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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – I – Introduction to OR – SMT1504

Operation Research:

1. OR is an aid for the executive in making decisions by providing him with the quantitative information, based on the scientific method analysis.

2. OR is the art of giving bad answers to problems, to which, otherwise worse answers are given.

3.OR is the art of wining wars without actually fighting them.

4.OR is the application of scientific methods by interdisciplinary teams to problems involving the control of organised (man-machine) systems so as to provide solutions which best serve the purpose of the organisation as a whole.

The characteristics of OR Model

A model is defined as idealised representation of the real life situation. It represents one or few aspects of reality.

Characteristics of OR—

1. The number of simplifying assumption should be as few as possible.

2. Model should be simple but close to reality.

3. It should be adaptable to parametric type of treatment.

4. It should be easy and economical to construct.

Main advantage and limitation of OR model.

Advantage

1 It provides a logical and systematic approach to the problem.

2. It indicate the scope .as well as limitation of a problem.

3. It makes the overall structure of the problem more comprehensible and helps in dealing with problem in its entirety.

Limitations—

Models are only idealised representation Of reality and should not be regarded as absolute in any case.

Distinguish between:

(i) Iconic or Physical Model and Analogue or schematic model.

(ii) Deterministic and Probabilistic model.

(i) Iconic Model—It represent the system by enlarging or reducing the size on some scale. In other words it is an image.

Example-toy aeroplane, photographs, drawings, maps etc.

Schematic Model—The models, in which one set of properties is used to

represent, another set of properties are called schematic or analogue models.

For example-graphs used to show different quantities.

(ii) Deterministic model—Such models assume conditions of complete certainty and perfect knowledge.

Example-LPP, transportation, assignment etc.

Probabilistic (or stochastic) Model—These type of models' usually handle such

situation in which the consequences or payoff of managerial actions cannot be predicted with certainty. However it is possible to forecast a pattern of events, based on which managerial decision can be made.

For example insurance companies are willing to insure against risk of fire, accident, sickness and so on. Here the pattern of events have been compiled in the form of probability distribution.

The objective of operation Research

The objective of OR is to provide a scientific basis to the managers of an organization for solving problems involving interaction of the components of the system, by employing a system approach by a team of scientists drawn from different disciplines, for finding a solution which is in the best interest of the organization as a whole.

The characteristics of operation research.

1. System Orientation

2. Use of interdisciplinary teams

3. Application of Scientific methOds

4. Uncovering of new problems

5. Improvement in the quality of decisions

6. Use of computer

7. Quantitative solutions

8. Human factors

The various phases of operation research

Or

The steps involved in the solution of OR Problem.

Operation research is based on scientific methodology which proceeds as:

1. Formulating the problem.

2 Constructing a model to represent the system under study

3. Deriving a solution from the model.

4. Testing the model and the solution derivq4 from it.

5. Establishing controls over the solution.

6. Putting the solution to work i.e. implementation.

(i) Assignment of job to machine

(ii) Product mix

(iii) Advertising media selection

(iv) Transportation.

2. Dynamic programming

(i) Capital budgeting

(ii) Employment smoothening

(iii) Cargo loading.

3. Inventory control

(i) Economic order quantity

MATHEMATICAL FORMULATION OF LPP

Egg contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and cost 12 paise per gram. Milk contains 8 units of vitamin A per gram and 12 units of vitamin B per gram and costs 20 paise per gram. The daily minimum requirement of vitamin A and vitamin B are 100 units and 120 units respectively. Find the optimal product mix. Formulate this problem as a LPP.

Let x_1 = number of grams of eggs to be consumed

x_2 = number of grams of milk to be consumed

The LPP is: Min Z = $12x_1 + 20x_2$

Subject to

$$6x_1 + 8x_2 \geq 100$$

$$7x_1 + 12x_2 \geq 120$$

$$x_1, x_2 \geq 0$$

GRAPHICAL METHOD

Working procedure

Step 1: Consider each inequality constraint as equation.

Step 2: Plot each equation on the graph, as each will geometrically represent a straight line.

Step 3: Mark the region. If the inequality constraint corresponding to a line is $\leq$ type then the region below the line lying in the first quadrant is shaded. For the inequality constraint $\geq$ type, the region above the line in the first quadrant is shaded. The points lying in the common region will satisfy all the constraints simultaneously. The common region is the feasible region.

Step 4: Plot the objective function by assuming Z = 0. This will be a straight line passing through the origin. As the value of Z is increased from zero, the line starts moving, parallel to itself. Move the line till it is farthest away from the origin for maximization of the objective function. For a minimization problem the line will be nearest to the origin. A point of the feasible region through which this line passes will be the optimal point.

Step 5: Alternatively find the co-ordinates of the extreme points of the feasible region and find the value of the objective function at each of these extreme points. The point at which the value is maximum (or minimum) is the optimal point and its coordinates give the optimal solution.

Solve the following LPP by graphical method.

Minimize Z = $20x_1 + 10x_2$

Subject to

$x_1 + 2x_2 \le 40$

$3x_1 + x_2 \ge 30$

$4x_1 + 3x_2 \ge 60$

& $x_1, x_2 \ge 0$

Solution:

Replace all the inequalities of the constraints by equation

$x_1 + 2x_2 = 40$ if $x_1 = 0 \Rightarrow x_2 = 20$

$x_2 = 0 \Rightarrow x_1 = 40$

$\therefore$ $x_1 + 2x_2 = 40$ passes through (0,20)(40,0)

$3x_1 + x_2 = 30$ passes through (0,30)(10,0)

$4x_1 + 3x_2 = 60$ passes through (0,20)(15,0)

Plot each equation on the graph. The feasible region is ABCD.

C and D are points of intersection of lines

$\therefore$ $x_1 + 2x_2 = 40$, $3x_1 + x_2 = 30$,

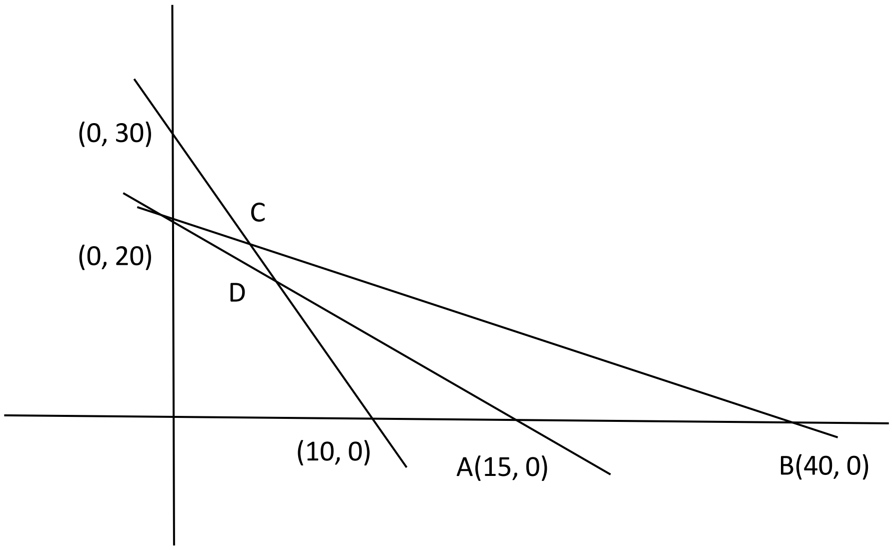
and $4x_1 + 3x_2 = 60$, $x_1 + x_2 = 30$

On solving we get C = (4, 18) D = (6, 12)

Corner points	Value of Z = 20 $x_1 + 10x_2$
A (15,0)	300
B (40,0)	800
C (4,18)	260
D (6,12)	240 minimum value

$\therefore$ The minimum value of Z occurs at D (6, 12).

Hence the optimal solution is $x_1 = 6, x_2 = 12$.



Problem. An advertising company wishes to plan its advertising strategy in three different media-television, radio and magazines. The purpose of advertising is to reach as large a number of potential customers as possible. Following data have been obtained from market survey-

	Television	Radio	Magazine I	Magazine II
Cost of an advertising unit	Rs. 30000	20000	15000	10000
No. of potential customer reached per unit	20000	600000	150000	100000
No. of female customer reached per unit	1, 50000	400000	70000	50000

The company wants to spend not more than Rs. 450000 on advertising. Following are the further requirements.

1. at least 1 million exposures take place among female customers.

2. advertising on magazines be limited to Rs 1, 50000

3. at least 3 advertising units to be bought on magazine 1 and 2 units on magazine II.

4. The number of advertising units on television and radio should each be between 5 and 10.

Formulate an LPP model for the problem.

Solution : Let x1- no. of advertising unit of television

no. of advertising unit of radio

x3- no. of advertising unit of Magazine I

x4— no. of advertising unit of Magazine II

Objective function

Maximize Z = i05 (2x1+ 6x2 + 1.5x3 + x4)

Constraints are

$$30000x_1 + 20000x_2 + 15000x_3 + 10000x_4 \leq 450000$$
$$150000x_1 + 400000x_2 + 70000x_3 + 50000x_4 \geq 1000000$$
$$15000x_3 + 10000x_4 \leq 150000$$
$$x_3 \geq 3$$
$$x_4 \geq 2$$
$$5 \leq x_1 \leq 10 \text{ or } x_1 \geq 5,\ x_1 \leq 10$$
$$5 \leq x_2 \leq 10 \text{ or } x_2 \geq 5,\ x_2 \leq 10$$
$$\text{where } x_1, x_2, x_3\ x_4 \text{ each } \geq 0$$

Problem. A person requires 10, 12 and 12 units of chemicals A, B and C respectively for herbal garden. A liquid product contains 5, 2 and 1 units of A,B and C respectively per Jar. A dry product contains 1, 2 and 4 units of A, B and C per cartoon. If the liquid product sells for Rs. 3 per Jar and dry product sells for Rs. 2 per cartoon, how many of each should be purchased to minimise the cost and meet the requirements.

Solution : Requirement

	A	B	C	
	10	12	12 units	
Liquid product	5	2	1	units Rs. 3/-per jar
Dry product	1	2	4	Rs. 2/-per Cartons

1. Select decision variable

   x_1 – no. of jars of liquid product

   x_2 – no. of cartoons of dry product

2. Objective function

   Minimize cost $(z) = 3x_1 + 2x_2$

3. Constraints :

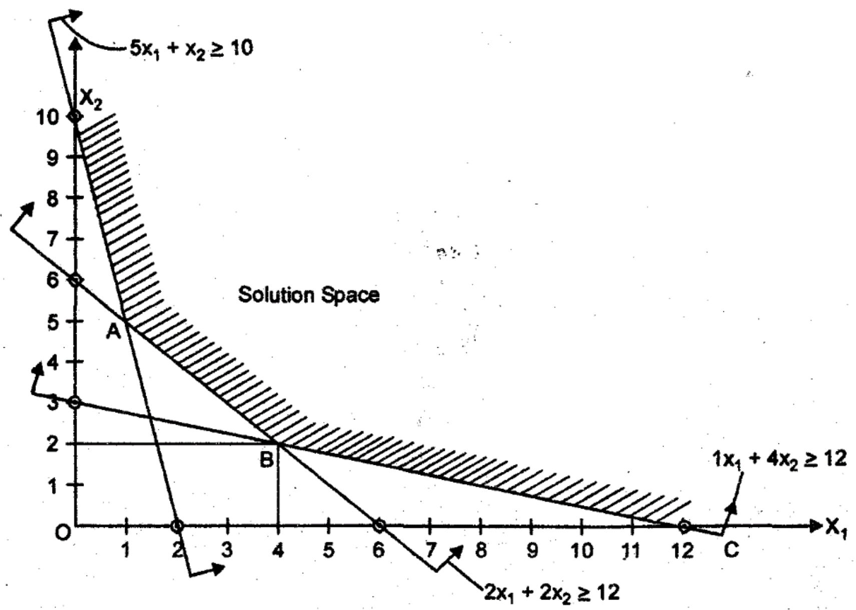
$$5x_1 + x_2 \geq 10$$
$$2x_1 + 2x_2 \geq 12$$
$$1x_1 + 4x_2 \geq 12$$

4. Add non negativity constraints :

$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Graphical Method :

$$5x_1 + x_2 = 10 \Rightarrow x_1 = 0; x_2 = 10 \text{ and } x_2 = 0; x_1 = 2$$
$$2x_1 + 2x_2 = 12 \Rightarrow x_1 = 0; x_2 = 6 \text{ and } x_2 = 0; x_1 = 6$$
$$1x_1 + 4x_2 = 12 \Rightarrow x_1 = 0; x_2 = 3 \text{ and } x_2 = 0; x_1 = 12$$



Point A (1,5) $Z(A) = 3 \times 1 + 2 \times 5 = 13$

Point B (4, 2) $Z(B) = 3 \times 4 + 2 \times 2 = 16$

Point C (12,0) $Z(C) = 3 \times 12 + 2 \times 0 = 36$

Minimum cost at point A *i.e.* Rs. 13

x_1 (no. of Jar of Liquid product) = 1

x_2 (no. of carton of dry product) = 5

Minimum cost (Z) = Rs. 13.

Problem. A firm manufactures pain relieving pills in two sizes A and B, size A contains 4 grains of element a, 7 grains of element b and 2 grains of element c, size B contains 2 grains of element a, 10 grains of element b and 8 grains of c. It is found by users that it requires at least 12 grains of element a, 74 grains of element b and 24 grains of element c to provide immediate relief It is required to determine that least no. of pills a patient should take to get immediate relief. Formulate the problem as standard LPP.

Solution : Pain relieving pills

	a	b	c
Size A	4	7	2
Size B	2	10	8
Min. requirement	12	74	24

Step 1. Select decision variable

x_1 – no. of pills of size A

x_2 – no. of pills of size B

Step 2. Objective function

Minimum (no. of pills) $z = x_1 + x_2$

Step 3. Constraints

$$4x_1 + 2x_2 \geq 12$$
$$7x_1 + 10x_2 \geq 74$$
$$2x_1 + 8x_2 \geq 24$$

Step. 4. Add non negativity constraints

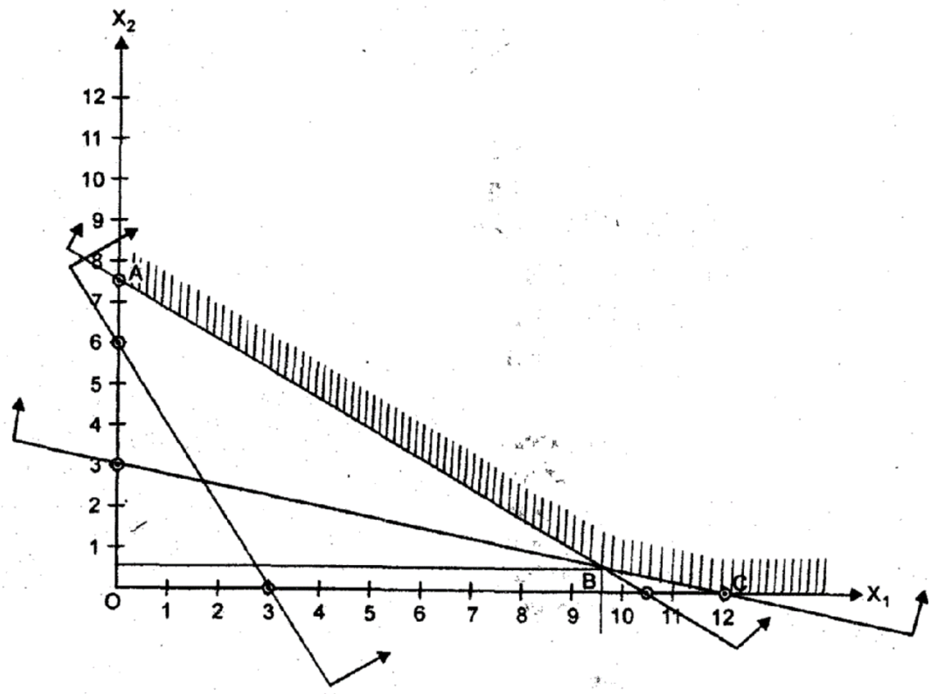
$$x_1 \geq 0 \quad ; \quad x_2 \geq 0$$

Determining the value of x_1 and x_2 by graphical method

$4x_1 + 2x_2 = 12$ $x_1 = 0; x_2 = 6$ and $x_2 = 0; x_1 = 3$

$7x_1 + 10x_2 = 74$ $x_1 = 0; x_2 = 7.4$ and $x_2 = 0; x_1 = 10.57$

$2x_1 + 8x_2 = 24$ $x_1 = 0; x_2 = 3$ and $x_2 = 0; x_1 = 12$



Point A (0, 7.4) Z(A) = 0 + 7.4 = 7.4 (Minimum)
Point C (12, 0) Z (C) = 12 + 0 = 12
Point B (9.6, 0.6) Z(B) = 9.6 + 0.6 = 10.2
No. of pills of size A = 0
No. of pills of size B = 7.4 ≃ 8 pills
Minimum no. of pills = 8 pills.

Problem. An automobile manufacturer makes automobiles and trucks in a factory that is divided into two shops. Shop A which perform the basic assy operation must work 5 man days on each truck but only 2 man days on each automobile. Shop B which perform finishing operations must work 3 man days for each automobile or truck that it produces. Because of men and machine limitations shop A has 180 man days per week available while shop B has 135.man days per week. If the manufacturer makes a profit of Rs. 300 on each truck and Rs. 200 on each automobile; how many of each should be produced to maximize his profit?

Solution :

	Shop A	Shop B	Profit
Automobile	2 man days	3 man days	Rs. 200
Trucks	5 man days	3 man days	Rs. 300
Availability	180 man days/week	135 man days/week	

1. Select decision variable

   x_1 – no. of automobile to be produced/week

   x_2 – no. of trucks to be produced/week

2. Objective function

   Maximize $Z = 200x_1 + 300x_2$

3. Constraints

$$2x_1 + 5x_2 \leq 180$$
$$3x_1 + 3x_2 \leq 135$$

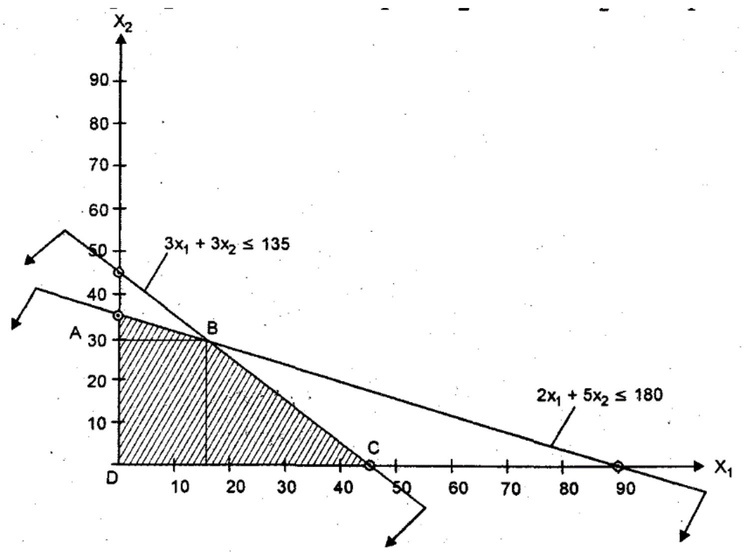
4. Add non negativity constraints

$$x_1 \geq 0; x_2 \geq 0$$

Determine the value of x_1 and x_2 by graphical method

$2x_1 + 5x_2 = 180$ $\quad x_1 = 0; x_2 = 36$ and $x_2 = 0; x_1 = 90$

$3x_1 + 3x_2 = 135$ $\quad x_1 = 0; x_2 = 45$ and $x_2 = 0; x_1 = 45$



Point D (0, 0) Z (D) $= 200 \times 0 + 300 \times 0 = 0$

Point A (0, 36) Z (A) $= 200 \times 0 + 300 \times 36 = 10800$

Point C (45, 0) Z (C) $= 200 \times 45 + 300 \times 0 = 9000$

Point B (15 , 30) Z (B) $= 200 \times 15 + 300 \times 30 = 3000 + 9000 = 12000$

Maximum Profit at Point B (15 , 30) *i.e.* Rs.12000/-

x_1 = no. of automobile/week = 15

x_2 = no. of trucks/week = 30

Maximum profit = 12000/-

Problem. On completing the construction of house a person discovers that J square feet of plywood scrap and 80 square feet of white pine scrap are in useable m for the construction of tables and book cases. It takes 16 square feet of plywood 8 square feet of white pine to make a table, 12 square feet of plywood and 16 Llare feet of white pine are required to contruct a book case. By selling the finishing duct to a local furniture store the person can realize a profit of Rs. 25 on each table d Rs. 290 on each book case. How may the man most profitably use the left over ood ? Use graphical method to solve problem.

Solution :

	Plywood	**White pine**	**Profit**	
Table	16	8	Rs. 25	each table
Book case	12	16	Rs. 290	each book case
Availability	100	80		

1. Select decision variable

   x_1 – no. of table

   x_2 – no. of book case

2. Objective function

   Maximize profit (Z) = $25x_1 + 290x_2$

3. Constraints

$$16x_1 + 12x_2 \leq 100$$
$$8x_1 + 16x_2 \leq 80$$

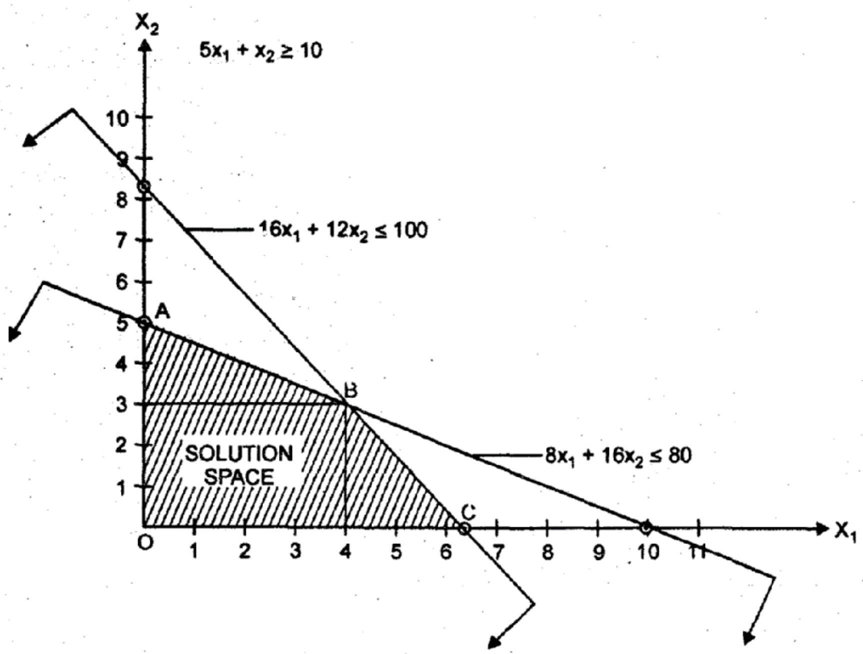
4. Add non negativity constraints

$$x_1 \geq 0$$
$$x_2 \geq 0$$

Determine the value of x_1 and x_2 using graphical method

$16x_1 + 12x_2 = 100$ $\quad$ $x_1 = 0; x_2 = 8.3$ and $x_2 = 0; x_1 = 6.25$

$8x_1 + 16x_2 = 80$ $\quad$ $x_1 = 0; x_2 = 5$ and $x_2 = 0; x_1 = 10$



Point O (0, 0) Z(O) = $25 \times 0 + 290 \times 0 = 0$

Point A (0, 5) Z (A) = $25 \times 0 + 290 \times 5 = 1450$

Point C (6.25, 0) Z (C) = $25 \times 6.25 + 290 \times 0 = 156.25$

Point B (4, 3) Z (B) = $25 \times 4 + 290 \times 3 = 100 + 870 = 970$

Maximum profit (Z) at point A *i.e.* Rs. 1450.

x_1 – no. of table = 0 Max. Profit (Z) = Rs. 1450/-

x_2 – no.of bookcase = 5.

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UNIT – II –Linear Programming Problem– SMT1504

Basic definition:

1) Define a feasible region.
Solution:

A region in which all the constraints are satisfied simultaneously is called a feasible region.

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2) Define a feasible solution.
Solution:

A solution to the LPP which satisfies the non-negativity restrictions of the LPP is called a feasible solution.

---

3) Define optimal solution.
Solution:

Any feasible solution which optimizes the objective function is called its optimal solution.

---

4) What is the difference between basic solution and basic feasible solution?
Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

---

5) Define unbounded solution.
Solution:

If the values of the objective function Z can be increased or decreased indefinitely, such solutions are called unbounded solutions.

---

6) What are slack and surplus variables?
Solution:

The non-negative variable which is added to LHS of the constraint to convert the inequality $\leq$ into an equation is called slack variable.

$\sum_{j=1}^{n} a_{ij}x_i + s_i = b_i (i = 1, 2, ..., m)$ where s_i are called slack variables.

The non-negative variable which is subtracted from the LHS of the constraint to convert the inequality $\geq$ into an equation is called surplus variable.

$\sum_{j=1}^{n} a_{ij}x_i - s_i = b_i (i = 1, 2, ..., m)$ where s_i are called surplus variables.

---

7) What is meant by optimality test in a LPP?
Solution:

By performing optimality test we can find whether the current feasible solution can be improved or not. This is possible by finding the $Z_j - C_j$ row. In the case of a maximization problem if all $Z_j - C_j$ are nonnegative, then the current solution is optimal.

---

8) What are the methods used to solve an LPP involving artificial variables?
Solution:

i) Big M method or penalty cost method
ii) Two-phase simplex method

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9) Define artificial variable
Solution:

Any non negative variable which is introduced in the constraint in order to get the initial basic feasible solution is called artificial variable.

---

10) When does an LPP posses a pseudo-optimal solution?
Solution:

An LPP possesses a pseudo-optimal solution if at least one artificial variable is in the basis at positive level even though the optimality conditions are satisfied.

---

11) What is degeneracy?
Solution:

The concept of obtaining a degenerate basic feasible solution in a LPP is known as degeneracy. In the case of a BFS, all the non basic variables have zero value. If some basic variables also have zero value, then the BFS is said to be a degenerate BFS.

---

12) How to resolve degeneracy in a LPP?
Solution:

a) Divide each element of the rows (with tie) by the positive coefficients of the key column in that row.
b) Compare the resulting ratios, column by column, first in the identity and then in the body from left to right.
c) The row which first contains the smallest ratio contains the leaving variable.

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13) State the characteristics of canonical form.
Solution:
The characteristics of canonical form are
i) The objective function is of maximization type
ii) All constraints are "$\leq$" type
iii) All variables X_i are non negative.

---

14) State the characteristics of standard form.
Solution:

The characteristics of standard form are

i) The objective function is of maximization type
ii) All constraints are expressed as equations
iii) RHS of each constraint is non- negative
iv) All variables X_i are non-negative.

---

15) Define basic feasible solution

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution. A basic solution in which all the basic variables are non-negative is called a basic feasible solution.

---

16) Define non-degenerate solution

Solution:

A non-degenerate basic feasible solution is the basic feasible solution which has exactly m positive X_i (i = 1, 2,------m) ie, none of the basic variables are zero.

---

17) Define degenerate solution

Solution:

A basic feasible solution is said to be degenerate if one or more basic variables are zero.

---

18) Write the general mathematical model of LPP in matrix form.

Solution:

Max or Min Z = CX
Subject to AX $(\leq = \geq)$ b
$X \geq 0$

---

19) Define basic solution:

Solution:

Given a system of m linear equations with n variables (m<n), the solution obtained by setting n-m variables equal to zero and solving for the remaining m variables is called a basic solution.

Simplex method(algorithm)

Step 1: Check whether the objective function of the given LPP is to be maximized or minimized. If it is to be minimized then we convert it into a problem of maximization by Min Z = - Max (-Z)

Step 2: Check whether all b_i are positive. If any one of b_i is negative then multiply the inequation of the constraint by -1 so as to get all b_i to be positive.

Step 3: Express the problem in standard form by introducing slack/surplus variables, to convert the inequality constraints into equations.

Step 4: Obtain an initial basic feasible solution to the problem in the form $X_B = \mathbf{B}^{-1}b$ and put it in the first column of the simplex table.

Step 5: Compute the net evaluations $Z_J - C_J$ by using the relation

$Z_J - C_J = C_B X_J - C_J$.

Examine the sign of $Z_J - C_J$.

(i) If all $Z_J - C_J \geq 0$, then the current BFS is the optimal solution.

(ii) If at least one $Z_J - C_J < 0$, then proceed to the next step.

Step 6: If there are more than one negative $Z_J - C_J$ choose the most negative

them. Let it be $Z_r - C_r$.

(i) If all $X_{ir} \leq 0 (i = 1, 2, ...m)$ then there is an unbounded solution to the given problem.

(ii) If at least one $X_{ir} > 0 (i = 1, 2, ...m)$ then the variable X_r (key column) enters the basis.

Step 7: Compute the ratio $\left\{ \frac{X_{Bi}}{X_{ir}} / X_{ir} > 0 \right\}$. Let the minimum of these ratios be

$\frac{X_{Bk}}{X_{kr}}$. Then choose the variable X_k (key row) to leave the basic. The element

at the intersection of the key column and the key row is called the key element.

Step 8: Form a new basis by dropping the leaving variable and introducing the

entering variable along with the associated value under C_B column. Convert the leading element to unity by dividing the key row by the key element and convert all other elements in the simplex table by using the formula

New element = Old element –

{(product of elements in key row and key column) / key element}

Go to Step 5 and repeat the procedure until either an optimal solution is obtained or there is an indication of unbounded solution.

The procedure of the big M method

Step 1: Express the problem in the standard form.

Step 2: Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type ≥ or = . Assign a very large penalty cost (-M for Maximization and M for Minimization) with artificial variables in the objective function.

Step 3: Solved the modified LPP by simplex method, until any one of the three cases that may arise.

1. If no artificial variable appears in the basis and the optimality conditions

are satisfied, then the current solution is an optimal basic feasible solution.

2. If at least one artificial variable in the basis is at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution (though degenerate).

3. If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied then the original problem has no feasible solution .The solution satisfies the constraints but does not optimize the objective function, since it contains very large penalty M and it is called pseudo optimal solution.

---

1. Solve the following LPP using simplex method

Max $Z = 15x_1 + 6x_2 + 9x_3 + 2x_4$

Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 \le 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 \le 24$$
$$7x_1 + x_4 \le 70$$
$$\& x_1, x_2, x_3, x_4 \ge 0$$

Solution:

Rewrite the inequality of the constraint into an equation by adding slack variables S_1, S_2 and S_3 the LPP becomes,

$$\text{Max } Z = 15x_1 + 6x_2 + 9x_3 + 2x_4 + 0.S_1 + 0.S_2 + 0.S_3$$

Subject to

$$2x_1 + x_2 + 5x_3 + 6x_4 + S_1 = 20$$
$$3x_1 + x_2 + 3x_3 + 25x_4 + S_2 = 24$$
$$7x_1 + x_4 + S_3 = 70$$
$$\& x_1, x_2, x_3, x_4, S_1, S_2, S_3 \ge 0$$

Initial basic feasible solution is

$S_1=20$
$S_2=24$
$S_3=70$

Initial simplex table

C_j	15	6	9	2	0	0	0

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_1}$
0	S_1	20	2	1	5	6	1	0	0	10
0	S_2	24	3	1	3	25	0	1	0	8
0	S_3	70	7	0	0	1	0	0	1	10
	Z_j	0	0	0	0	0	0	0	0	
	Z_J-C_J		-15	-6	-9	-2	0	0	0	

Since all $Z_J - C_J \le 0$ and the current basic feasible solution is not optimum.

First iteration: X_1 enters the basis and S_2 leaves the basis.

	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3	Min $\frac{X_B}{X_2}$
$\leftarrow 0$	S_1	4	0	1/3	3	$-32/3$	1	2/3	0	12
15	X_1	8	1	1/3	1	25/3	0	1/3	0	24
0	S_3	14	0	$-7/3$	-7	$-172/3$	0	$-7/3$	1	-
	Z_J	120	15	5	15	125	0	5	0	
	Z_J-C_J		0	-1 $\uparrow$	6	123	0	5		

Since some $Z_J - C_J \le 0$ the current basic feasible solution is not optimum

Second iteration:

X_2 enters the basis and S_1 leaves the basis.

The new table is

C_B	B	X_B	X_1	X_2	X_3	X_4	S_1	S_2	S_3
6	X_2	12	0	1	9	-32	3	-2	0
15	X_1	4	1	0	-2	57/3	-1	1	0
0	S_3	42	0	0	14	-132	7	-7	1
	Z_j	132	15	6	24	93	3	3	0
	Z_J-C_J		0	0	15	91	3	3	0

Since all $Z_J - C_J \ge 0$ and the current basic feasible solution is optimum

and is given by Max Z = 132, $X_1 = 4$, $X_2 = 12$, $X_3 = 0$, $X_4 = 0$.

----

2. Solve by the big M method

Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

Solution:

Given Minimize Z = $4x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

That is Max Z = $-4x_1 - 3x_2$

Subject to

$$2x_1 + x_2 \geq 10$$
$$-3x_1 + 2x_2 \leq 6$$
$$x_1 + x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

$$\text{Max Z} = -4x_1 - 3x_2 + 0.s_1 + 0.s_2 - MR_1 - MR_2$$

Subject to

$$2x_1 + x_2 - s_1 + R_1 = 10$$
$$-3x_1 + 2x_2 + s_2 = 6$$
$$x_1 + x_2 - s_3 + R_2 = 6$$
$$\& \ x_1, x_2, s_1, s_2, s_3, R_1, R_2 \geq 0$$

Initial basic feasible solution is

R_1=10
R_2=6
S_2=6

Initial iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_1	R_2	Min
-M	R_1	10	2	1	-1	0	0	1	0	5
0	S_2	6	-3	2	0	1	0	0	0	-
-M	R_2	6	1	1	0	0	-1	0	1	6
	$Z_j - C_j$	-16M	-3M+4	-2M+3	M	0	M	0	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j -4 -3 0 0 0 -M -M

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	R_2	Min
-4	x_1	5	1	1/2	-1/2	0	0	0	10
0	S_2	21	0	7/2	-3/2	1	0	0	42/7
-M	R_2	1	0	½	1/2	0	-1	1	2
	$Z_j - C_j$	-M-20	0	$\frac{-M+2}{2}$	$\frac{-M+4}{2}$	0	M	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

Second iteration:

C_j -4 -3 0 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3
-4	X_1	4	1	0	-1	0	1
0	S_2	14	0	0	-5	1	7
-3	x_2	2	0	1	1	0	-2
	$Z_j - C_j$	-22	0	0	1	0	2

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum.

and is given by Min Z = 22, $X_1 = 4$, $X_2 = 2$.

----

3. Use two phase simplex method to

Max Z = $5x_1 + 3x_2$

Subject to

$$2x_1 + x_2 \leq 1$$
$$x_1 + 4x_2 \geq 6$$
$$\& \ x_1, x_2 \geq 0$$

Solution:

By introducing the non negative slack, surplus and artificial variables, the standard form of the LPP is

Max Z = $5x_1 + 3x_2 + 0.s_1 + 0.s_2 - R_1$

Subject to

$$2x_1 + x_2 + s_1 = 1$$
$$x_1 + x_2 - s_2 + R_1 = 6$$
$$\& x_1, x_2, s_1, s_2, R_1 \geq 0$$

Initial basic feasible solution is

S_1=1

R1=6

Initial iteration:

C_j 0 0 0 0 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1	Min
0	S_1	1	2	1	1	0	0	1
-1	R_1	6	1	4	0	-1	1	6/4
	$Z_j - C_j$	-6	-1	-4	0	1	0	

Since some $Z_J - C_J \leq 0$ and the current basic feasible solution is not optimum.

First iteration:

C_j 0 0 0 -1 -1

C_B	B	X_B	X_1	X_2	S_1	S_2	R_1
0	x_2	1	2	1	1	0	0
-1	R_1	2	-7	0	-4	-1	1
	$Z_j - C_j$	-2	7	0	4	1	0

Since all $Z_J - C_J \geq 0$ and the current basic feasible solution is optimal to the auxiliary LPP. Since an artificial variable is in the current basis at positive level, the given LPP has no feasible solution.

SCHOOL OF SCIENCE AND HUMANITIES

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UNIT – III – Dual Simplex Method – SMT1504

Explain the meaning of duality in LPP

For every LP problem there is related unique L P problem mvolvmg the same data which also describes the original problem.

The given original problem is known as primal programme. The programme can be rewritten by transposing the rows and columns of the statement of the problem. Inverting the programme in this way results in dual programme. The two programmes have very closely related properties so that optimal solution of the dual problem gives complete information about the optimal solution of primal problem. Solving the problem by writing dual programme is known as duality in LP

If the dual of an LPP is solved, where will we get the value of decision variables of the primal LPP.

The value of decision variables of primal are given by the base row of the dual solution under the slack variable, neglecting the -ye sign if any, and under the artificial variables neglecting the —ye sign if any, after deleting the constant M.

What is the importance of duality?

1.If. the primal problem contains a large number of rows and a smaller number of columns, the computational procedure can be considerably reduced by converting it into dual and then solving it.

2. This can help managers in answer questions about alternative course of actions and their relative values.

3. Economic interpretation of the dual helps the management in making future decisions.

4. Calculation of the dual checks the accuracy of the primal solution.

Define dual of LPP.

For every LPP there is a unique LPP associated with it involving the same data and closely related optimal solution. The original problem is then called the primal problem while the other is called its dual problem. If the primal problem is

$$\begin{aligned} &\text{Maximize } Z = CX \\ &\text{subject to } AX \leq b \\ &\qquad\qquad X \geq 0 \end{aligned}$$

Then the dual is

$$\begin{aligned} &\text{Minimize } Z^* = b^T Y \\ &\text{subject to } A^T Y \geq C^T \\ &\qquad\qquad Y \geq 0 \end{aligned}$$

Problems

Problem 4.6. Write the dual form for the following :

$$\text{Min } z = x_1 + x_2$$

Subjected to

$$2x_1 + x_2 \geq 4$$
$$x_1 + 7x_2 \geq 7$$
$$x_1, x_2 \geq 0$$

Solution :

The dual of the given problem will be

$$\text{Max W} = 4y_1 + 7y_2$$

Subject to

$$2y_1 + 1y_2 \leq 1$$
$$y_1 + 7y_2 \leq 1$$
$$y_1 , y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.7. Write the dual form for the following :

$$\text{Max } z = 4x_1 + 2x_2$$

Subject to

$$x_1 + x_2 \geq 3$$
$$x_1 - x_2 \geq 2$$
$$x_1 , x_2 \geq 0$$

Solution : As the given problem is of maximization type, all constraints should by $\leq$ type; Multiply all constraints with –ve.

$$-x_1 - x_2 \leq -3$$
$$-x_1 + x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

The dual of the given problem will be

$$\text{Min W} = -3y_1 - 2y_2$$

Subject to

$$-y_1 - y_2 \geq 4$$
$$-y_1 + y_2 \geq 2$$
$$y_1 ; y_2 \geq 0$$

y_1 and y_2 are non negative dual variables.

Problem 4.8. Construct the dual of the problem.

Minimize z =$3x_1 - 2x_2 + 4x_3$

Subject to constraints

$$3x_1 + 5x_2 + 4x_3 \geq 7$$
$$6x_1 + x_2 + 3x_3 \geq 4$$
$$7x_1 - 2x_2 - x_3 \leq 10$$
$$x_1 - 2x_2 + 5x_3 \geq 3$$
$$4x_1 + 7x_2 - 2x_3 \geq 2$$
$$x_1, x_2, x_3 \geq 0$$

Solution: As the given problem is of minimization all constraints should be of type

$$-7x_1 + 2x_2 + x_3 \geq -10$$

The dual of the given problem will be

$$\text{Maximize } W = 7y_1 + 4y_2 - 10y_3 + 3y_4 + 2y_5$$

Subject to

$$3y_1 + 6y_2 - 7y_3 + y_4 + 4y_5 \leq 3$$
$$5y_1 + y_2 + 2y_3 - 2y_4 + 7y_5 \leq -2$$
$$5y_1 + 3y_2 + y_3 + 5y_4 - 2y_5 \leq 4$$
$$y_1, y_2, y_3, y_4, y_5 \geq 0.$$

Problem 4.9. Construct the dual of the problem

$$\text{maximize } z = 3x_1 + 10x_2 + 2x_3$$

Subject to

$$2x_1 + 3x_2 + 2x_3 \leq 7$$
$$3x_1 - 2x_2 + 4x_3 = 3$$
$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Solution : The equation $3x_1 - 2x_2 + 4x_3 = 3$ can be expressed as pair of inequality

$$3x_1 - 2x_2 + 4x_3 \leq 3$$
$$3x_1 - 2x_2 + 4x_3 \geq 3$$
$$-3x_1 + 2x_2 - 4x_3 \leq -3$$

Minimize

$$W = 7y_1 + 3(y_2' - y_2'')$$

Subject to

$$2y_1 + 3(y_2' - y_2'') \geq 3$$
$$3y_1 - 2(y_2' - y_2'') \geq 10$$
$$2y_1 + 4(y_2' - y_2'') \geq 2$$
$$y_1, y_2', y_2'' \geq 0$$

Subsituting $y_2' - y_2'' = y_2$

$$W = 7y_1 + 3y_2$$

Subject to

$$2y_1 + 3y_2 \geq 3$$
$$3y_1 - 2y_2 \geq 10$$
$$2y_1 + 4y_2 \geq 2$$
$$y_1 \geq 0$$

$y_2$unrestricted variable.

Solve by the dual simplex method the following LPP

Min Z = $5x_1 + 6x_2$.

Subject to $x_1 + x_2 \geq 2$

$4x_1 + x_2 \geq 4$

$x_1, x_2 \geq 0$

Solution:

By introducing slack variables S_1, S_2 we get the standard form of LPP as given below.

Max Z = $-5x_1 - 6x_2 + 0s_1 + 0s_2$

Subject to

$-x_1 - x_2 + s_1 = -2$

$-4x_1 - x_2 + s_2 = -4$

$x_1, x_2, s_1, s_2 \geq 0$

Initial table

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_1	-2	-1	-1	1	0
$\leftarrow 0$	S_2	-4	-4	-1	0	1
	Z_J	0	0	0	0	0
	$Z_J - C_J$		$5\uparrow$	6	0	0

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since all $X_{Bi} < 0$, the current solution is not a basic feasible solution.

Since X_{B2} = -4 is most negative, the basic variable S_2 leaves the basis.

Since Max {5/-4, 6/-1} = -5/4, X_1 enters the basis.

First iteration: Drop S_2 and introduce X_1

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
$\leftarrow 0$	S_1	-1	0	$-3/4$	1	$-1/4$
-5	x_1	1	1	$1/4$	0	$-1/4$
	Z_J	-5	-5	$-5/4$	0	$5/4$
	$Z_J - C_J$		0	$19/4$	0	$5/4$ $\uparrow$

Since all $Z_J - C_J \geq 0$ optimality conditions are satisfied. Since some $X_{Bi} < 0$ the current solution is not a basic feasible solution.

S_1 leaves the current basis. Since Max {19/-3, 5/-1} =5/-1, S_2 enters the basis.

Second iteration:

C_J -5 -6 0 0

C_B	B	X_B	X_1	X_2	S_1	S_2
0	S_2	4	0	3	-4	1

-5	x_1	2	1	1	-1	0
	Z_J	-10	-5	-5	5	0
	$Z_J - C_J$		0	1	5	0

Since all $Z_J - C_J \geq 0$ and all $X_{Bi} \geq 0$, the current basic feasible solution is optimum. The optimal solution is Z = 10, $x_1 = 2$.

Use dual simplex method to solve the LPP

Max Z = $-3x_1 - 2x_2$

Subject to

$x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10$
$x_2 \leq 3$
$\& \; x_1, x_2 \geq 0$

Solution:

The given LPP is

Max Z = $-3x_1 - 2x_2$

Subject to

$-x_1 - x_2 \leq 1$
$x_1 + x_2 \leq 7$
$-x_1 - 2x_2 \leq 10$
$x_2 \leq 3$
$\& \; x_1, x_2 \geq 0$

By introducing the non negative slack variables s_1, s_2, s_3 and s_4 the LPP becomes

Max Z = $-3x_1 - 2x_2 + 0.s_1 + 0.s_2 + 0.s_3 + 0.s_4$

Subject to

$$-x_1 - x_2 + s_1 = -1$$
$$x_1 + x_2 + s_2 = 7$$
$$-x_1 - 2x_2 + s_3 = -10$$
$$x_2 + s_4 = 3$$
$$\& \; x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$$

Initial iteration BFS is

$S_1 = -1$, $S_2=7$, $S_3=-10$, $S_4=3$

Initial Iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	-1	-1	-1	1	0	0	0
0	S_2	7	1	1	0	1	0	0
0	S_3	-10	-1	-2	0	0	1	0
0	S_4	3	0	1	0	0	0	1
	$Z_j - C_j$	0	3	2	0	0	0	0

First iteration

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	4	-1/2	0	1	0	-1/2	0
0	S_2	2	1/2	0	0	1	1/2	0
-2	x_2	5	1/2	1	0	0	-1/2	0
0	S_4	-2	-1/2	0	0	0	1/2	1
	$Z_j - C_j$	-10	2	0	0	0	1	0

Second iteration:

C_B	B	X_B	X_1	X_2	S_1	S_2	S_3	S_4
0	S_1	2	0	0	1	0	-1	-1
0	S_2	0	0	0	0	1	1	1
-2	x_2	3	0	1	0	0	0	1
-3	x_1	4	1	0	0	0	-1	-2
	$Z_j - C_j$	-18	0	0	0	0	3	4

Since all $Z_J - C_J \geq 0$ the current basic feasible solution is optimum and is given by Min Z = -18, $X_1 = 4$, $X_2 = 3$.

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UNIT – IV – Transportation and Assignment Problem – SMT1504

State transportation problem. Is this a special class of LPP ? When does it a unique solution ?

The transportation problem is to transport various amount of single object that are initially stored at various origins, to different destinations in such a way that the total transportation cost is minimum.

Yes it is a special class of LPP and may be solved by simplex method. Transportation problem always posses a feasible solution.

It has a unique solution when cell evaluation matrix has only positive values.

Write mathematical model for general transportation problem as LPP.

Mathematical formulation

Suppose that there are m sources and n destinations. Let a1be the number of supply units available at source i (i =1, 2, 3 m) and let b1 be the number of demand units required at destination j (f = 1, 2, 3 n). Let C, represent the unit transportation cost for transporting the units from source i to distination j. The objective is to determine the number of units to be transported from source i to destination j. So that total transportation cost is minimum.

If x_{ij} is the number of units shipped from source i to destination j, then
Find x_{ij} such that

Minimize $$z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

Subject to $$\sum_{J=1}^{n} x_{ij} = a_i \qquad i = 1, 2, 3, \ldots, m$$

and $$\sum_{i=1}^{m} . x_{ij} = b_j \qquad J = 1, 2, 3, \ldots, n$$

where $$x_{ii} \geq 0$$

List the various methods that can be used for obtaining an initial basic solution for transportation problem.

1. North west corner method

2. Row minimum method

3. Column minimum method

4. Least cost method

5. Vogal approximation method.

What is degeneracy in transportation problem?

In a transportation problem with m origins and n destinations if a basic feasible solution has less than (m + n—i) allocations, the problem is said to be a degenerate transportation problem.

What do you understand by a balanced and an unbalanced transportation problem ? How an unbalanced problem is tackled?

In a transportation problem if the total availability from all the origins is equal

to the total demand at all the destinations z $i.e. \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j.$ Such transportation

problems are known as balanced tansportation problems. (Total supply = Total demand) In many

situations, the total availability may not be equal to the total demand i.e. $\sum_{i=1}^{m} a_i \neq \sum_{J=1}^{n} b_j.$

Such problems are known as unbalanced tranportation problem.

The unbalanced problem could be tackled by adding a dummy destination or source depending upon the requirement and the costs of shipping to this destination (or from source) are set equal to zero. The zero cost cells are treated the same way as real cost cell and the problem is solved as a balanced problem. (Total supply   Total demand)

Describe the steps involved in vogal approximation method (VAM).

Ans.

Setp. 1. For each row of the transportation table identify the smallest and next to smallest cost. Determine the difference between them for each row. These are called penalities.' Similarly compute these penalities for each column.

Setp. 2. Identify the row or column with the largest penalty and allocate as much as possible within the restrictions of the rim conditions to the lowest cost cell in the row or column selected.

Setp. 3. Cross out of the row or column completely satisfied by the allocation.

Setp. 4. Repeat steps 1 to 3 untill all assignment have been made.

Define the following terms in transportation Problem.

(i) feasible solution (ii) Optimal solution

1. Feasible Solution. A feasible solution to a transportation problem is a set of non negative allocations, x that satisfy the rim conditions.

2. Optimal Solution. A feasible solution that minimize the transportation cost is called the optimal Solution.

Explain North west corner rule for finding initial solution for a transportation problem.

1. Start in the north west corner of the requirement table:

(a) If D1< S1, set x11 equal to find the balance supply and demand and proceed horizontally (cell 1, 2).

(b) If = S1 set x11 equal to D1, find the balance supply and demand and proceed diagonally (cell 2, 2).

(c) If D1>S1, set x11equal to compute the baiance supply and demand and proceed vertically (cell 2, 1).

2. Continue in this manner, step by step away from the north west corner until, finally a value is reached in the south east corner.

Give an algorithm to solve transportation problem.

Or

Describe the steps involved in solving transportation problem.

1. Make a transportation Model. For this enter the supply a., from the origin demand b1 at the destinations and unit cost C,, m the varous cells

2 Find initial basic feasible solution

3. Perform optimality test:

(a) Find dual variable (u_i, V_j)

(b) Make opportunity cost matrix $(C_{ij} = (u_i + V_j)$

(c) Compute the cell evaluation matrix $[C_{ij} - (u_i + V_j)]$ If all cell evaluation are positive or zero the current basic feasible solution is optimal.

(d) In case any cell evaluation is negative, select the vacant cell with the most negative evaluation. This isalled identified cell.

4. Iterate towards optimal solution. For this make as much allocation in the identified cell as possible so that it become basic.

5. Repeat step 3 and 4 till optimal solution is obtained.

State the Assignment model. Is assignment problem a special case of transportation?

Assignment Model Suppose there are n jobs to be performed and n person are available for doing these jobs. Assume that each person can do each job at a time, though with varying degree of efficiency.The problem is to find an assignment so that the total cost for performmg all jobs is minimum

Yes, the assignment problem is a special case of transportation problem when each

origin is associated with one and only one destination.

Give the mathematical formulation of an assignment problem

Ans Let = 0, if the facility is not assigned to 1th job

1, if the th facility is assigned to th job.

The model is given by

$$\text{minimize } z = \sum_{J=1}^{n} \sum_{i=1}^{n} C_{ij} x_{ij}$$

Subject to constraints

$$\sum_{J=1}^{n} x_{ij} = 1, i = 1, 2, 3, \ldots, n$$

$$\sum_{i=1}^{n} x_{ij} = 1 \; J = 1, 2, 3\ldots, n$$

$$x_{ij} = 0 \text{ or } 1.$$

What do you mean by restrictions an assignments?

Or

How a restriction problem tackled?

Or

How will you solve an assignment where a particular assignment is prohibited?

Sometime technical, space, legal'or other problems do not permit the assignment of a particular facility to a particularjob. Such problem are known restrictions an assignment problem. Such problem can be solved by assigning a very heavy cost to the corresponding cell. It will automatically excluded from further consideration.

What is the unbalanced assignment problem ? How is it solved by the Hungarian method?

When the number of facilities is not equal to the number of jobs, such problems

are known as unbalanced assignment problem.

Since the Hungarian methodof solution require a square matrix, fictitious facilities or jobs. Jobs may be added and zero costs be assigned to the corresponding cells of the matrix. These cells are then treated the same way as the real cost cells during the solution procedure.

How do you come to know that Assignment problem has alternate optimal solution?

Ans. Sometimes it is possible to have two or more ways to strike off all zero elements in the reduced matrix for a given problem. In such cases there will be an alternate optimal solution with same cost.

Describe the steps involved in solving assignment problem by Hungarian method.

1. Prepare a square matrix.

2. Reduce the matrix.

3. Check whether an optimal assignment can be made in the reduced matrix or not.

4. Find the minimum number of lines crossing all zeros. If this number of lines is equal to the order of matrix then it is an optimal solution. Otherwise gp to step 5.

5. Iterate towards the optimal solution.

6. Repeat step 3 through 5 until an optimal solution is obtained.

Compare assignment problem with transportation problem.

An assignment model may be regarded as special case of the transportation

model. Here facilities represent the sources and jobs represent the destination. Number of sources is equal to the number of destinations, supply at each source is unity and demand at each distination is unit. In assignment the number of units allocated to a cell be either one or zero.

The assignment problem is a completely degenerate form of transportation problem.

Distinguish between transportation, assignment and sequencing model what is sequencing model).

Ans. Transportation and assignment are allocation model (as explained above) Sequencing model. are applicable in situation in which the effectiveness measure

a function of order as sequence of performing a series of jobs. The selection of the apropriate order in which waiting customer/Job may be served is called sequencing.

Problem Find the optimum solution to the following problem.

	I	3	4	6	8	8	20
	II	2	10	1	5	30	30
From	III	7	11	20	40	15	15
	IV	2	1	9	14	18	13
		40	6	8	18	6	

Solution:

1. Make a transportation model

I	3	4	6	8	8	20
II	2	10	1	5	30	30
III	7	11	20	40	15	15
IV	2	1	9	14	18	13
	40	6	8	18	6	78

1. Find basic feasible solution (VAM method)
2.

$\boxed{14}$ 3	4	6	8	$\boxed{6}$ 8	20/14/0	[1] [1] [5]
$\boxed{4}$ 2	10	$\boxed{8}$ 1	$\boxed{18}$ 5	30	30/22/18	[1] [3] [3]
$\boxed{15}$ 7	11	20	40	15	15/0	[4] [4] ←
$\boxed{7}$ 2	$\boxed{6}$ 1	9	14	18	13/7/0	[1] [1] [12] ←
40/25 18/4/0	6/0	8/0	18/0	6/0	78	
[0]	[3]	[5]	[3]	[7]		
[1]	[3]	↑	[3]	↑		

Transportation cost = (14 × 3) + (6 × 8) + (4 × 2) + (8 × 1) +(18 × 5) + (15 × 7) + (7 × 2) + (6 × 1)

= 42 + 48 + 8 + 8 + 90 + 105 + 14 + 6

= Rs. 321.

3. Check for optimality (MODI Test) m (a) Cost matrix of allocated cell.

+ n - 1 = 8 (no. of allocation)

u_j \ V_i	1	2	3	4	5
1	3				8
2	2		1	5	
3	7				
4	2	1			

$$
\begin{array}{ll}
 & V_1 = 0 \\
u_1 + v_1 = 3 & u_1 = 3 \\
u_1 + v_5 = 8 & v_5 = 5 \\
u_2 + v_1 = 2 & u_2 = 2 \\
u_2 + v_3 = 1 & v_3 = -1 \\
u_2 + v_4 = 5 & v_4 = 3 \\
u_3 + v_1 = 7 & v_3 = 7 \\
u_4 + v_1 = 2 & u_4 = 2 \\
u_4 + v_2 = 1 & v_2 = -1
\end{array}
$$

(*b*) Opp. cost matrix

	0	–1	–1	3	5
3	·	2	2	6	·
2	·	1	·	·	7
7	·	6	6	10	12
2	·	·	1	5	7

(c) Cell evaluation matrix

·	2	4	2	·
·	9	·	·	23
·	5	14	30	3
·	·	8	9	11

Since all the elements of cell evaluation matrix are positive so optimality test is passed.

Minimum Transportation Cost = Rs. 321.

Solve the following cost-minimizing transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	Available
O_1	2	1	3	3	2	5	50
O_2	3	2	2	4	3	4	40
O_3	3	5	4	2	4	1	60
O_4	4	2	2	1	2	2	30
Required	30	50	20	40	30	10	180

Ans. 1. Make a transportation model.

1. Find basic feasible solution

2	$\boxed{50}$ 1	3	3	2	5	50/0	[1]
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4	40/20/0	[0][1][1][0]
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1	60/50/40/10/0	[1][1][1][1]
4	2	2	$\boxed{30}$ 1	2	2	30/0	[1][1]
30/0	50/0	20/0	40/10/0	30/20/0	10/0		
[1]	[1]	[0]	[1]	[0]	[1]		
[0]		[0]	[1]	[1]	[1]		
[0]		[2]	[2]	[1]	[3]		

1. Check for optimality test (m + n -1) > no. of allocation (8)

2	$\boxed{50}$ 1	3	3	$\boxed{\varepsilon}$ 2	5
3	2	$\boxed{20}$ 2	4	$\boxed{20}$ 3	4
$\boxed{30}$ 3	5	4	$\boxed{10}$ 2	$\boxed{10}$ 4	$\boxed{10}$ 1
			$\boxed{30}$		

m + n- 1 = no. of allocation 9

9. Cost matrix of allocated cell

	1			2	
		2		3	
3			2	4	1
			1		

$u_1 + v_2 = 1$ $\quad v_1 = 0$
$u_1 + v_5 = 2$ $\quad v_2 = 0$
$u_2 + v_3 = 2$ $\quad u_1 = 1$
$u_2 + v_5 = 3$ $\quad v_3 = 0$
$u_3 + v_1 = 3$ $\quad u_2 = 2$
$u_3 + v_4 = 2$ $\quad u_3 = 3$
$u_3 + v_5 = 2$ $\quad v_4 = -1$
$u_3 + v_6 = 1$ $\quad v_5 = 1$
$u_3 + v_4 = 1$ $\quad v_6 = -2$
$u_4 = 2$

(*b*) Opportunity cost matrix

	0	0	0	–1	1	–2
1	1	·	1	0	·	–1
2	2	2	·	1	·	0
3	·	3	3	·	·	·
2	2	2	2	·	3	0

(c) Cell evaluation matrix

1	·	2	3	·	6
1	0	·	3	·	4
·	2	1	·	·	·
2	0	0	·	–1√	2

identified cell

1. Iteration for optimal solution.
2.

	50			ε	
		20		20	
30			+10	10−	10
			−30	√+	

Check for optimality test

	50			ε	
		20		20	
30			20		10
			20	10	

2nd feasible solution

(a). Cost matrix of allocated cell.

	1			2	
		2		3	
3			2		1
			1	2	

(*b*) Opp. cost matrix

	0	−1	−1	−1	0	−2
2	2	·	1	1	·	0
3	3	2	·	2	·	1
3	·	2	2	·	3	·
2	2	1	1	·	·	0

(*c*) Cell evaluation matrix

0	·	2	2	·	5
0	0	·	2	·	3
·	3	2	·	1	·
2	1	1	·	·	2

$u_1 + v_2 = 1$ $\quad v_1 = 0$
$u_1 + v_5 = 2$ $\quad v_2 = -1$
$u_2 + v_3 = 2$ $\quad u_1 = 2$
$u_2 + v_5 = 3$ $\quad v_3 = -1$
$u_3 + v_1 = 3$ $\quad u_2 = 3$
$u_3 + v_6 = 1$ $\quad u_3 = 3$
$u_4 + v_4 = 1$ $\quad v_4 = -1$
$u_4 + v_5 = 2$ $\quad v_6 = -2$
$u_4 = 2$
$v_5 = 0$

Since all elements of cell evaluation matrix are non negative so 2hldI feasible solution is the optimum solution.

Transportation cost

$= 50 \times 1 + 20 \times 2 + 20 \times 3 + 30 \times 3 + 20 \times 2 + 10 \times 1 + 20 \times 1 + 10 \times 2$
$= 50 + 40 + 60 + 90 + 40 + 10 + 20 + 20$
$= 330/-$

Problem 3.21. Goods have to be transported from factories F_1, F_2, F_3 to ware house W_1, W_2, W_3 and W_4. The transportation cost per unit capacities and requirement of the ware house are given in the following table

	W_1	W_2	W_3	W_4	Capacity
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
Requrement	5	4	4	11	

Solution. 1. Make a transportation model

	W_1	W_2	W_3	W_4	
F_1	95	105	80	15	12
F_2	115	180	40	30	7
F_3	195	180	95	70	5
	5	4	4	11	24 Balanced Model

2. Find a basic feasible solution
VAM method

F_1	95	$\boxed{4}$ 105	80	$\boxed{8}$ 15	12/8/0	[65] ←
F_2	$\boxed{5}$ 115	180	$\boxed{2}$ 40	30	7/2/0	[10]
F_3	195	180	$\boxed{2}$ 95	$\boxed{3}$ 70	5/3/0	[25]
	5/0	4/0	4/2/0	11/3/0		
	[20]	[75]	[40]	[15]		
	[80]	↑	[55]	[50]		
	↑		↑			

3. Optimality test

$m + n - 1 =$ number of allocations
$6 = 6$

(a)Cost matrix of allocated cell

$u_i \backslash v_j$	1	2	3	4
1	·	105		15
2	115		40	
3		·	95	70

$$V_1 = 0$$
$$u_1 + V_2 = 105 \qquad V_2 = -10$$
$$u_1 + V_4 = 15 \qquad u_1 = 115$$
$$u_2 + V_1 = 115 \qquad u_2 = 115$$
$$u_2 + V_3 = 40 \qquad V_3 = -75$$
$$u_3 + V_3 = 95 \qquad u_3 = 170$$
$$u_3 + V_4 = 70 \qquad V_4 = -100$$

(*b*) Opp. cost matrix

	0	−10	−75	−100
115	115	·	40	·
115	·	105	·	15
170	170	160	·	·

(*c*) Cell evaluation matrix

−20	·	40	·
·	75	·	15
25	20	·	·

4. Iteration for optimal solution

✓ +	4		8 −
5 −		2 +	
		2 −	3 +

2	4		6
3		4	
			5

2nd feasible solution

5. Check for optimality
(*a*) Cost matrix of allocated cell

$u_i \backslash v_j$	1	2	3	4
1	95	105		15
2	115		40	
3				70

$$V_1 = 0$$
$$u_1 + V_1 = 95 \qquad u_1 = 95$$
$$u_1 + V_2 = 105 \qquad V_2 = 10$$
$$u_1 + V_4 = 15 \qquad V_4 = -80$$
$$u_2 + V_1 = 115 \qquad u_2 = 115$$
$$u_2 + V_3 = 40 \qquad V_3 = -75$$
$$u_3 + V_4 = 70 \qquad u_3 = 150$$

1. Opp. cost matrix

	0	10	–75	–80
95	·	·	20	·
115	·	125	·	35
150	150	160	75	·

Cell evaluation matrix

·	·	60	·
·	55	·	√ –5
45	20	20	·

Iteration for optimal solution.

+ 2	4		6 –
– 3		4	√ +
			5

5	4		3
		4	3
			5

3rd feasible solution

Check for optimality test (a)

Cost matrix of allocated cell

95	105		15
		40	30
			70

$$\begin{aligned}
u_1 + V_1 &= 95 & v_1 &= 0\\
u_1 + V_2 &= 105 & u_1 &= 95\\
u_1 + V_4 &= 15 & V_2 &= 10\\
u_2 + V_3 &= 40 & V_4 &= -80\\
u_2 + V_4 &= 30 & V_3 &= -70\\
u_3 + V_4 &= 70 & u_2 &= 110\\
& & u_3 &= 150
\end{aligned}$$

(*b*) Opp. cost matrix

	0	10	–70	–80
95	·	·	15	·
110	110	120	·	·
150	150	160	80	

(*c*) Cell evaluation matrix

·	·	65	·
5	60	·	·
45	20	15	·

Since all elements of cell evaluation matrix are non negative. Hence 3rd feasible solution is the optimum solution.

Transportation cost = (5 × 95) + (4 × 105) + (3 × 15) + (4 × 40) + (3 × 30) + (5 × 70) = 475 + 420 + 45 + 160 + 90 + 350 = Rs 1540/-

Problem 3.22. Solve the following assignment problem.

		1	2	3	4
	A	12	30	21	15
work	B	18	33	9	31
	C	44	25	24	21
	D	23	30	28	14

Solution:

1. Prepare a square matrix.

2. Reduce the matrix

0	5	12	1
6	8	0	17
32	0	15	7
11	15	19	0

3. Check if optimal assignment can be made in the current solution or not

	1	2	3	4
A	$\boxed{0}$	5	12	1
B	6	8	$\boxed{0}$	17
C	32	$\boxed{0}$	15	7
D	11	5	19	$\boxed{0}$

Since there is one assignment in each row and each column, the optimal assignment can be made in the current solution.

Minimum total cost = $12 \times 1 + 9 \times 1 + 25 \times 1 + 14 \times 1$

$= 12 + 9 + 25 + 14$

$= 60$

Ans. $A \longrightarrow 1$

$B \longrightarrow 3$

$C \longrightarrow 2$

$D \longrightarrow 4$

Minimum cost = Rs. 60.

Find the optimal assignment for the assignment problem with the following cost matrix.

	I	II	III	IV
A	5	3	1	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	6

Solution: 1. Prepare a square matrix

2. Prepare a reduced matrix.

0	0	0	2
2	6	1	0
1	1	4	1
0	4	6	0

0	0	0	2
2	6	1	0
0	0	3	0
0	4	6	0

3. Check if optimal assignment can be made in the current solution.

	I	II	III	IV
A	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	2
B	2	6	1	$\boxed{0}$
C	$\cancel{0}$	$\boxed{0}$	3	$\cancel{0}$
D	$\boxed{0}$	4	6	$\cancel{0}$

since each row and each column have assignment so optimal assignment can be made.

A – III
B – IV
C – II
D – I

Cost = 1+ 6 + 4 + 5 = 16.

Four different jobs are to be done on four different machines. Table below indicate the cost of producing job i on machine j in rupees.

Job \ m/c	1	2	3	4
1	5	7	11	6
2	8	5	9	6
3	4	7	10	7
4	10	4	8	3

Solution : 1. Reduced matrix

0	2	6	1
3	0	4	1
0	3	6	3
7	1	5	0

0	2	2	1
3	0	0	1
0	3	2	3
7	1	1	0

3. Check if optimal assignment can be made in the current solution or not

$\boxed{0}$	2	2	1	✓
3	$\boxed{0}$	$\cancel{0}$	1	
$\cancel{0}$	3	2	3	✓
7	1	1	$\boxed{0}$	

Cross marked column and unmaked row.
Since no. of lines
$\leq$ Rank of matrix
$3 \leq 4$

4. Iterate towards optimality

$\cancel{0}$	1	1	$\cancel{0}$	✓
4	$\boxed{0}$	$\boxed{0}$	1	
$\boxed{0}$	2	1	2	✓
8	1	1	$\boxed{0}$	✓
✓			✓	

number of lines (3) $\leq$ Rank of matrix (4)

		m/c 1	2	3	4
	1	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$
Job	2	5	$\boxed{0}$	$\cancel{0}$	2
	3	$\boxed{0}$	1	$\cancel{0}$	2
	4	8	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$

Since each row and column have assignment so optimality condition is satisfied.

Job 1 – M/c 3
Job 2 – M/c 2
Job 3 – M/c 1
Job 4 – M/c 4

Cost = 11 +5 + 4 +3 = 23

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SCHOOL OF SCIENCE AND HUMANITIES

DEPARTMENT OF MATHEMATICS

UNIT – V – Sequencing and Scheduling Problem – SMT1504

State the assumption made in sequencing model.

Ans 1 Only one operation is carried out on a m/c

2. Each operation once started, must be completed.

3. Only one rn/c of each type is available.

4. A job is processed as soon as possible but only in the order specified.

5. Processing time are independent of order f performing the operation.

6. Transportation time is negligible.

7. Cost of in process inventory is negligible.

Problem. 3.25. There are five jobs each of which just go through two machines A and B in the order of AB.

Processing times are given below. Determine a sequence for five jobs that will

minimize the elapse time and also calculate the total time.

Job	1	2	3	4	5
Time for A	5	1	9	3	10
Time for B	2	6	7	8	4

Determine the sequence for the jobs so as to minimize the process time. Find

total elapsed time.

Solution : Examine the columns of processing time on rn/c A and B and find the

smallest value. If this value falls in column A, schedule the job first on M/c, A, if this

value falls in column B, schedule the jobs last on M/c A. In this way sequence of jobs so as to minimize the process time is

2	4	3	5	1

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
2	0	1	1	7
4	1	4	7	15
3	4	13	15	22
5	13	23	23	27
1	23	28	28	30

Ans. Sequence

2	4	3	5	1

Total elapsed time = 30 hours. V

Problem 3.26. Find the sequence that minimize the total elapsed time to complete the following Jobs. Each Job is processed in the order of AB.

Job (Processing time in minutes)

		1	2	3	4	5	6	7
m/c	**A**	**12**	**6**	**5**	**11**	**5**	**7**	**6**
	B	**7**	**8**	**9**	**4**	**7**	**8**	**3**

Determine the sequence for the jobs so as to minimize the process time. Find the total elapsed time and idle time of M/c A and M/c B.

Solution : The sequence of jobs so as to minimize the process time is

5	3	2	6	1	4	7

Job	Machine A		Machine B	
	Time in	Time out	Time in	Time out
5	0	5	5	12
3	5	10	12	21
2	10	16	21	29
6	16	23	29	37
1	23	35	37	44
4	35	46	46	50
7	46	52	52	55

Min elapsed time = 55 mins
Idle time of M/c A = 3 mins
Idle time of M/c B = 9 mins.

Define (i) Network (ii) Path terms used in network.

(i) Network: It is the graphical representation of logically and sequentially connected arrows and nodes representing activities and events of a project.

(ii) Path : An unbroken chain of activity ,arrows connecting the initial event to

some other event is called path.

Define critical path and critical activities

Critical path the path containing critical activities (with zero float) is known as critical path.

Critical activity the activity, which can not be delayed without delaying the project duration, is known as critical activity

list four types of floats used in network analysis.

(a) Total float.

(b) Free float

(c) Independent float

(d) Interfering float

Define Free Float, Independent float, Interfering float as used in PERT chart.

Free float : Portion of the total float within which an activity can be manipulated without affecting the floats of subsequent activities.

Independent float: Portion of the total float within which an activity can be delayed

without affecting the floats of proceeding activities.

Interfering float : It is equal to the difference between the total float and the free float of the activity.

What do you mean by dummy activity?

Dummy activity : An activity, which only determines the dependency of one

activity on the other, but does not consume any time, is called a dummy activity.

Define dummy arrow used in network.

Dummy arrow: It represent the dummy activity in the network. It only represents the dependency of one activity on the other. It is denoted by dash/dotted line.

Define dangling and looping in net-work models.

Dangling : The disconnection of an activity before the completion of all the activities in a network diagram is known as dangling.

Looping (cycling) : Looping error is also known as cycling error in a network diagram. Drawing an endless loop in a network is known as error of looping.

Differentiate between event and activity.

Event: The beginning and end points of an activity are called events or nodes. Event is a point in time and does not consume any resources.

Activity : It is physically identifiable part of a project which require time and resources for its execution. An activity is represented by an arrow, the tail of which represents the start and the head, finish of the activity.

Differentiate between CPM and PERT.

CPM.:

1. CPM is activity oriented i.e., CPM network is built on the basis of activities.

2. CPM is a deterministic model. It does not take into account in uncertainties involved in the estimation of time.

3.

CPM places dual emphasis on project time as well as cost and finds the trade off. between project time and project cost.

4. CPM is primarily used for projects which are repetitive in nature and comparatively small in size.

PERT

1.PERT is event oriented.

2.PERT is a probabilitic model.

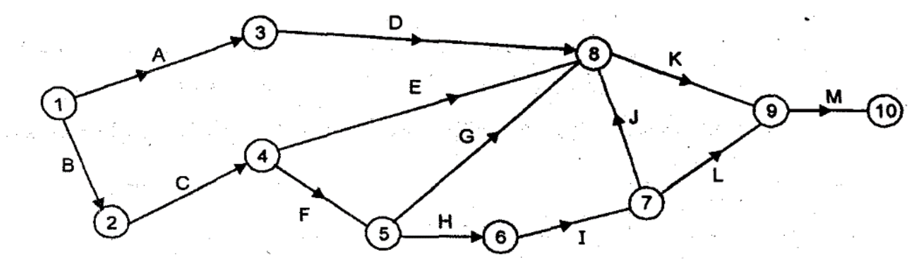
3.PERT is primarily concerned with time only.

4. PERT is used for large one timereserach and development type of projects.

Construct the network for the following activity data:

Activity	Preceded by	Activity	Preceded by
A	–	–	–
B	–	H	F
C	B	I	H
D	A	J	I
E	C	K	D,E,G,J
F	C	L	I
G	F	M	K,L

Solution. Network:

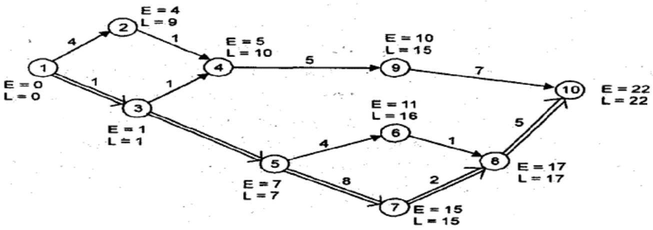


Problem 11.16. A project has the following time schedule :

Activity	1 – 2	1 – 3	2 – 4	3 – 4	3 – 5	4 – 9	5 – 6	5 – 7	6 – 8	7 – 8	8 – 9	8 – 10	9 – 10
Time weeks	4	1	1	1	6	5	4	8	1	2	1	5	7

1. Draw Network diagram and find the critical paths.
2. Calculate float on each activity.

Solution. (*i*)



2.

Activity	Duration (weeks)	Start Time		Finish Time		Total Float
		E	T_{LS}	T_{EF}	L	
1 – 2	4	0	5	4	9	5
1 – 3	1	0	0	1	1	0
2 – 4	1	4	9	5	10	5
3 – 4	1	1	9	2	10	8
3 – 5	6	1	1	7	7	0
4 – 9	5	5	10	10	15	5
5 – 6	4	7	12	11	16	5
5 – 7	8	7	7	15	15	0
6 – 8	1	11	16	12	17	5
7 – 8	2	15	15	17	17	0
8 – 10	5	17	17	22	22	0
9 – 10	7	10	15	17	22	5

Critical path 1—3—5—7—8—10 with project duration of 22 weeks.

The time estimate for the activities of a PERT network are given below :

Activity	t_0	t_m	t_p
1 – 2	1	1	7
1 – 3	1	4	7
1 – 4	2	2	8
2 – 5	1	1	1
3 – 5	2	5	14
4 – 6	2	5	8
5 – 6	3	6	15

(a) Draw the project network and identify all the path through it.

(b) Determine the expected project length.

(c) Calculate the standard deviation and variance of the project length.

(d) What is the probability that the project will be completed

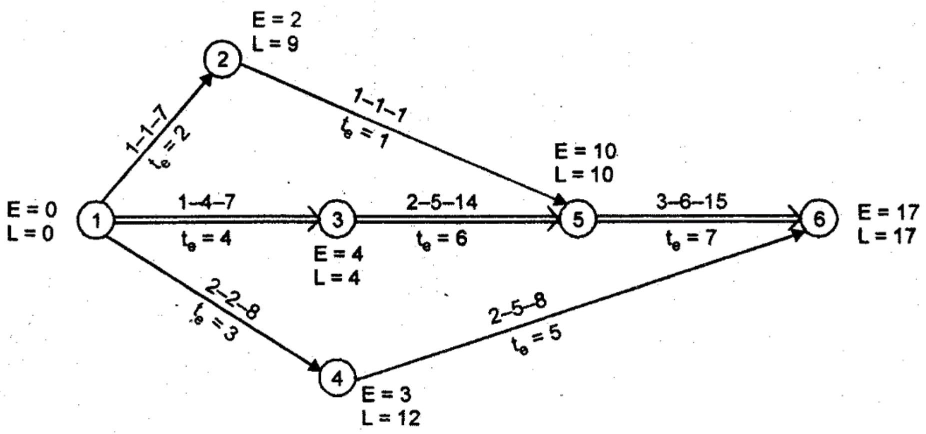
1. At least 4 weeks earlier than expected time.

2. No more than 4 weeks later than expected time.

(e) The probability that the project will be completed on schedule if the schedule completion time is 20 weeks.

(f) What should be the scheduled completion time for the probability of completion to be 90%.

Solution. (a) Network



Activity	t_0	t_m	t_p	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \frac{(t_p - t_0)^2}{6}$
1 – 2	1	1	7	2	1
1 – 3	1	4	7	4	1
1 – 4	2	2	8	3	1
2 – 5	1	1	1	1	0
3 – 5	2	5	14	6	4
4 – 6	2	5	8	5	1
5 – 6	3	6	15	7	4

Critical path—1 —3—5—6

Project duration = 17 weeks.

(c) Variance of the project length is the sum of the variance of the activities on the critical.

$$V_{cp} = V_{1\text{-}3} + V_{3\text{-}5} + V_{5\text{-}6} = 1 + 4 + 4 = 9$$
$$\sigma^2 = V \Rightarrow \sigma^2 = 9 \Rightarrow \sigma = 3 \text{ weeks.}$$

(d) (i) Probability that the project will be completed at least 4 week earlier than expected time

Expected time (E_p) = 17 weeks
Scheduled time = 17 – 4 = 13 weeks

$$Z = \frac{13-17}{3} = -1.33$$

$$P(-1.33) = 1 - 0.9082 = 0.0918$$

2. Probability that the project will be completed at least 4 weeks later than expected

Time

Expected time = 17 weeks Scheduled time =17+ 4 =21 weeks

$$Z = \frac{21-17}{3} = 1.33$$

$$P(1.33) = 0.9082 = 90.8\%.$$

(e) Scheduled time = 20 weeks

$$Z = \frac{20-17}{3} = 1$$

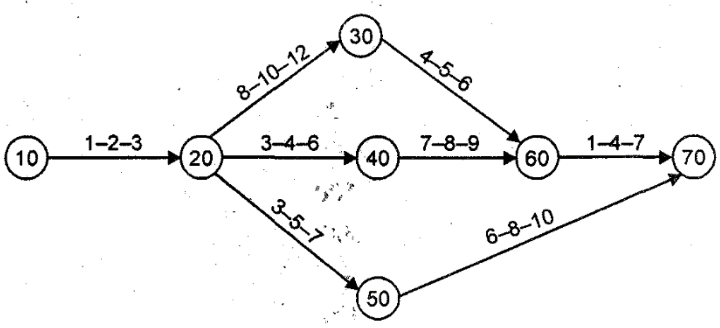
$$P(1) = 84.13\%$$

(f) Value of Z for P = 0.9 is 1.28 (from probability table)

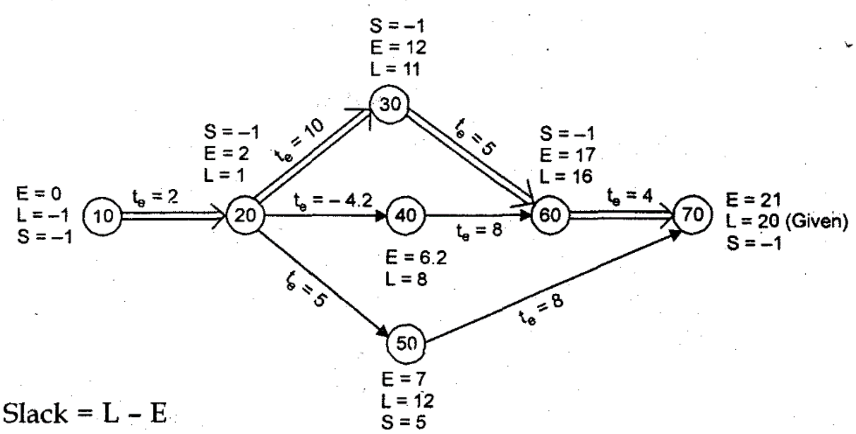
$$1.28 = \frac{T-17}{3}$$

$$T = 17 + 3.84 = 20.84 \text{ weeks.}$$

Problem 11.18. Consider the PERT network given in fig. Determine the float of each activity and identify the critical path if the scheduled completion time for the project is 20 weeks.



Solution.



(s) Slack = L – E

Activity	$t_e = \dfrac{t_0 + 4tm + t_p}{6}$	Start Time		Finish Time		Total Float
		E	T_{ES}	T_{EF}	L	
10 – 20	2	0	–1	2	1	–1
20 – 30	10	2	1	12	11	–1
20 – 40	4.2	2	3.8	6.2	8	1.8
20 – 50	5	2	7	7	12	5
30 – 60	5	12	11	17	16	–1
40 – 60	*8*	*6.2*	*8*	*14.2*	*16*	*1.8*
50 – 70	*8*	*7*	*12*	*15*	*20*	*5*
60 – 70	*4*	*17*	*16*	*21*	*20*	*-1*

Critical path 10 - 20- 30 - 60 — 70.

I.