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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – I – Introduction and Linear Programming– SPR1307

Operation Research:

1. OR is an aid for the executive in making decisions by providing him with the quantitative information, based on the scientific method analysis.

2. OR is the art of giving bad answers to problems, to which, otherwise worse answers are given.

3.OR is the art of wining wars without actually fighting them.

4.OR is the application of scientific methods by interdisciplinary teams to problems involving the control of organised (man-machine) systems so as to provide solutions which best serve the purpose of the organisation as a whole.

The characteristics of OR Model

A model is defined as idealised representation of the real life situation. It represents one or few aspects of reality.

Characteristics of OR—

1. The number of simplifying assumption should be as few as possible.

2. Model should be simple but close to reality.

3. It should be adaptable to parametric type of treatment.

4. It should be easy and economical to construct.

Main advantage and limitation of OR model.

Advantage

1 It provides a logical and systematic approach to the problem.

2. It indicate the scope .as well as limitation of a problem.

3. It makes the overall structure of the problem more comprehensible and helps in dealing with problem in its entirety.

Limitations—

Models are only idealised representation Of reality and should not be regarded as absolute in any case.

Distinguish between:

(i) Iconic or Physical Model and Analogue or schematic model.

(ii) Deterministic and Probabilistic model.

(i) Iconic Model—It represent the system by enlarging or reducing the size on some scale. In other words it is an image.

Example-toy aeroplane, photographs, drawings, maps etc.

Schematic Model—The models, in which one set of properties is used to

represent, another set of properties are called schematic or analogue models.

For example-graphs used to show different quantities.

(ii) Deterministic model—Such models assume conditions of complete certainty and perfect knowledge.

Example-LPP, transportation, assignment etc.

Probabilistic (or stochastic) Model—These type of models' usually handle such

situation in which the consequences or payoff of managerial actions cannot be predicted with certainty. However it is possible to forecast a pattern of events, based on which managerial decision can be made.

For example insurance companies are willing to insure against risk of fire, accident, sickness and so on. Here the pattern of events have been compiled in the form of probability distribution.

The objective of operation Research

The objective of OR is to provide a scientific basis to the managers of an organization for solving problems involving interaction of the components of the system, by employing a system approach by a team of scientists drawn from different disciplines, for finding a solution which is in the best interest of the organization as a whole.

The characteristics of operation research.

1. System Orientation

2. Use of interdisciplinary teams

3. Application of Scientific methOds

4. Uncovering of new problems

5. Improvement in the quality of decisions

6. Use of computer

7. Quantitative solutions

8. Human factors

The various phases of operation research

Or

The steps involved in the solution of OR Problem.

Operation research is based on scientific methodology which proceeds as:

1. Formulating the problem.

2 Constructing a model to represent the system under study

3. Deriving a solution from the model.

4. Testing the model and the solution derivq4 from it.

5. Establishing controls over the solution.

6. Putting the solution to work i.e. implementation.

(i) Assignment of job to machine

(ii) Product mix

(iii) Advertising media selection

(iv) Transportation.

2. Dynamic programming

(i) Capital budgeting

(ii) Employment smoothening

(iii) Cargo loading.

3. Inventory control

(i) Economic order quantity

MATHEMATICAL FORMULATION OF LPP

Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise per gram. Milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirement of vitamin A and vitamin B are 100 units and 120 units respectively. Find the optimal product mix. Formulate this problem as a LPP.

Let x_1 = number of grams of eggs to be consumed

x_2 = number of grams of milk to be consumed

The LPP is: Min Z = $12x_1 + 20x_2$

Subject to

$$6x_1 + 8x_2 \geq 100$$

$$7x_1 + 12x_2 \geq 120$$

$$x_1, x_2 \geq 0$$

GRAPHICAL METHOD

Working procedure

Step 1: Consider each inequality constraint as equation.

Step 2: Plot each equation on the graph, as each will geometrically represent a straight line.

Step 3: Mark the region. If the inequality constraint corresponding to a line is $\leq$ type then the region below the line lying in the first quadrant is shaded. For the inequality constraint $\geq$ type, the region above the line in the first quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region is the feasible region.

Step 4: Plot the objective function by assuming Z = 0. This will be a straight line passing through the origin. As the value of Z is increased from zero, the line starts moving, parallel to itself. Move the line till it is farthest away from the origin for maximization of the objective function. For a minimization problem the line will be nearest to the origin. A point of the feasible region through which this line passes will be the optimal point.

Step 5: Alternatively find the co-ordinates of the extreme points of the feasible region and find the value of the objective function at each of these extreme points. The point at which the value is maximum (or minimum) is the optimal point and its coordinates give the optimal solution.

Solve the following LPP by graphical method.

Minimize Z = $20x_1 + 10x_2$

Subject to

$x_1 + 2x_2 \le 40$
$3x_1 + x_2 \ge 30$
$4x_1 + 3x_2 \ge 60$
& $x_1, x_2 \ge 0$

Solution:

Replace all the inequalities of the constraints by equation

$$x_1 + 2x_2 = 40 \quad \text{if} \quad \begin{array}{l} x_1 = 0 \Rightarrow x_2 = 20 \\ x_2 = 0 \Rightarrow x_1 = 40 \end{array}$$

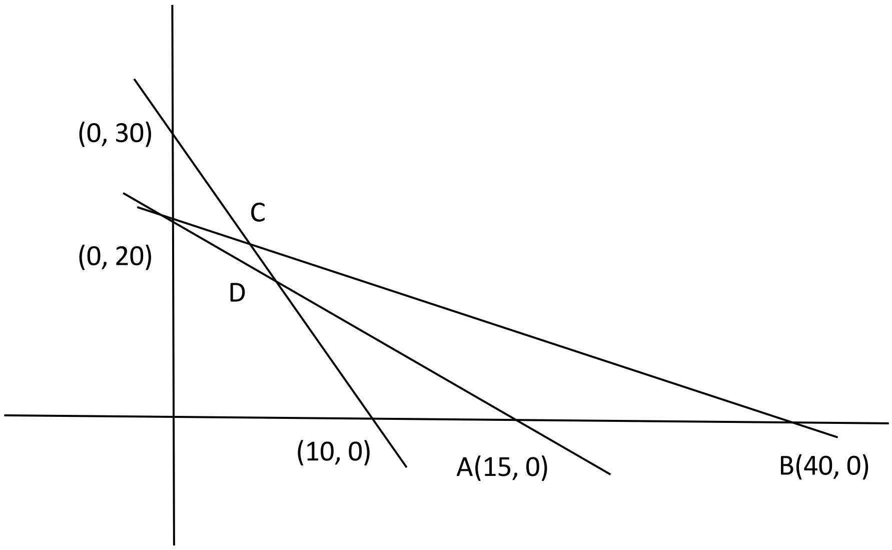
$\therefore$ $x_1 + 2x_2 = 40$ passes through (0,20)(40,0)
$3x_1 + x_2 = 30$ passes through (0,30)(10,0)
$4x_1 + 3x_2 = 60$ passes through (0,20)(15,0)

Plot each equation on the graph. The feasible region is ABCD.
C and D are points of intersection of lines

$\therefore$ $x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$,
and $4x_1 + 3x_2 = 60$, $x_1 + x_2 = 30$
On solving we get C = (4, 18) D = (6, 12)

Corner points	Value of Z = 20 $x_1 + 10x_2$
A (15,0)	300
B (40,0)	800
C (4,18)	260
D (6,12)	240 minimum value

$\therefore$ The minimum value of Z occurs at D (6, 12).
Hence the optimal solution is $x_1 = 6, x_2 = 12$.



Problem. An advertising company wishes to plan its advertising strategy in three different media-television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data have been obtained from market survey-

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	Rs. 30000	20000	15000	10000
No. of potential customer reached per unit	20000	600000	150000	100000
No. of female customer reached per unit	1, 50000	400000	70000	50000

The company wants to spend not more than Rs. 450000 on advertising. Following are the further requirements.

1. at least 1 million exposures take place among female customers.

2. advertising on magazines be limited to Rs 1, 50000

3. at least 3 advertising units to be bought on magazine 1 and 2 units on magazine II.

4. The number of advertising units on television and radio should each be between 5 and 10.

Formulate an LPP model for the problem.

Solution : Let x1- no. of advertising unit of television

no. of advertising unit of radio

x3- no. of advertising unit of Magazine I

x4— no. of advertising unit of Magazine II

Objective function

Maximize Z = i05 (2x1+ 6x2 + 1.5x3 + x4)

Constraints are

$$30000x_1 + 20000x_2 + 15000x_3 + 10000x_4 \le 450000$$
$$150000x_1 + 400000x_2 + 70000x_3 + 50000x_4 \ge 1000000$$
$$15000x_3 + 10000x_4 \le 150000$$
$$x_3 \ge 3$$
$$x_4 \ge 2$$
$$5 \le x_1 \le 10 \text{ or } x_1 \ge 5,\ x_1 \le 10$$
$$5 \le x_2 \le 10 \text{ or } x_2 \ge 5,\ x_2 \le 10$$
$$\text{where } x_1, x_2, x_3\ x_4 \text{ each } \ge 0$$

Problem. A person requires 10, 12 and 12 units of chemicals A, B and C respectively for herbal garden. A liquid product contains 5, 2 and 1 units of A,B and C respectively per Jar. A dry product contains 1, 2 and 4 units of A, B and C per cartoon. If the liquid product sells for Rs. 3 per Jar and dry product sells for Rs. 2 per cartoon, how many of each should be purchased to minimise the cost and meet the requirements.

Solution : Requirement

	A	B	C	
	10	12	12 units	
Liquid product	5	2	1	units Rs. 3/-per jar
Dry product	1	2	4	Rs. 2/-per Cartons

1. Select decision variable

   x_1 – no. of jars of liquid product

   x_2 – no. of cartoons of dry product

2. Objective function

   Minimize cost $(z) = 3x_1 + 2x_2$

3. Constraints :

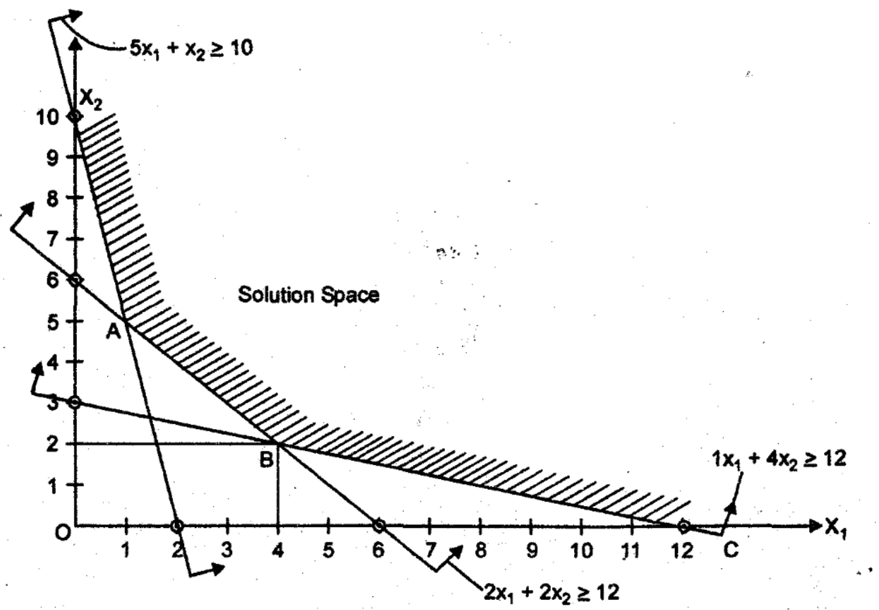
$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$1x_1 + 4x_2 \geq 12$$

4. Add non negativity constraints :

$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Graphical Method :

$$5x_1 + x_2 = 10 \Rightarrow x_1 = 0; x_2 = 10 \text{ and } x_2 = 0; x_1 = 2$$
$$2x_1 + 2x_2 = 12 \Rightarrow x_1 = 0; x_2 = 6 \text{ and } x_2 = 0; x_1 = 6$$
$$1x_1 + 4x_2 = 12 \Rightarrow x_1 = 0; x_2 = 3 \text{ and } x_2 = 0; x_1 = 12$$



Point A (1,5) $Z(A) = 3 \times 1 + 2 \times 5 = 13$

Point B (4, 2) $Z(B) = 3 \times 4 + 2 \times 2 = 16$

Point C (12,0) $Z(C) = 3 \times 12 + 2 \times 0 = 36$

Minimum cost at point A *i.e.* Rs. 13

x_1 (no. of Jar of Liquid product) = 1

x_2 (no. of carton of dry product) = 5

Minimum cost (Z) = Rs. 13.

Problem. A firm manufactures pain relieving pills in two sizes A and B, size A contains 4 grains of element a, 7 grains of element b and 2 grains of element c, size B contains 2 grains of element a, 10 grains of element b and 8 grains of c. It is found by users that it requires at least 12 grains of element a, 74 grains of element b and 24 grains of element c to provide immediate relief It is required to determine that least no. of pills a patient should take to get immediate relief. Formulate the problem as standard LPP.

Solution : Pain relieving pills

	a	b	c
Size A	4	7	2
Size B	2	10	8
Min. requirement	12	74	24

Step 1. Select decision variable

x_1 – no. of pills of size A

x_2 – no. of pills of size B

Step 2. Objective function

Minimum (no. of pills) $z = x_1 + x_2$

Step 3. Constraints

$$4x_1 + 2x_2 \geq 12$$
$$7x_1 + 10x_2 \geq 74$$
$$2x_1 + 8x_2 \geq 24$$

Step. 4. Add non negativity constraints

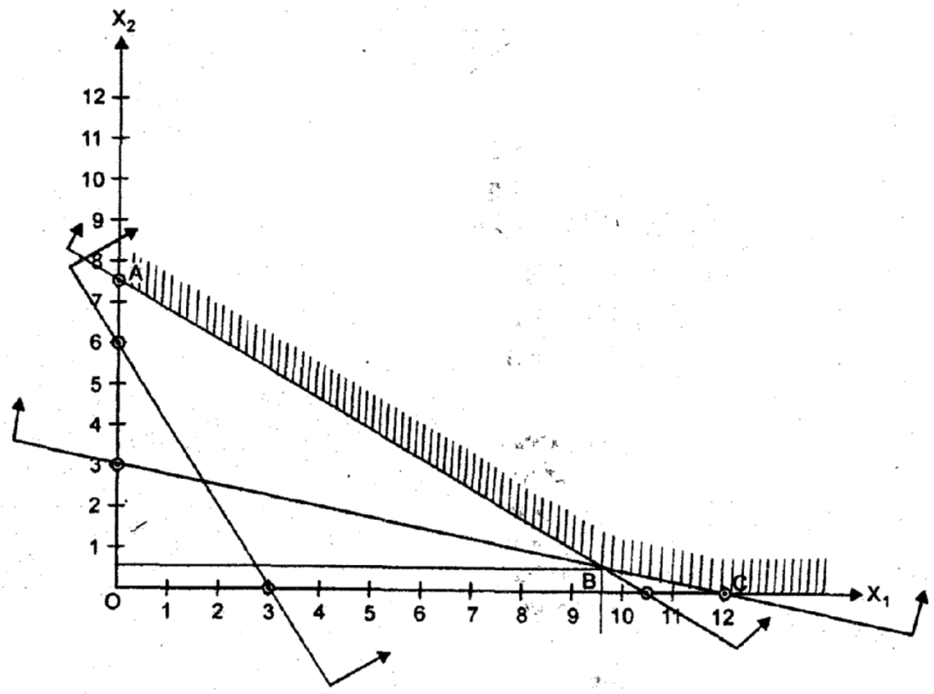
$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Determining the value of x_1 and x_2 by graphical method

$4x_1 + 2x_2 = 12$ $\quad x_1 = 0;\ x_2 = 6$ and $x_2 = 0;\ x_1 = 3$

$7x_1 + 10x_2 = 74$ $\quad x_1 = 0;\ x_2 = 7.4$ and $x_2 = 0;\ x_1 = 10.57$

$2x_1 + 8x_2 = 24$ $\quad x_1 = 0;\ x_2 = 3$ and $x_2 = 0;\ x_1 = 12$



Point A (0, 7.4) Z(A) = 0 + 7.4 = 7.4 (Minimum)

Point C (12, 0) Z (C) = 12 + 0 = 12

Point B (9.6, 0.6) Z(B) = 9.6 + 0.6 = 10.2

No. of pills of size A = 0

No. of pills of size B = 7.4 ≃ 8 pills

Minimum no. of pills = 8 pills.

Problem. An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A which perform the basic assy operation must work 5 man days on each truck but only 2 man days on each automobile. Shop B which perform finishing operations must work 3 man days for each automobile or truck that it produces. Because of men and machine limitations shop A has 180 man days per week available while shop B has 135.man days per week. If the manufacturer makes a profit of Rs. 300 on each truck and Rs. 200 on each automobile; how many of each should be produced to maximize his profit?

Solution :

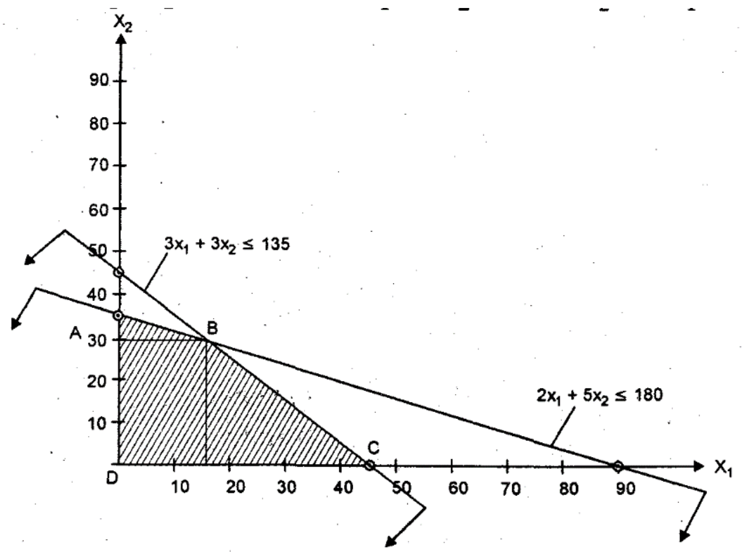
	Shop A	Shop B	Profit
Automobile	2 man days	3 man days	Rs. 200
Trucks	5 man days	3 man days	Rs. 300
Availability	180 man days/week	135 man days/week	

1. Select decision variable
   x_1 – no. of automobile to be produced/week
   x_2 – no. of trucks to be produced/week
2. Objective function
   Maximize $Z = 200x_1 + 300x_2$
3. Constraints
   $$2x_1 + 5x_2 \le 180$$
   $$3x_1 + 3x_2 \le 135$$
4. Add non negativity constraints
   $$x_1 \ge 0;\ x_2 \ge 0$$

Determine the value of x_1 and x_2 by graphical method

$2x_1 + 5x_2 = 180$ $\quad x_1 = 0;\ x_2 = 36$ and $x_2 = 0;\ x_1 = 90$

$3x_1 + 3x_2 = 135$ $\quad x_1 = 0;\ x_2 = 45$ and $x_2 = 0;\ x_1 = 45$



Point D (0, 0) Z (D) $= 200 \times 0 + 300 \times 0 = 0$

Point A (0, 36) Z (A) $= 200 \times 0 + 300 \times 36 = 10800$

Point C (45, 0) Z (C) $= 200 \times 45 + 300 \times 0 = 9000$

Point B (15 , 30) Z (B) $= 200 \times 15 + 300 \times 30 = 3000 + 9000 = 12000$

Maximum Profit at Point B (15 , 30) *i.e.* Rs.12000/-

x_1 = no. of automobile/week = 15

x_2 = no. of trucks/week = 30

Maximum profit = 12000/-

Problem. On completing the construction of house a person discovers that J square feet of plywood scrap and 80 square feet of white pine scrap are in useable m for the construction of tables and book cases. It takes 16 square feet of plywood 8 square feet of white pine to make a table, 12 square feet of plywood and 16 Llare feet of white pine are required to contruct a book case. By selling the finishing duct to a local furniture store the person can realize a profit of Rs. 25 on each table d Rs. 290 on each book case. How may the man most profitably use the left over ood ? Use graphical method to solve problem.

Solution :

	Plywood	White pine	Profit	
Table	16	8	Rs. 25	each table
Book case	12	16	Rs. 290	each book case
Availability	100	80		

1. Select decision variable

   x_1 – no. of table

   x_2 – no. of book case

2. Objective function

   Maximize profit (Z) = $25x_1 + 290x_2$

3. Constraints

$$16x_1 + 12x_2 \leq 100$$
$$8x_1 + 16x_2 \leq 80$$

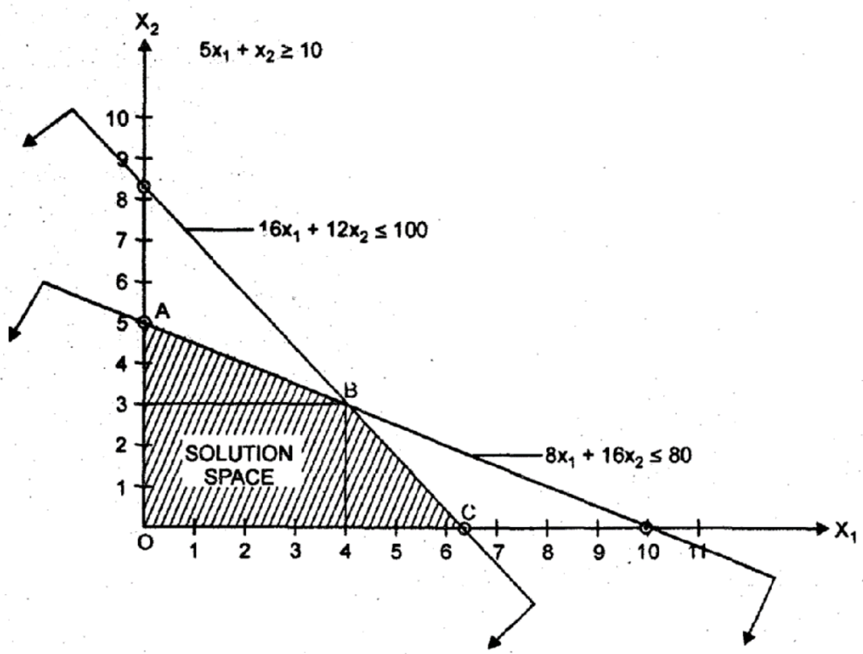
4. Add non negativity constraints

$$x_1 \geq 0$$
$$x_2 \geq 0$$

Determine the value of x_1 and x_2 using graphical method

$16x_1 + 12x_2 = 100 \qquad x_1 = 0; x_2 = 8.3$ and $x_2 = 0; x_1 = 6.25$

$8x_1 + 16x_2 = 80 \qquad x_1 = 0; x_2 = 5$ and $x_2 = 0; x_1 = 10$



Point O (0, 0) Z(O) = 25 × 0 + 290 × 0 = 0

Point A (0, 5) Z (A) = 25 × 0 + 290 × 5 = 1450

Point C (6.25, 0) Z (C) = 25 × 6.25 + 290 × 0 = 156.25

Point B (4, 3) Z (B) = 25 × 4 + 290 × 3 = 100 + 870 = 970

Maximum profit (Z) at point A *i.e.* Rs. 1450.

x_1 – no. of table = 0 Max. Profit (Z) = Rs. 1450/–

x_2 – no.of bookcase = 5.

Basic definition:

1) Define a feasible region.
Solution:

A region in which all the constraints are satisfied simultaneously is called a feasible region.

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2) Define a feasible solution.
Solution:

A solution to the LPP which satisfies the non-negativity restrictions of the LPP is called a feasible solution.

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3) Define optimal solution.
Solution:

Any feasible solution which optimizes the objective function is called its optimal solution.

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4) What is the difference between basic solution and basic feasible solution?
Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

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5) Define unbounded solution.
Solution:

If the values of the objective function Z can be increased or decreased indefinitely, such solutions are called unbounded solutions.

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6) What are slack and surplus variables?
Solution:

The non-negative variable which is added to LHS of the constraint to convert the inequality $\leq$ into an equation is called slack variable.

$\sum_{j=1}^{n} a_{ij} x_i + s_i = b_i (i = 1, 2, ..., m)$ where s_i are called slack variables.

The non-negative variable which is subtracted from the LHS of the constraint to convert the inequality $\geq$ into an equation is called surplus variable.

$\sum_{j=1}^{n} a_{ij} x_i - s_i = b_i (i = 1, 2, ..., m)$ where s_i are called surplus variables.

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7) What is meant by optimality test in a LPP?
Solution:

By performing optimality test we can find whether the current feasible solution can be improved or not. This is possible by finding the $Z_j - C_j$ row. In the case of a maximization problem if all $Z_j - C_j$ are nonnegative, then the current solution is optimal.

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8) What are the methods used to solve an LPP involving artificial variables?

Solution:

i) Big M method or penalty cost method
ii) Two-phase simplex method

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9) Define artificial variable

Solution:

Any non negative variable which is introduced in the constraint in order to get the initial basic feasible solution is called artificial variable.

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10) When does an LPP posses a pseudo-optimal solution?

Solution:

An LPP possesses a pseudo-optimal solution if at least one artificial variable is in the basis at positive level even though the optimality conditions are satisfied.

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11) What is degeneracy?

Solution:

The concept of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy. In the case of a BFS, all the non basic variables have zero value. If some basic variables also have zero value, then the BFS is said to be a degenerate BFS.

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12) How to resolve degeneracy in a LPP?

Solution:

a) Divide each element of the rows (with tie) by the positive coefficients of the key column in that row.
b) Compare the resulting ratios, column by column, first in the identity and then in the body from left to right.
c) The row which first contains the smallest ratio contains the leaving variable.

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13) State the characteristics of canonical form.

Solution:

The characteristics of canonical form are

i) The objective function is of maximization type
ii) All constraints are "$\leq$" type
iii) All variables X_i are non negative.

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14) State the characteristics of standard form.

Solution:

The characteristics of standard form are

i) The objective function is of maximization type
ii) All constraints are expressed as equations
iii) RHS of each constraint is non- negative
iv) All variables X_i are non-negative.

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15) Define basic feasible solution

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

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16) Define non-degenerate solution

Solution:

A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive X_i (i = 1, 2,------m) ie, none of the basic variables are zero.

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17) Define degenerate solution

Solution:

A basic feasible solution is said to be degenerate if one or more basic variables are zero.

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18) Write the general mathematical model of LPP in matrix form.

Solution:

Max or Min Z = CX

Subject to AX ($\leq = \geq$) b

$X \geq 0$

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19) Define basic solution:

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution.

Simplex method(algorithm)

Step 1: Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by Min Z = - Max (-Z)

Step 2: Check whether all b_i are positive. If any one of b_i is negative then multiply the inequation of the constraint by -1 so as to get all $b_i$to be positive.

Step 3: Express the problem in standard form by introducing slack/surplus variables, to convert the inequality constraints into equations.

Step 4: Obtain an initial basic feasible solution to the problem in the form $X_B = \mathbf{B}^{-1}b$ and put it in the first column of the simplex table.

Step 5: Compute the net evaluations $Z_J - C_J$ by using the relation $Z_J - C_J = C_B X_J - C_J$.

Examine the sign of $Z_J - C_J$.

(i) If all $Z_J - C_J \geq 0$, then the current BFS is the optimal solution.

(ii) If at least one $Z_J - C_J < 0$, then proceed to the next step.

Step 6: If there are more than one negative $Z_J - C_J$ choose the most negative them. Let it be $Z_r - C_r$.

(i) If all $X_{ir} \leq 0 (i = 1, 2, ...m)$ then there is an unbounded solution to the given problem.

(ii) If at least one $X_{ir} > 0 (i = 1, 2, ...m)$ then the variable X_r (key column) enters the basis.

Step 7: Compute the ratio $\left\{ \frac{X_{Bi}}{X_{ir}} / X_{ir} > 0 \right\}$. Let the minimum of these ratios be $\frac{X_{Bk}}{X_{kr}}$. Then choose the variable X_k (key row) to leave the basic. The element at the intersection of the key column and the key row is called the key element.

Step 8: Form a new basis by dropping the leaving variable and introducing the entering variable along with the associated value under C_B column. Convert the leading element to unity by dividing the key row by the key element and convert all other elements in the simplex table by using the formula

New element = Old element –

{(product of elements in key row and key column) / key element}

Go to Step 5 and repeat the procedure until either an optimal solution is obtained or there is an indication of unbounded solution.

The procedure of the big M method

Step 1: Express the problem in the standard form.

Step 2: Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type ≥ or = . Assign a very large penalty cost (-M for Maximization and M for Minimization) with artificial variables in the objective function.

Step 3: Solved the modified LPP by simplex method, until any one of the three cases that may arise.

1. If no artificial variable appears in the basis and the optimality conditions

are satisfied, then the current solution is an optimal basic feasible solution.

2. If at least one artificial variable in the basis is at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution (though degenerate).

3. If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied then the original problem has no feasible solution .The solution satisfies the constraints but does not optimize the objective function, since it contains very large penalty M and it is called pseudo optimal solution.

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1. Solve the following LPP using simplex method

$$\text{Max } Z = 15x_1 + 6x_2 + 9x_3 + 2x_4$$

Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$\& x_1, x_2, x_3, x_4 \ge 0$$

Solution:

Rewrite the inequality of the constraint into an equation by adding slack variables S_1, S_2 and S_3 the LPP becomes,

$$\text{Max } Z = 15x_1 + 6x_2 + 9x_3 + 2x_4 + 0.S_1 + 0.S_2 + 0.S_3$$

Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 + S_1 = 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 + S_2 = 24$$
$$7x_1 + x_4 + S_3 = 70$$
$$\& x_1, x_2, x_3, x_4, S_1, S_2, S_3 \ge 0$$

Initial basic feasible solution is

S_1=20
S_2=24
S_3=70

Initial simplex table

C_j 15 6 9 2 0 0 0

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_1}$
0	S_1	20	2	1	5	6	1	0	0	10
0	S_2	24	3	1	3	25	0	1	0	8
0	S_3	70	7	0	0	1	0	0	1	10
	Z_j	0	0	0	0	0	0	0	0	
	Z_J-C_J		-15	-6	-9	-2	0	0	0	

Since all $Z_J - C_J \le 0$ and the current basic feasible solution is not optimum.

First iteration: X_1 enters the basis and S_2 leaves the basis.

	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_2}$
$\leftarrow 0$	S_1	4	0	1/3	3	$-32/3$	1	2/3	0	12
15	X_1	8	1	1/3	1	25/3	0	1/3	0	24
0	S_3	14	0	$-7/3$	-7	$-172/3$	0	$-7/3$	1	-
	Z_J	120	15	5	15	125	0	5	0	
	Z_J-C_J		0	-1 $\uparrow$	6	123	0	5		

Since some $Z_J - C_J \le 0$ the current basic feasible solution is not optimum

Second iteration:

X_2 enters the basis and S_1 leaves the basis.

The new table is

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3
6	X_2	12	0	1	9	-32	3	-2	0
15	X_1	4	1	0	-2	57/3	-1	1	0
0	S_3	42	0	0	14	-132	7	-7	1
	Z_j	132	15	6	24	93	3	3	0
	Z_J-C_J		0	0	15	91	3	3	0

Since all $Z_J - C_J \ge 0$ and the current basic feasible solution is optimum

and is given by Max Z = 132, $X_1 = 4$, $X_2 = 12$, $X_3 = 0$, $X_4 = 0$.

----

2. Solve by the big M method

Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \; x_1, x_2 \geq 0$$

Solution:

Given Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \; x_1, x_2 \geq 0$$

That is Max Z = $-4x_1 - 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \; x_1, x_2 \geq 0$$

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

Max Z = $-4x_1 - 3x_2 + 0.s_1 + 0.s_2 - MR_1 - MR_2$

Subject to

$$2x_1 + x_2 - s_1 + R_1 = 10$$
$$-3x_1 + 2x_2 + s_2 = 6$$
$$x_1 + x_2 - s_3 + R_2 = 6$$
$$\& \; x_1, x_2, s_1, s_2, s_3, R_1, R_2 \geq 0$$

Initial basic feasible solution is

R_1=10
R_2=6
S_2=6

Initial iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_1	R_2	Min
-M 0 -M	R_1 S_2 R_2	10 6 6	2 -3 1	1 2 1	-1 0 0	0 1 0	0 0 -1	1 0 0	0 0 1	5 - 6
	$Z_j - C_j$	-16M	-3M+4	-2M+3	M	0	M	0	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_2	Min
-4 0 -M	x_1 S_2 R_2	5 21 1	1 0 0	1/2 7/2 ½	-1/2 -3/2 1/2	0 1 0	0 0 -1	0 0 1	10 42/7 2
	$Z_j - C_j$	-M-20	0	$\frac{-M+2}{2}$	$\frac{-M+4}{2}$	0	M	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

Second iteration:

C_j -4 -3 0 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3
-4 0 -3	X_1 S_2 x_2	4 14 2	1 0 0	0 0 1	-1 -5 1	0 1 0	1 7 -2
	$Z_j - C_j$	-22	0	0	1	0	2

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum.

and is given by Min Z = 22, X_1 = 4, X_2 = 2.

----

3. Use two phase simplex method to

Max Z = $5x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \leq 1$$
$$x_1 + 4x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

Solution:

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

Max Z = $5x_1 + 3x_2 + 0.s_1 + 0.s_2 - R_1$

Subject to

$$2x_1 + x_2 + s_1 = 1$$
$$x_1 + x_2 - s_2 + R_1 = 6$$
$$\& x_1, x_2, s_1, s_2, R_1 \geq 0$$

Initial basic feasible solution is

S_1=1

R1=6

Initial iteration:

C_j 0 0 0 0 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1	Min
0	S_1	1	2	1	1	0	0	1
-1	R_1	6	1	4	0	-1	1	6/4
	$Z_j - C_j$	-6	-1	-4	0	1	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j 0 0 0 -1 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1
0	x_2	1	2	1	1	0	0
-1	R_1	2	-7	0	-4	-1	1
	$Z_j - C_j$	-2	7	0	4	1	0

Since all $Z_J - C_J \geq 0$ and the current basic feasible solution is optimal to the auxiliary LPP. Since an artificial variable is in the current basis at positive level, the given LPP has no feasible solution.

Explain the meaning of duality in LPP

For every LP problem there is related unique L P problem mvolvmg the same data which also describes the original problem.

The given original problem is known as primal programme. The programme can be rewritten by transposing the rows and columns of the statement of the problem. Inverting the programme in this way results in dual programme. The two programmes have very closely related properties so that optimal solution of the dual problem gives complete information about the optimal solution of primal problem. Solving the problem by writing dual programme is known as duality in LP

If the dual of an LPP is solved, where will we get the value of decision variables of the primal LPP.

The value of decision variables of primal are given by the base row of the dual solution under the slack variable, neglecting the -ye sign if any, and under the artificial variables neglecting the —ye sign if any, after deleting the constant M.

What is the importance of duality?

1.If. the primal problem contains a large number of rows and a smaller number of columns, the computational procedure can be considerably reduced by converting it into dual and then solving it.

2. This can help managers in answer questions about alternative course of actions and their relative values.

3. Economic interpretation of the dual helps the management in making future decisions.

4. Calculation of the dual checks the accuracy of the primal solution.

Define dual of LPP.
For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem. If the primal problem is

$$\text{Maximize } Z = CX$$
$$\text{subject to } AX \leq b$$
$$X \geq 0$$

Then the dual is

$$\text{Minimize } Z^* = b^T Y$$
$$\text{subject to } A^T Y \geq C^T$$
$$Y \geq 0$$

Problems

Problem 4.6. Write the dual form for the following :

$$\text{Min } z = x_1 + x_2$$

Subjected to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

Solution :

The dual of the given problem will be

$$\text{Max W} = 4y_1 + 7y_2$$

Subject to

$$2y_1 + 1y_2 \leq 1$$
$$y_1 + 7y_2 \leq 1$$
$$y_1 , y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.7. Write the dual form for the following :

$$\text{Max } z = 4x_1 + 2x_2$$

Subject to

$$x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$x_1 , x_2 \geq 0$$

Solution : As the given problem is of maximization type, all constraints should by $\leq$ type; Multiply all constraints with –ve.

$$-x_1 - x_2 \leq -3$$
$$-x_1 + x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

The dual of the given problem will be

$$\text{Min W} = -3y_1 - 2y_2$$

Subject to

$$-y_1 - y_2 \geq 4$$
$$-y_1 + y_2 \geq 2$$
$$y_1 ; y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.8. Construct the dual of the problem.

Minimize $z = 3x_1 - 2x_2 + 4x_3$

Subject to constraints

$$3x_1 + 5x_2 + 4x_3 \geq 7$$
$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$4x_1 + 7x_2 - 2x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

Solution: As the given problem is of minimization all constraints should be of type

$$-7x_1 + 2x_2 + x_3 \geq -10$$

The dual of the given problem will be

$$\text{Maximize } W = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$$

Subject to

$$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$5y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0.$$

Problem 4.9. Construct the dual of the problem

$$\textbf{maximize } z = 3x_1 + 10x_2 + 2x_3$$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Solution : The equation $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as pair of inequality

$$3x_1 - 2x_2 + 4x_3 \leq 3$$
$$3x_1 - 2x_2 + 4x_3 \geq 3$$
$$-3x_1 + 2x_2 - 4x_3 \leq -3$$

Minimize

$$W = 7y_1 + 3(y_2' - y_2'')$$

Subject to

$$2y_1 + 3(y_2' - y_2'') \geq 3$$
$$3y_1 - 2(y_2' - y_2'') \geq 10$$
$$2y_1 + 4(y_2' - y_2'') \geq 2$$
$$y_1, y_2', y_2'' \geq 0$$

Subsituting $y_2' - y_2'' = y_2$

$$W = 7y_1 + 3y_2$$

Subject to

$$2y_1 + 3y_2 \geq 3$$
$$3y_1 - 2y_2 \geq 10$$
$$2y_1 + 4y_2 \geq 2$$
$$y_1 \geq 0$$

y_2 unrestricted variable.

Solve by the dual simplex method the following LPP

Min Z = $5x_1 + 6x_2$.

Subject to $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

Solution:

By introducing slack variables S_1, S_2 we get the standard form of LPP as given below.

Max Z = $-5x_1 - 6x_2 + 0s_1 + 0s_2$

Subject to

$$-x_1 - x_2 + s_1 = -2$$
$$-4x_1 - x_2 + s_2 = -4$$
$$x_1, x_2, s_1, s_2 \geq 0$$

Initial table

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_1	-2	-1	-1	1	0
$\leftarrow 0$	S_2	-4	-4	-1	0	1
	Z_J	0	0	0	0	0
	$Z_J - C_J$		$5\uparrow$	6	0	0

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since all $X_{Bi} < 0$, the current solution is not a basic feasible solution.

Since X_{B2} = -4 is most negative, the basic variable S_2 leaves the basis.

Since Max {5/-4, 6/-1} = -5/4, X_1 enters the basis.

First iteration: Drop S_2 and introduce X_1

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
$\leftarrow 0$	S_1	-1	0	$-3/4$	1	$-1/4$
-5	x_1	1	1	$1/4$	0	$-1/4$
	Z_J	-5	-5	$-5/4$	0	$5/4$
	$Z_J - C_J$		0	$19/4$	0	$5/4$ $\uparrow$

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since some $X_{Bi} < 0$ the current solution is not a basic feasible solution.

S_1 leaves the current basis. Since Max {19/-3, 5/-1} =5/-1, S_2 enters the basis.

Second iteration:

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_2	4	0	3	-4	1

-5	x_1	2	1	1	-1	0
	Z_J	-10	-5	-5	5	0
	$Z_J - C_J$		0	1	5	0

Since all $Z_J - C_J \geq 0$ and all $X_{Bi} \geq 0$, the current basic feasible solution is optimum. The optimal solution is Z = 10, $x_1 = 2$.

Use dual simplex method to solve the LPP

Max Z = $-3x_1 - 2x_2$

Subject to

$x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10$
$x_2 \leq 3$
$\& \; x_1, x_2 \geq 0$

Solution:

The given LPP is

Max Z = $-3x_1 - 2x_2$

Subject to

$-x_1 - x_2 \leq 1$
$x_1 + x_2 \leq 7$
$-x_1 - 2x_2 \leq 10$
$x_2 \leq 3$
$\& \; x_1, x_2 \geq 0$

By introducing the non negative slack variables s_1, s_2, s_3 and s_4 the LPP becomes

Max Z = $-3x_1 - 2x_2 + 0.s_1 + 0.s_2 + 0.s_3 + 0.s_4$

Subject to

$$-x_1 - x_2 + s_1 = -1$$
$$x_1 + x_2 + s_2 = 7$$
$$-x_1 - 2x_2 + s_3 = -10$$
$$x_2 + s_4 = 3$$
$$\& \ x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

Initial iteration BFS is

S_1 = -1, S_2=7, S_3=-10, S_4=3

Initial Iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	-1	-1	-1	1	0	0	0
0	S_2	7	1	1	0	1	0	0
0	S_3	-10	-1	-2	0	0	1	0
0	S_4	3	0	1	0	0	0	1
	$Z_j - C_j$	0	3	2	0	0	0	0

First iteration

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	4	-1/2	0	1	0	-1/2	0
0	S_2	2	1/2	0	0	1	1/2	0
-2	x_2	5	1/2	1	0	0	-1/2	0
0	S_4	-2	-1/2	0	0	0	1/2	1
	$Z_j - C_j$	-10	2	0	0	0	1	0

Second iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	2	0	0	1	0	-1	-1
0	S_2	0	0	0	0	1	1	1
-2	x_2	3	0	1	0	0	0	1
-3	x_1	4	1	0	0	0	-1	-2
	$Z_j - C_j$	-18	0	0	0	0	3	4

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum and is given by Min Z = -18, $X_1 = 4$, $X_2 = 3$.

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – II – Transportation and Assignment Model– SPR1307

State transportation problem. Is this a special class of LPP ? When does it a unique solution ?

The transportation problem is to transport various amount of single object that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum.

Yes it is a special class of LPP and may be solved by simplex method. Transportation problem always posses a feasible solution.

It has a unique solution when cell evaluation matrix has only positive values.

Write mathematical model for general transportation problem as LPP.

Mathematical formulation

Suppose that there are m sources and n destinations. Let a1be the number of supply units available at source i (i =1, 2, 3 m) and let b1 be the number of demand units required at destination j (f = 1, 2, 3 n). Let C, represent the unit transportation cost for transporting the units from source i to distination j. The objective is to determine the number of units to be transported from source i to destination j. So that total transportation cost is minimum.

If x_{ij} is the number of units shipped from source i to destination j, then
Find x_{ij} such that

Minimize $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to $$\sum_{J=1}^{n} x_{ij} = a_i \qquad i = 1, 2, 3, \ldots, m$$

and $$\sum_{i=1}^{m} . x_{ij} = b_j \qquad J = 1, 2, 3, \ldots, n$$

where $$x_{ii} \geq 0$$

List the various methods that can be used for obtaining an initial basic solution for transportation problem.

1. North west corner method

2. Row minimum method

3. Column minimum method

4. Least cost method

5. Vogal approximation method.

What is degeneracy in transportation problem?

In a transportation problem with m origins and n destinations if a basic feasible solution has less than (m + n—i) allocations, the problem is said to be a degenerate transportation problem.

What do you understand by a balanced and an unbalanced transportation problem ? How an unbalanced problem is tackled?

In a transportation problem if the total availability from all the origins is equal

to the total demand at all the destinations z $i.e. \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$ Such transportation

problems are known as balanced tansportation problems. (Total supply = Total demand) In many

situations, the total availability may not be equal to the total demand i.e. $\sum_{i=1}^{m} a_i \neq \sum_{J=1}^{n} b_j.$
Such problems are known as unbalanced tranportation problem.

The unbalanced problem could be tackled by adding a dummy destination or source depending upon the requirement and the costs of shipping to this destination (or from source) are set equal to zero. The zero cost cells are treated the same way as real cost cell and the problem is solved as a balanced problem. (Total supply   Total demand)

Describe the steps involved in vogal approximation method (VAM).

Ans.

Setp. 1. For each row of the transportation table identify the smallest and next to smallest cost. Determine the difference between them for each row. These are called penalities.' Similarly compute these penalities for each column.

Setp. 2. Identify the row or column with the largest penalty and allocate as much as possible within the restrictions of the rim conditions to the lowest cost cell in the row or column selected.

Setp. 3. Cross out of the row or column completely satisfied by the allocation.

Setp. 4. Repeat steps 1 to 3 untill all assignment have been made.

Define the following terms in transportation Problem.

(i) feasible solution (ii) Optimal solution

1. Feasible Solution. A feasible solution to a transportation problem is a set of non negative allocations, x that satisfy the rim conditions.

2. Optimal Solution. A feasible solution that minimize the transportation cost is called the optimal Solution.

Explain North west corner rule for finding initial solution for a transportation problem.

1. Start in the north west corner of the requirement table:

(a) If D1< S1, set x11 equal to find the balance supply and demand and proceed horizontally (cell 1, 2).

(b) If = S1 set x11 equal to D1, find the balance supply and demand and proceed diagonally (cell 2, 2).

(c) If D1>S1, set x11equal to compute the baiance supply and demand and proceed vertically (cell 2, 1).

2. Continue in this manner, step by step away from the north west corner until, finally a value is reached in the south east corner.

Give an algorithm to solve transportation problem.

Or

Describe the steps involved in solving transportation problem.

1. Make a transportation Model. For this enter the supply a., from the origin demand b1 at the destinations and unit cost C,, m the varous cells

2 Find initial basic feasible solution

3. Perform optimality test:

(a) Find dual variable (u_i, V_j)

(b) Make opportunity cost matrix $(C_{ij} = (u_i + V_j)$

(c) Compute the cell evaluation matrix $[C_{ij} - (u_i + V_j)]$ If all cell evaluation are positive or zero the current basic feasible solution is optimal.

(d) In case any cell evaluation is negative, select the vacant cell with the most negative evaluation. This isalled identified cell.

4. Iterate towards optimal solution. For this make as much allocation in the identified cell as possible so that it become basic.

5. Repeat step 3 and 4 till optimal solution is obtained.

State the Assignment model. Is assignment problem a special case of transportation?

Assignment Model Suppose there are n jobs to be performed and n person are available for doing these jobs. Assume that each person can do each job at a time, though with varying degree of efficiency.The problem is to find an assignment so that the total cost for performmg all jobs is minimum

Yes, the assignment problem is a special case of transportation problem when each

origin is associated with one and only one destination.

Give the mathematical formulation of an assignment problem

Ans Let = 0, if the facility is not assigned to 1th job

1, if the th facility is assigned to th job.

The model is given by

$$\text{minimize } z = \sum_{J=1}^{n} \sum_{i=1}^{n} C_{ij} x_{ij}$$

Subject to constraints

$$\sum_{J=1}^{n} x_{ij} = 1, i = 1, 2, 3, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \ J = 1, 2, 3\ldots, n$$

$$x_{ij} = 0 \text{ or } 1.$$

What do you mean by restrictions an assignments?

Or

How a restriction problem tackled?

Or

How will you solve an assignment where a particular assignment is prohibited?

Sometime technical, space, legal'or other problems do not permit the assignment of a particular facility to a particularjob. Such problem are known restrictions an assignment problem. Such problem can be solved by assigning a very heavy cost to the corresponding cell. It will automatically excluded from further consideration.

What is the unbalanced assignment problem ? How is it solved by the Hungarian method?

When the number of facilities is not equal to the number of jobs, such problems

are known as unbalanced assignment problem.

Since the Hungarian methodof solution require a square matrix, fictitious facilities or jobs. Jobs may be added and zero costs be assigned to the corresponding cells of the matrix. These cells are then treated the same way as the real cost cells during the solution procedure.

How do you come to know that Assignment problem has alternate optimal solution?

Ans. Sometimes it is possible to have two or more ways to strike off all zero elements in the reduced matrix for a given problem. In such cases there will be an alternate optimal solution with same cost.

Describe the steps involved in solving assignment problem by Hungarian method.

1. Prepare a square matrix.

2. Reduce the matrix.

3. Check whether an optimal assignment can be made in the reduced matrix or not.

4. Find the minimum number of lines crossing all zeros. If this number of lines is equal to the order of matrix then it is an optimal solution. Otherwise gp to step 5.

5. Iterate towards the optimal solution.

6. Repeat step 3 through 5 until an optimal solution is obtained.

Compare assignment problem with transportation problem.

An assignment model may be regarded as special case of the transportation

model. Here facilities represent the sources and jobs represent the destination. Number of sources is equal to the number of destinations, supply at each source is unity and demand at each distination is unit. In assignment the number of units allocated to a cell be either one or zero.

The assignment problem is a completely degenerate form of transportation problem.

Distinguish between transportation, assignment and sequencing model what is sequencing model).

Ans. Transportation and assignment are allocation model (as explained above) Sequencing model. are applicable in situation in which the effectiveness measure

a function of order as sequence of performing a series of jobs. The selection of the apropriate order in which waiting customer/Job may be served is called sequencing.

Problem Find the optimum solution to the following problem.

	I	3	4	6	8	8	20
	II	2	10	1	5	30	30
From	III	7	11	20	40	15	15
	IV	2	1	9	14	18	13
		40	6	8	18	6	

Solution:

1. Make a transportation model

I	3	4	6	8	8	20
II	2	10	1	5	30	30
III	7	11	20	40	15	15
IV	2	1	9	14	18	13
	40	6	8	18	6	78

1. Find basic feasible solution (VAM method)
2.

$\boxed{14}$ 3	4	6	8	$\boxed{6}$ 8	20/14/0	[1] [1] [5]
$\boxed{4}$ 2	10	$\boxed{8}$ 1	$\boxed{18}$ 5	30	30/22/18	[1] [3] [3]
$\boxed{15}$ 7	11	20	40	15	15/0	[4] [4] ←
$\boxed{7}$ 2	$\boxed{6}$ 1	9	14	18	13/7/0	[1] [1] [12] ←
40/25 18/4/0	6/0	8/0	18/0	6/0	78	
[0]	[3]	[5]	[3]	[7]		
[1]	[3]	↑	[3]	↑		

Transportation cost = (14 × 3) + (6 × 8) + (4 × 2) + (8 × 1) +(18 × 5) + (15 × 7) + (7 × 2) + (6 × 1)

= 42 + 48 + 8 + 8 + 90 + 105 + 14 + 6

= Rs. 321.

3. Check for optimality (MODI Test) m (a) Cost matrix of allocated cell.

+ n - 1 = 8 (no. of allocation)

u_j \ V_i	1	2	3	4	5
1	3				8
2	2		1	5	
3	7				
4	2	1			

	$V_1 = 0$
$u_1 + v_1 = 3$	$u_1 = 3$
$u_1 + v_5 = 8$	$v_5 = 5$
$u_2 + v_1 = 2$	$u_2 = 2$
$u_2 + v_3 = 1$	$v_3 = -1$
$u_2 + v_4 = 5$	$v_4 = 3$
$u_3 + v_1 = 7$	$v_3 = 7$
$u_4 + v_1 = 2$	$u_4 = 2$
$u_4 + v_2 = 1$	$v_2 = -1$

(*b*) Opp. cost matrix

	0	–1	–1	3	5
3	·	2	2	6	·
2	·	1	·	·	7
7	·	6	6	10	12
2	·	·	1	5	7

(c) Cell evaluation matrix

·	2	4	2	·
·	9	·	·	23
·	5	14	30	3
·	·	8	9	11

Since all the elements of cell evaluation matrix are positive so optimality test is passed.

Minimum Transportation Cost = Rs. 321.

Solve the following cost-minimizing transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	Available
O_1	2	1	3	3	2	5	50
O_2	3	2	2	4	3	4	40
O_3	3	5	4	2	4	1	60
O_4	4	2	2	1	2	2	30
Required	30	50	20	40	30	10	180

Ans. 1. Make a transportation model.

1. Find basic feasible solution

2	$\boxed{50}$ 1	3	3	2	5	50/0	[1]
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4	40/20/0	[0][1][1][0]
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1	60/50/40/10/0	[1][1][1][1]
4	2	2	$\boxed{30}$ 1	2	2	30/0	[1][1]
30/0	50/0	20/0	40/10/0	30/20/0	10/0		
[1]	[1]	[0]	[1]	[0]	[1]		
[0]		[0]	[1]	[1]	[1]		
[0]		[2]	[2]	[1]	[3]		

1. Check for optimality test (m + n -1) > no. of allocation (8)

2	$\boxed{50}$ 1	3	3	$\boxed{\varepsilon}$ 2	5
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1
			$\boxed{30}$		

m + n- 1 = no. of allocation 9

9. Cost matrix of allocated cell

	1			2	
		2		3	
3			2	4	1
·			1		

$u_1 + v_2 = 1$
$u_1 + v_5 = 2$
$u_2 + v_3 = 2$
$u_2 + v_5 = 3$
$u_3 + v_1 = 3$
$u_3 + v_4 = 2$
$u_3 + v_5 = 2$
$u_3 + v_6 = 1$
$u_3 + v_4 = 1$

$v_1 = 0$
$v_2 = 0$
$u_1 = 1$
$v_3 = 0$
$u_2 = 2$
$u_3 = 3$
$v_4 = -1$
$v_5 = 1$
$v_6 = -2$
$u_4 = 2$

(*b*) Opportunity cost matrix

	0	0	0	–1	1	–2
1	1	·	1	0	·	–1
2	2	2	·	1	·	0
3	·	3	3	·	·	·
2	2	2	2	·	3	0

(c) Cell evaluation matrix

1	·	2	3	·	6
1	0	·	3	·	4
·	2	1	·	·	·
2	0	0	·	–1√	2

identified cell

1. Iteration for optimal solution.
2.

	50			ε	
		20		20	
30			+10	10−	10
			−30	√+	

Check for optimality test

	50			ε	
		20		20	
30			20		10
			20	10	

2nd feasible solution

(a). Cost matrix of allocated cell.

	1			2	
		2		3	
3			2		1
			1	2	

$u_1 + v_2 = 1$ $v_1 = 0$
$u_1 + v_5 = 2$ $v_2 = -1$
$u_2 + v_3 = 2$ $u_1 = 2$
$u_2 + v_5 = 3$ $v_3 = -1$
$u_3 + v_1 = 3$ $u_2 = 3$
$u_3 + v_6 = 1$ $u_3 = 3$
$u_4 + v_4 = 1$ $v_4 = -1$
$u_4 + v_5 = 2$ $v_6 = -2$
$u_4 = 2$
$v_5 = 0$

(*b*) Opp. cost matrix

	0	−1	−1	−1	0	−2
2	2	·	1	1	·	0
3	3	2	·	2	·	1
3	·	2	2	·	3	·
2	2	1	1	·	·	0

(*c*) Cell evaluation matrix

0	·	2	2	·	5
0	0	·	2	·	3
·	3	2	·	1	·
2	1	1	·	·	2

Since all elements of cell evaluation matrix are non negative so 2hldI feasible solution is the optimum solution.

Transportation cost

$$= 50 \times 1 + 20 \times 2 + 20 \times 3 + 30 \times 3 + 20 \times 2 + 10 \times 1 + 20 \times 1 + 10 \times 2$$
$$= 50 + 40 + 60 + 90 + 40 + 10 + 20 + 20$$
$$= 330/-$$

Problem 3.21. Goods have to be transported from factories F_1, F_2, F_3 to ware house W_1, W_2, W_3 and W_4. The transportation cost per unit capacities and requirement of the ware house are given in the following table

	W_1	W_2	W_3	W_4	Capacity
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
Requrement	5	4	4	11	

Solution. 1. Make a transportation model

	W_1	W_2	W_3	W_4	
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
	5	4	4	11	24 Balanced Model

2. Find a basic feasible solution
VAM method

F_1	95	$\boxed{4}$ 105	80	$\boxed{8}$ 15	12/8/0	[65] ←
F_2	$\boxed{5}$ 115	180	$\boxed{2}$ 40	30	7/2/0	[10]
F_3	195	180	$\boxed{2}$ 95	$\boxed{3}$ 70	5/3/0	[25]
	5/0	4/0	4/2/0	11/3/0		
	[20]	[75]	[40]	[15]		
	[80]	↑	[55]	[50]		
	↑		↑			

3. Optimality test

$m + n - 1 =$ number of allocations

$6 = 6$

(a)Cost matrix of allocated cell

$u_i \backslash v_j$	1	2	3	4
1	·	105		15
2	115		40	
3			95	70

$V_1 = 0$

$u_1 + V_2 = 105$ $\quad V_2 = -10$

$u_1 + V_4 = 15$ $\quad u_1 = 115$

$u_2 + V_1 = 115$ $\quad u_2 = 115$

$u_2 + V_3 = 40$ $\quad V_3 = -75$

$u_3 + V_3 = 95$ $\quad u_3 = 170$

$u_3 + V_4 = 70$ $\quad V_4 = -100$

(*b*) Opp. cost matrix

	0	–10	–75	–100
115	115	·	40	·
115	·	105	·	15
170	170	160	·	·

(*c*) Cell evaluation matrix

–20	·	40	·
·	75	·	15
25	20	·	·

4. Iteration for optimal solution

+	4		– 8
– 5		+ 2	
		– 2	+ 3

2	4		6
3		4	
			5

2nd feasible solution

5. Check for optimality

(*a*) Cost matrix of allocated cell

$u_i \backslash v_j$	1	2	3	4
1	95	105		15
2	115		40	
3				70

$V_1 = 0$

$u_1 + V_1 = 95$ $\quad u_1 = 95$

$u_1 + V_2 = 105$ $\quad V_2 = 10$

$u_1 + V_4 = 15$ $\quad V_4 = -80$

$u_2 + V_1 = 115$ $\quad u_2 = 115$

$u_2 + V_3 = 40$ $\quad V_3 = -75$

$u_3 + V_4 = 70$ $\quad u_3 = 150$

1. Opp. cost matrix

	0	10	−75	−80
95	·	·	20	·
115	·	125	·	35
150	150	160	75	·

Cell evaluation matrix

·	·	60	·
·	55	·	√ −5
45	20	20	·

Iteration for optimal solution.

+ 2	4		6 −
− 3		4	√ +
			5

5	4		3
		4	3
			5

3rd feasible solution

Check for optimality test (a)

Cost matrix of allocated cell

95	105		15
		40	30
			70

$u_1 + V_1 = 95$ $\quad$ $v_1 = 0$
$u_1 + V_2 = 105$ $\quad$ $u_1 = 95$
$u_1 + V_4 = 15$ $\quad$ $V_2 = 10$
$u_2 + V_3 = 40$ $\quad$ $V_4 = -80$
$u_2 + V_4 = 30$ $\quad$ $V_3 = -70$
$u_3 + V_4 = 70$ $\quad$ $u_2 = 110$
$u_3 = 150$

(*b*) Opp. cost matrix

	0	10	-70	-80
95	· ·	·	15	·
110	110	120	·	·
150	150	160	80	

(*c*) Cell evaluation matrix

·	·	65	·
5	60	·	·
45	20	15	·

Since all elements of cell evaluation matrix are non negative. Hence 3rd feasible solution is the optimum solution.

Transportation cost = (5 × 95) + (4 × 105) + (3 × 15) + (4 × 40) + (3 × 30) + (5 × 70) = 475 + 420 + 45 + 160 + 90 + 350 = Rs 1540/-

Problem 3.22. Solve the following assignment problem.

		1	2	3	4
	A	12	30	21	15
work	B	18	33	9	31
	C	44	25	24	21
	D	23	30	28	14

Solution:

1. Prepare a square matrix.

2. Reduce the matrix

0	5	12	1
6	8	0	17
32	0	15	7
11	15	19	0

→

3. Check if optimal assignment can be made in the current solution or not

	1	2	3	4
A	$\boxed{0}$	5	12	1
B	6	8	$\boxed{0}$	17
C	32	$\boxed{0}$	15	7
D	11	5	19	$\boxed{0}$

Since there is one assignment in each row and each column, the optimal assignment can be made in the current solution.

Minimum total cost $= 12 \times 1 + 9 \times 1 + 25 \times 1 + 14 \times 1$

$= 12 + 9 + 25 + 14$

$= 60$

Ans.

$A \longrightarrow 1$

$B \longrightarrow 3$

$C \longrightarrow 2$

$D \longrightarrow 4$

Minimum cost = Rs. 60.

Find the optimal assignment for the assignment problem with the following cost matrix.

	I	II	III	IV
A	5	3	1	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	6

Solution: 1. Prepare a square matrix

2. Prepare a reduced matrix.

0	0	0	2
2	6	1	0
1	1	4	1
0	4	6	0

0	0	0	2
2	6	1	0
0	0	3	0
0	4	6	0

3. Check if optimal assignment can be made in the current solution.

	I	II	III	IV
A	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2
B	2	6	1	$\boxed{0}$
C	$\cancel{0}$	$\boxed{0}$	3	$\cancel{0}$
D	$\boxed{0}$	4	6	$\cancel{0}$

since each row and each column have assignment so optimal assignment can be made.

A – III
B – IV
C – II
D – I

Cost = 1+ 6 + 4 + 5 = 16.

Four different jobs are to be done on four different machines. Table below indicate the cost of producing job i on machine j in rupees.

Job \ m/c	1	2	3	4
1	5	7	11	6
2	8	5	9	6
3	4	7	10	7
4	10	4	8	3

Solution : 1. Reduced matrix

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

3. Check if optimal assignment can be made in the current solution or not

$\boxed{0}$	2	2	1	✓
3	$\boxed{0}$	$\cancel{0}$	1	
$\cancel{0}$	3	2	3	✓
7	1	1	$\boxed{0}$	

Cross marked column and unmaked row.
Since no. of lines
$\leq$ Rank of matrix
$3 \leq 4$
4. Iterate towards optimality

$\cancel{0}$	1	1	$\cancel{0}$	✓
4	$\boxed{0}$	$\boxed{0}$	1	
$\boxed{0}$	2	1	2	✓
8	1	1	$\boxed{0}$	✓
✓			✓	

number of lines (3) $\leq$ Rank of matrix (4)

		m/c 1	2	3	4
	1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
Job	2	5	$\boxed{0}$	$\cancel{0}$	2
	3	$\boxed{0}$	1	$\cancel{0}$	2
	4	8	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Since each row and column have assignment so optimality condition is satisfied.

Job 1 – M/c 3
Job 2 – M/c 2
Job 3 – M/c 1
Job 4 – M/c 4

Cost = 11 +5 + 4 +3 = 23

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – III – Resource Scheduling and Network Analysis– SPR1307

State the assumption made in sequencing model.

Ans 1 Only one operation is carried out on a m/c

2. Each operation once started, must be completed.

3. Only one rn/c of each type is available.

4. A job is processed as soon as possible but only in the order specified.

5. Processing time are independent of order f performing the operation.

6. Transportation time is negligible.

7. Cost of in process inventory is negligible.

Problem. 3.25. There are five jobs each of which just go through two machines A and B in the order of AB.

Processing times are given below. Determine a sequence for five jobs that will

minimize the elapse time and also calculate the total time.

Job	1	2	3	4	5
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

Determine the sequence for the jobs so as to minimize the process time. Find

total elapsed time.

Solution : Examine the columns of processing time on rn/c A and B and find the

smallest value. If this value falls in column A, schedule the job first on M/c, A, if this

value falls in column B, schedule the jobs last on M/c A. In this way sequence of jobs so as to minimize the process time is

2	4	3	5	1

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

Ans. Sequence

2	4	3	5	1

Total elapsed time = 30 hours. V

Problem 3.26. Find the sequence that minimize the total elapsed time to complete the following Jobs. Each Job is processed in the order of AB.

Job (Processing time in minutes)

		1	2	3	4	5	6	7
m/c	A	12	6	5	11	5	7	6
	B	7	8	9	4	7	8	3

Determine the sequence for the jobs so as to minimize the process time. Find the total elapsed time and idle time of M/c A and M/c B.

Solution : The sequence of jobs so as to minimize the process time is

5	3	2	6	1	4	7

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
5	0	5	5	12
3	5	10	12	21
2	10	16	21	29
6	16	23	29	37
1	23	35	37	44
4	35	46	46	50
7	46	52	52	55

Min elapsed time = 55 mins
Idle time of M/c A = 3 mins
Idle time of M/c B = 9 mins.

Define (i) Network (ii) Path terms used in network.

(i) Network: It is the graphical representation of logically and sequentially connected arrows and nodes representing activities and events of a project.

(ii) Path : An unbroken chain of activity ,arrows connecting the initial event to

some other event is called path.

Define critical path and critical activities

Critical path the path containing critical activities (with zero float) is known as critical path.

Critical activity the activity, which can not be delayed without delaying the project duration, is known as critical activity

list four types of floats used in network analysis.

(a) Total float.

(b) Free float

(c) Independent float

(d) Interfering float

Define Free Float, Independent float, Interfering float as used in PERT chart.

Free float : Portion of the total float within which an activity can be manipulated without affecting the floats of subsequent activities.

Independent float: Portion of the total float within which an activity can be delayed

without affecting the floats of proceeding activities.

Interfering float : It is equal to the difference between the total float and the free float of the activity.

What do you mean by dummy activity?

Dummy activity : An activity, which only determines the dependency of one

activity on the other, but does not consume any time, is called a dummy activity.

Define dummy arrow used in network.

Dummy arrow: It represent the dummy activity in the network. It only represents the dependency of one activity on the other. It is denoted by dash/dotted line.

Define dangling and looping in net-work models.

Dangling : The disconnection of an activity before the completion of all the activities in a network diagram is known as dangling.

Looping (cycling) : Looping error is also known as cycling error in a network diagram. Drawing an endless loop in a network is known as error of looping.

Differentiate between event and activity.

Event: The beginning and end points of an activity are called events or nodes. Event is a point in time and does not consume any resources.

Activity : It is physically identifiable part of a project which require time and resources for its execution. An activity is represented by an arrow, the tail of which represents the start and the head, finish of the activity.

Differentiate between CPM and PERT.

CPM.:

1. CPM is activity oriented i.e., CPM network is built on the basis of activities.

2. CPM is a deterministic model. It does not take into account in uncertainties involved in the estimation of time.

3.

CPM places dual emphasis on project time as well as cost and finds the trade off. between project time and project cost.

4. CPM is primarily used for projects which are repetitive in nature and comparatively small in size.

PERT

1.PERT is event oriented.

2.PERT is a probabilitic model.

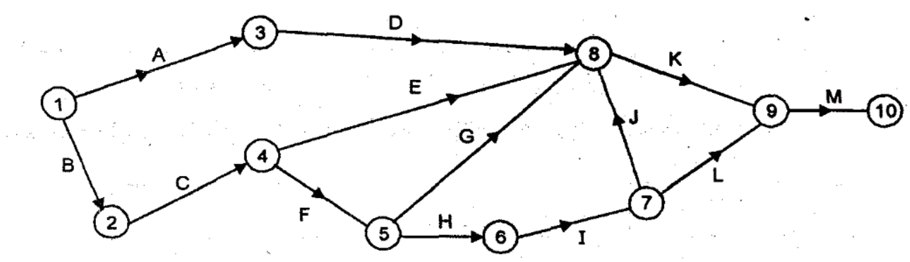
3.PERT is primarily concerned with time only.

4. PERT is used for large one timereserach and development type of projects.

Construct the network for the following activity data:

Activity	Preceded by	Activity	Preceded by
A	-	-	-
B	—	H	F
C	B	I	H
D	A	J	I
E	C	K	D,E,G,J
F	C	L	I
G	F	M	K,L

Solution. Network:

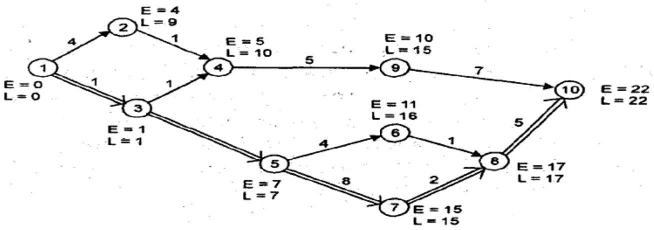


Problem 11.16. A project has the following time schedule :

Activity	1 – 2	1 – 3	2 – 4	3 – 4	3 – 5	4 – 9	5 – 6	5 – 7	6 – 8	7 – 8	8 – 9	8 – 10	9 – 10
Time weeks	4	1	1	1	6	5	4	8	1	2	1	5	7

1. **Draw Network diagram and find the critical paths.**
2. **Calculate float on each activity.**

Solution. (*i*)



2.

Activity	Duration (weeks)	Start Time		Finish Time		Total Float
		E	T_{LS}	T_{EF}	L	
1 – 2	4	0	5	4	9	5
1 – 3	1	0	0	1	1	0
2 – 4	1	4	9	5	10	5
3 – 4	1	1	9	2	10	8
3 – 5	6	1	1	7	7	0
4 – 9	5	5	10	10	15	5
5 – 6	4	7	12	11	16	5
5 – 7	8	7	7	15	15	0
6 – 8	1	11	16	12	17	5
7 – 8	2	15	15	17	17	0
8 – 10	5	17	17	22	22	0
9 – 10	7	10	15	17	22	5

Critical path 1—3—5—7—8—10 with project duration of 22 weeks.

The time estimate for the activities of a PERT network are given below :

Activity	t_0	t_m	t_p
1 - 2	1	1	7
1 - 3	1	4	7
1 - 4	2	2	8
2 - 5	1	1	1
3 - 5	2	5	14
4 - 6	2	5	8
5 - 6	3	6	15

(a) Draw the project network and identify all the path through it.

(b) Determine the expected project length.

(c) Calculate the standard deviation and variance of the project length.

(d) What is the probability that the project will be completed

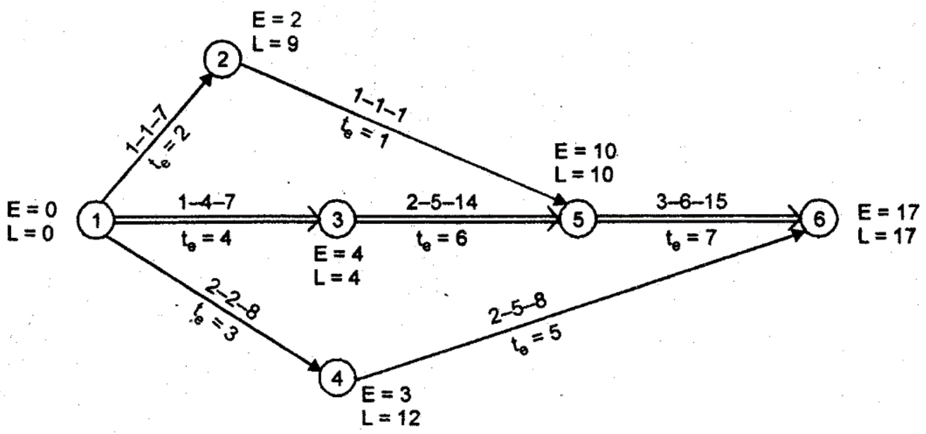
1. At least 4 weeks earlier than expected time.

2. No more than 4 weeks later than expected time.

(e) The probability that the project will be completed on schedule if the schedule completion time is 20 weeks.

(f) What should be the scheduled completion time for the probability of completion to be 90%.

Solution. (a) Network



Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \frac{(t_p - t_0)^2}{6}$
1 – 2	1	1	7	2	1
1 – 3	1	4	7	4	1
1 – 4	2	2	8	3	1
2 – 5	1	1	1	1	0
3 – 5	2	5	14	6	4
4 – 6	2	5	8	5	1
5 – 6	3	6	15	7	4

Critical path—1 —3—5—6

Project duration = 17 weeks.

(c) Variance of the project length is the sum of the variance of the activities on the critical.

$$V_{cp} = V_{1-3} + V_{3-5} + V_{5-6} = 1 + 4 + 4 = 9$$
$$\sigma^2 = V \Rightarrow \sigma^2 = 9 \Rightarrow \sigma = 3 \text{ weeks.}$$

(d) (i) Probability that the project will be completed at least 4 week earlier than expected time

Expected time (E_p) = 17 weeks

Scheduled time = 17 – 4 = 13 weeks

$$Z = \frac{13-17}{3} = -1.33$$

P(– 1.33) = 1 – 0.9082 = 0.0918

2. Probability that the project will be completed at least 4 weeks later than expected

Time

Expected time = 17 weeks Scheduled time =17+ 4 =21 weeks

$$Z = \frac{21-17}{3} = 1.33$$

$$P(1.33) = 0.9082 = 90.8\%.$$

(e) Scheduled time = 20 weeks

$$Z = \frac{20-17}{3} = 1$$

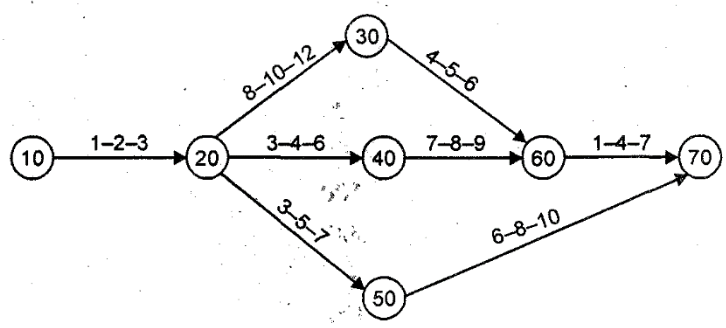
$$P(1) = 84.13\%$$

(f) Value of Z for P = 0.9 is 1.28 (from probability table)

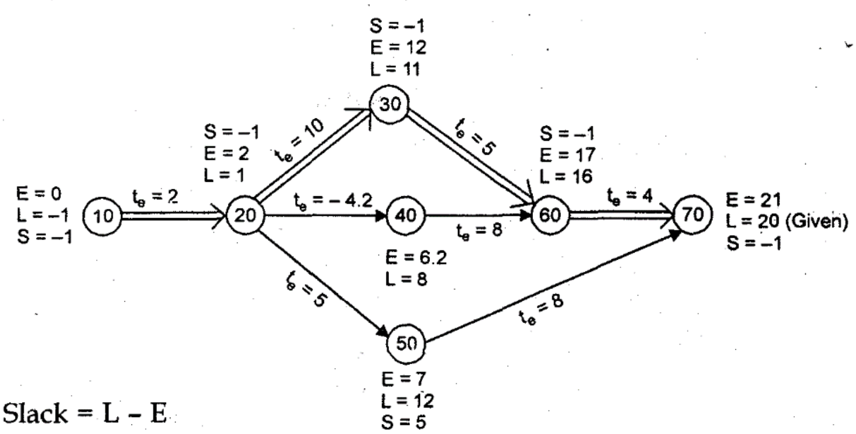
$$1.28 = \frac{T-17}{3}$$

$$T = 17 + 3.84 = 20.84 \text{ weeks.}$$

Problem 11.18. Consider the PERT network given in fig. Determine the float of each activity and identify the critical path if the scheduled completion time for the project is 20 weeks.



Solution.



(s) Slack = L – E

Activity	$t_e = \dfrac{t_0 + 4tm + t_p}{6}$	Start Time		Finish Time		Total Float
		E	T_{ES}	T_{EF}	L	
10 – 20	2	0	–1	2	1	–1
20 – 30	10	2	1	12	11	–1
20 – 40	4.2	2	3.8	6.2	8	1.8
20 – 50	5	2	7	7	12	5
30 – 60	5	12	11	17	16	–1
40 – 60	*8*	*6.2*	*8*	*14.2*	*16*	*1.8*
50 – 70	*8*	*7*	*12*	*15*	*20*	*5*
60 – 70	*4*	*17*	*16*	*21*	*20*	*–1*

Critical path 10 - 20- 30 - 60 — 70.

I.

SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – IV – Inventory Control– SPR1307

SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Subject Title: Resource Management Techniques **Subject Code: SPR 1307**

Course: B.E. **Semester: VIII**

Unit – 4: INVENTORY MODELS

Inventory Models- ABC Analysis, Cost involved in inventory management- EOQ Calculation, Deterministic Demand Inventory Models, Quantity Discount Models. Waiting Line Models- Feature of Waiting Line Models- Kendall Notations- M/M/1; M/M/C; FIFO/N/N Models only.

Prepared by Mrs. Meena Kumari. R

5.1. INVENTORY

Inventory may be defined a stock of goods, commodities or other economic resources that are stored or reserved for smooth and efficient running of business. The inventory may be kept in any one of the following forms:

1. Raw material
2. Work-in progress
3. Finished goods

If an order for a product is receive, we should have sufficient stock of materials required for manufacturing the item in order to avoid delay in production and supply. Also there should not be over stock of materials and goods as it involves storage cost and wastage in storing. Therefore inventory control is essential to promote business. Maintaining inventory helps to run the business smoothly and efficiently and also to provide adequate service to the customer. Inventory control is very useful to reduce the cost of transportation and storage.

A good inventory system, one has to address the following questions quantitatively and qualitatively.

- What to order?
- When to order?
- How much to order?
- How much to carry in an inventory?

5.1.1 Objectives of inventory management/Significance of inventory management

To maintain continuity in production.

To provide satisfactory service to customers.

To bring administrative simplicity.

To reduce risk.

To eliminate wastage.

To act as a cushion against high rate of usage.

To avoid accumulation of inventory.

To continue production even if there is a break down in few machinery.

To ensure proper execution of policies.

To take advantages of price fluctuations and buy economically.

5.1.2 Costs involved in inventory

1. **Holding Cost (Carrying or Storage Cost)**

   It is the cost associated with the carrying or holding the goods in stock. It includes storage cost, depreciation cost, rent for godown, interest on investment locked up, record keeping and administrative cost, taxes and insurance cost, deterioration cost, etc. It is denoted by ‘C’.

2. **Setup Cost/ Ordering Cost**

   Ordering cost is associated with cost of placing orders for procurement of material or finished goods from suppliers. It includes, cost of stationery, postage, telephones, travelling expenses, handling of materials, etc. (Purchase Model)Setup cost is associated with production. It includes, cost involved in setting up machines for production run. (Production Model). Both are denoted by ‘S’.

3. **Purchase Cost/Production Cost**

   When the organization purchases materials from other suppliers, the actual price paid for the material will be called the purchase cost.

   When the organization produces material in the factory, the cost incurred for production of material is called as production cost. Both are denoted by 'P'.

4. **Shortage Cost**

   If the inventory on hand is not sufficient to meet the demand of materials or finished goods, then it results in shortage of supply. The cost may include loss of reputation, loss of customer, etc.

   Total incremental cost = Holding Cost + Setup Cost/ Ordering Cost

   Total Cost = Purchase Cost/ Production Cost + Shortage Cost + Total Incremental cost.

5.2 *Demand* is one of the most important aspects of an inventory system.

Demand can be classified broadly into two categories:

5.2.1 Deterministic i.e., a situation when the demand is known with certainty. And, deterministic demand can either be *static* (where demand remains constant over time) or it could be *dynamic* (where the demand, though known with certainty, may change with time).

5.2.2 Probabilistic (Stochastic) refers to situations when the demand is *random* and is governed by a *probability density function* or *probability mass function.* Probabilistic demand can also be of two types - *stationary*(in which the demand probability density function remains unchanged over time),and *non-stationary,* where the probability densities vary over time.

Deterministic Inventory Models

i. Model I: Purchasing model without shortages
ii. Model II: Production model without shortages
iii. Model III: Purchasing model with shortages

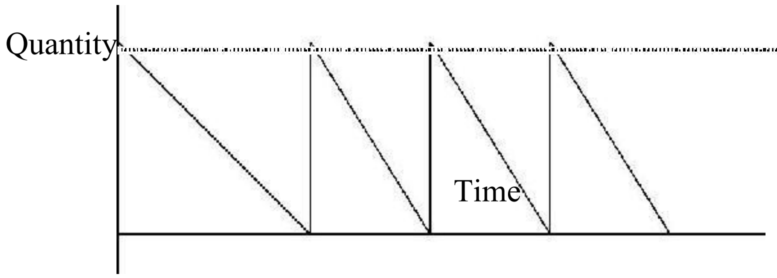
iv. Model IV: Production model with shortages

5.2.1.1 Model I: Purchasing model without shortages

Assumptions

- Demand(D) per year is known and is uniform
- Ordering cost(S) per order remains constant

- Carrying cost(C) per unit remains constant
- Purchase price(P) per unit remains constant
- No Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back. Lead time is Zero.

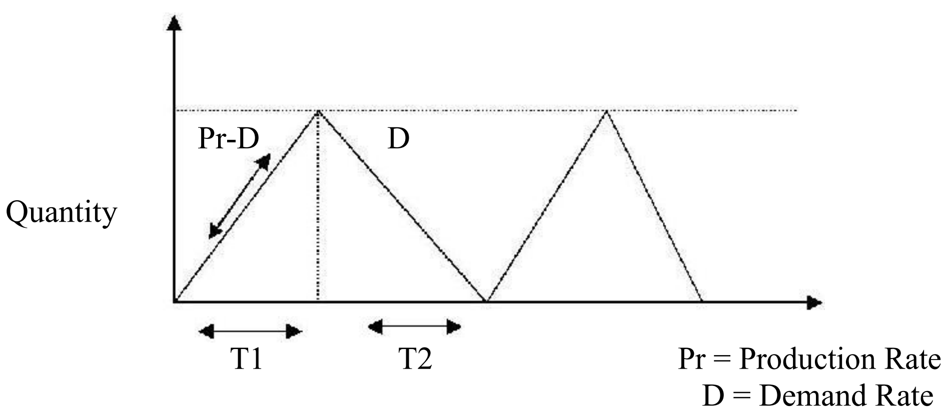


Inventory decreases at the rate of ‘D’ As soon as the level of inventory reaches zero, the inventory is replenished back

5.2.1.2 Model II: Production model without shortages

Assumptions

- Demand(D) per year is known and is uniform
- Setup cost (S) per production run remains constant
- Carrying cost(C) per unit remains constant
- Production cost per unit(P) per unit remains constant
- No Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back.



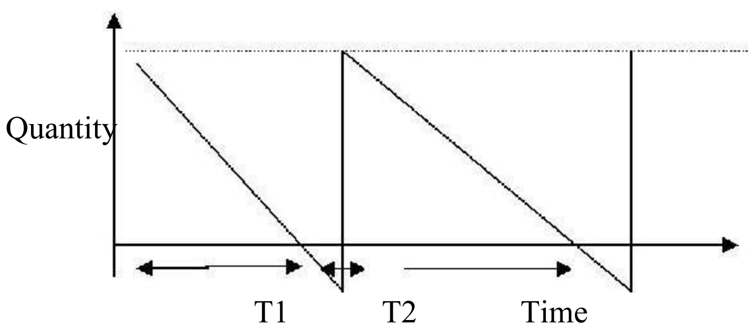
T1 is the time taken when manufacturing takes place at the rate of Pr and demand at the rate of

D. So the stock is built up at the rate of (Pr – D). During t2 there is no production only usage of stock. Hence, stock is decreased at the rate of ‘D’. At the end of t2, stock will be nil.

5.2.1.3 Model III: Purchasing model with shortages

Assumptions

- Demand(D) per year is known and is uniform
- Ordering cost(S) per order remains constant
- Carrying cost(C) per unit remains constant
- Purchase price(P) per unit remains constant
- Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back with lead time.
- Shortage cost (sh) per unit remains constant



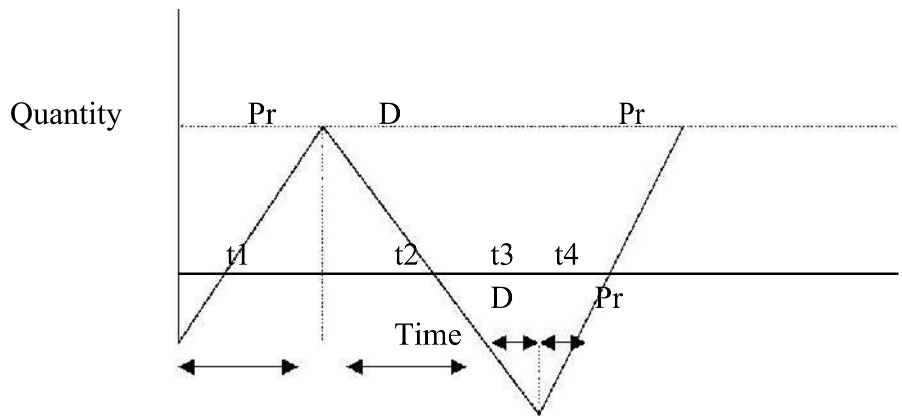
T1 is the time during which stock is nil.During T2 shortage occur and at the end of T2 stock is replenished back.

5.2.1.4 Model IV: Production model with shortages

Assumptions

Demand(D) per year is known and is uniform

- ❖ Setup cost (S) per production run remains constant
- ❖ Carrying cost(C) per unit remains constant
- ❖ Production cost per unit(P) per unit remains constant
- ❖ Shortages are allowed. As soon as the level of inventory reaches zero, the inventory is replenished back with lead time.
- ❖ Shortage cost (Sh) per unit remains constant



T1 is the time taken when manufacturing takes place at the rate of Pr and demand at the rate of D. So the stock is built-up at the rate of (Pr – D). During t2 there is no production only usage of stock. Hence, stock is decreased at the rate of 'D'. At the end of t2, stock will be nil. During T3 shortage exists at the rate of 'D'. During T4 production begins stock builds and shortage decreases at the rate of 'Pr-D'

5.2.2.1Inventory basic terminologies

- EOQ- Economic order quantity – The optimum order per order quantity for which total inventory cost is minimum.
- EBQ- Economic batch quantity – The optimum manufacturing quantity in one batch for which total inventory cost is minimum.
- Demand Rate – rate at which items are consumed
- Production rate- rate at which items are produced
- Stock replenishment rate
    - Finite rate – the inventory builds up slowly /step by step(production model)
    - Instantaneous rate – rate at which inventory builds up from minimum to maximum instantaneously (purchasing model)
- Lead time- Time taken by supplier to supply goods
- Lead time demand it is the demand for goods in the organization during lead time.
- Reorder level- the level between maximum and minimum inventory at which purchasing or manufacturing activities must start from replenishment.

  Reorder level = Buffer stock+ Lead time demand
- Buffer stock- to face the uncertainties in consumption rate and lead time , an extra stock is

maintained. This is termed as buffer stock:

Buffer stock = (Maximum Lead time – Average Lead time) x Demand per month

- Maximum Inventory Level: Maximum quantity that can be allowed in the stock: Maximum Inventory = EOQ + Buffer stock
- Minimum Inventory Level is the level that is expected to be available when thee supply is due: Minimum Inventory level = Buffer stock
- Average Inventory = (Minimum Inventory + Maximum Inventory)/2
- Order cycle is the period of time between two consecutive placements of orders.

5.3Inventory system followed in a organization:

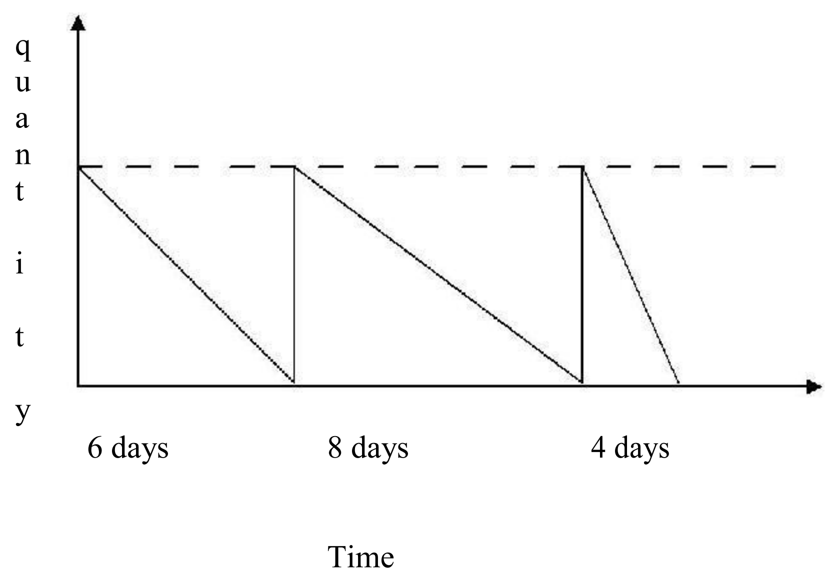
- Q – System (fixed order quantity system)
- P - System (fixed period system)

5.3.1Q – System

In a fixed order quantity system means every time an order is placed the quantity order is EOQ.

In Q – System, the period between the orders is not constant:

Ex. 1^{st} – 1 month –
EOQ 2^{nd} – 1 ½ month –
EOQ
3^{rd}- 2 month – EOQ
4^{th} – 15 days - EOQ
Whenever the stock reaches reorder level, next order is placed.



- Reorder level- the level between maximum and minimum inventory at which purchasing or manufacturing activities must start from replenishment.
  Reorder level = Buffer stock+ Lead time demand
- Lead time is the time taken by supplier to supply goods
- Lead time demand it is the demand for goods in the organization during lead time.
- Buffer stock: To face the uncertainties in consumption rate and lead time, an extra stock is maintained. This is termed as buffer stock:
  Buffer stock = (Maximum Lead time – Average Lead time) x Demand per month
- Maximum Inventory Level: Maximum quantity that can be allowed in the stock: Maximum Inventory = EOQ + Buffer stock
- Minimum Inventory Level is the level that is expected to be available when the supply is due:
  Minimum Inventory level = Buffer stock
  Average Inventory = (Minimum Inventory + Maximum Inventory)/2

5.3.2P – System

Time period between the orders is fixed; hence it is called as Fixed Period System. Period of order is fixed but the quantity will vary. Ex:

1^{st} – 1 month – 1000 units
2^{nd} – 1 month – 1200 units
3^{rd} - 1 month – 950 units

A predetermined level of inventory is fixed and thee order quantity is determined by deducting the level of stock at the time review from P determine level of inventory.

Order quantity = Predetermined level of inventory – level of stock at the time of review

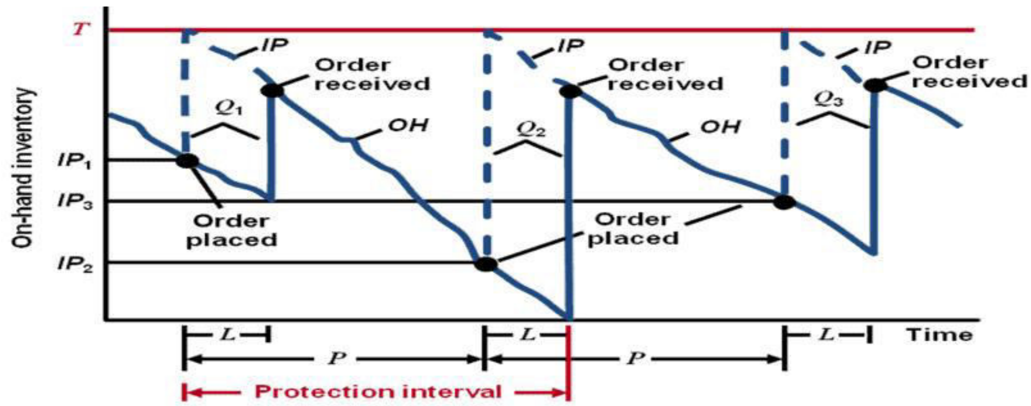


Figure 12.10 – P System When Demand Is Uncertain

5.4Inventory Selective Control Techniques

Every organization consumes several items of store. Since all the items are not of equal importance, a high degree control on inventories of each item is neither applicable nor useful. So it becomes necessary to classify items in group depending upon their utility importance. Such type of classification is name as the principle of selective control.

5.4.1ABC Analysis (Always Better Control)

- A - High value items
- B - Moderate value items
- C - Low value items

ABC analysis is one of the methods for classification of materials. It is based on Parelo's law that a few high usage value items constitute a major part of the inventory while a large bulk of items constitute to very low usage value.

5.4.1.1PROCEDURE FOR ABC ANALYSIS:

1. **Note down the material code.**
2. **Note down the annual usage in terms of units.**
3. **Note down the price per unit.**
4. **Calculate the Annual usage value.**

Annual usage value = Quantity used x Price per unit

5. Arrange the materials according to the value in descending order.
6. Find out the percentage contribution of each material to the total value.
7. Find out the percentage contribution of each material towards the total quantity.

8. Cumulate the % contribution towards value.
9. The classification is as follows.

A = 80% contribution B = 15% contribution C = 5% contribution.

5.4.1.2SIGNIFICANCE OF ABC ANALYSIS

ABC analysis is a very useful technique to classify the materials.

- The control procedure is based on which category the item belongs to. A = Tight control
  B = Moderate control
  C = Very little
  control.

- The inventory to be maintained is again based on the category A = Low Inventory
  B = Moderate
  Inventory C = High
  Inventory.

- The number of suppliers is also based on the category to which it belong s. A = Many suppliers
  B = Moderate No. of
  suppliers C = Few suppliers.

5.4.2VEDAnalysis

- V Vital items
- E Essential items
- D Desirable or Durable it ems

5.4.3HML Analysis

- High price items
- Moderate price items
- Low price items

5.4.4FNSD Analysis

- F Fast Moving items
- N Normal Moving items
- S Slow Moving items

- D Dead items

5.5Probabilistic Inventory Model

One such model is fixed order quantity model (FOQ).
In this model,

1. The demand (D) is uncertain, you can estimate the demand through any one of the forecasting techniques an d the probability of demand distribution is known.

2. Lead time (L) is uncertain, probability of lead time distribution is known.
3. Cost(C) all the costs are known.
   a. –Inventory holdin g costs C1

   b. –shortage cost C2
4. The optimum order level Z is determined by the following relationship

$$\sum_{d=0}^{z-1} p(d) < \frac{C2}{C1 + C2} < \sum_{d=0}^{z} p(d)$$

5.5.1Stock out Cost/Shortage cost

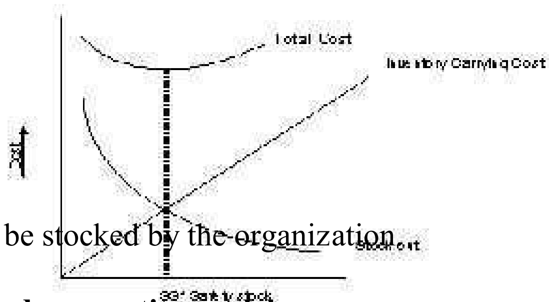
It is difficult to calculate stock out cost because it consists of components difficult to quantify so indirect way of handling stock out cost is through service levels. Service levels means ability of organization to meet the requirements of the customer as on when he demands for the product. It is measured in terms of percentage.

For example: if an organization maintains 90% service level, this means that 10% is “stock out” level. This way the stock out level is addressed.

5.5.2Safety stock

It is the extra stock or buffer stock or minimum stock. This is kept to take care of fluctuations in demand and lead time.

If you maintain more safety stock, this helps in reducing the chances of being “stock out”. But at the same time it increases the inventory carrying cost. Suppose the organization maintains less service level that results in more stock out cost but less inventory carrying cost. It requires a tradeoff between inventory carrying cost and stock out cost. This is explained through following Fig.



Safety stock (S.S*) is to be stocked by the organization.

5.5.3Working of fixed order quantity model

Fixed order quantity system is also known as continuous review system or perpetual inventory system or Q system.

In this system, the ordering quantity is constant. Time interval between the orders is the variable. The system is said to be defined only when if the ordering quantity and time interval between the orders are specified. EOQ provides answer for ordering quantity. Reorder level provides answers for time between orders.

The working and the fixed order quantity model is shown in the below Fig

5.5.4Application of Fixed Order Quantity System

1. It requires continuous monitoring of stock to know when the reorder point is reached.
2. This system could be recommended to" A" class because they are high consumption items. So we need to have fewer inventories. This system helps in keeping less inventory comparing to other inventory systems.

5.5.4.1Advantages:

1. Since the ordering quantity is EOQ, comparatively it is meaningful. You need to have less safety stock. This model relatively insensitive to the forecast and the parameter changes.
2. Fast moving items get more attention because of more usage.

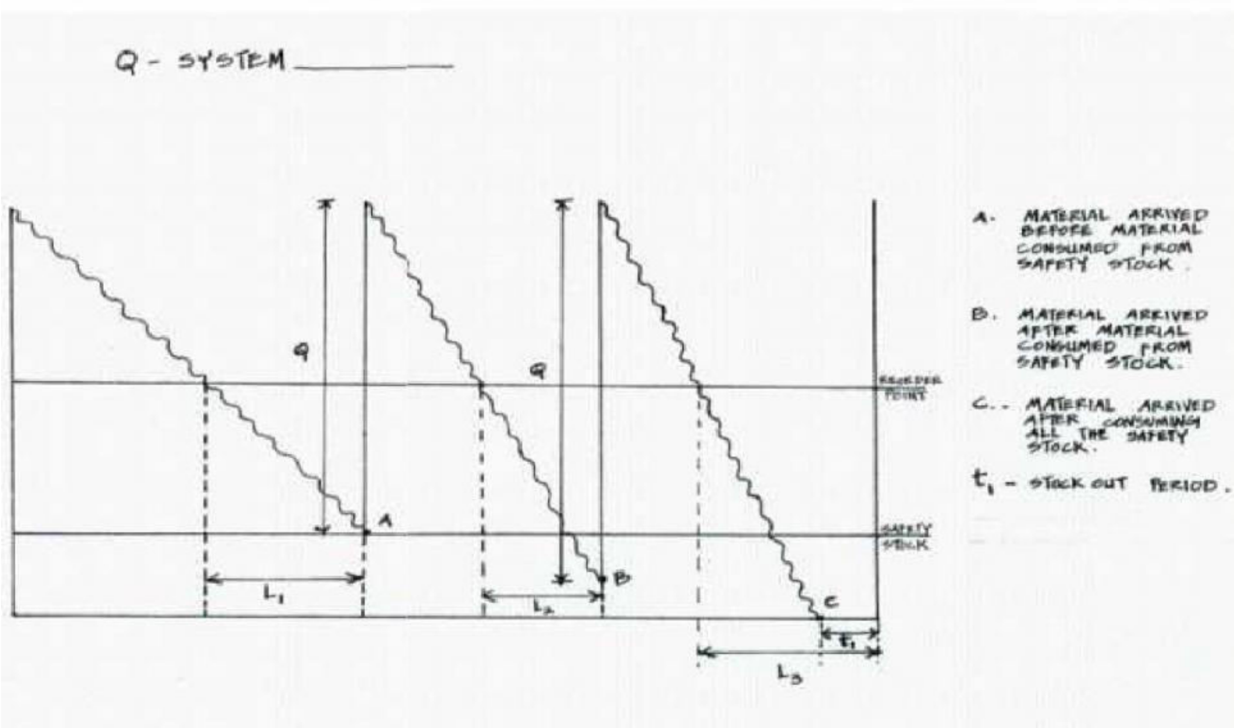
5.5.4.2Weakness:

1. We can't club the order for items which are to be procured from one supplier to reduce the ordering cost.

2. There is more chance for high ordering cost and high transaction cost for the items, which follow different reorder level.

3. You can not avail supplier discount. While the reorder level fall in different time periods.

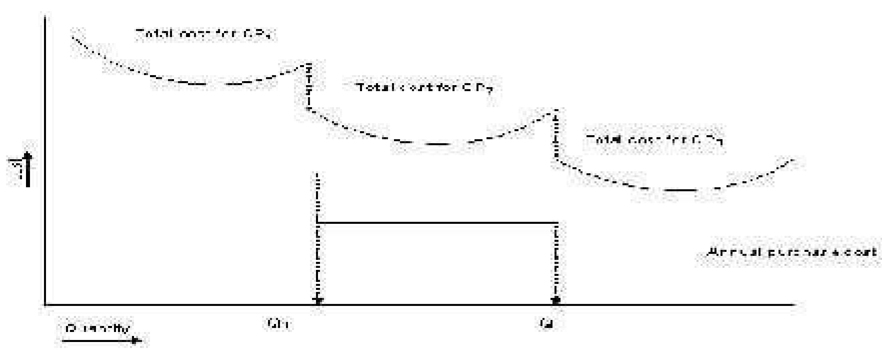
Figure –Fixed Order Quantity Model



5.5.5QUANTITY DISCOUNT MODEL

As it is mentioned already, the purchase cost becomes relevant with respect to the quantity of order only when the supplier offers discounts. Discounts means if the ordering quantity exceeds particular limit supplier offers the quantity at lesser price per unit. This is possible because the supplier produces more quantity. He could achieve the economy of scale the benefit achieved through economy of scale that he wants to pass it onto customer. This results in lesser price per unit if customer orders more quantity.

If you look at in terms of the customer's perspective customer has also to see that whether it is advisable to avail the discount offered, this is done through a trade off between his carrying inventory by the result of acquiring more quantity and the benefit achieved through purchase price. Suppose if the supplier offers discount schedule as follows,



If the ordering quantity is less than or equal to Q1 then purchase price is Cp1.

If the ordering quantity is more than Q1 and less than Q2 then purchase price is Cp2.

If the ordering quantity is greater than or equal to Q2 then purchase price is Cp3.

Then the curve you get cannot be a continuous total cost curve, because the annual purchase cost

breaks at two places namely at Q1 and Q2.

5.5.5.1STEPS TO FIND THE QUANTITY TO BE ORDERED

1. Find out EOQ for the all price break events. Start with lowest price
2. Find the feasible EOQ from the EOQ's we listed in step 1.
3. Find the total annual inventory cost using the formulae for feasible EOQ $=\sqrt{[2DSC]}+D*P$
4. Find the total annual inventory cost for the quantity at which price break took place using the following formula.

Total annual inventory cost = TC= $(D/Q)*S + (Q/2)*C + D*P$

5. Compare the calculated cost in steps 3 and 4. Choose the particular quantity as ordered Quantity at which the total annual inventory cost is minimum.

5.6QUEUING THEORY

Queuing theory concerns the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

The objective of the waiting line model is to minimize the cost of idle time & the cost of waiting time.

IDLE TIME COST: If an organization operates with many facilities and the demand from customers is very low, then the facilities are idle and the cost involved due to the idleness of the facilities is the *idle time cost*. The cost of idle service facilities is the payment to be made to the services for the period for which they remain idle.

*WAITING TIME COST:*If an organization operates with few facilities and the demand from customer is high and hence the customer will wait in queue. This may lead to dissatisfaction of

Problems

Problem 1. A particular item has a demand of 9000 units/year. The cost of one procurement is Rs. 100 and the holding cost per unit is Rs. 2.40 per year. The replacement is instantaneous and no shortance are allowed.
Determine
1. Economic lot size.
2. The no. of order per year.
3. The time between orders.
4. Total cost per year if the cost of one unit is Rs. 1.

Solution.

$$R = 9000 \text{ unit/year}$$
$$C_3 = \text{Rs. } 100/\text{procurement}$$
$$C_1 = \text{Rs. } 2.40/\text{unit/year}$$

1. $$q_0 = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 100 \times 9000}{2.40}} = 866 \text{ units/Procurement.}$$

2. $$n_0 = \frac{1}{t_0} = \sqrt{\frac{C_1R}{2C_3}} = \sqrt{\frac{2.40 \times 9000}{2 \times 100}} = \sqrt{108} = 10.4 \text{ order/year.}$$

3. $$t_0 = \frac{1}{n_0} = \frac{1}{10.4} = 0.0962 \text{ years} = 1.15 \text{ months between procurement.}$$

4. $$C_0 = 9000 \times 1 + \sqrt{2C_1C_3R}$$
$$= 9000 + \sqrt{2 \times 2.40 \times 100 \times 9000}$$
$$= \text{Rs. } 11080 \text{ per year.}$$

Problem 2. A manufacturing company purchases 9000 parts of a machine for its annual requirements, ordering one month usuage at a time. Each part cost Rs. 20. The order cost per ordering is Rs. 15 and the carrying charges are 15% of the average inventory per year. You have been asked to suggest a more economical purchasing policy for the company. What advice would you offer and how much would it same the company per year.

Solution. $R = 9000$ parts/year.

$$q = \frac{9000}{12} = 750 \text{ parts}$$

C = Rs. 20/ parts, C_3 = Rs. 15/order.

$$C_1 = \text{Rs.}20 \times \frac{15}{100} = \text{Rs. } 3/\text{part/year}$$

$$\text{Total annual variable cost} = \frac{q}{2}.C_1 + \frac{R}{q}.C_3$$

$$= \text{Rs.}\left[\frac{750}{2} \times 3 + \frac{9000}{750} \times 15\right]$$

$$= \text{Rs.}1305.$$

$$\text{E.O.Q. } (q) = \sqrt{\frac{2RC_3}{C_1}} = \sqrt{\frac{2 \times 9000 \times 15}{3}} = 300 \text{ units.}$$

$$\text{Total annual variable cost} = \sqrt{2RC_1C_3} = \sqrt{2 \times 9000 \times 3 \times 15}$$

(with E.O.Q.) = Rs. 900.

Hence if the company purchases 300 units each time and places 30 orders in the year, the net saving to the company will be Rs. (1305 — 900) = Rs. 405 a year.

Problem 3. You have to supply your customers 100 units of a certain product every monday- You obtain the product from a local supplier at Rs. 60 per unit. The cost of ordering and transportation from the supplier are Rs. 150 per order. The cost of carrying inventory is estimated at 15% per year of the cost of the product carried.

1. Find the lot size which will minimize the cost of the system.

2. Determine the optimal cost.

Solution. $R = 100$ units/week ; $C =$ Rs. 60.

$C_3 =$ Rs. 150 per order

$C_1 =$ 15% per year of the cost of the product.

$= (15 \times 60)/(100 \times 52)$ per unit per week.

$=$ Rs. 9/52 per unit per week.

1. $$q \text{ (EOQ)} = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 150 \times 100 \times 52}{9}} = 416 \text{ units.}$$

2. $$C_{min} = CR + \sqrt{(2C_1C_3R)}$$

$$= 60 \times 100 + \sqrt{2 \times \left(\frac{9}{52}\right) \times 150 \times 100}$$

$$= \text{Rs. } 6072.$$

Optimal cost $=$ Rs. 6072.

E.O.Q. $= 416$ units.

Problem 4. Daily demand for a product is normally distributed with mean, 60 units and a standard deviation of 6 units. The lead time is constant at 9 days. The cost of placing an order is Rs. 200, and the annual holding costs are 20% of the unit price of Rs. 50. A 95% service level is desired for the customers, who place orders during the reorder period. Determine the order quantity and the reorder level for the item in question, assuming that there are 300 working days during a year.

Solution.

(R) Demand/day $= 60$ units

(C_3) order cost $=$ Rs. 200/order

(C_1) holding cost $= 0.20 \times 50 =$ Rs. 10/- per unit per year

working day/year $= 300$

Demand/year $= 60 \times 300 = 18000$ units

$$\text{EOQ } q = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2 \times 200 \times 18000}{10}} = 848.52 \text{ units.}$$

Lead time $= 9$ days

Standard deviation of daily demand = 6 units

Now variance of demand during the lead time is equal to the sum of variance of daily demand during the lead time period

$$\text{variance} = 6^2 + 6^2 + 6^2 + \ldots\ldots + 6^2 \text{ (9 times)}$$
$$= 9 \times 6^2 = 324$$

Standard deviation of demand during the lead time

$$\text{Period} = \sqrt{324} = 18.$$

With E.O.Q. of 848.52 the no. of order during the year

$$(n_0) = \frac{18000}{848.52} = 21.21$$

Service level = 0.95

the normal deviate Z as found from probability table is 1.65.

$$\text{Safety stock} = Z \times \text{ standard deviation}$$
$$= 1.65 \times 18 = 29.7 \text{ units.}$$

$$\text{Reorder level} = \text{Expected demand during lead time + safety stock}$$
$$= 60 \times 9 + 29.7 = 569.7 \text{ units.}$$

Ans. Economic order quantity EOQ = 848.52 units.

Reorder level = 569.7 units.

Problem 5. The demand per month for a product is distributed normally with a mean of 100 and standard deviation 25. The lead time distribution is given below. What service level will be offorded by a reorder level of 500 units?

Lead time (months) :	1	2	3	4	5
Probability :	0.10	0.20	0.40	0.20	0.10

Solution. It is given that the demand is distributed normally with

Mean ($\overline{D}$) = 100 units   SD (σd) = 25 units

lead time (L) = 1, 2, 3, 4 and 5

Reorder level (M) = 500 units

We shall use iterative method of computing service level for the reorder level policy when the demand per unit time is distributed normally and distribution of lead time is known

$$Z = \frac{M - L\overline{D}}{\sigma d\sqrt{L}}$$

By iterative method

Lead time	Value of Z when M = 500	Probability of not running out of stock corresponding to the value of Z (from table)	Probability of this particular lead time occuring	Conditional Probability of not running out of stock
1.	$\frac{500-100\times1}{25\sqrt{1}}=16.0$	100	0.10	10
2.	$\frac{500-100\times2}{25\sqrt{2}}=8.49$	100	0.20	20
3.	$\frac{500-100\times3}{25\sqrt{3}}=4.49$	100	0.40	40
4.	$\frac{500-100\times4}{25\sqrt{4}}=2.00$	97.7	0.20	19.5
5.	$\frac{500-100\times5}{25\sqrt{5}}=0.00$	50.0	0.10	5.0

Total conditional probability of not running out of stock
=10 + 20+ 40+ 19.5 + 5= 94.5.
Hence a reorder level of 500 units will give 94.5% service level.

Problem 6. The annual demand for a product is 500 units. The cost of storage per unit per year is 10% of the unit cost, The ordering cost is Rs. 180 for each order. The unit cost depends upon the amount ordered. The range of amount ordered and the unit cost price are as follows

Range of amount ordered	$0 \le Q_1 \le 500$	$0 \le Q_2 \le 1500$	$1500 \le Q_3 \le 3000$	$3000 < Q_4$
Unit cost (Rs.)	25.00	24.80	24.60	24.4

Solution. Here $R = 500$ units

$I = 0.10$

$C_3 = \text{Rs. } 180$

$$\text{EOQ for unit price of Rs. 24.40} = \sqrt{\frac{2C_3R}{C_1}} = \sqrt{\frac{2\times180\times500}{24.40\times0.10}}$$

$$= 271.6 \text{ units.}$$

But this is not feasible because the unit price of Rs. 24.40 is not available for an order size of 271.6 units.

EOQ for unit price of Rs. 24.60 = $\sqrt{\dfrac{2 \times 180 \times 500}{24.60 \times 0.10}}$ = 270.5 units (infeasible)

EOQ for unit price of Rs. 24.80 = $\sqrt{\dfrac{2 \times 180 \times 500}{24.80 \times 0.10}}$ = 269.4 units (infeasible)

EOQ for unit price of Rs. 25.00 = $\sqrt{\dfrac{2 \times 180 \times 500}{25 \times 0.10}}$ = 268.3. units (feasible)

Total annual cost for order quantity of 268.3 units (optimal size)

$$= \sqrt{2C_1C_3R} + CR$$

$$= \sqrt{2 \times 25 \times 0.10 \times 180 \times 500} + 25 \times 500$$

= Rs. 13170.82.

Total annual cost for order quantity corresponding to cut off point of 500 units.

$$= \frac{q}{2}C_1 + C_3\frac{R}{q} + CR$$

$$= \frac{500}{2} \times 24.80 \times 0.10 + 180 \times \frac{500}{500} + 24.80 \times 500$$

= Rs. 13200.

Total annual cost for order quantity corresponding to cut of point of 3000 units.

$$= \text{Rs.}\left(\frac{1500}{2} \times 24.60 \times 0.10 + 180 \times \frac{500}{1500} + 24.60 \times 500\right)$$

= Rs. 14745.

Total annual cost for order quantity corresponding to cut off point of 3000 units.

$$= \text{Rs.}\left(\frac{3000}{2} \times 24.40 \times 0.10 + 180 \times \frac{500}{3000} + 24.40 \times 500\right)$$

= Rs. (3660 + 30 + 12200) = Rs. 15890.

Since the total cost is minimum at q_0 = 271.6 units. It represents the optimal order quantity.

List of Formulas

1. Expected number of units m the system (waiting + being served) (or)
Length of the system

$$\boxed{L_s = \frac{\lambda}{\mu - \lambda}}$$

SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – V – Queueing Theory and Replacement Models– SPR1307

SATHYABAMA INSTITUTE OF SCIENCE AND TECHNOLOGY

DEPARTMENT OF MATHEMATICS

Unit – 5: IQUEUING THEORY AND REPLACEMENT MODELS

Subject Title: Resource Management Techniques Subject Code: SPR 1307

Course: B.E. Semester: VIII

5.QUEUING THEORY

Queuing theory concerns the mathematical study of queues or waiting lines (seen in banks, post offices, hospitals, airports etc.). The formation of waiting lines usually occurs whenever the current demand for a service exceeds the current capacity to provide that service.

The objective of the waiting line model is to minimize the cost of idle time & the cost of waiting time.

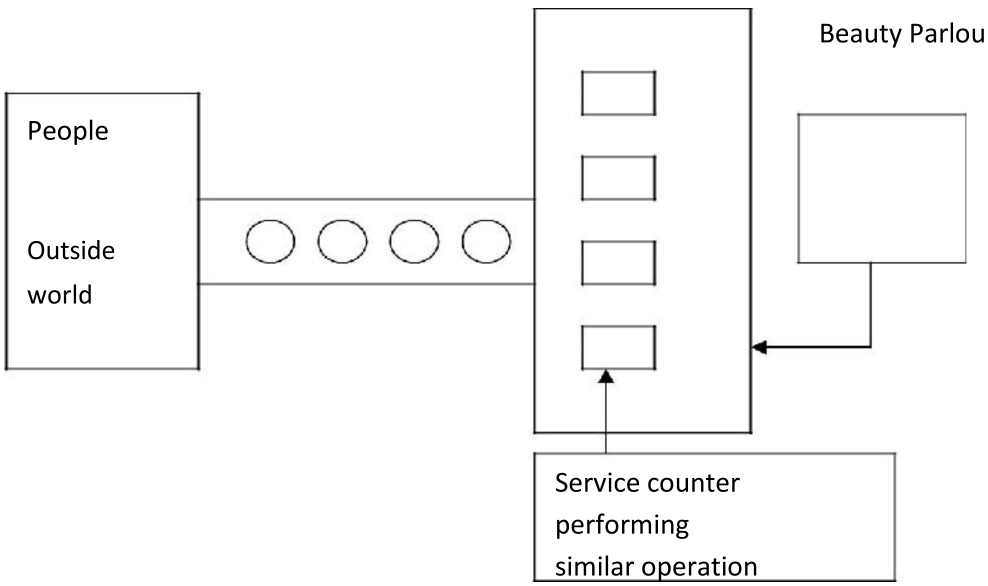
IDLE TIME COST: If an organization operates with many facilities and the demand from customers is very low, then the facilities are idle and the cost involved due to the idleness of the facilities is the *idle time cost*. The cost of idle service facilities is the payment to be made to the services for the period for which they remain idle.

*WAITING TIME COST:*If an organization operates with few facilities and the demand from customer is high and hence the customer will wait in queue. This may lead to dissatisfaction of customers, which leads to *waiting time cost*. The cost of waiting generally includes the indirect cost of lost business.

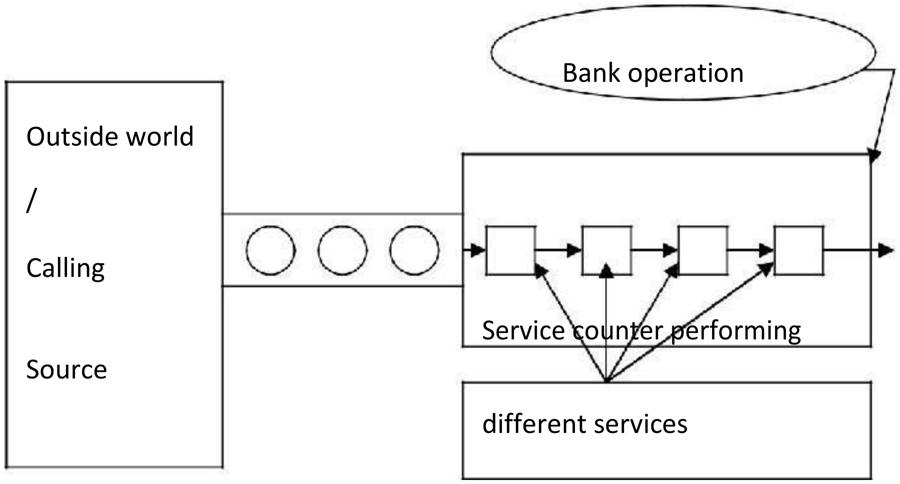
5.6.1TYPE OF QUEUE

a) Parallel queues. b) Sequential queues.

5.6.1.1PARALLEL QUEUES: If there is more than one server performing the same function, then queues are parallel.



5.6.1.2SEQUENTIAL QUEUES : If there is one server performing one particular function or many servers performing sequential operations then the queue will be sequential.



a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

a. **Infinite queue:** If the customer who arrives and forms the queue from a very large population the queue is referred to as infinite queue.

b. **Finite Queue:** if the customer who arrives and forms the queue from a small population then the queue is referred to as finite queue.

DEFINITIONS:

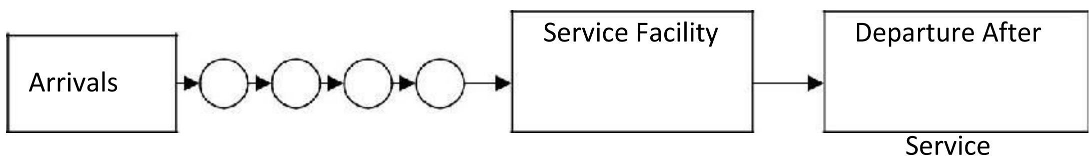
1. **The customer:** The arriving unit that requires some service to be provided.
2. **Server**: A server is one who provides the necessary service to the arrived customer.

3. **Queue (Waiting line):** The number of customers, waiting to be serviced. **The queue does not include the customer being serviced.**
4. **Service channel**: The process or system, which performs the service to the customer.

Based on the number of servers available.

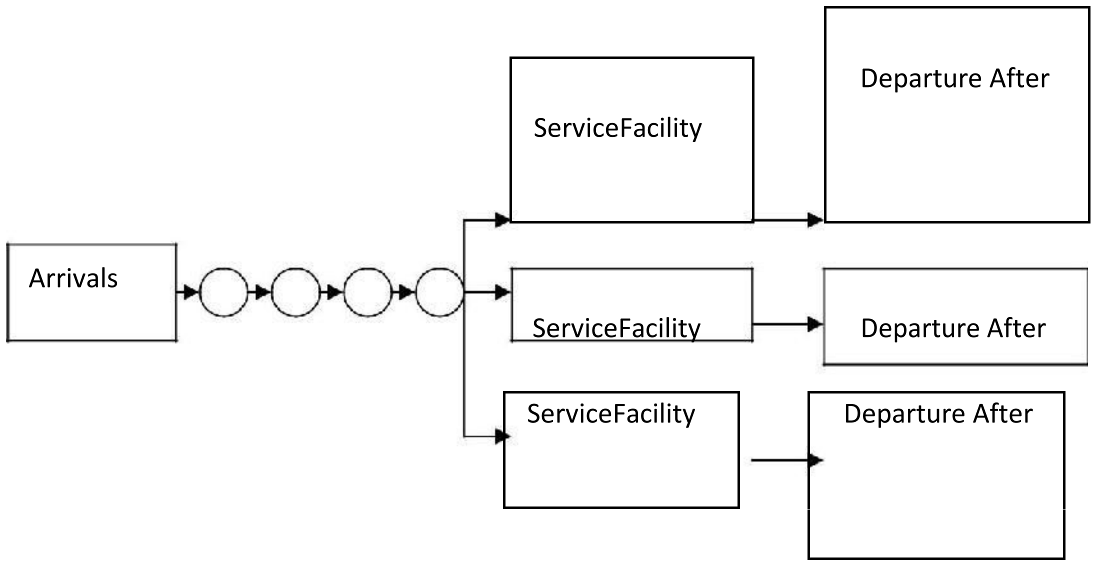
4A. **Single Channel:** If there is a single service station, customer arrivals from a single line to be serviced then the channel is said to Single Channel Model or Single Server Model.

Eg. Doctor's clinic



4B. **Multiple Channel Waiting Line Model:** If there are more than one service station to handle customer who arrive then it is called Multiple Channel Model. Symbol "c" is used.

E.g., Barber shop



5. **Arrival rate:** The rate at which the customers arrive to be serviced. It is denoted by λ. λ indicates take average number of customer arrivals per time period.

6. **Service rate:** The rate at which the customers are actually serviced. It in indicated by μ . μ indicates the mean value of customer serviced per time period.

7. **Infinite queue:** If the customers who arrive and form the queue from a very large population the queue is referred to as infinite queue.

8. **Priority:** This refers to method of deciding as to which customer will be serviced. Priority is said to occur when an arriving customer is chosen for service ahead of some other customer

already in the queue.

9. **Expected number in the queue**"**Lq**": This is average or mean number of customer waiting to be serviced. This is indicated by "Lq".

10. **Expected number in system Ls.:** This is average or mean number of customer either waiting to be serviced or being serviced. This is denoted by Ls.

11. **Expected time in queue Wq".:** This is the expected or mean time a customer spends waiting in the queue. This is denoted by "Wq".
12. **The Expected time in the system "Ws':** This is the expected time or mean time customers spends for waiting in the queue and for being serviced. This is denoted by "Ws'.
13. **Expected number in a non-empty queue:** Expected number of customer waiting in the line excluding those times when the line is empty.
14. **System utilization or traffic intensity**: This is ratio between arrival and service rate.
15. **Customer Behaviour:** The customer generally behaves in 4 ways:

a) **Balking:** A customer may leave the queue, if there is no waiting space or he has no time to wait.

b) **Reneging:** A customer may leave the queue due to impatience
c) **Priorities:** Customers are served before others regardless of their arrival
d) **Jockeying**: Customers may jump from one waiting line to another.

16. **Transient and Steady State**:

A system is said to be in Transient state when its operating characteristics are dependent on time.A system is said to be in Steady state when its operating characteristics are not dependent on time.

5.6.2CHARACTERISTICS OF QUEUING MODELS:

a) Input or arrival (inter –arrival) distribution.
b) Output or Departure (Service) distribution.
c) Service channel
d) Service discipline.
e) Maximum number of customers allowed in the system.
f) Calling source or Population.

a)ARRIVAL DISTRIBUTION:

It represents the rate in which the customer arrives at the system. Arrival rate/interval rate:

►Arrival rate is the rate at which the customers arrive to be serviced per unit of time.

►Inter-arrival time is the time gap between two arrivals.

►Arrival may be separated

1) By **equal** interval of time
2) By **unequal** interval of time which is **definitely known.**

3) Arrival may be **unequal** interval of time whose **probability is known.**

►Arrival rate may be

1. Deterministic (D)
2. Probabilistic
    a. Normal (N)
    b. Binomial (B)
    c. Poisson (M/N)
    d. Beta (β)
    e. Gama (g)
    f. Erlongian (Eh)

The typical assumption is that arrival rate is randomly distributed according to Poisson distribution it is denoted by λ. λ indicates average number of customer arrival per time period.

b) SERVICE OR DEPARTURE DISTRIBUTON:

It represents the pattern in which the customer leaves the system. Service rate at which the customer are actually serviced. It indicated by μ. μ indicates the mean value of service per time period. Interdeparture is the rate time between two departures.

Service time may be

- ► **Constant.**
- ► **Variable** with **definitely known** probability.
- ► **Variable** with **known probability**.

Service Rate Or Departure Rate may be:

1. Deterministic
2. Probabilistic.

   a. Normal (N)

   b. Binomial (B)

   c. Poisson (M/N)

   d. Beat (β)

   e. Gama (g)

   f. Erlongian (Ek)

   g. Exponential (M/N)

   The typical assumption used is that service rate is randomly distributed according to exponential distribution. Service rate at which the customer are actually serviced. It indicated by μ . μ indicates the mean value of serv ice per time period.

c) SERVICE CHANNELS:

The process or system, which is performing the service to the customer.

Based on the number of channels**:**

Single channel

If there is a single service station and customer arrive and from a single line to be serviced, the channel is said to single channel. **Single Channel – 1.**

Multiple channel

If there is more than one service station to handle customer who arrive, then it is called multiple channel model. **Multiple Channel - C.**

d) SERVICE DISCIPLINE: Service discipline or order of service is the rule by which customer are selected from the queue for service.

FIFO: First In First Out – Customer are served in the order of their arrival. Eg. Ticket counter, railway station, banks.
LIFO: Last In First Out – Items arriving last come out first.

Priority: is said to occur when a arriving customer is chosen ahead of some other customer for service in the queue.
SIRO: Service in random order

Here the common service discipline "First Come, First Served".

e) **MAXIMUM NUMBER OF CUSTOMER ALLOWED IN THE SYSTEM:**

Maximum number of customer in the system can be either finite or finite.

a. **Limited Queue:**

In some facilities, only a limited number of customers are allowed in the system and new arriving customers are not allowed to join the system unless the number below less the limiting value. (Number of appointments in hospitals)

b. **Unlimited Queue:**

In some facilities, there is no limit to the number of customer allowed in the system. (Entertainment centers).

f) POPULATION:

The arrival pattern of the customer depends upon the source, which generates them. a. **Finite population (<40):**

If there are a few numbers of potential customers the calling source is finite. b. **Infinite calling source or population:**

If there are large numbers of potential customer, it is usually said to be infinite.

5.6.3KENDALL'S NOTATION: a/b/c;

d/e/f. Where, a – Arrival rate.

b – Service rate.

c – Number of service s 1 or c.

d – Service discipline (FIFO)

e - Number of persons allowed in the queue (N or∞)

f - Number of people in the calling source (∞ or N)

1. **M/M/1, FIFO/**∞/∞**:**

Means Poisson arrival rate, Exponential service rate/one server /FIFO service discipline/Unlimited queues & Unlimited queue in the calling source.

2. **M/M/C, FIFO/**∞/∞**:**

Poisson arrival rate, Exponential service rate, more than one server, FIFO service discipline Unlimited queues and unlimited persons in the calling source.

3. **M/M/I, FIFO/N/**∞**:**

Means Poisson arrival rate, Exponential service rate, One server, FIFO, Limited queue & Unlimited population.

5.6.4SINGLE CHANNEL /MULTIPLE CHANNELPOPULATION MODEL:

1. Find an expression for probability of n customer in the system at time (Pn) in terms of λ and μ

2. Find an expression for probability of zero customers in the system at time t.(Po)
3. Having known Pn, find out the expected number of units in the Queue (Lq)
4. Find out the expected number of units in the system (Ls)
5. Expected waiting time in system (Ws)

6. Expected waiting time queue (Wq)

5.6.5 SOLUTION PROCESS

1. Determine what quantities you need to know.
2. Identify the server
3. Identify the queued items
4. Identify the queuing model
5. Determine the service time
6. Determine the arrival rate
7. Calculate ρ
8. Calculate the desired values

$$= \frac{1}{20} \times 60 = 3 \text{ min.}$$

(*c*) Avg. length of queue

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right)$$

$$L_q = \frac{10}{20}\left(\frac{10}{20-10}\right) = \frac{1}{2} = 0.5 \text{ vehicles}$$

(*d*) Probability that a customer arriving at the pump will have to wait

$$= \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

(*e*) The utilisation factor for the pump unit

$$= \frac{\lambda}{\mu} = \frac{10}{20} = 0.5$$

(*f*) Probability that the number of customer in the system is 2

$$P(n) = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

$$P(2) = \left(1 - \frac{10}{20}\right)\left(\frac{10}{20}\right)^2$$

$$= \left(1 - \frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0.125.$$

Problem. 8.14. Arrival at a telephone booth are considered are to be poission, with an average time of 10 minutes between one arrival and next. The length of phone call assumed to be distributed exponentially with mean 3 minutes then
(a) What is. the probability that a person arriving at the booth will have to wait?
(b) What is the average length of the queues that form from time to time.

Ans. Arrival rate $\lambda = \frac{1}{10}$ per minute

Service rate $\mu = \frac{1}{3}$ per minute

(*a*) Probability that a person arriving at the booth will have to wait

$$= \frac{\lambda}{\mu} = \frac{\frac{1}{10}}{\frac{1}{3}} = \frac{3}{10} = 0.3.$$

(b) Average queue length that is formed from time to time

$$= \frac{\mu}{\mu - \lambda} = \frac{\frac{1}{3}}{\frac{1}{3} - \frac{1}{10}} = \frac{\frac{1}{3}}{\frac{7}{30}}$$

$$= \frac{30}{21} = 1.42 \text{ customer.}$$

Problem. 8.15. Customers arrive at one-window drive according to a poission distribution with mean of 10 mm and service time per customer is exponential with mean of 6 minutes. The space in front of the window can accommodate only three vehicles including the serviced one. Other vehicles have wait outside the space.
Calculate.
(a) Probability that an arriving customer can drive directly to the space in front of the window.
(b) Probability that an arriving customer will have to wait outside the directed space. -
(c) How long is an arriving customer expected to wait before starting service?

Ans. Arrival rate $\lambda = \frac{1}{10}$ customers/minute

$= 6$ customers/hour

Service rate $\mu = \frac{1}{6}$ customers/minute

$= 10$ customers/hour

(a) The probability that an arriving customer can drive to the space in front of the window can be obtained by summing up the probabilities of the events in which this can happen. A customer can drive directly to the space if
(1) three is no. customer car already.
(2) there is already 1 customer car.
(3) there are 2 cars in the space.

Thus the required probability $= P_0 + P_1 + P_2$

$$= \left(1 - \frac{\lambda}{\mu}\right) + \frac{\lambda}{\mu}\left(1 - \frac{\lambda}{\mu}\right) + \left(\frac{\lambda}{\mu}\right)^2 \left(1 - \frac{\lambda}{\mu}\right)$$

$$= \left(1 - \frac{\lambda}{\mu}\right)\left[1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2}\right]$$

$$= \left(1-\frac{6}{10}\right)\left[1+\frac{6}{10}+\frac{36}{100}\right]$$

$$= \left(\frac{2}{5}\right)\left[\frac{196}{100}\right] = \frac{392}{500} = 0.78$$

(*b*) The probability that an arriving customer has to wait outside the directed space

$$= 1 - 0.78 = 0.22$$

(*c*) Avg. waiting time of a customer in the queue

$$= \frac{1}{\mu}\frac{\lambda}{\mu-\lambda} = \frac{1}{10}\left(\frac{6}{10-6}\right) = \frac{1}{10}\left(\frac{6}{4}\right) = \frac{6}{40} = \frac{3}{20}$$

$= 0.15$ hours $= 9$ minutes.

Problem 8.16. Arrival of machinists at a tool crib are considered to be poission distribution at an avg. rate of 6 per hour. The length of time the machinists must remain at the tool crib is exponentially distributed with an average time being 0.05 hours.

(a) What is the probability that the machinists arriving at tool crib will have to wait.

(b) What is the average number of machinists at the tool crib.

(c) The company will install a second tool crib when convinced that a machinist would expect to have spent at least 6 mins waiting and being serviced at the tool crib. By how much must the flow of machinists to toolcrib increase to justify the addition of second tool crib?

Ans. Arrival rate of machinist 2 = 6 per hour time spent by machinist at the tool crib = 0.05 hours.

Service rate to machinist $\mu = \frac{1}{0.05} = 20$ per hour

Probability that the machinists arriving at tool crib will have to wait

$$= \frac{\lambda}{\mu} = \frac{6}{20} = \frac{3}{10} = 0.3$$

Avg. no. of machinists at the tool crib

$$(L_S) = \frac{\lambda}{\mu-\lambda} = \frac{6}{20-6} = \frac{6}{14} = \frac{3}{7} \text{ machinists}$$

(*c*) Waiting time + Service time:

Time spent in the system $W_s = 6$ minutes $= \frac{1}{10}$ hour

λ_1 – new arrival rate of machinist

$$W_s = \frac{1}{\mu-\lambda_1} = \frac{1}{20-\lambda_1}$$

$$\frac{1}{10} = \frac{1}{20-\lambda_1} \Rightarrow 20-\lambda_1 = 10 \qquad \Rightarrow \lambda_1 = 10 \text{ machinist/hour}$$

Increase in the flow of machinists to toolcrib increase to justify the addition of a second tool crib =10 — 6 = 4/hour.

Problem 8.17 On an average 96 patients per 24 hours day require the service of an emergency clinic. Also an average a patient requires 10 miii. of active attention. Assume that the facility can handle one emergency at a time. Suppose that it cost the clinic Rs. 100 per patient treated to obtain an average servicing time of 10 minutes, and that each minute of decrease in his average time would cost Rs. 10/-per patient treated. How much would have to be budgeted by the clinic to decrease the average size of

the queue from $1\frac{1}{3}$ patients to $\frac{1}{2}$ patient.

Ans. $\lambda = \frac{96}{24} = 4$ patients/hour

$$\mu = \frac{1}{10} \times 60 = 6 \text{ patients/hour}$$

Avg. no. of patients in the queue.

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu-\lambda}\right) = \frac{4}{6}\left(\frac{4}{6-4}\right)$$

$$L_q = \frac{4}{2}(2) = \frac{8}{6} = 1\frac{1}{3}$$

This number is to be reduced from $1\frac{1}{3}$ to $\frac{1}{2}$. This can be achieved by increasing the service rate to say μ'

$$L_q' = \frac{\lambda}{\mu'}\left(\frac{\lambda}{\mu'-\lambda}\right)$$

$$\frac{1}{2} = \frac{4}{\mu'}\left(\frac{4}{\mu'-4}\right)$$

$$\mu'^2 - 4\mu' - 32 = 0 \text{ or } (\mu' - 8)(\mu' + 4) = 0$$

$\mu' = 8$ patients/hour ($\mu' = -4$ is illogical and hence neglected)

Avg. time required by each patient $= \frac{1}{8} hr$

$$= \frac{15}{2} \text{ minutes}$$

Therefore the budget required for each patient

$$= \text{Rs. } (100 + \frac{5}{2} \times 10) = \text{Rs. } 125/-$$

Thus to decrease the size of the queue, the budget per patient should be increased from Rs. 100 to Rs. 125/—

Problem 8.18. In a large maintenance department, fitters draw parts from the

parts stores which is at present staffed by one storeman. The maintenance foreman is concerned about the time spent by fitters getting parts and wants to know if the employment of a stores labourer to assist the storeman would be worth while. On investigation it is found that

(a) a simple queue situation exists.

(b) fitters cost Rs. 2.50 per hour.

(c) the storeman costs Rs. 2 per hour and can deal, on the avg. with 10 fitters per hour.

(d) a labourer could be employed at Rs. 1.75 per hour and would, increase the service capacity of the stores to 12 per hour.

(e) on the average 8 fitters visit the stores each hour.

Ans. We calculate the avg. number of customers in the system before and after the labouer is employed and compare the reduction in the resulting queuing cost with the increase in service cost.

Without labourer:

Number of customers in the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{8}{10-8} = 4$$

Cost/hr = 4 × Rs. 2.50 = Rs. 10/-.

With labourer :

$$\lambda = 8/\text{hr}, \ \mu = 12/\text{hr}.$$

Number of customer in the system

$$L_s = \frac{\lambda}{\mu-\lambda} = \frac{8}{12-8} = 2$$

Cost/hr = Cost of fitters per hour + cost of labourer per hour

= 2 * Rs. 2.50 + Rs. 1.75 = Rs. 6.75.

Since there is net saving of Rs. 3.25/- It is recommended to employs the labourer.

Problem 8.19. Customers arrive at the first class ticket counter of a theatre at the rate of 12 per hour. There is one clerk serving the customers at the rate of 30 per hour.

(a) What is the probability that there is no customer in the counter (i.e. that the system is idle) ?

(b) What is the probability that there are more than 2 customers in the counter?

(c) What is the probability that there is no customer waiting to be served?

(d) What is the probability that a customer is being served and no body is waiting.

Ans. Here λ = 12/hour, μ = 30/hour

(*a*) Probability that there is no customer in the system $P_0 = 1 - \frac{\lambda}{\mu} = 1 - \frac{12}{30} = 0.6$

Probability that there are more than two customers in the counter

$= P_3 + P_4 + P_5 + \ldots\ldots\ldots$

$= 1 - (P_0 + P_1 + P_2)$

$$= 1 - \left[\left(1-\frac{\lambda}{\mu}\right)+\frac{\lambda}{\mu}\left(1-\frac{\lambda}{\mu}\right)+\left(\frac{\lambda}{\mu}\right)^2\left(1-\frac{\lambda}{\mu}\right)\right]$$

$$= 1 - \left[\left(1-\frac{\lambda}{\mu}\right)+\left[1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}\right]\right]$$

$$= 1 - \left[0.6\left(1+\frac{12}{30}+\frac{144}{900}\right)\right]$$

$= 0.064$

Probability that there is no customer waiting to be served = Probability that there is at most one customer in the counter.

$$= P_0 + P_1 = 0.6 + 0.6\left(\frac{12}{30}\right) = 0.84$$

Probability that a customer is being served and no body is waiting.

$$= P_1 = \left(1-\frac{\lambda}{\mu}\right)\frac{\lambda}{\mu}$$

$$= 0.6\left(\frac{12}{30}\right) = 0.24.$$

Problem 8.20. In a bank there,is only one window, a solitary employee performs all the service required and the window remains continuously open from 7 am to 1 pm. It has been discovered that average number of clients is 54 during the day and the average service-time is of 5 mins per person.
Calculate
(a)Average number of clients in the system (including the one bring served)
(b)The average number of clients in the waiting line. (including the one being served)
(c) Average waiting time.
(d) Average time spends in the system. Ans. Working hours per day = 6 hrs.
Ans.

Arrival rate $\lambda = 54$ clients/day

$$= \frac{54}{6} = 9 \text{ clients/hr}$$

Service rate $\mu = 5$ min. per person

$$= \frac{1}{5} \text{ person/min}$$

$$= \frac{1}{5} \times 60 = 12 \text{ clients/hr.}$$

(*a*) Avg. no. of client in the system

$$L_s = \frac{\lambda}{\mu - \lambda} = \frac{9}{12-9} = \frac{9}{3} = 3 \text{ clients}$$

(*b*) Avg. no. of clients in the Queue

$$L_q = \frac{\lambda}{\mu}\left(\frac{\lambda}{\mu - \lambda}\right) = \frac{9}{12}(3) = \frac{9}{4} = 2.25 \text{ clients}$$

(*c*) Avg. waiting time $w_q = \dfrac{\lambda}{\mu(\mu - \lambda)} = \dfrac{1}{12}(3) = \dfrac{1}{4}$ hr.

$$= 15 \text{ min per client.}$$

(*d*) Avg. time spends in the system

$$W_s = \frac{1}{\mu - \lambda} = \frac{1}{12-9} = \frac{1}{3} \text{ hr.}$$

$$= \frac{1}{3} \times 60 = 20 \text{ min per client.}$$

QUESTION BANK

INVENTORY MANAGEMENT

INVENTORY MANAGEMENT

Deterministic cost Inventory Models

Model I: Purchasing model without shortages

(Demand rate Uniform, Production rate Infinite)

1. Find the economic order quantity and the number of orders if demand for the year is 2000 units. Ordering cost is Rs500 per order and the carrying cost for one unit per year is Rs2.50. calculate the Total Incremental Cost and Total cost if the purchase price of 1 unit is Rs25/-.

2. A manufacturing company uses an item at a constant rate of 4000 per year. Each unit costs Rs2. The company estimates that it will cost Rs50 to place an order and the carrying cost is 20% of stock value per year. Find economic order quantity and the Total Cost.

Model II: Production model without shortages

(Demand rate Uniform, Production rate finite)

3. A company needs 12000 units per year. The set up cost is Rs 400 per production run. Holding cost per unit per month is Rs15. The production cost is Rs4. The company can produce 2000 units per month. Find out the economic batch quantity, total incremental cost, total cost.

4. Demand = 2000 units/yr. The organization can produce @ 250 units per month. The set up cost is Rs1500/set up, running cost is 10% of average cost of the inventory pr year. If the organization incurs the cost of Rs100, determine how frequently the organization has to go for producing the required material.

Model III: Purchasing model with shortages

(Demand rate Uniform, Production rate Infinite, Shortages allowed)

5. The demand for an item is 20 units per month. The inventory carrying cost is Rs25 per item/month. The fixed cost (ordering cost) is Rs10 for each item a order is made. The purchase cost is Re.1 per item. The shortage cost is Rs15 per year. Determine how often a order should be made and what is the economic order quantity. Find the No. of orders, Total Incremental Cost and Total cost.

6. Demand = 9000 units. Cost of 1 procurement Rs100, holding cost – Rs2.40 per unit, shortage cost = Rs5 per unit. Find economic order quantity and how often should it be ordered. If price is Rs10 find Total Incremental Cost and Total Cost.

Model IV: Production model with shortages

(Demand rate Uniform, Production rate finite, Shortages allowed)

7. A company demands 12000 units per year. The set up cost is Rs 400 per production run. Holding cost per unit per month is Rs0.15. The shortage cost is Rs20 per year. The company can produce 2000 units per month. Find out the economic batch quantity, total incremental cost, total cost per year assuming cost of one unit is Rs 4.

8. The demand for an item in a company is 18,000 units per year, and the company can produce the item at a rate of 3000 per month. The cost of one set up is Rs.500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs.20 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set-ups.

Buffer Stock - Deterministic Model

9. A Company uses annually 50,000 units, Each order costs Rs.45 and inventory carrying costs are .18 per unit. i) Find economic order quantity ii) If the company operates 250 days a year and the procurement lead time is 10 days and safety stock is 500 units, find reorder level, maximum, minimum and average inventory.

10. Annual Demand = 12000, Ordering cost = Rs 12, Carrying cost = 10% of inventory per unit cost per unit is Rs 10. The company operates for 250 days per year .The procurement lead time in the past is 10 days, 8 days, 12 days, 13 days and 7 days. find EOQ, Buffer stock reorder level, maximum, minimum and average inventory.

PROBABILISTIC INVENTORY MODEL

1. The probability distribution of the demand for certain items is as follows

Monthly sales	0	1	2	3	4	5	6
Probability	.01	.06	.25	.35	.20	.03	.10

The cost of carrying inventory is Rs 30 per unit per month and cost of unit short is Rs 70 per month. Determine the optimum stock level that would minimize the total expected cost.

2. A news paper boy buys paper for Rs 1.40 and sells them for Rs 2.45 .He cannot return unsold news papers .Daily Demand for the following distribution is as follows

Customers	25	26	27	28	29	30	31	32	33	34	35	36
Probability	.03	.05	.05	.10	.15	.15	.12	.10	.10	.07	.06	.02

If the days demand is independent of the previous day, how many papers he should order each day?

3. The probability distribution of the demand for certain items is as follows

Monthly sales	0	1	2	3	4	5	6
Probability	.02	.05	.30	.27	.20	.10	.06

The cost of carrying inventory is Rs 10 per unit per month .The current policy is to maintain a stock of 4 items at the beginning of each month. Determine the shortage cost per one unit for one time unit.

4. A company orders a new machine after certain fixed time. It is observed that one of the parts of the parts of the machine is very expensive if it is ordered without the machine. The cost of spare part when ordered with the machine is Rs 500 and the cost of down time of the machine and cost of arranging the new part is Rs10, 000. From the past records it is observed that spare parts required with probabilities mentioned below

Demand	0	1	2	3	4	5	6
Probability	.90	.05	.02	.01	.01	.01	0.00

Find the optimal no of spare parts which should be ordered along with the machine.

QUANTITY DISCOUNT MODEL

5. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	**Unit Cost(Rs)**
$0 \leq Q1 < 50$	10
$50 \leq Q2 < 100$	9
$100 \leq Q3$	8

The monthly demand for the product is200 units, the cost of storage is 25%of the unit cost and ordering cost is Rs 20 per order.

6. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \leq Q1 < 500$	10
$500 \leq Q2$	9.25

The monthly demand for the product is200 units, the cost of storage is 2%of the unit cost and ordering cost is Rs 350 per order.

M/M/1, FIFO/∞/∞:

SINGLE CHANNEL/INFINITE POPULATION

Arrival Rate: Poisson

Service Rate: Exponential

No of Channels: Single

Service Discipline: FIFO

Queue Discipline: Infinite

Population: Infinite

1. Consider a self-service store with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the store.

2. In a public telephone booth, the arrivals are on an average 15 per hour. A call on the average takes 3 minutes. If there are just one phone (Poisson arrivals and exponential service), find the expected number of customer in the booth and the idle time of the booth.

M/M/1, FIFO/N/∞:

SINGLE CHANNEL/FINITE POPULATION

Arrival Rate: Poisson

Service Rate: Exponential

No of Channels: Single

Service Discipline: FIFO

Queue Discipline: finite

Population: Infinite

4. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 5 per hour and they are served according to exponential distribution with an average service rate of 10 minutes. There are only 5 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

5. Consider a single server queuing system with poisson input and exponential service times. Suppose mean arrival rate is 3 units per hour and expected service time is 0.25 hours and the maximum calling units in the system is two. Calculate expected number in the system .

5. In a railway marshalling yard, goods trains arrive at a rate of 30 trains per day. Assuming that the inter arrival time follows an exponential distribution and the service time distribution is also exponential with an average 36 minutes. the line capacity is 9 trains Calculate the following:

   a) The probability that the yard is empty
   b) Average queue length

MODEL QUESTION PAPER

PART – A

1. Write short notes on i) Re-order level ii) Safety stock
2. Explain the different models of inventory.
3. What do you mean by Buffer stock and write the formula to find buffer stock?
4. Discuss the various types of deterministic inventory models.
5. Define a) EOQ b) EBQ c) Lead time d) Shortage cost.
6. List out the inventory selective control techniques.
7. What do you mean by a) Parallel queues b) Sequential queues.
8. Briefly explain the characteristics of queuing model.
9. Explain the objectives of waiting line model.
10. Describe the queuing models M/M/1 and M/M/C.

PART – B

11. From the following information calculate EOQ, frequency of orders, Number of orders, Total cost, and Total incremental cost:

    Annual Demand - 20000 units/yr
    Ordering cost – Rs.30 per order
    Carrying cost – 12.5% on inventory cost
    Purchase price – Rs.1.50 per unit per year

12. A company orders a new machine after certain fixed time. It is observed that one of the parts of the parts of the machine is very expensive if it is ordered without the machine. The cost of spare part when ordered with the machine is Rs 500 and the cost of down time of the machine and cost of arranging the new part is Rs10, 000. From the past records it is observed that spare parts required with probabilities mentioned below

Demand	0	1	2	3	4	5	6
Probability	.90	.05	.02	.01	.01	.01	0.00

Find the optimal no of spare parts which should be ordered along with the machine.

13. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \le Q1 < 100$	20
$100 \le Q2$	19.25

The monthly demand for the product is100 units, the cost of storage is 2%of the unit cost and ordering cost is Rs 250 per order.

14. A news paper boy buys paper for 0.30p and sells them for 0.50p .He cannot return unsold news papers .Daily Demand for the following distribution is as follows

No. of copies sold	10	11	12	13	14	
Probability		0.1	0.15	0.20	0.25	0.30

If the days demand is independent of the previous day, how many papers he should order each day?

15. A Company uses annually 15,000 units, Each order costs Rs.25 and inventory carrying costs are .9 per unit. i) Find economic order quantity ii) If the company operates 200 days a year and the procurement lead time is 15days and safety stock is 250 units, find reorder level, maximum, minimum and average inventory.

18. Find the optimal order quantity for a product for which price break up is as follows :

Quantity	Unit Cost(Rs)
$0 \le Q1 < 25$	5
$25 \le Q2 < 50$	4
$50 \le Q3$	3

The monthly demand for the product is200 units, the cost of storage is 25%of the unit cost and ordering cost is Rs 20 per order.

19. The demand for an item in a company is 20,000 units per year, and the company can produce the item at a rate of 5000 per month. The cost of one set up is Rs.500 and the holding cost of one unit per month is 15 paise. The shortage cost of one unit is Rs.15 per month. Determine the optimum manufacturing quantity and the number of shortages. Also determine the manufacturing time and time between set-ups.

20. At a one-man barbershop, the customer arrives according to Poisson process at an average rate of 2 per hour and they are served according to exponential distribution with an average service rate of 5 minutes. There are only 4 seats available for waiting of the customer and customer do not wait if they find no seat available. Find the average number of customer in the system, average queue length and the average time a customer spends in the barbershop. Also find the idle time of the barber.

21. Consider a bank with one cashier. Assume Poisson arrival and exponential service times. Suppose 9 customers arrive on an average for every 5 minutes and the cashier can service 10 in 5 minutes. Find the average number of customer in the system and average time a customer spends in the bank.

4.1REPLACEMENT MODEL

If any equipment or machine is used for a long period of time, due to wear and tear, the item tends to worsen. A remedial action to bring the item or equipment to the original level is desired. Then the need for replacement becomes necessary. This may be due physical impairment, due to normal wear and tear, obsolescence etc. The resale value of the item goes on diminishing with the passage of time.

The depreciation of the original equipment is a factor, which is responsible not to favor replacement because the capital is being spread over a long time leading to a lower average cost. Thus there exists an economic trade-off between increasing and decreasing cost functions. We strike a balance between the two opposing costs with the aim of obtaining a minimum cost.

Replacement model aims at identifying the **time** at which the assets must be replaced in order to minimize the cost.

4.2REASONS FOR REPLACEMENT OF EQUIPMENT:

1. Physical impairment or malfunctioning of various parts refers to

- The physical condition of the equipment itself
- Leads to a decline in the value of service rendered by the equipment
- Increasing operating cost of the equipment
- Increased maintenance cost of the equipment
- Or a combination of the above.

2. Obsolescence of the equipment, caused due to improvement in the existing tools and machinery mainly when the technology becomes advanced.

3. When there is sudden failure or breakdown.

4.3REPLACEMENT MODELS:

- **Assets that fails Gradually:**

  Certain assets wear and tear as they are used. The efficiency of the assets decline with time. The maintenance cost keeps increasing as the years pass by eg. Machinery, automobiles, etc.

  1. Gradual failure without taking time value of money into consideration
  2. Gradual failure taking time value of money into consideration

- **Assets which fail suddenly**

  Certain assets fail suddenly and have to be replaced from time to time eg. bulbs.

  1. Individual Replacement policy (IRP)
  2. Group Replacement policy (GRP)

4.3.1Assests that fails Gradually

4.3.1.1Gradual failure without taking time value of money into consideration

As mentioned earlier the equipments, machineries and vehicles undergo wear and tear with the passage of time. The cost of operation and the maintenance are bound to increase year by year. A stage may be reached that the maintenance cost amounts prohibitively large that it is better and economical to replace the equipment with a new one. We also take into account the salvage value of the items in assessing the appropriate or opportune time to replace the item. We assume that the details regarding the costs of operation, maintenance and the salvage value of the item are already known

- **Procedure for replacement of an asset that fails gradually (without considering Time value of money):**

a) Note down the years

b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)

c) Calculate Cumulative the running cost '$\sum R$'

d) Note down the capital cost 'C'

e) Note down the scrap or resale value 'S'

f) Calculate Depreciation = Capital Cost – Resale value

g) Find the Total Cost

Total Cost = Cumulative Running cost + Depreciation

h) Find the average cost

Average cost = Total cost/No. of corresponding year

i) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

Year	Running Cost	Cumulative Running Cost	Capital cost	Salvage value Or Resale value	Depn. = Capital cost – salvage value	Total cost= Cumulative running cost + Depreciation	Average annual cost P_n = Total cost / no. of corresponding year
N	R_n	$\sum R_n$	C	S_n	C - S_n	$\sum R_n + C - S_n$	$(\sum R_n + C - S_n)$ /n
1	2	3	4	5	6 (4-5)	7 (3+6)	8 (7/1)

4.3.1.2Gradual failure taking time value of money into consideration

In the previous section we did not take the interest for the money invested, the running costs and resale value. If the effect of time value of money is to be taken into account, the analysis must be based on an equivalent cost. This is done with the present value or present worth analysis.

For example, suppose the interest rate is given as 10% and Rs. 100 today would amount to Rs. 110 after a year's time. In other words the expenditure of Rs. 110 in year's time is equivalent to Rs. 100 today. Likewise one rupee a year from now is equivalent to (1.1)-1 rupees today and one-rupee in '*n*' years from now is equivalent to (1.1)-n rupees today. This quantity (1.1)-n is called the present value or present worth of one rupee spent 'n' years from now.

- **Procedure for replacement of an asset that fails gradually (with considering Time value of money):**

Assumption:

i. Maintenance cost will be calculated at the beginning of the year
ii. Resale value at the end of the year

Procedure:

a) Note down the years
b) Note down the running cost 'R' (Running cost or operating cost or Maintenance cost or other expenses)
c) Write the present value factor at the beginning for running cost
d) Calculate present value for Running cost
e) Calculate Cumulative the running cost '$\sum$ R'
f) Note down the capital cost 'C'
g) Note down the scrap or resale value 'S'
h) Write the present value factor at the end of the year and also calculate present value for salvage or scrap or resale value.
i) Calculate Depreciation = Capital Cost – Resale value

j) Find the Total Cost = Cumulative Running cost + Depreciation
k) Calculate annuity factor (Cumulative present value factor at the beginning)

l) Find the Average cost = Total cost / Annuity

m) Replacement decision: Average cost is minimum (Average cost will decrease and reach minimum, later it will increase)

Year n	R_n	Pv^{n-1}	R_nPv^{n-1}	$\sum R_nPv^{n-1}$	C	S_n	Pv^n	S_nPv^n	C - S_nPv^n	$\sum R_nv^{n-1}$ + C - S_nPv^n	$\sum PV^{n-1}$	W_n
1	2	3	4(2*3)	5	6	7	8	9(7*8)	10	11(5+10)	12	13

4.3.2ITEMS THAT FAIL COMPLETELY AND SUDDENLY

There is another type of problem where we consider the items that fail completely. The item fails such that the loss is sudden and complete. Common examples are the electric bulbs, transistors and replacement of items, which follow sudden failure mechanism.

4.3.2.1INDIVIDUAL REPLACEMENT POLICY (IRP):

Under this strategy equipments or facilities break down at various times. Each breakdown can be remedied as it occurs by replacement or repair of the faulty unit.

Examples: Vacuum tubes, transistors

Calculation of Individual Replacement Policy (IRP): n

Average life of an item = $\sum_{i-1} i * Pi$

Pi denotes Probability of failure during that week **i** denotes no. of weeks

No. of failures = $\frac{\text{Total no. of items}}{\text{Average life of an item}}$

Total IRP Cost = No. of failures * IRP cost

4.3.2.2GROUP REPLACEMENT

As per this strategy, an optimal group replacement period '*P*' is determined and common preventive replacement is carried out as follows.

(a) Replacement an item if it fails before the optimum period '*P*'.

(b) Replace all the items every optimum period of '*P*' irrespective of the life of individual item. Examples: Bulbs, Tubes, and Switches.

Among the three strategies that may be adopted, the third one namely the group replacement policy turns out to be economical if items are supplied cheap when purchased in bulk quantities. With this policy, all items are replaced at certain fixed intervals.

4.3.4.1Procedure for Group Replacement Policy (GRP):

1. Write down the weeks
2. Write down the individual probability of failure during that week
3. Calculate No. of failures:

   N_0 - No. of items at the beginning

   N_1 - No. of failure during 1^{st} week (N_0P_1)

   N_2 - No. of failure during 2^{nd} week ($N_0P_2 + N_1P_1$)

   N_3 - No. of failure during 3^{rd} week ($N_0P_3 + N_1P_2 + N_2P_1$)
4. Calculate cumulative failures
5. Calculate IRP Cost = Cumulative no. of failures * IRP cost
6. Calculate and write down GRP Cost = Total items * GRP Cost
7. Calculate Total Cost = IRP Cost + GRP Cost
8. Calculate Average cost = Total cost / no. of corresponding year

Problems

Problem 1. The cost of a machine is Rs. 6100/- and its scrap The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance cost	100	250	400	600	900	1200	1600	2000

When should the machine be replaced ?

Ans. Let it is profitable to replace the machine after *n* years. The *n* is determined by the minimum value of T_{avg}.

Years service	Purchase price-scrap value	Annual maintenance cost	Summation of maintenance cost	Total cost	Avg. annual cost (T_{avg})
1.	6000	100	100	6100	6100
2.	6000	250	350	6350	3175
3.	6000	400	750	6750	2250
4.	6000	600	1350	7350	1837.50
5.	6000	900	2250	8250	1650
6.	6000	1200	3450	9450	$\boxed{1575}$ Min
7.	6000	1600	5050	11050	1578
8.	6000	2000	7050	13050	1631

The avg. annual cost is minimum Rs. should be replaced after 6 years of use.

(1575/-) during the sixth year. Hence the m/c

Problem 2. A machine owner finds from his past records that the costs per year of maintaining a machine whose purchase price is Ks. 6000 are as given below

Year	1	2	3	4	5	6	7	8
Maintenance cost	1000	1200	1400	1800	2300	2800	3400	4000
Cost Resale price	3000	1500	750	375	200	200	200	200

Determine at what age is a replacement due?

Ans. Capital cost C = 6000/-. Let it be profitable to replace the. machine after n

years. Then n should be determined by the minimum value of Tav•

Year of service	Resale value	Purchase Price Resale value	Annual Maintenance cost	Summation of maintenance cost	Total Cost	Average annual cost
1.	3000	3000	1000	1000	4000	4000
2.	1500	4500	1200	2200	6700	3350
3.	750	5250	1400	3600	8850	2950
4.	375	5625	1800	5400	11025	2756.25
5.	200	5800	2300	7700	13500	$\boxed{2700}$
6.	200	5800	2800	10500	16300	2716.66
7.	200	5800	3400	13900	19700	2814.28
8.	200	5800	3400	17300	23100	2887.5

We observe from the table that avg. annual cost is minimum (Rs. 2700/-). Hence the m/c should replace at the end of 5th year.
Type B. Replacement of items whose maintenance costs increase with time and value of money also changes with time.
The machine should be replaced if the next period's cost is greater than weighted average of previous cost.
Discount rate [Present worth factor (PWF)

$$V = \frac{1}{1+i}$$

$$V_n = (V)^{n-1}$$

n – no. of year

i – annual interest rate

V_n – PWF of n^{th} year.

Problem 3. A machine costs Rs. 500/— Operation and Maintenance cost are zero for the first year and increase by Rs. 100/— every year. If money. is worth 5% every year, determine the best age at which the machine should be replaced. The resale value of the machine is negligible small. What is the weighted average cost of owning and operating the machine?

Ans. Discount rate $V = \frac{1}{1+i} = \frac{1}{1+0.05} = 0.9524$

Discount rate for I^{st} year $V_n = \left(\frac{1}{1+i}\right)^{n-1}$

$$V_1 = (0.9524)^0 = 1$$
$$2^{nd} \text{ year } V_2 = (0.9524)^1 = 0.9524$$
$$3^{rd} \text{ year } V_3 = (0.9524)^2 = 0.9070$$
$$4^{th} \text{ year } V_4 = (0.9524)^3 = 0.8638$$
$$5^{th} \text{ year } V_5 = (0.9524)^4 = 0.8227$$

Years of service (n)	Maintenance cost (Rs)	Discount factor $(V)^{n-1}$	Discounted cost	Summation of cost of m/c and maint. Cost	Summation of discount factor	Weighted average cost
1	0	1.0000	0.00	500.00	1.0000	500
2	100	0.9524	95.24	595.24	1.9524	304.88
3	200	0.9070	181.40	776.64	2.8594	$\boxed{217.61}$ min
4	300	0.8638	259.14	1035.78	3.7232	278.20
5	400	0.8227	329.08	1364.86	4.5459	300.25

M/c su1d be replaced at the end of 3C1 year.

Problem 3. Purchase price of a machine is Rs. 3000/— and its running cost is given in the table below. If should be replaced. the discount rate is 0.90. Find at what age the machine

Year	1	2	3	4	5	6	7
Running cost (Rs.)	500	600	800	1000	1300	1600	2000

Ans. V (Discount rate) = 0.90

Year of service (n)	Running cost (Rs.)	Discount factor $(V)^{n-1}$	Discounted cost	Summation of cost of m/c and maint. cost	Summation of discount factor	Weighted average cost
1	500	1	500	3500	1	3500
2	600	0.90	540	4040	1.9	2126.31
3	800	0.81	648	4688	2.71	1729.88
4	1000	0.729	729	5417	3.439	1575.16
5	1300	0.6561	852.93	6269.93	4.0951	$\boxed{1531.08}$ min.
6	1600	0.59049	944.78	7214.71	4.6855	1539.79
7	2000	0.5314	1062.8	8277.51	5.2169	1586.6

M/c should be replaced at the end of 5th year.

Problem 4. The following mortality ratio have been observed for a certain type of light bulbs in an installation with 1000 bulbs

End of week	1	2	3	4	5	6
Probability of failure to date	0.09	0.25	0.49	0.85	0.97	1.00

There are a large no. of such bulbs which are to be kept in working order. If a bulb fails in service, it cost Rs. 3 to replace but if all the bulbs all replaced in the same operation it can be done for only Rs. 0.70/— a bulb. It is proposed to replace all bulbs at fixed intervals, whether or not they have burnt out and continue replacing burnt out bulb as they fail.

(a) What is the best interval between group replacement?

(b) Also establish if the policy, as determined by you is superior to the policy of replacing bulbs as and when they, fail, there being nothing like group replacement.

(c) At what group replacement price per bulb, would a policy of strictly individual replacement become preferable to the adopted policy?

Solution : Let p. be the probability that a new light bulbs fail during the 1th wek of the life.

$P_1 = 0.09$

$P_2 = 0.25 - 0.09 = 0.16$

$P_3 = 0.49 - 0.25 = 0.24$

$P_4 = 0.85 - 0.49 = 0.36$

$P_5 = 0.97 - 0.85 = 0.12$

$P_6 = 1.00 - 0.97 = 0.03$

Week	**Expected no. of failure (N)**	
0	$N_0 = N_0$	
1	$N_1 = 1000 \times 0.09$	$= 90$
2	$N_2 = 1000 \times 0.16 + 90 \times 0.09$	$= 168$
3	$N_3 = 1000 \times 0.24 + 90 \times 0.16 + 168 \times 0.09$	$= 269$
4	$N_4 = 1000 \times 0.36 + 90 \times 0.24 + 168 \times 0.16 + 269 \times 0.09$	$= 432$
5	$N_5 = 1000 \times 0.12 + 90 \times 0.36 + 168 \times 0.24 + 269 \times 0.16 + 432 \times 0.09$	$= 274$
6.	$N_6 = 1000 \times 0.03 + 90 \times 0.12 + 168 \times 0.36 + 269 \times 0.24 + 432 \times 0.16 + 274 \times 0.09$	$= 260$
and so on		

(*a*) Determination of optimum group replacement interval

Week	Total cost of group replacement	Avg cost/week
1.	$1000 \times 0.70 + 90 \times 3 = 970$	970.00
2.	$1000 \times 0.70 + 3\,(90+168) = 1474$	$\boxed{737.00}$
3.	$1000 \times 0.70 + 3\,(90 + 168 + 269) = 2281$	760.33