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SCHOOL OF SCIENCE AND HUMANTTIES

DEPARTMENT OF MATHEMATICS

UNIT – I - SENSITIVITYANALYSISSENSITIVITYANALYSIS – SMT5210

Introduction

The **Simplex Method** also called the ‘**Simplex Technique**’ or the **Simplex Algorithm** is an iterative procedure for solving a linear programming problem in a finite number of steps. The method provides an algorithm which consists in moving from one vertex / corner point of the region of feasible solutions to another vertex in such a manner that the value of the objective function at the succeeding vertex is improved [lesser in case of minimization and more in case of maximization] than at the preceding vertex. This procedure of jumping from one vertex to another is then repeated. Since the number of vertices is finite, the method leads to an optimal vertex in a finite number of steps or indicates the existence of an unbounded solution.
We will introduce various concepts / definitions that are related to simplex method in the following paragraphs.

Definations:

1. Objective Function
The function that is to be either minimized or maximized is called as objective function. For example, it may represent the cost that you are trying to minimize or total revenue that is to be maximized and so on.
2. Constraints
A set of equalities and inequalities that the feasible solution must satisfy is called as constraints of the problem.
3. Optimal Solution
A vector X, which is both feasible (satisfying all the constraints in the given problem) and optimal (obtaining the largest or smallest value for the objective function, depends upon the case) is known as optimal solution.
4. Feasible Solution
A solution vector, X, which satisfies all the constraints of the given problem is called feasible solution to the given LPP.
5. Basic Solution
X of (AX=b) is a basic solution if the n components of X can be partitioned into m “basic” and n-m “non-basic” variables in such a way that: the m columns of A corresponding to the basic variables form a nonsingular basis and the value of each “non-basic” variable is 0. The constraint matrix A has m rows (constraints) and n columns (variables).

6. Basis

The set of basic variables is called the basis for the given problem.

7. Basic Variables

Basic variables are set of variables, which are obtained by setting n-m variables values to zero, and are solving the resulting system.

8. Non-basic Variables

A variable not in the basic solution, not part of the solution is called non-basic variable.

9. Slack Variable

If we have a 'less or equal' to constraint, to convert that as an equation, a variable is added to the left hand side of the constraint; the new variable, which is added to the left hand side of the constraint is called as slack variable.

Ex: $2X1 + 5X2 \leq 10$

$2X1 + 5X2 + SX3 = 10$

The variable, SX3, is called as slack variable for the given constraint.

10. Surplus Variable

If we have a 'greater or equal' to constraint, to convert that as an equation, a variable is subtracted from the left hand side of the constraint; the new variable, which is subtracted, to the left hand side of the constraint is called as surplus variable.

Ex: $2X1 + 5X2 \geq 10$

$2X1 + 5X2 - SX4 = 10$

The variable, SX4, is called as surplus variable for the given constraint. Therefore, it is a variable added to the problem to eliminate greater-than constraints.

11. Artificial Variable

To get the initial basis in a 'greater than or' equal to constraint, additional variable is added in addition to the surplus variable. The additional variable added to a linear programming problem is called as 'artificial variable'.

12. Unbounded Solution

For some linear programs it is possible for the objective function to achieve infinitely high / low values, depends upon the objective. Such an LP is said to have an unbounded solution.

13. Standard form of LPP

Let the objective function be

Max Z = CX

And the set of constraints are represented as

AX<=b
Where, b- the vector obtained by collecting all the right hand side of the constraints.
If we add set of slack variables to all the constraints and if the constraints are equation, then that particular form is called as standard form of linear programming problem.
Therefore, Max Z = CX And the set of constraints are represented as
AX=b
Example
Consider the following LPP
Maximize Z = 15X1 + 10X2
Subject to constraints
4X1+6X2 <=360
3X1+0X2<=180
0X1+5X2 <=200
X1, X2>=0

The standard form of the given problem is obtained by adding slack variable X3 to the first constraint, X4 to the second and X5 to the third constraint.

4X1+6X2 + X3=360
3X1+0X2 +X4=180
0X1+5X2 +X5=200
X1, X2, X3, X4 & X5>=0
The modified objective function is
Maximize Z = 15X1 + 10X2 + 0X3 + 0X4 +OX5
14. Canonical form of LPP
Let the objective function be
Max Z = CX
And the set of constraints are represented as
AX<=b
Where, b- the vector obtained by collecting all the right hand side of the constraints.
This form, where all the constraints are ‘<=’ type for a maximization problem and ‘>=’ type for a minimization problem is known as canonical form of LPP.

Simplex Algorithm

Step 1
Check whether the objective function of the given L.P.P. is to be maximized or minimized.
If it is to be minimized then convert it into a problem of maximizing it by using the result
Minimum= -Maximum (-z)
Step 2

Check whether all ‘b’ values are non-negative. If any one of b is negative then multiply the corresponding inequality constraints by –1, so as to get all b values as non-negative.

Step 3

Convert all the in equations of the constraints into equations by introducing slack and/or surplus variables in the constraints. Put the costs of these variables equal to zero in the objective function, if the variables are slack variables. If surplus / artificial variables are added, then we need to use ‘Big M’ Method, which is a modified algorithm of the same simplex method.

Step 4

Obtain an initial basic feasible solution to the problem in the form Xb=B^-1 b and put it in the first column of the simplex table.

Step 5

Compute the net evaluations zj-cj (j=1,2…n) by using the relation,

Zj-Cj=CB yj-cj.

Examine the sign zj-cj.

i. If all values are >=0, then initial basic feasible solution is an optimum feasible solution.

ii. If at least one value < 0, go to next step.

Step 6

If there is more than one negative value, then choose most negative.

Step 7

Compute the ratio

{xb/yi, yi>0,I=1,2…. m} and choose the minimum of them.

The common element in the kth row and rth column is known as the leading element (pivotal element) of the table.

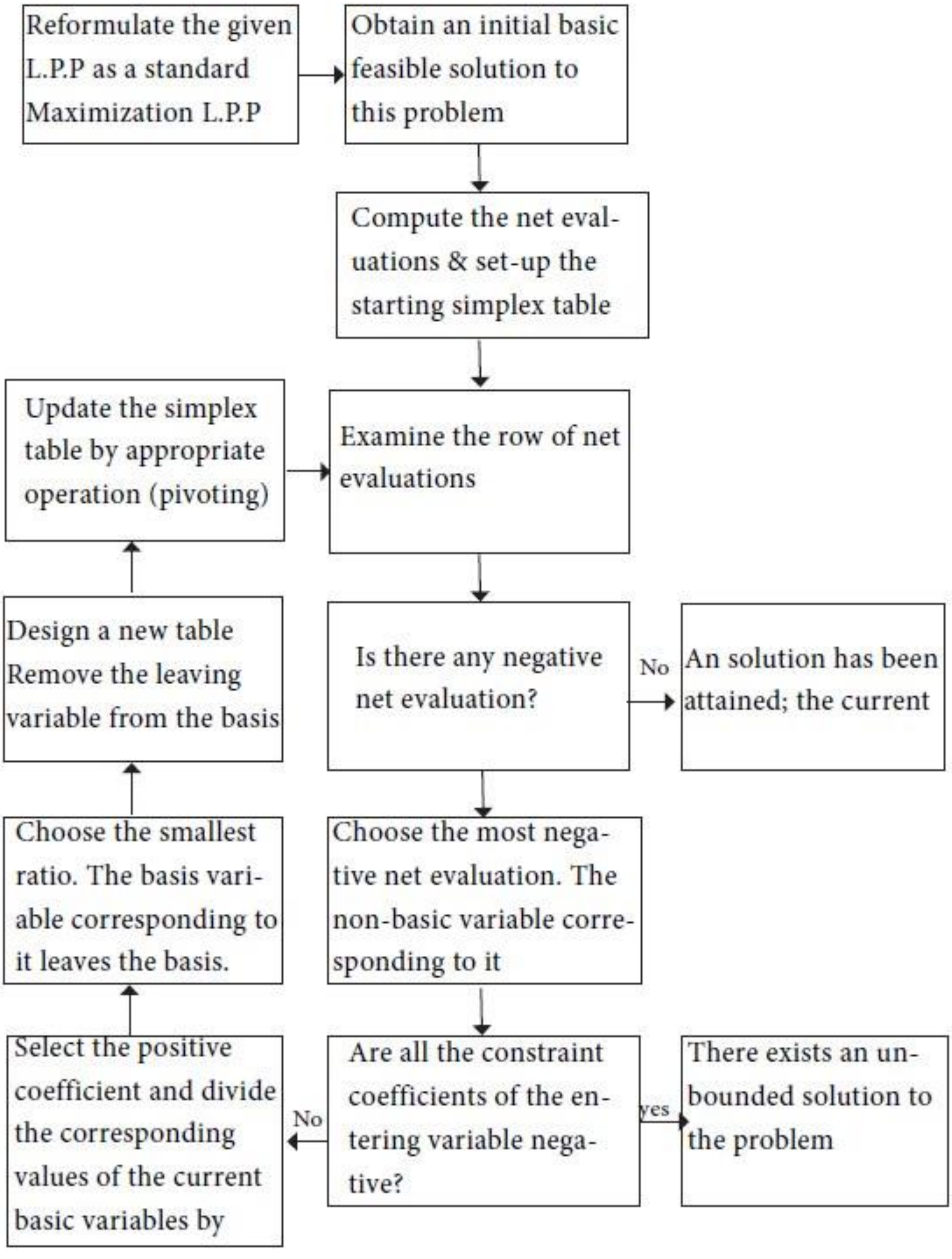
Step 8

Convert the leading element to unity by dividing its row by the leading element itself and all other elements in its column to zeros.

Step 9

Go to step 5 and repeat the process until either an optimum solution is obtained or there is an indication of unbounded solution.

Simplex Algorithm for Maximization L.P.P



Properties of The Simplex Method

1. The Simplex method for maximizing the objective function starts at a basic feasible solution for the equivalent model and moves to an adjacent basic feasible solution that does not decrease the value of the objective function. If such a solution does not exist, an optimal solution for the equivalent model has been reached. That

is, if all of the Coefficients of the non-basic variables in the objective function equation are greater than or equal to zero at some point, then an optimal solution for the equivalent model has been reached.

2. If an artificial variable is in an optimal solution of the equivalent model at a nonzero level, then no feasible solution for the original model exists. On the contrary, if the optimal solution of the equivalent model does not contain an artificial variable at a non-zero level, the solution is also optimal for the original model.

3. If all the slack, surplus, and artificial variables are zero when an optimal solution of the equivalent model is reached, then all of the constraints in the original model are strict "equalities" for the values of the variables that optimize the objective function.

4. If a non-basic variable has zero coefficients in the objective function equation when an optimal solution is reached, there are multiple optimal solutions. In fact, there is infinity of optimal solutions, the Simplex method finds only one optimal solution and stops.

5. Once an artificial variable leaves the set of basic variables (the basis), it will never enter the basis again, so all calculations for that variable can be ignored in future steps.

6. When selecting the variable to leave the current basis:

(a) If two or more ratios are smallest, choose one arbitrarily.

(b) If a positive ratio does not exist, the objective function in the original model is not bounded by the constraints. Thus a Finite optimal solution for The original model does not exist.

7. If a basis has a variable at zero level, it is called a degenerate basis.

8. Although cycling is possible, there have never been any practical problems for which the Simplex method failed to converge.

Example

Maximize z = X1+2X2
Subject to:
-X1+2X2<=8,
X1+2X2<=12,
X1-X2<=3;
X1>=0 and X2>=0.

Solution

Step 1

Introducing the slack Variable X3>=0, X4>=0 and X5>=0 to the first, second and third constraints respectively and convert the problem into standard form.

$-X_1+2X_2 + X_3 = 8, X_1+2X_2 +X_4 =12, X_1- \; X_2 +X_5 = 3;$

And the modified objective function is

$Z=X_1+2X_2+0X_3+0X_4+0X_5$

X_1, X_2, X_3, X_4 & $X_5 >0$

The constraints the given L.P.P are converted into the system of equations:

$$\begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 3 \end{pmatrix}$$

Step 2

An obvious initial basic feasible solution is given by $X_B=B^{-1}b$.
Where $B=I_3$ and $X_B=[X_3 \; X_4 \; X_5]$, & I_3 stands for Identity matrix of order of 3 (that is a 3X3 matrix). That is,
$[X_3 \; X_4 \; X_5] = I_3 \, [8 \quad 12 \quad 3] = [8 \quad 12 \quad 3]$

Step 3

We compute y_j and the net evaluations, z_j-c_j corresponding to the basic variables X_3, X_4 and X_5:

$y_1=B^{-1}a_1 = I_3 \, [-1 \;\; 1 \;\; 1] = [1 \;\; 1 \;\; 1]$

$y_2=B^{-1}a_1 = I_3 \, [2 \;\; 2 \;\; -1] = [2 \;\; 2 \;\; -1]$

$y_3 = B^{-1}e_1 = e_1, \; y_4 = B^{-1}e_2 = e_2$ and $y_5 = B^{-1}e_3 = e_3$.

$Z_1-C_1 = c_B \, y_1-c_1 = (0 \;\; 0 \quad 0) \, [-1 \quad 1 \, 1]-1 = -1.$

$Z_2-C_2 = c_B \, y_2-c_2 = (0 \;\; 0 \;\; 0) \, [2 \quad 2 \quad -1]-2 = -2.$

Z3-C3 = cB y3-c3 = (0 0 0) e1-0 = 0,

Z4-C4 = cB y4-c4 = (0 0 0) e2-0 = 0,

Z5-C5 = cB y5-c5 = (0 0 0) e3-0 = 0.

Step 4 - Deciding the entering variable

Making use of the above information, the starting simplex tableau is written as follows:

cB	yB	XB	y1	y2	y3	y4	y5
0	y3	8	-1	2	1	0	0
0	y4	12	1	2	0	1	0
0	y5	3	1	-1	0	0	1
	z	0	-1	-2	0	0	0

Simplex Table-1

From the starting tableau, it is apparent that there are two Zj-Cj values, which are having negative coefficients.

We choose the most negative of these, viz., -2. The corresponding column vector y2, therefore, enters the basis.

Step 5 - Deciding the leaving variable

Now, we will compute the ratios using the entering column elements and RHS of each constraint.

Each row of the table, the respective RHS coefficient of the constraint is divided by entering column, non-zero element and placed in the last column of the table. Then, the minimum among the value is chosen as leaving variable.
Min {XBi/Yi2, Yi2>0} = Min. {8/2, 12/2, no ratio for third row} = 4. Since the minimum ratio occurs for the first row, basis vector Y3 leaves the basis. The common intersection element y12 (=2) become the leading element for updating. We indicate the leading element in bold type with a star *.
Step 6

Convert the leading element y12 to unity and all other elements in its column (i.e.y2) to zero by the following transformations:

Y11 =Y11/Y12 =1/2, Y10 = Y10/Y12 =8/2 or 4, so on,

Y20=y20-(y10/y11) y22=12-(8/2) (2) =4.

Y30=y30-(y10/y12) y32=3-(8/2) (-2) =11.

Y21=y21-(y11/y12) y22=1-(-1/2) (2) =2.

Y31=y31-(y11/y12) y32=1-(-1/2) (-2) =0. And so on………

Step 7

Using the above computations, the following iterated simplex tableau is obtained:

The above simplex tableau yields a new basic feasible solution with the increased value of z.

Now since z1-c1 < 0, y1 enters the basis.

Also, since Min. {X Bi / yi >0} = Min {4/2, 7/ (1/2)} = 2, y4 leaves the basis.

Thus the leading element will be y21 (=2).

Converting the leadwing element to unity and all other elements of yi to zero by usual row transformations the next iterated tableau is obtained.

Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
2	Y2	X2=5	0	1	1/4	1/4	0	
1	Y1	X1=2	1	0	-1/2	1/2	0	
0	Y5	X5=6	0	0	3/4	-1/4	1	
Zj-Cj			0	0	0	1	0	

Since, all Zj-Cj>=0, we conclude that there is no more improvement possible and the problem is in its optimum stage.

Therefore, the optimal solution for the given problem is

X1= 2 X2= 5 Max Z = 12

Examples

1. Solve the following LP problem using Simplex method. And also obtain the variations in

C_j (j = 1,2) which are permitted without changing the optimal solution

Maximize $Z = 3x_1 + 5x_2$,

Subject to constraints

$x_1 + x_2 \leq 1$,

$2x_1 + x_2 \leq 1$

$x_1, x_2 \geq 0$

2. Solve the following LP problem using Simplex method.

Maximize $Z = 45x_1 + 80x_2$,

Subject to constraints

$5x_1 + 20x_2 \leq 400$,

$10x_1 + 15x_2 \leq 450$,

$x_1, x_2 \geq 0$

3. Determine the ranges for discrete changes in the components b2 of the requirement vector so as maintain the feasibility of the current optimal solution

Maximize $Z = -x_1 + 2x_2 - x_3$

Subject to constraints

$3x_1 + x_2 - x_3 \leq 10$

$-x_1 + 4x_2 + x_3 \geq 6$

$x_2 + x_3 \leq 4$

and $x_1, x_2 \geq 0$

4. Discuss the effect on the optimum solution of the discrete changes in

a) The availability of resources.

b) Change in the input- output coefficients.

c) Change in the coefficient of objective function.

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DEPARTMENT OF MATHEMATICS

UNIT – II - INTEGER LINEAR PROGRAMMING – SMT5210

INTRODUCTION:

Integer programming expresses the optimization of a linear function subject to a set of linear constraints over integer variables.

The statements presented in IPP are all linear programming models. However, linear programs with very large numbers of variables and constraints can be solved efficiently. Unfortunately, this is no longer true when the variables are required to take integer values. *Integer programming* is the class of problems that can be expressed as the optimization of a linear function subject to a set of linear constraints over integer variables. It is in fact NP-hard. More important, perhaps, is the fact that the integer programs that can be solved to provable optimality in reasonable time are much smaller in size than their linear programming counterparts. There are exceptions, of course, and this documentation describes several important classes of integer programs that can be solved efficiently, but users of OPL should be warned that discrete problems are in general much harder to solve than linear programs.

Example

Maximize z = X1+2X2
Subject to:
-X1+2X2<=8,
X1+2X2<=12,
X1-X2<=3;
X1>=0 and X2>=0.

Solution

Step 1

Introducing the slack Variable X3>=0, X4>=0 and X5>=0 to the first, second and third constraints respectively and convert the problem into standard form.

-X1+2X2 + X3 = 8,X1+2X2 +X4 =12,X1- X2 +X5 = 3;

And the modified objective function is

Z=X1+2X2+0X3+0X4+0X5

X1, X2, X3, X4 & X5 >0

The constraints the given L.P.P are converted into the system of equations:

$$\begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 3 \end{pmatrix}$$

Step 2

An obvious initial basic feasible solution is given by XB=B-1b.

Where B=I3 and XB=[X3 X4 X5], & I3 stands for Identity matrix of order of 3 (that is a 3X3 matrix). That is,

[X3 X4 X5] = I3 [8 12 3] = [8 12 3]

Step 3

We compute yj and the net evaluations, zj-cj corresponding to the basic variables X3, X4 and X5:

y1=B-1a1 = I3 [-1 1 1] = [1 1 1]

y2=B-1a1 = I3 [2 2 -1] = [2 2 -1]

y3 = B-1e1 = e1, y4 = B-1e2 = e2 and y5 = B-1e3 = e3.

Z1-C1 = cB y1-c1 = (0 0 0) [-1 1 1]-1 = -1.

Z2-C2 = cB y2-c2 = (0 0 0) [2 2 -1]-2 = -2.

Z3-C3 = cB y3-c3 = (0 0 0) e1-0 = 0,

Z4-C4 = cB y4-c4 = (0 0 0) e2-0 = 0,

Z5-C5 = cB y5-c5 = (0 0 0) e3-0 = 0.

Step 4 - Deciding the entering variable

Making use of the above information, the starting simplex tableau is written as follows:

cB	yB	XB	y1	y2	y3	y4	y5
0	y3	8	-1	2	1	0	0
0	y4	12	1	2	0	1	0
0	y5	3	1	-1	0	0	1
	z	0	-1	-2	0	0	0

Simplex Table-1

From the starting tableau, it is apparent that there are two Zj-Cj values, which are having negative coefficients.

We choose the most negative of these, viz., -2. The corresponding column vector y2, therefore, enters the basis.

Step 5 - Deciding the leaving variable

Now, we will compute the ratios using the entering column elements and RHS of each constraint.

Each row of the table, the respective RHS coefficient of the constraint is divided by entering column, non-zero element and placed in the last column of the table. Then, the minimum among the value is chosen as leaving variable.
Min {XBi/Yi2, Yi2>0} = Min. {8/2, 12/2, no ratio for third row} = 4. Since the minimum ratio occurs for the first row, basis vector Y3 leaves the basis. The common intersection element y12 (=2) become the leading element for updating. We

indicate the leading element in bold type with a star *.
Step 6

Convert the leading element y_{12} to unity and all other elements in its column (i.e.y_2) to zero by the following transformations:

$Y_{11} = Y_{11}/Y_{12} = 1/2$, $Y_{10} = Y_{10}/Y_{12} = 8/2$ or 4, so on,

$Y_{20}=y_{20}-(y_{10}/y_{11})\ y_{22}=12-(8/2)\ (2) =4$.

$Y_{30}=y_{30}-(y_{10}/y_{12})\ y_{32}=3-(8/2)\ (-2) =11$.

$Y_{21}=y_{21}-(y_{11}/y_{12})\ y_{22}=1-(-1/2)\ (2) =2$.

$Y_{31}=y_{31}-(y_{11}/y_{12})\ y_{32}=1-(-1/2)\ (-2) =0$. And so on………

Step 7

Using the above computations, the following iterated simplex tableau is obtained:

The above simplex tableau yields a new basic feasible solution with the increased value of z.

Now since $z_1-c_1 < 0$, y_1 enters the basis.

Also, since Min. $\{X_{Bi} / y_i >0\} = \text{Min}\ \{4/2, 7/ (1/2)\} = 2$, y_4 leaves the basis.

Thus the leading element will be y_{21} (=2).

Converting the leadwing element to unity and all other elements of y_i to zero by usual row transformations the next iterated tableau is obtained.

Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
2	Y2	X2=5	0	1	1/4	1/4	0	
1	Y1	X1=2	1	0	-1/2	1/2	0	
0	Y5	X5=6	0	0	3/4	-1/4	1	
Zj-Cj			0	0	0	1	0	

Since, all Zj-Cj>=0, we conclude that there is no more improvement possible and the problem is in its optimum stage.

Therefore, the optimal solution for the given problem is

X1= 2 X2= 5 Max Z = 12

Example:

1. Obtain integer solution to the all integer programming problem

   Maximize $Z = x_1 + 2x_2$

   Subject to constraints

   $3x_1 + 2x_2 \leq 5$

   $x_2 \leq 2$

   $x_1, x_2 \geq 0$ and are integer.

2. Obtain integer solution to the all integer programming problem

   Maximize $Z = x_1 + x_2$

   Subject to constraints

   $x_1 + 2x_2 \leq 4$

   $6x_1 + 2X_2 \leq 9$

   $x_1, x_2 \geq 0$ and are integer.

3. Solve the LPP by using Gomory's cutting plane method

Maximize $Z = x_1 + x_2$

Subject to constraints

$3x_1 + 2x_2 \le 5$

$x_2 \le 2$

$x_1, x_2 \ge 0$ and are integer.

4. Write algorithm on Fractional cut method-all integer programming problem.

5. Write algorithm on Fractional cut method-mixed integer programming problem.

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UNIT – III - GOALPROGRAMMING – SMT5210

INTRODUCTION:

Goal programming approach establishes a specific numeric goal for each of the objective and then
attempts to achieve each goal sequentially up to a satisfactory level rather than an optimal level.θ An important technique that has been developed to supplement LP is called goal programming θ It is not possible for LP to have multiple goals unless they are all measured in the same units, and this is a highly unusual situation θ In linear and integer programming methods the objective function is measured in one dimension only θ They may want to achieve several, sometimes contradictory, goals θ Firms often have more than one goal.

In GP, Slack and Surplus variables are known as Deviational Variables (di – and di +) (means under achievement In GP, instead of trying to minimize or maximize the objective function directly, as in case of an LP, the deviations from established goals within given set of constraints are minimized. θ Ijiri (1965) developed the concept of preemptive priority factors, assigning different priority levels to incommensurable goals and different weights to the goals at the same priority level. Examples of Multiple Conflicting Goals are: i. Maximize Profit and increase wages paid to employees ii. Upgrade product quality and reduce product cost θ GP; Channes and Cooper (1961); Suggested a method for solving an infeasible LP problem arising from various conflicting resource constraints (Goals). GOAL PROGRAMMING: AN INTRODUCTION (Cont…These deviational variables represent the extent to which target goals are not achieved. The objective function then becomes the minimization of a sum of these deviations, based on the relative importance within the preemptive structure assigned to each deviation.

OVERACHIEVEMENT

These deviations from each goal or sub-goal. Deviational variables are real “Satisficing” (instead of optimizing) Deviational variables minimized (instead of maximizing profit or minimizing cost of LP) Multiple goals (instead of one goal)

Example

Maximize z = X1+2X2
Subject to:
-X1+2X2<=8,
X1+2X2<=12,
X1-X2<=3;
X1>=0 and X2>=0.
Solution

Step 1

Introducing the slack Variable X3>=0, X4>=0 and X5>=0 to the first, second and third constraints respectively and convert the problem into standard form.

-X1+2X2 + X3 = 8,X1+2X2 +X4 =12,X1- X2 +X5 = 3;

And the modified objective function is

Z=X1+2X2+0X3+0X4+0X5

X1, X2, X3, X4 & X5 >0

The constraints the given L.P.P are converted into the system of equations:

$$\begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 3 \end{pmatrix}$$

Step 2
An obvious initial basic feasible solution is given by XB=B-1b.
Where B=I3 and XB=[X3 X4 X5], & I3 stands for Identity matrix of order of 3 (that is a 3X3 matrix). That is,
[X3 X4 X5] = I3 [8 12 3] = [8 12 3]

Step 3

We compute yj and the net evaluations, zj-cj corresponding to the basic variables X3, X4 and X5:

$$y1=B\text{-}1a1 = I3\ [-1\ \ 1\ \ 1] = [1\ \ 1\ \ 1]$$

$$y2=B\text{-}1a1 = I3\ [2\ \ 2\ \ -1] = [2\ \ 2\ -1]$$

$$y3 = B\text{-}1e1 = e1,\ \ y4 = B\text{-}1e2 = e2 \quad \text{and} \quad y5 = B\text{-}1e3 = e3.$$

$$Z1\text{-}C1 = cB\ y1\text{-}c1 = (0\ 0\ \ \ 0)\ [-1\ \ 1\ 1]\text{-}1 = -1.$$

$$Z2\text{-}C2 = cB\ y2\text{-}c2 = (0\ 0\ 0)\ [2\ \ 2\ \ -1]\text{-}2 = -2.$$

$$Z3\text{-}C3 = cB\ y3\text{-}c3 = (0\ 0\ 0)\ e1\text{-}0 = 0,$$

$$Z4\text{-}C4 = cB\ y4\text{-}c4 = (0\ 0\ 0)\ e2\text{-}0 = 0,$$

$$Z5\text{-}C5 = cB\ y5\text{-}c5 = (0\ 0\ 0)\ e3\text{-}0 = 0.$$

Step 4 - Deciding the entering variable

Making use of the above information, the starting simplex tableau is written as follows:

cB	yB	XB	y1	y2	y3	y4	y5
0	y3	8	-1	2	1	0	0
0	y4	12	1	2	0	1	0
0	y5	3	1	-1	0	0	1
	z	0	-1	-2	0	0	0

Simplex Table-1

From the starting tableau, it is apparent that there are two Zj-Cj values, which are having negative coefficients.

We choose the most negative of these, viz., -2. The corresponding column vector y2, therefore, enters the basis.

Step 5 - Deciding the leaving variable

Now, we will compute the ratios using the entering column elements and RHS of each constraint.

Each row of the table, the respective RHS coefficient of the constraint is divided by entering column, non-zero element and placed in the last column of the table. Then, the minimum among the value is chosen as leaving variable.
Min {XBi/Yi2, Yi2>0} = Min. {8/2, 12/2, no ratio for third row} = 4. Since the minimum ratio occurs for the first row, basis vector Y3 leaves the basis. The common intersection element y12 (=2) become the leading element for updating. We indicate the leading element in bold type with a star

Step 6

Convert the leading element y12 to unity and all other elements in its column (i.e.y2) to zero by the following transformations:

Y11 =Y11/Y12 =1/2, Y10 = Y10/Y12 =8/2 or 4, so on, Y20=y20-(y10/y11) y22=12-(8/2) (2) =4.

Y30=y30-(y10/y12) y32=3-(8/2) (-2) =11.

Y21=y21-(y11/y12) y22=1-(-1/2) (2) =2.

Y31=y31-(y11/y12) y32=1-(-1/2) (-2) =0. And so on………

Step 7

Using the above computations, the following iterated simplex tableau is obtained:

The above simplex tableau yields a new basic feasible solution with the increased value of z.

Now since z1-c1 < 0, y1 enters the basis.

Also, since Min. {X Bi / yi >0} = Min {4/2, 7/ (1/2)} = 2, y4 leaves the basis.

Thus the leading element will be y21 (=2).

Converting the leadwing element to unity and all other elements of yi to zero by usual row transformations the next iterated tableau is obtained.

Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
2	Y2	X2=5	0	1	1/4	1/4	0	
1	Y1	X1=2	1	0	-1/2	1/2	0	
0	Y5	X5=6	0	0	3/4	-1/4	1	
Zj-Cj			0	0	0	1	0	

Since, all Zj-Cj>=0, we conclude that there is no more improvement possible and the problem is in its optimum stage.

Therefore, the optimal solution for the given problem is

$X1= 2$ $X2= 5$ $\text{Max } Z = 12$

EXAMPLES:

1. Solve the GP problem

Minimize = $P_1 d_1^+ + P_2 d_2^- + P_3 d_3^-$

Subject to

$x_1 + x_2 + d_1^- - d_1^+ = 40$

$x_1 + d_2^- - d_3^+ = 20$

$x_1, x_2, d_i > o$

2. Use simplex method to solve the goal programming problem

Minimize $Z = P_1 d_1^- + P_2 (2d_2^- + d_3^-) + P_3 d_1^+$

Subject to constraints

$x_1 + x_2 + d_1^- - d_1^+ = 500$

$x_1 + d_2^- = 340$

$x_2 + d_3^- = 400$

$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$

3. Use simplex method to solve the goal programming problem

Minimize $Z = P_1 d_1^- + P_2 (2d_2^- + d_3^-) + P_3 d_1^+$

Subject to constraints

$x_1 + x_2 + d_1^- - d_1^+ = 400$

$x_1 + d_2^- = 240$

$x_2 + d_3^- = 300$

$x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$

4 Solve the goal programming problem by using Simplex method

Minimize $Z = P_1 d_1^- + P_2 d_4^+ + 5p_3 d_2^- + 3p_3 d_3^- + p_4 d_1^+$

Subject to constraints

$x_1 + x_2 + d_1^- - d_1^+ = 80$

$x_1 + x_2 + d_4^- - d_4^+ = 90$

$x_1 + d_2^- = 70$

$x_2 + d_3^- = 45$

$x_1, x_2, d_1^-, d_1^+ \; d_2^-, d_3^- \; d_4^- \; d_4^+ \geq 0$

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SCHOOL OF SCIENCE AND HUMANTTIES

DEPARTMENT OF MATHEMATICS

UNIT – IV - DECISIONANDGAMETHEORY – SMT5210

INTRODUCTION:

DECISION THEORY

Introduction to Decision Making process - Decision making under uncertainty - Maximin and Maximax criteria - Hurwicz criterion – Laplace criterion – Minimax Regret criterion – Decision tree analysis - Simulation - Nature and need for simulation - Monte Carlo method.

Decision theory deals with methods for determining the optimal course of action when a number of alternatives are available and their consequences cannot be forecast with certainty. It is difficult to imagine a situation which does not involve such decision problems, but we shall restrict ourselves primarily to problems occurring in business, with consequences that can be described in dollars of profit or revenue, cost or loss.

For these problems, it may be reasonable to consider as the best alternative that which results in the highest profit or revenue, or lowest cost or loss, on the average, in the long run. This criterion of optimality is not without shortcomings, but it should serve as a useful guide to action in repetitive situations where the consequences are not critical. (Another criterion of optimality, the maximization of expected ìutility,î provides a more personal and subjective guide to action for a consistent decision-maker.) The simplest decision problems can be resolved by listing the possible monetary consequences and the associated probabilities for each alternative, calculating the expected monetary values of all alternatives, and selecting the alternative with the highest expected monetary value.

The determination of the optimal alternative becomes a little more complicated when the alternatives involve sequences of decisions. In another class of problems, it is possible to acquire often at acertain cost additional information about an uncertain variable. This additional information is rarely entirely accurate. Its value hence, also the maximum amount one would be willing to pay to acquire it should depend on the difference between the best one expects to do with the help of this information and the best one expects to do without it. These are, then, the types of problems which we shall now begin to examine in more detail.

Very simply, the decision problem is how to select the best of the available alternatives. The elements of the problem are the possible alternatives (actions, acts), the possible events (states, outcomes of a random process), the probabilities of these events, the consequences associated with each possible alternative-event combination, and the criterion (decision rule) according to which the best alternative is selected.

The pay off (profit or loss) for the range of possible outcomes based on two factors:

1. Different decision choices
2. Different possible real world scenarios

For example, suppose Geoffrey Ramsbottom is faced with the following pay-off table. He has to choose how many salads to make in advance each day before he knows the actual demand.

- His choice is between 40, 50, 60 and 70 salads.
- The actual demand can also vary between 40, 50, 60 and 70 with the probabilities as shown in the table - e.g. P(demand = 40) is 0.1.
- The table then shows the profit or loss - for example, if he chooses to make 70 but demand is only 50, then he will make a loss of $60.

Daily supply						
Daily Demand		**Probability**	**40 salads**	**50 salads**	**60 salads**	**70 salads**
	40 salads	0.10	$80	$0	$(80)	$(160)
	50 salads	0.20	$80	$100	$20	$(60)
	60 salads	0.40	$80	$100	$120	$40
	70 salads	0.30	$80	$100	$120	$140

Maximax

The maximax rule involves selecting the alternative that maximises the maximum payoff available.

This approach would be suitable for an optimist, or 'risk-seeking' investor, who seeks to achieve the best results if the best happens. The manager who employs the maximax criterion is assuming that whatever action is taken, the best will happen; he/she is a risk-taker. So, how many salads will Geoffrey decide to supply?

Looking at the payoff table, the highest maximum possible pay-off is $140. This happens if we make 70 salads and demand is also 70. Geoffrey should therefore decide to supply 70 salads **every** day.

Minimax

The maximin rule involves selecting the alternative that maximises the minimum pay-off achievable. The investor would look at the worst possible outcome at each supply level, then selects the highest one of these. The decision maker therefore chooses the outcome which is guaranteed to minimise his losses. In the process, he loses out on the opportunity of making big profits.

This approach would be appropriate for a pessimist who seeks to achieve the best results if the worst happens.

So, how many salads will Geoffrey decide to supply? Looking at the payoff table,

- If we decide to supply 40 salads, the minimum pay-off is $80.
- If we decide to supply 50 salads, the minimum pay-off is $0.
- If we decide to supply 60 salads, the minimum pay-off is ($80).
- If we decide to supply 70 salads, the minimum pay-off is ($160).

The highest minimum payoff arises from supplying 40 salads. This ensures that the worst possible scenario still results in a gain of at least $80.

The miimax regret

The minimax regret strategy is the one that minimises the maximum regret. It is useful for a risk-neutral decision maker. Essentially, this is the technique for a 'sore loser' who does not wish to make the wrong decision.

'Regret' in this context is defined as the opportunity loss through having made the wrong decision.

To solve this a table showing the size of the regret needs to be constructed. This means we need to find the biggest pay-off for each demand row, then subtract all other numbers in this row from the largest number.

For example, if the demand is 40 salads, we will make a maximum profit of $80 if they all sell. If we had decided to supply 50 salads, we would achieve a nil profit. The difference or 'regret' between that nil profit and the maximum of $80 achievable for that row is $80.

Regrets can be tabulated as follows :

Daily supply					
Daily Demand		**40 salads**	**50 salads**	**60 salads**	**70 salads**
	40 salads	$0	$80	$160	$240
	50 salads	$20	$0	$80	$160
	60 salads	$40	$20	$0	$80
	70 salads	$60	$40	$20	$0

The maximum regrets for each choice are thus as follows (reading down the columns):

- If we decide to supply 40 salads, the maximum regret is $60.
- If we decide to supply 50 salads, the maximum regret is $80.
- If we decide to supply 60 salads, the maximum regret is $160.
- If we decide to supply 70 salads, the maximum regret is $240.

A manager employing the minimax regret criterion would want to minimise that maximum regret, and therefore supply 40 salads only.

Note that the above techniques can be used even if we do not have probabilities.

EXAMPLE:

1. The following Pay - off table is given

EVENTS

Action	E1	E2	E3	E4
A1	40	200	-200	100
A2	200	0	-200	0
A3	0	100	0	150
A4	-50	400	100	0

Determine the decision rule under i) Maximini ii) Regret iii) Minimax

iv) Laplace criterion.

2. Solve the game by using dominance property

		Group B			
		I	II	III	IV
	A	8	10	9	14
Group A	B	10	11	8	12
	C	13	12	14	13

3. Solve the following game graphically

Player B				
Player A	1	3	-3	7
	2	5	4	-6

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UNIT –V - DYNAMICPROGRAMMING – SMT5210

INTRODUCTION:

Dynamic programming is both a mathematical optimization method and a computer programming method. The method was developed by Richard Bellman in the 1950s and has found applications in numerous fields, from aerospace engineering to economics. Dynamic programming is a method for solving a complex problem by breaking it down into simpler sub problems, solving each of those sub problems just once, and storing their solutions – in an array (usually).

Now, every time the same sub-problem occurs, instead of recomputing its solution, the previously calculated solutions are used, thereby saving computation time at the expense of storage space.

Example

Maximize $z = X1+2X2$
Subject to:
$-X1+2X2<=8$,
$X1+2X2<=12$,
$X1-X2<=3$;
$X1>=0$ and $X2>=0$.

Solution

Step 1

Introducing the slack Variable $X3>=0$, $X4>=0$ and $X5>=0$ to the first, second and third constraints respectively and convert the problem into standard form.

$-X1+2X2 + X3 = 8, X1+2X2 +X4 =12, X1- X2 +X5 = 3$;

And the modified objective function is

$Z=X1+2X2+0X3+0X4+0X5$

X1, X2, X3, X4 & X5 >0

The constraints the given L.P.P are converted into the system of equations:

$$\begin{pmatrix} -1 & 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{pmatrix} = \begin{pmatrix} 8 \\ 12 \\ 3 \end{pmatrix}$$

Step 2

An obvious initial basic feasible solution is given by XB=B-1b.
Where B=I3 and XB=[X3 X4 X5], & I3 stands for Identity matrix of order of 3 (that is a 3X3 matrix). That is,

[X3 X4 X5] = I3 [8 12 3] = [8 12 3]

Step 3

We compute yj and the net evaluations, zj-cj corresponding to the basic variables X3, X4 and X5:

y1=B-1a1 = I3 [-1 1 1] = [1 1 1]

y2=B-1a1 = I3 [2 2 -1] = [2 2 -1]

y3 = B-1e1 = e1, y4 = B-1e2 = e2 and y5 = B-1e3 = e3.

Z1-C1 = cB y1-c1 = (0 0 0) [-1 1 1]-1 = -1.

Z2-C2 = cB y2-c2 = (0 0 0) [2 2 -1]-2 = -2.

Z3-C3 = cB y3-c3 = (0 0 0) e1-0 = 0,

Z4-C4 = cB y4-c4 = (0 0 0) e2-0 = 0,

Z5-C5 = cB y5-c5 = (0 0 0) e3-0 = 0.

Step 4 - Deciding the entering variable

Making use of the above information, the starting simplex tableau is written as follows:

cB	yB	XB	y1	y2	y3	y4	y5
0	y3	8	-1	2	1	0	0
0	y4	12	1	2	0	1	0
0	y5	3	1	-1	0	0	1
	z	0	-1	-2	0	0	0

Simplex Table-1

From the starting tableau, it is apparent that there are two Zj-Cj values, which are having negative coefficients.

We choose the most negative of these, viz., -2. The corresponding column vector y2, therefore, enters the basis.

Step 5 - Deciding the leaving variable

Now, we will compute the ratios using the entering column elements and RHS of each constraint.

Each row of the table, the respective RHS coefficient of the constraint is divided by entering column, non-zero element and placed in the last column of the table. Then, the minimum among the value is chosen as leaving variable.
Min {XBi/Yi2, Yi2>0} = Min. {8/2, 12/2, no ratio for third row} = 4. Since the minimum ratio occurs for the first row, basis vector Y3 leaves the basis. The common intersection element y12 (=2) become the leading element for updating. We

indicate the leading element in bold type with a star *.

Step 6

Convert the leading element y_{12} to unity and all other elements in its column (i.e.y_2) to zero by the following transformations:

$Y_{11} = Y_{11}/Y_{12} = 1/2$, $Y_{10} = Y_{10}/Y_{12} = 8/2$ or 4, so on,

$Y_{20}=y_{20}-(y_{10}/y_{11})\ y_{22}=12-(8/2)\ (2) =4$.

$Y_{30}=y_{30}-(y_{10}/y_{12})\ y_{32}=3-(8/2)\ (-2) =11$.

$Y_{21}=y_{21}-(y_{11}/y_{12})\ y_{22}=1-(-1/2)\ (2) =2$.

$Y_{31}=y_{31}-(y_{11}/y_{12})\ y_{32}=1-(-1/2)\ (-2) =0$. And so on………

Step 7

Using the above computations, the following iterated simplex tableau is obtained:

The above simplex tableau yields a new basic feasible solution with the increased value of z.

Now since $z_1-c_1 < 0$, y_1 enters the basis.

Also, since Min. $\{X_{Bi} / y_i >0\}$ = Min $\{4/2, 7/ (1/2)\} = 2$, y_4 leaves the basis.

Thus the leading element will be y_{21} (=2).

Converting the leadwing element to unity and all other elements of y_i to zero by usual row transformations the next iterated tableau is obtained.

Cb	YBa	Xba	Y1	Y2	Y3	Y4	Y5	Xbi/Yi1
2	Y2	X2=5	0	1	1/4	1/4	0	
1	Y1	X1=2	1	0	-1/2	1/2	0	
0	Y5	X5=6	0	0	3/4	-1/4	1	
Zj-Cj			0	0	0	1	0	

Since, all Zj-Cj>=0, we conclude that there is no more improvement possible and the problem is in its optimum stage.

Therefore, the optimal solution for the given problem is

X1= 2 X2= 5 Max Z = 12

EXAMPLE:

1. Obtain optimum solution by using dynamic programming problem

   Maximize $Z = 2x_1 + 5x_2$

   Subject to constraints

   $2x_1 + x_2 \leq 430$

   $2x_2 \leq 460$

   $x_1, x_2 \geq 0$

2. Solve the LPP by using dynamic programming problem

   Maximize $Z = 3x_1 + 5x_2$

   Subject to constraints

   $x_1 \leq 4$

   $x_2 \leq 6$

   $3x_1 + 2x_2 \leq 18$

   $x_1, x_2 \geq 0$

3. Use dynamic programming to solve the following

   Minimize $Z = y_1^2 + y_2^2 + y_3^2$

   Subject to constraints

$Y_1 + y_2 + y_3 = 10$ and $y_1, y_2, y_3 \geq 0$