

CLIQUE NUMBER AND CHROMATIC NUMBER OF A ROUGH ZERO DIVISOR GRAPH

Submitted in partial fulfilment of the requirements for the award of

Bachelor of Science Degree in Mathematics

By

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DEPARTMENT OF MATHEMATICS

SCHOOL OF SCIENCE AND HUMANITIES

SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

(DEEMED TO BE UNIVERSITY)

Accredited with a Grade “A” by

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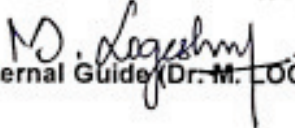
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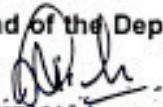
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BONAFIDE CERTIFICATE

This is to certify that this project report is the bonafide work of (M. BARATH) (40060001) & (K. DEEPIKA SRI) (40060002) who carried out the project entitled "CLIQUE NUMBER AND CHROMATIC NUMBER OF A ROUGH ZERO DIVISOR GRAPH" under our supervision from JANUARY 2023 to APRIL 2023.


Internal Guide (Dr. M. LOGESHWARI)

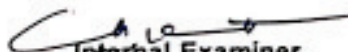
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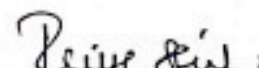

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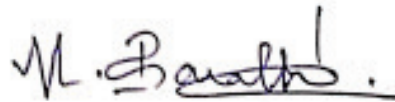
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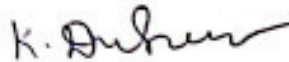
DECLARATION

We (M. BARATH) (40060001) & (K. DEEPIKA SRI) (40060002) hereby declare that the project report titled "CLIQUE NUMBER AND CHROMATIC NUMBER OF A ROUGH ZERO DIVISOR GRAPH" done by us under the guidance of (Dr. M. LOGESHWARI) (Internal) at SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY submitted in partial fulfilment of the requirements for the award of Bachelor of Science Degree in Mathematics



DATE: 29.4.2023

PLACE: Chennai



SIGNATURE OF THE CANDIDATE

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We would like to express our sincere and deep sense of gratitude to our Project Guide Dr. M. LOGESHWARI for her valuable guidance, suggestions and constant encouragement paved the way for the successful completion of our project work.

We wish to express our thanks to all Teaching and Non-teaching staff members of the Department of Mathematics who were helpful in many ways for the completion of the project.

Abstract

In this proposed work an approximation space $I = (U, R)$, where U denotes a nonempty finite set of objects and R denotes an arbitrary equivalence relation defined on U is considered. The Rough Zero Divisor Graph of a Rough Semiring (T, Δ, ∇) on I is taken for study.

Let $\{X_1, X_2, \dots, X_n\}$ be the n equivalence classes induced by R . Without loss of generality assume that $\{X_1, X_2, \dots, X_m\}$ be the m equivalence classes whose cardinality is greater than one and $\{X_{m+1}, X_{m+2}, \dots, X_n\}$ be the $n - m$ equivalence classes whose cardinality is equal to one. Let $B = \{\{x_1\}, \{x_2\}, \dots, \{x_m\}\}$ be the set of pivot elements chosen from the equivalence classes whose cardinality is greater than one. Also assume that M is the union of none, one or more of the equivalence classes whose cardinality is equal to one and M' is the union of one or more of the equivalence classes whose cardinality is equal to one.

The Rough Zero divisor graph consists of $2^{n-m}(3^m) - 2^m - 1$ vertices, $1 \leq m \leq n$, where n represents the number of equivalence classes in U and m represents the number of equivalence classes with cardinality greater than one. The study of this Rough Zero divisor graph is challenging for large values of m and n . The complicity in studying this Rough Zero Divisor Graph was made simpler by defining clustering on the vertices of the Rough Zero divisor graph. A partition on vertices of the Rough Zero divisor graph is obtained based on the degree of vertices.

The graph theoretical parameters of the Rough Zero divisor graph namely the Clique number and Chromatic number are computed with suitable illustrations.

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CHAPTER 1

INTRODUCTION

In 1736, Euler published a paper titled "Solutio problematis ad geometriam situs pertinentis" ("Solution to a Problem in the Geometry of Position"), in which he introduced the concept of a graph and used it to solve a famous problem known as the Seven Bridges of Königsberg problem.

The concepts of clique and chromatic number were introduced by Hassler Whitney, In a 1931 paper titled "On the abstract properties of linear dependence", Whitney defined the concept of a clique in a graph. In a 1932 paper titled "A theorem on graphs", Whitney defined the concept of the chromatic number of a graph.

Pawlak introduced the concept of rough set theory in 1982 to process incomplete information in the information system. The rough set theory will be used to discover fundamental patterns in data analysis, remove redundancies, generate decision rules, computational Intelligence and others.

The concept of semiring was first introduced by H.S. Vandiver in 1934. The Zero Divisor Graph (ZDG) was introduced by Irving Kaplansky, a prominent mathematician in algebra and algebraic geometry, in his 1970 paper "Fields and Rings". The concept of the Zero Divisor Graph (ZDG) was first introduced in a 1973 paper by Beck, which was published in the Journal of the London Mathematical Society. In the paper, Beck studied the ZDG of a commutative ring with identity, which is a graph whose vertices correspond to the nonzero zero-divisors of the ring, and two vertices are adjacent if their product is zero in the ring.

Praba et al. discussed the concept of the rough lattice. The authors considered an information system $I = (U, A)$ in this paper. In a partial ordering

relation, the operations Δ and ∇ were defined on $T = \{R S(X) \mid X \subseteq U\}$ using these two operations. The least upper bound and greatest lower bound were established. In the recent and past, rough set theory has triggered many researchers all around the world. The concept of rough set theory is the approximation space such as lower and upper approximations of a set determined by attributes.

The pair of lower and upper approximation is called a rough set also in rough set theory data can be represented in the form of an information system. An information system is a pair $I = (U, A)$ where U is a non-empty finite set of objects, called a universal set and A is a nonempty set of fuzzy attributes defined by $\mu_a: U \rightarrow [0, 1], a \in A$, is a fuzzy set. Indiscernibility is a core concept of rough set theory and it is defined as an equivalence between objects. Objects in the information system about which we have the same knowledge form an equivalence relation.

Formally any set $P \subseteq A$, there is an associated equivalence relation called P – Indiscernibility relation defined as follows,

$$IND(P) = \{(x, y) \in U^2 \mid \forall a \in P, \mu_a(x) = \mu_a(y)\}.$$

The partition induced by $IND(P)$ consists of equivalence classes defined by $[x]_P = \{y \in U \mid (x, y) \in IND(P)\}$. For any $X \subseteq U$, define the lower approximation space $P(X) = \{x \in U \mid [x]_P \subseteq X\}$. Also, define the upper approximation space $P(X) = \{x \in U \mid [x]_P \cap X \neq \varnothing\}$.

The concept of the Rough Zero Divisor Graph (RZDG) is a generalization of the ZDG, and it was introduced by Aladibi and Ashraf in a 2014 paper published in the Journal of Algebra and Its Applications. The RZDG is defined

in a similar way to the ZDG, but the vertices of the graph correspond to the nonzero zero-divisors of the ring, and two vertices are adjacent if their difference is a unit in the ring. The RZDG has been studied in various contexts, including algebraic geometry, algebraic number theory, and coding theory. For every subset $X \in \mathcal{U}$, there is an associated rough set $RS(X) = (\underline{P}(X), \overline{P}(X))$.

CHAPTER 2

PRELIMINARIES

Definition 2.1: **(Graph)** A graph can be defined as a pictorial representation or a diagram that represents data or values in an organized manner. The points on the graph often represent the relationship between two or more things.

Definition 2.2: **(Vertex)** A vertex is a point where multiple lines meet. It is also called a node. Similar to points, a vertex is also denoted by an alphabet.

Definition 2.3: **(Edges)** An edge is the mathematical term for a line that connects two vertices. Many edges can be formed from a single vertex. Without a vertex, an edge cannot be formed. There must be a starting vertex and an ending vertex for an edge

Definition 2.4: **(Degree of a vertex)** The number of edges incident to a vertex v is called the degree of the vertex and is denoted by $\deg(v)$. Equivalently, the degree of a vertex is the number of vertices adjacent to it.

Definition 2.5: **(Complete graph)** A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge. A complete digraph is a directed graph in which every pair of distinct vertices is connected by a pair of unique edges

Definition 2.6: **(Sub Graph)** A graph whose vertices and edges are subsets of another graph

Definition 2.7: **(complete subgraph)** A complete subgraph is a set of nodes for which all the nodes are connected to each other. Maximal complete subgraphs are then the largest (i.e. those containing most objects) of these complete subgraphs

Definition 2.8: **(Set)** A set is a well-defined collection of distinct objects considered a single entity. The objects in a set are called their elements or members. The elements of a set can be anything: numbers, letters, symbols, or even other sets. The elements of a set are listed inside braces $\{ \}$, separated by commas.

For example, the set of all vowels in the English alphabet can be written as $\{a, e, i, o, u\}$.

Definition 2.9: **(Subset)** A subset is a set that contains only elements that are also in another set. In other words, if every element of set A is also an element of set B, then A is a subset of B. We use the symbol $\subseteq$ (or sometimes $\subset$) to indicate that one set is a subset of another.

For example, if $A = \{2, 4, 6\}$ and $B = \{2, 3, 4, 6\}$, then A is a subset of B, and we write $A \subseteq B$. Note that A can also be equal to B (i.e., every element of A is also an element of B, and vice versa), in which case we write $A = B$.

Definition 2.11: **(Rough set)** Let $I=(U, A)$ be an information system and for any subset, X of U and $\underline{-(P(X))}, \overline{(P(X))}$ are the lower and upper approximations respectively

Definition 2.12: **(Semiring)** A semiring is an algebraic structure similar to a ring, but without requiring each element t have an additive inverse.

Definition 2.13: **(Rough Semiring)** A rough semiring is a triple $(S, +, \cdot)$, where S is a non-empty set, and $+$ and $\cdot$ are binary operations on S that satisfy the following axioms:

- a. $(S, +)$ is a commutative monoid with identity element 0
- b. $(S, \bullet)$ is a semiring with identity element 1
- c. Distributivity of $\bullet$ over $+$: for all a, b , and c in S , $a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
- d. Closure of $+$ under approximation: for all A, B in $P(S)$, the set $A + B = \{a + b \mid a \in A, b \in B\}$ is an upper approximation of $A \cup B$, and the set $A \bullet B = \{a \bullet b \mid a \in A, b \in B\}$ is a lower approximation of $A \cap B$

Definition 2.14: **(Zero Divisor Graph)** The Zero Divisor Graph of G is a graph $= (V, E)$, where $V = Z(G) - \{0\}$ is the set of nonzero zero divisors of G , and $E = \{(u, v) \mid uv = 0\}$ is the set of edges connecting pairs of vertices whose product is zero in the ring.

Definition 2.15: **(The Zero Divisor Graph of a Rough Semiring)** The zero divisor graph of a rough semiring is $T(Z(G)) = (V, E)$ where V is the set of vertices in $T(Z(G))$ consists of the non-empty zero divisors and E is the set of edges connecting the elements of V such that there is an edge connecting $RS(X)$ and $RS(Y)$ in V if $RS(X) \cap RS(Y) = RS(\emptyset)$. This graph $T(Z(G))$ is called a rough zero divisor graph of the rough semiring T .

Definition 2.16: **(Clique Number)** Maximum complete subgraph is the clique number of the graph.

Definition 2.21: **(Chromatic Number)** The chromatic number of a graph is the smallest number of colours needed to colour the vertices of a graph so that no two adjacent vertices share the same colour.

Definition 2.22: **(Clustering)** Clustering is a technique used in machine learning and data analysis to group together similar objects or data points based on some similarity or distance metric.

CHAPTER 3

CLIQUE AND CHROMATIC NUMBER OF ROUGH ZERO DIVISOR GRAPH

Theorem 3.1 Clique number of a Rough Zero Divisor Graph is equal to n .

Proof :

We know that the clique number of a graph is the maximum complete subgraph of a Rough Zero Divisor Graph.

We have to show that the number of equivalence classes forms a clique number

Let $RS(X_1)$ be connected to all the elements in

The form of $RS(x_2), RS(x_3) \dots RS(x_m), RS(X_2), \dots RS(X_m), RS(X_{m+1}) \dots RS(X_n)$. Similarly $RS(X_2)$ is connected to $RS(x_1), RS(x_3) \dots RS(x_m), RS(X_1), \dots RS(X_m), RS(X_{m+1}) \dots RS(X_n)$. correspondingly $RS(X_1), RS(X_2), RS(X_3) \dots RS(X_m), RS(X_{m+1}) \dots RS(X_n)$ are connected uniquely. so, n is the maximum connected vertices of the Rough Zero Divisor Graph, therefore, the clique number is n .

Theorem 3.2 Chromatic number of a Rough Zero Divisor Graph is equal to n .

Proof :

We know that the chromatic number of a graph is the minimum number of colours needed to colour a Rough Zero Divisor Graph such that no two adjacent vertices in a rough Zero Divisor Graph share the same colour.

We have to show that the number of equivalence classes forms a chromatic number

Let $RS(X_1)$ be connected to all the elements in

The form of $RS(x_2), RS(x_3) \dots RS(x_m), RS(X_2), \dots RS(X_m), RS(X_{m+1}) \dots RS(X_n)$ hence we

give one colour to $RS(X_1)$ Say(Green). Similarly $RS(X_2)$ is connected to $RS(x_1), RS(x_3) \dots \dots RS(x_m), RS(X_1), RS(X_3) \dots \dots RS(X_m), \dots \dots RS(X_n)$ we give Red to $RS(X_2)$. correspondingly there are n colours needed to colour the Rough Zero Divisor Graph therefore the chromatic number is n .

CHAPTER 4

ILLUSTRATION

Illustration 4.1

Information table

A/U	a ₁	a ₂	a ₃	a ₄
x ₁	0	0.1	0.3	0.2
x ₂	1	0.6	0.7	0.3
x ₃	0	0.1	0.3	0.2
x ₄	1	0.6	0.7	0.3
x ₅	0.8	0.5	0.2	0.4
x ₆	1	0.6	0.7	0.3

This is an information System (U, A) in this x_1 and x_3 rows contain the same value so, we are taking it into one Equivalence class. Then x_2, x_4 and x_6 rows contain the same value so, we are taking it into the 2nd equivalence class. Then x_5 is different from other rows so, we are taking it into the 3rd equivalence class. n is the number of Equivalence classes and m is the number of equivalence classes, whose coordinately is greater than one.

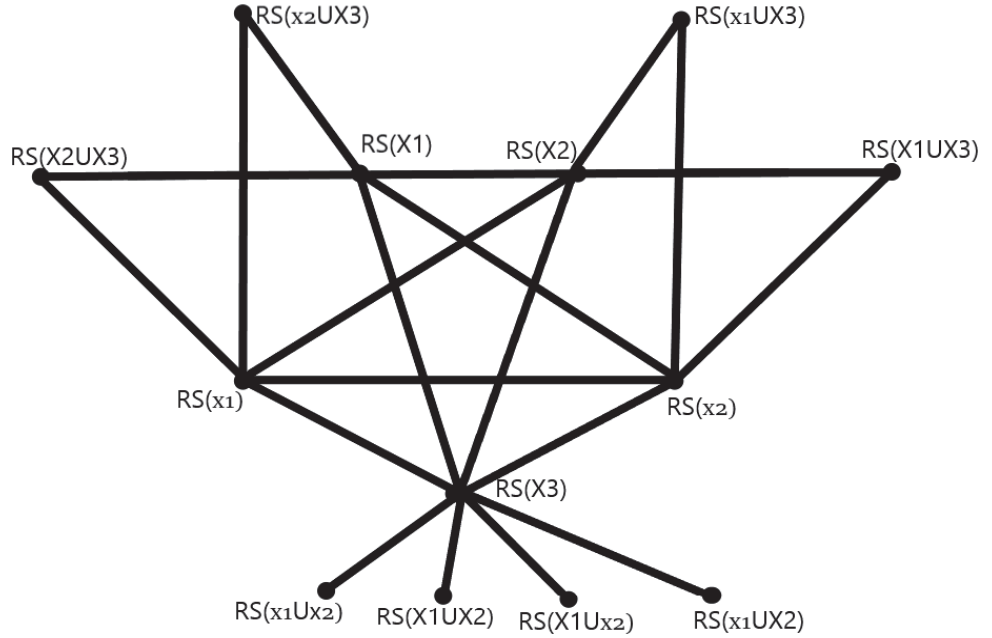
$$X_1 = [x_1]p = \{x_1, x_3\}$$

$$X_2 = [x_2]p = \{x_2, x_4, x_6\}$$

$$X_3 = [x_5]p = \{x_5\}$$

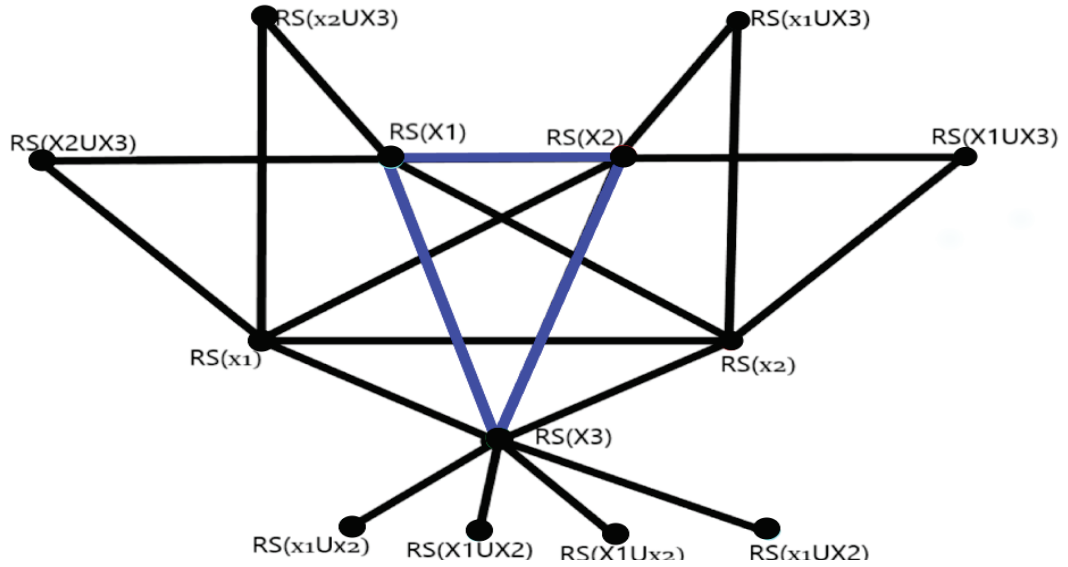
$$n = 3 \quad m = 2, \quad 2^{n-m} \cdot 3^m - 2^m - 1 = 13$$

$$V = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\})\}.$$



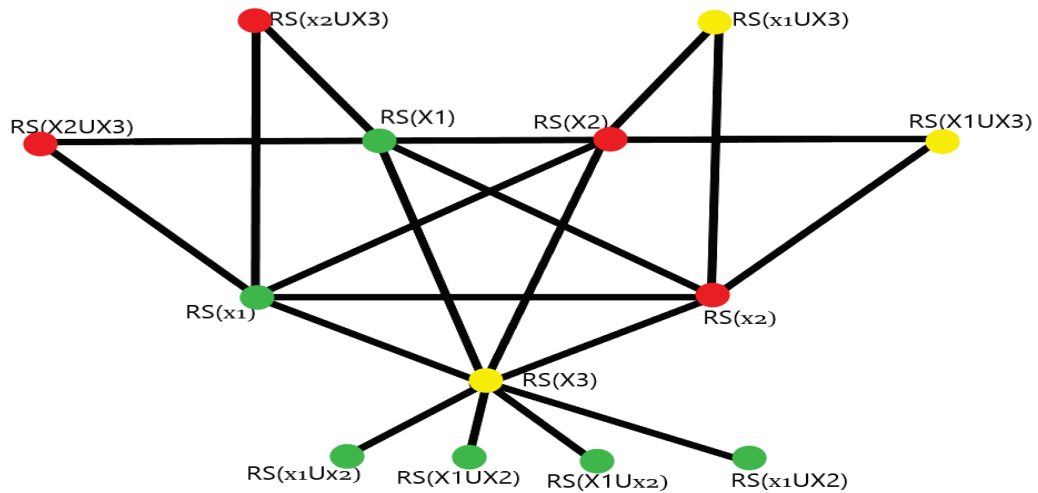
For $n = 3, m = 2$ Rough Zero Divisor Graph

The vertex $RS(X_1)$ is connected to $RS(X_2 \cup X_3)$ because $RS(X_1) \Delta RS(X_2 \cup X_3) = RS(\emptyset)$. but, The vertex $RS(X_1)$ is not connected to $RS(X_1 \cup X_3)$ because $RS(X_1) \Delta RS(X_1 \cup X_3) \neq RS(\emptyset)$



Clique number of $n = 3, m = 2$ Rough Zero Divisor Graph

As the clique number definition claims “the maximum complete subgraph “ we obtained that the clique number is equal to 3 .then it is equal to n.



The chromatic number of $n = 3, m = 2$ Rough Zero Divisor Graph

As chromatic number definition claims that the “smallest number of colours needed and no two adjacent has the same colour “.we obtained that the chromatic number is equal to 3 . which is equal to the equivalence class.

Illustration 4.2

Information table

A/U	a_1	a_2	a_3	a_4
x_1	0	0.1	0.3	0.2
x_2	1	0.6	0.7	0.3
x_3	0	0.1	0.3	0.2
x_4	1	0.6	0.7	0.3
x_5	0.8	0.5	0.2	0.4
x_6	1	0.6	0.7	0.3

This is an information System (U, A) in this x_1 and x_3 rows contain the same value so, we are taking it into one Equivalence class. Then x_2 and x_4 rows contain the same value so, we are taking it into the 2nd equivalence class. then x_5 and x_6 are different from other rows so, we are taking it into the 3rd equivalence class. n is the number of Equivalence classes and m is the number of equivalence classes, whose coordinately is greater than one.

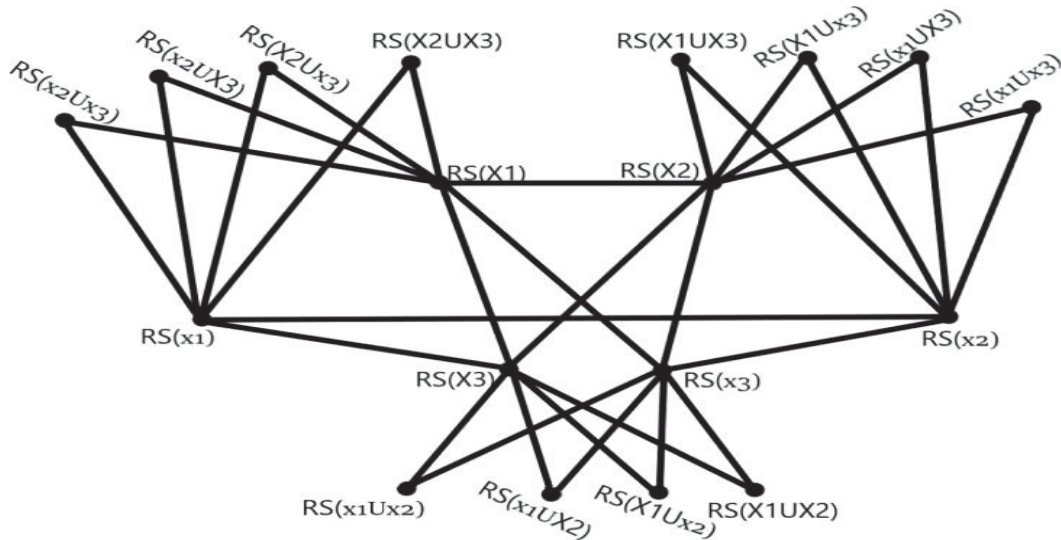
$$X_1 = [x_1]p = \{x_1, x_3\}$$

$$X_2 = [x_2]p = \{x_2, x_4\}$$

$$X_3 = [x_5]p = \{x_5, x_6\}$$

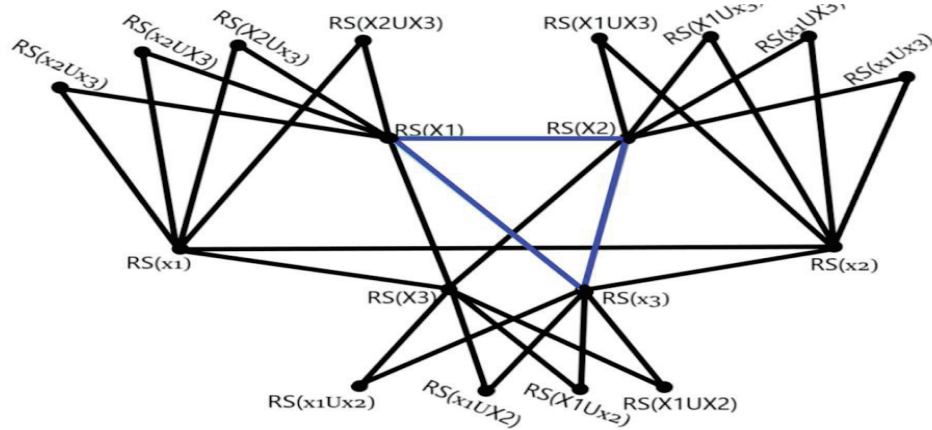
$$n = 3 \quad m = 3, \quad 2^{n-m} \cdot 3^m - 2^m - 1 = 18$$

$$V = \{RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\}), RS(X_1 \cup \{x_3\}), RS(\{x_2\} \cup \{x_3\}), RS(\{x_1\} \cup \{x_3\}), RS(\{x_3\}), RS(X_2 \cup \{x_3\})\}.$$



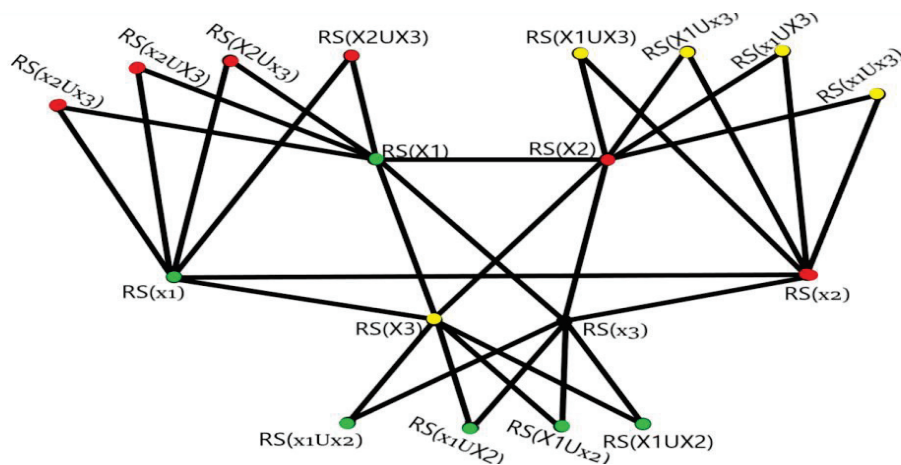
For $n = 3, m = 3$ Rough Zero Divisor Graph

The vertex $RS(X_1)$ is connected to $RS(X_2 \cup X_3)$ because $RS(X_1) \Delta RS(X_2 \cup X_3) = RS(\emptyset)$. but, The vertex $RS(X_1)$ is not connected to $RS(X_1 \cup X_3)$ because $RS(X_1) \Delta RS(X_1 \cup X_3) \neq RS(\emptyset)$



Clique number of $n = 3, m = 3$ Rough Zero Divisor Graph

As the clique number definition claims “the maximum complete subgraph “ we obtained that the clique number is equal to 3 .then it is equal to n.



The chromatic number of $n=3, m=3$ Rough Zero Divisor Graph

As chromatic number definition claims that the “smallest number of colours needed and no two adjacent has the same colour “.we obtained that the chromatic number is equal to 3 . which is equal to the equivalence class.

CHAPTER 5

APPLICATION

1. Firstly, let's see the application of a rough zero divisor graph

In covid situation, we were in lockdown then the covid cases has initiated in China. There is no transport from Asia to North and South America. So we can connect China to America because there is no connection .but, we can't connect China and the United Kingdom because there is a transport facility. Now if we want to check the covid cases all over the world. we can send the polymerase chain reaction (PCR) kit to another continent rather than North and South America. Because it is connected to China (Asia) which is one of the countries in Asia .m there is no transport between them.

2. Next is also the application of a rough Zero divisor graph

In the Tamil language, there are no words with the same letter repeated consecutively. So, it shows that the letter is always connected to the different letters it represents the rough zero divisor graph.

CHAPTER 6

CONCLUSION

The introduction section provides a basic understanding of Rough Set Theory and Graph Theory. Chapter 2 contains the definitions, theorems, and examples required to understand the remaining chapters.

In Chapter 3, the Rough Zero divisor graph. To simplify this study, the clustering graph corresponding to the Rough Zero divisor graph was introduced in which the vertices are partitioned into disjointed equivalence classes so that the vertices of a particular partition behave similarly. The graph theoretical parameters of the Rough Zero divisor graph are chromatic and clique numbers. These concepts are demonstrated using suitable examples.

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