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SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
BE PART TIME

GAS DYNAMICS AND JET PROPULSION

SMEA1602

UNIT – I FUNDAMENTALS OF COMPRESSIBLE FLOW – SMEA1602

UNIT-1

FUNDAMENTALS OF COMPRESSIBLE FLUID FLOW

1.1. Concept of Gas Dynamics

Gas dynamics mainly concerned with the motion of gases and its effects .It differ from fluid dynamics .Gas dynamics considers thermal or chemical effects while fluid dynamics usually does not.

Gas dynamics deals with the study of compressible flow when it is in motion. It analyses the high speed flows of gases and vapors' with considering its compressibility. The term gas dynamics is very general and alternative names have been suggested e.g.: Supersonic flow, compressible flow and aero thermodynamics etc.,

1.1.1 Significance with Applications:

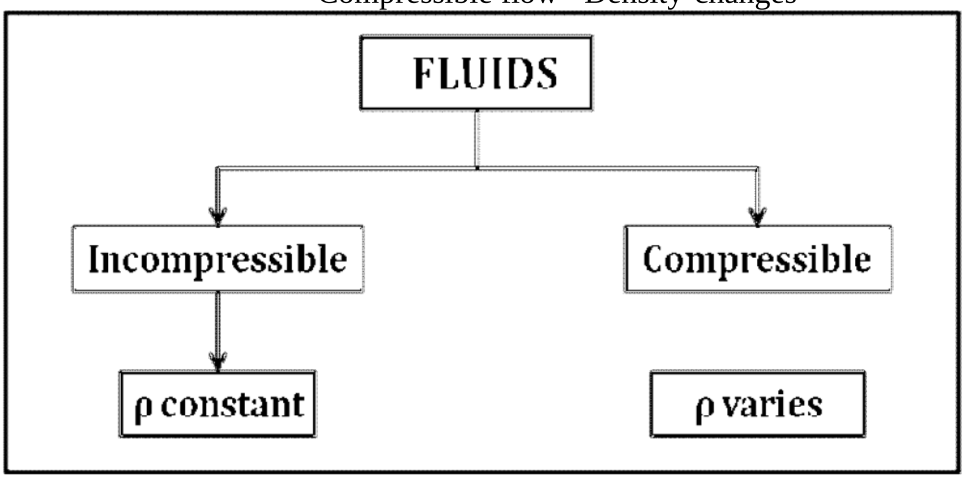
Gas dynamics is of interest to both mechanical and the aeronautical engineers but particular field of interest of the two different .It may be said that thermodynamicist is concerned with how an object in motion influenced as it flies through still air. In contrast to it the thermodynamicist in more interested in the cases in which the object in stationary and the fluid is in motion .The applications of gas dynamics are given below.

- It is used in Steam and Gas turbines
- High speed aero dynamics
- Jet and Rocket propulsion
- High speed turbo compressor

The fluid dynamics of compressible flow problems which involves the relation between forse, density, velocity and mass etc.Therfore the following laws are frequently used for solving the dynamic problems.

1. Steady flow energy equation
2. Entropy relations
3. Continity equation
4. Momentum equation

– Compressible flow - Density changes



We know that fluids, such as gas, are classified as Incompressible and Compressible fluids. Incompressible fluids do not undergo significant changes in density as they flow. In general, liquids are incompressible; water being an excellent example. In contrast compressible fluids do undergo density changes. Gases are generally compressible; air being the most common compressible fluid we can find. Compressibility of gases leads to many interesting features such as shocks, which are absent for incompressible fluids. Gas dynamics is the discipline that studies the flow of compressible fluids and forms an important branch of Fluid Mechanics.

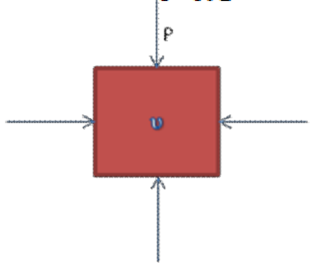
1.2.1 Compressible vs. Incompressible Flow

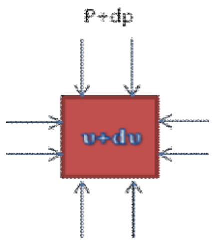
- A flow is classified as incompressible if the density remains nearly constant.
- Liquid flows are typically incompressible.
- Gas flows are often compressible, especially for high speeds.
- Mach number, Ma = V/c is a good indicator of whether or not compressibility effects are important.
    - Ma < 0.3 : Incompressible
    - Ma < 1 : Subsonic
    - Ma = 1 : Sonic
    - Ma > 1 : Supersonic
    - Ma >> 1 : Hypersonic

1.2.2 Compressibility:

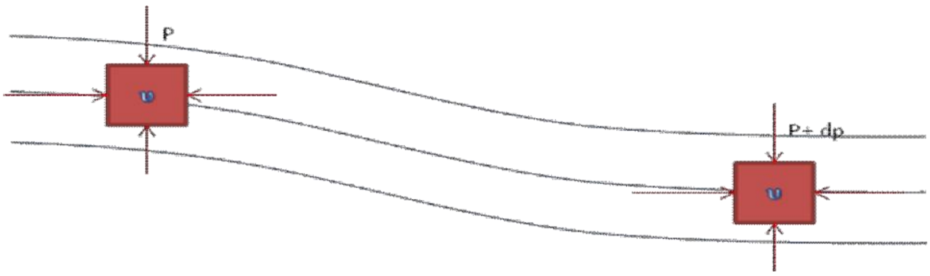
Measure of the relative volume change with pressure

$$\tau = -\frac{1}{v}\frac{dv}{dP} = \frac{1}{\rho}\frac{d\rho}{dP}$$





A measure of the relative volume change with pressure for a given process. Consider a small element of fluid of volume v, the pressure exerted on the sides of the element is p. Assume the pressure is now increased by an infinitesimal amount dp. The volume of the element will change by a corresponding amount dv , here the volume decrease so dv is a negative quantity. By definition, the compressibility of fluid is



The terms compressibility and incompressibility describe the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density, in other words, their "springiness." An incompressible fluid cannot be compressed and has relatively constant density throughout. Liquid is an incompressible fluid. A gaseous fluid such as air, on the other hand, can be either compressible or incompressible. Generally, for theoretical and experimental purposes, gases are assumed to be incompressible when they are moving at low speeds--under approximately 220 miles per hour. The motion of the object traveling through the air at such speed does not affect the density of the air. This assumption has been useful in aerodynamics when studying the behavior of air in relation to airfoils and other objects moving through the air at slower speeds.

In thermodynamics and fluid mechanics, compressibility is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial p}$$

Where *V* is volume and *p* is pressure. The above statement is incomplete, because for any object or system the magnitude of the compressibility depends strongly on whether the process is adiabatic or isothermal. Accordingly we define the isothermal compressibility as:

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

Where the subscript *T* indicates that the partial differential is to be taken at constant temperature. The adiabatic compressibility as:

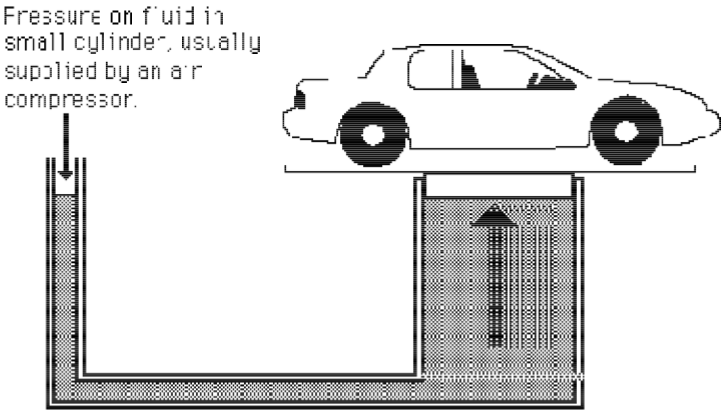
$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

Where *S* is entropy. For a solid, the distinction between the two is usually negligible. The inverse of the compressibility is called the bulk modulus, often denoted *K*(sometimes *B*).

1.2.3. Compressibility and Incompressibility

The terms compressibility and incompressibility describe the ability of molecules in a fluid to be compacted or compressed (made more dense) and their ability to bounce back to their original density, in other words, their "springiness." An incompressible fluid cannot be compressed and has relatively constant density throughout. Liquid is an incompressible fluid. A gaseous fluid such as air, on the other hand, can be either compressible or incompressible. Generally, for theoretical and experimental purposes, gases are assumed to be incompressible when they are moving at low speeds--under approximately 220 miles per hour. The motion of the object traveling through the air at such speed does not affect the density of the air. This assumption has been useful in aerodynamics when studying the behavior of air in relation to airfoils and other objects moving through the air at slower speeds.

However, when aircraft began traveling faster than 220 miles per hour, assumptions regarding the air through which they flew that were true at slower speeds were no longer valid. At high speeds some of the energy of the quickly moving aircraft goes into compressing the fluid (the air) and changing its density. The air at higher altitudes where these aircraft fly also has lower density than air nearer to the Earth's surface. The airflow is now compressible, and aerodynamic theories have had to reflect this. Aerodynamic theories relating to compressible airflow characteristics and behavior are considerably more complex than theories relating to incompressible airflow. The noted aerodynamicist of the early 20th century, Ludwig Prandtl, contributed the Prandtl-Glaubert rule for subsonic airflow to describe the compressibility effects of air at high speeds. At lower altitudes, air has a higher density and is considered incompressible for theoretical and experimental purposes.



Compressibility

- Compressibility of any substance is the measure of its change in volume under the action of external forces.
- The normal compressive stress on any fluid element at rest is known as hydrostatic pressure p and arises as a result of innumerable molecular collisions in the entire fluid.
- The degree of compressibility of a substance is characterized by the **bulk modulus of elasticity** E defined as

$$E = \lim_{\Delta\forall \to 0}\left(\frac{-\Delta p}{\Delta\forall / \forall}\right)$$

Where $\Delta\forall$ and Δp are the changes in the volume and pressure respectively, and $\forall$is the initial volume. The negative sign (-sign) is included to make E positive, since increase in pressure would decrease the volume i.e for $\Delta p>0$, $\Delta\forall<0$) in volume.

1.3 Steady Flow Energy Equation

From first law of Thermodynamics, we know that the total energy entering the system is equal to total energy leaving the system. This law is applicable to the steady flow systems.

Consider an open system-through which the working substance flows as a steady rate. The working substance entering the system at (1) and leaves the system at (2).

Let,

- P_1 - Pressure of the working substance entering the system *(N/m²)*
- V_1- Specific volume of the working substance entering the system (m^3/kg)
- C $_1$- Velocity of the, working substance entering the system *(m/s)*
- U_1 - Specific internal energy of the working substance entering the system *(Jlkg)*
- Z_1 - Height above the datum level for inlet in (m).

P_2, v_2 , c_2, U_2 and Z_2 - Corresponding values for the working substance leaving the system.

Q - Heat supplied to the system *(J/kg)*

W - Work delivered by the system (J/kg).

Total energy entering the system = Potential energy (gZ_1) + Kinetic energy ($c_1^2/2$) + Internal energy (U_1) + Flow energy ($p_1 v_1$) + Heat (Q)

Total energy leaving the system = Potential energy (gZ_2) + Kinetic energy ($c_2^2/2$) + Internal energy (U_2) + Flow energy ($p_2 v_2$) + Work (W)

From first law of Thermodynamics,

Total energy entering the system = Total energy leaving the system

$$gZ_1 + (c_1^2/2) + U_1 + p_1 v_1 + Q = gZ_2 + (c_2^2/2) + U_2 + p_2 v_2 + W$$

$$\mathbf{gZ_1 + (c_1^2/2) + h_1 + Q = gZ_2 + (c_2^2/2) + W}$$

[i.e h = U + pv]

The above equation is known as steady flow energy equation.

1.4 Momentum Principle for a Control Volume

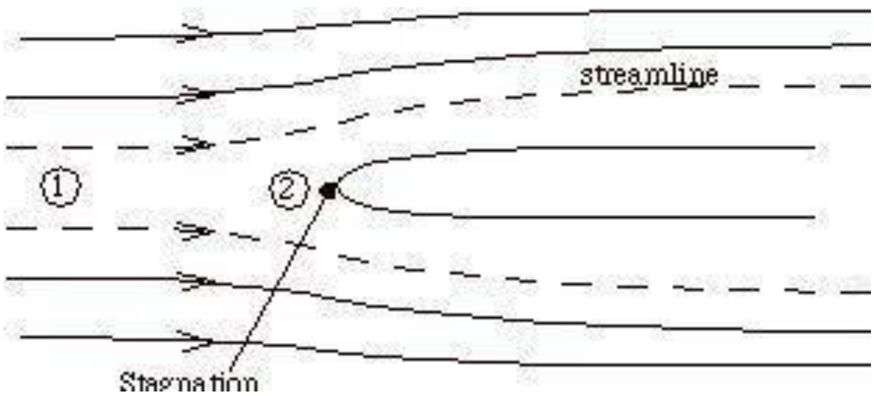
For a finite control volume between Sections 1 and 2 the momentum principle is,

$$p_1 A_1 + p_2 A_2 + F = \dot{m} V_2 - \dot{m} V_1$$
$$= \rho_2 V_2^2 A_2 - \rho_1 V_1^2 A_1$$

Where F is the x-component of resultant force exerted on the fluid by the walls. Note that the momentum principle is applicable even when **there are frictional dissipative processes** within the control volume.

1.5 Stagnation Enthalpy

Suppose that our steady flow control volume is a set of streamlines describing the flow up to the nose of a blunt object, as in Figure



Streamlines and a stagnation region; a control volume can be drawn between the dashed streamlines and points 1 and 2. The streamlines are stationary in space, so there is no external work done on the fluid as it flows. If there is also no heat transferred to the flow (adiabatic),

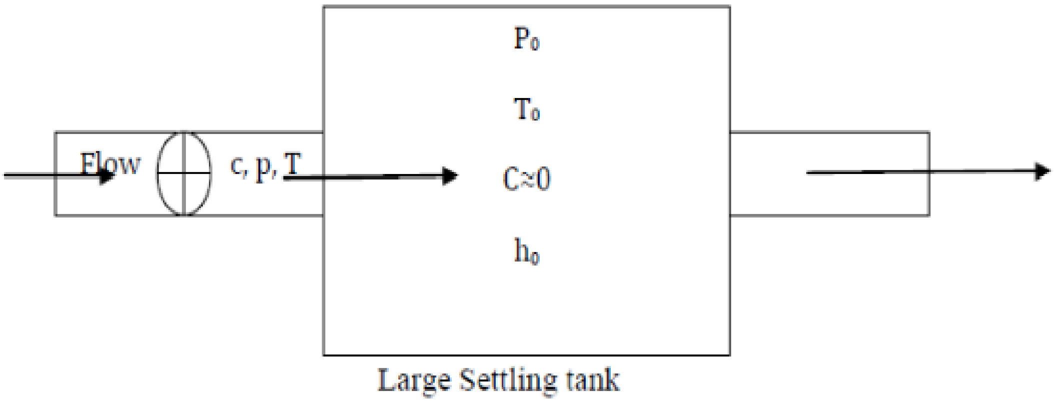


Fig: Deceleration of gas to Stagnation state

$$h_1 + \frac{1}{2}C_1^2 = h_2 + \frac{1}{2}C_2^2$$

Put,

- $h_1 = h$,
- $c_1 = c$
- for the initial state $h_2 = h_0$
- for the final state $c_2 = 0$

We have the energy equation for a nozzle and diffuser is

By substituting this in above equation,

$$h + \frac{1}{2}C^2 = h_0$$

Where,
h_0 = Stagnation enthalpy
h = Static enthalpy
c = Fluid velocity m / s

In an adiabatic energy transformation process the stagnation enthalpy remain constant.

1.6 Stagnation Temperature (or) Total temperature (T_0)

Stagnation temperature of a gas when its isentropically decelerated to zero velocity at zero elevation.

We know that, Stagnation enthalpy

$$h + \frac{1}{2}C^2 = h_0$$

We have stagnation enthalpy and static enthalpy for a perfect gas is,

$$h_0 = c_p T_0$$

$$h = c_p T$$

By substituting this in above equation,

$$C_p T_0 = C_p T + \frac{C^2}{2}$$

Divide by C_p through out the eqn.

$$T_0 = T + \frac{C^2}{2C_p}$$

Divided by T

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2C_p T}$$

We know that

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$a = \sqrt{\gamma R T}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2\frac{\gamma RT}{\gamma - 1}}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{C^2}{2\frac{a^2}{\gamma - 1}}$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times \frac{C^2}{a^2}$$

$$where \frac{C^2}{a^2} = M^2$$

$$\Rightarrow \frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times M^2$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} \times M^2$$

Where

- To= stagnation temperature
- T = static temperature
- M = Mach number (C/a)

1.7 Stagnation Pressure, [Po] or total pressure

Stagnation pressure of a gas when it is isentropically decelerated to zero velocity at zero elevation. For isentropic flow.

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma - 1}}$$

For stagnation condition,

$$P_2 = P_0$$

$$T_2 = T_0$$

$$P_1 = P$$

$$T_1 = T$$

$$\Rightarrow \frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} \times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P} = \left[1 + \frac{\gamma-1}{2} \times M^2\right]^{\frac{\gamma}{\gamma-1}}$$

1.8 Stagnation velocity of sound (a_0):

We know that the acoustic velocity of sound

$a = \sqrt{\gamma RT}$.

For a given value of Stagnation temperature, Stagnation velocity of sound, For stagnation condition, Put, $a = a_0$, $T = T_0$

$$a_0 = \sqrt{\gamma R T_0}$$

$$a_0 = \sqrt{\frac{C_p(\gamma-1)\gamma T_0}{\gamma}}$$

Where

$$R = \frac{C_p(\gamma-1)}{\gamma}$$

$$a_0 = \sqrt{C_p(\gamma-1)T_0}$$

Where

$$h_0 = C_p T_0$$

$$a_0 = \sqrt{h_0(\gamma-1)}$$

1.9 Various regions of flow

The adiabatic energy equation for a perfect gas is derived in terms of velocity of fluid(C) and Velocity of sound $[a_0]$

We have stagnation enthalpy and static enthalpy for a perfect gas is

$$h_0 = c_p T_0$$

$$h = c_p T$$

$$C_p = \frac{\gamma R}{\gamma - 1}$$

$$a = \sqrt{\gamma R T}$$

$$h + \frac{1}{2}C^2 = h_0$$

Put

$a = a_0$

$T = T_0$

$a_0 = \sqrt{\gamma R T_0}$

$$R = \frac{\gamma C_p T}{\gamma - 1}$$

$$a = \sqrt{\gamma R T}$$

$$a^2 = \gamma R T$$

$$h = \frac{a^2}{\gamma - 1}$$

$$h + \frac{1}{2}C^2 = h_0$$

$$\Rightarrow h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2}C^2$$

Case

At

$T = 0,$

$h = o$

Put

$$c = c_{max}$$

$$h_0 = C_p T_0 + \frac{1}{2}C^2{}_{max}$$

$$h_0 = \frac{1}{2} C^2{}_{max}$$

Put

at

$C = 0$

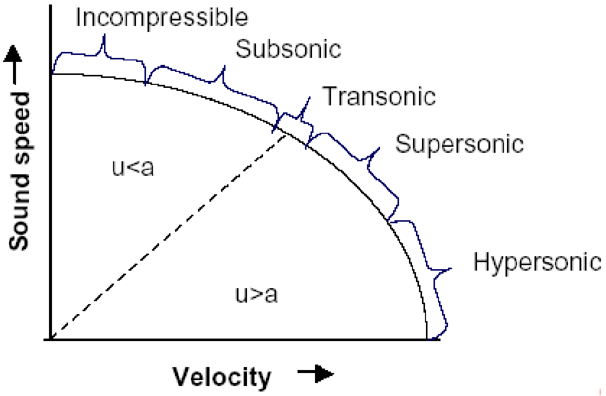
$a = a_0$

$h_0 = h$

$$h_0 = \frac{a_0{}^2}{\gamma - 1}$$

$$h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{a_0{}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}$$

$$\boxed{h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2} C^2 = \frac{a_0{}^2}{\gamma - 1} = \frac{1}{2} C^2{}_{max}}$$



- Subsonic Flow ($0.8 < M_o$)
- Transonic Flow ($0.8 > M_0 > 1.2$)
- Supersonic Flow ($M_0 > 1.2$)
- Hypersonic Flow ($M_0 > 5$)

- **Incompressible region**

In incompressible flow region fluid velocity (c) is much smaller than the sound velocity (a). Therefore the Mach number (M = c/a) is very low.

Eg: flow through nozzles

- **Subsonic flow region**

The subsonic flow region is on the right of the incompressible flow region. In subsonic flow, fluid velocity (c) is less than the sound velocity (a) and the Mach number in this region is always less than unity.
i.e. $M = c/a < 1$.
Eg: passenger air craft

- **Sonic flow region**

If the fluid velocity (c) is equal to the sound velocity (a), that type of flow is known as sonic flow. In sonic flow Mach number value is unity.
$M = c/a = 1 \Rightarrow c = a$. Eg: Nozzle throat

- **Transonic flow region**

If the fluid velocity close to the speed of sound, that type of flow is known as transonic flow .In transonic flow, Mach number value is in between 0.8 and 1.2. i.e.$0.8 < M < 1.2$.

- **Supersonic flow region**

The supersonic region is in the right of the transonic flow region. In supersonic flow, fluid velocity (c) is more than the sound velocity (a) and the Mach number in this region is always greater than unity.
i.e. $M = c/a > 1$.Eg: military air crafts

- **Hypersonic flow region**

In hypersonic flow region, fluid velocity (c) is much greater than sound velocity (a).In this flow, Mach number value is always greater than 5.
i.e. $M = c/a > 5$. Eg: rockets

1.11 Reference Velocities

In compressible flow analysis it is often convenient to express fluid velocities is non dimensional forms**.**

- Local velocity of sound
- Stagnation velocity of sound
- Maximum velocity of fluid
- Critical velocity of fluid/sound. $C^* = a^*$

1.11.1 Maximum velocity of fluid:

From adiabatic energy equation has two components of the total energy: the enthalpy h and the kinetic energy .If kinetic energy is absent the total energy is entirely energy represented by the stagnation enthalpy h0 .the other extreme conditions which can be conceived is when the entire energy is made up of kinetic energy only=0 and $C = C_{max}$. The fluid velocity (C_{max}) corresponding to this condition is the maximum velocity that would be achieved by the fluid when it is accelerated to absolute zero temperature (T = 0, T = 0) in an imaginary adiabatic expansion process.

$$C\max = \sqrt{2h_0}$$

For a perfect gas

$$C\max = \sqrt{2C_pT_0}$$

$$= \sqrt{\frac{2\gamma}{\gamma-1}RT_0}$$

Equation $h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{a_0{}^2}{\gamma-1} = \frac{1}{2}C^2{}_{\max}$ and equation

$$= \sqrt{\frac{2\gamma}{\gamma-1}RT_0}$$ Yield

$$\frac{C_{\max}}{a_0} = \sqrt{\frac{2}{\gamma-1}} = 2.24(for\,\gamma = 1.4)$$

1.11.2 Critical velocity of sound

It is the velocity of flow that would exist if the flow is isentropically accelerated or decelerated to unit Mach number (critical condition).

We have

Considering the *section (where M = 1) and its stagnation section

$M \Rightarrow 1$

$T \Rightarrow T^*$

$p \Rightarrow p^*$

$\rho \Rightarrow \rho^*$

$$M_{critical} = \frac{C^*}{a^*}$$

$$\Rightarrow \frac{T_0}{T^*} = \left[\frac{\gamma+1}{2}\right]$$

Multiplying both sides with γR

$$T_0 = T^*\left[\frac{\gamma+1}{2}\right]$$

$$a^2{}_0 = a^{*2} \times \frac{\gamma+1}{2}$$

$$\frac{a^*}{a_0} = \frac{C^*}{a_0} = \sqrt{\frac{2}{\gamma+1}}$$

From Equations

$$\frac{C_{max}}{a_0} = \sqrt{\frac{2}{\gamma-1}} \text{ and } \frac{a^*}{a_0} = \frac{C^*}{a_0} = \sqrt{\frac{2}{\gamma+1}}$$

$\frac{C_{max}}{a^*} = \frac{C_{max}}{a^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45(for\gamma)$ Finally we can written the equation is

$$C_{max} = a^* \times \sqrt{\frac{\gamma+1}{\gamma-1}}$$

$$= C^2{}_{max} = a^{2*} \times \frac{\gamma+1}{\gamma-1}$$

$$h_0 = \frac{a^2}{\gamma-1} + \frac{1}{2}C^2 = \frac{1}{2}C^2{}_{max} = \frac{1}{2}a^{2*} \times \frac{\gamma+1}{\gamma-1}$$

Expressions for $\frac{T_o}{T^*}, \frac{p_o}{p^*}$ and $\frac{\rho_o}{\rho^*}$

$\rho \Rightarrow \rho^*$

We already have

$$\frac{T_o}{T} = 1 + \frac{\gamma-1}{2}M^2$$

$$\therefore \frac{T_o}{T^*} = \frac{\gamma+1}{2}$$

Similarly

$$\frac{p_o}{p^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_o}{p^*} = \left[1 + \frac{\gamma + 1}{2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_o}{\rho^*} = \left[1 + \frac{\gamma - 1}{2}\right]^{\frac{1}{\gamma-1}}$$

$$= \left[\frac{\gamma - 1}{2}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{\rho_o}{\rho^*} = \left[\frac{\gamma + 1}{2}\right]^{\frac{1}{\gamma-1}}$$

Expressions for $\frac{T}{T^*}, \frac{p}{p^*}$ and $\frac{\rho}{\rho^*}$

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

At $M = 1 \quad T = T^*$

$$\therefore \frac{T_o}{T^*} = \frac{\gamma + 1}{2}$$

$$\frac{T}{T^*} = \frac{T_o/T^*}{T_o/T}$$

$$\frac{p}{p^*} = \frac{\left[\frac{\gamma + 1}{2}\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma - 1}{2} M^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{\rho}{\rho^*} = \frac{\rho_o/\rho^*}{\rho_o/\rho}$$

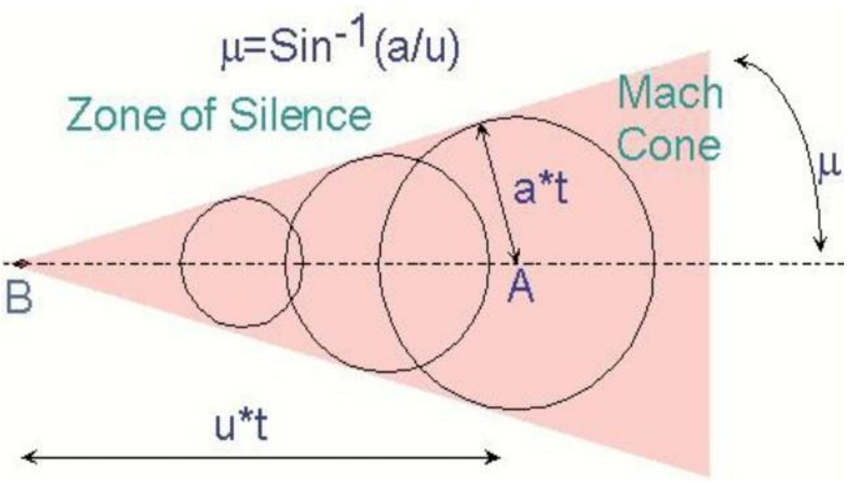
$$\frac{\rho}{\rho^*} = \frac{\left[\frac{\gamma+1}{2}\right]^{\frac{1}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M^2\right]^{\frac{1}{\gamma-1}}}$$

1.12 Mach number

In fluid mechanics, Mach number (M or Ma) is a dimensionless quantity representing the ratio of speed of an object moving through a fluid and the local speed of sound.

$$\mathrm{M} = \frac{v}{v_{\mathrm{sound}}}$$

1.13 Mach Cone



1.14 Reference Mach number M*

In the analysis of high speed flows, another Mach number called M^* is employed. It is defined as the non dymensionlizing the fluid velocity by the critical fluid velocity or the sound velocity.

That is,

$$M^* = \frac{C}{a^*} = \frac{C}{C^*}$$

$$M^{*2} = \frac{C^2}{a^{*2}} = \frac{C}{a^2} \times \frac{a^2}{a^{*2}} = M^2 \frac{a^2}{a^{*2}}$$

Some it is more convenient to use M* instead of M because

(i) at high fluid velocities M approaches infinity

(ii) M is not proportional to the velocity alone

It should be pointed out here that M*does not mean M = 1this only other type of Mach number.

We have $h_0 = \frac{a^2}{\gamma - 1} + \frac{1}{2}C^2 = \frac{1}{2}C^2{}_{max} = \frac{1}{2}a^{2*} \times \frac{\gamma+1}{\gamma-1}$

$= \frac{1}{2}a^{2*} \times \frac{\gamma+1}{\gamma-1} =$

Multiplying by 2

$$\frac{2a^2}{\gamma-1} + C^2 = \frac{\gamma+1}{\gamma-1}a*2$$

Divided by $a*2$

$\div$by$\frac{\gamma-1}{2}$

$$M^{*2}\left(1 + \frac{2}{\gamma-1}\frac{1}{M^2}\right) = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2}\left(\frac{2}{\gamma-1}\right)\left(\frac{\gamma-1}{2} + \frac{1}{M^2}\right) = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} = \frac{\frac{1}{2}(\gamma+1)M^2}{1 + \frac{1}{2}(\gamma-1)M^2}$$

When M* =0 at M = o

M* = 1 at M = 1

Eqn.we have

$$M^{*2} + \frac{2}{\gamma-1}\frac{M^{*2}}{M^2} = \frac{\gamma+1}{\gamma-1}$$

It gives

$$\frac{2}{\gamma-1}\frac{M^{*2}}{M^2}=\frac{\gamma+1}{\gamma-1}-M^{*2}$$

$$\frac{M^{*2}}{M^2}=\frac{\gamma+1}{2}-\frac{\gamma-1}{2}M^{*2}$$

$$\frac{C^2}{a^{*2}}+\frac{2}{\gamma-1}\frac{a^2}{a^{*2}}=\frac{\gamma+1}{\gamma-1}$$

$$M^{*2}+\frac{2}{\gamma-1}\frac{M^{*2}}{M^2}=\frac{\gamma+1}{\gamma-1}$$

$$M^2=\frac{\left\{\frac{2}{\gamma+1}\right\}M^{*2}}{1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2}}$$

$$1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2}=0$$

$$M^*_{\max}\frac{C_{\max}}{C^*}=\sqrt{\frac{\gamma+1}{\gamma-1}}=2.45(for\gamma=1.4)$$

$$C_r=\frac{C}{C_{\max}}.$$

At M = ∝

$$1-\left\{\frac{\gamma+1}{\gamma-1}\right\}M^{*2}=0$$

$$M^*_{\max}\frac{C_{\max}}{C^*}=\sqrt{\frac{\gamma+1}{\gamma-1}}=2.45(for\gamma=1.4)$$

1.15 Crocco number

A non -dimensional fluid velocity can be defined by using the Maximum fluid velocity,

$$C_r = \frac{flowvelocity}{\max.fluidvelocity}$$

$$C_r = \frac{C}{C_{\max}.}$$

1.16 Isothermal Flow

- Adiabatic and Reversible.
- No energy added, No energy losses.
- Small and gradual change in flow variables.

The earliest reference to is other mal flow was found in Shapiro's Book. The model Suggests that the choking occurs at 1 /√k and it appears that Shapiro was the first one to realize this

$$\sum F_{xx} = \frac{d}{dt}(\text{momentum})\Big]_{sys} = \frac{d}{dt}(mv_{xx})\Big]_{sys}$$

$$\therefore \sum F_{xx} = F_{1xx} + F_{2xx} + F_{3xx} + F_{4xx}$$

$$F_{1xx} = F_1 \, . \, pA$$

$$F_{2xx} = -F_2 = -(p + dp)(A + dA)$$

$$= -[pA + pdA + Adp] \text{ (neglecting HOT)}$$

$$F_{3xx} = F_3 \cos(90 - \alpha) = F_3 \sin \alpha$$

$$= \text{(Average presure)} \times \text{(curved wall area)} \sin \alpha$$

$$= \left(P + \frac{dp}{2}\right) A_w \sin\alpha$$

But $A_w \sin\alpha$ is the projection of the curved wall area on to a plane perpendicular to flow direction and equal to dA.

$$F_{3xx} = \left(P + \frac{dp}{2}\right) dA = pdA \quad \text{(neglecting HOT)}$$

$$F_{4xx} = - F4 \cos(90 - \theta)$$

$$= - F4 \sin \theta$$

$$= - mg \sin \theta$$

$\left.\frac{d}{dt}(mv_{xx})\right]_{sys}$ is the total change of momentum and represents change of momentum with regard to time and change of momentum with respect to position.

The system and CV are interrelated through Reynold's transport theorem.

$$\left.\frac{d}{dt}(mv_{xx})\right]_{sys} = \frac{\partial}{\partial t}(mv_{xx})_{CV} + (\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in}$$

$$\therefore \sum F_{xx} = \frac{\partial}{\partial t}(mv_{xx})_{CV} + (\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in} \quad \text{........(i)}$$

1.17.1 Assumptions

- 1 - D flow
- Steady flow
- Higher order terms (Hot's) are neglected
- The fluid is invicid (M = 0)
- Compressible fluid

$\sum F_{XX}$ - This represents the summation of all forces acting on CV and includes surface and body forces. Surface forces are of normal and tangential forms. Since the fluid is invicid shear force (tangential) = 0

Figure above shows a flow through CV. All fluid properties at inlet and exit are represented. Various forces act ing are also represented in their respective direction at the appropriate locations.

$$F_{4xx} = -\left[\left\{\left(A + \frac{dA}{2}\right)dx\left(\rho + \frac{d\rho}{2}\right)\right\}g\,dz\right]$$

= - ρ Ag dz Neglecting HOT's ....

$$\therefore \sum F_{xx} = pA - pA - pdA - Adp + pdA - \rho g\, Adz$$

$$\sum F_{xx} = - Adp - \rho g\, Ag\, dz$$

RHS of (1)

$$= \frac{\partial}{\partial t}(mv_{xx})_{CV} + (\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in}$$

Since steady flow

$$\frac{\partial}{\partial t}(mv_{xx})_{CV} = 0$$

$$(\dot{m}v_{xx})_{out} - (\dot{m}v_{xx})_{in} = \dot{m}\left(v_{xx_{out}} - v_{xx_{in}}\right)$$

$$= \rho AV[(V + dV) - V]$$

$$= \rho AV\, dV$$

$$\therefore RHS = \rho AV\, dV$$

Equating LHS and RHS of (1)

$$-Adp - \rho\, Ag\, dz = \rho\, AV\, dV$$

$$Adp + \rho\, AVdV + \rho\, Agdz = 0$$

$\frac{d\rho}{\rho} + VdV + gdz = 0$ Momentum equation - Euler's equation

1.18 Problems

1) Atm air at 1.1×10^5 N/m^2 and 65^oC is accelerated isentropically to a mach number of 1. Find final temp, pressure and flow velocity.

Ans. Assume, the given state is the stagnation state, From data book, is entropic table, K = 1.4

$T = T^* = 8.834 \times T_o$

$= 0.834 \times 338$

$T = 281.9$ K

Final temperature $T^* = 281.9$ K

$$\left.\frac{P}{P_o}\right]_{M=1} = 0.528$$

Stagnation properties (P_o, T_o)

From isentropic tables (K = 1.4)

$$\left.\frac{T_1}{T_o}\right]_{M_1=0.708} = 0.908$$

$$T_o = \frac{T_1}{0.908} = \frac{310}{0.908}$$

$T_o = 341.41$ K

Stagnation temperature $T_o = 341.41$ K

Final pressure $P^* = 0.581 \times 10^5$ N/m^2

$$M = \frac{V}{C}$$

Final mach number = 1

$$1 = \frac{V_{final}}{C_{final}} = \frac{V^*}{C^*}$$

$$V^* = C^* = \sqrt{KRT^*}$$

$$= 20.05\sqrt{281.9}$$

Final velocity $V^* = 336.6$ ms^{-1}

Q2) The pressure, temperature velocity of air at a point in a flow field are $1.2 \times 10^5 N/m^2$, 37^oC and $250\ ms^{-1}$ respectively. Find the stagnation pressure and stagnation temperature corresponding to given condition

Ans

Match number (M_1)

$$M_1 = \frac{V_1}{C_1} = \frac{c_1}{a_1}$$ Fig.

$$\left.\frac{P_1}{P_o}\right]_{M_1=0.708} = 0.714$$

$$P_o = \frac{P_1}{0.714} = \frac{1.2 \times 10^5}{0.714}$$

$= 1.681 \times 10^5\ N/m^2$

Stagnation pressure, $P_o = 1.681 \times 10^5\ N/m^2$

Q 3) Determine velocity of sound in air at 38^oC.

Velocity of sound $C = \sqrt{KRT}$

$= \sqrt{1.4 \times 287 \times 311}$

$C = 353.5\ ms^{-1}$

Velocity of sound at 30^oC

$= 353.5\ ms^{-1}$

Q 4) Air flows through a duct with a velocity 300 ms^{-1} pressure 1 bar, temp. 30°C. Find (i) Stagnation pressure and temperature (ii) Velocity of sound in dynamic and stagnation condition

Ans.

Velocity of sound at (1)

$C_1 = \sqrt{KRT_1} = 20.05\sqrt{303}$ Fig.

$C_1 = 349\ ms^{-1}$

$$M_1 = \frac{V_1}{C_1} = \frac{300}{349}$$

$M_1 = 0.86$

Stagnation properties (P_o and T_o) from Tables

$$\left.\frac{T_1}{T_o}\right]_{M=0.86} = 0.871$$

$$T_o = \frac{T_1}{0.871} = \frac{300}{0.871} = 347.9\ K$$

Stagnation temperature $T_o = 347.9$ K

$$\left.\frac{P_1}{P_o}\right]_{M=0.86} = 0.617$$

$$P_o = \frac{P_1}{0.617} = \frac{1}{0.617} = 1.62$$

Stagnation pressure = 1.62 bar

Velocity pressure = 1.62 bar

Velocity of sound at stagnation temperature (C_o)

$C_o = \sqrt{KRT_o}$

$= 20.05\sqrt{347.9}$

$C_o = 373.97\ ms^{-1}$

Q 5) Air at stagnation condition has a temperature of 700 K. Find the max. possible velocity. What would be the sonic velocity if flow velocity is 1/2 of the maximum velocity.

$$h + \frac{V^2}{2} = h_o$$

$$\frac{V^2}{2} = (h_o - h) = C_p (T_o - T)$$

V becomes maximum when T = 0

$$\frac{V^2_{max}}{2} = C_p T_o = 1005 \times 700 \ . \ \frac{J}{KgK} . K$$

$$V_{max} = \sqrt{2 \times 1005 \times 700} = 1186.2 \text{ ms}^{-1}$$

Maximum possible flow velocity $V_{max} = 1186.2 \text{ ms}^{-1}$

New flow velocity $V_1 = \frac{V_{max}}{2} = \frac{1186.2}{2} = 593.1 \text{ ms}^{-1}$

We have,

$$h_1 + \frac{V_1^2}{2} = h_o$$

$$C_p T_1 + \frac{(593.1)^2}{2} = C_p T_o$$

$$T_1 + \frac{(593.1)^2}{1005 \times 2} = T_o$$

$$T_1 + 700 - \frac{(593.1)^2}{2010}$$

$T_1 = 524.99$ K (525 K)

Sonic velocity

$$C_1 = \sqrt{KRT_1} = 20.05 \sqrt{525}$$

$C_1 = 459.4 \text{ ms}^{-1}$

Q 6) Air is discharged into the atmosphere from a big vessel through a nozzle. The atmospheric pressure and temperature are 1.1×10^5 N/m^2 and 40°C and the jet velocity is 100 ms^{-1}. Find the chamber pressure and temperature.

Ans.

Mach number

$$M_1 = \frac{V_1}{C_1}$$

$$C_1 = \sqrt{KRT_1}$$

Fig.

$$= 20.05\sqrt{313} = 354.72 \text{ ms-1}$$

$$M_1 = \frac{V_1}{C_1}$$

$$= \frac{100}{354.72}$$

$$M_1 = 0.28$$

Stagnation properties (P_o, T_o)

From Gas tables we have,

$$\left.\frac{P_1}{P_o}\right]_{M=0.28} = 0.999$$

$$P_o = \frac{P_1}{0.999} = \frac{1.1\times10^5}{0.999} = 1.101 \times 10^5 \text{ N/m}^2$$

Chamber pressure $P_o = 1.101 \times 10^5$ N/m^2

$$\left.\frac{T_1}{P_o}\right]_{M=0.28} = 0.999$$

$$T_o = \frac{T_1}{0.999} = \frac{313}{0.999} = 313.33 \text{ K}$$

Chamber temperature $T_o = 313.33$ K

Q 7) Air stored in a big container t 400 K is allowed to expand adiabatically through a pipe. At a station (A) down stream in the pipe line the air attains a velocity of 250 m/s. Find velocity of sound in the container and at station A in the pipes.

Answer

Assumption - Friction losses are neglected

We have from SFEE,

$$h + \frac{V^2}{2} = h_o$$

Fig.

$$C_p T_1 + \frac{V^2}{2} = C_p T_o$$

$$C_p T_A + \frac{V^2_A}{2} = C_p T_o$$

$T_A = 368.9$ K

Temperature at section (A) = 368.9 K

Velocity pof sound at station (A) (C_A)

$$C_A = \sqrt{KRT_A} = 20.05\sqrt{368.9} = 385.1\ \text{ms}^{-1}$$

$$\text{Mach number } M_A = \frac{V_A}{C_A} = \frac{250}{385.1}$$

$M_A = 0.65$

To find the temperature in the chamber

From Gas tables,

$$\left.\frac{T_A}{T_o}\right]_{M=0.68} = 0.915$$

$$T_o = \frac{T_A}{0.915} = \frac{368.9}{0.915}$$

Chamber temperature $T_o = 403.17$ K

Velocity of sound in the container (C_o)

$$C_o = \sqrt{KRT_o} = 20.05 \sqrt{403.17}$$

$$C_o = 402.6 \text{ ms}^{-1}$$

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GAS DYNAMICS AND JET PROPULSION

SMEA1602

UNIT – II FLOW THROUGH VARIABLE AREA DUCTS – SMEA1602

UNIT-2

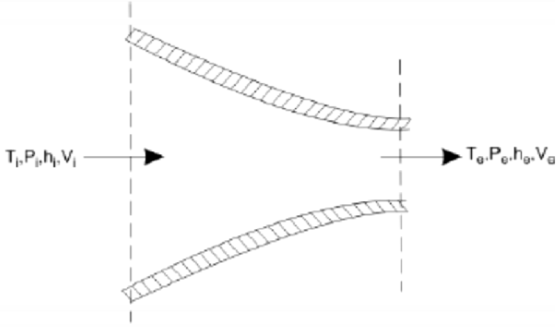
FLOW THROUGH VARIABLE AREA DUCTS

2.1 Flow through Nozzles

A nozzle is a duct that increases the velocity of the flowing fluid at the expense of pressure drop. A duct which decreases the velocity of a fluid and causes a corresponding increase in pressure is a diffuser. The same duct may be either a nozzle or a diffuser depending upon the end conditions across it. If the cross-section of a duct decreases gradually from inlet to exit, the duct is said to be convergent. Conversely if the cross section increases gradually from the inlet to exit, the duct is said to be divergent. If the cross-section initially decreases and then increases, the duct is called a convergent-divergent nozzle. The minimum cross-section of such ducts is known as throat. A fluid is said to be compressible if its density changes with the change in pressure brought about by the flow. If the density does not changes or changes very little, the fluid is said to be incompressible. Usually the gases and vapors are compressible, whereas liquids are incompressible.

Nozzle:

A nozzle is primarily used to increase the flow velocity.



The first law reduces to

$$h_e + \frac{V_e^2}{2} = h_i + \frac{V_i^2}{2}$$

Or

$$V_e^2 - V_i^2 = 2(h_i - h_e)$$

If the inlet velocity is negligible $V_i \approx 0$ *and then*

$$V_e \sqrt{2(h_i - h_e)}$$

$$V_e \sqrt{2(h_i - h_e) + V_i^2}$$

The velocity is increased at the cost of drop in enthalpy. If an ideal gas is flowing through the nozzle, the exit velocity V_e can be expressed in terms of inlet and outlet pressure and temperatures by making use of the relations

$$P\upsilon = RT$$

$$dh = c_p dT$$

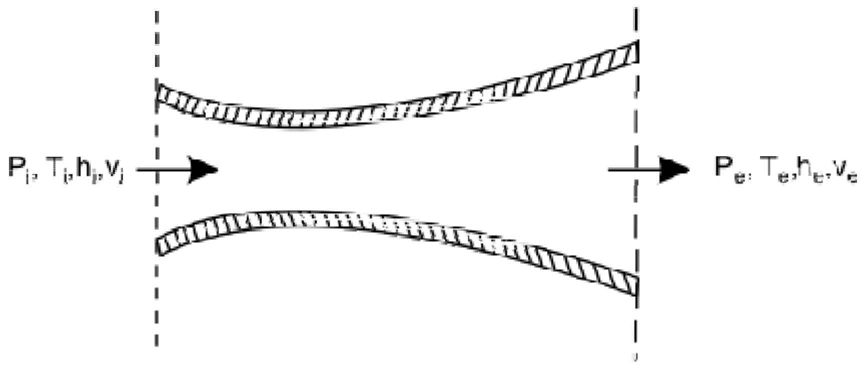
$$V_e^2 = 2c_p(T_i - T_e) = 2c_p T_i\left(1 - \frac{T_e}{T_i}\right)$$

$$\frac{T_e}{T_i} = \left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}}$$

$$V_e = \sqrt{2C_p T_i\left[1 - \left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

2.2 Diffuser:

A diffuser can be thought of as a nozzle in which the direction of flow is reversed.



For an adiabatic diffuser, Q and W_{sh} are zero and the first law reduces to

$$h_e + \frac{V_e^2}{2} = h_i + \frac{V_i^2}{2}$$

$$V_e^2 - V_i^2 = 2(h_i - h_e)$$

The diffuser discharges fluid with higher enthalpy. The velocity of the fluid is reduced.

- **Example of a reversible process:**

– Slow compression of air in a balloon does work on the air inside the balloon, and takes away energy from the surroundings - When the balloon is allowed to expand, the air inside and the surrounding air are both restored to original conditions
Example of an irreversible process:
Heat flows from hot to cold, never in the opposite direction; Most conductive and viscous processes are irreversible Stagnation point is thus when fluid is brought to stagnant state (eg, reservoir)

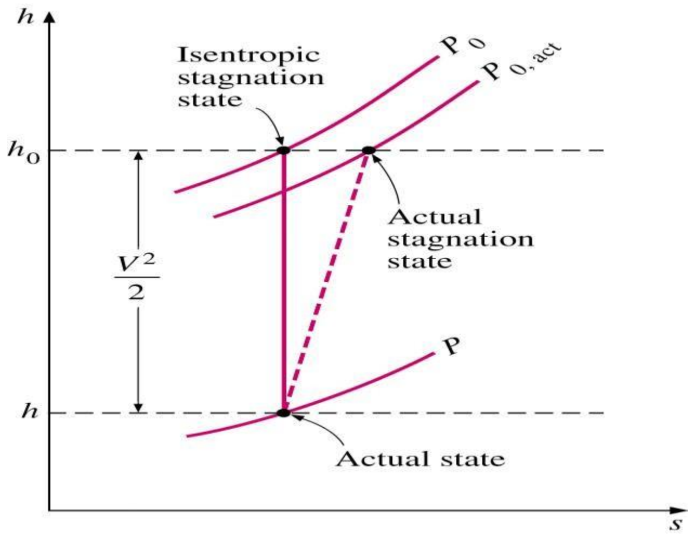
Stagnation properties can be obtained at any point in a flow field if the fluid at that point were decelerated from local conditions to zero velocity following an isentropic (frictionless, adiabatic) process

Pressure:	p_0
Temperature:	T_0
Density:	ρ_0

- If a fluid were brought to a complete stop ($C_2 = 0$)
- Therefore, h_0 represents the enthalpy of a fluid when it is brought to rest adiabatically.
- During a stagnation process, kinetic energy is converted to enthalpy.
- Properties at this point are called **stagnation properties** (which are identified by subscript 0)

If the process is also reversible, the stagnation state is called the **isentropic stagnation state.**

Stagnation enthalpy is the same for isentropic and actual stagnation states



Q 8) A conical diffuser has entry diameter 20 cm, the match number temp and pressure are 0.6, 120 KN/m^2 and 340 K. The mach number at exist is 0.2. For 1 - D isentropic flow, calculate the following.

i) Pressure, temp and velocity at exist

ii) Mass flow rate, and exit diameter

iii) Change in impulse function

From the gas tables,

$$\left.\frac{T_1}{T_o}\right]_{M=0.6} = 0.933$$

$$T_o = \frac{340}{0.933} = 364.4 \text{ K}$$

Stagnation temperature $T_o = 364.4$ K

$$\left.\frac{P_1}{P_o}\right]_{M=0.6} = 0.784$$

$$P_o = \frac{1.2 \times 10^5}{0.784}$$

$$P_o = 1.53 \times 10^5 \ N/m^2$$

To find temperature at the exit

We have,

$$\left.\frac{T_2}{T_o}\right]_{M=0.2} = 0.992$$

$$T_2 = 364.4 \times 0.992$$

$$T_2 = 361.5 \ K$$

To find pressure at the exit

We have,

$$\left.\frac{P_2}{P_o}\right]_{M=0.2} = 0.973$$

$$P_2 = 1.53 \times 10^5 \times 0.973$$

$$= 1.48 \times 10^5 \ N/m^2$$

To find velocity at the exit

Sonic velocity at (1)

$$C_2 = \sqrt{KRT_2} = 20.05 \ \sqrt{361.5}$$

$$C_2 = 381.2 \ m/s$$

Velocity $V_2 = M_2 \times C_2$

$$= 0.2 \times 381.2$$

$$V_2 = 76.2 \ ms^{-1}$$

To find mass flow rate

Velocity at inlet

$$V_1 = C_1 \times M_1 = . \sqrt{KRT_1} \times M_1$$

We have $\left.\frac{A_2}{A^*}\right]_{0.2} = 2.914$

$\left.\frac{A_1}{A^*}\right]_{0.6} = 1.188$

$$\frac{A_2}{A_1} = \frac{2.964}{1.188} = \frac{(d_2)^2}{(d_1)^2}$$

$= 20.05 \sqrt{340} \times 0.6$

$V_1 = 221.82\ ms^{-1}$

$$r_1 = \frac{P_1}{RT_1} = \frac{1.2 \times 10^5}{287 \times 340} = 1.23\ Kg/m^3$$

Mass flow rate = r_1 A V_1

$\dot{m} = 1.23 \times \frac{\pi}{4}(0.2)^2 \times 221.82 = 8.57\ Kg/s$

$\therefore$ Exit diameter, $d_2 = 31.5$ cm

Impulse function

$$F = PA + rAV^2 = PA\left[1 + \frac{\rho}{P}V^2\right]$$

$$= PA\left[1 + \frac{1}{RTP}V^2\right] = PA\left[1 + \frac{KV^2}{KRT}\right]$$

i.e., $F = PA\left[1 + KM^2\right]$

$DF = F_2 - F_1$

$DF = P_1A_1\left[1 + KM_1^2\right] - P_2A_2\left[1 + KM_2^2\right]$

DF = 6.503 KN

Q 9) Air is discharged from a reservoir at Po = 6.91 Kgf/cm^2 and to = 325^oC through a nozzle to an exit pressure of 0.98 Kgf/cm^2 . If the flow rate is 1 kg/s, find the throat area, pressure and velocity. Also find the exit area, exit temperature and exit velocity.

Ans

Exit Mach number M_e] = 1.93

$$\frac{P_e}{P_o} = \frac{0.98}{6.91} = 0.142$$

To find exit temperature

$$\left.\frac{T_e^*}{T_o}\right]_{M=1.93} = 0.573$$

$T_e = 598 \times 0.573$

$= 342.65K$

$$r_e = \frac{P_e}{R\,T_e} = \frac{0.98 \times 9.81 \times 10^4}{287 \times 342.65}$$

$r_e = 0.975\ Kg/m^3$

To find velocity at exit (V_e)

Sonic Velocity

$C_e = \sqrt{KRT_e} = 20.05\sqrt{342.65}$

$C_e = 371.14\ ms^{-1}$

$V_e = M_c\,C_e = 1.93 \times 371.14 = 716.3\ ms^{-1}$

To find exit area

We have

$\dot{m} = r_e\,A_e\,V_e$

$$A_e = \frac{\dot{m}}{\rho_e V_e} = \frac{1}{0.978 \times 716.3} = ... 0.0014 \text{ m}^2$$

To find throat pressure (P*)

$$\left.\frac{P^*}{P_o}\right]_{M=1} = 0.528$$

$$P^* = 0.528 \times 6.91 = 3.65 \text{ K gf/cm}^2$$

To find throat tempeature (T*)

$$\left.\frac{T^*}{T_o}\right]_{M=1} = 0.834$$

$$T^* = 0.834 \times 598 = 498.732 \text{ K}$$

$$\left.\frac{A}{A^*}\right]_{M=1} = 1.000$$

$$\left.\frac{A}{A^*}\right]_{M=1.93} = 1.593$$

$$A^* = \frac{A_e}{1.593} = \frac{0.0014}{1.593} = 0.00088 \text{ m}^2$$

$$V^* = \sqrt{KRT^*} = 20.05\sqrt{498.732} = 447.76 \text{ ms}^{-1}$$

Q 10) Air flows isentropically through a C.D. The inlet conditions are pressure 700 KN/m^2, temperature 320^{o}C, velocity 50 m/s. The exit pressure is 10^{5}KN/m^2 and the exist area is 6.25 cm^2. Calculate

2.2.1 Mach number, temperature and velocity at exit

2.2.2 Pressure, temperature and velocity at throat

2.2.3 Mass flow rate

2.2.4 Throat area

Ans: To find the exit mach number (M2)

Sonic velocity at inlet

$$C_1 = \sqrt{KRT_1}$$

$$= 20.05\sqrt{593}$$

$= 488.25 \ ms^{-1}$

Inlet mach number

$$M_1 = \frac{V_1}{C_1}$$

$$= \frac{50}{488.25} = 0.1$$

We have

$$\left.\frac{P_1}{P_o}\right]_{M=0.1} = 0.993$$

$$P_o = \frac{700}{3.993}$$

$$= 7.04 \times 10^5 \ N/m^2$$

Exit mach number

$$M_2\Big]_{\frac{P_2}{P_o} = \frac{105}{704} = 0.149} = 1.9$$

We have

$$\left.\frac{T_1}{T_o}\right]_{M=0.1} = 0.998$$

$$To = \frac{593_1}{0.998}$$

$$= 594.2 \ K$$

We have

$$\left.\frac{T_2}{T_o}\right]_{M_2=1.9} = 0.581$$

$$T_2 = 594.2 \times 0.581$$

Exit temperature $T_2 = 345.22$ K

Sonic velocity

$C_2 = \sqrt{KRT_2}$

$= 20.05 \sqrt{345.22}$

$= 372.5 \text{ ms}^{-1}$

Final velocity

$V_2 = M_2 \times C_2$

$= 1.9 \times 372.5$

$= 707.8 \text{ ms}^{-1}$

To find throat parameters

$$\left.\frac{P^*}{P_o}\right]_{M=1} = 0.528$$

Throat pressure

$P^* = 7.04 \times 10^5 \times 0.528$

$P^* = 371.7 \text{ KN/m}^2$

$$\left.\frac{T^*}{T_o}\right]_{M=1} = 0.834$$

Throat temperature

$T^* = 594.2 \times 0.834$

$T^* = 495.5 \text{ K}$

Velocity

$V^* = C^*$

$= \sqrt{KRT^*}$

$= 20.05 \sqrt{495.5}$

Throat velocity $V^* = 446.3 \text{ ms}^{-1}$

$$\left.\frac{A_2}{A^*}\right]_{M=1.9} = 1.555$$

$$A^* = \frac{6.25 \times 10^{-4}}{1.555}$$

Throat area

$$A^* = 4.019 \times 10^{-4}\ m^2$$

2.3 Tutorial Problems:

1.The pressure, temperature and Mach number at the entry of a flow passage are 2.45 bar, 26.5° C and 1.4 respectively. If the exit Mach number is 2.5 determine for adiabatic flow of perfect gas (γ =1.3, R=0.469 KJ/Kg K).

2. Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 k. Taking γ =1.4 and R =287J/Kg K determine: 1) Stagnation pressure and temperature, 2) Velocity of sound in the dynamic and stagnation conditions, 3) Stagnation pressure assuming constant density.

3. A nozzle in a wind tunnel gives a test –section Mach number of 2.0 .Air enters the nozzle from a large reservoir at 0.69 bar and 310 k .The cross –sectional area of the throat is1000cm^2 .Determine the following quantites for the tunnel for one dimensional isentropic flow 1) Pressures,temperature and velocities at the throat and test sections, 2) Area of cross- sectional of the test section , 3) Mass flow rate, 4) Power rate required to drive the compressor.

4. Air is discharged from a reservoir at Po =6.91bar and To =325° c through a nozzle to an exit pressure of 0.98 bar .If the flow rate is 3600Kg/hr determine for isentropic flow: 1)Throat area, pressure,and velocity, 2)Exit area,Mach number ,and 3)Maximum velocity.

5. Air flowing in a duct has a velocity of 300 m/s ,pressure 1.0 bar and temperature 290 K. Taking γ =1.4 and R =287J/Kg K determine: 1)Stagnation pressure and temperature, 2)Velocity of sound in the dynamic and stagnation conditions 3)Stagnation pressure assuming constant density.

6. A conical diffuser has entry and exit diameters of 15 cm and 30cm respectively. Pressure ,temperature and velocity of air at entry are 0.69bar,340 k and 180 m/s respectively . Determine 1) The exit pressure, 2) The exit velocity and 3) The force exerted on the diffuser walls.Assume isentropic flow, γ =1.4,Cp =1.00 KJ Kg-K

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GAS DYNAMICS AND JET PROPULSION

SMEA1602

UNIT – III FANNO FLOW AND RAYLEIGH FLOW – SMEA1602

UNIT – 3

FANNO FLOW AND RAYLEIGH FLOW

3.1 Introduction

Friction is present in all real flow passages. There are many practical flow situations where the effect of wall friction is small compared to the effect produced due to other driving potential like area, transfer of heat and addition of mass. In such situations, the result of analysis with assumption of frictionless flow does not make much deviation from the real situation. Nevertheless; there are many practical cases where the effect of friction cannot be neglected in the analysis in such cases the assumption of frictionless flow leads to unrealistic influence the flow. In high speed flow through pipe lines for long distances of power plants, gas turbines and air compressors, the effect of friction on working fluid is more than the effect of heat transfer ,it cannot be neglected An adiabatic flow with friction through a constant area duct is called fanno flow when shown in h-s diagram, curves ,obtained are fanno lines. Friction induces irreversibility resulting in entropy increase. The flow is adiabatic since no transfer of heat is assumed.

3.2 Fanno Flow

A steady one-dimensional flow in a constant area duct with friction in the absence of Work and heat transfer is known as “fanno flow”.

3.2.1 Applications

Fanno flow occurs in many practical engineering applications of such flow includes

- Flow problems in aerospace propulsion system.
- Transport of fluids in a chemical process plants.
- Thermal and nuclear power plants.
- Petrochemical and gas industries.
- Various type of flow machineries.
- Air conditioning systems.
- High vacuum technology.
- Transport of natural gas in long pipe lines.
- Emptying of pressured container through a relatively short tube
- Exhaust system of an internal combustion engine
- Compressed air systems

When gases are transported through pipe over a long distances. It is also a practical importance when equipment handling gases are connected to high pressure reservoirs which may be located some distance away. Knowledge of this flow will allow us to determine the mass flow rate that can be candled, pressure drop etc…

In real flow, friction at the wall arises due to the viscosity of the fluid and this appears in the form of shear stress at the walls far in our discussion, we have assumed the fluid to be calorically perfect in viscid as well. Thus, strictly speaking, viscous effects cannot be accounted for in this formulation.However,in reality, viscous effects are confined to very thin region (boundary layer)near the walls. Effects such as viscous dissipation are also usually negligible.Hence, we can still assume the fluid to be inviscid and take the friction force exerted by the wall as an externally imposed force. The origin of this force is of significance to the analysis.

The following are the main assumptions employed for analyzing the frictional flow problem. in fanno flow

- One dimensional steady flow.
- Flow takes place in constant sectional area.
- There is no heat transfer or work exchange with the surroundings.
- The gas is perfect with constant specific heats.
- Body forces are negligible.
- Wall friction is a sole driving potential in the flow.
- There is no obstruction in the flow.
- There is no mass addition or rejection to or from the flow.

In thermodynamics coordinates, the fanno flow process can be described by a curve know as Fanno line and it's defined as the locus of the state which satisfies the continuity and energy and entropy equation for a frictional flow is known as "fanno line".

3.3 Fanno line or Fanno curve (Governing equation)

Flow in a constant area duct with friction and without heat transfers is described by a curve is known as Fanno line or Fanno curve

Weknow that,
From continuity equation,

$$m=\rho Ac$$

$$\frac{m}{A} = \rho c$$

$$G=mA=\rho c$$

Where

G- Mass flow density.
c- Velocity of sound.
ρ - Density of fluid.

$$G = \rho c$$

$$c = \frac{G}{\rho}$$

$$ho=h+12\times G2\rho 2$$

$$h=ho-12\times G2\rho 2$$

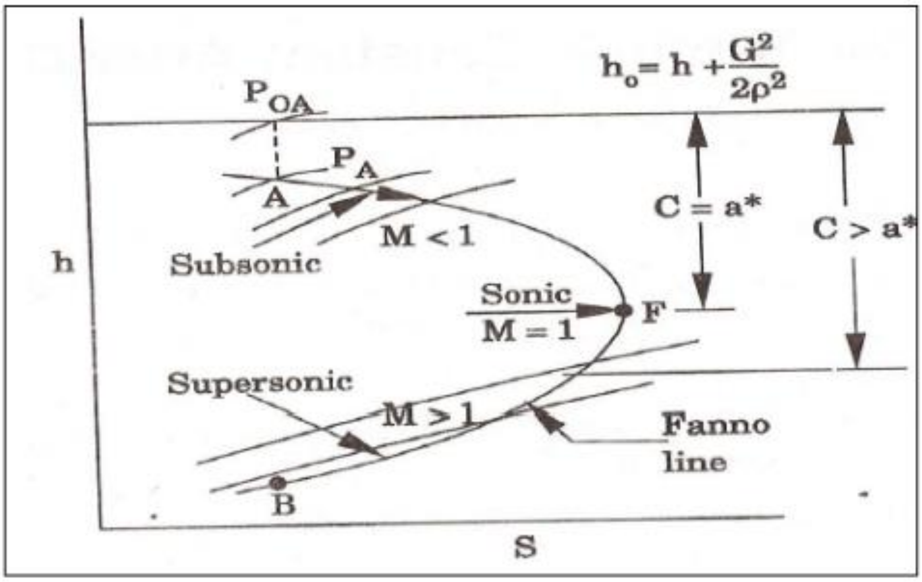
Density ρ is a function of entropy and enthalpy.

$\rho = f(s, h)$

Substitute the value for ρ in the equation for 'h'

We get, $h=ho-12*G22\{fs,h\}2$

The above equation can be used to show a fanno-line in h-s diagram.



In the line

- ➢ Point F is the sonic point
- ➢ Point lying below are super sonic points
- ➢ Points lying above are subsonic flow

Since entropy can only increase the processes that happen will always coverage to the sonic point F.The curve consists of two branches AF and FB. At point F the flow is sonic i,e, M=1

The flow A to F is subsonic (M<1) and B to F is Supersonic (M>1)

In subsonic flow region (A to F), the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas.

In supersonic flow region (B to F), the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas.

We know by the second law o thermodynamics that for an adiabatic flow, the entropy may increase but cannot decrease. So the processes in the direction F to A and F to B are not possible because they lead to decrease in entropy.

Fanno curves are drawn for different vales of mass flow density (G).When G increases, the velocity increases and pressure decreases in the sub sonic region. When G increases, the pressure increases and velocity decreases in the super sonic region

3.4 Important features of Fanno curve

- From the second law of thermodynamics, the entropy of the adiabatic flow increases but not decreases. Thus, the path of states along the Fanno curve must be toward the right.
- In the subsonic region, the effects of friction will be to increase the velocity and Mach number and to decrease the enthalpy and pressure of the stream .
- In the supersonic region, the effects of friction will be to decrease the velocity and Mach number and to increase the enthalpy and pressure of the stream.
- A subsonic flow can never become supersonic, due to the limitation of second law of thermodynamics, but in can approach to sonic i.e,M=1.
- A supersonic flow can never become subsonic, unless a discontinuity (shock)is present.
- In the case of isentropic stagnation, pressure is reduced whether the flow is subsonic or supersonic.

3.5 Chocking in Fanno flow

In a fanno flow, subsonic flow region, the effect of friction will increase the velocity and Mach number and to decrease the enthalpy and pressure of the gas. In supersonic flow region, the effect of friction will decrease the velocity and Mach number and to increase the enthalpy and pressure of the gas. In both cases entropy increases up to limiting state where the Mach number is one (M=1) and it is constant afterwards. At this point flow is said to be chocked flow.

3.6 Adiabatic Flow of a Compressible Fluid Through a Conduit

Flow through pipes in a typical plant where line lengths are short, or the pipe is well insulated can be considered adiabatic. A typical situation is a pipe into which gas enters at a given pressure and temperature and flows at a rate determined by the length and diameter of the pipe and downstream pressure. As the line gets longer friction losses increase and the following occurs:

- Pressure decreases
- Density decreases
- Velocity increases
- Enthalpy decreases
- Entropy increases

The question is “will the velocity continue to increasing until it crosses the sonic barrier?” The answer is NO. The maximum velocity always occurs at the end of the pipe and continues to increase as the pressure drops until reaching Mach 1. The velocity cannot cross the sonic barrier in adiabatic flow through a conduit of constant cross section. If an effort is made to decrease downstream pressure further, the velocity, pressure, temperature and density remain constant at the end of the pipe corresponding to Mach 1 conditions. The excess pressure drop is dissipated by shock waves at the pipe exit due to sudden expansion. If the line length is increased to drop the pressure further the mass flux decreases, so that Mach 1 is maintained at the end of the pipe.

The effects of friction on the properties of Fanno flow

Property	Subsonic	Supersonic
Velocity, V	Increase	Decrease
Mach number, Ma	Increase	Decrease
Stagnation temperature, T_0	Constant	Constant
Temperature, T	Decrease	Increase
Density, ρ	Decrease	Increase
Stagnation pressure, P_0	Decrease	Decrease
Pressure, P	Decrease	Increase
Entropy, s	Increase	Increase

The effect of friction in supersonic flow of the following parameters

a) velocity   b) pressure   c) temperature

$\frac{dc}{c}$ = 12 M2(1+ Υ−12M2) dM^2

Integrating between M=1 and M= M,

c∗cdcc = 1MdM22M2(1+Υ−12M2)

ln$\frac{c}{c^*}$ = lnM [Υ−12(1+Υ−12M2)]$^{1/2}$

$\frac{c}{c^*}$ = M [Υ+12(1+Υ−12M2)]$^{1/2}$

Applying this equation for section 1 and 2 of the duct

$\frac{c1}{c2}$ = M1M1[1+Υ−12M121+Υ−12M22]1/2

b) Pressure

p∗pdpp = - −1 M1+Υ−1M22M2+Υ−1M2 dM^2

lnp p∗ = ln$\frac{1}{M}$ [Υ+12(1+Υ−12M2)]

$\frac{p}{p^*}$ = 1M [Υ+12(1+Υ−12M2)]1/2

Applying this equation for section 1 and 2 of the duct

$\frac{p2}{p1}$ = $\frac{M1}{M2}$ [1+Υ−12M121+Υ−12M22]1/2

c) Temperature

T∗TdTT = -1MΥ−12(1+Υ−12M2) dM^2

lnTT∗ = ln Υ+12(1+Υ−12M2)

$\frac{T}{T^*}$ = Υ+12(1+Υ−12M2)

Applying this equation for section 1 and 2

$\frac{T2}{T1}$ = 1+Υ−12M121+Υ−12M22

2.7 Variation of flow properties

The flow properties (P,T,ρ,C) at M=M* =1are used as reference values for non-dimensionalizing various properties at any section of the duct.

Temperature

Stagnation temperature –Mach number relation

At critical state

M=1

$T_0 = T_0*$

$T = T^*$

For fanno flow T0= T0∗ constant

$$\Rightarrow \boxed{\frac{T}{T^*} = \frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}}$$

Applying this equation for section 1 and 2

$$\Rightarrow \frac{T_2}{T_1} = \frac{\frac{T_{01}}{T_1}}{\frac{T_{02}}{T_2}} = \frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_1^{\ 2}}$$

$$\Rightarrow \boxed{\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_1^{\ 2}}}$$

Velocity

Mach number M

$$M = \frac{c}{a}\ , c = M \times \sqrt{RT\gamma}$$

At critical state

M=1

C = C*

$T = T^*$

$$c^* = \sqrt{R\gamma T^*}$$

$$\Rightarrow \frac{C}{C^*} = \frac{M \times \sqrt{R\gamma T}}{\sqrt{R\gamma T^*}}$$

$$= M\left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= M\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{C}{C^*} = M\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$\frac{C_2}{C_1} = \frac{M_2 a_2}{M_1 a_1}$$

$$= \frac{M_2}{M_1} \times \frac{\sqrt{\gamma R T_2}}{\sqrt{\gamma R T_1}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{T}{T^*}\right]^{\frac{1}{2}}$$

$$= \frac{M_2}{M_1} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_1^{\,2}} \right]^{\frac{1}{2}}$$

$$\frac{C_2}{C_1} = \frac{M_2}{M_1} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^{\,2}}{1+\frac{\gamma-1}{2}M_2^{\,2}} \right]$$

Density

$$\rho = \frac{1}{c}$$

$$\frac{\rho}{\rho^*} = \frac{1}{\frac{c}{c^*}}$$

$$= \frac{1}{M\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)} \right]^{\frac{1}{2}}}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1} \right]^{\frac{1}{2}}$$

$$= \frac{1}{M}\left[\frac{2+(\gamma-1)M^2}{\gamma+1} \right]^{\frac{1}{2}}$$

$$= \left[\frac{2+(\gamma-1)M^2}{(\gamma+1)M^2} \right]^{\frac{1}{2}}$$

$$\frac{\rho^*}{\rho} = \left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2} \right]^{\frac{1}{2}}$$ Applying this equation for section 1 and 2

$$\frac{\rho_2}{\rho_1} = \frac{1}{\frac{C_2}{C_1}}$$

$$= \frac{1}{\frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1^2\right)}{\left(1+\frac{\gamma-1}{2}M2^2\right)}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{\rho_2}{\rho_1} = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{1}{2}}}$$

Pressure

Pressure= ρRT

M=1 Expressions for $\frac{T_0}{T*}, \frac{p_0}{p*}$ and $\frac{\rho_0}{\rho*}$

Considering the *section (where M = 1) and its stagnation section

We have

$M \Rightarrow 1$

$T \Rightarrow T*$

$p \Rightarrow p*$

$\rho \Rightarrow \rho*$

$$P^* = \rho R T^*$$

$$\frac{P}{P^*} = \frac{\rho R T}{\rho R T^*}$$

$$= \frac{\rho}{\rho^*} \times \frac{T}{T^*}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}} \times \left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]$$

$$\frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}\times\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}\times\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P}{P^*}=\frac{1}{M}\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{\frac{1}{2}}}$$

Applying this equation for section 1 and 2

$$=\frac{P_2}{P_1}=\frac{\rho_2 RT_2}{\rho_1 RT_1}$$

$$=\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{1}{2}}\times\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{2}}\times\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{2}}$$

$$\boxed{\frac{P_2}{P_1}=\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{2}}}$$

$$\frac{P_0}{P}=\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\boxed{P_0=P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}$$

Considering the *section (where M = 1) and its stagnation section

We have

$p_0^* = p_0$

$p = p^*$

$T_0^* = T$

$T = T^*$

$$p_0^* = P^*\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_0}{P_0^*} = \frac{P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}{P^*\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{P_0}{P_0^*} = \frac{P}{P^*}\frac{\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}{\left(\frac{T_o^*}{T^*}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{P}{P^*}\times\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}\times\left(\frac{T^*}{T_o^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P}{P^*}\times\left(\frac{T_o^*}{T}\right)^{\frac{\gamma}{\gamma-1}}\times\left(\frac{T^*}{T_0^*}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P}{P^*}\times\left(\frac{T^*}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M}\left[\frac{\gamma+1}{2\left(1+\frac{\gamma-1}{2}M^2\right)}\right]^{-\frac{1}{2}}\times\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{-\frac{1}{2}}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{-\frac{1}{2}+\frac{\gamma}{\gamma-1}}$$

$$P_{02} = P_1 \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_{01}} = \frac{P_2 \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}{P_1 \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$= \frac{P_2}{P_1} \times \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}} \times \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{P_2}{P_1} \times \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

Applying this equation for section 1 and 2

$$\boxed{P_0 = P\left(\frac{T_o}{T}\right)^{\frac{\gamma}{\gamma-1}}}$$

$$P_{02} = P_2 \left(\frac{T_{o2}}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{1}{2}} \times \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{-\gamma}{\gamma-1}}$$

$$= \frac{M_1}{M_2} \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma-1-2\gamma}{2(\gamma-1)}}$$

$$= \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}{M_1}^2}{1+\frac{\gamma-1}{2}{M_2}^2}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}$$

$$\boxed{\frac{P_{O2}}{P_{O1}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}{M_1}^2}{1+\frac{\gamma-1}{2}{M_2}^2}\right]^{\frac{(\gamma+1)}{2(\gamma-1)}}}$$

Impulse Function

Impulse function =PA (1+ γ M^2)

Considering the *section (where M = 1) and its stagnation section

We have

$F = F^*$

$$\frac{F}{F^*} = \frac{PA\left(1+\gamma M^2\right)}{P^* A^* (1+\gamma)}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{(1+\gamma)}}{\frac{P^*}{P}}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{(1+\gamma)}}{\frac{P^*}{P}}$$

$$= \frac{\frac{\left(1+\gamma M^2\right)}{(1+\gamma)}}{M\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$= \frac{\left(1+\gamma M^2\right)}{M(1+\gamma)\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}$$

$$\boxed{\frac{F}{F^*} = \frac{\left(1+\gamma M^2\right)}{M(1+\gamma)\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}}}$$

Applying this equation for section 1 and 2

We know that

$$\frac{F_2}{F_1}=\frac{P_2A_2}{P_1A_1}\times\frac{\left(1+\gamma M_2^{\ 2}\right)}{\left(1+\gamma M_1^{\ 2}\right)}$$

$$=\frac{P_2}{P_1}\times\frac{\left(1+\gamma M_2^{\ 2}\right)}{\left(1+\gamma M_1^{\ 2}\right)}$$

$$=\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right]^{\frac{1}{2}}\times\frac{\left(1+\gamma M_2^{\ 2}\right)}{\left(1+\gamma M_1^{\ 2}\right)}$$

$$\frac{F_2}{F_1}=\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^{\ 2}}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right]^{\frac{1}{2}}\times\frac{\left(1+\gamma M_2^{\ 2}\right)}{\left(1+\gamma M_1^{\ 2}\right)}$$

$$\frac{s-s^*}{R}=-\ln\left[\frac{p_0}{P_0^{\ *}}\right]$$

$$=-\ln\left[\frac{p_0}{P_0^{\ *}}\right]\times\frac{1}{M}\left[\frac{2\left(1+\frac{\gamma-1}{2}M^2\right)}{\gamma+1}\right]^{\frac{1}{2}}$$

Changing entropy for section 1 and 2

$$\frac{S_2-S_1}{R}=-\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\boxed{\frac{S_2-S_1}{R}=\ln\frac{M_2}{M_1}\left[\frac{\left(1+\frac{\gamma-1}{2}M_1^{\ 2}\right)}{1+\frac{\gamma-1}{2}M_2^{\ 2}}\right]^{\frac{(\gamma+1)}{2(\gamma+1)}}}$$

3.8 Variation of Mach number with duct length

The duct length required for the flow to pass from a given initial mach number M1 to a given final mach number m2can be obtained from the following expression.

Mean friction coefficient with respect to duct length is given by

$$\bar{f} = \frac{1}{L_{MAX}} \int_0^{L_{MAX}} \frac{fdx}{fdx}$$

from

$$4\bar{f}\frac{dx}{D} = \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

integrating limits are x=0 to x= L_{MAX}

M=M to M=1

$$\int_0^{L_{MAX}} 4\bar{f}\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma M^2\left(1+\frac{\gamma-1}{2}\right)}\frac{dM^2}{M^2}$$

$$4\bar{f}\frac{dx}{D} = \int_0^1 \frac{1-M^2}{\gamma\left(1+\frac{\gamma-1}{2}\right)}\frac{1}{M^4}dM$$

$$= 4\bar{f}\frac{dx}{D} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\frac{1}{M^4}\ln\frac{(\gamma+1)M^2}{2\left(1+\frac{\gamma-1}{2}M^2\right)}$$

The distance (L) between two section of duct where the ach numbers M1 &M2 are given by

$$\boxed{4\bar{f}\frac{L_{Max}}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_1} - \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_2}}$$

3.9 Problems

1. Air flows through a pipe of 300 mm diameter. At inlet temperature is 35 ^{0}C, pressure is 0.6 bar and stagnation pressure is 12 bar.At a location 2 m down stream, the static pressure is 0.89 bar.Estimate the average friction coefficient between two section.

Given data:

D =300 mm; T1 = 35 ^{0}C P_{o1}= 12 bar ; P2 =12 bar

$$\frac{P_1}{P_{O1}} = \frac{0.6}{12} = 0.05$$

From Fanno table

$$[M]_{\frac{P_1}{P_{O1}}} = 2.6$$

$$\left[4f\frac{L_{Max}}{D}\right]_{M=2.6_1} = 0.453$$

To find M_2

$$\frac{P_2}{P_1} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{\left(\frac{P_2}{P^*}\right)_{M_1=2.6}}$$

$$\frac{0.89}{0.62} = \frac{\left(\frac{P_2}{P^*}\right)_{M_2}}{0.275}$$

$$\left(\frac{P_2}{P^*}\right)_{M_2} = 0.275 \times \frac{0.89}{0.62} = 0.4079$$

$$[M_2]_{0.479} = 2$$

$$\left[4f\frac{L_{Max}}{D}\right]_{M=2} = 0.305$$

$$\boxed{4f\frac{L_{Max}}{D} = \left[4f\frac{L_{Max}}{D}\right]_{M1=2.6} - \left[4f\frac{L_{Max}}{D}\right]_{M_2=2}}$$

$$= \left[4f\frac{L_{Max}}{D}\right]_{M1=2.6} - \left[4f\frac{L_{Max}}{D}\right]_{M_2=2} = 0.453 - 0.305 = 0.148$$

$$\left[4f\frac{2}{D}\right]_{M=2} = 0.148$$

$$f = \frac{0.148\times 3}{8} = 5.5\times 10^{-3}$$

2 A circular duct passes 8.25 Kg / S of air at an exit Mach number of 0.5. The entry pressure and temperature are 345 KPa and 38 ^{0}C respectively and the co – efficient of friction 0.005. If the Mach number at entry is 0.15, determine,

(i) The diameter of the duct,
(ii) Length of the duct,
(iii) Pressure and temperature at exit and

(iv) Stagnation pressure loss.

Given Data:
m = 8.25 Kg / S ; M_2 = 0.5 ; P_1 = 345 KPa ; T_1 = 311K ; f =0.005 ;

M_1= 0.15

(i) **Diameter of the pipe**

$$M = \rho_1 A_1 C_1$$

$$= \frac{P_1}{RT_1} A_1 M_1 \sqrt{\gamma R T_1}$$

$$m = \frac{P_1 A_1 \sqrt{\gamma} M_1}{\sqrt{RT_1}}$$

$$A_1 = \frac{m\sqrt{RT_1}}{\sqrt{\gamma} M_1 P_1} = \frac{8.25\times\sqrt{287\times 311}}{345\times 10^3\times 0.15\sqrt{1.4}}$$

$$= 0.040253m$$

$$A = \frac{\pi}{4}d^2$$

$$d = 0.226389$$

(ii) Length of the pipe
From Isentropic table M_1 = 0.15,γ =1.4

$$\frac{4fL}{D} = \left[\frac{4fL_{Max}}{D}\right]_{M1} - \left[\frac{4fL_{Max}}{D}\right]_{M_2}$$

=28.354-1.069

$$\frac{40.005 \times L}{0.226389} = 27.285$$
$$L = 308.85m$$

Pressure and temperature at the exit

$$\left(\frac{P_2}{P_1}\right) = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} = \frac{2.138}{7.3193}$$

$$\left(\frac{P_2}{P_1}\right) = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} = \frac{2.138}{7.3193}$$

$$= 100.773 \text{ KPa}$$

$$T_2 = \frac{T_2 T_1^*}{T_2 T_1^*} \times T_1$$
$$= \frac{1.43}{1.1945} \times 311 = 297.59\text{K}$$

$$P_{02} = \frac{P_{02}}{P_{02}*} \times \frac{P_{01}}{P_{01}*} P_{01}$$
$$= \frac{1.34}{3.928} \times 350.609 = 231.009 \text{ кPA}$$

$$P_2 = \frac{\left(\frac{p_2}{p^*}\right)_{M2}}{\left(\frac{p_1}{p^*}\right)_{M1}} \times P_1 = \frac{2.629}{4.3615} \times 700$$
$$T_2 = \frac{1.161}{1.185} \times 333 = 326.2557\text{K}$$

$$C_2 = \frac{0.4415}{0.272} \times 0.25 \times \sqrt{1.4 \times 287 \times 333}$$
$$= 148.432\text{m/sec}$$

$$\frac{T}{T*} = 1.185;$$

Question – 3 :- Air is flowing in an insulated duct with a Mach number of M1 = 0.25. At a section downstream the entropy is greater by amount 0.124 units, as a result of friction. What is the Mach number of this section? The static properties at inlet are 700KPa and 60^0C. Find velocity, temperature and pressure at exit. Fin d the properties at the critical section.

Given Data: M1 = 0.25; (S2 – S1) = 0.124 KJ/ Kg K; P1 = 700 KPa;
T1 = 60 +273 = 333K
We know that,

$$\frac{S_2 - S_1}{R} = -\ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\frac{S_2 - S_1}{R} = \ln\left[\frac{p_{01}}{P_{02}}\right]$$

$$\ln\left[\frac{p_{01}}{P_{02}}\right] = \frac{0.124}{0.287} = 0.4320557$$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

From isentropic table $M_1 = 0.25, \gamma = 1.4$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$$\frac{T_1}{T_{01}} = 0.987 : \frac{p_1}{P_{01}} = 0.957$$

$$T_{01} = 337.386k : P_{01} = 731.452kpa$$

From fanno table $\gamma = 1.4$

$$\frac{p_{01}}{P_{02}} = 1.540421$$

$$\frac{\frac{p_{01}}{P_0{}^*}}{\frac{p_{02}}{P_0{}^*}} = 1.540421$$

$$\frac{p_{02}}{P_{01}} = \frac{2.4065}{1.540421} = 1.5622$$

From fanno table $\frac{p_0}{P_0 *}$ =1.5622, the corresponding Mach number M_2=0.41

b) Velocity ,pressure and Temperature at the exit section

$$T^* = \frac{T_1}{1.185} = \frac{333}{1.185}$$

$$T^* = 281.01266K$$

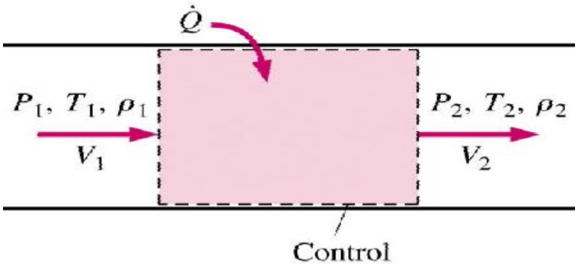
$$\frac{P}{P^*} = 4.3615$$

$$P^* = 160.495Ka$$

$$C^* = \sqrt{\gamma RT} = 336.022m/\sec$$

3.10 Rayleigh Flow (Duct Flow with Heat Transfer and Negligible Friction)

Flow in a constant area duct with heat transfer and without friction is known as Rayleigh flow . Many compressible flow problems encountered in practice involve chemical reactions such as combustion, nuclear reactions, evaporation, and condensation as well as heat gain or heat loss through the duct wall Such problems are difficult to analyze Essential features of such complex flows can be captured by a simple analysis method where generation/absorption is modeled as heat transfer through the wall at the same rate



In certain engineering processes, heat is added either by external sources across the system boundary by heat exchangers or internally by chemical reactions in a combustion chamber. Such process are not truely adiabatic, they are called adiabatic processes.

- **Applications**

The combustion chambers inside turbojet engines usually have a constant area and the fuel mass addition is negligible. These properties make the Rayleigh flow model applicable for heat addition to the flow through combustion, assuming the heat addition does not result in dissociation of the air-fuel mixture. Producing a shock wave inside the combustion chamber of an engine due to thermal choking is very undesirable due to the decrease in mass flow rate and thrust. Therefore, the Rayleigh flow model is critical for an initial design of the duct geometry and combustion temperature for an engine.

The Rayleigh flow model is also used extensively with the Fanno flow model. These two models intersect at points on the enthalpy-entropy and Mach number-entropy diagrams, which is meaningful for many applications. However, the entropy values for each model are not equal at the sonic state. The change in entropy is 0 at M = 1 for each model, but the previous statement means the change in entropy from the same arbitrary point to the sonic point is different for the Fanno and Rayleigh flow models.

- Combustion processes.
- Regenerator,
- Heat exchangers.
- Inter coolers.

The following are the assumptions that are made for analyzing the such flow problem.

- One dimensional steady flow.
- Flow takes place in constant area duct.
- The frictional effects are negligible compared to heat transfer effects..
- The gas is perfect.
- Body forces are negligible.
- There is no external shaft work.
- There is no obstruction in the flow.
- There is no mass addition or rejection during the flow.
- The composition of the gas doesn't change appreciably during the flow.

3.11 Rayleigh line (or) curve

The frictionless flow of a perfect gas through a constant area duct in which heat transfer to or from the gas is the dominant factor bringing about changes in the flow is referred to as Rayleigh flow or diabetic flow. In thermodynamic coordinates, the Rayleigh flow process can be described by a curve known as Rayleigh line and is defined as the locus of quasi- static thermodynamic state points traced during the flow. The Rayleigh line satisfies the equation of state along with simple forms of continuity and momentum equation.

3.12 Governing Equations

In order to formulate the equation for the Rayleigh line, let us consider steady flow of a perfect gas through a constant area passage in which transfer of heat with the surroundings is the major factor responsible for changes in fluid properties. The simple form of continuity equation for steady one dimensional flow in a constant area duct is

$$m = \rho A c$$

$$\frac{m}{A} = \rho c$$

$$G = mA = \rho c$$.

Where

G- Mass flow density.
c- Velocity of sound.
- ρ Density of fluid.

$$G = \rho c$$

$$c = \frac{G}{\rho}$$

Momentum equation is given by

$$p + \rho c^2 = cont.$$

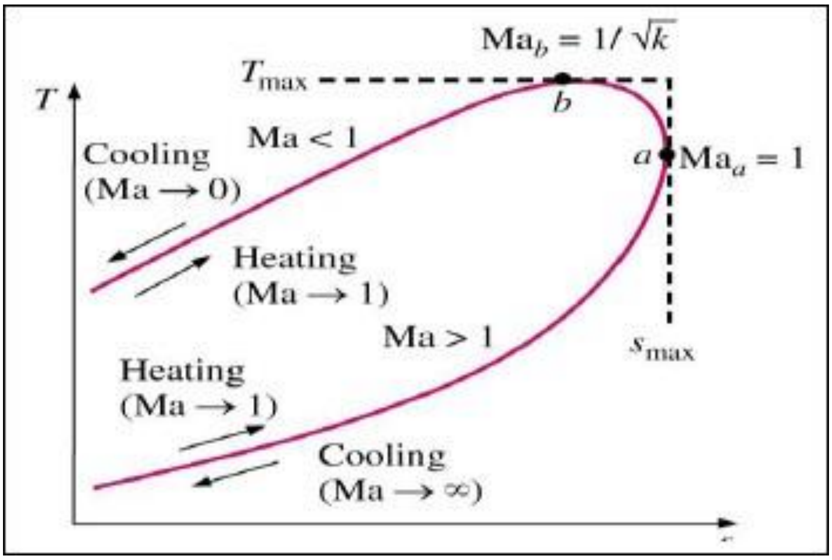
substitute

$$c = \frac{G}{\rho}$$

$$p + \frac{G^2}{\rho^2} = C$$

$p + G^2 v = c$ (Specific volume, v=1/ρ) -.....................................1

Equation 1 may be used for representing Rayleigh line on the h- s diagram, as illustrated in fig shown in below. In general, most of the fluids in practical use have Rayleigh curves of the general form shown in fig.



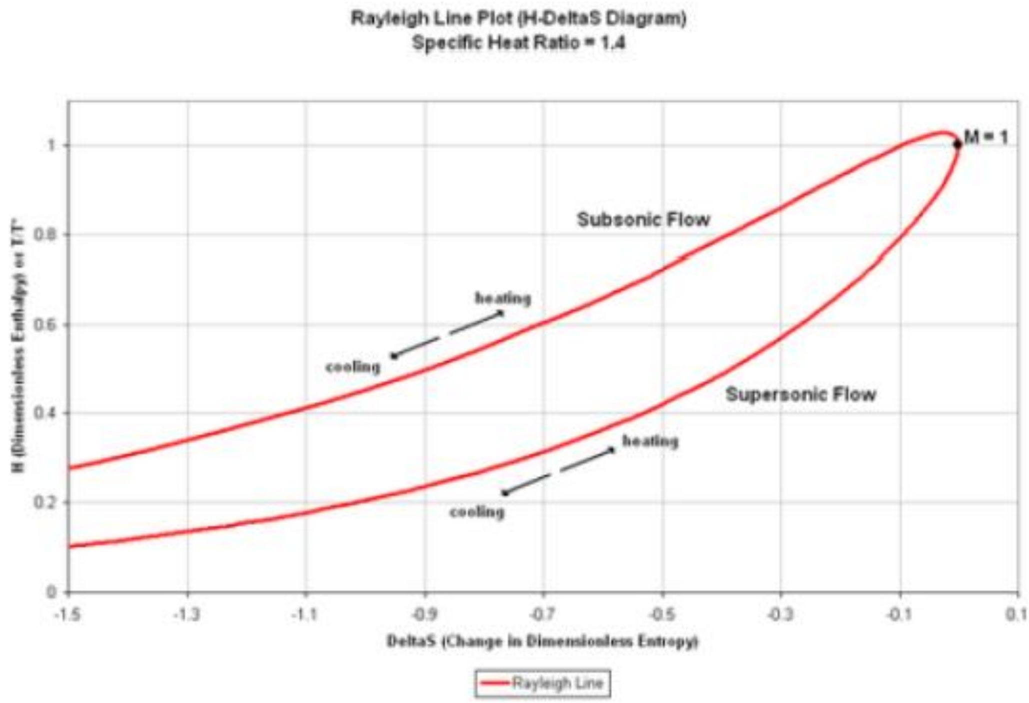
The portion of the Rayleigh curve above the point of maximum entropy usually represents subsonic flow (M<1) and the portion below the maximum entropy point represents supersonic flow (M>1).

An entropy increases due to heat addition and entropy decreases due to heat rejection. Therefore, the Mach number is increased by heating and decreased by cooling at subsonic speeds. On the other hand, the Mach number is decreased by heating and increased by cooling at supersonic speeds. Therefore, like friction, heat addition also tends to make the Mach number in the duct approach unity. Cooling causes the Mach number to change in the direction away from unity.

Rayleigh Flow

Rayleigh flow refers to adiabetic flow through a constant area duct where the effect of heat addition or rejection is considered. Compressibility effects often come into consideration, although theRayleigh flow model certainly also applies to incompressible flow. For this model, the duct area remains constant and no mass is added within the duct. Therefore, unlike Fanno flow, the stagnation temperature is a variable. The heat addition causes a decrease in stagnation pressure which is known as the Rayleigh effect and is critical in the design of combustion systems. Heat addition will cause both supersonic and subsonic Mach numbers to approach Mach 1, resulting in choked flow. Conversely, heat rejection decreases a subsonic Mach number and increases a supersonic Mach number along the duct. It can be shown that for calorically perfect flows the maximum entropy occurs at M = 1. Rayleigh flow is named after John Strutt, 3rd Baron Rayleigh.

Theory



A Rayleigh Line is plotted on the dimensionless H-ΔS axis.

The Rayleigh flow model begins with a differential equation that relates the change in Mach number with the change in stagnation temperature, T_0. The differential equation is shown below.

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2}{1-M^2}\left(1+\frac{\gamma-1}{2}M^2\right)\frac{dT_0}{T_0}$$

Solving the differential equation leads to the relation shown below, where $T0^*$ is the stagnation temperature at the throat location of the duct which is required for thermally choking the flow.

$$\frac{T_0}{T_0^*} = \frac{2\,(\gamma+1)\,M^2}{(1+\gamma M^2)^2}\left(1+\frac{\gamma-1}{2}M^2\right)$$

3.13 Fundamental Equations

The following fundamental equations will be used to determine the variation of flow parameters in Rayleigh flows.

Continuity equation

We know that

Mass flow rate,

$$m = \rho_1 A_1 C_1 = \rho_2 A_2 C_2$$

For constant area duct $A_1 = A_2$

$$m = \rho_1 C_1 = \rho_2 C_2$$

$$\rho_1 C_1 = \rho_2 C_2$$

$$\frac{C_1}{C_2} = \frac{\rho_2}{\rho_1}$$

Where ;-

C1 –Velocity of fluid at inlet-m/s

C2 –Velocity of fluid at outlet-m/s

ρ_1 - Density of fluid at inlet-kg/m3

ρ_2 - Density of fluid at out let-kg/m3

Momentum equation

Momentum equation between state 1nad 2 is given by

$$p_1 A + mc_1 = p_2 A + mc_2$$

$$P_1 A - P_2 A = mc_2 - mc_1$$

$$(P_1 - P_2) A = m (c_2 - c_1)$$

Mass flow rate; $m = \rho A c$

$$(P_1 - P_2) A = \rho A C (c_2 - c_1)$$

$$= \rho_2 A C^2{}_2 - \rho_1 A C^2{}_1$$

$$(P_1 - P_2) A = \rho_2 A C^2{}_2 - \rho_1 A C^2{}_1$$

$$(P_1 - P_2) = \rho_2 C^2{}_2 - \rho_1 C^2{}_1$$

$$\left[\rho = \frac{P}{RT}\right]$$

$$= \frac{P_2}{RT_2} \times M^2{}_2 \times a_2{}^2 - \frac{P_1}{RT_1} \times M^2{}_1 \times a_1{}^2$$

$$= \frac{P_2 \times M^2{}_2 \times \sqrt{\gamma RT_2}}{RT_2} - \frac{P_1 \times M^2{}_1 \times \sqrt{\gamma RT_1}}{RT_1}$$

$$(P_1 - P_2) = P_2 M^2{}_2 \gamma - P_1 M^2{}_1 \gamma$$

$$P_1 + P_1 M^2{}_1 \gamma = P_2 + P_2 M^2{}_2 \gamma$$

$$P_1\left[1 + M^2{}_1 \gamma\right] = P_2\left[1 + M^2{}_2 \gamma\right]$$

$$\boxed{\frac{P_2}{P_1} = \frac{\left[1 + M^2{}_1 \gamma\right]}{\left[1 + M^2{}_2 \gamma\right]}}$$

- **Mach Number**

The Mach number at the two states are

$$= \frac{C_2}{C_1} \times \frac{a_1}{a_2}$$

$$\frac{C_2}{C_1} \times \frac{\sqrt{\gamma RT_1}}{\sqrt{\gamma RT_2}}$$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^{\frac{1}{2}}$$

- **Impulse Fuction**

$$F = P\left[1 + M^2 \gamma\right]$$

$$F_1 = P_1\left[1 + M_1{}^2 \gamma\right]$$

$$F_2 = P_2\left[1 + M_2{}^2 \gamma\right]$$

$$\frac{F_2}{F_1} = \frac{1 + M_2{}^2 \gamma}{1 + M_1{}^2 \gamma} \times \frac{P_2}{P_1}$$

$$\frac{P_2}{P_1} = \frac{1+M_1^2\gamma}{1+M_2^2\gamma}$$

$$\frac{F_2}{F_1} = \frac{P_1}{P_2} \times \frac{P_2}{P_1}$$

$$\frac{F_2}{F_1} = 1$$

Stagnation Pressue

Stagnation pressure-Mach number relation is given by

$$\frac{P_{01}}{P_1} = \left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = \left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{02}}{P_2} = \left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\frac{P_{02}}{P_2}}{\frac{P_{01}}{P_1}} = \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\frac{P_{02}}{P_2} \times \frac{P_1}{P_2} = \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

$$\boxed{\frac{P_{02}}{P_2} = \frac{P_2}{P_1} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}}$$

$$\frac{P_{02}}{P_2} = \frac{1+M_1^2}{1+M_2^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1+\frac{\gamma-1}{2}M_1^2\right]^{\frac{\gamma}{\gamma-1}}}$$

Static Temperature

ROM EQN. $\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$

$$\frac{M_2}{M_1} = \frac{C_2}{C_1} \times \left[\frac{T_1}{T_2}\right]^2$$

WE KNOW THAT

$$\rho_1 = \frac{P_1}{RT_1} \qquad \rho_2 = \frac{P_2}{RT_2}$$

$$\frac{\rho_2}{\rho_1} = \frac{\frac{P_2}{RT_2}}{\frac{P_1}{RT_1}}$$

$$\frac{\rho_2}{\rho_1} = \frac{P_2}{P_1} \times \frac{T_1}{T_2}$$

$$\frac{T_1}{T_2} = \frac{\rho_2}{\rho_1} \times \frac{P_1}{P_2}$$

$$\frac{T_1}{T_2} = \frac{C_1}{C_2} \times \frac{P_1}{P_2}$$

$$\frac{C_1}{C_2} = \frac{P_2}{P_1} \times \frac{T_1}{T_2} \qquad \left[\frac{\rho_2}{\rho_1} = \frac{C_1}{C_2}\right]$$

$$\frac{T_1}{T_2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1} \times \frac{T_1}{T_2}\right]^2$$

$$\frac{T_1}{T_2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2 \times \frac{T_1^2}{T_2^2}$$

$$\frac{T_1}{T_2} \times \frac{T_1^2}{T_2^2} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \times \frac{P_2}{P_1}\right]^2$$

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1} \times \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right] \qquad \left[\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]$$

$$\frac{T_2}{T_1} = \frac{M_2}{M_1} \times \left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2$$

Stagnation Temperature

Stagnation Temperature – Ach Number Relation is Given by

$$\frac{T_0}{T} = \left[1 + \frac{\gamma - 1}{2} M_2^2\right]$$

$$\frac{T_{01}}{T_1} = \left[1 + \frac{\gamma - 1}{2} M_1^2\right]$$

$$\frac{T_{02}}{T_2} = \left[1 + \frac{\gamma - 1}{2} M_2^2\right]$$

$$\frac{\frac{T_{02}}{T_2}}{\frac{T_{01}}{T_1}} = \frac{\left[1 + \frac{\gamma - 1}{2} M_2^2\right]}{\left[1 + \frac{\gamma - 1}{2} M_1^2\right]}$$

$$\frac{T_{02}}{T_{01}} = \frac{M_2^2}{M_1^2} \times \frac{\left(1+\gamma M_1^2\right)^2}{\left(1+\gamma M_2^2\right)^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\frac{\gamma-1}{2}M_1^2\right]}$$

$$\left[\frac{T_2}{T_1} = \frac{M_2^2}{M_1^2} \times \frac{\left(1+\gamma M_1^2\right)^2}{\left(1+\gamma M_2^2\right)^2}\right]$$

$$\boxed{\frac{T_{02}}{T_{01}} = \frac{M_2^2}{M_1^2} \times \frac{\left(1+\gamma M_1^2\right)^2}{\left(1+\gamma M_2^2\right)^2} \times \frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\frac{\gamma-1}{2}M_1^2\right]}}$$

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] - C_p \ln\left[\frac{P_2}{P_1}\right]^{\frac{\gamma+1}{\gamma}}$$

$$S_2 - S_1 = C_p \ln\left[\frac{\left[\frac{M_2}{M_1} \times \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2}{\left[\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^{\frac{\gamma+1}{\gamma}}}\right]$$

$$S_2 - S_1 = C_p \ln\left[\left(\frac{M_2^2}{M_1^2}\right) \times \left(\frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right)^{\left(\frac{\gamma+1}{\gamma}\right)}\right]$$

- **Heat Transfer**

We have

$$Q = mc_p \left(T_{02} - T_{01}\right)$$

$$Q = mc_p T_{01}\left(\frac{T_{02}}{T_{01}} - 1\right)$$

÷ BY c_p T1

$$\frac{Q}{c_pT_1}=\frac{T_{01}}{T_1}\left(\frac{T_{02}}{T_{01}}-1\right) = =\left[1+\frac{\gamma-1}{2}M_1^2\right]\left[\frac{T_2}{T_1}\frac{\left[1+\frac{\gamma-1}{2}M_2^2\right]}{\left[1+\frac{\gamma-1}{2}M_1^2\right]}\right]$$

$$\frac{Q}{c_pT_1}=\frac{(M_2^2-M_1^2)\left(2-2\gamma M_2^2M_1^2+[\gamma-1](M_2^2+M_1^2)\right)}{2M_1^2(1+\gamma M_2^2)^2}$$

Expression for Heat Transfer

we have

$Q = mc_p(T_{02}-T_{01})$

the condition 1 is fixed but the value of T_{02} attains its maximum when $T_{02} = T_0*$

3.14 Problems based on Rayleigh flow

1.The condition of gas in a combustion chamber at entry are M1=0.28, T01=380 K, P01=4.9 bar. The heat supplied in the combustion chamber is 620 kJ/kg.Determine Mach number, pressure and temperature of the gas at exit and also determine the stagnation pressure loss during heating. Take γ = 1.3, cp=1.22 kJ/Kg K.

Given,

$M_1 = 0.28$, $T_{01} = 380$ K,

$P_{01} = 4.9$ bar $= 4.9\times10^5$ N/m^2

Q = 620 kJ/kg = 620×10^3 J/kg

Take γ = 1.3, c_p=1.22 kJ/Kg K.= 1.22×10^3 J/kg K

To find

1. Mach number, pressure and temperature of the gas at exit,(M_2,P_2 and T_2)
2. Stagnation pressure loss (Δp_0)

Solution

Refer Isentropic flow table for γ=1.3 and M_1=0.28

$\frac{T_2}{T_{01}} = 0.988$ [From gas table]

$\frac{P_2}{P_{02}} = 0.951$

$P_1 = P_{01}\times0.951$

$= 4.9\times10^5\times0.951$

$P_1 = 4.659 \times 10^5$ N/m²

$T_1 = T_{01} \times 0.988$

$= 380 \times 0.988$

$T_1 = 375.44$ K

Refer Rayleigh flow table for $\gamma = 1.3$ and $M_1 = 0.28$

$$\frac{T_1}{T^*_1} = 0.342$$

$$\frac{T_{01}}{T^*_{01}} = 0.300$$

$$P^* = \frac{P_1}{2.087} = \frac{4.659 \times 10^5}{2.087} = 2.23 \times 10^5\, N/m^2$$
$$= 2.23 \times 10^5\, N/m^2$$

$$P^*_1 = P^*_2$$
$$P^*_{01} = \frac{P_{01}}{1.198}$$

$$T^*_1 = \frac{T_1}{0.342} = \frac{375.44}{0.342} = 1097.77K = T^*_2$$

$$T^*_{01} = \frac{T_{01}}{0.300} = \frac{380}{0.300} = 1266.6K = T^*_{02}$$

$$P^*_{01} = 4.09 \times 10^5\, N/m^2 = P^*_{02}$$

$Q = mc_p\,(T_{02} - T_{01})$

For unit mass

$Q = c_p\,(T_{02} - T_{01})$

$620 \times 10^3 = 1.22 \times 10^3\, [T_{02} - 380]$

$[T_{02}-380]=508.19$

$T_{02}=888.19$ K

$$\frac{T_{02}}{T^*_{02}}=\frac{888.19}{1266.6}=0.701$$

Refer Rayleigh flow table for $\gamma = 1.3$ and M2=0.52

[From gas table]

$M_2=0.52$

$$\frac{P_2}{P^*_2}=1.702$$

$$\frac{T_2}{T^*_2}=0.783$$

$$\frac{P_{02}}{P^*_{02}}=1.702$$

[**Note**: Mach no $M_1 < 1$,so $M_2 < 1$]

$P_2 = P^*_2 \times 1.702$

$= 2.23\times10^5\times1.702$

$P_2=3.79 \times10^5$ N/m^2

$T_2=T_2^* \times 0.783$

$=1097.77\times0.783$

$T_2=859.55$ K

$P_{02} = p_{02}^* \times 1.103$

$= 4.09\times10^5\times1.103$

$P_{02} = 4.511\times10^5$ N/m^2

Stagnation pressure loss

$\Delta p_0 = p_{01}-p_{02}$

$= 4.9\times10^5- 4.511\times10^5$

$\Delta p_0=0.389\times10^5$ N/m^2

Result,

1) $M_2=0.52$, $P_2=3.79 \times10^5$ $T_2=859.55$ K

2) $\Delta p_0=0.389 \times10^5$ N/m^2

2.A gas (γ=1.3 and R = 0.46 KJ / Kg K) at a pressure of 70 Kpa and temperature of 295 K enters a combustion chamber at a velocity of 75 m / sec. The heat supplied in a combustion chamber is 1250 KJ / Kg .Determine the Mach number, pressure and temperature of gas at exit.

Given: γ = 1.3: R = 0.46 KJ / Kg K $^{-1}$= 70 Kpa : T1 =295 K
C1 = 75 m/sec: Q = 1250 KJ / Kg

$$C_P = \frac{\gamma}{\gamma - 1} \quad = \ 1.999333 \ \text{KJ / Kg K}$$

$$M_1 = \frac{C_1}{\sqrt{\gamma R T_1}} \ \frac{75}{\sqrt{1.3 \times 460 \times 295}} = 01.785 \ \approx \ 0.18$$

From isentropic table M_1 = 0.18 , γ=1.3.

$$\frac{T_1}{T_0} = 0.995 : \qquad \frac{P}{P_0} = 0.979$$

$$T_{01} = 296.4824 K \qquad P_{01} = 71.5015 \ K pa$$

From Rayleigh table M1 = 0.18 , γ=1.3.

$$\frac{P}{P^*} = 2.207 \qquad \frac{P_0}{P^*_0} = 1.23 \quad \frac{T}{T^*} = 0.138$$

$$T_0{}^* = \frac{T_{01}}{0.138} = 2148.4231 K$$

We know that

Heat transfer

$$T_0{}^* = \frac{T_{01}}{0.138} = 2148.4231 K$$

$$Q = C_P \ (T_{01} - T_{01})$$

$$T_{02} = \frac{Q}{C_p} + T_{01} = \frac{1250}{1.92333} + 296.4824$$

$$= 923.5727\,K$$

From Rayleigh table corresponding to $M_2 = 0.35$

$$\frac{P}{P^*} = 1.9835 \qquad \frac{T}{T^*} = 0.482$$

$$P_2 = \frac{P_2}{P^*} \times \frac{P^*}{P_1} \times P_1$$

$$= \frac{1.9835}{2.207} \times 70 \quad = 62.911\,Kpa$$

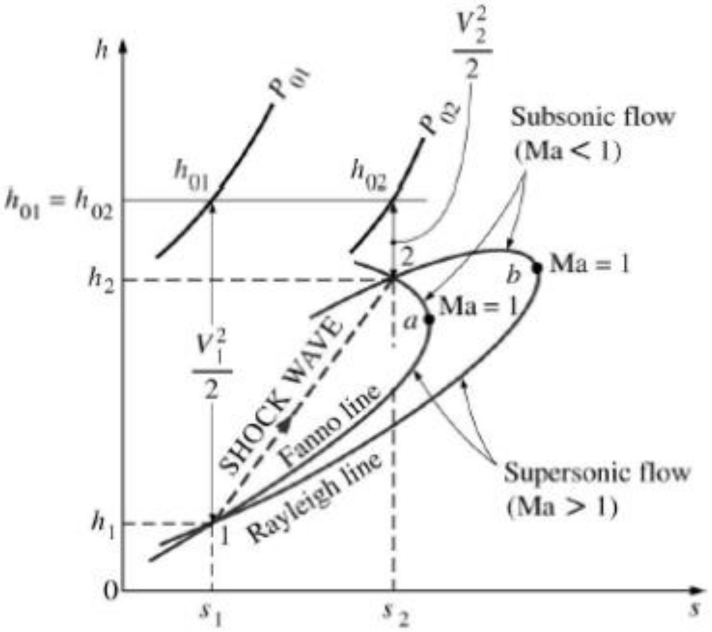
$$T_2 = \frac{0.4802}{0.158} \times 295 \quad = 900\,K$$

Result:

1. Mach number at the exit $M_2 = 0.35$
2. Pressure of the gas at the exit $P_2 = 62.99111$Kpa
3. Temperature of the gas at the exit $T_2 = 900$ K

3.15 Intersection of Fanno and a Rayleigh Line

Fanno and Rayleigh line, when plotted on h-s plane, for same mass velocity G, intersect at 1 and 2.as shown in fig. All states of Fanno line have same stagnation temperature or stagnation enthalpy, and all states of Rayleigh line have same stream thrust F / A. Therefore, 1 and 2 have identical values of G, h_0 and F / A. from 1 to 2 possible by a compression shock wave without violating Second Law Thermodynamics. A shock is a sudden compression which increases the pressure and entropy of the fluid but the velocity is decrease from supersonic to subsonic.



A change from states 2 to 1 from subsonic to supersonic flow is not possible in view Second Law Thermodynamics. (Entropy can not decrease in a flow process)

3.The pressure temperature and Mach no. of air in combustion chamber are 4 bar,100°C and 0.2 respectively. The stagnation temperature of air in combustion chamber is increased 3 times the initial value. Calculate:

1. The mach no., pressure and temperature at exit.
2. Stagnation pressure
3. Heat supplied per Kg of air

Solution

Refer isentropic flow table for γ=1.4and m1=.2

$\frac{T_1}{T_{01}} = 0.992$

$\frac{P_1}{P_{01}} = 0.973$

$= T_{01} = \frac{T_1}{0.992} = \frac{373}{0.992} = 376K$

$T_{01} = 376K$

$P_{01} = \frac{P_1}{0.973} = \frac{0.4\times10^5}{0.973}. = 4.1\times10^5 N/m^2$

$P_{01} = \frac{P_1}{0.973}$

$T_{01} = \frac{T_1}{0.992}$

$P_{01} = 4.1x10^5 N/m^2$

From Rayleigh's flow table for γ=1.4and M_1=.2

$$\frac{P_{01}}{P_{01}*} = 1.235$$

$$\frac{T_1}{T*_1} = 0.207$$

$$P_{1*} = \frac{P_1}{2.273} = 1.795 \text{ x}10^{\,5}\text{N/m2}$$

$$P_{01*} = \frac{P_{01}}{1.235} = \frac{4.11 \text{ x}10^{\,5}}{1.235} = 3.327\text{x}10^{\,5} = N/m^2$$

$$T_1* = \frac{T_1}{0.207}$$

$$= \frac{373}{0.273}$$

$$= 1801.93 = T_2*$$

$$T*_{01} = \frac{T_{01}}{0.174} = 2160.91 = T_{02}*$$

From given data we know that

$$T_{02} = 3 \times T_{01}$$

$$= 3 \text{ x } 376$$

$$= 1128 \text{ K}$$

From Rayleigh's flow table for $\gamma=1.4$and $\frac{T_{02}}{T*_{02}} = 0.522$

$M_2=.4$

$$\frac{P_2}{P_{2*}} = 1.961$$

$$\frac{P_{02}}{P_{02*}} = 1.961$$

$$\frac{T_2}{T_2*} = 0.615$$

$$P_2 = P^*_2 \times 1.196$$

$$= 1.759 \times 10^5 \times 1.196$$

$$= 3.44 \times 10^5 N/m^2$$

$T_2 = T_2^* \times .615$

$= 1801.18K$

$P_{02} = 3.849 \times 10^5 N/m^2$

Stagnation pressure loss

$\Delta P_0 = P_{01} - P_{02}$

$= 4.11 \times 10^5 - 3.84 \times 10^5$

$= .261 \times 10^5 N/m2$

Heat supplied $Q = C_P(T_{02} - T_{01})$

$= 1005(1128 - 376)$

$= 755.7 \times 10^3 J/kg$

3.16 Tutorial Problems:

1.A circular duct passes 8.25Kg/s of air at an exit Mach number of 0.5. The entry pressure temperature are 3.45 bar and 38° C respectively and the coefficient of friction 0.005.If the Mach number at entry is 0.15, determine : I. The diameter of the duct,
II. Length of the duct, III. Pressure and temperature at the exit, IV. Stagnation pressure loss, and V. Verify the exit Mach number through exit velocity and temperature.

1) A gas (γ =1.3,R=0.287 KJ/KgK) at p1 =1bar, T1 =400 k enters a 30cm diameter duct at a Mach number of 2.0.A normal shock occurs at a Mach number of 1.5 and the exit Mach number is1.0, If the mean value of the friction factor is 0.003 determine: 1)Lengths of the duct upstream and downstream of the shock wave, 2)Mass flow rate of the gas and downstream of the shock.

2) Air enters a long circular duct (d =12.5cm,f=0.0045) at a Mach number 0.5, pressure 3.0 bar and temperature 312 K.If the flow is isothermal throughout the duct determine (a) the length of the duct required to change the Mach number to 0.7,(b) pressure and temperature of air at M =0.7 (c) the lengthof the duct required to attain limiting Mach number, and (d) state of air at the limiting Mach umber.compare these values with those obtained in adiabatic flow.

4. Show that the upper and lower branches of a Fanno curve represent subsonic and supersonic flows respectively . prove that at the maximum entropy point Mach number is unity and all processes approach this point .How would the state of a gas in a flow change from the supersonic to subsonic branch ? Flow in constant area ducts with heat transfer(Rayleigh flow)

5) The Mach number at the exit of a combustion chamber is 0.9. The ratio of stagnation temperature at exit and entry is 3.74. If the pressure and temperature of the gas at exit are 2.5 bar and 1000° C respectively determine (a) Mach number, pressure and temperature of the gas at entry, (b) the heat supplied per kg of the gas and (c) the maximum heat that can be supplied. Take γ = 1.3, Cp= 1.218 KJ/KgK

6) The conditions of a gas in a combuster at entry are: P1=0.343 bar ,T1 = 310K ,C1=60m/s.Detemine the Mach number ,pressure ,temperature and velocity at the exit if the increase in stagnation enthalpy of the gas between entry and exit is 1172.5KJ/Kg. Take Cp=1.005KJ/KgK, γ =1.4

7) A combustion chamber in a gas turbine plant receives air at 350 K ,0.55bar and 75 m/s .The air –fuel ratio is 29 and the calorific value of the fuel is 41.87 MJ/Kg .Taking γ=1.4 and R =0.287 KJ/kg K for the gas determine.

a) The initial and final Mach numbers

b) Final pressure, temperature and velocity of the gas

c) Percent stagnation pressure loss in the combustion chamber, and

d) The maximum stagnation temperature attainable.

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GAS DYNAMICS AND JET PROPULSION

SMEA1602

UNIT – IV NORMAL AND OBLIQUE SHOCKS – SMEA1602

UNIT – 4

NORMAL AND OBLIQUE SHOCKS

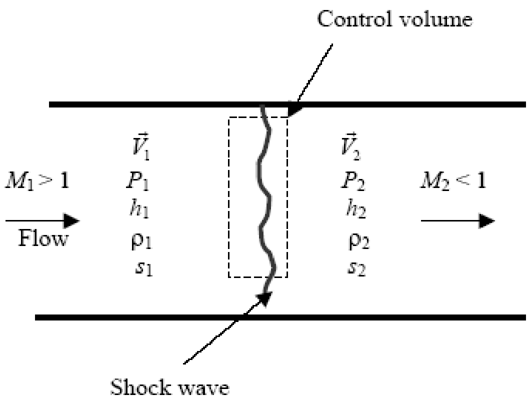
4.1 Normal Shocks

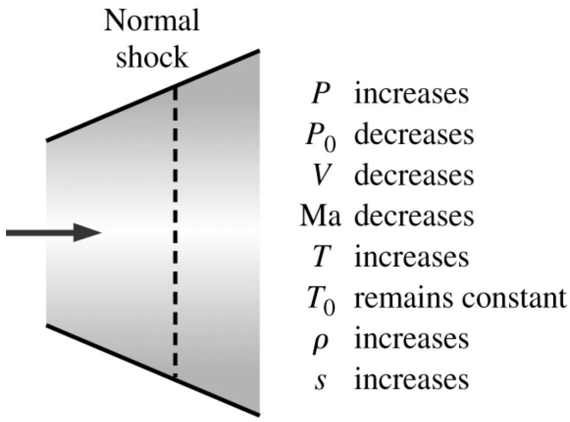
When there is a relative motion between a body and fluid, the disturbance is created if the disturbance is of an infinitely small amplitude, that disturbance is transmitted through the fluid with the speed of sound. If the disturbance is finite shock waves are created.

4.2 Shock Waves and Expansion Waves Normal Shocks

Shocks which occur in a plane normal to the direction of flow are called **normal shock waves.** Flow process through the shock wave is highly irreversible and *cannot* be approximated as being isentropic. Develop relationships for flow properties before and after the shock using conservation of mass, momentum, and energy.

In some range of back pressure, the fluid that achieved a sonic velocity at the throat of a converging-diverging nozzle and is accelerating to supersonic velocities in the diverging section experiences a *normal shock*. The normal shock causes a sudden rise in pressure and temperature and a sudden drop in velocity to subsonic levels. Flow through the shock is highly irreversible, and thus it cannot be approximated as isentropic. The properties of an ideal gas with constant specific heats before (subscript 1) and after (subscript 2) a shock are related by





4.2.1 Assumptions

- Steady flow and one dimensional
- dA = 0, because shock thickness is small
- Negligible friction at duct walls since shock is very thin
- Zero body force in the flow direction
- Adiabatic flow (since area is small)
- No shaft work
- Potential energy neglected

4.3 Governing Equations:

(i) Continuity

$$\dot{m}_x = \dot{m}_y$$

$$(\rho A V)_x = (\rho A V)_y$$

$\rho_x V_x = \rho_y V_y$ (Shock thickness being small $A_x = A_y$) $G_x = G_y$ (Mass velocity). Mass velocity remains constant across the shock.

(ii) Energy equation

$$\text{SFEE: } \dot{q} - \dot{\omega}_{sh} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2}$$

Across the shock

$$h_y + \frac{V_y^2}{2} = h_x + \frac{V_x^2}{2}$$

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy}$$

T_O remains constant across the shock.

(ii) Momentum

Newton's second law

$$\Sigma F_{xx} = \frac{\partial}{\partial t}(\dot{m}V_{xx})_{cv} + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$P_xA - P_yA = 0 + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$= \dot{m}(V_y - V_x) = \rho AV(V_y - V_x) = \rho_y AV_y^2 - \rho_x AV_x^2$$

Momentum gives

$$P_xA - P_yA = \rho_y AV_y^2 - \rho_x AV_x^2$$

$$P_xA + \rho_x AV_x^2 = P_yA + \rho_y AV_y^2$$

$$\therefore F_x = F_y$$

Impulse function remains constant across the shock.

4.3.1Property relations across the shock.

$$(1)\ \frac{T_y}{T_x}$$

Energy

$h_{ox} = h_{oy}$

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy} \quad (1)$$

For the isentropic x – ox

$$\frac{T_{ox}}{T_x} = 1 + \frac{K-1}{2}M_x^2$$

$$T_{ox} = T_x\left[1 + \frac{K-1}{2}M_x^2\right] \qquad \text{.......... (2)}$$

Similarly

$$T_{oy} = T_y\left[1+\frac{K-1}{2}M_y^2\right] \qquad \text{............ (3)}$$

Combining (1), (2) and (3)

$$T_x\left[1+\frac{K-1}{2}M_x^2\right] = T_y\left[1+\frac{K-1}{2}M_y^2\right]$$

$$\frac{T_y}{T_x} = \left[\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}\right]$$

(ii) $\frac{P_y}{P_x}$

Momentum

$$F_x = F_y$$

$$P_x A + \rho_x A V_x^2 = P_y A + \rho_y A V_y^2$$

$$P_x\left[1+\frac{\rho_x V_x^2}{P_x}\right] = P_y\left[1+\frac{\rho_y V_y^2}{P_y}\right]$$

$$P_x\left[1+KM_x^2\right] = P_y\left[1+KM_y^2\right]$$

$$\frac{P_y}{P_x} = \left[\frac{1+KM_x^2}{1+KM_y^2}\right]$$

(iii) $\frac{\rho_y}{\rho_x}$

Equation of state $P = \rho RT$

$$P_x = \rho_x R T_x$$

$$P_y = \rho_y R T_y$$

$$\frac{\rho_x}{\rho_y} = \frac{\left(\frac{P_x}{RT_x}\right)}{\left(\frac{P_y}{RT_y}\right)} = \frac{P_x T_y}{T_x P_y}$$

$$\frac{\rho_x}{\rho_y} = \frac{1 + KM_y^2}{1 + KM_x^2}\left[\frac{1 + \frac{K-1}{2}M_x^2}{1 + \frac{K-1}{2}M_y^2}\right]$$

(iv) $\frac{V_y}{V_x}$

Continuity equation

$$\rho_x V_x = \rho_y V_y$$

$$\frac{V_y}{V_x} = \frac{\rho_x}{\rho_y}$$

Equation of state

$$P = \rho RT$$

$$P_x = \rho_x RT_x$$

$$P_y = \rho_y RT_y$$

$$\frac{V_y}{V_x} = \left[\frac{1 + KM_y^2}{1 + KM_x^2}\right]\left[\frac{1 + \frac{K-1}{2}M_x^2}{1 + \frac{K-1}{2}M_y^2}\right]$$

(v) $\frac{P_{oy}}{P_{ox}}$

For the isentropic x – ox

$$\frac{P_{ox}}{P_x} = \left(1 + \frac{K-1}{2}M_x^2\right)^{\frac{K}{K-1}}$$

For the isentropic (y – oy)

$$\frac{P_{oy}}{P_y} = \left(1 + \frac{K-1}{2} M_y^2\right)^{\frac{K}{K-1}}$$

$$\frac{P_{oy}}{P_{ox}} = \frac{P_y}{P_x}\left[\frac{1 + \frac{K-1}{2} M_y^2}{1 + \frac{K-1}{2} M_x^2}\right]^{\frac{K}{K-1}}$$

But $\dfrac{P_y}{P_x} = \dfrac{1 + KM_x^2}{1 + KM_y^2}$

$$\therefore \frac{P_{oy}}{P_{ox}} = \left[\frac{1 + KM_x^2}{1 + KM_y^2}\right]\left[\frac{1 + \frac{K-1}{2} M_y^2}{1 + \frac{K-1}{2} M_x^2}\right]^{\frac{K}{K-1}}$$

(vi) Entropy change (ΔS)

$$\Delta S = S_y - S_x$$

$$= C_p \ln \frac{T_y}{T_x} - R \ln \frac{P_y}{P_x} = C_p \ln \frac{T_{oy}}{T_{ox}} - R \ln \frac{P_{oy}}{P_{ox}} = -R \ln \frac{P_{oy}}{P_{ox}}$$

$$\therefore S_y - S_x = -R \ln \left\{\left[\frac{1 + KM_x^2}{1 + KM_y^2}\right]\left[\frac{1 + \frac{K-1}{2} M_y^2}{1 + \frac{K-1}{2} M_x^2}\right]^{\frac{K}{K-1}}\right\} \quad [\because T_{oy} - T_{ox}]$$

(vii) Relation between M_x^2 and M_y^2

Prove that $M_y^2 = \left[\dfrac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1} M_x^2 - 1}\right]$

We have

$$(1)\ \frac{T_y}{T_x} = \left[\frac{1 + \frac{K-1}{2} M_x^2}{1 + \frac{K-1}{2} M_y^2}\right]$$

(2) $\frac{P_y}{P_x} = \left[\frac{1+KM_x^2}{1+KM_y^2}\right]$

(3) $\frac{P_x M_x}{\sqrt{T_x}} = \frac{P_y M_y}{\sqrt{T_y}}$

Equation (3) $\Rightarrow \frac{P_y^2}{P_x^2} \cdot \frac{M_y^2}{M_x^2} = \frac{T_y}{T_x}$

$$\left[\frac{1+KM_x^2}{1+KM_y^2}\right]^2 \left(\frac{M_y}{M_x}\right)^2 = \frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}M_y^2}$$

$$\left(1+KM_x^2\right)^2 M_y^2 \left(1+\frac{K-1}{2}M_y^2\right) = \left(1+KM_y^2\right)^2 M_x^2 \left(1+\frac{K-1}{2}M_x^2\right)$$

$$\text{L.H.S.} = \left(1+2KM_x^2+K^2M_x^4\right)M_y^2\left(1+\frac{K-1}{2}M_y^2\right)$$

$$= M_y^2\left(1+2KM_x^2+K^2M_x^4+\frac{K-1}{2}M_y^2+K(K-1)M_x^2M_y^2+\frac{K^2(K-1)}{2}M_x^4M_y^2\right)$$

$$= M_y^2+2KM_x^2M_y^2+K^2M_x^4M_y^2+\frac{K-1}{2}M_y^4+K(K-1)M_x^2M_y^4+\frac{K^2(K-1)}{2}M_x^4M_y^4$$

Similarly R.H.S.

$$= M_x^2+2K_x^2M_y^2+K^2M_y^2M_x^2+\frac{K-1}{2}M_x^4+K(K-1)M_y^2M_x^4+\frac{K^2(K-1)}{2}M_y^4M_x^4$$

$$= \left(M_y^2-M_x^2\right)+K^2\left(M_x^4M_y^2-M_y^4M_x^2\right)+\frac{K-1}{2}\left(M_y^4-M_x^4\right)+K(K-1)\left(M_x^2M_y^4-M_y^2M_x^4\right)$$

$$= \left(M_y^2-M_x^2\right)\left[1+\frac{K-1}{2}\left(M_y^2+M_x^2\right)-KM_x^2M_y^2\right] = 0$$

Since $M_y \neq M_x$

So $\left(M_y^2-M_x^2\right)$ cannot be equal to zero

$$\text{Hence } 1+\frac{K-1}{2}\left(M_y^2+M_x^2\right)=KM_x^2M_y^2$$

$$\frac{K-1}{2}\left(M_y^2+M_x^2\right)=KM_x^2M_y^2-1$$

$$\left(M_y^2+M_x^2\right)=\frac{2}{K-1}\left(KM_x^2M_y^2-1\right)$$

$$M_y^2=\frac{2K}{K-1}M_x^2M_y^2-\frac{2}{K-1}-M_x^2$$

$$M_y^2\left[1-\frac{2K}{K-1}M_x^2\right]=-\frac{2}{K-1}-M_x^2$$

$$M_y^2=\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}$$

Property relations in terms of incident Mach Number M_X

$$\text{(1) Prove } \frac{P_y}{P_x}=\frac{2K}{K-1}M_x^2-\frac{K-1}{K+1}$$

$$\text{Momentum } F_x=F_y$$

$$P_x\left(1+KM_x^2\right)=P_y\left(1+KM_y^2\right)$$

$$\frac{P_y}{P_x}=\frac{1+KM_x^2}{1+KM_y^2}$$

$$\text{but } M_y^2=\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}$$

$$\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+K\left[\dfrac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}\right]} = \frac{\left(1+KM_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}{\left(\dfrac{2K}{K-1}M_x^2-1\right)+K\left[\dfrac{2}{K-1}+M_x^2\right]} \qquad (1)$$

$$\text{Dinominator } D_r = \left(\frac{2K}{K-1}M_x^2-1\right)+K\left[\frac{2}{K-1}+M_x^2\right]$$

$$= M_x^2\left[\frac{2K}{K-1}+K\right]+\frac{2K}{K-1}-1 \quad = KM_x^2\left[\frac{K+1}{K-1}\right]+\frac{K+1}{K-1}$$

$$= \frac{K+1}{K-1}\left(1+KM_x^2\right) \text{ substitute in equation (1).}$$

$$\frac{P_y}{P_x} = \frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}$$

(2) Prove that $\dfrac{T_y}{T_x} = \dfrac{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}{\dfrac{(K+1)^2}{2(K-1)}M_x^2}$

Energy

$$h_{ox} = h_{oy}$$

$$T_{ox} = T_{oy}$$

$$\frac{T_y}{T_x} = \left[\frac{1+\dfrac{K-1}{2}M_x^2}{1+\dfrac{K-1}{2}M_y^2}\right]$$

$$M_y^2 = \frac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}$$

$$\frac{T_y}{T_x}=\frac{1+\frac{K-1}{2}M_x^2}{1+\frac{K-1}{2}\left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]}=\frac{1+\frac{K-1}{2}M_x^2\left(\frac{2K}{K-1}M_x^2-1\right)}{\frac{2K}{K-1}M_x^2-1+\frac{K-1}{2}\left(\frac{2}{K-1}+M_x^2\right)}$$

Denominator, $D_r=\frac{2K}{K-1}M_x^2-1+1+\left(\frac{K-1}{2}\right)M_x^2$

$$=M_x^2\left[\frac{2K}{K-1}+\frac{K-1}{2}\right]=M_x^2\left[\frac{4K+(K-1)^2}{2(K-1)}\right]=M_x^2\frac{(K+1)^2}{2(K-1)}$$

Hence $\frac{T_y}{T_x}=\frac{\left(1+\frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K-1}M_x^2-1\right)}{\frac{(K+1)^2}{2(K-1)}M_x^2}$

(3) Prove that $\frac{P_{oy}}{P_{ox}}=\left[\frac{\frac{K+1}{2}M_x^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}}\left[\frac{2}{K+1}KM_x^2-\frac{K-1}{K+1}\right]^{\frac{1}{K-1}}$

For the isentropic x – ox,

$$\frac{P_{ox}}{P_x}=\left[1+\frac{K-1}{2}M_x^2\right]^{\frac{K}{K-1}}$$

$$P_{ox}=P_x\left[1+\frac{K-1}{2}M_x^2\right]^{\frac{K}{K-1}}$$

$$P_{oy}=P_y\left[1+\frac{K-1}{2}M_y^2\right]^{\frac{K}{K-1}}$$

$$\frac{P_{oy}}{P_{ox}}=\frac{P_y}{P_x}\left[\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2}\right]^{\frac{K}{K-1}} \quad \text{......... (1)}$$

But $\frac{P_y}{P_x}$ is obtained from momentum as

$$\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+KM_y^2}$$

But $M_y^2 = \left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]$

$$\frac{P_y}{P_x} = \frac{1+KM_x^2}{1+K\left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]}$$

$$\frac{P_y}{P_x} = \frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}$$

$$\frac{1+\frac{K-1}{2}M_y^2}{1+\frac{K-1}{2}M_x^2} = \frac{1+\frac{K-1}{2}\left[\frac{\frac{2}{K-1}+M_x^2}{\frac{2K}{K-1}M_x^2-1}\right]}{1+\frac{K-1}{2}M_x^2} = \frac{\frac{(K+1)^2}{2(K-1)}M_x^2}{\left(1+\frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K-1}M_x^2-1\right)}$$

Equation (1) becomes,

$$\frac{P_{oy}}{P_{ox}} = \left[\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right]\left[\frac{\frac{(K+1)^2}{2(K-1)}M_x^2}{\left(1+\frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K-1}M_x^2-1\right)}\right]^{\frac{K}{K-1}}$$

$$= \left[\frac{2K}{K+1}M_x^2 - \frac{K-1}{K+1}\right]\left[\frac{\frac{K+1}{2}M_x^2}{\left(1+\frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K+1}M_x^2-\frac{K-1}{K+1}\right)}\right]^{\frac{K}{K-1}}$$

$$= \left(\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1} \right)^{1 - \frac{K}{K-1}} \left[\frac{\frac{K+1}{2} M_x^2}{1 + \frac{K-1}{2} M_x^2} \right]^{\frac{K}{K-1}}$$

$$\therefore \frac{P_{oy}}{P_{ox}} = \left(\frac{2K}{K+1} M_x^2 - \frac{K-1}{K+1} \right)^{\frac{-1}{K-1}} \left[\frac{\frac{K+1}{2} M_x^2}{1 + \frac{K-1}{2} M_x^2} \right]^{\frac{K}{K-1}}$$

(4) Prove that $\frac{V_y}{V_x} = \frac{1 - \frac{K-1}{2} M_x^2}{\frac{K+1}{2} M_x^2}$

$$M_x^2 = \frac{V_x^2}{C_x^2}$$

$$V_x^2 = M_x^2 C_x^2$$

$$V_y^2 = M_y^2 C_y^2$$

$$\frac{V_y^2}{V_x^2} = \frac{M_y^2}{M_x^2} \cdot \frac{T_y}{T_x} \qquad \text{......... (1)}$$

But $\frac{T_y}{T_x} = \frac{1 + \frac{K-1}{2} M_x^2}{1 + \frac{K-1}{2} M_y^2}$

But $M_y^2 = \frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1} M_x^2 - 1}$

Substituting and simplifying

$$\frac{V_y^2}{V_x^2}=\frac{\left[\dfrac{\dfrac{2}{K-1}+M_x^2}{\dfrac{2K}{K-1}M_x^2-1}\right]}{M_x^2}\times\frac{\left(1+\dfrac{K^2-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}{\dfrac{(K+1)^2}{2(K-1)}M_x^2}$$

Substituting (2) in (1), we have

$$\frac{T_y}{T_x}=\frac{\left(1+\dfrac{K-1}{2}M_x^2\right)\left(\dfrac{2K}{K-1}M_x^2-1\right)}{\dfrac{(K+1)^2}{2(K-1)}M_x^2} \qquad \text{............ (2)}$$

$$=\frac{\left(\dfrac{2}{K-1}+M_x^2\right)\left(1+\dfrac{K-1}{2}M_x^2\right)}{\dfrac{(K+1)^2}{2(K-1)}M_x^4}$$

$$=\frac{\left(\dfrac{2}{K-1}\right)\left(1+\dfrac{K-1}{2}M_x^2\right)^2}{\dfrac{(K+1)^2}{2(K-1)}M_x^4}=\frac{\left(1+\dfrac{K-1}{2}M_x^2\right)^2}{\dfrac{(K+1)^2 M_x^4}{4}}$$

$$\therefore \frac{V_y}{V_x}=\frac{1+\dfrac{K-1}{2}M_x^2}{\dfrac{K+1}{2}M_x^2}$$

4.4 Prandtl-Meyer relationship

$V_x.V_y=C^{*2}$ or

$M_x^2 M_y^2=1$

4.5 Governing relations for a normal shock

(1) Continuity

$$\rho_x V_x = \rho_y V_y \qquad \text{....... (1)}$$

(2) Energy

$$h_{ox} = h_{oy}$$

$$h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} = h_0 \qquad \text{............ (2)}$$

(3) Momentum

$$\Sigma F_{xx} = \frac{\partial}{\partial t}(\dot{m}V_{xx})_{CV} + (\dot{m}V_{xx})_{out} - (\dot{m}V_{xx})_{in}$$

$$P_x A - P_y A = \dot{m}(V_y - V_x)$$

$$(P_x - P_y) = G(V_y - V_x)$$

$$P_x - P_y = \rho V(V_y - V_x) \qquad \text{.............. (3)}$$

From (2) we have

$$h_x + \frac{V_x^2}{2} = h_0$$

$$C_p T_x + \frac{V_x^2}{2} = C_p T_o$$

$$\frac{KR}{K-1} T_x + \frac{V_x^2}{2} = \frac{KR}{K-1} T_0$$

$$\frac{C_x^2}{K-1} + \frac{V_x^2}{2} = \frac{C_0^2}{K-1}$$

$$\therefore\ C_x^2 = C_0^2 - \frac{K-1}{2} V_x^2 \qquad \text{.......... (4)}$$

Similarly $C_y^2 = C_0^2 - \frac{K-1}{2} V_y^2$ .... (5)

Equation (3) $\Rightarrow P_x - P_y = \rho V(V_y - V_x)$

$$\frac{P_x - P_y}{\rho V} = \left(V_y - V_x\right)$$

$$\frac{P_x}{\rho_x V_x} - \frac{P_y}{\rho_y V_y} = \left(V_y - V_x\right)$$

$$\frac{RT_x}{V_x} - \frac{RT_y}{V_y} = \left(V_y - V_x\right)$$

$$\frac{KRT_x}{V_x} - \frac{KRT_y}{V_y} = K\left(V_y - V_x\right)$$

$$\frac{C_x^2}{V_x} - \frac{C_y^2}{V_y} = K\left(V_y - V_x\right) \qquad \text{............. (6)}$$

Using (4) and (5) in (6)

$$\frac{1}{V_x}\left[C_0^2 - \frac{K-1}{2}V_x^2\right] - \frac{1}{V_y}\left[C_0^2 - \frac{K-1}{2}V_y^2\right] = K\left(V_y - V_x\right)$$

$$C_0^2\left[\frac{1}{V_x} - \frac{1}{V_y}\right] + \frac{K-1}{2}\left(V_y - V_x\right) = K\left(V_y - V_x\right)$$

$$\frac{C_0^2\left(V_y - V_x\right)}{V_x V_y} + \frac{K-1}{2}\left(V_y - V_x\right) = K\left(V_y - V_x\right)$$

$$\frac{C_0^2}{V_x V_y} = K - \frac{K-1}{2}$$

$$C_0^2 = \left(\frac{K+1}{2}\right)V_x V_y \qquad \text{.......... (7)}$$

$$\frac{C^{*2}}{C_0^2} = \frac{KRT^*}{KRT_0} = \frac{T^*}{T_0} = \frac{2}{K+1} \qquad \left[\frac{T_0}{T} = 1 + \frac{K-1}{2}M^2\right]$$

$$\left[\frac{T_0}{T^*} = \frac{K+1}{2}\right]$$

$$C_0^2 = \left(\frac{K+1}{2}\right)C^{*2} \qquad \text{............ (8)}$$

Substitution (8) in (7) we have

$$\left(\frac{K+1}{2}\right)C^{*2} = \left(\frac{K+1}{2}\right)V_x V_y$$

$$C^{*2} = V_x V_y \qquad \frac{V_x V_y}{C^{*2}} = 1$$

for a shock, $C_x^* = C_y^* = C^*$

$$\frac{V_x}{C_x^*}\frac{V_y}{C_y^*} = 1$$

$$M_x^* . M_y^* = 1 \qquad \text{By definition} \quad M^* = \frac{V}{C^*}$$

4.6 The Rankine – Hugoniot Equatios

Density Ratio Across the Shock

We know that density, $\rho = \frac{P}{RT}$

For upstream shock,

$$\rho x = \frac{P_x}{R_x T_x}$$

For down stream shock

$$\rho_y = \frac{P_y}{R_y T_y}$$

$$\frac{\rho_y}{\rho x} = \frac{\frac{P_y}{R_y T_y}}{\frac{P_x}{R_x T_x}}$$

$$\frac{\rho_y}{\rho x} = \frac{p_y}{p_x} \times \frac{T_X}{T_Y}$$

We know that,

$$\boxed{\frac{P_Y}{P_X} = \frac{2\gamma}{\gamma+1} M_X{}^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$\frac{2\gamma}{\gamma+1} M_X{}^2 = \frac{P_Y}{P_X} + \frac{\gamma-1}{2\gamma}$$

We know that,

$$\frac{T_Y}{T_X} = \frac{\left[\frac{2\gamma}{\gamma+1} M_X{}^2 - 1\right]\left[1 + \frac{\gamma-1}{2} M_X{}^2\right]}{\frac{M_X{}^2}{2(\gamma-1)} \times (\gamma+1)^2}$$

$$\frac{T_Y}{T_X} = \frac{\frac{2\gamma}{\gamma-1}\left[\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right) + \frac{\gamma-1}{2\gamma}\right] - 1\left[1 + \frac{\gamma-1}{2}\left(\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right) + \frac{\gamma-1}{2\gamma}\right)\right]}{\left[\frac{\gamma+1}{2\gamma}\left(\frac{P_Y}{P_X}\right) + \frac{\gamma-1}{2\gamma}\right] \times \frac{(\gamma+1)^2}{2(\gamma-1)}}$$

$$\frac{T_Y}{T_X} = \frac{\left[\frac{(\gamma+1)}{(\gamma-1)}\frac{P_Y}{P_X} + 1 - 1\right] \times \left[1 + \frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_Y}{P_X} + \frac{(\gamma-1)^2}{4\gamma}\right]}{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right) + \frac{(\gamma-1)^2}{4\gamma}}$$

Taking out $\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right)$

$$= \frac{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X}\right)\left[\frac{4\gamma}{(\gamma+1)^2} + \frac{(\gamma-1)}{(\gamma+1)}\frac{P_Y}{P_X} + \frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\frac{(\gamma-1)^3}{4\gamma(\gamma-1)}\left(\frac{P_Y}{P_X} + \frac{(\gamma-1)}{(\gamma+1)}\right)}$$

$$= \frac{\left(\frac{P_Y}{P_X}\right)\left[\frac{4\gamma}{(\gamma+1)^2} + \frac{(\gamma-1)}{(\gamma+1)}\frac{P_Y}{P_X} + \frac{(\gamma-1)^2}{(\gamma+1)^2}\right]}{\left(\frac{P_Y}{P_X} + \frac{(\gamma-1)}{(\gamma+1)}\right)}$$

$$= \frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)} \times \frac{P_Y}{P_X} + \frac{4\gamma + (\gamma-1)^2}{(\gamma+1)^2}\right]}{\frac{(\gamma-1)}{(\gamma+1)} + \frac{P_Y}{P_X}}$$

$$= \frac{\frac{P_Y}{P_X}\left[\left(\frac{\gamma-1}{\gamma+1}\right) \times \frac{P_Y}{P_X} + \frac{4\gamma + (\gamma^2 - 2\gamma + 1)^2}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X} + \frac{\gamma-1}{\gamma+1}}$$

$$= \frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)} \times \frac{P_Y}{P_X} + \frac{(\gamma^2 + 2\gamma + 1)}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X} + \frac{\gamma-1}{\gamma+1}}$$

$$= \frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)} \times \frac{P_Y}{P_X} + \frac{(\gamma+1)^2}{(\gamma+1)^2}\right]}{\frac{P_Y}{P_X} + \frac{\gamma-1}{\gamma+1}}$$

$$\frac{T_Y}{T_x} = \frac{\frac{P_Y}{P_X}\left[\frac{(\gamma-1)}{(\gamma+1)} \times \frac{P_Y}{P_X} + 1\right]}{\frac{P_Y}{P_X} + \frac{\gamma-1}{\gamma+1}}$$

$$\frac{P_Y}{P_X} = \frac{\frac{T_Y}{T_x} \times \left(\frac{P_Y}{P_X} + \frac{\gamma-1}{\gamma+1}\right)}{\left[1 + \frac{P_Y}{P_X} \times \frac{\gamma-1}{\gamma+1}\right]}$$

$$\frac{P_Y}{P_X}\times\frac{T_x}{T_Y}=\frac{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\times\frac{P_Y}{P_X}}$$

We have,

$$\frac{\rho_y}{\rho_x}=\frac{P_Y}{P_X}\times\frac{T_x}{T_Y}$$

$$\frac{\rho_y}{\rho_x}=\frac{\frac{P_Y}{P_X}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\times\frac{P_Y}{P_X}}$$

$$\frac{\rho_y}{\rho_x}=\frac{\frac{\gamma-1}{\gamma+1}\left[1+\frac{\gamma-1}{\gamma+1}\times\frac{P_Y}{P_X}\right]}{\frac{\gamma-1}{\gamma+1}\left[\frac{\gamma-1}{\gamma+1}+\frac{P_Y}{P_X}\right]}$$

$$\boxed{\frac{\rho_y}{\rho_x}=\frac{\left[1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}\right]}{\left[\frac{\gamma+1}{\gamma-1}+\frac{P_Y}{P_X}\right]}}$$

$$\frac{\rho_y}{\rho_x}\left[\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}\right]=\left[1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}\right]$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right)+\left(\frac{\rho_y}{\rho_x}\right)\frac{P_Y}{P_X}=1+\frac{\gamma+1}{\gamma-1}\times\frac{P_Y}{P_X}$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) = 1 + \frac{\gamma+1}{\gamma-1} \times \frac{P_Y}{P_X} - \left(\frac{\rho_y}{\rho_x}\right)\frac{P_Y}{P_X}$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) = 1 + \frac{P_Y}{P_X}\left[\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}\right]$$

$$\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1 = \frac{P_Y}{P_X}\left[\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}\right]$$

$$\boxed{\frac{P_Y}{P_X} = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\left[\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}\right]}}$$

the above eqn.s is knows Rankine - Hugoniot equations.

4.7 Strength of a Shock Wave

It is defined as the ratio of difference in down stream and upstream shock pressures (p_y - p_x) to upstream shock pressures (p_x). It is denoted by ξ.

$$\xi = \frac{p_y - p_x}{p_x}$$

$$\boxed{\xi = \frac{p_y}{p_x} - 1}$$

Substituting for p_y/p_x

$$\xi = \left[\frac{2\gamma}{\gamma+1} M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right] - 1$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - (\gamma-1) - (\gamma+1)\right]$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - \gamma + 1 - \gamma - 1\right]$$

$$= \frac{1}{\gamma+1} \left[2\gamma M_x^2 - 2\gamma\right]$$

$$\boxed{\xi = \frac{2\gamma}{\gamma+1} [M_x^2 - 1]}$$

From the above equation;

$$\boxed{\xi \propto M_x^2 - 1}$$

The strength of shock wave may be expressed in another form using Rankine-Hugoniot equation.

$$\frac{p_y}{p_x} = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

We know that

$$\xi = \frac{p_y}{p_x} - 1$$

$$\Rightarrow \quad \xi = \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}} - 1$$

$$= \frac{\frac{\rho_y}{\rho_x}\left(\frac{\gamma+1}{\gamma-1}\right) - 1 - \frac{\gamma+1}{\gamma-1} + \frac{\rho_y}{\rho_x}}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x} - 1\right] + \left[\frac{\rho_y}{\rho_x} - 1\right]}{\frac{\gamma+1+1-1}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\left[\frac{\rho_y}{\rho_x} - 1\right] + \left[\frac{\rho_y}{\rho_x} - 1\right]}{\frac{\gamma-1}{\gamma-1} + \frac{2}{\gamma-1} - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x} - 1\right]\left[\frac{\gamma+1}{\gamma-1} + 1\right]}{\frac{2}{\gamma-1} + 1 - \frac{\rho_y}{\rho_x}}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x} - 1\right]\left[\frac{\gamma+1+\gamma-1}{\gamma-1}\right]}{\frac{2}{\gamma-1} - \left[\frac{\rho_y}{\rho_x} - 1\right]}$$

$$= \frac{\left[\frac{\rho_y}{\rho_x} - 1\right]\left[\frac{2\gamma}{\gamma-1}\right]}{\frac{2}{\gamma-1} - \left[\frac{\rho_y}{\rho_x} - 1\right]}$$

$$\Rightarrow \boxed{\xi = \frac{\left[\frac{2\gamma}{\gamma-1}\right]\left[\frac{\rho_y}{\rho_x} - 1\right]}{\frac{2}{\gamma-1} - \left[\frac{\rho_y}{\rho_x} - 1\right]}}$$

From this equation we came to know strength of shock wave is directly proportional to;

$$\left[\frac{\rho_y}{\rho_x} - 1\right]$$

$$\Rightarrow \boxed{\xi \propto \left[\frac{\rho_y}{\rho_x} - 1\right]}$$

4.8 PROBLEMS

1. The state of a gas (γ =1.3,R =0.469 KJ/KgK.) upstream of normal shock wave is given by the following data: M_X =2.5, P_X =2 bar. T_X =275 K calculate the Mach number,pressure,temperatureand velocity of a gas down stream of shock: check the calculated values with those given in the gas tables.**Take K = γ.**

Ans

$$M_y^2 = \frac{\frac{2}{K-1} + M_x^2}{\frac{2K}{K-1}M_x^2 - 1} = \frac{\frac{2}{1.3-1} + 2.5^2}{\frac{2\times 1.3}{1.3-1}\times 2.5^2 - 1} = \frac{12.92}{53.19} = 0.243$$

$$M_Y = 0.4928$$

$$\boxed{\frac{P_Y}{P_X} = \frac{2\gamma}{\gamma+1}M_X{}^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}$$

$$= \frac{2\times 1.3}{1.3+1}\times 1.5^2 - \left(\frac{1.3-1}{1.3+1}\right)$$

$$\frac{P_Y}{P_X} = 7.065 - 0.130 = 6.935$$

$$P_Y = 6.935\times 2 = 13.870\ bar$$

$$\frac{T_y}{T_x} = \frac{\left(1+\frac{K-1}{2}M_x^2\right)\left(\frac{2K}{K-1}M_x^2 - 1\right)}{\frac{(K+1)^2}{2(K-1)}M_x^2}$$

$$= \frac{\left(1+\frac{1.3-1}{2}\times 2.5^2\right)\left(\frac{2\times 1.3}{1.3-1}2.5^2 - 1\right)}{\frac{(1.3+1)^2}{2(1.3-1)}2.5^2}$$

$$= \frac{\left(1+\frac{03}{2}\times 6.25\right)\times 53.19}{\frac{(2.3)^2}{2(0.3)}\times 6.25} = \frac{1.937\times 53.19}{55.104}$$

$$\frac{T_y}{T_x} = 1.869$$

$$T_Y = 1.869 \times 275 = 513.975\text{K}$$

$$\frac{C_Y}{C_x} = \frac{3}{1.3+1}\times\frac{1}{6.25}+\frac{0.3}{2.3} = 0.269$$

$$C_Y = 0.269C_x \quad .0269 M_x a_x$$

$$C_Y = 0.269 M_x \sqrt{KRT_x}$$
$$C_Y = 0.269\, 2.5_x \sqrt{1.3\times 469\times 275}$$

$$C_y = M_y \sqrt{KRT_y}$$
$$= 0.4928\sqrt{1.3\times 469\times 513.975}$$
$$C_y = 275.16\, m/s.$$

2. An Aircraft flies at a Mach number of 1.1 at an altitude of 15,000 metres.The compression in its engine is partially achieved by a normal shock wave standing at the entry of the diffuser. Determine the following for downstream of the shock.

1. Mach number
2. Temperature of the air
3. Pressure of the air
4. Stagnation pressure loss across the shock.

Given

$M_X = 1.1$

Altitude, Z =15, 000 m

Refer gas tables for Altitude, $Z = 15,000\, m$

$$T_x = 216.6\, K$$

$$P_x = 0.120\, bar$$

$P_x = 0.120 \times 10^5 \, N/m^2$

Re *fer Normal shocks gas tables for* $M_x = 1.1$ *and* $\gamma = 1.4$

$M_y = 0.911$

$$\frac{P_y}{P_x} = 1.245$$

$$\frac{T_y}{T_x} = 1.065$$

$$\frac{P_{0y}}{P_{0x}} = 0.998$$

$$\frac{P_{0y}}{P_x} = 2.133$$

$$P_y = 1.24 \times 5P_x$$
$$= 1.24 \times 0.120 \times 10^5 \, N/m^2$$

$P_y = 0.149 \times 10^5 \, N/m^2$

$$T_y = 1.067 \times T_x$$
$$= 1.065 \times 216.6$$

$T_y = 230.67K$

$$P_{0y} = 2.133 \times P_x$$
$$= 2.133 \times 0.120 \times 10^5$$

$P_{0y} = 0.259 \times 10^5 \, N/m^2$

$$P_{0x} = \frac{P_{0y}}{0.998}$$

$$P_{0x} = \frac{0.259 \times 10^5}{0.998}$$

$$P_{0x} = 0.2564 \times 10^5\, N/m^2$$

$Pr\,essure\ loss$

$$\Delta P_0 = P_{0x} - P_{0y}$$

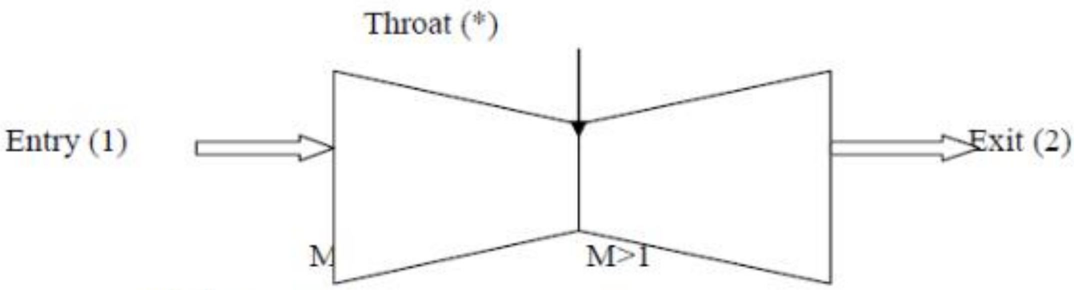
$$= 0.2564 \times 10^5 - 0.259 \times 10^5$$

$$\Delta P_0 = 50\, N/m^2$$

3. Supersonic nozzle is provided with a constant diameter circular duct at its exit. The duct diameter is same as the nozzle diameter. Nozzle exit cross ection is three times that of its throat. The entry conditions of the gas (γ=1.4, R= 287J/KgK) are P_0 =10 bar, T_0 =600K. Calculate the static pressure, Mach number and velocity of the gas in duct.
(a) When the nozzle operates at its design condition. (b) When a normal shock occurs at its exit. (c) When a normal shock occurs at a section in the diverging part where the area ratio, A/A* =2.

Given:
$A_2 = 3A^*$
Or $A_2/A^* = 3$
$\gamma = 1.4$
R= 287 J/KgK
P_0= 10 bar = 10^6 Pa
T_0= 600K

For nozzle



Solution:

Case (i)

Refer isentropic flow table for $A_2/A^*=3$ and $\gamma = 1.4$

$M_2 = 2.64$

$T_2/T_{02} = 0.417$

$p_2/p_{02} = 0.0417$

{Note: For A_2/A^* =3, we can refer gas tables page no.30 and 36. But we have to take M >1 corresponding values since the exit is divergent nozzle}

i.e. $T_2 = 0.417 \times T_{02}$

$= 0.417 \times 600$ {since $T_0=T_{01}=T_{02}$} $T_2 = 250.2\,K$

$P_2 = 0.0471 \times p_{02}$

$= 0.0471 \times 10 \times 10^5$ {since $p_0 = p_{01} = p_{02}$}

$p_2 = 0.471 \times 10^5\ N/m^2$

We know

$C_2 = M_2 \times a_2$

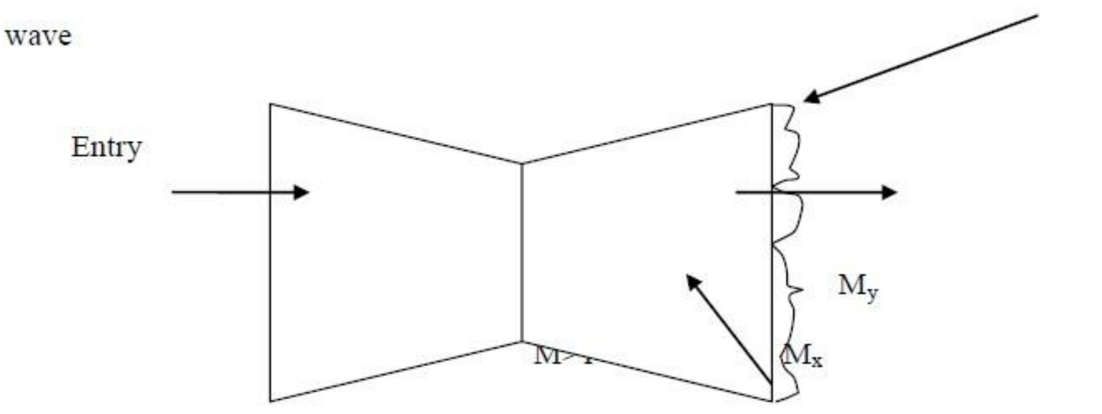
$= M_2 \times \sqrt{\gamma R T_2}$

$= 2.64 \times (1.4 \times 287 \times 250.2)^{.5}$

C_2 =837.05 m/s

Case (ii)

Normal shock occurs at exit.



$A_2/A^* = A_x/A_x^* = 3$ [since in this case $A_2 = A_x$]

Refer isentropic table for A_x/A_x^* =3 and γ =1.4.

M_x =2.64

$T_x/T_x^* = 0.417$

$P_x/P_{0x} = 0.0417$ [From gas tables page no. 36]

i.e. $T_x = 0.417 \times T_{0x}$

$= 0.417 \times 600$

i.e. $T_x = 250.2$ K [Since $T_0 = T_{0x}$]

So $p_x = 0.0417 \times P_{0x}$

$= 0.0417 \times 10 \times 10^5$ [Since $p_0 = p_{0x}$]

i.e. $p_x = 0.417 \times 10^5$ N/m^2

Refer Normal shock table for $M_x = 2.64$ and $\gamma = 1.4$

$M_y = 0.5$

$P_y/P_x = 7.965$

$T_y/T_x = 2.279$

i.e. $P_y = 7.695 \times p_x$

$= 7.695 \times 0.471 \times 10^5$

So $p_y = 3.75 \times 10^5$ N/m^2

Also $T_y = 2.279 \times T_x$

$= 2.279 \times 250.2$

i.e. $T_y = 570.2$ K

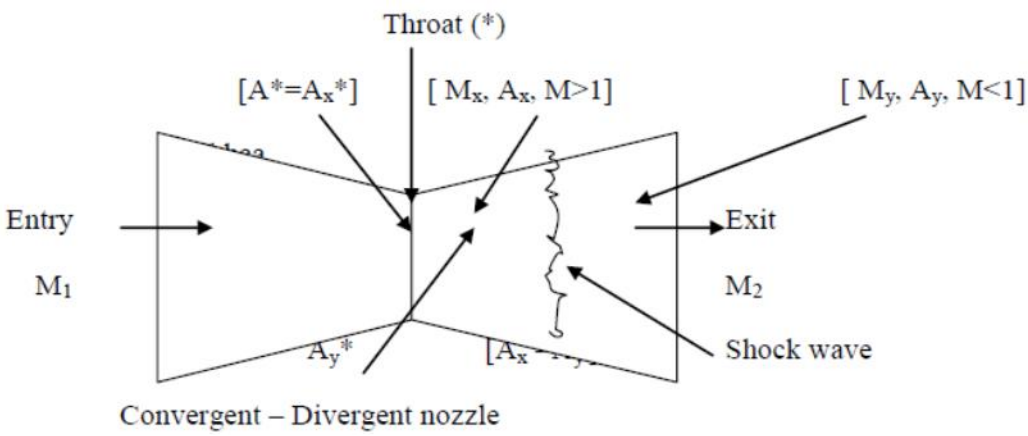
Also $C_y = M_y \times a_y$

$= M_Y \times \sqrt{\gamma R T_Y}$

$= 0.5 \times (1.4 \times 287 \times 570.2)$

$C_y = 239.32$ m/s

Case (iii)
Area ratio $A/A^* = 2$
i.e . $A_x/A_x^* = 2$
Refer isentropic flow table for $A_x/A_x^* = 2$ and $\gamma = 1.4$
$M_x = 2.2$ [from gas tables page no. 35]
Refer Normal shocks table for $M_x = 2.2$ and $\gamma = 1.4$
$M_y = 0.547$
$P_{0y}/P_{0x} = 0.628$
$P_{0y} = 0.628 \times P_{0x}$
$= 0.628 \times 10 \times 10^5$
$P_{0y} = 6.28 \times 10^5\ N/m^2$



Refer isentropic flow table for $M_y = 0.547$ and $\gamma = 1.4$
$A_y/A_y^* = 1.255$
We know that
[Since $A_x = A_y$, $A^* = A_x^*$] $A_2/A_y^* = (A_2/A_x^*) \times (A_x^*/A_x) \times (A_y/A_y^*)$
i.e. $A_2/A_y^* = 3 \times .5 \times 1.255$ or $A_2/A_y^* = 1.8825$
Refer isentropic table for $A_2/A_y^* = 1.8825 = 1.871$ and $\gamma = 1.4$
$M_2 = 0.33$ [From gas tables page no. 29] $T_2/T_{0y} = 0.978$
$P_2/p_{0y} = 0.927$

i.e. $T_2 = 0.978 \times T_{0y}$
$= 0.978 \times 600$ or $T_2 = 586.8\ K$
So $P_2 = 0.927 \times p_{0y}$
$= 0.927 \times 6.28 \times 10^5$
or $P_2 = 5.82 \times 10^5\ N/m^2$

$C_2 = M_2 \times a_2$
$= M_2 \times \sqrt{\gamma R T_2} = 0.33 \times (1.4 \times 287 \times 586.8)^{.0.5}$

C_2= 160.23 m/s

Result

Case (i): p_2=0.471 × 10^5 N/m^2, M_2= 2.64, c_2= 837.05 m/s

Case (ii): p_y= 3.75 × 10^5 N/m^2, M_y=0.5, c_y= 239.32 m/s

Case (iii): p_2= 5.82 × 10^5 N/m^2, M_2= 0.33, c_2= 160.23 m/s

4.9 Tutorial Problems:

1) The state of a gas (γ =1.3,R =0.469 KJ/Kg K) upstream of a normal shock is given by the following data: Mx =2.5, px= 2bar ,Tx =275K calculate the Mach number, pressure,temperature and velocity of the gas downstream of the shock;check the calculated values with those give in the gas tables.

2) The ratio of th exit to entry area in a subsonic diffuser is 4.0 .The Mach number of a jet of air approaching the diffuser at p0=1.013 bar, T =290 K is 2.2 .There is a standing normal shock wave just outside the diffuser entry. The flow in the diffuser is isentropic . Determine at the exit of the diffuser.a) Mach number , b) Temperature, and c) Pressure d) What is the stagnation pressure loss between the initial and final states of the flow ?

3) The velocity of a normal shock wave moving into stagnant air (p=1.0 bar, t=17^0C) is 500 m/s . If the area of cross- section of the duct is constant determine (a) pressure (b) temperature (c) velocity of air (d) stagnation temperature and (e) the mach number imparted upstream of the wave front.

4) The following data refers to a supersonic wind tunnel:
Nozzle throat area =200cm^2,Test section cross- section =337.5cm^2, Working fluid ;air (γ =1.4, Cp =0.287 KJ/Kg K) Determine the test section Mach number and the diffuser throat area if a normal shock is located in the test section.

5) A supersonic diffuser for air (γ =1.4) has an area ratio of 0.416 with an inlet Mach number of 2.4 (design value). Determine the exit Mach number and the design value of the pressure ratio across the diffuser for isentropic flow. At an off- design value of the inlet Mach number (2.7) a normal shock occurs inside the diffuser .Determine the upstream Mach number and area ratio at the section where the shock occurs, diffuser efficiency and the pressure ratio across the diffuser. Depict graphically the static pressure distribution at off design.

6) Starting from the energy equation for flow through a normal shock obtain the following relations (or) prandtl – meyer relation Cx Cy =a^{*2} M*x M*y =1

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GAS DYNAMICS AND JET PROPULSION

SMEA1602

UNIT – V JET AND SPACE PROPULSION – SMEA1602

UNIT – 5

JET AND SPACE PROPULSION

5.1 Jet Propulsion System

It is the propulsion of a jet aircraft (or) Rocket engines which do not use atmospheric air other missiles by the reaction of jet coming out with high velocity. The jet propulsion in used when the oxygen is obtained from the surrounding atmosphere.

Jet propulsion is based on Newton's second and third low of motion. Newton's second law states that 'the range of momentum in any direction is proportional to the force acting in that direction'. Newton's third low states that for every action there is an equal and opposite reaction.

In propulsion momentum is imparted to a mass of fluid in such a manner that the reaction of the imparted momentum furnishes a propulsive force. The jet aircraft draws in air and expels it to the rear at a markedly increased velocity; the rocket greatly changes the velocity of its fuel which it ejects rearward in the form of products of combustion. In each case the action of accelerating the mass of fluid in a given direction created a reaction in the opposite direction in the form of a propulsive force. The magnitude of this propulsive force is defined as thrust.

5.2 Types of Jet Propulsion System:

The jet propulsion engines are classified basically as to their method of operation. The two main categories of jet propulsion engines are the atmospheric jet engines and the rockets. The atmospheric jet engines require oxygen from the atmospheric air for the combustion of fuel. As a result, their performance depends to a great degree on the forward speed of the engine and upon the atmospheric pressure and temperature.

The rocket engine differs from the atmospheric jet engines in that the entire mass of jet is generated from the propellants carried within the engine, i.e., the rocket engine carries its own oxidant for the combustion of the fuel and is therefore, independent of the atmospheric air. The performance of this type of power plant is independent of the forward speed and affected to a maximum of about 10% by changes in altitude.

5.2.1 Air Breathing Engines

Air breathing engines can further be classified as follows:

1. Reciprocating engines (Air screw)
2. Gas Turbine engines
   (i) Turbojet
   (ii) Turbojet with after burner (also known as turbo ramjet, turbojet with tail pipe burning and turbojet with reheater)
   (iii) Turboprop (also known as propjet).
3. Athodyds (Aero Thermodynamics Ducts)
   (i) Steady combustion system, continuous air flow – Ramjet (also known as Lorin tube)

(ii) Intermittent combustion system, intermittent air flow – Pulse jet (also known as aero pulse, retro jet, Schmidt tube and intermittent jet).

The reciprocating engine develops its thrust by accelerating the air with the help of a propeller driven by it, the exhaust of engine imparting almost negligible amount of thrust to that developed by the propeller.

The turbojet, turbojet with afterburner and turboprop are modified simple open cycle gas turbine engines. The turbojet engine consists of an open cycle gas turbine engine (compressor, combustion chamber and turbine) with an entrance air diffuser added in front of the compressor and an exit nozzle added aft of the turbine. The turbojet with afterburner is a turbojet engine with a reheater added to the engine so the extended tail pipe acts as a combustion chamber. The turboprop is a turbojet engine with extra turbine stages, a reduction gear train and a propeller added to the engine. Approximately 80 to 905 of the thrust of the turboprop is produced by acceleration of the air outside the engine by the propeller and about 10 to 20% of the thrust is produced by the jet exit of the exhaust gases. The ramjet and the pulsejet are athodyd, i.e., a straight duct type of jet engine without compressor and turbine wheels.

5.2.2 Rocket Engines

The necessary energy and momentum which must be imparted to a propellant as it is expelled from the engine to produce a thrust can be given in many ways. Chemical, nuclear or solar energy can be used and the momentum can be imparted by electrostatic or electromagnetic force.

Chemical rockets depend up on the burning of the propellant inside the combustion chamber and expanding it through a nozzle to obtain thrust. The propellant may be solid, liquid, gas or hybrid.

The vast store of atomic energy is utilized incase of nuclear propulsion. Radioactive decay or Fission or Fusion can be used to increase the energy of the propellant.

In electrical rockets electrical energy from a separate energy source is used and the propellant is accelerated by expanding in a nozzle or by electrostatic or electromagnetic forces.

In solar rockets solar energy is used to propel spacecraft.

5.3 The Ramjet Engine

The ramjet engine is an air breathing engine which operates on the same principle as the turbojet engine. Its basic operating cycle is similar to that of the turbojet. It compresses the incoming air by ram pressure, adds the heat energy to velocity and produces thrust. By converting kinetic energy of the incoming air into pressure, the ramjet is able to operate without a mechanical compressor. Therefore the engine requires no moving parts and is mechanically the simplest type of jet engine which has been devised. Since it depends on the velocity of the incoming air for the needed compression, the ramjet will not operate statistically. For this reason it requires a turbojet or rocket assist to accelerate it to operating speed.

At supersonic speeds the ramjet engine is capable of producing very high thrust with high efficiency. This characteristic makes it quite useful on high speed aircraft and missiles, where its great power and low weight make flight possible in regions where it would be impossible with any other power plant except the rocket. Ramjets have also been used at subsonic speeds where their low cost and light weight could be used to advantage.

5.3.1 Principle of Operation:

The ramjet consists of a diffuser, fuel injector, flame holder, combustion chamber and exit nozzle (Ref figure 9). The air taken in by the diffuser is compressed in two stages.

The external compression takes place takes place because the bulk of the approaching engine forces the air to change its course. Further compression is accomplished in the diverging section of the ramjet diffuser. Fuel is injected into and mixed with air in the diffuser. The flame holder provides a low velocity region favourable to flame propagation, and the fuel-air mixture recirculates within this sheltered area and ignites the fresh charge as it passes the edge of the flame holder. The burning gases then pass through the combustion chamber, increasing in temperature and therefore in volume. Because the volume of air increases, it must speed up to get out of the way off the fresh charge following behind it, and a further increase in velocity occurs as the air is squeezed out through the exit nozzle. The thrust produced by the engine is proportional to this increase in velocity.

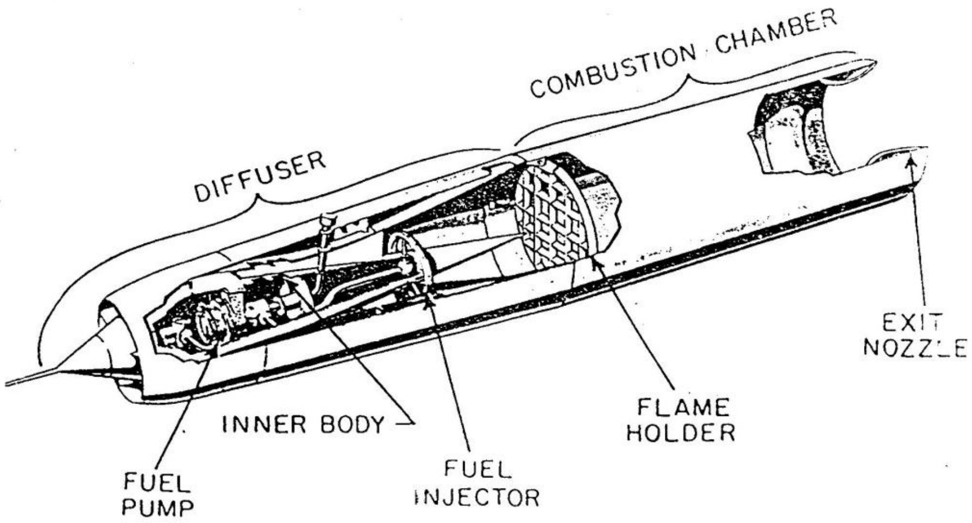
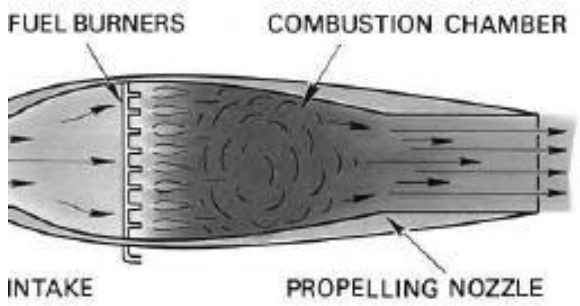
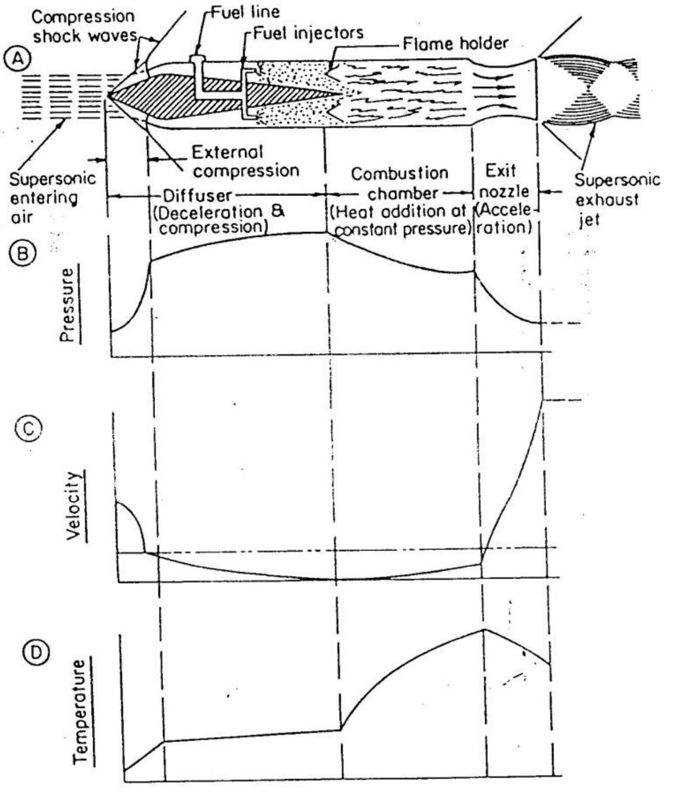


Figure 9 b Supersonic ramjet, showing components.



1-6 A ram Jet engine.



Ramjet operating principle

Figure — 9c· Operating conditions within a supersonic ramjet.

5.3.2 Advantages

- Ramjet is very simple and does not have any moving part. It is very cheap and requires almost no maintenance.
- Since turbine is not used the maximum temperature which can be allowed in ramjet is very high, about 2000 ^{0}C as compared to about 1000 ^{0}C in turbojets. This allows a greater thrust to be obtained by burning fuel at A/F ratio of about 15.1 which gives higher temperatures.
- The SFC is better than turbojet engines at high speed and high altitudes.
- There seems to be no upper limit to the flight speed of the ramjet.

5.3.3 Disadvantages

- Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start a ramjet without an external launching device.
- The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds.
- Due to high air speed, the combustion chamber requires flame holder to stabilise the combustion.
- At very high temperature of about 2000^0 C dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion.

5.3.4 Application:

- Due to its high thrust at high operational speed, it is widely used in high speed aircrafts and missiles.
- Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

5.4 Pulse Jet Engine

The pulse jet engine is an intermittent, compressor less aerodynamic power plant, with few or none of the mechanical features of conventional aviation power plants. In its simplest form, the operation of the pulse jet depends only on the properties of atmospheric air, a fuel, a shaped tube and some type of flow-check valve, and not on the interposition of pistons, impellers, blades or other mechanical part whose geometry and motion are controllable. The pulse jet differs from other types of air breathing engines, in that the air flow through it is intermittent. It can produce static thrust.

5.4.1 Operations:

During starting compressed air is forced into the inlet which opens the spring loaded flapper valves. In practice this may done by blowing compressed air though the valve box or by the motion of the engine through the air. The air that enters the engine passes by the fuel injector and is mixed with the fuel(Fig. A)

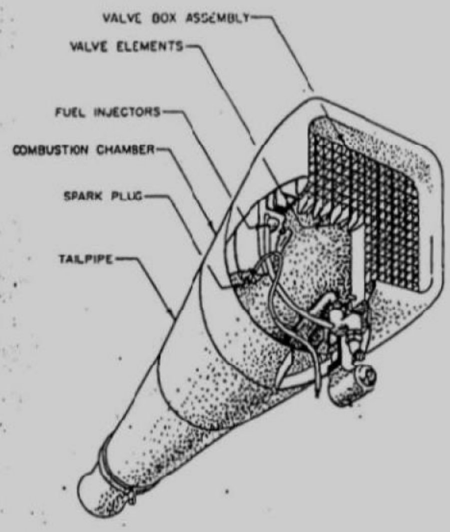


Figure ·10 Typical pulsejet engine.

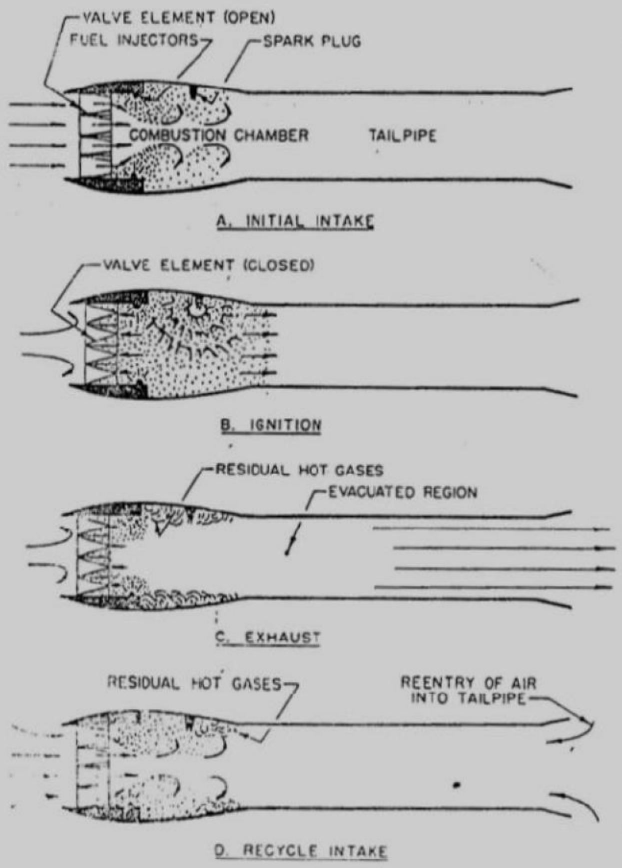


Figure ·10 Pulsejet-engine operating cycle.

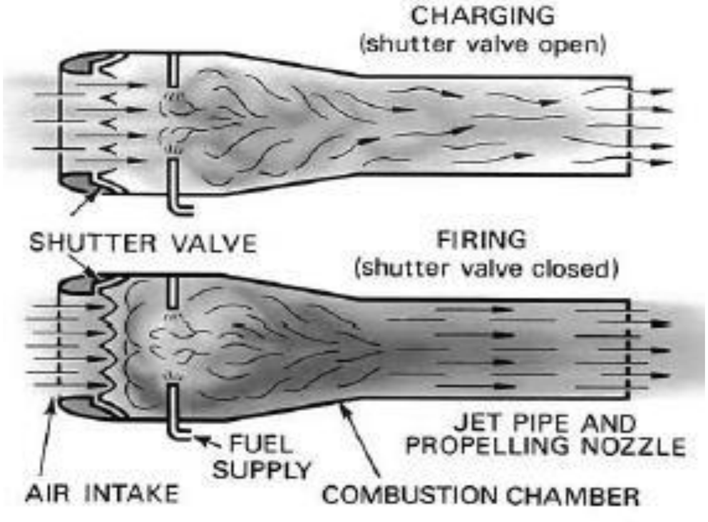


Fig. 1-7 A pulse jet engine.

When the fuel-air mixture reaches the proper proportion to burn, it is ignited by a spark plug. The burning takes place with explosive force, thus causing a very rapid rise in pressure, the increase in pressure forces the flapper valves shut and propels the charge of burned gases out of the tail pipe, as in B of the figure.

The momentum of the gases leaving the tailpipe causes the air to continue t flow out even after the pressure within the engine has reached atmospheric pressure. The pressure within the engine is therefore evacuated to below atmosphere, part C in figure.

After the pressure has reached its lowest point, atmospheric pressure (and the ram pressure if the engine is in flight) forces air into the engine through the flapper valves. At the same time, air will also be drawn in through tailpipe, since the pressure within the tailpipe is low and has nothing to prevent the entry of air, At this point, part D in figure, the engine is ready to begin another cycle. The fequency of cycles depends upon the duct shape and working temperature in V-1 rocket it was about 40 c/s which corresponds to about 2400 rpm of a two stroke reciprocating engine.

Once the engine operation has become established, the spark plug is no longer necessary. The reignition between each cycle is accomplished when the fresh charge of fuel and air is ignited by some residual flame which is left over from the preceding cycle. The air flow which reenters the tailpipe is important from both the engine operation and thrust stadpoints. Experiments have shown that the amount of air which flows into the tailpipe can be several times as much as that which flows into the inlet. This mass flow of air increases the thrust of the engine by providing additional mass for the explosion pressure to work on. It also increases the pressure within the engine at the beginning of each explosion cycle, resulting in a more efficient burning process. Reentry of air into the tailpipe is made more difficult as the airspeed surrounding the engine increases. The thrust of the engine, therefore, tends to decrease with speed. As the speed increases, the amount of reentering air flow decreases to the point where the internal pressure is eventually too low to support combustion and the engine will no longer operate.

5.4.2 Characteristics :

The chief advantages of the pulse jet are its simplicity, light weight, low cost and good zero speed thrust characteristic. Its particular disadvantages are its 650-800 km/h. operating speed limit, rather limited altitude range and somewhat unpredictable valve life.

One interesting and sometimes objectionable, feature of the pulse jet engine is the sound it makes when in operation. The sound is a series of loud reports caused by the firing of the individual charges of fuel and air in the combustion chamber. The frequency of the reports depends upon the length of the engine form the inlet valves to the end of the tailpipe and upon the temperature of the gases within the engine. The resulting sound is a continuous, loud, and vibratory note that can usually be heard for several kilometers.

The pulse jet has low thermal efficiency. In early designs the efficiency obtained was about 2 to 3% with a total flight life of 30 to 60 minutes. The maximum operating speed is seriously limited by tow factors: (i) It is possible to design a good diffuser at high speeds. (ii) The fiepper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if resonance with the cycle frequency occurs then the valve may remain open and no compression will take place. Also, as the speed increases it is difficult for air to flow back. This reduces total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases.

5.4.3 Advantages :

- This is very simple device only next to ramjet and is light in weight. It requires very small and occasional maintenance.
- Unlike ramjet, it has static thrust because of the compressed air starting, thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust.
- It can run on an almost any type of liquid fuel without much effect on the performance. It can also operate on gaseous fuel with little modifications.
- Pulse jet is relatively cheap.

5.4.4 Disadvantages :

- 1.The biggest disadvantage is very short life of flapper valve and high rates of fuel consumption. The SFC is as high as that of ramjet.
- The speed of the pulse jet is limited within a very narrow range of about 650-800 km/h because of the limitations in the aerodynamic design of an efficient diffuser suitable for a wide range.
- The high degree of vibrations due to intermittent nature of the cycle and the buzzing noise has made it suitable for pilotless crafts only.
- It has lower propulsive efficiency that turbojet engine.
- The operational range of the pulse jet is limited in altitude range.

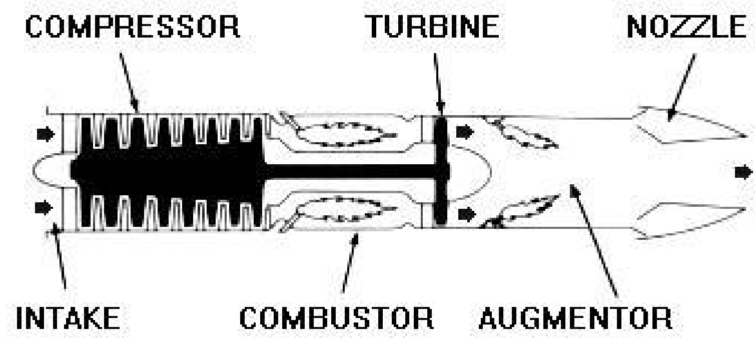
5.4.5 **Applications**:

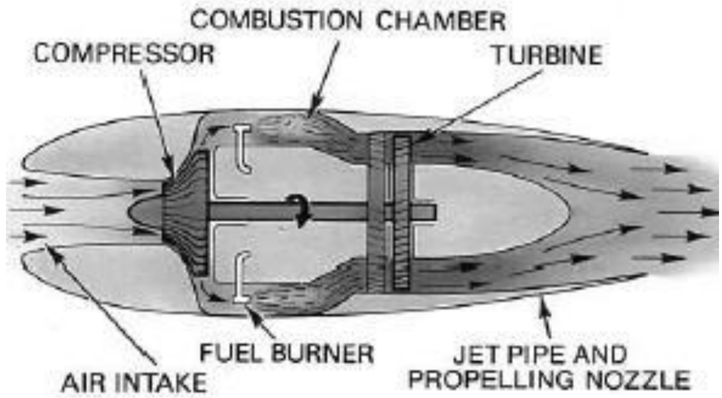
- German V-1 buzz bomb,
- American Helicopter company’s Jet Jeep Helicopter,
- Auxiliary power plant for sail planes.

5.5 The Turbojet Engine

The turbojet engine consists of diffuser which shows down the entrance air and thereby compresses it, a slows down the entrance air and thereby compresses it, a simple open cycle gas turbine and an exist gas into kinetic energy. The increased velocity, of air thereby produces thrust.

Figure 2 shows the basic arrangement of the diffuser, compressor, combustion chamber, turbine and the exhaust nozzle of a turbojet engine. Of the total pressure rise of air, a part is obtained by the rain compression in the diffuser and rest in the compressor. The diffuser converts kinetic energy of the air into pressure energy. In the ideal diffuser, the air is diffused isentopically down to zero velocity. In the actual diffuser the process is irreversible adiabatic and the air leaves the diffuser at a velocity between 60 and 120 m/s.

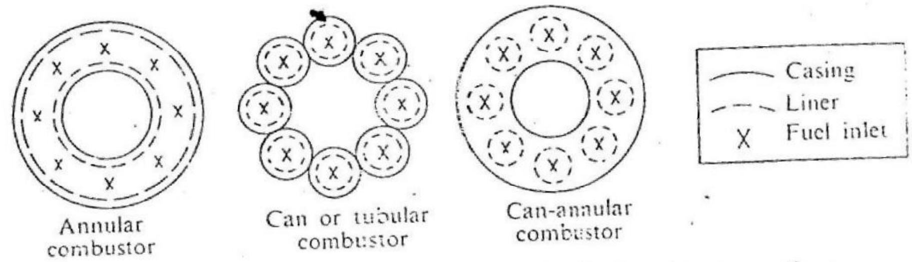




The centrifugal compressor gives a pressure ratio of about 4:1 to 5:1 in a single stage and usually a double-sided rotor is used. The turbojet using centrifugal compressor has a short and sturdy appearance. The advantages of centrifugal compressor are high durability, ease of manufacture and low cost and good operation under adverse conditions such as icing and when sand and small foreign particles are inhaled in the inlet duct. The primary disadvantage is the lack of straight-through airflow. Air leaves compressor in radial direction and ducting with the attendant pressure losses is necessary to change the direction. The axial flow is more efficient than the centrifugal type and gives the turbojet a long slim, streamlined appearance. The engine diameter is reduced which results in low aircraft drag. A multistage axial flow compressor can develop a pressure ratio as high as 6:1 or more. The air handled by it is more than that handled by a centrifugal compressor of the same diameter.

A variation of the axial compressor, the twin-spool (dual spool, split spool or coaxial) compressor has two or more sections, each revolving at or near the optimum speed for its pressure ratio and volume of air. A very high-pressure ratio of about 9:1 to 13:1 is obtained. The use of high-pressure ratio gives very good specific fuel consumption and is necessary for thrust ratings in the region of 50000 N or greater.

In the combustion chamber heat is added to the compressed air nearly at constant pressure. The three types being 'can', 'annular' and 'can-annular' (ref.fig.3). In the can type individual burners, or cans, are mounted in a circle around the engine axis with each one receiving air through its own cylindrical shroud. One of the main disadvantages of can type burners is that they do not make the best use of available space and this results in a large diameter engine. On the other hand, the burners are individually removable for inspection and air-fuel patterns are easier to control than in annular designs. The annular burner is essentially a single chamber made of concentric cylinders mounted co-axially about the engine axis. This arrangement makes more complete use of available space, has low pressure loss, fits well with the axial compressor and turbine and form a technical viewpoint has the highest efficiency, but has a disadvantage in that structural problems may arise due to the large diameter, thin-wall cylinder required with this type of chamber. The problem is more severe for larger engines. There is also some disadvantage in that the entire combustor must be removable from the engine for inspection and repairs. The can- annular design also makes good use of available space, but employs a number of individually replaceable cylindrical inner liners that receive air through a common annular housing for good control of fuel and air flow patterns. The can-annular arrangement has the added advantage of greater structural stability and lower pressure loss than that of the can type.



Combustion chamber, longitudinal and cross sections.

Figure 3

The heated air then expands through the turbine thereby increasing its velocity while losing pressure. The turbine extracts enough energy to drive the compressor and the necessary auxiliary equipments. Turbines of the impulse, reaction and a combination of both types are used. In general, it may be stated that those engines of relatively low thrust and simple design employ the impulse type, while those of large thrust employ the reaction and combination types.

The hot gas is then expended through the exit nozzle and the energy of the hot gas is converted into as much kinetic energy as is possible. This change in velocity of the air passing through the engine multiplied by the mass flow of the air is the change of momentum, which produces thrust. The nozzle can be a fixed jet or a variable area nozzle. The variable area nozzle permits the turbojet to operate at maximum efficiency over a wide range of power output. Clamshell, Finger or Iris, Centre plug with movable shroud, annular ring, annular ring with movable shroud are the various types of variable area nozzle for turbojet engines. The advantage of variable area nozzle is the increased cost, weight and complexity of the exhaust system.

The needs and demands being fulfilled by the turbojet engine are

- Low specific weight – ¼ to ½ of the reciprocating engine
- Relative simplicity – no unbalanced forces or reciprocating engine
- Small frontal area, reduced air cooling problem- less than ¼ th the frontal area of the reciprocating engine giving a large decrease in nacelle drag and consequently giving a greater available excess thrust or power, particularly at high speeds.
- Not restricted in power output - engines can be built with greatly increased power output over that of the reciprocating engine without the accompanying disadvantages.
- Higher speeds can be obtained – not restricted by a propeller to speeds below 800 km/h.

5.6 Turboprop Engine (Propeller turbine, turbo-propeller, prop jet, turbo-prop)

For relatively high take-off thrust or for low-speed cruise applications, turboprop engines are employed to accelerate a secondary propellant stream, which is much larger than the primary flow through the engine. The relatively low work input per unit mass of secondary air can be adequately transmitted by a propeller. Though a ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan could also be used for this purpose, a propeller is generally lighter compared to ducted fan engine and with variable pitch, it is capable of a wider range of satisfactory performance.

In general, the turbine section of a turboprop engine is very similar to that of a turbojet engine. The main difference is the design and arrangement of the turbines. In the turbojet engine the turbine is designed to extract only enough power from the high velocity gases to drive the compressor, leaving the exhaust gases with sufficient velocity to produce the thrust required of the engine. The turbine of the turboprop engine extracts enough power from the gases to drive both the compressor and the propeller. Only a small amount of power is left as thrust. Usually a turboprop engine has two or more turbine wheels. Each wheel takes additional power from the jet stream, with the result that the velocity of the jet is decreased substantially.

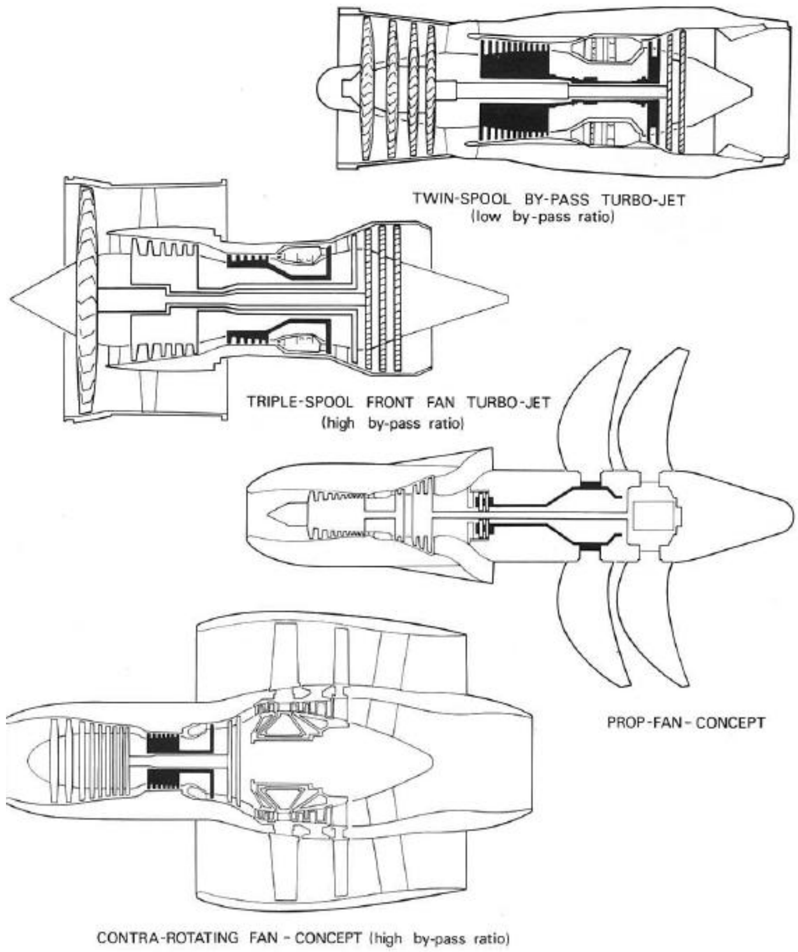
Figure 6 shows a schematic diagram of a turboprop engine. The air enters the diffuser as in a turbojet and is compressed in a compressor before passing to the combustion chamber. The compressor in the turboprop is essentially an axial flow compressor. The products of combustion expand in a two-stage or multistage turbine. One stage of the turbine drives the compressor and the other drives the propeller.

Thus the turbine expansion is used to drive both compressor as well as propeller and less energy is available for expansion in the nozzle. Due to lower speeds of propeller a reduction gear is necessary between turbine and the propeller. About 80 to 90% of the available energy in exhaust is extracted by the turbine while rest, about 10 to 20%, contributes the thrust by increasing the exhaust jet velocity.

Total thrust = jet thrust + propeller thrust

Turboprop engines combine in them the high take-off thrust and good propeller efficiency of the propeller engines at speeds lower than 800 km/h and the small weight, lower frontal area and reduced vibration and noise of the pure turbojet engine.

Its operational range is between that of the propeller engines and turbojets though it can operate in any speed up to 800 km/h.



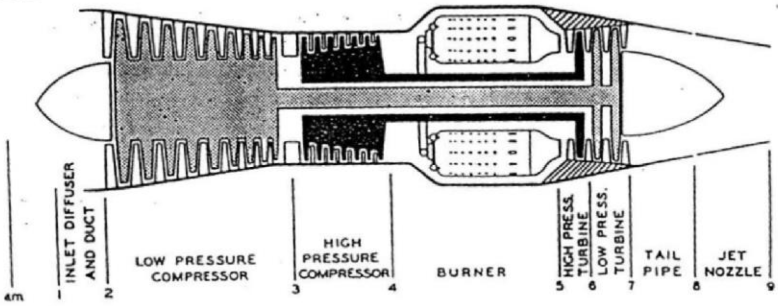


Figure 4 Arrangement of a split-compressor, axial-flow engine.

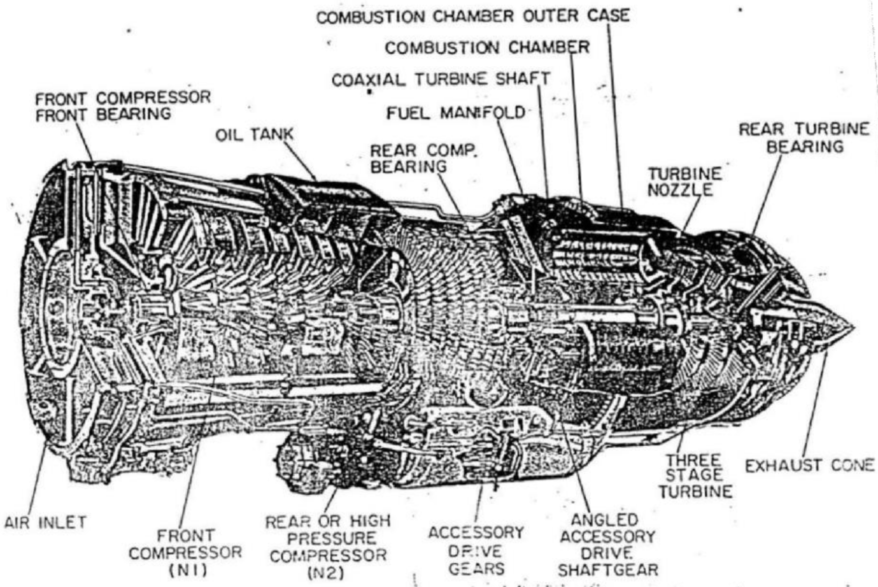


Figure 5 Pratt & Whitney J57 turbojet engine.

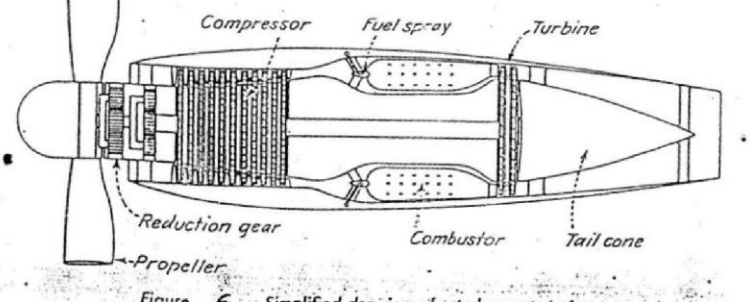
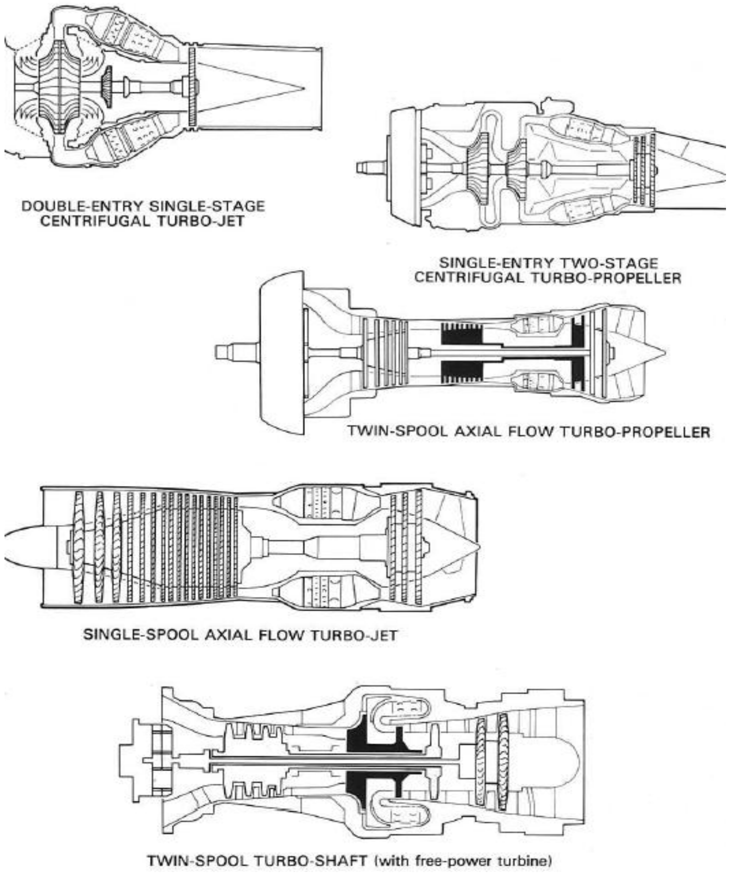


Figure 6 Simplified drawing of a turboprop engine.



The power developed by the turboprop remains almost same at high altitudes and high speeds as that under sea-level and take-off conditions because as speed increases ram effect also increases. The specific fuel consumption increases with increase in speed and altitude. The thrust developed is high at take-off and reduces at increased speed.

5.6.1 Advantages

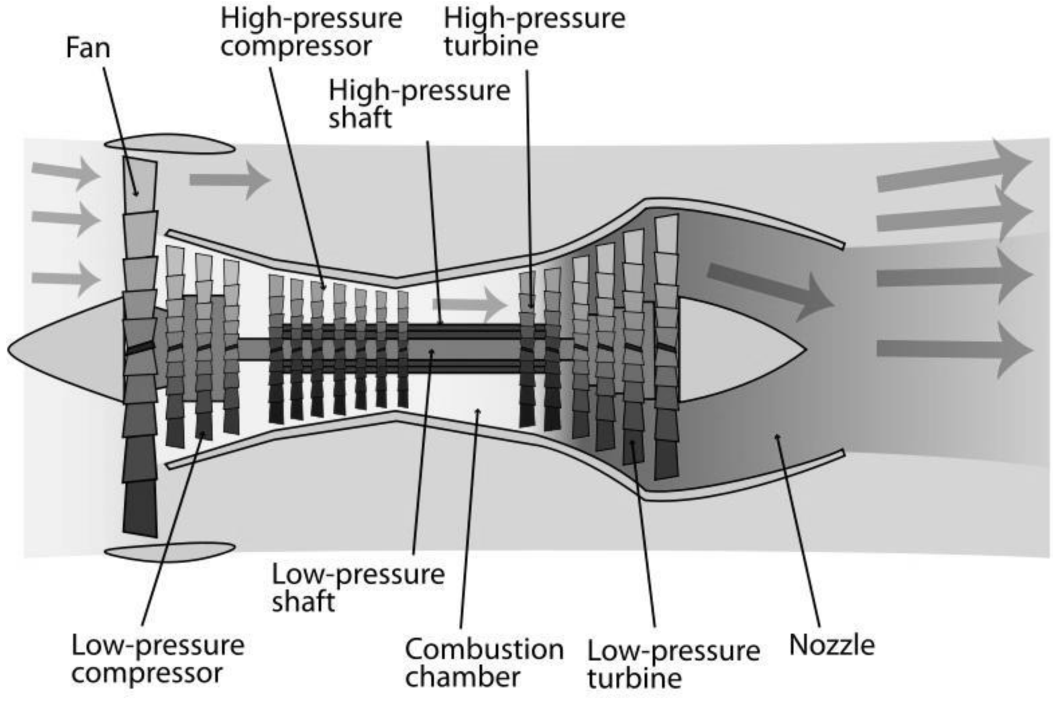
- Turboprop engines have a higher thrust at take-off and better fuel economy.
- The frontal area is less than air screw so that drag is reduced.
- 3.The turboprop can operate economically over a wide range of speeds ranging from low speeds, where pure jet engine is uneconomical, to speeds of about 800 km/h where the propeller engine efficiency islow.
- It is easy to maintain and has lower vibrations and noise.
- The power output is not limited as in the case of propeller engines (air screw).
- The multicast arrangement allows a great flexibility of operation over a wide range of speeds.

5.6.2 Disadvantages

- The main disadvantage is that at high speeds due to shocks and flow separation, the propeller efficiency decreases rapidly, thereby, putting up a maximum speed limit on the engine.
- It requires a reduction gear which increases the cost and also consumes certain energy developed by the turbine in addition to requiring more space.

5.7 The Turbofan Engine

The turboprop is limited to mach number of about 0.7 because of the sharp decrease in propeller efficiency encountered above that mach number. However, the turboprop concept of increasing mass flow rate without producing an excessive increment in exhaust velocity is valid at any mach number and the use of a ducted fan combined with a jet turbine provides more economical operation at mach numbers close to unity than does the simple jet turbine. If a duct or shroud is placed around a jet engine and air is pumped through the annular passage by means of one or more sets of compressor blades, the resulting engine is called a turbofan, and is capable of producing (under proper conditions) somewhat better thrust specific fuel consumption characteristics than the turbojet itself. Basically, the air passing through the fan bypasses the combustion process but has energy added to it by the compressor fan, so that a sizable mass flow can be shunted through the fan. The air which bypasses the combustion process leaves the engine with a lower amount of internal energy and a lower exhaust speed than the jet exhaust. Yet, the thrust is not decreased since the turbofan can pump more air per unit time than a conventional jet at subsonic speeds. Accordingly, the average exhaust velocity of the turbofan (averaging the turbine flow and the bypass flow) can be made smaller at a given flight speed than that of a comparable turbojet and greater efficiency can be obtained. In turbofan engine the fan cannot be designed for all compressor ratios which is efficient at all mach numbers, thus, the turbofan is efficient over a rather limited range of speeds. Within this speed range, however, its improved cruise economy makes it a desirable unit for jet transport aircraft.



The turbofan engine has a duct enclosed fan mounted at the front or rear of the engine and driven either mechanically geared down or at the same speed as the compressor, or by an independent turbine located to the rear of the compressor drive turbine (Ref. Figure 7). There are two methods of handling the fan air. Either the fan can exit separately from the primary engine air, or it can be ducted back to mix with the primary engine's air at the rear. If the fan air is ducted to the rear, the total fan pressure must be higher than the static pressure in the primary engine's exhaust, or air will not flow. Similarly, the static fan discharge pressure must be less than the total pressure the primary engine's exhaust, or the turbine will not be able to extract the energy required to drive the compressor and fan. By closing down the area of flow of the fan duct, the static pressure can be reduced and the dynamic pressure is increased.

The efficiency of the fan engine is increased over that of the pure jet by converting more of the fuel energy into pressure energy rather than the kinetic energy of a high velocity exhaust gas stream. The fan produces additional force or thrust without increasing fuel flow. As in the turboprop primary engine exhaust gas velocities and pressures are low because of the extra turbine stages needed to drive the fan, and as a result this makes the turbofan engine much quieter. One fundamental difference between the turbofan and the turboprop engine is that the air flow through the fan is controlled by design so that the air velocity relative to the fan blades is unaffected by the aircraft's speed. This eliminates the loss in operational efficiency at high air speeds which limits the maximum air speed of propeller driven aircraft.

Fan engines show a definite superiority over the pure jet engines at speeds below Mach 1. The increased frontal area of the fan presents a problem for high- speed aircraft which, of course require small frontal areas. At high speeds air can be offset at least partially by burning fuel in the fan discharge air. This would expand the gas, and in order to keep the fan discharge air at the same pressure, the area of the fan jet nozzle is increased. This action results in an increase in gross thrust due to an increase in pressure times an area (PA), and an increase in gross thrust specific fuel consumption.

5.8 Nozzle and diffuser efficiencies

In ideal case, flow through nozzle and diffuser is isentropic. But in actual case, friction exists and affects in following ways:
i) Reduces the enthalpy drop reduces the final velocity of steam iii) Increases the final dryness fraction iv) Increases specific volume of the fluid v) Decreases the mass flow rate

5.8.1 Nozzle performance

The isentropic operating conditions are very easy to determine. Frictional losses in the nozzle can be accounted by several methods.

(1) Direct information on the entropy change could be given although this is usually not available.
(2) Some times equivalents information is provided in the form of stagnation pressure ratio. Normally nozzle performance is indicated by efficiency parameter defined as

$$\eta_N = \frac{\text{actual change in KE}}{\text{total change in KE}}$$

$$\text{i.e., } \eta_N = \left.\frac{(\Delta\ KE)_{actual}}{(\Delta\ KE)_{ideal}}\right]_{Pb}$$

From SFEE

$$h + \frac{V^2}{2} = h_o$$

$$\Delta\ KE]_{actual} = h_o - h_a$$

$$\Delta\ KE]_{ideal} = h_o - h_i$$

$$\text{i.e., } \eta_N = \left.\frac{h_o - h_a}{h_o - h_i}\right]_{Pb}$$

$$\eta_N = \left.\frac{T_o - T_a}{T_o - T_i}\right]_{Pb}$$

Note : If the nozzle inlet is not stagnant.

$$\eta_N = \frac{\frac{V_{2a}^2}{2} - \frac{V_i^2}{2}}{\frac{V_{2i}^2}{2} - \frac{V_i^2}{2}}$$

$$= \frac{h_1 - h_{2a}}{h_1 - h_{2i}}$$

$$\eta_N = \frac{T_1 - T_{2a}}{T_1 - T_{2i}}$$

5.9 Problems:

1. Air is discharged from a C - D nozzle. Pilot-tube readings at inlet and exist of the nozzle ae 6.95 × 105 N/m^2 and 5.82 × 10^5 N/m^2 respectively. The inlet stagnation temp 250^{o}C and exit static pressure, 1.5 × 10^5 N/m^2. Find the inlet and exit stagnation pressure, exit Mach numbers, exit flow velocity and nozzle efficiency.

Note:

i) (Pitot - tube reading gives stagnation pressure)

ii) (Since stagnation pr. vlaues are different for inlet and exit the flow is no longer isentropic) Mach number at exist (M_a). Consider the isentropic deceleration process shown (a - oa)

$$M_a \Big]_{\frac{P_a}{P_{oa}} = \frac{1.5\times10^5}{5.82\times10^5} = 0.257} = 1.54$$

Exit flow velocity (V_a)

$$Ma = \frac{V_a}{C_a}$$

$$Ca = \sqrt{KRT_a}$$

$$\frac{T_a}{T_{oa}}\Big]_{M=1.54} = 0.678$$

$$T_a = 6.23 \times 0.678$$

$$= 422.4 \text{ K}$$

$$C_a = 20.05 \sqrt{422.4}$$

$$= 412.1 \text{ ms}^{-1}$$

$$V_a = M_a \times C_a$$

$$= 412.1 \times 1.54$$

$$= 634.6 \text{ ms}^{-1}$$

Nozzle efficiency (h_N)

$$h_N = \frac{T_o - T_a}{T_o - T_i}$$

To get T_i

For the isentropic i - oi

$$M_i \Big]_{\frac{P_i}{P_{oi}} = \frac{1.5\times10^5}{6.75\times10^5} = 0.222}$$

$$\frac{T_i}{T_{oi}}\Big]_{M=1.64} = 0.650$$

$T_i = 623 \times 0.65$

$= 404.95$ K

Nozzle efficiency

$$(h_N) = \frac{T_o - T_a}{T_o - T_i} \times 100$$

$$= \left(\frac{623 - 422.4}{623 - 404.95}\right) \times 100$$

$h_N = 91.99$ %

2. A converging, nozzle operating with air and inlet conditions of P = 4 Kg/cm^2, T_0 = 450°C and T = 400°C is expected to have an exit static pressure of 2.5 Kgf/cm^2 under ideal conditions. Estimate the exit temperature and mach number, assuming a nozzle efficiency = 0.92 when the expansion takes place to the same back pressure.

$$M\Big]_{\frac{T}{T_0} = \frac{673}{723} = 0.93} = 0.61$$

$$\frac{P}{P_0}\Big]_{M=0.61} = 0.778$$

$$P_0 = \frac{4}{0.778} = 5.14 \text{ Kgf / cm}^2$$

$$M_{2i}\Big]_{\frac{P_{2i}}{P_0} = \frac{2.5}{5.14} = 0.486} = 1.07$$

$$\frac{T_{2i}}{T_0}\Big]_{M=1.07} = 0.814$$

$T_{2i} = 723 \times 0.814 = 588.5$ K

Nozzle efficiency

E $$h_N = \frac{T_1 - T_{2a}}{T_1 - T_{2i}}$$

$$0.92 = \frac{673 - T_{2a}}{673 - 588.5}$$

$$T_{2a} = 595.3 \text{ K}$$

$$M_{2a}\Big]_{\frac{p_{2a}}{p_{20}} = \frac{595.3}{7.23} = 0.823} = 1.04$$

3. An aircraft flies at a speed of 520 kmph at an altitude of 8000 m. The diameter of the propeller of an aircraft is 2.4 m and flight to jet speed ratio is 0.74. Find the following:

(i) The rate of air flow through the propeller

(ii) Thrust produced

(iii) Specific thrust

(iv) Specific impulse

(v) Thrust power Given:

Air craft speed (or) Flight speed = 520 kmph

$$= \frac{520 \times 10^3}{3600\ s}$$

$= 144.44\ m/s$

Altitude z = 8000 m

Diameter of the propeller $d = 2.4\ m$

Flight to jet speed ratio $\sigma = \frac{u}{c_j} = 0.74$

Where c_j – *jet speed (or) Speed of exit gases from the engine*

Solution : Area of the propeller disc $A = \frac{\pi}{4} d^2$

$$= \frac{\pi}{4}(2.4)^2$$

$$= \boxed{A = 4.52\ m^2}$$

From gas tubes at $Z = 8000 m$

$$\rho = 0.525\ kg/m^3$$

Effective speed ratio $, \sigma = \frac{u}{c_j}$

$$0.74 = \frac{144.44}{c_j}$$

$\boxed{Velocity\ of\ jet, c_j = 195.19\ m/s}$

Velocity of air flow at the propeller

$$c = \frac{1}{2}\left[u + c_j\right]$$

$$= \frac{1}{2}\left[144.44 + 195.19\right]$$

$$\boxed{c = 169.81\ m/s}$$

Mass flow rate of air – fuel mixture

$$m^{\bullet} = \rho\ A\ c$$

$$= 0.525 \times 4.52 \times 169.81$$

$$\boxed{m^{\bullet} = 402.96\ kg/s}$$

We know that

$$m^{\bullet} = m^{\bullet}_a + m^{\bullet}_f$$

since *mass flow rate of fuel* $(m^{\bullet}_f)$ *is not given, let us take*

$$m^{\bullet} = m^{\bullet}_a$$

$\boxed{Mass\ flow\ rate\ of\ air\ m^{\bullet}_a = 402.96\ kg/s}$

Thrust produced, $F = m^{\bullet}_a\left[c_j - u\right]$

$$= 402.96\left[195.19 - 144.44\right]$$

$$\boxed{F = 20.45 \times 10^3\ N}$$

Specific thrust $F_{sp} = \dfrac{F}{m^{\bullet}}$

$$= \frac{F}{m^{\bullet}_a}$$

$$= \frac{20.45 \times 10^3}{402.96}$$

$$\boxed{F_{sp} = 50.75\ N/(kg/s)}$$

$$Specific\ impulse\ (I_{sp}) = \frac{F}{W}$$

$$= \frac{F}{m^{\bullet} g}$$

$$= \frac{F}{m^{\bullet}_a \times g}$$

$$= \frac{20.45\times10^3}{402.96\times9.81}$$

$$\boxed{I_{sp} = 5.17\ s}$$

$$Thrust\ \ power\ ,\ P = Thrust(F)\times\ Flight\ \ speed\ (u)$$

$$= 20.45\times10^3\times144.44$$

$$P = 2.95\times10^6\ W$$

$$\text{Result}:\ \ (i)\ m^{\bullet}_a\ = 402.96\ kg/s$$

$$(ii)\ F\ = 20.45\times10^3\ N$$

$$(iii)\ \boxed{F_{sp} = 50.75\ N/(kg/s)}$$

$$(iv)\ \boxed{I_{sp} = 5.17\ s}$$

$$(v)\ \boxed{p = 2.95\times10^6 W}$$

5.10 Tutorial Problems:

1. The diameter of the propeller of an aircraft is 2.5m; It flies at a speed of 500Kmph at an altitude of 8000m. For a flight to jet speed ratio of 0.75 determine (a) the flow rate of air through the propeller, (b) thrust produced (c) specific thrust, (d) specific impulse and (e) the thrust power.

2. Aircraft speed of 525 Kmph. The data for the engine is given below
Inlet diffuser efficience =0.875
Compressor efficieny =0.790
Velocity of air at compressor entry =90m/s
Properties of air : γ =1.4, Cp =1.005 KJ/kg K

3. An aircraft flies at 960Kmph. One of its turbojet engines takes in 40 kg/s of air and expands the gases to the ambient pressure .The air ‑fuel ratio is 50 and the lower calorific value of the fuel is 43 MJ/Kg .For maximum thrust power determine (a)jet velocity (b) thrust (c) specific thrust (d) thrust power (e) propulsive, thermal and overall efficiencies and (f) TSFC

4. A turboprop engine operates at an altitude of 3000 meters above mean sea level and an aircraft speed of 525 Kmph. The data for the engine is given below

Inlet diffuser efficience =0.875,Compressor efficieny =0.790. Velocity of air at compressor entry =90m/s Properties of air :_ =1.4, Cp =1.005 KJ/kg K

5. A turbo jet engine propels an aircraft at a Mach number of 0.8 in level flight at an altitude of 10 km .The data for the engine is given below: Stagnation temperature at the turbine inlet =1200K Stagnation temperature rise through the compressor =175 K Calorific value of the fuel =43 MJ/Kg Compressor efficiency =0.75. Combustion chamber efficiency =0.975, Turbine efficiency =0.81,Mechanical efficiency of the power transmission between turbine and compressor =0.98, Exhaust nozzle efficiency=0.97, Specific impulse =25 seconds.Assuming the same properties for air and combustion gases calculate,

i. Fuel ‑air ratio
ii. Compressor pressure ratio,
iii. Turbine pressure ratio
iv. Exhaust nozzles pressure ratio ,and
v. Mach number of exhaust jet

SPACE PROPULSION

5.1 Rocket Propulsion

In the section about the rocket equation we explored some of the issues surrounding the performance of a whole rocket. What we didn't explore was the heart of the rocket, the motor. In this section we'll look at the design of motors, the factors which affect the performance of motors, and some of the practical limitations of motor design. The first part of this section is necessarily descriptive as the chemistry, thermodynamics and maths associated with motor design are beyond the target audience of this website.

5.2 General Principles of a Rocket Motor

In a rocket motor a chemical reaction is used to generate hot gas in a confined space called the combustion chamber. The chamber has a single exit through a constriction called the throat. The pressure of the hot gas is higher than the surrounding atmosphere, thus the gas flows out through the constriction and is accelerated.

5.3 Propellants

The chemical reaction in model rocket motors is referred to as an "exothermal redox" reaction. The term "exothermal" means that the reaction gives off heat, and in the case of rocket motors this heat is mainly absorbed by the propellants raising their temperature.

The term "redox" means that it is an oxidation/reduction reaction, in other words one of the chemicals transfers oxygen atoms to another during the reaction. The two chemicals are called the oxidising agent and the reducing agent.

The most popular rocket motors are black powder motors, where the oxidising agent is saltpetre and the reducing agents are sulphur and carbon. Other motors include Potassium or ammonium perchlorate as the oxidising agent and mixtures of hydrocarbons and fine powdered metals as the reducing agents. Other chemicals are often added such as retardants to slow down the rate of burn, binding agents to hold the fuel together (often these are the hydrocarbons used in the reaction), or chemicals to colour the flame or smoke for effects. In hybrid motors a gaseous oxidiser, nitrous oxide, reacts with a hydrocarbon, such as a plastic, to produce the hot gas.

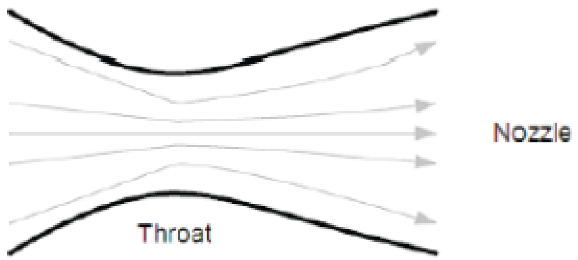
5.4 Energy Conversion

This reaction releases energy in the form of heat, and by confining the gas within the combustion chamber we give it energy due to its pressure. We refer to the energy of this hot pressurised gas as its "enthalpy". By releasing the gas through the throat the rocket motor turns the enthalpy of the gas into a flow of the gas with kinetic energy. It is this release of energy which powers the rocket. So the energy undergoes Two conversions:

- Chemical energy to enthalpy
- Enthalpy to kinetic energy

The conversion from chemical energy to enthalpy takes place in the combustion chamber. To obtain the maximum enthalpy it is clearly important to have a reaction which releases lots of heat and generates lots of high energy molecules of gas to maximise pressure here is clearly a limit to the temperature & pressure, as the combustion chamber may melt or split if these are too high. The designer has a limitation placed on his choice of reagents in that the reaction must not heat the combustion chamber to a point where it is damaged, nor must the pressure exceed that which the chamber can survive.

Changing enthalpy to kinetic energy takes place in the throat and the nozzle. Our mass of hot gas flows into the throat, accelerating as the throat converges. If we reduce the chameter of the throat enough, the flow will accelerate to the speed of sound, at which pint something unexpected occurs. As the flow diverges into the nozzle it continues to accelerate beyond the speed of sound, the increase in velocity depending on the increase in area. This type of nozzle is called a De Laval nozzle.



5.5 Kinetic energy of a body:

If we consider a small volume of gas, it will have a very low mass. As we accelerate this gas it gains kinetic energy proportional to the square of the velocity, so if we double the velocity we get four times the kinetic energy. The velocity of the supersonic flow increases proportional to the increase in area of the nozzle, thus the kinetic energy increases by the fourth power of the increase in nozzle diameter. Thus doubling the nozzle diameter increases the kinetic energy by 16 times! The De Laval nozzle make rocket motors possible, as only such high velocity flows can generate the energy required to accelerate a rocket.

In model rockets the reaction is chemical generally short lived, a few seconds at most, sothe amount of heat transferred to the structural parts of the motor is limited. Also, the liner of the motor casing acts to insulate the casing from the rapid rise in temperature which would result from a reaction in direct contact with the metal casing. Model rocket motors also run at quite low pressure, well below the limits if the motor casing, further protecting the casing. It can be seen that the enthalpy of a model rocket motor is thus quite low. In large launch vehicles such as Ariane, the pressure and temperature are high, the burn may last several minutes, and the mass budget for the designer is very tight. Designing motors for these purposes is highly complex.

5.6 Thrust

The basic principles of a rocket motor are relatively straightforward to understand. In rocketry the motor exists to accelerate the rocket, and thus it has to develop a force called “thrust”. One of several definitions of force is that: Force = rate of change of momentum If we ignore (for a few paragraphs) any external effects we can say that the thrust is entirely due to the momentum of the propellant, a force called the “momentum thrust”. If we denote the thrust as F and the momentum as P, then mathematically:

$$F = \frac{dP}{dt}$$

Sometimes for mathematical clarity we us the notation of P with a dot on top to denote the first derivative of P, and with 2 dots for the second derivative. Thus, in this new notation:

$$F = \frac{dP}{dt} = \dot{P}$$

You may also recall from the section on the rocket equation that momentum is the product of the mass and velocity. Thus we can say that the momentum of the flow from the nozzle of the rocket has a momentum:

$$P = mv_e$$

Thus:

$$F = \frac{dP}{dt} = \dot{P} = \frac{d(mv_e)}{dt}$$

If the exhaust velocity remains constant, which is a reasonable assumption, we arrive at the equation:

$$F = v_e \frac{d(m)}{dt} = \dot{m}v_e$$

The term “m-dot” is known as the mass flow rate, in other words the rate at which mass is ejected through the nozzle in kg/sec. In other words this is the rate at which the rocket burns fuel. This is an interesting relationship, which can be expressed in words as: Momentum

Thrust = mass flow rate x exhaust velocity

5.7 Flow expansion

The propellant is accelerated into the atmosphere. As it leaves the nozzle the propellant has an exit pressure Pexit and enters an atmosphere which has a pressure Patm. The transition from one pressure to the other cannot happen instantaneously as any pressure difference will cause a flow of high pressure fluid into the low pressure region.

So the force (a component of thrust called "pressure thrust") depends on the pressure difference and the area

$$F = \dot{m} v_e + A(P_{exit} - P_{atm})$$

5.8 Types of Rocket Engines:

Rocket or rocket vehicle is a missile, spacecraft, aircraft or other vehicle which obtains thrust from a rocket engine. In all rockets, the exhaust is formed entirely from propellants carried within the rocket before use. Rocket engines work by action and reaction. Rocket engines push rockets forwards simply by throwing their exhaust backwards extremely fast.

Rockets for military and recreational uses date back to the 13th century. Significant scientific, interplanetary and industrial use did not occur until the 20th century, when rocketry was the enabling technology of the Space Age, including setting foot on the moon.

Rockets are used for fireworks, weaponry, ejection seats, launch vehicles for artificial satellites, human spaceflight and exploration of other planets. While comparatively inefficient for low speed use, they are very lightweight and powerful, capable of generating large accelerations and of attaining extremely high speeds with reasonable efficiency.

Chemical rockets are the most common type of rocket and they typically create their exhaust by the combustion of rocket propellant. Chemical rockets store a large amount of energy in an easily released form, and can be very dangerous. However, careful design, testing, construction and use minimize risks.

Rocket vehicles are often constructed in the archetypal tall thin "rocket" shape that takes off vertically, but there are actually many different types of rockets including, tiny models such as balloon rockets, water rockets, skyrockets or small solid rockets that can be purchased at a hobby store missiles space rockets such as the enormous Saturn V used for the Apollo program rocket cars rocket bike, rocket powered aircraft (including rocket assisted takeoff of conventional aircraft- JATO), rocket sleds rocket trains rocket torpedos, rocket powered jet packs, rapid escape systems such as ejection seats and launch escape systems space probes

Propellants:

A propellant is a material that is used to move ("propel") an object. The material is usually expelled by gas pressure through a nozzle. The pressure may be from a compressed gas, or a gas produced by a chemical reaction. The exhaust material may be a gas, liquid, plasma, or, before the chemical reaction, a solid, liquid or gelled. Common chemical propellants consist of a fuel; like gasoline, jet fuel, rocket fuel, and an oxidizer. Propellant used for propulsion Technically, the word propellant is the general name for chemicals used to create thrust.

For vehicles, the term propellant refers only to chemicals that are stored within the vehicle prior to use, and excludes atmospheric gas or other material that may be collected in operation.

Amongst the English-speaking laymen, used to having fuels propel vehicles on Earth, the word fuel is inappropriately used. In Germany, the word Treibstoff—literally "drive-stuff"—is used; in France, the word ergols is used; it has the same Greek roots as hypergolic, a term used in English for propellants which combine spontaneously and do not have to be set ablaze by auxiliary ignition system.

In rockets, the most common combinations are bipropellants, which use two chemicals, a fuel and an oxidiser. There is the possibility of a tripropellant combination, which takes advantage of the ability of substances with smaller atoms to attain a greater exhaust velocity, and hence propulsive efficiency, at a given temperature. Although not used in practice, the most developed tripropellant systems involves adding a third propellant tank containing liquid hydrogen to do this.

5.8 Solid propellant:

In ballistics and pyrotechnics, a propellant is a generic name for chemicals used for propelling projectiles from guns and other firearms. Propellants are usually made from low explosive materials, but may include high explosive chemical ingredients that are diluted and burned in a controlled way (deflagration) rather than detonation. The controlled burning of the propellant composition usually produces thrust by gas pressure and can accelerate a projectile, rocket, or other vehicle. In this sense, common or well known propellants include, for firearms, artillery and solid propellant rockets: Gun propellants, such as:

- Gunpowder (black powder)
- Nitrocellulose-based powders
- Cordite
- Ballistite
- Smokeless powders

Composite propellants made from a solid oxidizer such as ammonium perchlorate or ammonium nitrate, a rubber such as HTPB, or PBAN (may be replaced by energetic polymers such as polyglycidyl nitrate or polyvinyl nitrate for extra energy) , optional high explosive fuels (again, for extra energy) such as RDX or nitroglycerin, and usually a powdered metal fuel such as aluminum.

Some amateur propellants use potassium nitrate, combined with sugar, epoxy, or other fuels / binder compounds.

Potassium perchlorate has been used as an oxidizer, paired with asphalt, epoxy, and other binders.

5.9 Grain

Propellants are used in forms called grains. A grain is any individual particle of propellant regardless of the size or shape. The shape and size of a propellant grain determines the burn time, amount of gas and rate produced from the burning propellant and consequently thrust vs time profile.

There are three types of burns that can be achieved with different grains.

- Progressive Burn:
  Usually a grain with multiple perforations or a star cut in the center providing a lot of surface area.

- Digressive Burn
  Usually a solid grain in the shape of a cylinder or sphere.
- Neutral Burn
  Usually a single perforation; as outside surface decreases the inside surface increases at the same rate.

5.10 Composition

There are four different types of solid propellant compositions:

Single Based Propellant:

A single based propellant has nitrocellulose as its chief explosives ingredient. Stabilizers and other additives are used to control the chemical stability and enhance the propellant's properties.

Double Based Propellant:

Double based propellants consist of nitrocellulose with nitroglycerin or other liquid organic nitrate explosives added. Stabilizers and other additives are used also. Nitroglycerin reduces smoke and increases the energy output. Double based propellants are used in small arms, cannons, mortars and rockets.

Triple Based Propellant

Triple based propellants consist of nitrocellulose, nitroquanidine, nitroglycerin or other liquid organic nitrate explosives. Triple based propellants are used in cannons.

5.11 Composite

Composites contain no nitrocellulose, nitroglycerin, nitroquanidine or any other organic nitrate. Composites usually consist of a fuel such as metallic aluminum, a binder such as synthetic rubber, and an oxidizer such as ammonium perchlorate. Composite propellants are used in large rocket motors.

5.12 Liquid propellant

Common propellant combinations used for liquid propellant rockets include:

- Red fuming nitric acid (RFNA) and kerosene or RP-1
- RFNA and Unsymmetrical dimethyl hydrazine (UDMH) Dinitrogen tetroxide and UDMH, MMH and/or hydrazine Liquid oxygen and kerosene or RP-1
- Liquid oxygen and liquid hydrogen
- Liquid oxygen and ethanol
- Hydrogen peroxide and alcohol or RP-1
- Chlorine pentafluoride and hydrazine

Common monopropellant used for liquid rocket engines include:

- Hydrogen peroxide
- Hydrazine
- Red fuming nitric acid (RFNA)

Introducing propellant into a combustion chamber

Rocket propellant is mass that is stored, usually in some form of propellant tank, prior to being ejected from a rocket engine in the form of a fluid jet to produce thrust.

Chemical rocket propellants are most commonly used, which undergo exothermic chemical reactions which produce hot gas which is used by a rocket for propulsive purposes. Alternatively, a chemically inert reaction mass can be heated using a high- energy power source via a heat exchanger, and then no combustion chamber is used.

A solid rocket motor:

Solid rocket propellants are prepared as a mixture of fuel and oxidizing components called 'grain' and the propellant storage casing effectively becomes the combustion chamber. Liquid-fueled rockets typically pump separate fuel and oxidiser components into the combustion chamber, where they mix and burn. Hybrid rocket engines use a combination of solid and liquid or gaseous propellants. Both liquid and hybrid rockets use injectors to introduce the propellant into the chamber. These are often an array of simple jets- holes through which the propellant escapes under pressure; but sometimes may be more complex spray nozzles. When two or more propellants are injected the jets usually deliberately collide the propellants as this breaks up the flow into smaller droplets that burn more easily.

5.13 Rocket Ignition :

Rocket fuels, hypergolic or otherwise, must be mixed in the right quantities to have a controlled rate of production of hot gas. A hard start indicates that the quantity of combustible propellant that entered the combustion chamber prior to ignition was too large. The result is an excessive spike of pressure, possibly leading to structural failure or even an explosion (sometimes facetiously referred to as "spontaneous disassembly").

Avoiding hard starts involves careful timing of the ignition relative to valve timing or varying the mixture ratio so as to limit the maximum pressure that can occur or simply ensuring an adequate ignition source is present well prior to propellant entering the chamber.

Explosions from hard starts often cannot happen with purely gaseous propellants, since the amount of the gas present in the chamber is limited by the injector area relative to the throat area, and for practical designs propellant mass escapes too quickly to be an issue.

A famous example of a hard start was the explosion of Wernher von Braun's "1W" engine during a demonstration to General Dornberger on December 21, 1932. Delayed ignition allowed the chamber to fill with alcohol and liquid oxygen, which exploded violently. Shrapnel was embedded in the walls, but nobody was hit.

5.14 Rocket Combution:
Combustion chamber

For chemical rockets the combustion chamber is typically just a cylinder, and flame holders are rarely used. The dimensions of the cylinder are such that the propellant is able to combust thoroughly; different propellants require different combustion chamber sizes for this to occur. This leads to a number called L

L = Vc/At   where:Vc is the volume of the chamber

At is the area of the throat, L* is typically in the range of 25–60 inches (0.63–1.5 m).

The combination of temperatures and pressures typically reached in a combustion chamber is usually extreme by any standards. Unlike in air-breathing jet engines, no atmospheric nitrogen is present to dilute and cool the combustion, and the temperature can reach true stoichiometric. This, in combination with the high pressures, means that the rate of heat conduction through the walls is very high.

5.15 Rocket nozzles:

Typical temperatures (T) and pressures (p) and speeds (v) in a De Laval Nozzle. The large bell or cone shaped expansion nozzle gives a rocket engine its characteristic shape.

In rockets the hot gas produced in the combustion chamber is permitted to escape from the combustion chamber through an opening (the "throat"), within a high expansion- ratio 'de Laval' nozzle.

Provided sufficient pressure is provided to the nozzle (about 2.5-3x above ambient pressure) the nozzle chokes and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy.

The exhaust speeds vary, depending on the expansion ratio the nozzle is designed to give, but exhaust speeds as high as ten times the speed of sound of sea level air are not uncommon.

Rocket thrust is caused by pressures acting in the combustion chamber and nozzle. From Newton's third law, equal and opposite pressures act on the exhaust, and this accelerates it to high speeds.

About half of the rocket engine's thrust comes from the unbalanced pressures inside the combustion chamber and the rest comes from the pressures acting against the inside of the nozzle (see diagram). As the gas expands (adiabatically) the pressure against the nozzle's walls forces the rocket engine in one direction while accelerating the gas in the other.

5.16 Propellant efficiency:

For a rocket engine to be propellant efficient, it is important that the maximum pressures possible be created on the walls of the chamber and nozzle by a specific amount of propellant; as this is the source of the thrust. This can be achieved by all of: Heating the propellant to as high a temperature as possible (using a high energy fuel, containing hydrogen and carbon and sometimes metals such as aluminium, or even using nuclear energy)

Using a low specific density gas (as hydrogen rich as possible). Using propellants which are, or decompose to, simple molecules with few degrees of freedom to maximize translational velocity. Since all of these things minimise the mass of the propellant used, and since pressure is proportional to the mass of propellant present to be accelerated as it pushes on the engine, and since from Newton's third law the pressure that acts on the engine also reciprocally acts on the propellant, it turns out that for any given engine the speed that the propellant leaves the chamber is unaffected by the chamber pressure (although the thrust is proportional). However, speed is significantly affected by all three of the above factors and the exhaust speed is an excellent measure of the engine propellant efficiency. This is termed exhaust velocity, and after allowance is made for factors that can reduce it, the effective exhaust velocity is one of the most important parameters of a rocket engine (although weight, cost, ease of manufacture etc. are usually also very important). For aerodynamic reasons the flow goes sonic ("chokes") at the narrowest part of the nozzle, the 'throat'. Since the speed of sound in gases increases with the square root of temperature, the use of hot exhaust gas greatly improves performance. By comparison, at room temperature the speed of sound in air is about 340 m/s while the speed of sound in the hot gas of a rocket engine can be over 1700 m/s; much of this performance is due to the higher temperature, but additionally rocket propellants are chosen to be of low molecular mass, and this also gives a higher velocity compared to air.

Expansion in the rocket nozzle then further multiplies the speed, typically between 1.5 and 2 times, giving a highly collimated hypersonic exhaust jet. The speed increase of a rocket nozzle is mostly determined by its area expansion ratio—the ratio of the area of the throat to the area at the exit, but detailed properties of the gas are also important. Larger ratio nozzles are more massive but are able to extract more heat from the combustion gases, increasing the exhaust velocity.

Nozzle efficiency is affected by operation in the atmosphere because atmospheric pressure changes with altitude; but due to the supersonic speeds of the gas exiting from a rocket engine, the pressure of the jet may be either below or above ambient, and equilibrium between the two is not reached at all altitudes (See Diagram).

5.17 Back pressure and optimal expansion:

For optimal performance the pressure of the gas at the end of the nozzle should just equal the ambient pressure: if the exhaust's pressure is lower than the ambient pressure, then the vehicle will be slowed by the difference in pressure between the top of the engine and the exit; on the other hand, if the exhaust's pressure is higher, then exhaust pressure that could have been converted into thrust is not converted, and energy is wasted.

To maintain this ideal of equality between the exhaust's exit pressure and the ambient pressure, the diameter of the nozzle would need to increase with altitude, giving the pressure a longer nozzle to act on (and reducing the exit pressure and temperature). This increase is difficult to arrange in a lightweight fashion, although is routinely done with other forms of jet engines. In rocketry a lightweight compromise nozzle is generally used and some reduction in atmospheric performance occurs when used at other than the 'design altitude' or when throttled. To improve on this, various exotic nozzle designs such as the plug nozzle, stepped nozzles, the expanding nozzle and the aerospike have been proposed, each providing some way to adapt to changing ambient air pressure and each allowing the gas to expand further against the nozzle, giving extra thrust at higher altitudes.

When exhausting into a sufficiently low ambient pressure (vacuum) several issues arise. One is the sheer weight of the nozzle- beyond a certain point, for a particular vehicle, the extra weight of the nozzle outweighs any performance gained. Secondly, as the exhaust gases adiabatically expand within the nozzle they cool, and eventually some of the chemicals can freeze, producing 'snow' within the jet. This causes instabilities in the jet and must be avoided.

On a De Laval nozzle, exhaust gas flow detachment will occur in a grossly over-expanded nozzle. As the detachment point will not be uniform around the axis of the engine, a side force may be imparted to the engine. This side force may change over time and result in control problems with the launch vehicle.

5.18 Thrust vectoring:

Many engines require the overall thrust to change direction over the length of the burn. A number of different ways to achieve this have been flown: The entire engine is mounted on a hinge or gimbal and any propellant feeds reach the engine via low pressure flexible pipes or rotary couplings.

Just the combustion chamber and nozzle is gimbled, the pumps are fixed, and high pressure feeds attach to the engine multiple engines (often canted at slight angles) are deployed but throttled to give the overall vector that is required, giving only a very small penalty fixed engines with vernier thrusters high temperature vanes held in the exhaust that can be tilted to deflect the jet

5.19 Overall rocket engine performance:

Rocket technology can combine very high thrust (meganewtons), very high exhaust speeds (around 10 times the speed of sound in air at sea level) and very high thrust/weight ratios (>100) simultaneously as well as being able to operate outside the atmosphere, and while permitting the use of low pressure and hence lightweight tanks and structure.

Rockets can be further optimised to even more extreme performance along one or more of these axes at the expense of the others.

Specific impulse:

The most important metric for the efficiency of a rocket engine is impulse per unit of propellant, this is called specific impulse (usually written Isp). This is either measured as a speed (the effective exhaust velocity Ve in metres/second or ft/s) or as a time (seconds).

An engine that gives a large specific impulse is normally highly desirable. The specific impulse that can be achieved is primarily a function of the propellant mix (and ultimately would limit the specific impulse), but practical limits on chamber pressures and the nozzle expansion ratios reduce the performance that can be achieved. Space flight: Spaceflight is the act of travelling into or through outer space. Spaceflight can occur with spacecraft which may, or may not, have humans on board. Examples of human spaceflight include the Russian Soyuz program, the U.S. Space shuttle program, as well as the ongoing International Space Station. Examples of unmanned spaceflight include space probes which leave Earth's orbit, as well as satellites in orbit around Earth, such as communication satellites.

5.20 Space Flights

Spaceflight is used in space exploration, and also in commercial activities like space tourism and satellite telecommunications. Additional non-commercial uses of spaceflight include space observatories, reconnaissance satellites and other earth observation satellites.

A spaceflight typically begins with a rocket launch, which provides the initial thrust to overcome the force of gravity and propels the spacecraft from the surface of the Earth. Once in space, the motion of a spacecraft—both when unpropelled and when under propulsion—is covered by the area of study called astrodynamics. Some spacecraft remain in space indefinitely, some disintegrate during atmospheric reentry, and others reach a planetary or lunar surface for landing or impact.

5.20.1 Types of spaceflight

Human spaceflight:

The first human spaceflight was Vostok 1 on April 12, 1961, on which cosmonaut Yuri Gagarin of the USSR made one orbit around the Earth. In official Soviet documents, there is no mention of the fact that Gagarin parachuted the final seven miles.[3] The international rules for aviation records stated that "The pilot remains in his craft from launch to landing". This rule, if applied, would have "disqualified" Gagarins space-flight. Currently the only spacecraft regularly used for human spaceflight are Russian Soyuz spacecraft and the U.S. Space Shuttle fleet. Each of those space programs have used other spacecraft in the past. Recently, the Chinese Shenzhou spacecraft has been used three times for human spaceflight, and SpaceshipOne twice.

Sub-orbital spaceflight

On a sub-orbital spaceflight the spacecraft reaches space and then returns to the atmosphere after following a (primarily) ballistic trajectory. This is usually because of insufficient specific orbital energy, in which case a suborbital flight will last only a few minutes, but it is also possible for an object with enough energy for an orbit to have a trajectory that intersects the Earth's atmosphere, sometimes after many hours. Pioneer 1 was NASA's first space probe intended to reach the Moon. A partial failure caused it to instead follow a suborbital trajectory to an altitude of 113,854 kilometers (70,746 mi) before reentering the Earth's atmosphere 43 hours after launch.

The most generally recognized boundary of space is the Kármán line (actually a sphere) 100 km above sea level. (NASA alternatively defines an astronaut as someone who has flown more than 50 miles or 80 km above sea level.) It is not generally recognized by the public that the increase in potential energy required to pass the Kármán line is only about 3% of the orbital energy (potential plus kinetic energy) required by the lowest possible earth orbit (a circular orbit just above the Kármán line.) In other words, it is far easier to reach space than to stay there.

On May 17, 2004, Civilian Space eXploration Team launched the GoFast Rocket on a suborbital flight, the first amateur spaceflight. On June 21, 2004, SpaceShipOne was used for the first privately-funded human spaceflight.

Orbital spaceflight

A minimal orbital spaceflight requires much higher velocities than a minimal sub-orbital flight, and so it is technologically much more challenging to achieve. To achieve orbital spaceflight, the tangential velocity around the Earth is as important as altitude. In order to perform a stable and lasting flight in space, the spacecraft must reach the minimal orbital speed required for a closed orbit.

Interplanetary spaceflight

An artist's imaginative impression of a vehicle entering a wormhole for interstellar travel Interplanetary travel is travel between planets within a single planetary system. In practice, the use of the term is confined to travel between the planets of the Solar System. Interstellar spaceflight five spacecraft are currently leaving the Solar System on escape trajectories. The one farthest from the Sun is Voyager 1, which is more than 100 AU distant and is moving at 3.6 AU per year.[4] In comparison Proxima Centauri, the closest star other than the Sun, is 267,000 AU distant. It will take Voyager 1 over 74,000 years to reach this distance. Vehicle designs using other techniques, such as nuclear pulse propulsion are likely to be able to reach the nearest star significantly faster.

Another possibility that could allow for human interstellar spaceflight is to make use of time dilation, as this would make it possible for passengers in a fast-moving vehicle to travel further into the future while aging very little, in that their great speed slows down the rate of passage of on-board time. However, attaining such high speeds would still require the use of some new, advanced method of propulsion.

Intergalactic spaceflight

Intergalactic travel involves spaceflight between galaxies, and is considered much more technologically demanding than even interstellar travel and, by current engineering terms, is considered science fiction.

5.21 Effective Speed Ratio (σ):

The ratio of flight speed to jet velocity is known as effective speed ratio (σ)

$$\sigma = \frac{Flight\ speed}{Jet\ velocity\ (or)\ Velocity\ of\ exit\ gases}$$

$$\boxed{\sigma = \frac{u}{c_j}}$$

We know that, Thrust $F = m^{\bullet}_a [c_j - u]$

$$= m^{\bullet}_a \times c_j \left[1 - \frac{u}{c_j}\right]$$

$$\boxed{F = m^{\bullet}_a \times c_j\ [1 - \sigma]}$$

SPECIFIC THRUST (F_{sp})

The thrust developed per unit mass flow rate is known as specific thrust

$$\boxed{F_{sp} = \frac{F}{m^{\bullet}}}$$

Thrust Specific Fuel Consumption (TSFC)

The fuel consumption rate per unit thrust is known as Thrust Specific Fuel Consumption

$$\boxed{TSFC = \frac{m^{\bullet}_f}{F}}$$

Specific Impulse (I_{sp})

The thrust developed per unit weight flow rate is known as specific impulse

$$I_{sp} = \frac{F}{w}$$

$$= \frac{m^{\bullet}(c_j - u)}{m^{\bullet} \times g}$$

$$\frac{c_j - u}{g} = \frac{u}{g}\left[\frac{c_j}{u} - 1\right]$$

$$I_{sp} = \frac{u}{g}\left[\frac{1}{\sigma} - 1\right]$$

$$Where, \quad \sigma - effective\ speed\ ratio = \frac{u}{c_j}$$

propulsive efficiency:

It is defined as the ratio of Propulsive power (or) thrust power to the power output of the engine.

$$\eta_P = \frac{Propulsive\ power\ (or)\ Thrust\ power}{Power\ output\ of\ the\ engine}$$

We know that

$$Thrust\ power = Thrust(F) \times Flight\ speed(u)$$

$$\boxed{Thrust\ power = m^{\bullet}(c_j - u) \times u}$$

At the outlet of the engine , the power is

$$\boxed{Power\ output = \frac{1}{2} m^{\bullet}\left[c^2{}_j - u^2\right]}$$

$$\eta_P = \frac{m^{\bullet}\left[c_j - u\right] \times u}{\frac{1}{2} m^{\bullet}\left[c^2{}_j - u^2\right]}$$

$$\eta_P = \frac{\left[c_j - u\right] \times u}{\frac{1}{2}\left[c^2{}_j - u^2\right]} = \frac{2u\left[c_j - u\right]}{c^2{}_j - u^2}$$

$$= \frac{2u\left[c_j - u\right]}{(c_j + u)\,(c_j - u)}$$

$$\boxed{\eta_P = \frac{2u}{c_j + u}}$$

Divide the numerator and denominator by c_j.

$$\eta_P = \frac{\frac{2u}{c_j}}{\frac{c_j + u}{c_j}} = \frac{\frac{2u}{c_j}}{1+\frac{u}{c_j}}$$

$$\boxed{\eta_P \; \frac{2\sigma}{1+\sigma}} \quad \textit{Where}$$

$$\sigma - \textit{Effective speed ratio} = \frac{u}{c_j}$$

Thermal Efficiency

It is defined as the ratio of power output of the engine to the power input to the engine.

$$\eta_t = \frac{\textit{Power output of the engine}}{\textit{Power input to the engine through fuel}}$$

Power is given as the input by burning the fuel

So, $\quad$ *Power input* $= m^{\bullet}{}_f \times C \cdot V$

We know that

$$\textit{Power output} = \frac{1}{2} m^{\bullet}\left[c^2{}_j - u^2\right]$$

$$\eta_t = \frac{\frac{1}{2} m^{\bullet}\left[c^2{}_j - u^2\right]}{m^{\bullet}{}_f \times C \cdot V}$$

Where,

m^* – Mass of air fuel mixture

c_j – Velocity of jet

u – Flight velocity

$*m_f$ – Mass of fuel

C ·V– Calorific value of fuel

If efficiency of combustion is considered ,

$$\eta_t = \frac{\frac{1}{2} m^{\bullet} \left[c^2_j - u^2 \right]}{\eta_B \times m^{\bullet}_f \times C \cdot V}$$

Overall Efficiency

It is defined as the ratio of propulsive power to the power input to the engine.

$$\eta_0 = \frac{\text{Propulsive Power (or) Thrust Power}}{\text{Power input to the engine}}$$

We know that ,

$$\text{Thrust power} = m^{\bullet} \left[c_j - u \right] \times u$$

$$\text{Power input} = m^{\bullet}_f \times C \cdot V$$

$$\eta_0 = \frac{m^{\bullet} \left[c_j - u \right] \times u}{m^{\bullet}_f \times C \cdot V}$$

$$= \frac{m^{\bullet} \left[c_j - u \right] \times u}{\frac{1}{2} m^{\bullet} \left[c^2_j - u^2 \right]} \times \frac{\frac{1}{2} m^{\bullet} \left[c^2_j - u^2 \right]}{m^{\bullet}_f \times C \cdot V}$$

$$\eta_P \times \eta_\tau$$

$$\boxed{\eta_0 = \eta_P \times \eta_\tau}$$

5.22 Problems:

1.A rocket moves with a velocity of 10,000 km/hr with an effective exhaust velocity of 1400 m/sec, the propellant flow rate is 5 kg/sec and the propellant mixture has a heating value of 6500 kJ/kg. Find

1. Propulsion efficiency
2. Engine output power
3. Thermal efficiency
4. Overall efficiency

Sol)

Rocket speed, u = 10,000km/hr

$$= \frac{10,000 \times 1000}{3600} \text{ m/sec}$$

$$= 2777.7 \text{ m/sec}$$

Jet velocity, C_j = 1400 m/sec

Calorific value (C.V.) = 6600 kJ/kg

Propellant flow rate m_p = 5 kg/sec

$$\text{Speed ratio, } \sigma = \frac{u}{c_j} = \frac{2777.7}{1400} = 1.984$$

$$\text{Propulsion efficiency, } \eta_p = \frac{2 \times \sigma}{1+\sigma^2}$$

$$= \frac{2 \times 1.984}{1+(1.984)^2} = .804$$

$$\eta_p = 80.4\ \%$$

$$\text{Thrust, } F = m_p \times c_j$$

$$= 5 \times 1400$$

$$= 7000 \text{ N}$$

$$\text{Propulsive power, } P = F \times u$$

$$= 7000 \times 2777.7$$

$$P = 19.44 \times 10^6 \text{ W}$$

$$\text{Propulsive efficiency} = \frac{\text{Propulsive power}}{\text{Power output of engine}}$$

$$.804 = \frac{19.44\times10^6}{\text{Power output of engine}}$$

$$\text{Engine output} = 24.18 \times 10^6\,\text{W}$$

$$\text{Thermal efficiency, } \eta_t = \frac{\text{Power output of engine}}{\text{Power input to engine}}$$

$$= \frac{24.18\times10^6}{m_p \times C.V}$$

$$= \frac{24.18\times10^6}{5\times6600\times10^3} = .733$$

$$\eta_t = 73.3\ \%$$

$$\text{Overall efficiency, } \eta_o = \eta_t \times \eta_p$$

$$= .733 \times .804$$

$$= .589$$

$$\eta_o = 58.9\ \%$$

5.23 Tutorial problems:

1. Calculate the orbital and escape velocities of a rocket at mean sea level and an altitude of 300km from the following data:
Radius of earth at mean sea level =6341.6Km
Acceleration due to gravity at mean sea level =9.809 m/s^2

2. A missile has a maximum flight speed to jet speed ratio of 0.2105 and specific impulse equal to 203.88 seconds .Determine for a burn out time of 8 seconds
a) Effective jet velocity
b) Mass ratio and propellant mass functions
c) Maximum flight speed, and
d) Altitude gain during powered and coasting flights

3. A ramjet engine operates at M=1.5 at an altitude of 6500m.The diameter of the inlet diffuser at entry is 50cm and the stagnation temperature at the nozzle entry is 1600K.The

calorific value of the fuel used is 40MJ/Kg .The properties of the combustion gases are same as those of air (y =1.4, R=287J/Kg K). The velocity of air at the diffuser exit is negligible. Calculate

(a) the efficiency of the ideal cycle,

(b) flight speed

(c) air flow rate

(d) diffuser pressure ratio

(e) fuel –ratio

(f)nozzle pressure ratio

(g) nozzle jet Mach number

(h) propulsive efficiency

(i) and thrust.

Assume the following values:0D =0.90,0B =0.98, 0 j= 0.96.

Stagnation pressure loss in the combustion chamber =0.002Po2.