

A NEW OPERATIONS AND RANKING FOR HEPTAGONAL FUZZY NUMBER AND ITS APPLICATIONS

Submitted in partial fulfillment of the requirements for the award of

Bachelor of Science Degree in Mathematics

By

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DEPARTMENT OF MATHEMATICS

SCHOOL OF SCIENCE AND

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SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

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CHENNAI - 600 119

APRIL – 2023



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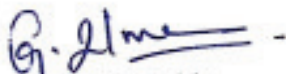
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BONAFIDE CERTIFICATE

This is to certify that this project report is the bonafide work of **JAYASHREE J (40060005) & KAVIYA K (40060006)** who carried out the project entitled "**A NEW OPERATIONS AND RANKING FOR HEPTAGONAL FUZZY NUMBER AND ITS APPLICATIONS**" under our supervision from **JANUARY 2023** to **APRIL 2023**.

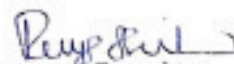

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We **JAYASHREE J (40060005) & KAVIYA K (40060006)** hereby declare that the project report entitled "**A NEW OPERATIONS AND RANKING FOR HEPTAGONAL FUZZY NUMBER AND ITS APPLICATIONS**" done by us under the guidance of **Dr.G.UMA** (Internal) at **SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY** submitted in partial fulfillment of the requirements for the award of Bachelor of Science Degree in Mathematics.

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ABSTRACT

Fuzzy numbers can be used for expert system reasoning. Fuzzy systems include fuzzy set theory and fuzzy logic which has many successful applications. In this work we introduced a basic operation (+, -, *) and ranking for the heptagonal fuzzy number. To find the ranking for the heptagonal fuzzy number, we used a novel method called centroid of the centroids. We applied the new operations and ranking to solve the heptagonal fuzzy transportation problem. A north-west corner method, least cost method, and Vogel's approximation method have been used to solve the heptagonal fuzzy transportation problem

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CHAPTER 1

INTRODUCTION

Fuzzy logic is a form of many-valued logic. It deals with reasoning that is approximate rather than fixed and gives an exact solution. In classical set theory where binary sets have two-valued logic true or false. In fuzzy logic, the true value of variables ranges between 0 and 1. The idea of fuzzy logic was first introduced by Zadeh. Klir et al. apply his logic in fuzzy sets and fuzzy logic.

Today, fuzzy logic is used in many applications to make human life easier. It is used in washing machines, smartphones, air conditioners, traffic control systems, automatic gear control systems, cameras, TV sets, automobile engines, vacuum cleaners, etc.

The ranking method is used in many decision problems to express fuzzy numbers in crisp value. Due to the overlapping of fuzzy numbers, there is difficult to find which number is smaller and which is larger. We determine the correct ordering of fuzzy numbers by using the ranking function.

The two contributions developed by Hitchcock and Koopmans helped in the development of transportation methods that involve a number of plants and a number of destinations. Afterward, Charnes and Cooper (1954) developed a linear programming calculation method to solve transportation problems.

CHAPTER 2

LITERATURE SURVEY

An essential component of our Modern life is the shipping of goods from where they are produced to Markets worldwide. Nationally, companies spend billions of dollars annually in transporting goods. The Transportation Problem (TP) is a special class of network optimization problem that deals with transporting a homogenous product from multiple sources (e.g. factories) to multiple destinations (e.g. warehouses).

The TP was formalized by the French Mathematician Gaspard Monge (1781). Major advantages were made in the field during World War II by the Russian Mathematician and economist Leonid Kantorovich. Tolstoi was the one of the first to study the transportation problem mathematically. An article named "Method of finding the minimal total kilometrage in Cargo transportation planning in space, in the collection of transportation planning", Volume I at the national commissariat of transportation of the union was published by Tolstoi (1930). Koopmans has presented an independent study not related to Hitchcock's and called "Optimum utilization of transportation systems". The two contributions which developed by Hitchcock [1] and Koopmans [2] helped in the development of transportation methods which involve a number of plants and a number of destinations. After that Charnes and Cooper (1954) [3], have published the stepping stone method of explaining linear programming calculations in transportation problems. In 1951, Dantzig applied the concept of linear programming in solving the transportation models. Reinfeld and Vogel (1958) [4] have published a book named Mathematical Programming. In the beginning of the 20th century, Veena Adlakha and Krzysztof Kowalski (2000) [5] introduced an alternative solutions analysis for transportation problems. Also Gen, Choi and Ida (2000) [6] improved genetic algorithm for generalized transportation problem. Kulkarni and Datar (2010) [7] published a paper which is named "On solution to modified unbalanced transportation problem". In the same year Pandian and Natarajan [8] obtained a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Korukoglu and Balli (2001) [9] improved Vogel's approximation method for the transportation problem. All the parameters of the transportation problem may not be precisely due to

uncontrollable factors in real world applications. The parameters of the transportation problem are not always exactly known and stable. The imprecision may follow from the lack of exact information, uncertainty in judgement etc. In 1965, Zadeh [10] introduced the notion of fuzziness that was reinforced by Bellman and Zadeh[11]. Zimmermann [12] has discussed about the effective solutions of fuzzy set theory, fuzzy linear programming with several objective functions. Bit A.K, Biswal M.P., Alam S.S [13] have introduced the fuzzy programming approach to multi-criteria decision making transportation problem. Chanas and Kuchta [14]

This work contains seven chapters. Chapter 3 deals with types of fuzzy numbers. In Chapter 4, we discussed about the heptagonal fuzzy number and their operations. In Chapter 5, discussed about the ranking on Heptagonal fuzzy numbers using centroid of centroids. In Chapter 6, we discussed three methods to solve Heptagonal fuzzy Transportation Problem.

Basic Definitions

A classical (crisp) set is normally defined as a collection of elements or objects $x \in X$ that can be finite, countable, or over countable. Each single element can either belong to or not belong to a set A , $A \subseteq X$. In the former case, the statement “ x belongs to A ” is true, whereas in the latter case this statement is false.

Such a classical set can be described in different ways: one can either enumerate (list) the elements that belong to the set; describe the set analytically, for instance, by stating conditions for membership

$A = \{x / x \leq 5\}$; or define the member elements by using the characteristic function, in which 1 indicates membership and 0 non-membership. For a fuzzy set, the characteristic function allows various degrees of membership for the elements of a given set.

Definition 2.1

If X is a collection of objects denoted generically by x , then a fuzzy set $\tilde{A}$ in X is a set of ordered pairs: $\tilde{A} = \{x, \mu_{\tilde{A}}(x) / x \in X\}$
 $\mu_{\tilde{A}}(x)$ is called the membership function or grade of membership (also degree of

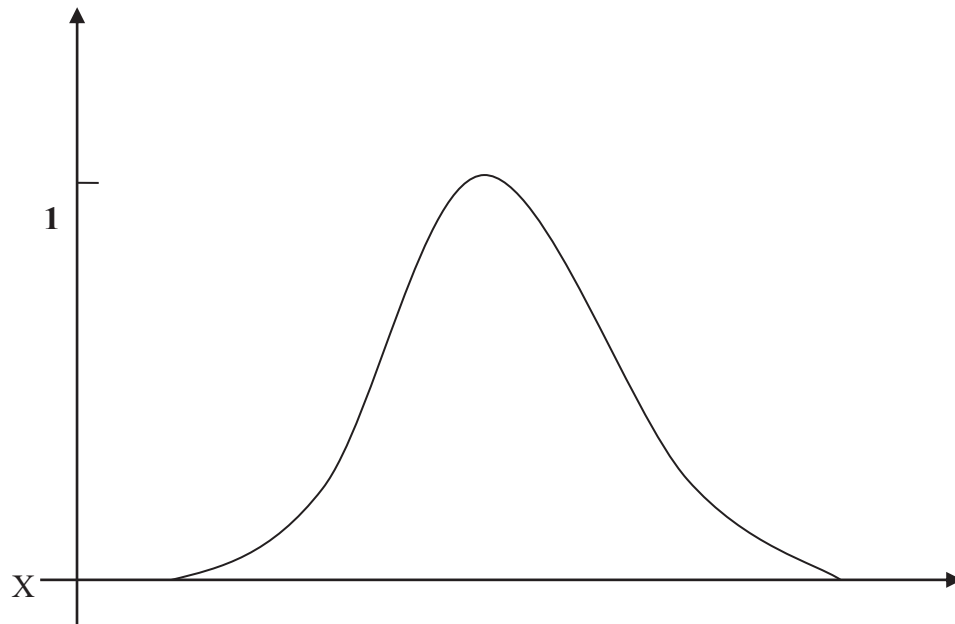
compatibility or degree of truth) of x in A that maps X to the membership space M (When M contains only the two points 0 and 1. $\tilde{A}$ is non fuzzy and $\mu_{\tilde{A}}(x)$ is identical to the characteristic function of a non fuzzy set). The range of the membership function is a subset of the nonnegative real numbers whose supremum is finite. Elements with a zero degree of membership are normally not listed.

Definition 2.2

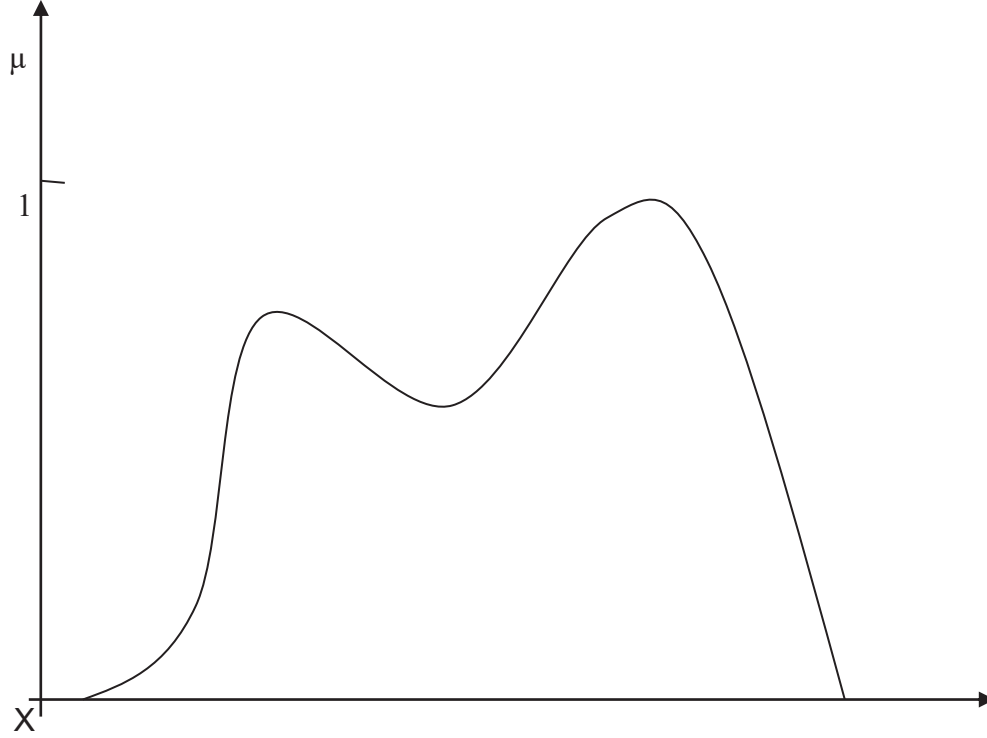
A fuzzy set $\tilde{A}$ is convex is

$\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min \{\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\}, x_1, x_2 \in X, \lambda \in [0,1]$ Alternatively, a fuzzy set is convex if all α – level sets are convex.

Graphical representation of convex fuzzy set



Graphical representation of non-convex fuzzy set



Definition 2.3

The membership function $\mu_{\tilde{C}}(x)$ of the *intersection* $\tilde{C} = \tilde{A} \cap \tilde{B}$ is pointwise defined by

$$\mu_{\tilde{C}}(x) = \min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X$$

Definition 2.4

The membership function $\mu_{\tilde{D}}(x)$ of the *union* $\tilde{D} = \tilde{A} \cup \tilde{B}$ is pointwise defined by

$$\mu_{\tilde{D}}(x) = \max\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\}, \quad x \in X$$

Definition 2.5

The membership function of the *complement* of a normalized fuzzy set $\tilde{A}$, $\mu_{\complement\tilde{A}}(x)$ is defined by

$$\mu_{\complement\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x), \quad x \in X$$

Fuzzy set

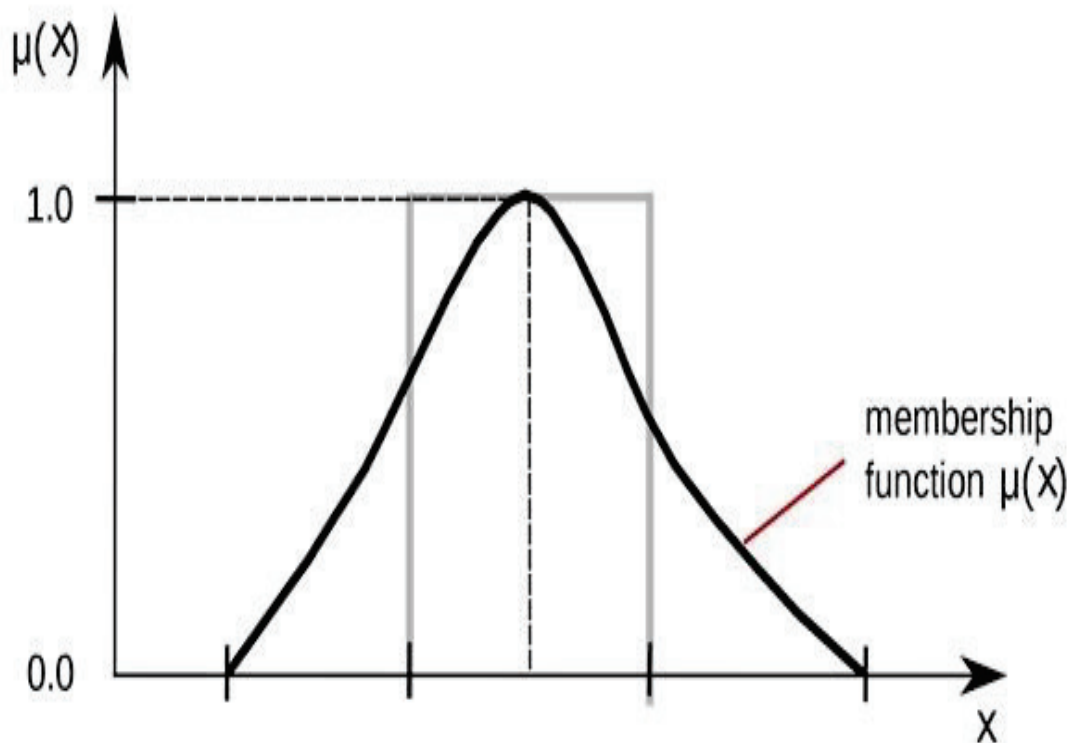
A fuzzy set is characterized by a membership function mapping element of a domain space or the universe of discourse X to the unit interval $[0, 1]$.

(i.e.) $\mu_A : X \rightarrow [0,1]$

Fuzzy logic

Fuzzy logic is a form of many valued logic in which the truth values of variables may be any real number between 0 and 1 both inclusive. It is employed to handle the concept of partial truth, where the truth value may range between completely true and completely false. The truth values of variables may only be the integer values 0 or 1.

Graphical representation of fuzzy logic



CHAPTER 3

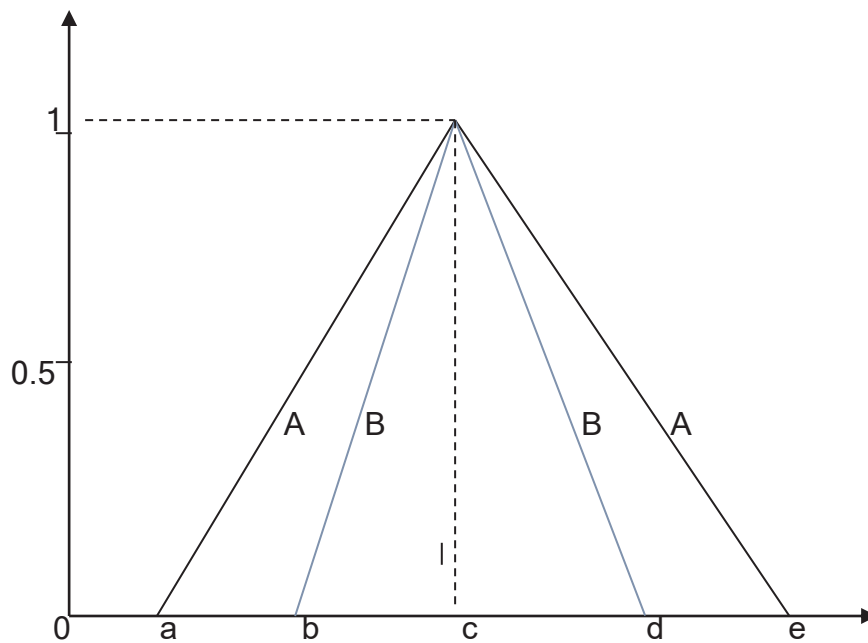
TYPES OF FUZZY NUMBERS

3.1 Fuzzy Number

A fuzzy set A defined on the set of real numbers R is said to be a fuzzy number if its membership function $\mu_A : R \rightarrow [0,1]$ has the following properties

- i) A must be a normal fuzzy set.
- ii) α_A must be a closed interval for every $\alpha \in (0,1]$
- iii) The support of A , O^+_A must be bounded.

Graphical representation of fuzzy number

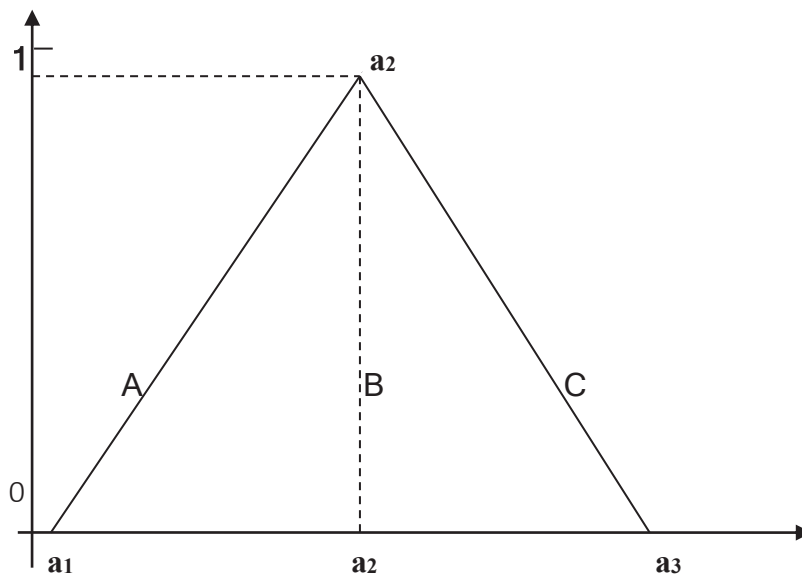


3.2 Triangular fuzzy number

Triangular Fuzzy Number A fuzzy number $A_T = (a_1, a_2, a_3: 1)$ is called triangular Fuzzy number if its membership function is given by,

$$\mu_{A_T}(x) = \begin{cases} 0 & \text{for } 0 < a_1, \\ \left[\frac{x - a_1}{a_2 - a_1} \right] & \text{for } a_1 \leq x \leq a_2, \\ \left[\frac{a_3 - x}{a_3 - a_2} \right] & \text{for } a_2 \leq x \leq a_3, \\ 0 & \text{for } 0 > a_3 \end{cases}$$

Graphical representation of triangular fuzzy number



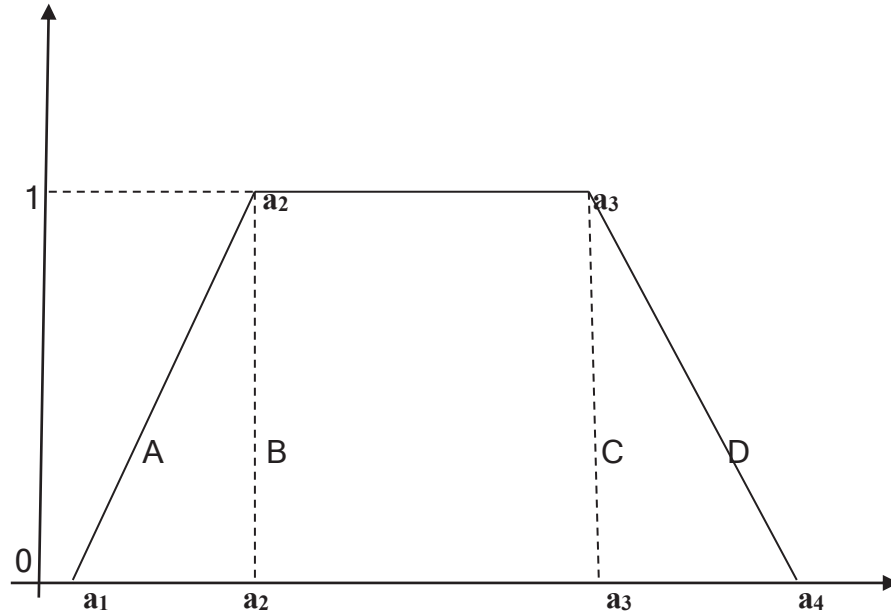
3.3 Trapezoidal Fuzzy Number

A Fuzzy number A defined on the universal set of real numbers R denoted as

$\tilde{A}_{Tr} = (a_1, a_2, a_3, a_4; 1)$ is said to be a trapezoidal fuzzy number its membership function $\mu_{\tilde{A}_{Tr}}(x)$ is given by,

$$\mu_{\tilde{A}_{Tr}}(x) = \begin{cases} 0 & \text{for } x < a_1, \\ \left[\frac{x - a_1}{a_2 - a_1} \right] & \text{for } a_1 \leq x \leq a_2, \\ 1 & \text{for } a_2 \leq x \leq a_3, \\ \left[\frac{a_4 - x}{a_4 - a_3} \right] & \text{for } a_3 \leq x \leq a_4, \\ 0 & \text{for } x > a_4 \end{cases}$$

Graphical representation of trapezoidal fuzzy number

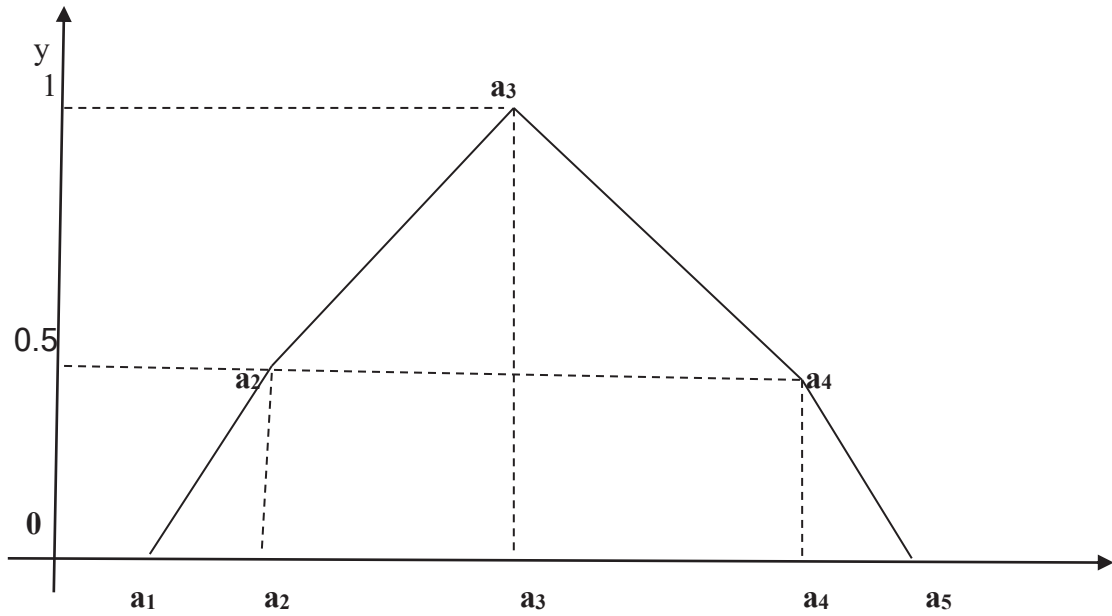


3.4 Pentagonal Fuzzy Numbers

A pentagon fuzzy number $\tilde{A}_p = (a_1, a_2, a_3, a_4, a_5; 1)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership is given below,

$$\mu_{\tilde{A}_p}(x) = \begin{cases} 0 & \text{for } 0 < a_1, \\ \frac{1}{2} \left[\frac{x - a_1}{a_2 - a_1} \right] & \text{for } a_1 \leq x \leq a_2, \\ \frac{1}{2} + \frac{1}{2} \left[\frac{x - a_2}{a_3 - a_2} \right] & \text{for } a_2 \leq x \leq a_3, \\ 1 - \frac{1}{2} \left[\frac{a_4 - x}{a_4 - a_3} \right] & \text{for } a_3 \leq x \leq a_4, \\ \frac{1}{2} \left[\frac{a_5 - x}{a_5 - a_4} \right] & \text{for } a_4 \leq x \leq a_5, \\ 0 & \text{for } 0 > a_5 \end{cases}$$

Graphical representation of Pentagonal fuzzy number



3.5 Hexagonal Fuzzy Number

A fuzzy number $\tilde{A}_H$ is a hexagonal fuzzy number denoted by $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6; 1)$ where $a_1, a_2, a_3, a_4, a_5, a_6$ are real numbers and its membership function is given below.

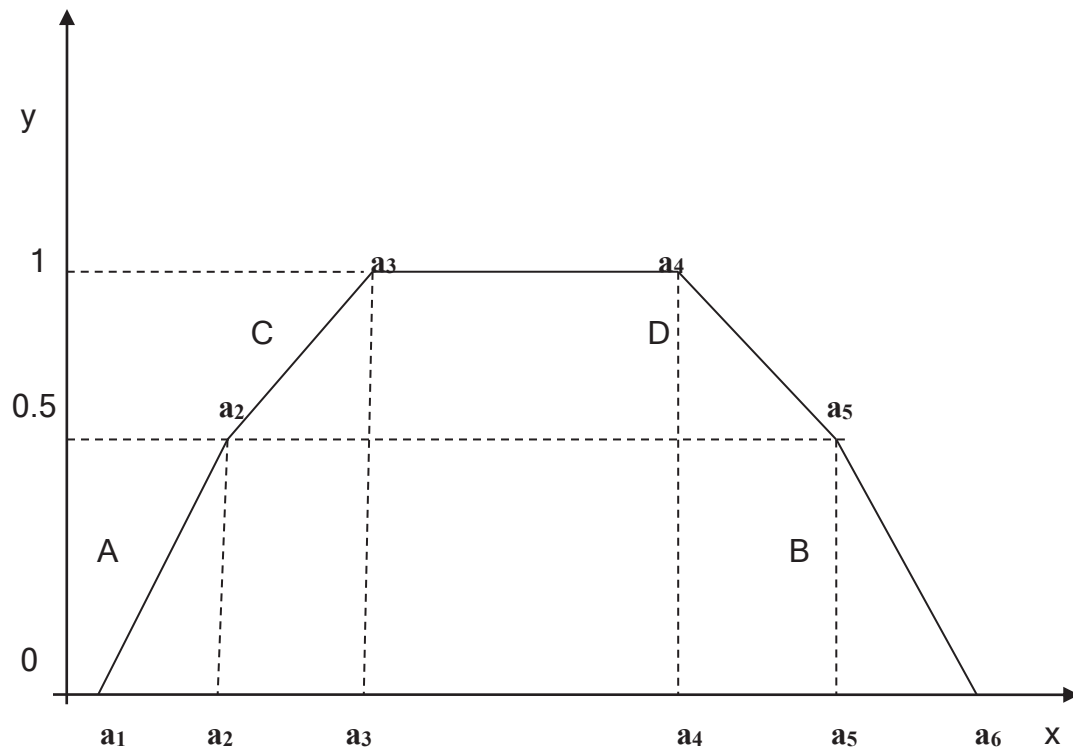
Membership function is

$$\mu_{\tilde{A}_H}(x) = \begin{cases} 0 & \text{for } 0 < a_1, \\ \frac{1}{2} \left[\frac{x - a_1}{a_2 - a_1} \right] & \text{for } a_1 \leq x \leq a_2, \\ \frac{1}{2} + \frac{1}{2} \left[\frac{x - a_2}{a_3 - a_2} \right] & \text{for } a_2 \leq x \leq a_3, \\ 1 & \text{for } a_3 \leq x \leq a_4, \\ 1 - \frac{1}{2} \left[\frac{x - a_4}{a_5 - a_4} \right] & \text{for } a_4 \leq x \leq a_5, \\ \frac{1}{2} \left[\frac{a_6 - x}{a_6 - a_5} \right] & \text{for } a_5 \leq x \leq a_6, \\ 0 & \text{for } 0 > a_6 \end{cases}$$

An Hexagonal fuzzy number denoted by $\tilde{A}_H$ is defined as (A, B, C, D) ; for $A, B \in [0, 0.5]$ and $C, D \in [0.5, 1]$ where,

- (i) A is a bounded left continuous increasing function over $[0, 0.5]$
- (ii) C is a bounded left continuous increasing function over $[0.5, 1]$
- (iii) D is a bounded right continuous decreasing function over $[1, 0.5]$
- (iv) B is a bounded right continuous decreasing function over $[0.5, 0]$

Graphical representation of Hexagonal fuzzy number



CHAPTER 4

HEPTAGONAL FUZZY NUMBERS AND THEIR OPERATIONS

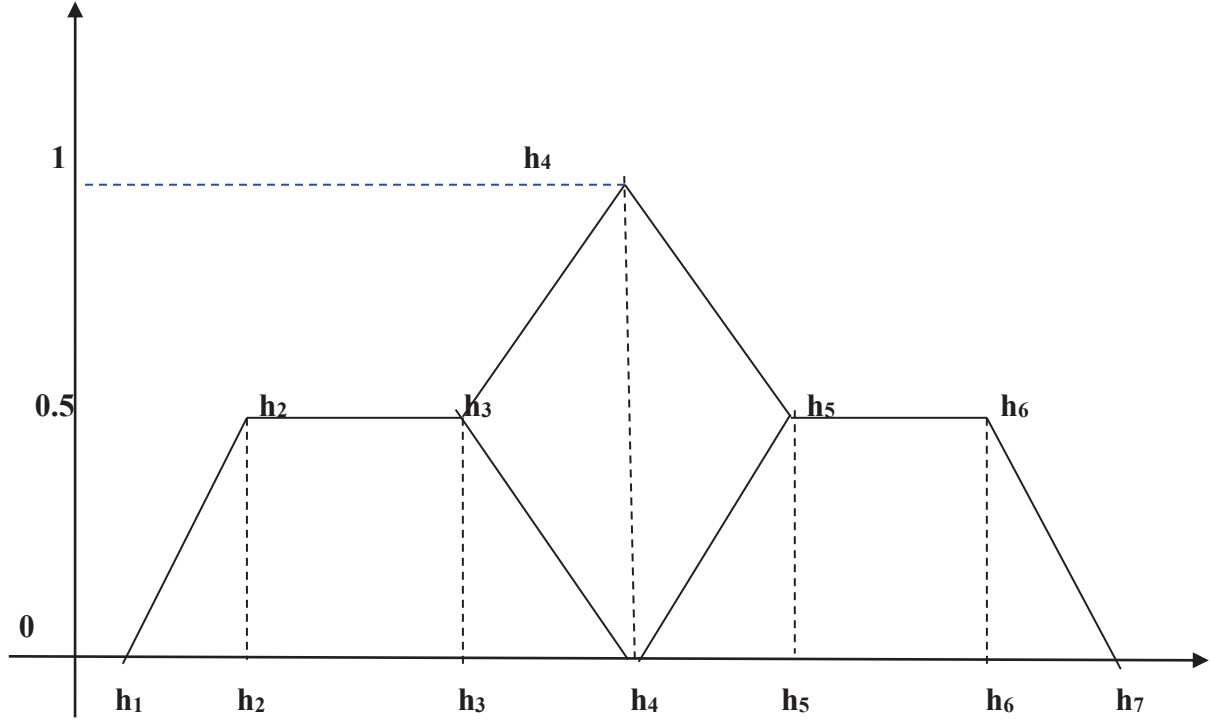
4.1 Heptagonal Fuzzy Numbers

A new form of fuzzy number called as Heptagonal Fuzzy number (HFN) is introduced which can be effectively used in solving many decision making problem. A fuzzy number $\tilde{H}$ in R is said to be a heptagonal fuzzy number if its membership function $\mu_{\tilde{H}}: R \rightarrow [0,1]$ has the following characteristics.

We denote the heptagonal fuzzy number by $\tilde{H} = (h_1, h_2, h_3, h_4, h_5, h_6, h_7)$

$$\mu_{\tilde{H}}(x) = \begin{cases} 0 & \text{for } x < h_1, \\ \frac{1}{2} \left[\frac{x - h_1}{h_2 - h_1} \right] & \text{for } h_1 \leq x \leq h_2, \\ \frac{1}{2} & \text{for } h_2 \leq x \leq h_3, \\ \frac{1}{2} + \frac{1}{2} \left[\frac{x - h_3}{h_4 - h_3} \right] & \text{for } h_3 \leq x \leq h_4, \\ \frac{1}{2} + \frac{1}{2} \left[\frac{h_5 - x}{h_5 - h_4} \right] & \text{for } h_4 \leq x \leq h_5, \\ \frac{1}{2} & \text{for } h_5 \leq x \leq h_6, \\ \frac{1}{2} \left[\frac{h_7 - x}{h_7 - h_6} \right] & \text{for } h_6 \leq x \leq h_7, \\ 0 & \text{for } x \geq h_7 \end{cases}$$

Graphical representation of Heptagonal fuzzy number



4.2 Arithmetic operations on Heptagonal fuzzy numbers

Let $\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ & $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ be two heptagonal fuzzy numbers then addition, subtraction, multiplication and division can be performed as,

Addition

$$\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$$

EXAMPLE: 1

Let $\tilde{A}_H = (8, 11, 13, 14, 15, 17, 20)$ & $\tilde{B}_H = (6, 8, 10, 12, 14, 16, 18)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$$

$$\tilde{A}_H + \tilde{B}_H = (14, 19, 23, 26, 29, 33, 38)$$

EXAMPLE: 2

Let $\tilde{A}_H = (8,10,12,14,16,18,20)$ & $\tilde{B}_H = (4,6,8,10,12,14,16)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H + \tilde{B}_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$$

$$\tilde{A}_H + \tilde{B}_H = (12,16,20,24,28,32,36)$$

Subtraction

$$\tilde{A}_H - \tilde{B}_H = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$$

EXAMPLE: 1

Let $\tilde{A}_H = (8,11,13,14,15,17,20)$ & $\tilde{B}_H = (6,8,10,12,14,16,18)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H - \tilde{B}_H = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$$

$$\tilde{A}_H - \tilde{B}_H = (-10, -5, -1, 2, 5, 9, 14)$$

EXAMPLE: 2

Let $\tilde{A}_H = (8,10,12,14,16,18,20)$ & $\tilde{B}_H = (4,6,8,10,12,14,16)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H - \tilde{B}_H = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$$

$$\tilde{A}_H \sim \tilde{B}_H = (-8, -4, 0, 4, 8, 12, 16)$$

Multiplication

$$\tilde{A}_H \times \tilde{B}_H = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6, a_7 \times b_7)$$

EXAMPLE: 1

Let $\tilde{A}_H = (9,10,11,12,13,14,15)$ & $\tilde{B}_H = (4,5,6,7,8,9,10)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H \times \tilde{B}_H = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6, a_7 \times b_7)$$

$$\tilde{A}_H \times \tilde{B}_H = (36,50,66,84,104,126,150)$$

EXAMPLE: 2

Let $\tilde{A}_H = (8,9,10,11,12,13,14)$ & $\tilde{B}_H = (2,4,8,9,10,14,16)$ are two heptagonal fuzzy numbers then

$$\tilde{A}_H \times \tilde{B}_H = (a_1 \times b_1, a_2 \times b_2, a_3 \times b_3, a_4 \times b_4, a_5 \times b_5, a_6 \times b_6, a_7 \times b_7)$$

$$\tilde{A}_H \times \tilde{B}_H = (16,36,80,99,120,182,224)$$

CHAPTER 5

RANKING ON HEPTAGONAL FUZZY NUMBERS

RANKING METHOD

The Centroid of Heptagonal fuzzy numbers is considered to be the balancing point of the heptagon. Firstly, divide the heptagon into two trapezoidal and one rhombus AHIC, DKLG and IJKD respectively. Then find the centroid of these plane figures. Let the centroid of these plane figures be G_1, G_2 and G_3 respectively. The Centroid of centroids G_1, G_2 and G_3 is taken as the point of reference to define the ranking of generalized heptagonal fuzzy numbers. The centroid of centroids G_1, G_2 and G_3 would be a better point than the centroid point of the Heptagonal.

Consider the generalized heptagonal fuzzy number

$$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; w)$$

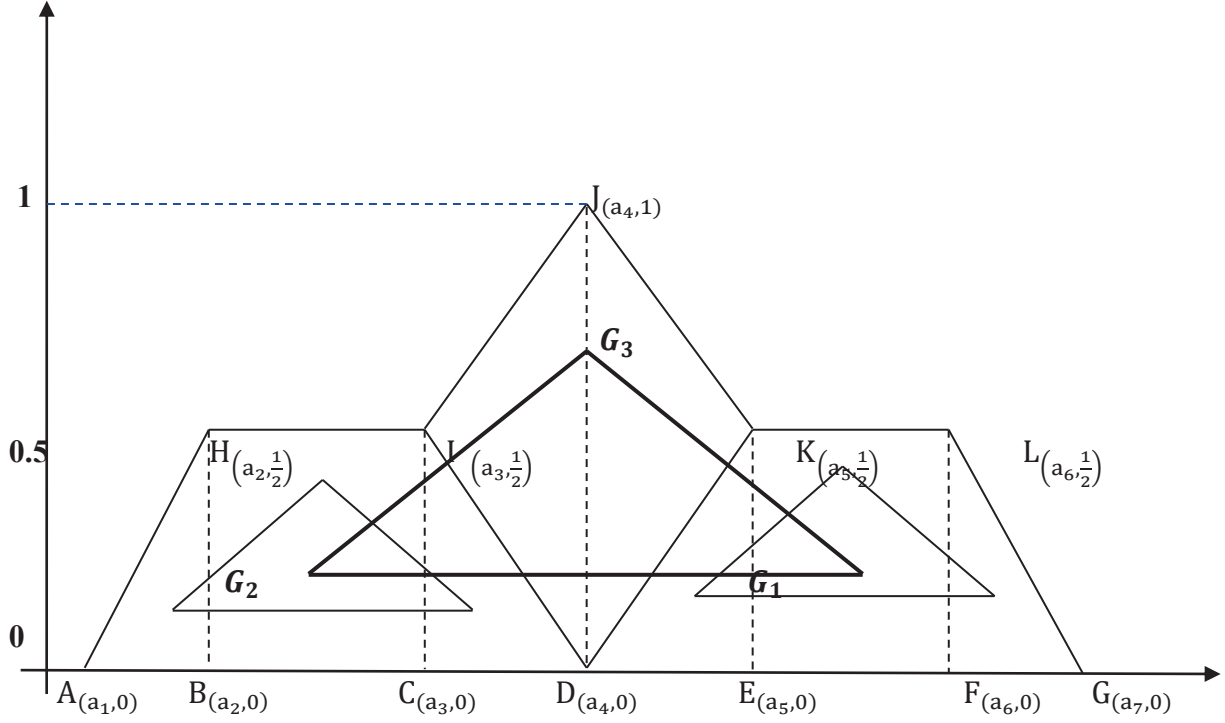
The centroid of these triangles are

$$G_1 = \left(\frac{2a_1 + 7a_2 + 7a_3 + 2a_4}{18}, \frac{7(w)}{36} \right)$$

$$G_2 = \left(\frac{2a_4 + 7a_5 + 7a_6 + 2a_7}{18}, \frac{7(w)}{36} \right)$$

$$G_3 = \left(a_4, \frac{w}{2} \right)$$

Graphical representation of proposed ranking method for Heptagonal fuzzy number



As G_1, G_2 and G_3 are non collinear and they form a triangle. Therefore Centroid

$G_{A_H} = (x_0, y_0)$ of a triangle whose vertices are G_1, G_2 and G_3 of generalized heptagonal fuzzy numbers is

$$G_{A_H}(x_0, y_0) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 22a_4 + 7a_5 + 7a_6 + 2a_7}{54}, \frac{16(w)}{54} \right)$$

The Ranking function of the generalized heptagonal fuzzy numbers is defined as:

$$R_{A_H}(x_0, y_0) = x_0, y_0 = \left(\left(\frac{2a_1 + 7a_2 + 7a_3 + 22a_4 + 7a_5 + 7a_6 + 2a_7}{54} \right), \left(\frac{16(w)}{54} \right) \right)$$

The Ranking function of the generalized heptagonal fuzzy number is defined as:

$$R_{A_H}(x_0, y_0) = \sqrt{x_0^2 + y_0^2}$$

RANKING OF GENERALISED HEPTAGONAL FUZZY NUMBERS

Using the above definitions proposed in above section. The ranking procedure for two generalized heptagonal fuzzy numbers is as follows:

$\tilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6, a_7 : w)$ and $\tilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6, b_7 : w)$ be two generalized heptagon fuzzy numbers then

Find $\check{R}(\tilde{A})$ and $\check{R}(\tilde{B})$

- 1) If $\check{R}(\tilde{A}) > \check{R}(\tilde{B})$ then $\tilde{A} > \tilde{B}$
- 2) If $\check{R}(\tilde{A}) < \check{R}(\tilde{B})$ then $\tilde{A} < \tilde{B}$
- 3) If $\check{R}(\tilde{A}) = \check{R}(\tilde{B})$ then $\tilde{A} = \tilde{B}$

EXAMPLE 1

Let $\tilde{A}_H = (4, 5, 6, 7, 8, 9, 10 : 1)$ & $\tilde{B}_H = (9, 10, 11, 12, 13, 14, 15 : 1)$ be two generalized heptagonal fuzzy numbers

$$\text{Then } G_{\tilde{A}_H}(\bar{x}_0, \bar{y}_0) = (12, 0.2037)$$

$$G_{\tilde{B}_H}(\bar{x}_0, \bar{y}_0) = (7, 0.2037)$$

where $R_{\tilde{A}_H} = 12.0169$ and $R_{\tilde{B}_H} = 7.0029$

Thus $R_{\tilde{A}_H} < R_{\tilde{B}_H}$ and $\tilde{A}_H < \tilde{B}_H$

EXAMPLE 2

Let $\tilde{A}_H = (4, 5, 6, 7, 8, 9, 10 : 1)$ & $\tilde{B}_H = (9, 10, 11, 12, 13, 14, 15 : 1)$ be two generalized heptagonal fuzzy numbers

$$\text{Then } G_{\tilde{A}_H}(\bar{x}_0, \bar{y}_0) = (9, 0.1629)$$

$$G_{\tilde{B}_H}(\bar{x}_0, \bar{y}_0) = (9, 0.1629)$$

Find $R_{\tilde{A}_H}$ and $R_{\tilde{B}_H}$

$$R_{\tilde{A}_H} = 9.00146$$

$$R_{\tilde{B}_H} = 9.00147$$

Thus $R_{\tilde{A}_H} = R_{\tilde{B}_H}$.

CHAPTER 6

SOLVING HEPTAGONAL FUZZY TRANSPORTATION PROBLEMS

6.1 Generalized Fuzzy North – West corner method (GFNWCM)

The steps for finding the IFBFS using GFNWCM

Step1: Select the NWC cell of the fuzzy transportation table (FTT) and find minimum of $\tilde{a}_i$ and $\tilde{b}_j$. There may arise the following three cases:

Case (i) If minimum $(\tilde{a}_i, \tilde{b}_j) = \tilde{a}_i$ then allocate $\tilde{x}_{ij} = \tilde{a}_i$ in the NWC of $m \times n$ FTT. Ignore the i th row to obtain a new FTT of order $(m - 1) \times n$. Replace $\tilde{b}_j - \tilde{a}_i$ in the obtained FTT. Go to step 2.

Case (ii) If minimum $(\tilde{a}_i, \tilde{b}_j) = \tilde{b}_j$ then allocate $\tilde{x}_{ij} = \tilde{b}_j$ in the NWC of $m \times n$ FTT. Ignore the j th column to obtain a new FTT of order $m \times (n - 1)$. Replace $\tilde{a}_i - \tilde{b}_j$ in the obtained FTT. Go to step 2.

Case (iii) If $\tilde{a}_i = \tilde{b}_j$, then either follow case (i) or case (ii) but not both, simultaneously. Go to step 2.

Step 2: Repeat step 1 for the obtained FTT, until the FTT is reduced into a FTT of order 1×1 .

Step3: Allocate all $\tilde{x}_{ij}$ in the (i, j) th cell of given FTT.

Step4: The obtained IFBFS and initial fuzzy transportation cost $\tilde{x}_{ij}$ and

$$\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \times \tilde{x}_{ij} \text{ respectively.}$$

6.1.1 Numerical Example:

Consider a fully fuzzy transportation problem in which supply at sources, demand at destinations and fuzzy unit transportation costs, etc... are assumed to be heptagonal fuzzy numbers.

Destinations							
		$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	$\tilde{D}_5$	Fuzzy Supply ($\tilde{S}$)
S O U R C E S	$\tilde{S}_1$	63,66,69,73,77, 80,83	15,20,25,40,55, 60,65	3,5,8,9,10,13,15	70,73,75,79,83, 85,88	12,16,17,20,23, 24,28	2,3,5,8,11,13,14
		$\tilde{S}_2$	56,59,61,62,63, 65,68	87,89,92,93,94, 97,99	87,90,92,96,100, 102,105	0,4,5,8,11,12,16	7,8,10,13,16,18, 19
	$\tilde{S}_3$	90,92,95,96,97, 100,102	56,59,61,65,69, 71,74	72,76,77,80,83, 84,88	44,45,47,50,53, 55,56	59,62,64,65,66, 68,71	3,5,8,9,10,13,15
		$\tilde{S}_4$	48,51,53,57,61, 63,66	50,54,55,58,61, 62,66	23,24,26,29,32, 34,35	6,9,11,12,13,15, 18	81,83,86,87,88, 91,93
	$\tilde{S}_5$	48,52,53,57,61, 63,66	17,18,20,23,26, 28,29	81,8,84,86,87,88, 90,93	12,14,17,18,19, 22,24	43,42,41,12, -17,-18,-19	-3,1,2,5,8,9,13
		Fuzzy Demand ($\tilde{D}$)	0,1,3,6,9,11,12	2,5,7,8,9,11,14	4,6,9,10,11,14,16	-5,-2,0,4,8,10, 13	-4,0,1,4,7,8,12

The heptagonal fuzzy transportation problem is balanced

$$\sum \tilde{a}_i = \sum \tilde{b}_j = (-3,10,20,32,44,54,67)$$

By using North – West Corner method,

Destinations							
		$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	$\tilde{D}_5$	Fuzzy Supply ($\tilde{S}$)
sources	$\tilde{S}_1$	0,1,3,6,9,11,12	-10,-8,- 4,2,8,12,14				2,3,5,8,11,13,14
	$\tilde{S}_2$		-12,-7,- 1,6,13,19,24	-23,-15,- 7,1,9,17,25			1,4,6,7,8,10,13
	$\tilde{S}_3$			-21,- 11,0,9,18,29,39			3,5,8,9,10,13,15
	$\tilde{S}_4$				-6,-3,- 1,3,7,9,12		-6,-3,-1,3,7,9,12
	$\tilde{S}_5$				-17,-11,- 7,1,9,13,19	-22,-12,- 7,4,15,20,29	-3, 1,2,5,8,9,13
Fuzzy Demand($\tilde{D}$)		0,1,3,6,9,11,12	2,5,7,8,9,11,14	4,6,9,10,11,14,16	-5,- 2,0,4,8,10,13	- 4,0,1,4,7,8,12	

FUZZY TOTAL TRANSPORTATION COST

$$= (-4027, -3584, -1613, 1994, 4772, 7696, 10813)$$

6.2 Generalized Fuzzy Least – Cost method (GFLCM)

The steps for finding the IFBFS using GFLCM

Step1: Determine the smallest fuzzy cost in fuzzy transportation table (FTT). Let it be $\tilde{c}_{ij}$. Find $\tilde{x}_{ij} = \text{minimum}(\tilde{a}_i, \tilde{b}_j)$. There may arise the following three cases:

Case (i) If $\text{minimum}(\tilde{a}_i, \tilde{b}_j) = \tilde{a}_i$ then allocate $\tilde{x}_{ij} = \tilde{a}_i$ in the (i,j)th cell of $m \times n$ FTT. Ignore the ith row to obtain a new FTT of order $(m - 1) \times n$. Replace $\tilde{b}_j$ by $\tilde{b}_j - \tilde{a}_i$ in the obtained FTT. Go to step 2.

Case (ii) If $\text{minimum}(\tilde{a}_i, \tilde{b}_j) = \tilde{b}_j$ then allocate $\tilde{x}_{ij} = \tilde{b}_j$ in the (i,j)th cell of $m \times n$ FTT. Ignore the jth column to obtain a new FTT of order $m \times (n - 1)$. Replace $\tilde{a}_i$ and $\tilde{a}_i - \tilde{b}_j$ in the obtained FTT. Go to step 2.

Case (iii) If $\tilde{a}_i = \tilde{b}_j$, then either follow case (i) or case (ii) but not both, simultaneously. Go to step 2.

Step 2: Repeat step1 for the obtained FTT, until the FTT is reduced into a FTT of order 1×1 .

Step3: Allocate all $\tilde{x}_{ij}$ in the (i,j)th cell of given FTT.

Step4: The obtained IFBFS and initial fuzzy transportation cost $\tilde{x}_{ij}$ and

$$\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \times \tilde{x}_{ij} \text{ respectively.}$$

6.2.1 Numerical Example:

The above fully fuzzy transportation problem is solved by using Generalized Fuzzy Least – Cost method.

		Destinations					Fuzzy Supply ($\tilde{S}$)
		$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	$\tilde{D}_5$	
Sources	$\tilde{S}_1$			2,3,5,8,11,13,14			2,3,5,8,11,13,14
	$\tilde{S}_2$	-11,-4,- 1,3,7,10,17			-5,- 2,0,4,8,10,13		1,4,6,7,8,10,13
	$\tilde{S}_3$	-26,-13,- 6,2,10,17,30	-15,- 4,0,7,14,18,29				3,5,8,9,10,13,15
	$\tilde{S}_4$	-20,-14,- 7,1,9,16,22		-10,-7,- 2,2,6,11,14			-6,-3,-1,3,7,9,12
	$\tilde{S}_5$		-15,-7,- 5,1,7,9,17			- 4,0,1,4,7,8,12	-3, 1,2,5,8,9,13
Fuzzy Demand($\tilde{D}$)		0,1,3,6,9,11,12	2,5,7,8,9,11,14	4,6,9,10,11,14,16	-5,- 2,0,4,8,10,13	- 4,0,1,4,7,8,12	

FUZZY TOTAL TRANSPORTATION COST

$$= (-5417, -2669, -1139, 1123, 3398, 5407, 9107)$$

6.3 Generalized Fuzzy Vogel's Approximation method (GFVAM)

The steps for finding the IFBFS using GFVAM

Step1: Take the first row and choose its smallest entry and subtract this from next smallest entry, and write in from of the row on the right. This is the fuzzy penalty for first row below corresponding columns.

Step2: Select the highest fuzzy penalty and observe the row or column for which this corresponds. Let it be $\tilde{c}_{ij}$.

Find $\tilde{x}_{ij} = \text{minimum}(\tilde{a}_i, \tilde{b}_j)$. There may arise the following three cases:

Case (i) If $\text{minimum}(\tilde{a}_i, \tilde{b}_j) = \tilde{a}_i$ then allocate $\tilde{x}_{ij} = \tilde{a}_i$ in the (i,j)th cell of $m \times n$ FTT. Ignore the ith row to obtain a new FTT of order $(m - 1) \times n$. Replace $\tilde{b}_j$ by $\tilde{b}_j - \tilde{a}_i$ in the obtained FTT. Go to step 3.

Case (ii) If $\text{minimum}(\tilde{a}_i, \tilde{b}_j) = \tilde{b}_j$ then allocate $\tilde{x}_{ij} = \tilde{b}_j$ in the (i,j)th cell of $m \times n$ FTT. Ignore the jth row to obtain a new FTT of order $m \times (n - 1)$. Replace $\tilde{a}_i$ and $\tilde{a}_i - \tilde{b}_j$ in the obtained FTT. Go to step 3.

Case (iii) If $\tilde{a}_i = \tilde{b}_j$, then either follow case (i) or case (ii) but not both, simultaneously. Go to step 3.

Step 3: Repeat step1 for the obtained FTT, until the FTT is reduced into a FTT of order 1×1 .

Step 4: Allocate all $\tilde{x}_{ij}$ in the (i, j)th cell of given FTT.

Step 5: The obtained IFBFS and initial fuzzy transportation cost $\tilde{x}_{ij}$ and $\sum_{i=1}^m \sum_{j=1}^n \tilde{c}_{ij} \times \tilde{x}_{ij}$ respectively.

6.3.1 Numerical Example:

The above fully fuzzy transportation problem is solved by using Generalized Fuzzy Vogel's Approximation method then

Destinations							
		$\tilde{D}_1$	$\tilde{D}_2$	$\tilde{D}_3$	$\tilde{D}_4$	$\tilde{D}_5$	Fuzzy Supply ($\tilde{S}$)
Sources	$\tilde{S}_1$			2,3,5,8,11,13,14			2,3,5,8,11,13,14
	$\tilde{S}_2$				-11,-4,- 1,3,7,10,17	- 4,0,1,4,7,8,12	1,4,6,7,8,10,13
	$\tilde{S}_3$	0,1,3,6,9,11,12	-9,-6,- 1,3,7,12,15				3,5,8,9,10,13,15
	$\tilde{S}_4$			-10,-7,- 2,2,6,11,14	-20,-14,- 7,1,9,16,22		-6,-3,-1,3,7,9,12
	$\tilde{S}_5$		-3,1,2,5,8,9,13				-3, 1,2,5,8,9,13
Fuzzy Demand($\tilde{D}$)		0,1,3,6,9,11,12	2,5,7,8,9,11,14	4,6,9,10,11,14,16	-5,- 2,0,4,8,10,13	- 4,0,1,4,7,8,12	

FUZZY TOTAL TRANSPORTAION COST

$$= (-927, -539, -390, 1104, 2172, 3251, 4364)$$

CHAPTER 7

CONCLUSION

The primary objective of this study was to develop new operations for heptagonal fuzzy numbers and to develop a new ranking method for heptagonal fuzzy numbers called centroid of centroids. These methods are necessary depending on the nature of the various decision-making problems. Secondary focus has been on the application of new operations and ranking for heptagonal fuzzy transportation problems. In order to solve the heptagonal fuzzy transportation problem, we have used the north-west corner method, the least cost method, and Vogel's approximation method.

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