

# **OPTIMIZATION OF LINEAR PROGRAMMING PROBLEM BY FOURIER MOTZKIN ELIMINATION ALGORITHM USING MATLAB**

Submitted in partial fulfillment of the requirements for the award of  
**Master of Science degree in Mathematics**

By

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**DEPARTMENT OF MATHEMATICS  
SCHOOL OF SCIENCE AND HUMANITIES**

## **SATHYABAMA**

**INSTITUTE OF SCIENCE AND TECHNOLOGY  
(DEEMED TO BE UNIVERSITY)**

**Accredited with Grade “A++” by NAAC**

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**APRIL - 2024**



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## **DEPARTMENT OF MATHEMATICS**

### **BONAFIDE CERTIFICATE**

This is to certify that this Project Report is the bonafide work of **RUBY SHALINI T (42630006)** who carried out the project entitled **"Optimization of Linear Programming Problem by Fourier Motzkin Elimination Algorithm using MATLAB"** under our supervision from **November 2023 to April 2024**.

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**Submitted for Viva voce Examination held on April 20-2024**

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## DECLARATION

I **RUBY SHALINI T (42630006)** hereby declare that the Project Report entitled **“Optimization of Linear Programming Problem by Fourier Motzkin Elimination Algorithm using MATLAB ”** done by me under the guidance of **Dr.H.GIRIJA BAI** at **Sathyabama Institute of Science and Technology** is submitted in partial fulfillment of the requirements for the award of Master of Science degree in Mathematics.

**DATE : 20-04-2024**

**PLACE : CHENNAI**

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**SIGNATURE OF THE CANDIDATE**

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## **ABSTRACT**

This study uses the Fourier Motzkin elimination approach to tackle linear programming problems. We usually utilize simplex, dual, and graphical methods to handle linear programming problems, but we employ the Fourier Motzkin Elimination Algorithm to solve linear programming problems. The advantage of this approach is that it is quite easy to achieve convergence when compared to other ways. The challenge is solved by describing the real life optimization problem as a linear programming problem and utilizing the Fourier Motzkin Elimination Algorithm to solve the Linear Programming Problem and Formulating MATLAB code to reduce the workload and compute the optimal solution.

### **KEYWORDS :**

- Linear Programming Problem
- Fourier Motzkin Elimination
- Optimization
- Optimal Solution
- MATLAB

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# **CHAPTER - 1**

## **INTRODUCTION**

### **1.1 OPERATION RESEARCH**

- Operation Research is an analytical method for problem-solving and decision-making which is very useful in the management of an organization.
- Operations Research is the application of analytical methods designed to help the decision makers.

#### **1.1.1 METHODS OF OPERATIONS RESEARCH**

##### ***OPTIMIZATION***

The process of Decision-making involves a lot of data collection and analysis of possible outcomes.

##### ***SIMULATION***

Simulation is a method of solving decision making problem by designing, constructing and manipulating a model of the real system.

##### ***PROBABILITY AND STATISTICS***

Mathematical algorithms are the best way to evaluate potential risks and predict the possible outcomes. This method is very useful in public health.

#### **1.1.2 PROCESS OF OPERATION RESEARCH**

The process of operation research can be broadly broken down into the following steps:

- Identify a problem that needs to be solved.
- Constructing a model around the problem that resembles the real word and variables.
- Using the model to derive solution to the problem.
- Testing each solution on the model and analyzing its success.
- Implementing the solution to the actual solution.

## **1.2 LINEAR PROGRAMMING**

- This is a constrained optimization technique, which optimize some criterion within some constraints.
- Linear programming the objective function and constraints are linear.

### **1.2.1 LINEAR PROGRAMMING PROBLEM FORMULATION**

The linear programming problem formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products in this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints.

maximize or minimize  $Z = C_{11}X_{11} + C_{12}X_{12} + \dots + C_{nn}X_{nn}$

### **1.2.2 CONSTRAINED OPTIMIZATION MODELS**

#### ***DECISION VARIABLES***

Decision variables are physical quantities controlled by the decision maker and represented by mathematical symbols

#### ***OBJECTIVE FUNCTION***

It is a mathematical function of the decision variables that converts a solution into numerical evaluation of that solution.

#### ***THE CONSTRAINTS***

There are always certain constraints on the use of resources, that limit the degree to which an objective can be achieved. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. The solution of an LP model must satisfy these constraints

## **CHAPTER - 2**

### **LITERATURE SURVEY**

IN 2017, K.Arumugam, R.Abdul Saleem and K.Sugantha began to discuss the fourier motzkin elimination algorithm of hexagonal fuzzy numbers for solving fuzzy linear programming problems [1]. S.I. Bastrakov, A.V. Churkin and N.Yu.Zolotykh discussed about accelerating fourier-motzkin elimination using Bit Pattern tress in 2020 [2]. Beyza Ahlatcioglu ozkok discussed in finding the fuzzy optimal solution to fully fuzzy linear programming problems with trapezoidal fuzzy numbers [3]. H.P.Williams discussed about fourier motzkin elimination extension to integer programming problems in 1976 [4]. Rui Juan Jing, Marc Moreno-Maza and Delaram Talaashrafi proceeded with a discussion on complexity estimates for fourier motzkin elimination in 2020 [5]. Ivan Simecek, Richard Fritish and Robert Lorencz discussed parallel solvers of larger systems of linear inequalities using fourier motzkin elimination [6].

## **CHAPTER – 3**

### **AIM AND SCOPE**

In this we bring forth the optimization of a linear programming problem by “Fourier Motzkin Elimination Algorithm”. The main objective of my project is solve a linear programming problem and formulate a MATLAB code for it. Using this method to find the optimal solution of linear programming problem.

## CHAPTER – 4

### ALGORITHM

#### 4.1 MATHEMATICAL FORMULATION OF LINEAR PROGRAMMING

The mathematical expression of a general linear programming problem is as follows:

$$\text{Maximize or Minimize } Z = \sum_{j=1}^n c_j x_j$$

$$\text{Subject to } \sum_{j=1}^n a_{ij} x_j \{ \leq, =, \geq \} b_i; i=1, 2, \dots, m$$

$$x_j \geq 0; b_i \geq 0$$

where one and only one of the signs ( $\leq, =, \geq$ ) holds for each constraint and the sign may vary from one constraint to another. Here  $c_j$  ( $j=1, 2, \dots, n$ ) are called profit or cost coefficients,  $x_j$  ( $j=1, 2, \dots, n$ ) are called decision variables. The set of feasible solution to the linear programming problem is

$$S = \{ (x_1, x_2, \dots, x_n)^T : (x_1, x_2, \dots, x_n)^T \in R^n \}$$

The set  $S$  is called the constraints set, feasible set or feasible region of linear programming.

In matrix vector notation the above problem can be expressed as :

$$\text{Maximize or Minimize } Z = cx$$

$$\text{Subject to } Ax (\leq, =, \geq) b$$

$$x \geq 0; b \geq 0$$

where  $A$  is an  $m \times n$  matrix,  $x$  is an  $n \times 1$  column vector,  $b$  is an  $m \times 1$  column vector and  $c$  is a  $1 \times n$  row vector,  $b \in R^m$ ;  $x, c \in R^n$ .

#### 4.2 MATHEMATICAL FORMULATION OF FOURIER MOTZKIN ELIMINATION

We briefly review the Fourier-Motzkin elimination method. Consider again a linear system  $Ax \leq b$  where  $A \in R^{m,n}$ ,  $b \in R^m$  and let  $I := \{1, 2, \dots, m\}$ . We write the system in component form.

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n &\leq b_1 \\
a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n &\leq b_2 \\
&\vdots \\
&\vdots \\
&\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n &\leq b_m
\end{aligned}$$

Say that we want to eliminate  $x_1$  from the system (1). For each  $i$  where  $a_{i1} \neq 0$  we multiply the  $i$ 'th inequality  $\sum_{j=1}^n a_{ij} x_j \leq b_i$  by  $1/|a_{i1}|$ . This gives an equivalent system

$$\begin{aligned}
x_1 + a'_{i2}x_2 + \dots + a'_{in}x_n &\leq b'_i \quad (i \in I^+) \\
a_{i2}x_2 + \dots + a_{in}x_n &\leq b_i \quad (i \in I^0) \\
-x_1 + a'_{i2}x_2 + \dots + a'_{in}x_n &\leq b'_i \quad (i \in I^-)
\end{aligned}$$

where  $I^+ = \{i : a_{i1} > 0\}$ ,  $I^0 = \{i : a_{i1} = 0\}$ ,  $I^- = \{i : a_{i1} < 0\}$ ,  $a'_{ij} = a_{ij}/|a_{i1}|$  and  $b'_i = b_i/|a_{i1}|$ . Thus, the row index set  $I = \{1, 2, \dots, m\}$  is partitioned into subsets  $I^+$ ,  $I^0$  and  $I^-$ , some of which may be empty. It follows that  $x_1, x_2, \dots, x_n$  is a solution of the original system (1) if and only if  $x_2, x_3, \dots, x_n$  satisfy

$$\sum_{j=2}^n a'_{kj} x_j - b'_k \leq b'_i - \sum_{j=2}^n a'_{ij} x_j \quad (i \in I^+, k \in I^-)$$

$$\sum_{j=2}^n a_{ij} x_j \leq b_i \quad (i \in I^0)$$

and  $x_1$  satisfies

$$\max_{k \in I^-} (\sum_{j=2}^n a'_{kj} x_j - b'_k) \leq x_1 \leq \min_{i \in I^+} (b'_i - \sum_{j=2}^n a'_{ij} x_j)$$

If  $I^-$  (resp.  $I^+$ ) is empty, the first set of constraints in (3) vanishes and the maximum (resp. minimum) in (4) is interpreted as  $\infty$  (resp.  $-\infty$ ). If  $I^0$  is empty and either  $I^-$  or  $I^+$  is empty too, then we terminate: the general solution of  $Ax \leq b$  is obtained by choosing  $x_2, x_3, \dots, x_n$  arbitrarily and choosing  $x_1$  according to (4). The constraint in (4) says that  $x_1$  lies in a certain interval which is determined by  $x_2, x_3, \dots, x_n$ . The

polyhedron defined by (3) is the projection of  $P$  along the  $x_1$ -axis, i.e., into the space of the variables  $x_2, x_3, \dots, x_n$ . then proceed and similarly eliminate  $x_2, x_3$  etc. Eventually one obtains a system  $l \leq x_n \leq u$ .

If  $l > u$  one concludes that  $Ax \leq b$  has no solution, otherwise one may choose  $x_n \in [l, u]$ , and then choose  $x_{n-1}$  in an interval which depends on  $x_n$  etc. This back Substitution produces a solution  $x=(x_1, x_2, \dots, x_n)$  to  $Ax \leq b$ . Moreover, every solution of  $Ax \leq b$  may be produced in this way. (If the system is inconsistent, this might possibly be discovered at an early stage and one terminates).

The number of constraints may grow exponentially fast as variables are eliminated using fourier motzkin elimination. Actually, a main problem in practice is that the number of inequalities becomes “too large” during the elimination process, even when redundant inequalities are removed. It is therefore of interest to know situation where the project linear system is not very large or, at least ,have some interesting,structure. These questions are discussed in the remaining part of the page. We refer to [7] for a further discussion of fourier motzkin elimination algorithm and a collection of references on this method.

#### 4.3 METHODOLOGY:

##### **FOURIER MOTZKIN ELIMINATION ALGORITHM**

**STEP 1:** Formulate the linear programming problem from the given problem.

**STEP 2:** Change the objective function as inequality and join it with the constraints. Now, we get the linear system of the equation. For maximization use  $\leq$  and for minimization use  $\geq$  inequality.

**STEP 3:** Change all the inequalities of the fuzzy system into  $\geq$  for minimization problem and  $\leq$  for maximization problem.

**STEP 4:** Eliminate one by one in  $x_1, x_2, x_3, x_4, \dots, x_n$

- Divide each equation by its modulus value of  $x_1$  coefficient for all the equations.
- Now, we have three classes of  $x_1$  coefficient i.e, “-1”, or “+1” or “0” in the system of linear equation.
- Adding or subtracting any two classes of equations to eliminate  $x_1$ .

**STEP 5:** Repeat step 4 until all the 'n' variables are eliminated.

**STEP 6:** After eliminating all the 'n' variables, we get the z values and substitute the z in above, we get the values of variables in back substitution.

**STEP 7:** If we cannot able to eliminate all 'n' variables. Then the solution is unbounded and has infinitely many solution. There are some problems with no solution.

#### **4.4 NUMERICAL EXAMPLE:**

##### **EXAMPLE 1 :**

*FORMATION :*

Maximize  $z = x_1 + x_2$

Subject to

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

**STEP 1:** Change objective as  $\leq$

$$z \leq x_1 + x_2$$

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1 \geq 0$$

$$x_2 \geq 0$$

**STEP 2:** In objective & non-negative constraints bring to the form of linear inequality ( $\leq$ )

$$-x_1 - x_2 + z \leq 0$$

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0$$

**STEP 3 :** Divide the coefficient of  $x_1$

$$-x_1 - x_2 + z \leq 0$$

$$x_1 + \frac{2}{3}x_2 \leq \frac{5}{3}$$

$$x_2 \leq 2$$

$$-x_1 \leq 0$$

$$-x_2 \leq 0$$

**STEP 4:** Now, we have first class, second class , third class equations.

First class : Coefficient of  $x_1$  is -1

Second class : Coefficient of  $x_1$  is +1

Third class : Coefficient of  $x_1$  is 0

**STEP 5:** Adding first class and second class equations to eliminate  $x_1$

$$-\frac{1}{3}x_2 + z \leq \frac{5}{3}$$

$$\frac{2}{3}x_2 \leq \frac{5}{3}$$

**STEP 6:** Divide the coefficient of  $x_2$

$$-x_2 + 3z \leq \frac{15}{3}$$

$$x_2 \leq \frac{5}{2}$$

**STEP 7:** Adding first class and second class equation to eliminate  $x_2$

$$3z \leq \frac{45}{6}$$

$$z = \frac{45}{18}$$

$$z = 2.5$$

**STEP 8:** Substitute z in above equation

$$-x_2 + 3z \leq \frac{15}{3}$$

$$-x_2 + 3(2.5) = \frac{15}{3}$$

$$-x_2 + 7.5 = \frac{15}{3}$$

$$x_2 = 2.5$$

**STEP 9:** Substitute  $x_2 = 2.5$  in above equation

$$x_1 + \frac{2}{3}x_2 \leq \frac{5}{3}$$

$$x_1 + \frac{2}{3}(2.5) = \frac{5}{3}$$

$$x_1 + \frac{5}{3} = \frac{5}{3}$$

$$x_1 = 0$$

The Optimal Solution  $x_1 = 0$  and  $x_2 = 2.5$

Maximize  $z = x_1 + x_2$

Maximize  $z = 2.5$

## CHAPTER – 5

### RESULT AND DISCUSSION

#### 5.1 RESULT

##### **EXAMPLE 1 :**

Maximize  $z = x_1 + x_2$

Subject to

$$3x_1 + 2x_2 \leq 5$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

##### **5.1.1 MATLAB CODE :**

```
A=[-1 -1 1;3 2 0;0 1 0;-1 0 0;0 -1 0]
```

```
b=[0;5;2;0;0]
```

```
nvar=3
```

```
[m,d] = size(A)
```

```
R = [A,b]
```

```
R = sortrows(R,1)
```

```
nneg = sum(R(:,1)<0)
```

```
nzer = sum(R(:,1)==0)
```

```
npos = sum(R(:,1)>0)
```

```
J = R(nneg+nzer+1:m,:)
```

```
K = r(1:nneg,:)
```

```
L = r(nneg+1:nneg+nzer,:)
```

```
R=[J; K; L]
```

```
abs(R(1:npos+nneg,1))
```

```
ones(1,d+1)
```

```
abs(R(1:npos+nneg,1))*ones(1,d+1)
```

```

R(1:npos+nneg,:) = R(1:npos+nneg,:)/abs(R(1:npos+nneg,1)*ones(1,d+1))
R = sortrows(R,1)
nneg = sum(R(:,1)<0)
nzer = sum(R(:,1)==0)
npos = sum(R(:,1)>0)
J= R(nneg+nzer+1:m,:)
K = R(1:nneg,:)
L= R(nneg+1:nneg+nzer,:)
a= J+K
[x,y]=size(a)
a = sortrows(a,2)
anneg = sum(a(:,2)<0)
anzer = sum(a(:,2)==0)
anpos = sum(a(:,2)>0)
J= a(anneg+anzer+1:x,:)
O = a(1:anneg,:)
abs(a(1:anpos+anneg,2))
ones(1,d+1)
l=abs(a(1:anpos+anneg,2))*ones(1,d+1)
P= a(1:anpos+anneg,:)
a = P./l
W= sum(a,1)
[e,d]=size(W)
U=W(:, 1:(d-1))
V=W(:,d)
syms x1 x2 Z
X= [x1 x2 Z]

```

```

E1=U.*X
E1=sum(E1)-V
Z=solve(E1,Z)
Z=double(Z)
q= a(1,:)
Q1=q(:, 1:(d-1))
Q3=q(:,d)
syms x1 x2
X= [x1 x2 Z]
Q=Q1.*X
Q=sum(Q)-Q3
E=solve(Q,x2)
x2=double(E)
f= R(1,:)
S1=f(:,1:(d-1))
S2=f(:,d)
X= [x1 x2 Z]
X1=S1.*X
H=sum(X1)-S2
X1=solve(H,x1)
x1=double(X1)
% Generate a grid of points for plotting
[x1, x2, z] = meshgrid(linspace(0,3,4),linspace(0,3,4),linspace(0,3,4));
% Flatten the matrices for linprog
x1 = x1(:);
x2= x2(:);
z = z(:);

```

```

% Plot the feasible region in 3D

scatter3(x1,x2,z,'cyan');

% Plot the optimal solution

scatter3(x1, x2,z, 'ro', 'LineWidth',2);

% Add labels and title

xlabel('X-axis');

ylabel('Y-axis');

zlabel('Z-axis');

title('Feasible Region and Optimal Solution for LPP');

% Display the grid

grid on;

hold off;

```

### ***OUTPUT :***

A =

```

-1 -1 1
3 2 0
0 1 0
-1 0 0
0 -1 0

```

b =

```

0
5
2
0
0

```

nvar =

3

R =

-1 -1 1 0

3 2 0 5

0 1 0 2

-1 0 0 0

0 -1 0 0

Z =

2.5000

x2 =

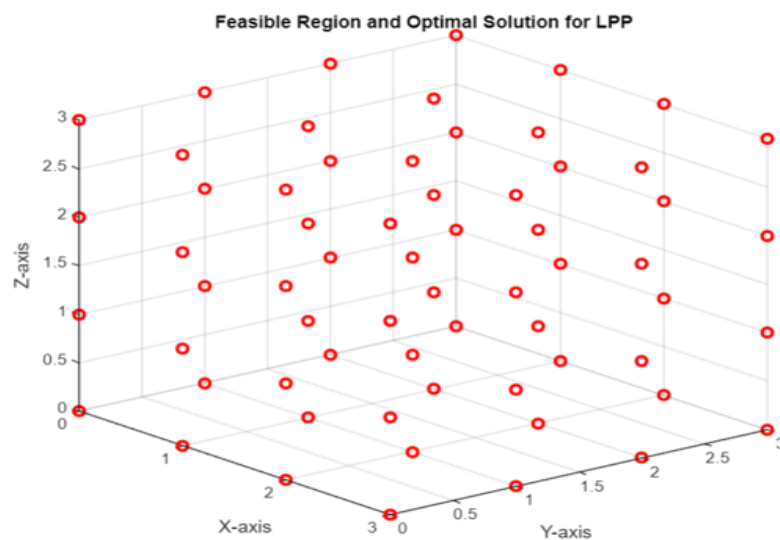
2.5000

x1 =

0

The optimal solution  $x_1 = 0$ ,  $x_2 = 2.5$  and Max  $z = 2.5$

### 5.1.2 GRAPH



**FIG 5.1.2**

## 5.2 EXAMPLES :

### EXAMPLE 2 :

The N.Dustrious company produces two products A and B . The raw material requirements are space needed for storage, production rates and selling prices for these products.

TABLE 5.2.1

| Requirements    | A  | B  |
|-----------------|----|----|
| Storage space   | 4  | 5  |
| Raw material    | 5  | 3  |
| Production rate | 60 | 30 |
| Selling price   | 13 | 11 |

The total amount of raw material available per day for both products is 1575. The total storage space for all products is 1500 feet and a maximum of 7 hours per day can be used for production. The company wants to determine how many units of each product to produce per day to maximize its total income.

$$\text{Max } z = 13x_1 + 11x_2$$

Subject to

$$4x_1 + 5x_2 \leq 1500$$

$$5x_1 + 3x_2 \leq 1575$$

$$x_1 + 2x_2 \leq 420$$

$$x_1 \geq 0, x_2 \geq 0$$

Solved by Fourier Motzkin Elimination method using MATLAB coding, we obtained the optimal solution as

$$x_1 = 270 \text{ and } x_2 = 75$$

Max  $z = 4335$ .

**EXAMPLE 3 :**

A Pharmaceutical company is developing a new cancer treatment that involves a combination of three drugs. Drug A ,Drug B and Drug C. The company needs to determine the optimal dosage of each drug to maximize treatment effectiveness.

**TABLE 5.2.2**

| Effects  | DRUG A | DRUG B | DRUG C | Minimum side effect |
|----------|--------|--------|--------|---------------------|
| Efficacy | 2      | 3      | 2      | 120                 |
| Toxicity | 1      | 2      | 3      | 150                 |
| Dosage   | 3      | 2      | 1      | 100                 |

Treatment effectiveness of Drug A is 5 ml , Drug B is 4 ml and Drug C is 6 ml. The optimal dosage to each drug to maximize treatment effectiveness.

$$\text{Max } z = 5x_1 + 4x_2 + 6x_3$$

subject to

$$2x_1 + 3x_2 + 2x_3 \geq 120$$

$$x_1 + 2x_2 + 3x_3 \geq 150$$

$$3x_1 + 2x_2 + x_3 \geq 100$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

Solved by Fourier Motzkin Elimination method using MATLAB coding, we obtained the optimal solution as

$$x_1 = 15 \text{ and } x_2 = 0 \text{ and } x_3 = 45$$

Max  $z = 345$ .

**EXAMPLE 4 :**

A City government aims to reduce carbon emissions from transportation by promoting the use of electric vehicles and public transportation. The city needs to determine the optimal allocation of resources between investing in electric vehicle infrastructure and improving public transportation services to minimize carbon emissions while considering budget constraints.

**TABLE 5.2.3**

| Carbon emissions | Electric vehicle | Public transportation | Available |
|------------------|------------------|-----------------------|-----------|
| Budget           | 2                | 3                     | 80        |
| Reduction target | 1                | 2                     | 100       |

The impact of electric vehicle infrastructure is 5 and public transportation improvement is 4.

$$\text{Min } z = 5x_1 + 4x_2$$

Subject to

$$2x_1 + 3x_2 \leq 80$$

$$x_1 + 2x_2 \leq 100$$

$$x_1 \geq 0, x_2 \geq 0$$

Solved by Fourier Motzkin Elimination method using MATLAB coding, we obtained the optimal solution as

$$x_1 = 40 \text{ and } x_2 = 0$$

Min  $z = 200$ .

**EXAMPLE 5 :**

Tamilnadu, like many other region in India, faces challenges related to water scarcity and efficient management of water resources. The government of Tamilnadu aims to optimize the allocation of water resources between agriculture and urban consumption to ensure sustainable water usage and minimize water wastage.

**TABLE 5.2.4**

| Water resources           | Sustainable water | Water wastage | Minimum water requirement |
|---------------------------|-------------------|---------------|---------------------------|
| Agricultural water demand | 2                 | 3             | 40                        |
| Urban water demand        | 3                 | 2             | 30                        |

Allocation of water to agriculture is 10 and Allocation of water to urban is 8. To find minimum water wastage.

$$\text{Min } z = 10x_1 + 8x_2$$

Subject to

$$2x_1 + 3x_2 \geq 40$$

$$3x_1 + 2x_2 \geq 30$$

$$x_1 \geq 0, x_2 \geq 0$$

Solved by Fourier Motzkin Elimination method using MATLAB coding, we obtained the optimal solution as

$$x_1 = 2 \text{ and } x_2 = 12$$

Min  $z = 116$ .

### **5.3 DISCUSSION**

In this project, we compared our method with all other methods and we got the accurate result and formulated a MATLAB code for the method Fourier Motzkin elimination.

#### **5.3.1 ADVANTAGES :**

- Its computational steps are so easy from other methods.
- We can solve all types of LPP.
- The final result converges quickly in this method.
- We get the accurate result that coincide with analytical solution.
- Using MATLAB , we get the optimal solution of LPP very quickly.

#### **5.3.2 DISADVANTAGES :**

- There are some exceptional problems that get unbounded solution and no solution.

#### **5.3.3 EXAMPLE :**

A Goldsmith produces two product such as 8 gm of earrings and 2 gm of ring.

The production of both made by two processes, process -1 include melting, wheeling and dying . Process - 2 include fitting , soldering and polishing.

**TABLE 5.3.3**

| PRODUCT  | PROCESS -1 | PROCESS -2 | AVAILABLE<br>RESOURCES |
|----------|------------|------------|------------------------|
| Earrings | 2          | 4          | 180                    |
| Rings    | 1          | 2          | 180                    |

Each earring gives a profit of Rs. 3600 and each ring gives a profit of Rs. 900 . Find the optimal solution.

The solution of this problem is unbounded and has infinitely many solution.

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## **CHAPTER – 6**

### **CONCLUSION**

In this project, we describe the optimization problem as a linear programming problem using Fourier Motzkin Elimination Algorithm. The model is to find the optimum solution of linear programming problem. The linear programming problem has been solved using the Fourier Motzkin Elimination Algorithm. In this project, We conclude that the Fourier Motzkin Elimination Algorithm is one of the method to solve the Linear Programming Problem. Using MATLAB coding we found the optimal solution. Then We conclude that, there are some exceptional Linear programming problem that cannot be solved by this method.

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