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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT – I - Geometrical Optics – SPH1214

GEOMETRICAL OPTICS

LensFormula

In optics, therelationshipbetweenthedistanceofanimage (v), thedistanceofanobject(u) andthefocallength(f)ofthe lens is given by the formula known as Lens formula.Lens formula is applicable for convex as well as concave lenses. These lenses have negligible thickness. The formula is as follows*:*

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Lens Formula Derivation

- ConsideraconvexlenswithobjectABkeptontheprincipal axis.Tworaysareconsideredsuch thatonerayisparallelto theprincipalaxisandafterreflection,itpassesthroughthe focus.The second rayis towards the optical centre such that it passesdeviated.
- A' is the point where the two rays intersect and also the imageformedbypointA.PointB imageisobtainedonthe principalaxisasthepointBisontheprincipalaxis.

As the object is perpendicular to the principal axis, even the object is perpendicular to the principalaxis.Togetthe locationoftheimageformedbypoint B,weneedtodrawa perpendicular frompointA'totheprincipalaxis.Following arethethingsobtainedafterdrawingthefigure:

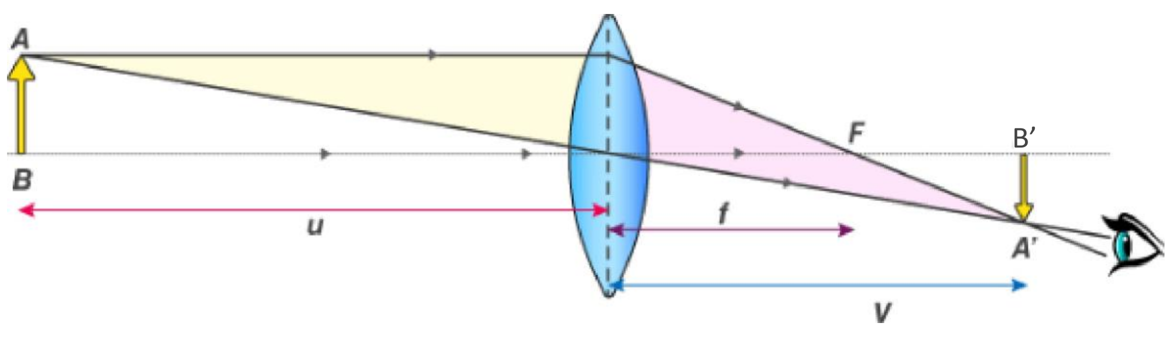


FIG.1.1

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\frac{AB}{A'B'} = \frac{BO}{OB'}$ (from similar ΔABO and ΔA'B'O) (equ. 1)

$\frac{PO}{OF} = \frac{A'B'}{FB'}$ (from similar ΔP0F and ΔFB'A')

$\therefore \frac{AB}{OF} = \frac{A'B'}{FB'}$ (from figure P0 = AB)

$\frac{AB}{A'B'} = \frac{OF}{FB'}$ (equ. 2)

$\frac{BO}{OB'} = \frac{OF}{FB'}$ (from equ. 1 and equ. 2)

$\therefore \frac{BO}{OB'} = \frac{OF}{OB'-OF} \; \frac{-u}{+v} = \frac{+f}{v-f}$ (substituting optical distance values)

$\therefore -uv + uf = fv - \frac{1}{f} + \frac{1}{v} = \frac{1}{u}$ (dividing by u,v and f on both the sides)

$$\frac{1}{v} = \frac{1}{u} + \frac{1}{f} \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

Therefore, this is known as Lens formula.

Compound Lens

A simple lens system consists of the use of only one lens as opposed to compound lens system where multiple lenses can be used with a single common axis. The simplest of these compound lens systems are two thin lenses kept in contact with each other. Let's study about compound lenses and properties of them.

The common focal length for a system, where two thin lenses are sharing an axis are kept in contact with each other, is given by the following formula.

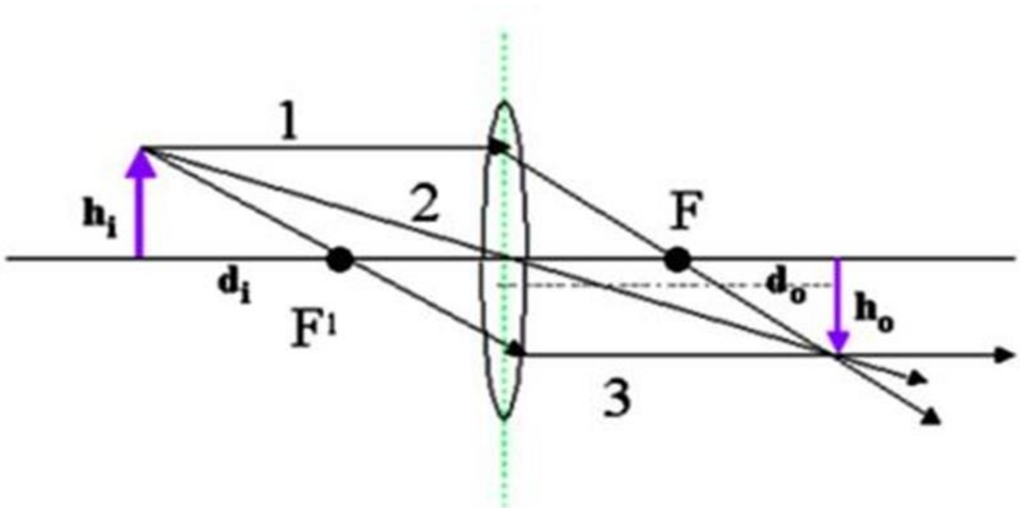


FIG.1.2

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2}$$

$$\frac{1}{f} = \frac{1}{f1} + \frac{1}{f2} - \frac{d}{f1.f2}$$

where,
f is the combined focal length

f_1 is the focal length of the first lens

f_2 is the focal length of the second lens

Sincethepoweristhereciprocalofthefocallength,whatisveryevidentinthis Case. Forthinlensesincontact,itisprettyclearthatthecombinedpowerofthe system is given by adding the powers of the individual lenses.

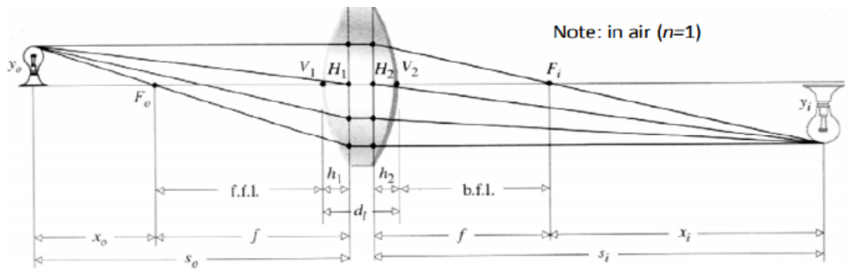
But what if the lenses aren't in contact with each other. If the lenses are separated by a distance'd', then in this case the combined focal length can be calculated using the following formula.

Thick Lenses

- Eliminate theintermediate image distance Si1.
- Focal points:
- Rays passing throughthefocalpointarerefractedparallel to the optical axis by both surfacesof the lens
- Rays parallel to the optical axis are refracted through the focal point
- For athinlens, wecandraw thepoint where refraction occurs in a common plane
- Forathicklens,refractionforthetwotypesofrayscan occur at different planes

Thick Lens: equations

The **lens equation** allows us to understand geometric optic in a quantitative way where 1/d0 + 1/di = 1/f. The **lens equation** essentially states that the magnification of the object = - distance of the image over distance of the object. **lens equation**.



$$\frac{1}{s_o}+\frac{1}{s_i}=\frac{1}{f} \qquad x_o x_i = f^2$$

effective focal length:
$$\frac{1}{f}=(n_l-1)\left[\frac{1}{R_1}-\frac{1}{R_2}+\frac{(n_l-1)d_l}{n_l R_1 R_2}\right]$$

Principal planes:
$$h_1=-\frac{f(n_l-1)d_l}{n_l R_2} \qquad h_2=-\frac{f(n_l-1)d_l}{n_l R_1}$$

Magnification:
$$M_T \equiv \frac{y_i}{y_o}=-\frac{s_i}{s_o}=-\frac{x_i}{f}=-\frac{f}{x_o}$$

FIG. 1.3

Spherical aperture and Stops

• Two important aspects of any imaging system are the amount of radiation passed by the system and the extent of an object that is seen by the system. Stops and apertures limit the brightness of an image and the field of view of an optical system. The aperture stop (AS) is defined to be the stop or lens ring, which physically limits the solid angle of rays passing through the system from an on- axis object point. The aperture stop limits the brightness of an image.

• For system (a) in the figure on the right the aperture is the aperture stop, for system (b) the first lens is the aperture stop and for system (c) the second lens is the aperture stop.

• The exit pupil is the image of the aperture stop formed by the light rays after they have passed through the optical system, i.e. it is the image of the aperture stop as seen through all the optics beyond the aperture stop. It can be a real or virtual image, depending on the location of the aperture stop.

• The entrance pupil is the opening an observer would identify as the limitation on the solid angle of rays diverging from an on-axis object point, i.e. it is the image of the aperture stop in as seen through all the optics before the aperture stop. Again, it can be a real or virtual image, depending on the location of the aperture stop.

Field stop

- For an off-axis object, the **chief ray** (CR) is the ray that passes through the center of the aperture stop. Rays that pass through the edge of the aperture stop are**marginal rays** (MR).
- The aperture stop determines the solid angle of the transmitted light cone for an on-axis object. It limits the brightness of an image. The **field stop** determines the solid angle formed by chief rays from off**- axis** objects. It limits the field of view of an optical instrument. Theimage of the field stops as seen through all the optics before the field stop is called the **entrance window**. The image as seen through all the optics after the field stop is called the **exit window**.
- The following is simply a recipe for finding the field stop, entrance window, and exit window, given an optical system.
- Image all optical elements in the system into object space.Find the angle subtended by each image at the on-axis position of the entrance pupil.The element image with the smallest angle is the entrance windowThe physical object corresponding to this image is the field stop.Theimageof the field stop in image space is the exit window.

Single lens Magnifier

- Field of view of a simple magnifier
- Field of view=area seen throughthe magnifier
- Power increase=lens diameter and fieldofview decrease
- H max = max extent of the object that is seen through the magnifier;

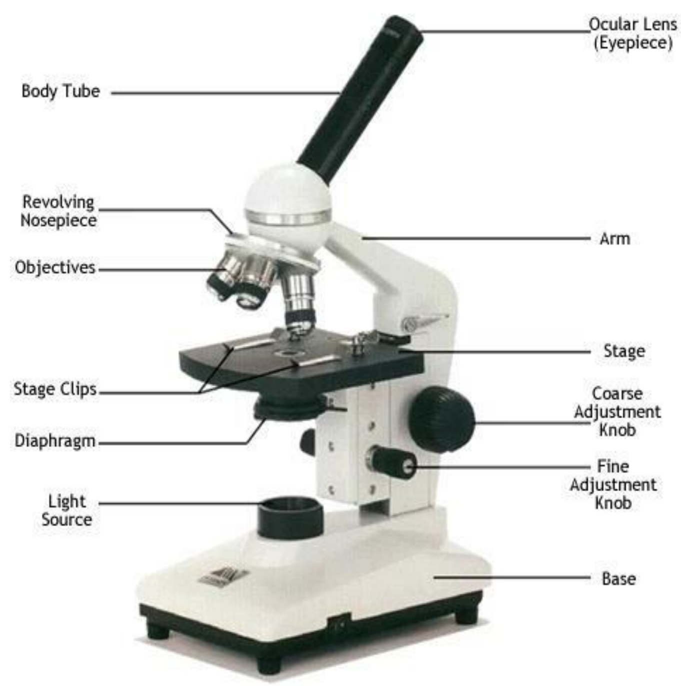
Single lens Camera

Photography is undoubtedly one of the most important inventions in history -- it has truly transformedhowpeople conceiveoftheworld.Nowwecan"see"allsortsofthings thatareactuallymany miles--andyears--awayfromus. Photographyletsuscapturemomentsintimeandpreserve them for years tocome.

The basic technology that makes all of this possible is fairly simple.Astillfilmcameraismadeofthree basicelements: anopticalelement(thelens),achemicalelement(thefilm) and amechanical element(the camera body itself). As we'll see, the only trick to photography is calibrating and combining these elementsinsuchawaythattheyrecorda crisp, recognizableimage.

MICROSCOPE

Thecompoundmicroscopeisausefultoolformagnifyingobjectsuptoasmuchas 1000 times their normal size.Usingthe microscope takes lots of practice.Follow the procedures below both to get the best results and to avoid damaging the equipment.



Parts of the compound microscope

FIG.1.4

- Theeye piece,also called the ocular lens,is a low powerlens.
- Theobjectivelensesofcompoundmicroscopesare**parfocal**. Youdo notneedtorefocus(except forfineadjustment)whenswitchingto ahigherpowerifthe objectisinfocusonalowerpower.
- The field of view is widest on the lowest power objective. When you switch to a higher power,the**field of view** isclosesin.Youwill see more of an object on low power.
- The**depth of focus**isgreatestonthelowestpowerobjective.Each time you switch to a higher power, the depth of focus is reduced. Therefore, a smaller part of the specimen is in focus at higher power.
- The amount of light transmitted to your eye is greatest at the low power. When you switch to a higher power, light (and therefore **resolving power**, or the ability to distinguish two nearby objects as separate) is reduced. Compensate with the light control (sometimes called the**iris diaphragm**).

Depth of Focus

- The depth of focus is greatest on the lowest power objective. Each time you switch to a higherpower,the depthoffocusisreduced.Therefore asmallerpartof the specimen is in focus at higher power. Again, this makes it easier to find an object on low power, and then switch to higher power after it is in focus. A common exercise to demonstrate depth of focus involveslayingthreedifferentcoloredthreadsoneon top of the other. As the observer focuses down,first thetopthreadcomesintofocus,thenthemiddleone, andfinallythebottom one.On higer power objectives onemay gooutoffocusas another comes into focus.
- **Telescope**
- A telescope is an instrument used to see objectsthatarefaraway.Telescopesareoften used to view the planets and stars. Some of the same optical technology that is used in telescopes isalsousedtomakebinocularsand cameras.

FIG.1.5

Construction

- In contrast to a telephoto lens, for any given focal length a simple lens of non-telephoto design is constructed from one lens (which can, to minimize aberrations, consist of several elements to form an achromatic lens). To focus on an object at infinity, the distance from this single lens to focal plane of the camera (where the sensor or film is respectively) has to be adjusted to this focal length. For example, given a focal length of 500 mm, the distance between lens and focal plane is 500 mm. The farther the focal length is increased, the more the physical length of such a simple lens makes it unwieldy.
- But such simple lenses are not telephoto lenses, no matter how extreme the focal length – they are known as *long-focus lenses*.[1] While the optical centre of a simple ("non-telephoto") lens is within the construction, the telephoto lens moves the optical centre in front of the construction. While the length of a long-focus lens approximates its focal length, a telephoto lens manages to be shorter than its focal length. E.g., a telephoto lens might have a focal length of 400mm, while it is shorter than that.
- A telephoto lens works by having the outermost (i.e. light gathering) element of a much shorter focal length than the equivalent long-focus lens and then incorporating a second set of elements close to the film or sensor plane that extend the cone of light so that it appears to have come from a lens of much greater focal length. The basic construction of a telephoto lens consists of front lens elements that, as a group, have a positive focus. The focal length of this group is shorter than the effective focal length of the lens. The converging rays from this group are intercepted by the rear lens group, sometimes called the "telephoto group," which has a negative focus. The simplest telephoto designs could consist of one element in each group, but in practice, more than one element is used in each group to correct for various aberrations. The combination of these two groups produces a lens assembly that is physically shorter than a long-focus lens producing the same image size.

QUESTION BANK

Part A

Delineate Thin Lens and Draw thin lens
Demarcate Thick lens and the role of its thickness
Describe the power of a thick lens and write its expression
Examine the term Critical thickness in lens
Elucidate the necessity of numerical aperture and stop.
Where does the single lens magnifier can be used
Explain the application of single lens camera
Differentiate Thin lens and Thick lens
When the focal length of convex lens is 15 cm and a point source of light is located at 20 cm . Find the position of image point
A radius of curvature of concave lens is 100 cm and refractive indices of two opposite faces are 1.5 and 1.2. The object is located at a distance 30 cm find the position of the image.

PART B

Derive Lens equation for thin lens and mention its applications
Derive focal length for combination of thin lens with neat sketch
Prove $1/f = (\mu -1) [1/R_1 – 1/R_2 + (\mu -1)/R_1R_2 (t/ \mu)$ in thick lens
State and explain the working of compound microscope with neat sketch
Explain the application of combination of lens with neat sketch and necessary expression (i) Telephoto lens and (ii) Telescope lens
Prove $\mu_1/u + \mu_2/v = \mu_2 - \mu_1/R$ by using law of refraction at a spherical refracting surface
Derive the expression of focal length when two thick lens are in combination

References

1. Physics of Light and Optics (Black & White) Book by Justin Peatross and Michael Ware

2. Optics Textbook by Eugene Hecht

3. Fundamentals of Physics II: Electromagnetism, Optics, and Quantum Mechanics Textbook by Ramamurti Shankar

UNIT – II – Lens Aberration – SPH1214

Lens Aberrations

Spherical Aberration

Spherical Aberration is an optical problem that occurs when all incoming light rays end up focusing at different points after passing through a spherical surface. Light rays passing through a lens near its horizontal axis are refracted less than rays closer to the edge or "periphery" of the lens and as a result, end up in different spots across the optical axis. In other words, the parallel light rays of incoming light do not converge at the same point after passing through the lens. Because of this, Spherical Aberration can affect resolution and clarity, making it hard to obtain sharp images. Here is an illustration that shows Spherical Aberration:

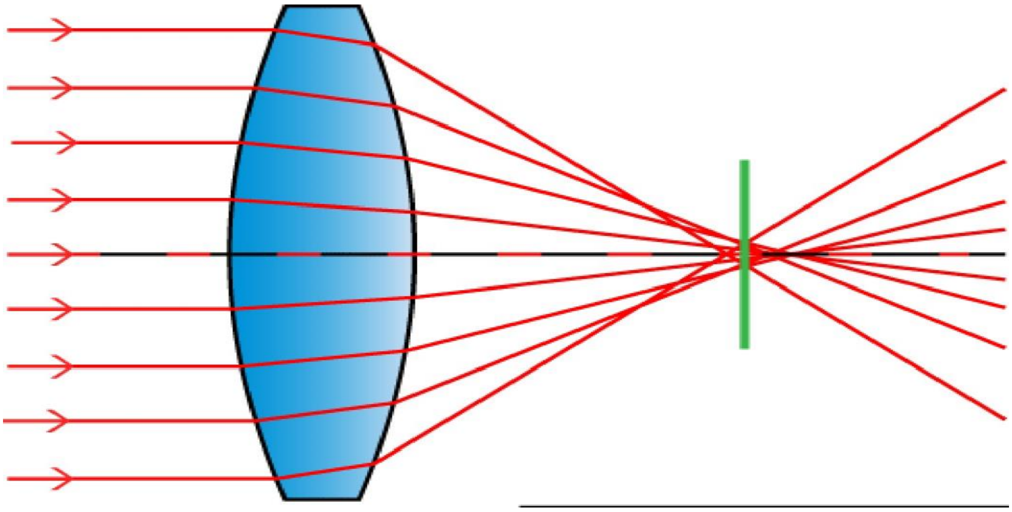


Fig. 2.1

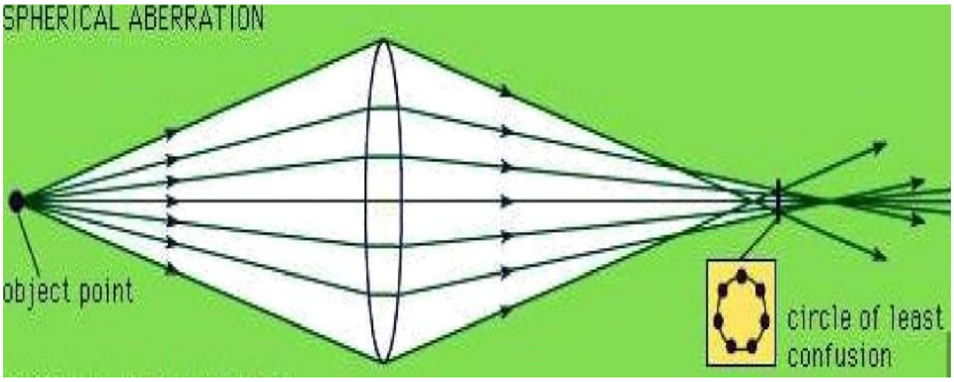


Fig.2.2

Chromatic Aberration

- Chromatic aberration (also known as color fringing or dispersion) is a common problem in lenses which occurs when colors are incorrectly refracted (bent) by the lens, resulting in a mismatch at the focal point where the colors do not combine as they should.
- To help understand this a bit better, remember that the focal plane is your sensor's point of focus; where all the light from your lens should rejoin together to be correctly detected by your

sensor. The thing is, depending on the construction of your lens, your chosen focal length, and even the aperture that you've used, certain wavelengths (colors) may arrive at points before or after where the focal plane sits.

- **Why does chromatic aberration occur?**
- Chromatic aberration happens because your lens acts as a prism; bending light depending on the various properties of the glass, and much like the triangle-shaped one made famous by Pink Floyd, colors passing through it are split at different angles.
- **Avoiding chromatic aberration defects**
  Chromatic aberration can actually be effectively removed in post-processing if you are shooting in RAW. However, good practice states that you should try and remove issues in-camera first, rather than creating more work down the line.
  The good news is that if you are stuck working with a lens that exhibits some form of visible chromatic aberration there are several easy-to-understand strategies which can help you to remove or minimize the visible effect of it in your photos.

Coma

Coma is an aberration which causes rays from an off-axis point of light in the object plane to create a trailing "comet-like" blur directed away from the optic axis. A lens with considerable coma may produce a sharp image in the center of the field, but become increasingly blurred toward the edges. For a single lens, coma can be partially corrected by bending the lens. More complete correction can be achieved by using a combination of lenses symmetric about a central stop.

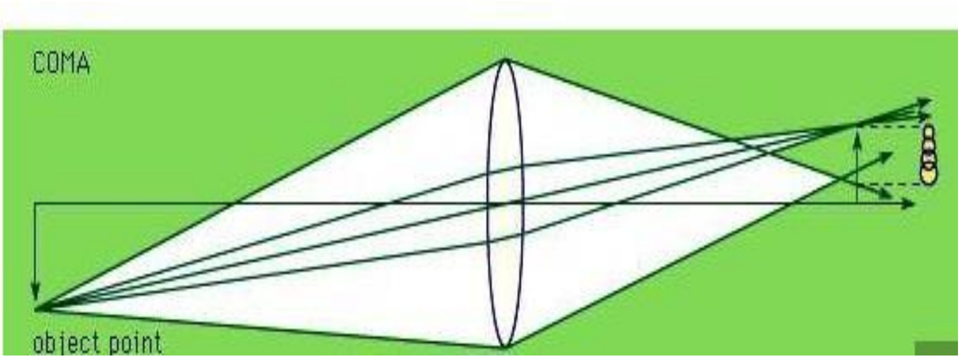


Fig. 2.3

Astigmatism

- Unlike spherical aberration and coma, results from the failure of a single zone of a lens to focus the image of an off-axis point at a single point. As shown in the three-dimensional schematic the two planes at right angles to one another passing through the optical axis are the meridian plane and the sagittal plane, the meridian plane being the one containing the off-axis object point. Rays not in the meridian plane, called skew rays, are focused farther away from the lens than those lying in the plane. In either case the rays do not meet in a point focus but as lines perpendicular to each other. Intermediate between these two positions the images are elliptical in shape.

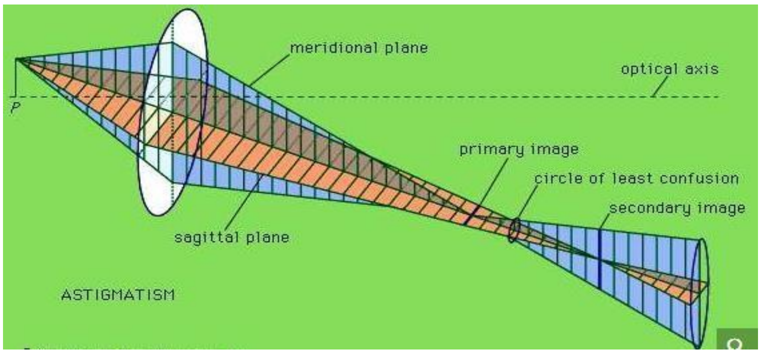


Fig. 2.4

Curvature of Field

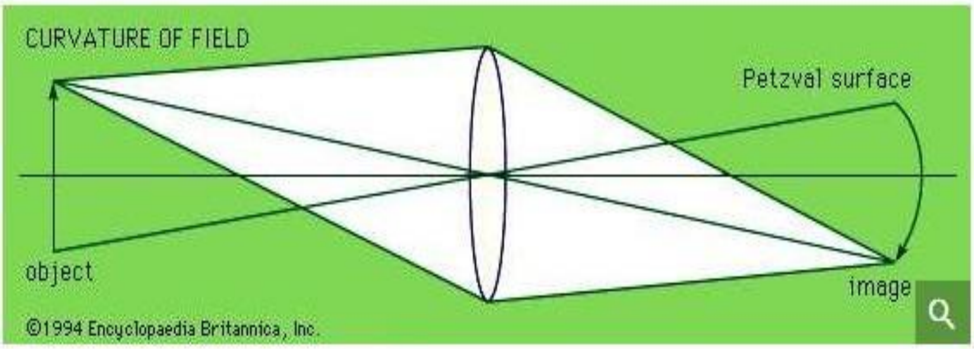


Fig.2.5

Curvature of field and distortion refer to the location of image points with respect to one another. Even though the former three aberrations may be corrected for in the design of a lens, these two aberrations could remain. In curvature of field, the image of a plane object perpendicular to the optical axis will lie on a paraboloidal surface called the Petzval surface (after József Petzval, a Hungarian mathematician). Flat image fields are desirable in photography in order to match the film plane and projection when the enlarging paper or projection screen lie on a flat surface. Distortion refers to deformation of an image. There are two kinds of distortion, either of which may be present in a lens: barrel distortion, in which magnification decreases with distance from the axis, and pincushion distortion, in which magnification increases with distance from the axis.

Question Bank

Part A

Compare Chromatic and Achromatic aberration
Differentiate longitudinal and lateral chromatic aberration
How to reduce the chromatic aberration by using lens
Describe monochromatic aberration and how they are classified
Discuss some remedies to reduce the monochromatic aberration
Compare coma and astigmatism
Designate the term of crossed lens
Mention the importance of aplanatic lens
In which chromatic defect Circle of least confusion is appeared and it can be minimized
Compare Ramsden and Huygen's eyepeiece
State Merits and Demerits of Huygen's eyepeiece

Part B

Explicate Achromatism and Explain the conditions of achromatism (i) when two lens are in contact (ii) When two lens are kept at a distance

State and explain Spherical aberration and list how the remedies with neat sketch

(i) State and explain COMA and list how the remedies with neat sketch
(ii) State and explain Astigmatism and list how the remedies with neat sketch

Describe Construction and working mechanism of Huygen's eye piece with neat diagram
Explain Construction and working mechanism of Ramsden's eye piece with neat diagram

References

1.Aberrations, Donald Dilworth ,Published July 2018 • Copyright © IOP Publishing Ltd 2018
2. Handbook of Optical Systems: Vol. 3. Aberration Theory and Correction of Optical Systems. Edited by Herbert Gross Copyright © 2007 WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim ISBN: 978-3-527-40379-0

UNIT – II – Light Waves– SPH1214

LIGHT WAVES

Maxwell's Equations

Maxwell's four equations describe the electric and magnetic fields arising from distributions of electric charges and currents, and how those fields change in time. They were the mathematical distillation of decades of experimental observations of the electric and magnetic effects of charges and currents, plus the profound intuition of Michael Faraday. Maxwell's own contribution to these equations is just the last term of the last equation—but the addition of that term had dramatic consequences. It made evident for the first time that varying electric and magnetic fields could feed off each other—these fields could propagate indefinitely through space, far from the varying charges and currents where they originated. Previously these fields had been envisioned as tethered to the charges and currents giving rise to them. Maxwell's new term (called the displacement current) freed them to move through space in a self-sustaining fashion, and even predicted their velocity—it was the velocity of light!

1. Gauss' Law for electric fields:

$$\int \vec{E} \cdot d\vec{A} = q/\varepsilon_0.$$

(The integral of the outgoing electric field over an area enclosing a volume equals the total charge inside, in appropriate units.)

2. The corresponding formula for magnetic fields:

$$\int \vec{B} \cdot d\vec{A} = 0.$$

(No magnetic charge exists: no "monopoles".)

3. Faraday's Law of Magnetic Induction:

$$\oint \vec{E} \cdot d\vec{\ell} = -d/dt\left(\int \vec{B} \cdot d\vec{A}\right).$$

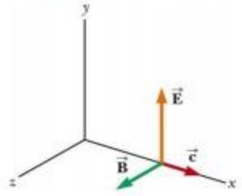
The first term is integrated round a closed line, usually a wire, and gives the total voltage change around the circuit, which is generated by a varying magnetic field threading through the circuit.

4. Ampere's Law plus Maxwell's displacement current:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0\left(I + \frac{d}{dt}\left(\varepsilon_0 \int \vec{E} \cdot d\vec{A}\right)\right).$$

Plane wave equation-Homogenous and inhomogenous waves

Assume that the electric and magnetic fields are constrained to the y and z directions, respectfully, and that they are both functions of only x and t. This will result in a linearly polarized plane wave travelling in the x direction at the speed of light c.



$$\vec{E}(x,t) = E(x,t)\hat{j} \text{ and } \vec{B}(x,t) = B(x,t)\hat{k}$$

We will derive the wave equation from Maxwell's Equations in free space where I and Q are both zero.

$$\nabla \cdot E = 0 \qquad \nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times B = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$$

Start with Faraday's Law. Take the curl of the E field: $\nabla \times E(x,t)\hat{j} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E(x,t) & 0 \end{vmatrix} = \frac{\partial E}{\partial x}\hat{k}$

Equating magnitudes in Faraday's Law: $\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$ (1)

This means that the spatial variation of the electric field gives rise to a time-varying magnetic field, and visa-versa. In a similar fashion we derive a second equation from Ampere-Maxwell's Law:

Take the curl of B: $\nabla \times B(x,t)\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & B(x,t) \end{vmatrix} = -\frac{\partial B}{\partial x}\hat{j} \rightarrow \frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$ (2)

This means that the spatial variation of the magnetic field gives rise to a time-varying electric field, and visa-versa.

Take the partial derivative of equation (1) with respect to x and combining the results from (2):

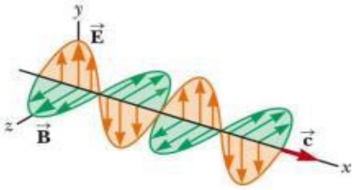
$$\frac{\partial^2 E}{\partial x^2} = -\frac{\partial}{\partial x}\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}\frac{\partial B}{\partial x} = -\frac{\partial}{\partial t}\left(-\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\boxed{\frac{\partial^2 E}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2}} \quad (3)$$

In a similar manner, we can derive a second equation for the magnetic field: $\boxed{\frac{\partial^2 B}{\partial x^2} = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2}}$ (4)

Both equations (3) and (4) have the form of the general wave equation for a wave $\psi(x,t)$ traveling in the x direction with speed v: $\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$. Equating the speed with the coefficients on (3) and (4) we derive the speed of electric and magnetic waves, which is a constant that we symbolize with "c":

$$\boxed{c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 2.997x10^8 m/s}$$



The simplest solutions to the differential equations (3) and (4) are sinusoidal wave functions:

$$E(x,t) = E_{\max}\cos(kx - \omega t) \quad (5)$$
$$B(x,t) = B_{\max}\cos(kx - \omega t) \quad (6)$$

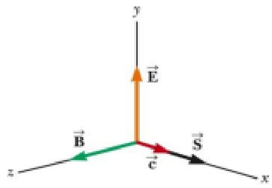
where k = 2π/λ is the wavenumber , ω = $2\pi f$ is the angular frequency, λ is the wavelength, f is the frequency and $\omega / k = \lambda f = v = c.$. Taking partial derivatives of E and B and substituting into (1):

$$\frac{\partial E}{\partial x} = -kE_{\max}\sin(kx - \omega t) \quad \text{and} \quad \frac{\partial B}{\partial t} = \omega B_{\max}\sin(kx - \omega t)$$

$$-kE_{\max}\sin(kx - \omega t) = -\omega B_{\max}\sin(kx - \omega t)$$

$$E_{\max} = \frac{\omega}{k}B_{\max} = cB_{\max}$$

$$\boxed{\frac{E}{B} = c.}$$



At every instant, the ratio of the electric field to the magnetic field in an electromagnetic wave equals the speed of light. The rate of energy transfer by an electromagnetic wave is described by the *Poynting* vector, **S**, defined as the rate at which energy passes through a unit surface area perpendicular to the direction of wave propagation (W/m^2): $\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$. For a plane electromagnetic wave:

$S = \frac{EB}{\mu_0} = \frac{E^2}{\mu_0 c} = \frac{cB^2}{\mu_0}$. The time average of S over one or more cycles is called the wave intensity, *I*, which gives the very important result that the intensity of a light wave is proportional to the square of the amplitude of the electric or magnetic fields:

$$\boxed{I = S_{ave} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{B_{\max}^2}{2c\mu_0}}$$

(The ½ comes from averaging the cosine squared terms.) Intensity is a measureable "real" quantity that has primary significance in all wave phenomena. Intensity is the loudness of a sound or the brightness of a light. Intensity will serve as an important conceptual analog in our study of quantum physics in the following way: "Intensity is to the electric field as probability is to the wave function." Yum!

Cylindrical wave equation

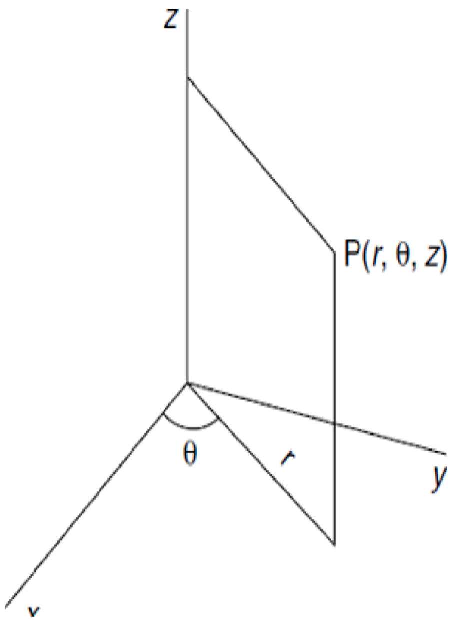
$$\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2\psi}{\partial t^2}.$$

Solution

We shall solve by direct substitution.

We have y = r Sin θ, z = z and $r^2 = x^2 + y^2$, θ = $\tan^{-1}$ (y/x). The following solution is lengthy. So we use subscripts to denote partial derivatives and write a = Sin θ = y/r and b = Cos θ = x/r.

We shall require the derivatives:



$$\partial r/\partial x = r_x = x/r = \cos\theta = b,$$
$$\partial r/\partial y = r_y = y/r = \sin\theta = a;$$
$$\partial\theta/\partial x = \theta_x = \left[\frac{\partial}{\partial x}(y/x)\right]/[1+(y/x)^2] = -\frac{y}{x^2}\left(\frac{1}{1+(\frac{y}{x})^2}\right)$$
$$= -(\sin\theta)/r = -a/r,$$
$$\partial\theta/\partial y = \theta_y = \left[\frac{\partial}{\partial y}(y/x)\right]/[1+(y/x)^2] = \frac{1}{x}\left(\frac{1}{1+(\frac{y}{x})^2}\right)$$
$$= (\cos\theta)/r = b/r.$$

To replace ψ_{xx} and ψ_{yy} with derivatives with respect to r and θ, we write:

$$\psi_x = \psi_r r_x + \psi_\theta \theta_x = \psi_r b - \psi_\theta a/r,$$
$$\psi_y = \psi_r r_y + \psi_\theta \theta_y = \psi_r a + \psi_\theta b/r.$$

Then,

$$\psi_{xx} = \frac{\partial}{\partial r}(\psi_r b - \psi_\theta a/r)\, r_x + \frac{\partial}{\partial\theta}(\psi_r b - \psi_\theta a/r)\,\theta_x$$
$$= \left(\psi_{rr} b - \psi_{r\theta} a/r + \psi_\theta a/r^2\right) b$$
$$+ \left(\psi_{r\theta} b - \psi_r a - \psi_{\theta\theta} a/r - \psi_\theta b/r\right)(-a/r)$$
$$= \left[\psi_{rr} b^2 - \psi_{r\theta}(2ab/r) + \psi_\theta\left(2ab/r^2\right) + \psi_r\left(a^2/r\right) + \psi_{\theta\theta}(a/r)^2\right],$$

$$\psi_{yy} = \frac{\partial}{\partial r}\left(\psi_r a + \psi_\theta b/r\right) r_y + \frac{\partial}{\partial \theta}\left(\psi_r a + \psi_\theta b/r\right)\theta_y$$
$$= \left(\psi_{rr} a + \psi_{r\theta} b/r - \psi_\theta, b/r^2\right) a$$
$$+ \left(\psi_{r\theta} a + \psi_r b + \psi_{\theta\theta} b/r - \psi_\theta a/r\right)(b/r)$$
$$= \left(\psi_{rr} a^2 + \psi_{r\theta} 2ab/r - \psi_\theta 2ab/r^2\right) + \left(\psi_r b^2/r + \psi_{\theta\theta} b^2 r^2\right).$$

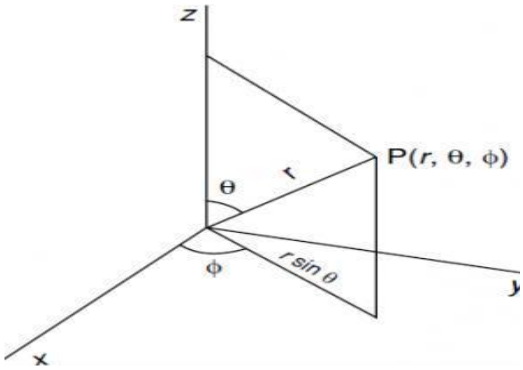
Thus

$$\nabla^2 \psi = \psi_{rr}\left(a^2 + b^2\right) + \psi_r\left(a^2 + b^2\right)/r + \psi_{\theta\theta}\left(a^2 + b^2\right)/r^2 + \psi_{zz},$$

so

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r}\frac{\partial \psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \psi}{\partial \theta^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}.$$

Spherical wave equation



Transform the wave equation into spherical coordinates (see Figure 2.6b), showing that it be

$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial \psi}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial \psi}{\partial \theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 \psi}{\partial \phi^2}\right] = \frac{1}{V^2}\frac{\partial^2 \psi}{\partial t^2}.$$

Solution

Spherical coordinates $(r,\ \theta,\ \phi)$ and rectangular coordinates are related as follows (see Figu

$$\begin{aligned} x &= r\sin\theta\cos\phi, \\ y &= r\sin\theta\sin\phi, \\ z &= r\cos\theta, \\ r &= \left(x^2 + y^2 + z^2\right)^{1/2}, \\ \theta &= \cos^{-1}(z/r), \\ \phi &= \tan^{-1}(y/x). \end{aligned}$$

$$
\begin{aligned}
a &= \sin\theta, & b &= \cos\theta,\\
m &= \sin\phi, & n &= \cos\phi,\\
a_\theta &= b, & b_\theta &= -a,\\
m_\phi &= n, & n_\phi &= -m.
\end{aligned}
$$

The derivatives of r, θ, and ϕ now become:

$$r_x = x/r = an,$$

$$
\begin{aligned}
&r_y = y/r = am,\\
&r_z = z/r = b;\\
&\theta_x = \frac{\partial}{\partial x}\left[\cos^{-1}(z/r)\right] = -\frac{\partial}{\partial r}(z/r)\left[1-(z/r)^2\right]^{-1/2} = \frac{Z}{r^2}r_x(1/a) = bn/r,\\
&\theta_y = -\frac{\partial}{\partial y}(z/r)\left[1-(z/r)^2\right]^{-1/2} = -\frac{Z}{r^2}r_y(1/a) = bm/r,\\
&\theta_z = -\frac{\partial}{\partial z}(z/r)\left[1-(z/r)^2\right]^{-1/2} = -\left(1/r - z/r^2 r_z\right)\left[1-(z/r)^2\right]^{-1/2}\\
&\quad = (-1/r)(1-zb/r)(1/a) = (-1/r)\left(1-b^2\right)(1/a) = -a/r,\\
&\phi_x = \frac{\partial}{\partial x}\left[\tan^{-1}(y/x)\right] = \frac{\partial}{\partial x}(y/x)\left[1+(y/x)^2\right]^{-1} = -\left(y/x^2\right)\left[1+(y/x)^2\right]^{-1}\\
&\quad = -(y/x)(1/x)\left[1+(y/x)^2\right]^{-1} = -(m/n)(1/ran)\left(1/n^2\right)^{-1} = -m/ar,\\
&\phi_y = \frac{\partial}{\partial y}(y/x)\left[1+(y/x)^2\right]^{-1} = (1/x)\left[1+(y/x)^2\right]^{-1} = (1/ran)\left(1/n^2\right)^{-1} = n/ar,\\
&\phi_z = 0 \text{ (because } \phi \text{ is not a function of } z).
\end{aligned}
$$

Summarizing these results, we have

$$
\begin{aligned}
r_x &= an, & r_y &= am, & r_z &= b,\\
\theta_x &= bn/r, & \theta_y &= bm/r, & \theta_z &= -a/r,\\
\phi_x &= -m/ar, & \phi_y &= n/ar, & \phi_z &= 0.
\end{aligned}
$$

We now calculate the derivatives ϕ_{xx}, etc.:

$$
\begin{aligned}
\psi_x &= \psi_r r_x + \psi_\theta \theta_x + \psi_\phi \phi_x \\
&= \psi_r an + \psi_\theta bn/r - \psi_\phi m/ar; \\
\psi_{xx} &= \frac{\partial}{\partial r}\left(\psi_r an + \psi_\theta bn/r - \psi_\phi m/ar\right)(an) \\
&\quad + \frac{\partial}{\partial \theta}\left(\psi_r an + \psi_\theta bn/r - \psi_\phi m/ar\right)(bn/r) \\
&\quad + \frac{\partial}{\partial \theta}\left(\psi_r an + \psi_\theta bn/r - \psi_\phi m/ar\right)(-m/ar), \\
&= \left(\psi_{rr} an + \psi_{r\theta} bn/r - \psi_\theta bn/r^2 - \psi_{\gamma\phi} m/ar + \psi_\phi m/ar^2\right) an \\
&\quad + \left(\psi_{r\theta} an + \psi_r bn + \psi_{\theta\theta} bn/r - \psi_\theta an/r - \psi_{\theta\phi} m/ar \right.\\
&\quad \left. + \psi_\phi mb/a^2 r\right)(bn/r) + \left(\psi_{\gamma\phi} an - \psi_r am + \psi_{\theta\phi} bn/r \right.\\
&\quad \left. - \psi_\theta bm/r - \psi_{\phi\phi} m/ar - \psi_\phi n/ar\right)(-m/ar) \\
&= \psi_{rr} a^2 n^2 + \psi_{r\theta}\left(2abn^2/r\right) - \psi_{\gamma\phi}\left(2mn/r\right) + \psi_r\left(\frac{b^2 n^2 + m^2}{r}\right) \\
&\quad + \psi_{\theta\theta}\left(\frac{bn}{r}\right)^2 + \psi_{\theta\phi}\left(\frac{b}{a}\right)\left(\frac{2mn}{r}\right) + \psi_\theta\left(\frac{bm^2}{ar^2} - \frac{2abn}{r^2}\right) \\
&\quad + \psi_{\phi\phi}\left(\frac{m}{ar}\right)^2 + \psi_\phi\left(\frac{2mn}{a^2 r^2}\right); \\
\psi_y &= \psi_r r_y + \psi_\theta \theta_y + \psi_\phi \phi_y \\
&= \psi_r am + \psi_\theta bm/r + \psi_\phi n/ar; \\
\psi_{yy} &= \frac{\partial}{\partial r}\left(\psi_r am + \psi_\theta bm/r + \psi_\phi n/ar\right)(am) \\
&\quad + \frac{\partial}{\partial \theta}\left(\psi_r am + \psi_\theta bm/r + \psi_\phi n/ar\right)(bm/r)
\end{aligned}
$$

Charge density and Poynting Vector

The plane wave equation solutions are,

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

$$\mathbf{B}(\mathbf{r},t) = \frac{1}{c}\left(\hat{\mathbf{k}} \times \mathbf{E}_0\right)\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)$$

$$u(\mathbf{r},t) = \frac{1}{c}\sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)\right]^2,$$

and

$$\mathbf{S}(\mathbf{r},t) = \sqrt{\frac{\varepsilon_0}{\mu_0}}\left[E_0 \cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi)\right]^2 \hat{\mathbf{k}},$$

Question Bank

Part A

Examine how the wave equation varied for spherical and cylindrical solution
Define phase velocity
Compare homogenous and inhomogenous waves
Analyze the term of dispersion
Write a short note on Cauchy's constant
According to Cauchy, state the relation between dispersive power and wavelength
Write differential and integral form of Maxwell equations
State few significance of Maxwell's equations
Define charge density and state the relation between Electric field and Magnetic field
Analyze the relation between charge density and Poynting vector
What are the dependent factors of Poynting vector

Part B

When a wave front is moving in a plane wave, derive the solution for a plane wave

Derive the expression for a plane wave solution for a homogenous and in homogenous medium

State and derive Cauchy's dispersion formula and explain how the Cauchy's constants can be obtained from the λ and μ relation

State Charge density and derive the expression for Poynting vector with respect to charge density

References

1. Wave Physics, Stephen Nettel, Springer, 2003
2. Physics of Oscillations and Waves With use of Matlab and Python, Arnt Inge Vistnes, Springer, 2018

UNIT – IV – Interference – SPH1214

Interference

Air Wedge

An air wedge is one of the simplest designs of shearing interferometers used to visualize the disturbance of the wave front after propagation through a test object. An air wedge can be used with nearly any light source, including non-coherent white light. The thickness of the film gradually increases from O to X. Such a film of non-uniform thickness is known as wedge shaped film. The point O at which the thickness is zero is known as the edge of the wedge. The angle θ between the surfaces OX and OX′ is known as the angle of wedge.

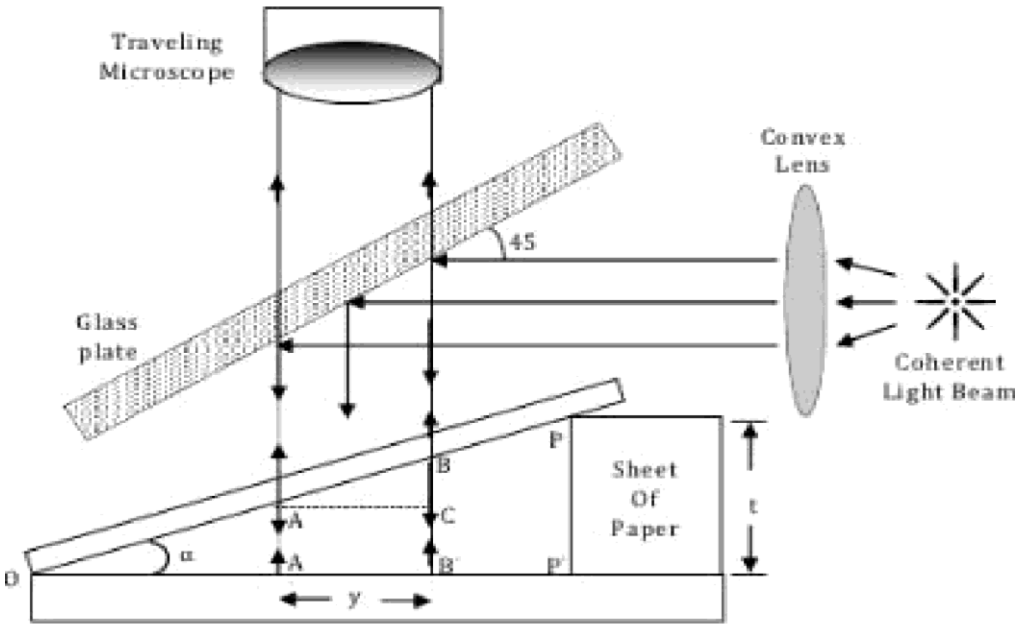


Fig.4.1

The principle of air-wedge method

To get such interference fringes the phase difference between the set of two rays is /2. alternative light and dark fringes is /2. To get bright and dark fringes the path difference between the rays must equal /2. An air wedge is formed by placing a sheet of foil between the edges of two glass plates 75 mm from their point of contact. When the wedge is illuminated with light of wavelength 5.8 x 10-7 m the fringes are 1.30 mm apart

In wedge shaped film the thickness of the air is constant over a straight line along the width of the wedge. Hence the fringes are straight . ... Because the thickness of the wedge-shaped film along certain lines is constant and so the path difference for the light passing through the points of one such line is the same.

The angle between the incoming ray and the **glass plate** is **45 degree** to make turns the light rays to 90 **degrees** and that's why the rays fall normally on the plano-convex lens. Finally forms circular rings.

Haidinger fringe

Haidinger fringes are interference fringes formed by the interference of monochromatic and coherent light to form visible dark and bright fringes. Fringe localization is the region of space where fringes with reasonably good contrast are observed.

Haidinger fringes are fringes localized at infinity. Also known as fringes of equal inclination, these fringes result when light from an extended source falls on a thin film made of an optically denser medium. These fringes indicate the positions where light interferes, emerging from the medium at an equal angle .

They are also observed in Fabry-Pérot and Michelson interferometers. They can be observed by introducing a converging lens between the film and observation plane with focus of the lens lying in observation plane.

Michelson's Interferometer

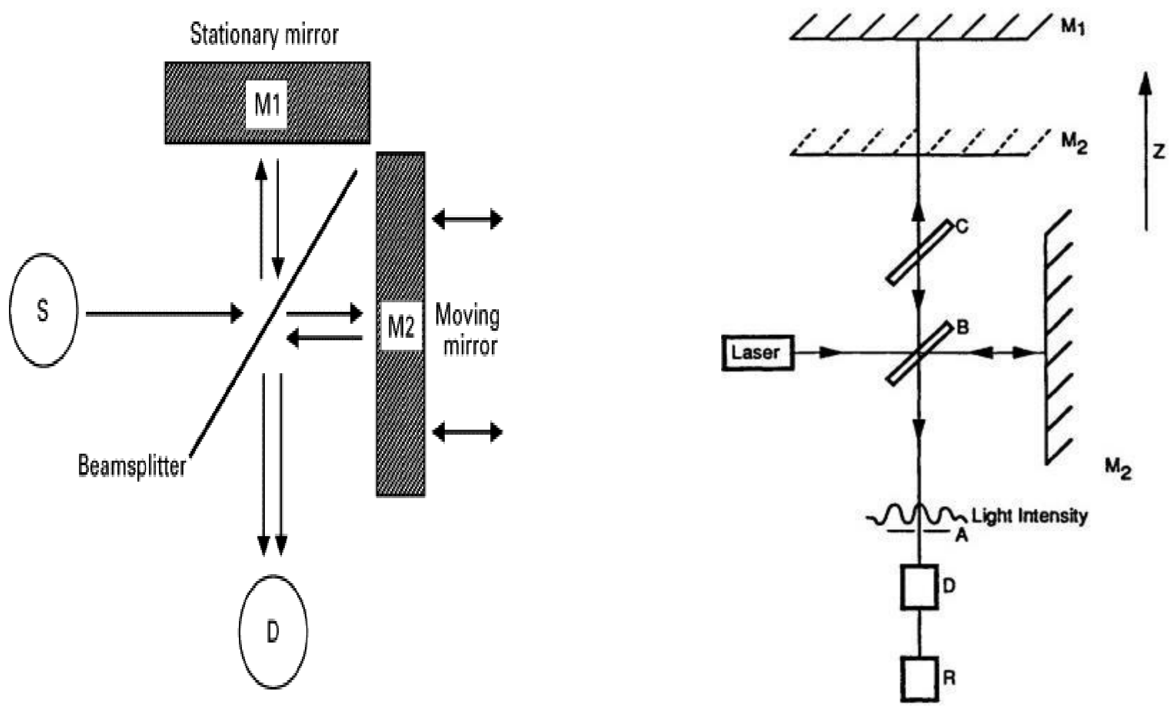


Fig.4.2

Procedure

The Michelson interferometer consists of two mirrors, M1 and M2, arranged as shown in Figure , with a beamsplitter inclined at 45° to the mirrors. The collimated beam of laser light is incident on the beamsplitter, and it is divided into two beams when it strikes the partially reflecting surface on the beamsplitter.

Part of the light follows the path to mirror M1, is reflected by M1, and retraces the path to the beamsplitter. Some of this light will pass through the semitransparent beamsplitter. Another light ray originally passes through the beamsplitter to mirror M2, is reflected back to the beamsplitter, and is partially reflected by the surface of the beamsplitter, so as to be superimposed on the first ray. These two beams contain only part of the original light, the rest having been lost by reflection or transmission at the beamsplitter and returned back toward the laser. A

compensating plate, which has the same thickness and is made of the same glass as the beamsplitter, is sometimes placed in the path of the first ray. The first ray traverses the beamsplitter only once, whereas the second ray traverses the beamsplitter three times. Insertion of the compensating plate provides for equal paths in glass.

The phase difference δ between the two beams is

$$\delta = 2\pi \frac{2S\cos\theta}{\lambda} \qquad (9.1)$$

The formation of fringes is a result of the wave nature of light. A light wave moving in the negative *z* direction may be described by the function

$$E(z,\ t) = E_0\ \exp\ i(kz + \omega t + \phi) \qquad (9.2)$$

where E_0 represents the amplitude of the wave, ω the angular frequency, and $k = 2\pi/\lambda$, with λ the wavelength. The function φ represents the phase of the wave, and it is generally a function of position and time. For monochromatic and coherent laser light, we may consider φ as a constant.

Brewster Fringe

David Brewster (1781-1868) is perhaps best known for his invention of the Kaleidoscope, about which he wrote a treatise published in 1819. In the first part of his scientific career he did a good deal of research in optics, but later turned to writing and editing of scientific works as his main source of income. In 1815 he observed the fringes produced when two thick, parallel-sided plates of glass of identical thickness were placed close to each other. The plates are located at one end of a brass tube that has a narrow slit at the other end; the inside of the tube is blackened to prevent reflections. The two plates can be seen at the end of the apparatus shown at the right. An adjusting screw for changing the angle of inclination of the two plates can be seen. The theory of the apparatus may be found in Thomas Preston, The Theory of Light, 2nd ed (London, Macmillan Co., 1895) pp 194-197.

This apparatus is in the collection of the United States Military Academy at West Point, New York, and was purchased from Duboscq of Paris about 1844. The brass tube is 30 cm in length.

Fig.4.3

Application:

Refractive index of gases and liquids:

The instrument has a measuring accuracy of 1/100th wavelength and permits differences of refractive index in liquids to be determined to 1 × 10-7 and in gases to 1 × 10-8. Measurements are made visually.

Disadvantages:

Each apparatus to be individually calibrated with mercury light; Taking outside readings on the drum was cumbersome. There were also short comings in temperature control. The most troublesome factor was the lack of brightness, which is a source of difficulty when photoelectric adjustments and measurements aremade.

Jamin Interferrometer

The Jamin interferometer is a type of interferometer, related to the Mach– Zehnder interferometer. It was developed in 1856 by the French physicist Jules Jamin.

The interferometer consists of two mirrors, made of the thickest glass possible. The Fresnel reflection from the first surface of the mirror acts as a beam splitter. The incident light is split into two rays, parallel to each other and displaced by an amount depending on the thickness of the mirror. The rays are recombined at the second mirror and ultimately imaged onto a screen.

If a phase-shifting element is added to one arm of the interferometer, then the displacement it causes can be determined by simply counting the interference fringes, i.e., the minima.
The Jamin interferometer allows very exact measurements of the refractive index and dispersion of gases; a transparent pressure chamber can be positioned in the instrument. The phase shift due to changes in pressure is quite easy to measure.

Fig.4.4

Rayleigh interferometer

The Rayleigh interferometer employs two beams of light from a single source, and determines the difference in optical path length between the two paths using interference between the two beams when they are recombined following traversal of the paths. An example is shown in the figure.[1] Light from a source (top) is collimated by a lens and split into two beams using slits. The beams are sent through two different paths and pass through compensating plates. They are brought to a focus by a second lens (bottom) where an interference pattern is observed to determine the optical path difference in terms of wavelengths of the light.

The advantage of the Rayleigh interferometer is its simple construction. Its drawbacks are (i) it requires a point or line source of light for good fringe visibility, and (ii) the fringes must be viewed with high magnification

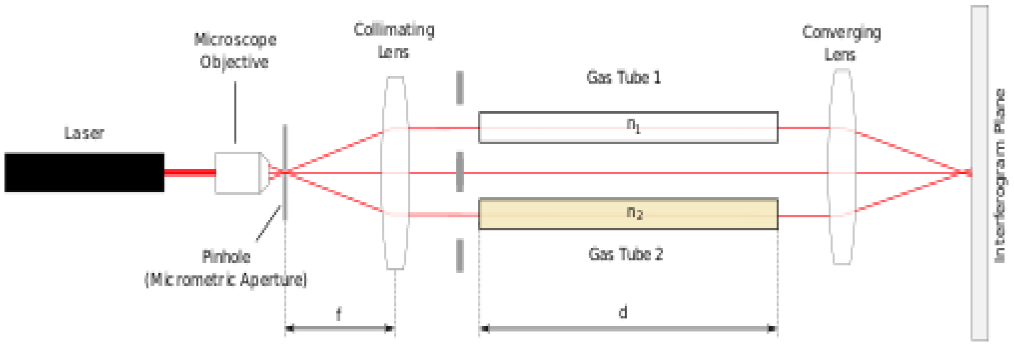


Fig.4.5

Holography

Holography is based on the principle of interference. A hologram captures the interference pattern between two or more beams of coherent light (i.e. laser light). One beam is shone directly on the recording medium and acts as a reference to the light scattered from the illuminated scene. The hologram captures light as it interests the whole area of the film, hence being described as a ‘window with memory’. By contrast a photograph captures a single small area ‘aperture’ of perspective, the photographic image being created by focusing this light onto film or a digital sensor.The physical medium of holographic film is photo-sensitive with a fine grain structure. Common materials used are silver-halide emulsions, dichromate gelatins and photopolymers – each having their own characteristics and require different processing.

Holograms can also be embossed ‘stamped into a foil’ with applications including in security identification, such as on passports, credit cards, tickets and packaging, as they are difficult to copy without the master hologram. The hologram is the recorded interference pattern of constructive (intensity peaks) and destructive (elimination) of the superimposed light-wave fronts (the electromagnetic field). By using a coherent laser light-source and a stable geometry (or short ‘pulse’ duration) the interference pattern is stationary and can be recorded into the hologram’s photosensitive emulsion. The hologram is then chemically processed so that the emulsion has a modulated density, freezing the interference pattern into ‘fringes’.

Question bank

Part A

State the Principle of Michelson’s interferrometer
How to find good optical flatness of the lens
Write a brief note on Brewster fringes
Analyze the principle of Holography
Scrutinize the principle of colour photography
Examine the cause of destructive and constructive interference in thin films
How to find wavelength of light by using Air wedge
Mention some salient features of interference patterns
Write few advantages and disadvantages of Rayleigh interferrometer
Write some applications of Michelson interferrometer
Write a short note on Haidinger fringe
Define interference and mention its applications

Part B

Explain the construction and working of Jamin and Rayleigh interferometer with neat diagram

Explain the construction and working of Micheson’s interferometer with neat diagram

Apply Michelson interferometer to find (i) resolution of spectral line and (ii) thickness of thin transparent sheet

How to determine the path difference in Haidinger fringes

Explain the determination of wavelength of light by using Air wedge

References

1. Interference of Light Waves by Oleg Marchenko, Sergei Kazantsev and Laurentius Windholz, Springer, 2007

UNIT – V – Polarisation and Diffraction – SPH1214

Polarisation and Diffraction

Huygens'Theory of Double Refraction

This is the Phenomenon of double refraction was explained by Huygens' for which he extended his principle of secondary wavelets and made the following assumptions.

• When a light wave strikes the surface of a doubly refracting crystal, each point of the crystal becomes the origin of two secondary wavelets, named as ordinary ray and extraordinary ray. These two wavelets spread out into the crystal.

• The wave front corresponding to ordinary ray is spherical as the velocity of ordinary ray remains the same in all the directions.

The wave front corresponding to extraordinary ray is an ellipsoid of revolution with the optic axis as its axis of revolution. This is due to the fact that the velocity of E-ray is different in different directions in the crystal.

• The two wave fronts corresponding to O-ray and E-ray touch each other along the optic axis since both the rays travel with the same velocity along the direction of optic axis.

For negative uniaxial crystals (like calcite) in which the velocity of O-ray is less than the velocity of E-ray, sphere lies inside the ellipsoid. However, for positive uniaxial crystals (like quartz) the ellipsoid lies inside the sphere since in this case the velocity of O-ray is greater than the velocity of E-ray

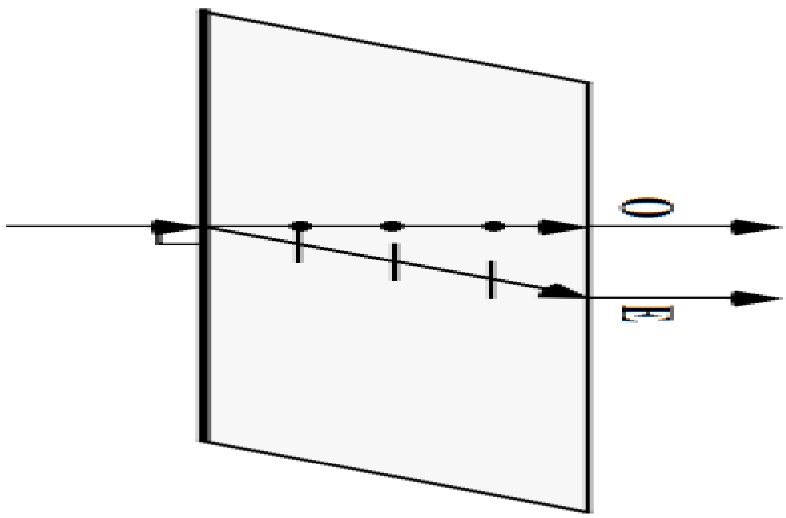


Fig.5.1

NICOL PRISM

Nicol prism is an optical device which is used for producing and analyzing plane polarized light in practice.

Principle

Nicol Prism is based upon phenomenon of Double refraction.

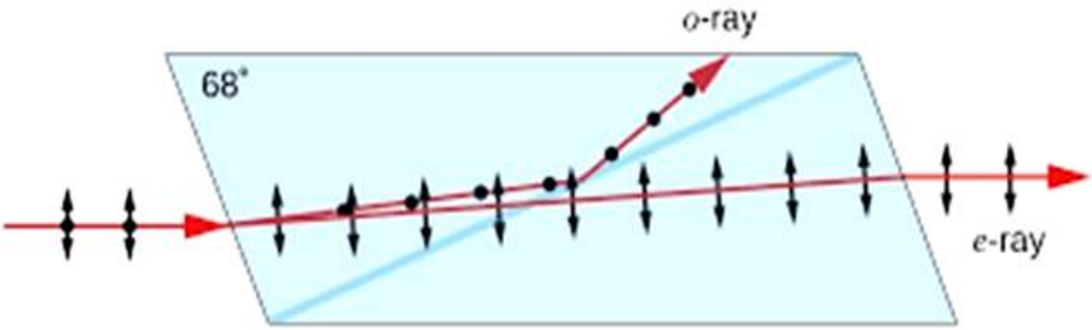


Fig.5.2

Construction

It is constructed from the calcite crystal PQRS having length three times of its width. • Its end faces PQ and RS are cut such that the angles in the principal section become 68° and 112° in place of 71° and 109° • The crystal is then cut diagonally into two parts. The surfaces of these parts are grinded to make optically flat and then these are polished. • Thus polished surfaces are connected together with a special cement known as Canada Balsam.

Working

When a beam of unpolarised light is incident on the face P′Q, it gets split into two refracted rays, named O-ray and E-ray. • These two rays are plane polarised rays, whose vibrations are at right angles to each other. The refractive index of Canada balsam cement being 1.55 lies between those of ordinary and extraordinary and 1.4864, respectively. • It is clear from the above discussion that Canada Balsam layer acts as an optically rarer medium for the ordinary ray and it acts as an optically denser medium for the extraordinary ray. • When ordinary ray of light travels in the calcite crystal and enters the Canada balsam cement layer, it passes from denser to rarer medium. Moreover, the angle of incidence is greater than the critical angle, the incident ray is totally internally reflected from the crystal and only extraordinary ray is transmitted through the prism. • Therefore, fully plane polarised wave is generated with the help of Nicol prism

Nicol Prism as Polariser and Analyser

In order to produceand analysethe planepolarized light,we arrangetwo nicolprisms.

- When a beam of unpolarised light is incident on the nicol prism, emergent beam from the prism is obtained as plane polarised, and which has vibrations parallel to the principal section. • This prism is therefore known as polariser. If this polarised beam falls on another parallel nicol prism P2, whose principal section is parallel to that of P1, then the incident beam will behave as Eray inside the nicol prism P2 and gets completely transmitted through it. • This way the intensity of emergent light will be maximum.

- Now the nicol prism P2 is rotated about its axis, then we note that the intensity of emrging light decreases and becomes zero at 90° rotation of the second prism (Fig. b). • In this position, the vibrations of E-ray become perpendicular to the principal section of the analyser (nicol prism P2). • Hence, this ray behaves as O-ray for prism P2 and it is totally internally reflected by Canada balsam layer. This fact can be used for detecting the plane polarised light and the nicol prism P2 acts as an analyser. • If the nicol prism P2 is further rotated about its axis, the intensity of the light emerging from it increases and becomes maximum for the position when principal section of P2 is again parallel to that of P1 (Fig. c). • Hence, the nicol prisms P1 and P2 acts as polariser and analyser, respectively.
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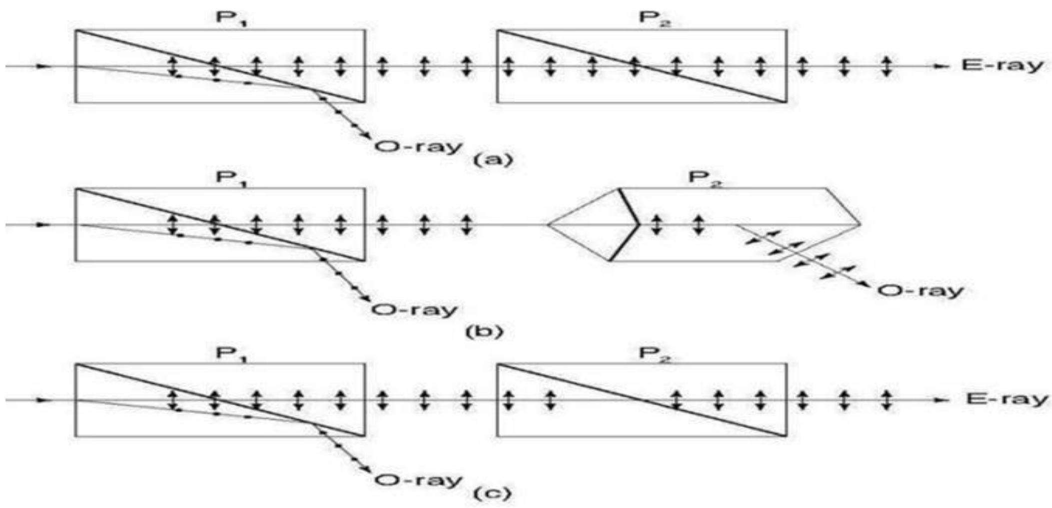


Fig.5.3

Dichroism

- Dichroism is caused by the dependence of the optical absorption spectrum on the polarization .When the absorption changes within the whole optical wavelength window, the crystal can be used as a polarizer since only the component with the lowest absorption will pass a sufficiently thick crystal.
- Dichroism is closely related to birefringence (double refraction). When the absorption band lies outside the optical frequency window, i.e., the material is transparent, the real part of the complex refractive index still shows variations with polarization, causing different propagation velocities and diffracting properties of the crystal for different polarization directions.
- Isotropic systems or crystals with cubic symmetry never show dichroism. For dichroism, anisotropy is necessary. Minerals with a one-axis anisotropy such as hexagonal or tetragonal systems may show dichroism. Crystals with lower symmetry can even exhibit trichroism (or pleochroism).

Polaroids

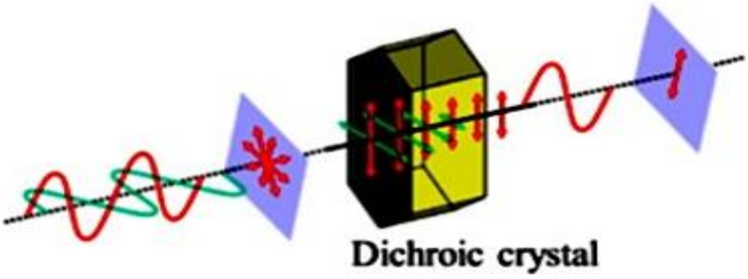


Fig.5.4

Polaroid is a large sheet of synthetic material packed with tiny crystals of a dichroic substance oriented parallel to one another so that it transmits light only in one direction of the electric vector.

Uses of polaroids are:

- Used to produce and analyse polarised light as they are cheaper.
- To view three dimensional pictures.
- Used in motor car head lights and wind screen.
- Used in sunglasses to avoid glare of light.
- In CD players, calculators, LCD,s.

Huygens-Fresnel diffraction

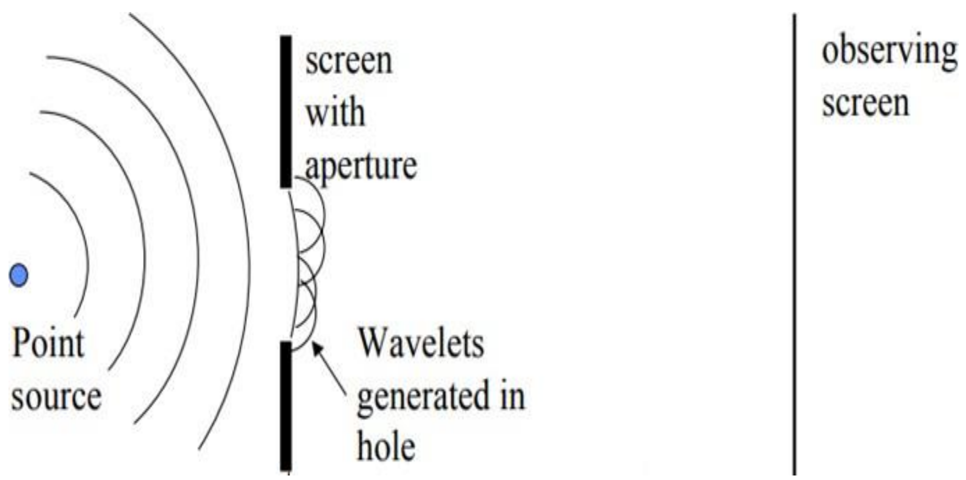


Fig.5.5

Eachwavelet illuminatesthe observingscreen

- Theamplitudesproducedbythevariouswaves attheobserving screencanaddwithdifferent phases

- Finalresultobtainedbytakingsquareofall amplitudes added up – Zero in shadow area – Non-zero in illuminated area.

Fresnel Diffraction (Near-field Diffraction)

- Fresnel diffraction also referred to as near-field diffraction is a form of diffraction which occurs when a wave passes through an aperture and diffracts in the near field, causing any diffraction pattern observed to differ in size and shape depending on the distance between the sources of the obstruction (aperture) to the screen (projection).
- When either the distance from the source to the obstruction or the distance from the obstruction to the screen is comparable to the size of the obstruction, Fresnel diffraction occurs.
- What You Need To Know About Fresnel Diffraction
- If the source of light and screen are at finite distance from the obstacle, then the diffraction is referred to as Fresnel Diffraction.
- Fresnel diffraction patterns occur on flat surfaces.
- To obtain Fresnel diffraction, zone plates are used.
- Shape and intensity of diffraction pattern change as the waves propagate downstream of the scattering source.
- Diffraction pattern move along the corresponding shift in the object.
- In Fresnel diffraction, wavefronts leaving the obstacle are also spherical.
- In Fresnel diffraction, source and screen are far away from each other.
- In Fresnel diffraction, incident wavefronts are spherical.
- In Fresnel diffraction, the convex lens is not required to converge the spherical wavefronts.

Diffraction from circular apertures

•Whathappensifanaperturethediameter of the first Fresnel zone is inserted in the beam?

- Amplitude is twice as high as before insertingaperture!!–Intensityfourtimesas large
- This only applies to intensity on axis

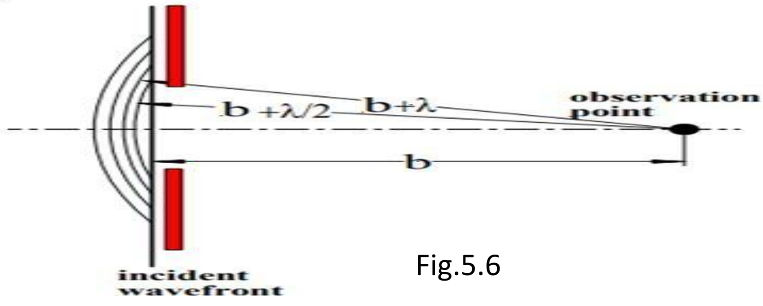


Fig.5.6

- Suppose aperture size and observation distance chosen so that aperture allows just light from first Fresnel zone to pass – Only the term A1 will contribute – Amplitude will be twice as large as case with no aperture!
- If distance or aperture size changed so two Fresnel zones are passed, then there is a dark central spot – alternate dark and light spots along axis – circular fringes off the axis

Fresnel diffraction by circular obstacle— Arago's spot

- Construct Fresnel zones just as before except start with first zone beginning at edge of aperture
- Carrying out the same reasoning as before, we find that the intensity on axis (in the geometrical shadow) is just what it would be in the absence of the obstacle

Close to diffracting screen (near field) – Intensity pattern closely resembles shape of aperture, just like you would expect from geometrical optics – Close examination of edges reveals some fringes

- Farther from screen (intermediate) – Fringes more pronounced, extend into center of bright region – General shape of bright region still roughly resembles geometrical shadow, but edges very fuzzy
- Large distance from diffracting screen (far field) – Fringe pattern gets larger – bears little resemblance to shape of aperture (except symmetries) – Small features in hole lead to larger features in diffraction pattern – Shape of pattern doesn't change with further increase in distance, but it continues to get larger.

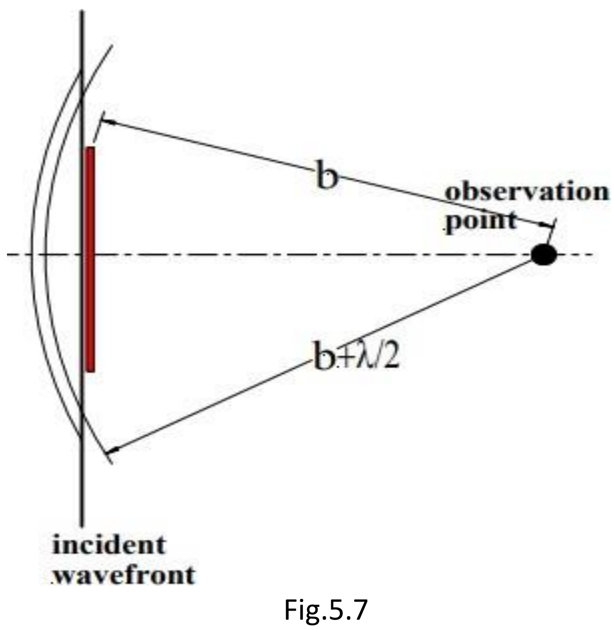


Fig.5.7

Fraunhofer Diffraction (Far-field Diffraction)

- Fraunhofer diffraction also referred to as **Far-field diffraction**, is a form of diffraction in which light source and the reception screen are considered as at infinite distances or at great distance from the diffracting object, so that the resultant wavefronts are considered as planar rather than spherical.

- **What You Need To Know About Fraunhofer Diffraction**
- Ifthesourceoflightandscreenareatinfinitedistancefromtheobstaclethenthediffraction is referred to as Fraunhofer diffraction.
- Fraunhofer diffraction patterns occur on spherical surfaces.
- ToobtainFraunhoferdiffraction,thesingle-doubleplanediffractiongraftingisused.
- Shape and intensity of a Fraunhofer diffraction remains constant.
- Diffraction pattern remains in a fixed position.
- InFraunhoferdiffraction,diffractionobstaclegivesrisetowavefrontswhicharealsoplane.
- In fraunhofer diffraction, source and the screen are far away from each other.
- In Fraunhofer diffraction, incident wavefronts on the diffracting obstacle are plane.
- In Fraunhofer diffraction, Plane diffracting wavefronts are converged by means of a convex lens to produce a diffraction pattern.

Fraunhofer diffraction at a slit

- Traditional (pre laser) setup – source is nearly monochromatic
- Condenser lens collects light
- Source slit creates point source – produces spatial coherence at the second slit
- Collimating lens images source back to infinity – laser, a monochromatic, spatially coherent source, replaces all this second slit is diffracting aperture whose pattern we want
- Focusing lens images Fraunhofer pattern (at infinity) onto screen

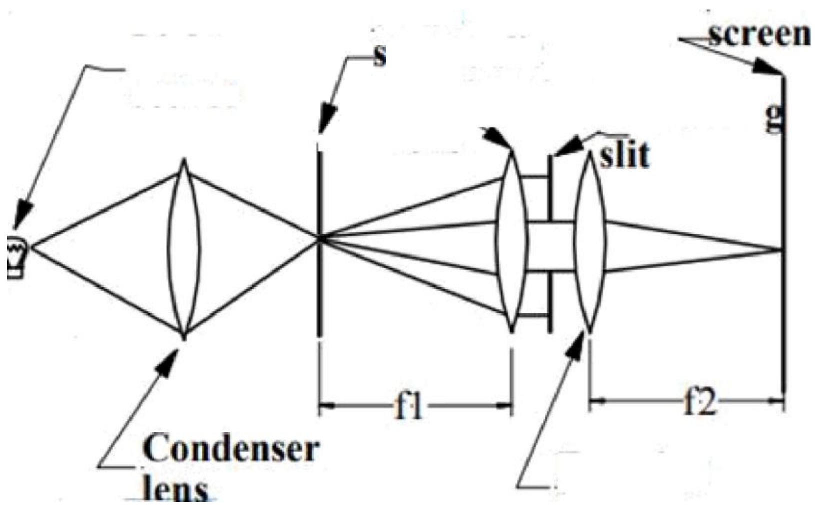


Fig.5.8

Multiple slit diffraction

- In multiple slit patterns discussed earlier, each slit produces a diffraction pattern
- Result: Multiple slit interference pattern is superimposed over single slit diffraction pattern

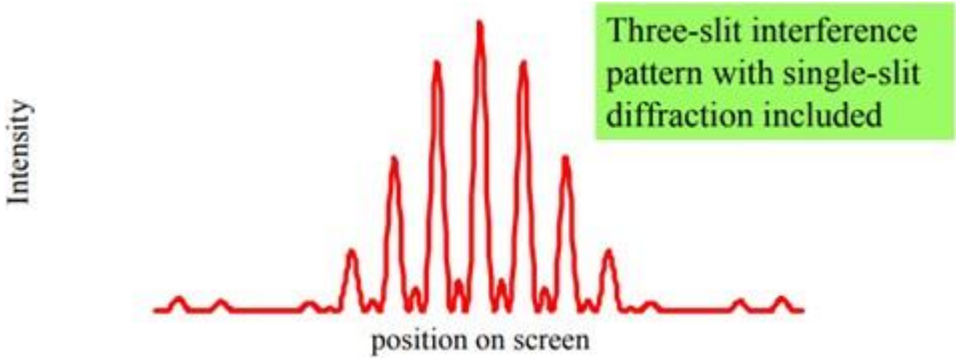


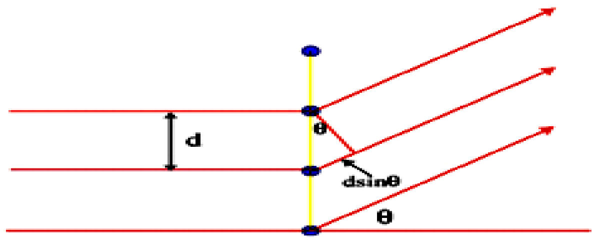
Fig.5.9

Measuring wavelengths with a diffraction grating

- Theoretical Discussion The diffraction of classical waves refers to the phenomenon wherein the waves encounter an obstacle that fragments the wave into components that interfere with one another.
- Interference simply means that the wavefronts add together to make a new wave which can be significantly different than the original wave.
- For example, a pair of sine waves having the same amplitude, but being 180◦ out of phase will sum to zero, since everywhere one is positive, the other is negative by an equal amount.
- Although the diffraction of light waves ostensibly appears the same as the diffraction of classical waves such as water or sound waves, it is an intrinsically quantum mechanical process. Indeed, while the diffraction pattern of a wave of water requires the simultaneous presence of a macroscopic number of water molecules, (of the order of 1024), an optical diffraction pattern can be built up over time by permitting photons to transit the diffracting obstacle one at a time!. This is really pretty amazing if you think about it.
- It is important to understand the physical processes that are occuring that give rise to the diffraction phenomenon.
- For the sake of concreteness, we will consider the diffraction of light through a diffraction grating, which is the device that we will be using in today's lab.
- A diffraction grating consists of a transparent material into which a very large number of uniformly spaced wires have been embedded. One section of such a grating is shown in figure 1. As the light impinges on the grating, the light waves that fall between the wires propagate straight on through.
- The light that impinges on the wires, however, is absorbed or reflected backward. At certain points in the forward direction the light passing through the spaces (or slits) in between the wires will be in phase, and will constructively interfere. The condition for constructive interference can be understood by studying figure 1: Whenever the difference in path length between the light passing through different slits is an integral number of wavelengths of the incident light, the light from each of these slits will be in phase, and the

it will form an image at the specified location.

- Mathematically, the relation is simple: $d \sin \theta = m\lambda$ where d is the distance between adjacent slits (which is the same is the distance between adjacent wires), θ is the angle the re- created image makes with the normal to the grating surface, λ is the wavelength of the light, and m = 0, 1, 2, . . . is an integer.



: Geometry determining the conditions for diffraction from a multi-wire grating

Fig.5.10

- Diffraction gratings can be used to split light into its constituent wavelengths (colors).
- In general, it gives better wavelength separation than does a prism, although the output light intensity is usually much smaller. By shining a light beam into a grating whose spacing d is known, and measuring the angle θ where the light is imaged, one can measure the wavelength λ.
- This is the manner in which the atomic spectra of various elements were first measured. Alternately, one can shine a light of known wavelength on a regular grid of slits, and measure their spacing. You can use this technique to measure the distance between grooves on a CD or the average spacing between the feathers on a bird's wing.
- Consider figure 2, which shows the set-up for a diffraction grating experiment. If a monochromatic light source shines on the grating, images of the light will appear at a number of angles—θ1, θ2, θ3 and so on. The value of θm is given by the grating equation shown above, so that

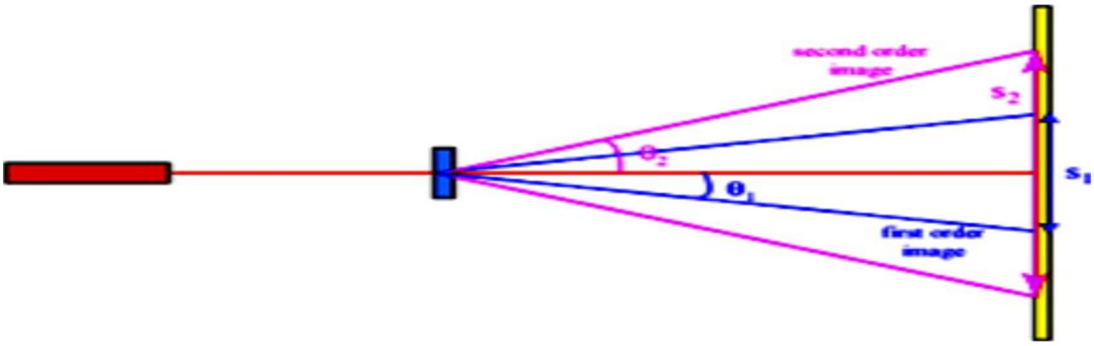


Fig.5.11

Question Bank

Part B

Explain the Fresnel diffraction in a circular aperture and analyze how the bright and dark fringes are depend the distance

Explain the Fresnel diffraction in a straight edge and analyze how the half period strips played a role towards intensity of the image

Derive the expression of width of central maxima in Fraunhoffer diffraction at a single slit

Examine the interference and diffraction at maxima and minima in Fraunhoffer diffraction at double slit.

Explain and analyze how Nicol prism act as a polarizer and analyzer with neat sketc

Part A

State and explain Fresnel Diffraction

State and explain Fraunhoffer Diffraction

Differentiate Interference and Diffraction

Compare the Fresnel Diffraction in a narrow wire and thick wire

Differentiate polarizer and analyser

State positive and negative crystal in view of Huygens double refraction phenomenon

Mention the application of Polaroid sheets

Clarify double refraction phenomenon

Enlighten the role of optic axis

How to identify polarized and un polarized light waves

References

1. Crystal Optics: Properties and Applications by Ashim Kumar Baines, Wiley publisher, 2019
2. Interferences, diffraction and polarisation by M. Françon, Springer, Volume 5/24
3. Optics by Brij lal, M N Avadhanulu & N Subrahmanyam S.chand Publications, 2012