

**UNSTEADY MAGNETOHYDRODYNAMIC STOKES FLOW OF
VISCOUS FLUID BETWEEN TWO PARALLEL PLATES**

Submitted in partial fulfillment of the requirements for the award of

MASTER OF SCIENCE degree in **MATHEMATICS**

by

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**DEPARTMENT OF MATHEMATICS
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SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

Accredited with Grade “A⁺⁺” by NAAC

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of **DEEPIKA .M (42630001)** Who carried out the project entitled "**UNSTEADY MAGNETOHYDRODYNAMIC STOKES FLOW OF VISCOUS FLUIDS BETWEEN TWO PARALLEL PLATES**" under our supervision from **NOVEMBER 2023** to **APRIL 2024**.

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DECLARATION

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SIGNATURE OF THE CANDIDATE

ACKNOWLEDGEMENT

I am pleased to acknowledge my sincere thanks to the **Board of Management** of **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** for their kind encouragement in doing this project and for completing it successfully. I am grateful to them.

I convey my thanks to **Dr. REKHA CHAKARAVATHI**, Dean, School of Science and Humanities and **Dr. M. NIRMALA**, Head of the Department, Dept. of Mathematics for providing me necessary support and details at the right time during the progressive reviews.

I would like to express my sincere and deep sense of gratitude to my Project Guide **Dr. R. DELHI BABU** for his valuable guidance, suggestions and constant encouragement paved way for the successful completion of my Project work.

I wish to express my thanks to all Teaching and Non-teaching staff members of the Department of Mathematics who were helpful in many ways for the completion of the project.

ABSTRACT

The objective of the project is to analyze the Unsteady Magnetohydrodynamic stokes flow of viscous fluid between two parallel porous plates when the fluid is being withdrawn through both the walls of the channel at the same rate. The problem is reduced to a third order nonlinear differential equation which depends on a Suction Reynolds number R and a Hartmann number M for which an exact solution is obtained.

KEYWORDS:

Fluid flow

Parallel plates

Magnetohydrodynamic flow

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CHAPTER - 1

INTRODUCTION

FLUID DYNAMICS:

Fluid Mechanics is that branch of science which deals with the behaviour of fluids (liquids or gases) at rest as well as in motion. Thus, this branch of science deals with the static, kinematic and dynamic aspects of fluids. The study of fluids at rest is called fluid statics. The study of fluids in motion, where pressure force is not considered is called fluid kinematics and if the pressure forces are also considered for the fluids in motion, that branch of science is called Fluid Dynamics. Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, predicting weather patterns, the extraction of oils from underground reservoirs; influence of wind upon building structures, etc. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid.

1.1 IMPORTANCE OF FLUID FLOW

Fluids are divided into two categories. Those which undergo appreciable variations in density and volume under the impressed forces fall under the category of 'compressible fluids'. Then, there are those which undergo no noticeable changes in density and volume during motion. They are termed 'incompressible fluids'. Compressibility is thus a measure of the change in density and consequently, the change in volume of a fluid under the effect of external forces.

1.2 MAGNETOHYDRODYNAMIC FLOWS

MHD fluid flow in a parallel plate channel is an interesting area in the study of fluid mechanics because of its relevance to various engineering applications. The MHD flow in the planar channels leads to a startup process implying thereby a viscous layer at the boundary is suddenly set into motion and becomes important in the application of various branches of geophysics, astrophysics and fluid engineering. Magnetohydrodynamic flow has many applications in aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, accelerators, fluid droplets, MHD pumps, power generators and purification of crude oil.

1.3 FLOW THROUGH POROUS MEDIUM

A medium which is a solid body containing pores is called a porous medium. Flow through porous media is also of interest in chemical engineering (adsorption, filtration, and flow in packed columns), petroleum engineering, hydrology, soil physics, biophysics and geophysics. Two macroscopic properties of porous media which may be used to describe fluid flow are porosity (ϵ) and permeability (k_1). The porosity of a porous medium describes how densely the material is packed. The porosity of a porous medium is defined as the fraction of the total volume of the medium that is occupied by the void space. Thus $1-\epsilon$ is the fraction that is occupied by solid. Porosity is a fraction between 0 and 1, typically ranging from less than 0.01 for solid granite to more than 0.5 for peat and clay. For manmade materials such as metallic frames, ϵ can approach to the value 1. For natural media, ϵ does not normally exceed 0.6. The permeability is related to pore-size distribution since the distribution of the sizes of entrances, exits and lengths of the pore walls make up the major resistance to flow. The permeability is the single parameter that reflects the conductance of a given pore structure. The dimensions of the permeability are length squared.

1.4 NON-NEWTONIAN FLOWS

Fluids can be defined as substances that flow or deform under the application of shear stress and these include liquids and gases. They are part of engineering study in many tertiary institutions. Basically, in the study of science, fluids are divided into two broad groups. These divisions in this write-up are known as types of fluids which are Newtonian and non-Newtonian fluids. Newtonian fluids are those fluids that obey Newton Law of viscosity. The Newtonian viscosity is a physical property which is only a function of pressure and temperature. The flow properties of an incompressible Newtonian fluid can be characterized solely in terms of the density and viscosity.

CHAPTER - 2

LITERATURE SURVEY

Flow through porous media has many geophysical and engineering applications.

Berman SM [1] analyzed the Navier-Stokes equations have been solved to obtain a complete description of the fluid flow in a channel having a rectangular cross section and two equally porous walls. The scope has been limited to two-dimensional incompressible steady-state laminar flow. The solution of the flow equations leads to detailed expressions for the dependence of the velocity components and the pressure on position coordinates, channel dimensions, and fluid properties.

Hartmann and Lazarus [2] investigated the effect of a transversely uniform MHD on the flow of conductive fluid between two infinitely parallel and fixed insulating plates. The equation describing MHD flow is a synthesis of the continuous equation and the Navier-Stokes equation. Governing equations are differential equations that require analytical or numerical solutions.

Cobble, M. H [3] considered the magnetic field in the laminar flow of conductive liquids.

Hassaninen I. A. and Mansour M.A. [4] Analyzed the two-dimensional unsteady flow of a viscous, incompressible electrically- conducting fluid through a porous medium bounded by two infinite parallel plates under the action of a transverse magnetic field is presented. The lower plate is fixed while the other is oscillating in its own plane. Expressions for the transient velocity, the amplitude, the phase angle, and the skin-friction are derived. The effects of the magnetic parameters, permeability of the porous medium, and the frequency parameter are discussed.

Hamza E.A [5] A similarity solution has been obtained for a flow between two parallel plates (rectangular or circular) approaching or receding from each other with suction or injection at the porous plate. Approximate analytic solutions based on the regular perturbation technique are given and a numerical solution of the resulting nonlinear

differential equations is presented. The effects of injection and suction on the velocity profiles, pressure distribution and shearing stress on the wall are studied. In the case of the flow between circular plates the effects on load capacity due to mass transfer are investigated.

Soundalgekar V. and Uplekar [6] investigated an exact analysis of a generalized MHD Couette flow, on taking into account the Hall current, is presented. Expressions for the axial and transverse components of velocity, the skin friction, are derived. These are shown in graphs. The exact solution to the energy equation, assuming the linear variation of the temperature along the plate, is also derived and this is shown graphically. The effects of different parameters are discussed.

Sing & Ram [7] solved the differential equation by Laplace transform method, considering the laminar flow of conductive fluid laminar flow through a channel with a transverse magnetic field under a periodic pressure gradient.

Attia, H. A. and Kotb, N. A., [8] studied the steady flow of an electrically conducting, viscous, incompressible fluid bounded by two parallel infinite insulated horizontal plates and the heat transfer through it are studied. The upper plate is given a constant velocity while the lower plate is kept stationary. The viscosity of the fluid is assumed to vary with temperature. The effect of an external uniform magnetic field as well as the action of an inflow perpendicular to the plates together with the influence of the pressure gradient on the flow and temperature distributions are reported. A numerical solution for the governing non-linear ordinary differential equations is developed.

Chiam T.C. [9] investigated the boundary layer equations arising from the effects of a continuously moving flat plate under the action of a transverse applied magnetic field are converted into a system of ordinary differential equations subject to two-point boundary conditions by using the Local Non similarity Method. An iterative method incorporating the shooting method is presented for solving both the two-equation and three-equation models.

R. Delhi Babu, S. Ganesh & C. K. Kirubhashankar [10] studied the impact of unsteady magnetohydrodynamic flow of dust particles on the velocity of electrically directed

viscous fluid between parallel porous plates with an angular velocity at the sight of a transverse magnetic field. The differential equations managing the flow are explained by similarity transformation. The velocity of the fluid, velocity of the dust particles and pressure distribution are obtained. The outcomes are the velocity of the dust particle is higher than that velocity of the fluid particles for each of the parameters examined.

CHAPTER -3

RESEARCH METHODOLOGY

3.1 AIM AND SCOPE

The main objectives of the work reported are:

- To study the effect of individual factors like Magnetic field, Hartmann number and Reynolds number.
- To study the solution for the unsteady MHD fluid flow through the parallel plate channel with heat source and external transverse magnetic field is presented.
- The effects of velocity are studied for the parameter magnetic field parameter.

The scope of the work reported is:

- It not only helps the people working in the field of Physiological fluid dynamics but also to the medical practitioners.

3.2 DESCRIPTION OF RESEARCH WORK

This research work focuses on the MATLAB implementation of interpretation of analytical solutions obtained for various fluid flow problems.

Research Problem

The main objective of the research work is to analyze the flow of fluids through parallel porous plates in the presence/absence of magnetic effects.

Research Methodologies

Analytical solutions are obtained by transforming the partial differential equation concerning the flow into an ordinary linear differential equation with constant coefficients by using similarity transformation. The arbitrary constant is then calculated by using boundary condition of the respective problem and the results are interpreted by using MATLAB applications.

CHAPTER – 4

MATHEMATICAL FORMULATION

The unsteady laminar flow of an incompressible viscous fluid between two parallel perpendicular to the walls.

The origin is taken at the centre of the channel and let x and y be the coordinate axes parallel and perpendicular to the channel walls.

The length of the channel is assumed to be L and 2h is the distance between the two plates. Let u and v be the velocity components in the x and y directions respectively

The equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (4.1)$$

The equation of momentum is

$$\rho \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} \right) + \left(\frac{\partial^2 u}{\partial y^2} \right) - \sigma_e B_0^2 u \quad (4.2)$$

$$\rho \frac{\partial v}{\partial t} = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} \right) + \left(\frac{\partial^2 v}{\partial y^2} \right) \quad (4.3)$$

4.1 Notations:

σ_e - electrical conductivity

$B_0 = \mu_e H_0$

B_0 – *electromagnetic induction*

μ - Coefficient of viscosity

μ_e - magnetic permeability

Ψ – Stream function

η – Dimensionless variable

H_0 – Transverse magnetic field

4.2 Boundary Condition:

The boundary conditions are

$$u=0 \text{ on } y=h \text{ and } y=-h;$$

$$v=v_0 e^{i\omega t} \text{ on } y=h \text{ and } v=-v_0 e^{i\omega t} \text{ on } y=-h$$

where v_0 is the velocity of suction at the walls of the channel.

The dimensionless variable be $\eta = \frac{y}{h}$,

$$u=u(x,y) e^{i\omega t}, \quad v=v(x,y) e^{i\omega t}, \quad p=p(x,y) e^{i\omega t}, \text{ where } \omega \text{ is the frequency.}$$

The equation (4.1), (4.2) and (4.3) becomes

$$\frac{\partial u}{\partial x} + \frac{1}{h} \frac{\partial v}{\partial \eta} = 0 \quad (4.4)$$

$$\rho i \omega u = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 u}{\partial \eta^2} \right) - \sigma_e B_0^2 u \quad (4.5)$$

$$\rho i \omega v = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{1}{h^2} \frac{\partial^2 v}{\partial \eta^2} \right) \quad (4.6)$$

Let $\nu = \frac{\mu}{\rho}$ = Kinematic viscosity,

ρ - the density of the fluids

μ - the coefficient of the viscosity

p - the pressure

The boundary condition is converted into

$$u(x,1) = 0, \quad u(x,-1) = 0 \quad (4.7)$$

$$v(x,1) = v_0, \quad v(x,-1) = -v_0 \quad (4.8)$$

let ψ be the stream function such that

$$u = \frac{1}{h} \frac{\partial \psi}{\partial \eta} \quad (4.9)$$

$$v = -\frac{\partial \psi}{\partial x} \quad (4.10)$$

The equation of continuity can be satisfied by a stream function of the form

$$\psi(x, \eta) = (h U(0) - v_0 x) f(\eta) \quad (4.11)$$

Where $U(0)$ is the average entrance velocity at $x=0$. From equation (4.11), the velocity components (4.9) and (4.10) are given by

$$u = \frac{1}{h} (h U(0) - v_0 x) f'(\eta) \quad (4.12)$$

$$v = v_0 f(\eta) \quad (4.13)$$

Where the prime denotes the differentiation with respect to the dimensionless variable $\eta = \frac{y}{h}$. Since the fluid is being withdrawn at constant rate from both the walls, v_0 is independent of x .

Using (4.12) and (4.13) in (4.5) and (4.6), the equation of momentum reduces to

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \left(i\omega f'(\eta) - \frac{v}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} \right) \quad (4.14)$$

$$-\frac{1}{h\rho} \frac{\partial p}{\partial \eta} = i\omega v_0 f(\eta) - \frac{v \cdot v_0 f''(\eta)}{h^2} \quad (4.15)$$

Now differentiating (4.15) w.r.t. x' , the following equation is obtained.

$$\frac{\partial^2 p}{\partial x \partial \eta} = 0 \quad (4.16)$$

differentiating (4.14) w.r.t. ' η ', it follows that

$$-\frac{1}{\rho} \frac{\partial^2 p}{\partial x \partial \eta} = \left(U(0) - \frac{v_0 x}{h} \right) \cdot \frac{d}{d\eta} \left(i\omega f'(\eta) - \frac{v}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right) \quad (4.17)$$

From (4.16), equation (4.17) can be written as

$$\frac{d}{d\eta} \left(i\omega f'(\eta) - \frac{v}{h^2} f'''(\eta) + \frac{\sigma_e B_0^2}{\rho} f'(\eta) \right) \quad (4.18)$$

Let $R = \text{Suction Reynolds number} = \frac{h v_0}{\nu}$

$$M = \text{Hartmann number} = B_0 h \left(\frac{\sigma_e}{\nu \rho} \right)^{\frac{1}{2}}$$

Integrating (4.18) w.r.t.' η ' and substituting the above expressions, we get

$$f'''(\eta) - \frac{i\omega h^2}{v} f'(\eta) - \frac{\sigma_e B_0^2 h^2}{\rho v} f'(\eta) = K$$

$$f'''(\eta) - \alpha^2 h^2 f'(\eta) - a_1 R f'(\eta) = K \quad (4.19)$$

where $R = \frac{h v_0}{v}$ and $a_1 = \sigma_e h B_0^2$ and K is an arbitrary constant.

$$f(1) = 1, f(-1) = -1, f'(1) = 0 \text{ and } f'(-1) = 0 \quad (4.20)$$

4.3 GENERAL SOLUTION OF EQUATION OF MOTION AND CONTINUITY

Hence the solution of the equations of motion and continuity is given by a nonlinear third order differential equation (4.19) subject to the boundary conditions (4.20)

$\therefore$ Equation (4.19) can be rewritten as

$$f'''(\eta) - (\alpha^2 h^2 + M^2) f'(\eta) = K \text{ where } a_1 R = M^2 \text{ and } \alpha^2 = \frac{i\rho\omega}{\mu}$$

$$\text{i.e. } (D^3 - (\alpha^2 h^2 + M^2)D) f(\eta) = K \text{ where } D = \frac{d}{d\eta} \text{ and } D^2 = \frac{d^2}{d\eta^2}$$

The roots of the A.E for the above equation are $m=0$,

$$m = \pm \sqrt{\alpha^2 h^2 + M^2}$$

$$\therefore f(\eta) = A + B e^{\sqrt{\alpha^2 h^2 + M^2} \eta} + C e^{-\sqrt{\alpha^2 h^2 + M^2} \eta} - \frac{K\eta}{\alpha^2 h^2 + M^2} \quad (4.21)$$

Applying the boundary condition, we get,

$$f(\eta) = \frac{1}{(1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2})} \cdot \left(\frac{\sinh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \eta \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2} \right) \quad (4.22)$$

4.4 EXPRESSION OF VELOCITY COMPONENTS

Hence the expression for the velocity components is

$$\begin{aligned}
 u &= \frac{1}{h} [hU(0) - v_0 x] f'(\eta) e^{i\omega t} \\
 &= \left(U(0) - \frac{v_0 x}{h} \right) \cdot \frac{\sqrt{\alpha^2 h^2 + M^2}}{(1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2})} \cdot \\
 &\quad \left(\frac{\cosh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \coth \sqrt{\alpha^2 h^2 + M^2} \right) \cdot e^{i\omega t}
 \end{aligned}$$

$$\begin{aligned}
 v &= v_0 f(\eta) e^{i\omega t} \\
 &= v_0 \left(\frac{1}{(1 - \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2})} \right) \cdot \\
 &\quad \left(\frac{\sinh \sqrt{\alpha^2 h^2 + M^2} \eta}{\sinh \sqrt{\alpha^2 h^2 + M^2}} - \eta \sqrt{\alpha^2 h^2 + M^2} \coth \sqrt{\alpha^2 h^2 + M^2} \right) \cdot e^{i\omega t}
 \end{aligned}$$

CHAPTER – 5

PERFORMANCE ANALYSIS

5.1 FIGURES AND DISCUSSION

The graphs of the axial velocity and radial velocity profiles have been drawn for different values of M . Figures 5.1 and 5.2 represents the axial velocity profiles at different cross sections of the channel at $x = 0$, $x = 5$ when the average entrance velocity is $u_0 = 0.5$ and $h = 1.0$. The magnitude of the axial velocity increases as x increases from $x = 0$ to $x = 5$ for different values of ω_t namely $\omega_t = 0, \frac{\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \pi$ and respectively. When the Hartmann number M is increased, we see that the magnitude of the axial velocity profiles decreases as seen in Figure 5.3. The Figure 5.4 represent the radial velocity profiles of v at $v_0 = 0.5$, $h = 1.0$, $\alpha = 1.0$ and for different values of M . As M increases from $M = 1.0$ to $M = 5.0$, it is seen that there is a marginal increase in the magnitude of the radial velocity. The Figure 5.5 represent the axial velocity profiles of u at $x = 0$, $h = 1.0$, $u_0 = 0.5$, $\omega_t = 0$ and for different values of M . Here we see that the profiles are parabolic.

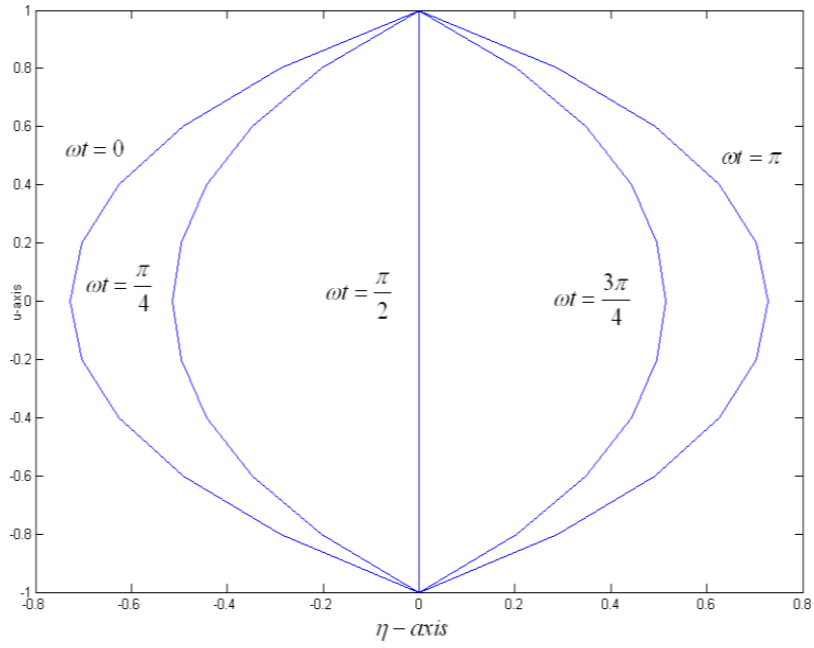


Figure 5.1 Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

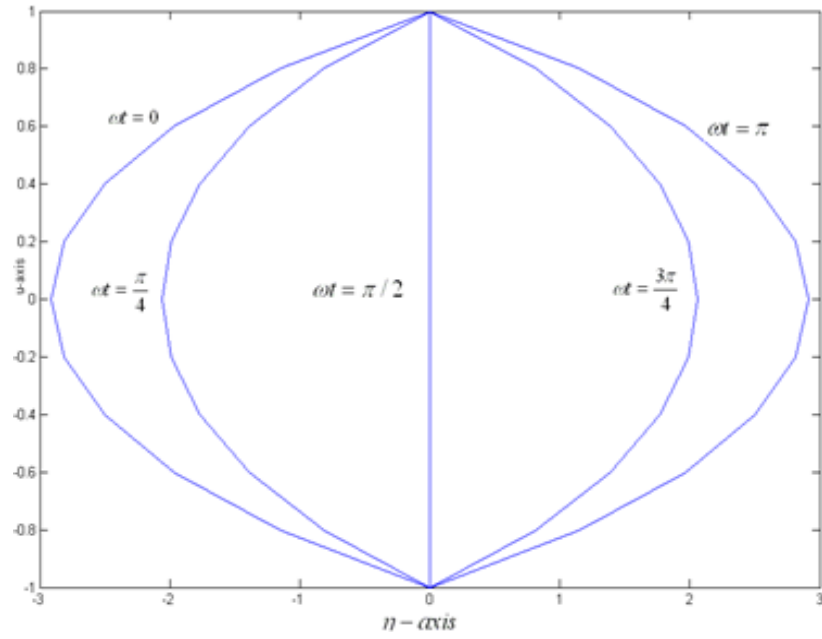


Figure 5.2 Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=5$ and for different values of ωt

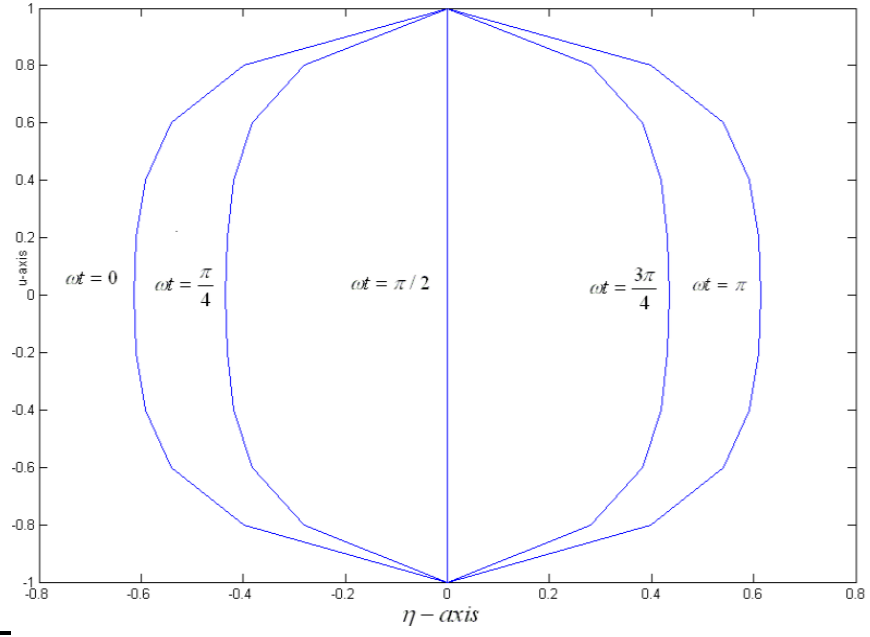


Figure 5.3 Axial velocity profiles when $u_0=0.5$, $v_0=0.5$, $h=1.0$, $M=5.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

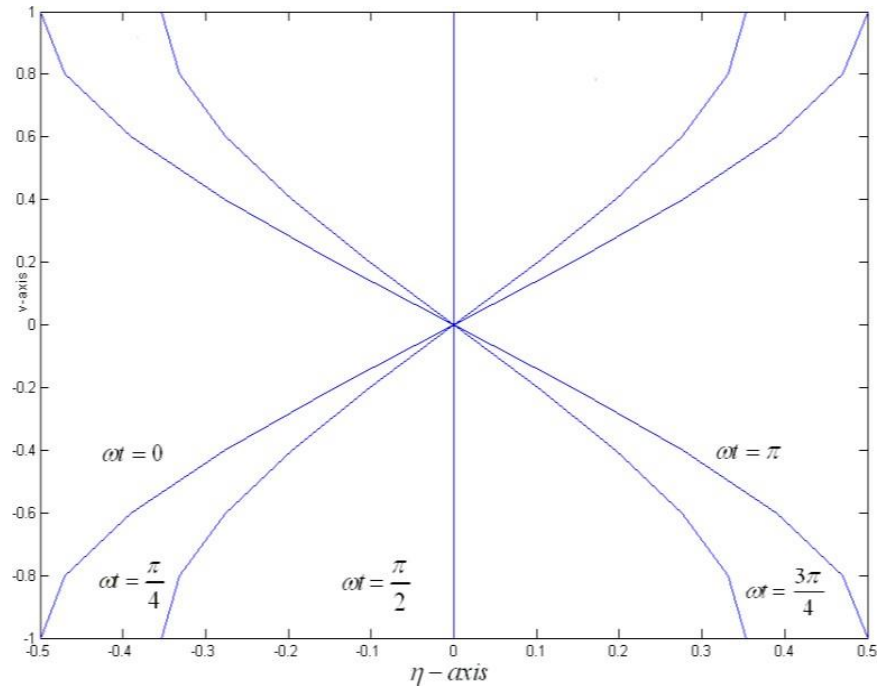


Figure 5.4 Radial velocity profiles when $v_0=0.5$, $h=1.0$, $M=1.0$, $\alpha=1.0$, $x=0$ and for different values of ωt

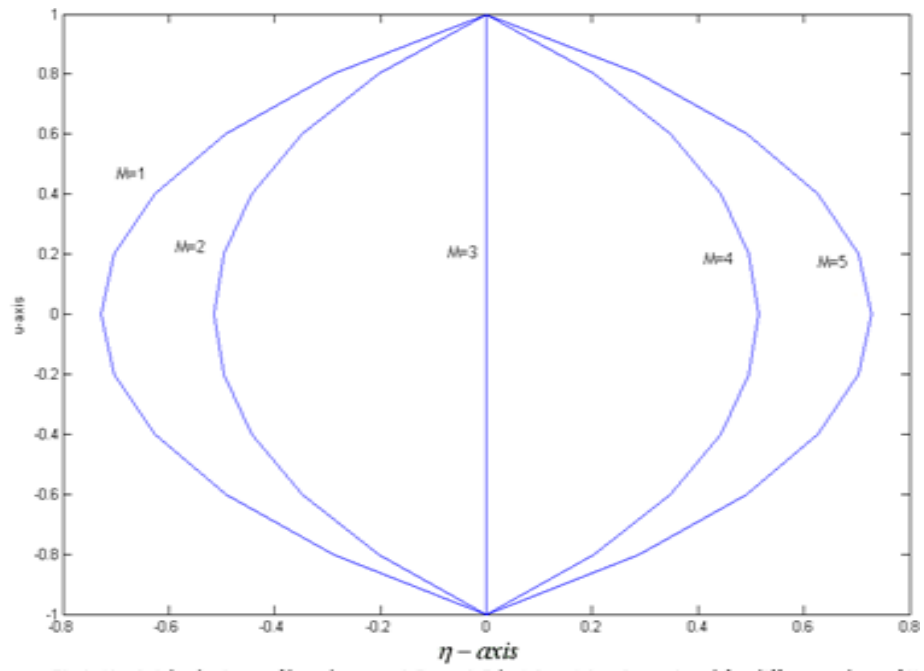


Figure 5.5 Axial velocity profiles when $u_0 = 0.5$, $v_0 = 0.5$, $h = 1.0$, $\alpha = 1.0$, $x = 0$, $\omega t = 0$ and for different values of M

CHAPTER - 6

SUMMARY AND CONCLUSIONS

6.1 CONCLUSION

The study of unsteady magnetohydrodynamic (MHD) Stokes flow of a viscous fluid between two parallel plates provides valuable insights into the dynamic behavior of fluid flow under the influence of magnetic fields. Through analytical and numerical investigations, we have examined the complex interplay between fluid viscosity, magnetic field strength, and the unsteady nature of the flow. Our findings underscore the significant impact of MHD effects on the flow dynamics, particularly in scenarios where unsteadiness plays a crucial role. The presence of magnetic fields alters the flow patterns and velocity profiles, leading to non-trivial changes in the transport properties of the fluid. Moreover, the unsteady nature of the flow introduces additional complexities, such as time-dependent variations in the flow behavior and the development of transient phenomena. The results obtained from this study have implications for various engineering applications, including materials processing, electromagnetic systems, and microfluidic devices. Understanding the dynamics of unsteady MHD Stokes flow is essential for optimizing process efficiency, enhancing device performance, and predicting system behavior under transient conditions. Despite the progress made in this study, several avenues for future research warrant attention. Further investigation into more complex fluid models, boundary conditions, and geometries can provide deeper insights into the behavior of MHD flows in practical scenarios. Additionally, experimental validation of theoretical predictions is necessary to validate the accuracy and applicability of the proposed models. In summary, the study of unsteady magnetohydrodynamic Stokes flow of a viscous fluid between two parallel plates contributes to the advancement of fluid dynamics knowledge and offers valuable insights for the development of innovative technologies in various engineering fields. In the above analysis a class of solutions of unsteady magnetohydrodynamics stokes flow of viscous fluid between two parallel porous plates is presented, in the presence of a transverse magnetic field when the fluid is being withdrawn through both the walls of the channel at the same rate.

6.2 FUTURE WORKS

- **Nonlinear Effects:** Investigate the impact of nonlinear effects on the flow behaviour, considering factors such as variable viscosity, thermal conductivity, and electric conductivity.
- **Boundary Conditions:** Explore different boundary conditions, including slip boundary conditions or mixed boundary conditions, and analyze their influence on flow characteristics.
- **External Forces:** Examine the effects of external forces, such as gravitational forces or electromagnetic forces, on the flow dynamics and stability.
- **Instabilities and Transition:** Study the occurrence of instabilities and transition phenomena in the flow regime, including the development of instability modes and their dependence on parameters such as Reynolds number and magnetic field strength.
- **Numerical Simulations:** Develop advanced numerical techniques, such as finite element methods or spectral methods, to accurately simulate the unsteady MHD Stokes flow and validate theoretical predictions.
- **Applications:** Investigate practical applications of unsteady MHD Stokes flow, such as in microfluidic devices, magnetohydrodynamic power generation, or industrial processes, and assess their feasibility and efficiency.
- **Optimization:** Explore optimization strategies to enhance the performance of MHD systems, considering factors like maximizing energy conversion efficiency or minimizing flow resistance.

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