

A COMPARATIVE ANALYSIS OF ASSIGNMENT PROBLEM

Submitted in partial fulfillment of the requirements for the award of

Bachelor of Science Degree in Mathematics

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(DEEMED TO BE UNIVERSITY)

Accredited with Grade "A" by

NAAC I2B Status by UGC Approved by AICTE

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BONAFIDE CERTIFICATE

This is to certify that this project report is the bonafide work of **KAVIYA.R (40060007) AND MADHESWARI.A (40060008)** who carried out the project entitled "**A COMPARATIVE ANALYSIS OF ASSIGNMENT PROBLEM**" under our supervision from **JANUARY 2023 to APRIL 2023**.

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ABSTRACT

The objective of this project is to solve Assignment problem using the Hungarian method and Average total opportunity cost method. They are two distinct mathematical techniques used for different purposes.

The Hungarian method is an algorithm used to solve assignment problems in mathematics, while the average total opportunity cost method is a Decision-making tool used to allocate resources

KEY WORDS

- Hungarian method
- Average total opportunity cost method
- Optimal solution
- Assignment problem
- Total cost or time
- Optimization

CONTENTS

CHAPTER 1	PAGE NO
1. INTRODUCTION	2
1.1 OPERATION RESEARCH	2
1.1.1 METHODS	2
1.1.2 PROCESS	3
1.2 LINEAR PROGRAMMING	3
1.2.1 LINEAR PROGRAMMING PROBLEM FORMULATION	3
1.2.2 CONSTRAINED OPTIMIZATION MODELS	4
 CHAPTER 2	
LITERATURE SURVEY	6
 CHAPTER 3	
3. ASSIGNMENT PROBLEM	9
3.1 AIM AND SCOPE	9
3.1.1 OBJECTIVE	9
3.1.2 APPLICATION	9
3.1.3 MATHEMATICAL FORMULATION	10
3.1.4 CATEGORIES	11
3.1.5 OPTIMAL SOLUTION	11
3.2 THE AVERAGE TOTAL OPPORTUNITY COST METHOD(ATOC)	12
3.2.1 ADVANTAGE	12
3.2.2 LIMITATIONS	12

CHAPTER 4

RESEARCH METHODOLOGY

4.1 PROCEDUREIN HUNGARIAN METHOD	14
4.2 PROCEDUREIN AVERAGE TOTAL OPPORTUNITY COST METHOD	15
4.3 MATLAB	15

CHAPTER 5

RESULT AND DISCUSSION

5.1 CASE STUDY	17
5.2 CASE STUDY	23
5.3 CASE STUDY	28

CHAPTER 6

6.1 CONCLUSION	33
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REFERENCE	34
------------------	-----------

	PAGE NO
LIST OF TABLES	
Table 3.1.5 ASSIGNMENT PROBLEM IN OPTIMAL SOLUTION	11
Table 5.1 CASE STUDY 1	17
Table 5.2 CASE STUDY 2	28
Table 5.3 CASE STUDY 3	32
TABLE 5.4 COMPARATIVE STUDY OF HUNGARIAN METHOD, AVERAGE TOTAL OPPORTUNITY COST METHOD AND MATLAB FOR ALL THREE CASES	36

CHAPTER 1

1. INTRODUCTION

The Assignment problem is a special case of linear programming problem; it is one of the fundamental combinational optimization problems in the branch of optimization or operations research in mathematics. The most effective way to assigning a set of workers to a set of jobs. The objective is to minimize the total cost or maximize the total profit of the assignments. There are various methods to solve the assignment problem, and one of them is the Average Total Opportunity Cost (ATOC) method.

In this project, we will discuss the ATOC method in detail and explain how it can be used to find the optimal solution for the Assignment problem.

Some of the problems where the assignment technique may be useful are

- (i) workers to machines,
- (ii) salesmen to different sales areas,
- (iii) clerks to various checkout counters,
- (iv) classes to rooms,
- (v) vehicles to routes, etc.,

1.1 OPERATION RESEARCH

Operations Research is the application of analytical methods designed to help the decision makers.

1.1.1 METHODS OF OPERATIONS RESEARCH

OPTIMIZATION

The process of Decision-making involves a lot of data collection and analysis of possible outcomes.

SIMULATION

Simulation is a method of solving decision making problem by designing, constructing and manipulating a model of the real system.

PROBABILITY AND STATISTICS

Mathematical algorithms are the best way to evaluate potential risks and predict the possible outcomes. This method is very useful in public health.

1.1.2 PROCESS OF OPERATION RESEARCH

The process of operation research can be broadly broken down into the following steps:

- Identify a problem that needs to be solved.
- Constructing a model around the problem that resembles the real word and variables.
- Using the model to derive solution to the problem.
- Testing each solution on the model and analyzing its success.
- Implementing the solution to the actual solution.

1.2 LINEAR PROGRAMMING

- This is a constrained optimization technique, which optimize some criterion within some constraints.
- Linear programming the objective function and constraints are linear

1.2.1 LINEAR PROGRAMMING PROBLEM FORMULATION

The linear programming problem formulation is illustrated through a product mix problem. The product mix problem occurs in an industry where it is possible to manufacture a variety of products in this case the linear programming technique identifies the products combination which will maximize the profit subject to the availability of limited resource constraints.

maximize or minimize $Z = C_{11}X_{11} + C_{12}X_{12} + \dots\dots\dots + C_{nn}X_{nn}$

1.2.2 CONSTRAINED OPTIMIZATION MODELS

DECISION VARIABLES

Decision variables are physical quantities controlled by the decision maker and represented by mathematical symbols

OBJECTIVE FUNCTION

It is a mathematical function of the decision variables that converts a solution into a numerical evaluation of that solution.

THE CONSTRAINTS

There are always certain constraints on the use of resources, that limit the degree to which an objective can be achieved. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. The solution of an LP model must satisfy these constraints

In this project, Assignment problem was solved using,

- i. Hungarian method
- ii. Average Total Opportunity Cost Method and Results were analyzed and Comparative study were made.

CHAPTER 2

LITERATURE SURVEY

R. Sophia porchelvi and m. Anitha, 2018 has given “optimal solution for assignment problem by average total opportunity cost method”. It minimizes the total cost of certain problems and for few problems it gives a more cost compared to existing Hungarian method

Syakinahfaudzi, syarizaAbdulrahmanandRosshairyAbdRahman 2018 were analyzed “An assignment problem and its application in education domain: a review and potential path” . Problems related to assignment arise in a range of fields, for example, healthcare, transportation, education, and sports. In fact, this is a well-studied topic in combinatorial optimization problems under optimization or operations research branches.

Dhanashri a. Munot and kirtiwant p. Ghadle 2022 has proposed “a new approach to solve assignment problem using congruence modulo and its coding in MATLAB”. In this paper authors have proposed a new algorithm using congruence modulo approach to solve ap which is also demonstrated by illustrating numerical examples.

Humayradilafroz , dr. Mohammad anwarhossen 2017 has proposed “new proposed method for solving assignment problem and comparative study with the existing methods”. Assignment problem is an important problem in mathematics and is also discuss in real physical world. In this paper we attempt to introduce a new proposed approach for solving assignment problem with algorithm and solution steps.

Eliasmunapo, 2020 has given “Development of an accelerating Hungarian method for assignment problems”. Assignment model and the Hungarian method have application in addressing the weapon target assignment (WTA) problem.

Aminurrahman khan, adrianvilcu, nahid sultana and syedsabbirahmed 2015 has proposed “Determination of initial basic feasible solution of a transportation problem: a TOCM-sum approach”. A new heuristic for obtaining an initial basic feasible solution of a transportation problem, comparison of findings obtained by the new heuristic and the existing heuristics show that the method presented herein gives a better result.

S.M. Abul kalam azad, Md. Bellelhossain, md.Mizanurrahman, 2017 has proposed “An algorithmic approach to solve transportation problems with the average total opportunity cost method”. The current new algorithmic approach to solve the transportation problem (TP) is based upon the total opportunity cost (TOC) of a transportation table (TT). Opportunity cost in each cell along each row of a TT is the difference of the corresponding cell value from the lowest cell value of the corresponding row.

CHAPTER 3

3. ASSIGNMENT PROBLEM

3.1 AIM AND SCOPE

The assignment problem arises in various real-world situations, such as assigning jobs to workers, allocating resources to projects, or matching medical residents to hospitals. It can be formulated as a mathematical model and solved using different algorithms, in such a way Hungarian method that the time or cost involved in minimised and the sale or profit is maximized.

- One common variation of the assignment problem is the bipartite matching problem, in which tasks and resources are divided into two disjoint sets.
- The assignment problem is a special case of transportation problem when each origin is associated with one and only destination
- Suppose there are n jobs to be performed and n workers are available for doing these jobs.
- The total cost or time for performing all jobs is minimum

3.1.1 OBJECTIVE

- Minimize the cost and maximize the profit.
- Success in competition and market leadership.
- Decision making and improve quality.
- To develop more effective approaches to the programming

3.1.2 APPLICATION AREAS OF ASSIGNMENT PROBLEM

- Assignment of employees to machines.
- Assignment of operators to jobs.
- Effectiveness of teachers and subjects.
- Allocation of machines for optimum utilization of space.

3.1.3 MATHEMATICAL FORMULATION OF AN ASSIGNMENT PROBLEM

- The assignment problem can be started that in the form of $M \times N$ matrix c_{ij} called a Cost Matrix (or) Effectiveness Matrix where c_{ij} is the cost of assigning i^{th} workers to j^{th} job.

$$\begin{bmatrix} c_{11} & c_{12} & \dots & \dots & \dots & \dots & c_{1n} \\ c_{21} & c_{22} & \dots & \dots & \dots & \dots & c_{2n} \\ \vdots & \vdots & & & \vdots & & \vdots \\ c_{m1} & c_{m2} & \dots & \dots & \dots & \dots & c_{mn} \end{bmatrix}$$

- When a workers i ($=1,2,\dots,n$) is assigned to jobs j ($=1,2, \dots,n$) that incurs a cost c_{ij} .
- Total cost = $X_{11} * C_{11} + X_{12} * C_{12} + \dots + X_{nn} * C_{nn}$
- The objective function is Minimize $C_{11}X_{11} + C_{12}X_{12} + \dots + C_{nn}X_{nn}$
- The general aim of assignment problem is to optimize the allocation of resources to demand points where both resources and demand point share equal number [1]. The problem, hence, can be mathematically presented as follows:

Optimize

$$\sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij} \quad (1)$$

subject to

$$\sum_{j=1}^n x_{ij} = 1, i = 1,2 \dots n \quad (2)$$

$$\sum_{i=1}^n x_{ij} = 1, j = 1,2 \dots n \quad (3)$$

$$x_{ij} = 0 \text{ or } 1, i=1,2\dots n, j=1,2\dots n \quad (4)$$

- Where x_{ij} is the cost or effectiveness of assigning i_{th} works to j_{th} jobs, x_{ij} is 0 or 1 (as presented in (4)), and n is the number of workers or jobs. The constraints of the problem are defined as (2) and (3). Equation (2) indicates that each workers i needs to be assigned to only one j jobs, while (3) shows that each jobs j needs to be assigned to only one workers i .

- Let $x_{ij} = 1$, if j^{th} jobs is assigned to i^{th} workers.

$x_{ij} = 0$, if j^{th} jobs is not assigned to i^{th} workers.

3.1.4 CATEGORIES OF ASSIGNMENT PROBLEMS

- Balanced minimization $m=n$
- Unbalanced minimization $m \neq n$

3.1.5 ASSIGNMENT PROBLEM IN OPTIMAL SOLUTION

5	6	8	9
6	8	10	6
9	5	8	5
9	8	7	10

- Subtract the smallest cost element of each row from all the elements in the row of the given cost matrix. See that each row contains at least one zero.
- Examine the rows and columns successively until a row and columns with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this encircled zero, as these will not be considered for any future assignment

0	1	3	4
0	2	4	0
4	0	3	0
2	1	0	3

Total cost = $5+5+7+6=23$ so we the optimal solution

3.2 The Average Total Opportunity Cost Method(ATOC)

It is based on the concept of opportunity cost, which refers to the cost of the next best alternative when a particular decision is made.

The ATOC method works by calculating the average opportunity cost for each agent-task pair and then assigning the agent to the task with the lowest average opportunity cost. This process is repeated until all agents have been assigned to a task.

3.2.1 Advantages of the ATOC Method

- It is relatively simple and easy to understand.
- It does not require any complex mathematical formulas or algorithms, making it accessible to a wide range of users.
- It can handle large-scale assignment problems with many agents and tasks.
- It is also flexible enough to accommodate different types of costs and constraints, such as time constraints or capacity constraints.

3.2.2 Limitations of the ATOC Method

- It may not always provide the optimal solution to the assignment problem. In some cases, other methods such as the Hungarian algorithm may be more suitable.
- The opportunity cost is constant and does not change over time.
- This may not be true in real-world scenarios where the cost of an assignment may vary depending on external factors.

CHAPTER 4

4. RESEARCH METHODOLOGY

4.1 PROCEDURE IN HUNGARIAN METHOD

First check whether the number of rows is equal to number of columns, if it is so, the assignment problem is said to be balanced. Then proceed to step 1. If it is not balanced, then it should be balanced before applying the algorithm.

Step 1: Subtract the smallest cost element of each row from all the elements in the row of the given cost matrix. See that each row contains at least one zero.

Step 2: Subtract the smallest cost element of each column from all the elements in the column of the resulting cost matrix obtained by step 1 and make sure each column contains at least one zero.

Step 3: (Assigning the zeros)

(a) Examine the rows and columns successively until a row and columns with exactly one unmarked zero is found. Make an assignment to this single unmarked zero by encircling it. Cross all other zeros in the column of this encircled zero, as these will not be considered for any future assignment. Continue in this way until all the rows and columns have been examined

Step 4: (Apply Optimal Test)

(a) If each row and each column contain exactly one encircled zero, then the current assignment is Optimal solution obtained.

(b) If at least one row or column is without an assignment (i.e., if there is at least one row or column is without one encircled zero), then the current assignment is Not a optimal solution.

HUNGARIAN METHOD

It was developed and published in 1955 by Harold Kuhn, who gave the name Hungarian method.

It is a combinatorial optimization algorithm that solves the assignment problem in polynomial time and which anticipated later prime dual methods.

Step 5: Cover all the zeros by drawing a minimum number of straight lines as follows:

- (a) Mark the rows that do not have assignment.
- (b) Mark the columns (not already marked) that have zeros in marked rows.
- (c) Mark the rows (not already marked) that have assignments in marked columns.
- (d) Repeat (b) and (c) until no more marking is required.
- (e) Draw lines through all unmarked rows and marked columns. If the number of these lines is equal to the order of the matrix then it is an optimum solution otherwise not.

Step 6: Determine the smallest cost element not covered by the straight lines. Subtract this smallest cost element from all the uncovered elements and add this to all those elements which are lying in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines.

Step 7: Repeat steps (1) to (6), until an optimum assignment is obtained.

4.2PROCEDUREIN AVERAGE TOTAL OPPORTUNITY COST METHOD

Here are the steps to solve the Assignment Problem using the Average Total Opportunity Cost Method:

1. Calculate the opportunity cost matrix: For each pair of objects, calculate the opportunity cost of assigning one to the other. The opportunity cost can be calculated as the difference between the cost of assigning the object to its best available match and the cost of assigning it to the second-best match.
2. Find the minimum opportunity cost in each row and subtract it from all the elements in that row. This is called row reduction.
3. Find the minimum opportunity cost in each column and subtract it from all the elements in that column. This is called column reduction.

4. If the number of zeros in the reduced matrix is equal to the number of rows or columns, then an optimal solution has been found. Otherwise, select the row or column with the most zeros, and choose one of the zeros as the starting point for a new path.
 5. Follow the path by alternating between rows and columns, selecting zeros as you go, until a new zero cannot be found. Then, find the smallest number in the rows and columns that have been crossed out and add it to all the remaining numbers. Subtract it from all the numbers in the rows and columns that have not been crossed out. This is called the Hungarian method.
 6. Repeat steps 4 and 5 until an optimal solution is found.
 7. Calculate the total opportunity cost of the optimal solution and divide it by the number of assignments to get the average total opportunity cost.
 8. Assign the objects to their matches based on the optimal solution.
- Note that the Average Total Opportunity Cost Method can be computationally intensive for large problems, and there are other methods available that may be more efficient.

4.3 SOFTWARE IN MATLAB

The assignment problem can be solved in MATLAB using different techniques, such as the Hungarian method implemented by the “munkres” function. which finds the optimal assignment in polynomial time that minimizes the total cost or maximizes the total profit, given a cost matrix representing the cost of assigning each task to each agent.

CODE

```
clc,  
clear screen,  
C=[ cost of matrix ]  
CostUnmatched = max(size(C)) * max(C,[],'all')  
M= matchpairs(C, cost unmatched)  
Totaltime = sum(C(sub2ind(size(C), M(:,1), M(:,2))))
```

CHAPTER 5

5. RESULT AND DISCUSSION

5.1 CASE STUDY 1

Assign 4 jobs to 10 persons with minimum cost

PERSON	1	2	3	4
A	11	8	9	8
B	4	5	29	33
C	10	5	29	33
D	1	18	25	31
E	23	22	33	30
F	3	9	13	19
G	6	8	27	32
H	32	30	39	38
I	36	35	31	21
J	15	11	10	28

By Hungarian method

person	1	2	3	4
A	3	0	1	0
B	0	1	25	29

C	5	0	24	28
D	0	17	24	30
E	1	0	11	8
F	0	6	10	16
G	0	2	21	26
H	2	1	7	6
I	15	14	10	0
J	5	1	0	18

SOLUTION OF HUNGARIAN METHOD

B	1	4
I	4	21
J	3	10
A	2	8
	COST	43

SOLUTION OF AVERAGE TOTAL OPPORTUNITY COST METHOD

STEP 1

PERSON	1	2	3	4
A	3	0	1	0
B	0	1	25	29
C	5	0	24	28
D	0	17	24	30
E	1	0	11	8
F	0	6	10	16
G	0	2	21	26
H	2	1	7	6
I	15	14	10	0
J	5	1	0	18

STEP 2

person	1	2	3	4
A	10	3	0	0
B	3	0	20	25
C	9	0	20	25
D	0	13	16	23

E	22	17	24	22
F	2	4	4	11
G	5	3	18	24
H	31	25	30	30
I	35	30	22	13
J	14	6	1	20

STEP 3

PERSON	1	2	3	4
A	13	3	1	0
B	3	1	45	54
C	14	0	44	53
D	0	30	40	53
E	23	17	35	30
F	2	10	14	27
G	5	5	39	50
H	33	25	37	36
I	50	44	32	13
J	19	7	1	38

STEP 4

person	1	2	3	4	AVERAGE
A	13	3	1	0	4.25
B	3	1	45	54	25.75
C	14	0	44	53	24
D	0	30	40	53	30.75
E	23	17	35	30	26.25
F	2	10	14	27	13.25
G	5	5	39	50	24.75
H	33	25	37	36	25
I	50	44	32	13	34
J	19	7	1	38	16.25
AVERAGE	16.2	14.2	28.8	34.4	

SOLUTION OF AVERAGE TOTAL OPPORTUNITY COST METHOD

D	1	1
J	3	10
B	2	5
A	4	8
	COST	24

MATLAB

```
clc,  
clear screen,  
C=[11,8,9,8,  
    4,5,29,33,  
    10,5,29,33,  
    1,18,25,31,  
    23,22,33,30,  
    3,9,13,19,  
    6,8,27,32,  
    32,30,39,38,  
    36,35,31,21,  
    15,11,10,28]  
M= matchpairs(C,60)  
Totaltime = sum(C(sub2ind(size(C), M(:,1), M(:,2))))
```

OUTPUT

```
M =  
  
     4     1  
     2     2  
    10     3  
     1     4
```

```
Totaltime = 24
```

Using the Algorithm of Average Total Opportunity Cost Method, the minimum cost is Rs.24. By using the Hungarian method, we get the optimum cost is 43 and MATLAB we get minimum cost 24.

Hence the difference is Rs.19. So, the ATOC method gives the least cost compared to Hungarian method.

5.2 CASE STUDY 2

Priya has used online shopping in the past two years, and recently she decided to purchase medicine, groceries, furniture, clothes through online platforms and she choose Filpkart, Amazon, Jio, Snap Deal, as which one is best platform to purchase the items and save time also find which one the fast delivery

Shops platforms	Medicine	Groceries	Stationery	Toys & Baby items
Filpkart	2 hours	8 hours	27 hours	5 hours
Amazon	6 hours	13 hours	21 hours	15 hours
Jio	1 hour	10 hours	40 hours	24 hours
Snap deal	4 hours	3 hours	7 hours	18 hours

SOLUTION

HUNGARIAN METHOD

Shops platforms	Medicine	Groceries	Stationery	Toys & Baby items
Filpkart	6	5	20	0
Amazon	0	0	4	0
Jio	0	2	28	14
Snap deal	8	0	0	13

SOLUTION OF HUNGARIAN METHOD

Filpkart	Toys and baby items	5 hours
Amazon	Groceries	13 hours
Jio	medicine	1 hour
Snap deal	Stationery	7 hours
	TOTAL	26 hours

BY AVERAGE TOTAL OPPORTUNITY COST METHOD

STEP 1

Shops platforms	Medicine	Groceries	Stationery	Toys & Baby items
Filpkart	0	6	25	3
Amazon	0	7	15	9
Jio	0	9	39	23
Snap deal	1	0	4	15

STEP 2

Shops platforms	Medicine	Groceries	Furniture	Clothes
Filpkart	1	5	20	0
Amazon	5	10	14	10
Jio	0	7	33	19
Snap deal	3	0	0	13

STEP 3

Shops platforms	Medicine	Groceries	Stationery	Toys & Baby items
Filpkart	1	11	45	3
Amazon	5	17	29	19
Jio	0	16	72	42
Snap deal	4	0	4	28

STEP 4

Shops platforms	Medicine	Groceries	Stationery	Toys & Baby items	AVERAGE
Filpkart	1	11	45	3	15
Amazon	5	17	29	19	17.5
Jio	0	16	72	42	32.5
Snap deal	4	0	4	28	9
AVERAGE	2.5	11	37.5	23	

SOLUTION OF AVERAGE TOTAL OPPORTUNITY COST METHOD

Filpkart	Toys and baby items	5 hours
Amazon	Groceries	13 hours
Jio	medicine	1 hour
Snap deal	Stationery	7 hours
	TOTAL	26 hours

SOFTWARE IN MATLAB

```
clc,  
clear screen,  
C=[2,8,27,5,  
6,13,21,15,  
1,10,40,24,  
4,3,7,18]  
M= matchpairs(C,60)  
Totaltime = sum(C(sub2ind(size(C), M(:,1), M(:,2))))
```

OUTPUT

C =

```
2 8 27 5  
6 13 21 15  
1 10 40 24  
4 3 7 18
```

M =

```
3 1  
2 2  
4 3  
1 4
```

Totaltime =

26

The Hungarian method time is 26 hours, Average total opportunity cost method is 26 hours the MATLAB code also 26 hours.

From this case study, we can see that both the Average total opportunity cost method the Hungarian method and MATLAB yield the same optimal assignment and total cost for this particular problem instance. However, in general, the ATOC method may result in sub-optimal solutions.

5.3 CASE STUDY 3

Assign 3 patients to 5 doctors with minimum cost

person	Patient 1	Patient 2	Patient 3
Doctor 1	10	5	7
Doctor 2	7	6	5
Doctor 3	8	7	6
Doctor 4	3	2	6
Doctor 5	1	2.6	4

Hungarian method

person	Patient 1	Patient 2	Patient 3
Doctor 1	4	0	3
Doctor 2	1	1	0
Doctor 3	1	1	0
Doctor 4	0	0	3
Doctor 5	0.6	0	3

SOLUTION OF HUNGARIAN METHOD

Doctor 1	Patient 1	5
Doctor 2	Patient 2	5
Doctor 4	Patient 3	3
	total	13

SOLUTION OF AVERAGE TOTAL OPPORTUNITY COST METHOD

STEP 1

person	Patient 1	Patient 2	Patient 3
Doctor 1	5	0	3
Doctor 2	2	1	0
Doctor 3	2	1	0
Doctor 4	1	0	3
Doctor 5	1.6	0	3

STEP 2

person	Patient 1	Patient 2	Patient 3
Doctor 1	9	3	3
Doctor 2	6	4	1
Doctor 3	7	5	2
Doctor 4	2	0	2
Doctor 5	0	0.6	0

STEP 3

Person	Patient 1	Patient 2	Patient 3
Doctor 1	14	3	6
Doctor 2	8	5	1
Doctor 3	9	6	2
Doctor 4	3	0	5
Doctor 5	1.6	0.6	3

STEP 4

Person	Patient 1	Patient 2	Patient 3	Average
Doctor 1	14	3	6	7.6
Doctor 2	8	5	1	4.6
Doctor 3	9	6	2	5.6
Doctor 4	3	0	5	2.6
Doctor 5	1.6	0.6	3	1.7
Average	7.12	2.92	3.4	

SOLUTION OF AVERAGE TOTAL OPPORTUNITY COST METHOD

Doctor 5	Patient 1	1
Doctor 1	Patient 2	5
Doctor 3	Patient 3	6
	Total	12

SOFTWARE IN MATLAB

```
clc,  
clear screen,  
C=[10,5,7  
    7,6,5  
    8,7,6  
    3,2,6  
    1,2,6,4]  
M= matchpairs(C,10)  
Totaltime = sum(C(sub2ind(size(C), M(:,1), M(:,2))))
```

OUTPUT

```
M =  
  
    5    1  
    4    2  
    2    3
```

Total time = 12

Using the Algorithm of Average Total Opportunity Cost Method, and the minimum cost is Rs.12 and MATLAB also same cost. By using the Hungarian method, we get the optimum cost is 13.

Hence the difference is Rs.1. So, the ATOC method gives the least cost compared to Hungarian method

TABLE 5.4 COMPARATIVE STUDY BASED ON HUNGARIAN, AVERAGE TOTAL OPPORTUNITY COST METHOD AND MATLAB FOR ALL THREE CASE STUDY GIVEN BELOW

PROBLEMS	AVERAGE TOTAL OPPORTUNITY COST METHOD	HUNGARIAN METHOD	MATLAB	DIFFERENCE
CASE STUDY 5.1	24	43	24	19
CASE STUDY 5.2	26	26	26	0
CASE STUDY 5.3	12	13	12	1

CHAPTER 6

6.1 CONCLUSION

In this paper, Comparative study were made on Hungarian method with average total opportunity cost method. we had found an optimal solution for the assignment problem by using Average Total Opportunity Cost Method (ATOC). For any kind of assignment problem, whether its function to be maximized or minimized balanced or unbalanced this method can be employed to solve. It gives minimum cost as compared to existing Hungarian method and also gives the same cost in some problems. It is simple, flexible, and can handle large-scale problems. However, it may not always provide the optimal solution and may have limitations in certain situations.

Overall, the ATOC method is a valuable tool for decision-making and optimization in a wide range of industries and applications

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