

**NUMERICAL STUDY OF POPULATION GROWTH USING
RUNGE KUTTA METHOD IN MATLAB**

Submitted in partial fulfilment of the requirements for the award of

Bachelor of Science Degree in Mathematics

By

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INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited with Grade “A” by

NAAC I 12B Status by UGC I Approved by AICTE

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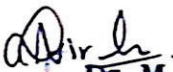
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DEPARTMENT OF MATHEMATICS BONAFIDE CERTIFICATE

This is to certify that this project report is the bonafide work of PAVUNRAJ.P (40060010) & PONSWEETHA.S (40060010) who carried out the project entitled "NUMERICAL STUDY OF POPULATION GROWTH USING RUNGE KUTTA METHOD IN MATLAB" under our supervision from JANUARY 2023 to APRIL 2023.

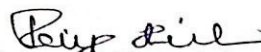

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DECLARATION

We are PAVUNRAJ.P (40060010) & PONSWETHA.S(40060011) hereby declare that the project report entitled "NUMERICAL STUDY OF POPULATION GROWTH USING RUNGE KUTTA METHOD IN MATLAB" done has us under the guidance of Dr. M. PRASANNA JEYANTHI (Internal) at SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY submitted in partial fulfilment of the requirements for the award of Bachelor of Science Degree in Mathematics

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Abstract

This numerical study explores the population growth by using the Runge-Kutta method in MATLAB software. The Runge-Kutta method is a popular algorithm used to approximate solutions of differential equations. In this study, we apply the method to a population growth model with different allee effect, which is described by a first-order ordinary differential equation. Using the Runge-Kutta method, we simulate the population growth over time and analyse the impact of various parameters on the population dynamics. The result shows the graph for Runge Kutta method and different allee values. Overall, this study demonstrates the effectiveness of the Runge-Kutta method in solving complex population growth models and provides a useful tool for understanding the dynamics of populations in various in environments.

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Chapter -1

INTRODUCTION:

Differential equations have been a major branch of pure and applied mathematics and their inauguration is from the mid-17th century. The study of the rate equation in calculus is called differential equation. If there is only one independent variable with its derivatives is called ordinary differential equation.

- **INITIAL VALUE PROBLEM-** Ordinary differential equation is together with initial condition which specifies the value of unknown function at a given point in the domain.
- **BOUNDARY VALUE PROBLEM-** A system of ordinary differential equations with solution and derivative are specified at more than one point.

A modeling is an abstract description of a concrete system using mathematical concepts and language. The process of developing a mathematical model is termed as modelling.

ODE is a mathematical tool used to measure the change of a quantity which is determined by another quantity. The change of a quantity is called derivative.

Population modeling is a common application of ordinary differential equations and can be studied even the linear case. In this study we will investigate some cases of population growth models.

The population of any country does not remain constant. It generally increases. The relative increase of population over an Interval of time is called **population growth**. Growth per unit time is called growth rate. The growth of the population is also affected by the number of deaths, in-migrants, and out-migrants at the end of the given period of time. The two types of population growth models are Exponential growth, logistic growth .in the present study we will concentrate on the numerical solution of logistic growth model with Allee effect.

Population Model:

In 1798 the English economist THOMAS ROBERT MALTHUS proposed a mathematical model of population growth model. A population growth can also be defined as a collection of individuals that can be described by a set of parameters such as the population size, growth rate, carrying capacity, birth and death rates, and migration rates. These parameters can be used to construct mathematical models that simulate the dynamics of the population over time.

$$P_T = P \cdot (1 + (R/100))^T - D + \text{Min} - M_{\text{out}}$$

Where,

D -----> Number of deaths

Min -----> In-migrants

M out -----> Out-migrants

Exponential growth:

Exponential growth is a function that shows an increase within a population that occurs at the same rate over time. When populations experience doubling or tripling in numbers, you can assume the data increases exponentially. The opposite of exponential growth is exponential decay, where data shrinks rather than grows. The pattern an exponential function shows appears as an upward curve when you visualize your data on a graph. You can also calculate exponential growth using the formula $f(x) = a(1 + r)^x$.

Logistic growth:

Logistic growth is a type of population growth that occurs when the growth rate of a population decreases as the population approaches its carrying capacity. The carrying capacity is the maximum number of individuals that a given environment can support sustainably. In logistic growth, the population initially grows exponentially at a constant rate, but eventually reaches a point where the growth rate slows down due to resource limitations.

Density-independent growth: At times, populations invade new habitats that contain abundant resources. For a while at least, these populations can grow rapidly because the initial number of individuals is small and there is no competition for resources. This is called density-independent growth because the density of individuals does not have any effect on future growth. As you can imagine, this cannot continue indefinitely. The first person to mathematically describe a population's potential to reproduce was by Thomas Malthus, and his writings would influence by the ideas of Charles Darwin.

Comparison of Logistic growth and Exponential growth:

There are two population growth models, exponential and logistic growth. Exponential growth is seen only in ideal conditions. If plotted on a graph, it would show a J shaped curve. Logistic mode of growth is the one that is seen in the real world. This kind of growth is limited due to various factors such as limited resources, unfavourable conditions etc. Logistic growth forms a sigmoid or S-shaped graph. Exponential growth depends on the size of the population and occur when the resources are plentiful. Logistic Growth depends on the size of population competition and the amount of resources or it occurs when the resources are limited. Exponential growth does not frequently reaches a stationary phase and does not have an upper limit. Logistic growth reaches a stationary phase and it has an upper limit called the carrying capacity. Exponential growth has two phases namely lag and log phase. It causes an explosion of the population. The population crash occurs due to mass mortality. Logistic growth has four phases namely lag, log, deceleration and stationary. It causes relatively constant growth rate in the population. This population crash occurs very rarely. Exponential growth commonality does not occur. Logistical growth commonality often occurs.

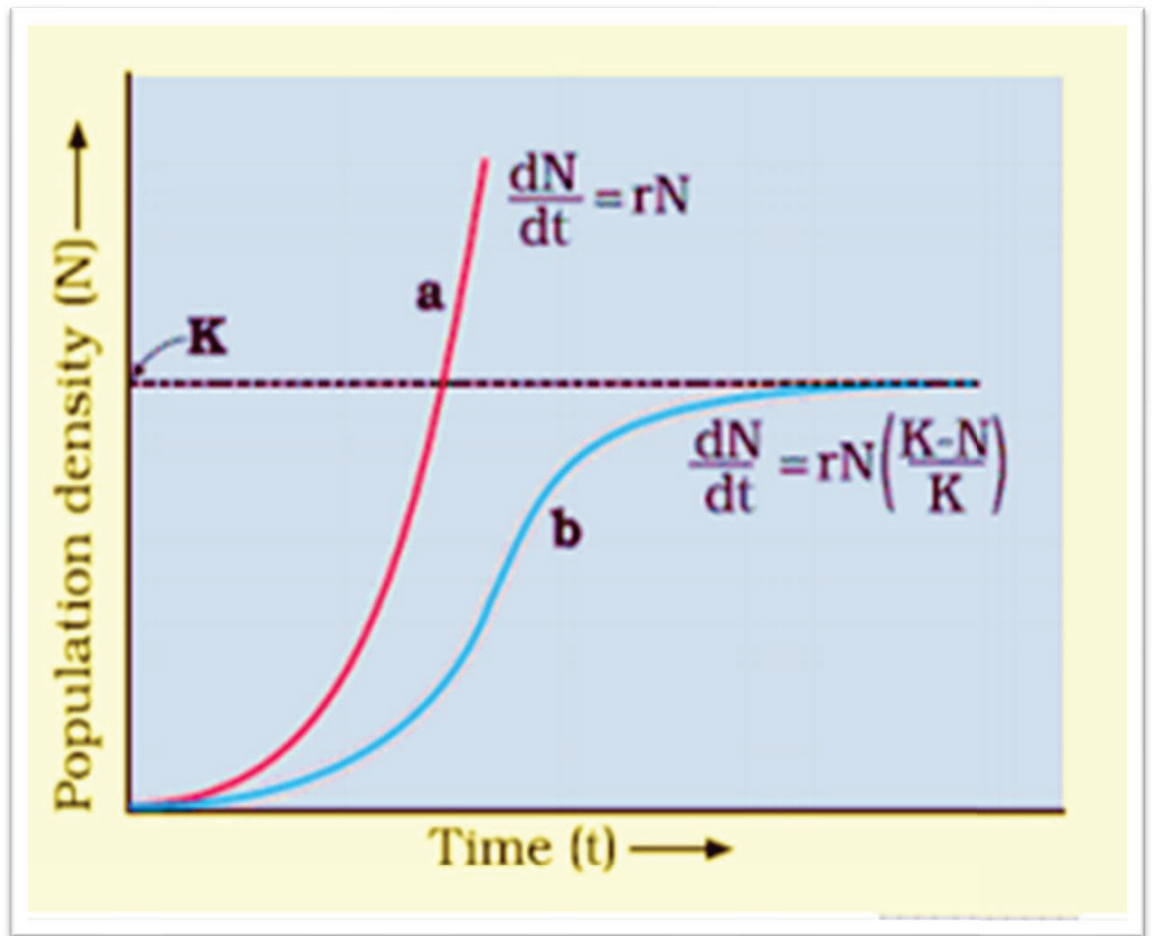


Fig :

Line A - Exponential growth

Line B - Logistic growth

CHAPTER – 2

Literature survey of Runge kutta methods:

The Runge-Kutta methods continue to be an active area of research, and new developments and improvements to the methods are being published regularly in various journals and conference proceedings.

Here are some recent developments in the field of Runge-Kutta methods:

Improved Error Analysis: Researchers have been working on improving the error analysis of Runge-Kutta methods, with the goal of better understanding the accuracy and stability of the methods.

High-Order Methods: Researchers have developed high-order Runge-Kutta methods, which offer improved accuracy over the classical RK4 method. These high-order methods are particularly useful for solving problems with rapidly changing solutions.

Adaptive Step-Size Methods: Researchers have developed adaptive step-size Runge-Kutta methods, which automatically adjust the step size during the solution process to balance accuracy and computational cost. These methods have proven to be particularly useful for solving problems with complex solutions.

Parallel Computing: Researchers have been exploring the use of parallel computing to improve the computational performance of Runge-Kutta methods. This work has led to the development of parallel algorithms for solving ODEs using Runge-Kutta methods.

Applications: The Runge-Kutta methods continue to be widely used in various fields, and researchers have been exploring new applications of the methods, including problems in finance, biology, and engineering.

These are some of the recent developments in the field of Runge-Kutta methods. To stay up-to-date on the latest research, I recommend regularly reviewing journals such as the Journal of Numerical Mathematics, Numerical Algorithms, and the Journal of Scientific Computing.

CHAPTER -3

ALLEE EFFECT:

Allee effect is positive density dependence, or the positive correlation between population density and individual fitness. The Allee effect is a process in Biology identified by a correspondence between density or population size and the mean individual fitness. Allee effect is categorized by the essence of dependence on density which is relatively lower.

When the population shrinks to lower densities and a weak Allee effect when the proliferation rate is high and positive. The null hypothesis is when the proliferation rates are decreasing at low density but are still positive. The difference between the two types is distinctly based on the property of showing critical population density or size by a specific region.

A population showing a sturdy Allee effect will have a critical density or population size where an increase becomes negative. On the other hand, the Allee-effect exhibiting population will have a decreased per capita rate of growth every week at low population size which always exhibits a positive per capita rate of growth. Hence a population will become extinct when the population size hits a level below the edge.

Allee witnessed that aggregation was at positive effects on land isopods and their survival which when subjected to isolation desiccated rapidly. His fact-based instances of the benefits of aggregation conflicted with the Logistic model and the Malthusian paradigm. Taking into consideration both kinds of Allee effects, the following view can be generalized: An Allee effect is indeed a positive link between the absolute average individual fitness and population size over a period of time. This association may however it produces a critical population density to a point where the population cannot persist.

Types of Allee effect:

Demographic allee effect - the positive relationship between the overall individual fitness and population density.

Component Allee Effect -The positive relationship between any measurable component of individual fitness and population density

The difference between the 2 effects is often understood by the measure of the allee effect. For a demographic Allee effect to exist, there should be a minimum of one component allee effect. The demographic Allee effect won't necessarily exist if component Allee effects exist. For eg., collaborative hunting and therefore the ability to seek out mates are component Allee effects, since they influence the fitness of people during a population.

When the population is low, the sum of component Allee effects produces demographic allee effects. But in the case of increased population density, the negative density dependence is compensated by the component Allee effects through competition for resources, this results in depleting the demographic Allee effect.

The obvious conclusion that the growth rate of a population (dN/dt , where $N(t)$ is the population size at time t) depended on (i) the birth rate b and (ii) the mortality rate m , both of which would vary in direct proportion to the size of the population N :

$$\frac{dN}{dt} = bN - mN \quad \text{----- (1)}$$

After combining b and m into one parameter r , called the intrinsic rate of natural increase—or more usually by biologists without the time to get their tongues around that, just r —equation (1) becomes

$$\frac{dN}{dt} = rN \quad \text{----- (2)}$$

This model of population growth has a problem, which should be clear if not, plot dN/dt for increasing values of N . It is a straightforward exponential growth curve. Eventually it has to step in and slow down dN/dt . A differential equation for the population $N(t)$ that included both r and the carrying capacity K :

$$\frac{dN}{dt} = rN \left(1 - \frac{N}{K}\right), \quad r > 0. \quad \text{----- (3)}$$

Equation (3) is called the logistic equation.. Hopefully, it is clear that the term $(1 - N/K)$, which is Verhulst's contribution to equation (2), is $(1 - N/K) = 1$ when $N = 0$, leading to exponential growth, and $(1 - N/K) = 0$ as $N \rightarrow K$, hence it causes the growth curve of $N(t)$ to approach the horizontal asymptote $N(t)=K$. Thus the size of the population cannot exceed the carrying capacity of the environment.

The logistic equation (3) gives the overall growth rate of the population, if we consider per capita growth rate that is, the growth rate of the population per the number of individuals in the population. To get per capita growth rate, we just divide each side of equation (3) by N :

$$\frac{1}{N} \frac{dN}{dt} = r \left(1 - \frac{N}{K}\right) = r - \frac{r}{K} N$$

This second version of (3) immediately shows (or plot it) that this relationship is a straight line with a maximum value of $\frac{1}{N} \cdot \frac{dN}{dt}$ at $N = 0$ (assuming that negative population sizes are not relevant) and $\frac{dN}{dt} = 0$ at $N = K$. As a result of Allee's work, a population where the per capita growth rate declines at low population size is said to show an Allee effect. The Allee effects can be modelled in many ways. One of the simplest mathematical models, a variation of the logistic equation, is:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)\left(\frac{N}{A} - 1\right) . \quad \text{----- (4)}$$

where A is called the Allee threshold. The value $N(t) \leq A$ is the population size below which the population growth rate becomes negative due to an Allee effect—situated a value of N somewhere between $N \leq 0$ and $N \leq K$, that is, $0 \leq A \leq K$, depending on the species.

CHAPTER -4

RUNGE KUTTA METHODS:

Runge–Kutta method is an effective and widely used method for solving the initial-value problems of differential equations. Runge–Kutta method can be used to construct high order accurate numerical method by functions self without needing the high order derivatives of functions.

Origin Of Runge Kutta Method:

The Runge-Kutta methods are a family of numerical methods for solving ordinary differential equations (ODEs). The methods were developed by the German mathematician Carl Runge and the German physicist Wilhelm Kutta in the late 19th and early 20th centuries

Runge first published his work on numerical methods for solving ODEs in 1895. He developed an early form of the Runge-Kutta method, which was a direct extension of Euler's method for solving ODEs. This early form of the Runge-Kutta method was based on the arithmetic mean of intermediate estimates and had limited accuracy.

In 1901, Kutta further developed Runge's work and introduced a more accurate form of the method, which is now known as the "classical" Runge-Kutta method (RK4). The RK4 method uses a weighted average of four intermediate estimates, which significantly improves the accuracy of the method.

Since the development of the classical Runge-Kutta method by Runge and Kutta, the family of methods has been further developed and extended to include a variety of methods that balance accuracy and computational cost. The Runge-Kutta methods are now widely used in various fields, including physics, engineering, finance, and biology, to solve ODEs and other related problems.

Advantages of Runge kutta method:

The Runge-Kutta method is a numerical method used for solving ordinary differential equations (ODEs) and is widely used in many scientific and engineering fields. It is popular because it is both efficient and accurate, and it can handle a wide range of different types of ODEs.

Compared to other existing methods, such as the Euler method, the Runge-Kutta method generally provides more accurate results for the same number of computational steps. This is because it uses multiple evaluations of the derivative at different points within a time step, which helps to capture the behaviour of the solution more accurately.

Another advantage of the Runge-Kutta method is that it allows for adaptive step sizes, which can improve the efficiency and accuracy of the method. By using smaller steps in regions where the solution is changing more rapidly and larger steps where the solution is changing more slowly, the method can optimize the balance between accuracy and computational efficiency.

Additionally, the Runge-Kutta method can handle stiff differential equations, which are difficult to solve with other methods. Stiff differential equations are those where the solution varies rapidly in some regions and slowly in others, and numerical methods that are not designed to handle these types of equations can be unstable or require extremely small step sizes.

Overall, the Runge-Kutta method is a well-established and versatile method for solving ODEs and has proven to be more accurate and efficient than many other existing methods, especially for complex and stiff equations.

ORDERS IN RUNGE KUTTA METHOD:

Here are some of the commonly used Runge-Kutta methods based on different approaches:

Classic Runge-Kutta Methods:

- RK4 (Classical Runge-Kutta Method)
- RK3/RK3(2)
- RK2 (Heun's Method)
- RK1 (Euler's Method)

Runge-Kutta-Fehlberg Method (RKF45):

- A variable step-size method that balances accuracy and computational cost.

Adaptive Step-Size Runge-Kutta Methods:

- Dormand-Prince (RK45) Method
- Bogacki-Shampine (RK23) Method

Runge-Kutta-Nyström Methods:

- Gaussian Quadrature RKN (GRKN)
- Extended Trapezoidal RKN (ETRKN)

Runge-Kutta-Chebyshev Methods:

- Runge-Kutta-Chebyshev (RKC) Method

Embedded Runge-Kutta Methods:

- Cash-Karp (RK5(4)) Method
- Fehlberg (RKF7(8)) Method

These methods vary in terms of their accuracy, stability, and computational cost. The choice of the method depends on the specific problem being solved and the desired trade-off between accuracy and computational cost. In this we are going to concentrate only on fourth order RK method

The Runge-Kutta methods are typically based on weighted averages of intermediate estimates, and the choice of weights depends on the particular method being used. Some common weighting schemes used in the development of Runge-Kutta methods include:

Arithmetic Mean: The most basic weighting scheme, where the intermediate estimates are simply averaged together.

Harmonic Mean: The inverse of the average of the reciprocals of the intermediate estimates.

Geometric Mean: The n th root of the product of the intermediate estimates.

Midpoint Rule: A specific type of arithmetic mean, where the intermediate estimates are taken at the midpoint of the interval being considered.

Trapezoidal Rule: A specific type of arithmetic mean, where the intermediate estimates are taken at the endpoints of the interval being considered and weighted by their proximity to the endpoints.

Simpson's Rule: A specific type of arithmetic mean, where the intermediate estimates are taken at the endpoints and midpoint of the interval being considered and weighted by their proximity to the endpoints and midpoint.

These are some of the most common weighting schemes used in the development of Runge-Kutta methods. The choice of the weighting scheme depends on the specific problem being solved and the desired trade-off between accuracy and computational cost.

CHAPTER 5

NUMERICAL EXAMPLE:

In this chapter the first order ordinary differential equations are taken into consideration for testing the population problem with different Allee effect in Runge Kutta method.

The equation $\frac{dN}{dt} = rN(1 - \frac{N}{K})(\frac{N}{A} - 1)$ can be solved explicitly for N(t) using the technique of partial fractions. Do this, and plot N(t), where appropriate values for r, K, and N (0) are r=1, K=1, N (0)=0.01

```
% Define the time range
```

```
tspan = [0 50];
```

```
% Define the initial condition
```

```
N0 = 0.01;
```

```
% Define the differential equation
```

```
dNdt = @(t,N) r*N*(1-N/k)*(N/a-1);
```

```
% Solve the differential equation using ode45
```

```
[t,N] = ode45(dNdt, tspan, N0);
```

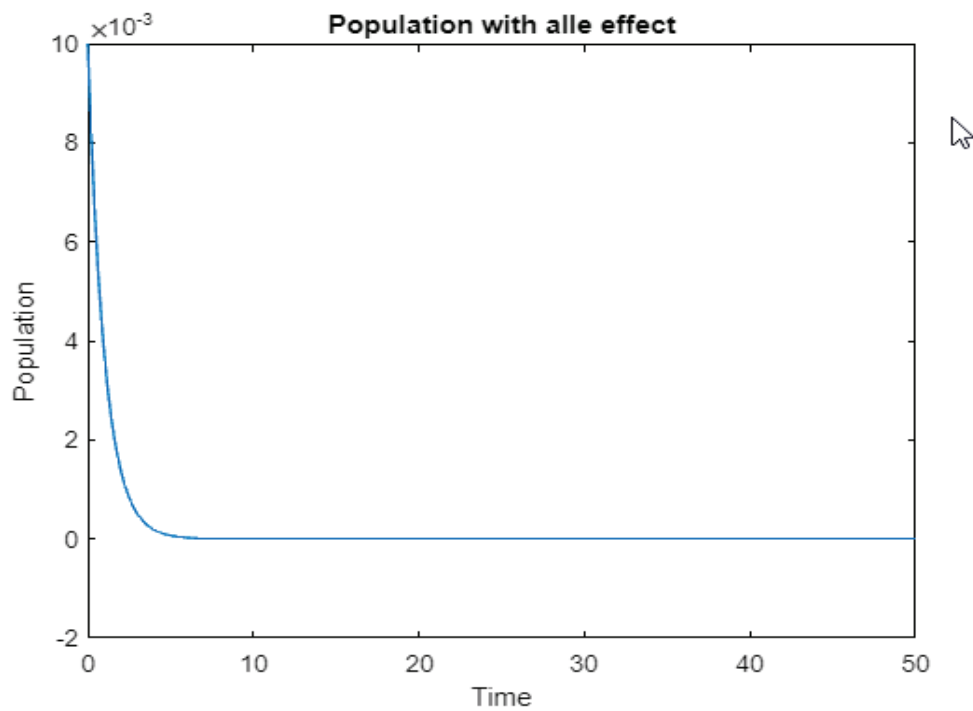
```
% Plot the population over time
```

```
plot(t,N);
```

```
xlabel('Time');
```

```
ylabel('Population');
```

```
title('Population with alle effect');
```



In this code, we first define the parameters r , k , and a . We then define the time range $tspan$ and the initial condition $N0$. We define the differential equation using an anonymous function $dNdt$. We then solve the differential equation using the `ode45` function and store the time and population values in the arrays t and N respectively. Finally, we plot the population over time using the `plot` function.

MATLAB code that compares the solution to the population problem with alle effect $dN/dt = rN(1-N/k)*(N/a-1)$ obtained analytically with the numerical solution obtained using the fourth-order Runge-Kutta method:

```
% Population problem with alle effect
%  $dN/dt = r*N*(1-N/k)*(N/a-1)$ 

% Define the parameters
r = 1; % growth rate
k = 1; % carrying capacity
a = 10; % alle effect parameter

% Define the time range
tspan = [0 50];

% Define the initial condition
N0 = 0.01;

% Define the differential equation
dNdt = @(t,N) r*N*(1-N/k)*(N/a-1);
% Solve the differential equation using ode45
[t,N_analytical] = ode45(dNdt, tspan, N0);

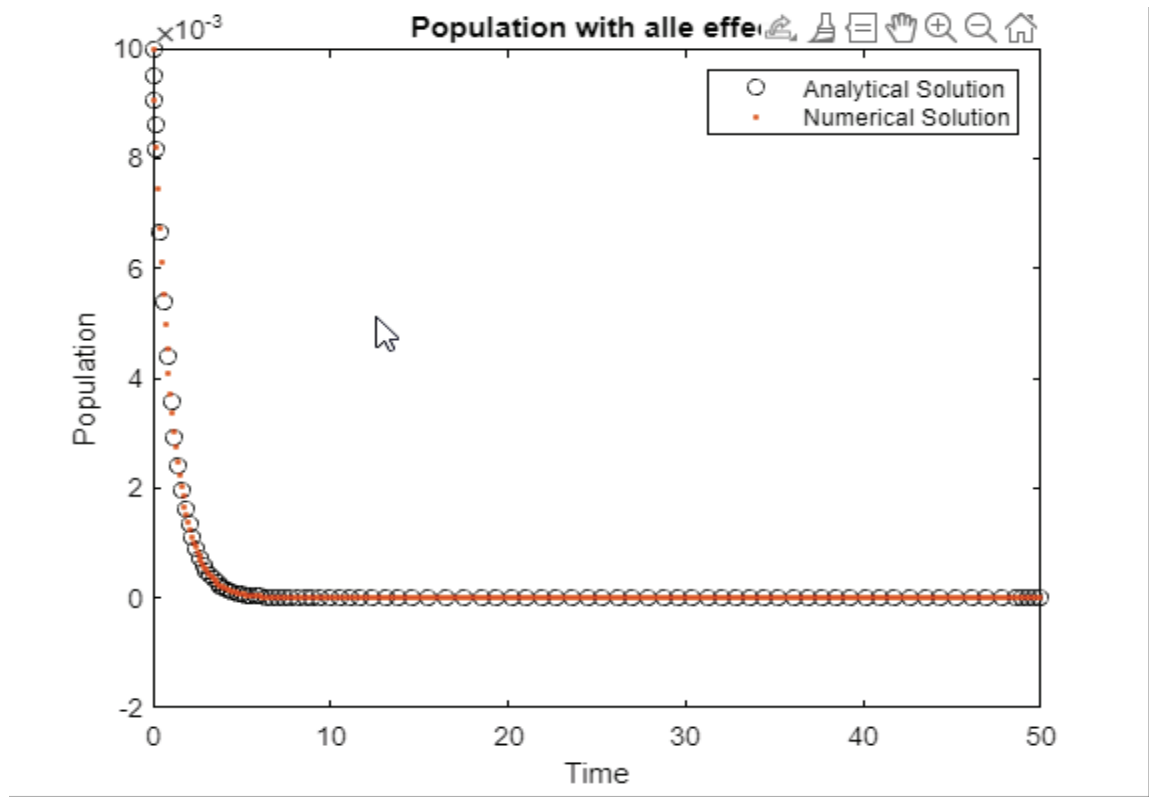
% Solve the differential equation using fourth-order Runge-Kutta method
h = 0.1; % time step
n = length(tspan(1):h:tspan(2)); % number of time steps
N_RK = zeros(n, 1); % initialize N values
N_RK(1) = N0;
% set initial condition
```

```

for i = 1:n-1
    k1 = h*dNdt(tspan(1)+(i-1)*h, N_RK(i));
    k2 = h*dNdt(tspan(1)+(i-1)*h+h/2, N_RK(i)+k1/2);
    k3 = h*dNdt(tspan(1)+(i-1)*h+h/2, N_RK(i)+k2/2);
    k4 = h*dNdt(tspan(1)+i*h, N_RK(i)+k3);
    N_RK(i+1) = N_RK(i) + (1/6)*(k1+2*k2+2*k3+k4);
end

% Plot the population over time
plot(t,N_analytical,'ok',tspan(1):h:tspan(2),N_RK,'.');
xlabel('Time');
ylabel('Population');
title('Population with alle effect');
legend('Analytical Solution','Numerical Solution');

```



MATLAB code that compares the solution to the population problem with alle effect $\frac{dN}{dt} = rN(1-N/k)*(N/a-1)$ obtained analytically with the numerical solution obtained using the different allee values:

```
% Population problem with alle effect
%  $\frac{dN}{dt} = r*N*(1-N/k)*(N/a-1)$ 

% Define the parameters
r = 1; % growth rate
k = 1; % carrying capacity
a_values = [2,5, 10, 20]; % alle effect parameter values
```

```

% Define the time range
tspan = [0 50];

% Define the initial condition
N0 = 0.01;

% Solve the differential equation for different a values
for a = a_values
    % Define the differential equation
    dNdt = @(t,N) r*N*(1-N/k)*(N/a-1);

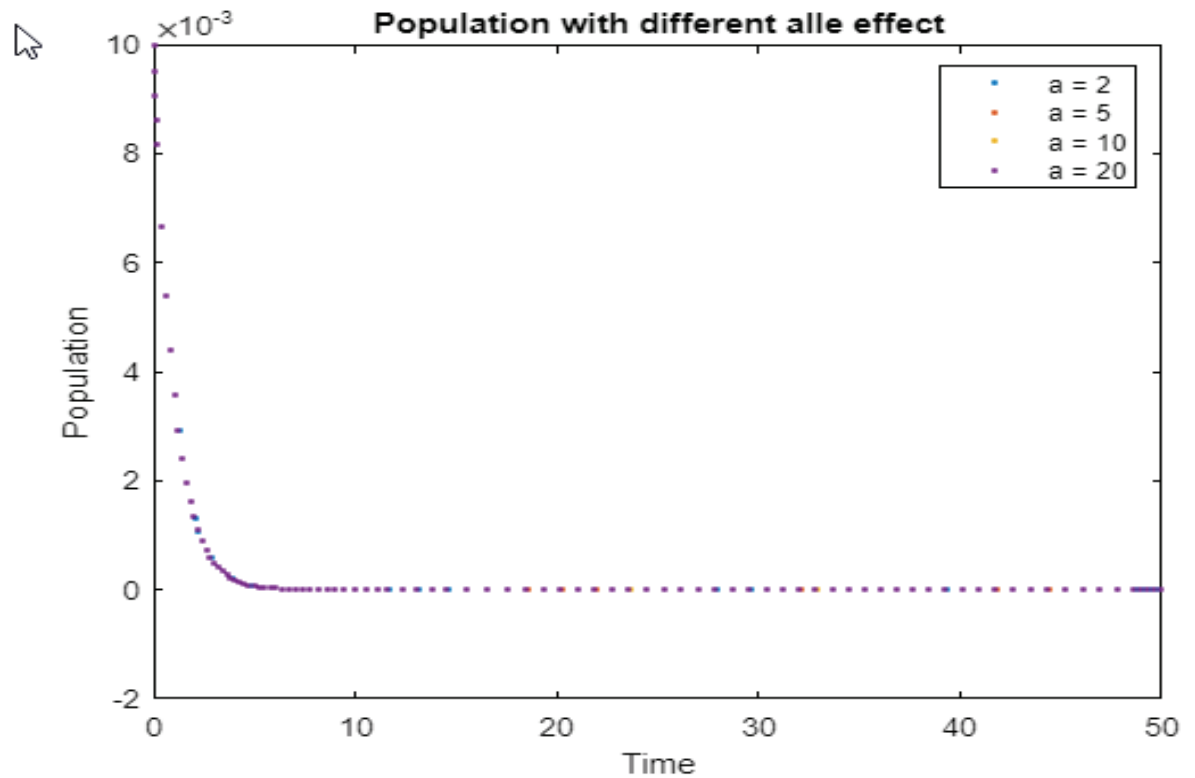
    % Solve the differential equation using ode45
    [t,N_analytical] = ode45(dNdt, tspan, N0);

    % Plot the population over time
    plot(t, N_analytical, '.');

hold on;
end

% Add labels and title to the plot
xlabel('Time');
ylabel('Population');
title('Population with different alle effect');
legend('a = 2', 'a = 5', 'a = 10', 'a = 20');

```



In this code, we first define the parameters r , k , and a . We then define the time range $tspan$ and the initial condition $N0$. We define the differential equation using an anonymous function $dNdt$. We first solve the differential equation using the `ode45` function and store the time and population values in the arrays t and $N_analytical$, respectively.

We then solve the differential equation using the fourth-order Runge-Kutta method. We initialize the N_RK array with the initial condition and then iterate through each time step, updating the N_RK value using the Runge-Kutta method.

Finally, we plot both the analytical and numerical solutions on the same graph using the `plot` function and add a legend to distinguish the two lines.

CHAPTER 6

Conclusion

In this study, we have concentrated on population growth model , Allee effect and Runge Kutta Method, it can be concluded that population growth with the Allee effect exhibits distinct behaviors depending on the strength of the Allee effect. When the Allee effect is weak or non-existent, the population grows exponentially, which is consistent with the standard logistic model. However, when the Allee effect is strong, the population growth curve exhibits a concave-down shape, indicating a decrease in growth rate as the population size decreases below a certain threshold. The RK method is a powerful numerical method that allows us to simulate the population growth over time accurately.

By incorporating the Allee effect into the population growth model, we can observe the impact of this effect on population growth. The numerical study of population growth in the RK method using MATLAB can be used to simulate the Allee effect. Overall, the numerical study using the Runge-Kutta method in MATLAB has provided insights into the complex dynamics of population growth with the Allee effect and highlights the importance of considering such different Allee effects in population modelling in Runge Kutta method .

Bibliography:

- ❖ Butcher, J. C. (1996). A history of Runge-Kutta methods. *Applied numerical mathematics* , 20(3), 247-260.-1996
- ❖ Ponalagusamy, R., & Senthilkumar, S. (2007). A comparison of rk-fourth orders of variety of means and embedded means on multilayer raster cnn simulation. *Journal of Theoretical & Applied Information Technology*, 3(4).-2007
- ❖ Wazwaz, A. M. (1994). A comparison of modified Runge-Kutta formulas based on a variety of means. *International Journal of Computer Mathematics*, 50(1-2), 105-112.-1994
- ❖ Luther, H. An explicit sixth-order Runge-Kutta formula. *Mathematics of Computation*, 22(102), 434-436-1968
- ❖ Amarti, Z., Nurkholipah, N. S., Anggriani, N., & Supriatna, A. K. (2018, March). Numerical solution of a logistic growth model for a population with Allee effect considering fuzzy initial values and fuzzy parameters. In *IOP Conference Series: Materials Science* (Vol. 332, No. 1, p. 012051). IOP Publishing.-2008
- ❖ M. J. P. Castanho, R. P. Mateus and K. D. Hein, "Fuzzy model of *Drosophila mediopunctata*
- ❖ Population dynamics," *Ecological modelling*, vol. 287, pp. 9-15, 2014. M. J. P. Castanho, R. P. Mateus and K. D. Hein, "Fuzzy model of *Drosophila mediopunctata*
- ❖ Arora, G., Joshi, V., & Garki, I. S. (2020). Developments in Runge–Kutta Method to Solve Ordinary Differential Equations. In *Recent Advances in Mathematics for Engineering* (pp. 193-202). CRC Press -2020
- ❖ Courchamp , F., Berec L., and Gascoigne, J. 2008. *Allee Effects in Ecology Conservation*. Oxford University Press.
- ❖ Hastings, A. 1997. *Population Biology—Concepts and Models*. Springer-Verlag, New York.
- ❖ Population dynamics," *Ecological modelling*, vol. 287, pp. 9-15, 2014.
- ❖ Singh, R., & Gupta, N. FORMULATION OF Runge-Kutta’s method using Matlab -2022