

SUPERNOVAE DATA AND THE ACCELERATING UNIVERSE

Submitted in partial fulfillment of the requirements for the award of

Master of Science degree in Physics

By

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DEPARTMENT OF PHYSICS

SCHOOL OF SCIENCE AND HUMANITIES

SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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BONAFIDE CERTIFICATE

This is to certify that this Project Report is the bonafide work of Stalin Marivalan K. (41590021) who carried out the project entitled "Supernovae Data and The Accelerating Universe" under my supervision from November 2022 to May 2023.

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Submitted for Viva voce Examination held on 9.5.2023 2023

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DECLARATION

I, Stalin Marivalan K (41590021) hereby declare that the Project Report entitled Supernovae Data and The Accelerating Universe done by me under the guidance of Dr. S. Murugesan (Internal), Department of Physics, Sathyabama Institute of Science and Technology, Chennai is submitted in partial fulfilment of the requirements for the award of Master of Science Degree in Physics.

DATE: 09-05-2023

PLACE: CHENNAI

A handwritten signature in black ink, appearing to read 'Stalin Marivalan K', enclosed within a rectangular box.

SIGNATURE OF THE CANDIDATE

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ABSTRACT

Dynamics of our Universe can be determined by measuring density parameters of different components through various observations. Based on the the density parameters, the rate of expansion of our Universe could vary greatly. To determine the rate of expansion effectively, we need standard candles which can be seen at large redshifts i.e. luminosity distance greater than 1000 Mpc. One of the main obstacles is that objects in our Universe evolve with time, so the property which we are measuring could change as time evolves. We need a standard candle whose property can be determined at large redshifts. The main objective of this work is to use Type-1A supernovae as standard candles to determine the density parameters from the distance modulus-redshift relation.

Keywords: Density parameters, Supernovae, Accelerating Universe

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1 INTRODUCTION

Cosmology is the study of the Universe as a whole. From the famous efforts of cosmologists such as Edwin Hubble and others, we know that the universe is expanding. From the discovery of the Cosmic Microwave Background (CMB), it became clear that the Universe was hot and dense in early times. The work of a cosmologist revolves around the question of the past, present and future of the Universe i.e, the contents of the Universe and its evolution.

1.1 EXPANSION OF THE UNIVERSE

Around the early twentieth century, cosmologists found that galaxies which are farther away have their absorption spectrum shifted to longer wavelengths in their spectrum. Redshift ¹ or shift in the spectrum is written as

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \quad (1.1)$$

i.e.,

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}$$

where,

λ_{obs} - wavelength at the time of observation,

λ_{em} - wavelength at the time of emission.

In his famous work, Edwin Hubble showed that the farther the galaxies

¹Stretching of the photons due to the expansion of the Universe, also known as cosmological redshift

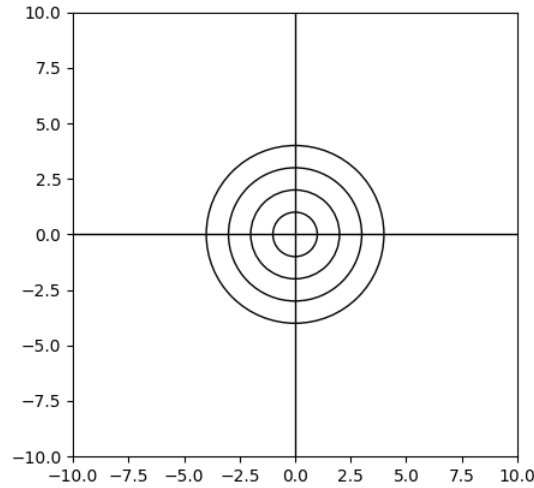


Figure 1.1: concentric circles with radius 1,2,3,4

higher their receding velocities from us. This can be efficiently expressed as

$$\vec{v} = H_0 \vec{r}. \quad (1.2)$$

we can imagine the expansion as the enlargement of concentric circles around each observer who are themselves comoving with the expansion. From figure (1.1) and (1.2) where we have plotted objects at different distances, it is clear that the farther the circle larger the distance it moves away.

In a more abstract way, expansion can also be imagined as a multiplication of the number line $S = 1, 2, 3, 4, 5, \dots$ by any number ² for example: $2 * S = S^0 = 2, 4, 6, 8, 10, \dots$ we can see that the larger the number in set S larger the multiplied result in the set S^0 , where the numbers in the set correspond to the distance from the observer.

Also, over large scales, every direction looks the same around us. The above two statements directly imply that the Universe is homogeneous and isotropic at a large scale which is known as cosmological principle.

²corresponds to the Hubble parameter which changes over time

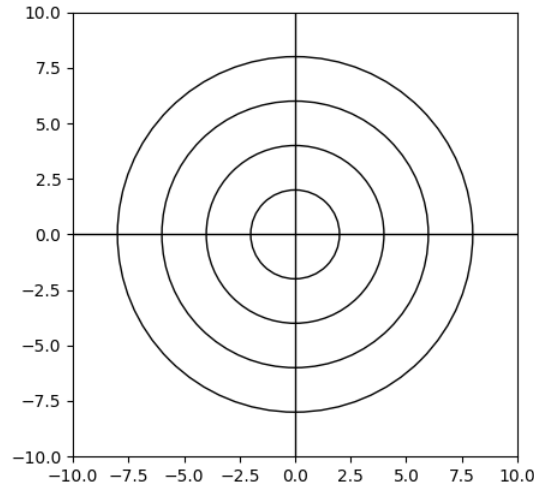


Figure 1.2: concentric circles expanded by a factor of 2

1.2 HOT BIG BANG

If we trace back the expansion of the universe we can come to a conclusion that the Universe was closer together in the earlier times.

1.2.1 *Cosmic Microwave Background (CMB)*

The main finding which supported the theory of the hot big bang model over the static universe was the discovery of CMB. Around 1965 CMB was detected, and experiments were conducted at different wavelengths to plot the distribution of total energy density of CMB. Finally, it was found that the distribution exactly matches that of the black-body spectrum, in fact, it was such a perfect Planck spectrum distribution that it can't even be recreated in the lab.

The total energy density of the black-body spectrum is described by the Stefan-Boltzmann law

$$\epsilon_{rad} = \rho_{rad} * c^2 = \alpha T^4. \quad (1.3)$$

From observations, we know that the current temperature ³ of CMB is

$$T_0 = 2.725k \approx 3k$$

³Andrew Liddle, 2015, An introduction to modern cosmology, third edition, John Wiley sons

using this in the equation(1.3) we can obtain the energy density of CMB as $\rho_{rad} * c^2 = 4.17 * 10^{-14} Jm^{-3}$.

1.2.2 ORIGIN OF CMB

In early times, the Universe was in thermal equilibrium and highly interacting. Due to the high temperature, photons ionized the hydrogen atom effectively creating an opaque Universe. As the universe expands, the energy of the photon decreased since the wavelength of the photon is directly proportional to the expansion ⁴. when it reduced lower than 13.6ev, the photon couldn't ionize the hydrogen and all electrons got bounded with the proton. Then the photon started to travel through the universe without impediment and interaction between the photon and matter ceased which is known as decoupling.

The boundary where the photon got lastly scattered around each observer at the time of decoupling is known as *surface of the last scattering*. Since then the energy of the photon decreased with the expansion, but the distribution of total energy density got preserved.

Having discussed the big bang and the expansion of the Universe, our main objective is to find the density parameters. There are many methods to constrain the density parameter using CMB and Baryon Acoustic Oscillation (BAO), We are going to use supernovae data especially a category of supernovae called Type-1 A. Limitations of observing Type-1 A Supernovae is that they are rare, approximately one event per century in our galaxy ⁵. With the advent of advanced space and high-altitude telescopes, now it is relatively easy to observe them.

Finally, we are going to use supernovae data and Chi-Squared minimization technique to constrain the density parameters.

⁴the statement implies that the temperature of the CMB is inversely proportional to expansion

⁵Barbara Ryden, 2003, Introduction to Cosmology, first edition, Pearson Addison Wesley

2 LITERATURE SURVEY

2.1 MAJOR EQUATIONS OF COSMOLOGY

We are going to derive an equation which describes the motion of any test particle in a homogeneous medium of mass¹ using Newtonian gravity.

2.1.1 Comoving coordinate

The physical distance $\vec{r}$ between any two objects can be expressed in terms of comoving coordinate $\vec{x}$. Consider a grid embedded in the fabric of space, the grid expands with the expansion of the Universe. Such that the position/coordinate $\vec{x}$ of an object in the Hubble flow² doesn't change during expansion. Then the physical distance is written as

$$\vec{r} = a(t)\vec{x}, \quad (2.1)$$

where "a" is scale factor, denotes the expansion of the universe.

2.1.2 Friedmann equation

The total energy of any particle can be written as the sum of kinetic and potential energy.

$$U = T + V. \quad (2.2)$$

According to Newton, force on any particle of mass 'm' relative to an observer centred at distance $\vec{r}$ in a homogeneous medium is only due to the mass which is inside the sphere of radius $\vec{r}$. Thus from force $-\frac{dV}{dx} = \frac{GMm}{r^2}$ the potential energy becomes

$$V = -\frac{GMm}{r}. \quad (2.3)$$

¹approximates the Universe at large scale, invoking Cosmological Principle

²objects that are moving with the expansion alone without any peculiar velocity

where $M = \frac{4\pi\rho r^3}{3}$ and ρ = mass density of the medium.

The kinetic energy of the particle is

$$T = \frac{1}{2}m\dot{r}^2, \quad (2.4)$$

expressing these equations in terms of comoving coordinates and applying conservation of energy in eqn. (2.2), we obtain the famous Friedmann equation which explains how the distance between any two objects evolves with time for a given parameters of k and ρ

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \quad (2.5)$$

where $k = \frac{-2U}{mc^2x^2}$, denotes the curvature of space.

By differentiating the eqn.(2.1) and relating it to eqn.(1.2), we define Hubble parameter ³ as

$$H(t) = \frac{\dot{a}}{a}.$$

2.1.3 Fluid equation

The density of components evolves differently with time for different components. We can derive the relation between density and scale factor using the first law of thermodynamics, viz.

$$TdS = dE + pdV \quad (2.6)$$

where

S - entropy within the sphere of unit comoving radius,

p - pressure inside the sphere.

Consider a volume "V" of unit comoving radius expanding, energy ⁴ "E" contained inside the volume is

$$E = \frac{4\pi}{3}a^3\rho c^2,$$

and

$$V = \frac{4\pi a^3}{3}.$$

³Hubble parameter at present time is known as Hubble constant H_0 .

⁴using $E = mc^2$

Using these two equations in eqn.(2.6); For reversible adiabatic expansion⁵ $dS = 0$ we obtain the fluid equation,

$$\dot{\rho} = -3H\left(\rho + \frac{p}{c^2}\right) \quad (2.7)$$

from the above equation, we can see that there are two factors which contribute to the evolution of the density, volume and pressure of a component.

2.1.4 Equation of state

A parameterised equation describing the relation between pressure and density, here different components are described by its equation of parameter w . For matter and radiation, the equation of state parameter is $w = 0$ and $w = \frac{1}{3}$ respectively and using observational data we can determine whether cosmological constant $w = -1$ is actually constant or quintessence with different value of w .

Substituting the equation of parameter in eqn.(2.7) we get,

$$\dot{\rho} = -3\frac{\dot{a}}{a}(1+w)\rho_w$$

and solving it gives,

$$\rho_w(a) = \rho_{w,0}a^{-3(1+w)} \quad (2.8)$$

From the above equation, we can determine that the matter density ($w = 0$) evolves as

$$\rho_m(a) = \frac{\rho_{m,0}}{a^3}$$

.

From ideal gas law, considering the particles as photons the pressure relates to the energy as $p = \frac{1}{3}n\bar{E}$, where n - number of particles per unit volume and $\bar{E}$ - mean energy of photons. Thus the equation of state parameter for radiation is $w = \frac{1}{3}$, then the radiation density evolves as

$$\rho_{rad}(a) = \frac{\rho_{rad,0}}{a^4}.$$

⁵no transfer of heat with the surroundings

2.2 CURVATURE OF SPACE TIME

Even though the curvature term " k " comes in the Friedmann equation as energy per particle, according to General Relativity it represents the curvature of four-dimensional space time. The properties of different k values which correspond to different geometry of the Universe are as follows,

Flat geometry ($k = 0$): This is the Universe where Euclidean geometry holds; angles of triangles add up to 180° and parallel lines remain parallel. It is infinite in extent to hold the cosmological principle.

Spherical Geometry ($k > 0$): It is best explained by the surface of a sphere as an analogy since it is hard for us to imagine 3d space to be curved. Parallel lines passing through the equator on the surface of the sphere do meet at the poles and triangles large enough drawn on the sphere add up to more than 180° . Spherical geometry is finite in extent and has no edge; a person travelling in a straight line on the surface of a sphere would end up where he started his journey.

Hyperbolic Geometry ($k < 0$): The main property of this curvature is that parallel lines diverge away from each other, due to this the Universe with negative " k " is infinite in extent. This geometry can be imagined as a curved surface of a saddle. Triangles drawn on this surface have a total angle less than 180° .

One of the properties of these geometries is cosmological principle is valid in all of them.

2.3 FRIEDMANN EQUATION IN TERMS OF DENSITY PARAMETERS

Instead of defining each component with its true density, we describe by their relative density; relative to the density (critical density) which would be required to make the Universe flat for a given Hubble parameter. Setting

$k = 0$ in eqn.(2.5) and rearranging it will give us the critical density ⁶,

$$\rho_c(t) = \frac{3H^2}{8\pi G}. \quad (2.9)$$

Now, we can represent any component in terms of its density parameter as

$$\Omega(t) = \frac{\rho}{\rho_c}. \quad (2.10)$$

To obtain the Friedmann eqn in terms of density parameters we do the following rearrangements in eqn.(2.5),

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \sum_w \rho_{w,0} a^{-3(1+w)} - \frac{kc^2}{a^2} \quad (2.11)$$

$$1 = \frac{8\pi G}{3H^2} \sum_w \rho_{w,0} a^{-3(1+w)} - \frac{kc^2}{H^2 a^2}$$

using equation eqn.(2.9) and (2.10) in the above equation,

$$1 = \sum_w \Omega_w a^{-3(1+w)} + \Omega_k \quad (2.12)$$

where $\Omega_k = -\frac{kc^2}{H^2 a^2}$

2.4 DECELERATION PARAMETER " q_0 "

The scale factor $a(t)$ can be approximated around our present time using Taylor series expansion,

$$a(t) = a(t_0) + \left(\frac{da}{dt}\right)_{t=t_0} (t - t_0) + \frac{1}{2} \left(\frac{d^2a}{dt^2}\right)_{t=t_0} (t - t_0)^2 + \dots \quad (2.13)$$

By taking the first three terms alone, we can approximate the scale factor for the near future. Now, dividing the whole equation by a_0 gives us,

$$\frac{a(t)}{a(t_0)} \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2, \quad (2.14)$$

where $q_0 = -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0}$ known as deceleration parameter.

A positive value of q_0 corresponds negative value of $\ddot{a}$ and represents decelerating Universe. A negative value of q_0 corresponds positive value of $\ddot{a}$ and represents the accelerating Universe.

⁶ $\rho_c(t)$ is a function of time, so as Hubble parameter

We can determine the deceleration parameter " q_0 " by representing it in terms of density parameters. Differentiating (Friedmann) eqn.(2.5) with respect to time and with the use of (Fluid) eqn.(2.7) we can obtain the acceleration equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} \sum_w \rho_w (1 + 3w) \quad (2.15)$$

Dividing the whole equation by H^2 and rearranging it gives,

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2} \sum_w \Omega_w (1 + 3w) \quad (2.16)$$

and evaluating the same at the present moment gives the deceleration parameter q_0 .

2.5 DISTANCES IN COSMOLOGY

As the Universe is expanding the distance between any two points is continuously increasing, so the distance between an observer and an object at the time of emission (t_e) of light was much shorter than the distance at present. In a Universe described by Robertson-walker metric, we define proper distance⁷ as the distance between source and observer at the time of observation (t_o) and it is written as,

$$d_p(t_0) = c \int_{t_e}^{t_0} \frac{dt}{a(t)} \quad (2.17)$$

As we cannot measure the proper distance due to expansion, we resort to other practical ways such as parallax distance, angular diameter distance and luminosity distance. For our purposes, we use the latter.

The luminosity distance is calculated for an object of given luminosity and observed flux as,

$$d_L = \left(\frac{L}{4\pi f} \right)^{\frac{1}{2}} \quad (2.18)$$

Unlike proper distance, luminosity distance is affected by curvature as well as expansion. Consider the surface area "A" covered by the light pulse

⁷shortest path followed by a light source(null geodesic)

emitted from a point source, for a positively curved Universe the surface area would be less than $4\pi r^2$, where r is the distance⁸ covered by the light in the flat Universe. For a negatively curved Universe, the surface area would be larger than $4\pi r^2$.

The photons from an object are affected by the expansion in two ways, one from the photon being stretched by the expansion and the second from the increase in time between the observation of photons. These two factors cause the flux to be reduced by a factor of $(1 + z)^{-2}$. Then the luminosity distance d_L in a flat universe is written as

$$d_L = r(1 + z)$$

where r is also a function of redshift, $r(z)$ and in terms of proper distance,

$$d_L = d_p(t_0)(1 + z) = \left(\frac{L}{4\pi f} \right)^{\frac{1}{2}} \quad (2.19)$$

Distance modulus: Astronomers commonly use apparent magnitude "m" and absolute magnitude "M" instead of using flux and luminosity. So, the difference between the apparent magnitude and absolute magnitude is used in terms of luminosity distance,

Apparent magnitude: It denotes the apparent brightness of an object with flux f relative to an object with flux f_x ,

$$m = -2.5 \log_{10} \frac{f}{f_x} \quad (2.20)$$

Absolute magnitude: It denotes the apparent brightness of an object with luminosity L relative to an object with luminosity L_x corresponding to flux f_x if it were kept a distance of 1 parsec, in this case, 1Mpc.

$$M = -2.5 \log_{10} \frac{L}{L_x} \quad (2.21)$$

Then the distance modulus is,

$$M - m = -5 \log_{10} \left(\frac{d_L}{1 \text{Mpc}} \right) - 25 \quad (2.22)$$

⁸where r is equal to the proper distance in a spatially flat universe ($r = d_p(t_0)$)

2.6 STANDARD CANDLES

If we are to measure luminosity distance using the eqn.(2.19), we would need objects with known absolute luminosity "L", this is where the standard candles come in. Standard candles are objects for which the absolute luminosity can be measured from one of the properties of the object and it should also be seen at high redshifts.

Supernovae as standard candles: There are few objects which can be measured at large distances, one of them are supernovae. They are categorized into three groups according to their spectra such as Type-IA, Type-IB and Type II ⁹.

Table 2.1: Supernovas categorized as per their spectra.

Supernovae	Hydrogen lines	Singly ionized silicon lines	Helium lines
Type-1A	No	Yes	No
Type-1B	No	No	Yes
Type-2	Yes	No	No

Type II supernovas are explosions of massive stars at the end of their life when their core collapse into a black hole or neutron star. Type-Ib supernova explosions are similar to Type II but their outer layer of hydrogen is ripped off by a stellar wind thus containing no hydrogen lines. But, Type-IA supernovae are made up of binary systems of which one of them is a white dwarf. As time progresses the white dwarf accretes mass from its companion, eventually crossing Chandrasekhar's limit of 1.4 solar mass and exploding as a supernova.

All Type-IA supernovae have a characteristic peak brightness due to their similar path to the explosion. But, more study into Type-IA supernova has found that there is a spread in their characteristic peak brightness, this difficulty was solved by using the correlation between the light curve and the peak luminosity; they had found shorter peak luminosity corresponds to

⁹Data source: Bernard Jean Trefor Jones, 2017, Precision Cosmology: The First Half Million Years, first edition, Cambridge University Press

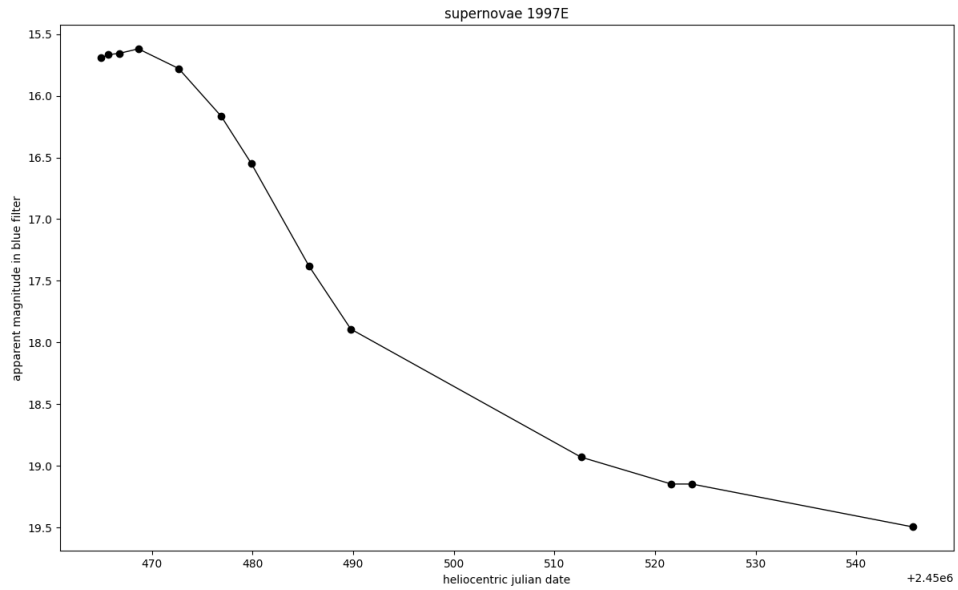


Figure 2.1: plotted the apparent magnitude in blue filter for supernovae 1997E

shorter rise and fall time and higher peak luminosity corresponds to larger rise and fall time which is used to standardize supernovae as standard candles. Figure (2.1) shows a similar light curve¹⁰ of a supernova.

¹⁰Data source: <https://supernova.lbl.gov/Union/>

3 MODELS OF UNIVERSES

The universe in its evolution was dominated by different components at different periods of its life. In the early universe, radiation was the dominant contributor to the energy density and then the universe went from radiation to matter domination. So, evaluating the Friedmann equation for universes with different components lets us know how the Universe would have evolved at that time. For example, in the radiation-dominated era, the universe can be approximated as containing only radiation and around matter-radiation equality the universe can be approximated as having only matter and radiation. Here, we are going to see Universe with different components to demonstrate how the densities of those components affect the luminosity distance.

3.1 FLAT, SINGLE-COMPONENT UNIVERSES

Assuming the Universe to be spatially flat $k = 0$, the Friedmann equation (2.5) becomes,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_w}{3}. \quad (3.1)$$

Solving this using power law for the scale factor and using eqn.(2.8) gives,

$$a(t) = \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}. \quad (3.2)$$

Using the above equation, the Hubble constant is written as

$$H_0 = \frac{2}{3(1+w)}t_0^{-1}. \quad (3.3)$$

Then the age of the Universe can be represented as,

$$t_0 = \frac{2}{3(1+w)}H_0^{-1} \quad (3.4)$$

since the redshift relates to the scale factor as follows,

$$1 + z = \frac{a(t_0)}{a(t_e)} = \left(\frac{t_0}{t_e} \right)^{\frac{2}{3(1+w)}}$$

we can get the time at which the light with redshift z was emitted as,

$$t_e = \frac{2}{3(1+w)H_0(1+z)^{\frac{3(1+w)}{2}}} \quad (3.5)$$

Now using t_e, t_0 and eqn.(3.2), We write the luminosity distance d_L to an object with redshift z as,

$$\begin{aligned} d_L &= d_P(t_0)(1+z) \\ &= \left[c \int_{t_e}^{t_0} \left(\frac{t_0}{t} \right)^{\frac{2}{3(1+w)}} dt \right] (1+z) \\ d_L &= \frac{c}{H_0} \frac{2}{1+3w} \left[1 - (1+z)^{-\frac{(1+3w)}{2}} \right] (1+z) \end{aligned} \quad (3.6)$$

by substituting the corresponding parameter " w " (matter or radiation), we can obtain each of the above values for the single-component universe.

Matter only Universe: For $w = 0$ the age of the Universe and the scale factor is,

$$t_0 = \frac{2}{3H_0}; a_m(t) = \left(\frac{t}{t_0} \right)^{\frac{2}{3}}.$$

Then the luminosity distance becomes,

$$d_L = \frac{2c(1+z)}{H_0} \left[1 - \frac{1}{(1+z)^{\frac{1}{2}}} \right]. \quad (3.7)$$

Cosmological constant only Universe Λ : For a flat lambda only Universe the Friedmann equation becomes,

$$\dot{a}^2 = \frac{8\pi G\rho_\Lambda}{3}a^2,$$

where ρ_Λ is constant and solving this scale factor we get,

$$\begin{aligned} H_0 &= \frac{\dot{a}}{a} = \left(\frac{8\pi G\rho_\Lambda}{3} \right)^{\frac{1}{2}} \\ a_\Lambda(t) &= e^{H_0(t-t_0)}. \end{aligned}$$

Using the above equation to obtain luminosity distance as,

$$d_L = \frac{cz(1+z)}{H_0}. \quad (3.8)$$

3.2 FLAT MULTI-COMPONENT UNIVERSE(Matter and Cosmological constant)

When considering matter and cosmological constant contained flat Universe, the Friedmann equation (2.12) becomes,

$$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} + (1 - \Omega_{m,0}) \quad (3.9)$$

since for flat Universe

$$\Omega_{\Lambda,0} = 1 - \Omega_{m,0}.$$

Different combinations of matter and lambda density parameters can give a flat universe in principle. For example, $\Omega_{m,0} = 1.1$ and $\Omega_{\Lambda,0} = -0.1$ where the expansion would cease eventually due to the negative value of lambda ¹ and start to contract. Our Universe appears to have a positive cosmological constant and a flat Universe, then the scale factor at which the energy density of matter and cosmological constant were equal is written using eqn.(2.8) as,

$$a_{m\Lambda} = \left(\frac{\Omega_{m,0}}{1 - \Omega_{m,0}} \right)^{\frac{1}{3}} \quad (3.10)$$

Integrating the Friedmann equation (3.9) to obtain the Age of the Universe gives,

$$H_0 t = \frac{2}{3(1 - \Omega_{m,0})^{\frac{1}{2}}} \ln \left[\left(\frac{a}{a_{m\Lambda}} \right)^{\frac{3}{2}} + \sqrt{1 + \left(\frac{a}{a_{m\Lambda}} \right)^3} \right] \quad (3.11)$$

Since our universe is currently going through a matter and lambda-dominated era which we will see in the next chapter, the above model closely approximates our present Universe.

3.3 χ^2 TEST

Chi-square is essentially a way of determining the unknown parameters by minimizing the difference between the observed values and theoretical values.

¹negative lambda provides an attractive force

Initially, we construct a function $f(x; a)$ that corresponds to the theory which predicts the observed value, where " x " is a variable and " a " is a parameter. We take the squared difference between the theoretical and observed values for a set of measurements with error " σ ". Finally, the χ^2 over all data points is written as,

$$\chi^2 = \sum_{i=1}^N \left[\frac{y_i - f(x_i; a)}{\sigma_i} \right]^2 \quad (3.12)$$

A generalisation of the above equation is written as,

$$\chi^2 = (\Delta m)^T C^{-1} \Delta m \quad (3.13)$$

where,

Δm is difference between observed and theoretical value,

C^{-1} is inverse covariance matrix.

Using the supernovae's observed luminosity distance and predicted value in the above equation and minimizing it using a Python code gives the values of the unknown parameters Ω_Λ and Ω_m .

4 RESULT AND CONCLUSION

The minimized value of χ^2 using Python is

$$\Omega_{m,0} = 0.2710; \Omega_{\Lambda,0} = 0.6469$$

Using eqn.(2.16) deceleration parameter for the above-minimized values with the given theoretical radiation¹ density parameter $\Omega_{rel,0} = 8.4 * 10^{-5}$ is $q_0 = -0.511$.

The flux of an object received by an observer depends on the expansion. Thus in an accelerating Universe, the flux received by an observer is less than the flux that would be received in a flat decelerating universe. This difference is reflected in the distance modulus of the figure which shows the comparison between the minimized values and the model universes. Since the density parameter of cosmological constant is the dominant component in our Universe, we conclude that the Universe expansion is *accelerating*.

¹where I have considered radiation as photon and neutrino combined

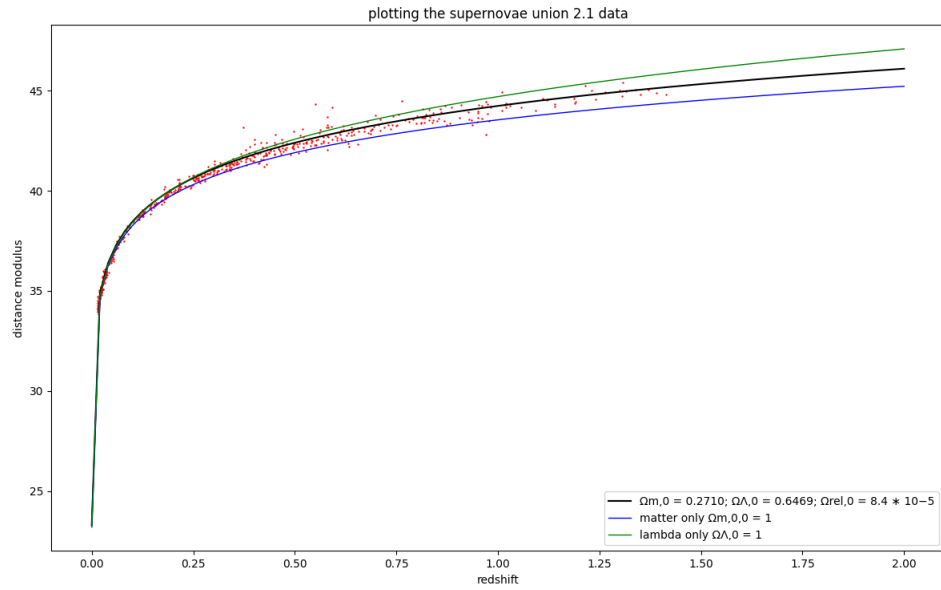


Figure 4.1: Distance modulus versus Redshift.

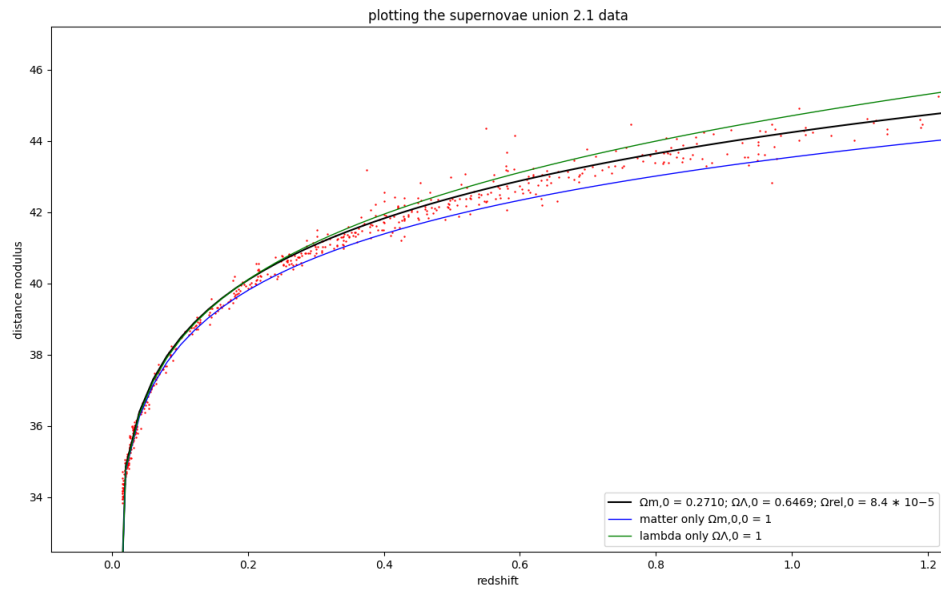


Figure 4.2: Distance modulus vs redshift in the low redshift region

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- Data source: <https://supernova.lbl.gov/Union/>

Appendices

A Code for chi-square test

```
import pandas as pd

import numpy as np

import sympy as smp

from scipy.integrate import quad

from scipy.optimize import minimize


def f(z, om_m, om_r, om_l, om_k):

    return 1 / (-(om_k) * (1 + z) ** 2 + om_m * (1 + z) ** 3 +

om_r * (1 + z) ** 4 + om_l) ** smp.Rational(1, 2)


def p(z, om_m, om_r, om_l, om_k):

    c = float(2.99 * 10 ** 8) # speed of light in meters

    meter_Mpc = float(3.24 * 10 ** -23) # meter to Megaparsec

    H_0 = float(2.2197 * 10 ** -18) # hubble constant


    integral = quad(f, 0, z, args=(om_m, om_r, om_l, om_k))

    l_m = c * meter_Mpc * (1 + z) * integral[0] / H_0

    distance_modulus = float(5 * smp.log(l_m, 10) + 25)

    return distance_modulus


# taking the observed data
```

```

data = pd.read_csv("supernovae data.txt", sep="\t",
names=['name', 'redshift', 'distance modulus', 'upper bound error',
'lower bound error'])
z_data = np.array(data['redshift'].tolist())
dm_data = np.array(data['distance modulus'].tolist())

# importing inverse covariance matrix
icov_data = pd.read_csv("icov.txt", sep="\t")
columns = list(range(len(icov_data.columns)))
icov = pd.read_csv("icov.txt", sep="\t", names=[str(col) for col in columns])
# name argument takes only string not integers
icov0 = icov.dropna(axis=1, how='all')
print(icov0)

# chi square test

def chi(G):
    om_m = G[0]
    om_l = G[1]
    om_r = 1 - om_l - om_m
    om_k = 0

    # taking the theoretical value from function p(z)
    dm_theory = [] # creating an empty array
    for i in range(len(z_data)):
        dm_theory.append(p(z_data[i], om_m, om_r, om_l, om_k))

    difference = dm_data - dm_theory
    difference_tanspose = difference.transpose()
    y = np.matmul(icov0, difference)
    x = np.matmul(difference_tanspose, y)
    return x

```

```
G_initial = [0.3, 0.6]
```

```
chi_min = minimize(chi, G_initial, method='Nelder-Mead')
```

```
print(chi_min)
```

```
luminosity-distance-versus-redshift > chi.py
44
45
46 # taking the theoretical value from function p(z)
47 dm_theory = [] # creating an empty array
48 for i in range(len(z_data)):
49     dm_theory.append(p(z_data[i], om_m, om_r, om_l, om_k))
50
51
52 difference = dm_data - dm_theory
53 difference_tanspose = difference.transpose()
54 y = np.matmul(icov0, difference)
55 x = np.matmul(difference_tanspose, y)
56 return x
57
58 G_initial = [0.3, 0.6]
59
60 chi_min = minimize(chi, G_initial, method='Nelder-Mead')
61 print(chi_min)
```

```
Run: chi
message: Optimization terminated successfully.
success: True
status: 0
fun: 545.1104548980579
x: [ 2.710e-01  6.469e-01]
nit: 24
nfev: 47
final_simplex: (array([[ 2.710e-01,  6.469e-01],
 [ 2.710e-01,  6.478e-01],
 [ 2.710e-01,  6.469e-01]]), array([ 5.451e+02,  5.451e+02,  5.451e+02]))
Process finished with exit code 0
```

Figure A.1: Result.