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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT - 1

Oscillations and Waves – SPH1212

Introduction

Introduction to Oscillations

In your school science courses you must have learnt about different types of motions. You are familiar with the motion of falling bodies, planets and satellites. A body released from rest and falling freely (under the action of gravity) moves along a straight line. But an object dropped from an aeroplane or a ball thrown up in the air follows a curved path (except when it is thrown exactly vertically). You must have also observed the motion of the pendulum of a wall clock and vibrating string of a violin or some other string instrument. These arc examples of oscillatory motion. The simplest kind of oscillatory motion which can be analysed mathematically is the Simple Harmonic Motion (SHM). We can analyse oscillatory motions of systems of entirely different physical nature in terms of SHM. For example, the equation of motion that we derive for a pendulum will be similar to the equation of motion of a charge in a circuit containing an inductor and a capacitor. The form of solutions of these equations and the time variation of energy in these systems show remarkable similarities. However, there are many important phenomena which arise due to superposition of two or more harmonic oscillations. For example, our ear drum vibrates under a complex combination of harmonic vibrations. But we shall discuss this aspect in the next unit.

In this unit we will study oscillatory systems using simple mathematical techniques. Our emphasis would be on highlighting the similarities between different systems.

Objectives

After studying this unit you should be able to

- state the basic criteria for the simple harmonic motion of a system
- establish the differential equation for a system executing SHM and solve it
- define the terms amplitude, phase and time period
- compute potential, kinetic and total energies of a body executing SHM

You all know that each hand of a clock comes buck to a given position after the lapse of certain time. This is a familiar example of *periodic motion*. When a body in periodic motion moves to-and-fro (or back and forth) about its position, the motion is called *vibratory* or *oscillatory*. Oscillatory motion is a common phenomenon. Well known examples of oscillatory motion are: oscillating bob of a pendulum clock. piston of an engine. vibrating strings of a musical instrument, oscillating uranium nucleus before it fissions. Even large scale buildings and bridges may at times undergo oscillatory motion. Many stars exhibit periodic variations in brightness, You must have observed that normally such oscillations, left to themselves, do not continue indefinitely, i.e., they gradually die down due to various damping factors like friction and air resistance, etc. Thus, in actual practice, the oscillatory motion may be quite complex, as for instance, the vibrations of a violin string. We begin our study with the discussion of the essential features of SHM. For this we consider an idealised model of a spring-mass system, as an example of a *simple harmonic oscillator*.

Oscillations of a Spring-mass 'System

A spring-mass system consists of a spring of negligible mass whose one end is fixed to a rigid support S and the other end carries a block of mass m which lies flat on a horizontal frictionless table (Fig. 1.1a). Let us take the x-axis to be along the length of the spring. When the mass is at rest. we mark a point on it and we define the origin of the axis by this point. That is. at equilibrium the mark lies at $x = 0$.

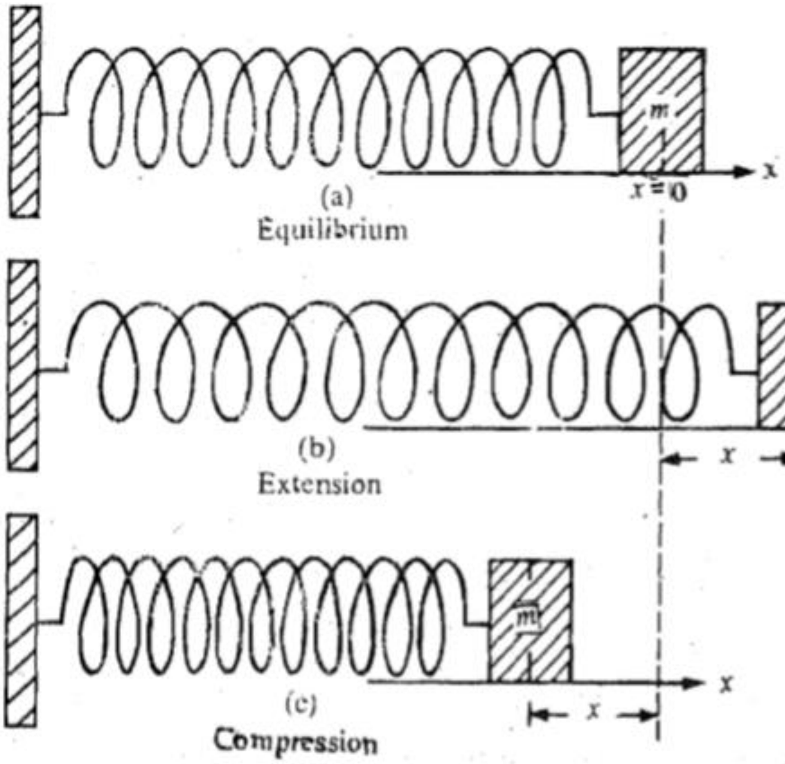


Fig. 1.1 A Spring-mass System as an ideal oscillator (a) The equilibrium configuration, (b) An extended configuration, (c) A compressed configuration.

If the spring is stretched by pulling the mass longitudinally, due to elasticity a restoring force comes into play which tends, to bring the mass back towards the equilibrium position (Fig 1.1b). If the spring were compressed the restoring force would tend to extend the spring and restore the mass to its equilibrium position

(Fig.1.1c). More you stretch/compress the spring, more will be the restoring farce. ***So the direction of the restoring force is always opposite to the displacement.*** If total change in the length is small compared to the original length, then the magnitude of restoring force is linearly proportional to the displacement. Mathematically, we can write

$$F = -kx \tag{1.1}$$

The negative sign signifies that the restoring force opposes the displacement. The quantity k is called the ***spring constant*** or the ***force constant*** of the spring. It is numerically equal to the magnitude of restoring force exerted by the spring for unit extention. Its SI unit is Nm^{-1}

SAQ 1

The spring in Fig. 1.1a is stretched by 5 cm when a force of 2 N is applied. Calculate the spring constant. How much will this spring be compressed by a force of 2.5 N?

How does the spring-mass system oscillate? To answer this question, we note that when we pull the mass, the spring is stretched. The restoring force tends to bring the mass back to its equilibrium position ($x = 0$). Therefore, on being released, the mass moves towards the equilibrium position. In this process it acquires kinetic energy and overshoots the equilibrium position. Do you know why? It is because of inertia. Once it overshoots and moves to the other side, the spring is compressed and the mass is acted upon by a restoring force but in the opposite direction. Thus we can say that oscillatory motion results from two intrinsic properties of the system: (i) elasticity and (ii) inertia.

What is the direction of the restoring force vis-a-vis the equilibrium position of an oscillating body?

The restoring force is always directed towards the equilibrium position of the oscillating body.

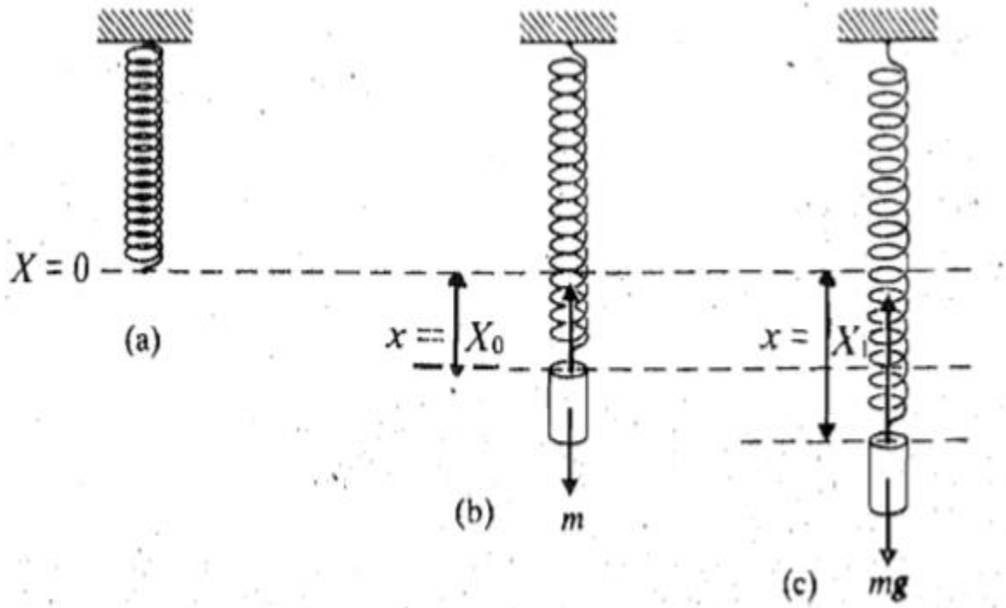
In discussing the spring-mass system we observed two important points:

(i) The restoring force is linearly proportional to the displacement of mass from its equilibrium position.

(ii) The restoring force is always directed towards the equilibrium position.

Any oscillatory motion which satisfies both these conditions is called simple harmonic motion. The study of **SMM** is important because, as you will see, oscillatory motion of systems of entirely different physical nature can be analysed in terms of it.

Let us now study the effect of gravity on oscillations of spring-mass system. Consider a spring of negligible mass suspended from a rigid support with a mass m attached to its lower end (Fig.1.2).



Let us choose the X-axis along the length of the spring. We take the bottom of the spring as our reference point, X = 0, when no weight is attached to it (Fig. 1.2a). When a mass m is suspended from the spring, let the reference point move to $X = X_0$ (Fig. 1.2b). At equilibrium, the weight, mg, balances the spring force, kX_0. Since the net force is zero: we have

$$mg - kX_0 = 0$$

or

$$mg = kX_0. \qquad (1.2)$$

Now if the mass is pulled downwards so that the reference mark shifts to X_1 (Fig 1.2c), then the total restoring force will be kX_1 and point in the upward direction. The net downward force will therefore be (using Eq. (1.2))

$$mg - kX_1 = k(X_0 - X_1) = -kx$$

where $x = X_1 - X_0$.

Thus, the resulting restoring force on the mass is

$$F = -kx$$

where x is its displacement from the equilibrium position, X_0. This result is of the same form as Eq. (1.1) for the horizontal arrangement. It is thus clear that gravity has no effect on the frequency of oscillation's of a mass hanging vertically from a spring; i only displaces the equilibrium.

Let us now find the differential equation which describes the oscillatory motion of a spring-mass system. The equation of motion of such a system is given by equating the two forces acting on the mass:

mass × acceleration = restoring force

or

$$m \frac{d^2x}{dt^2} = -kx$$

where $\frac{d^2x}{dt^2}$ is the acceleration of the body,

It is important to note that in this equation, the equilibrium position of the body is taken as the origin, $x = 0$.

You will note that the quantity k/m has units of $Nm^{-1}\ kg^{-1} = (kg.ms^{-2})\ kg^{-1}m^{-1} = s^{-2}$ Hence we can replace k/m by ω_0^2 where ω_0 is called ***angular frequency***. Then the above equation takes the form

$$\frac{d^2x}{dt^2} + \omega_0^2\, x = 0 \qquad (1.3)$$

It may be remarked here that Eq. (1.3) is the differential form of Eq. (1.1) and describes simple harmonic motion in one dimension.

Solution of the differential equation for SHM

To find the displacement of the mass at any time t, we have to solve Eq. (1.3) subject to given initial conditions. A close inspection of Eq. (1.3) shows that x should be such a function that its second derivative with respect to time is the negative of the function itself, except for a multiplying factor ω_0^2. From elementary calculus, we know that sine and cosine functions have this property.

You can check that this property does not change even if sine and cosine functions have a constant multiplying factor.

A general solution for x (t) can thus be expressed as a linear combination of both sine and cosine terms, i.e.

$$x(t) = A_1 \cos at + A_2 \sin \alpha t \qquad (1.4)$$

Putting $A_1 = A \cos \phi$ and A2 = $- A \sin \phi$, we get

$$x(t) = A \cos(at + \phi)$$

Differentiating this equation twice with respect to time and comparing the resultant expression with Eq. (1.3), we obtain $\alpha = \pm \omega_0$. The negative sign is dropped as it gives negative frequency which is a physically absurd quantity.

Substituting $\alpha = \omega_0$ in the above equation, we get

$$x(t) = A \cos(\omega_0 t + \phi) \qquad (1.5)$$

Let us assume that the mass is held steady at some distance a from the equilibrium position and then released at $t = 0$. Thus the initial conditions are: at $t = 0$, $\mathbf{x} = a$

and $\frac{dx}{dt} = 0$. Then, from Eq. (1.5) we would have

$$x \text{ (at } t = 0) = A \cos \theta = a$$

and

$$\frac{dx}{dt} \text{ (at } t = 0) = - A \omega_0 \text{ Sin } 9 = 0$$

These conditions are sufficient to **fix** A and ϕ. The second conditidn tells us that ϕ is either zero or $n\pi$ ($n = 1, 2, ...$). We reject the second option because the first condition requires $\cos \phi$ to be positive. Thus with the above initial conditions, Eq. (1.5) has the simple form

$$\mathbf{x} = a \cos \omega_0 t. \qquad \textbf{(1.6)}$$

SAQ 2

Take $A_1 = B \sin \theta$ and $A_2 = B \cos \theta$ in Eq.(1.4). In this case show that the solution is

$$x(t) = B \sin (\omega_0 t + \theta)$$

We therefore observe that both cosine and sine forms are valid solutions of Eq. (1.3). If you plot Eq. **(1.5)**, the graph-will be a cosine curve with a definite initial **phase** (Fig. 1.3)

Phase and Amplitude

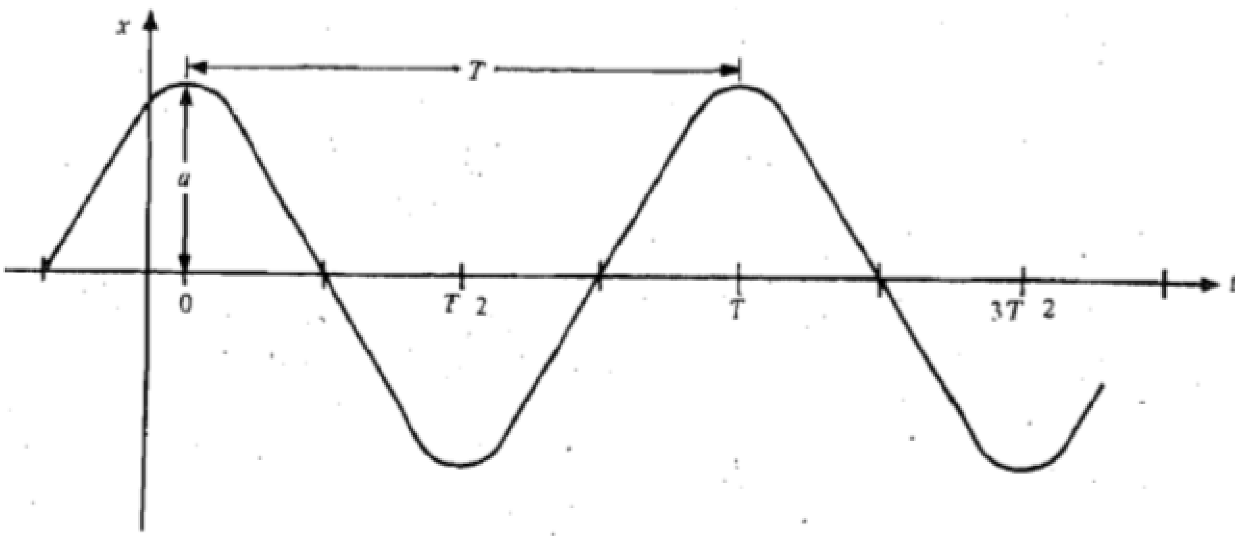


Fig. 1.3 Displacement-time graph of simple harmonic motion with an initial phase ϕ.

$$x_0 = a \cos \phi$$

We know that the value of the sine and cosine functions lies between 1 and −1. When cos $(\omega_0 t + \phi) = 1$ or −1, the displacement has the maximum value. Let us denote it by a or −a. The quantity a is called the *amplitude* of oscillation.

We can, therefore, rewrite Eq. (1.5) as

$$x(t) = a \cos(\omega_0 t + \phi) \tag{1.7}$$

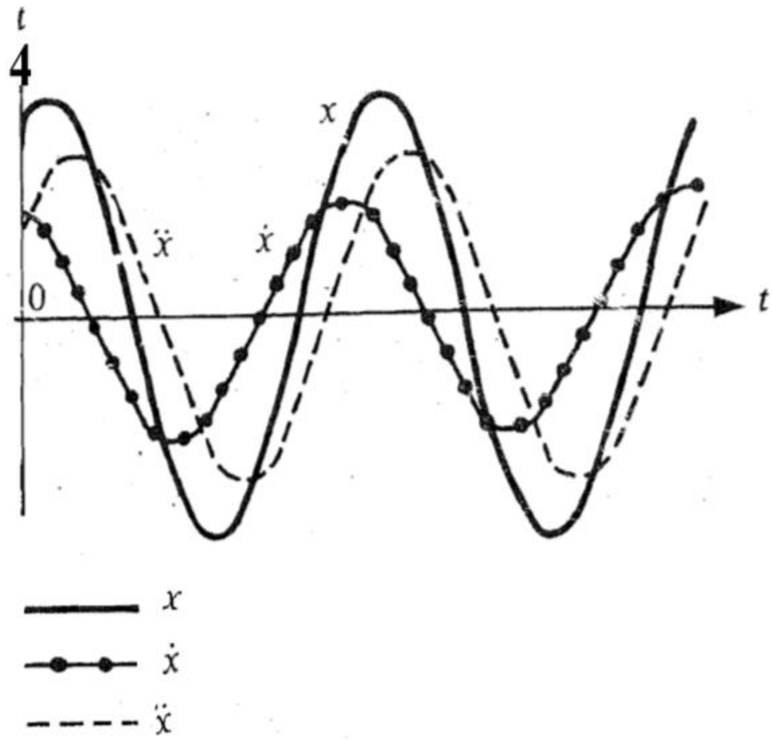


Fig. 1.5 Time variation of displacement, velocity and acceleration of a body executing SHM ($\phi = 8$)'

Velocity and Acceleration

We know that the displacement of a mass executing simple harmonic motion is given by

$$x = a \cos(\omega_0 t + \phi)$$

Therefore, the instantaneous velocity, which is the first time derivative of displacement, is given by

$$v = \frac{dx}{dt} = -\omega_0 a \sin(\omega_0 t + \phi) \qquad (1.11)$$

We can rewrite it as

$$v = \omega_0 a \cos(\pi/2 + \omega_0 t + \phi) \qquad (1.12a)$$

You may also like to know the value of v at any point x. To this end, we rewrite Eq. (1.11) as

$$v = -\omega_0 [a^2 - a^2 \cos^2(\omega_0 t + \phi)]^{1/2}$$
$$= -\omega_0 (a^2 - x^2)^{1/2} \qquad \text{for } -a \leq x \leq a \qquad (1.12b)$$

We also know that acceleration is the first time derivative of velocity. From Eq. (1.11) it readily follows that

$$\frac{dv}{dt} = -\omega_0^2 a \cos(\omega_0 t + \phi)$$
$$= \omega_0^2 a \cos(\pi + \omega_0 t + \phi) \qquad (1.13a)$$

Obviously, in terms of displacement

$$\frac{dv}{dt} = -\omega_0^2 x \qquad (1.13b)$$

TRANSFORMATION OF ENERGY IN OSCILLATING SYSTEMS : POTENTIAL AND KINETIC ENERGIES

Consider the spring-mass system shown in Fig .1.1. When the mass is pulled, the spring is elongated. The amount of energy required to elongate the spring through a distance dx is equal to the work done in bringing about this change. It is given by $dW = dU = F_0\, dx$, where F_0 is the applied force (such as by hand). This force is balanced by the restoring force. That is, its magnitude is same as that of F and we can write $F_0 = kx$. Therefore, the energy required to elongate the spring through a distance x is

$$U = \int_0^x F_0\, dx = k \int_0^x x dx = \frac{1}{2} kx^2 \tag{1.14}$$

This energy is stored in the spring in the form of potential energy and is responsible for oscillations of the spring-mass system.

On substituting for the displacement from Eq. (1.7) in Eq. (1.14), we get

$$U = ½\, ka^2 \cos^2 (\omega_0 t + \phi) \tag{1.15}$$

Note that at $t = 0$, the potential energy is

$$U_0 = ½\, ka^2 \cos^2 \phi \tag{1.16}$$

As the mass is released, it moves towards the equilibrium position and the potential energy starts changing into kinetic energy ($K.E$). The kinetic energy at any time t is given by $K.E = ½ mv^2$. Using Eq. (1.11), we get

$$K.E = ½ m \omega_0^2 a^2 \sin^2 (\omega_0 t + \phi)$$

$$= ½ ka^2 \sin^2 (\omega_0 t + \phi) \qquad (1.17)$$

since $\omega_0^2 = k/m$.

One can also express $K.E$ in terms of the displacement by writing

$$K.E = ½ ka^2 [1 - \cos^2 (\omega_0 t + \phi)]$$

$$= ½ ka^2 - ½ ka^2 \cos^2 (\omega_0 t + \phi)$$
$$= ½ ka^2 - ½ kx^2 = ½ k (a^2 - x^2) \qquad (1.18)$$

This shows that when **an** oscillating body passes through the equilibrium position ($x = 0$), its kinetic energy is maximum and equal to $½ ka^2$.

SAQ 5
Show that the periods of potential and kinetic energies are one-half of the period of vibration.

It is thus clear from the explicit time dependence of Eqs. (1.15) and (1.17) that in a spring-mass system the mass and the spring alternately exchange energy. Let us consider that the initial phase $\phi = 0$. At $t = 0$, potential energy stored in the spring is maximum and $K.E$ of the mass is zero. 'At $t = T/4$, the potential energy is zero and $K.E$ is maximum. As the mass oscillates, energy oscillates from kinetic form to potential form and vice versa. At any instant, the total energy, E, of the oscillator will be sum of both these energies. Hence, from Eqs. (1.15) and (1.17), we can write

$$E = U + K.E = ½ ka^2 \cos^2 (\omega_0 t + \phi) + ½ ka^2 \sin^2 (\omega_0 t + \phi) = ½ ka^2 \qquad (1.19)$$

This means that the total energy remains constant (independent of time) and is proportional to the square of the amplitude. As long as there are no dissipative forces like friction, the total mechanical energy will be conserved.

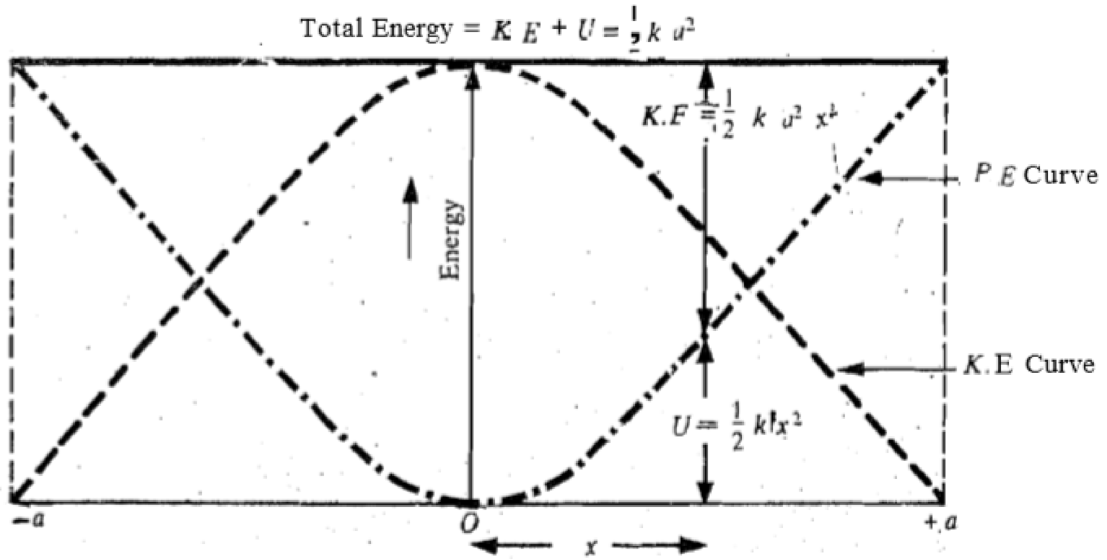


Fig. 1.6 Variation of potential energy (U), kinetic energy ($K.E$) and total energy (E) with displacement according to Eqs. (1.14), (1.18) and (1.19).

The plots of U and $K.E$ as a function of x as obtained from Eqs. (1.14) and (1.18) are shown in Fig 1.6. You will note that

(i) the shape of these curves is parabolic, (ii) the shape is symmetric about the origin, and (iii) the potential and kinetic energy curves are inverted with respect to one another. Why? This is due to the phase difference of $\pi/2$ between the displacement and velocity of a harmonic oscillator. At any value of x, the total energy is the sum of kinetic and potential energies and is equal to $\frac{1}{2} ka^2$. This is represented by the horizontal line.

The points where this horizontal line intersects the potential energy curve are called the 'turning points'. The oscillating particle cannot go beyond these and turns back towards the equilibrium position. At these points, the total energy of the oscillator is entirely potential ($E = U = \frac{1}{2} ka^2$) and $K.E$ is zero. At the equilibrium position ($x = 0$) the energy is entirely kinetic ($K.E = E = \frac{1}{2} ka^2$) so that the maximum speed, v_{max} is given by the relation $\frac{1}{2}mv^2_{max} = E$, i.e. $v_{max} = \sqrt{2E/m}$.

At any intermediate position, energy is partly kinetic and partly potential, but the total energy always remains the same. The transformation of energy in a spring-mass system is shown in Fig. 1.7

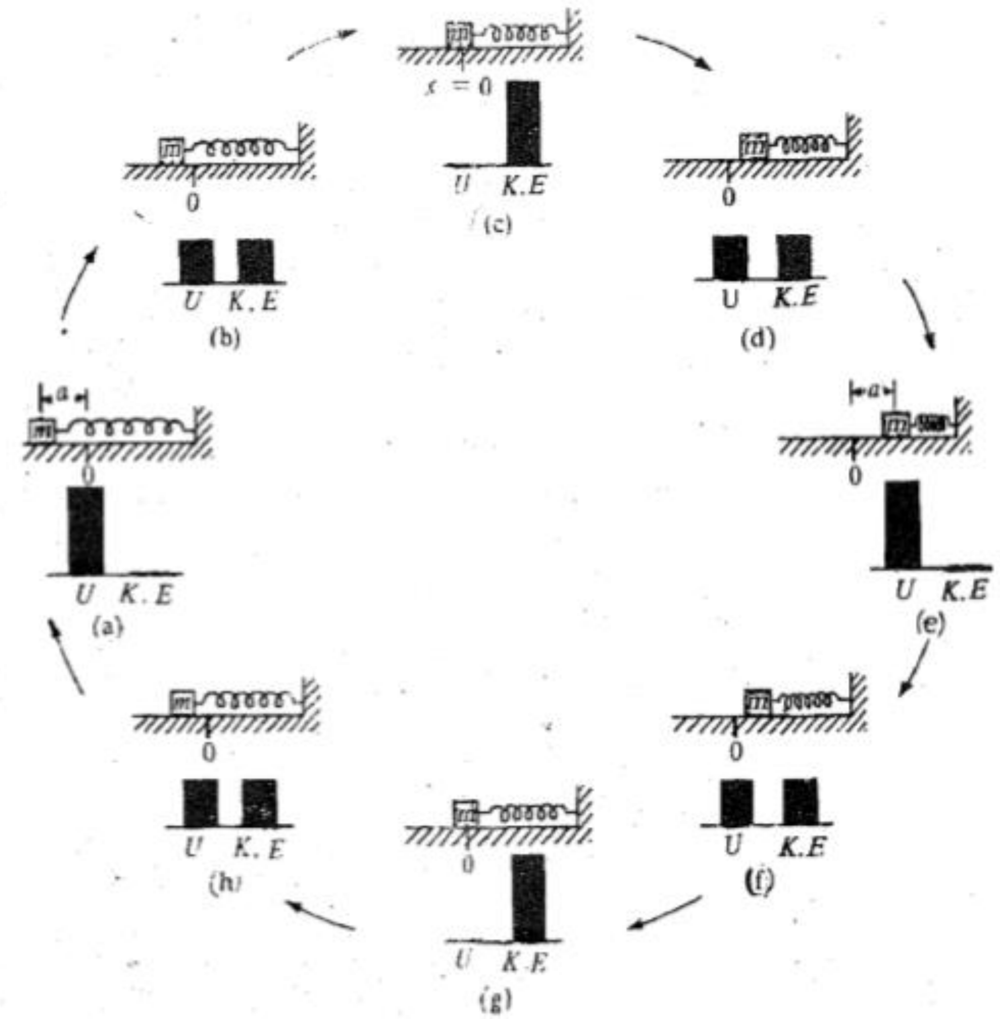


Fig. 1.7 Energy transformation in a spring-mass system at various times. The bars indicating potential and kinetic energies are shown at intervals of $t = T/8$.

CALCULATION OF AVERAGE VALUES OF QUANTITIES ASSOCIATED WITH SHM

In Fig. 1.5 we have plotted displacement, velocity and acceleration as a function of time. You will note that for any complete cycle in each case, the area under the curve for the first half is exactly equal to the area under the curve in the second half and the two are opposite in sign. Thus over one complete cycle the algebraic sum of these areas is zero. This means that average values of displacement, velocity and acceleration over one complete cycle are zero. If we plot x^2 (or v^2) versus t, the curves would lie in the upper half only so that the total area will be positive during one complete cycle. This suggests that we can talk about average values of kinetic and potential energies.

The time average of kinetic energy over one complete cycle is defined as

$$< K.E > = \frac{\int_0^T K.E\, dt}{T} \tag{1.20a}$$

On substituting for $K.E$ from Eq. (1.17), we get

$$< K.E > = \frac{ka^2}{2T} \int_0^T \sin^2 (\omega_0 t + \phi)\, dt \tag{1.20b}$$

On solving the integral in Eq. (1.20b) you will find that its value is $T/2$. So, the expression for average kinetic energy reduces to

$$< K.E > = \frac{ka^2}{4} \tag{1.21}$$

Similarly, one can show that the average value of potential energy over one cycle is

$$< U > = \frac{ka^2}{4} \tag{1.22}$$

That is, the average kinetic energy of a harmonic oscillator is equal to the average potential energy over one complete period.

Thus the sum of average kinetic and average potential energies is equal to the total

An Acoustic Oscillator

Consider a flask of volume V with a narrow neck of length l and area of cross-section A such that $V >> lA$ (Fig.1.12). Such a system is also called ***Helmholtz resonator*** because the system can resonate when the frequency of sound incident on it coincides with its natural frequency. We will here calculate the expression for the natural frequency of the resonator.

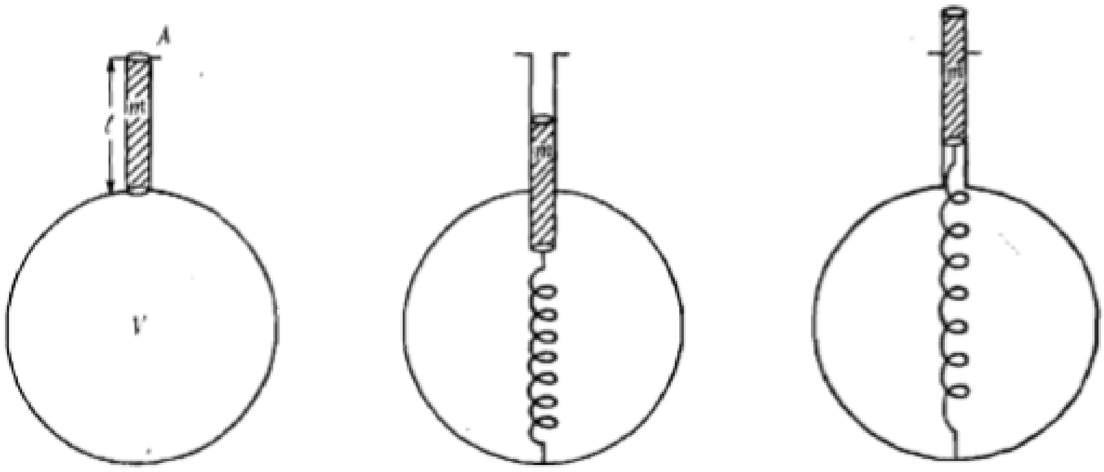


Fig. 1.12 (a) An Acoustic oscillator, (b) As air in the neck is pushed, air in the flask is compressed, and (c) Due to elasticity, air in the flask exerts a restoring force on the air in the neck.

We consider free vibrations of air in the neck of the flask. **As** the air in the neck moves in, the air in the flask is compressed. If air in the neck goes out, the air in the flask is rarefied. So the air in the neck behaves like the mass and the air in the flask behaves like the spring in a mechanical oscillator.

Suppose that the air in the neck moves inward through a distance x from the equilibrium position. The change in the volume of the air in the bulb A V = **x** ***A*.** Let the increase in pressure over the atmospheric pressure be ***Ap*.** We know that the volume of a gas depends on the pressure as well as the temperature. Therefore, the pressure changes in acoustic vibrations should alternately heat and cool the air in the flask as it gets compressed and rarefied. We assume that the pressure changes are so rapid that they do not permit any exchange of heat. That is, the process is adiabatic. Hence, we can write

$$\Delta p = - E_\gamma \frac{\Delta V}{V} = - E_\gamma \frac{Ax}{V} \qquad (1.41)$$

where E_γ is the ***adiabatic elasticity*** of the gas. It is defined as the ratio of the stress to volume strain. Numerically, stress is same as pressure. So we can write

$$E_\gamma = - \frac{\Delta p}{(\Delta V/V)}$$

The negative sign signifies the fact that as pressure increases, volume decreases and vice-versa.

This excess pressure Δp of air inside the bulb provides the restoring force F, which acts upward. We can therefore write

$$F = \Delta p \, A = - \frac{E_\gamma A^2}{V} x$$

If ρ is the density of air, the mass of the air in the neck $m = \ell A \rho$. Hence, the equation of motion of air in the neck can be written as

$$\ell A \rho \frac{d^2x}{dt^2} = - \frac{E_\gamma A^2}{V} x$$

or

$$\frac{d^2x}{dt^2} + \frac{E_\gamma A}{V \ell \rho} x = 0 \qquad (1.42)$$

This equation has the standard form for simple harmonic motion. Hence, the frequency of oscillation of air in the neck is

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{E_\gamma A}{V l \rho}} = \frac{v_s}{2\pi}\sqrt{\frac{A}{V l}} \qquad (1.43)$$

where $v_s = \sqrt{E_\gamma/\rho}$ is the speed of sound. We know that v_s is proportional to square root of temperature. So the frequency of vibration of air in a flask is also proportional to the square root of temperature.

SAQ 10

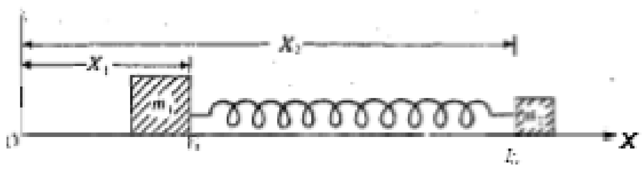
A flask has a neck of radius 1 cm and length 10 cm. If the capacity of the flask is 2 litres, determine the frequency at which the system will resonate (speed of sound in air = 350 ms^{-1}).

A Diatomic Molecule: Two-Body Oscillations

A diatomic molecule like HCl is an example of a two-body system which can oscillate along the line joining the two atoms. The atoms of a diatomic molecule are coupled through forces which have electrostatic origin. The bonding between them may be likened to a spring. Thus we may consider a diatomic molecule as a system of two masses connected by a spring. We will now consider the oscillations of such a system.

Suppose that two masses m_1 and m_2 are connected by a spring of force constant k. The masses are constrained to oscillate along the axis of the spring (Fig. 1.13a). Let r_0 be the normal length of the spring. We choose X-axis along the line joining the two masses. If X_1 and X_2 are the coordinates of the two ends of the spring at time t, the change in length is given by

$$x = (X_2 - X_1) - r_0 \qquad (1.44)$$



(b)

Fig. 1.13 (a) A two-body oscillator (b) An equivalent one-body oscillator.

For $x > 0$; $x = 0$ and $x < 0$, the spring is extended, normal and compressed respectively. Suppose that at a given instant of time the spring is extended, i.e. $x > 0$. Though the spring exerts the same force (kx) on the two masses but the force F_1 (= kx) acting on m_1 opposes the force F_2 (= $-kx$) on m_2, i.e.

$$F_1 = kx \quad \text{and} \quad F_2 = -kx$$

According to Newton's second law, above equation can be written as

$$m_1 \frac{d^2X_1}{dt^2} = kx$$

and

$$m_2 \frac{d^2X_2}{dt^2} = -kx$$

On rearranging terms, we obtain

$$\frac{d^2X_1}{dt^2} = \frac{kx}{m_1} \tag{1.45 a}$$

and

$$\frac{d^2X_2}{dt^2} = -\frac{kx}{m_2} \tag{1.45 b}$$

On subtracting one from the other, we get

$$\frac{d^2(X_2 - X_1)}{dt^2} = -\left(\frac{1}{m_1} + \frac{1}{m_2}\right)kx$$

Since r_0 denotes a constant length of the spring, Eq. **(1.44)** tells us that

$$\frac{d^2x}{dt^2} = \frac{d^2(X_2 - X_1)}{dt^2}$$

Hence, the equation of motion of a diatomic molecule reduces to

$$\frac{d^2x}{dt^2} + \frac{k}{\mu}x = 0 \tag{1.46}$$

where $\mu \equiv \left(\frac{1}{m_1} + \frac{1}{m_2}\right)^{-1} = \frac{m_1m_2}{m_1 + m_2}$ is called the **reduced mass** of **the** system.

Eq. (1.46) describes simple harmonic oscillation of frequency

$$\nu_0 = \frac{1}{2\pi}\sqrt{k/\mu} \tag{1.47}$$

This means that a diatomic molecule behaves as a single object of mass μ, connected by a spring of force constant k (Fig. 1.13 b).

SAQ 11

For an HCl molecule, $r_0 = 1.3$ Å . Find the value of the force constant and frequency of oscillation. Given that $m_H = 1.67 \times 10^{-27}$ kg and $m_{cl} = 35\ m_H$.

Use $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ Nm^2 C^{-2}

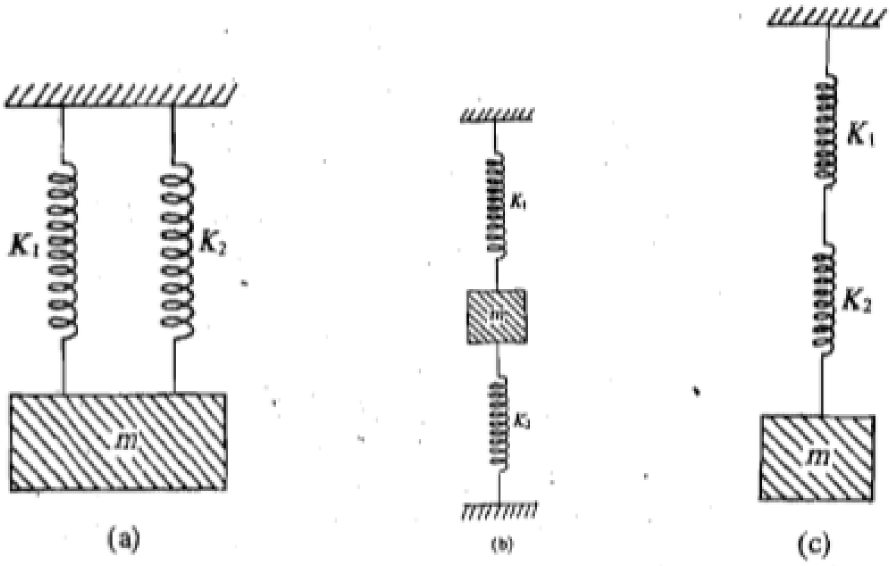
PRACTICE QUESTIONS

1 In Figs Fig. 1.14 **a,** b and c, three combinations of two springs of force constants k_1 and k_2 are given. Show that the periods of oscillation in the three cases are given

a). $2\pi\sqrt{m/(k_1+k_2)}$

b) $2\pi\sqrt{m/(k_1+k_2)}$

c) $2\pi\sqrt{m(1/k_1+1/k_2)}$



2 A smooth tunnel is bored through the earth along one of its diameters and a ball is dropped into it. Show that the ball will execute simple harmonic motion with period $T=2\pi\sqrt{R/g}$ where R is radius of the earth and g is acceleration due to gravity at the surface of the earth. Assume the earth to be a homogeneous sphere of uniform density.

3 Find the angular frequency and the amplitude of harmonic oscillations of a particle if at distances x_1 and x_2 from the equilibrium position its velocity equals v_1 and v_2 respectively.

4 Show that the centres of suspension and oscillations in a compound pendulum are mutually interchangeable.

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT - 2

Oscillations and Waves – SPH1212

Reflection, Refraction of Diffraction and Attenuation of Sound waves

In Unit I you learnt that SHM is a universal phenomenon. Now you also know that in the ideal case the total energy of a harmonic oscillator remains constant in time and the displacement follows a sine curve. This implies that once such a system is set in motion it will continue to oscillate forever. Such oscillations are said to befree or ***undamped.*** Do you know of any physical system in the real world which experiences no damping? Probably there is none. You must have observed that oscillations of a swing, a simple or torsional pendulum and a spring-mass system when left to themselves, die down gradually. Similarly, the amplitude of oscillation of charge in an LCR circuit or of the coil in a suspended type galvanometer becomes smaller and smaller. This implies that every oscillating system loses some energy as time elapses. The question now arises: Where does this energy go? To answer this, we note that when a body oscillates in a medium it experiences resistance to its motion. This means that damping force comes into play. Damping force can arise within the body itself, as well as due to the surrounding medium (air or liquid). The work done by the oscillating system against the damping forces leads to dissipation of energy of the system. That is, the energy of an oscillating body is used up in overcoming damping. But in some engineering systems we knowingly introduce damping. A familiar example is that of brakes—we increase friction to reduce the speed of a vehicle in a short time. In general, damping causes wasteful loss of energy. Therefore, we invariably try to minimise it.

Many a time it is desirable to maintain the oscillations of a system. For this we have to feed energy from an outside agency to make up for the energy losses due to damping. Such oscillations are called ***forced oscillations.*** You will learn various aspects of such oscillations in the next unit.

In this unit you will learn to establish and solve the equation of motion of a damped harmonic oscillator. Damping may be quantified in terms of logarithmic decrement, relaxation time and quality factor. You will also learn to compute expressions for the logarithmic decrement, power dissipated in one cycle and the quality factor.

PRINCIPLE OF SUPERPOSITION

We know that for small oscillations, a simple pendulum executes simple harmonic motion. Let us reconsider this motion and release the bab at the instant $t = 0$ when it has initial displacement a_1. Let the displacement at a subsequent time t be x_1. Let us repeat the experiment with an initial displacement a_2. Let the displacement after the same interval of time t be x_2. Now if we take the initial displacement to be the sum of the earlier displacemen'ts, viz. $a_1 + a_2$, then according to the superposition principle, the displacement x_3 after the same interval of time t will be

$$x_3 = x_1 + x_2.$$

You can perform this activity by taking three identical simple pendulums. Release all three bobs simullaneously such that their initial velocities are zero and initial displacements of the first, second and the third pendulum are a_1, a_2 and $a_1 + a_2$, respectively. You will find that at any time the displacement x_3 of the third pendulum will be the algebraic sum of the displacements of the other two. In general, the initial velocities may be non-zero. Thus, the principle of superposition can be stated as follows:

When we superpose the initial conditions corresponding to velocities and amplitudes, the resultant displacement of two (or more) harmonic displacements will be simply the algebraic sum of the individual displacements at all subsequent times.

You will note that the principle of superposition holds for any number of simple harmonic ascillations. These may be in the same or mutually perpendicular directions, i.e. in two dimensions.

In Unit 1, we observed that Eq. (1.3) describes SHM:

$$\frac{d^2x}{dt^2} = -\omega_0^2 x \qquad (2.1)$$

This is a linear homogeneous equation of second order.

Such an equation has an important property that the sum of its two linearly independent solutions is itself a solution. We have already used this property in Unit 1 while writing Eq. (1.4).

Let $x_1(t)$ and $x_2(t)$ respectively satisfy equations

$$\frac{d^2x_1}{dt^2} = -\omega_0^2 x_1 \tag{2.2}$$

and

$$\frac{d^2x_2}{dt^2} = -\omega_0^2 x_2 \tag{2.3}$$

Then by adding Eqs. (2.2) and (2.3), we get

$$\frac{d^2(x_1 + x_2)}{dt^2} = -\omega_0^2 (x_1 + x_2) \tag{2.4}$$

According to the principle of superposition, the sum of two displacements given by

$$x(t) = x_1(t) + x_2(t), \tag{2.5}$$

also satisfies Eq. (2.1). In other words, the superposition of two displacements satisfies the same linear homogeneous differential equation which is satisfied individually by x_1 and x_2.

SUPERPOSITION OF TWO HARMONIC OSCILLATIONS OF THE SAME FREQUENCY ALONG THE SAME LINE

Let us superpose two collinear (along the same line) harmonic oscillations of amplitudes a_1 and a_2 having frequency ω_0 and a phase difference of π. The displacements of these oscillations are given by

$$x_1 = a_1 \cos \omega_0 t \tag{2.6}$$

and

$$\begin{aligned} x_2 &= a_2 \cos (\omega_0 t + \pi) \\ &= -a_2 \cos \omega_0 t \end{aligned} \tag{2.7}$$

According to the principle of superposition, the resultant displacement is given by

$$\begin{aligned} x(t) &= x_1(t) + x_2(t) \\ &= a_1 \cos \omega_0 t - a_2 \cos \omega_0 t \\ &= (a_1 - a_2) \cos \omega_0 t \end{aligned} \tag{2.8}$$

This represents a simple harmonic motion of amplitude $(a_1 - a_2)$. In particular, if two amplitudes are equal, i.e. $a_1 = a_2$ the resultant displacement will be zero at all times. Displacement-time graph depicting this situation is shown in Fig. 2.1.

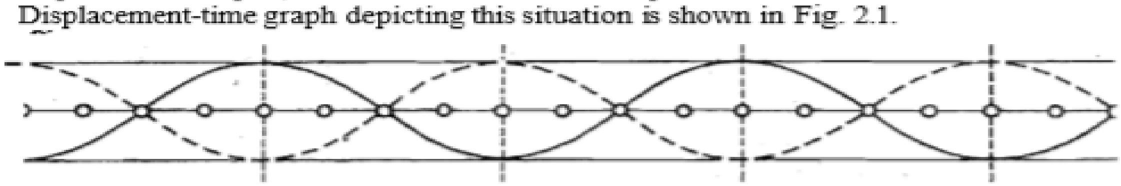


Fig. 2.1 Superposition of twa collinear harmonic oscillations of equal amplitude but out of phase by π.

Using the expression for the cosine of the sum of two angles, this can be written as

$$x(t) = a_1 \cos \omega_0 t \cos \phi_1 - a_1 \sin \omega_0 t \sin \phi_1 \\ + a_2 \cos \omega_0 t \cos \phi_2 - a_2 \sin \omega_0 t \sin \phi_2$$

Collecting the coefficients of $\cos \omega_0 t$, and $\sin \omega_0 t$, we get

$$x(t) = (a_1 \cos \phi_2 + a2 \cos \phi_1) \cos \omega_0 t \\ - (a_1 \sin \phi_1 + a_2 \sin \phi_2) \sin \omega_0 t \qquad (2.11)$$

Since a_1, a_2, ϕ_1 and ϕ_2 are constant, wc can set

and $$a \cos \phi = a_1 \cos \phi_1 + a_2 \cos \phi_2 \qquad (2.12)$$

$$a \sin \phi = a_1 \sin \phi_1 + a_2 \sin \phi_2 \qquad (2.13)$$

where a and 4 have to be determined. Then, we can rewrite **Eq.** (2.11) in the form

$$x(t) = a \cos \phi \cos \omega_0 t - a \sin \phi \sin \omega_0 t,$$

It has the form of the cosine of the sum of two angles and can be expressed as

$$x(t) = a \cos (\omega_0 t + \phi) \qquad (2.14)$$

This equation has the same form as either of our original equations for separate harmonic oscillations. Hence, we have the important result that ***the sum of two collinear harmonic oscillations of the same frequency is also a harmonic oscillarion of the samefrequency and along the same line. But if has a new amplitude*** a ***and a new phase constant*** ϕ. The amplitude can easily be calculated by squaring Eqs. (2.12) and (2.13) and adding the resultant expressions. On simplification we have

$$a^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos (\phi_1 - \phi_2) \qquad (2.15)$$

Similarly, the phase ϕ is determined by dividing Eq. (2.13) by Eq. (2.12) :

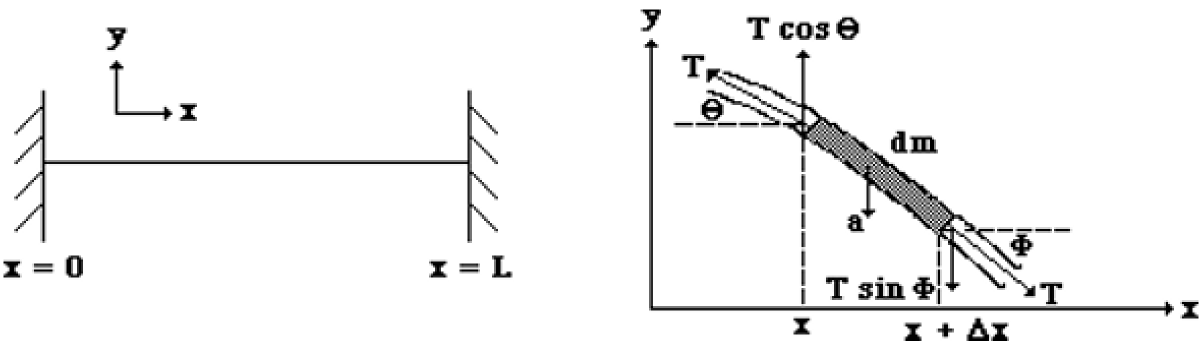
$$\phi = \tan^{-1} \left[\frac{a_1 \sin \phi_1 + a_2 \sin \phi_2}{a_1 \cos \phi_1 + a_2 \cos \phi_2} \right] \qquad (2.16)$$

Two harmonic oscillations of frequency ω_0 have initial phases ϕ_1 and ϕ_2 and amplitudes a_1 and a_2. Their resultant has the phase

(a) $\phi_1 - \phi_2 = 2n\pi$

and (b) $\phi_1 - \phi_2 = (2n + 1)\pi$

where n is an integer. Using Eq. **(2.15), show that the** amplitudes **of** the resultant oscillations are equal to $(a_1 + a_2)$ and $(a_1 - a_2)$, respectively, Consider a thin, flexible string (piano wire, rope, etc.) of length L, linear mass density μ, under tension T, which is fixed at both ends as shown in figure 1. Two questions we might ask are whether waves can exist in such a system and if so what is the form of the function y(x,t) which describes the propagation of the wave?



(a) (b)

Figure 1

If a system will support waves, then the equation describing the behavior of the system will have the form of the classical wave equation,

$$\frac{\partial^2 y}{\partial x^2} - 1/v^2 \frac{\partial^2 y}{\partial t^2} = 0 \qquad . \qquad (1)$$

Therefore, to answer the first question posed above, we need to derive the equation of motion for our string and compare its form to that of equation (1). To answer the second question, we need to look at the effect the fixed ends have on waves traveling down the string. In what follows we

Consider a small section of the string dm which has been displaced in the vertical direction as shown in figure 1a. The displacement is y = y(x,t). We will assume that the displacement is small and that θ and ϕ are everywhere small so that we can use the approximations $\cos\theta \approx \cos\phi \approx 1$, $\theta \approx \sin\theta \approx \tan\theta$ and $\phi \approx \sin\phi \approx \tan\phi$. The element dm is acted upon by two forces, the tension T at both ends. (Since the string is thin, gravitational forces can be neglected). The forces in the horizontal and vertical directions are

$$F_x = T\cos\phi - T\cos\theta \qquad (2a)$$

$$F_y = T\sin\theta - T\sin\phi \qquad (2b)$$

Since $\cos\theta \approx \cos\phi \approx 1$, the horizontal forces cancel leaving a net force only in the y direction. Applying the small angle approximation to equation (2b) yields

$$F_y = T(\tan\theta - \tan\phi) \quad . \quad (3)$$

But $\tan\theta = -\partial y/\partial x|_x$ and $\tan\phi = -\partial y/\partial x|_{x+\Delta x}$. Substituting these expressions into equation (3) gives

$$F_y = T\left(\left(-\frac{\partial y}{\partial x}\right)_x - \left(-\frac{\partial y}{\partial x}\right)_{x+\Delta x}\right) \quad . \quad (4)$$

To make the notation simpler, we define a function $g(x) = \partial y/\partial x|_x$. Substituting this into equation (4) and rearranging terms yields

$$F_y = T\left[g(x + \Delta x) - g(x)\right] \quad . \quad (5)$$

Applying Newton's second law gives

$$ma_y = T\left[g(x + \Delta x) - g(x)\right] \quad . \quad (6)$$

But $m = \mu\Delta x$ and $a_y = \partial^2 y/\partial t^2$. Substituting these expressions into equation (6) and rearranging terms yields

$$\frac{g(x + \Delta x) - g(x)}{\Delta x} - \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = 0 \quad . \quad (7)$$

Realizing that $\frac{g(x + \Delta x) - g(x)}{\Delta x} = \frac{\partial g(x)}{\partial x} = \frac{\partial^2 y}{\partial x^2}$, allows us to rewrite equation (7) in its final form,

$$\frac{\partial^2 y}{\partial x^2} - \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2} = 0 \quad . \quad (8)$$

If we let $\mu/T = 1/v^2$, then we see that the equation governing the motion of the string has the same form as the classical wave equation. Therefore, waves can exist in our system. The waves will travel with velocity $v = (T/\mu)^{1/2}$ and the function y(x,t) will be numerically equal to the y displacement at a time t of a point on the string at position x.

Now that we know that waves can exist in our system, we can turn our attention to the question of the form of the function y (x,t). The fact that the ends are fixed means that the y amplitude must always be zero at the ends. Therefore, only those functions for which y (0,t) = 0 = y (L,t) are suitable solutions. It can be shown by substitution that functions of the form y = A sin (Kx ± ωt) and y = B cos (Kx ± ωt) [where K(wave number) and 2π/λ and ω (angular frequency) = 2πν = v K], are solutions to equation (8). However, since the functions y = Bcos (Kx ± ωt) cannot always be zero when x = 0 we can eliminate that set of functions. To determine the form of the function y (x,t) for our system we must use waves of the form y = A sin (Kx ± ωt).

The function y_1 = A sin (Kx - ωτ) represents a continuous sine wave traveling to the right down the string, and y_2 = A sin (Kx + ωt) represents one traveling to the left. If these waves are perfectly reflected at the ends, we have two waves of equal frequency, amplitude and speed traveling in opposite directions on the same string. The principle of superposition of waves states that the resulting wave will be the algebraic sum of the individual waves,

$$y = y_1 + y_2 = A[\sin(Kx - \omega t) + \sin(Kx + \omega t)] \quad ;$$

or using the trigonometric identity for the sum of the sines of two angles (sin B + sin C = 2 sin 1/2(B+C) cos1/2(C-B)), we obtain

$$y = 2A \sin Kx \cos \omega t \quad . \quad (9)$$

This function obviously satisfies the boundary conditions at x = 0, but will only satisfy the boundary condition at x = L when K = nπ/L (where n = 1,2,3...). Limiting the values of K to only certain values also limits the wavelength, frequency and speed of the waves to certain discrete values. Therefore, unlike traveling waves on an infinite string which can have any wave- length or frequency, waves on a bounded string are quantized, restricted to only certain wavelengths and frequencies. To note this quantization, equation (9) can be rewritten as

$$y_n = 2A_n \sin K_n x \cos \omega_n t \quad (10)$$

Where $K_n = n\pi / L$, $\omega_n = K_n v = \frac{n\pi}{L}\sqrt{T/\mu}$ and $n = 1, 2, 3, \cdots$.

Equation (10) is the equation of a standing wave. Note that a particle at any particular point x executes simple harmonic motion as time passes and that all particles vibrate with the same frequency. Note also that the amplitude is not the same for different particles, but varies with the location x of the particle. The amplitude, $2 A_n \sin K_n x$, has a maximum value of $2A_n$ at positions where $Kx = \pi/2, 3\pi/2, 5\pi/2$ etc. or where $x = \lambda/4, 3\lambda/4, 5\lambda/4$, etc. These points are called antinodes and are spaced one half wavelength apart. The amplitude has a minimum value of zero at positions where $Kx= \pi, 2\pi , 3\pi$, etc. or $x = \lambda/2, , 3\lambda/2, 2\lambda,$ etc. These points are called nodes and are also spaced one half wavelength apart.

Finally, it should be noted that although equation (10) is a form of wave which can exist in the bounded string system, it is not the most general form. The most general form is

$$y_n(\mathbf{x}, t) = \sum_{n=1}^{\alpha} (a_n \cos \omega_n t + B_n \sin \omega_n t) \quad . \qquad (11)$$

Now that we have determined that waves can exist in our system and how they can be represented mathematically, we might ask what we would expect to see if we tried to create the waves in an actual string. Consider then a string fixed at both ends which is being driven by a force $F \cos \omega t$. If the driving frequency is such that the distance L between the ends is neither an integral or half-integral number of wavelengths, the initial and reflected waves will be "out of phase" and will destructively interfere with each other. No clear pattern will be set up. If however the string is driven with a frequency near ω_n so that L is an integral or half-integral number of wavelengths, the initial and reflected waves will be "in phase" and will constructively interfere. The standing wave $y_n(x,t)$ will be produced and will attain a large amplitude. If $n = 1$ then $L = \lambda/2$ and the string is said to be vibrating at its fundamental frequency. This is the lowest frequency for which a standing wave pattern can be set up in the string. If the string is driven at a frequency which an integral multiple of the fundamental frequency, standing waves with different patterns will be set up. The patterns for the first four frequencies are shown in figure 2.

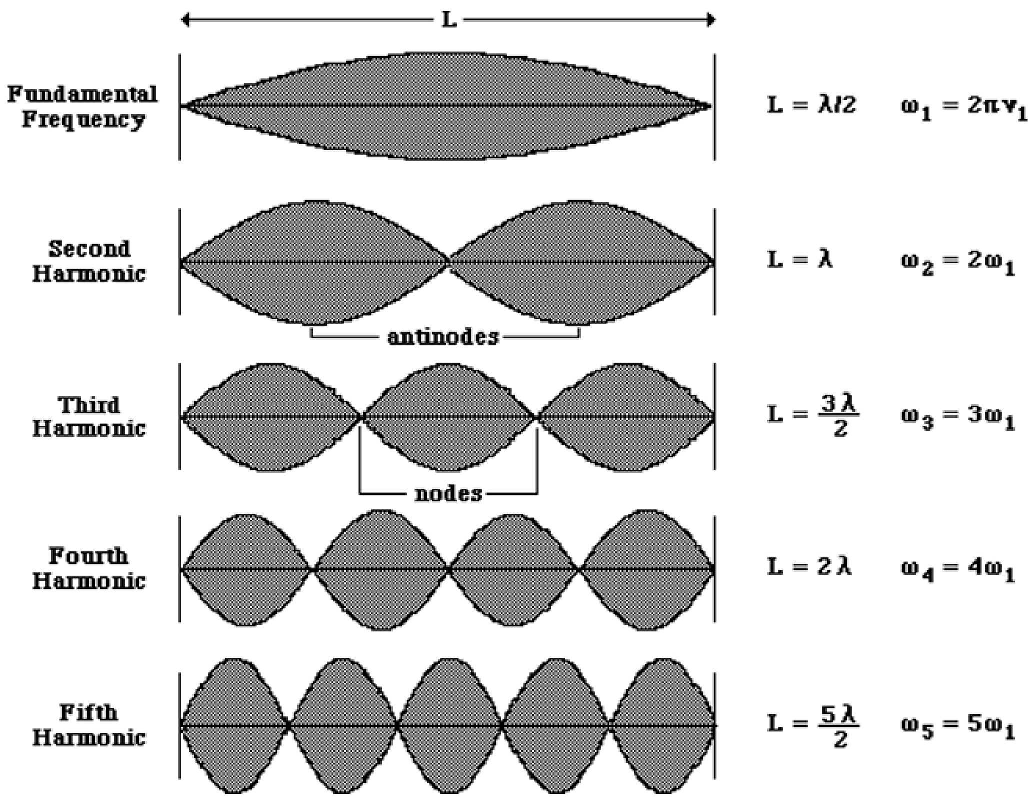


Figure 2

Finally it should be noted that if the string is plucked rather than driven by a periodic force, then in general the response y(x,t) will not be a single natural frequency but a sum of many natural frequencies.

$$y_n(x, t) = \sum_n (A_n \cos \omega_n t + B_n \sin \omega_n t) \sin K_n x \qquad . \qquad (11)$$

The observed pattern is very complicated in general. However, it is possible to pluck the string so as to have one natural frequency dominate.

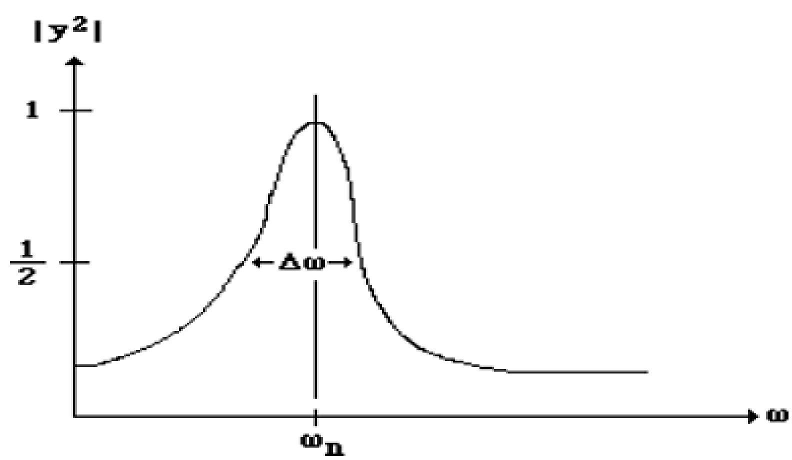
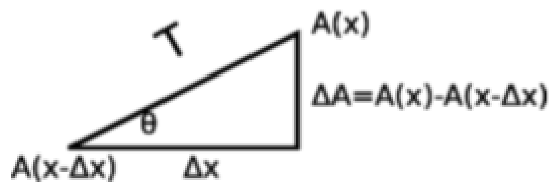


Figure 3

Consider a string of tension T. We define the amplitude of the string at a point x at time t as $A(x,t)$. In this section, we'll sometimes write $A(x,t)$ just as $A(x)$ to avoid clutter. Let us treat the string as a bunch of massless test probes connected by a elastic strings. Then we can draw the picture as

What is the force acting on the test mass at position x (in red)?

First, consider the *downward* component of the force pulling on the test mass at x from the mass to the left (at $x-\Delta x$). We can draw a triangle:



The force is given by

$$F_{\text{downwards, from left mass}} = T\sin\theta = T\frac{\Delta A}{\sqrt{\Delta A^2+\Delta x^2}} \tag{3}$$

If the system is close to equilibrium, then the slope will be small. That is, $\Delta A \ll \Delta x$. In this case, we can approximate $\sqrt{\Delta A^2+\Delta x^2}\approx \Delta x$ and so

$$F_{\text{downwards, from left mass}} = T\frac{\Delta A}{\Delta x} = T\frac{A(x)-A(x-\Delta x)}{\Delta x} = T\frac{\partial A}{\partial x} \tag{4}$$

where we have taken $\Delta x \to 0$ in the last step turning the difference into a derivative.

Similarly, the *downward* force from the mass on the right is

$$F_{\text{downwards, from right mass}} = T\frac{A(x)-A(x+\Delta x)}{\Delta x} = -T\frac{\partial A(x+\Delta x)}{\partial x} \tag{5}$$

Thus,

$$F_{\text{total downwards}} = -T\left[\frac{\partial A(x+\Delta x)}{\partial x} - \frac{\partial A}{\partial x}\right] \tag{6}$$

Now we use $F=ma$, where $a=-\frac{\partial^2 A}{\partial t^2}$ is the downward acceleration. So we should have

$$F_{\text{total downwards}} = -m\frac{\partial^2 A}{\partial t^2} = -\mu\Delta x\frac{\partial^2 A}{\partial t^2} \tag{7}$$

Plugging this into Eq. (6) we find

$$\frac{\partial^2 A}{\partial t^2} = \frac{T}{\mu}\left[\frac{\frac{\partial A(x+\Delta x)}{\partial x} - \frac{\partial A}{\partial x}}{\Delta x}\right] = \frac{T}{\mu}\frac{\partial^2 A}{\partial x^2} \tag{8}$$

Thus,

$$\boxed{\left[\frac{\partial^2}{\partial t^2} - v^2\frac{\partial^2}{\partial x^2}\right]A(x,t) = 0 \quad \text{with} \quad v=\sqrt{\frac{T}{\mu}}} \tag{9}$$

So the wave equation is again satisfied with a wave speed $v=\sqrt{\frac{T}{\mu}}$.

Note that the acceleration is due to a **difference of forces**. The force pulling up from the right has to be different from the force pulling down from the left to get an acceleration. Each force is proportional to a first derivative, thus the acceleration is proportional to a second derivative.

Now lets talk about standing wave solutions in more detail. Again, we consider the wave equation

$$\left[\frac{\partial^2}{\partial t^2} - v^2 \frac{\partial^2}{\partial x^2}\right] A(x,t) = 0 \tag{27}$$

and we would like solutions of fixed frequency ω. These are solutions which are periodic in time. We can write the general such solution as a sum of terms of the form

$$A(x,t) = A_0 \sin(kx + \phi_1)\sin(\omega t + \phi_2) \tag{28}$$

In this solution, A_0 is the **amplitude** and k the **wavenumber**. The frequency determined from the wavenumber through the dispersion relation

$$\omega = vk \tag{29}$$

There are two phases ϕ_1 and ϕ_2. Instead of using phases, we could write the general solution as

$$A(x,t) = A_0 \sin(kx)\sin(\omega t) + A_1 \sin(kx)\cos(\omega t) + A_2 \cos(kx)\cos(\omega t) + A_3 \cos(kx)\sin(\omega t) \tag{30}$$

The two forms are equivalent and we will go back and forth between them as convenient.

Consider first the case where one of the boundary conditions is that the string is fixed at $x = 0$. That is

$$\boxed{A(0,t) = 0} \tag{31}$$

This is known as a fixed, closed, or **Dirichlet** boundary condition. If there were a $A_3 \cos(kx)\sin(\omega t)$ component, then the $x = 0$ point would oscillate as $x(0,t) = A_3 \sin(\omega t)$ meaning it is not fixed. Thus $A_3 = 0$. Similarly, $A_2 = 0$. Thus the general solution with $A(0,t) = 0$ is

$$A(x,t) = A_0 \sin(kx)\sin(\omega t + \phi) \tag{32}$$

If we fix the other end of the string at $x = L$ then we must have $\sin(kL) = 0$ which implies

$$k = \frac{\pi}{L} n, \quad n = 1, 2, 3, \cdots \tag{33}$$

This tells us which frequencies can be produced

$$\boxed{\omega_n = v k_n = v \frac{\pi}{L} n, \quad n = 1, 2, 3, \cdots} \quad \text{both ends fixed} \tag{34}$$

This is the spectrum for 2 Dirichlet boundary conditions.

In this case if we take Δx to 0 we see that $\frac{\partial A}{\partial x} \to 0$. Thus a free end must satisfy

$$\boxed{\frac{\partial A(L,t)}{\partial x} = 0} \tag{37}$$

This is known as a free, open, or **Neumann** boundary condition.

Now using the $x = 0$ fixed solution, Eq. (32), the Neumann condition at $x = L$ implies

$$0 = \frac{\partial A(L,t)}{\partial x} = k A_0 \cos(kL)\sin(\omega t + \phi) \tag{38}$$

For this to hold at all times, $\cos(kL)$ must be at a zero of the cosine curve. Now, $\cos(x) = 0$ when $x = \left(n + \frac{1}{2}\right)\pi$. Thus,

$$\boxed{\omega_n = v \frac{n + \frac{1}{2}}{L}\pi, \quad n = 0, 1, 2, 3}, \quad \text{one fixed end, one free end} \tag{39}$$

This solution says that the lowest frequency is

$$\nu_0 = \frac{\omega_1}{2\pi} = \frac{1}{2} v \frac{\frac{1}{2}}{L} = \frac{1}{4} \frac{v}{L} \tag{40}$$

the next frequency up is

$$\nu_1 = \frac{1}{2} v \frac{1 + \frac{1}{2}}{L} = \frac{3}{4} \frac{\nu}{L} = 3\nu_0 \tag{41}$$

and so on. Thus the even harmonics are missing!! This has dramatic consequences for instruments like the trumpet and the clarinet.

Finally, if $x = 0$ is free, we must have $A(x, t) = A_0 \cos(kx) \sin(\omega t + \phi)$. Then, if $x = L$ is also free, we find $\sin(kL) = 0$ which implies

$$\boxed{\omega_n = v \frac{n}{L} \pi, \quad n = 0, 1, 2, 3}, \quad \text{both ends free} \tag{42}$$

The only difference between both free ends solution and both fixed end solution is that for free ends $n = 0$ is allowed. However, the $n = 0$ solution is $A(x, t) = \text{const}$ which has $k = \omega = 0$ thus it is not physically interesting.

Here are the lowest harmonics with different boundary conditions

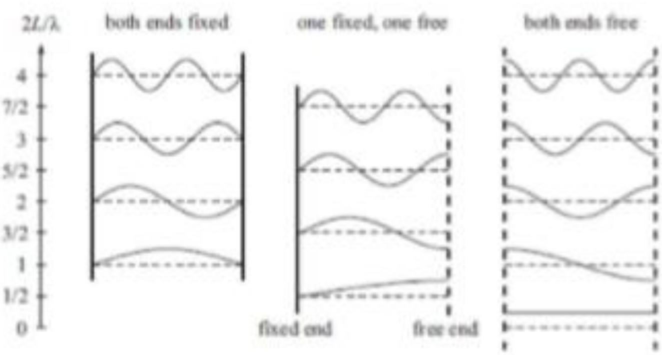


Figure 2. Frequencies allowed for different boundary conditions

If the fundamental note (lowest frequency) is ν, then we find

	lowest	next	second	third
both fixed	ν	2ν	3ν	4ν
one fixed, one free	ν	3ν	5ν	7ν
both free	ν	2ν	3ν	4ν

To work out the resonant frequency for a Helmholtz resonator we can use $\omega = \sqrt{\frac{k}{m}}$. We can extract the spring constant k from $F = -k\Delta x$. For pressure, $F = A \cdot dp$, where A is the area, in this case the cross sectional area of the neck. Now, $\rho = \frac{m}{V}$ so

$$d\rho = \frac{d}{dV}\left(\frac{m}{V}\right)dV = -\frac{m}{V^2}dV = -\rho\frac{dV}{V} \tag{44}$$

Also using Eq. (16), $\frac{dp}{d\rho} = \gamma\frac{p}{\rho}$ for sound waves, we have

$$dp = \frac{dp}{d\rho}d\rho = \gamma\frac{p}{\rho}d\rho = -\gamma\frac{p}{V}dV \tag{45}$$

Now, $dV = A\Delta x$ and so

$$F = A \cdot dp = -\gamma A^2\frac{p}{V}\Delta x \tag{46}$$

Thus

$$k = \gamma A^2\frac{p}{V} = A^2 c_s^2\frac{\rho}{V} \tag{47}$$

The mass on which the spring acts is the air in the neck. It has mass $m = \rho A L$, thus

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{A^2 c_s^2 \rho / V}{\rho A L}} = c_s\sqrt{\frac{A}{VL}} \tag{48}$$

Thus Helmholtz resonators resonate at a single frequency

$$\boxed{\nu = \frac{c_s}{2\pi}\sqrt{\frac{A}{VL}}} \tag{49}$$

where A is the area of the opening, L is the length of the neck, and V is the volume of the cavity.

For example, consider a 10 cm wide jar with a 10 cm long neck. Using $v = 343\,\frac{m}{s}$, $A = 1\text{cm}^2$, $L = 1\text{cm}$, $V = 1L = 1000\text{cm}^3$, we find $\nu = 172\,\text{Hz}$. The associated wavelength in air is $\lambda = \frac{c_s}{\nu} = 2m$. Note that the wavelength of sound in the resonator is much larger than the size of the resonator.

Since Helmholtz resonators have only one frequency, they have no harmonics (no overtones). However, they can have low Q values. Indeed, if you blow on a bottle, you see that the sound does not resonate for long at all. This is good, if you are building an instrument, since you want all the audible frequencies to resonate. On a violin, the vibrations are produced on the strings, transmitted to the wooden body of the violin through the bridge (the part of the violin which connects the strings to the body). The body then vibrates, exciting the air in the body which emits sound through the holes. I can't describe the function of the body of a violin better than Heller. Here's his description [Heller, p. 267]

> Helmholtz resonators can be used as transducers, turning mechanical energy into sound energy. A prime example is the violin. The violin body is basically a box containing air, with the f-holes opening to the outside. It functions deliberately as a Helmholtz resonator, enhancing the low frequency response of the violin, giving it much of its richness of tone...
>
> The violin body's broad Helmholtz resonance peaks around 300 Hz. No doubt the shape thin but long holes serve to increase air friction and thus lower the Q of the Helmholtz mode, spreading the resonance over a broader frequency range. This props up the transduction of string vibrations into sound down to the frequency of the open D string [$\nu \sim 300\text{Hz}$].

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT – 3

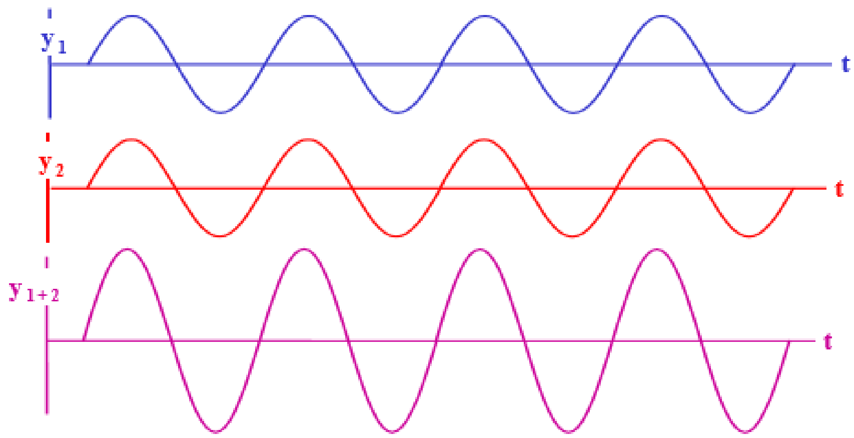
Oscillation and waves applications

Oscillations and Waves – SPH1212

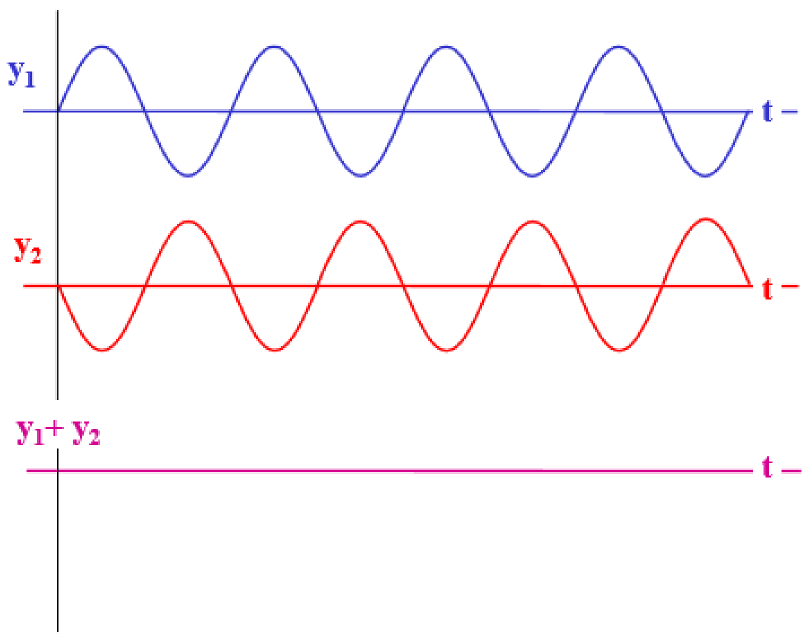
Oscillation and waves applications

Reflection of Waves

Waves obey something we call the **principle of linear superposition**. That is, if two waves are in the same region of space at the same time, they will interact with each other. Linear superposition allows us to describe how they interact fairly easily. If we were to plot the two waves as a function of time, they might look like the top two waves in the picture to the right. Linear superposition means that we just add the value of each wave and plot that as the sum. So the result of this combination of waves is another sine wave, but with an amplitude that is the sum of the amplitudes of the two starting waves.



Let's repeat this, but shift one of the waves by 180°



This time, the maximum of the first wave is at the minimum of the second and vice-versa, so when you add them up, you get zero.

When the waves interact so that the sum is larger than the original waves, we call that **constructive interference**. When they interact so that the sum is smaller, we call that **destructive interference**. You can have everything

One of the reasons we care about how the waves interact with each other is because there are a number of places where waves travel into an object – like an organ pipe – and travel out again. Use the rope as an example. If I shake the rope, a pulse traveling down the rope will reach the fixed end and will reflect back inverted. If I keep shaking the rope, I set up a wave train such that, when one pulse reaches the end of the line and turns around, it will interfere with one of the pulses still heading toward the wall. There will be some points along the rope where the waves interact constructively and some points where they interact destructively. The result is that there are some points on the rope that are always standing still. We call these **nodes**. There are other points at which the wave has maximum values, which we call **anti-nodes**. The waves that result from this are called standing waves.

If I move my hand faster up and down, you see that I can change the number of nodes and antinodes. The length of the rope limits the configurations I can set up. The **fundamental** is the configuration in which there are no nodes (except the two at the end). When you pluck a guitar string, for example, you are exciting the fundamental. If you change the length of the string by holding it at one of the frets, you change the wavelength and thus the frequency heard.

Some nomenclature:

Any frequency that is an integral multiple of the fundamental is called a harmonic.

The first harmonic is the frequency, which we'll denote as f_1.

The second harmonic has a frequency exactly twice the fundamental, so $f_2 = 2f_1$

The first harmonic is the situation in which there is one node. Two nodes denote the second harmonic, etc.

The other piece of nomenclature is the idea of an overtone. Overtones are the harmonics above the fundamental frequency. The first overtone for a wave on a string is the second harmonic. The second overtone for a wave on a string is the third harmonic

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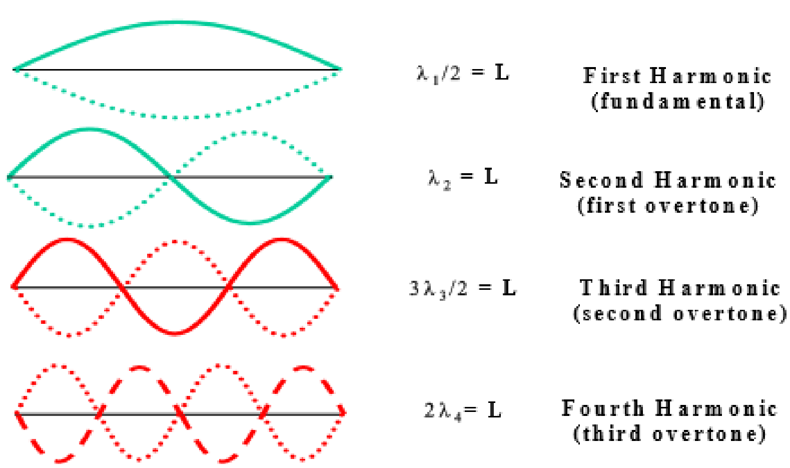
You'll notice that we don't have many options here. There are either one, two, three, etc. nodes on our string. This limits the number of patterns we can have. Let's investigate how many patterns are possible and the conditions under which they are produced. The chart at left shows that there is a pattern. The n^{th} harmonic is related to the length as

$$L = \frac{n\lambda_n}{2}$$

where n = 1 for the first harmonic, 2 for the second harmonic, etc.



The wavelengths for each harmonic are given by:

$$L = \frac{n\lambda_n}{2}$$
$$\frac{2L}{n} = \lambda_n$$

The n^{th} harmonic will always have n loops in the wave pattern.

Note that the frequency and the wavelength of each wave on the string is different, but that the all the waves have the same velocity.

$$v = f_1\lambda_1 = f_2\lambda_2 = f_3\lambda_3$$

and so on.

We can related the harmonics to the fundamental as follows:

$$f_n = \frac{v}{\lambda_n}; \text{substitute in } \lambda_n = \frac{2L}{n}$$
$$f_n = \frac{v}{\frac{2L}{n}}$$
$$f_n = n\left(\frac{v}{2L}\right) = nf_1$$

EXAMPLE : A guitar string has a fundamental frequency of 440 Hz and a length of 0.50 m.

a) Draw the picture of the first five overtones and find their frequencies.

b) What are the wavelengths of the waves on the string?

c) What is the velocity of waves on the string?

d) What is the velocity of the sound waves produced by the string?

Solution a: The first three overtones are given by the picture above. The fourth overtone (which is the same as the fifth harmonic) will have four nodes/five loops. The fifth overtone (which is the same as the sixth harmonic) will have five nodes and six loops

A harmonic is an integral multiple of the fundamental. We will always have that $f_n = nf_1$, so

$$f_2 = 2f_1 = 2(440\,\text{Hz}) = 880\,\text{Hz}$$
$$f_3 = 3f_1 = 3(440\,\text{Hz}) = 1320\,\text{Hz}$$
$$f_4 = 4f_1 = 4(440\,\text{Hz}) = 1760\,\text{Hz}$$
$$f_5 = 5f_1 = 5(440\,\text{Hz}) = 2200\,\text{Hz}$$
$$f_6 = 6f_1 = 6(440\,\text{Hz}) = 2640\,\text{Hz}$$

Notice that the difference between any two harmonics that differ by one will always be equal to the fundamental frequency.

$$f_4 - f_3 = 4f_1 - 3f_1 = f_1$$
$$f_3 - f_2 = 3f_1 - 2f_1 = f_1$$

Solution b: The wavelength of the waves can be found from

$$\lambda_n = \frac{2L}{n}$$
$$\lambda_1 = \frac{2L}{1} = 2(0.50\,m) = 1.0\,m$$
$$\lambda_2 = \frac{2L}{2} = 0.50\,m$$
$$\lambda_3 = \frac{2L}{3} = \frac{2}{3}0.50\,m = 0.33\,m$$
$$\lambda_4 = \frac{2L}{4} = \frac{1}{2}0.50\,m = 0.25\,m$$
$$\lambda_5 = \frac{2L}{5} = \frac{2}{5}0.50\,m = 0.20\,m$$

Solution c: The velocity of waves on the string is given by

$$v = f_1\lambda_1 = (440\,Hz)(1.00\,m) = 440\,\tfrac{m}{s}$$

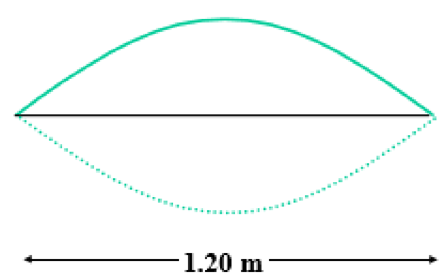
Note that you get the same thing if you multiply any f_n and λ_n!

Solution d: The velocity of the sound waves produced will be 340 m/s, which is the general speed of sound at 15°C. Don't confuse the two velocities!

EXAMPLE A nylon string is stretched between supports 1.20 m apart.

a) what is the wavelength of waves on this string?

b) If the speed of transverse waves on a string is 850 m/s, what is the frequency of the first harmonic and the first two overtones?



a) To determine the wavelength, draw the fundamental.

The fundamental is one half of a wavelength. The wavelength is therefore twice the length of the string, or 2.40 m.

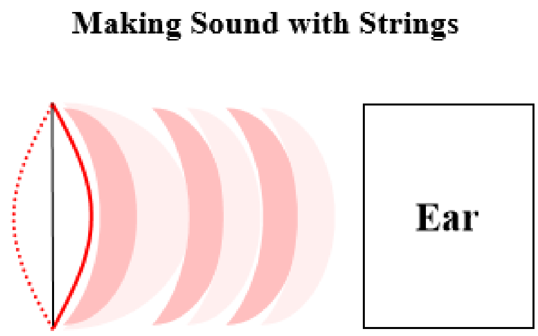
b) The frequency of the first harmonic is given by

$$f_1 = \frac{v}{\lambda_1}$$
$$f_1 = \frac{850\,\tfrac{m}{s}}{2.40\,m} = 350\,Hz$$

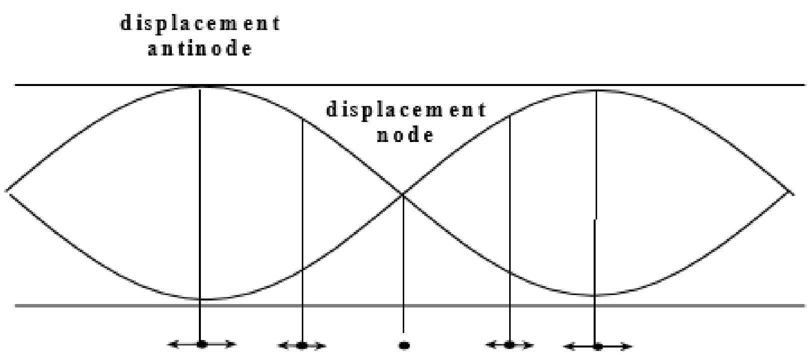
The frequency of the first two overtones are given by $f_2 = 2f_1 = 700$ Hz and $f_3 = 3f_1 = 1050$ Hz

Waves in Tubes

String instruments produce sound by causing vibrations in the string. These vibrations excite the air around the string, causing the air around the strings to be alternately compressed and rarefied creating sound waves. Note that the velocity of waves on a string is not the same thing as the velocity of sound waves.



In wind instruments, the pressure variations are controlled by using a column. Consider a tube of length L that is open at both ends. When you blow into the tube, you create a longitudinal wave. The sound wave is thus created directly. In a string instrument, you create a transverse wave on the string, which then excites the air surrounding the string and creates the sound wave (which is longitudinal). The sound wave is created directly by wind instruments.



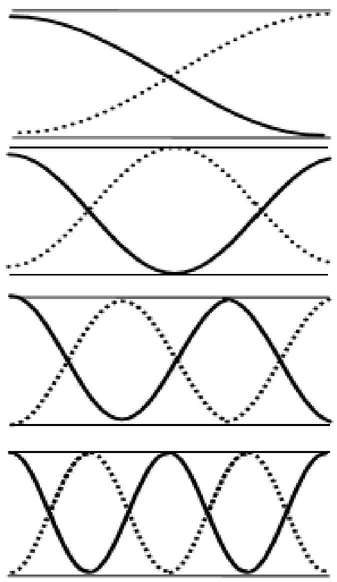
Longitudinal waves are variations in the density of the air in a given part of the tube. If you set up longitudinal standing waves, we find an analogous situation to the transverse standing waves seen on a string. If we could take a picture of the movement of the air molecules in each part of the standing wave, we would find the following: at some points along the standing wave, there is no motion of the molecules at that position. This is called displacement node, exactly like the node along a string when the string doesn't move. Similarly, there are points along the tube where the molecules oscillate at their maximum amplitudes.

These are displacement antinodes. We can plot the amplitude of the motion of the molecules to illustrate this. Note that the diagrams for the production of sound by wind instruments are different, because we're plotting displacement waves and not the actual shape of the air.

Waves in a pipe open at both ends.

We're going to be working in the limit of the tube length being much greater than the diameter of the tube. This allows us to ignore effects at the ends of the tube that would complicate our description.

At the open end of a column of air, the air molecules can move freely, so there will be a displacement antinode at the open end of a pipe. We can use the same approach to determine the modes of a tube of length L open at both ends are as we did in finding the waves on a string - draw the possibilities.



First Harmonic

$\lambda_1/2 = L$

$f_1 = v/\lambda_1 = v/2L$

Second Harmonic

$\lambda_2 = L$

$f_2 = v/\lambda_2 = v/L = 2v/2L$

Third Harmonic

$3\lambda_3/2 = L$

$f_3 = v/\lambda_3 = 3v/2L$

Fourth Harmonic

$2\lambda_4 = L$

$f_4 = v/\lambda_4 = 2v/L = 4v/2L$

For the first harmonic (fundamental), we have half of a wavelength in the tube

$$L = \frac{\lambda}{2}$$

For the second mode, we have a full wavelength in the tube

$$L = \lambda = \frac{2\lambda}{2}$$

For the third mode,

$$L = \frac{3\lambda}{2}$$

so in general, we can extrapolate this to:

$$L = n\frac{\lambda_n}{2}$$

$$f_n = n\frac{v}{2L}$$

These formulas are good for waves in a **tube open on both ends.**

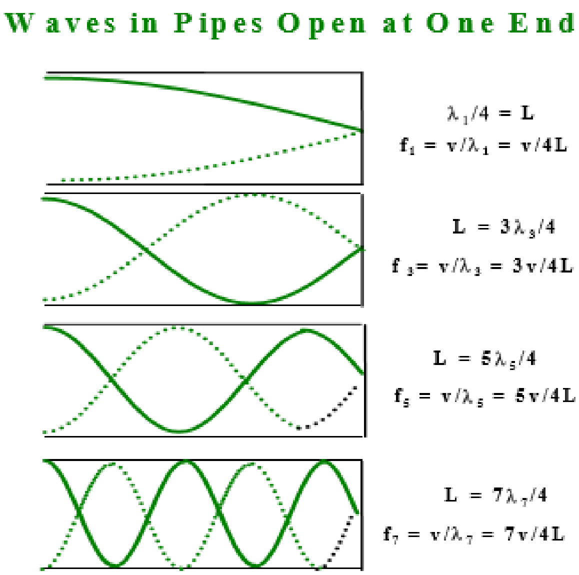
Examples of instruments with pipes open at both ends:

- flute
- trumpet
- organ pipes

You change the length of the tube by pressing keys. In a flute, closing a key elongates the tube. In a trumpet or French horn, pressing keys adds additional lengths of tubing to the pipe.

Standing Waves in a Pipe Open on One End

We can also have pipes that are closed on one end and open on the other. (Closed on two ends wouldn't make any sense.) This is a slightly different case, because at the closed end, we're required to have a displacement **node**, which will change the wave patterns allowed. Although the frequencies of waves in a pipe open at two ends are the same as those of a string with the same length, the case of waves in a pipe open at only one end will be quite different. We can draw the allowed patterns as shown.



In general,

$$L = n\frac{\lambda_n}{4}$$

$$f_n = n\frac{v}{4L}$$

but n can only be odd! Therefore, we talk about this case having only odd harmonics. There are only λ_1, λ_3, λ_5…We call λ_3 the first overtone, λ_5 the second overtone, etc.

Examples of instruments with pipes closed at one end include organ pipes

EXAMPLE 36-4: a) Calculate the fundamental frequency and the first three overtones of a hollow pipe open at one end having length 30.0 cm. b) Calculate the wavelength of each wave.

We have

$$f_n = n\frac{v}{4L}\text{, but we are restricted to n odd}$$

so

$$f_1 = \frac{v}{4L} = \frac{340\,\frac{m}{s}}{4(0.30\,m)} = 283\,Hz$$

$$f_3 = 3\frac{v}{4L} = 3\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 850\,Hz$$

$$f_5 = 5\frac{v}{4L} = 5\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 1420\,Hz$$

$$f_7 = 7\frac{v}{4L} = 7\frac{340\,\frac{m}{s}}{4(0.30\,m)} = 1980\,Hz$$

Bars Chladni in 1787, and Biot in 1816, conducted experiments on the longitudinal vibration of rods. In 1824, Navier, presented an analytical equation and its solution for the longitudinal vibration of rods.

Shafts Charles Coulomb did both theoretical and experimental studies in 1784 on the torsional oscillations of a metal cylinder suspended by a wire [5]. By assuming that the resulting torque of the twisted wire is proportional to the angle of twist, he derived an equation of motion for the torsional vibration of a suspended cylinder. By integrating the equation of motion, he found that the period of oscillation is independent of the angle of twist. The derivation of the equation of motion for the torsional vibration of a continuous shaft was attempted by Caughy in an approximate manner in 1827 and given correctly by Poisson in 1829. In fact, Saint-Venant deserves the credit for deriving the torsional wave equation and finding its solution in 1849.

Beams The equation of motion for the transverse vibration of thin beams was derived by Daniel Bernoulli in 1735, and the first solutions of the equation for various support conditions were given by Euler in 1744. Their approach has become known as the *Euler–Bernoulli* or *thin beam theory*. Rayleigh presented a beam theory by including the effect of rotary inertia. In 1921, Stephen Timoshenko presented an improved theory of beam vibration, which has become known as the *Timoshenko* or *thick beam theory*, by considering the effects of rotary inertia and shear deformation.

Membranes In 1766, Euler, derived equations for the vibration of rectangular membranes which were correct only for the uniform tension case. He considered the rectangular membrane instead of the more obvious circular membrane in a drumhead, because he pictured a rectangular membrane as a superposition of two sets of strings laid in perpendicular directions. The correct equations for the vibration of rectangular and circular membranes were derived by Poisson in 1828. Although a solution corresponding to axisymmetric vibration of a circular membrane was given by Poisson, a nonaxisymmetric solution was presented by Pagani in 1829.

The method of placing sand on a vibrating plate to find its mode shapes and to observe the various intricate modal patterns was developed by the German scientist Chladni in 1802. In his experiments, Chladni distributed sand evenly on horizontal plates. During vibration, he observed regular patterns of modes because of the accumulation of sand along the nodal lines that had no vertical displacement. Napoléon Bonaparte, who was a trained military engineer, was present when Chladni gave a demonstration of his experiments on plates at the French Academy in 1809. Napoléon was so impressed by Chladni's demonstration that he gave a sum of 3000 francs to the French Academy to be presented to the first person to give a satisfactory mathematical theory of the vibration of plates. When the competition was announced, only one person, Sophie Germain, entered the contest by the closing date of October 1811 [8]. However, an error in the derivation of Germain's differential equation was noted by one of the judges, Lagrange. In fact, Lagrange derived the correct form of the differential equation of plates in 1811. When the academy opened the competition again, with a new closing date of October 1813, Germain entered the competition again with a correct form of the differential equation of plates. Since the judges were not satisfied, due to the lack of physical justification of the assumptions she made in deriving the equation, she was not awarded the prize. The academy opened the competition again with a new closing date of October 1815. Again, Germain entered the contest. This time she was awarded the prize, although the judges were not completely satisfied with her theory. It was found later that her differential equation for the vibration of plates was correct but the boundary conditions she presented were wrong. In fact, Kirchhoff, in 1850, presented the correct boundary conditions for the vibration of plates as well as the correct solution for a vibrating circular plate.

The great engineer and bridge designer Navier (1785–1836) can be considered the originator of the modern theory of elasticity. He derived the correct differential equation for rectangular plates with flexural resistance. He presented an exact method that transforms the differential equation into an algebraic equation for the solution of plate and other boundary value problems using trigonometric series. In 1829, Poisson extended Navier's method for the lateral vibration of circular plates.

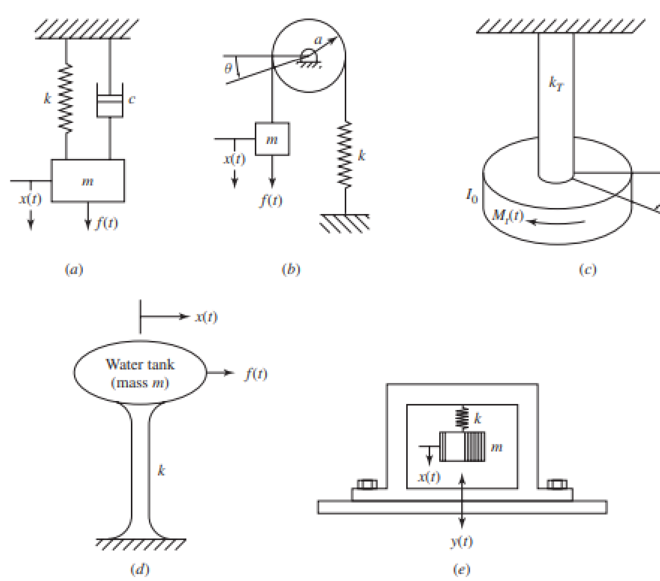


Figure 2.1 Single-degree-of-freedom systems.

VIBRATION OF A SINGLE-DEGREE-OF-FREEDOM SYSTEM

The number of degrees of freedom of a vibrating system is defined by the minimum number of displacement components required to describe the configuration of the system during vibration. Each system shown in Fig. 2.1 denotes a single-degree-of-freedom system. The essential features of a vibrating system include (1) a mass m, producing an inertia force: $m\ddot{x}$; (2) a spring of stiffness k, producing a resisting force: kx; and (3) a damping mechanism that dissipates the energy. If the equivalent viscous damping coefficient is denoted as c, the damping force produced is $c\dot{x}$.

Free Vibration

In the absence of damping, the equation of motion of a single-degree-of-freedom system is given by

$$m\ddot{x} + kx = f(t) \tag{2.1}$$

where $f(t)$ is the force acting on the mass and $x(t)$ is the displacement of the mass m. The free vibration of the system, in the absence of the forcing function $f(t)$, is governed by the equation

$$m\ddot{x} + kx = 0 \tag{2.2}$$

The solution of Eq. (2.2) can be expressed as

$$x(t) = x_0 \cos \omega_n t + \frac{\dot{x}_0}{\omega_n} \sin \omega_n t \tag{2.3}$$

where ω_n is the natural frequency of the system, given by

$$\omega_n = \sqrt{\frac{k}{m}} \tag{2.4}$$

$x_0 = x(t = 0)$ is the initial displacement and $\dot{x}_0 = dx(t = 0)/dt$ is the initial velocity of the system. Equation (2.3) can also be expressed as

$$x(t) = A \ \cos(\omega_n t - \phi) \tag{2.5}$$

Forced Vibration under Harmonic Force

For an undamped system subjected to the harmonic force $f(t) = f_0 \cos \omega t$, the equation of motion is

$$m\ddot{x} + kx = f_0 \cos \omega t \tag{2.22}$$

where f_0 is the magnitude and ω is the frequency of the applied force. The steady-state solution or the particular integral of Eq. (2.22) is given by

$$x_p(t) = X \cos \omega t \tag{2.23}$$

where

$$X = \frac{f_0}{k - m\omega^2} = \frac{\delta_{st}}{1 - (\omega/\omega_n)^2} \tag{2.24}$$

denotes the maximum amplitude of the steady-state response and

$$\delta_{st} = \frac{f_0}{k} \tag{2.25}$$

indicates the static deflection of the mass under the force f_0. The ratio

$$\frac{X}{\delta_{st}} = \frac{1}{1 - (\omega/\omega_n)^2} \tag{2.26}$$

represents the ratio of the dynamic to static amplitude of motion and is called the *amplification factor*, *magnification factor*, or *amplitude ratio*. The variation of the amplitude

Properties of Eigenvalues and Eigenfunctions

The fundamental properties of eigenvalues and eigenfunctions of Sturm–Liouville problems are given below.

1. Regular and periodic Sturm–Liouville problems have an infinite number of distinct real eigenvalues $\lambda_1, \lambda_2, \ldots$ which can be arranged as

$$\lambda_1 < \lambda_2 < \cdots$$

The smallest eigenvalue λ_1 is finite and the largest one is infinity:

$$\lim_{n\to\infty} \lambda_n = \infty$$

2. A unique eigenfunction exists, except for an arbitrary multiplicative constant, for each eigenvalue of a regular Sturm–Liouville problem.

3. The infinite sequence of eigenfunctions $w_1(x), w_2(x), \ldots$ defined over the interval $a \le x \le b$ are said to be orthogonal with respect to a weighting function $r(x) \ge 0$ if

$$\int_a^b r(x) w_m(x) w_n(x)\, dx = 0, \qquad m \neq n \tag{6.27}$$

When $m = n$, Eq. (6.27) defines the *norm* of $w_n(x)$, denoted $||w_n(x)||$, as

$$||w_n(x)||^2 = \int_a^b r(x) w_m^2(x)\, dx > 0 \tag{6.28}$$

By normalizing the function $w_m(x)$ as

$$\overline{w}_m(x) = \frac{w_m(x)}{||w_m(x)||}, \qquad m = 1, 2, \ldots \tag{6.29}$$

FLEXURAL WAVES IN BEAMS

The equation of motion for the transverse motion of a thin uniform beam, according to Euler–Bernoulli theory, is given by

$$\frac{\partial^4 w(x,t)}{\partial x^4} + \frac{1}{c^2}\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{16.84}$$

where

$$c = \sqrt{\frac{EI}{\rho A}} \tag{16.85}$$

It can be observed that Eq. (16.84) differs from the one-dimensional wave equation, Eq. (16.1), studied earlier in terms of the following:

1. Equation (16.84) contains a fourth derivative with respect to x instead of the second derivative.
2. The constant c does not have the dimensions of velocity; its dimensions are in^2/sec and not the in./sec required for velocity.

Thus, the general solution of the wave equation,

$$w(x,t) = f(x - ct) + g(x + ct) \tag{16.86}$$

will not be a solution of Eq. (16.84). As such, we will not be able to state that the motion given by Eq. (16.84) consists of waves traveling at constant velocity and without alteration of shape. Consider the solution of Eq. (16.84) for an infinitely long beam in the form of a harmonic wave traveling with velocity v in the positive x direction:

$$w(x,t) = A\cos\frac{2\pi}{\lambda}(x - vt) \equiv A\cos(kx - \omega t) \tag{16.87}$$

where A is a constant, λ is the wavelength, v is the phase velocity, k is the wave number, and ω is the circular frequency of the wave, with the following interrelationships:

$$\omega = 2\pi f = kv \tag{16.88}$$

$$k = \frac{2\pi}{\lambda} \tag{16.89}$$

Substitution of Eq. (16.87) into Eq. (16.84) yields the velocity, also called the *wave velocity* or *phase velocity*, as

$$v = \frac{2\pi}{\lambda}c = \frac{2\pi}{\lambda}\sqrt{\frac{EI}{\rho A}} \tag{16.90}$$

Thus, unlike in the case of transverse vibration of a string, the velocity of propagation of a harmonic flexural wave is not a constant but varies inversely as the wavelength. The material or medium in which the wave velocity v depends on the wavelength is called a *dispersive medium*. Physically, it implies that a nonharmonic flexural pulse (of arbitrary shape) can be considered as the superposition of a number of harmonic waves of different wavelengths. Since each of the component harmonic waves has different phase velocity, a flexural pulse of arbitrary shape cannot propagate along the beam without dispersion, which results in a change in the shape of the pulse.

A pulse composed of several or a group of harmonic waves is called a *wave packet*, and the velocity with which the group of waves travel is called the *group velocity* [4, 5]. The group velocity, denoted by v_g, is the velocity with which the energy is propagated, and its physical interpretation can be seen by considering a wave packet composed of two simple harmonic waves of equal amplitude but slightly different frequencies $\omega + \Delta\omega$ and $\omega - \Delta\omega$. The waves can be described as

$$w_1(x, t) = A\cos(k_1 x - \omega_1 t) \tag{16.91}$$

$$w_2(x, t) = A\cos(k_2 x - \omega_2 t) \tag{16.92}$$

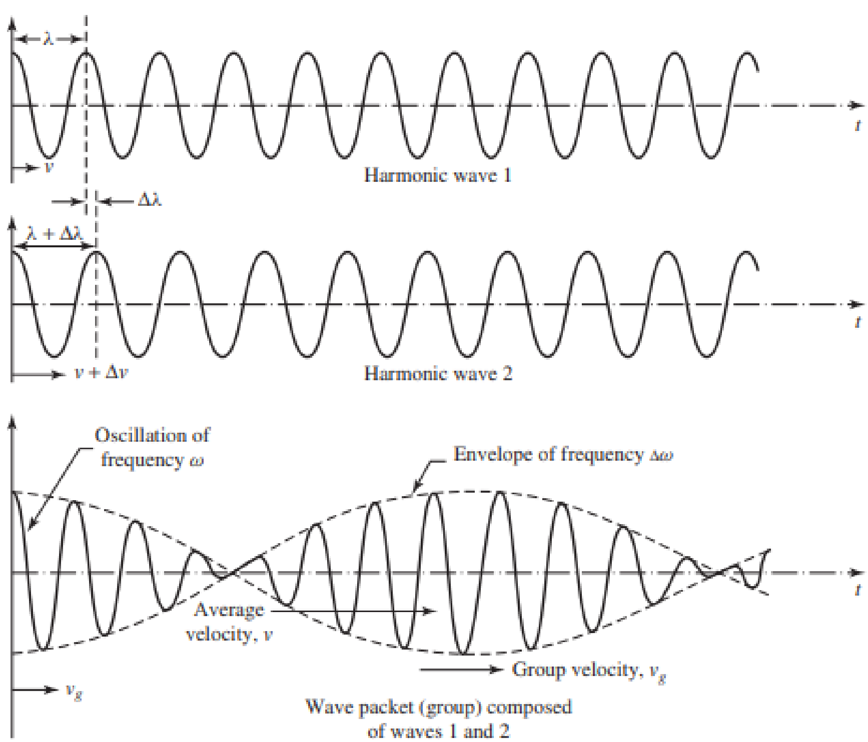


Figure 16.13 Wave packet and group velocity.

Longitudinal Waves A theory of propagation of longitudinal stress waves in a cylindrical rod with several step changes in the cross-sectional area was developed by Beddone [10]. The analysis obtained a transient solution of the one-dimensional wave equation by means of Laplace transform methods based on the concepts of traveling waves and reflection and transmission coefficients.

Wave Propagation in Periodic Structures The problem of free coupled longitudinal and flexural waves of a periodically supported beam was studied by Lee and Yeen [11]. It was shown that the characteristic or dispersion equation can be factorized into product form, which simplifies the analysis and classification of the dynamic nature of the system. Sen Gupta [12] studied the propagation of flexural waves in doubly periodic structures consisting of the repetition of a basic unit that is a periodic structure in itself. The analysis is simplified by introducing a direct and a cross-chain receptance for multispan structures and by utilizing the concept of the equivalent internal restraint.

Wave Propagation Under Moving Loads Ju used the three-dimensional finite element method to simulate the soil vibrations due to high-speed trains moving across bridges in Ref. [13]. He first analyzed a bridge system passed by trains. Then the pier forces and moments calculated were applied to a pile cap to simulate wave propagation in the soil.

Waves Through Plate or Beam Junctions In a study of elastic wave transmission through plate–beam junctions by Langley and Heron [14], a generic plate–beam junction was considered to be composed of an arbitrary number of plates either coupled through a beam or coupled directly along a line. The effects of shear deformation, rotary inertia, and warping were included in the analysis of the beam, and due allowance was made for offsets between the plate attachment lines and the shear axis of the beam.

Vibration Analysis Using a Wave Equation Langley showed that the vibrations of beams and plates may be analyzed in the frequency domain by using a wave equation instead of the conventional differential equations of motion provided that certain assumptions are made regarding the response of the system in the vicinity of a structural discontinuity [15].

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT - 4

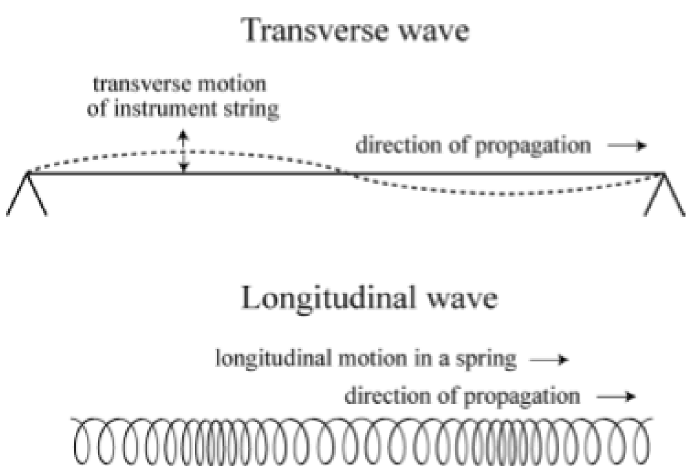
Oscillations and Waves – SPH1212

UNIT-IV

Acoustics

Acoustics: the study of sound waves

Sound is the phenomenon we experience when our ears are excited by vibrations in the gas that surrounds us. As an object vibrates, it sets the surrounding air in motion, sending alternating waves of compression and rarefaction radiating outward from the object. Sound information is transmitted by the amplitude and frequency of the vibrations, where amplitude is experienced as loudness and frequency as pitch. The familiar movement of an instrument string is a transverse wave, where the movement is perpendicular to the direction of travel (See Figure 1). Sound waves are longitudinal waves of compression and rarefaction in which the air molecules move back and forth parallel to the direction of wave travel centered on an average position, resulting in no net movement of the molecules. When these waves strike another object, they cause that object to vibrate by exerting a force on them.



Examples of transverse waves:

vibrating strings
water surface waves
electromagnetic waves
seismic S waves

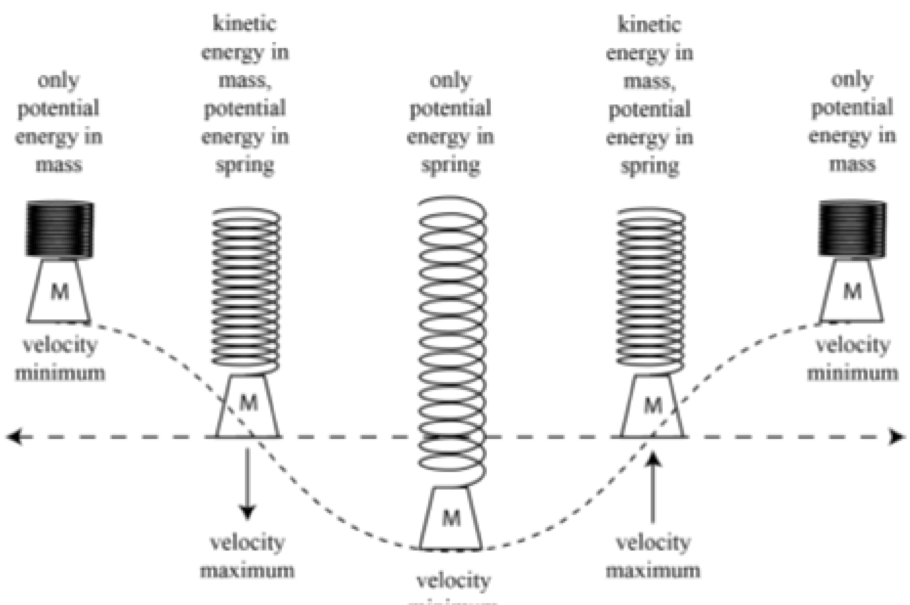
Examples of longitudinal waves:

waves in springs
sound waves
tsunami waves
seismic P waves

Figure 1: Transverse and longitudinal waves

The forces that alternatively compress and stretch the spring are similar to the forces that propagate through the air as gas molecules bounce together. (Springs are even used to simulate reverberation, particularly in guitar amplifiers.) Air molecules are in constant motion as a result of the thermal energy we think of as heat. (Room temperature is hundreds of degrees above absolute zero, the temperature at which all motion stops.) At rest, there is an average distance between molecules although they are all actively bouncing off each other. Regions of compression (also called condensation) and rarefaction (expansion) radiate away from a sound source in proportion to the movement of the source. It is the net force exerted by the moving air that acts on our ears and on transducers like microphones to produce the sensation of hearing and the electrical signals that become sound recordings. The same physics that describe oscillating mechanical systems like springs can be used to describe the behavior of gases like air: the equations derived to describe weights on springs can also be used to describe acoustics. Furthermore, electrical circuits exhibit similar behavior and can be described using very similar mathematical equations. This helps unify the field of sound recording, since mechanical, acoustical and electrical systems are all employed in the recording of sound.

Figure 2 shows how energy is interchanged between kinetic and potential energy in an oscillating system. Potential energy is energy that is capable of doing work, while kinetic energy is the result of active motion. As a mass suspended on a spring bounces up and down, it exchanges the potential energy of a raised mass and tension stored in a spring with the kinetic energy of the moving mass back and forth until friction depletes the remaining energy. At the top of its vertical motion, the mass has only potential energy due to the force of gravity while the spring is relaxed and contains no energy. As the mass falls, it acquires kinetic energy while tension builds in the spring. At the mid-point of its fall, the mass reaches its maximum velocity and then begins to slow as the force exerted by the spring's expansion builds to counter the force of gravity. At the bottom of its travel, the mass stops moving and therefore no longer has kinetic energy while the spring is maximally stretched and its potential energy is at its maximum, pulling the mass back upwards. Since air has both mass and springiness, it behaves much the same way as the mechanical spring and mass.



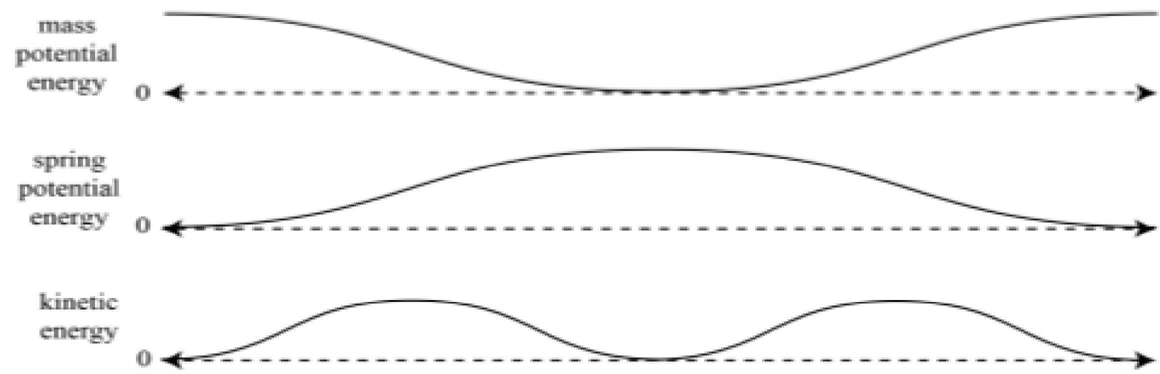


Figure 2: Oscillating mechanical systems interchange kinetic and potential energy - the same principle applies to acoustic systems.

Figure 3 shows how the time-varying characteristics of a sound wave may be measured. The amplitude can be measured as either pressure, velocity, or particle displacement of the air. Pressure is often used because it is predominantly what is perceived by the ear and by many microphones. Peaks of increased pressure and troughs of reduced pressure alternate as they radiate away from the source. Their wavelength and period can be used to describe the flow of the wave. The reciprocal of the time between peaks or troughs is the frequency (f) in cycles/second or Hertz (Hz). The distance between peaks as they move outward is the wavelength (λ). The two quantities are related by the speed of sound (c), about 340 m/s or 1130 ft/s.

$$\lambda = c / f$$

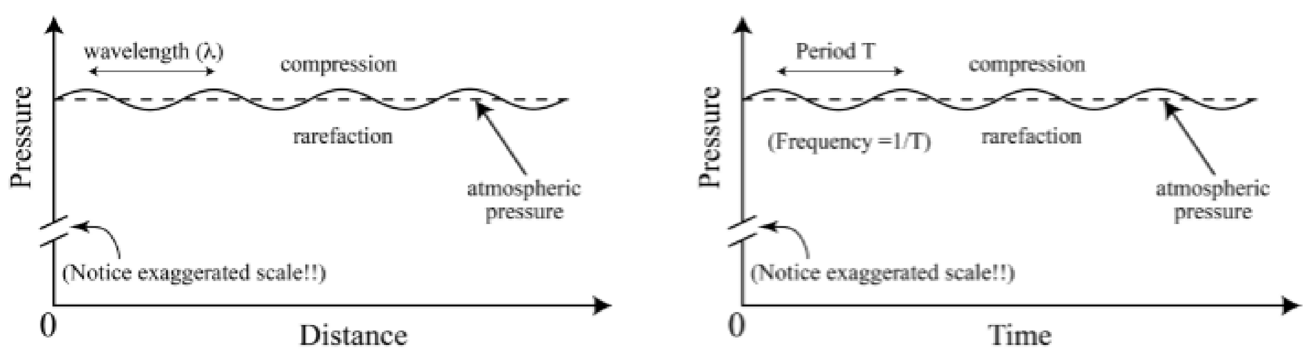


Figure 3: The period of a sinusoidal sound wave is the time between subsequent pressure peaks or troughs. The wavelength is the distance between those same peaks and troughs. As long as the speed of sound is constant, the two are related.

The pressure variations associated with sound are extremely small compared to the average air pressure. Barometric pressure at sea level is about 101 kPa [Pa (pascal) = N/m^2] or 14.7 lb/in^2. The pressure variation considered the threshold of audibility is 20 μPa, on the order of one billionth of atmospheric pressure! The sensitivity of the ear is quite impressive. This also begins to explain why sound transmission is so hard to control. Further any device used to convert sound into electricity must be similarly sensitive and able to respond over a huge range of pressures and frequencies.

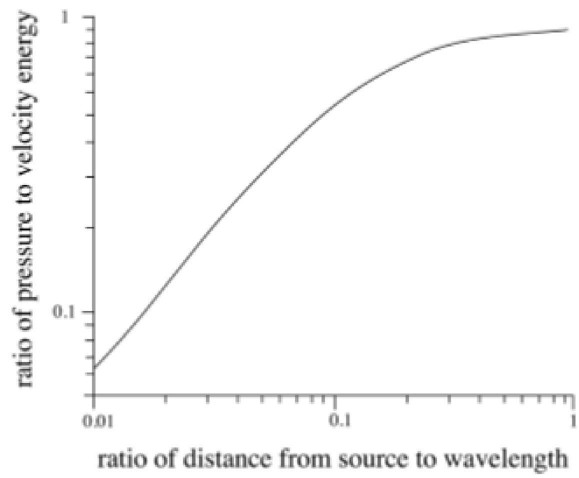


Figure 4: The ratio of energy contained in pressure to that of velocity as a function of the ratio of the distance from the source to the wavelength of the sound wave.

As a vibrating object begins to move, it forces the air molecules in contact with its surface to move. Thus, the air is accelerated by the sound source, causing a net increase in particle velocity. Particle velocity refers to the movement of a hypothetical small mass of air rather than to the turbulent individual air molecules that vibrate locally with extreme velocities but only over infinitesimal distances. (*Volume velocity* is also used to describe the velocity component - it's the flow of bulk fluid - air in this case.) The dimensions of nitrogen and oxygen molecules are on the order of about 3 Angstroms (10^{-10} m). This is many orders of magnitude smaller than wavelengths of sound so considering air as a bulk mass is sensible. As the movement continues outward from the source, the molecules are forced together, increasing the local pressure. Very near a sound source, most of the energy is contained in the form of particle velocity while far from the source the energy is transmitted predominantly in the form of pressure (See Figure 4.) Close to a small source, the sound wavefront expands in two dimensions as the spherical surface area grows with the square of the distance from the origin (See Figure 5.) Far from the source, the wavefront is practically planar and the energy radiates through the same area as it flows outward hence there is no decrease in sound pressure due to geometric dispersion. This distinction affects how sensors respond to the sound, as some are sensitive only to pressure while others are sensitive to the velocity of the sound wave that is driven by the pressure difference along the axis of movement.

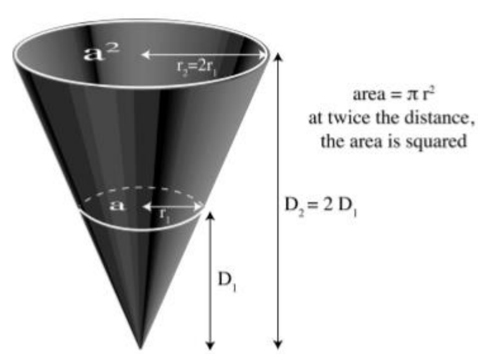


Figure 5: Spherical waves propagate through increasingly large area *a* proportional to πr^2.

The ear functions mainly as a pressure sensor as do omnidirectional microphones, designed as pressure sensors. Other types of microphones are sensitive to the air pressure gradient (often called velocity microphones.) These behave differently than pure pressure sensors, as we will see when we examine microphone design and performance. The core of this difference depends on the fact that pressure is a scalar quantity while velocity is a vector quantity. [A scalar quantity has only a magnitude while a vector has both magnitude and direction.] In order for a microphone to respond differently with respect to the direction from which a sound originates, it must be at least somewhat sensitive to the velocity vector while pressure microphones respond only to the pressure of a sound wave and not to the direction from which it originates.

In directional microphones, it is the pressure difference between two points (the front and back of the microphone), the pressure gradient, that is responsible for the forces that are converted to electrical signals in the transducer. While some directional microphones do respond to the velocity directly, like dynamic and ribbon microphones, others use multiple pressure-sensitive elements to produce a signal proportional to the pressure difference without actually moving at the equivalent velocity. For this reason, it is preferable to refer to directional microphones in general as pressure-gradient microphones.

Air is a gas mixture, predominantly nitrogen and oxygen. The molecules are in constant motion. The hotter the gas, the more frequent and energetic the molecular collisions. The kinetic energy of the gas exerts a force on other objects in contact with the air, which we call pressure. The relationship between the pressure and volume occupied by the gas in a closed system is reflected in the basic law of gases, Boyle's Law:

$$PV=\mathrm{n}RT$$

where P is pressure, V is volume, n is the amount of gas in moles, R is a constant and T is the absolute temperature. The most important aspect of this relationship is that the product of pressure and volume is constant - if one goes up, the other goes down. (Related changes in temperature may complicate this relationship slightly.) The pressure goes up when the gas is compressed by reducing the volume it occupies because the molecules are forced closer together where their collisions become more frequent. The compressibility of air can be observed using a syringe. With the plunger half-way into the syringe body, block the open end of the syringe and push or pull on the plunger. When you release the plunger, it returns to its original position much like air molecules do during a passing sound wave.

There is plenty of confusion about how to measure sound amplitude. Sound *intensity* is the product of pressure and velocity and reflects the power (energy/time) of the sound wave:

$$\text{Intensity} = \text{pressure x velocity} = \text{power / area}$$

Near the sound source, intensity decreases with the square of the distance from the source. Intensity measures the total power of the sound wave - its ability to do work. What we hear as loudness is more closely related to the pressure of the sound wave, which decreases linearly with distance from the source. When we are concerned with our perception of loudness, we need to consider the sound pressure level rather than the intensity. We can also consider only the pressure when we use omnidirectional pressure microphones. When using directional microphones, however, we will need to also consider the velocity component of the sound wave. Directional microphones have the ability to respond differently to sounds originating from different directions because they are sensitive to the sound wave pressure gradient, which changes as a function of the angle from which the sound originates. It is more complicated to measure the velocity of a sound wave than it is to measure its pressure, therefore most audio systems use sound pressure as a measure of amplitude. Most often, amplitude is measured as sound pressure level (SPL).

The distinction between spherical and plane waves is of practical importance because it explains how different microphone types respond depending on their distance from the sound source. For spherical waves, the intensity decreases with the square of the distance but the sound pressure level decreases linearly with distance. A spherical wave sound pressure decreases by 1/2 or 6 dB for a doubling of distance. In a more distant plane wave, any decrease in sound pressure is due to absorption and scattering rather than geometric considerations. In theory, plane wave pressures do not decrease with distance as they do for spherical waves. Equations for spherical and plane waves are presented below. These equations show how pressure *p* and velocity *u* change as a function of both time and distance from the source and as a function of wavelength.

Plane wave	Spherical wave
$p = akc^2\rho \sin k(ct - x)$	$p = \frac{S\rho ck}{4\pi r}\sin k(ct - r)$
$u = akc \sin k(ct - x)$	$u = -\frac{Sk}{4\pi r}\left[\frac{1}{kr}\cos k(ct - r) - \sin k(ct - r)\right]$

a = particle displacement (cm)
$k = 2\pi/\lambda$
λ = wavelength (cm)
c = velocity of sound (cm/sec)
ρ = density of air (g/cm^3)
t = time (sec)
x = distance from source (cm)
S = maximum rate of fluid (gas) emission from source (cm^3/sec)
r = radius or distance from source (cm)

Though somewhat complicated, the significant difference in the equations relates to the two terms in the velocity expression for spherical waves. There, the cosine term contributes when *r* is small in contrast to the case when *r* is very large. For large *r*, the spherical wave velocity equation reduces to the plane wave equation. One important feature of the behavior of sound fields near the source is that the pressure and velocity are not in phase as they are in the more distant plane wave. In spherical waves, the velocity leads the pressure by up to 90º for low frequency waves. Figure 6 shows the relationships between displacement, pressure and velocity for spherical and plane waves. The relationship is simple for plane waves, but for spherical waves the relationship depends on the proximity to the source in a complex way. For the purposes of understanding microphone behavior, it is only necessary to recognize the difference between spherical and plane waves in a general way.

$$Z_s = \frac{p}{u} = \rho_0 c$$

The ratio of pressure *p* to velocity *u*, known as the specific acoustic impedance Z_s, remains constant in plane waves but varies with distance in a spherical wave. (Velocity here is a complex number as it represents a vector quantity while pressure is a scalar, thus Z_s is a complex number.) In a plane wave, the acoustic impedance is also equal to the product of the density of the medium ρ_0 and the velocity of sound *c*. In a spherical wave, the acoustic impedance is a function of the distance from the source. As we will see when we discuss directional microphones, this phenomenon is responsible for the low frequency boost known the as proximity effect.

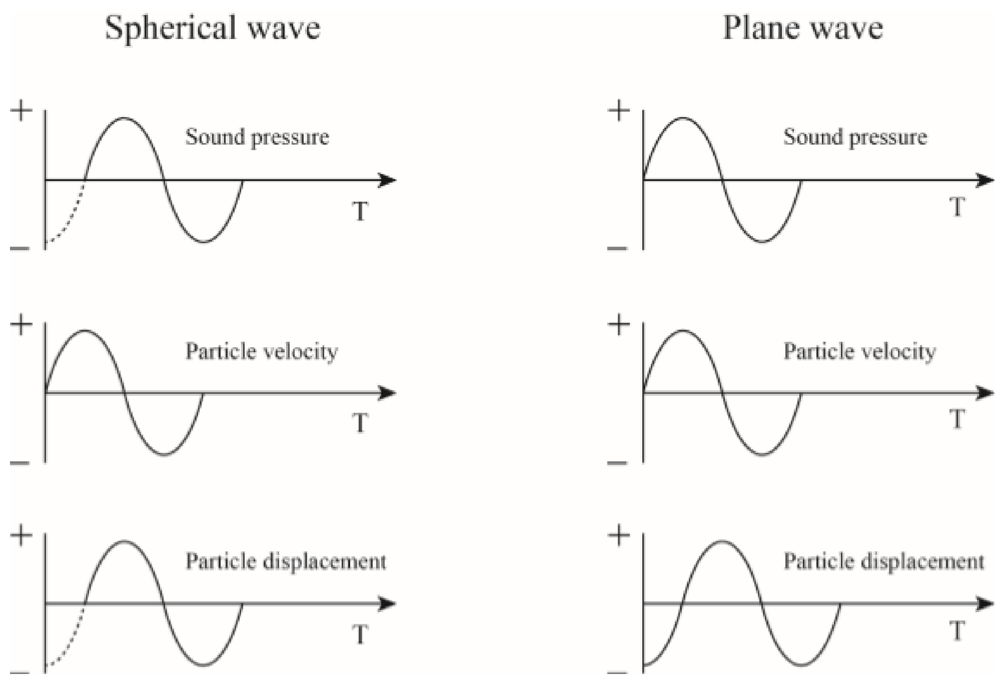


Figure 6: Sound wave pressure, velocity and displacement relationships for spherical and plane waves. Exact relationship in spherical waves depends on distance from sound source.

Plane waves can be assumed to occur at a distance from the source that depends on the relative wavelength of the sound itself. Low frequency sound has long wavelengths, about 17 m at 20 Hz while the wavelength at 20 kHz is only 1.7 cm. High frequency waves can be considered planar a few centimeters from the source while low frequency sounds require many meters of separation from the source to be close to planar. Of course, real sounds contain many component frequencies so the issue gets complicated. The difference in behavior between spherical and plane waves does pertain to the differences we observe between close and distant microphone placement. Pressure gradient microphones are more sensitive to their distance from the source than are omnidirectional microphones, particularly when placed close to the source.

Frequency (Hz)	Loss [@ 25° C, 50%RH] (dB/km)
50	0.67
500	3.2
5000	37
10000	131

Figure 7: Air absorption of sound pressure varies significantly with frequency.

For very large distances, the absorption of sound energy by the air becomes the dominant cause of sound pressure decrease. (See Figure 7) This is more pronounced for high frequencies than for low, a phenomenon sometimes observed at large outdoor concerts. A 1 kHz wave loses 6 dB at a distance of 20 miles purely through absorption while a 10 kHz loses 6 dB at about 1000 feet and 20 kHz loses 6 dB at just under 300 feet. Fortunately, these distances rarely if ever affect microphone placement.

The sound field intercepted by a microphone is dependent on its distance from the source. There are alternative ways of characterizing the qualities of the sound field created by a source. The terms near field and far field apply to individual sound sources and are based on the balance between pressure and velocity at a given distance. The near field conditions involve spherical waves and the consequent acoustical behavior while the far field relates to plane wave behavior. The dividing line betwen the two conditions is wavelength (frequency) dependent. The terms free field (direct field) and diffuse field (reverberant field) apply to sounds measured in a room. Near the source, the direct sound is dominant while further away the reverberant sound dominates. In the direct field, sound pressure decreases linearly with distance while there is no decrease in sound pressure with distance in the diffuse field. The distance at which the direct and reverberant sounds are at equal pressure is called the critical distance.

Unfortunately, there is no direct correspondence between near field and direct field and between far field and diffuse field. Both sets of terms are used but they are not interchangeable. While it can be helpful to understand how the behavior of sound changes with the distance from the source, the decisions about microphone placement ultimately depend on how the converted signal sounds to the engineer. The physical acoustic relationships can help to explain why microphones sound as they do but they cannot tell you where exactly to place a microphone in order to get the desired sound. That relies more on experience and careful listening.

REVERBERATION AND REVERBERATION TIME

A room with hard walls no furniture and no drapes , echoes. To hear well in a room or auditorium it may seem that the walls should not absorb and sound .This would lead to echoing .A room especially a large room with an excess of sound absorbing materials such as large soft drapes and soft stuffed furniture may have a quality referred to as dead. Too much absorption results in too low an intensity. Both these types of rooms are called poor acoustics. A balance must be there fore obtained.

When the sound is switched on intensity slowly builds up when it is switched off the intensity drops slowly . The prolongation of sound inside a room or hall even after the source producing the sound is turned off is called reverberation this is due to multiple reflections from the walls ceiling floor and other reflecting materials present in the hall.

The reverberation time for a room is the time required for the intensity to drop to one millionth (10^{-6}) of its initial value.

Reverberation time can be expressed interms of sound levels (in dB) rather than intensity . If the initial intensity is I_i and the final intensity I_f is 10^{-6} I_i then

$$dB_i=10 \log I_i / I \text{ (standard)}$$

$$dB_f=10 \log I_f / I \text{ (standard)}$$

$$dB_i-dB_f=10 \log(I_i/I_f)$$

since $I_i / I_f = 10^6$

$$dB_i - dB_f = 10 \log 10^6$$

$$=6*10=60$$

The reverberation time is the time required for the intensity to drop by 60 decibels . I t depends on the volume V of the room.It also depends on the absorption of all parts of the room of the room walls furniture people and so forth . Some parts may be highly absorbent and some absorb only little.

BASIC REQUIREMENTS OF ACOUSTICALLY GOOD HALL

The reverberation of sound in an auditorium is mainly due to multiple reflections at various surfaces inside. The volume and the shape of the auditorium and the sound absorption inside influence the behaviour of sound. By varying the absorption of sound inside the hall the reverberation time can be brought to optimum value. The following are the basic requirements of acoustically good hall.

1) The volume of the auditorium is decided by the type of programme to be conducted there and also the number of seats to be accommodated. A musical hall requires a large volume whereas a lecture hall requires a smaller volume. In deciding the volume of the hall its height plays an important role than its length and breadth. The ratio between the ceiling height and breadth should in deciding the volume of the hall the following guidelines may be followed.

i) In cinema theatres – 3.74 to4.2 m^3 per seat
ii) In lecture halls – 2.8 to 3.7 m^3 per seat
iii) In musical halls – 4.2 to 5.6 m^3 per seat

2) The shape of the wall and ceiling should be so as to provide uniform distribution of sound throughout the hall. The design of a hall requires smooth decay and growth of sound . To ensure these factors the hall should have scattering objects walls should have irregular surface and walls must be fixed with absorptive materials . In fig 5.1 a design which enables uniform distribution of sound is presented.

3) The reverberation should be optimum i.. e. neither too large nor too small . The reverberation time should be 1 to 2 seconds for music and 0.5 to 1 second for speech to control the reverberation the sound absorbing materials are to be chosen carefully.

4) The sound heard must be sufficiently loud in every part of the hall and no echoes should be present.

5) The total quality of the speech and music must be unchanged i.e. the relative intensities of the several components of a complex sound must be maintained .

6) For the sake of clarity the successive syllables spoken must be clear and distinct i.e. there must be no confusion due to overlapping of syllables.

7) There should be no concentration of sound in any part of the hall.

8) The boundaries should be sufficiently sound proof to exclude extraneous noise.

9) The should be no Echelon effect.

10) There should be no resonance within the building .

11) The hall must be full of audience.

ABSORPTION COEFFICIENT

Since different materials absorb sound energy differently absorption of all the materials are expressed in terms of absorption coefficient.

The coefficient of absorption of a material is defined as the ratio of sound energy absorbed by the surface to that of the total sound energy incident on the surface.

$$\text{i.e. Abdortion coefficient a} = \frac{\text{Sound energy absorbed by the surface}}{\text{Total sound energy incident on the surface}}$$

As all sound waves falling on an open window a pass through an open window is taken to be a ;perfect absorber of sound and absorption coefficient of all substance are measured in terms of open window unit(O. W. U) Absorption coefficient of a surface is also defined as the reciprocal of its area which absorbs the same sound energy as absorbed by unit area of an open window. The absorption coefficient of a given material depends on the frequency of the sound also . It is generally higher at higher frequencies.

Sound absorption coefficient of some materials (at 500 H_z frequency range)

Meterial	Absorption coefficient O.W.U
Marble	0.01
Concrete	0.17
Cork	0.23
Asbestos	0.26
Carpet	0.30
Fibre board	0.50
Heavy curtains	0.50
Fibre glass	0.75
Perforated cellulose fibre tiles	0.80
Human body	0.50
Open window	1.0

SABINE'S FORMULA FOR REVERBERATION TIME

Now we are going to derive an expression for reverberation time inside a room of volume V. Sound is produced by a source inside the room. There sound waves spread and fall on the walls they are partly absorbed and pertly reflected . The sound energy inside the room at any instant is given by

Rate of growth of energy in the space Inside the room = rate of supply of energy by the source – rate of absorption by all the surface

After getting an expression for the above if we switch off the source supplying energy then due to absorption of energy by all the surfaces energy inside the room will decay and from the decay rate reverberation time can be calculated. This derivation is based on the assumption that there is a uniform distribution of sound energy inside the room.

1) Rate of supply of energy by the source

Rate of supply of energy by the source is nothing but the power of the source P.

2) Rate of absorption of sound inside the room

In order to calculate the absorption by the wall we consider a small element ds on a plane wall AB as shown in fig 5.2 this element receives sound energy from the volume infront of it Energy received by this element per second can be calculated by constructing a hemisphere around this element with radius 'υ' where υ is the velocity of sound . Energy from every volume element with in this hemisphere will reach the element ds per elements ds . From the same centre with radii γ and γ+dγ two circles are drawn in the plane containing the normal. At angles θ and θ +dθ with respect to the normal two radii are and the area (shaded in the figure) enclosed by these two radii between the circle is considered.

This surface element is rotated about the normal through an angle dΦ and the circumferential distance moved by this elements is r sinθdΦ.

Volume traced out by this area element = area of the element * distance moved

$$dV = r\,d\theta\,dr(r \sin\theta \, d\Phi)$$
$$= r^2 \sin\theta\, d\theta\, dr\, d\Phi$$

If E is the sound energy density i.e. energy per unit volume then energy present in the is volume =EdV. Since the sound energy from this volume element propagates in all directions (i.e. through solid angle 4π)

The energy traveling per unit solid angle $= \dfrac{EdV}{4\pi}$

The energy traveling towards surface element ds alone falls on ds.
The energy traveling towards ds =energy traveling per unit solid anle *solid angle
Subtended by ds at the volume element dV

The solid angle subtended by area ds at this elements of volume dV $= \dfrac{ds\cos\theta}{r^2}$

Hence energy traveled towards ds from the Volume element dV $= \dfrac{EdV}{4\pi} \cdot \dfrac{ds\cos\theta}{r^2}$

$$= \frac{E.r^2 \sin\theta d\theta dr d\Phi}{4\pi} \cdot \frac{ds\cos\theta}{r^2}$$

$$= \frac{Eds}{4\pi} \sin\theta \cos\theta \, d\theta d\Phi dr$$

Total energy received by ds in one second from the whole Volume in its front $= \dfrac{Eds}{4\pi} \int \sin\theta \cos\theta d\theta d\Phi dr$

This equation has three variable sincewe consider the energy received per second r varies between 0 and v where is the velocity of sound θ varies between 0 and π / 2 , Φ varies between 0 and 2π.

Hence energy received by ds per second

$$= \frac{Eds}{4\pi} \int_0^{\pi/2} \sin\theta\cos\theta d\theta \int_0^{\pi/2} d\Phi \int_0^{v} dr$$

$$= \frac{Eds}{4\pi} v*2\pi*\int_0^{\pi/2} \sin\theta\cos\theta d\theta$$

$$= \frac{Evds}{4} \int_0^{\pi/2} 2\sin\theta\cos\theta d\theta$$

$$= \frac{Evds}{4} \quad (\text{since} \int_0^{\pi/2} 2\sin\theta\cos\theta d\theta = 1)$$

If a is the absorption of coefficient of the material of the wall ABthen energy absorbed by the surface element ds per second

$$= \frac{Evds}{4} a$$

Hence total rate of absorption by all the surfaces of the wall

$$= \frac{Ev}{4} \Sigma ads$$

$$= \frac{EvA}{4}$$

Where Σads = A the total absorption a on all the surface on which sound falls.

3) The growth and decay of sound energy in the room

Let p be the power output i.e. rate of emission of energy from the surface and V the total volume of the room . Then the total energy in the room at the instant when energy density is E will be EV

$$\text{Rate of growth of energy} = \frac{d}{Dt}(EV) = V \frac{dE}{dt}$$

But at any instant rate of growth of energy in space= rate of supply of energy from the furface – rate of absorption by all the surfaces.

When steady state is attained dE/dt =0 and if the steady state energy density is denoted by E_m then its value is given by

$$0 = p - \frac{E_m vA}{4}$$

$$E_m = \frac{4p}{vA}$$

from equation

$$\frac{dE}{Dt} = \frac{p}{v} - \frac{vA}{4v}.E$$

(Let $\frac{vA}{4v} = \alpha$ and hence $\frac{1}{v} = \frac{4\alpha}{vA}$)

$$\frac{dE}{dt} = \frac{4p\alpha}{vA} - \alpha E$$

or

$$\frac{dE}{dt} + \alpha E = \frac{4p}{vA}.\alpha$$

multiplying both sides by $e^{\alpha t}$ we have

$$\left(\frac{dE}{Dt} + \alpha E\right) e^{\alpha t} = \frac{4p}{vA} \alpha e^{\alpha t}$$

Or

$$\frac{d}{dt}(Ee^{\alpha t}) = \frac{4p}{vA} \alpha e^{\alpha t}$$

integrating the above equation we get

$$Ee^{\alpha t} = \frac{4p}{vA} e^{\alpha t} + k$$

where k is a constant of integration . Using the boundary conditions we can find the value of K

i) Growth of the energy density:- If t is measured from the instant the source start emitting sound . then initial condition is that at t=0 E=0, Applying this condition to equation we get

$$K = \frac{-4p}{vA}$$

substituting this value in equation we get

$$E e^{\alpha t} = \frac{4p}{vA} e^{\alpha t} - \frac{4p}{vA} \text{ or } E = \frac{4p}{vA} - \frac{4p}{vA} e^{-\alpha t}$$

$$E = \frac{4p}{vA}(1-e^{-\alpha t})$$

or

$$E = E_m(1-e^{-\alpha t})$$

The equation shows the growth of energy with time t. The growth is along the exponential curve shown in fig 5.3 which shows that E increases along the curve with time At t= 0,E = 0 and at

ii) Decay of energy density:- Let the source be cut off when E has reached the maximum value E_m . Now at t=0, p=0, E= E_m from equation K =E_m

substituting this value of K in equation

$E e^{\alpha t}$ E_m (since p=0)

Equation show s the decay of the energy density with time after the source is cut off. This decay is shown by the exponential curve .

4) Deduction of standard reverberation time (T) i.e. sabine's formula : We know that the persistence of audible sound in the room even after the source has stopped producing the sound is called reverberation and the standard time of rev3ereration T is defined as the time taken for the sound energy density inside a room to fall to one millionth of its initial maximum value hence to calculate T we put $E_m /E = 10^6$ and t= T in equation

$$\frac{E}{E_m} = e^{\alpha T} \; 10^{-6} \text{ (or) } e^{\alpha T} = 10^6$$

$$\alpha T = 6 \log_e 10 = 2.3026*6$$

substituting for α

$$T = \frac{4*2.3026*6}{330} \quad \frac{V}{A} \quad (\text{ v=330m/s})$$

Or

$$T = \frac{0.165V}{A} = \frac{0.165V}{\Sigma aS}$$

This equation is in good agreement with the experimental values obtained by sabine .this is sabine's formula for reverberation time .

i) Directly proportional to the volume of the auditorium
ii) Inversely proportional to the areas of sound absorbing surfaces such as ceiling wall floor and other materials present inside the hall and
iii) Inversely proportional to the total absorption

It has been experimentally found that the reverberation time of 1.03 second is most suitable for all room having approximately a volume less that 350 meters.

The first method is based on the determination of standard time of reverberation in the room without and with the sample of the material inside the room , If T_1 is the reverberation time without the sample inside the room then applying sabine's formula

$$\frac{1}{T_1} = \frac{A}{0.165V} = \frac{\Sigma aS}{0.165V}$$

Now with the sample inside the room reveration time T_2 is measured.

$$\frac{1}{T_2} = \frac{\Sigma aS + a_1 s_1}{0.165V}$$

Where a_1 is the absorption coefficient of the area S_1. From the above equation we have

$$\frac{a_1 S_1}{0.165V} = \left(\frac{1}{T_2} - \frac{1}{T_1}\right)$$

FACTORS AFFECCTING THE ARCHITECTURAL ACOUSTICS AND THEIR REMEDIES

By an acoustically good hall we mean that every syllable or musical note reches an audible level of loudness at every point of the hall and then quickly dies away to make Room for for the next syllable or group of note. The deviation from this makes the hall defective acoustically . Following factors affect the architectural acoustics.
1) In a hall when reverberation is large there is overlapping of successive sound which results in loss of clarity in hearing.On the other hand if the reverberation is very small the loudness is inadequate .Thus the time of reverberation for a hall should neither be too large nor too small . It must have a definite value which may be satisfactory both to the speaker as well as to the audience . The preferred value of the time or reverberation is called the optimum reverberation time . A formula for standard time of reverberation was given by W. C sabine which is

$$T = \frac{0.165V}{A} = \frac{0.165V}{\Sigma aS}$$

Where A is the total absorption of the hall V its volume in cubic metre and S is the surface area in square metre.

Experimentally its is observed that the time of reverberation depends upon the size of the hall loudness of sound and on the kind of the music fro which the hall is used . For a frequency of 512 H_z the best time of reverberation lies between 1 and 1.5 sec for small halls and up to 2-3 seconds for larger ones.

REMEDY:- The reverberation can be controlled by the following factors

1) By providing windows and ventilators which can be opened and closed to make the value of the4 time of reverberation optimum
2) Decorating the walls by pictures and maps
3) Using heavy curtains with folds
4) By lining the walls with absorbent materials such as felt celotex fibre board glass wool etc
5) Having full capacity of audience (please remember that empty hall reverberarate each person is equivalent to about 0.50 sq metre area of an open window)
6) By covering the floor with carpets
7) By providing acoustic tiles.

LOUDNESS:- With large absorption the time of reverberation will be smaller. This will minimise the chances of confusion between the different syllables by the intensity of sound may go ;below the level of intelligibility of hearing . Sufficient loudness at every point in the hall is an important factor for satisfactory hearing .

REMEDY:- The loudness may be increased by

1) Using large sounding boards behind the speaker and facing the audience large polished wooden reflecting surfaces immedialtely above the speaker are also helpful
2) Low ceiling are also of great help in reflecting the sound energy towards the audience.
3) By providing additional sound energy with the help of equipments like loud speakers. To achieve uniform distribution of intensity through out the hall loudspeakers are to be positioned carefully .

FOCUSSING:-If there are focusing surface such as concave spherical cylindrical or parabolic ones on the walls or ceiling of the hall they produce concentration of sound in particular regions while in come other parts no sound reaches at all. In this way there will be regions of silence or poor audibility . If there are extensive reflecting surfaces in the hall the reflected and direct sound waves may form stationary wave system thus making the sound intensity distribution bad and uneven.

REMEDY:- For uniform distribution of sound energy in the hall

1) There should be no curved surfaces . If such surfaces are present they should be covered with absorbent material
2) Ceiling should be low .
3) A paraboloidal reflected surface arranged with the speaker at the focus is also helpful in sending a uniform reflected beam of sound in the hall.

ECHOES:- An echo is heard when direct sound waves coming from the source and its reflected wave reach the listener with a time interval of about 1/7 second/ The reflected sound arriving earlier than this helps in raising the loudness whole those arriving later produce echoes and cause confusion

REMEDY :- Echoes may be avoided by covering the long distant walls and high ceiling with absorbent material

ECHELON EFFECT:- A set of railing s or any regular spacing of reflecting surfaces may produce a musical note due to the regular succession of echoes of the original sound to the listerner. This makes the original sound confusing or unintelligible .

REMEDY:- So this type of surface should be avoided or covered with proper sound absorbing materials.

RESONANCE:- Sometimes the window panes sections of the wooden portions and walls lacking in rigidity ate thrown into forced vibrations and create sound. For some note of audio frequency the frequencies of forced vibrations any be the same thus resulting in the resonance . Moreover if the frequency of the of the created sound is not equal to the original sound at least certain tone s of the original music will be reinforced. Due to the interference between original sound and created sound the original sound is distorted . Thus the intensity of the note is entirely different from the original one enclosed air in the hall also causes resonance.

REMEDY:- Such resonant vibrations should be suitably damped.

NOISE:- Generally there are three types of noises which are very troublesome they are

1) Air- borne noise
2) Structure borne noise and
3) Inside noise

The prevention of the transmission of noise inside or outside the hall is known as sound insulation . This is also known as sound proofing . The method of sound insulation depends on the type of noise to be treated . Here we shall discuss the different types of noises and their sound insulation.

AIR BORNE NOISE:- The noise which commonly reaches the hall from outside through open window doors and ventilators is known as air born noise since this noise is transmitted through the air it is called so.

REMEDY :- Sound insulation for the reduction of air borne noise can be achieved by the following methods

1) By allotting proper places for doors and windows
2) By making arrangements for perfectly shutting doors and windows
3) Using heavy glass in doors windows and ventilators
4) Using double doors and window with separate frames and having insulating material between them
5) By providing double wall construction floating floor construction suspended ceiling construction box type construction etc.
6) By avoiding opening s for pipes and ventilators

STRUCTURE BORNE NOISE:- The noise which are conveyed through the structures of the building are known as structural noise. The noises may be caused due to structure . The most common sources of this type of sound are foot steps street traffic operating machinery moving of furniture etc

REMEDY:- Sound insulation for the reduction of structures borne noise is done in the following ways

1) Breaking the continuity by interposing layers of some acoustical insulators
2) Using double walls with air space between them
3) Using anti vibration mounts
4) Soft floor finish (carpet rubber etc)
5) Mechanical equipments such as refrigerators lifts fans etc. Produce vibrations in the structure. These vibrations can be checked by insulating the equipment properly

INSIDE NOISE:- The noise which are produced inside the hall or rooms in big offices are called as inside noises . They are produced due to machinery like air conditioners type writers in the hall.

CONCLUSION:- Acoustics of buildings is a very important fields. If proper care is not taken to achieve required acoustical properties in a building the building becomes unusable . Hence at the planning stage itself it is essential to take necessary to achieve optimum reverberation time.

SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF PHYSICS

UNIT - 5

Oscillations and Waves – SPH1212

Introduction

- The human ear can hear the sound waves having frequencies in between 20 Hz to 20 kHz. These frequencies are known as audible frequencies.
- The sound waves having frequencies less than 20 Hz are known as infrasonic waves o
- The sound waves having frequencies greater than 20 kHz are known as ultrasonic waves.
- The wavelength of ultrasonic waves are very much less than the wavelengths of audible sound waves.
- So they applications in non-destructive testing of materials, medical diagnostics, military and marine.
- Ultrasonic method is widely used in industries to find the size, shape, and location of flaws such as cracks, voids, laminations, and inclusions of foreign materials, walls thickness of produced pipes and vessels.
- The wall thickness measurements are very important in corrosion studies.

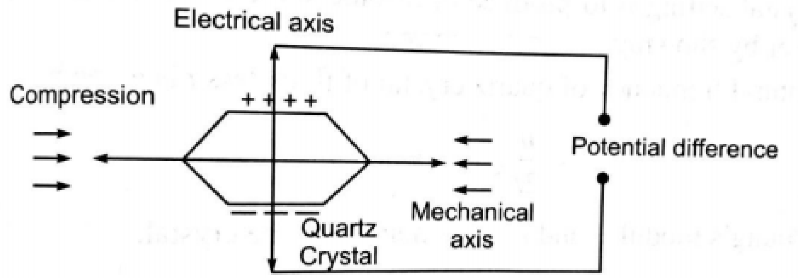
<u>Properties of ultrasonic waves</u>

a. Ultrasonic waves are high frequency and high energetic sound waves.
b. Ultrasonic waves produce negligible diffraction effects because of their small wavelength.
c. Ultrasonic wave travels longer distances without any energy loss.
d. The speed of propagation of ultrasonic waves increases with the frequency of the waves.
e. At room temperature, ultrasonic welding is possible.
f. Ultrasonic waves produce cavitation effects in liquids.
g. Ultrasonic waves produce acoustic diffraction in liquids.
h. Ultrasonic waves cannot travel through the vacuum.
i. Ultrasonic waves travel with speed of sound in a given medium.
j. Ultrasonic waves require one material medium for its propagation.
k. Ultrasonic waves can produce vibrations in low viscosity liquids.
l. Ultrasonic wave's produces heat effect passes through the medium.
m. Ultrasonic waves obey reflection, refraction, and absorption properties similar to sound waves.
n. Ultrasonic waves produce stationary wave pattern in the liquid while passing through it.
o. When the ultrasonic wave is absorbed by a medium, it generates heat. They are able to drill and cut thin metals.

<u>Piezoelectric effect</u>

✓ The piezoelectric effect was discovered in 1880 by two French physicists, brothers Pierre and Paul-Jacques Curie, in crystals of quartz, tourmaline, and Rochelle salt (potassium sodium tartrate).
✓ This phenomenon is observable in many naturally available crystalline materials, including quartz, Rochelle salt and even human bone.
✓ Engineered material, such as lithium niobate and lead zirconate titanate (PZT), exhibit a more pronounced piezoelectric effect.
✓ **When a crystals like (calcite or quartz) under goes mechanical deformation along the mechanical axis then electric potential difference is produced along the electrical axis perpendicular to mechanical axis. This phenomenon is known as piezoelectric effect.**

- ✓ Piezoelectricity (also called the piezoelectric effect) is the appearance of an electrical potential (a voltage, in other words) across the sides of a crystal when you subject it to mechanical stress (by squeezing it).



I. *Production of ultrasonics*

Ultrasonics waves are produced by the following methods.

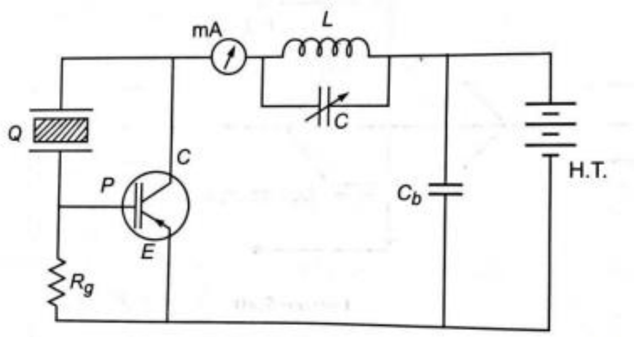
- ✓ Piezo electric method
- ✓ Magneto-striction method

I. *Piezo electric method*

- ✓ The sound waves having frequencies greater than 20 kHz are known as ultrasonic waves.
- ✓ **When a crystals like (calcite or quartz) under goes mechanical deformation along the mechanical axis then electric potential difference is produced along the electrical axis perpendicular to mechanical axis. This phenomenon is known as piezoelectric effect.**
- ✓ The converse of the effect is also possible.
- ✓ When an alternative potential is applied along the electrical axis, the crystal will set into electric vibrations along the mechanical axis.

Construction

- ✓ The experimental setup for the production of ultrasonic waves using Piezo electric method is shown in figure



- ✓ The quartz crystal between the metal plates is connected to collector and base of transistor.
- ✓ Collector is also connected to LC circuit and high tension source shunted a by pass capacitor C_b.
- ✓ C_b is used to stop high frequency currents from passing through battery.
- ✓ The capacity of variable capacitor is adjusted so that the frequency of the oscillating circuits is equal to the natural frequency of the crystal. R_g provided necessary biasing for base and emitter circuit.

'orking:-

- ✓ When the circuit is starts functioning slowly an alternative potential difference is built across the quartz crystal which sets the crystal into vibrations.
- ✓ By varying the capacitor of capacitor C, at a particular stage the frequency of the alternating potential across the crystal coincides with the natural frequency of the quartz crystal it to produce ultrasonic waves.
- ✓ This stage is indicated by milli ammeter by showing maximum current.
- ✓ The natural frequency of quartz crystal of thickness t is given by

$$f = \frac{n}{2t}\sqrt{\frac{y}{\rho}}$$

- ✓ Where y is young's modulus and ρ is the density of crystal

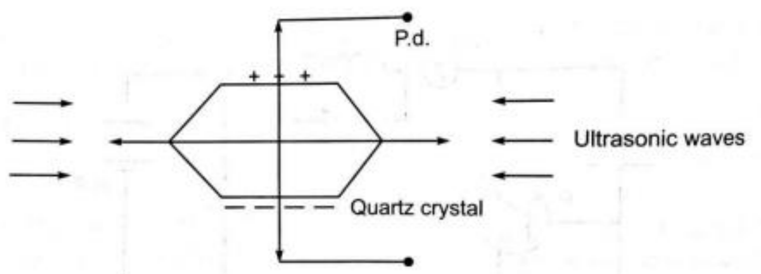
Detection of ultrasonics

The presence of ultrasonic waves can be detected by the following methods.

- ✓ Piezo electric method
- ✓ Kundt's tube method
- ✓ Sensitive flame method
- ✓ Thermal detection method

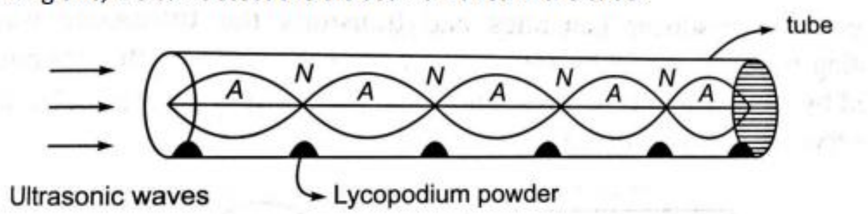
iezo electric method

- ✓ Piezo electric effect can also used for the detection of ultrasonics.
- ✓ When ever the ultrasonic waves are incident along the mechanical axis of the crystal a certain potential difference is developed across the faces.
- ✓ This potential difference indicates the ultrasonic waves.



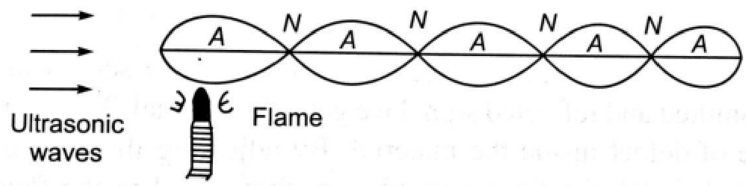
Kundt's tube method

- ✓ As shown in figure Kundt's tube filled with lycopodium power in the bottom portion of the tube can also be used for detecting ultrasonic waves whose length is of the order of a few millimeters.
- ✓ When ultrasonic waves pass through tube then stationary waves are formed due to super position of incident and reflected waves. The power will be collected as leaps at nodes and dispended at anti nodes.
- ✓ By observing this, we can detect the ultrasonic waves in the tube.



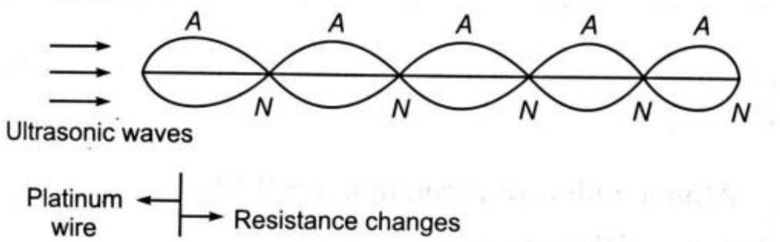
Sensitive flame method

- ✓ A narrow sensitive flame is moved along the medium.
- ✓ At the positions of antinodes, the flame is steady.
- ✓ At the positions of nodes, the flame flickers because there is a change in pressure.
- ✓ In this way, positions of nodes and antinodes can be found out in the medium.
- ✓ By observing this, we can detect the ultrasonic waves



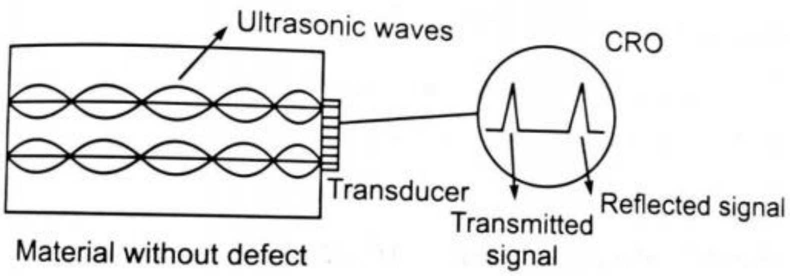
Thermal detection method

- ✓ When ultrasonic waves pass through a medium, then alternative compressions and rare factors are formed. At compression, particles of medium are brought closer and collisions between them increases. As a result of this the temperature of medium increases at compressions. On the other hand, the temperature of medium decreases at rarefaction due to the fact that particles of medium go move away from each other and frequency of collisions is decreased
- ✓ When platinum wire is moved in the medium consists of standing waves of ultrasonics due to variations of temperature at nodes and antinodes, the resistance of the wire changes. By noticing the changing of resistance of wire one can detect the presence of ultrasonic waves.

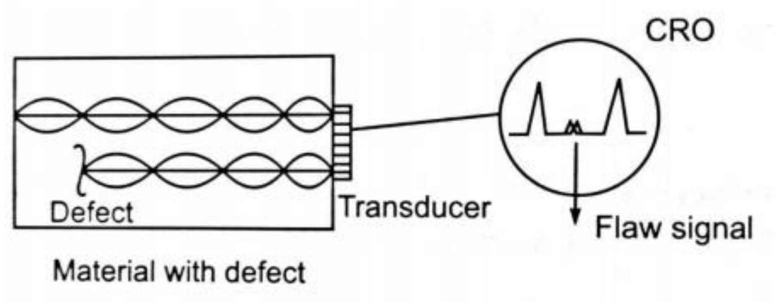


Application in Nondestructive testing

- ✓ Ultrasonic waves are extremely used for nondestructive testing of the material i.e., detecting the defects (flaws) inside the material without disturbing material properties.
- ✓ Nondestructive testing systems consist of transducers for generation and transmission of ultrasonic waves into the material and also to receive the reflected waves from the flaws or defects.



To identify the defects cathode ray oscilloscope is used.
When the transducer generates and transmits the ultrasonic waves into the testing material it will be reflected by the other end of the material and is received by the transducer.
Corresponding to the transmitted and reflected waves, we can observe two well-resolved signals on the screen of CRO.



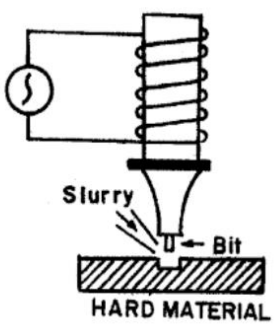
When the material having defect, then in the CRO screen in addition to regular transmitted and reflected signal, we get a flaw signal.
This signal indicates the presence of defect inside the material.
By knowing the velocity and time taken by the ultrasonic waves the flaw location can be identified

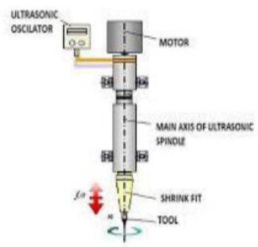
Applications of Ultrasonic waves

- Depth of sea
- SONAR
- NDT-Non Destructive Testing
- US welding/cutting drilling
- US cleaning
- US soldering

US waves in the field of Industry:

Cutting, drilling, welding,coining,grinding…

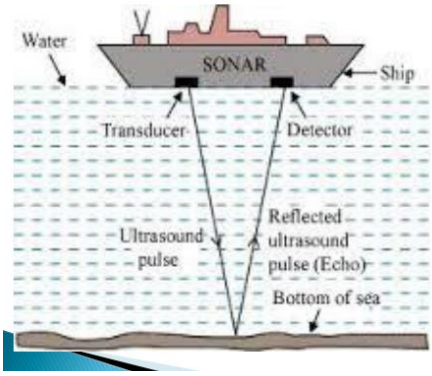


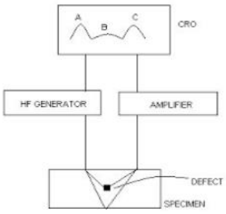


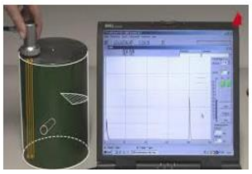
ULTRASONIC DRILLING PROCESS

Applications of Ultrasonic waves-

SONAR & NDT







As a sub category of acoustics, ultrasonics deals with the acoustics above the human hearing range (the audio frequency limit) of 20 kHz. Unlike audible sound waves, the ultrasonic waves are not sensed by human ear due to the limitations on the reception of vibrations of high frequency and energies by the membrane. Ultrasonic wave exhibits all the characteristic properties of sound. Ultrasonic vibrations travel in the form of wave, similar to the way light travels. However, unlike light waves, which can travel in vacuum, ultrasonic wave requires elastic medium such as a liquid or a solid. The wavelength of this wave changes from one medium to another medium due to the elastic properties and induced particle vibrations in the medium. This wave can be reflected off with very small surfaces due to having much shorter wavelength. It is the property that makes ultrasound useful for the non-destructive characterization/testing of materials. The knowledge of generation/detection of ultrasonic wave and its characteristics is important for its precise and suitable application.

US waves in the field of Medicine:

- Diagnostic sonography
- Ultrasound cardiograph
- Obstetric ultrasound
- Ultrasound therapeutic
- Ultrasonic guidance for blind

Material characterization techniques (NDT & DT)

The two major classification of material characterization technique are non-destructive testing (NDT) and destructive testing (DT). Under destructive technique (such as: tensile testing, creep testing, impact testing, torsion testing, hardness testing etc.) of characterization the tested material or product can not be used again. The destruction of test object usually makes this type of test more costly. Non-destructive testing technique is a specific procedure whereby the service ability of materials or components is not impaired by testing process. The various methods like visual testing, liquid penetrant testing, magnetic particle testing, eddy current testing, radiographic testing, ultrasonic testing, leak testing, thermography and neutron radiography are the NDT technique of material characterization. Among the various non-destructive testing and evalution (NDT&E) plays a key role in material characterization.Ultrasonic properties provide important diagnostic for microstructural properties as well as deformation processes in a material, controlling material behaviour based on the physical mechanism to predict future performance of the materials.

Classification of ultrasonic application and testing

The ultrasonic testing involves both the low intensity and high intensity ultrasonic wave for the characterization, that belongs in non-destructive and destructive techniques of characterization respectively. Uses of high intensity and low frequency ultrasonic wave includes medical therapy and surgery, atomization of liquids, machining of materials, cleaning and wielding of plastics and metals, disruption of biological cells, and homogenization of materials. The low intensity and high frequency ultrasonic waves are applied for medical diagnosis, acoustical holography, material characterization etc. The low

intensity ultrasoud measurements provides a good diagnosis of material property and process control in industrial apllication (Alers, 1965; Green,1973; Lowrance, 1975; Renolds, 1978; Teagle, 1983; Smith, 1987; Varry, 1987; Thompson,1996; Jayakumar, 1998; Kumar, 2001; Raj, 2003; Roth, 2003; Blodgett, 2005).

Ultrasonic NDT as a material characterization

There are four mode of propagation by which an ultrasonic wave can propagate in a medium, as: longitudinal or compressnal wave, transverse or shear wave, surface or Rayleigh wave and plate or lamb wave. The most common methods of ultrasonic examination utilize the longitudinal waves or shear waves.

Ultrasonic velocity or attenuation are the parameters that correlate to structural inhomogenities or flaw size atomistic (interstitials), elastic parameters, precipitates, dislocations, ordering of molecules in liquid crystals, phase transformations, porosity and cracks, concentration of different components of alloys or mixed crystal system, vacancies in lattice sites, size of the nanoparticles in nano-structured materials, electrical resistivity, specific heat, thermal conductivity and other thermophysical properties of the materials depending upon the different physical conditions like temperature, pressure, crystallographic orientation, magnetization etc. Thus, ultrasonic study of a material provides information about elastic constants, microstructure, discountinuty, and mechanical properties under different condition.

Ultrasonic velocity

On the basis of mode of propagation there are four types of ultrasonic velocities, as longitudinal, shear, surface and lamb wave velocity. Longitudinal and shear wave velocities are more important for the material characterization because they are well related to elastic constants and density. However, it is independent of frequency of wave and dimension of the given material. The mechanical behaviour and anisotropic properties of the material can be well defined on the knowledge of ultrasonic velocity. The mathematical formulations and measurement techniques for ultrasonic velocity are detailed in following heads.

Measurement techniques of ultrasonic velocity

The study of the propagation of ultrasonic waves in materials determines the elastic constants, which provides better understanding of the behaviour of the engineering materials. The elastic constants of material are related with the fundamental solid state phenomenon such as specific heat, Debye temperature and Grüneisen parameters. The elastic constants in the materials can be determined by measuring the velocity of longitudinal and shear waves. Elastic constants are related to interatomic forces, co-ordination changes etc., and also with the impact shock, fracture, porosity, crystal growth and microstructural factors (grain shape, grain boundaries, texture and precipitates etc.). So, the study of ultrasonic velocity is useful not only for characterization of the structured materials, engineering materials, porous materials, composites, glasses, glass ceramics but also bioactive glasses, nanomaterials, nanofluids etc.

Interferometer or continuous wave method and pulse technique are the general electrical method for the measurement of ultrasonic velocity. In CW method, the wavelength of wave in the test material is measured, which in turn provides the ultrasonic velocity with relation $V = \nu \lambda$. While in the Pulse technique, transit time (t: the time needed for a signal to travel between the front and back surface of the specimen or concerned medium) is measured with the help of echo pattern. If x is thickness of the material then ultrasonic velocity becomes equal to 2x/t.

For precise measurement, the Pulse technique has been improved in the form of following techniques (Papadakis, 1976, Raj, 2004).

a. Sing around
b. Pulse superposition method
c. Pulse echo overlap method
d. Cross-correlation method an
e. Phase slop method
f. Pulse transmission method

The pulse echo-overlap, pulse transmission and pulse superposition techniques are widely used techniques due to their absolute accuracy and precision respectively. Now a day, computer controlled devices of pulse echo overlap and pulse superposition techniques are being used. Resonance ultrasound spectroscopy and Laser interferometry are the recent techniques for the measurement of ultrasonic velocity in thin film, crystal, textured alloy etc.

Ultrasonic attenuation

The intensity of ultrasonic wave decreases with the distance from source during the propagation through the medium due to loss of energy. These losses are due to diffraction, scattering and absorption mechanisms, which take place in the medium. The change in the physical properties and microstructure of the medium is attributed to absorption while shape and macroscopic structure is concerned to the diffraction and scattering. The absorption of ultrasonic energy by the medium may be due to dislocation damping (loss due to imperfection), electron-phonon interaction, phonon-phonon interaction, magnon-phonon interaction, thermoelastic losses, and bardoni relaxation. Scattering loss of energy is countable in case of polycrystalline solids which have grain boundaries, cracks, precipitates, inclusions etc. The diffraction losses are concerned with the geometrical and coupling losses, that are little or not concerned with the material properties. Thus in single crystalline material, the phenomenon responsible to absorption of wave is mainly concerned with attenuation. An addition of scattering loss to the absorption is required for knowledge of attenuation in polycrystalline materials. So, the rate of ultrasonic energy decay by the medium is called as ultrasonic attenuation.

The ultrasonic intensity/energy/amplitude decreases exponentially with the source. If I_X is the intensity at particular distance x from source to the medium inside then:

On solving the equations (10) and (11), one can easily obtain the following expression of ultrasonic attenuation.

$$\alpha = \frac{1}{(x_2 - x_1)} \log_e \frac{I_{X_1}}{I_{X_2}} \tag{12}$$

The ultrasonic attenuation or absorption coefficient (α) at a particular temperature and frequency can be evaluated using equation (12). In pulse echo-technique the (X_2-X_1) is equal to twice of thickness of medium because in this technique wave have to travel twice distance caused by reflection, while is equal to medium thickness in case of pulse transmission technique. Attenuation coefficient is defined as attenuation per unit length or time. i.e. The α is measured in the unit of Np cm^{-1} or Np t^{-1}. The expression of α in terms of decibel (dB) unit are written in following form.

$$\alpha = \frac{1}{(x_2 - x_1)} 20\log_{10} \frac{I_{X_1}}{I_{X_2}}; \text{ in unit of dB/cm} \tag{13a}$$

$$\alpha = \frac{V}{(x_2 - x_1)} 20\log_{10} \frac{I_{X_1}}{I_{X_2}}; \text{ in unit of dB/}\mu\text{s} \tag{13b}$$

Source of ultrasonic attenuation

The attenuation of ultrasonic wave in solids may be attributed to a number of different causes, each of which is characteristic of the physical properties of the medium concerned. Although the exact nature of the cause of the attenuation may not always be properly understood. However, an attempt is made here to classify the various possible causes of attenuation that are as.

a. Loss due to thermoelastic relaxation
b. Attenuation due to electron phonon interaction
c. Attenuation due to phonon phonon interaction

a. Loss due to thermoelastic relaxation

A polycrystalline solid may be isotropic because of the random orientation of the constituent grains although the individual grains may themselves be anisotropic. Thus, when a given stress is applied to this kind of solid there will be variation of strain from one grain to another. A compression stress causes a rise in temperature in each crystallite. But because of the inhomogeneity of the resultant strain, the temperature distribution is not uniform one. Thus, during the compression half of an acoustic cycle, heat will flow from a grain that has suffered the greater strain, which is consequently at high temperature, to one that has suffered a lesser strain, which as a result is at lower temperature. A reversal in the direction of heat flow takes place during the expansion half of a cycle. The process is clearly a relaxation process. Therefore, when an ultrasonic wave propagates in a crystal, there is a relaxing flow of thermal energy from compressed (hot region) towards the expanded (cool region) regions associated with the wave. This thermal conduction between two regions of the wave causes thermoelastic attenuation. The loss is prominent for which the thermal expansion coefficient and the thermal conductivity is high and it is not so important in case of insulating or semi-conducting crystals due to less free electrons. The thermoelastic loss $(\alpha)_{Th}$ for longitudinal wave can be evaluated by the Mason expression (Bhatia, 1967; Mason, 1950, 1965) .

$$\alpha_{Th.} = \frac{\omega^2 < \gamma_i^j >^2 KT}{2dV_L^5} \tag{14a}$$

$$(\alpha / f^2)_{Th.} = \frac{4\pi^2 < \gamma_i^j >^2 KT}{2dV_L^5} \tag{14b}$$

where ω and V_L are the angular frequency and longitudinal velocity of ultrasonic wave. d, K and T are the density, thermal conductivity and temperature of the material. γ_i^j is the Grüneisen number, which is the direct consequence of the higher order elastic constants (Mason, 1965; Yadawa 2009). In the case of shear wave propagation, no thermoelastic loss occurs because of no any compression & rarefaction and also for the shear wave, average of the Grüneisen number is zero.

b. Attenuation due to electron-phonon interaction

Debye theory of specific heat shows that energy exchanges occur in metals between free electrons and the vibrating lattice and also predicts that the lattice vibrations are quantized in the same way as electromagnetic vibrations, each quantum being termed as phonon.Ultrasonic absorption due to electron-phonon interaction occurs at low temperatures because at low temperatures mean free path of electron is as compared to wavelength of acoustic phonon. Thus a high probability of interaction occurs between free electrons and acoustic phonons. The fermi energy level is same along all directions for an electron gas in state of equilibrium, i.e. the fermi surface is spherical in shape. When the electron gas is compressed uniformly, the fermi surface remains spherical. The passage of longitudinal ultrasonic wave through the electron gas gives rise to a sudden compression (or rarefaction) in the direction of the wave and the electron velocity components in that direction react immediately, as a result fermi surface becomes ellipsoidal. To restore the spherical distribution, collision between electron and lattice occur. This is a relaxational phenomenon because the continuous varying phase of ultrasonic wave upsets this distribution.

$$(\alpha)_{Shear} = \frac{\omega^2}{2dV_S^3}\eta_e \tag{15b}$$

where η_e and χ represent the electronic shear and compressional viscosities of electron gas.

c. Attenuation due to phonon–phonon interaction

The energy quanta of mechanical wave is called as phonon. With the passage of ultrasound waves (acoustic phonons), the equilibrium distribution of thermal phonons in solid is disturbed. The re-establishment of the equilibrium of thermal phonons are maintained by relaxation process. The process is entropy producing, which results absorption. The concept of modulated thermal phonons provides following expression for the absorption coefficient of ultrasonic wave due to phonon–phonon interaction in solids $(\alpha)_{Akh}$ (Bhatia, 1967; Mason, 1950, 1958, 1964, 1965; Yadav & Singh 2001; Yadawa, 2009) .

$$\alpha_{Akh} = \alpha_{PP} = \frac{\omega^2 \tau\, \Delta C}{2dV^3(1+\omega^2\tau^2)} \tag{16a}$$

Where τ is the thermal relaxation time (the time required for the re-establishment of the thermal phonons) and V is longitudinal or shear wave velocity. ΔC is change in elastic modulli caused by stress (by passage of ultrasonic wave) and is given as:

$$\Delta C = 3E_0 < (\gamma_i^j)^2 > - < \gamma_i^j >^2 C_v T \tag{16b}$$