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# SCHOOL OF MECHANICAL ENGINEERING

# DEPARTMENT OF MECHANICAL ENGINEERING

## UNIT- I -Static Stresses and Variable Stresses in Machine Components-SMEA1502

# I. Introduction

## STATIC STRESSES AND VARIABLE STRESSES IN MACHINE

The subject Machine Design is the creation of new and better machines and improving the existing ones. A new or better machine is one which is more economical in the overall cost of production and operation. The process of design is a long and time consuming one.From the study of existing ideas, a new idea has to be conceived. The idea is then studied keeping in mind its commercial success and given shape and form in the form of drawings.In the preparation of these drawings, care must be taken of the availability of resources in money, in men and in materials required for the successful completion of the new idea into an actual reality. In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

**Classifications of Machine Design**

The machine design may be classified as follows:

**1. Adaptive design.** In most cases, the designer's work is concerned with adaptation of existing designs. This type of design needs no special knowledge or skill and can be attempted by designers of ordinary technical training. The designer only makes minor alternation or modification in the existing designs of the product.

**2. Development design.** This type of design needs considerable scientific training and design ability in order to modify the existing designs into a new idea by adopting a new material or different method of manufacture. In this case, though the designer starts from the existing design, but the final product may differ quite markedly from the original product.

**3. New design.** This type of design needs lot of research, technical ability and creative thinking. Only those designers who have personal qualities of a sufficiently high order can take up the work of a new design. The designs, depending upon the methods used, may be classified as follows:

**(*a*) Rational design.** This type of design depends upon mathematical formulae of principle of mechanics.

**(*b*) Empirical design.** This type of design depends upon empirical formulae based on the practice and past experience.

**(*c*) Industrial design.** This type of design depends upon the production aspects to manufacture any machine component in the industry.

**(*d*) Optimum design.** It is the best design for the given objective function under the specified constraints. It may be achieved by minimizing the undesirable effects.

**(*e*) System design.** It is the design of any complex mechanical system like a motor car.

**(*f*) Element design.** It is the design of any element of the mechanical system like piston, crankshaft, connecting rod, etc.

**(*g*) Computer aided design.** This type of design depends upon the use of computer systems to assist in the creation, modification, analysis and optimization of a design.

## General Considerations in Machine Design

Following are the general considerations in designing a machine component:

**1. Type of load and stresses caused by the load.** The load, on a machine component, may act in several ways due to which the internal stresses are set up. The various types of load and stresses are discussed later.

**2. Motion of the parts or kinematics of the machine.** The successful operation of any machine depends largely upon the simplest arrangement of the parts which will give the motion required.

The motion of the parts may be:

(*a*) Rectilinear motion which includes unidirectional and reciprocating motions.

(*b*) Curvilinear motion which includes rotary, oscillatory and simple harmonic.

(*c*) Constant velocity

(*d*) Constant or variable acceleration

**3. Selection of materials.** It is essential that a designer should have a thorough knowledge of the properties of the materials and their behavior under working conditions. Some of the important characteristics of materials are: strength, durability, flexibility, weight, resistance to heat and corrosion, ability to cast, welded or hardened, mach inability, electrical conductivity, etc. The various types of engineering materials and their properties are discussed later.

4. **Form and size of the parts.** The form and size are based on judgment. The smallest practicable cross-section may be used, but it may be checked that the stresses induced in the designed cross-section are reasonably safe. In order to design any machine part for form and size, it is necessary to know the forces which the part must sustain. It is also important to anticipate any suddenly applied or impact load which may cause failure.

**5. Frictional resistance and lubrication.** There is always a loss of power due to frictional resistance and it should be noted that the friction of starting is higher than that of running friction. It is, therefore, essential that a careful attention must be given to the matter of lubrication of all surfaces which move in contact with others, whether in rotating, sliding, or rolling bearings.

**6. Convenient and economical features.** In designing, the operating features of the machine should be carefully studied. The starting, controlling and stopping levers should be located on the basis of convenient handling. The adjustment for wear must be provided employing the various take up devices and arranging them so that the alignment of parts is preserved. If parts are to be changed for different products or replaced on account of wear or breakage, easy access should be provided and the necessity of removing other parts to accomplish this should be avoided if possible. The economical operation of a machine which is to be used for production or for the processing of material should be studied, in order to learn whether it has the maximum capacity consistent with the production of good work.

**7. Use of standard parts.** The use of standard parts is closely related to cost, because the cost of standard or stock parts is only a fraction of the cost of similar parts made to order. The standard or stock parts should be used whenever possible; parts for which patterns are already in existence such as gears, pulleys and bearings and parts which may be selected from regular shop stock such as screws, nuts and pins. Bolts and studs should be as few as possible to avoid the delay caused by changing drills, reamers and taps and also to decrease the number of wrenches required.

**8. Safety of operation.** Some machines are dangerous to operate, especially those which are speeded up to insure production at a maximum rate. Therefore, any moving part of a machine which is within the zone of a worker is considered an accident hazard and may be the cause of an injury. It is, therefore, necessary that a designer should always provide safety devices for the safety of the operator. The safety appliances should in no way interfere with operation of the machine.

**9. Workshop facilities.** A design engineer should be familiar with the limitations of this employer's workshop, in order to avoid the necessity of having work done in some other workshop. It is sometimes necessary to plan and supervise the workshop operations and to draft methods for casting, handling and machining special parts.

**10. Number of machines to be manufactured.** The number of articles or machines to be manufactured affects the design in a number of ways. The engineering and shop costs which are called fixed charges or overhead expenses are distributed over the number of articles to be manufactured. If only a few articles are to be made, extra expenses are not justified unless the machine is large or of some special design. An order calling for small number of the product will not permit any undue expense in the workshop processes, so that the designer should restrict his specification to standard parts as much as possible.

**11. Cost of construction.** The cost of construction of an article is the most important consideration involved in design. In some cases, it is quite possible that the high cost of an article may immediately bar it from further considerations. If an article has been invented and tests of handmade samples have shown that it has commercial value, it is then possible to justify the expenditure of a considerable sum of money in the design and development of automatic machines to produce the article, especially if it can be sold in large numbers. The aim of design engineer under all conditions should be to reduce the manufacturing cost to the minimum.

**12. Assembling.** Every machine or structure must be assembled as a unit before it can function. Large units must often be assembled in the shop, tested and then taken to be transported to their place of service. The final location of any machine is important and the design engineer must anticipate the exact location and the local facilities for erection.

## Factors influencing in Machine Design

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows (Fig.1.1):

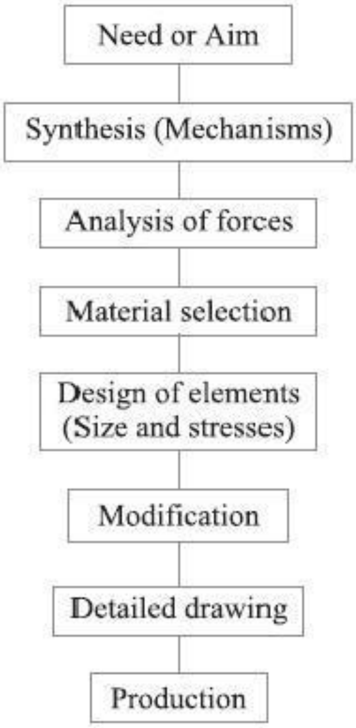



Fig.1.1: General Machine Design Procedure

**1. Recognition of need:**

First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

**2. Synthesis (Mechanisms):**

Select the possible mechanism or group of mechanisms which will give the desired motion.

**3. Analysis of forces:**

Find the forces acting on each member of the machine and the energy transmitted by each member.

**4. Material selection:**

**5. Design of elements (Size and Stresses):** Find the size of each member of the machine by considering the force acting on t he member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than th e permissible limit.

**6. Modification:**

Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

**7. Detailed drawing:**

Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

**8. Production*:*** The component, as per the drawing, is manufactured in the workshop.

The flow chart for the general procedure in machine design is shown in Fig.

**Note:** When there are number of components in the market having the same qualities of efficiency, durability and cost, then the customer will naturally attract towards the most appealing product. The aesthetic and ergonomics are a very important feature which gives grace and lustre to product and dominates the market.

**Selection of engineering materials based on mechanical properties:**

The knowledge of materials and their properties is of great significance for a design engineer. The machine elements should be made of such a material which has properties suitable for the conditions of operation. In addition to this, a design engineer must be familiar with the effects which the manufacturing processes and heat treatment have on the properties of the materials. Now, we shall discuss the commonly used engineering materials and their properties in Machine Design.

**Classification of Engineering Materials**

The engineering materials are mainly classified as:

**1.** Metals and their alloys, such as iron, steel, copper, aluminum, etc.

2. Non-metals, such as glass, rubber, plastic, etc.
   *(A)* Ferrous metals and (*B*) Non-ferrous metals.

**Ferrous metals** are those which have the iron as their main constituent, such as cast iron, wrought iron and steel.

(B) **Non-ferrous** metals are those which have a metal other than iron as their main constituent, such as copper, aluminum, brass, tin, zinc, etc.

## Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problems for the designer. The best material is one which serves the desired objective at the minimum cost. The following factors should be considered while selecting the material:

**1.** Availability of the materials,

**2.** Suitability of the materials for the working conditions in service, and

**3.** The cost of the materials.

The important properties, which determine the utility of the material, are physical, chemical and mechanical properties. We shall now discuss the physical and mechanical properties of the material in the following articles.

## Physical Properties of Metals

The physical properties of the metals include luster, color, size and shape, density, electric and thermal conductivity, and melting point. The following table shows the important physical properties of some pure metals.

## Mechanical Properties of Metals

The mechanical properties of the metals are those which are associated with the ability of the material to resist mechanical forces and load. These mechanical properties of the metal include strength, stiffness, elasticity, plasticity, ductility, brittleness, malleability, toughness, resilience, creep and hardness. We shall now discuss these properties as follows:

### 1. Strength:

It is the ability of a material to resist the externally applied forces without breaking or yielding. The internal resistance offered by a part to an externally applied force is called stress.

### 2. Stiffness:

It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness.

**3. Elasticity:**

It is the property of a material to regain its original shape after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber.

**4. Plasticity:**

It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

**5. Ductility:**

It is the property of a material enabling it to be drawn into wire with the application of a tensile force. A ductile material must be both strong and plastic. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area. The ductile material commonly used in engineering practice (in order of diminishing ductility) are mild steel, copper, aluminum, nickel, zinc, tin and lead.

**6. Brittleness:**

It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. Brittle materials when subjected to tensile loads snap off without giving any sensible elongation. Cast iron is a brittle material.

**7. Malleability:**

It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. A malleable material should be plastic but it is not essential to be so strong. The malleable materials commonly used in engineering practice (in order of diminishing malleability) are lead, soft steel, wrought iron, copper and aluminum.

**8. Toughness:**

It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated. It is measured by the amount of energy that a unit volume of the material has absorbed after being stressed up to the point of fracture. This property is desirable in parts subjected to shock and impact loads.

**9. Machinability:**

It is the property of a material which refers to a relative case with which a material can be cut. The machinability of a material can be measured in a number of ways such as comparing the tool life for cutting different materials or thrust required to remove theIt may be noted that brass can be easily machined than steel.

**10. Resilience:**

It is the property of a material to absorb energy and to resist shock and impact loads. It is measured by the amount of energy absorbed per unit volume within elastic limit. This property is essential for spring materials.

**11. Creep:**

When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called **creep.** This property is considered in designing internal combustion engines, boilers and turbines.

**12. Fatigue:**

When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as ***fatigue**. The failure is caused by means of a progressive crack formation which are usually fine and of microscopic size. This property is considered in designing shafts, connecting rods, springs, gears, etc.

**13. Hardness:**

It is a very important property of the metals and has a wide variety of meanings. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal. The hardness is usually expressed in numbers which are dependent on the method of making the test. The hardness of a metal may be determined by the following tests:

**(a)** Brinell hardness test,

**(b)** Rockwell hardness test,

**(c)** Vickers hardness (also called Diamond Pyramid) test, and

**(d)** Shore scleroscope.

The plain carbon steels varying from 0.06% carbon to 1.5% carbon are divided into the following types depending upon the carbon content.

**1.** Dead mild steel — up to 0.15% carbon

**2.** Low carbon or mild steel — 0.15% to 0.45% carbon

**3.** Medium carbon steel — 0.45% to 0.8% carbon

**4.** High carbon steel — 0.8% to 1.5% carbon

According to Indian standard *[IS: 1762 (Part-I)–1974], a new system of designating the steel is recommended. According to this standard, steels are designated on the following two basis: (*a*) On the basis of mechanical properties, and (*b*) On the basis of chemical composition. We shall now discuss, in detail, the designation of steel on the above two basis, in the following pages.

**Steels Designated on the Basis of Mechanical Properties**

These steels are carbon and low alloy steels where the main criterion in the selection and inspection of steel is the tensile strength or yield stress. According to Indian standard IS: 1570 (Part–I)- 1978 (Reaffirmed 1993), these steels are designated by a symbol 'Fe' or 'Fe E' depending on whether the steel has been specified on the basis of minimum tensile strength or yield strength, followed by the figure indicating the minimum tensile strength or yield stress in N/mm2. For example 'Fe 290' means a steel having minimum tensile strength of 290 N/mm2 and 'Fe E 220' means a steel having yield strength of 220 N/mm$^2$.

**Steels Designated on the Basis of Chemical Composition**

According to Indian standard, IS : 1570 (Part II/Sec I)-1979 (Reaffirmed 1991), the carbon steels are designated in the following order :

***(a)*** Figure indicating 100 times the average percentage of
carbon content, ***(b)*** Letter 'C'.

***(c)*** Figure indicating 10 times the average percentage of manganese content. The figure after multiplying shall be rounded off to the nearest integer.

For example 20C8 means a carbon steel containing 0.15 to 0.25 per cent (0.2 per cent on average) carbon and 0.60 to 0.90 per cent (0.75 per cent rounded off to 0.8 per cent on an average) manganese.

**Manufacturing considerations in Machine design:**

The knowledge of manufacturing processes is of great importance for a design engineer. The following are the various manufacturing processes used in Mechanical Engineering.

**1.** ***Primary shaping processes*****.** The processes used for the preliminary shaping of the machine component are known as primary shaping processes. The common operations used for this process are casting, forging, extruding, rolling, drawing, bending, shearing, spinning, powder metal forming, squeezing, etc.

**2.** ***Machining processes*****.** The processes used for giving final shape to the machine component, according to planned dimensions are known as machining processes. The common operations used for this process are turning, planning, shaping, drilling, boring, reaming, sawing, broaching, milling, grinding, hobbling, etc.

**3.** ***Surface finishing processes*****.** The processes used to provide a good surface finish for the machine component are known as surface finishing processes. The common operations used for this process are polishing, buffing, honing, lapping, abrasive belt grinding, barrel tumbling, electroplating, super finishing, sheradizing, etc.

**4.** ***Joining processes*****.** The processes used for joining machine components are known as joining processes. The common operations used for this process are welding, riveting, soldering, brazing, screw fastening, pressing, sintering, etc.

5. ***Processes effecting change in properties*****.** These processes are used to impart certain specific properties to the machine components so as to make them suitable for particular operations or uses. Such processes are heat treatment, hot-working, cold-working and shot penning.

**Other considerations in Machine design**

1. Workshop facilities.
2. Number of machines to be manufactured
3. Cost of construction

**References:**


1. Machine Design - V.Bandari .
2. Machine Design – R.S. Khurmi

Design Data hand Book - S MD Jalalud


**Torsional Stress:**

When a machine member is subjected to the action of two equal and opposite couples acting in parallel planes (or torque or twisting moment), then the machine member is said to be subjected to ***torsion.*** The stress s et up by torsion is known as ***tensional stress.*** It is zero at the censorial axis and maximum at the outer surface. Consider a shaft fixed a t one end and subjected to a torque (*T*) at the other end as shown in Fig.1.2. As a result of this torque, every cross-section of the shaft is subjected to tensional shear stress. We have discussed above that the tensional shear stress is zero at the censorial axis and maximum at the outer surface. The maximum tensional stress at the outer surface of the shaft may be obtained from the following equation:

$$\frac{\tau}{r} = \frac{T}{J} = \frac{C \cdot \theta}{l} \quad \text{-------- (i)}$$

Where τ = Torsional stress induced at the outer surface of the shaft or maximum stress, *r* = Radius of the shaft,

*T* = Torque or twisting moment,

*J* = Second moment of area of the section about its polar axis or polar moment of inertia,

*C* = Modulus of rigidity for the shaft material, *l* = Length of the shaft, and

θ = Angle of twist in radians on a length *l*.

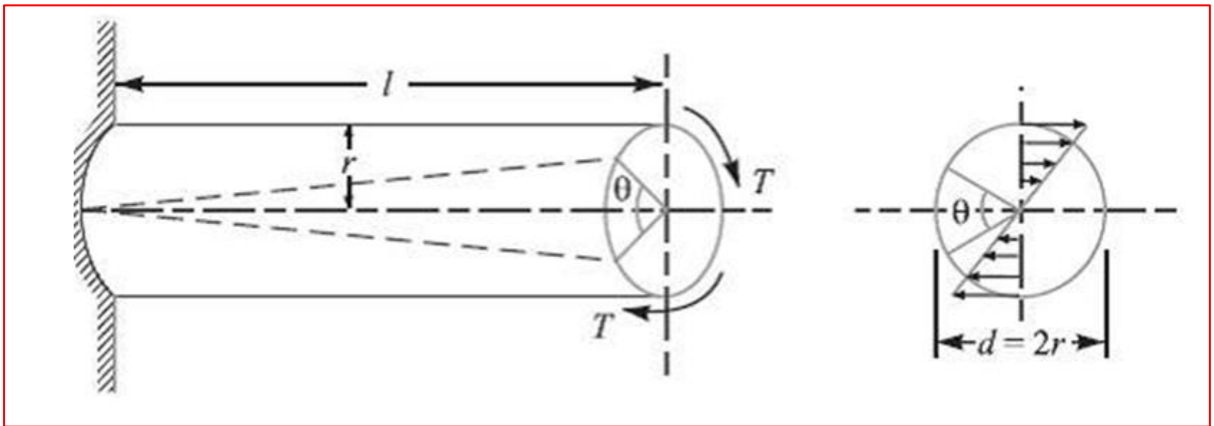



**Fig.1.2:Twisting moment of cylinder**

The above equation **is** known as ***torsion equation.*** It is based on the following a assumptions:

**1.** The material of the shaft is uniform throughout.

**2.** The twist along the length of the shaft is uniform.

**3.** The normal cross-sections of the shaft, which were plane and circular before twist, remain plane and circular after twist.

**4.** All diameters of the normal cross-section which were straight before twist, remain straight with their magnitude unchanged, after twist.

**5.** The maximum stress induced in the shaft due to the twisting moment does not exceed its elastic limit value.

**Note: 1.** Since the tensional stress on any cross-section normal to the axis is directly proportional to the distance from the centre of the axis, therefore the tensional stress at a distance *x* from the centre of the shaft is given by

$$\frac{\tau_x}{x} = \frac{\tau}{r}$$

**2.** From equation (*i*), we know that

$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$

For a solid shaft of diameter (*d*), the polar moment of inertia,

$$J - I_{XX} + I_{YY} - \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$

Therefore,

$$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$

In case of a hollow shaft with external diameter ($d_o$) and internal diameter ($d_i$), the polar moment of inertia,

$$J - \frac{\pi}{32} [(d_o)^4 \quad (d_i)^4] \text{ and } r - \frac{d_o}{2}$$

$$T = \tau \times \frac{\pi}{32} [(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[ \frac{(d_o)^4 - (d_i)^4}{d_o} \right]$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4) \qquad \ldots \left( \text{Substituting, } k = \frac{d_i}{d_o} \right)$$

**3.** The expression ($C \times J$) is called ***torsional rigidity*** of the shaft.

**4.** The strength of the shaft means the maximum torque transmitted by it. Therefore, in order to design a shaft for strength, the above equations are used. The power transmitted by the shaft (in watts) is given by

$$P = \frac{2 \pi N . T}{60} = T . \omega \qquad \ldots \left( \because \omega = \frac{2 \pi N}{60} \right)$$

Where $T$ = Torque transmitted in N-m, and

$\omega$ = Angular speed in rad/s.

**Problem 1.**

A shaft is transmitting 100 kW at 160 rpm. Find a suitable diameter for th e shaft, if the maximum torque transmitted exceeds the mean by 25%. Take maximum allowable shear stress as 70 MPa.

**Solution.** Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 160$ r.p.m ; $T_{max} = 1.25\ T_{mean}$ ; $\tau = 70$ MPa $= 70$ N/mm$^2$

Let $T_{mean}$ = Mean torque transmitted by the shaft in N-m, and

$d$ = Diameter of the shaft in mm.

We know that the power transmitted ($P$),

$$100 \times 10^3 = \frac{2\,\pi\,N\,.\,T_{mean}}{60} = \frac{2\pi \times 160 \times T_{mean}}{60} = 16.76\ T_{mean}$$

$$\therefore \quad T_{mean} = 100 \times 10^3/16.76 = 5966.6 \text{ N-m}$$

and maximum torque transmitted,

$$T_{max} = 1.25 \times 5966.6 = 7458 \text{ N-m} = 7458 \times 10^3 \text{ N-mm}$$

We know that maximum torque ($T_{max}$),

$$7458 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 70 \times d^3 = 13.75\, d^3$$

$$\therefore \quad d^3 = 7458 \times 10^3/13.75 = 542.4 \times 10^3 \text{ or } d = 81.5 \text{ mm Ans.}$$

**Bending Stress**

In engineering practice, the machine parts of structural members may be subjected to static or dynamic loads which cause bending stress in the sections besides other types of stresses such as tensile, compressive and shearing stresses. Consider a straight beam subjected to a bending moment $M$ as shown in Fig.1.3.The following assumptions are usually made while deriving the bending formula.

**1.** The material of the beam is perfectly homogeneous (*i.e.* of the same material throughout) and isotropic (*i.e.* of equal elastic properties in all directions).

**2.** The material of the beam obeys s Hooke's law.

**3.** The transverse sections (*i.e. BC* or *GH*) which were plane before bending remain plane after bending also.

**4.** Each layer of the beam is free to expand or contract, independently, of the layer, above or below it.

**5.** The Young's modulus (*E*) is t he same in tension and compression.

**6.** The loads are applied in the plane of bending.

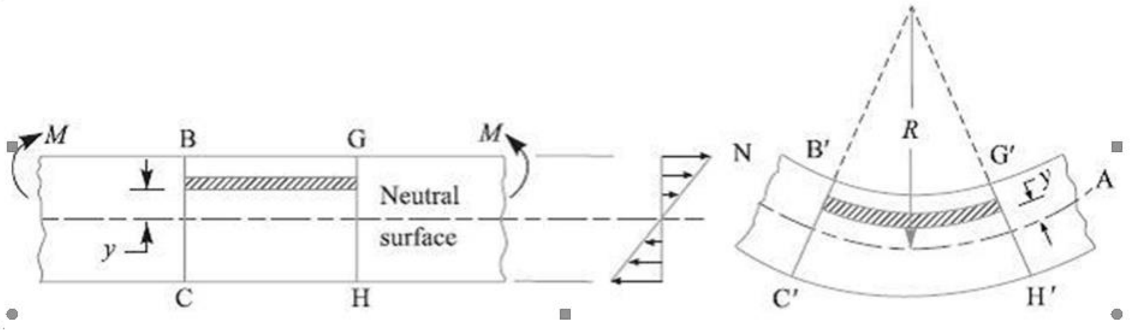



**Fig.1.3: Bending moment of Beam**

A little consideration will show that when a beam is subjected to the bending moment, the fibers on the upper side of the beam will be shortened due to compression and those on the lower side will be elongated du e to tension. It may be seen that somewhere between the top and bottom fibers there is a surface at which the fibers are neither shortened no r lengthened. Such a surface is called ***neutral surface*****.** The intersection of the neutral surf ace with any normal cross-section of the beam is known *as* ***neutral axis*****.** The stress distribution of a beam is shown in Fig. The bending equation is given by

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

Where *M* = Bending moment ac ting at the given section, σ = Bending stress,

*I* = Moment of inertia of the cross-section about the neutral axis, *y* = Distance from the neutral axis to the extreme fiber, *E* = Young's modulus of the material of the beam,and *R* = Radius of curvature o f the beam.

From the above equation, the bending stress is given by

$$\sigma = y \times \frac{E}{R}$$

Since *E* and *R* are constant, therefore within elastic limit, the stress at any point is directly proportional to *y*, *i.e*. the distance of the point from the neutral axis.Also from the above equation, the bending stress,

$$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$$

The ratio *I*/*y* is known as ***section modulus*** and is denoted by *Z*.

**Notes: 1.** the neutral axis of a section always passes through its centric.

2. In case of symmetrical sections such as circular, square or rectangular, the neutral axis passes through its geometrical centre and the distance of extreme fiber from the neutral axis is $y = d / 2$, where *d* is the diameter in case of circular section or depth in case of square or rectangular section.

3. In case of unsymmetrical sections such as L-section or T-section, the neutral axis does not pass through its geometrical centre. In such cases, first of all the centroid of the section is calculated and then the distance of the extreme fibres for both lower and upper side of the section is obtained. Out of these two values, the bigger value is used in bending equation.

**Problem:2.**

A beam of uniform rectangular cross-section is fixed at one end and carries an electric motor weighing 400 N at a distance of 300 mm from the fixed end. The maximum bending stress in the beam is 40 MPa. Find the width and depth of the beam, if depth is twice that of width.

**Solution. Given:** $W = 400$ N ; $L = 300$ mm ; $\sigma_b = 40$ MPa $= 40$ N/mm$^2$ ; $h = 2b$

The beam is shown in Fig. 5.7.

Let $b$ = Width of the beam in mm, and

$h$ = Depth of the beam in mm.

∴ Section modulus,

$$Z = \frac{b \cdot h^2}{6} = \frac{b\,(2b)^2}{6} = \frac{2\,b^3}{3} \text{ mm}^3$$

Maximum bending moment (at the fixed end),

$$M = W.L = 400 \times 300 = 120 \times 10^3 \text{ N-mm}$$

We know that bending stress ($\sigma_b$),

$$40 = \frac{M}{Z} = \frac{120 \times 10^3 \times 3}{2\,b^3} = \frac{180 \times 10^3}{b^3}$$

∴ $b^3 = 180 \times 10^3/40 = 4.5 \times 10^3$ or $b = 16.5$ mm **Ans.**

and $h = 2b = 2 \times 16.5 = 33$ mm **Ans.**

**Problem:3.**

A cast iron pulley transmits 10 kW at 400 rpm. The diameter of the pulley is 1.2 metre and it has four straight arms of elliptical cross-section, in which the major axis is twice the minor axis. Determine the dimensions of the arm if the allowable bending stress is 15 MPa.

**Solution.** Given : $P = 10$ kW $= 10 \times 10^3$ W ; $N = 400$ r.p.m ; $D = 1.2$ m $= 1200$ mm or $R = 600$ mm ; $\sigma_b = 15$ MPa $= 15$ N/mm$^2$

Let $T$ = Torque transmitted by the pulley.

We know that the power transmitted by the pulley ($P$),

$$10 \times 10^3 = \frac{2\,\pi\,N\,.\,T}{60} = \frac{2\pi \times 400 \times T}{60} = 42\,T$$

$\therefore$ $T = 10 \times 10^3/42 = 238$ N-m $= 238 \times 10^3$ N-mm

Since the torque transmitted is the product of the tangential load and the radius of the pulley, therefore tangential load acting on the pulley

$$= \frac{T}{R} = \frac{238 \times 10^3}{600} = 396.7 \text{ N}$$

Since the pulley has four arms, therefore tangential load on each arm,

$$W = 396.7/4 = 99.2 \text{ N}$$

and maximum bending moment on the arm,

$$M = W \times R = 99.2 \times 600 = 59\,520 \text{ N-mm}$$

Let $2b$ = Minor axis in mm, and

$2a$ = Major axis in mm $= 2 \times 2b = 4b$ ...(Given)

$\therefore$ Section modulus for an elliptical cross-section,

$$Z = \frac{\pi}{4} \times a^2 b = \frac{\pi}{4}\,(2b)^2 \times b = \pi\,b^3 \text{ mm}^3$$

We know that bending stress ($\sigma_b$),

$$15 = \frac{M}{Z} = \frac{59\,520}{\pi\,b^3} = \frac{18\,943}{b^3}$$

or $b^3 = 18\,943/15 = 1263$ or $b = 10.8$ mm

$\therefore$ Minor axis, $2b = 2 \times 10.8 = 21.6$ mm **Ans.**

and major axis, $2a = 2 \times 2b = 4 \times 10.8 = 43.2$ mm **Ans.**

**Principal Stresses and Principal Planes**

But it has been observed that at any point in a strained material, there are three planes, mutually perpendicular to each other which carry direct stresses only and no shear stress. It may be noted that out of these three direct stresses, one will be maximum and the other will be minimum. These perpendicular planes which have no shear stress are known as ***principal planes*** and the direct stresses along these plane are known as ***principal stresses*.**

**Determination of Principal Stresses for a Member Subjected to Bi-axial Stress**

When a member is subjected to bi-axial stress (*i.e.* direct stress in two mutually perpendicular planes accompanied by a simple shear stress), then the normal and shear stresses are obtained as discussed below:

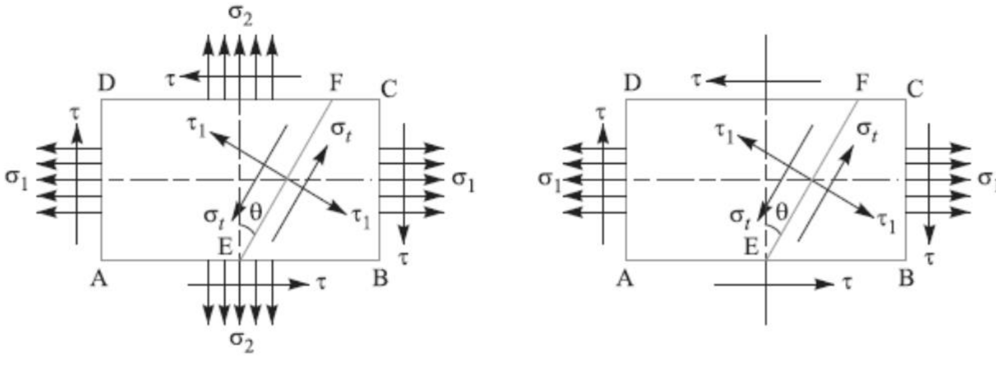
Consider a rectangular body *ABCD* of uniform cross-sectional area and unit thickness subjected to normal stresses σ1 and σ2 as shown in Fig. (*a*).And tangential stress (*i.e.* shear stress) across the section *EF*,

$$\tau_1 = \frac{1}{2}(\sigma_1 - \sigma_2)\sin 2\theta - \tau\cos 2\theta \qquad \ldots(ii)$$

$$\sigma_t = \frac{\sigma_1 + \sigma_2}{2} + \frac{\sigma_1 + \sigma_2}{2}\cos 2\theta + \tau\sin 2\theta \qquad \ldots(i)$$

Since the planes of maximum and minimum normal stress (*i.e.* principal planes) have no shear stress, therefore the inclination of principal planes is obtained by equating τ1 = 0 in the above equation **(*ii*),** *.e.*

$$\tan 2\theta = \frac{2\tau}{\sigma_1 - \sigma_2} \qquad \ldots(iii)$$




(*a*) Direct stress in two mutually prependicular planes accompanied by a simple shear stress.

(*b*) Direct stress in one plane accompanied by a simple shear stress.

**Fig.1.4:Principal stresses for a member subjected to bi-axial stress**

We know that there are two principal planes at right angles to each other. Let θ1 and θ2 be the inclinations of these planes with the normal cross-section. From the following Fig.1.4, we find that

$$\sin 2\theta - \pm \frac{2\tau}{\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2}}$$

The maximum and minimum principal stresses may now be obtained by substituting the values of sin 2θ and cos 2θ in equation ***(i)***. So, Maximum principal (or normal) stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2} \qquad ...(iv)$$

And minimum principal (or nor

al) stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2} \qquad ...(v)$$

The planes of maximum shear stress are at right angles to each other and are inclined at 45° to the principal planes. The maximum shear stress is given by ***one-half the algebraic difference between the principal stresses,*** *i.e*.

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{1}{2}\sqrt{(\sigma_1 - \sigma_2)^2 + 4\tau^2} \qquad ...(vi)$$

Notes: 1. when a member is subjected to direct stress in one plane accompanied by a simple shear stress, then the principal stresses are obtained by substituting σ2 = 0 in equation ***(iv)***, ***(v)*** and ***(vi)***.

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

$$\frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\tau^2}\right]$$

In many shafts such as propeller shafts, C-frames etc., there are direct tensile or compressive stresses due to the external force and shear stress due to torsion, which acts normal to direct tensile or compressive stresses. The shafts like crank shafts, are subjected simultaneously to torsion and bending. In such cases, the maximum principal stresses, due to the combination of tensile or compressive stresses with shear stresses may be obtained. The results obtained in the previous article may be written as follows:

**1.** Maximum tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

**2.** Maximum compressive stress,

$$\sigma_{c(max)} = \frac{\sigma_c}{2} + \frac{1}{2}\left[\sqrt{(\sigma_c)^2 + 4\,\tau^2}\right]$$

**3.** Maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_t)^2 + 4\,\tau^2}\right]$$

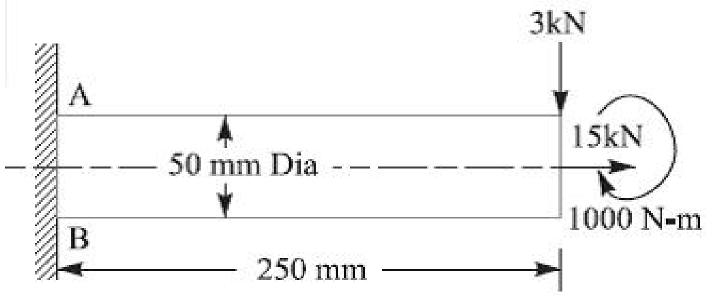
Where $\sigma t$ = Tensile stress due to direct load and bending,

$\sigma c$ = Compressive stress, and

$\tau$ = Shear stress due to torsion.

**Problem.4**

**A shaft, as shown in Fig., is subjected to a bending load of 3 kN, pure torque of 1000 N-m and an axial pulling force of 15 kN. Calculate the stresses at A and B.**



**<u>Solution</u>**

Given data

W= 3kN= 3000 N

T= 1000 Nm =1 x $10^6$ N mm

P = 15 kN =15 x
$10^3$N d= 50 mm
x = 250 mm

Area of the shaft=A = π4 d2

$$= \frac{\pi}{4}(50)^2 = 1964\ mm^2$$

∴ Tensile stress due to axial pulling at points *A* and *B*,

$$\sigma_o - \frac{P}{A} - \frac{15\times10^3}{1964} - 7.64\ N/mm^2 - 7.64\ MPa$$

Bending moment at points *A* and *B*,

$$M = W.x = 3000 \times 250 = 750 \times 10^3\ N\text{-}mm$$

Section modulus for the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(50)^3$$

$$= 12.27 \times 10^3\ mm^3$$

∴ Bending stress at points *A* and *B*,

$$\sigma_b = \frac{M}{Z} = \frac{750\times10^3}{12.27\times10^3}$$

$$= 61.1\ N/mm^2 = 61.1\ MPa$$

This bending stress is tensile at point *A* and compressive at point *B*.

∴ Resultant tensile stress at point *A*,

$$\sigma_A = \sigma_b + \sigma_o = 61.1 + 7.64$$
$$= 68.74\ MPa$$

and resultant compressive stress at point *B*,

$$\sigma_B - \sigma_b \quad \sigma_o - 61.1 \quad 7.64 - 53.46\ MPa$$

We know that the shear stress at points *A* and *B* due to the torque transmitted,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\times1\times10^6}{\pi\,(50)^3} = 40.74\ N/mm^2 = 40.74\ MPa \qquad \ldots\left(\because\ T = \frac{\pi}{16}\times\tau\times d^3\right)$$

*Stresses at point A*

We know that maximum principal (or normal) stress at point *A*,

$$\sigma_{A(max)} = \frac{\sigma_A}{2} + \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right]$$

$$= \frac{68.74}{2} + \frac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 34.37 + 53.3 = 87.67 \text{ MPa (tensile)}$$ **Ans.**

Minimum principal (or normal) stress at point *A*,

$$\sigma_{A(min)} = \frac{\sigma_A}{2} - \frac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = 34.37 - 53.3 = -18.93 \text{ MPa}$$

$$= 18.93 \text{ MPa (compressive)}$$ **Ans.**

and maximum shear stress at point *A*,

$$\tau_{A(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_A)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(68.74)^2 + 4\,(40.74)^2}\right]$$

$$= 53.3 \text{ MPa}$$ **Ans.**

*Stresses at point B*

We know that maximum principal (or normal) stress at point *B*,

$$\sigma_{B(max)} = \frac{\sigma_B}{2} + \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right]$$

$$= \frac{53.46}{2} + \frac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 26.73 + 48.73 = 75.46 \text{ MPa (compressive)}$$ **Ans.**

Minimum principal (or normal) stress at point *B*,

$$\sigma_{B(min)} = \frac{\sigma_B}{2} - \frac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\tau^2}\right]$$

$$= 26.73 - 48.73 = -22 \text{ MPa}$$

$$= 22 \text{ MPa (tensile)}$$ **Ans.**

and maximum shear stress at point *B*,

$$\tau_{B(max)} = \tfrac{1}{2}\left[\sqrt{(\sigma_B)^2 + 4\,\tau^2}\right] = \tfrac{1}{2}\left[\sqrt{(53.46)^2 + 4\,(40.74)^2}\right]$$

$$= 48.73 \text{ MPa}$$ **Ans.**

**Problem 5:**

The load on a bolt consist of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of the bolt required according to. Maximum principal stress theory, Maximum shear stress theory and Maximum distortion energy theory.

**3. *According to maximum principal strain theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-\,2.635}{d^2}\ \text{kN/mm}^2$$

$$= \frac{-\,2635}{d^2}\ \text{N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \text{ or } \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7 \text{ mm}$$ **Ans.**

**4. *According to maximum strain energy theory***

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$\therefore \quad d^4 = 26\,724 \text{ or } d = 12.78 \text{ mm}$$ **Ans.**

**5. *According to maximum distortion energy theory***

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-\,2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-\,2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$\therefore \quad d^4 = 32\,391 \text{ or } d = 13.4 \text{ mm}$$ **Ans.**

**Effect of Loading on Endurance Limit—Load Factor**

The endurance limit ($\sigma e$) of a material as determined by the rotating beam method is for reversed bending load. There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading. The endurance limit depending upon the type of loading may be modified as discussed below:

Let $Kb$ = Load correction factor for the reversed or rotating bending load. Its value is usually taken as unity.$Ka$ = Load correction factor for the reversed axial load. Its value may be taken as 0.8.$Ks$ = Load correction factor for the reversed torsional or shear load. Its value may be taken as 0.55 for ductile materials and 0.8 for brittle materials.

∴ Endurance limit for reversed bending load, $\sigma_{eb} = \sigma_e K_b = \sigma_e$

Endurance limit for reversed axial load, $\sigma_{ea} = \sigma_e K_a$

and endurance limit for reversed torsional or shear load, $\tau_e = \sigma_e K_s$

**Effect of Surface Finish on Endurance Limit—$K_{sf}$**

When a machine member is subjected to variable loads, the endurance limit of the material for that member depends upon the surface conditions. the values of surface finish factor for the various surface conditions and ultimate tensile strength (Fig.1.5).

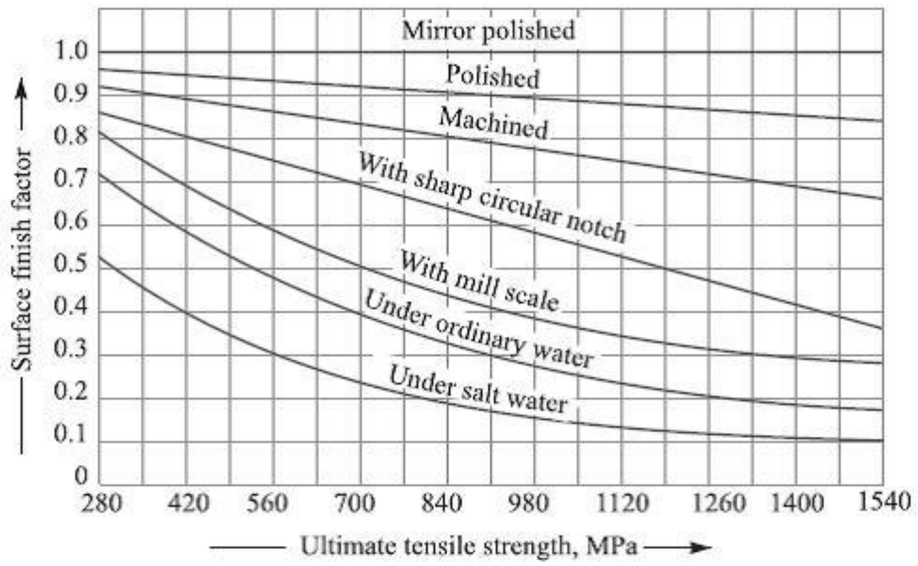



**Fig.1.5: Surface finish factor Curve**

When the surface finish factor is known, then the endurance limit for the material of the machine member may be obtained by multiplying the endurance limit and the surface finish factor. We see that for a mirror polished material, the surface finish factor is unity. In other words, the endurance limit for mirror polished material is maximum and it goes on reducing due to surface condition. Let *Ksur* = Surface finish.

**Factor of Safety for Fatigue Loading**

When a component is subjected to fatigue loading, the endurance limit is the criterion for failure. Therefore, the factor of safety should be based on endurance limit. Mathematically,

$$\text{Factor of safety } (F.S.) = \frac{\text{Endurance limit stress}}{\text{Design or working stress}} = \frac{\sigma_e}{\sigma_d}$$

For steel, $\sigma_e = 0.8 \text{ to } 0.9\, \sigma_y$

$\sigma_e$ = Endurance limit stress for completely reversed stress cycle, and

$\sigma_y$ = Yield point stress.

**Fatigue Stress Concentration Factor**

When a machine member is subjected to cyclic or fatigue loading, the value of fatigue stress concentration factor shall be applied instead of theoretical stress concentration factor. Since the determination of fatigue stress concentration factor is not an easy task, therefore from experimental tests it is defined as Fatigue stress concentration

factor, $$K_f = \frac{\text{Endurance limit without stress concentration}}{\text{Endurance limit with stress concentration}}$$

**Notch Sensitivity**

The term ***notch sensitivity*** is applied to this behaviour. It may be defined as the degree to which the theoretical effect of stress concentration is actually reached. The stress gradient depends mainly on the radius of the notch, hole or fillet and on the grain size of the material. Since the extensive data for estimating the notch sensitivity factor ($q$) is not available, therefore the curves, as shown in Fig., may be used for determining the values of $q$ for two steals.

$K_{fs} = 1 + q\,(K_{ts} - 1)$ ...[For shear stress]

**Problem 6.**

Determine the thickness of a 120 mm wide uniform plate for safe continuous operation if the plate is to be subjected to a tensile load that has a maximum value of 250 KN and a minimum value of 100 kN. The properties of the plate material are asfollows: Endurance limit stress = 225 MPa, and Yield point stress = 300 MPa. The factor of safety based on yield point may be taken as 1.5.

Let $t$ = Thickness of the plate in mm.

$\therefore$ Area, $A = b \times t = 120\, t \text{ mm}^2$

We know that mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{250 + 100}{2} = 175 \text{ kN} = 175 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{175 \times 10^3}{120t} \text{ N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{250 - 100}{2} = 75 \text{ kN} = 75 \times 10^3 \text{ N}$$

$$\therefore \quad \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{75 \times 10^3}{120t} \text{ N/mm}^2$$

According to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v}{\sigma_e}$$

$$\frac{1}{1.5} = \frac{175 \times 10^3}{120t \times 300} + \frac{75 \times 10^3}{120t \times 225} = \frac{4.86}{t} + \frac{2.78}{t} = \frac{7.64}{t}$$

$\therefore$ $t = 7.64 \times 1.5 = 11.46$ say $11.5$ mm **Ans.**

**Problem 7:**

Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σe = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from Wmin = – 300 × $10^3$ N to Wmax = 700 × $10^3$ N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.

Let $d$ = Diameter of the circular rod in mm.

$$\therefore \quad \text{Area, } A = \frac{\pi}{4} \times d^2 = 0.7854\ d^2\ \text{mm}^2$$

We know that the mean or average load,

$$W_m = \frac{W_{max} + W_{min}}{2} = \frac{700 \times 10^3 + (-300 \times 10^3)}{2} = 200 \times 10^3\ \text{N}$$

$$\therefore \quad \text{Mean stress, } \sigma_m = \frac{W_m}{A} = \frac{200 \times 10^3}{0.7854\ d^2} = \frac{254.6 \times 10^3}{d^2}\ \text{N/mm}^2$$

$$\text{Variable load, } W_v = \frac{W_{max} - W_{min}}{2} = \frac{700 \times 10^3 - (-300 \times 10^3)}{2} = 500 \times 10^3\ \text{N}$$

$$\therefore \text{Variable stress, } \sigma_v = \frac{W_v}{A} = \frac{500 \times 10^3}{0.7854\ d^2} = \frac{636.5 \times 10^3}{d^2}\ \text{N/mm}^2$$

We know that according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e}$$

$$\frac{1}{2} = \frac{254.6 \times 10^3}{d^2 \times 350} + \frac{636.5 \times 10^3 \times 1.8}{d^2 \times 265} = \frac{727}{d^2} + \frac{4323}{d^2} = \frac{5050}{d^2}$$

$$\therefore \quad d^2 = 5050 \times 2 = 10\,100 \quad \text{or} \quad d = 100.5\ \text{mm}\ \textbf{Ans.}$$

**Problem 8 :**

A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety of 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar are given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.

**Solution.** Given : $l$ = 500 mm ; $W_{min}$ = 20 kN = 20 × $10^3$ N ; $W_{max}$ = 50 kN = 50 × $10^3$ N ; $F.S.$ = 1.5 ; $K_{sz}$ = 0.85 ; $K_{sur}$ = 0.9 ; $\sigma_u$ = 650 MPa = 650 N/mm$^2$ ; $\sigma_y$ = 500 MPa = 500 N/mm$^2$ ; $\sigma_e$ = 350 MPa = 350 N/mm$^2$

Let $d$ = Diameter of the bar in mm.

We know that the maximum bending moment,

$$M_{max} = \frac{W_{max} \times l}{4} = \frac{50 \times 10^3 \times 500}{4} = 6250 \times 10^3\ \text{N-mm}$$

and minimum bending moment,

$$M_{min} = \frac{W_{min} \times l}{4} = \frac{20 \times 10^3 \times 500}{4} = 2550 \times 10^3\ \text{N-mm}$$

∴ Mean or average bending moment,

$$M_m = \frac{M_{max} + M_{min}}{2} = \frac{6250 \times 10^3 + 2500 \times 10^3}{2} = 4375 \times 10^3\ \text{N-mm}$$

and variable bending stress,

$$\sigma_v = \frac{M_v}{Z} = \frac{1875 \times 10^3}{0.0982\, d^3} = \frac{19.1 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that according to Goodman's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_u} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$


$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 650} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$

$$= \frac{68 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{139 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 139 \times 10^3 \times 1.5 = 209 \times 10^3$ or $d = 59.3$ mm

and according to Soderberg's formula,

$$\frac{1}{F.S.} = \frac{\sigma_m}{\sigma_y} + \frac{\sigma_v \times K_f}{\sigma_e \times K_{sur} \times K_{sz}}$$

$$\frac{1}{1.5} = \frac{44.5 \times 10^6}{d^3 \times 500} + \frac{19.1 \times 10^6 \times 1}{d^3 \times 350 \times 0.9 \times 0.85} \qquad \text{...(Taking } K_f = 1\text{)}$$


$$= \frac{89 \times 10^3}{d^3} + \frac{71 \times 10^3}{d^3} = \frac{160 \times 10^3}{d^3}$$

$\therefore \quad d^3 = 160 \times 10^3 \times 1.5 = 240 \times 10^3$ or $d = 62.1$ mm

Taking larger of the two values, we have $d = 62.1$ mm **Ans.**

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**SCHOOL OF MECHANICAL ENGINEERING**

**DEPARTMENT OF MECHANICAL ENGINEERING**

# Unit-II Design of Shafts and Couplings-SMEA1502

# Introduction

A shaft is a rotating machine element which is used to transmit power from one place to other place.

**Materials**

Carbon steels of grade 40C8, 45C8, 50C4, 50C12 are normally used as shaft materials. Material properties

It should have high strength

It should have good machinability.

It should have low notch sensitivity factor ,It should have good heat treatment roperties. It should have high wear resistance.

**Types of Shaft**

**1. Transmission shaft:**

These shafts transmit power between the source and machines absorbing power. The counter shafts, line shafts, overhead shafts all shafts are transmission shafts.

**2. Machine shafts:**

These shafts from an integral part of the machine itself.

**Following stresses are induced in the shaft**

1. Shear stress due to transmission of torque
2. Bending stress due to forces acting upon machine elements like gears, pulleys etc.
3. Stresses due to combined torsional and bending loads.

**DESIGN OF SHAFTS**

The shaft may be designed on the basis of

1. Strength
2. Rigidity and
3. Stiffness

**In designing shaft on the basis of strength the following cases may be consider**

1. Shaft subjected to bending moment only
2. Shaft subjected to combined twisting moment and bending moment
3. Shaft subjected to fluctuating loads

4. Shafts subjected to twisting moment only

**1. SHAFTS SUBJECTED TO TWISTING MOMENT ONLY**

$$\frac{T}{J}=\frac{\tau}{r}$$

$$T=\frac{\pi}{16}\times\tau\times d^3$$

For hollow section

$$T=\frac{\pi}{16}\times\tau\times d_o{}^3\times\left(1-k^4\right)$$

$k=\frac{d_i}{d_o}$ Where $d_i$=inside diameter, $d_o$= outside diameter

Twisting moment may be obtained by using the following relation

$$T=\frac{P\times 60}{2\times\pi\times N}$$

In case of belt drives

$$T=(T_1-T_2)\,R$$

$T_1$- Tension in the tight side

$T_2$- Tension in the slack side

R- Radius of the pulley

**2. SHAFT SUBJECTED TO BENDING MOMENT ONLY**

The bending moment equation is

$$\frac{M}{I}=\frac{\sigma_b}{y}$$

M- Bending moment

I- moment of inertia of cross sectional area of the shaft about the axis of rotation

$\sigma_b$- Bending stress

For round solid shaft

$$M = \frac{\pi}{32} \times \sigma_b \times d^3$$

For hollow shaft

$$M = \frac{\pi}{32} \times \sigma_b \times d_o^{\,3} \times \left(1 - k^4\right)$$

**3. SHAFT SUBJECTED TO COMBINED TWISTING MOMENT AND BENDING MOMENT**

When the shaft is subjected to combined twisting moment ad bending moment then the shaft must be designed on the basic of two moments simultaneously

For solid shaft

$$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{\left(M^2 + T^2\right)}$$

$$\frac{\pi}{32} \times \sigma_{b(max)} \times d^3 = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

For hollow shaft

$$\frac{\pi}{16} \times \tau_{max} \times d_o^{\,3} \times \left(1 - k^4\right) = \sqrt{\left(M^2 + T^2\right)}$$

$$\frac{\pi}{32} \times \sigma_{b(max)} \times d_o^{\,3} \times \left(1 - k^4\right) = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)$$

## KEY

A key is a piece of mild steel inserted between the shaft and hub or boss of the pulley to connect these together in order to prevent relative motion between them. It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses. A keyway is a slot or recess in a shaft and hub of the pulley to accommodate a key

. **TYPES OF KEYS**

1. Sunk key, 2.Saddle key, 3.Tangent key, 4. Round key 5.Splines

## KEYWAYS

The keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross sectional area of the shaft.

## DESIGN OF COUPLING

Shaft couplings are used in machinery for several purposes

1. To provide for connection of shaft of units those are manufactured separately.
2. To provide for misalignment of the shaft or to introduce mechanical flexibility.
3. To reduce the transmission of shock loads from one shaft to another.
4. To introduce protection against over loads.

## REQUIREMENT OF A GOOD SHAFT COUPLING

1. It should be easy to connect or disconnect.
2. It should transmit the full power from one shaft to the other shaft without losses.
3. It should hold the shaft in perfect alignment.
4. . It should have no projecting parts.

## TYPES OF SHAFT COUPLINGS

**1. Rigid coupling**

It is used to connect two shafts which are perfectly aligned. The types are

i. Sleeve (or) muff coupling
ii. Clamp(or) split muff type
iii. Flange coupling

2. **Flexible coupling**

It is used to connect two shafts having lateral and angular misalignments. The types are

Bushed pin type

Universal coupling

Oldham coupling

**SLEEVE (or) MUFF COUPLING**

It is made of cast iron. It consists of a hollow cylinder whose inner diameter is that same as that of the shaft. It is fitted over the ends of two shafts by means of a gib head key. The power transmitted from one shaft two other shafts by means of a key and a sleeve.

Outer diameter of sleeve D=2d+13mm

Length of sleeve L=3.5d

d- diameter of shaft

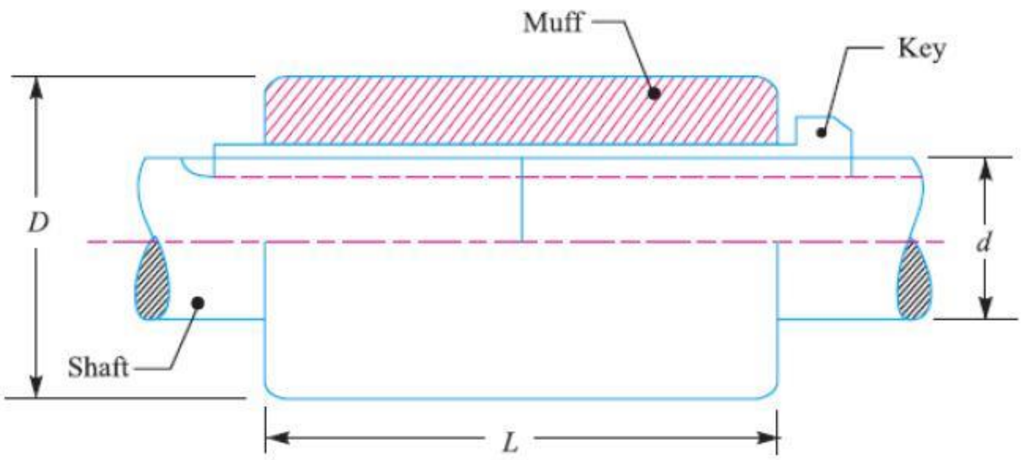



Figure 2.1 muff coupling

**DESIGN OF MUFF COUPILNG**

1. Design for sleeve

The sleeve is designed by considering it as a hollow shaft

$$T = \frac{\pi}{16} \times \tau \times \left( \frac{D^4 - d^4}{D} \right)$$

2. Design for key

The length of coupling key is at least equal to the length of the sleeve. The coupling key is usually made into two parts so that the length of key in each shaft

$$l = \frac{L}{2}$$

after that the induced shearing and crushing stresses may be checked.

$$T = l \times w \times \tau \times \frac{d}{2}$$

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

**CLAMP (or) COMPRESSION COUPLING**

In this case the muff or sleeve is made into two halves are bolted together. The halves of the muff are made of cast iron. The shaft end is made to abut each other and a single key is fitted directly in the keyway of both the shaft. Both the halves are held together by means of mildsteel bolts and nuts. The number of bolt may be two or four or six.

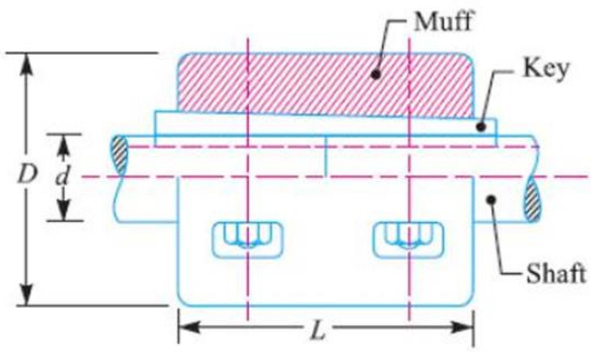



Fig.2.2:Compressive coupling

Diameter of muff D=2d+13mm

Length of muff or sleeve L=3.5d

**DESIGN OF CLAMP (or) COMPRESSION COUPILNG**

1. Design for sleeve

The sleeve is designed by considering it as a hollow shafts

$$T = \frac{\pi}{16} \times \tau \times \left( \frac{D^4 - d^4}{D} \right)$$

2. Design for key

The length of coupling key is at least equal to the length of the sleeve. The coupling key is usually made into two parts so that the length of key in each shaft

$$l = \frac{L}{2}$$

after that the induced shearing and crushing stresses may be checked.

$$T = l \times w \times \tau \times \frac{d}{2}$$

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$

3. Design of clamping bolts

Force exerted by each bolt

$$=\frac{\pi}{4}\times d_b^{\,2}\times\sigma_t$$

Force exerted by each side of the shaft

$$=\frac{\pi}{4}\times d_b^{\,2}\times\sigma_t\times\frac{n}{2}$$

Torque transmitted by the coupling

$$T=\frac{\pi}{16}\times\mu\times d_b^{\,2}\times\sigma_t\times n\times d$$

Where

T-torque transmitted by the shaft

d-diameter of shaft

$d_b$- root or effective dia of bolt

n- number of bolt

σ-Permissible stress for bolt

μ-coefficent of friction between the muff and shaft

**FLANGE COUPLING**

A flange coupling usually applied to a coupling having two separate cast iron flanges. Each flange is mounted on the shaft and keyed to it. The faces are turned up at right angle to the axis of the shaft. One of the flange has a projected portion and the other flange has a corresponding recess. This helps to bring the shaft into line and to maintain alignment. The two flanges are coupled together by means of bolt and nuts.

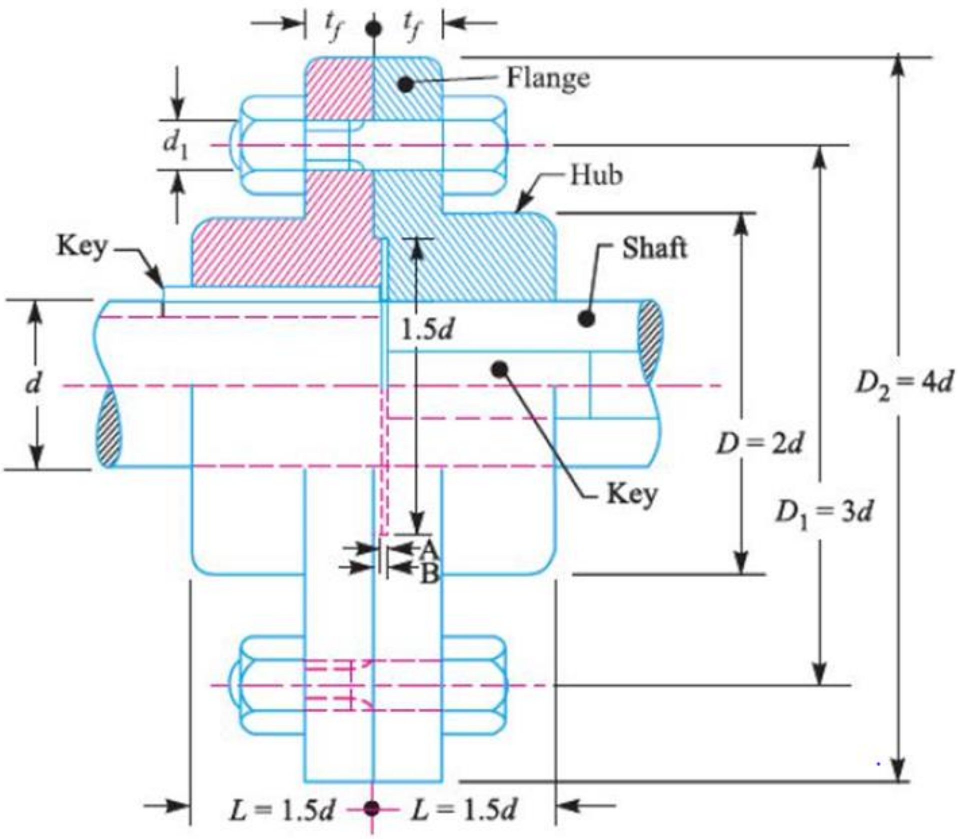



Fig.2.3:Flange Coupling

1. Design for hub

The hub is designed by considering it as a hollow shaft

$$T = \frac{\pi}{16} \times \tau_c \times \left( \frac{D^4 - d^4}{D} \right)$$

D=2*d

Length of hub L=1.5d

2. Design for key

The key is designed with equal properties and then checked for shearing and crushing stress. The length of key is taken equal to the length of hub

3. Design for flange

The key is designed with equal properties and then checked for shearing and crushing stress. The length of key is taken equal to the length of hub

3. Design for flange

$$T = \frac{\pi D^2}{2} \times \tau_c \times t_f$$

$t_f$- thickness of flange(d/2)

4. Design for bolt

The bolts are subjected to shear stress due to torque transmitted. The number of bolts (n) depends upon the diameter of shaft and pitch circle diameter is taken

$D_1=3d$

Torque transmitted

$$T = \frac{\pi}{4} \times d_1^2 \times \tau_b \times n \times \frac{D_1}{2}$$

$d_1$- diameter of bolt

for crushing

$$T = (n \times d_1 \times t_f \times \sigma_{cb})\frac{D_1}{2}$$

**PROBLEM 1 :**

Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

**Solution.** Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $\tau_u = 360$ MPa $= 360$ N/mm² ; $F.S. = 8$ ; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

***Diameter of the solid shaft***

Let $d$ = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft ($T$),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

***Diameter of hollow shaft***

Let $d_i$ = Inside diameter, and

$d_o$ = Outside diameter.

We know that the torque transmitted by the hollow shaft ( $T$ ),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

$\therefore$ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

**Problem 2.**

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

**Solution.** Given : $M$ = 3000 N-m = $3 \times 10^6$ N-mm ; $T$ = 10 000 N-m = $10 \times 10^6$ N-mm ; $\sigma_{tu}$ = 700 MPa = 700 N/mm$^2$ ; $\tau_u$ = 500 MPa = 500 N/mm$^2$

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let $d$ = Diameter of the shaft in mm.

According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment ($T_e$),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$\therefore$ $d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6$ or $d = 86$ mm

According to maximum normal stress theory, equivalent bending moment,

$$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \tfrac{1}{2}\,(M + T_e)$$

$$= \tfrac{1}{2}\,(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment ($M_e$),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$\therefore$ $d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6$ or $d = 83.7$ mm

Taking the larger of the two values, we have

$d = 86$ say 90 mm **Ans.**

**Problem 3.**

mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?

**Solution.** Given : $P = 20$ kW $= 20 \times 10^3$ W ; $N = 200$ r.p.m. ; $W = 900$ N ; $L = 2.5$ m ; $\tau = 42$ MPa $= 42$ N/mm$^2$ ; $\sigma_b = 56$ MPa $= 56$ N/mm$^2$

***Size of the shaft***

Let $d$ = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$
$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment ($T_e$),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3$ or $d = 51.2$ mm

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \tfrac{1}{2}(M + T_e)$$
$$= \tfrac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment ($M_e$),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$\therefore$ $d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3$ or $d = 53.4$ mm

Taking the larger of the two values, we have

$d = 53.4$ say 55 mm **Ans.**

**Problem 5.**

hoisting drum 0.5 m in diameter is keyed to a shaft which is supported in two bearings and driven through a 12 : 1 reduction ratio by an electric motor. Determine the power of the driving motor, if the maximum load of 8 kN is hoisted at a speed of 50 m/min and the efficiency of the drive is 80%. Also determine the torque on the drum shaft and the speed of the motor in r.p.m. Determine also the diameter of the shaft made of machinery steel, the working stresses of which are 115 MPa in tension and 50 MPa in shear. The drive gear whose diameter is 450 mm is mounted at the end of the shaft such that it overhangs the nearest bearing by 150 mm. The combined shock and fatigue factors for bending and torsion may be taken as 2 and 1.5 respectively.

**Solution.** Given : $D = 0.5$ m or $R = 0.25$ m ; Reduction ratio = 12 : 1 ; $W = 8$ kN = 8000 N ; $v = 50$ m/min ; $\eta = 80\% = 0.8$ ; $\sigma_t = 115$ MPa = 115 N/mm$^2$ ; $\tau = 50$ MPa = 50 N/mm$^2$ ; $D_1 = 450$ mm or $R_1 = 225$ mm = 0.225 m ; Overhang = 150 mm = 0.15 m ; $K_m = 2$ ; $K_t = 1.5$

***Power of the driving motor***

We know that the energy supplied to the hoisting drum per minute

$$= W \times v = 8000 \times 50 = 400 \times 10^3 \text{ N-m/min}$$

∴ Power supplied to the hoisting drum

$$= \frac{400 \times 10^3}{60} = 6670 \text{ W} = 6.67 \text{ kW} \qquad ...(\because 1 \text{ N-m/s} = 1 \text{ W})$$

Since the efficiency of the drive is 0.8, therefore power of the driving motor

$$= \frac{6.67}{0.8} = 8.33 \text{ kW}$$ **Ans.**

***Torque on the drum shaft***

We know that the torque on the drum shaft,

$$T = W.R = 8000 \times 0.25 = 2000 \text{ N-m}$$ **Ans.**

***Speed of the motor***

Let $N$ = Speed of the motor in r.p.m.

We know that angular speed of the hoisting drum

$$= \frac{\text{Linear speed}}{\text{Radius of the drum}} = \frac{v}{R} = \frac{50}{0.25} = 200 \text{ rad / min}$$

Since the reduction ratio is 12 : 1, therefore the angular speed of the electric motor,

$$\omega = 200 \times 12 = 2400 \text{ rad/min}$$

and speed of the motor in r.p.m.,

$$N = \frac{\omega}{2\pi} = \frac{2400}{2\pi} = 382 \text{ r.p.m.}$$ **Ans.**

***Diameter of the shaft***

Let $d$ = Diameter of the shaft.

Since the torque on the drum shaft is 2000 N-m, therefore the tangential tooth load on the drive gear,

$$F_t = \frac{T}{R_1} = \frac{2000}{0.225} = 8900 \text{ N}$$

Assuming that the pressure angle of the drive gear in 20°, therefore the maximum bending load on the shaft due to tooth load

$$= \frac{F_t}{\cos 20°} = \frac{8900}{0.9397} = 9470 \text{ N}$$

Since the overhang of the shaft is 150 mm = 0.15 m, therefore bending moment at the bearing,

$$M = 9470 \times 0.15 = 1420 \text{ N-m}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1420)^2 + (1.5 \times 2000)^2} = 4130 \text{ N-m} = 4130 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment ($T_e$),

$$4130 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 50 \times d^3 = 9.82\, d^3$$

$\therefore$ $d^3 = 4130 \times 10^3 / 9.82 = 420.6 \times 10^3$ or $d = 75$ mm

Again we know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}(K_m \times M + T_e)$$

$$= \tfrac{1}{2}(2 \times 1420 + 4130) = 3485 \text{ N-m} = 3485 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment ($M_e$),

$$3485 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 115 \times d^3 = 11.3\, d^3$$

$\therefore$ $d^3 = 3485 \times 10^3 / 11.3 = 308.4 \times 10^3$ or $d = 67.5$ mm

Taking the larger of the two values, we have

$d = 75$ mm **Ans.**

Problem 6.

Design and make a neat dimensioned sketch of a muff coupling which is used to connect two steel shafts transmitting 40 kW at 350 r.p.m. The material for the shafts and key is plain carbon steel for which allowable shear and crushing stresses may be taken as 40 MPa and 80 MPa respectively. The material for the muff is cast iron for which the allowable shear stress may be assumed as 15 MPa.

**Solution.** Given : $P$ = 40 kW = 40 × 103 W; $N$ = 350 r.p.m.; $\tau s$ = 40 MPa = 40 N/mm$^2$; $\sigma cs$ = 80 MPa =80 N/mm2; $\tau c$ = 15 MPa = 15 N/mm$^2$.

The muff coupling is designed as discussed below :

**1. *Design for shaft***

Let $d$ = Diameter of the shaft.

We know that the torque transmitted by the shaft,

key and muff,

$$T = \frac{P \times 60}{2\pi N} = \frac{40 \times 10^3 \times 60}{2\pi \times 350} = 1100 \text{ N-m}$$
$$= 1100 \times 10^3 \text{ N-mm}$$

We also know that the torque transmitted ($T$),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$\therefore$ $d^3 = 1100 \times 10^3/7.86 = 140 \times 10^3$ or $d = 52$ say 55 mm **Ans.**

**2. *Design for sleeve***

We know that outer diameter of the muff,

$$D = 2d + 13 \text{ mm} = 2 \times 55 + 13 = 123 \text{ say } 125 \text{ mm}$$ **Ans.**

and length of the muff,

$$L = 3.5\, d = 3.5 \times 55 = 192.5 \text{ say } 195 \text{ mm}$$ **Ans.**

Let us now check the induced shear stress in the muff. Let $\tau_c$ be the induced shear stress in the muff which is made of cast iron. Since the muff is considered to be a hollow shaft, therefore the torque transmitted ($T$),

$$1100 \times 10^3 = \frac{\pi}{16} \times \tau_c \left(\frac{D^4 - d^4}{D}\right) = \frac{\pi}{16} \times \tau_c \left[\frac{(125)^4 - (55)^4}{125}\right]$$
$$= 370 \times 10^3\, \tau_c$$

$\therefore$ $\tau_c = 1100 \times 10^3/370 \times 10^3 = 2.97 \text{ N/mm}^2$

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm$^2$, therefore the design of muff is safe.

Since the induced shear stress in the muff (cast iron) is less than the permissible shear stress of 15 N/mm$^2$, therefore the design of muff is safe.

**3. *Design for key***

From Table 13.1, we find that for a shaft of 55 mm diameter,

Width of key, $w = 18$ mm **Ans.**

Since the crushing stress for the key material is twice the shearing stress, therefore a square key may be used.

$\therefore$ Thickness of key, $t = w = 18$ mm **Ans.**

We know that length of key in each shaft,

$$l = L/2 = 195/2 = 97.5 \text{ mm}$$ **Ans.**

Let us now check the induced shear and crushing stresses in the key. First of all, let us consider shearing of the key. We know that torque transmitted ($T$),

$$1100 \times 10^3 = l \times w \times \tau_s \times \frac{d}{2} = 97.5 \times 18 \times \tau_s \times \frac{55}{2} = 48.2 \times 10^3\, \tau_s$$

$\therefore$ $\tau_s = 1100 \times 10^3 / 48.2 \times 10^3 = 22.8 \text{ N/mm}^2$

Now considering crushing of the key. We know that torque transmitted (*T*),

$$1100 \times 10^3 = l \times \frac{t}{2} \times \sigma_{cs} \times \frac{d}{2} = 97.5 \times \frac{18}{2} \times \sigma_{cs} \times \frac{55}{2} = 24.1 \times 10^3 \sigma_{cs}$$

$$\therefore \quad \sigma_{cs} = 1100 \times 10^3 / 24.1 \times 10^3 = 45.6 \text{ N/mm}^2$$

Since the induced shear and crushing stresses are less than the permissible stresses, therefore the design of key is safe.

**Problem 7.**

Design a cast iron protective type flange coupling to transmit 15 kW at 900 r.p.m. from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used :

Shear stress for shaft, bolt and key material = 40 MPa

Crushing stress for bolt and key = 80 MPa

Shear stress for cast iron = 8 MPa

Draw a neat sketch of the coupling.

**Solution.** Given : P = 15 kW = 15 × 103 W ; N = 900 r.p.m. ; Service factor = 1.35 ; $\tau_s = \tau b = \tau_k$ = 40 MPa = 40 N/mm$^2$ ; $\sigma_{cb} = \sigma_{ck}$ = 80 MPa = 80 N/mm$^2$ ; $\tau_c$ = 8 MPa = 8 N/mm$^2$

The protective type flange coupling is designed as discussed below

**1. *Design for hub***

First of all, let us find the diameter of the shaft (*d*). We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{15 \times 10^3 \times 60}{2 \pi \times 900} = 159.13 \text{ N-m}$$

Since the service factor is 1.35, therefore the maximum torque transmitted by the shaft,

$$T_{max} = 1.35 \times 159.13 = 215 \text{ N-m} = 215 \times 10^3 \text{ N-mm}$$

We know that the torque transmitted by the shaft (*T*),

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\, d^3$$

$$\therefore \quad d^3 = 215 \times 10^3 / 7.86 = 27.4 \times 10^3 \quad \text{or} \quad d = 30.1 \text{ say } 35 \text{ mm}$$ **Ans.**

We know that outer diameter of the hub,

$$D = 2d = 2 \times 35 = 70 \text{ mm}$$ **Ans.**

and length of hub, $$L = 1.5\, d = 1.5 \times 35 = 52.5 \text{ mm}$$ **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron. Considering the hub as a hollow shaft. We know that the maximum torque transmitted ($T_{max}$).

$$215 \times 10^3 = \frac{\pi}{16} \times \tau_c \left[\frac{D^4 - d^4}{D}\right] = \frac{\pi}{16} \times \tau_c \left[\frac{(70)^4 - (35)^4}{70}\right] = 63\ 147\ \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3 / 63\ 147 = 3.4\ \text{N/mm}^2 = 3.4\ \text{MPa}$

Since the induced shear stress for the hub material (*i.e.* cast iron) is less than the permissible value of 8 MPa, therefore the design of hub is safe.

### 2. *Design for key*

Since the crushing stress for the key material is twice its shear stress (*i.e.* $\sigma_{ck} = 2\tau_k$), therefore a square key may be used. From Table 13.1, we find that for a shaft of 35 mm diameter,

Width of key, $w = 12$ mm **Ans.**

and thickness of key, $t = w = 12$ mm **Ans.**

The length of key ( $l$ ) is taken equal to the length of hub.

$\therefore$ $l = L = 52.5$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing.

Considering the key in shearing. We know that the maximum torque transmitted ($T_{max}$),

$$215 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 52.5 \times 12 \times \tau_k \times \frac{35}{2} = 11\ 025\ \tau_k$$

$\therefore$ $\tau_k = 215 \times 10^3 / 11\ 025 = 19.5\ \text{N/mm}^2 = 19.5\ \text{MPa}$

Considering the key in crushing. We know that the maximum torque transmitted ($T_{max}$),

$$215 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 52.5 \times \frac{12}{2} \times \sigma_{ck} \times \frac{35}{2} = 5512.5\ \sigma_{ck}$$

$\therefore$ $\sigma_{ck} = 215 \times 10^3 / 5512.5 = 39\ \text{N/mm}^2 = 39\ \text{MPa}$

Since the induced shear and crushing stresses in the key are less than the permissible stresses, therefore the design for key is safe.

### 3. *Design for flange*

The thickness of flange ($t_f$) is taken as 0.5 $d$.

$\therefore$ $t_f = 0.5\ d = 0.5 \times 35 = 17.5$ mm **Ans.**

Let us now check the induced shearing stress in the flange by considering the flange at the junction of the hub in shear.

We know that the maximum torque transmitted ($T_{max}$),

$$215 \times 10^3 = \frac{\pi D^2}{2} \times \tau_c \times t_f = \frac{\pi (70)^2}{2} \times \tau_c \times 17.5 = 134\ 713\ \tau_c$$

$\therefore$ $\tau_c = 215 \times 10^3 / 134\ 713 = 1.6\ \text{N/mm}^2 = 1.6\ \text{MPa}$

Since the induced shear stress in the flange is less than 8 MPa, therefore the design of flange is safe.

### 4. *Design for bolts*

Let $d_1$ = Nominal diameter of bolts.

Since the diameter of the shaft is 35 mm, therefore let us take the number of bolts,

$n = 3$

and pitch circle diameter of bolts,

$D_1 = 3d = 3 \times 35 = 105$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted ($T_{max}$),

$$215 \times 10^3 = \frac{\pi}{4}(d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}(d_1)^2\ 40 \times 3 \times \frac{105}{2} = 4950\ (d_1)^2$$

$\therefore$ $(d_1)^2 = 215 \times 10^3/4950 = 43.43$ or $d_1 = 6.6$ mm

Assuming coarse threads, the nearest standard size of bolt is M 8. **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 35 = 140 \text{ mm}$$ **Ans.**

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 35 = 8.75 \text{ say } 10 \text{ mm}$$ **Ans.**

**Problem 8.**

Design and draw a protective type of cast iron flange coupling for a steel shaft transmitting 15 kW at 200 r.p.m. and having an allowable shear stress of 40 MPa. The working stress in the bolts should not exceed 30 MPa. Assume that the same material is used for shaft and key and that the crushing stress is twice the value of its shear stress. The maximum torque is 25% greater than the full load torque. The shear stress for cast iron is 14 MPa.

**Solution.** Given : P = 15 kW = 15 × 103 W ; N = 200 r.p.m. ; $\tau s$ = 40 MPa = 40 N/mm$^2$ ; $\tau_b$ = 30 MPa = 30 N/mm$^2$ ; $\sigma_{ck} = 2\tau k$ ; Tmax = 1.25 $T_{mean}$ ; $\tau c$ = 14 MPa = 14 N/mm$^2$

The protective type of cast iron flange coupling is designed as discussed below :

**1. *Design for hub***

First of all, let us find the diameter of shaft ( $d$ ). We know that the full load or mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

and maximum torque transmitted,

$$T_{max} = 1.25\,T_{mean} = 1.25 \times 716 \times 10^3 = 895 \times 10^3 \text{ N-mm}$$

We also know that maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_s \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\,d^3$$

$\therefore$ $d^3 = 895 \times 10^3 / 7.86 = 113\ 868$ or $d = 48.4$ say 50 mm **Ans.**

We know that the outer diameter of the hub,

$$D = 2\,d = 2 \times 50 = 100 \text{ mm}$$ **Ans.**

and length of the hub, $L = 1.5\,d = 1.5 \times 50 = 75$ mm **Ans.**

Let us now check the induced shear stress for the hub material which is cast iron, by considering it as a hollow shaft. We know that the maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = \frac{\pi}{16} \times \tau_c \left( \frac{D^4 - d^4}{D} \right) = \frac{\pi}{16} \times \tau_c \left( \frac{(100)^4 - (50)^2}{100} \right) = 184\ 100\ \tau_c$$

$\therefore \quad \tau_c = 895 \times 10^3/184\ 100 = 4.86\ N/mm^2 = 4.86\ MPa$

Since the induced shear stress in the hub is less than the permissible value of 14 MPa, therefore the design for hub is safe.

**2. *Design for key***

Since the crushing stress for the key material is twice its shear stress, therefore a square key may be used.

From Table 13.1, we find that for a 50 mm diameter shaft,

Width of key, $w = 16$ mm **Ans.**

and thickness of key, $t = w = 16$ mm **Ans.**

The length of key ( $l$ ) is taken equal to the length of hub.

$\therefore \quad l = L = 75$ mm **Ans.**

Let us now check the induced stresses in the key by considering it in shearing and crushing. Considering the key in shearing. We know that the maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = l \times w \times \tau_k \times \frac{d}{2} = 75 \times 16 \times \tau_k \times \frac{50}{2} = 30 \times 10^3\ \tau_k$$

$\therefore \quad \tau_k = 895 \times 10^3 / 30 \times 10^3 = 29.8\ N/mm^2 = 29.8\ MPa$

Considering the key in crushing. We know that the maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = 75 \times \frac{16}{2} \times \sigma_{ck} \times \frac{50}{2} = 15 \times 10^3\ \sigma_{ck}$$

$\therefore \quad \sigma_{ck} = 895 \times 10^3 / 15 \times 10^3 = 59.6\ N/mm^2 = 59.6\ MPa$

Since the induced shear and crushing stresses in key are less than the permissible stresses, therefore the design for key is safe.

**3. *Design for flange***

The thickness of the flange ( $t_f$) is taken as 0.5 $d$.

$\therefore \quad t_f = 0.5 \times 50 = 25$ mm **Ans.**

Let us now check the induced shear stress in the flange, by considering the flange at the junction of the hub in shear. We know that the maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = \frac{\pi\ D^2}{2} \times \tau_c \times t_f = \frac{\pi\ (100)^2}{2} \times \tau_c \times 25 = 392\ 750\ \tau_c$$

$\therefore \quad \tau_c = 895 \times 10^3/392\ 750 = 2.5\ N/mm^2 = 2.5\ MPa$

Since the induced shear stress in the flange is less than the permissible value of 14 MPa, therefore the design for flange is safe.

**4. *Design for bolts***

Let $\quad d_1$ = Nominal diameter of bolts.

Since the diameter of shaft is 50 mm, therefore let us take the number of bolts,

$n = 4$

and pitch circle diameter of bolts,

$D_1 = 3\ d = 3 \times 50 = 150$ mm

The bolts are subjected to shear stress due to the torque transmitted. We know that the maximum torque transmitted ($T_{max}$),

$$895 \times 10^3 = \frac{\pi}{4}\ (d_1)^2\ \tau_b \times n \times \frac{D_1}{2} = \frac{\pi}{4}\ (d_1)^2\ 30 \times 4 \times \frac{150}{4} = 7070\ (d_1)^2$$

$(d_1)^2 = 895 \times 10^3 / 7070 = 126.6 \qquad$ or $\qquad d_1 = 11.25$ mm

Assuming coarse threads, the nearest standard diameter of the bolt is 12 mm (M 12). **Ans.**

Other proportions of the flange are taken as follows :

Outer diameter of the flange,

$$D_2 = 4\,d = 4 \times 50 = 200 \text{ mm} \quad \textbf{Ans.}$$

Thickness of the protective circumferential flange,

$$t_p = 0.25\,d = 0.25 \times 50 = 12.5 \text{ mm} \quad \textbf{Ans.}$$

SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

# UNIT- III-Temporary and Permanent Joints-SMEA1502

# Introduction

## Temporary and Permanent Joints

**<u>Threaded Fasteners:</u>**

**1. Threaded Joints:**

It is defined as a separable joint of two or more machine parts that are held together by means of a threaded fastening such as a bolt and nut.

**2. Bolts:**

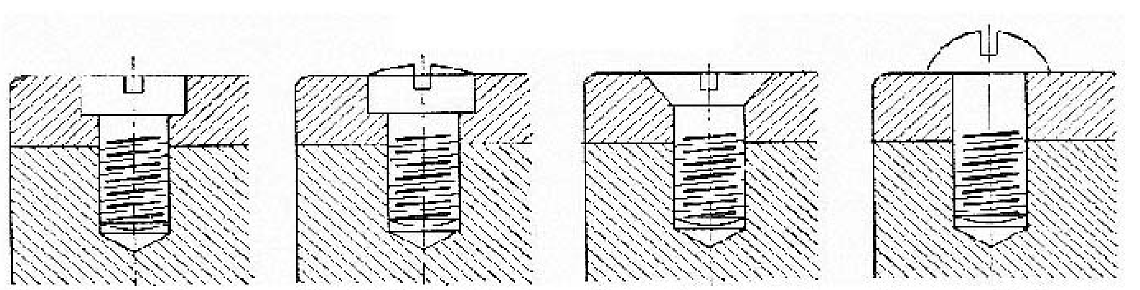
They are basically threaded fasteners normally used with bolts and nuts.

**3. Screws:**

They engage either with preformed or self-made internal threads.

**4. Studs:**

They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut. There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type as shown in Fig 3.1.




**Fig, 3.1 :Studs**

**5. Tapping screws:**

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used. There are two types of tapping screws. They are known as thread forming which displaces or forms the adjacent materials and thread cutting which have cutting edges and chip cavities which create a mating thread.

**6. Set Screws:**

These are semi-permanent fasteners which hold collars, pulleys, gears etc on a shaft. Different heads and point styles are available.

**7. Thread forms:**

Basically, when a helical groove is cut or generated over a cylindrical or conical section, threads are formed. When a point moves parallel to the axis of a rotating cylinder or cone held between centers, a helix is generated. Screw threads formed in this way have two functions to perform in general:

(a) To transmit power - Square. ACMES, Buttress, Knuckle types of thread forms are useful for this purpose.

(b) To secure one member to another- V-threads are most useful for this purpose. V-threads are generally used for securing because they do not shake loose due to the wedging action provided by the thread. Square threads give higher efficiency due to a low friction. The various threads as shown in Fig 3.2 to 3.4.

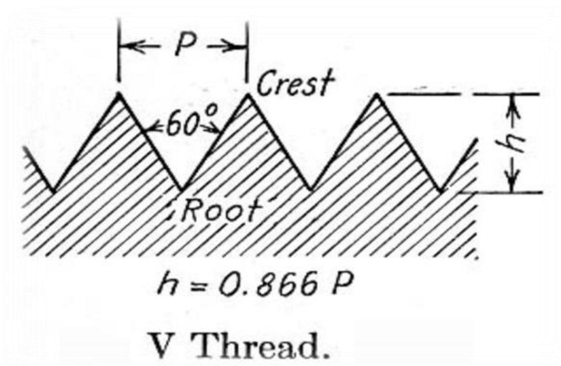



**Fig.3.2:Thread**

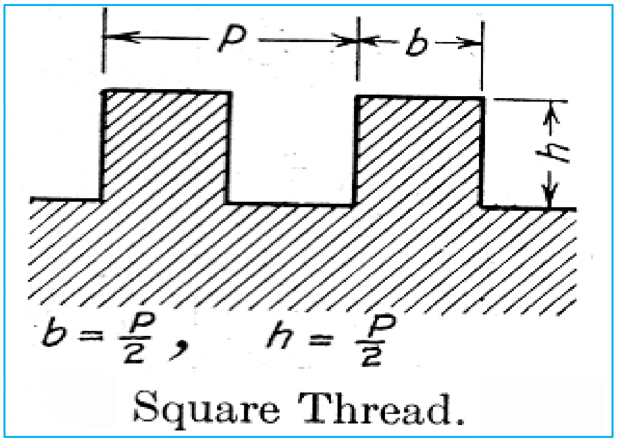



**Fig. 3.3**

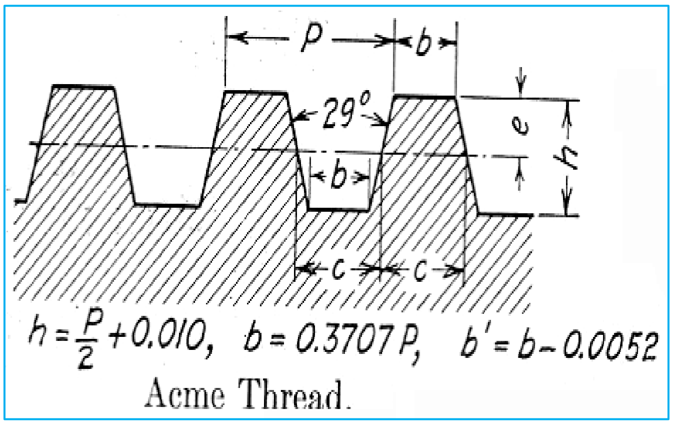



**Fig.3 4:Screw thread**

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They are externally threaded headless fasteners. One end usually meets a tapped component and the other with a standard nut. There are different forms of bolt and screw heads for a different usage. These include bolt heads of square, hexagonal or eye shape and screw heads of hexagonal, Fillister, button head, counter sunk or Phillips type. As shown in Fig.3.5.

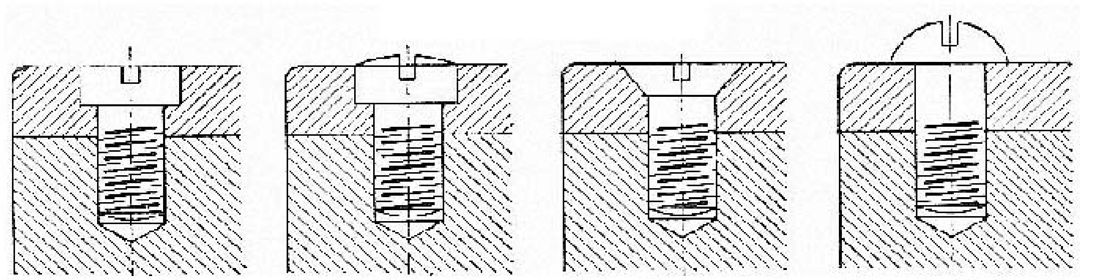


**Fig. 3.5 Screw thread**

***5.Tapping screws:***

These are one piece fasteners which cut or form a mating thread when driven into a preformed hole. These allow rapid installation since nuts are not used. There are two types of tapping screws. They are known as thread forming which displaces or forms the adjacent materials and thread cutting which have cutting edges and chip cavities which create a mating thread.

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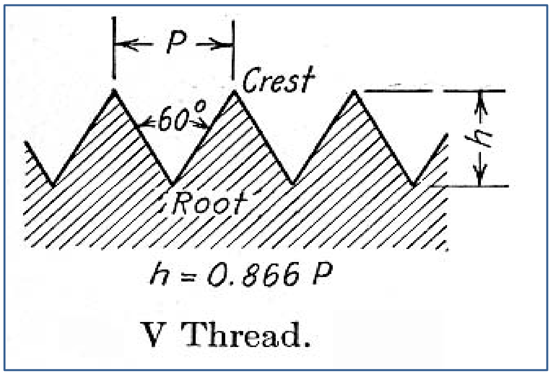
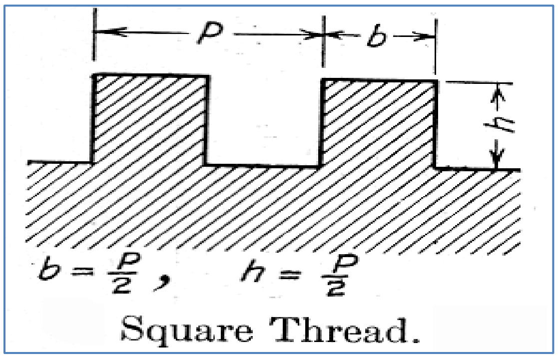



**Fig, 3.6**




**Fig, 3.7**

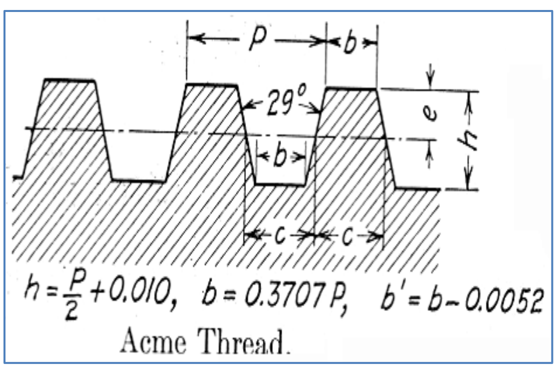



**Fig. 3.8:Thrad efficiency**

***Advantages of Threaded joints:***

- No loosening of the parts, Therefore it called Reliable Joints.
- Due to usage of spanners more mechanical advantage and force required is less
- Compact construction
- Threads are self-locking
- Manufacturing is simple and economic
- It is standardized and used widely for different operations and applications.

A part from transmitting motion and power the threaded members are also used for fastening or jointing two elements. The threads used in power screw are square or Acme while threads used in fastening screws have a v profile. Because of large transverse inclination the effective friction coefficient between the screw and nut increases by equation $= \mu = \mu / \cos \theta$ where $\mu$ is the basic coefficient of friction of the pair of screw and nut, $\theta$ is the half of thread angle and $'\mu$ is the effective coefficient of friction. The wedging effect of transverse inclination of the thread surface was explained. According to IS : 1362-1962 the metric thread has a thread angle of 60o. The other proportions of thread profile are shown in Figure 5.17. IS:1362 designates threads by M followed by a figure representing the major diameter, d. For example a screw or bolt having the major diameter of 2.5 mm will be designated as M 2.5.The standard describes the major (also called nominal) diameter of the bolt and nut, pitch, pitch diameter, minor or core diameter, depth of bolt thread and area resisting load (Also called stress area). Pitch diameter in case of V-threads corresponds to mean diameter in square or Acme thread. Inequality of dm and dp is seen from Figure 3.9 and Fig. 3.10.

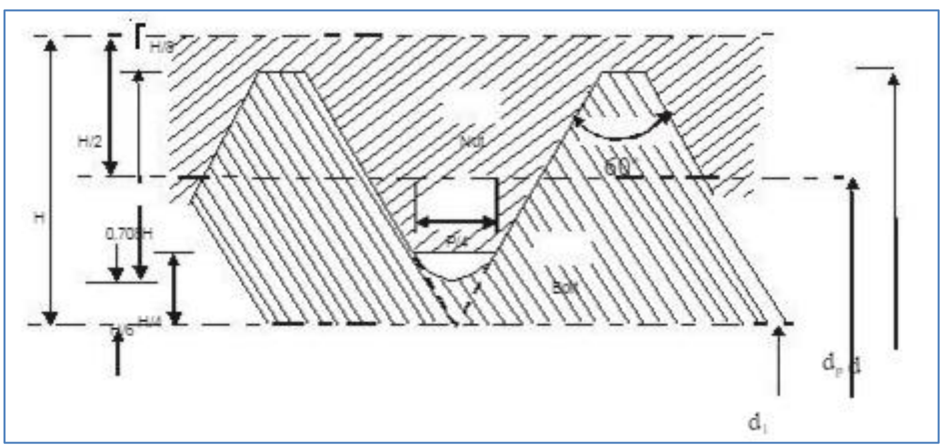



**Fig :3.9 Profiles of Fastener Threads on Screw (Bolt) and Nut**

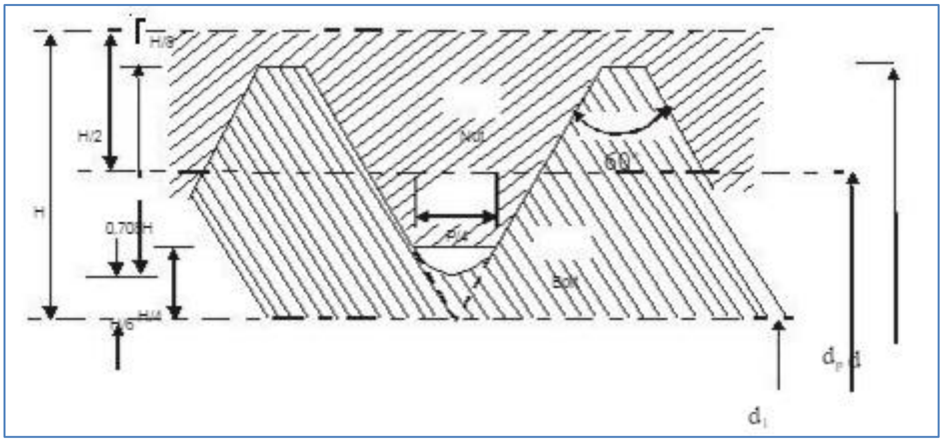
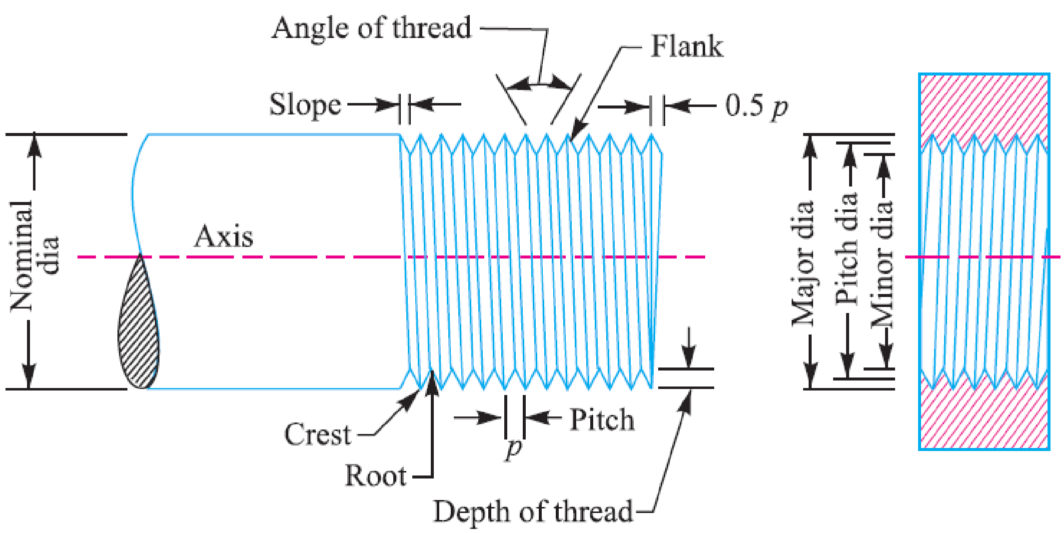



**Fig. 3.10 Profiles of Fastener Threads on Screw (Bolt) and Nut**

## Important Terms Used in Screw Threads

The following terms used in screw threads, as shown in Fig. 3.12, are important from the subject point of view :




**Fig, 3.12 Terms used in screw threads.**

**1. *Major diameter*.** It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as ***outside*** or ***nominal diameter*.**

**2. *Minor diameter*.** It is the smallest diameter of an external or internal screw thread. It is also known as ***core*** or ***root diameter*.**

**3. *Pitch diameter*.** It is the diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an ***effective diameter*.** In a nut and bolt assembly, it is the diameter at which the ridges on the bolt are in complete touch with the ridges of the corresponding nut.

**4. *Pitch*.** It is the distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane. Mathematically, Pitch =1/ No. of threads per unit length of screw

**5. *Lead*.** It is the distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.

**6. *Crest*.** It is the top surface of the thread.

**7. *Root*.** It is the bottom surface created by the two adjacent flanks of the thread.

**8. *Depth of thread*.** It is the perpendicular distance between the crest and root.

**9. *Flank*.** It is the surface joining the crest and root.

**10. *Angle of thread*.** It is the angle included by the flanks of the thread.

**11. *Slope*.** It is half the pitch of the thread

**Example 1.** *Two machine parts are fastened together tightly by means of a 24 mm tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in the bolt by the initial tightening.*

**Solution.** Given: $d$ = 24 mm

we find that the core diameter of the thread corresponding to

Let $\sigma_t$ = Stress set up in the bolt.

We know that initial tension in the bolt,

$$P = 2840\, d = 2840 \times 24 = 68\,160 \text{ N}$$

We also know that initial tension in the bolt ($P$),

$$68\,160 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(20.30)^2\,\sigma_t = 324\,\sigma_t$$

$$\therefore \quad \sigma_t = 68\,160 / 324 = 210 \text{ N/mm}^2 = 210 \text{ MPa}$$ **Ans.**

## Stresses due to External Forces

The following stresses are induced in a bolt when it is subjected to an external load.

**1. *Tensile stress*.** The bolts, studs and screws usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt. Let $d_c$ = Root or core diameter of the thread, and $\sigma_t$ = Permissible tensile stress for the bolt material.

Let $d_c$ = Root or core diameter of the thread, and

$\sigma_t$ = Permissible tensile stress for the bolt material.

We know that external load applied,

$$P = \frac{\pi}{4}(d_c)^2 \sigma_t \quad \text{or} \quad d_c = \sqrt{\frac{4P}{\pi \sigma_t}}$$

Now from Table 11.1, the value of the nominal diameter of bolt corresponding to the value of $d_c$ may be obtained or stress area $\left[\frac{\pi}{4}(d_c)^2\right]$ may be fixed.

**Notes:** **(*a*)** If the external load is taken up by a number of bolts, then

$$P = \frac{\pi}{4}(d_c)^2 \sigma_t \times n$$

(*b*) In case the standard table is not available, then for coarse threads, $d_c = 0.84\, d$, where $d$ is the nominal diameter of bolt.

***2. Shear stress.*** Sometimes, the bolts are used to prevent the relative movement of two or more parts, as in case of flange coupling, then the shear stress is induced in the bolts. The shear stresses should be avoided as far as possible. It should be noted that when the bolts are subjected to direct shearing loads, they should be located in such a way that the shearing load comes upon the body (*i.e.* shank) of the bolt and not upon the threaded portion. In some cases, the bolts may be relieved of shear load by using shear pins. When a number of bolts are used to share the shearing load, the finished bolts should be fitted to the reamed holes.

Let $d$ = Major diameter of the bolt, and

$n$ = Number of bolts.

Let $d$ = Major diameter of the bolt, and

$n$ = Number of bolts.

∴ Shearing load carried by the bolts,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau \times n \quad \text{or} \quad d = \sqrt{\frac{4P_s}{\pi \tau n}}$$

***3. Combined tension and shear stress.*** When the bolt is subjected to both tension and shear loads, as in case of coupling bolts or bearing, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load. A diameter slightly larger than that required for either shear or tension may be assumed and stresses due to combined load should be checked for the following principal stresses. Maximum principal shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

and maximum principal tensile stress,

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

**Example 11.4.** *Two shafts are connected by means of a flange coupling to transmit torque of 25 N-m. The flanges of the coupling are fastened by four bolts of the same material at a radius of 30 mm. Find the size of the bolts if the allowable shear stress for the bolt material is 30 MPa.*

**Solution.** Given : $T = 25$ N-m $= 25 \times 10^3$ N-mm ; $n = 4$; $R_p = 30$ mm ; $\tau = 30$ MPa $= 30$ N/mm$^2$

We know that the shearing load carried by flange coupling,

$$P_s = \frac{T}{R_p} = \frac{25 \times 10^3}{30} = 833.3 \text{ N} \quad \text{...}(i)$$

Let $d_c$ = Core diameter of the bolt.

$\therefore$ Resisting load on the bolts

$$= \frac{\pi}{4}(d_c)^2 \tau \times n = \frac{\pi}{4}(d_c)^2 \; 30 \times 4 = 94{\cdot}26\,(d_c)^2 \quad \text{...}(ii)$$

From equations *(i)* and *(ii)*, we get

$(d_c)^2 = 833.3 / 94.26 = 8.84$ or $d_c = 2.97$ mm

From Table 11.1 (coarse series), we find that the standard core diameter of the bolt is 3.141 mm the corresponding size of the bolt is M 4. **Ans.**

## WELDED JOINTS

### Introduction

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium *e.g.* to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

## Advantages and Disadvantages of Welded Joints over Riveted Joints

Following are the advantages and disadvantages of welded joints over riveted joints.

***Advantages***

**1.** The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.

**2.** The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.

**3.** Alterations and additions can be easily made in the existing structures.

**4.** As the welded structure is smooth in appearance, therefore it looks pleasing.

**5.** In welded connections, the tension members are not weakened as in the case of riveted joints.

**6.** A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.

**7.** Sometimes, the members are of such a shape (*i.e.* circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.

**8.** The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.

**9.** It is possible to weld any part of a structure at any point. But riveting requires enough clearance.

**10.** The process of welding takes less time than the riveting.

***Disadvantages***

**1.** Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.

**2.** It requires a highly skilled labour and supervision.

**3.** Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.

**4.** The inspection of welding work is more difficult than riveting work.

## Welding Processes

The welding processes may be broadly classified into the following two groups:

**1.** Welding processes that use heat alone *e.g.* fusion welding.

**2.** Welding processes that use a combination of heat and pressure *e.g.* forge welding.

These processes are discussed in detail, in the following pages.

## Fusion Welding

In case of fusion welding, the parts to be jointed are held in position while the molten metal is supplied to the joint. The molten metal may come from the parts themselves (*i.e.* parent metal) or filler metal which normally have the composition of the parent metal. The joint surface become plastic or even molten because of the heat from the molten filler metal or other source. Thus, when the molten metal solidifies or fuses, the joint is formed.

The fusion welding, according to the method of heat generated, may be classified as:

**1.** Thermit welding, **2.** Gas welding, and **3.** Electric arc welding.

## Types of Welded Joints

Following two types of welded joints are important from the subject point of view as shown in Fig. 3.13:

**1.** Lap joint or fillet joint, and **2.** Butt joint.

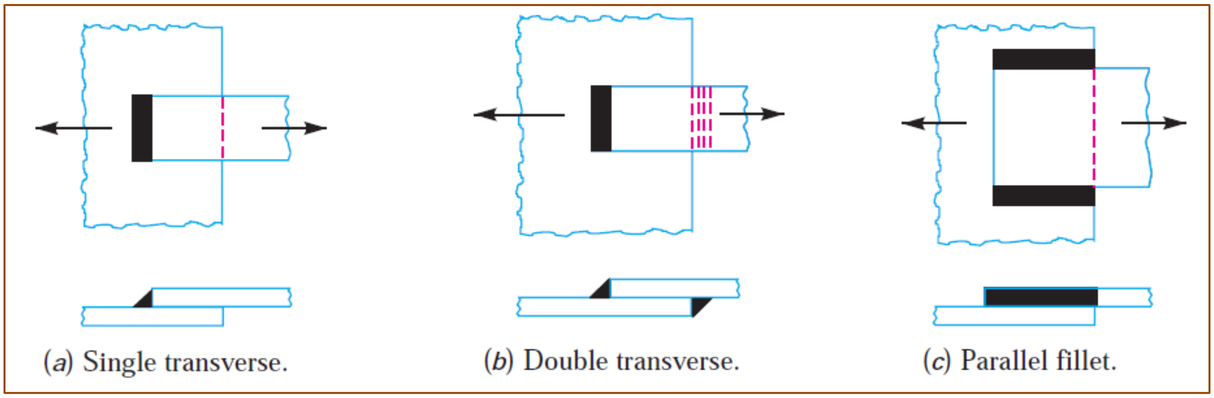



**Fig. 3.13:Types of Joints**

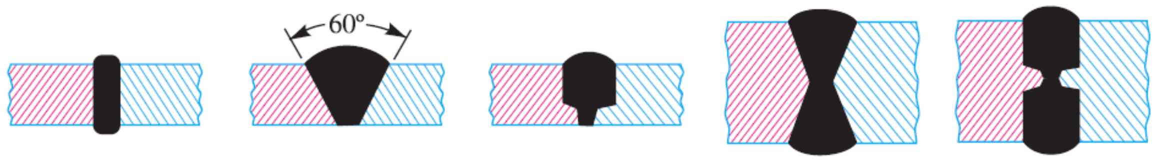
## Lap Joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be **1.** Single transverse fillet, **2.** Double transverse fillet, and **3.** Parallel fillet joints.

The fillet joints are shown in Fig. 3.14 A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

## Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. 10.3. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U-groove on both sides.




(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

**Fig. 3.14 :Welded joints**

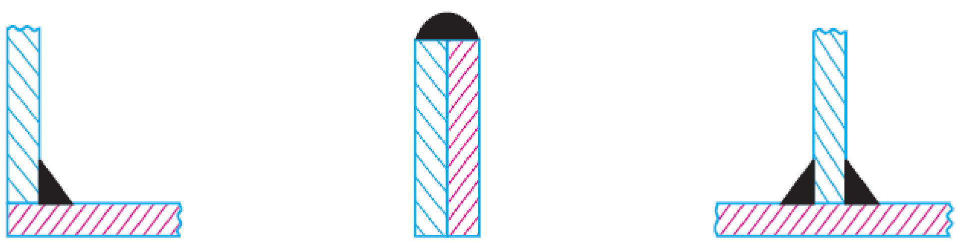
The butt joints may be

**1.** Square butt joint, **2.** Single V-butt joint **3.** Single U-butt joint,

**4.** Double V-butt joint, and **5.** Double U-butt joint.

These joints are shown in Fig.3.15.

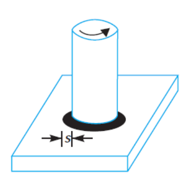
The other type of welded joints are corner joint, edge joint and T-joint as shown in Fig.



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

**Fig. 3.15 Welded joints**

**Example: A** *50 mm diameter solid shaft is welded to a flat plate by 10 mm fillet weld as shown in Fig.. Find the maximum torque that the welded joint can sustain if the maximum shear stress intensity in the weld material is not to exceed 80 MPa.*



**Solution.** Given : $d$ = 50 mm ; $s$ = 10 mm ; $\tau_{max}$ = 80 MPa = 80 N/mm$^2$

Let $T$ = Maximum torque that the welded joint can sustain.

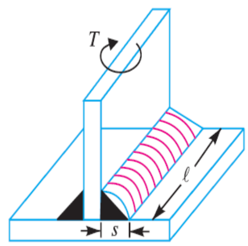
We know that the maximum shear stress ($\tau_{max}$),

$$80 = \frac{2.83\,T}{\pi s \times d^2} = \frac{2.83\,T}{\pi \times 10\,(50)^2} = \frac{2.83\,T}{78550}$$

$\therefore$ $T = 80 \times 78\,550/2.83$

$= 2.22 \times 10^6$ N-mm = 2.22 kN-m **Ans.**

**Example.** *A plate 1 m long, 60 mm thick is welded to another plate at right angles to each other by 15 mm fillet weld, as shown in Fig. Find the maximum torque that the welded joint can sustain if the permissible shear stress intensity in the weld material is not to exceed 80 MPa.*




**Solution.** Given: $l$ = 1m = 1000 mm ; Thickness = 60 mm ; $s$ = 15 mm ; $\tau max$ = 80 MPa = 80 N/mm2

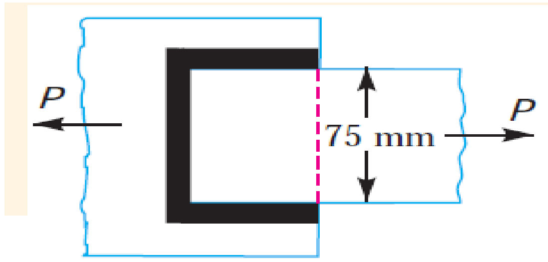
Let $T$ = Maximum torque that the welded joint can sustain

We know that the maximum shear stress ($\tau_{max}$),

$$80 = \frac{4.242\,T}{s \times l^2} = \frac{4.242\,T}{15\,(1000)^2} = \frac{0.283\,T}{10^6}$$

$\therefore$ $T = 80 \times 10^6 / 0.283 = 283 \times 10^6$ N-mm = 283 kN-m **Ans.**

**Example 10.5.** *A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.*




**Solution.** Given : Width = 75 mm ; Thickness = 12.5 mm ; $\sigma\tau$ = 70 MPa = 70 N/mm2 ; $\tau$ = 56 MPa = 56 N/mm2.

The effective length of weld ($l1$) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

$$\therefore l1 = 75 - 12.5 = 62.5 \text{ mm}$$

***Length of each parallel fillet for static loading***

Let $l2$ = Length of each parallel fillet.

We know that the maximum load which the plate can carry is

$$P = \text{Area} \times \text{Stress} = 75 \times 12.5 \times 70 = 65\,625 \text{ N}$$

Load carried by single transverse weld,

$$P1 = 0.707\, s \times l1 \times \sigma t = 0.707 \times 12.5 \times 62.5 \times 70 = 38\,664 \text{ N}$$

and the load carried by double parallel fillet weld,

$$P2 = 1.414\, s \times l2 \times \tau = 1.414 \times 12.5 \times l2 \times 56 = 990\, l2 \text{ N}$$

Load carried by the joint ($P$),

$65\,625 = P1 + P2 = 38\,664 + 990\, l2$ or $l2 = 27.2$ mm

Adding 12.5 mm for starting and stopping of weld run, we have

$l2 = 27.2 + 12.5 = 39.7$ say 40 mm **Ans.**

***Length of each parallel fillet for fatigue loading***

From Table 10.6, we find that the stress concentration factor for transverse welds is 1.5 and for parallel fillet welds is 2.7.

∴ Permissible tensile stress,

$\sigma t = 70 / 1.5 = 46.7$ N/mm2

and permissible shear stress,

$\tau = 56 / 2.7 = 20.74$ N/mm2

Load carried by single transverse weld,

$P_1 = 0.707\ s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\ 795$ N

and load carried by double parallel fillet weld,

$P_2 = 1.414\ s \times l_2 \times \tau = 1.414 \times 12.5\ l_2 \times 20.74 = 366\ l_2$ N

∴ Load carried by the joint ($P$),

$65\ 625 = P_1 + P_2 = 25\ 795 + 366\ l_2$ or $l_2 = 108.8$ mm

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm}$$ **Ans.**

***Example:*** *Determine the length of the weld run for a plate of size 120 mm wide and 15 mm thick to be welded to another plate by means of*

*1. A single transverse weld; and*

*2. Double parallel fillet welds when the joint is subjected to variable loads.*

**Solution.** Given : Width = 120 mm ; Thickness = 15 mm

In Fig. *AB* represents the single transverse weld and *AC* and *BD* represents double parallel fillet welds.

**1. *Length of the weld run for a single transverse weld***

The effective length of the weld run ($l_1$) for a single transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

∴ $l_1 = 120 - 12.5 = 107.5$ mm **Ans.**

**2. *Length of the weld run for a double parallel fillet weld subjected to variable loads***

Let $l_2$ = Length of weld run for each parallel fillet, and

$s$ = Size of weld = Thickness of plate = 15 mm

Assuming the tensile stress as 70 MPa or $N/mm^2$ and shear stress as 56 MPa or $N/mm^2$ for static loading. We know that the maximum load which the plate can carry is

$P = \text{Area} \times \text{Stress} = 120 \times 15 \times 70 = 126 \times 10^3$ N

we find that the stress concentration factor for transverse weld is 1.5 and for parallel fillet welds is 2.7. ∴ Permissible tensile stress,

$\sigma_t = 70 / 1.5 = 46.7\ N/mm^2$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

∴ Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 15 \times 107.5 \times 46.7 = 53\,240 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l_2 \times \tau = 1.414 \times 15 \times l_2 \times 20.74 = 440\, l_2 \text{ N}$$

∴ Load carried by the joint ($P$),

$$126 \times 10^3 = P_1 + P_2 = 53\,240 + 440\, l_2 \quad \text{or} \quad l_2 = 165.4 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

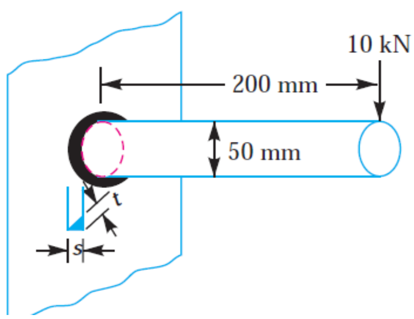
$$l_2 = 165.4 + 12.5 = 177.9 \text{ say } 178 \text{ mm}$$ **Ans.**

**Example 10.10.** *A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig.. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.*

**Solution.** Given : $D$ = 50 mm ; $s$ = 15 mm ; $P$ = 10 kN = 10 000 N ; $e$ = 200 mm

Let $t$ = Throat thickness.

The joint, as shown in Fig. is subjected to direct shear stress and the bending stress. We know that the throat area for a circular fillet weld,




$$A = t \times \pi D = 0.707\, s \times \pi D$$
$$= 0.707 \times 15 \times \pi \times 50$$
$$= 1666 \text{ mm}^2$$

∴ Direct shear stress,

$$\tau = \frac{P}{A} = \frac{10\,000}{1666} = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

We know that bending moment,

$$M = P \times e = 10\,000 \times 200 = 2 \times 10^6 \text{ N-mm}$$

*Maximum shear stress*

We know that the maximum shear stress,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2} = 48.4 \text{ MPa} \quad \textbf{Ans.}$$

$$Z = \frac{\pi\, t\, D^2}{4} = \frac{\pi \times 0.707\ s \times D^2}{4} = \frac{\pi \times 0.707 \times 15\ (50)^2}{4} = 20\ 825 \text{ mm}^3$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2 \times 10^6}{20\ 825} = 96 \text{ N/mm}^2 = 96 \text{ MPa}$$

*Maximum normal stress*

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2} \times 96 + \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2}$$

$$= 48 + 48.4 = 96.4 \text{ MPa} \quad \textbf{Ans.}$$

## Knuckle Joint

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs. Its use may be found in the link of a cycle chain, tie rod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridge structure and lever and rod connections of various types.

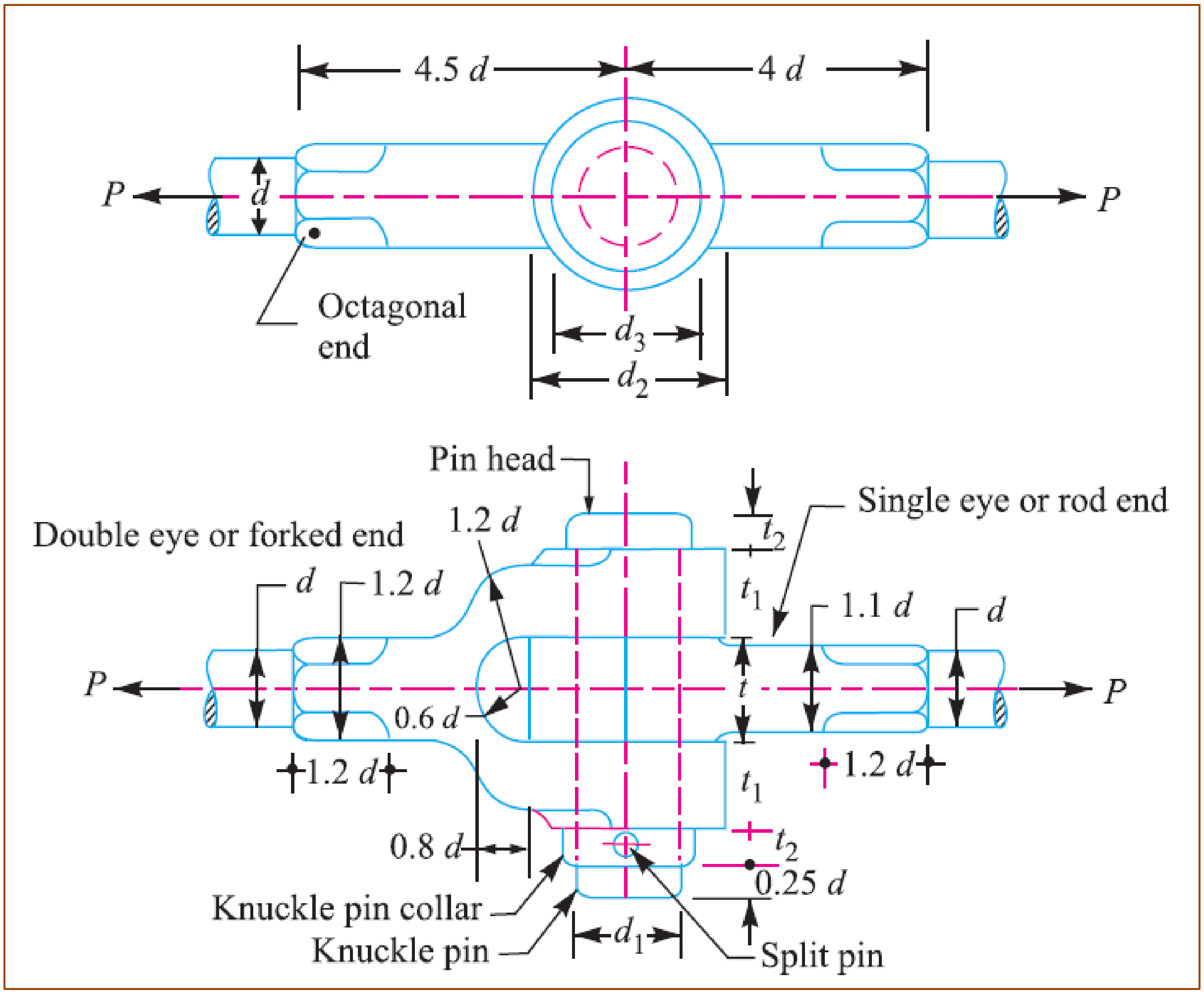



**Fig. 3.16 :Knuckle joint**

## Methods of Failure of Knuckle Joint

Consider a knuckle joint as shown in Fig.

Let $P$ = Tensile load acting on the rod,

$d$ = Diameter of the rod,

$d1$ = Diameter of the pin,

$d2$ = Outer diameter of eye,

$t$ = Thickness of single eye,

$t1$ = Thickness of fork.

$\sigma t$ , $\tau$ and $\sigma c$ = Permissible stresses for the joint material in tension, shear and crushing respectively.

In determining the strength of the joint for the various methods of failure, it is assumed that

**1.** There is no stress concentration, and

**2.** The load is uniformly distributed over each part of the joint.

Due to these assumptions, the strengths are approximate, however they serve to indicate a well-proportioned joint. Following are the various methods of failure of the joint:

**1. *Failure of the solid rod in tension***

Since the rods are subjected to direct tensile load, therefore tensile strength of the rod,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to the load ($P$) acting on the rod, we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rod ( $d$ ) is obtained.

**2. *Failure of the knuckle pin in shear***

Since the pin is in double shear, therefore cross-sectional area of the pin under shearing

$$= 2 \times \frac{\pi}{4} (d_1)^2$$

and the shear strength of the pin

$$= 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

Equating this to the load ($P$) acting on the rod, we have

$$P = 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

**3. *Failure of the single eye or rod end in tension***

The single eye or rod end may tear off due to the tensile load. We know that area resisting tearing $= (d_2 - d_1)\, t$

$\therefore$ Tearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \sigma_t$$

Equating this to the load ($P$) we have

$$P = (d_2 - d_1)\, t \times \sigma_t$$

**4. *Failure of the single eye or rod end in shearing***

The single eye or rod end may fail in shearing due to tensile load. We know that area resisting shearing $= (d_2 - d_1)\, t$

$\therefore$ Shearing strength of single eye or rod end

$$= (d_2 - d_1)\, t \times \tau$$

Equating this to the load ($P$), we have

$$P = (d_2 - d_1)\, t \times \tau$$

From this equation, the induced shear stress ($\tau$) for the single eye or rod end may be checked.

**5. *Failure of the single eye or rod end in crushing***

The single eye or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times t$

$\therefore$ Crushing strength of single eye or rod end

$$= d_1 \times t \times \sigma_c$$

Equating this to the load ($P$), we have

$$\therefore \quad P = d_1 \times t \times \sigma_c$$

**6. *Failure of the forked end in tension***

The forked end or double eye may fail in tension due to the tensile load. We know that area resisting tearing

$$= (d_2 - d_1) \times 2\, t_1$$

$\therefore$ Tearing strength of the forked end

$$= (d_2 - d_1) \times 2\, t_1 \times \sigma_t$$

Equating this to the load ($P$), we have

$$P = (d_2 - d_1) \times 2t_1 \times \sigma_t$$

From this equation, the induced tensile stress for the forked end may be checked.

**7. *Failure of the forked end in shear***

The forked end may fail in shearing due to the tensile load. We know that area resisting shearing

$$= (d_2 - d_1) \times 2t_1$$

$\therefore$ Shearing strength of the forked end

$$= (d_2 - d_1) \times 2t_1 \times \tau$$

Equating this to the load ($P$), we have

$$P = (d_2 - d_1) \times 2t_1 \times \tau$$

**8. *Failure of the forked end in crushing***

The forked end or pin may fail in crushing due to the tensile load. We know that area resisting crushing $= d_1 \times 2\, t_1$

$\therefore$ Crushing strength of the forked end

$$= d_1 \times 2\, t_1 \times \sigma_c$$

Equating this to the load ($P$), we have

$$P = d_1 \times 2\, t_1 \times \sigma_c$$

**Example 12.7.** *Design a knuckle joint to transmit 150 kN. The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in compression.*

**Solution.** Given : $P$ = 150 kN = 150 × 103 N ; $\sigma t$ = 75 MPa = 75 N/mm2 ; $\tau$ = 60 MPa = 60 N/mm2 ; $\sigma c$ = 150 MPa = 150 N/mm2

**1. *Failure of the solid rod in tension***

Let $d$ = Diameter of the rod.

We know that the load transmitted ($P$),

$$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 75 = 59\, d^2$$

$\therefore$ $d^2 = 150 \times 10^3 / 59 = 2540$ or $d = 50.4$ say 52 mm **Ans.**

Now the various dimensions are fixed as follows :

Diameter of knuckle pin,

$$d_1 = d = 52 \text{ mm}$$

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar,

$$d_3 = 1.5\,d = 1.5 \times 52 = 78 \text{ mm}$$

Thickness of single eye or rod end,

$$t = 1.25\,d = 1.25 \times 52 = 65 \text{ mm}$$

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\,d = 0.5 \times 52 = 26$ mm

**2. *Failure of the knuckle pin in shear***

Since the knuckle pin is in double shear, therefore load ($P$),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times (d_1)^2\, \tau = 2 \times \frac{\pi}{4} \times (52)^2\, \tau = 4248\, \tau$$

$\therefore$ $\tau = 150 \times 10^3 / 4248 = 35.3 \text{ N/mm}^2 = 35.3$ MPa

**3. *Failure of the single eye or rod end in tension***

The single eye or rod end may fail in tension due to the load. We know that load ($P$),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \sigma_t = (104 - 52)\, 65 \times \sigma_t = 3380\, \sigma_t$$

$\therefore$ $\sigma_t = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4$ MPa

**4. *Failure of the single eye or rod end in shearing***

The single eye or rod end may fail in shearing due to the load. We know that load ($P$),

$$150 \times 10^3 = (d_2 - d_1)\, t \times \tau = (104 - 52)\, 65 \times \tau = 3380\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

**5. *Failure of the single eye or rod end in crushing***

The single eye or rod end may fail in crushing due to the load. We know that load ($P$),

$$150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 3380 = 44.4 \text{ N/mm}^2 = 44.4 \text{ MPa}$$

**6. *Failure of the forked end in tension***

The forked end may fail in tension due to the load. We know that load ($P$),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \sigma_t = (104 - 52)\, 2 \times 40 \times \sigma_t = 4160\, \sigma_t$$

$$\therefore \quad \sigma_t = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

**7. *Failure of the forked end in shear***

The forked end may fail in shearing due to the load. We know that load ($P$),

$$150 \times 10^3 = (d_2 - d_1)\, 2\, t_1 \times \tau = (104 - 52)\, 2 \times 40 \times \tau = 4160\, \tau$$

$$\therefore \quad \tau = 150 \times 10^3 / 4160 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

**8. *Failure of the forked end in crushing***

The forked end may fail in crushing due to the load. We know that load ($P$),

$$150 \times 10^3 = d_1 \times 2\, t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

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**DEPARTMENT OF MECHANICAL ENGINEERING**

# UNIT- IV -Design of Springs -SMEA1502

# Introduction

A spring is defined as an elastic body, whose function is to distort when loaded and to recover its original shape when the load is removed. The various important applications of springs are as follows :

**1.** To cushion, absorb or control energy due to either shock or vibration as in car springs, railway buffers, air-craft landing gears, shock absorbers and vibration dampers.

**2.** To apply forces, as in brakes, clutches and spring loaded valves.

**3.** To control motion by maintaining contact between two elements as in cams and followers.

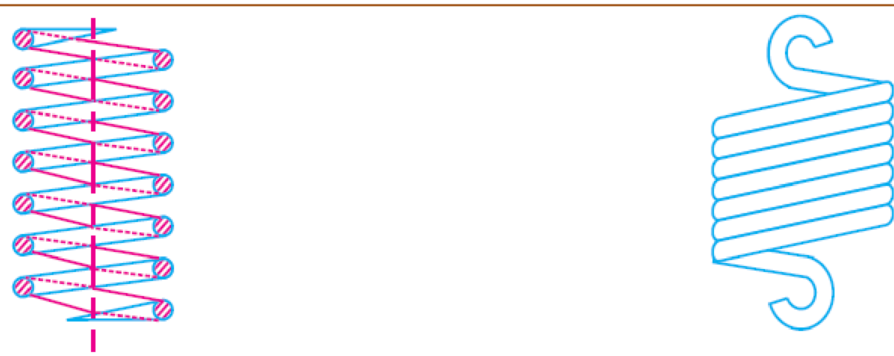
**4.** To measure forces, as in spring balances and engine indicators.

**5.** To store energy, as in watches, toys, etc.

## Types of Springs

Though there are many types of the springs, yet the following, according to their shape, are important from the subject point of view.

**1. *Helical springs*.** The helical springs are made up of a wire coiled in the form of a helix and is primarily intended for compressive or tensile loads. The cross-section of the wire from which the spring is made may be circular, square or rectangular. The two forms of helical springs are ***compression helical spring*** as shown in Fig. and ***tension helical spring*** as shown in Fig. 4.1.



(*a*) Compression helical spring. (*b*) Tension helical spring.

**Fig 4.1:Helical spring**

The helical springs are said to be ***closely coiled*** when the spring wire is coiled so close that the plane containing each turn is nearly at right angles to the axis of the helix and the wire is subjected to torsion. In other words, in a closely coiled helical spring, the helix angle is very small, it is usually less than 10°. The major stresses produced in helical springs are shear stresses due to twisting. The load applied is parallel to or along the axis of the spring.

In ***open coiled helical springs,*** the spring wire is coiled in such a way that there is a gap between the two consecutive turns, as a result of which the helix angle is large. Since the application of open coiled helical springs are limited, therefore our discussion shall confine to closely coiled helical springs only.

The helical springs have the following advantages:

**(*a*)** These are easy to manufacture.

**(*b*)** These are available in wide range.

**(*c*)** These are reliable.

**(*d*)** These have constant spring rate.

**(*e*)** Their performance can be predicted more accurately.

**(*f*)** Their characteristics can be varied by changing dimensions.

**2. *Conical and volute springs*.** The conical and volute springs, as shown in Fig., are used in special applications where a telescoping spring or a spring with a spring rate that increases with the load is desired. The conical spring, as shown in Fig. 4.2 is wound with a uniform pitch whereas the volute springs, as shown in Fig. 4.2 , are wound in the form of paraboloid with constant pitch.

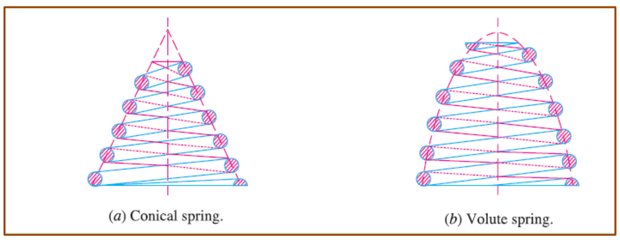



**Fig. 4.2: Conical spring**

and lead angles. The springs may be made either partially or completely telescoping. In either case, the number of active coils gradually decreases. The decreasing number of coils results in an increasing spring rate. This characteristic is sometimes utilised in vibration problems where springs are used to support a body that has a varying mass. The major stresses produced in conical and volute springs are also shear stresses due to twisting.

**3. *Torsion springs*.** These springs may be of ***helical*** or ***spiral*** type as shown in Fig. The **helical type** may be used only in applications where the load tends to wind up the spring and are used in various electrical mechanisms. The spiral type is also used where the load tends to increase the number of coils and when made of flat strip are used in watches and clocks.

The major stresses produced in torsion springs are tensile and compressive due to bending.

(*a*) Helical torsion spring. (*b*) Spiral torsion spring. Fig. 4.3   Torsion springs.

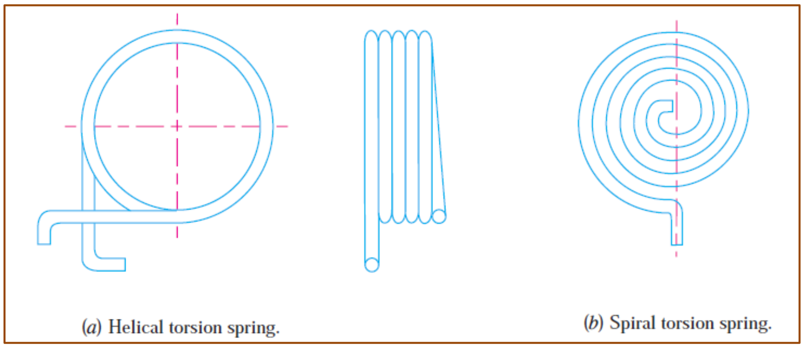



**Fig. 4.3:Spiral spring**

**4. *Laminated or leaf springs*.** The laminated or leaf spring (also known as ***flat spring*** or ***carriage spring***) consists of a number of flat plates (known as leaves) of varying lengths held together by means of clamps and bolts, as shown in Fig.4.5. These are mostly used in automobiles. The major stresses produced in leaf springs are tensile and compressive stresses.

**Fig.** Laminated or leaf springs. **Fig.**  Disc or bellevile springs.

**5. *Disc or bellevile springs*.** These springs consist of a number of conical discs held together against slipping by a central bolt or tube as shown in Fig. 23.5. These springs are used in applications where high spring rates and compact spring units are required.

The major stresses produced in disc or bellevile springs are tensile and compressive stresses.

***6. Special purpose springs.*** These springs are air or liquid springs, rubber springs, ring springs etc. The fluids (air or liquid) can behave as a compression spring. These springs are used for special types of application only.

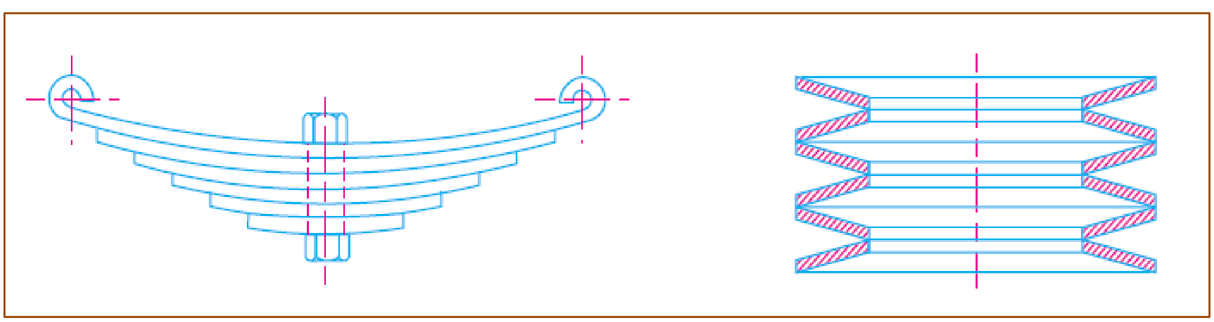


**Fig. 4.5: Leaf spring**

## Material for Helical Springs

The material of the spring should have high fatigue strength, high ductility, high resilience and it should be creep resistant. It largely depends upon the service for which they are used *i.e.* severe service, average service or light service.

***Severe service*** means rapid continuous loading where the ratio of minimum to maximum load (or stress) is one-half or less, as in automotive valve springs.

***Average service*** includes the same stress range as in severe service but with only intermittent operation, as in engine governor springs and automobile suspension springs.

***Light service*** includes springs subjected to loads that are static or very infrequently varied, as in safety valve springs.

Problem 1:

*A compression coil spring made of an alloy steel is having the following specifications :*

*Mean diameter of coil = 50 mm ; Wire diameter = 5 mm ; Number of active coils = 20.*

*If this spring is subjected to an axial load of 500 N ; calculate the maximum shear stress (neglect the curvature effect) to which the spring material is subjected.*

**Solution.** Given : $D$ = 50 mm ; $d$ = 5 mm ; *$n$ = 20 ; $W$ = 500 N

We know that the spring index,

$$C = \frac{D}{d} = \frac{50}{5} = 10$$

∴ Shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 10} = 1.05$$

and maximum shear stress (neglecting the effect of wire curvature),

$$\tau = K_S \times \frac{8W.D}{\pi d^3} = 1.05 \times \frac{8 \times 500 \times 50}{\pi \times 5^3} = 534.7 \text{ N/mm}^2$$

$$= 534.7 \text{ MPa}$$ **Ans.**

Problem 2:

*A helical spring is made from a wire of 6 mm diameter and has outside diameter of 75 mm. If the permissible shear stress is 350 MPa and modulus of rigidity 84 kN/mm2, find the axial load which the spring can carry and the deflection per active turn.*

**Solution.** Given : $d$ = 6 mm ; $D_o$ = 75 mm ; $\tau$ = 350 MPa = 350 N/mm$^2$ ; $G$ = 84 kN/mm$^2$ = $84 \times 10^3$ N/mm$^2$

We know that mean diameter of the spring,

$$D = D_o - d = 75 - 6 = 69 \text{ mm}$$

∴ Spring index,

$$C = \frac{D}{d} = \frac{69}{6} = 11.5$$

Let $W$ = Axial load, and

$\delta / n$ = Deflection per active turn.

**1.** ***Neglecting the effect of curvature***

We know that the shear stress factor,

$$K_S = 1 + \frac{1}{2C} = 1 + \frac{1}{2 \times 11.5} = 1.043$$

and maximum shear stress induced in the wire ($\tau$),

$$350 = K_S \times \frac{8\,W.D}{\pi\, d^3} = 1.043 \times \frac{8\,W \times 69}{\pi \times 6^3} = 0.848\,W$$

∴ $W = 350 / 0.848 = 412.7$ N **Ans.**

We know that deflection of the spring,

$$\delta = \frac{8\,W.D^3.n}{G.d^4}$$

∴ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\,W.D^3}{G.d^4} = \frac{8 \times 412.7\,(69)^3}{84 \times 10^3 \times 6^4} = 9.96 \text{ mm}$$ **Ans.**

**2. *Considering the effect of curvature***

We know that Wahl's stress factor,

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} = \frac{4 \times 11.5 - 1}{4 \times 11.5 - 4} + \frac{0.615}{11.5} = 1.123$$

We also know that the maximum shear stress induced in the wire ($\tau$),

$$350 = K \times \frac{8W.C}{\pi\, d^2} = 1.123 \times \frac{8 \times W \times 11.5}{\pi \times 6^2} = 0.913\, W$$

$\therefore$ $W = 350 / 0.913 = 383.4$ N **Ans.**

and deflection of the spring,

$$\delta = \frac{8\, W.D^3.n}{G.d^4}$$

$\therefore$ Deflection per active turn,

$$\frac{\delta}{n} = \frac{8\, W.D^3}{G.d^4} = \frac{8 \times 383.4\,(69)^3}{84 \times 10^3 \times 6^4} = 9.26 \text{ mm} \quad \textbf{Ans.}$$

Problem 3 :

*Design a helical compression spring for a maximum load of 1000 N for a deflection of 25 mm using the value of spring index as 5. The maximum permissible shear stress for spring wire is 420 MPa and modulus of rigidity is 84 kN/mm2. Take Wahl's factor, K, where C = Spring index.*

**Solution.** Given : $W = 1000$ N ; $\delta = 25$ mm ; $C = D/d = 5$ ; $\tau = 420$ MPa $= 420$ N/mm$^2$ ; $G = 84$ kN/mm$^2 = 84 \times 10^3$ N/mm$^2$

***1. Mean diameter of the spring coil***

Let $D$ = Mean diameter of the spring coil, and

$d$ = Diameter of the spring wire.

We know that Wahl's stress factor,

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4 \times 5 - 1}{4 \times 5 - 4} + \frac{0.615}{5} = 1.31$$

and maximum shear stress ($\tau$),

$$420 = K \times \frac{8\,W.C}{\pi\, d^2} = 1.31 \times \frac{8 \times 1000 \times 5}{\pi\, d^2} = \frac{16\,677}{d^2}$$

$\therefore$ $d^2 = 16\,677 / 420 = 39.7$ or $d = 6.3$ mm

From Table 23.2, we shall take a standard wire of size *SWG* 3 having diameter ($d$) = 6.401 mm.

$\therefore$ Mean diameter of the spring coil,

$$D = C.d = 5\,d = 5 \times 6.401 = 32.005 \text{ mm } \textbf{Ans.} \qquad \ldots (\because C = D/d = 5)$$

and outer diameter of the spring coil,

$$D_o = D + d = 32.005 + 6.401 = 38.406 \text{ mm } \textbf{Ans.}$$

***2. Number of turns of the coils***

Let $n$ = Number of active turns of the coils.

We know that compression of the spring ($\delta$),

$$25 = \frac{8W.C^3.n}{G.d} = \frac{8 \times 1000\,(5)^3 n}{84 \times 10^3 \times 6.401} = 1.86\,n$$

$\therefore$ $n = 25 / 1.86 = 13.44$ say 14 **Ans.**

For squared and ground ends, the total number of turns,

$$n' = n + 2 = 14 + 2 = 16 \textbf{ Ans.}$$

***3. Free length of the spring***

We know that free length of the spring

$$= n'.d + \delta + 0.15\,\delta = 16 \times 6.401 + 25 + 0.15 \times 25$$

$$= 131.2 \text{ mm } \textbf{Ans.}$$

***4. Pitch of the coil***

We know that pitch of the coil

$$= \frac{\text{Free length}}{n' - 1} = \frac{131.2}{16 - 1} = 8.75 \text{ mm } \textbf{Ans.}$$

Problem 4:

*A closely coiled helical spring is made of 10 mm diameter steel wire, the coil consisting of 10 complete turns with a mean diameter of 120 mm. The spring carries an axial pull of 200 N. determine the shear stress induced in the spring neglecting the effect of stress concentration. Determine also the deflection in the spring, its stiffness and strain energy stored by it if the modulus of rigidity of the material is 80 kN/mm2.*

**Solution.** Given : $d = 10$ mm ; $n = 10$ ; $D = 120$ mm ; $W = 200$ N ; $G = 80$ kN/mm$^2$ = $80 \times 10^3$ N/mm$^2$

***Shear stress induced in the spring neglecting the effect of stress concentration***

We know that shear stress induced in the spring neglecting the effect of stress concentration is,

$$\tau = \frac{8\,W.D}{\pi\,d^3}\left(1+\frac{d}{2D}\right) = \frac{8 \times 200 \times 120}{\pi\,(10)^3}\left[1+\frac{10}{2 \times 120}\right] \text{ N/mm}^2$$

$$= 61.1 \times 1.04 = 63.54 \text{ N/mm}^2 = 63.54 \text{ MPa } \textbf{Ans.}$$

***Deflection in the spring***

We know that deflection in the spring,

$$\delta = \frac{8\,W.D^3 n}{G.d^4} = \frac{8 \times 200\,(120)^3\,10}{80 \times 10^3\,(10)^4} = 34.56 \text{ mm } \textbf{Ans.}$$

***Stiffness of the spring***

We know that stiffness of the spring

$$= \frac{W}{\delta} = \frac{200}{34.56} = 5.8 \text{ N/mm}$$

***Strain energy stored in the spring***

We know that strain energy stored in the spring,

$$U = \frac{1}{2}\,W.\delta = \frac{1}{2} \times 200 \times 34.56 = 3456 \text{ N-mm} = 3.456 \text{ N-m } \textbf{Ans.}$$

## Helical Torsion Springs

The helical torsion springs as shown in Fig. may be made from round, rectangular or square wire. These are wound in a similar manner as helical compression or tension springs but the ends are shaped to transmit torque. The primary stress in helical torsion springs is bending stress whereas in compression or tension springs, the stresses are torsional shear stresses. The helical torsion springs are widely used for transmitting small torques as in door hinges, brush holders in electric motors, automobile starters etc. A little consideration will show that the radius of curvature of the coils changes when the twisting moment is applied to the spring. Thus, the wire is under pure bending. According to A.M. Wahl, the bending stress in a helical torsion spring made of round wire is

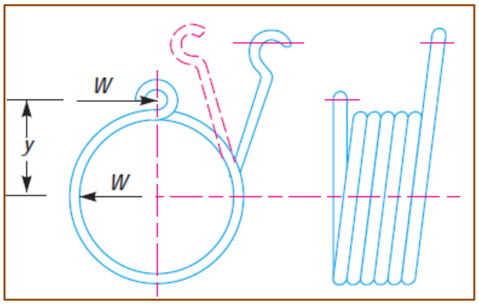



**Fig. 4.6:Torsion spring**

$$\sigma_b = K \times \frac{32 M}{\pi d^3} = K \times \frac{32 W.y}{\pi d^3}$$

where $K$ = Wahl's stress factor = $\frac{4C^2 - C - 1}{4C^2 - 4C}$,

$C$ = Spring index,

$M$ = Bending moment = $W \times y$,

$W$ = Load acting on the spring,

$y$ = Distance of load from the spring axis, and

$d$ = Diameter of spring wire.

and total angle of twist or angular deflection,

$$*\theta = \frac{M.l}{E.I} = \frac{M \times \pi D.n}{E \times \pi d^4 / 64} = \frac{64 M.D.n}{E.d^4}$$

where $l$ = Length of the wire = $\pi.D.n$,

$E$ = Young's modulus,

$I$ = Moment of inertia = $\frac{\pi}{64} \times d^4$,

$D$ = Diameter of the spring, and

$n$ = Number of turns.

and deflection, $\delta = \theta \times y = \frac{64 M.D.n}{E.d^4} \times y$

When the spring is made of rectangular wire having width $b$ and thickness $t$, then

$$\sigma_b = K \times \frac{6 M}{t.b^2} = K \times \frac{6 W \times y}{t.b^2}$$

where $K = \frac{3C^2 - C - 0.8}{3C^2 - 3C}$

Angular deflection, $\theta = \frac{12\,\pi M.D.n}{E.t.b^3}$; and $\delta = \theta.y = \frac{12\,\pi M.D.n}{E.t.b^3} \times y$

In case the spring is made of square wire with each side equal to $b$, then substituting $t = b$, in the above relation, we have

$$\sigma_b = K \times \frac{6\,M}{b^3} = K \times \frac{6W \times y}{b^3}$$

$$\theta = \frac{12\,\pi M.D.n}{E.b^4}; \quad \text{and} \quad \delta = \frac{12\,\pi M.D.n}{E.b^4} \times y$$

Problem:

*A helical torsion spring of mean diameter 60 mm is made of a round wire of 6 mm diameter. If a torque of 6 N-m is applied on the spring, find the bending stress induced and the angular deflection of the spring in degrees. The spring index is 10 and modulus of elasticity for the spring material is 200 kN/mm2. The number of effective turns may be taken as 5.5.*

**Solution.** Given : $D = 60$ mm ; $d = 6$ mm ; $M = 6$ N-m $= 6000$ N-mm ; $C = 10$ ; $E = 200$ kN/mm$^2$ $= 200 \times 10^3$ N/mm$^2$ ; $n = 5.5$

***Bending stress induced***

We know that Wahl's stress factor for a spring made of round wire,

$$K = \frac{4C^2 - C - 1}{4C^2 - 4C} = \frac{4 \times 10^2 - 10 - 1}{4 \times 10^2 - 4 \times 10} = 1.08$$

∴ Bending stress induced,

$$\sigma_b = K \times \frac{32\,M}{\pi\,d^3} = 1.08 \times \frac{32 \times 6000}{\pi \times 6^3} = 305.5 \text{ N/mm}^2 \text{ or MPa} \quad \textbf{Ans.}$$

***Angular deflection of the spring***

We know that the angular deflection of the spring (in radians),

$$\theta = \frac{64\,M.D.n}{E.d^4} = \frac{64 \times 6000 \times 60 \times 5.5}{200 \times 10^3 \times 6^4} = 0.49 \text{ rad}$$

$$= 0.49 \times \frac{180}{\pi} = 28^\circ \quad \textbf{Ans.}$$

## Leaf Springs

Leaf springs (also known as **flat springs**) are made out of flat plates. The advantage of leaf spring over helical spring is that the ends of the spring may be guided along a definite path as it deflects to act as a structural member in addition to energy absorbing device. Thus the leaf springs may carry lateral loads, brake torque, driving torque etc., in addition to shocks.

## Materials for Leaf Springs

The material used for leaf springs is usually a plain carbon steel having 0.90 to 1.0% carbon.

The leaves are heat treated after the forming process. The heat treatment of spring steel produces greater strength and therefore greater load capacity, greater range of deflection and better fatigue properties.

According to Indian standards, the recommended materials are :

**1.** For automobiles: 50 Cr 1, 50 Cr 1 V 23, and 55 Si 2 Mn 90 all used in hardened and tempered state.

**2.** For rail road springs: C 55 (water-hardened), C 75 (oil-hardened), 40 Si 2 Mn 90 (water hardened) and 55 Si 2 Mn 90 (oil-hardened).

**3.** The physical properties of some of these materials are given in the following table. All values are for oil quenched condition and for single heat only.

Problem 1:

*Design a leaf spring for the following specifications : Total load = 140 kN ; Number of springs supporting the load = 4 ; Maximum number of leaves = 10; Span of the spring = 1000 mm ; Permissible deflection = 80 mm. Take Young's modulus, E = 200 kN/mm2 and allowable stress in spring material as 600 MPa.*

**Solution.** Given : Total load = 140 kN ; No. of springs = 4; $n$ = 10 ; $2L$ = 1000 mm or $L$ = 500 mm ; $\delta$ = 80 mm ; $E$ = 200 kN/mm$^2$ = 200 × 10$^3$ N/mm$^2$ ; $\sigma$ = 600 MPa = 600 N/mm$^2$

We know that load on each spring,

$$2W = \frac{\text{Total load}}{\text{No. of springs}} = \frac{140}{4} = 35 \text{ kN}$$

$\therefore$ $W = 35 / 2 = 17.5 \text{ kN} = 17\,500 \text{ N}$

Let $t$ = Thickness of the leaves, and

$b$ = Width of the leaves.

We know that bending stress ($\sigma$),

$$600 = \frac{6\,W.L}{n.b.t^2} = \frac{6 \times 17\,500 \times 500}{n.b.t^2} = \frac{52.5 \times 10^6}{n.b.t^2}$$

$\therefore$ $n.b.t^2 = 52.5 \times 10^6 / 600 = 87.5 \times 10^3$ ...(*i*)

and deflection of the spring ($\delta$),

$$80 = \frac{6\,W.L^3}{n.E.b.t^3} = \frac{6 \times 17\,500\,(500)^3}{n \times 200 \times 10^3 \times b \times t^3} = \frac{65.6 \times 10^6}{n.b.t^3}$$

$\therefore$ $n.b.t^3 = 65.6 \times 10^6 / 80 = 0.82 \times 10^6$ ...(*ii*)

Dividing equation (*ii*) by equation (*i*), we have

$$\frac{n.b.t^3}{n.b.t^2} = \frac{0.82 \times 10^6}{87.5 \times 10^3} \quad \text{or} \quad t = 9.37 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

Now from equation (*i*), we have

$$b = \frac{87.5 \times 10^3}{n.t^2} = \frac{87.5 \times 10^3}{10\,(10)^2} = 87.5 \text{ mm}$$

and from equation (*ii*), we have

$$b = \frac{0.82 \times 10^6}{n.t^3} = \frac{0.82 \times 10^6}{10\,(10)^3} = 82 \text{ mm}$$

Taking larger of the two values, we have width of leaves,

$b$ = 87.5 say 90 mm **Ans.**

Problem 2:

*A truck spring has 12 number of leaves, two of which are full length leaves. The spring supports are 1.05 m apart and the central band is 85 mm wide. The central load is to be 5.4 kN with a permissible stress of 280 MPa. Determine the thickness and width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflection of the spring.*

**Solution.** Given : $n = 12$ ; $n_F = 2$ ; $2L_1 = 1.05$ m $= 1050$ mm ; $l = 85$ mm ; $2W = 5.4$ kN $= 5400$ N or $W = 2700$ N ; $\sigma_F = 280$ MPa $= 280$ N/mm$^2$

*Thickness and width of the spring leaves*

Let $t$ = Thickness of the leaves, and

$b$ = Width of the leaves.

Since it is given that the ratio of the total depth of the spring ($n \times t$) and width of the spring ($b$) is 3, therefore

$$\frac{n \times t}{b} = 3 \quad \text{or} \quad b = n \times t / 3 = 12 \times t / 3 = 4\,t$$

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1050 - 85 = 965 \text{ mm}$$

$$\therefore \quad L = 965 / 2 = 482.5 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 12 - 2 = 10$$

Assuming that the leaves are not initially stressed, therefore maximum stress or bending stress for full length leaves ($\sigma_F$),

$$280 = \frac{18\,W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 2700 \times 482.5}{4\,t \times t^2\,(2 \times 10 + 3 \times 2)} = \frac{225\,476}{t^3}$$

$$\therefore \quad t^3 = 225\,476 / 280 = 805.3 \quad \text{or} \quad t = 9.3 \text{ say } 10 \text{ mm } \textbf{Ans.}$$

and $\quad b = 4\,t = 4 \times 10 = 40$ mm **Ans.**

*Deflection of the spring*

We know that deflection of the spring,

$$\delta = \frac{12\,W.L^3}{E.b.t^3\,(2n_G + 3n_F)}$$

$$= \frac{12 \times 2700 \times (482.5)^3}{210 \times 10^3 \times 40 \times 10^3\,(2 \times 10 + 3 \times 2)} \text{ mm}$$

$= 16.7$ mm **Ans.** ... (Taking $E = 210 \times 10^3$ N/mm$^2$)

Problem 3:

*A semi-elliptical laminated vehicle spring to carry a load of 6000 N is to consist of seven leaves 65 mm wide, two of the leaves extending the full length of the spring. The spring is to be 1.1 m in length and attached to the axle by two U-bolts 80 mm apart. The bolts hold the central portion of the spring so rigidly that they may be considered equivalent to a band having a width equal to the distance between the bolts. Assume a design stress for spring material as 350MPa. Determine : 1. Thickness of leaves, 2. Deflection of spring, 3. Diameter of eye, 4. Length of leaves, and 5. Radius to which leaves should be initially bent. Sketch the semi-elliptical leaf-spring arrangement. The standard thickness of leaves are : 5, 6, 6.5, 7, 7.5, 8, 9, 10, 11 etc. in mm.*

**Solution.** Given : $2W = 6000$ N or $W = 3000$ N ; $n = 7$ ; $b = 65$ mm ; $n_F = 2$ ; $2L_1 = 1.1$ m $= 1100$ mm or $L_1 = 550$ mm ; $l = 80$ mm ; $\sigma = 350$ MPa $= 350$ N/mm$^2$

**1.** ***Thickness of leaves***

Let $t$ = Thickness of leaves.

We know that the effective length of the spring,

$$2L = 2L_1 - l = 1100 - 80 = 1020 \text{ mm}$$

$$\therefore \quad L = 1020 / 2 = 510 \text{ mm}$$

and number of graduated leaves,

$$n_G = n - n_F = 7 - 2 = 5$$

Assuming that the leaves are not initially stressed, the maximum stress ($\sigma_F$),

$$350 = \frac{18\, W.L}{b.t^2\,(2n_G + 3n_F)} = \frac{18 \times 3000 \times 510}{65 \times t^2\,(2 \times 5 + 3 \times 2)} = \frac{26\,480}{t^2} \quad \dots (\sigma_F = \sigma)$$

$$\therefore \quad t^2 = 26\,480 / 350 = 75.66 \quad \text{or} \quad t = 8.7 \text{ say } 9 \text{ mm} \quad \textbf{Ans.}$$

**2.** ***Deflection of spring***

We know that deflection of spring,

$$\delta = \frac{12\, W.L^3}{E.b.t^3\,(2n_G + 3n_F)} = \frac{12 \times 3000\,(510)^3}{210 \times 10^3 \times 65 \times 9^3\,(2 \times 5 + 3 \times 2)}$$

$$= 30 \text{ mm} \quad \textbf{Ans.} \quad \dots (\text{Taking } E = 210 \times 10^3 \text{ N/mm}^2)$$

**3.** ***Diameter of eye***

The inner diameter of eye is obtained by considering the pin in the eye in bearing, because the inner diameter of the eye is equal to the diameter of the pin.

Let $d$ = Inner diameter of the eye or diameter of the pin,

$l_1$ = Length of the pin which is equal to the width of the eye or leaf (*i.e.* $b$) = 65 mm ...(Given)

$p_b$ = Bearing pressure on the pin which may be taken as 8 N/mm$^2$.

We know that the load on pin ($W$),

$$3000 = d \times l_1 \times p_b$$
$$= d \times 65 \times 8 = 520\, d$$
$$\therefore \quad d = 3000 / 520$$
$$= 5.77 \text{ say } 6 \text{ mm}$$

Maximum bending moment on the pin,

$$M = \frac{W \times l_2}{4} = \frac{3000 \times 69}{4} = 51\,750 \text{ N-mm}$$

and section modulus, $Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3$

We know that bending stress ($\sigma_b$),

$$80 = \frac{M}{Z} = \frac{51\,750}{0.0982\, d^3} = \frac{527 \times 10^3}{d^3} \qquad \text{... (Taking } \sigma_b = 80 \text{ N/mm}^2\text{)}$$

$\therefore \quad d^3 = 527 \times 10^3 / 80 = 6587 \quad \text{or} \quad d = 18.7$ say 20 mm **Ans.**

We shall take the inner diameter of eye or diameter of pin ($d$) as 20 mm **Ans.**

Let us now check the pin for induced shear stress. Since the pin is in double shear, therefore load on the pin ($W$),

$$3000 = 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4}\,(20)^2\ \tau = 628.4\ \tau$$

$\therefore \quad \tau = 3000 / 628.4 = 4.77$ N/mm$^2$, which is safe.

**4. *Length of leaves***

We know that ineffective length of the spring

$= l = 80$ mm ... ($\because$ *U*-bolts are considered equivalent to a band)

$\therefore$ Length of the smallest leaf $= \dfrac{\text{Effective length}}{n-1} + \text{Ineffective length}$

$$= \frac{1020}{7-1} + 80 = 250 \text{ mm} \quad \textbf{Ans.}$$

Length of the 2nd leaf $= \dfrac{1020}{7-1} \times 2 + 80 = 420$ mm **Ans.**

Length of the 3rd leaf $= \dfrac{1020}{7-1} \times 3 + 80 = 590$ mm **Ans.**

Length of the 4th leaf $= \dfrac{1020}{7-1} \times 4 + 80 = 760$ mm **Ans.**

Length of the 5th leaf $= \dfrac{1020}{7-1} \times 5 + 80 = 930$ mm **Ans.**

Length of the 6th leaf $= \dfrac{1020}{7-1} \times 6 + 80 = 1100$ mm **Ans.**

The 6th and 7th leaves are full length leaves and the 7th leaf (*i.e.* the top leaf) will act as a maste leaf.

We know that length of the master leaf

$$= 2L_1 + \pi\,(d + t)\,2 = 1100 + \pi\,(20 + 9)2 = 1282.2 \text{ mm} \quad \textbf{Ans.}$$

**5.** ***Radius to which the leaves should be initially bent***

Let $R$ = Radius to which the leaves should be initially bent, and

$y$ = Camber of the spring.

We know that

$$y\,(2R - y) = (L_1)^2$$

$$30(2R - 30) = (550)^2 \quad \text{or} \quad 2R - 30 = (550)^2/30 = 10\,083 \qquad \ldots (\because y = \delta)$$

$$\therefore \quad R = \frac{10\,083 + 30}{2} = 5056.5 \text{ mm} \quad \textbf{Ans.}$$

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**SCHOOL OF MECHANICAL ENGINEERING**

**DEPARTMENT OF MECHANICAL ENGINEERING**

# UNIT- V -Design of bearings and flywheel -SMEA1502

**introduction**

**DESIGN OF BEARINGS**

## Introduction

A bearing is a machine element which support another moving machine element (known as journal). It permits a relative motion between the contact surfaces of the members, while carrying the load. A little consideration will show that due to the relative motion between the contact surfaces, a certain amount of power is wasted in overcoming frictional resistance and if the rubbing surfaces are in direct contact, there will be rapid wear. In order to reduce frictional resistance and wear and in some cases to carry away the heat generated; a layer of fluid (known as lubricant) may be provided. The lubricant used to separate the journal and bearing is usually a mineral oil refined from petroleum, but vegetable oils, silicon oils, greases etc., may be used.

## Classification of Bearings

Though the bearings may be classified in many ways, yet the following are important from the subject point of view:

Depending upon the direction of load to be supported. The bearings under this group are classified as:

(a) Radial bearings, and (b) Thrust bearings.

In radial bearings, the load acts perpendicular to the direction of motion of the moving Element as shown in Fig. (a) and (b). In thrust bearings, the load acts along the axis of rotation as shown in Fig. (c).

Note : These bearings may move in either of the directions as shown in Fig. 5.1

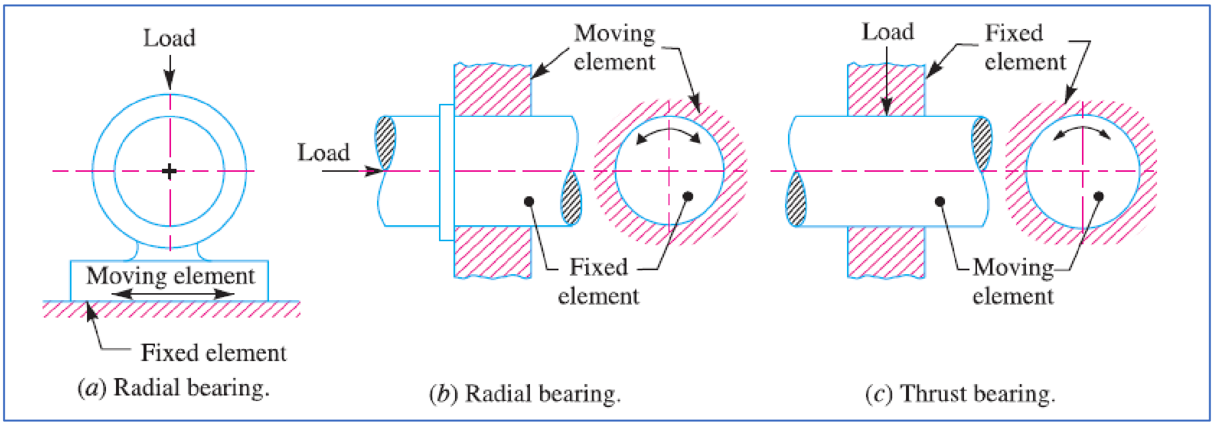



**FIG. 5.1.Types of Bearings**

2. Depending upon the nature of contact. The bearings under this group are classified as :

(a) Sliding contact bearings, and (b) Rolling contact bearings.

In sliding contact bearings, as shown in Fig. 26.2 (a), the sliding takes place along the urfaces of contact between the moving element and the fixed element. The sliding contact bearings are also known as plain bearings.

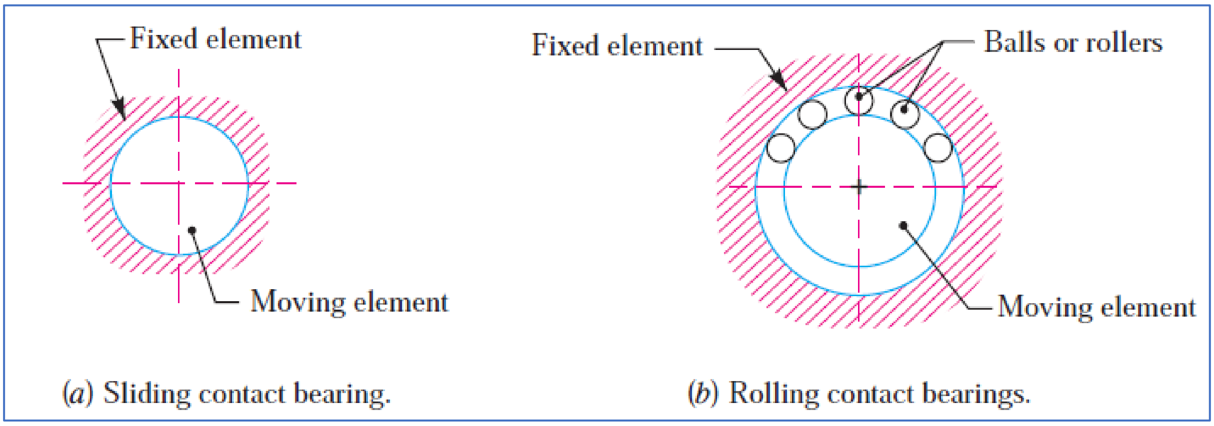



**FIG. 5.2.Sliding contact Bearing**

In rolling contact bearings, as shown in Fig. 5.2 (b), the steel balls or rollers, are interposed between the moving and fixed elements. The balls offer rolling friction at two points for each ball or roller.

**Types of Sliding Contact Bearings**

The sliding contact bearings in which the sliding action is guided in a straight line and carrying radial loads, as shown in Fig.5.3 (a), may be called slipper or guide bearings. Such type of bearings are usually found in cross-head of steam engines.

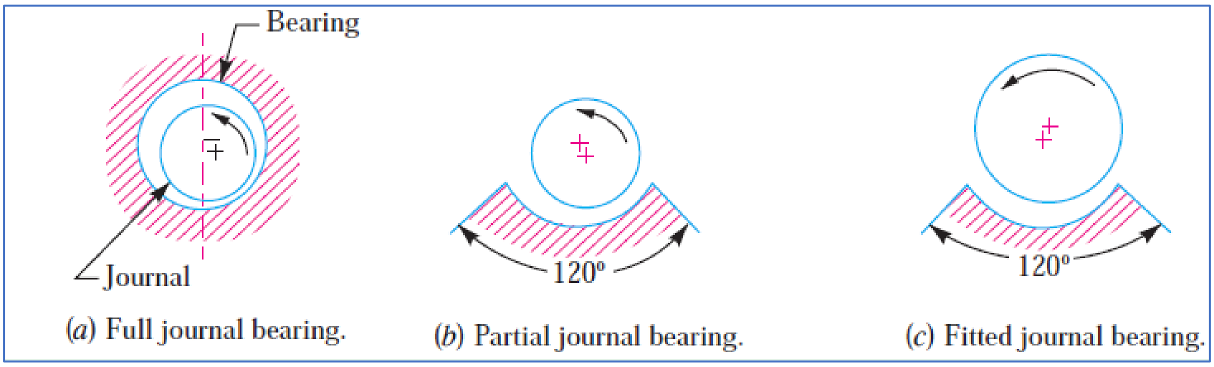



**FIG. 5.3. Sliding contact Bearing**

The sliding contact bearings in which the sliding action is along the circumference of a circle or an arc of a circle and carrying radial loads are known as journal or sleeve bearings. When the angle of contact of the bearing with the journal is 360° as shown in Fig. (a), then the bearing is called a full journal bearing. This type of bearing is commonly used in industrial machinery to accommodate bearing loads in any radial direction. When the angle of contact of the bearing with the journal is 120°, as shown in Fig. (b), then the bearing is said to be partial journal bearing. This type of bearing has less friction than full journal bearing, but it can be used only where the load is always in one direction. The most common application of the partial journal bearings is found in rail road car axles. The full and partial journal bearings may be called as clearance bearings because the diameter of the journal is less than that of bearing.

When a partial journal bearing has no clearance i.e. the diameters of the journal and bearing are equal, then the bearing is called a fitted bearing, as shown in Fig. 5.3(c). The sliding contact bearings, according to the thickness of layer of the lubricant between the bearing and the journal, may also be classified as follows:

**1. Thick film bearings.** The thick film bearings are those in which the working surfaces are completely separated from each other by the lubricant. Such type of bearings are also called as hydrodynamic lubricated bearings.

**2. Thin film bearings.** The thin film bearings are those in which, although lubricant is present, the working surfaces partially contact each other at least part of the time. Such type of bearings are also called boundary lubricated bearings.

**3. Zero film bearings.** The zero film bearings are those which operate without any lubricant present.

4. Hydrostatic or externally pressurized lubricated bearings. The hydrostatic bearings are those which can support steady loads without any relative motion between the journal and the bearing. This is achieved by forcing externally pressurized lubricant between the members.

## Hydrodynamic Lubricated Bearings

We have already discussed that in hydrodynamic lubricated bearings, there is a thick film of lubricant between the journal and the bearing. A little consideration will show that when the bearing is supplied with sufficient lubricant, a pressure is build up in the clearance space when the journal is rotating about an axis that is eccentric with the bearing axis. The load can be supported by this fluid pressure without any actual contact between the journal and bearing.

The load carrying ability of a hydrodynamic bearing arises simply because a viscous fluid resists being pushed around. Under the proper conditions, this resistance to motion will develop a pressure distribution in the lubricant film that can support a useful load. The load supporting pressure in hydrodynamic bearings arises from either

1. the flow of a viscous fluid in a converging channel (known as wedge film lubrication), or
2. the resistance of a viscous fluid to being squeezed out from between approaching surfaces (known as squeeze film lubrication).

**Assumptions in Hydrodynamic Lubricated Bearings:**

The following are the basic assumptions used in the theory of hydrodynamic lubricated bearings:

1. The lubricant obeys Newton's law of viscous flow.
2. The pressure is assumed to be constant throughout the film thickness.
3. The lubricant is assumed to be incompressible.
4. The viscosity is assumed to be constant throughout the film.
5. The flow is one dimensional, i.e. the side leakage is neglected.

PROBLEM 1

*Design a journal bearing for a centrifugal pump from the following data :*

*Load on the journal = 20 000 N; Speed of the journal = 900 r.p.m.; Type of oil is SAE 10, for which the absolute viscosity at 55°C = 0.017 kg / m-s; Ambient temperature of oil = 15.5°C ; Maximum bearing pressure for the pump = 1.5 N / mm2.*

*Calculate also mass of the lubricating oil required for artificial cooling, if rise of temperature of oil be limited to 10°C. Heat dissipation coefficient = 1232 W/m2/°C.*

**Solution.** Given : $W = 20\,000$ N ; $N = 900$ r.p.m. ; $t_0 = 55°C$ ; $Z = 0.017$ kg/m-s ; $t_a = 15.5°C$ ; $p = 1.5$ N/mm$^2$ ; $t = 10°C$ ; $C = 1232$ W/m$^2$/°C

The journal bearing is designed as discussed in the following steps :

**1.** First of all, let us find the length of the journal ( $l$ ). Assume the diameter of the journal ( $d$ ) as 100 mm. From Table 26.3, we find that the ratio of $l / d$ for centrifugal pumps varies from 1 to 2. Let us take $l / d = 1.6$.

$$\therefore \quad l = 1.6\, d = 1.6 \times 100 = 160 \text{ mm } \textbf{Ans.}$$

**2.** We know that bearing pressure,

$$p = \frac{W}{l.d} = \frac{20\,000}{160 \times 100} = 1.25$$

Since the given bearing pressure for the pump is 1.5 N/mm$^2$, therefore the above value of $p$ is safe and hence the dimensions of $l$ and $d$ are safe.

**3.** $$\frac{Z.N}{p} = \frac{0.017 \times 900}{1.25} = 12.24$$

From Table 26.3, we find that the operating value of

$$\frac{Z.N}{p} = 28$$

$$3\ K = \frac{ZN}{p}$$

∴ Bearing modulus at the minimum point of friction,

$$K = \frac{1}{3}\left(\frac{Z.N}{p}\right) = \frac{1}{3} \times 28 = 9.33$$

Since the calculated value of bearing characteristic number $\left(\frac{Z.N}{p} = 12.24\right)$ is more than 9.33, therefore the bearing will operate under hydrodynamic conditions.

**4.** From Table 26.3, we find that for centrifugal pumps, the clearance ratio $(c/d)$

$= 0.0013$

**5.** We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8} \times 12.24 \times \frac{1}{0.0013} + 0.002$$

$= 0.0031 + 0.002 = 0.0051$ ... [From Art. 26.13, $k = 0.002$]

**6.** Heat generated,

$$Q_g = \mu\ W\ V = \mu\ W\left(\frac{\pi\, d.N}{60}\right)\ \text{W} \qquad \ldots\left(\because V = \frac{\pi\, d.N}{60}\right)$$

$$= 0.0051 \times 20\,000\left(\frac{\pi \times 0.1 \times 900}{60}\right) = 480.7\ \text{W}$$

... ($d$ is taken in metres)

7. Heat dissipated,

$$Q_d = C.A\,(t_b - t_a) = C.l.d\,(t_b - t_a)\ \text{W} \qquad \ldots(\because A = l \times d)$$

We know that

$$(t_b - t_a) = \tfrac{1}{2}\,(t_0 - t_a) = \tfrac{1}{2}\,(55° - 15.5°) = 19.75°\text{C}$$

∴ $Q_d = 1232 \times 0.16 \times 0.1 \times 19.75 = 389.3$ W

We know that the amount of artificial cooling required

= Heat generated – Heat dissipated = $Q_g - Q_d$

= 480.7 – 389.3 = 91.4 W

***Mass of lubricating oil required for artificial cooling***

Let $m$ = Mass of the lubricating oil required for artificial cooling in kg / s.

We know that the heat taken away by the oil,

$$Q_t = m.S.t = m \times 1900 \times 10 = 19\ 000\ m\ \text{W}$$

... [∵ Specific heat of oil ($S$) = 1840 to 2100 J/kg/°C]

Equating this to the amount of artificial cooling required, we have

$19\ 000\ m = 91.4$

∴ $m = 91.4 / 19\ 000 = 0.0048$ kg / s = 0.288 kg / min **Ans.**

PROBLEM

*A 150 mm diameter shaft supporting a load of 10 kN has a speed of 1500 r.p.m. The shaft runs in a bearing whose length is 1.5 times the shaft diameter. If the diametral clearance of the bearing is 0.15 mm and the absolute viscosity of the oil at the operating temperature is 0.011 kg/m-s, find the power wasted in friction.*

**Solution.** Given : $d$ = 150 mm = 0.15 m ; $W$ = 10 kN = 10 000 N ; $N$ = 1500 r.p.m. ; $l$ = 1.5 $d$ ; $c$ = 0.15 mm ; Z = 0.011 kg/m-s

We know that length of bearing,

$$l = 1.5\,d = 1.5 \times 150 = 225 \text{ mm}$$

∴ Bearing pressure,

$$p = \frac{W}{A} = \frac{W}{l.d} = \frac{10\,000}{225 \times 150} = 0.296 \text{ N/mm}^2$$

We know that coefficient of friction,

$$\mu = \frac{33}{10^8}\left(\frac{ZN}{p}\right)\left(\frac{d}{c}\right) + k = \frac{33}{10^8}\left(\frac{0.011 \times 1500}{0.296}\right)\left(\frac{150}{0.15}\right) + 0.002$$

$$= 0.018 + 0.002 = 0.02$$

and rubbing velocity,

$$V = \frac{\pi d.N}{60} = \frac{\pi \times 0.15 \times 1500}{60} = 11.78 \text{ m/s}$$

We know that heat generated due to friction,

$$Q_g = \mu.W.V = 0.02 \times 10\,000 \times 11.78 = 2356 \text{ W}$$

∴ Power wasted in friction

$$= Q_g = 2356 \text{ W} = 2.356 \text{ kW}$$ **Ans.**

# Rolling Contact Bearings

In rolling contact bearings, the contact between the bearing surfaces is rolling instead of sliding as in sliding contact bearings. We have already discussed that the ordinary sliding bearing starts from rest with practically metal-to-metal contact and has a high coefficient of friction.

It is an outstanding advantage of a rolling contact bearing over a sliding bearing that it has a low starting friction. Due to this low friction offered by rolling contact bearings, these are called ***antifriction bearings.***

## Rolling Contact Bearings Over Sliding Contact Bearings

The following are some advantages and disadvantages of rolling contact bearings over sliding contact bearings.

**Advantages**

1. Low starting and running friction except at very high speeds.
2. Ability to withstand momentary shock loads.
3. Accuracy of shaft alignment.
4. Low cost of maintenance, as no lubrication is required while in service.
5. Small overall dimensions.
6. Reliability of service.
7. Easy to mount and erect.
8. Cleanliness.

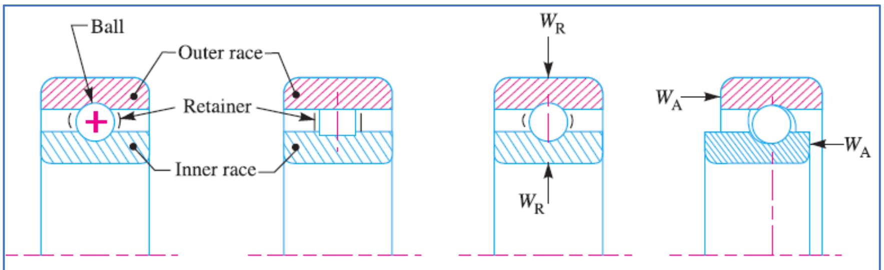
**Disadvantages**

1. Noisier at very high speeds.
2. Low resistance to shock loading.
3. More initial cost.
4. Design of bearing housing complicated.

**Types of Rolling Contact Bearings**

**Following are the two types of rolling contact bearings:**

1. Ball bearings; and 2. Roller bearing




(*a*) Ball bearing. (*b*) Roller bearing. (*a*) Radial ball bearing. (*b*) Thrust ball bearing.

**Fig. 5.4 . Rolling Contact Bearings**

The ball and roller bearings consist of an inner race which is mounted on the shaft or journal and an outer race which is carried by the housing or casing. In between the inner and outer race, there are balls or rollers as shown in Fig. A number of balls or rollers are used and these are held at proper distances by retainers so that they do not touch each other. The retainers are thin strips and is usually in two parts which are assembled after the balls have been properly spaced. The ball bearings are used for light loads and the roller bearings are used for heavier loads.

The rolling contact bearings, depending upon the load to be carried, are classified as:

(a) Radial bearings, and (b) Thrust bearings.

The radial and thrust ball bearings are shown in Fig. (a) and (b) respectively. When a ball bearing supports only a radial load (WR), the plane of rotation of the ball is normal to the centre line of the bearing, as shown in Fig. (a). The action of thrust load (WA) is to shift the plane of rotation of the balls, as shown in Fig. (b). The radial and thrust loads both may be carried simultaneously

PROBLEM.1.

A shaft rotating at constant speed is subjected to variable load. The bearings supporting the shaft are subjected to stationary equivalent radial load of 3 kN for 10 per cent of time, 2 kN for 20 per cent of time, 1 kN for 30 per cent of time and no load for remaining time of cycle. If the total life expected for the bearing is 20 × 106 revolutions at 95 per cent reliability, calculate dynamic load rating of the ball bearing.

**Solution.** Given : $W_1 = 3$ kN ; $n_1 = 0.1\,n$ ; $W_2 = 2$ kN ; $n_2 = 0.2\,n$ ; $W_3 = 1$ kN ; $n_3 = 0.3\,n$ ; $W_4 = 0$ ; $n_4 = (1 - 0.1 - 0.2 - 0.3)\,n = 0.4\,n$ ; $L_{95} = 20 \times 10^6$ rev

Let $L_{90}$ = Life of the bearing corresponding to reliability of 90 per cent,

$L_{95}$ = Life of the bearing corresponding to reliability of 95 per cent

$= 20 \times 10^6$ revolutions ... (Given)

We know that

$$\frac{L_{95}}{L_{90}} = \left[\frac{\log_e (1/R_{95})}{\log_e (1/R_{90})}\right]^{1/b} = \left[\frac{\log_e (1/0.95)}{\log_e (1/0.90)}\right]^{1/1.17} \quad \text{... } (\because b = 1.17)$$

$$= \left(\frac{0.0513}{0.1054}\right)^{0.8547} = 0.54$$

$\therefore$ $L_{90} = L_{95} / 0.54 = 20 \times 10^6 / 0.54 = 37 \times 10^6$ rev

We know that equivalent radial load,

$$W = \left[\frac{n_1 (W_1)^3 + n_2 (W_2)^3 + n_3 (W_3)^3 + n_4 (W_4)^3}{n_1 + n_2 + n_3 + n_4}\right]^{1/3}$$

$$= \left[\frac{0.1n \times 3^3 + 0.2\,n \times 2^3 + 0.3n \times 1^3 + 0.4\,n \times 0^3}{0.1\,n + 0.2n + 0.3n + 0.4n}\right]^{1/3}$$

$$= (2.7 + 1.6 + 0.3 + 0)^{1/3} = 1.663 \text{ kN}$$

We also know that dynamic load rating,

$$C = W\left(\frac{L_{90}}{10^6}\right)^{1/k} = 1.663\left(\frac{37 \times 10^6}{10^6}\right)^{1/3} = 5.54 \text{ kN}$$ **Ans.**

**PROBLEM.2.**

*Select a single row deep groove ball bearing for a radial load of 4000 N and an axial load of 5000 N, operating at a speed of 1600 r.p.m. for an average life of 5 years at 10 hours per day. Assume uniform and steady load.*

**Solution.** Given : $WR = 4000$ N ; $WA = 5000$ N ; $N = 1600$ r.p.m.

Since the average life of the bearing is 5 years at 10 hours per day, therefore life of the bearing in hours,

$$L_H = 5 \times 300 \times 10 = 15\,000 \text{ hours} \quad \text{... (Assuming 300 working days per year)}$$

and life of the bearing in revolutions,

$$L = 60\,N \times L_H = 60 \times 1600 \times 15\,000 = 1440 \times 10^6 \text{ rev}$$

We know that the basic dynamic equivalent radial load,

$$W = X.V.W_R + Y.W_A \quad \text{...}(i)$$

In order to determine the radial load factor ($X$) and axial load factor ($Y$), we require $W_A / W_R$ and $W_A / C_0$. Since the value of basic static load capacity ($C_0$) is not known, therefore let us take $W_A / C_0 = 0.5$. Now from Table 27.4, we find that the values of $X$ and $Y$ corresponding to $W_A / C_0 = 0.5$ and $W_A / W_R = 5000 / 4000 = 1.25$ (which is greater than $e = 0.44$) are

$$X = 0.56 \quad \text{and} \quad Y = 1$$

Since the rotational factor ($V$) for most of the bearings is 1, therefore basic dynamic equivalent radial load,

$$W = 0.56 \times 1 \times 4000 + 1 \times 5000 = 7240 \text{ N}$$

From Table 27.5, we find that for uniform and steady load, the service factor ($K_S$) for ball bearings is 1. Therefore the bearing should be selected for $W = 7240$ N.

We know that basic dynamic load rating,

$$C = W\left(\frac{L}{10^6}\right)^{1/k} = 7240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3} = 81\ 760 \text{ N}$$

$$= 81.76 \text{ kN} \qquad \text{... } (\because k = 3, \text{ for ball bearings})$$

From Table 27.6, let us select the bearing No. 315 which has the following basic capacities,

$$C_0 = 72 \text{ kN} = 72\ 000 \text{ N} \quad \text{and} \quad C = 90 \text{ kN} = 90\ 000 \text{ N}$$

Now $\quad W_A / C_0 = 5000 / 72\ 000 = 0.07$

$\therefore$ From Table 27.4, the values of $X$ and $Y$ are

$$X = 0.56 \quad \text{and} \quad Y = 1.6$$

Substituting these values in equation (*i*), we have dynamic equivalent load,

$$W = 0.56 \times 1 \times 4000 + 1.6 \times 5000 = 10\ 240 \text{ N}$$

$\therefore$ Basic dynamic load rating,

$$C = 10\ 240\left(\frac{1440 \times 10^6}{10^6}\right)^{1/3} = 115\ 635 \text{ N} = 115.635 \text{ kN}$$

# DESIGN OF FLYWHEEL

## Introduction

A flywheel used in machines serves as a reservoir which stores energy during the period when the supply of energy is more than the requirement and releases it during the period when the requirement of energy is more than supply. In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke. For example, in I.C. engines, the energy is developed only during power stroke which is much more than the engine load, and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines. The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed. A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.

### Coefficient of Fluctuation of Energy

It is defined as the ratio of the maximum fluctuation of energy to the work done per cycle. It is usually denoted by CE. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Work done per cycle}}$$

The workdone per cycle may be obtained by using the following relations:

1. Workdone / cycle $= T_{mean} \times \theta$

where $T_{mean}$ = Mean torque, and

$\theta$ = Angle turned in radians per revolution

= $2\pi$, in case of steam engines and two stroke internal combustion engines.

= $4\pi$, in case of four stroke internal combustion engines.

The mean torque ($T_{mean}$) in N-m may be obtained by using the following relation *i.e.*

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where $P$ = Power transmitted in watts,

$N$ = Speed in r.p.m., and

$\omega$ = Angular speed in rad/s = $2\pi N / 60$

**2.** The workdone per cycle may also be obtained by using the following relation:

$$\text{Workdone / cycle} = \frac{P \times 60}{n}$$

where $n$ = Number of working strokes per minute.

= $N$, in case of steam engines and two stroke internal combustion engines.

= $N/2$, in case of four stroke internal combustion engines.

The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

**Problem:**

The turning moment diagram for a petrol engine is drawn to the following scales:

Turning moment, 1 mm = 5 N-m; Crank angle, 1 mm = 1º. The turning moment diagram repeats itself at every half revolution of the engine and the areas above and below the mean turning moment line, taken in order are 295, 685, 40, 340, 960, 270 mm2. Determine the mass of 300 mm diameter flywheel rim when the coefficient of fluctuation of speed is 0.3% and the engine runs at 1800 r.p.m. Also determine the cross-section of the rim when the width of the rim is twice of thickness. Assume density of rim material as 7250 kg / m3.

**Solution.** Given : $D$ = 300 mm or $R$ = 150 mm = 0.15 m ; $C_S$ = 0.3% = 0.003 ; $N$ = 1800 r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ; $\rho = 7250$ kg / m$^3$

*Mass of the flywheel*

Let $m$ = Mass of the flywheel in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram is shown in Fig. 22.6.

Since the scale of turning moment is 1 mm = 5 N-m, and scale of the crank angle is 1 mm = 1° = $\pi$ / 180 rad, therefore 1 mm$^2$ on the turning moment diagram.

$$= 5 \times \pi / 180 = 0.087 \text{ N-m}$$

Let the total energy at $A = E$. Therefore from Fig. 22.6, we find that

Energy at $B = E + 295$

Energy at $C = E + 295 - 685 = E - 390$

Energy at $D = E - 390 + 40 = E - 350$

Energy at $E = E - 350 - 340 = E - 690$

Energy at $F = E - 690 + 960 = E + 270$

Energy at $G = E + 270 - 270 = E$ = Energy at $A$

From above we see that the energy is maximum at $B$ and minimum at $E$.

∴ Maximum energy $= E + 295$

and minimum energy $= E - 690$

We know that maximum fluctuation of energy,

$$\Delta E = \text{Maximum energy} - \text{Minimum energy}$$
$$= (E + 295) - (E - 690) = 985 \text{ mm}^2$$
$$= 985 \times 0.087 = 86 \text{ N-m}$$

We also know that maximum fluctuation of energy ($\Delta E$),

$$86 = m.R^2.\omega^2.C_S = m\,(0.15)^2\,(188.5)^2\,(0.003) = 2.4\,m$$

$\therefore$ $m = 86 / 2.4 = 35.8$ kg **Ans.**

*Cross-section of the flywheel rim*

Let $t$ = Thickness of rim in metres, and

$b$ = Width of rim in metres = $2\,t$ ...(Given)

$\therefore$ Cross-sectional area of rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim ($m$),

$$35.8 = A \times 2\pi R \times \rho = 2t^2 \times 2\pi \times 0.15 \times 7250 = 13\,668\,t^2$$

$\therefore$ $t^2 = 35.8 / 13\,668 = 0.0026$ or $t = 0.051$ m = 51 mm **Ans.**

and $b = 2\,t = 2 \times 51 = 102$ mm **Ans.**

**Problem:**

*The intercepted areas between the output torque curve and the mean resistance line of a turning moment diagram for a multicylinder engine, taken in order from one end are as follows: – 35, + 410, – 285, + 325, – 335, + 260, – 365, + 285, – 260 mm2. The diagram has been drawn to a scale of 1 mm = 70 N-m and 1 mm = 4.5°. The engine speed is 900 r.p.m. and the fluctuation in speed is not to exceed 2% of the mean speed. Find the mass and cross-section of the flywheel rim having 650 mm mean diameter. The density of the material of the flywheel may be taken as 7200 kg / m3. The rim is rectangular with the width 2 times the thickness. Neglect effect of arms, etc.*

**Solution.** Given : $N = 900$ r.p.m. or $\omega = 2\pi \times 900 / 60 = 94.26$ rad/s ; $\omega_1 - \omega_2 = 2\%\ \omega$ or

$$\frac{\omega_1 - \omega_2}{\omega} = C_S = 2\% = 0.02 \ ; D = 650 \text{ mm or } R = 325 \text{ mm} = 0.325 \text{ m} \ ; \rho = 7200 \text{ kg / m}^3$$

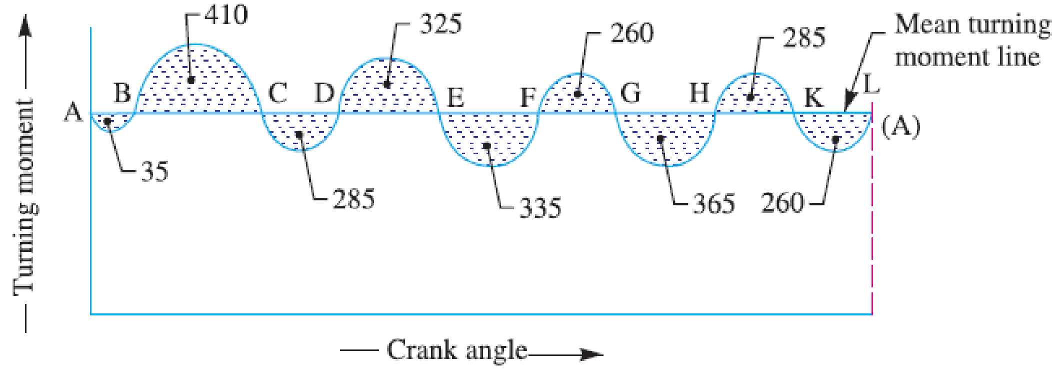
*Mass of the flywheel rim*

Let $m$ = Mass of the flywheel rim in kg.

First of all, let us find the maximum fluctuation of energy. The turning moment diagram for a multi-cylinder engine is shown in Fig. 22.7.

Since the scale of turning moment is 1 mm = 70 N-m and scale of the crank angle is 1 mm = 4.5° = $\pi$ / 40 rad, therefore 1 mm$^2$ on the turning moment diagram.

$$= 70 \times \pi / 40 = 5.5 \text{ N-m}$$




Let the total energy at $A = E$. Therefore from

$$\text{Energy at } B = E - 35$$
$$\text{Energy at } C = E - 35 + 410 = E + 375$$
$$\text{Energy at } D = E + 375 - 285 = E + 90$$
$$\text{Energy at } E = E + 90 + 325 = E + 415$$
$$\text{Energy at } F = E + 415 - 335 = E + 80$$
$$\text{Energy at } G = E + 80 + 260 = E + 340$$
$$\text{Energy at } H = E + 340 - 365 = E - 25$$
$$\text{Energy at } K = E - 25 + 285 = E + 260$$
$$\text{Energy at } L = E + 260 - 260 = E = \text{Energy at } A$$

From above, we see that the energy is maximum at $E$ and minimum at $B$.

$\therefore$ Maximum energy $= E + 415$

and minimum energy $= E - 35$

We know that maximum fluctuation of energy,

$$= (E + 415) - (E - 35) = 450 \text{ mm}^2$$
$$= 450 \times 5.5 = 2475 \text{ N-m}$$

We also know that maximum fluctuation of energy ($\Delta E$),

$$2475 = m.R^2.\omega^2.C_S = m\,(0.325)^2\,(94.26)^2\,0.02 = 18.77\,m$$

$\therefore$ $m = 2475 / 18.77 = 132$ kg **Ans.**

*Cross-section of the flywheel rim*

Let $t$ = Thickness of the rim in metres, and

$b$ = Width of the rim in metres = $2\,t$

$\therefore$ Area of cross-section of the rim,

$$A = b \times t = 2\,t \times t = 2\,t^2$$

We know that mass of the flywheel rim ($m$),

$$132 = A \times 2\,\pi\,R \times \rho = 2\,t^2 \times 2\,\pi \times 0.325 \times 7200 = 29\,409\,t^2$$

$\therefore$ $t^2 = 132 / 29\,409 = 0.0044$ or $t = 0.067$ m = 67 mm **Ans.**

and $b = 2t = 2 \times 67 = 134$ mm **Ans.**

**Problem**

*A single cylinder double acting steam engine develops 150 kW at a mean speed of 80 r.p.m. The coefficient of fluctuation of energy is 0.1 and the fluctuation of speed is ± 2% of mean peed. If the mean diameter of the flywheel rim is 2 metres and the hub and spokes provide 5 percent of the rotational inertia of the wheel, find the mass of the flywheel and cross- ectional area of the rim. Assume the density of the flywheel material (which is cast iron) as 7200 kg / m3.*

**Solution.** Given : $P = 150$ kW $= 150 \times 10^3$ W ; $N = 80$ r.p.m. ; $C_E = 0.1$; $\omega_1 = \pm 2\%\ \omega$ ; $D = 2$ m or $R = 1$ m ; $\rho = 7200$ kg/m$^3$

*Mass of the flywheel rim*

Let $m$ = Mass of the flywheel rim in kg.

We know that the mean angular speed,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 80}{60} = 8.4 \text{ rad / s}$$

Since the fluctuation of speed is ± 2% of mean speed (ω), therefore total fluctuation of s

$$\omega_1 - \omega_2 = 4\%\ \omega = 0.04\ \omega$$

and coefficient of fluctuation of speed,

$$C_S = \frac{\omega_1 - \omega_2}{\omega} = 0.04$$

We know that the work done by the flywheel per cycle

$$= \frac{P \times 60}{N} = \frac{150 \times 10^3 \times 60}{80} = 112\,500 \text{ N-m}$$

We also know that coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone / cycle}}$$

∴ Maximum fluctuation of energy,

$$\Delta E = C_E \times \text{Workdone / cycle}$$
$$= 0.1 \times 112\,500 = 11\,250 \text{ N-m}$$

Since 5% of the rotational inertia is provided by hub and spokes, therefore the max fluctuation of energy of the flywheel rim will be 95% of the flywheel.

∴ Maximum fluctuation of energy of the rim,

$$(\Delta E)_{rim} = 0.95 \times 11\,250 = 10\,687.5 \text{ N-m}$$

We know that maximum fluctuation of energy of the rim $(\Delta E)_{rim}$,

$$10\,687.5 = m.R^2.\omega^2.C_S = m \times 1^2\,(8.4)^2\,0.04 = 2.82\,m$$

$$\therefore \quad m = 10\,687.5 / 2.82 = 3790 \text{ kg} \quad \textbf{Ans.}$$

*Cross-sectional area of the rim*

Let $A$ = Cross-sectional area of the rim.

We know that the mass of the flywheel rim ($m$),

$$3790 = A \times 2\pi R \times \rho = A \times 2\pi \times 1 \times 7200 = 45\,245\,A$$

$$\therefore \quad A = 3790 / 45\,245 = 0.084 \text{ m}^2 \quad \textbf{Ans.}$$