

UNDERSTANDING THE ANISOTROPIES IN THE COSMIC MICROWAVE BACKGROUND

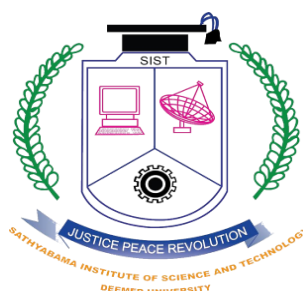
Submitted in partial fulfillment of the requirements for the award of

Master of Science Degree in Physics

By

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DEPARTMENT OF PHYSICS

SCHOOL OF SCIENCE AND HUMANITIES

SATHYABAMA

**INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)**

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BONAFIDE CERTIFICATE

This is to certify that the dissertation entitled "**Understanding the Anisotropies of Cosmic Microwave Background**" is the bonafide work carried out under my direct supervision by **R. Rajesh, (Reg No.41590015)** and jointly with **Dr.L SriramKumar** supervisor from IITM, submitted in partial fulfillment for the award of degree of Master of Science in Physics from Sathyabama Institute of Science and Technology, Chennai - 600 119. No part of the dissertation has been submitted for any degree/diploma or any other academic award anywhere before.

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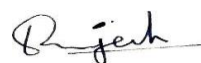
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DECLARATION

I hereby declare that the work entitled here as “**Understanding the Anisotropies in the Cosmic Microwave Background**” is submitted in partial fulfillment of the requirement for the award of the degree in M.Sc, Sathyabama Institute of Science and Technology, Chennai is a record of my own work carried out by me during the academic year 2021–2023 under the supervision and guidance of **Dr.S. Murugesan**, Department of Physics, Sathyabama Institute of Science and Technology, Chennai and **Dr. L Sriramkumar**, Professor, Department of Physics, Indian Institute of Technology Madras, Chennai. The extent and source of information are derived from the existing literature and have been indicated through the dissertation at the appropriate places. The matter embodied in this work is original and has not been submitted for the award of any other degree or diploma, either in this or any other University.

Date: 09-05-2023

Place: Chennai



Signature of the student

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ABSTRACT

Presenting an acceptable narrative of our universe is arguably one of the most significant accomplishments of contemporary cosmology. The project is an attempt to understand the Anisotropies in the Cosmic Microwave Background (CMB). We present the concept of modern cosmology through various equations such as Friedmann, fluid and acceleration equation. Through the lens of these equations, cosmologists have been able to differentiate between the different phases of the universe, such as the matter-dominated era, radiation-dominated era. We try to understand Cosmic Microwave Background including its origin, characteristics and the physics behind it. We also learn how spherical harmonics is used to study the temperature fluctuations of cosmic background map using correlation function.

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LIST OF ABBREVIATIONS

- z - Redshift
- H - Hubble Parameter
- K_B - Boltzmann Constant
- H_0 - Hubble Constant

Chapter 1

INTRODUCTION

The Big Bang theory suggests that the Universe started as a singularity, a point of infinite density and temperature, and has been expanding ever since. Initially, the Steady State Universe theory was proposed as an alternative to the Big Bang theory. This theory argued that the Universe is in a constant state of expansion, with new matter being continuously created to fill the gaps as the Universe expands.

However, the Big Bang theory has become the most widely accepted theory of the Universe's origins, as it is supported by a vast number of arguments, including Cosmic Microwave Background. In this report we try to discuss the origin of the Cosmic Microwave Background, its Anisotropies and the analysis of it.

Chapter 2

Early Universe

2.1 Bigbang Model

The Bigbang model is a cosmological model that describes the origin and evolution of the universe. It is based on the principles of homogeneity and isotropy. According to the Big Bang model, the universe began as an extremely hot and dense state, and has been expanding and cooling down ever since.

The idea of a Big Bang was first proposed by Belgian astronomer Georges Lemaitre in 1927. However, it wasn't until the discovery of the cosmic microwave background radiation in 1965 by Dr. Arno A. Penzias and Dr. Robert W. Wilson at Bell laboratories that the Big Bang model became widely accepted. One of the biggest challenges in big bang model is explaining the observed large-scale structure of the universe, such as the distribution of galaxies and galaxy clusters.

2.2 Expansion of Universe

According to Hubble Law, the farther the galaxy is, the faster it recedes. This is measured by the redshift. It is based on the Doppler shift,(i.e) galaxies have absorption and emission spectrum, if the galaxy is moving towards us blue shift is observed and if it is moving away from us redshift is observed. Redshift of an object can be measured

by

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}} \quad (2.1)$$

Hubble later stated that recessional velocity was proportional to the distance of object from us

$$\vec{v} = H_0 \vec{r} \quad (2.2)$$

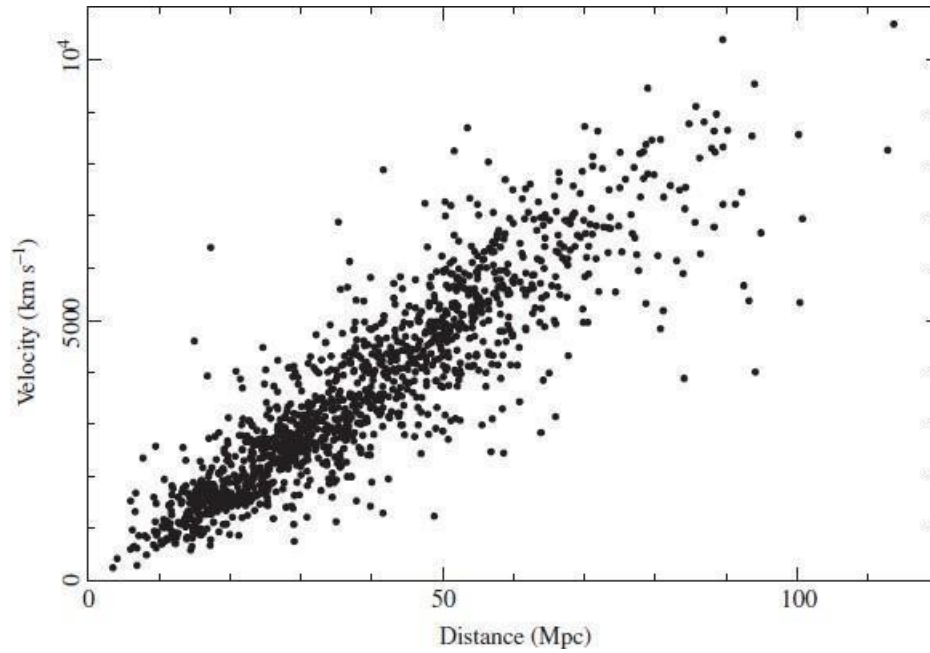


Figure 2.1: Hubble's law implied by a straight line. For a set of 1355 galaxies a plot of velocity vs distance is shown here. Source : An Introduction to modern Cosmology

2.2.1 Friedmann Equation

Friedmann Equation helps in finding the evolution of the universe. Total energy of any particle is derived from the sum of kinetic energy and the potential energy given as,

$$U = T + V \quad (2.3)$$

By Newton's theorem, we know that potential energy is,

$$V = -\frac{GMm}{r} = -\frac{4\pi G\rho r^2 m}{3} \quad (2.4)$$

We know that Kinetic Energy of a particle is taken as,

$$T = \frac{1}{2}mr'^2 \quad (2.5)$$

Solving the above equations and by using energy conservation, we get Friedmann Equation.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \quad (2.6)$$

where, k is a constant, which describes the curvature of space.

2.2.2 Fluid Equation

By First Law of Thermodynamics we know that,

$$dE + pdV = T dS \quad (2.7)$$

For an expanding volume V of unit comoving radius with $V = \frac{4\pi a^3}{3}$

Energy is given by

$$E = \frac{4\pi a^3 \rho c^2}{3} \quad (2.8)$$

Rearranging and using the above equations for adiabatic expansion $dS=0$, we obtain,

$$\rho' + 3 \frac{\dot{a}}{a} \rho + \frac{\dot{p}}{c^2} = 0 \quad (2.9)$$

The above equation is known as Fluid Equation. It describes how the density evolves with time for different components.

2.2.3 Curvature and Geometry

The term "k" signifies the curvature of four-dimensional space-time. The different "k" values correspond to different geometries of the Universe.

Flat Geometry: A Universe with flat geometry ($k = 0$) follows Euclidean geometry, where triangles angles add up to 180 degrees, and parallel lines remain parallel throughout. It is infinite in extent and adheres to the cosmological principle.

Spherical Geometry: A Universe with spherical geometry ($k > 0$) can be best explained by the surface of a sphere, where parallel lines passing through the equator meet at the poles, and triangles drawn on the sphere add up to more than 180 degrees. It has no edge and is finite in extent. If a person travels in a straight line on the surface of a sphere, they would end up where they started their journey.

Hyperbolic Geometry: A Universe with hyperbolic geometry ($k < 0$) has parallel lines diverge away from each other. Due to this, the Universe with negative "k" is infinite in extent. It can be visualized as a curved surface of a saddle, where triangles drawn on this surface have a total angle less than 180 degrees.

2.3 Matter Dominated

Universe at different time period has different component dominating over each other. The expansion rate depends upon the component dominated. By solving Fluid equation with $p=0$, then we have

$$\rho' + 3 \frac{\dot{a}}{a} (\rho) = 0 \quad (2.10)$$

$$\frac{d}{dt} \rho a^3 = 0 \quad (2.11)$$

Integrating we get,

$$\rho \propto \frac{1}{a^3} \quad (2.12)$$

This explains that density of universe is inversely proportional to the volume of the universe.

2.4 Radiation Dominated

For Radiation dominated, we use $p = \frac{\rho c^2}{3}$ in the fluid equation and we

get,

$$\rho' + 4 \frac{\dot{a}}{a} (\rho) = 0 \quad (2.13)$$

By Integrating we get,

$$\rho \propto \frac{1}{a^4} \quad (2.14)$$

This explains that universe will expand more slower in radiation dominated than the matter dominated universe.

Chapter 3

Cosmic Microwave Background

3.1 Discovery of CMB

In the early 1960s, physicists Arno Penzias and Robert Wilson were working at Bell Labs in New Jersey, using a giant, 20-foot horn-shaped antenna to map radio signals from the Milky Way. Their goal was to better understand the structure of the galaxy and its radio emissions. However, when they analyzed their data, they found a uniform signal in the microwave range that seemed to come from all directions, and interfered with their observations.

Penzias and Wilson tried to eliminate all potential sources of the excess radiation, including urban interference, radiation from our own galaxy, and extraterrestrial radio sources, but couldn't find a satisfactory explanation. Eventually, they decided that the problem might be due to droppings from pigeons roosting in the horn-shaped antenna, and spent hours removing the pigeon dung.

Even after this, the excess radiation remained, and Penzias and Wilson started to look for theoretical explanations. At around the same time, physicist Robert Dicke at Princeton University had been working on the idea that if the universe began with the Big Bang, there would be a low-level background radiation throughout the universe at

a temperature of around 3 degrees Kelvin. He had been looking for evidence to support his theory, and when Penzias and Wilson got in touch with his laboratory, he visited Bell Labs and confirmed that the mysterious radio signal was indeed the cosmic microwave background (CMB) radiation, providing proof of the Big Bang.

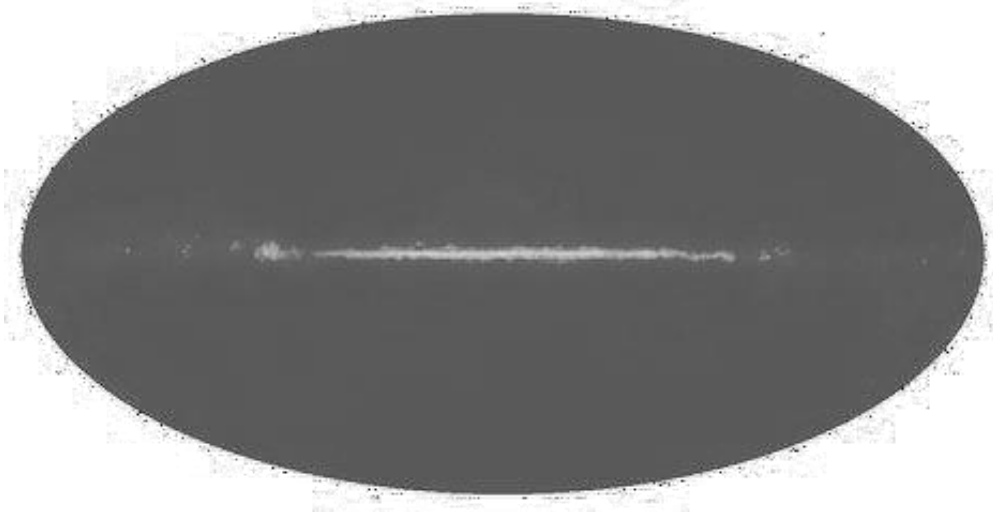


Figure 3.1: CMB Map Found By Penzias and Wilson : Source: Bell Labs

Penzias and Wilson published their results in *Astrophysical Journal Letters*, and were awarded the Nobel Prize in physics in 1978 for their discovery of the CMB. The discovery of the CMB was a pivotal moment in the history of cosmology, as it provided evidence for the Big Bang theory and enabled scientists to study the early universe in new ways. In the decades since, NASA has sent satellites to investigate the CMB in great detail, producing the first detailed maps of the small irregularities, or "ripples," in the microwave background.

The giant radio antenna at Holmdel where Penzias and Wilson made their discovery was designated a National Historic Landmark in 1990, and the lowly pigeon trap they used to remove the pigeons is now one of the key artifacts on permanent display in the Smithsonian

Institute's National Air and Space Museum in Washington, D.C. The discovery of the CMB by Penzias and Wilson was a classic example of a scientific discovery, rather than by intent, and it opened up new possibilities for understanding the universe and its origins.

As Earth is surrounded by uniform microwave radiation, with a black-body distribution, at a temperature of approximately 3K, from all directions, We will be discussing it in more details in the section on recombination. It is confirmed that this observation has an extremely good accuracy to black body from observations made by the COBE (Cosmic Background Explorer) satellite.

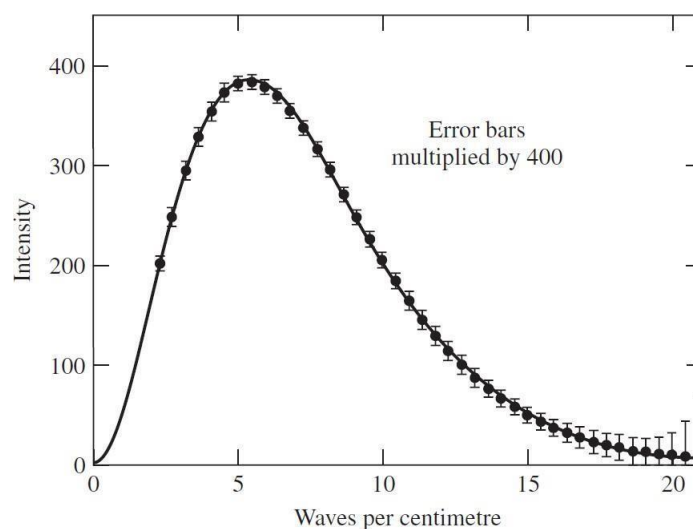


Figure 3.2: CMB Map by COBE Satellite depicting blackbody spectrum Source: Introduction to Cosmology Barbara Ryden

3.2 Origin of CMB

The CMB is essentially the afterglow of the Big Bang, which was created when the universe was still extremely hot and dense. At that time, the universe was filled with a hot, dense plasma of particles that scattered light in all directions. As the universe expanded and cooled,

this plasma began to cool and neutralize, allowing light to travel freely through space. The light from this early stage of the universe has been stretched and redshifted by the expansion of the universe over time, resulting in a faint glow of microwave radiation that is visible in all directions.

The CMB provides a unique window into the early universe, allowing us to study the conditions that existed just 380,000 years after the Big Bang. By analyzing the subtle fluctuations in the CMB, scientists have been able to learn a great deal about the composition and evolution of the universe. For example, these fluctuations have helped to confirm the existence of dark matter and dark energy, which together make up about 95% of the universe's total mass-energy.

In addition to providing insights into the early universe, the CMB has also been used to measure other important cosmological parameters, such as the age of the universe and the rate of its expansion. By analyzing the CMB, scientists have been able to create a detailed model of the universe's evolution, which is consistent with a range of other astronomical observations.

Overall, the accidental discovery of the CMB has turned out to be a key piece of evidence for the Big Bang theory, providing crucial insights into the early universe and its evolution over time. It has become an important tool for cosmologists, enabling them to test their theories and refine our understanding of the universe. CMB has exact blackbody temperature.

Event	Redshift	Temp (K)	Time(MYr)
Radiation-Matter Equality	3440	9390	0.050
Recombination	1380	3760	0.25
Decoupling	1090	2970	0.37
Last Scattering	1090	2970	0.37

Table 3.1: Events of Universe, Source: Introduction to Cosmology, Barbara Ryden

3.3 Recombination

Epoch of Recombination give us a basic understanding of how baryonic matter varies from ionised state to neutral state As the ionisation energy of the hydrogen is 13.6 eV, any photon with greater energy than the ionisation energy of hydrogen, then it is capable of photo ion- ize an hydrogen. During Recombination the baryonic component goes from ionised state to neutral state.Fractional Ionization X is defined as the degree to which baryonic component is ionised

$$X = \frac{n_p}{n_p + n_H} = \frac{n_e}{n_H} = \frac{n_e}{n_H} \quad (3.1)$$

The value varies from X=1 Fully Ionised and X=0 Fully Neutral.

Before scattering occurs,the mean distance a photon travels is given by

$$\lambda = \frac{1}{n_e \sigma_e} \quad (3.2)$$

Scattering rate of photons travelling with speed c is given by,

$$\Gamma = \frac{c}{\lambda} = n_e \sigma_e c \quad (3.3)$$

As we know early universe was fully ionised , the electron density was,

$$n_e = \frac{n_{bar,0}}{a^3} \quad (3.4)$$

Substituting above equation, We get

$$\Gamma = \frac{n_{bar,0} \sigma_e c}{a^3} \quad (3.5)$$

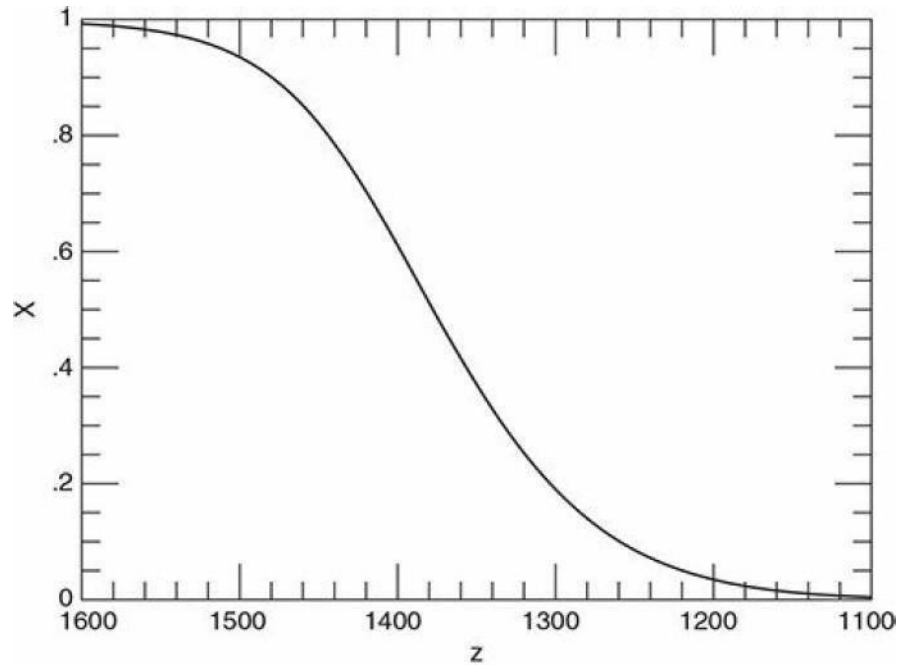


Figure 3.3: Graph depicting fractional Ionization X vs Redshift z

At $a = 10^{-5}$, $\Gamma = 5.0 \times 10^{-6} \text{s}^{-1}$ As long as the scattering rate Γ is larger than H , the Hubble Parameter the photons remained coupled to the electrons. When Γ becomes lesser than H then photons decouple from electrons and the universe becomes transparent. During radiation dominated time the Friedmann Equation was

$$\frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} \quad (3.6)$$

Thus the Hubble Parameter

$$H = \frac{H_0 \Omega_r^{1/2}}{a^2} \quad (3.7)$$

This means at $a = 10^{-5}$ The Hubble Parameter H is much smaller than scattering rate Γ , Therefore the photons were well coupled with electrons and protons

3.3.1 Physics Behind Recombination

The temperature at the time of recombination is given by $E = 3K_B T$.

$$T = \frac{E}{3K_B} \quad (3.8)$$

Then taking the ionization of energy and the value of boltzmann constant into account, the temperature at the time of recombination is calculated as 60,000 k approximately. Assuming $n_x(p)dp$ as the number density of x particles with momentum $p \rightarrow p+dp$ and the particles are in kinetic equilibrium then

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \frac{p^2 dp}{\exp([E - \mu_x]/kT) \pm 1} \quad (3.9)$$

By substituting the values and integrating the above equation for all photon frequencies we get,

$$n_\gamma = 0.2436 \frac{kT^3}{\hbar c} \quad (3.10)$$

At the time of Recombination $mc^2 \gg KT$ that is having high non relativistic thermal speeds, So we can use the approximation $p \approx m_x v$ Therefore the equation

$$E \approx m_x c^2 + \frac{m_x v^2}{2} \quad (3.11)$$

becomes,

$$E \approx m_x c^2 + \frac{p^2}{2m_x} \quad (3.12)$$

Substituting this values and by further integration we get

$$n_x = g_x \frac{m_x kT}{2\pi \hbar^2}^{3/2} \exp \frac{-m_x c^2 + \mu_x}{kT} \quad (3.13)$$

By assuming the following equation to be in chemical equilibrium at the time of recombination,

$$H + \gamma \rightleftharpoons p + e^- \quad (3.14)$$

Using the above equation and further simplifying the equations we get to find n_H, n_p, n_e , the solution is Saha Equation.

$$\frac{n_H}{n_p n_e} = \frac{m_e kT}{2\pi \hbar^2}^{-3/2} \exp \frac{Q}{kT} \quad (3.15)$$

Converting Saha Equation into relation between X, T, η and further calculations we finally get the equation for X in terms of T and η

$$\frac{1-X}{X^2} = 3.84\eta \frac{kT}{m_e c^2}^{3/2} \exp \frac{Q}{kT} \quad (3.16)$$

By solving the above quadratic equation we finally get,

$$S(T, \eta) = 3.84\eta \frac{kT}{m_e c^2}^{3/2} \exp \frac{Q}{kT} \quad (3.17)$$

Since S depends on the exponential of Temperature, value of η doesn't affect Recombination Temperature.

Chapter 4

Anisotropies of CMB

The CMB is a nearly uniform glow of microwave radiation that is visible in all directions, but it is not perfectly uniform. There are small variations, or anisotropies, in the temperature of the CMB across the sky, which contain valuable information about the early universe.

The Anisotropies in the CMB are caused by fluctuations in the density of matter in the early universe, which were imprinted on the CMB as it traveled through space. These fluctuations were the result of quantum fluctuations in the early universe, which were amplified by the process of inflation, a period of rapid expansion that occurred in the first fractions of a second after the Big Bang.

The Anisotropies in the CMB are extremely small, with temperature variations of only a few parts in 100,000. However, they contain a wealth of information about the early universe. By analyzing these anisotropies, scientists have been able to learn about the composition and structure of the early universe, as well as its evolution over time.

One of the most important features of the anisotropies in the CMB is their scale dependence. The temperature variations are not evenly distributed across all scales, but are concentrated at certain angular

scales. This pattern of variation is known as the angular power spectrum, which describes how much power is contained in the temperature fluctuations at different scales.

The angular power spectrum of the CMB Anisotropies has been measured with great precision by a number of experiments, including the Planck satellite mission. The spectrum contains several peaks and troughs, which correspond to different physical scales in the early universe. The first peak, for example, is thought to correspond to the size of the "sound horizon" at the time when the CMB was emitted, which is determined by the speed of sound and the temperature of the plasma in the early universe.

By comparing the observed angular power spectrum of the CMB with theoretical models, scientists have been able to constrain a range of cosmological parameters, such as the density of matter and the rate of expansion of the universe. This has helped to refine our understanding of the early universe and to test the validity of the Big Bang theory.

Overall, the anisotropies in the CMB provide a powerful tool for studying the early universe and for testing our models of cosmology. The precision measurements of the CMB by experiments such as Planck have enabled cosmologists to make remarkable progress in understanding the composition and evolution of the universe.

4.1 Causes of Anisotropies

The main peak is due to the sound horizon at the time of recombination which depends on the baryonic density. The second peak is due to the baryonic dark matter and the third peak is because of general dark matter.

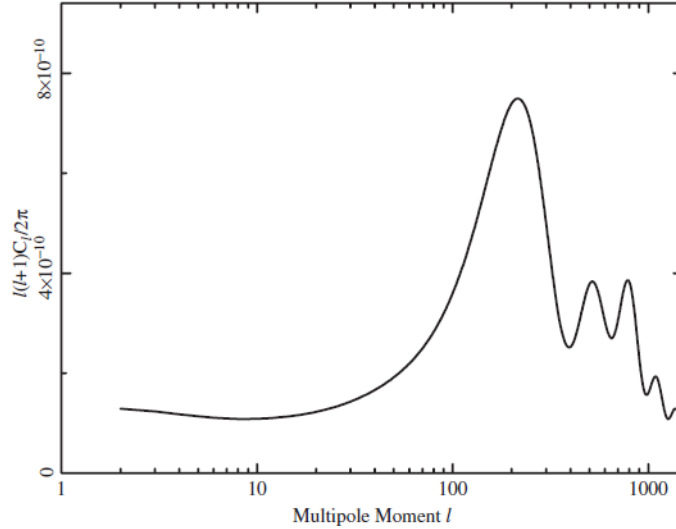


Figure 4.1: WMAP Data of CMB with errorbars Source: Introduction to Cosmology

4.2 Temperature Fluctuations

Cosmic Microwave Background anisotropies are so small to measure. However, the temperature fluctuations $T(\theta, \phi)$ can be measured on a given direction on the sky. The mean temperature over the sky is taken as,

$$\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - T^-}{T^-} \quad (4.1)$$

By expanding the above equation using spherical harmonics, We get,

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^l a_{lm} Y_{lm}(\theta, \phi) \quad (4.2)$$

The Y_m^l here is the spherical Harmonic Function. This is used in defining a function on a sphere.

$$Y_m^l(\theta, \phi) = \frac{\sqrt{(2l+1)(l-m)!}}{4\pi(l+m)!} \exp(im\phi) P_m^l(\cos \theta) \quad (4.3)$$

Our aim is to find the correlation function over the sky map. To find it, we first consider two points over the sky in the directions $\hat{n}$ and $\hat{n}'$ separated by an angle θ with the relation $\hat{n}_1 \cdot \hat{n}_2 = \cos \theta$. Then the

Correlation Function $C(\theta)$ is given by,

$$C(\theta) = \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \quad \hat{n}_1 \cdot \hat{n}_2 = \cos \theta \quad (4.4)$$

Using Spherical Harmonics to expand the above equation,

$$C(\theta) = \sum_{l_1=0}^{\infty} \sum_{m_1=-l_1}^{l_1} a_{l_1 m_1} Y_{l_1 m_1}(\hat{n}_1) \sum_{l_2=0}^{\infty} \sum_{m_2=-l_2}^{l_2} a_{l_2 m_2} Y_{l_2 m_2}(\hat{n}_2) \quad \hat{n}_1 \cdot \hat{n}_2 = \cos \theta \quad (4.5)$$

$$C(\theta) = \sum_{l_1} \sum_{l_2} \sum_{m_1} \sum_{m_2} a_{l_1 m_1} a_{l_2 m_2}^* Y_{l_1 m_1}(\hat{n}_1) Y_{l_2 m_2}^*(\hat{n}_2) \quad (4.6)$$

$$= \sum_{l_1} \sum_{l_2} \sum_{m_1} \sum_{m_2} a_{l_1 m_1} a_{l_2 m_2}^* Y_{l_1 m_1}(\hat{n}_1) Y_{l_2 m_2}^*(\hat{n}_2) \quad (4.7)$$

Since we Know

$$a_{l_1 m_1} a_{l_2 m_2}^* = C_l \delta_{l_1 l_2} \delta_{m_1 m_2} \quad (4.8)$$

Here δ_{xy} are Kronecker Delta. Therefore the above becomes

$$C(\theta) = \sum_{l_1} C_{l_1} \sum_{m_1} Y_{l_1 m_1}(\hat{n}_1) Y_{l_1 m_1}^*(\hat{n}_2) \quad (4.9)$$

With Legendre Polynomials, We know

$$P_l(\cos \theta) = \frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{n}_1) Y_{lm}(\hat{n}_2) \quad (4.10)$$

$$\sum_m Y_{lm}(\hat{n}_1) Y_{lm}(\hat{n}_2) = P_l(\cos \theta) \frac{2l+1}{4\pi} \quad (4.11)$$

Substituting the desired equations, We get

$$C(\theta) = \frac{1}{4\pi} \sum_{l=0} (2l+1) C_l P_l(\cos \theta) \quad (4.12)$$

The above equation is the Correlation function. If we define the mean temperature precise then the monopole ($l=0$) can be neglected. For larger values of l the term C_l is the measure of fluctuations in temperature on an angular scale that is $\theta \sim \frac{1}{80}^\circ$

The multipole moment ($l>2$) gives us the data of fluctuations present at the time of last scattering. By measuring this temperature fluctuations, we may obtain a wealthy information about universe at the time of Photon Decoupling.

Chapter 5

Conclusions

So at the end of the thesis we can check what we have gone through. We started from the base of cosmological principle and then have gone through the equations governing the expansion of the universe. Then we saw Cosmic Microwave Background, a golden piece of evidence for modern cosmology and how does it arise by observing the physics of recombination behind it. We analysed the anisotropies caused in the microwave background and why does it arise and how the correlation function is derived from Spherical harmonics. I am ending the thesis here after discussing the various causes for the anisotropies and the physics behind it.

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