

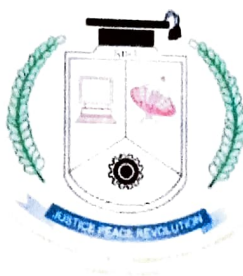
EIGEN VALUES OF A ROUGH ZERO DIVISOR GRAPH ON A ROUGH SEMIRING

Submitted in partial fulfillment of the requirements for the award of a
Bachelor of science degree in Mathematics

By

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**DEPARTMENT OF MATHEMATICS
SCHOOL OF SCIENCE AND HUMANITIES**

SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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APRIL – 2024



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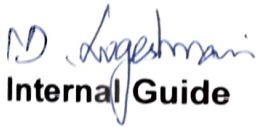
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BONAFIDE CERTIFICATE

This is to certify that this project report is the bonafide work of **(S. Karthika) (41060008) & (S. Kowsalya) (41060009)** who carried out the project entitled **"EIGEN VALUES OF A ROUGH ZERO DIVISOR GRAPH ON A ROUGH SEMIRING"** under our supervision from **JANUARY 2024** to **APRIL 2024**.

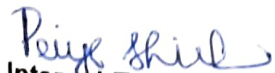

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ABSTRACT

In this proposed work an approximation space $I = (U, R)$, where U denotes a nonempty finite set of objects and R denotes an arbitrary equivalence relation defined on U is considered. The Rough Zero divisor graph of a Rough Semiring (T, Δ, ∇) on I is taken for study.

Let $\{X_1, X_2, \dots, X_n\}$ be the n equivalence classes induced by R . Without loss of generality assume that $\{X_1, X_2, \dots, X_m\}$ be the m equivalence classes whose cardinality is greater than one and $\{X_{m+1}, X_{m+2}, \dots, X_n\}$ be the $n - m$ equivalence classes whose cardinality is equal to one. Let $B = \{\{x_1\}, \{x_2\}, \dots, \{x_m\}\}$ be the set of pivot elements chosen from the equivalence classes whose cardinality is greater than one. Also assume that M is the union of none, one or more of the equivalence classes whose cardinality is equal to one and M' is the union of one or more of the equivalence classes whose cardinality is equal to one.

The Rough Zero divisor graph consists of $2^{n-m}(3^m) - 2^m - 1$ vertices, $1 \leq m \leq n$, where n represents the number of equivalence classes in U and m represents the number of equivalence classes with cardinality greater than one. The study of this Rough Zero divisor graph is challenging for large values of m and n . The complicity in studying this Rough Zero Divisor Graph was made simpler by defining clustering on the vertices of the Rough Zero divisor graph. A partition on vertices of the Rough Zero divisor graph is obtained based on the degree of vertices.

Keywords: Rough set, Rough Zero Divisor Graph, Eigen values, Python

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CHAPTER 1

INTRODUCTION

1.1 INTRODUCTION

The concept of semiring was first introduced by H.S. Vandiver in 1934. The Zero Divisor Graph was introduced by Irving Kaplansky, a prominent mathematician in algebra and algebraic geometry, in his 1970 paper "Fields and Rings". The concept of the Zero Divisor Graph was first introduced in a 1973 paper by Beck, which was published in the Journal of the London Mathematical Society. In the paper, Beck studied the of a commutative ring with identity, which is a graph whose vertices correspond to the nonzero zero-divisors of the ring, and two vertices are adjacent if their product is zero in the ring.

Pawlak introduced the concept of rough set theory in 1982 to process incomplete information in the information system. The rough set theory will be used to discover fundamental patterns in data analysis, remove redundancies, generate decision rules, computational Intelligence and other. Praba et al. discussed the concept of the rough lattice. The authors considered an information system $I = (U, A)$ in this paper. In a partial ordering relation, the operations Praba Δ and Praba ∇ were defined on $T = \{R S(X) \mid X \subseteq U\}$ using these two operations. The least upper bound and greatest lower bound were established. In the recent and past, rough set theory has triggered many researchers all around the world. The concept of rough set theory is the approximation space such as lower and upper approximations of a set determined by attributes.

The pair of lower and upper approximation is called a rough set also in rough set theory data can be represented in the form of an information system. An information system is a pair $I = (U, A)$ where U is a non-empty finite set of objects, called a universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a: U \rightarrow [0, 1]$, $a \in A$, is a fuzzy set. Indiscernibility is a core concept of rough set theory and it is defined as an

equivalence between objects. Objects in the information system about which we have the same knowledge form an equivalence relation.

Formally any set $P \subseteq A$, there is an associated equivalence relation called P – Indiscernibility relation defined as follows,

$$IND(P) = \{(x, y) \in U_2 | \forall a \in P, \mu a(x) = \mu a(y)\}.$$

The partition induced by $IND(P)$ consists of equivalence classes defined by $[x]_P = \{y \in U | (x, y) \in IND(P)\}$. For any $X \subseteq U$, define the lower approximation space $\underline{P}(X) = \{x \in U | [x]_P \subseteq X\}$. Also, define the upper approximation space $\overline{P}(X) = \{x \in U | [x]_P \cap X \neq \varphi\}$.

The concept of the Rough Zero Divisor Graph is a generalization of the zero divisor graph, and it was introduced by Aladibi and Ashraf in a 2014 paper published in the Journal of Algebra and Its Applications. The Rough zero divisor graph is defined in a similar way to the zero divisor graph, but the vertices of the graph correspond to the nonzero zero divisors of the ring, and two vertices are adjacent if their difference is a unit in the ring. The Rough zero divisor graph has been studied in various contexts, including algebraic geometry, algebraic number theory, and coding theory. For every subset $X \in U$, there is an associated rough set $RS(X) = (\underline{P}(X), \overline{P}(X))$.

The paper is organized as follows. The essential definitions of Rough Zero Divisor Graph, Eigenvalues and are given in Sect. 4. In Sect. 3, we stated Aim and Scope ; in Sect. 5, the concepts are explored with examples. Sect 5 is the application. Finally, Sect. 6 gives the conclusion.

CHAPTER 2

LITERATURE SURVEY

2.1 LITERATURE SURVEY

For a Commutative ring with non-zero divisors. In this note we provide counter example to the eigenvalues, the energy and second Zagreb index related to zero divisor graph of rings obtained in matlab we give a topological indices of ring [1].

In Computer science, Rough set ,first described by Polish Computer Scientist Zdzislaw I. Pawlak, is a formal approximation of a crisp set in terms of a pair of sets which give the lower approximation of the original set. In the standard version of rough set theory(Pawlak 1991), the lower- and upper- approximation sets are crisp sets ,but in other variations, the approximating sets may be fuzzy sets [8].

B. Praba, V.M. Chandrasekaran and A. Manimaran discuss initiate to deal the concepts of ideals on rough sets. Also we introduce the notations rough pivot set and semiring on the set of all rough sets for the given information system together with the operation Praba Δ and Praba ∇ . Illustrate these concepts through examples [6].

In this paper discussed eigen values of the zero divisor graph of finite commutative rings and study the interplay between these eigen values, the ring theoretic properties of R and the graph theoretic properties of zero divisor graph [3]

CHAPTER 3

AIM AND SCOPE

1. We have consider an information system $I = (U,A)$. Where U is non-empty finite set of object called Universe and A is a fuzzy set of attributes.
2. In our study, the rough semiring T is taken as a underlying algebraic structure together with rough ideal J .
3. The main objective of this study to obtain graph theoretical parameters of a Rough zero divisor graph denoted by $T(G)$.
4. We have obtained eigen values and eigen vectors of Rough zero divisor graph by using Python code.
5. First we will take degrees of each vertex by adjacence property. After getting the degrees of each vertex we form the adjacency matrix by using Laplacian matrix we get the eigen values. Equally we will generate the eigen values through python coding.
6. Eigenvalue analysis is also employed in the design of car stereo systems, where it aids in the reproduction of car vibration caused by music.
7. Many applications of matrices in both engineering and science utilize eigenvalues and, sometimes, eigenvectors. Control theory, vibration analysis, electric circuits, advanced dynamics and quantum mechanics are just a few of the application areas.

CHAPTER 4

PRELIMINARIES

4.1 PRELIMINARIES

Definition 4.1.1: (Graph) A graph can be defined as a pictorial representation or a diagram that represents data or values in an organized manner. The points on the graph often represent the relationship between two or more things.

Definition 4.1.2: (Vertex) A vertex is a point where multiple lines meet. It is also called a node. Similar to points, a vertex is also denoted by an alphabet.

Definition 4.1.3: (Edges) An edge is the mathematical term for a line that connects two vertices. Many edges can be formed from a single vertex. Without a vertex, an edge cannot be formed. There must be a starting vertex and an ending vertex for an edge

Definition 4.1.4: (Degree of a vertex) The number of edges incident to a vertex v is called the degree of the vertex and is denoted by $\deg(v)$. Equivalently, the degree of a vertex is the number of vertices adjacent to it.

Definition 4.1.5: (set) A set is a well-defined collection of distinct objects considered a single entity. The objects in a set are called their elements or members. The elements of a set can be anything: numbers, letters, symbols, or even other sets. The elements of a set are listed inside braces $\{ \}$, separated by commas.

For example, the set of all vowels in the English alphabet can be written as $\{a, e, i, o, u\}$.

Definition 4.1.6: (Subset) A subset is a set that contains only elements that are also in another set. In other words, if every element of set A is also an element of set B , then

A is a subset of B . We use the symbol $\subseteq$ (or sometimes $\subset$) to indicate that one set is a subset of another .

For example, if $A = \{2, 4, 6\}$ and $B = \{2, 3, 4, 6\}$, then A is a subset of B , and we write $A \subseteq B$. Note that A can also be equal to B (i.e., every element of A is also an element of B , and vice versa), in which case we write $A = B$.

Definition 4.1.7: (Rough graph) The theory of rough graphs is an attempt to unify rough set theory and graph theory. Graph theory, where objects are represented by vertices and relations by edges, is a convenient way of representing information involving relationships between objects.

Definition 4.1.8: (Rough set) Let $I = (U, A)$ be an information system and for any subset, X of U and $(\underline{P}(X), \overline{P}(X))$ are the lower and upper approximations respectively

Definition 4.1.9: (Monoid) A monoid is a set M together with a binary operation $\cdot$ that satisfies the following axioms:

- 1 . Associativity: For any elements a, b , and c in M $(a \cdot b) \cdot c = a \cdot (b \cdot c)$,
- 2 . Identity element: There exists an element e in M such that for any element a in M , $a \cdot e = e \cdot a = a$

The binary operation $\cdot$ is often called multiplication, and the identity element e is often called the unit element or identity

Examples of monoids include natural numbers (with addition as the operation), a set of strings (with concatenation as the operation), and the set of all endomorphisms of a vector space (with function composition as the operation).

Definition 4.1.10: (Commutative monoid) A commutative monoid is a monoid in which the binary operation is also commutative (i.e., $a \cdot b = b \cdot a$ for all a, b in M). Commutative monoids arise naturally in many areas of mathematics, including algebra, geometry, and number theory.

Examples of commutative monoids include natural numbers (with addition or multiplication as the operation), the set of polynomials with real coefficients (with multiplication as the operation)

Definition 4.1.11: (Group) A group is a set G together with a binary operation " $\bullet$ " (usually called multiplication) that satisfies the following axioms:

1. Closure: For all a, b in G , the product $a \bullet b$ is also in G
2. Associativity: For all a, b , and c in G , the product $(a \bullet b) \bullet c$ is equal to $a \bullet (b \bullet c)$
3. Identity: There exists an element e in G (called the identity element or unit element) such that for all a in G , $a \bullet e = e \bullet a = a$
4. Inverse: For every element a in G , there exists an element b in G (called the inverse of a) such that $a \bullet b = b \bullet a = e$

Examples of groups include the integers under addition, the real numbers (excluding zero) under multiplication, and the symmetric group on n letters (which consists of all permutations of n objects).

Definition 4.1.12: (Ring) A ring is a set R together with two binary operations, usually denoted by " $+$ " and " $\cdot$ ", that satisfy the following axioms:

1. R is an additive group under " $+$ "
2. The multiplication " $\cdot$ " is associative, and there exists an element 1 in R such that $1 \cdot a = a \cdot 1 = a$ for all a in R
3. The distributive laws hold: $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$ for all a, b , and c in R

Definition 4.1.13: (Semiring) A semiring is an algebraic structure similar to a ring, but without requiring each element t have an additive inverse.

Definition 4.1.14:(Rough semiring) A rough semiring is a triple $(S, +, \cdot)$, where S is a non-empty set, and $+$ and $\cdot$ are binary operations on S that satisfy the following axioms:

$(S, +)$ is a commutative monoid with identity element 0

$(S, \cdot)$ is a semiring with identity element 1

3. Distributivity of $\cdot$ over $+$: for all a, b , and c in S , $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$

Closure of $+$ under approximation: for all A, B in $P(S)$, the set $A + B = \{a + b \mid a \in A, b \in B\}$ is an upper approximation of $A \cup B$, and the set $A \cdot B = \{a \cdot b \mid a \in A, b \in B\}$ is a lower approximation of $A \cap B$

Definition 4.1.15: (Zero Divisor graph) The Zero Divisor Graph of G is a graph $= (V, E)$, where $V = Z(G) - \{0\}$ is the set of nonzero zero divisors of G , and $E = \{(u, v) \mid uv = 0\}$ is the set of edges connecting pairs of vertices whose product is zero in the ring.

Definition 4.1.16: (The Zero Divisor Graph of a Rough Semiring) The zero divisor graph of a rough semiring is $T(Z(G)) = (V, E)$ where V is the set of vertices in $T(Z(G))$ consists of the non-empty zero divisors and E is the set of edges connecting the elements of V such that there is an edge connecting $RS(X)$ and $RS(Y)$ in V if $RS(X) \cap RS(Y) = RS(\emptyset)$. This graph $T(Z(G))$ is called a rough zero divisor graph of the rough semiring T .

Definition 4.1.17: (Degree matrix) The degree matrix is a diagonal matrix where each diagonal element represents the degree of a vertex in the graph. For an undirected graph, the degree of a vertex is the number of edges incident to it. The degree matrix is often denoted as D and can be defined as:

$$D = \begin{cases} \deg(v_i) & \text{if } i = j \\ 0 & \text{Otherwise} \end{cases}$$

Where $\deg(v_i)$ is the degree of vertex v_i

Definition 4.1.18: (Adjacency matrix) The adjacency matrix of a graph is a square matrix used to represent connections between vertices. In a graph with n vertices, the adjacency matrix is an $n \times n$ matrix where the entry in row i and column j represents whether there is an edge and 0 if there isn't, but in weighted graphs, it can represent the weight of the edge.

Definition 4.1.19: (Laplacian matrix) The Laplacian matrix, often denoted as L , is a square matrix derived from a graph. It is defined as the difference between the degree matrix and adjacency matrix of the graph. In mathematical terms for an undirected graph G with n vertices, the laplacian matrix L is given by $L=D-A$, where D is the degree matrix and A is the adjacency matrix

Definition 4.1.20: (Eigen value) Eigenvalues are the special set of scalar associated with the system of linear equations. It is mostly used in matrix equations. 'Eigen' is a German word that means 'Proper' or Characteristic.

$$Ax = \lambda X$$

They are solutions to the characteristic equation of the matrix, $|A - \lambda I| = 0$ where A is the matrix, λ is the eigen value, and I is the identity matrix.

CHAPTER 5

RESULTS AND DISCUSSION

5.1 EIGEN VALUES OF A GRAPH

In this Graph we find the eigen values manually compare with python coding

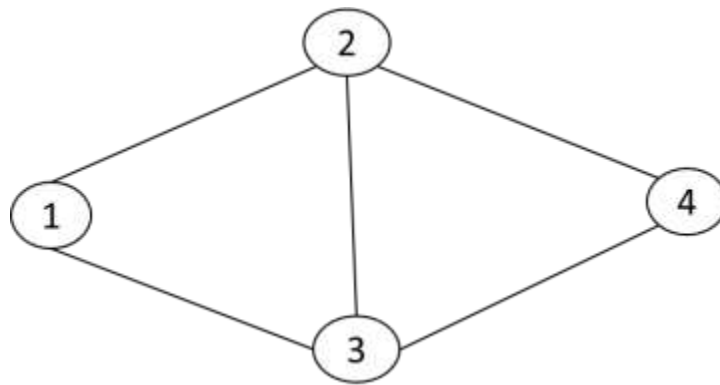


Fig 5.1.1

DEGREE MATRIX of D

$$D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

ADJACENCY MATRIX of A

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

LAPLACIAN MATRIX of L

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 1 & 3 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

To find characteristic equation

$$|A - \lambda I| = 0$$

$$A = \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ 1 & 3-\lambda & -1 & 0 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

The determined of the obtained matrix

$$\begin{vmatrix} 3-\lambda & -1 & 0 \\ 2-\lambda & -1 & 3-\lambda \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

$$= (3-\lambda) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) - (-1)(-1)$$

$$= (3-\lambda)(2-\lambda) + 1$$

$$\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} = (-1)(2-\lambda) - (-1)(-1)$$

$$= -(2-\lambda) + 1$$

$$\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} = (3-\lambda)(2-\lambda) + 1$$

$$= (3 - \lambda)(2 - \lambda) + 1$$

$$= 6 - 5\lambda + \lambda^2 + 1$$

$$= \lambda^2 - 5\lambda + 7$$

$$\begin{vmatrix} -1 & -1 \\ -1 & 2 - \lambda \end{vmatrix} = -(2 - \lambda) + 1$$

$$= 2 + \lambda + 1$$

$$= \lambda - 1$$

$$|A| = (2 - \lambda)(\lambda^2 - 5\lambda + 7) - (-1)(\lambda - 1) + (-1) \cdot 0$$

$$= (2 - \lambda)(\lambda^2 - 5\lambda + 7) + (\lambda - 1)$$

$$= -\lambda^3 + 7\lambda^2 - 10\lambda + \lambda + 14 - 1$$

$$= \lambda^3 + 7\lambda^2 - 9\lambda + 13$$

$$|A| = -\lambda^3 + 7\lambda^2 - 9\lambda + 13$$

$$-\lambda^3 + 7\lambda^2 - 9\lambda + 13 = 0$$

$$(\lambda - 4)(\lambda - 3)(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda = 1$$

$$\lambda = 2$$

$$\lambda = 3$$

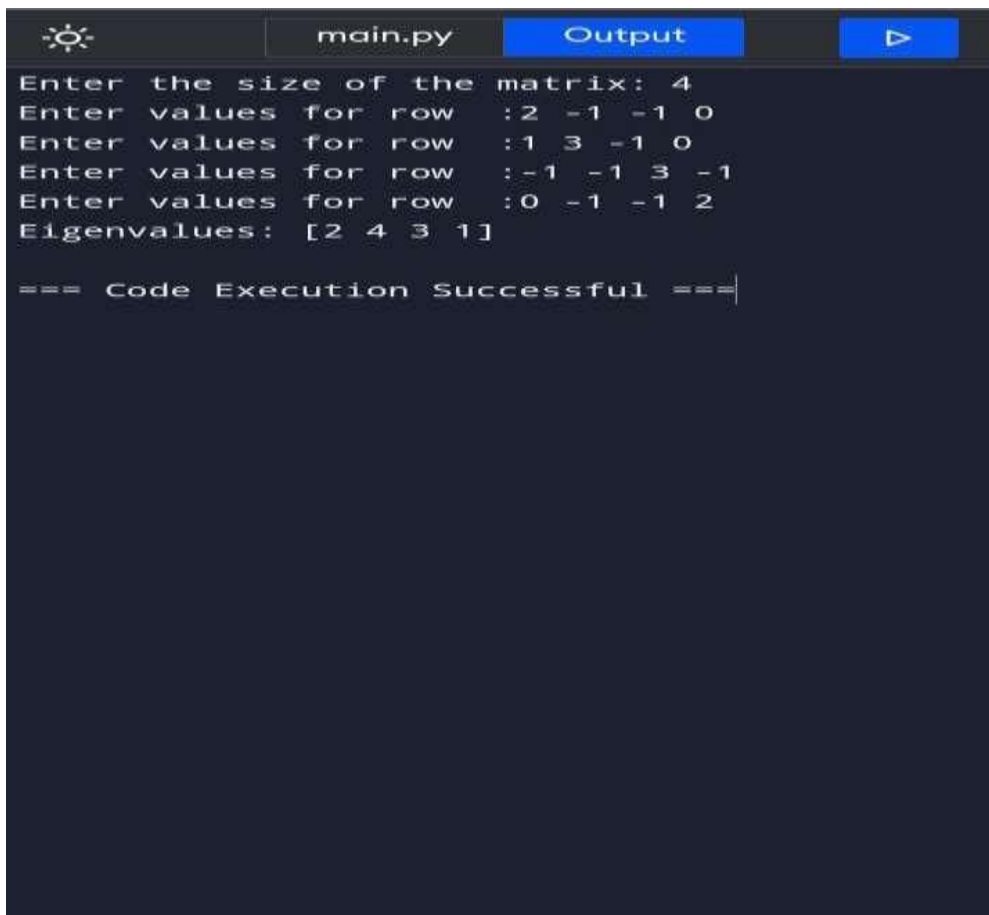
$$\lambda = 4$$

The Eigen values of a Graph 1, 2 , 3, 4 .

PYTHON CODING FOR ROUGH ZERO DIVISOR GRAPH

```
1 import numpy as np
2
3
4 def calculate_eigen(matrix):
5     matrix = np.array(matrix)
6     eigenvalues, ev= np.linalg.eig(matrix)
7     return np.round(eigenvalues).astype(int)
8
9
10 n = int(input("Enter the size of the matrix: "))
11 matrix = []
12 for i in range(n):
13     row = [int(x) for x in input("Enter values for row :").split()]
14     matrix.append(row)
15 eigenvalues= calculate_eigen(matrix)
16 print("Eigenvalues:", eigenvalues)
```

INPUT AND OUTPUT OF A ROUGH ZERO DIVISOR GRAPH



```
Enter the size of the matrix: 4
Enter values for row :2 -1 -1 0
Enter values for row :1 3 -1 0
Enter values for row :-1 -1 3 -1
Enter values for row :0 -1 -1 2
Eigenvalues: [2 4 3 1]

=== Code Execution Successful ===
```

We have taken 3×3 we obtained the eigen values in manually and also using Python coding the solution will be same. In this way we have taken the matrix of greater value we will having the same solution in manually as well as in Python coding.

5.2.ILLUSTRATION

Table 5.2.1 Information table

A/U	a ₁	a ₂	a ₃	a ₄
x ₁	0	0.1	0.3	0.2
x ₂	1	0.6	0.7	0.3
x ₃	0	0.1	0.3	0.2
x ₄	1	0.6	0.7	0.3
x ₅	0.8	0.5	0.2	0.4
x ₆	1	0.6	0.7	0.3

This is an information System (U, A) in this x_1 and x_3 rows contain the same value so, we are taking it into one Equivalence class. Then x_2 , x_4 and x_6 rows contain the same value so, we are taking it into the 2nd equivalence class. then x_5 is different from other rows so, we are taking it into the 3rd equivalence class. n is the number of Equivalence classes and m is the number of equivalence classes, whose coordinately is greater than one.

$$X_1 = [x_1]_p = \{x_1, x_3\}$$

$$X_2 = [x_2]_p = \{x_2, x_4, x_6\}$$

$$X_3 = [x_5]_p = \{x_5\}$$

$$n = 3 \quad m = 2, \quad 2^{n-m} \cdot 3^m - 2^m - 1 = 13$$

$$T = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\emptyset), RS(U), RS(x_1), \\ RS(x_1 \cup X_2), RS(x_1 \cup X_3), RS(x_1 \cup X_2 \cup X_3), RS(x_2), RS(X_1 \cup x_2), RS(x_2 \cup X_3), \\ RS(X_1 \cup x_2 \cup X_3), RS((x_1 \cup x_2 \cup X_3), RS((x_1 \cup x_2))\}.$$

$$Z(T) = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(x_1), \\ RS(x_1 \cup X_2), RS(x_1 \cup X_3), RS(x_2), RS(X_1 \cup x_2), RS(x_2 \cup X_3), RS((x_1 \cup x_2))\}.$$

$n = 3, m = 2$, Rough zero divisor graph

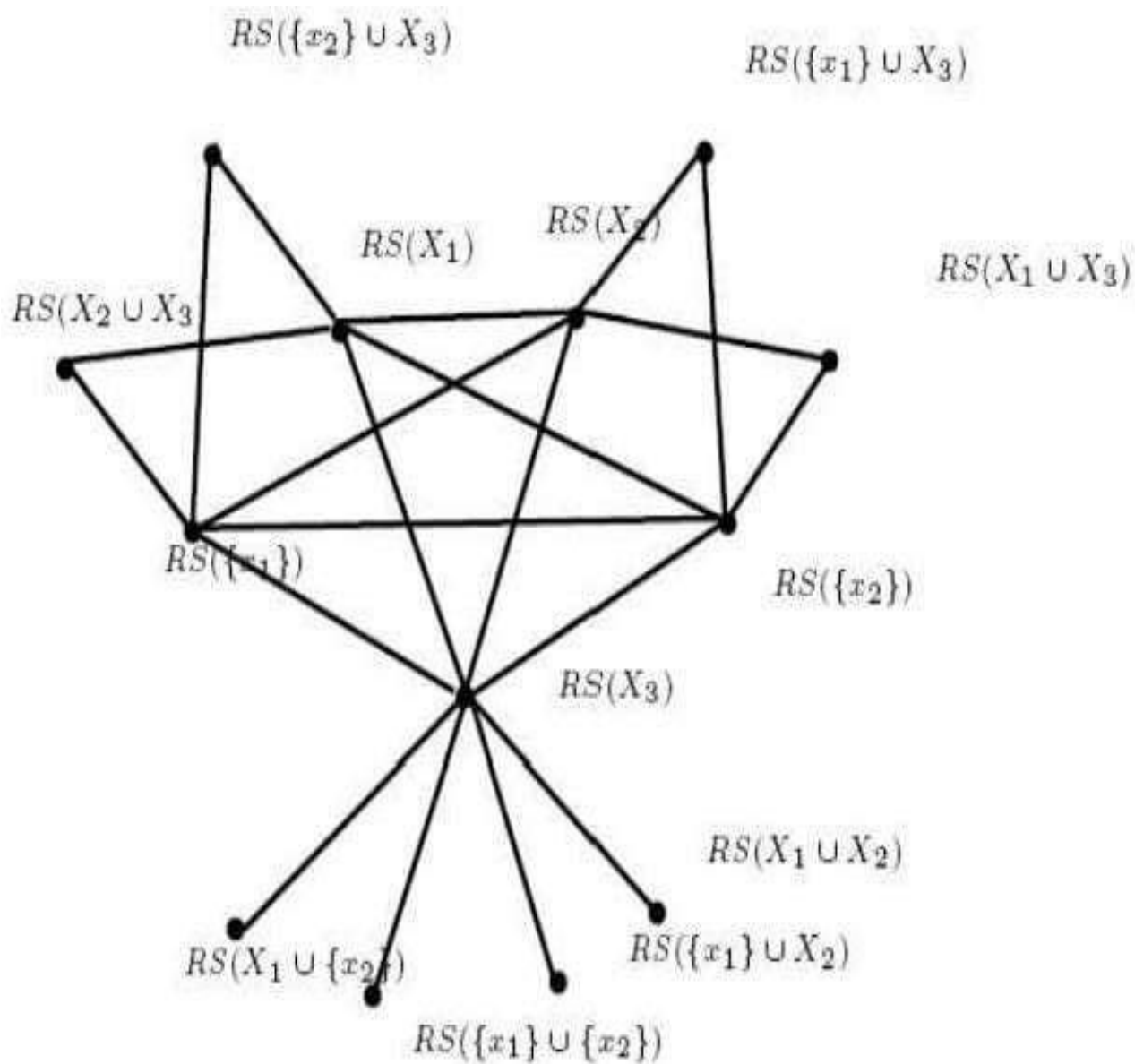


Fig : 5.2.1

DEGREE MATRIX of D

[illegible]

ADJACENCY MATRIX of A

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

LAPLACIAN MATRIX of L

$$L = \begin{bmatrix} 5 & -1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 5 & -1 & -1 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 8 & -1 & -1 & 0 & 0 & -1 & -1 & -1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 5 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 \\ -1 & 0 & -1 & -1 & 5 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Now we obtained the eigen values of $n = 3, m = 2$ of a Rough zero divisor graph of the rough semiring using Python Language.

PYTHON CODING FOR EIGEN VALUES

```
1 import numpy as np
2
3
4 def calculate_eigen(matrix):
5     matrix = np.array(matrix)
6     eigenvalues, ev= np.linalg.eig(matrix)
7     return np.round(eigenvalues).astype(int)
8
9
10 n = int(input("Enter the size of the matrix: "))
11 matrix = []
12 for i in range(n):
13     row = [int(x) for x in input("Enter values for row :").split()]
14     matrix.append(row)
15 eigenvalues= calculate_eigen(matrix)
16 print("Eigenvalues:", eigenvalues)
```

INPUT AND OUTPUT OF EIGEN VALUES OF A ROUGH ZERO
DIVISOR GRAPH OF A ROUGH SEMIRING $n = 3, m = 2$

```
C:\Users\Barathraj\PycharmProjects\STUDENT\venv\Scripts\python.exe C:\Us
Enter the size of the matrix: 13
Enter values for row :5 -1 -1 0 -1 0 0 0 0 0 0 -1 -1
Enter values for row :-1 5 -1 -1 0 -1 -1 0 0 0 0 0 0
Enter values for row :-1 -1 8 -1 -1 0 0 -1 -1 -1 -1 0 0
Enter values for row :0 -1 -1 5 -1 0 0 0 0 0 0 -1 -1
Enter values for row :-1 0 -1 -1 5 -1 -1 0 0 0 0 0 0
Enter values for row :0 -1 0 0 -1 2 0 0 0 0 0 0 0
Enter values for row :0 -1 0 0 -1 0 2 0 0 0 0 0 0
Enter values for row :0 0 -1 0 0 0 0 1 0 0 0 0 0
Enter values for row :0 0 -1 0 0 0 0 0 1 0 0 0 0
Enter values for row :0 0 -1 0 0 0 0 0 0 1 0 0 0
Enter values for row :0 0 -1 0 0 0 0 0 0 0 1 0 0
Enter values for row :-1 0 0 -1 0 0 0 0 0 0 0 2 0
Enter values for row :-1 0 0 -1 0 0 0 0 0 0 0 0 2
Eigenvalues: [9 8 0 1 1 4 5 5 2 2 1 1 1]

Process finished with exit code 0
```

5.3 ILLUSTRATION

Table 5.3.1 Information table

A/U	a ₁	a ₂	a ₃	a ₄
x ₁	0	0.1	0.3	0.2
x ₂	1	0.6	0.7	0.3
x ₃	0	0.1	0.3	0.2
x ₄	1	0.6	0.7	0.3
x ₅	0.8	0.5	0.2	0.4
x ₆	1	0.6	0.7	0.3

This is an information System (U, A) in this x_1 and x_3 rows contain the same value so, we are taking it into one Equivalence class. Then x_2 and x_4 rows contain the same so, we are taking it into the 2nd equivalence class. then x_5 and x_6 are different from other rows so, we are taking it into the 3rd equivalence class. n is the number of Equivalence classes and m is the number of equivalence classes, whose coordinately is greater than one.

$$\begin{aligned}
 X_1 &= [x_1]p = \{x_1, x_3\} \\
 X_2 &= [x_2]p = \{x_2, x_4\} \\
 X_3 &= [x_5]p = \{x_5, x_6\} \\
 n &= 3 \quad m = 3, \quad 2^{n-m} \cdot 3^m - 2^m - 1 = 18
 \end{aligned}$$

$T = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\emptyset), RS(U), RS(x_1),$
 $RS(x_1 \cup X_2), RS(x_1 \cup X_3), RS(x_1 \cup X_2 \cup X_3), RS(x_2), RS(X_1 \cup x_2), RS(x_2 \cup X_3),$
 $RS(X_1 \cup x_2 \cup X_3), RS((x_1 \cup x_2 \cup X_3), RS((x_1 \cup x_2))\}.$

$Z(T) = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(x_1),$
 $RS(x_1 \cup X_2), RS(x_1 \cup X_3), RS(x_2), RS(X_1 \cup x_2), RS(x_2 \cup X_3), RS(x_1 \cup x_2),$
 $RS(X_2 \cup x_3), RS(x_2 \cup x_3), RS(x_3), RS(X_2 \cup x_3)\}.$

$n = 3, m = 3$, Rough zero divisor graph

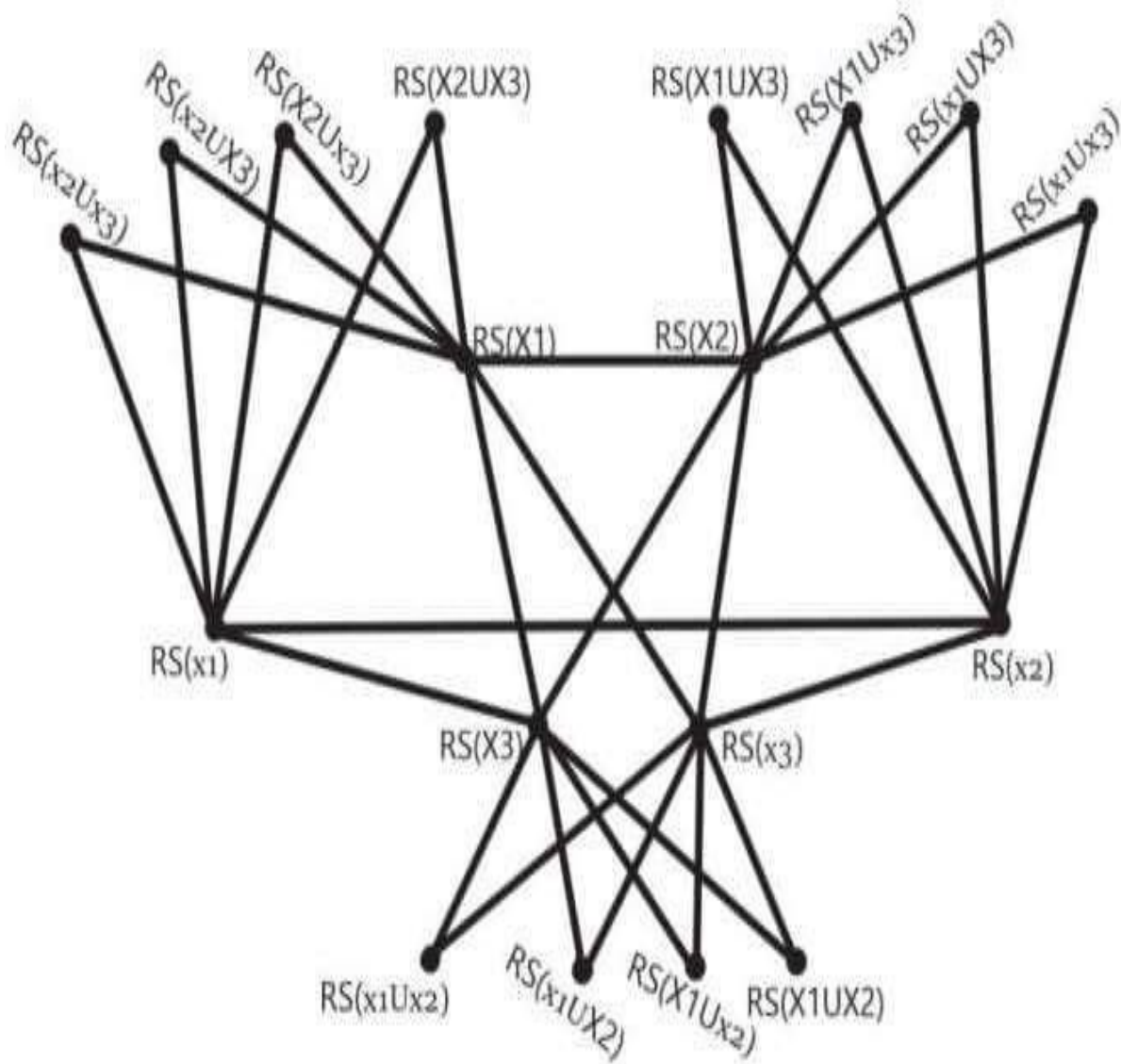


Fig : 5.3.1

DEGREE MATRIX of D

[illegible]

ADJACENCY MATRIX of A

[illegible]

LAPLACIAN MATRIX of L

[illegible]

PYTHON CODING FOR EIGEN VALUES

```
1 import numpy as np
2
3
4 def calculate_eigen(matrix):
5     matrix = np.array(matrix)
6     eigenvalues, ev= np.linalg.eig(matrix)
7     return np.round(eigenvalues).astype(int)
8
9
10 n = int(input("Enter the size of the matrix: "))
11 matrix = []
12 for i in range(n):
13     row = [int(x) for x in input("Enter values for row :").split()]
14     matrix.append(row)
15 eigenvalues= calculate_eigen(matrix)
16 print("Eigenvalues:", eigenvalues)
```

INPUT AND OUTPUT OF A ROUGH ZERO DIVISOR GRAPH

$n = 3, m = 3$

```

C:\Users\Barathraj\PycharmProjects\STUDENT\venv\Scripts\python.exe C:\
Enter the size of the matrix: 18
Enter values for row :7 -1 0 0 0 0 0 -1 0 0 0 0 -1 0 -1 -1 -1 -1
Enter values for row :-1 7 -1 -1 -1 -1 0 -1 0 0 0 0 -1 0 0 0 0 0
Enter values for row :0 -1 2 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
Enter values for row :0 -1 0 2 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
Enter values for row :0 -1 0 0 2 0 -1 0 0 0 0 0 0 0 0 0 0 0
Enter values for row :0 -1 0 0 0 2 -1 0 0 0 0 0 0 0 0 0 0 0
Enter values for row :0 0 -1 -1 -1 -1 6 -1 0 0 0 0 0 -1 0 0 0 0
Enter values for row :-1 0 0 0 0 0 -1 7 -1 -1 -1 -1 0 0 0 0 0 0
Enter values for row :0 0 0 0 0 0 0 -1 2 0 0 0 -1 0 0 0 0 0
Enter values for row :0 0 0 0 0 0 0 -1 0 2 0 0 -1 0 0 0 0 0
Enter values for row :0 0 0 0 0 0 0 -1 0 0 2 0 -1 0 0 0 0 0
Enter values for row :0 0 0 0 0 0 0 -1 0 0 0 2 -1 0 0 0 0 0
Enter values for row :-1 -1 0 0 0 0 0 0 -1 -1 -1 -1 7 -1 0 0 0 0
Enter values for row :0 0 0 0 0 0 -1 0 0 0 0 0 -1 6 -1 -1 -1 -1
Enter values for row :-1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 2 0 0 0
Enter values for row :1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 2 0 0
Enter values for row :1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 2 0
Enter values for row :1 0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 2
Eigenvalues: [9 8 8 6 6 6 0 1 1 2 2 2 2 2 2 2 2 2]

```

5.4.APPLICATION

If you look at the construction of suspension bridges... Each such bridge has a "natural frequency" and when the physical system that models the bridge is linearized, this frequency corresponds to the eigenvalue of the smallest magnitude.

$$\text{Natural frequency } n = (1/2\pi)\sqrt{\lambda}$$

Where f is the natural frequency. λ is the eigen value. Engineers want this frequency to be reasonably far away from any frequencies that occur naturally, such as local wind conditions

Engineers want that this frequency to be reasonably far away from any frequencies that occur naturally, such as local wind conditions. When this fails to be the case, i.e. when the natural frequency of the bridge corresponds to wave frequencies induced by the environment, the frequencies become (nearly) additive, as opposed to the desired state of the frequency (nearly) cancelling out. The result is, for example, what occurred in the 1940 collapse of the Tacoma Narrows Bridge. Hence, Eigenvalue analysis or vibration analysis is very useful for the design of bridges.



CHAPTER 6

CONCLUSION AND FUTURE WORK

In the Introduction section we provide a basic understanding of Rough set theory and Graph theory and in chapter 2, Preliminaries chapter 3, The Eigenvalues of Rough zero divisor graph described the present study relates the Eigenvalues of the Rough zero divisor graph and it is established through Python coding.

Here we have established the eigen value of a Rough zero divisor graph of a Rough semiring T . Also we have embellished the Python coding for the Rough zero divisor graph. These concepts are illustrated through examples. The future work in this direction is to explore more applications of Eigen values and Eigen vectors of Rough zero divisor graph of a Rough semiring.

REFERENCE

- [1] Bilal Ahamad Rather, Jan 2024, A note on the Eigen values of Zero Divisor Graphs Associated with Commutative Rings, Journal of Applied Mathematics and Computation , Vol 54, pages 787-802
- [2] Clement Johnson Rayer, Jan 2023 Ravi Shankar Jeyaraj, Applications on Topological Indices of Zero-Divisor Graph Associated with Commutative Rings, Mathematics Symmetry, Vol 15, Issue 2
- [3] Katja Monius , Nov 2021, Eigenvalues of Zero-Divisor Graphs of finite commutative rings, Journal of Algebraic Combinatorics, Volume 64 pages 787-802
- [4] Manimaran A , Praba B , March 2017 Characterization of Rough Semiring, Afrika Matematika ,vol 28 pages 945-956
- [5] Nurhidayah Zaid, Nor Haniza Sarmin, Sanhan Khasraw, Ibrahim Gambo, Nur Athirah Farhana Omar Zai , Aug 2020 Zero divisor Graph of a Finite Ring of Matrices and its Perfect Codes, Conference paper
- [6] Praba B, Chandrasekaram VM, Manimaran A, 2015, Semiring on Roughsets Indian Journal of science and technology, Volume: 8 280-286
- [7] Selvam .V ,SenthilKumar G, 2012, Rough ideals in semirings , International Journal of Mathematical science and Applications ,vol .2 no.2
- [8] Zdzislaw pawlak, 1982 Rough sets, International Journal of Compute and Information science, Volume 11, pages 341-356

