2} (\alpha\beta - \beta^2 + \beta c_n + \beta c_r - 2\alpha c_r + 2b\alpha y_{r_3}^* - b\beta y_{r_3}^*)^2 - \frac{1}{2} c y_{r_3}^{*2}$$

CHAPTER 5

RESULTS AND NUMERICAL ANALYSIS

5.1 COMPARISON OF OPTIMAL VALUES

In this section, the expressions for the optimal parameters are compared for the three cases which was presented in section 3.2. The results relative to price, demand and profit presented by the following proposition.

Proposition 1: The remanufacturer invests a product warranty in case 2, as compared to case 1, we have:

$$1) p_{n_2}^* \leq p_{n_1}^*, \quad p_{r_2}^* \geq p_{r_1}^*$$

$$2) q_{n_2}^* \leq q_{n_1}^*, \quad q_{r_2}^* \geq q_{r_1}^*$$

$$3) \Pi n_2^* \leq \Pi n_1^*$$

$$4) \Pi r_2^* \geq \Pi r_1^*$$

Proof: 1) and 2): The price of new products in case 2 is less than price of new products in case 1 as in case 2 the remanufacturer provides warranty.

As the remanufacturer provides warranty in case 2 the price of remanufactured products in case 2 is greater than price of remanufactured products in case 1.

The demand of the new products in case 2 is less than demand of new products in case 1 because in case 2 warranty is provided to the remanufactured products.

The demand for the remanufactured products is higher in case 2 as the remanufactured products have warranty.

3): If $\Pi n_2^* - \Pi n_1^* \leq 0$

$$\begin{aligned} \Pi n_2^* \leq \Pi n_1^* &= \left[\frac{(2\alpha^2 - 2\alpha\beta - 2\alpha c_n + \beta c_n + \alpha c_r - b\alpha y_{r_2}^*)^2}{(\alpha - \beta)(4\alpha - \beta)^2} \right] - \left[\frac{(2\alpha^2 - 2\alpha\beta - 2\alpha c_n + \beta c_n + \alpha c_r)^2}{(\alpha - \beta)(4\alpha - \beta)^2} \right] \\ &= \frac{(-b\alpha y_{r_2}^*)^2}{(\alpha - \beta)(4\alpha - \beta)^2} \leq 0 \end{aligned}$$

4): If $\Pi_{r_2}^* - \Pi_{r_1}^* \geq 0$

$$\begin{aligned}\Pi_{r_2}^* \geq \Pi_{r_1}^* &= \frac{\alpha(\alpha\beta - \beta^2 + \beta c_n - 2\alpha c_r + 2b\alpha y_{r_2}^{*2})^2}{\beta(\alpha - \beta)(4\alpha - \beta)^2} - \frac{1}{2}cy_{r_2}^* - \frac{\alpha(\alpha\beta - \beta^2 + \beta c_n + \beta c_r - 2\alpha c_r + \beta c_r)^2}{\beta(\alpha - \beta)(4\alpha - \beta)^2} \\ &= \frac{y_{r_2}^{*2}[(2b\alpha - b\beta)^2 - \frac{1}{2}c]}{\beta(\alpha - \beta)(4\alpha - \beta)^2} \geq 0\end{aligned}$$

This proposition indicates that the warranty for remanufactured products results in an increase in the sales price and sales quantities of the remanufactured products, which it leads to a decrease in the sales price and sales quantities of the new products.

Obviously, warranty policy can promote the development of the remanufacturing industry. This finding shed significant insights for supply chain managers who are under increasing competitive pressure. It might well be the case of finding a right trade-off between warranty service and economic performances.

Proposition 2: The optimal warranty level decreases as the discount factor for remanufactured products increasing in case 2.

Proof: If, $\frac{\partial y_{r_2}}{\partial \beta} \leq 0$

$$\begin{aligned}y_{r_2} &= \frac{2b\alpha(2\alpha - \beta)(\alpha\beta - \beta^2 + \beta c_n - 2\alpha c_r + \beta c_r)}{c\beta(\alpha - \beta)(4\alpha - \beta)^2 - 2b^2\alpha(2\alpha - \beta)^2} \\ \frac{\partial y_{r_2}}{\partial \beta} &= \frac{1}{(c\beta(\alpha - \beta)(4\alpha - \beta)^2 - 2b^2\alpha(2\alpha - \beta)^2)^2} \times (-2b\alpha)c\beta(\alpha - \beta)(4\alpha - \beta)^2 - \\ &\quad 2b^2\alpha(2\alpha - \beta)^2 - (c\alpha - 2\beta c + 2\beta - 8\alpha + 8\alpha^2 b - \\ &\quad 4b^2\alpha\beta)2b^2\alpha(2\alpha - \beta)^2 - (2b\alpha(2\alpha - \beta)\alpha\beta - \beta^2 + \beta c_n - \\ &\quad 2\alpha c_r + \beta c_r) \leq 0\end{aligned}$$

The more trust on the product, the less to be guarantee. The product warranty is a legal contract that it needs the manufacturer to be responsible for the case of the product failure.

Proposition 3: When both manufacturers provide warranty, we have

$$\begin{aligned}\text{(i)} \quad &\frac{\partial p_{n_2}^*}{\partial y_{r_2}} < 0, \frac{\partial p_{r_2}^*}{\partial y_{r_2}} > 0, \frac{\partial p_{n_3}^*}{\partial y_{n_3}} > 0 \\ \text{(ii)} \quad &\frac{\partial p_{n_3}^*}{\partial y_{r_3}} < 0, \frac{\partial p_{r_3}^*}{\partial y_{n_3}} < 0, \frac{\partial p_{r_3}^*}{\partial y_{r_3}} > 0\end{aligned}$$

Proof: If, $\frac{\partial p_{n_2}^*}{\partial y_{r_2}} < 0$

$$p_{n_2}^* = \frac{2\alpha^2 - 2\alpha\beta + 2\alpha c_n + \alpha c_r - b\alpha y_{r_2}}{4\alpha - \beta}$$

$$= \frac{\partial p_{n_2}^*}{\partial y_{r_2}} - \frac{b\alpha}{4\alpha - \beta} < 0$$

If, $\frac{\partial p_{r_2}}{\partial y_{r_2}} > 0$

$$p_{r_2} = \frac{\alpha\beta - \beta^2 + \beta c_n + 2\alpha c_r + 2b\alpha y_{r_2} - b\beta y_{r_2}}{4\alpha - \beta}$$

$$\frac{\partial p_{r_2}}{\partial y_{r_2}} = \frac{2b\alpha - b\beta}{4\alpha - \beta} > 0$$

If, $\frac{\partial p_{n_3}^*}{\partial y_{n_3}} > 0$

$$p_{n_3} = \frac{(2\alpha^2 - \det \beta + 2\alpha c_n + \alpha c_r - b\alpha y_{r_3} + e(2\alpha - \beta)y_{n_3})}{(4\alpha - \beta)}$$

$$\frac{\partial p_{n_3}^*}{\partial y_{n_3}} = \frac{2e\alpha - \beta e}{4\alpha - \beta} > 0$$

Similarly, (ii) can be verified in a same manner as (i).

Proposition 3 implies that the prices in the final market increase in its own service level while decrease in its competitive service level. The more invest in the product warranty, the higher WTP for the correspondingly product.

5.2 NUMERICAL ANALYSIS

This section illustrates the results through a number of numerical examples to examine the effect of the parameter β on the optimal price and profit. Let the parameter values be $\alpha = 1$, $c_n = 0.6$, $c_r = 0.2$, $b=0.3$, $e=0.1$, $c=1$, $\beta \in [0.4, 0.7]$.

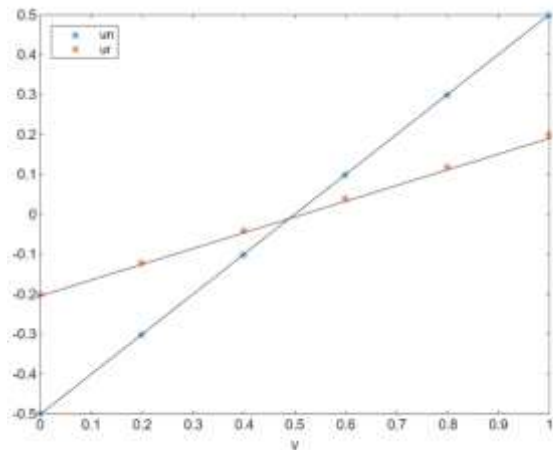


Fig 5.1. The effect of customer types on the customer surplus

Fig 5.1, shows that different customer has different value for new products and remanufactured products. High-end customers utility is higher for new products, and of course, they will choose to buy new products. On the contrary, only low-end customers would buy remanufactured products because the remanufactured products are viewed as inferior in quality to the new products.

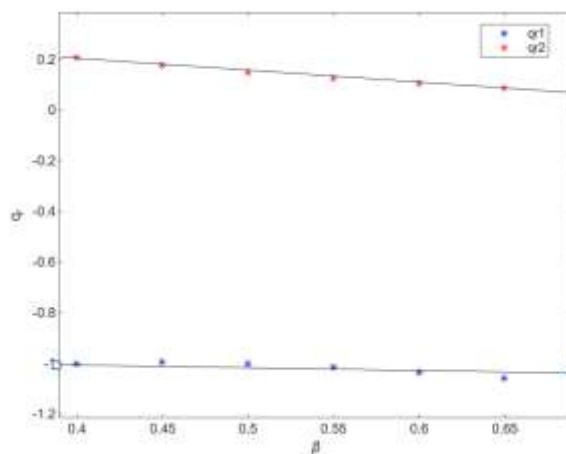


Fig 5.2. The effect of valuation discount factor on the demand of remanufactured products

Fig 5.2, the demand of remanufactured products is highest in case 2. That is, warranty can stimulate the demand, whereas Original Equipment Manufacturer does not, it is best for the sales of remanufactured products in this case.

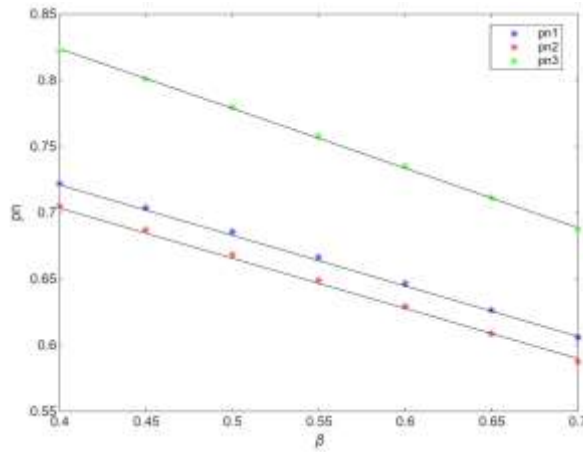


Fig 5.3. The effect of valuation discount factor on the price of new products

Fig 5.3, indicates that the price of new product decreases as the discount factor for remanufactured products increasing. That is to say the price of new product adversely affected by the discount factor β .

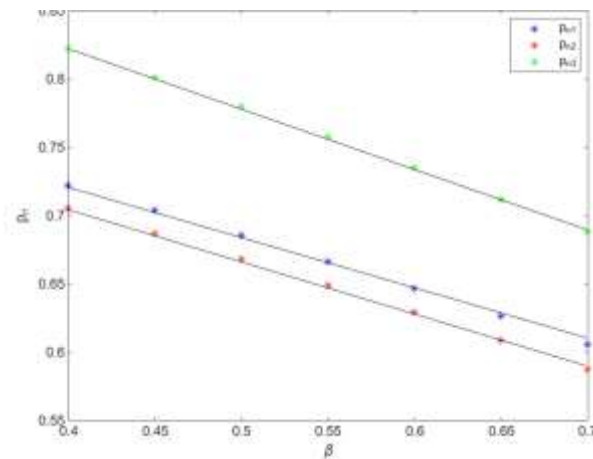


Fig 5.4. The effect of valuation discount factor on the price of remanufactured products

From fig. 5.4, it is inferred that the discount factor has a positive impact on the price of the remanufactured products. Meanwhile, the curves mean that customers are willing to pay more for the warranty service. Clearly, it is always better for the remanufacturer to offer warranty policy. The existence of the warranty service increases the, the insurance price of the product. This result confirms that when the customers are utility maximisers provided to the customers against a product failure allows the charge of the premium.

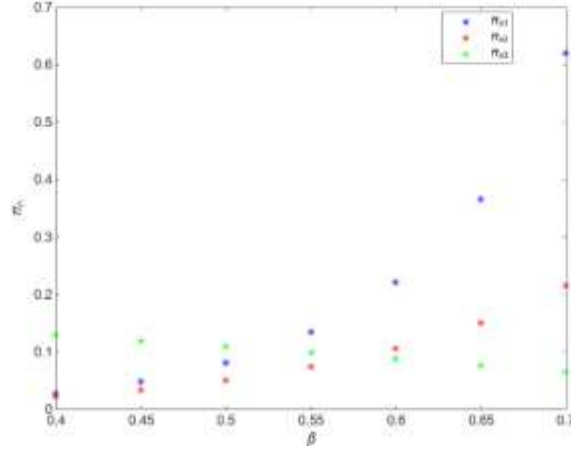


Fig 5.5. The effect of valuation discount factor on the OEM's profit

Fig 5.5, shows that the Original Equipment Manufacturer prefers to adopt case 1. As long as the remanufactured products provide warranty, the Original Equipment Manufacturer's profit decreases. At the same time, it is observed that the Original Equipment Manufacturer's warranty policy can eliminate losses in case 3.

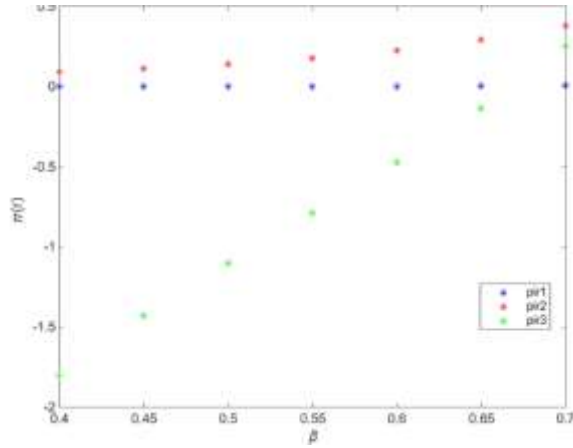


Fig 5.6. The effect of valuation discount factor on the price of remanufacturer's profit

Fig 5.6, that the warranty policy as part of the business is always beneficial for the remanufacturer comparatively to the case when it is not. The warranty policy increases the efficiency of the remanufacturer creating the higher demand, and consequently, higher profit.

Therefore, the remanufacturer is interested in providing the warranty policy to push customers to buy the products to increase profit. For all admissible parameter values, the remanufacturer is always economically better off when providing the warranty service. In a word, the warranty policy is an effective way to improve profits.

CHAPTER 6

CONCLUSIONS

6.1 CONCLUSIONS

In this report the manufacturers' warranty policy in a supply chain which comprises two competing manufacturers is examined. The manufacturers set retail price and their warranty policy: no warranty service for the two types of products (Case 1), remanufacturer provides warranty, whereas Original Equipment Manufacturer does not (Case 2), both the two types of products provide warranty at the same time (Case 3). The optimal retail price and the optimal profit associated with different cases are derived. The roles of warranty in a manufacturing industry have been discussed in this report. The results are summarized as follows:

- (1) The stronger warranties are significantly associated with higher Willingness to pay for the correspondingly products. On the contrary, the consumers have lower Willingness to pay for the competitive products.
- (2) The warranty policy benefits the correspondingly manufacturer's profit and injures the competitive manufacture's profit.
- (3) The discount factor β has a positive impact on the price of the remanufactured products and the remanufacturer's profit. It adversely affects the price of new products and the Original Equipment Manufacturer's profit.
- (4) Original Equipment Manufacturer has the highest profit in case 1, so case 1 is optimal for Original Equipment Manufacturer. While remanufacturer prefer case 2.

The future extension of this study is to analyse an extended warranty policy for remanufactured products in an uncertain environment concerning demand and quality of the remanufactured products.

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