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SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – I - MECHANISMS AND CAMS – SMEA1402

MECHANICS OF MACHINES (SMEA1402)

2019 REGULATIONS

B.E. MECHANICAL ENGINEERING

UNIT 1 MECHANISMS AND CAMS

INTRODUCTION

Mechanics may be divided into two branches as Statics and Dynamics. Statics deals with forces acting on and in a body at rest. Dynamics deals with forces acting on and in a body at motion. Dynamics is further divided into Kinematics and Kinetics. Kinematics is the branch of mechanics which tells us about the motion without considering the cause of motion. In this portion, we study the displacement, speed and acceleration without bothering about the input force or torque. It describes the possible motions of a body or

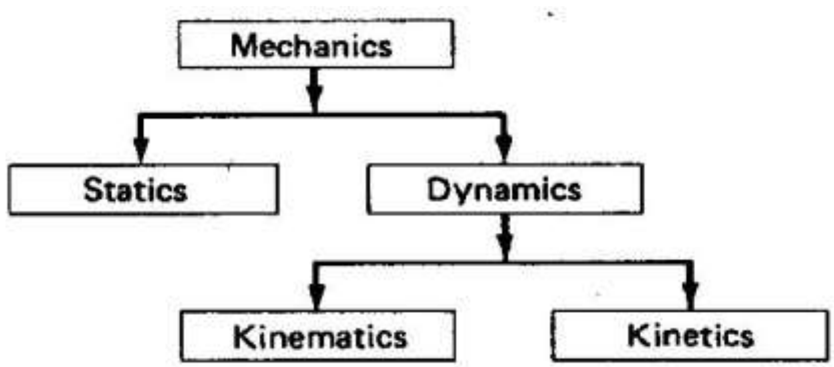


Fig. 1 Classification of Engineering Mechanics

system of bodies. Kinetics attempts to explain or predict the motion that will occur in a given situation without considering the force which is responsible for it.

KINEMATIC LINK OR ELEMENT

Each part of a machine, which moves relative to some other part, is known as a kinematic link or element. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another. The link should have the following two characteristics

1. It should have relative motion, and
2. It must be a resistant body

They are 4 types of links: Rigid, flexible, Fluid and Floating Links.

- Rigid link is one which does not undergo any deformation while transmitting motion. Ex: Connecting rod, crank etc.
- Flexible link is partly deformed in a manner not to affect the transmission of motion. Ex: Belts, ropes, springs, chains and wires are flexible links and transmit tensile forces only.
- Fluid link one which is formed by having a fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.
- Floating Link is a link which is not connected frame.

TYPES OF JOINTS

The usual types of joints in a chain are

- Binary joint
- Ternary joint
- Quaternary joint

Binary Joint

If two links are joined at the same connection, it is called a binary joint.

Ternary Joint

If three links are joined at a connection, it is known as a ternary joint. It is considered equivalent to two binary joints since fixing of any one link constitutes two binary joints with each of the other two links.

Quaternary Joint

If four links are joined at a connection, it is known as a quaternary joint. It is considered equivalent to three binary joints since fixing of any one link constitutes three binary joints.

In general, if 'n' number of links that are connected at a joint, it is equivalent to (n-1) binary joints.

KINEMATIC PAIRS

Pairing elements: the geometrical forms by which two members of a mechanism are joined together, so that the relative motion between these two is consistent are known as pairing elements and the pair so formed is called kinematic pair. Each individual link of a mechanism forms a pairing element.

KINEMATIC CONSTRAINTS

Two or more rigid bodies in space are collectively called a rigid body system. We can hinder the motion of these independent rigid bodies with kinematic constraints. Kinematic constraints are constraints between rigid bodies that result in the decrease of the degrees of freedom of rigid body system.

The three main types of constrained motion in kinematic pair are,

Completely constrained motion: If the motion between a pair of links is limited to a definite direction, then it is completely constrained motion. E.g.: Motion of a shaft or rod with collars at each end in a hole as shown in fig.

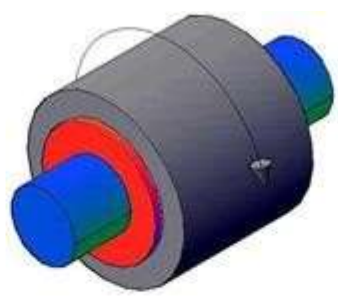


Fig. 2 Completely constrained motion

Incompletely Constrained motion: If the motion between a pair of links is not confined to a definite direction, then it is incompletely constrained motion. E.g.: A spherical ball or circular shaft in a circular hole may either rotate or slide in the hole as shown in fig.

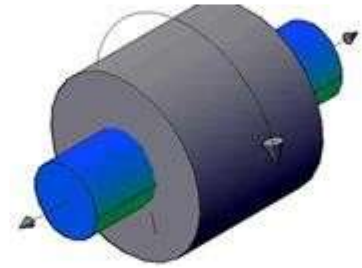


Fig. 3 Incompletely constrained motion

Successfully constrained motion or partially constrained motion: If the motion in a definite direction is not brought about by itself but by some other means, then it is known as successfully constrained motion. E.g.: Foot step Bearing shown in fig.

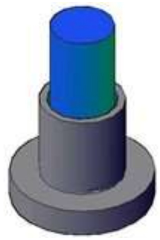


Fig. 4 Successfully constrained motion

TYPES OF KINEMATIC PAIRS

Kinematic pairs according to nature of relative motion.

Sliding pair: If two links have a sliding motion relative to each other, they form a sliding pair. A rectangular rod in a rectangular hole in a prism is an example of a sliding pair.

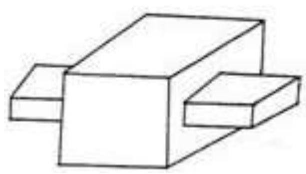


Fig. 5 Sliding pair

Turning Pair: When on link has a turning or revolving motion relative to the other, they constitute a turning pair or revolving pair.

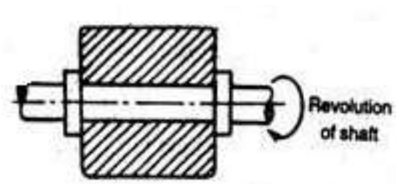


Fig. 6 Turning pair

Rolling pair: When the links of a pair have a rolling motion relative to each other, they form a rolling pair. e.g. A rolling wheel on a flat surface, ball and roller bearings.

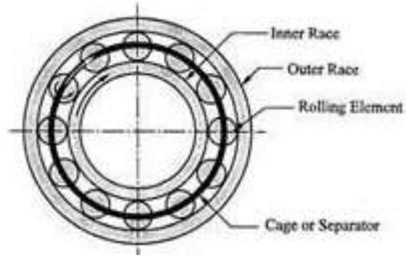


Fig. 7 Rolling pair

Screw pair (Helical Pair): if two mating links have a turning as well as sliding motion between them, they form a screw pair. This is achieved by cutting matching threads on the two links. The lead screw and the nut of a lathe is a screw Pair.

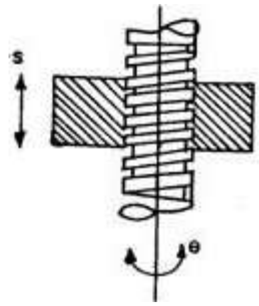


Fig. 8 Screw pair

Spherical pair: When one link in the form of a sphere turns inside a fixed link, it is a spherical pair. The ball and socket joint is a spherical pair.

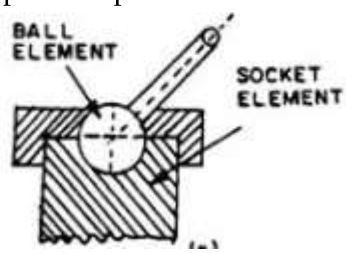


Fig. 9 Spherical pair

Kinematic pairs according to nature of contact:

Lower Pair: A pair of links having surface or area contact between the members is known as a lower pair. The contact surfaces of the two links are similar. e.g.: Nut turning on a screw, shaft rotating in a bearing, all pairs of a slider-crank mechanism, universal joint.

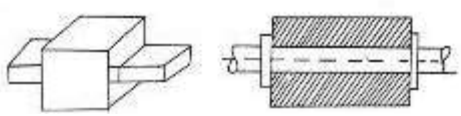


Fig. 10 Lower pair

Higher Pair: When a pair has a point or line contact between the links, it is known as a higher pair. The contact surfaces of the two links are dissimilar.

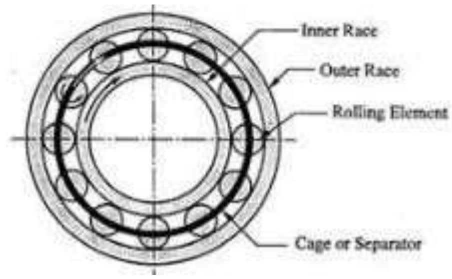


Fig. 11 Higher pair

e.g.: Wheel rolling on a surface cam and follower pair, tooth gears, ball and roller bearings, etc
Kinematic pairs according to nature of mechanical constraint:

Closed pair: When the elements of a pair are held together mechanically, it is known as a closed pair. The contact between the two can only be broken only by the destruction of at least one of the members. All the lower pairs and some of the higher pairs are closed pairs.

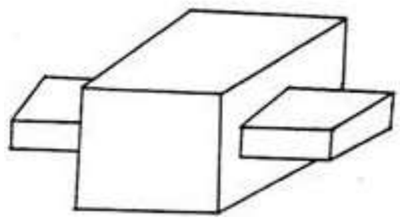


Fig. 12 Closed pair

Open pair: When two links of a pair are in contact either due to force of gravity or some spring action, they constitute an unclosed pair. In this the links are not held together mechanically. Ex.: Cam and follower pair.

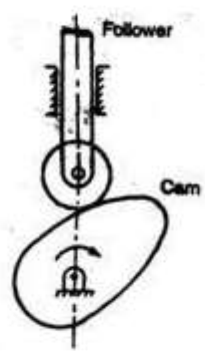


Fig. 13 Open pair

MECHANISM AND MACHINES

Mechanism: A mechanism is a constrained kinematic chain. This means that the motion of any one link in the kinematic chain will give a definite and predictable motion relative to each of the others. Usually one of the links of the kinematic chain is fixed in a mechanism.

If, for a particular position of a link of the chain, the positions of each of the other links of the chain cannot be predicted, then it is called as unconstrained kinematic chain and it is not mechanism.

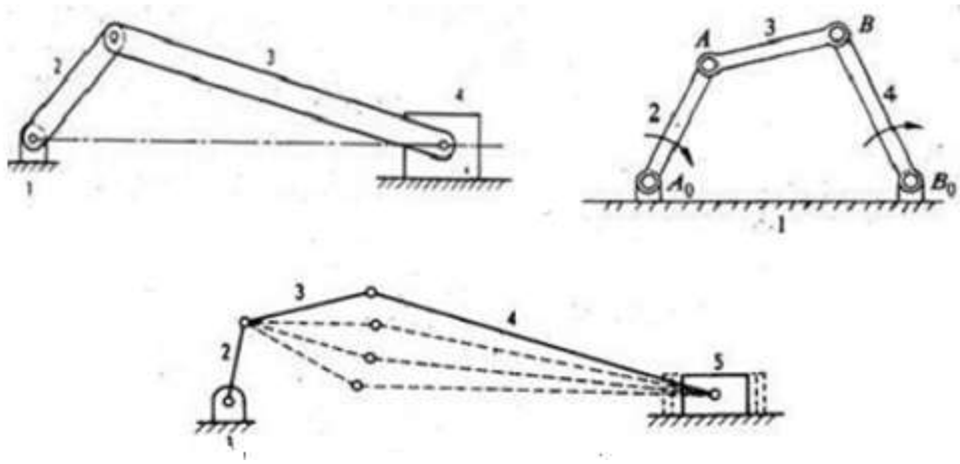


Fig. 14 Four bar Mechanism

Machine: A machine is a mechanism or collection of mechanisms, which transmit force from the source of power to the resistance to be overcome. Though all machines are mechanisms, all mechanisms are not machines. Many instruments are mechanisms but are not machines, because they do no useful work nor do they transform energy. Eg. Mechanical clock, drafter.

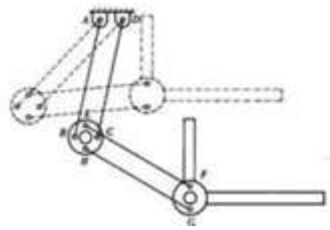


Fig.15 Mini drafter

Difference between a Machine and a Structure

The following differences between a machine and a structure are important from the subject point of view :

1. The parts of a machine move relative to one another, whereas the members of a structure do not move relative to one another.

2. A machine transforms the available energy into some useful work, whereas in a structure no energy is transformed into useful work.

3. The links of a machine may transmit both power and motion, while the members of a structure transmit forces only.

PLANAR MECHANISMS

When all the links of a mechanism have plane motion, it is called as a planar mechanism. All the links in a planar mechanism move in planes parallel to the reference plane.

KINEMATIC CHAIN

A kinematic chain is a group of links either joined together or arranged in a manner that permits them to move relative to one another. If the links are connected in such a way that no motion is possible, it

results in a locked chain or structure.
It is a combination of several successively arranged joints constituting a complex motor system.
Kinematic chain is when a number of links are united in series.
Relation between Links, Pairs and Joints

l = No of links p = No of Pairs j = No of Joints
L.H.S >R.H.S = Locked chain
L.H.S = R.H.S = Constrained Kinematic Chain
L.H.S <R.H.S = Unconstrained Kinematic Chain

l = 3;, p = 3;j = 3
l = 2p – 4
3 = 2 × 3 – 4 = 2
i.e. L.H.S. > R.H.S.

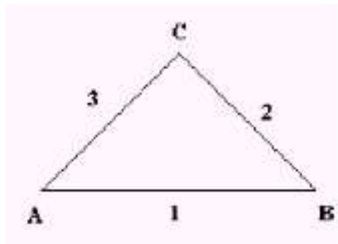


Fig. 16 Three bar Chain- If any one link is fixed- Structure

l=2p-4 j=(3/2) l – 2

j=(3/2) l – 2 3=(3/2)3 – 2
= 2.5
i.e. L.H.S. > R.H.S. ABC does not form a Kinematic chain but forms a structure.

DEGREES OF FREEDOM

An unconstrained rigid body moving in space can describe the following independent motions. Translational Motions along any three mutually perpendicular axes x, y and z, Rotational motions along these axes.

Thus a rigid body possesses six degrees of freedom. The connection of a link with another imposes certain constraints on their relative motion. The number of restraints can never be zero (joint is disconnected) or six (joint becomes solid).

Degrees of freedom of a pair is defined as the number of independent relative motions, both translational and rotational, a pair can have.

Degrees of freedom = 6 – no. of restraints.

To find the number of degrees of freedom for a plane mechanism we have an equation known as Grubler's equation and is given by

$$F = 3 (n – 1) – 2 j_1 – j_2$$

F = Mobility or number of degrees of freedom n = Number of links including frame.
j_1 = Joints with single (one) degree of freedom. J_2= Joints with two degrees of freedom.
If
F > 0, results a mechanism with 'F' degrees of freedom. F = 0,
results in a statically determinate structure.

F < 0, results in a statically indeterminate structure.

KINEMATIC INVERSION

Inversions of mechanism: A mechanism is one in which one of the links of a kinematic chain is fixed. Different mechanisms can be obtained by fixing different links of the same kinematic chain. These are called as inversions of the mechanism. By changing the fixed link, the number of mechanisms which can be obtained is equal to the number of links. Excepting the original mechanism, all other mechanisms will be known as inversions of original mechanism. The inversion of a mechanism does not change the motion of its links relative to each other.

Kinematic Inversions of Four Bar Chain, Slider Crank and Double Slider Crank Mechanism

Four bar chain

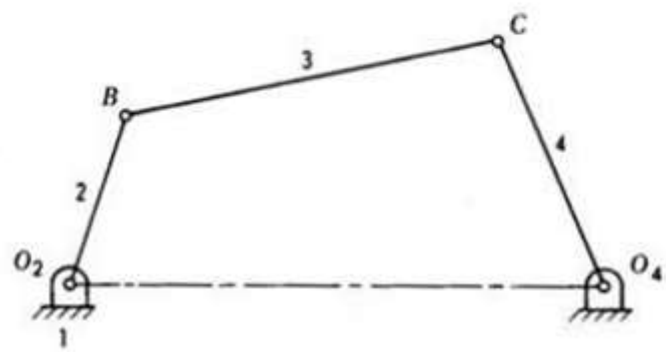


Fig. 17 Four bar Mechanism

One of the most useful and most common mechanisms is the four-bar linkage. In this mechanism, the link which can make complete rotation is known as crank (link 2). The link which oscillates is known as rocker or lever (link 4). And the link connecting these two is known as coupler (link 3). Link 1 is the frame.

Inversions of four bar chain

- Beam Engine or Crank and lever mechanism.
- Coupling rod of locomotive or double crank mechanism.
- Watt's straight line mechanism or double lever mechanism.

Beam Engine: When the crank AB rotates about A, the link CE pivoted at D makes vertical reciprocating motion at end E. This is used to convert rotary motion to reciprocating motion and vice versa. It is also known as Crank and lever mechanism. This mechanism is shown in the figure below.

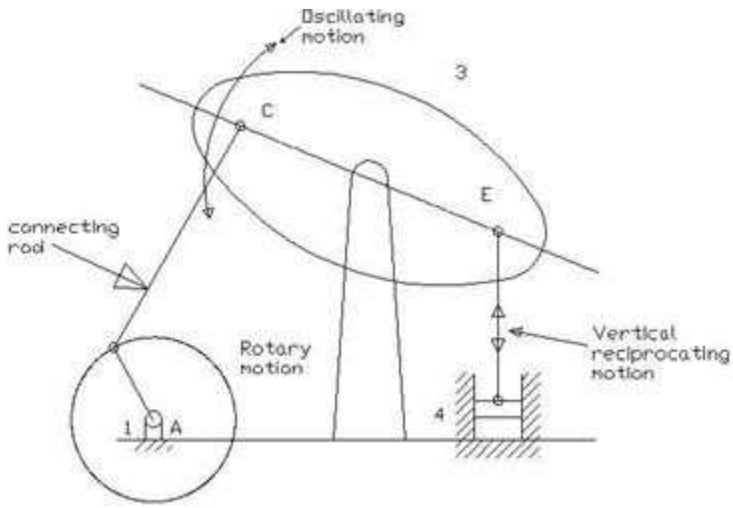


Fig. 18 Beam Engine Mechanism

Coupling rod of locomotive: In this mechanism the length of link AD = length of link C. Also length of link AB = length of link CD. When AB rotates about A, the crank DC rotates about D. this mechanism is used for coupling locomotive wheels. Since links AB and CD work as cranks, this mechanism is also known as double crank mechanism. This is shown in the figure below.

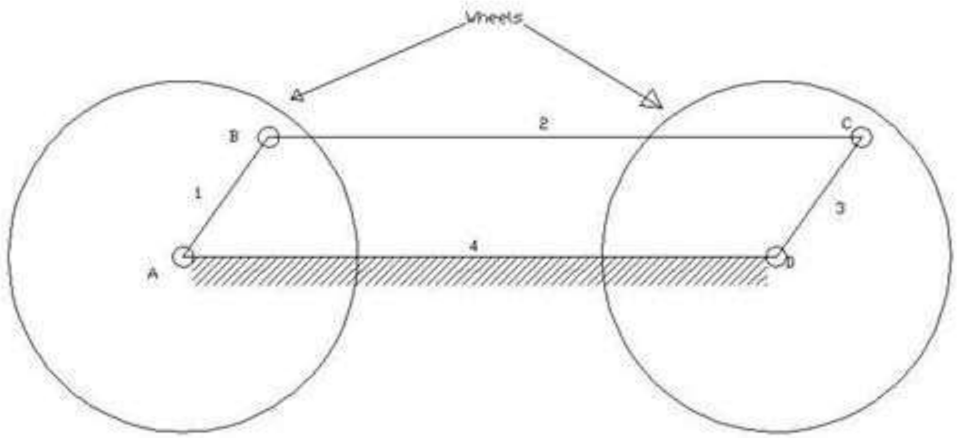


Fig. 19 Coupling rod of locomotive Mechanism

Watt's straight line mechanism or Double lever mechanism: In this mechanism, the links AB & DE act as leversat the ends A & E of these levers are fixed. The AB & DE are parallel in the mean position of the mechanism and coupling rod BD is perpendicular to the levers AB & DE. On any small displacement of the mechanism the tracing point 'C' traces the shape of number '8', a portion of which will be approximately straight. Hence this is also an example for the approximate straight line mechanism. This mechanism is shown below.

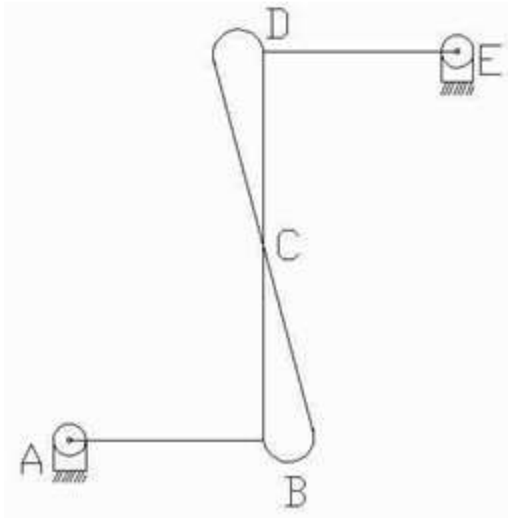


Fig. 20 Watt's straight line Mechanism

SLIDER CRANK CHAIN

It is a four bar chain having one sliding pair and three turning pairs. It is shown in the figure below the purpose of this mechanism is to convert rotary motion to reciprocating motion and vice versa.

Inversions of a Slider crank chain

There are four inversions in a single slider chain mechanism. They are

- Reciprocating engine mechanism (1st inversion)
- Oscillating cylinder engine mechanism (2nd inversion)
- Crank and slotted lever mechanism (2nd inversion)
- Whitworth quick return motion mechanism (3rd inversion)
- Rotary engine mechanism (3rd inversion)
- Bull engine mechanism (4th inversion)
- Hand Pump (4th inversion)

Reciprocating engine mechanism: In the first inversion, the link 1 i.e., the cylinder and the frame is kept fixed. The fig below shows a reciprocating engine.

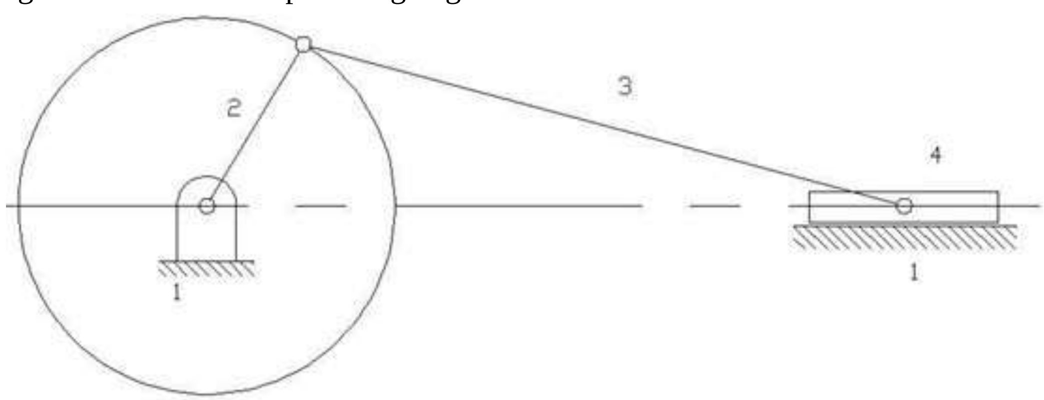


Fig. 21 Reciprocating engine Mechanism

A slotted link 1 is fixed. When the crank 2 rotates about O, the sliding piston 4 reciprocates in the slotted link 1. This mechanism is used in steam engine, pumps, compressors, I.C. engines, etc.

Crank and slotted lever mechanism: It is an application of second inversion. The crank and slotted lever mechanism is shown in figure below.

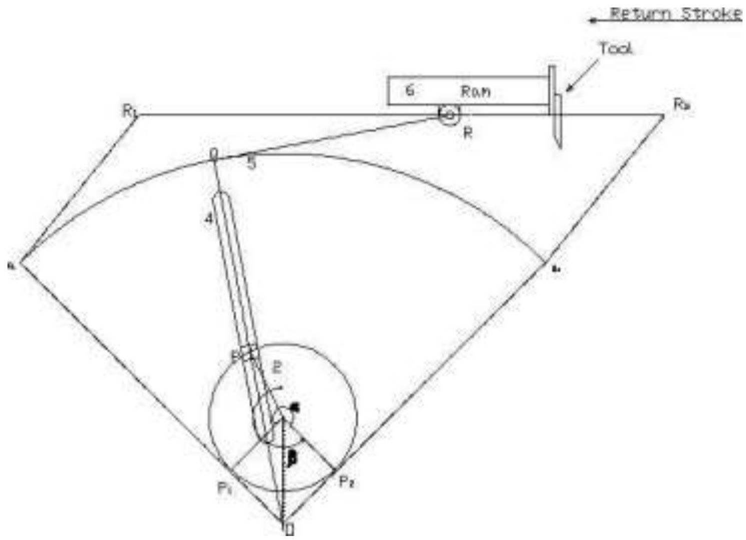


Fig. 22 Crank and slotted lever Mechanism

In this mechanism link 3 is fixed. The slider (link 1) reciprocates in oscillating slotted lever (link 4) and crank (link 2) rotates. Link 5 connects link 4 to the ram (link 6). The ram with the cutting tool reciprocates perpendicular to the fixed link 3. The ram with the tool reverses its direction of motion when link 2 is perpendicular to link 4. Thus the cutting stroke is executed during the rotation of the crank through angle α and the return stroke is executed when the crank rotates through angle β or 360–α. Therefore, when the crank rotates uniformly, we get,

$$\frac{\text{Time to cutting}}{\text{Time of return}} = \frac{\alpha}{\beta} = \frac{\alpha}{360 - \alpha}$$

This mechanism is used in shaping machines, slotting machines and in rotary engines.

Whitworth quick return motion mechanism: Third inversion is obtained by fixing the crank

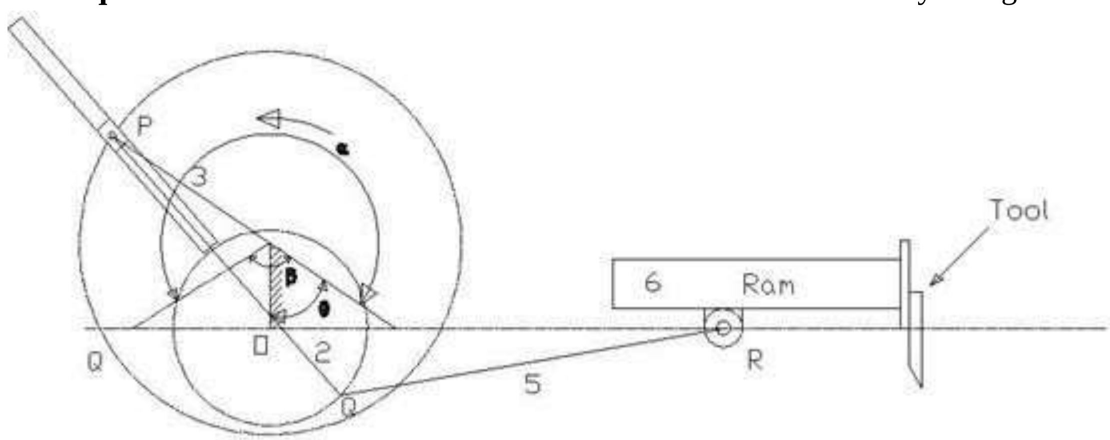


Fig. 23 Whitworth quick return motion Mechanism

i.e. link 2. Whitworth quick return mechanism is an application of third inversion. This mechanism is shown in the figure below. The crank OC is fixed and OQ rotates about O. The slider slides in the slotted link and generates a circle of radius CP. Link 5 connects the extension OQ provided on the opposite side of the link 1 to the ram (link 6). The rotary motion of P is taken to the ram R which reciprocates. The quick return motion mechanism is used in shapers and slotting machines.The angle covered during cutting stroke from P1 to P2 in counter clockwise direction is α or 360 - 2θ. During the return stroke, the angle covered is 2θ or β.

$$\frac{\text{Time to cutting}}{\text{Time of return}} = \frac{360 - 2\theta}{2\theta} = \frac{\alpha}{\beta}$$

Rotary engine mechanism or Gnome Engine: Rotary engine mechanism or gnome engine is another application of third inversion. It is a rotary cylinder V – type internal combustion engine used as an aero engine. But now Gnome engine has been replaced by Gas turbines. The Gnome engine has generally seven cylinders in one plane. The crank OA is fixed and all the connecting rods from the pistons are connected to A. In this mechanism when the pistons reciprocate in the cylinders, the whole assembly of cylinders, pistons and connecting rods rotate about the axis O, where the entire mechanical power developed, is obtained in the form of rotation of the crank shaft. This mechanism is shown in the figure below.

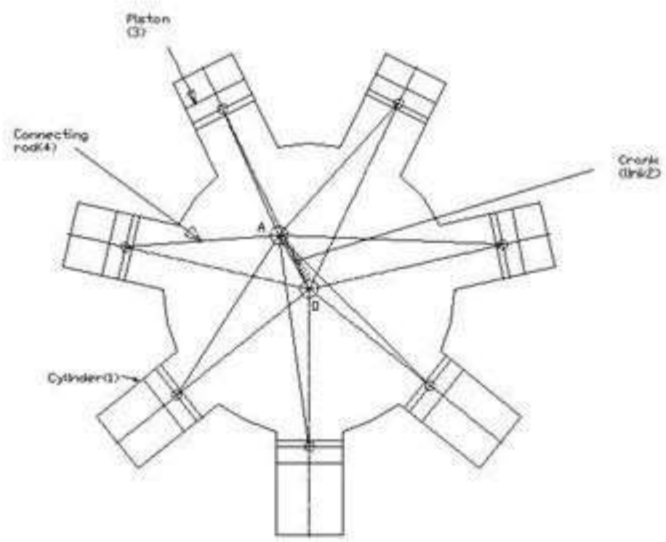


Fig. 24 Rotary engine mechanism or Gnome Engine

DOUBLE SLIDER CRANK CHAIN

A four bar chain having two turning and two sliding pairs such that two pairs of the same kind are adjacent is known as double slider crank chain.

Inversions of Double slider Crank chain

It consists of two sliding pairs and two turning pairs. They are three important inversions of double slider crank chain.

- Elliptical trammel.
- Scotch yoke mechanism.
- Oldham's Coupling.

Elliptical Trammel

This is an instrument for drawing ellipses. Here the slotted link is fixed.

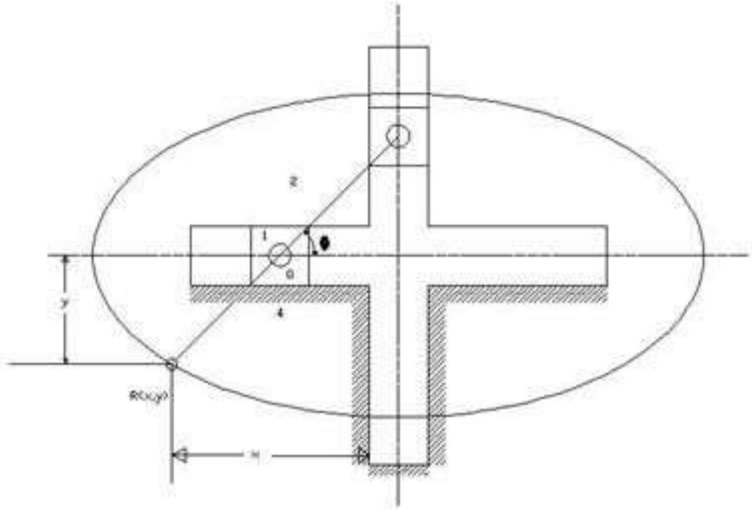


Fig. 25 Elliptical Trammel Mechanism

The sliding block P and Q in vertical and horizontal slots respectively. The end R generates an ellipse with the displacement of sliders P and Q.

The co-ordinates of the point R are x and y.

From the fig. cos θ= x/PR

and Sin θ =

y/QR

Squaring and adding (i) and (ii) we get

$$\frac{x2}{(PR)^2} + \frac{y2}{(QR)^2} = \cos^2\theta + \sin^2\theta$$

$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = 1$$

The equation is that of an ellipse, Hence the instrument traces an ellipse.

Path traced by mid-point of PQ is a circle. In this case PR = PQ.

$$\frac{x^2}{(PR)^2} + \frac{y^2}{(QR)^2} = 1$$

Its an equation of circle with PR = QR = radius of a circle.

Scotch yoke mechanism: This mechanism is used for converting rotary motion into a reciprocating motion. The inversion is obtained by fixing either the link 1 or link 3. In Fig., link 1 is fixed. In this mechanism, when the link 2 (which corresponds to crank) rotates about *B* as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

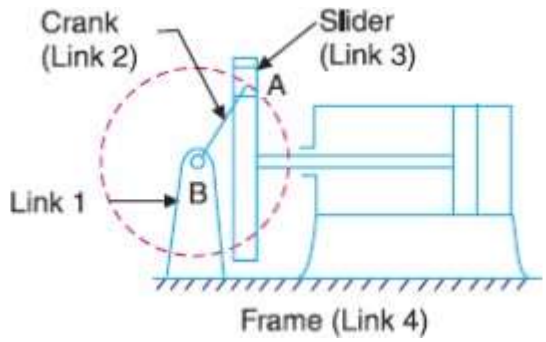


Fig. 26 Scotch yoke Mechanism

Oldham's coupling: An oldham's coupling is used for connecting two parallel shafts whose axes are at a small distance apart. The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. The link 1 and link 3 form turning pairs with link 2. These flanges have diametrical slots cut in their inner faces. The intermediate piece (link 4) which is a circular disc, have two tongues (*i.e.* diametrical projections) *T*1 and *T*2 on each face at right angles to each other. The tongues on the link 4 closely fit into the slots in the two flanges (link 1 and link 3). The link 4 can slide or reciprocate in the slots in the flanges.

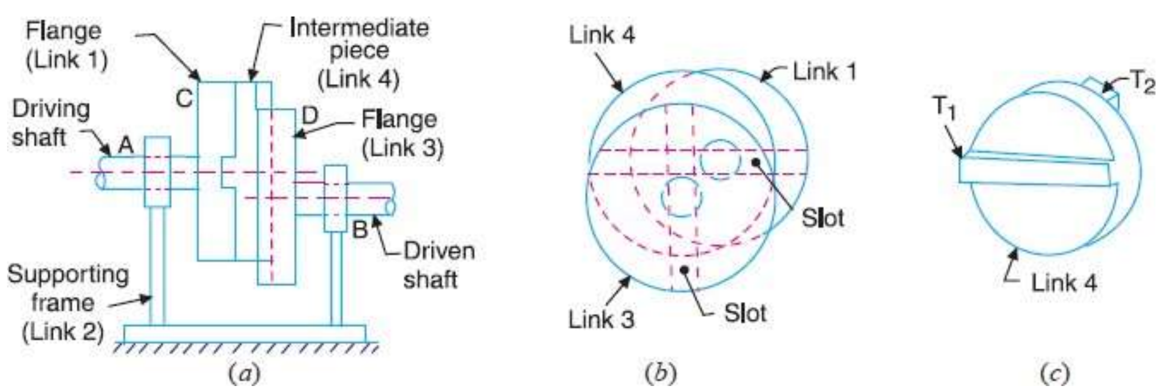


Fig. 27 Coupling rod of locomotive Mechanism

When the driving shaft *A* is rotated, the flange *C* (link 1) causes the intermediate piece (link 4) to rotate at the same angle through which the flange has rotated, and it further rotates the flange *D* (link 3) at the same angle and thus the shaft *B* rotates. Hence links 1, 3 and 4 have the same angular velocity at every instant. A little consideration will show, that there is a sliding motion between the link 4 and each of the other links 1 and 3.

PANTOGRAPH

A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point. It consists of a jointed parallelogram *ABCD* . It is made up of bars connected by turning pairs. The bars *BA* and *BC* are extended to *O* and *E* respectively, such that

OA/*OB* = *AD*/*BE*.Thus, for all relative positions of the bars, the triangles *OAD* and *OBE* are similar and the points *O*, *D* and *E* are in one straight line. It may be proved that point *E* traces out the same path as described by point *D*.

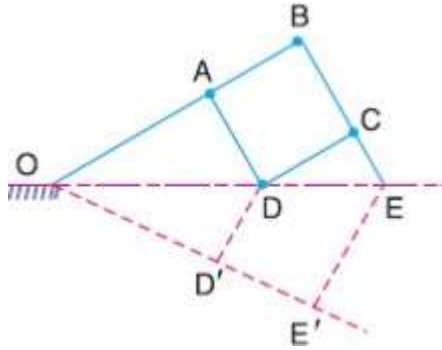


Fig. 28 Pantograph Mechanism

A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales. It is, sometimes, used as an indicator rig in order to reproduce to a small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine. It is also used to guide cutting tools. A modified form of pantograph is used to collect power at the top of an electric locomotive.

PEAUCELLIER MECHANISM

It consists of a fixed link *OO*1 and the other straight links *O*1*A*, *OC*, *OD*, *AD*, *DB*, *BC* and *CA* are connected by turning pairs at their intersections. The pin at *A* is constrained to move along the circumference of a circle with the fixed diameter *OP*, by means of the link *O*1*A*.

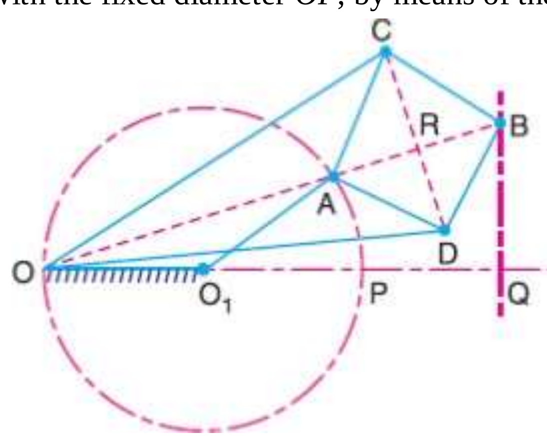


Fig. 29 Peaucellier Mechanism

AC = *CB* = *BD* = *DA* ;*OC* = *OD* ; and *OO*1 = *O*1*A*. It may be proved that the product *OA* × *OB* remains constant, when the link *O*1*A* rotates. Join *CD* to bisect *AB* at *R*. Now from right angled triangles *ORC* and *BRC*,

$$OC^2 = OR^2 + RC^2$$

and $$BC^2 = RB^2 + RC^2$$

Subtracting equation (*ii*) from (*i*), we have

$$OC^2 - BC^2 = OR^2 - RB^2$$

$$= (OR + RB)\,(OR - RB)$$

$$= OB \times OA$$

Since *OC* and *BC* are of constant length, therefore the product *OB* × *OA* remains constant. Hence the point *B* traces a straight path perpendicular to the diameter *OP*.

WATT'S MECHANISM: The approximate straight line motion mechanisms are the modifications of the four-bar chain mechanisms. Following mechanisms to give approximate straight line motion. It is a crossed four bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximate straight line motion.

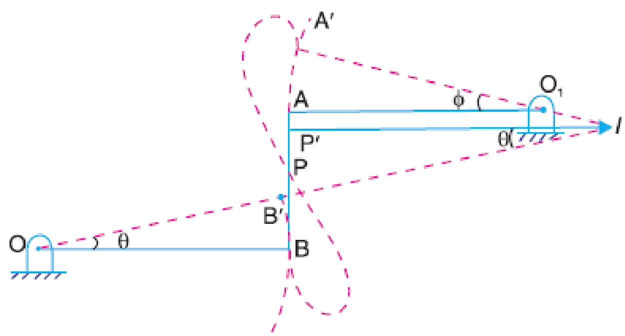


Fig. 30 Watt's Mechanism

QUESTIONS

1. Explain the term kinematic link. Give the classification of kinematic link.
2. What is a machine ? Giving example, differentiate between a machine and a structure.
3. Write notes on complete and incomplete constraints in lower and higher pairs, illustrating your answer with neat sketches.
4. Explain Grubler's criterion for determining degree of freedom for mechanisms.
5. Explain the terms : 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion.
6. In what way a mechanism differ from a machine ?
7. What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism?
8. Explain different kinds of kinematic pairs giving example for each one of them.
9. Sketch and explain the various inversions of a four bar chain mechanism?
10. Sketch and explain the various inversions of a slider crank chain
11. Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.
12. Sketch and explain any two inversions of a double slider crank chain.
13. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

UNIT I KINEMATICS OF CAM MECHANISMS

INTRODUCTION

A cam is a mechanical device used to transmit motion to a follower by direct contact. The driver is called the cam and the driven member is called the follower. In a cam follower pair, the cam normally rotates while the follower may translate or oscillate. A familiar example is the cam shaft of an automobile engine, where the cams drive the push rods (the followers) to open And close the valves in synchronization with the motion of the pistons.

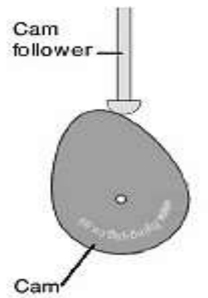


Fig. 31 Cam

Type of cams, Type of followers, Displacement, Velocity and acceleration time curves for cam profiles, Disc cam with reciprocating follower having knife edge, roller follower, Follower motions including SHM, Uniform velocity, Uniform acceleration and retardation and Cycloidal motion.

Cams are used to convert rotary motion into reciprocating motion. The motion created can be simple and regular or complex and irregular. As the cam turns, driven by the circular motion, the cam follower traces the surface of the cam transmitting its motion to the required mechanism. Cam follower design is important in the way the profile of the cam is followed. A fine pointed follower will more accurately trace the outline of the cam. This more accurate movement is at the expense of the strength of the cam follower.

Types of cams: Cams can be classified based on their physical shape.

a) **Disk or plate cam** The disk (or plate) cam has an irregular contour to impart a specific motion to the follower. The follower moves in a plane perpendicular to the axis ofrotation of the cam shaft and is held in contact with the cam by springs or gravity.

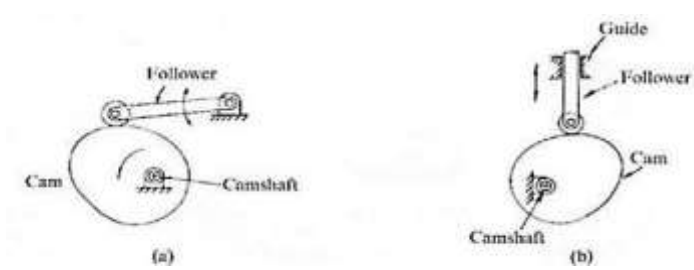


Fig 32 Plate or disk cam.

b) **Cylindrical cam:**The cylindrical cam has a groove cut along its cylindrical surface.The roller follows the groove, and the follower moves in a plane parallel to the axis of rotation of the cylinder.

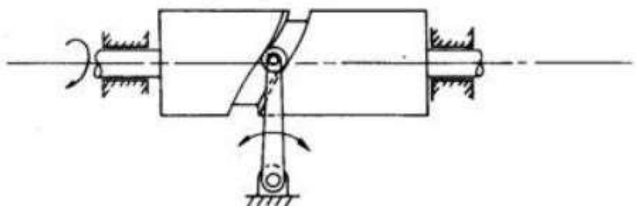


Fig. 33 Cylindrical cam.

c)Translating cam. The translating cam is a contoured or grooved plate sliding on a guiding surface(s).The follower may oscillate (Fig.3 a)or reciprocate (Fig. 3b).The contour or the shape of the groove is determined by the specified motion of the follower.

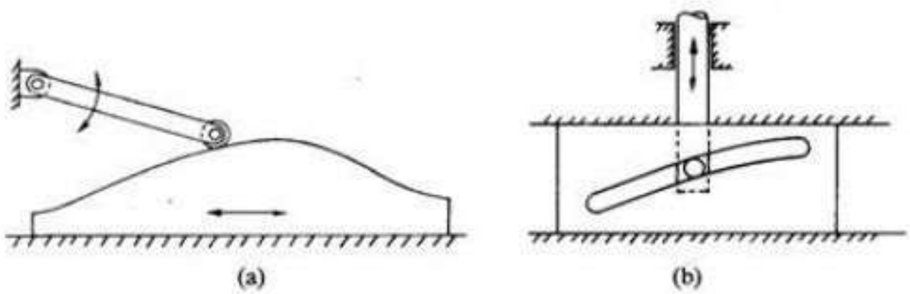


Fig. 34 Translating cam

Types of followers

(i) Basedonsurfaceincontact.

(a) Knifeedgefollower

(b)Rollerfollower

(c)Flatfacedfollower

(d)Sphericalfollower

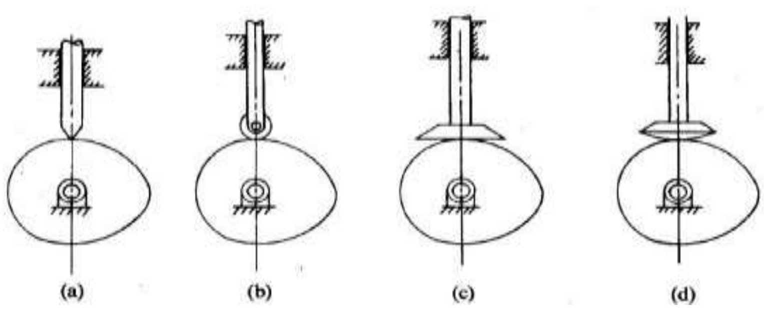


Fig. 35 Types of followers

(ii) Based on type of motion

a)Oscillating follower (b)Translating follower

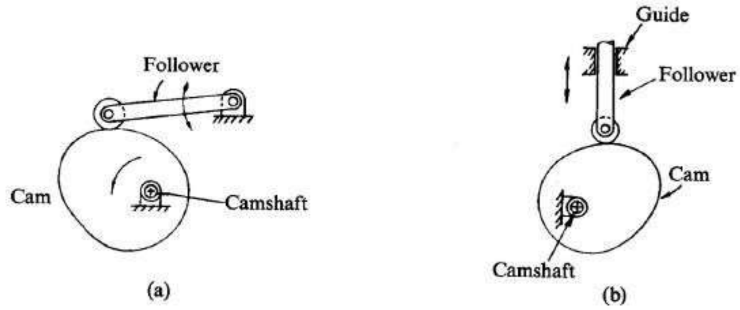


Fig. 36 Motion Type of followers

(ii) Based on line of motion

(a) Radial follower:Thelines of movement of in-line cam followers pass through the centers of the camshafts

(b) Off-set follower:For this type,the lines of movement are off set from the centers of the Cam shafts

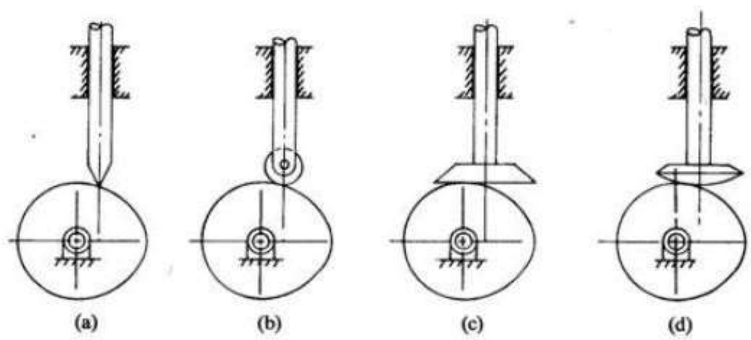


Fig. 37 Line of motion of followers

NOMENCLATURE OF CAMS

Cam Profile Thecontouroftheworkingsurfaceofthecam.

Trace Point The point at the knife edge of a follower, or the center of a roller, or the center of a spherical face.

Pitch Curve The path of the tracer point.

Base Circle The smallest circle drawn, tangential to the cam profile, with its center on the axis of the cam shaft.The size of the base circle determines the size of the cam.

Prime Circle The smallest circle drawn, tha can be drawn from the center of the cam and tangent to the pitch curve.

Prime circle radius = Base circle radius for knife edge and flat faced follower

Prime circle radius = Base ciircle radius + radius of roller for roller follower

Pressure Angle The angle between the normal to the pitch curve and the direction of Motion of the follower at the point of contact

Lift of stroke: It is the maximum travel of the follower from its lowest position to the topmost position. The maximum rise is called lift

Pitch Point: It is a point on the curve having maximum pressure angle

Pitch Circle: It is the circle drawn from the center of the cam through the pitch points

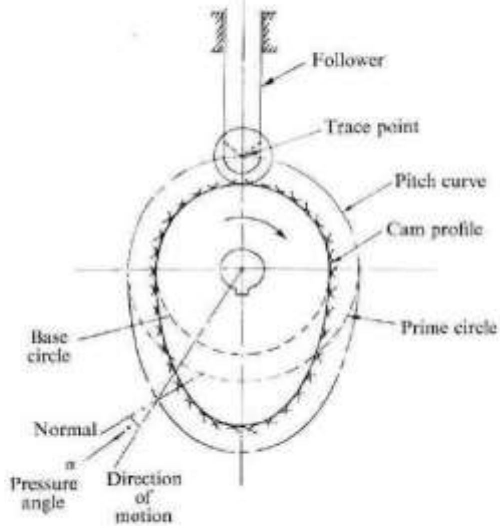


Fig. 38 Cam nomenclature

Types of follower motion:

Cam follower systems are designed to achieve a desired oscillatory motion. Appropriate displacement patterns are to be selected for this purpose, before designing the cam surface. The cam is assumed to rotate at a constant speed and the follower raises, dwells, returns to its original position and dwells again through specified angles of rotation of the cam, during each revolution of the cam. Some of the standard follower motions are as follows:

They are, follower motion with,

(a)Uniform velocity

(b)Modified uniform velocity

(c)Uniform acceleration and deceleration

(d)Simple harmonic motion

(e)Cycloidal motion

Displacement diagrams:

In a cam follower system, the motion of the follower is very important. Its displacement can be plotted against the angular displacement θ of the cam and it is called as the displacement diagram. The displacement of the follower is plotted along they-axis and angular displacement θ of the cam is plotted along x-axis. From the displacement diagram, velocity y and acceleration of the follower can also be plotted for different angular displacements θ of the cam. The displacement, velocity and acceleration diagrams are plotted for one cycle of operation i.e., one rotation of the cam. Displacement diagrams are basic requirements for the construction of cam profiles. Construction of displacement diagrams and calculation of velocities and accelerations of followers with different types of motions are discussed in the following sections.

(a)Follower motion with Uniform velocity:

Fig.3.8shows the displacement, velocity and acceleration patterns of a follower having uniform velocity

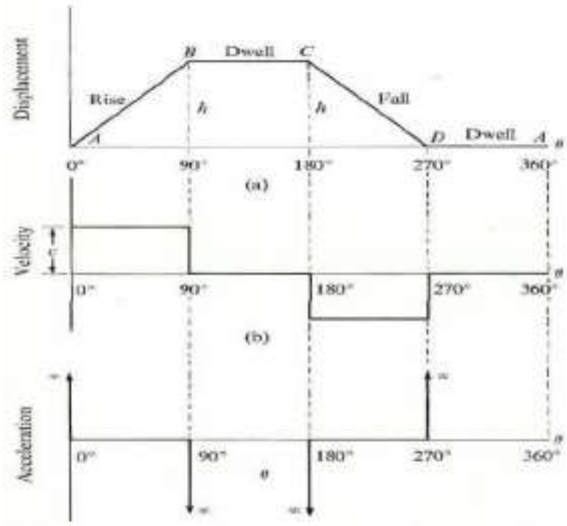


Fig. 39 Displacement, velocity and acceleration patterns of a follower

type of motion. Since the follower moves with constant velocity, during rise and fall, the displacement varies linearly with θ.Also,since the velocity changes from zero to a finite value, with in no time, theoretically,the acceleration be comes in finite at the beginning and end of rise and fall.

Follower motion with modified uniform velocity:

Itis observedin thedisplacement diagrams ofthefollower with uniform velocity that the acceleration nof the follower becomes in finit eat the beginning and ending of rise and return strokes. Inorder to prevent this,the displacement diagrams are slightly modified.In the modified form,thevelocity of the follower changes uniformly during the beginning and end of each stroke. Accordingly,the displacemen to the follower varies parabolically during the seperiods. With this modification, the acceleration becomes constant during the seperiods, instead of being infinite a sin the uniform velocity type of motion.The displacement, velocity and acceleration patterns shown in fig 9

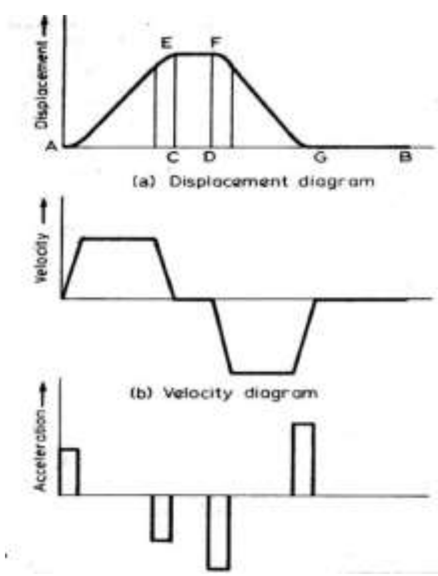


Fig. 40 Displacement, velocity and acceleration patterns

b)Simple Harmonic Motion: In fig 10, the motion executed by point Pl, which is the projection of point P on the vertical diameter is called simple harmonic motion. Here, P moves with uniform angular velocity ωp, along a circle of radius r (r=s/2).

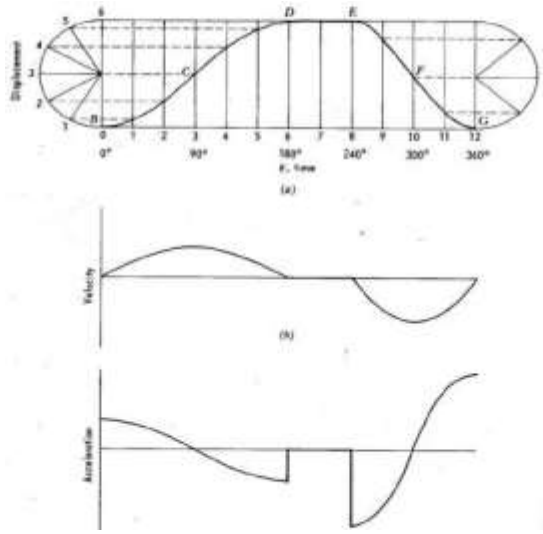
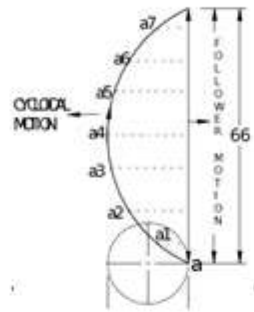


Fig. 41 Displacement, velocity and acceleration patterns

(c) Cycloidal motion: Cycloid is the path generated by a point on the circumference of a circle, as the circle rolls without slipping, on a straight/flat surface.The motion executed by the follower here, is similar to that of the projection of a point moving along a cyloidal curve on a vertical lineas shown in figure. 11



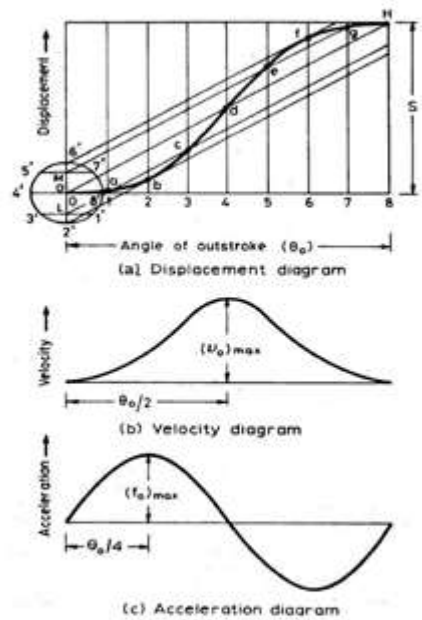


Fig. 42 Displacement, velocity and acceleration patterns

Draw the cam profile for following conditions:

Follower type = Knife edged, in-line; lift = 50mm; base circle radius = 50mm; outstroke with SHM,for 60°cam rotation; dwell for 45°cam rotation; return stroke with SHM, for 90° cam rotation; dwell for the remaining period. (2) Draw the cam profile for the same operating condition so f with the follower offset by 10mm to the left of cam center**.**

Displacement diagram:

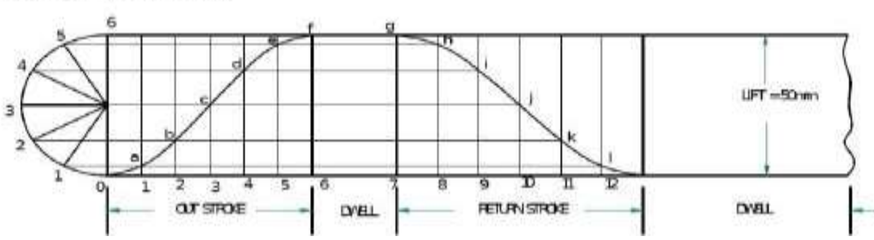
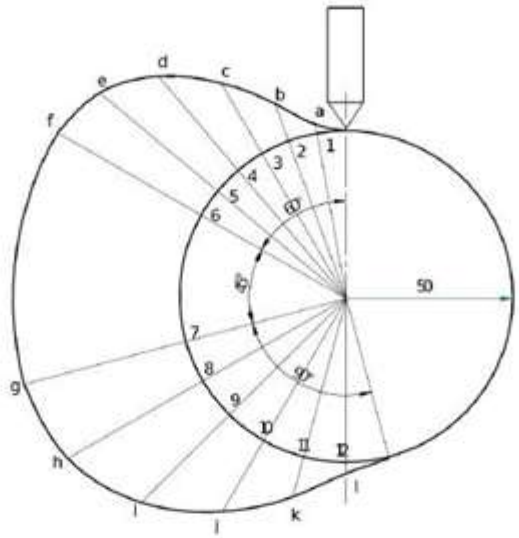


Fig 43 Displacement Diagram

Cam profile:



Cam profile with 10 mm

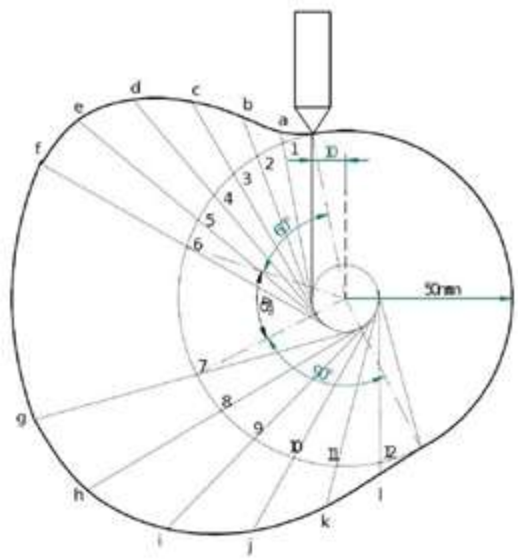


Fig. 44 Cam Profile

Draw the cam profile for following conditions:

Follower type=roller follower, in-line; lift=25mm; base circle radius = 20mm; roller radius = 5mm; out stroke with Uniform acceleration and retardation, for 120° cam rotation;dwell for 60° cam rotation; return stroke with Uniform acceleration and retardation,for 90° cam rotation; dwell for the remaining period. Draw the cam profile for conditions same with follower off set to right of cam center by 5mm and cam rotating counter clockwise.

Displacement Diagram:

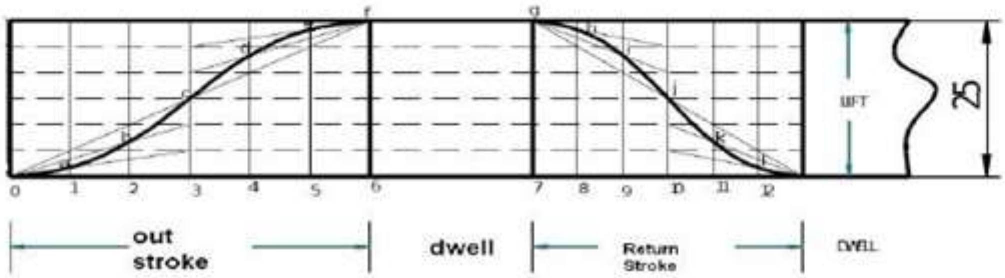
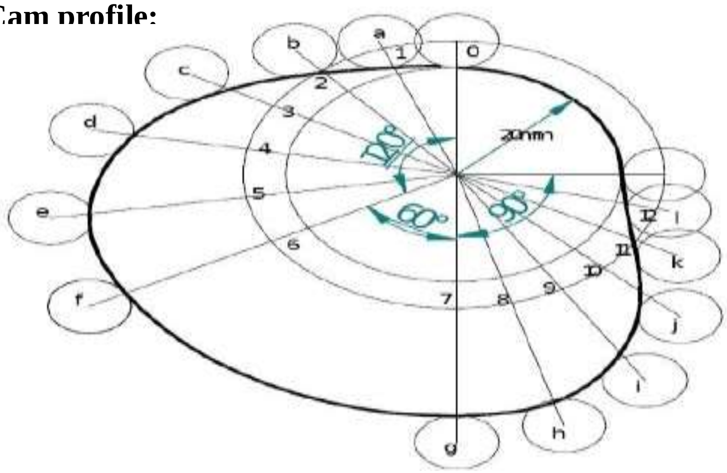


Fig. 45. Displacement diagram

Cam profile:



Cam profile with 5 mm offset

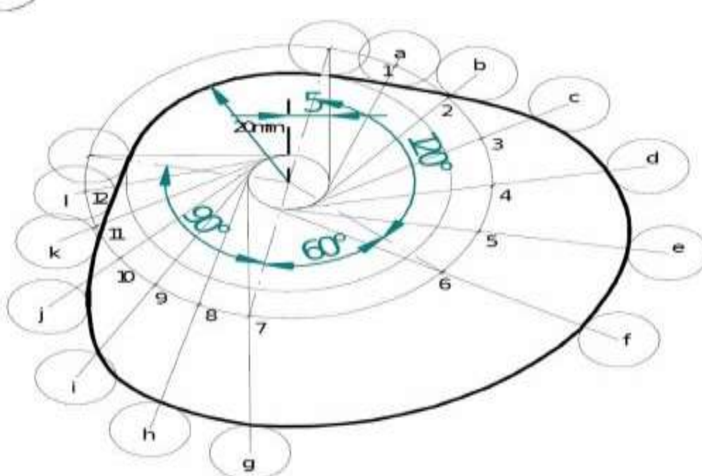


Fig. 46 Cam Profile

Draw the cam profile for following conditions:

Follower type=knife edge d follower, in line; lift=30mm;base circle radius =20mm;outstroke with uniform velocity in 120°of cam rotation; dwell for 60°; return stroke with uniform velocity, during 90°of cam rotation; dwell for the remaining period.

Displacement Diagram

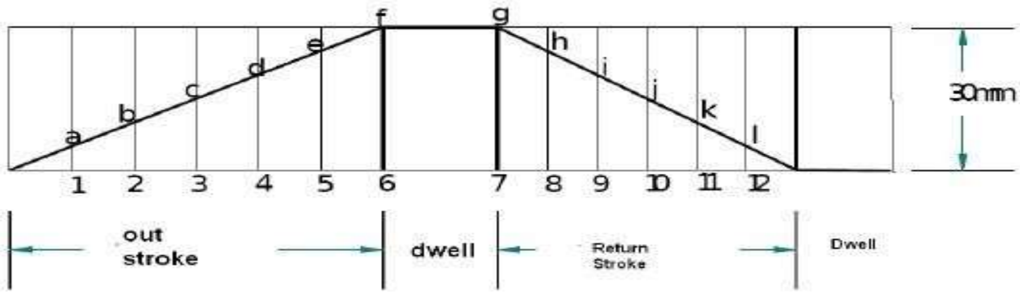


Fig. 47 Displacement Diagram

Cam profile

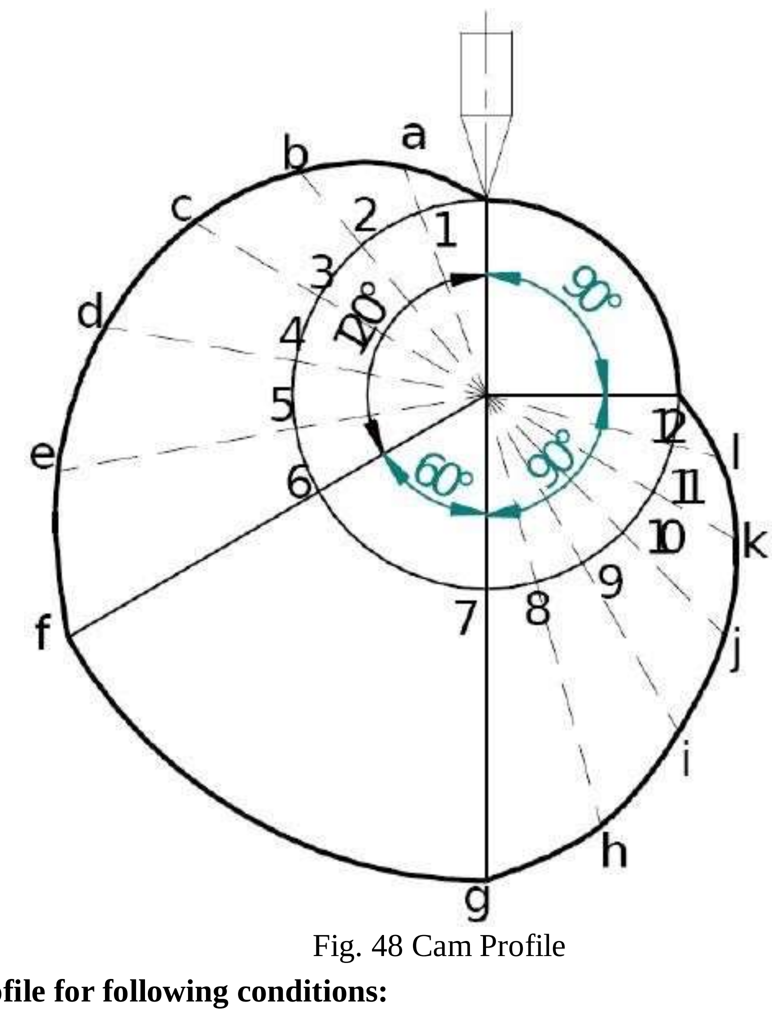
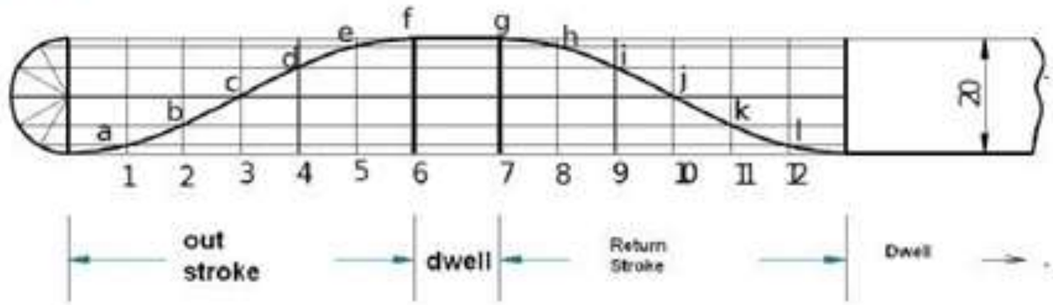


Fig. 48 Cam Profile

Draw the cam profile for following conditions:

Follower type = flat faced follower, inline; follower rises by 20mm with SHM in 120° of cam rotation, dwells for 30° of cam rotation; returns with SHM in 120° of cam rotation and dwells during the remaining period. Base circle radius =25mm.

Displacement Diagram:



Cam profile

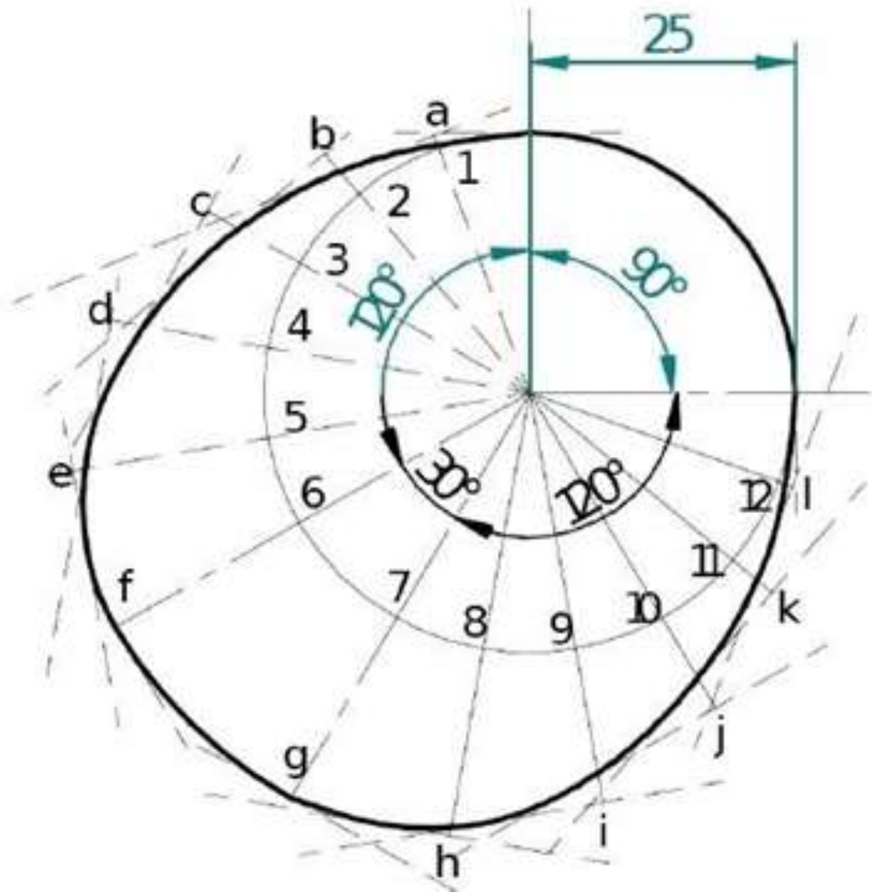


Fig.49 Displacement diagram and Cam profile

Layout of plate cam profiles:

- Drawing the displacement diagrams for the different kinds of the motions and the plate cam profiles for these different motions and different followers.
- SHM, Uniform velocity, Uniform acceleration and retardation and Cycloidal motions
- Knife-edge, Roller, Flat-faced and Mushroom followers.

Derivatives of Follower motion:

- Velocity and acceleration of the followers for various types of motions.
- Calculation of Velocity and acceleration of the followers for various types of motions.

Circular arc and Tangent cams:

- Circular arc
- Tangent cam

Standard cam motion:

- Simple Harmonic Motion
- Uniform velocity motion
- Uniform acceleration and retardation motion
- Cycloidal motion

Pressure angle and undercutting:

- Pressure angle
- Undercutting

A cam, with a minimum radius of 25 mm, rotating clockwise at a uniform speed is to be designed to give a roller follower, at the end of a valve rod, motion described below:

1. To raise the valve through 50 mm during 120º rotation of the cam:
2. To keep the valve fully raised though next 30º.
3. To lower the valve during next 60º and
4. To keep the valve closed during rest of the revolution i.e. 150º

The diameter of the roller is 20 mm and the diameter of the cam shaft is 25 mm. Draw the profile of the cam when

1. The line of stroke of the valve rod passes through the axis of the cam, and
2. The line of stroke is offset 1.5 mm from the axis of the cam shaft.

The displacement of the valve, while being raised and lowered, is to take place with simple harmonic moton. Determine the maximum accelaration of the valve rod when the cam shaft rotated at 100 r.p.m. Draw the displacemnet, velocity and the accelaration diagrams for one complete revolution of the cam.
Solution: Given: S = 50 mm = 0.05 m; θ_O = 120º 2×π/3 rad = 2.1 rad; θ_R = 60º = π/3 rad =1.047 rad; N = 100 r.p.m

Profile of the cam when the line of stroke of the valve rod passes through the axis of the cam shaft

The profile of the cam, as shown in Fig, is drawn as discussed in the following steps:

a. Draw a base circle with center O and radius equal to the minimum radius of the cam (i.e. 25 mm)
b. Draw a prime circle with center O and radius,
c. OA = Minimum radius of cam + $\frac{1}{2}$ Diameter of roller = 25 + $\frac{1}{2}$ × 20 = 35 mm
d. Draw angle AOS = 120º to represent raising or out stoke of the valve, angle SOT = 30º to represent dwell and angle TOP = 60º to represent lowering or return stroke of the valve.
e. Divide the angular displacements of the cam during raising and lowering of the valve (i.e, angle AOS and TOP) into same number of equal even parts as in displacement diagram.
f. Join the points 1, 2, 3, etc. with the centre O and produce the lines beyond prime circle as shown in Fig 22.
g. Set off 1B, 2C, 3D etc. equal to the displacements from displacement diagram.
h. Join the points A, B, C …. N, P, A. The curve drawn through these points is known as pitch curve.
i. From the points A, B, C …. N, P, draw circles of radius equal to the radius of the roller.
j. Join the bottom of the circles with a smooth curve as shown in fig 22. This is the required profile of the cam.

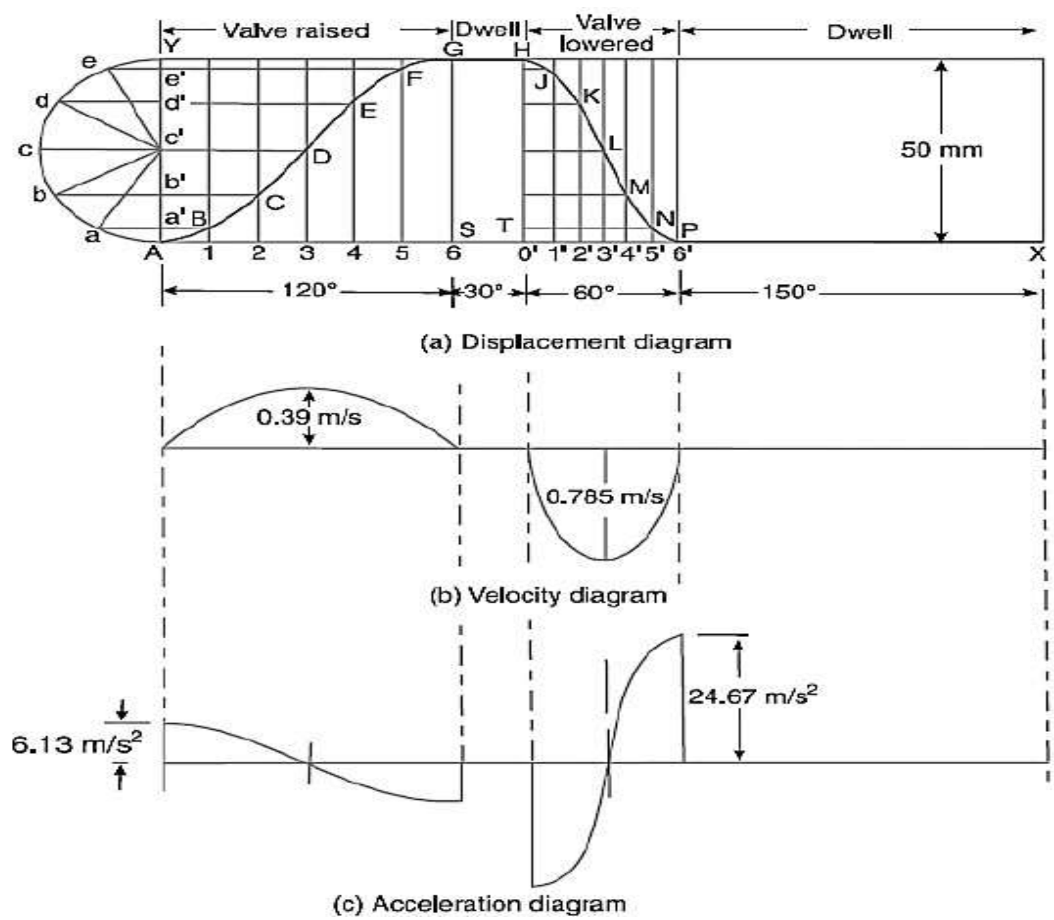


Fig. 50 Cam diagram inputs

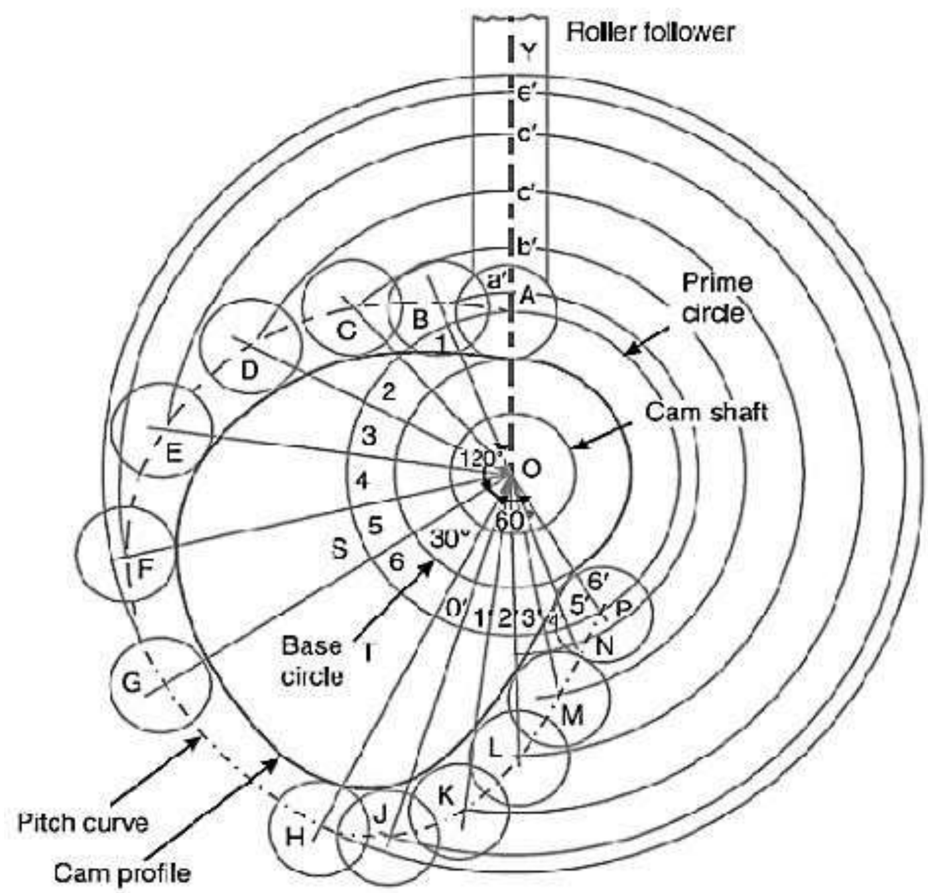


Fig. 51 Cam Profile

(a) profile of the cam when the line of stroke is offset 15 mm from the axis of the cam shaft:

The profile of the cam when the line of stroke is offset from the axis of the cam shaft, as shown in fig 24 may be drawn as discussed in the following steps:

a. Draw a base circle with center O and radius equal to 25 mm.
b. Draw a prime circle with center O and radius OA = 35 mm
c. Draw an offset circle with center O and radius equal to 15 mm.
d. Join OA. From OA draw the angular displacement of cam i.e. draw angle AOS = 120º. Angle SOT = 30º angle TOP = 60º
e. Divide the angular displacements of the cam during raising and lowering of the valve into the same number of equal even parts (i.e. six parts) as in displacement diagrams.
f. From points 1, 2, 3 …. Etc on the prime circle draw tangents to the offset circle.
g. Set off 1B, 2C, 3D … etc. equal to displacements as measured from the displacement diagram.
h. By joining the points A, B, C … M, N, P. with a smooth curve, we get pitch curve.
i. Now A, B. C, etc. as center draw circles with a radius equal to the radius of roller.
j. Join the bottoms of the circles with a smooth curve as shown in fig 24. This is the required profile of the cam.

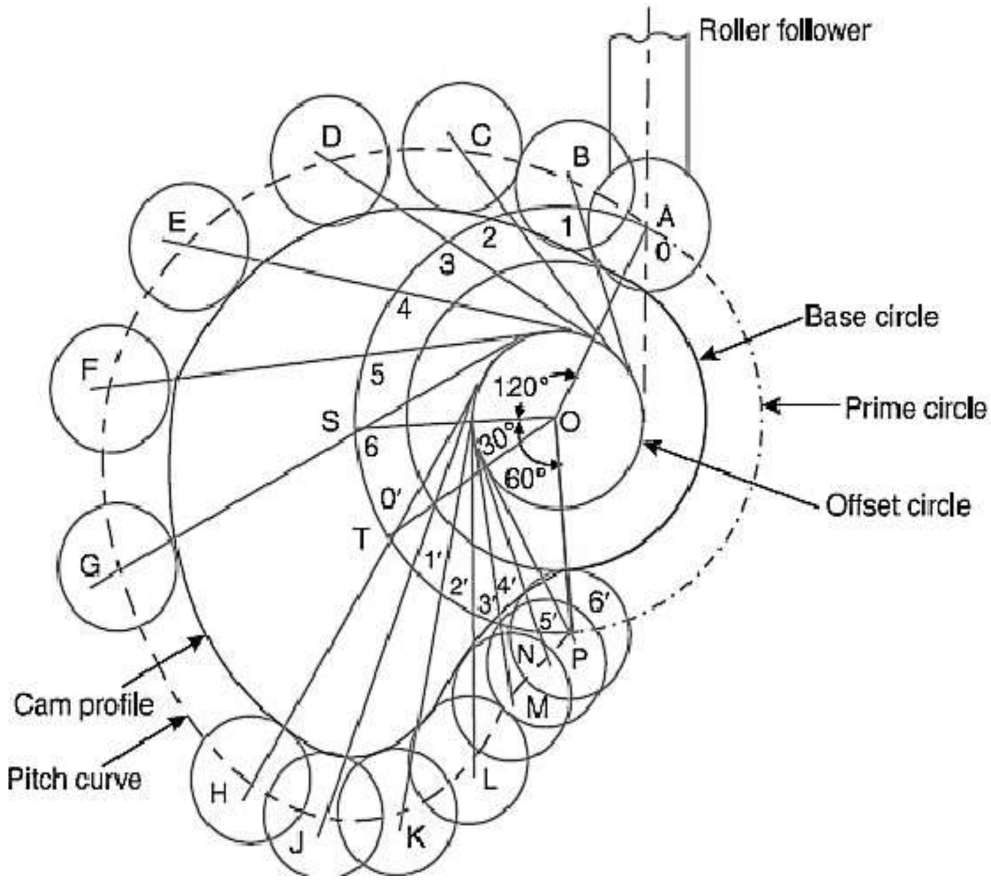


Fig. 52 Cam Profile

Maximum acceleration of the valve rod

We know that angular velocity of the cam shaft.

$\omega = 2\pi N/60 = 2\pi \times 100/60 = 10.47$ rad/s

We also know that maximum velocity of the valve rod to raise valve.

$V_O = \pi\ \omega S/2\theta_O = \pi \times 10.47 \times 0.05/2 \times 2.1 = 0.39$ m/s

And maximum velocity of the valve rod to lower the valve.

$V_R = \pi\ \omega\ S/2\theta_R = \pi \times 10.47 \times 0.05/2 \times 1.047 = 0.785$ m/s

The velocity diagram for one complete revolution of the cam is shown fig 24

We know that the maximum acceleration of the valve rod to raise the valve.

$a_{O\ =}\ \pi^2\ \omega^2\ S/2(\theta_O)^2 = \pi^2 \times (10.47)^2 \times 0.05/\ 2 \times (201)^2 = 0.785$ m/s^2

and maximum acceleration of the valve rod to lower the valve,

$a_{O\ =}\ \pi^2\ \omega^2\ S/2(\theta_R)^2 = \pi^2 \times (10.47)^2 \times 0.05/\ 2 \times (1.047)^2 = 0.785$ m/s^2

A cam is to give the following motion to a <u>KNIFE-EDGED FOLLOWER</u> :

1. Outstroke during 60° of cam rotation ; 2. Dwell for the next 30° of cam rotation ; 3. Return stroke during next 60° of cam rotation, and 4. Dwell for the remaining 210° of cam rotation. The stroke of the follower is 40 mm and the minimum radius of the cam is 50 mm. The follower moves with <u>UNIFORM VELOCITY</u> during both the outstroke and return strokes. Draw the profile of the cam when

(a) the axis of the follower passes through the axis of the cam shañ, and

(b) the axis of the follower is offset by 20 mm from the axis of the cam shañ.

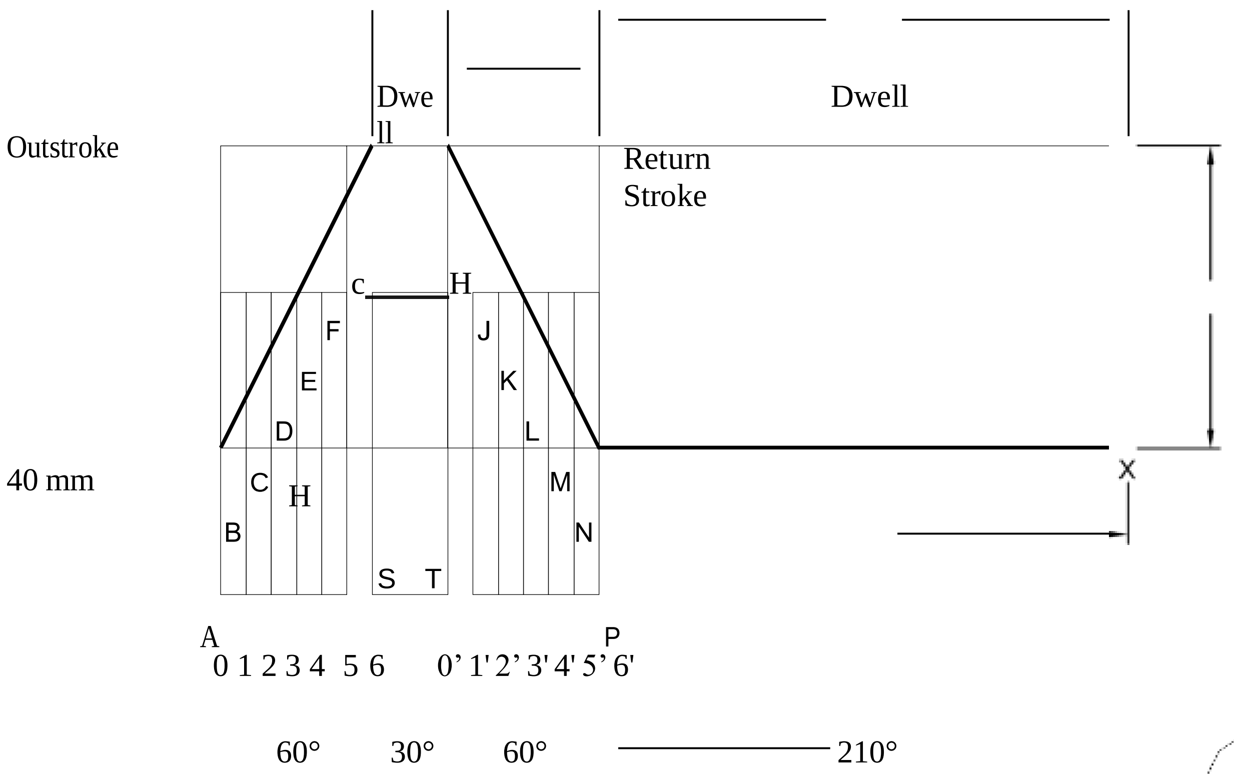
<u>Given:</u>

Outstroke Angle = 60° (20mm)
Dwell (Outstroke) Angle = 30° (10mm) Return Stroke angle = 60° (20mm)
Dwell (Return Stroke) Angle = 210° (70mm) Stroke = 40mm
Radius of CAM = 50mm
Offset = 20mm

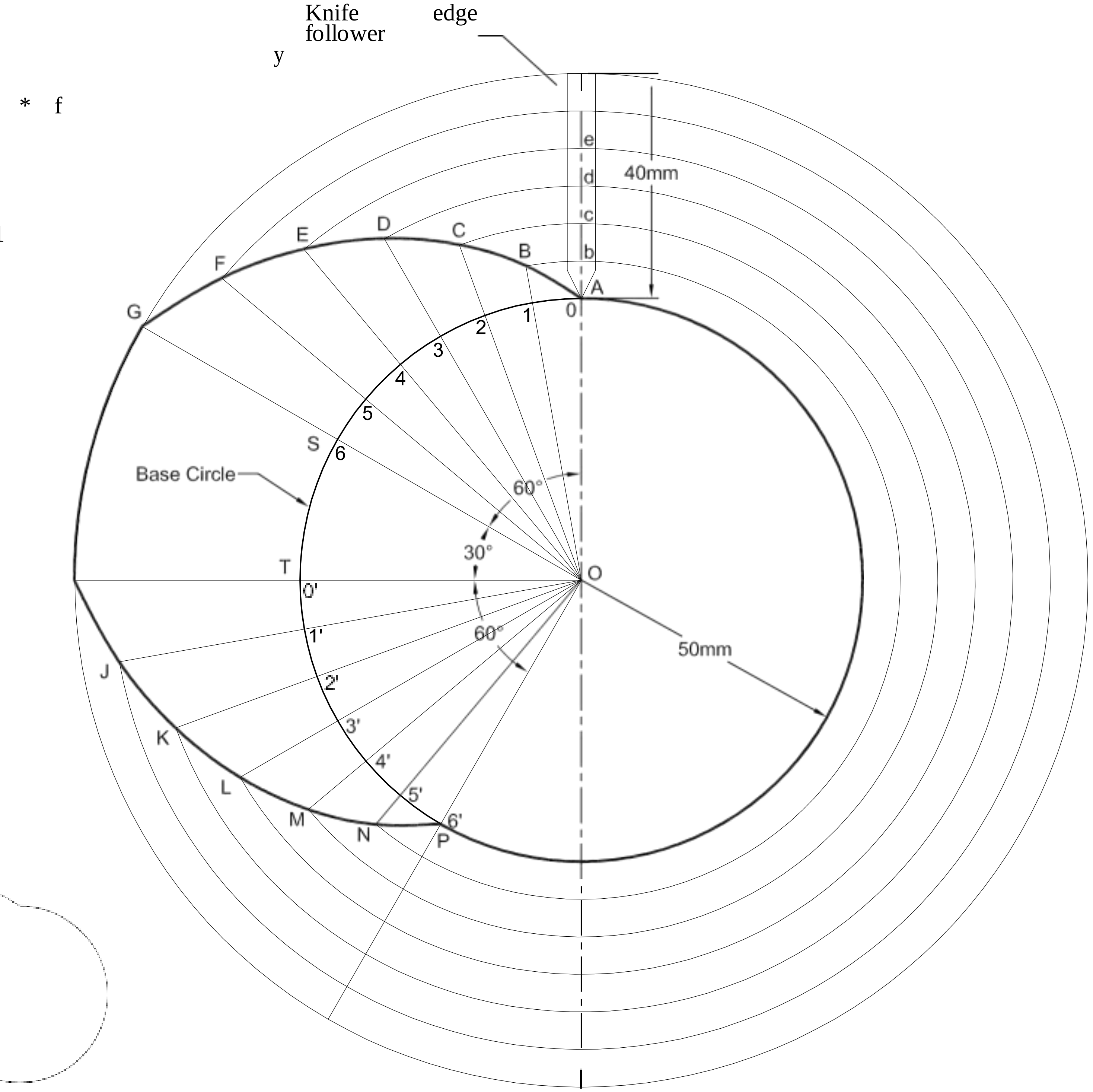
Assume

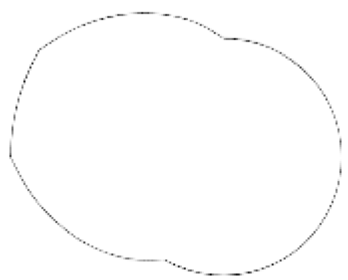
30° = 10mm
360°=120mm

Scale-1:1



<u>Space or Displacement Diagram</u>





Surface in Contact : Knife Edge

Path of Motion of Follower Radial

/Motion of the Follower Uniform Velocity

<u>CAM Profile</u>

<u>Profile of the cam when the axis of follower passes through the axis of cam shaft</u>

A cam is to be designed for a KNIFE EDGE FOLLOWER with the following data :

1. Cam lift = 40 mm during 90° of cam rotation with simple harmonic motion. 2. Dwell for the next 30°.
3. During the next 60° of cam rotation, the follower returns to its original position with SIMPLE HARMONIC MOTION. 4. Dwell during the remaining 180°.
Draw the profile of the cam when

(a) the line of stroke of the follower passes through the axis of the cam shañ, and

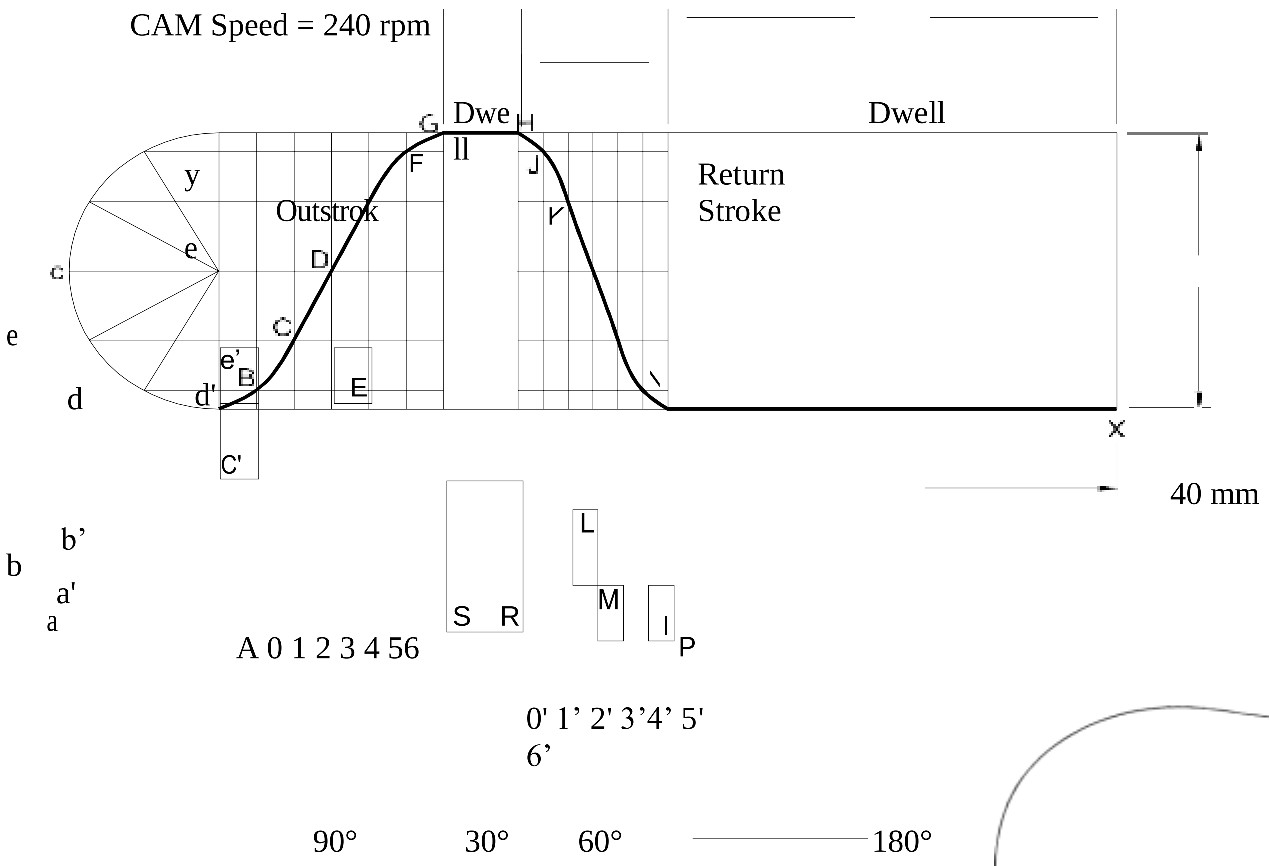
(b) the line of stroke is offset 20 mm from the axis of the cam shaft. The radius of the base circle of the cam is 40 mm. Determine the maximum velocity and acceleration of the follower during its ascent and descent, if the cam rotates at 240 r.p.m.

Given:

Outstroke Angle = 90° (30mm)
Dwell (Outstroke) Angle = 30° (10mm) Return Stroke angle = 60° (20mm)
Dwell (Return Stroke) Angle = 180° (60mm) Stroke = 40mm
Radius of CAM = 40mm
Offset = 20mm
CAM Speed = 240 rpm

Assume
30° = 10mm
360° = 120mm

Scale -1:1

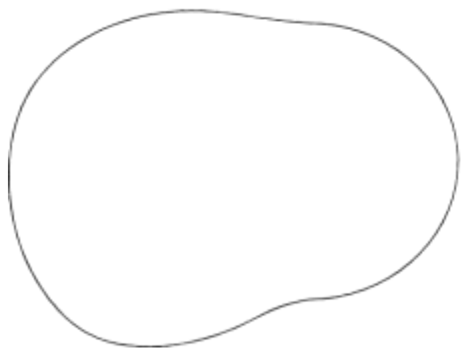


Space or Displacement Diagram

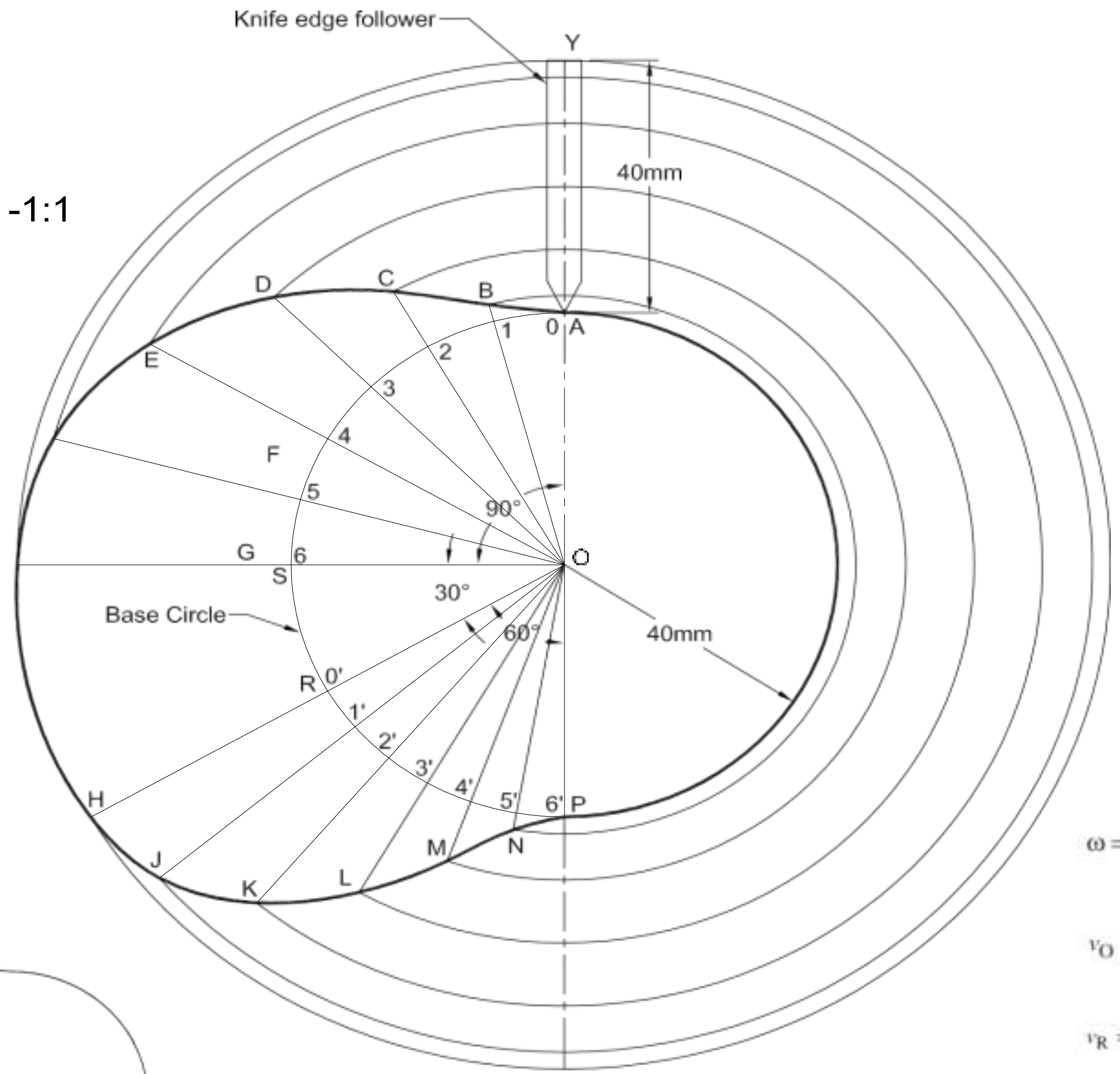
Surface in Contact : Knife Edge Path of Motion of

Follower Radial

Motion of the Follower Simple Harmonic



CAM Profile



Profile of the cam when the line of stroke of the follower passes through the axis of the camshaft

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.14 \text{ rad/s}$$

$$v_O = \frac{\pi \omega . S}{2\theta_O} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.571} = 1 \text{ m/s}$$

$$v_R = \frac{\pi \omega . S}{2\theta_R} = \frac{\pi \times 25.14 \times 0.04}{2 \times 1.047} = 1.51 \text{ m/s}$$

$$a_O = \frac{\pi^2 \omega^2 . S}{2(\theta_O)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.571)^2} = 50.6 \text{ m/s}^2$$

$$a_R = \frac{\pi^2 \omega^2 . S}{2(\theta_R)^2} = \frac{\pi^2 (25.14)^2 0.04}{2(1.047)^2} = 113.8 \text{ m/s}^2$$

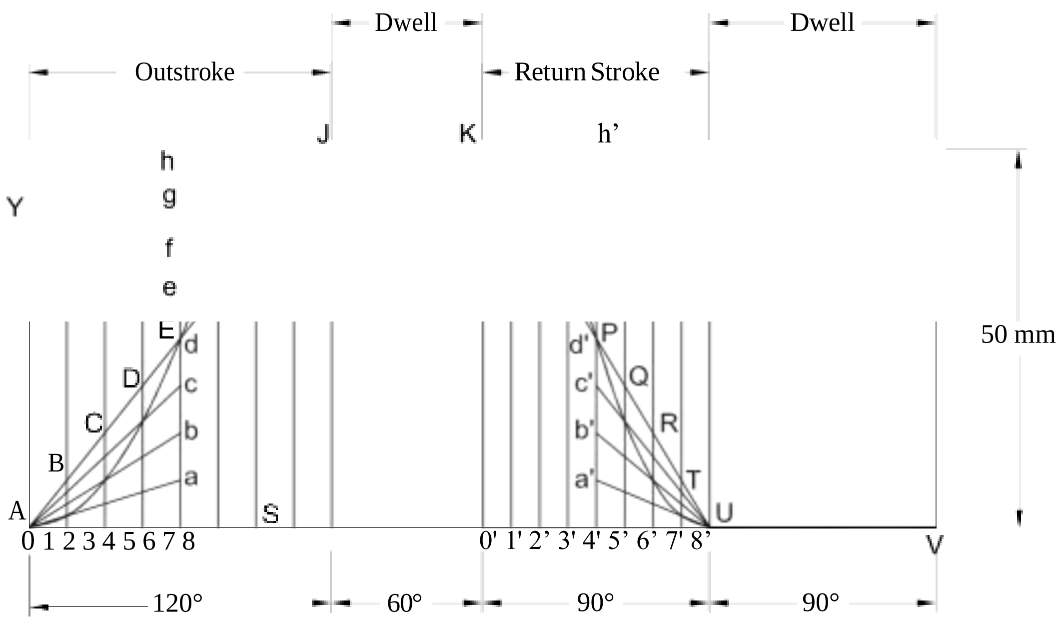
A cam rotating clockwise at a uniform speed of 1000 r.p.m. is required to give a <u>ROLLER FOLLOWER</u> the motion defined below :
1. Follower to move outwards through 50 mm during 120° of cam rotation, 2. Follower to dwell for next 60° of cam rotation,
3. Follower to return to its starting position during next 90° of cam rotation, 4. Follower to dwell for the rest of the cam rotation.
The minimum radius of the cam is 50 mm and the diameter of roller is 10 mm. The line of stroke of the follower is off-set by 20 mm from the axis of the cam shaft. If the displacement of the follower takes place with <u>UNIFORM AND EQUAL ACCELERATION AND RETARDATION</u> on both the outward and return strokes, draw profile of the cam and find the maximum velocity and acceleration during out stroke and return stroke.

<u>Given:</u>

Outstroke Angle = 120° (40mm)
Dwell (Outstroke) Angle = 60° (20mm)
Return Stroke angle = 90° (30mm)
Dwell (Return Stroke) Angle = 90° (30mm)
Stroke = 50mm
Radius of CAM = 50mm
Offset = 20mm
Roller Dia = 10mm Speed = 1000 rpm

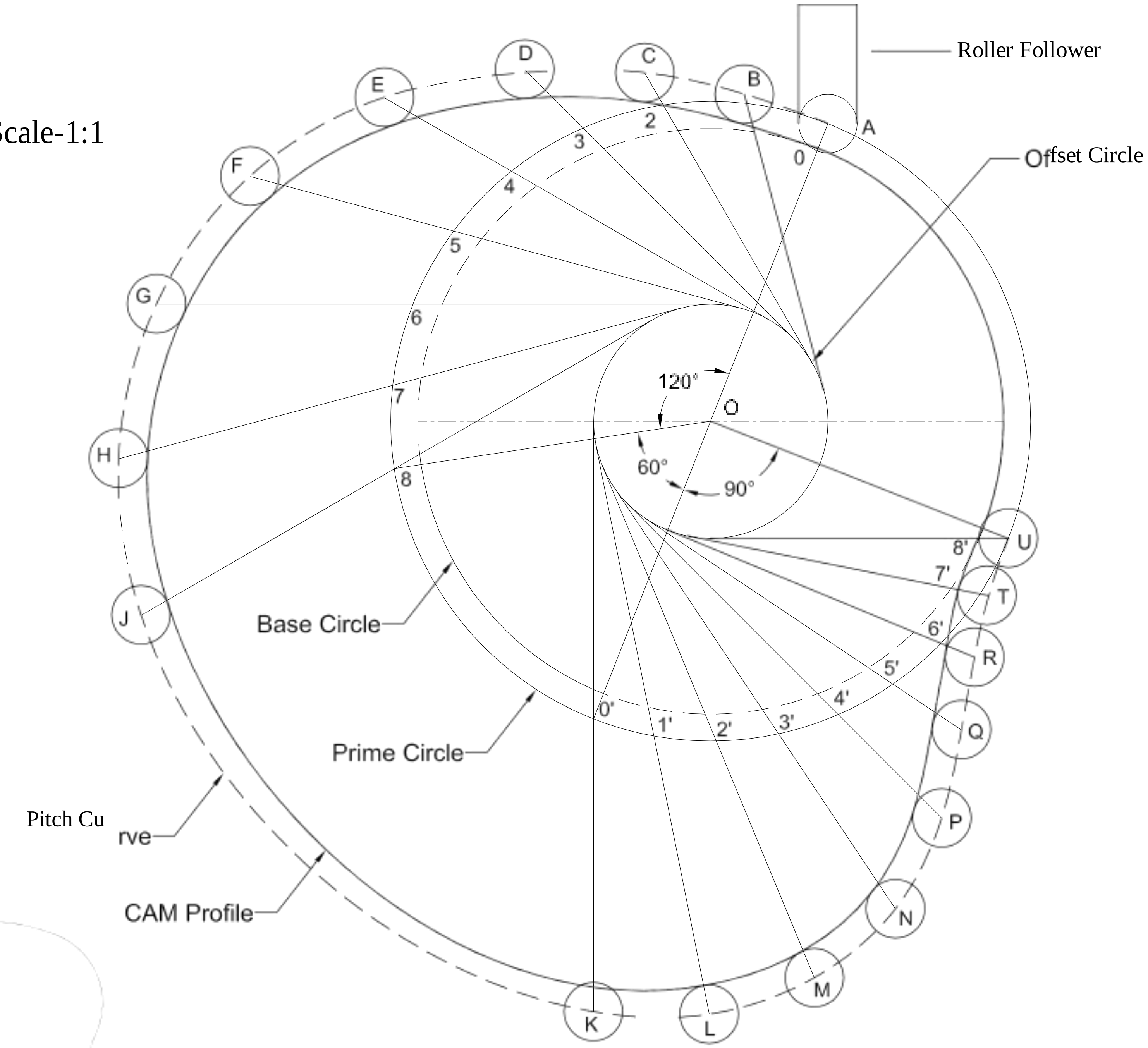
Assume

30° = 10mm
360° = 120mm



<u>Space or Displacement Diagram</u>

Scale-1:1



Surface in Contact : Roller

Path of Motion of Follower : Ohset

Motion of the Follower **Uniform Acceleration & Retardation**

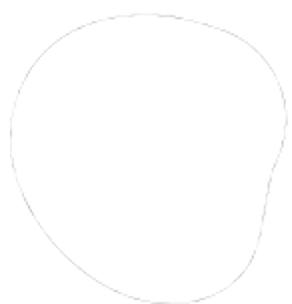
$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 1000}{60} = 104.7 \text{ rad/s.}$$

$$v_O = \frac{2\omega S}{\theta_O} = \frac{2 \times 104.7 \times 0.05}{2.1} = 5 \text{ m/s}$$

$$v_R = \frac{2\omega S}{\theta_R} = \frac{2 \times 104.7 \times 0.05}{1.571} = 6.66 \text{ m/s}$$

$$a_O = \frac{4\omega^2 . S}{(\theta_O)^2} = \frac{4(104.7)^2 0.05}{(2.1)^2} = 497.2 \text{ m/s}^2$$

$$a_R = \frac{4\omega^2 . S}{(\theta_R)^2} = \frac{4(104.7)^2 0.05}{(1.571)^2} = 888 \text{ m/s}^2$$



<u>CAM Profile</u>

Draw the profile of the cam when the ROLLER FOLLOWER moves with CYCLOIDAL MOTION during out stroke and return stroke, as given below :
1. Out stroke with maximum displacement of 31.4 mm during 180° of cam rotation, 2. Return stroke for the next 150° of cam rotation,
3. Dwell for the remaining 30° of cam rotation.
The minimum radius of the cam is 15 mm and the roller diameter of the follower is 10 mm. The axis of the roller follower is offset by 10 mm towards right from the axis of cam sha2.

Given:

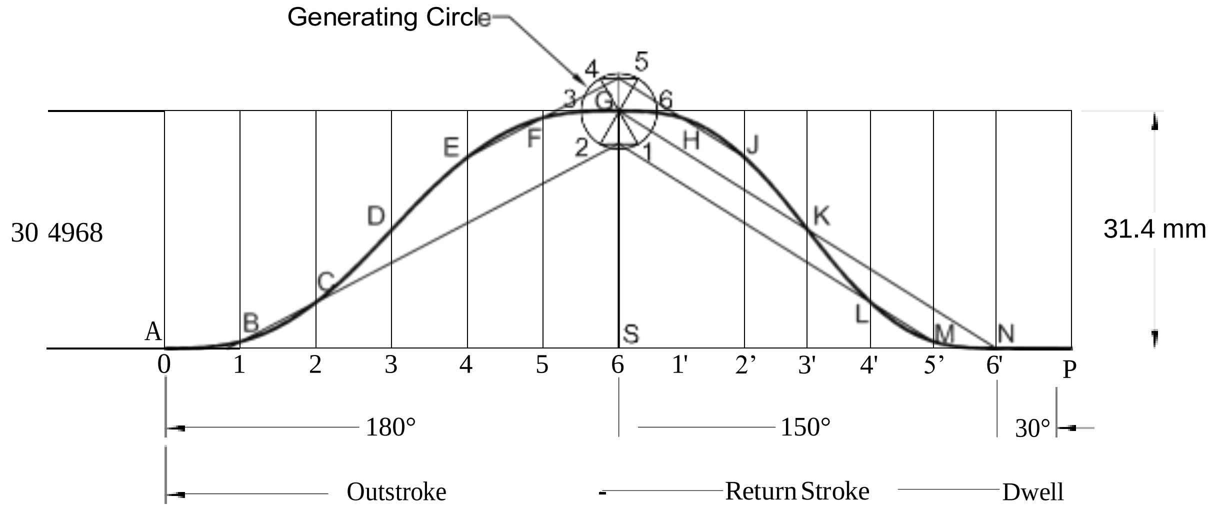
Outstroke Angle = 180° (60mm)
Return Stroke angle = 150° (50mm)
Dwell Angle = 30° (10mm)
Stroke = 31.4mm Radius
of CAM = 15mm Offset =
10mm
Roller Dia = 10mm

Assume

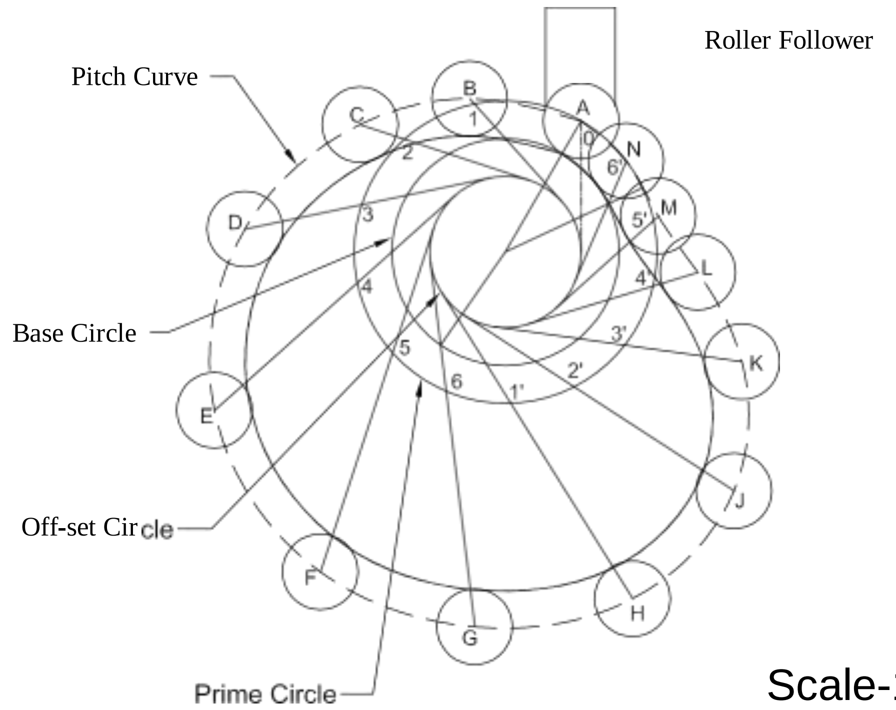
30° = 10mm
360° = 120mm

$$r = \frac{\text{Stroke}}{2\pi} = \frac{31.4}{2\pi} = 5 \text{ mm}$$

Generating **Circle**



Space or Displacement Diagram



Scale-1:1

Surface in Contact : Roller

Path of Motion of Follower : O€set

Motion of the Follower : Cycloidal

CAM Profile

QUESTIONS

14. Explain the term kinematic link. Give the classification of kinematic link.
15. What is a machine ? Giving example, differentiate between a machine and a structure.
16. Write notes on complete and incomplete constraints in lower and higher pairs, illustrating your answer with neat sketches.
17. Explain Grubler's criterion for determining degree of freedom for mechanisms.
18. Explain the terms : 1. Lower pair, 2. Higher pair, 3. Kinematic chain, and 4. Inversion.
19. In what way a mechanism differ from a machine ?
20. What is the significance of degrees of freedom of a kinematic chain when it functions as a mechanism?
21. Explain different kinds of kinematic pairs giving example for each one of them.
22. Sketch and explain the various inversions of a four bar chain mechanism?
23. Sketch and explain the various inversions of a slider crank chain
24. Sketch and describe the working of two different types of quick return mechanisms. Give examples of their applications. Derive an expression for the ratio of times taken in forward and return stroke for one of these mechanisms.
25. Sketch and explain any two inversions of a double slider crank chain.
26. In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the inclination of the slotted bar with the vertical in the extreme position and the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – II - FLY WHEELS AND BALANCING – SMEA1402

FLYWHEEL AND TURNING MOMENT DIAGRAMS

INTRODUCTION

Application of slider-crank mechanism can be found in reciprocating (steam) engines in the power plant i.e. internal combustion engines, generators to centrifugal pumps, etc. Output is non-uniform torque from crankshaft; accordingly there will be fluctuation is speed and subsequently in voltage generated in the generator that is objectionable or undesirable. Output torque at shaft is required to be uniform. Other kind of applications can be in punch press. It requires huge amount of power for small time interval. Remaining time of cycle it is ideal. Large motor that can supply huge quantity of energy for a small interval is required. Output power at piston is required to be non-uniform. These can be overcome by using flywheel at the crank- shaft. This will behave like a reservoir of energy. This will smoothen out the non-uniform output torque from crankshaft. Also it will store energy during the ideal time and redistribute during the deficit period.

Turning moment diagrams and fluctuations of the crank shaft speed:

A turning moment (crank torque) diagram for a four-stroke internal combustion engine is shown in Figure 1. The complete cycle is of 720º. From the static and inertia force analyses T $-\theta$ can be obtained (at interval of 15º or 5º preferably).

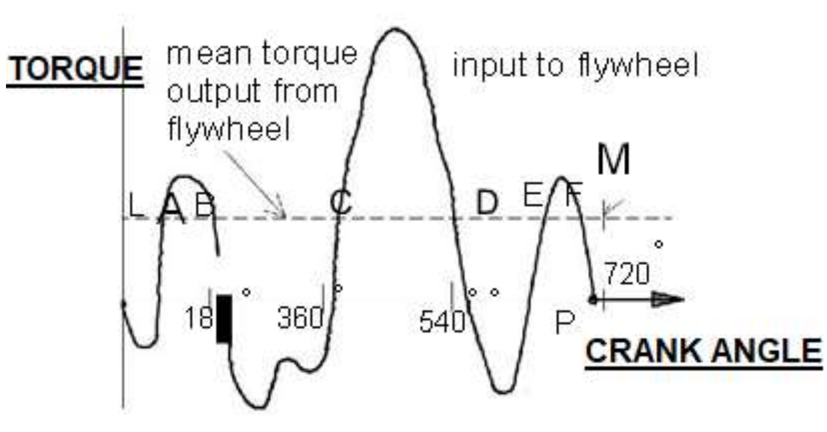


Figure 1 Turning moment diagram

Torque is negative in some interval of the crank angle, it means energy is supplied to engine during this period i.e. during the compression of the gas and to overcome inertia forces of engine members. This is supplied by the flywheel (and inertia of engine members), which is attached to the crankshaft. When flywheel is attached to the crankshaft. LM in diagram shown is the mean torque line. It is defined as

$$T_m = \frac{\sum_{i=1}^{N} i}{N} \tag{1}$$

If, $T_m = 0$ then no net energy in the system, $T_m \rangle 0$ then there is an excess of the net energy in the system and $T_m \langle 0$ then there is a deficit of the net energy in the system. Area OLMP = net (energy) area of turning moment diagram = $T_m (4\pi)$. During interval AB, CD and EF , the crank torque is more than the mean torque means hence excess of energy is supplied to crank i.e. it will accelerate (ω $\uparrow$). During other interval i.e LA, BC, DE and FM, the crank torque T is less than the mean torque T_m i.e. there is deficit in energy i.e. crank will decelerate ($\omega \downarrow$) .

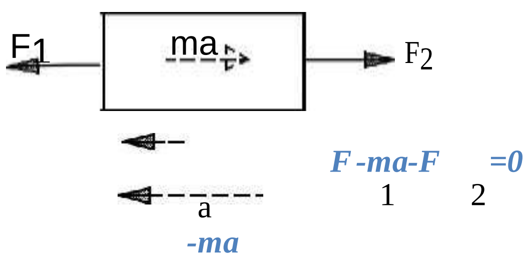


Figure 2 Linear acceleration of a body

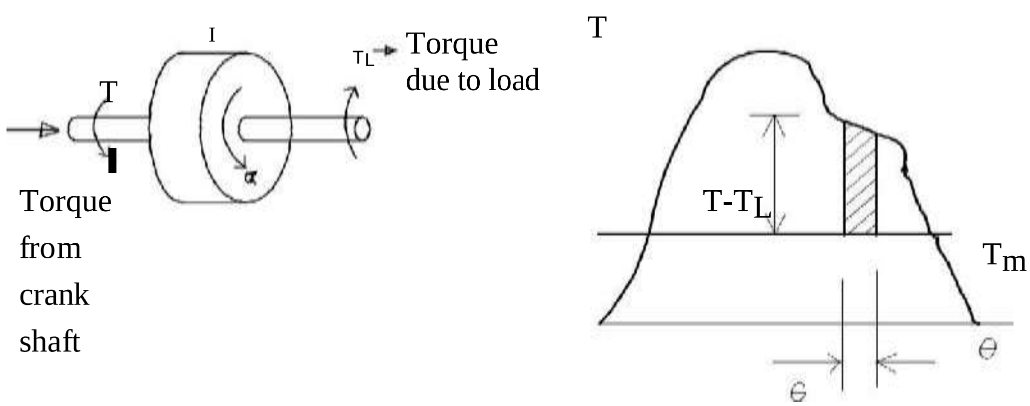


Figure 3 Angular acceleration of a body

Figure 4 T-θ diagram

From Newton’s second law, we have

$$\sum T = I\alpha$$

$$T - T_L = I\alpha \tag{2}$$

with

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta} \tag{3}$$

Substituting eqn. (3) in (2), we get

$$T - T_L = I\omega\frac{d\omega}{d\theta} \qquad \text{or} \qquad (T - T_L)d\theta = I\omega\, d\omega \tag{4}$$

Integrating θ from $\theta_{\omega=\omega_{min}}$ to $\theta_{\omega=\omega_{max}}$ and ω from ω_{min} to ω_{max}, we get

$$E = \int_{\theta_{\omega_{min}}}^{\theta_{\omega_{max}}} (T - T_L)d\theta = \int_{\omega_{min}}^{\omega_{max}} I\omega d\omega = \tfrac{1}{2} I\left(\omega_{max}^2 - \omega_{min}^2\right) \tag{5}$$

where E is the net area in T-θ diagram between θω max and θω min , and I is the polar mass moment of inertia. A plot of shaft torque versus crank angle θ shows a large variation in magnitude and sense of torque as shown in Figure 4. Since in same phases the torque is in the same sense as the crank motion and in other phases the torque is opposite to the crank motion. It would seem that the assumption of constant crank speed is invalid since a variation in torque would produce a variation in crank speed in the cycle. However, it is usual and necessary to fix a flywheel to the crankshaft and a flywheel of relatively small moment of inertia will reduce crank speed variations to negligibly small values (1 or 2% of the crank speed). We cannot change out put torque from the engine (it is fixed) but by putting flywheel we can regulate speed variation of crankshaft in cycle.

Our interest is to find maximum and minimum speeds and its positions in Figure 5. Points A, B, C, D, E and F are the points where T $-\theta$ diagram cuts the mean torque line. These points are transition points from deficit to extra energy or vice versa. So crank starts accelerate from deceleration from such points or vice versa. For example at points: A, C, E → accelerate and at B, D, F → decelerate.

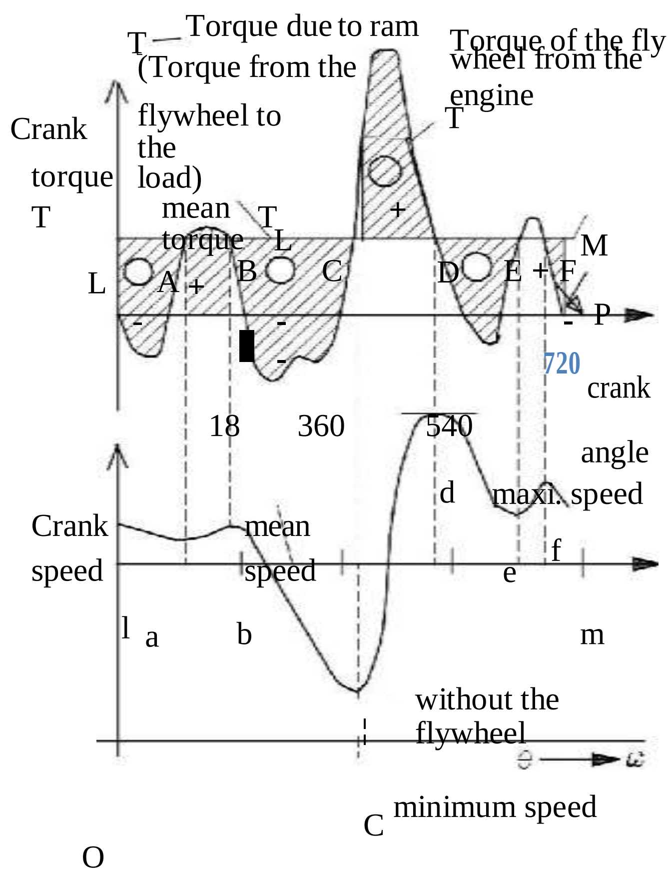


Figure 5 Fluctuation of the energy

At all such points have zero velocity slope i.e. having velocity maximum or minimum. Crank speed diagram can be drawn qualitatively (approximately) as shown in Figure 5, where c is the minimum speed location. Area of turning moment diagram represents energy for a particular period. Net energy between the maximum speed and the minimum speed instant is termed as fluctuation of energy. For this case area of diagram between C and D or between D and C through points E, F, M, L, A, B and C. Turning moment diagram for multi cylinder engine can be obtained by T $-\theta$ of individual engine by super imposing them in properphase. The coefficient of fluctuation of speed is defined as

$$\delta_s = \frac{\omega_{max} - \omega_{min}}{\omega} \qquad (6)$$

with

$$\omega = \frac{\omega_{max} + \omega_{min}}{2} \qquad (7)$$

where ω is the average speed. The fluctuation of energy, E, is represented by corresponding area in $T - \theta$ diagram as

$$=1/2I(\omega_{max} - \omega_{min})^2 = 1/2\, I(\omega_{max} + \omega_{min})\,(\omega_{max} - \omega_{min}) = I\{(\omega_{max} + \omega_{min})/2\}\{(\omega_{max} - \omega_{min})/\omega\}\omega$$

$$= I\omega\ \{(\omega_{max} - \omega_{min})/\omega\}\omega = I\omega^2\delta_s \qquad \underline{(8)}$$

By making I as large as possible, the fluctuation of speed can be reduced for the same fluctuation of energy.

For the disc type flywheel the diameter is constrained by the space and thinness of disc by stress

$$I = \frac{1}{2}Mr^2 \qquad \text{with} \qquad k = r/[\text{ Sq root of } 2] \qquad (9)$$

where r is the radius of the disc and k is the radius of gyration. For rim type flywheel diameter is restricted by centrifugal stresses at rim

$$I = Mr_m^2 \qquad \text{with} \qquad k = r_m \qquad (10)$$

Equation (9) or (10) gives the mass of rim. The mass of the hub and the arm also contribute by small amount to I, which in turn gives the fluctuation of speed slightly less than required. By experience equation (10) gives total mass of the flywheel with 90% of the rim & 10% for the hub and the arm. Typical values of the coefficient of fluctuation are δ_S = 0.002 to 0.006 for electric generators and 0.2 for centrifugal pumps for industrial applications.

Flywheel:

A rigid body rotating about a fixed point with an angular velocity ω (rad/s) and having mass moment of inertia I (kg-m^2) about the same point, the kinetic energy will be

$$T = \frac{1}{2} I\omega^2 \qquad (11)$$

For a flywheel having the maximum speed is ω_{max} and the minimum speed is ω_{min} the change in the kinetic enegy or fluctuation of energy $E = I(\omega_{max}^2 - \omega_{min}^2)/2$. Let V is the linear velocity of a point at a radius r from the center of rotation of flywheel E can be written as

$$E = 0.5Ir^2 (V_{max}^2 - V_{min}^2) \qquad (12)$$

Also coefficient of fluctuation can be written as

$$\delta_s = \frac{\omega_{max} - \omega_{min}}{\omega} = \frac{V_{max} - V_{min}}{V} \qquad (13)$$

with

$$V = \frac{V_{max} + V_{min}}{2} \qquad (14)$$

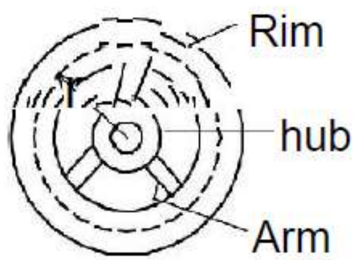


Figure 6 A rim type flywheel

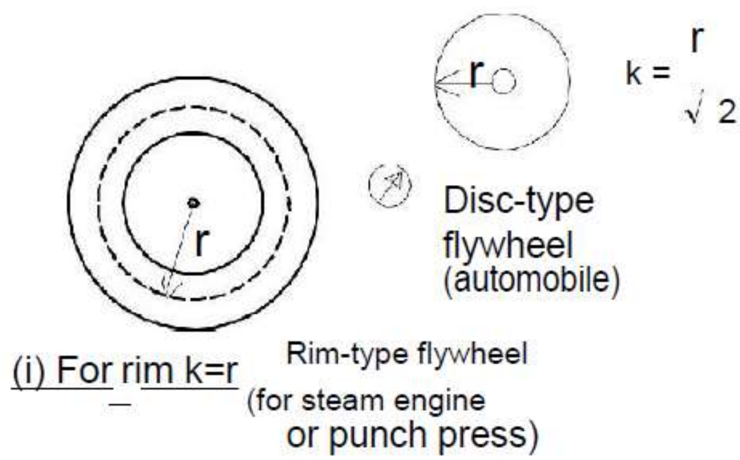


Figure 7 Polar mass moment of inertia of rim and disc type flywheel

Combining equations (13) and (14) with equation (12), we get

Figure 6 A rim type flywheel

or punch press)

Figure 7 Polar mass moment of inertia of rim and disc type flywheel

Combining equations (13) and (14) with equation (12), we get

$$E = I\delta^{2}\omega^{2} = \frac{I\delta_s V^2}{r^2} \qquad \text{with } I = mk^2 \tag{15}$$

Equation (15) becomes

$$E = m\delta_s k^2\omega^2 = m\frac{\delta_s k^2 V^2}{r^2} \tag{16}$$

Mass of flywheel (or polar mass moment of inertia) can be obtained as

$$\boxed{M = \frac{E}{\delta_s k^2\omega^2} = \frac{Er^2}{\delta_s k^2 V^2}} \qquad \boxed{I = \frac{E}{\delta_s\omega^2} = \frac{Er^2}{\delta_s\omega^2}} \tag{17}$$

On neglecting the effect of arm and hub, k can be taken as the mean radius of rim r_m. Taking $r = r_m$, and $k = r_m$, we get

$$M = \frac{E}{\delta_s V^2} \tag{18}$$

On using equation (13), we get

$$M = \frac{2E}{\left(V_{max}^2 - V_{min}^2\right)} \tag{19}$$

Since $V_{max}^2 - V_{min}^2 = \frac{\left(V_{max} + V_{min}\right)}{2} 2\left(V_{max} - V_{min}\right) = V\left(2\delta_S V\right)$, hence

$$\delta_S V^2 = 0.5\left(V_{max}^2 - V_{min}^2\right) \tag{20}$$

Equations (18) or (19) can be used for finding mass of the flywheel. The 90% of M will be distributed at rim and 10% for the hub and arms. By experience the maximum velocity V_{max} is limited by the material and centrifugal stresses at the rim.

Flywheel of a Punch Press

Let d be the diameter of hole to be punched, t is the thickness of plate to be punched, f_{smax} is the resistance to shear (shear stress), T is the time between successive punch (punching period), t_p is the time for the actual punching operation. ($\approx 0.1T$) and N is speed of motor in rpm to which the flywheel is attached. Experiments show that: (i) the maximum force P occurs at time = $(3/8)t_p$ and (ii) the area under the actual force curve i.e. the energy required to punch a hole is equal to rectangular area (shaded area), hence

Energy required for punching a hole $W = Pt/2$ (21)

In other words the average force is half the maximum force.

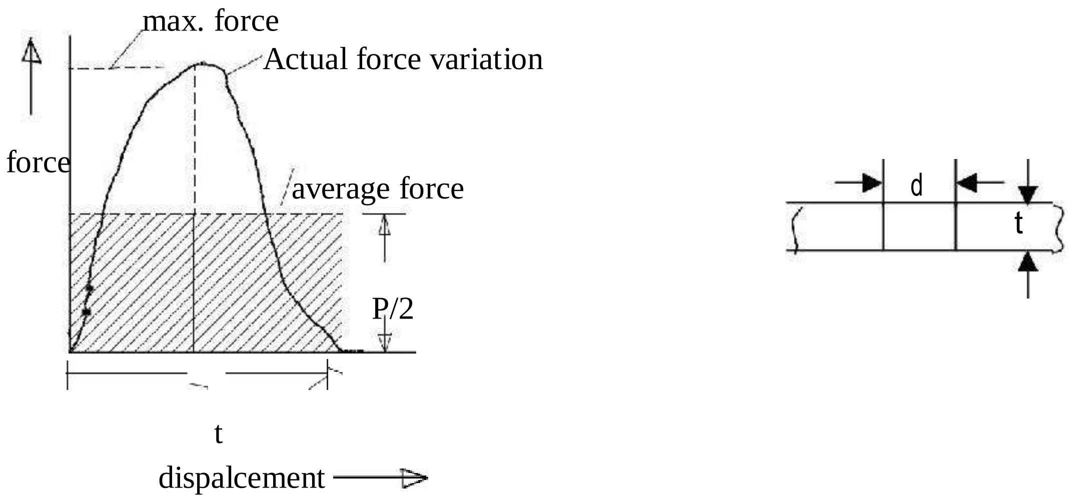


Figure 8 Punching force variation with deformations

Maximum force required to punch a hole

$$P = f_{s\,max}\,\pi dt \tag{22}$$

Combining equations (21) and (22), it gives

$$W = 0.5Pt = 0.5\left(f_{Smax}\,\pi\, dt\right)t \tag{23}$$

where f_{Smax} is in N/m^2, d in m, t in m, W in N-m and t_p in sec. Average power during punching

$$\text{W/(time for actual punching)} = \frac{0.5\left(f_{s\,max}\,\pi\, d\, t^2\right)}{t_p}\ \text{Watt} \tag{24}$$

Hence, in absence of the flywheel the motor should be capable of supplying large power instantly as punching is done almost instantaneously. If flywheel is attached to the motor shaft, then the flywheel store energy during ideal time and will give back during the actual punching operation

$$\text{Average power required from motor} = W/(\text{Punching interval}) = \frac{0.5\left(f_{s\,max}\,\pi\, d\, t^2\right)}{T}\ \text{Watt} \qquad (25)$$

Average power from eqn. (25) will be for less than that from equation (24) (e.g. of the order of 1/10).

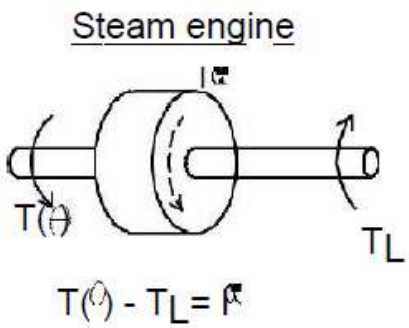


Figure 9 (a) Steam engine

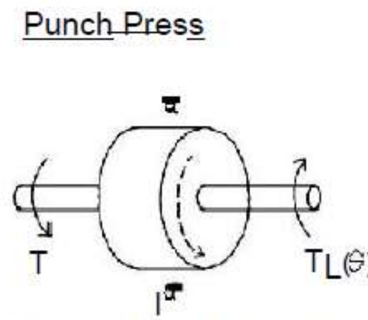


Figure 10(a) Punch press

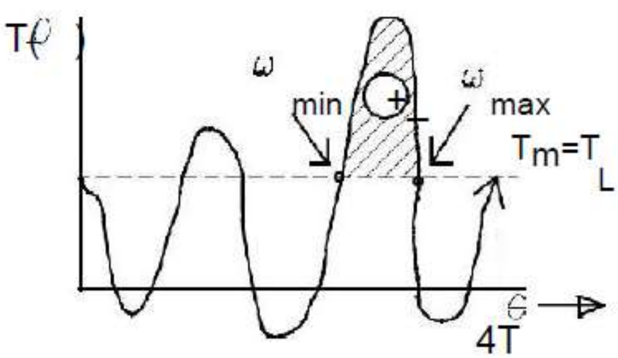


Figure 9(b) A turning moment diagram

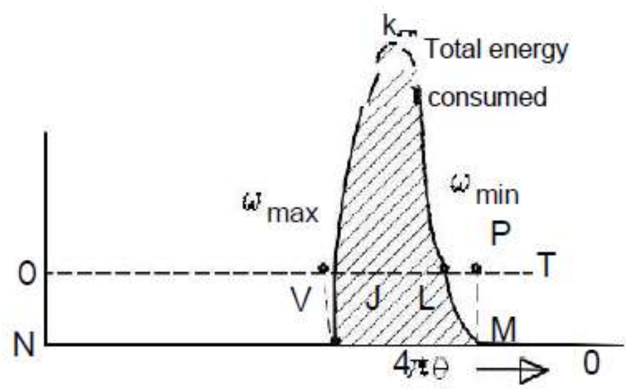


Figure 10(b) A turning moment diagram

In Figure 10(b) the total energy consumed during ω_{max} and ω_{min} = area IJLM.

The total energy supplied in period during same period (ω_{max} to ω_{min})

=area IVPM Hence, the fluctuation of the energy E= IJLM-IVPM = area

NOPM – area IVPM.

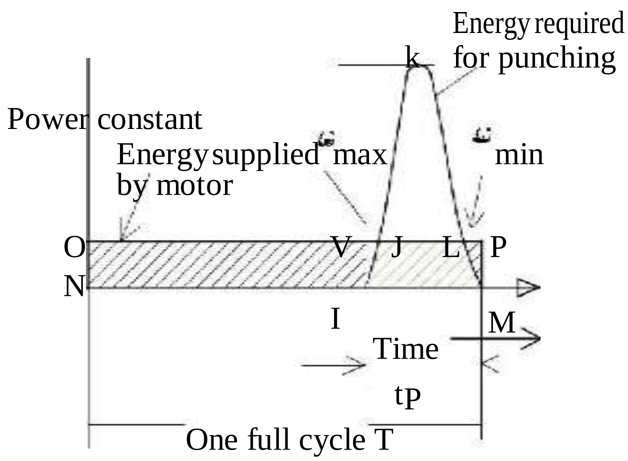


Figure 11 Turning moment diagram if a punch press

Hence,

$$\boxed{E = \left(\frac{1/2\, f_S \pi dt}{T}\right)^2 \times T - \left(\frac{1/2\, f_S \pi dt}{T}\right)^2 {}_p}$$

The fluctuation of energy will be the power supplied by the motor during the ideal period. Whatever energy is supplied during the actual punching will also be consumed in the punching operation. Maximum speed will occurs just before the punching and minimum speed will occur just after the punching. The net energy gained by the flywheel during this period i.e. from the minimum speed to the maximum speed (or vice versa) will be the fluctuation of energy. The mass of flywheel can be obtained by:

$\boxed{M = E/\delta_S V^2}$ for given δ_S and V , once E is calculated from eqn. (26).

Location of the maximum and minimum speeds:

Let A_i be the area of the respective loop, ω_o is the speed at start of cycle (datum value). We will take datum at starting point and will calculate energy after every loop. At end of cycle total energy should be zero. The maximum speed is at the maximum energy (7 units) and the minimum speed is at the minimum

energy (-2 units). E = Net area between ω_{max} and ω_{min} (-4+2-7=-9 units) or between ω_{min} and ω_{max} (4-3+2-1+7=9 units) $= 0.5I(\omega_{max}^2 - \omega_{min}^2)$.

Location of maximum and minimum speeds:

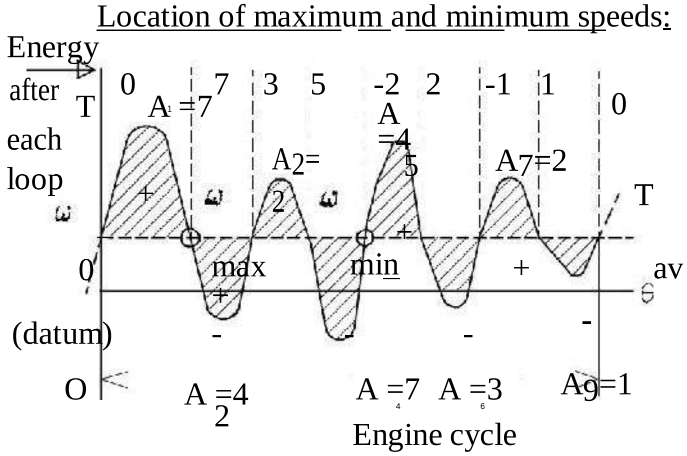


Figure 12 Turning moment diagram

Analytical expressions for turning moment: The crankshaft torque is periodic or repetitive in nature (over a cycle), so we can express torque as a sum of harmonics by Fourier analysis

$$T = T(\theta) = C_0 + A_1 \sin\theta + A_2 \sin 2\theta + \cdots + A_n \sin n\theta + \cdots + B_1 \cos\theta + B_2 \cos 2\theta + \cdots + B_n \cos n\theta + \cdots \quad (27)$$

With the knowledge of $T(\theta)$, C_0, A_1, A_2 $\cdots$ can be obtained. For all practical purpose first few harmonics will give a sufficient result. This will be very useful in analysis of torsional vibration of engine rankshaft. We will use this analysis for finding mass of flywheel. Let period of $T(\theta)$ is 360°, then

$$\text{Work done per revolution} = \int_0^{2\pi} T(\theta)\,d\theta = C_0\, 2\pi \quad (28)$$

and

$$\text{Mean torque} = T_m = \frac{1}{2\pi}\int_0^{2\pi} T(\theta)\,d\theta = \frac{1}{2\pi} C_0\, 2\pi = C_0 \quad (29)$$

Now we have to obtain the intersection point of "T(θ)- θ " curve with T_m line. Putting $T - T_m = 0$ in (27), we can get θ, as

$$T(\theta) - T_m = 0 = A_1 \sin\theta + A_2 \sin 2\theta + \cdots + A_n \sin n\theta + \cdots + B_1 \cos\theta + B_2 \cos 2\theta + \cdots + B_n \cos n\theta + \cdots$$

which gives

$$A_1 \sin\theta + A_2 \sin 2\theta + \cdots + A_n \sin n\theta + \cdots + B_1 \cos\theta + B_2 \cos 2\theta + \cdots + B_n \cos n\theta + \cdots = 0 \qquad (30)$$

Equation (30) is a transdental (non-linear) eqn. in terms of θ, from which we can get $\theta = \theta_1, \theta_2 \cdots$. Let during period of 360° two intersections θ_1 and θ_2 are there, then the fluctuation of energy can be obtained as (Figure 13):

$$\boxed{E = \int_{\theta_1}^{\theta_2} (T(\theta) - T_m)\, d\theta} \qquad (31)$$

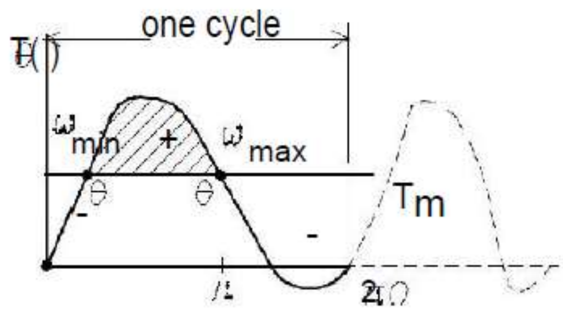


Figure 13 TM Diagram

Example 1: A single-cylinder, four- stroke oil engine develops 25 kW at 300 rpm. The work done by the gases during expansion stroke is 2.3 times the work done on the gases during compression stroke and the work done during the suction and exhaust strokes is negligible. If the turning moment diagram during expansion is assumed to be triangular in shape and the speed is to be maintained within 1% of the mean speed, find the moment of inertia of the flywheel.

Solution: Given data are: $\delta_S = 0.02$; $P = 25$ kW; $W_{exp} = 2.3W_{comp}$;

$\omega = 3000$ rpm $= 2\pi\ 300/60 = 100\pi$ rad/s $= 31.41$ rad/s ;

$T_{av} = P/\omega = 25\times 10^3/(100\pi) = 2500/\pi$ Nm $= 795.8$ Nm (In Figure 14 height: AC)

Total work done in one cycle (i.e. 4π rad. rotation) $W_{total} = T_{av}\,4\pi = (2500/\pi)\,4\pi = 10000$ Nm

We have,

$W_{total} = W_{exp} - W_{comp}$ hence $10000 = 2.3W_{comp} - W_{comp}$

which gives

$W_{comp} = 7692.3$ Nm and $W_{exp} = 17692.3$ Nm

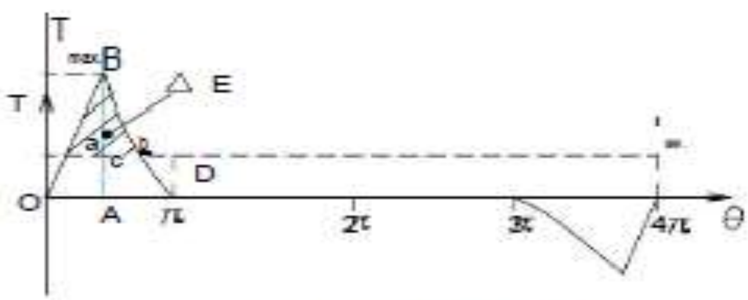


Figure 14 Turning moment diagram

Work done during expansion stroke: $(1/2)T_{max}\pi = W_{exp} = 17692.3$, which gives $T_{max} = 11263.3$ N.m = AB .

BC = maxl. excess turning moment $= T_{max} - T_{av} = AB - AC = 11263.3 - 795.8 = 10467.5$ Nm .

Hence the fluctuation of energy is $E = (1/2)\,BC\times ab$

ODB & abB are similar, hence $ab/\pi = BC/AB$ or $ab = \pi\,(10467.5/11263.3)$ or $ab = 2.92$.

Which gives $E = (1/2)10467.5\times 2.92 = 15280.6$ Nm

We have $E = I\delta_s W_{av}$; or $I = 15280.6/\{0.02\times(31.40.6)^2\} = 774.4$ kg-m^2

Or

$$E = W_{exp}\left(1 - AC/AB\right)^2 = 17.692\left[1 - (0.795733/11.7631)\right]^2 = 15.28 \text{ Nm}$$

Or

From similar OBD & aBD : ab / OD = BC / AB

$$\frac{\text{Area Bab}}{\text{Area OBD}} = \frac{(1/2)\,ab \times BC}{(1/2)\,OD \times AB} = \frac{ab}{OD}\,\frac{BC}{AB} = \frac{BC}{AB}\,\frac{BC}{AB} = \left(\frac{BC}{AB}\right)^2 = \left(\frac{10467.5}{11263.3}\right)^2$$

where Area Bab $= E$ and Area OBD $= W_{exp}$.

Area Bab = $E = (10467.5/11263.3)^2 \times 17692.3 = 15280.55$ Nm

Example 2. The vertical scale of the turning moment diagram for a multi-cylinder engine, shown in Figure 15, is 1 cm = 7000 Nm of torque, and horizontal scale is 1 cm = 30^0 of crank rotation. The areas (in cm^2) of the turning moment diagram above and below the mean resistance line, starting from A in Figure @ and taken in order, are 0.5, +1.2, -0.95, +1.45, -0.85, +0.71, -1.06. The engine speed is 800 rpm and it is desired that the fluctuation from minimum to maximum speed should not be more then 2% of average speed. Determine the moment of inertia of the flywheel.

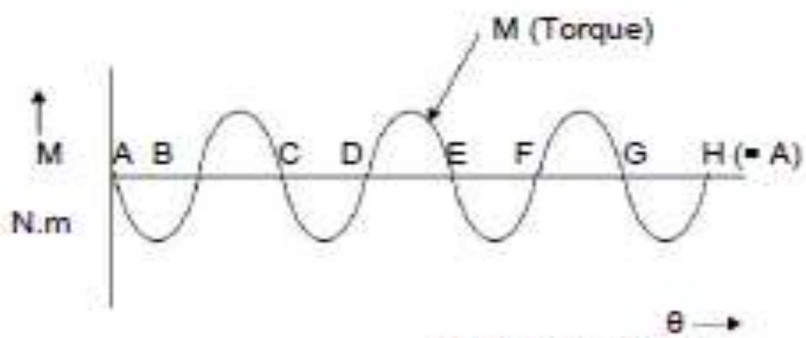


Figure 15 Example 2

Solution:

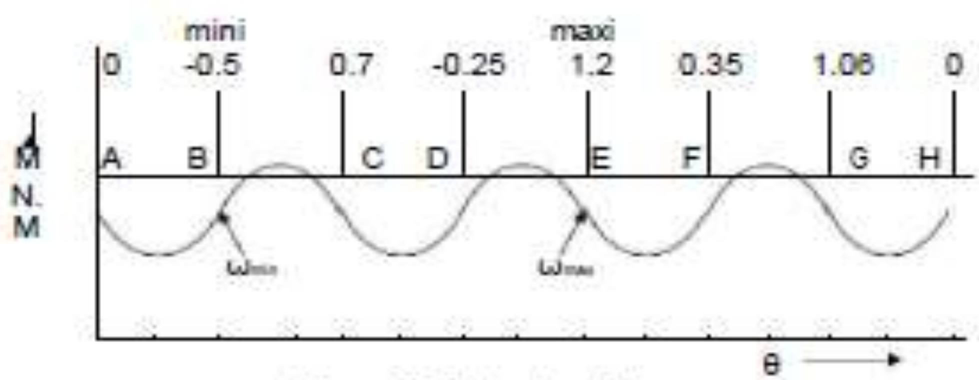


Figure 16 Fluctuation of the energy

$$E = E_{max} - E_{min} = 1.2 - (-0.5) = 1.7\ \text{cm}^2 = 1.7 \times 7000 \times \frac{\pi}{180}\,30 = 6230.825\text{N-m}$$

ω = 800 rpm = 83.776 rad/s and δ_s = 0.02

Hence,

$$I = \frac{E}{\omega^2 \delta_s} = 44.39 \text{ kgm}^2$$

Exercise Problems:

(1) Twenty 1-cm holes are to be punched every minute in a 1.5 cm plate whose resistance to shear is 35316 N/cm^2. The actual punching takes place in one-fifth of the interval between successive operations. The speed of the flywheel is 300 rpm. Making the usual assumptions specify the dimensions of a suitable CI rimmed flywheel. Use coefficient of fluctuation of speed = 0.01 and V = 60 m/s.

(2) The equation of a turning moment curve of an IC engine running at 300 rpm is given by

T = [25000 + 8500 sin3θ]. A flywheel coupled to the crankshaft has a moment of inertia 450 kg m^2 about the axis of rotation. Determine (a) Horse power of the engine (b) total percentage fluctuation of speed (c) maximum angle by which the flywheel leads or lags an imaginary flywheel running at a constant speed of 300 rpm.

(3) The turning moment diagram for a multi cylinder IC engine is drawn to the

following scales 1 cm = 15^o crank angle

1 cm = 3 k Nm

During one revolution of the crank the areas with reference to the mean torque line are 3.52, (□) 3.77, 3.62, (□) 4.35, 4.40 and (–) 3.42 cm^2. Determine mass moment of inertia to keep the fluctuation of mean speed within 2.5% with reference to mean speed. Engine speed is 200 rpm.

(4) A single cylinder four-stroke petrol engine develops 18.4 kW power at a mean speed of 300 rpm. The work done during suction and exhaust strokes can be neglected. The work done by the gases during explosion strokes is three times the work done on the gases during the compression strokes and they can be represented by the triangles. Determine the mass of the flywheel to prevent a fluctuation of speed greater than 2 per cent from the mean speed. The flywheel diameter may be taken as 1.5 m.

(5)A three cylinder two-stroke engine has its cranks 120^{o} apart. The speed of the engine is 600 rpm. The turning moment diagram for each cylinder can be represented by a triangle for one expansion stroke with

a maximum value of one stroke with a maximum value of 600 Nm at 60^{o} from the top dead centre. The

turning moment in other stroke is zero for all the cylinders. Determine :

(a) the power developed by the engine,

(b) the coefficient of fluctuation of speed with a flywheel having mass 10 kg and radius of gyration equal to 0.5 m,

(c) the coefficient of fluctuation of energy, and

(d) the maximum angular acceleration of the flywheel.

BALANCING

Balancing is the process of eliminating or at least reducing the ground forces and moments. It is achieved by changing the location of the mass centers of links. Balancing of rotating parts is a well known problem. A rotating body with fixed rotation axis can be fully balanced i.e. all the inertia forces and moments. For mechanism containing links rotating about axis which are not fixed, force balancing is possible, moment balancing by itself may be possible, but both not possible. We generally try to do force balancing. A fully force balance is possible, but any action in force balancing severe the moment balancing.

Balancing of rotating masses:

The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

Static balancing:

The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of the rotation. This is the condition for static balancing.

Dynamic balancing:

The net couple due to dynamic forces acting on the shaft is equal to zero. The algebraic sum of the moments about any point in the plane must be zero.

Various cases of balancing of rotating masses:

- ✓ Balancing of a single rotating mass by single mass rotating in the same plane.
- ✓ Balancing of a single rotating mass by two masses rotating in the different plane.
- ✓ Balancing of a several masses rotating in single plane.
- ✓ Balancing of a several masses rotating in different planes.

Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider r_1 be the radius of rotation of the mass m_1 attached to a shaft rotating at ω rad/s . The centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \omega^2 r_1$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.

r_2 = Radius of rotation of the balancing mass m_2

Centrifugal force due to mass m_2, $F_{C2} = m_2\omega^2 r_2$

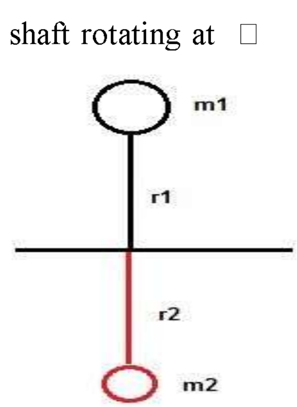


Figure 16 Balancing of Single mass

$m_1 \omega^2 r_1 =$

$m_2\omega^2 r_2$

$m_1 r_1 = m_2 r_2$

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

A disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes Q and P . Let r, r_1 and r_2 be the radii of rotation of the masses in planes R, Q and P respectively.

l_1 = Distance between the planes R and Q,
l_2 = Distance between the planes R and P, and
l = Distance between the planes Q and P

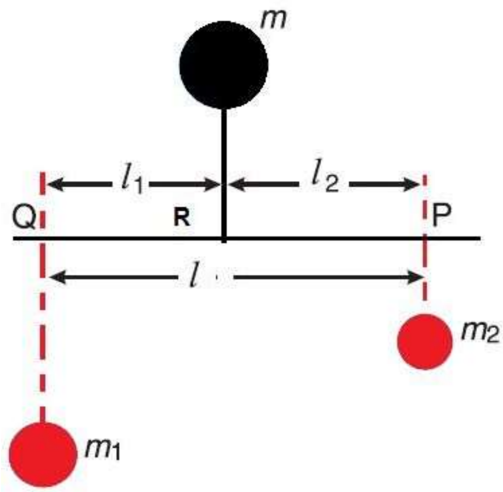


Figure 17 Balancing of Single mass by two masses

The centrifugal force exerted by the mass m on the shaft,

$F_C = m \omega^2 r$

Similarly for mass m_1 and mass m_1

$F_{C1} = m_1 \omega^2 r_1$ & $F_{C2} = m_2\omega^2 r_2$

$F_C = F_{C1} = F_{C2}$

$m \omega^2 r = m_1 \omega^2 r_1 + m_2\omega^2 r_2$

To dynamic balancing, take moments about Q and P,

F_{C1} x $l = F_C$ x

l_2 $m_1 r_1 l = mr$

l_2 Similarly,

F_{C2} x $l = F_C$ x

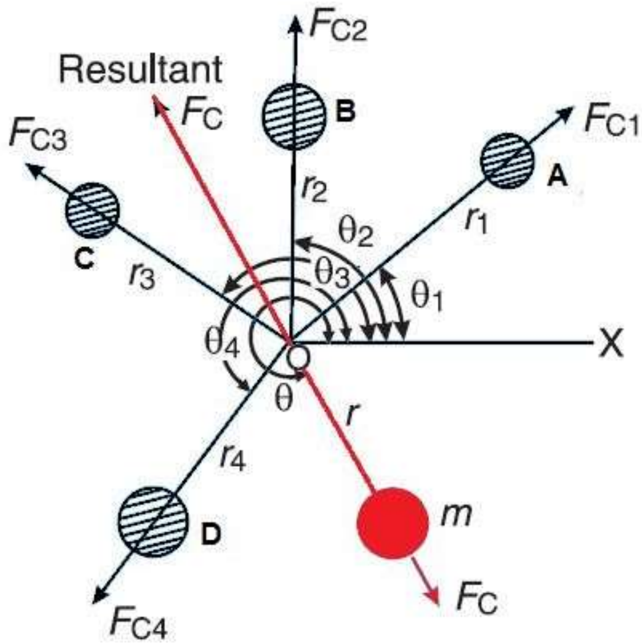
l_1 m_2 r_2 l

$=mrl_1$

Balancing of Several Masses Rotating in the Same Plane

Consider any number of masses of magnitude *A, B, C and D* at distances of r_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let □1 , □2 , □3 and □4 be the angles of these masses with the horizontal line. The magnitude and position of the balancing mass may be found out graphically:

Angular Position diagram



Vector diagram

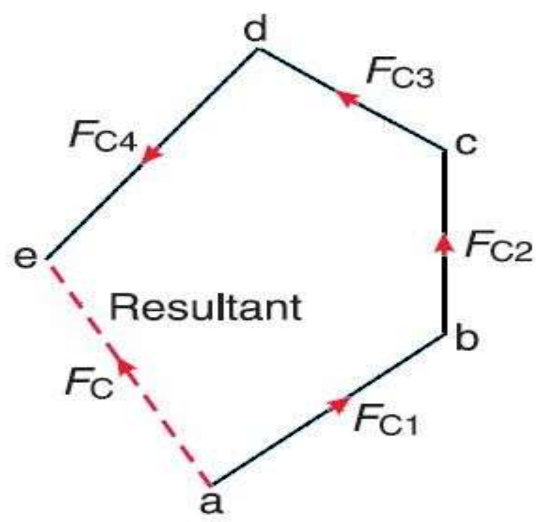
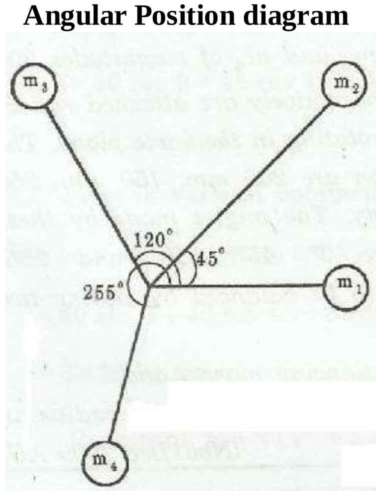
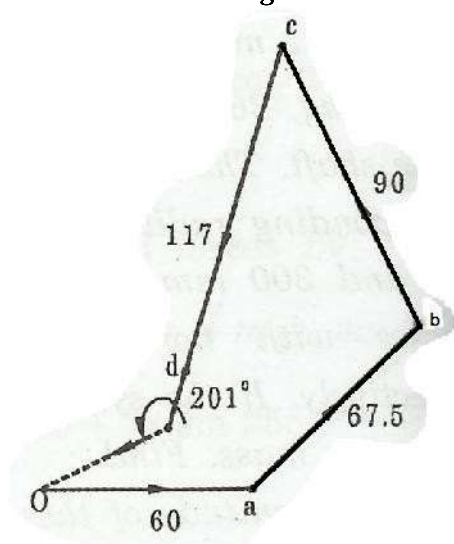


Figure 18 Position and Force diagram

Problem 1:

A shaft is rotating at a uniform speed with four masses m_1,m_2,m_3 and m_4 of magnitudes 300 kg, 450kg, 360kg and 390 kg respectively. The masses are rotating in the same plane. The corresponding radii of rotation are 200mm, 150mm, 250mm and 300mm. The angles made by these masses with respect to horizontal are 0□, 45□, 120□ and 255□ respectively. Find the magnitude and position of balance mass if it radius of rotation is 200mm.

Mass (m) kg	***Radius (r) m***	***Cent.force □ □²* *(mr) kg-m***
300	0.2	60
450	0.15	67.5
360	0.25	90
390	0.3	117

Angular Position diagram	**Vector diagram**
	

mr = od = 38 kg-m

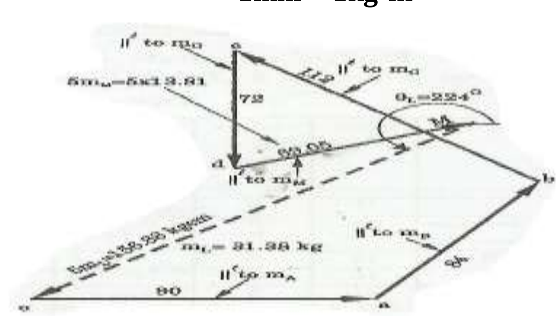
m =38/0.2 = **190**

kg

Balancing mass 190 kg at 201□ w.r.t to horizontal.

Force Polygon

1mm = 1kg-m



Problem 2:

Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

Analytical method

Resolving $m_1.r_1$, $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$ horizontally,

$\Sigma H = m_1.r_1 \cos\Theta 1 + m_2.r_2 \cos\Theta 2 + m_3.r_3 \cos\Theta 3 + m_4.r_4 \cos\Theta 4$

$= 40 \cos 0° + 45\cos 45° + 60 \cos 120° + 78\cos 255°$

$= 40 + 31.8 - 30 - 20.2 = 21.6$ kg-m

Resolving vertically,

$\Sigma V = m_1.r_1 \sin\Theta 1 + m_2.r_2 \sin\Theta 2 + m_3.r_3 \sin\Theta 3 + m_4.r_4 \sin\Theta 4$

$= 40 \sin 0° + 45\sin 45° + 60 \sin 120° + 78\sin 255°$

$= 0 + 31.8 + 52 - 75.3 = 8.5$ kg-m

Resultant, $R = \sqrt{\Sigma H^2 + \Sigma V^2} = \sqrt{21.6^2 + 8.5^2} = 23.2$ *kg-m*

$mr = R = 23.2$ or $m = 23.2 / r = 23.2 / 0.2 =$ **116 kg**

$\tan\Theta = \frac{\Sigma V}{\Sigma H} = \frac{8.5}{21.6} = 0.3985$

$\mathbf{\Theta = 21.5^0}$

Angle of the balancing mass from the horizontal mass is $\Theta = 180^0 + 21.5^0 = \mathbf{201.5^0}$

Problem 3:

A rotating shaft carries four unbalanced mass 18kg, 14kg, 16kg and 12kg at radii 50mm, 60mm, 70mm and 60mm respectively. The second, third and fourth mass revolve in planes 80mm , 160mm , 280mm respectively from the first mass and angularly at 60°, 135° and 270° respectively in ACW from first mass. The shaft dynamically balanced by adding two masses at radii 50mm and first mass revolving in mid way between first and second and second mass revolving in mid way between third and fourth. Determine the angular position and magnitude of the balance mass required.

Plane	***Mass (m) kg***	***Radius (r) m***	***Cent.force/ω^2 (mr) kg-m***	***Distance from ref. plane (l) m***	***Couple/ω^2 (m.r.l) kg-m^2***
A	18	5	90	-4	-360
L (R.P)	m_L	5	5 m_L	0	0
B	14	6	84	4	336

C	16	7	112	12	1344
M	m_M	5	5 m_M	18	90 m_M
D	12	6	72	24	1728
Plane Position diagram			**Angular Position diagram**		

First draw the couple polygon, From that, vector od = 90 m_M od = 1243 kg-m^2

90 m_M = 1243 □ m_M = **13.81 kg.**

Angle of m_M w.r. t A is 24□ in ACW.

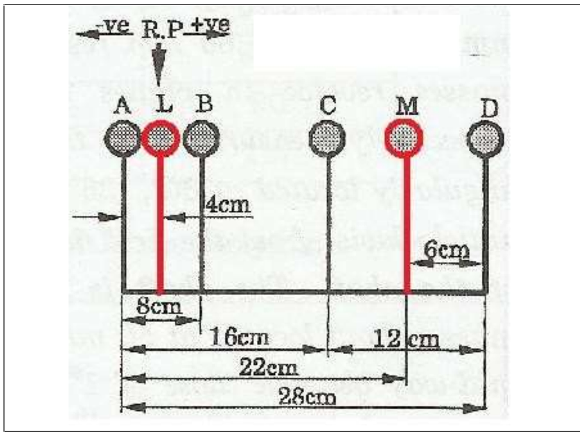
Substitute m_M in force and draw force polygon,

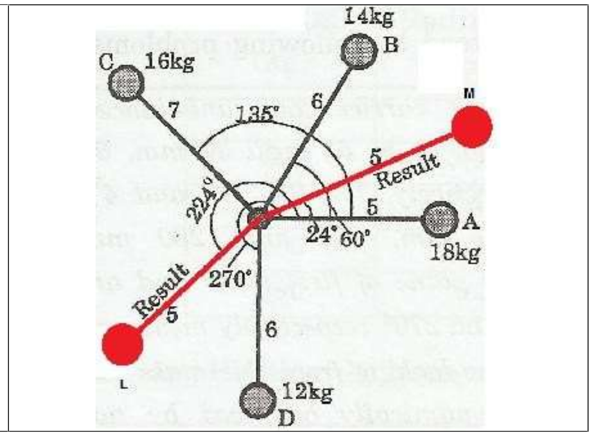
Vector oM =5 m_L

oM = 157 kg-m

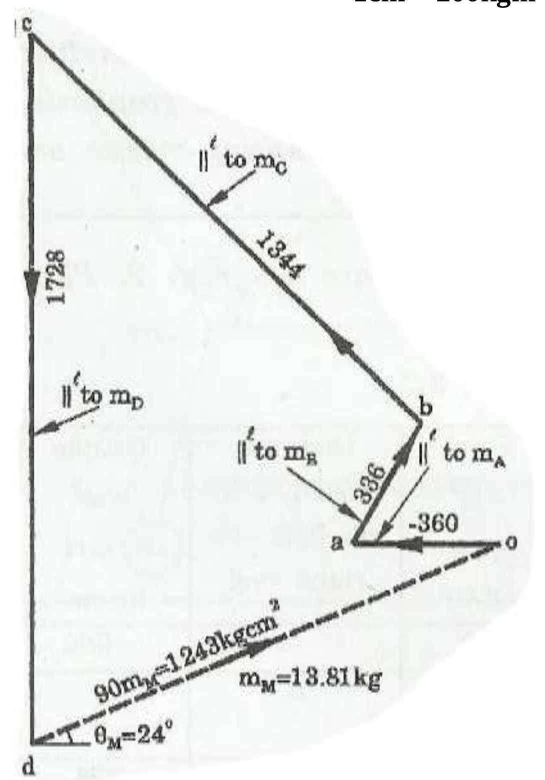
5 m_L = 157 □ m_L = **31.4 kg.**

Angle of m_L w.r. t A is 224□ in ACW.



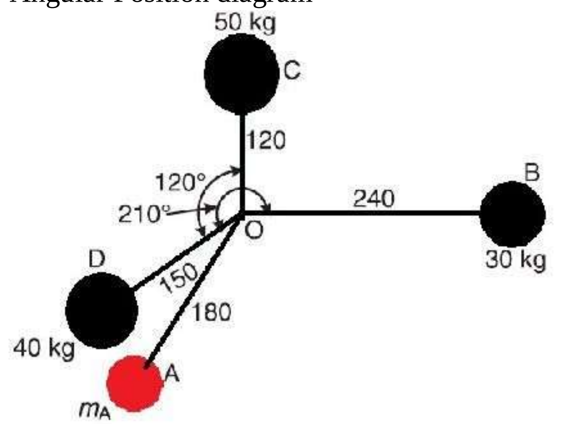


Couple polygon
1cm = 200kgm^2

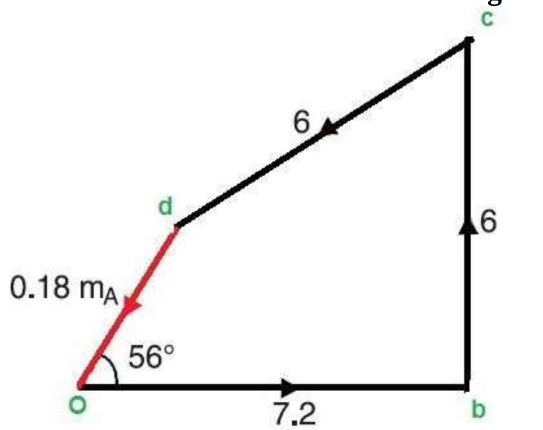
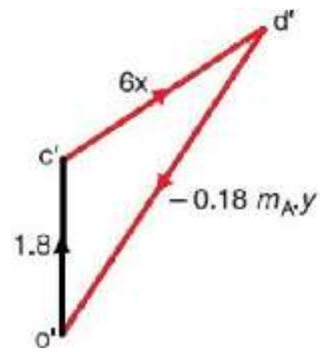


Problem 4:

Four masses A, B, C and D are to be completely balanced. Mass B, C and D are mass 30kg, 50kg and 40kg at radii 240mm, 120mm and 150mm respectively. The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find : The magnitude and the angular position of mass A if its radius is 180mm ; and The position of planes A and D.

Plane Position diagram	Angular Position diagram 

Plane	***Mass (m) kg***	***Radius (r) m***	***Cent.force □ □^{2} (mr) kg-m***	***Distance from ref. plane (l) m***	***Couple □□^{2} (m.r.l) kg-m^2***
A	m_A	0.18	0.08 m_A	-y	-0.18 m_Ay
B	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	6x

Force Polygon 1cm = 1kg-m	**Couple polygon** 5cm = 1kgm^2
	

First draw force polygon, from that,

$0.18\, m_A$ = Vector *do* = 3.6 kg-m or $\boldsymbol{m_A}$ **= 20 kg**

Angular position of mass *A* from mass *B* in the anticlockwise direction is **236°**

To draw couple polygon, Draw vector *o' c'* parallel to *OC* and equal to 1.8 kg-m^2. From points *c'* and *o'* draw lines parallel to *OD* and *OA* respectively, such that they intersect at point *d'* ,

$6\,x$ = vector $c'\,d'$ = 2.3 kg-m^2 or $\boldsymbol{x}$ **= 0.383 m**

The plane of mass *D* is 0.383 m or 383 mm towards left of plane *B*.

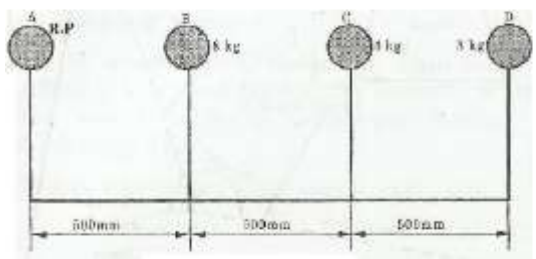
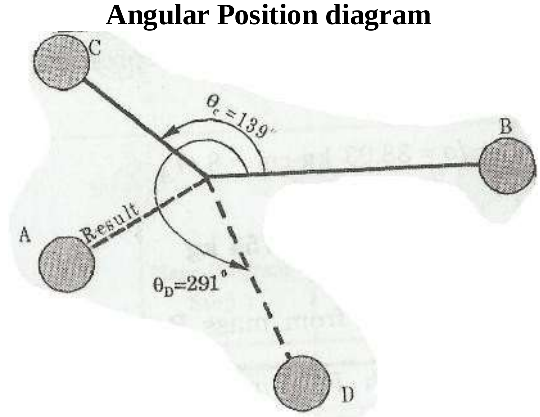
Similarly,
$-0.18\, m_A.y$ = vector *o'd'* = 3.6 kg-m^2

$-0.18 \times 20\, y = 3.6$ or $y =$ **– 1 m**

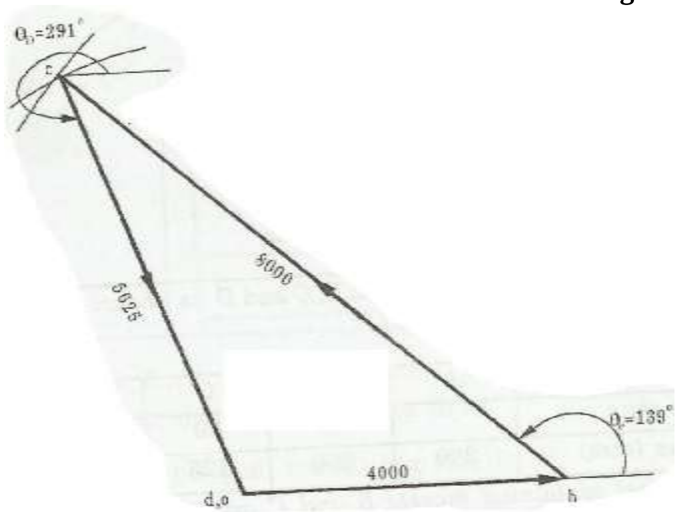
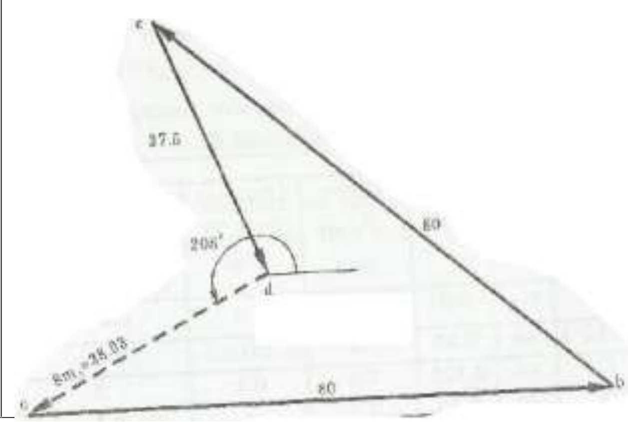
Plane A is 1000 mm towards right of plane *B*.

Problem 5:

Four masses A,B,C and D carried by a rorating shaft at radii 80mm, 100mm, 200mm and 125mm respectively. The planes of rotation are equi spaced by 500mm. The masses B,C and D are 8kg, 4kg and 3kg respectively. The magnitude of mass A and the angular position of entire system, if it completely balanced.

Plane Position diagram	**Angular Position diagram**
	

Plane	***Mass (m) kg***	***Radius (r) m***	***Cent.force □ □²*** ***(mr) kg-m***	***Distance from ref. plane (l) m***	***Couple □□²*** ***(m.r.l) kg-m²***
A (R.P)	m_A	0.08	0.08 m_A	0	0
B	8	0.1	0.8	0.5	0.4
C	4	0.2	0.8	1	0.8
D	3	0.125	0.375	1.5	0.5625

Couple polygon 1mm = 0.001kgm²	**Force Polygon** 1mm = 0.01kg-m
	

First draw the couple polygon with the mrl values. Assuming angle of mass B as horizontal, form a triangle to find the angular position for C and D.

Next draw couple polygon to find A,
Vector od = 0.08 m_A

od = 0.381 kg-m

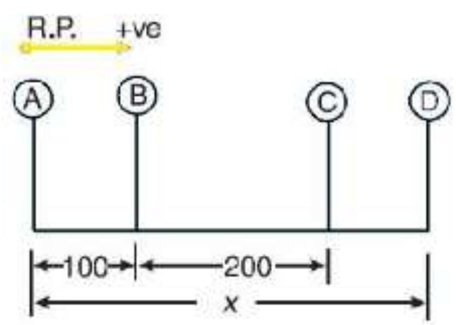
0.08 m_A = 0.381 kg-m **m_A = 4.8kg**

Angular position of mass *A* from mass *B* in the clockwise direction is **208**

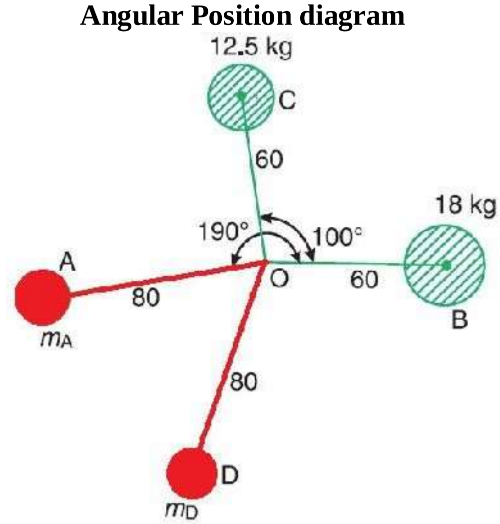
Problem 6:

A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine : **1.** The magnitude of the masses at A and D ; **2.** the distance between planes A and D ; and **3.** the angular position of the mass at D.

Plane Position diagram

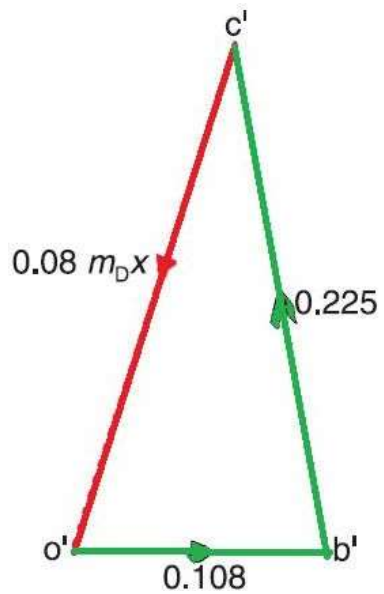


Angular Position diagram

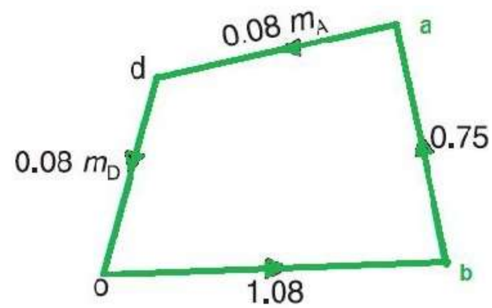


Plane	Mass (m) kg	Radius (r) m	Cent.force □ □² (mr) kg-m	Distance from ref. plane (l) m	Couple □□² (m.r.l) kg-m²
A	m_A	0.08	0.08 m_A	0	0
B	18	0.06	1.08	0.1	0.108
C	12.5	0.06	0.75	0.3	0.225
D	m_D	0.08	0.08 m_D	x	0.08 m_Dx

Couple polygon
1mm = 0.001kgm^2



Force Polygon
1mm = 0.01kg-m



First draw the couple polygon, assuming angle of mass B as horizontal.

0.08 $m_D.x$ = vector $c'o'$ = 0.235 kg-m^2

Then, draw the force polygon, Draw vector *ob* parallel to *OB* and equal to 1.08 kg-m. From point *b*, draw vector *bc* parallel to *OC* and equal to 0.75 kg-m. From point *c*, draw vector *cd* parallel to *OA* and from point *o* draw vector *od* parallel to *OD*. The vectors *cd* and *od* intersect at *d*.

0.08 m_A = vector *cd* = 0.77 kg-m or **m_A = 9.625 kg**

0.08 m_D = vector *do* = 0.65 kg-m or **m_D = 8.125 kg**

0.08 $m_D.x$ = 0.235 kg-m^2
0.08 × 8.125 × x = 0.235 kg-m^2
0.65 x = 0.235

x=0.3615m

Angular position of mass at *D* from mass *B* in the anticlockwise direction **is 251°**

Exercises Problems:

1. Four masses *A*, *B*, *C* and *D* are attached to a shaft and revolve in the same plane. The masses are 12 kg, 10 kg, 18 kg and 15 kg respectively and their radii of rotations are 40 mm, 50 mm, 60 mm and 30 mm. The angular position of the masses *B*, *C* and *D* are 60°, 135° and 270° from the mass *A*. Find the magnitude and position of the balancing mass at a radius of 100 mm.
   [Ans. 7.56 kg ; 87° clockwise from *A*]

2. A shaft carries five masses *A*, *B*, *C*, *D* and *E* which revolve at the same radius in planes which are equidistant from one another. The magnitude of the masses in planes *A*, *C* and *D* are 50 kg, 40 kg and 80 kg respectively. The angle between *A* and *C* is 90° and that between *C* and *D* is 135°. Determine the magnitude of the masses in planes *B* and *E* and their positions to put the shaft in complete rotating balance.
   [Ans. 12 kg, 15 kg ; 130° and 24° from mass *A* in anticlockwise direction]

3. *A*, *B*, *C* and *D* are four masses carried by a rotating shaft at radii 100 mm, 150 mm, 150 mm and 200 mm respectively. The planes in which the masses rotate are spaced at 500 mm apart and the magnitude of the masses *B*, *C* and *D* are 9 kg, 5 kg and 4 kg respectively. Find the required mass *A* and the relative angular settings of the four masses so that the shaft shall be in complete balance.
   [Ans. 10 kg ; Between *B* and *A* 165°, Between *B* and *C* 295°, Between *B* and *D* 145°]

4. Four masses A, B, C and D revolve at equal radii and are equally spaced along a shaft. The mass B is 7 kg and the radii of C and D make angles of 90° and 240° respectively with the radius of B. Find the magnitude of the masses A, C and D and the angular position of A so that the system may be completely balanced.
   [Ans. 5 kg ; 6 kg ; 4.67 kg ; 205° from mass B in anticlockwise direction]

5. A rotating shaft carries four masses A, B, C and D which are radially attached to it. The mass centres are 30 mm, 38 mm, 40 mm and 35 mm respectively from the axis of rotation. The masses A, C and D are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of A and B is 400 mm and between B and C is 500 mm. The masses A and C are at right angles to each other. Find for a complete balance,
   1. the angles between the masses B and D from mass A,
   2. the axial distance between the planes of rotation of C and D,
   3. the magnitude of mass B. [Ans. 162.5°, 47.5° ; 511 mm : 9.24 kg]

Unit II

Balancing

The Partial Balance of Two-cylinder Locomotives

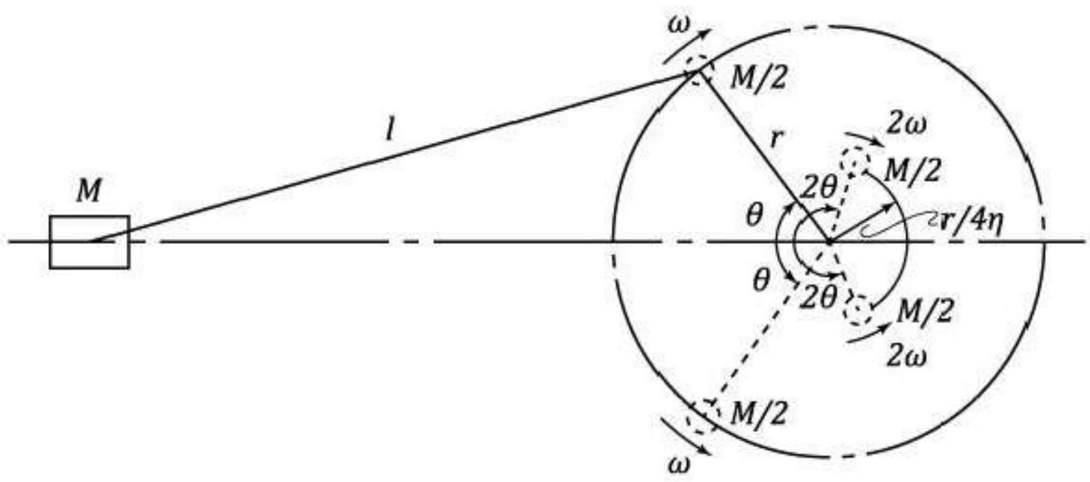
It is normal for the cranks to be at right angles and as a result the secondary forces are small and in opposite directions. As a result they are usually neglected and only the primary forces and couples are considered.

It is usual to balance about two-thirds of the reciprocating parts with masses fixed to the wheels.The unbalanced vertical components of the reciprocating masses give rise to a variation of rail pressure known as Hammer Blow and a Rocking Couple about a fore and aft horizontal axis.The unbalanced reciprocating masses cause a variation in draw-bar pull and

a swaying couple about a vertical axis

Radial Engines - Direct and Reverse Cranks

The primary force for a reciprocating mass M is equivalent to the resultant of the centrifugal forces of two masses $\frac{M}{2}$ rotating at a crank radius r and at a speed ω, one in the forward direction of motion and the other in the reverse direction. Note that the "direct" and "reverse" cranks are equally inclined to the dead centre position.



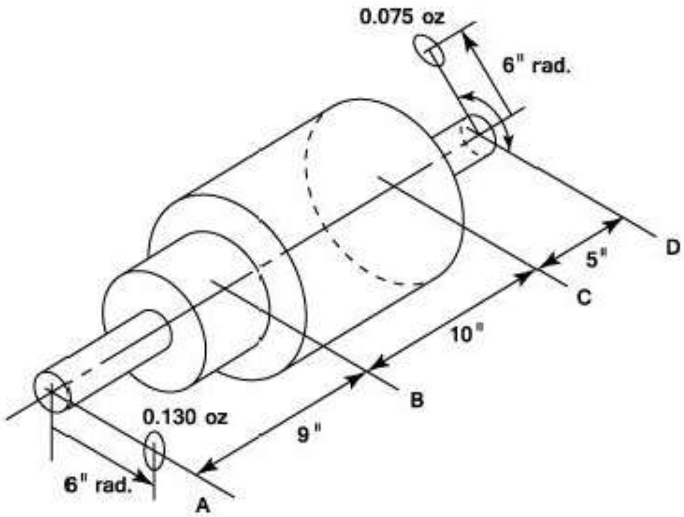
Similarly, the secondary force can be represented by direct and reverse cranks inclined 2θ at to the inner dead centre and each carrying a mass $\frac{M}{2}$ m at a radius of r/4n rotating at a speed of 2ω. This method is particularly useful for examining the balance of radial engines with a number of connecting rods attached to the same crank. It is usually assumed that the crank and connecting rod lengths are the same for each cylinder, though from a practical consideration of design this is not generally true.

Example 1

A motor armature is in running balance when weights of 0.130 oz. and 0.075 oz. (There are 16 oz. in 1 lb.) are added temporarily in the positions shown in the planes A D

and

diagram.n the



If the actual balancing is to be carried out by the permanent addition of masses in the planes B C and each at 4 in radius, find their respective **magnitudes** and **angular positions** to the radius shown in plane A.

The problem is to determine the masses in the planes B C and which will provide the same resultant force and couple, when rotating, as the given masses in the planes A and D.
It is possible to eliminate one of the "unknowns" (say B) by taking B as the reference plane. The following table can now be constructed. The figures in bracket are added as and when they are calculated.

The polygon can now be drawn. Note that is negative.

It is the couple which tends to make the leading wheels sway from side to side, to separation of unbalanced primary forces along the line of stroke by some dist

Planes	*M* oz.	*r* in.	x in.	*Mr*	*Mrx*	θ
A	0·13	6	-9	0·78	-7·02	0°
B	0·4	4	0	1·6	0	353°
C	0·291	4	10	1·166	11·66	148°
D	0·075	6	15	0·45	6·75	115°

C is the couple

The Mr value of C is now Mrx for C divided by the appropriate value of .
With this information the Force polygon B can be constructed in which and give the same resultant as and .
The Mr value and angular position of B can now be added to the table.
The required magnitudes of B and C are calculated by dividing Mr by the corresponding value for .

Tractive force

The term **tractive force** can either refer to the total traction a vehicle exerts on a surface, or the amount of the total traction that is parallel to the direction of motion.

Tractive effort

The term **tractive effort** is often qualified as **starting tractive effort**, **continuous tractive effort** and **maximum tractive effort**. These terms apply to different operating conditions, but are related by common mechanical factors: input torque to the driving wheels, the wheel diameter, coefficient of friction (**μ**) between the driving wheels and supporting surface, and the weight applied to the driving wheels (**m**). The product of **μ** and **m** is the factor of adhesion, which determines the maximum torque that can be applied before the onset of wheelspin or wheelslip.

Starting tractive effort: Starting tractive effort is the tractive force that can be generated at a standstill. This figure is important on railways because it determines the maximum train weight that a locomotive can set into motion.

Maximum tractive effort: Maximum tractive effort is defined as the highest tractive force that can be generated under any condition that is not injurious to the vehicle or machine. In most cases, maximum tractive effort is developed at low speed and may be the same as the starting tractive effort.

Continuous tractive effort: Continuous tractive effort is the tractive force that can be maintained indefinitely, as distinct from the higher tractive effort that can be maintained for a limited period of time before the power transmission system overheats. Due to the relationship between power (P), velocity (v) and force (F), described as:

$P = vF$ or $P/v = F$

tractive effort inversely varies with speed at any given level of available power. Continuous tractive effort is often shown in graph form at a range of speeds as part of a **tractive effort curve**.[2]

Vehicles having a hydrodynamic coupling, hydrodynamic torque multiplier or electric motor as part of the power transmission system may also have a **maximum continuous tractive effort** rating, which is the highest tractive force that can be produced for a short period of time without causing component harm. The period of time for which the maximum continuous tractive effort may be safely generated is usually limited by thermal considerations. such as temperature rise in a traction motor.

Hammer blow

Hammer blow, in rail terminology, refers to a vertical force which alternately adds to and subtracts from the locomotive's weight on a wheel.

It is transferred to the track by the driving wheels of many steam locomotives. It is an out-of- balance force on the wheel (known as overbalance. It is the result of a compromise when a locomotive's wheels are unbalanced to off-set horizontal reciprocating masses, such as connecting rods and pistons, to improve the ride. The hammer blow may cause damage to the locomotive and track if the wheel/rail force is high enough. 'Dynamic augment' is the US term for the same force.

Principles The addition of extra weights on the wheels reduces the unbalanced reciprocating forces on the locomotive but causes it to be out of balance vertically creating hammer blow.

Locomotives were balanced to their individual cases, especially if several of the same design were constructed (a class). Each class member was balanced for its normal operating speed. Between 40% and 50% of the reciprocating weights on each side were balanced by rotating weights in the wheels.

Causes

While the coupling rods of a locomotive can be completely balanced by weights on the driving wheels since their motion is completely rotational, the reciprocating motions of the pistons, piston rods, main rods and valve gear cannot be balanced in this way. A two- cylinder locomotive has its two cranks "quartered" — set at 90° apart — so that the four power strokes of the double-acting pistons are evenly distributed around the cycle and there are no points at which both pistons are at top or bottom dead centre simultaneously.

A four-cylinder locomotive can be completely balanced in the longitudinal and vertical axes, although there are some rocking and twisting motions which can be dealt with in the locomotive's suspension and centring; a three-cylinder locomotive can also be better balanced, but a two-cylinder locomotive only balanced for rotation will surge fore and aft.

Additional balance weight — "overbalance" — can be added to counteract this, but at the cost of adding vertical forces, hammer blow. This can be extremely damaging to the track, and in extreme cases can actually cause the driving wheels to leave the track entirely.

The heavier the reciprocating machinery, the greater these forces are, and the greater a problem this becomes. Except for a short period early in the twentieth century when balanced compound locomotives were tried, American railroads were not interested in locomotives with inside cylinders, so the problem of balance could not be solved by adding more cylinders per coupled wheel set. As locomotives got larger and more powerful, their reciprocating machinery had to get stronger and thus heavier, and thus the problems posed by imbalance and hammer blow became more severe. Higher speeds also increased unbalanced forces which rise with the square of the wheel rotational speed.

Swaying couple is produced due to unbalanced parts of the primary disturbing forces acting at a distance between the line of stroke of the cylinders. Hammer blow is the maximum value of the unbalanced vertical force of the balance weights.

Examples

1. A single cylinder engine has stroke of 50 cm and runs at 300 rpm. The reciprocating masses are 60 kg and revolving masses are equivalent to 35 kg at a radius of 20 cm at a radius of 20 cm. Determine the balancing mass to be placed opposite to the crank at a radius of 40 cm which is equivalent to mass of revolving masses and two third of reciprocating masses. Find the magnitude of the remaining unbalanced force when the crank has turned through 30 degrees from inner dead centre.

Given Data:

r2= 0.5/2= 0.25 m

N = 300 rpm
ω = 2πN/60 = 2 X π X 300 / 60 = 31.42 rad / s

Balancing masses to be placed at a radius of b (0.4) =
We know that,
$Bb = m_1 r_1 + C\, m_2 r_2$ When all the masses rotating in same plane.
Where,
m_1 = revolving masses = 35 kg
m_2 = reciprocating masses = 60 kg

r_1 = 0.2 m
C = fraction of reciprocating masses to be balanced = 2/3
Bb = (35 X 0.2) + $\frac{2 \times 60 \times 0.25}{3}$
B X 0.4 = 17
B = 42.5 kg
Balancing mass (B) = 42.5 kg
To find the remaining unbalanced force, We know that,
∑H = Horizontal component of unbalanced force (or) Unbalanced force along the line of stroke
$= (1 - C)\, m_2 (\omega)^2 r \cos \Theta$
∑V = Vertical component of unbalanced force (or) Unbalanced force perpendicular to the line of stroke
$= C\, m_2 (\omega)^2 r \sin \Theta$

R = Resultant of unbalanced force
$= \sqrt{\sum H^2 + \sum V^2}$
$= \sqrt{((1 - C)\, m_2 (\omega)^2 r \cos \Theta)^2 + (C\, m_2 (\omega)^2 r \sin \Theta)^2}$
$= m_2 (\omega)^2 r \sqrt{\ldots}$
60 x ... ^{2} x 0.2 ... $\sqrt{0.0\ldots}$

$= 60 \text{ x } 31.42^2 \text{ x } 0.25 \text{ x} \sqrt{0.0833 + 0.1111}$
$= 6529.2$ N

2. The following particulars relate to an outside cylinder of an uncoupled locomotive Rotating mass / cylinder = 250 kg
Length of each crank = 0.35 m
Distance between wheels = 1.6 m
Distance between cylinder centers = 1.9 m Dia of driving wheel = 1.85 m
Radius of balancing mass = 0.8 m

If all the rotating masses and 2/3rd of the reciprocating masses are to be balanced, determine the magnitude and position of the balancing mass in the plane of the wheel. The angle between the cranks of the cylinder is 90 degrees.

Solution

Balancing masses are fitted in the wheel A and B. Since the cylinders are outside of the wheel, these are called outside cylinder locomotive.

M_1 = mass to be balanced on cylinder 1

$$m_1 = 200 + \frac{2}{3} X\, 250 = 366.7 kg$$

M_2 = mass to be balanced on cylinder 2

$$m_2 = 200 + \frac{2}{3} X\, 250 = 366.7 kg$$

Take A as reference plane

By using given data we can draw a table,

Plane	Mass in kg	r in m	Force/ω^2 = mr kg m	Distance from wheel A l m	Couple /ω^2 = mrlkg m^2
Cylinder 1	$200 + \frac{2}{3} X\, 250 = 366.7 kg$	0.35	128.345	-0.15	-19.253
R.P wheel A	m_A	0.8	0.8 m_A	0	0
Wheel B	m_B	0.8	0.8 m_B (141.14)	1.6	1.28 m_B
Cylinder 2	$200 + \frac{2}{3} X\, 250 = 366.7 kg$	0.35	128.345	1.75	225

–

Draw a couple polygon first, since it contains only one unknown and measure 1.28 m_B

1.28 m_B = 222.72 kg

$$m_b \square \frac{222.72}{1.28} \square 174kg$$

Substitute m_B value in force column and draw force polygon

Measure

0.8 m_A = 139.2

m_A = 139.2/0.8

= 174 kg

Measure =

175 degrees

from

cylinder1

3. The Centre distance between the cylinders of inner cylinder locomotive is 0.8m. It has a stroke of 0.5m. The rotating mass per cylinder is equivalent to 200kg at the crank pin and the reciprocating mass per cylinder is 240kg. The wheel Centre lines are 1.7m apart. The cranks are at right angle.

The whole of the rotating mass and 2/3rd of the reciprocating masses are balanced by masses at a radius of 0.6m. Find the magnitude and direction of the balancing masses.

Find the hammer blow, variation in tractive effort and the magnitude of the swaying couple at a speed of 300 rpm

Solution

Since whole of the rotating masses (m1) and 2/3rd of the reciprocating masses (m2) are balanced,

The total balancing mass / cylinder = m1 + Cm2

= 200 + 2/3 X 240

= 320 kg

To balance the above masses in cylinder B and C, we place balancing masses mA and m_D at a radius of 0.6m at an angle of Θ_A and Θ_D from the first crank B.

Procedure

1. Draw space diagram. Assume crank B as horizontal position and crank C is at 90 degrees to B.
2. Draw plane diagram. Take wheel A as reference plane (R.P)
3. Since couple column (mrl) contains only one unknown value (1.02mD m_D) we can draw couple polygon first to find m_D.

Plane	Mass m kg	Radius r m	Centrifugal force/	Distance from plane A l m	Couple/
A (R.P)	m_A	0.6	$0.6m_A$	0	0
B	320	0.25	80	1.45	36
C	320	0.25	80	1.25	100
D	m_D	0.6	$0.6\ m_D$	1.7	$1.02\ m_D$

In couple polygon,

The closing side CO

represents $1.02m_D = 106.3$

kg m ,

$m_D = 104.2$ kg

$\theta_D = 71 + 180 = 251\,^{\circ}$from B .

By knowing $m_D$value substitute in mr column and draw force polygon. In force polygon the closing side do represents $0.6\ m_A$ = 62.52 kg m

$$mA = \frac{62.52}{0.6} = 104.2kg$$

$\theta_A = 199$ degrees from B

In couple polygon,

The closing side CO

represents $1.02m_D = 106.3$

kg m ,

$m_D = 104.2$ kg

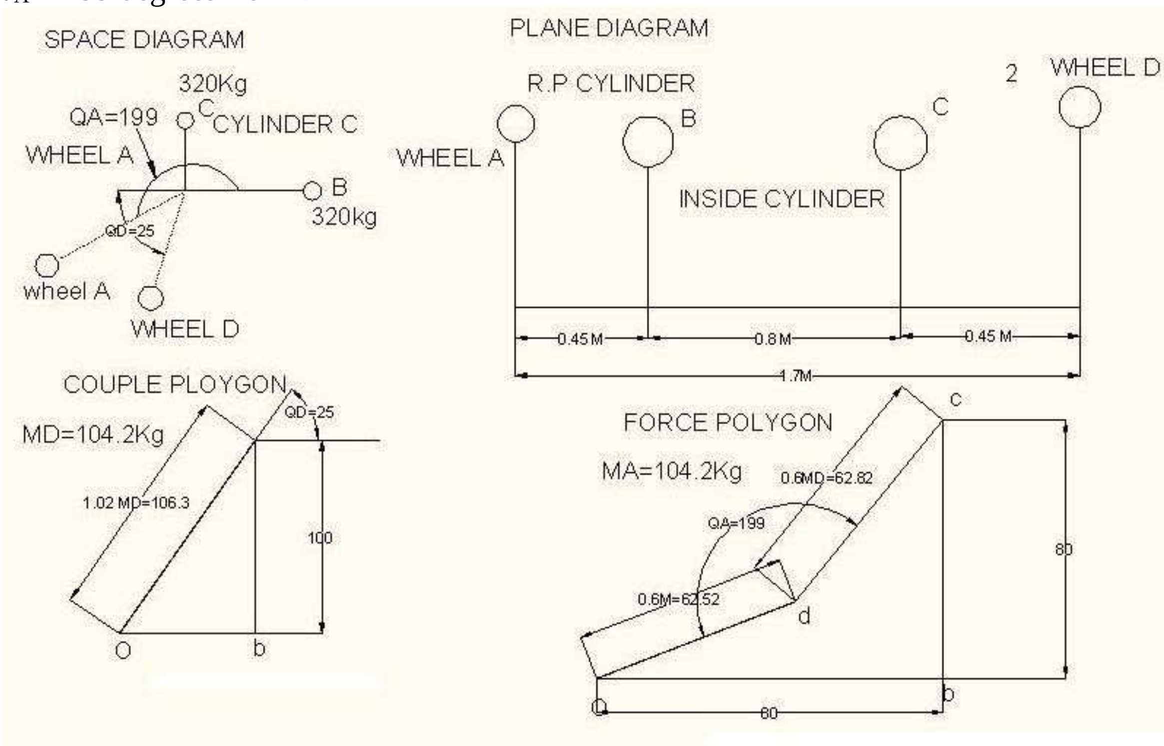
$\theta_D = 71 + 180 = 251\,^{\circ}$from B .

By knowing $m_D$value substitute in mr column and draw force polygon. In force polygon the closing side do represents $0.6\ m_A$

= 62.52 kg m

$$mA = \frac{62.52}{0.6} = 104.2kg$$

θ_A = 199 degrees from B



To find hammer blow,

Each balancing mass is 104.2 kg.

Total 320 kg is balanced by 104.2 kg . In this 104.2 kg we have to find out the contribution of reciprocating mass alone.

2/3 x 240 = 160 kg of reciprocating mass is balanced in 320 kg. So 320 kg mass contains 160 kg of reciprocating mass alone.

104.2 kg mass contains 160/320 X 104.2 = 52.1 kg of reciprocating mass alone.

Hence B = 52.1 kg ; b = 0.6 m

$\omega = 2\pi N/60 = 2x\pi x300/60 = 31.42$ rad/s

Hammer blow per cylinder = $B\omega^2 b$

$= 52.1 \times 31.42^2 \times 0.6$

$= 30852.4$ N

Variation in tractive effort

M_2 = reciprocating mass = 240 kg

Max variation of tractive effort

$$= \sqrt{2}(1 - C) m \omega^2 r$$

$$= \sqrt{2}\left(1 - \frac{2}{3}\right) 240 x 31.42^2 x 0.25$$

= 27923 N

Swaying couple

Max swaying couple

$$= \sqrt{2}(1 - C) m \omega^2 r \frac{a}{2}$$

$$= \sqrt{2}\left(1 - \frac{2}{3}\right) 240 x 31.42^2 x 0.25 x \frac{0.8}{2}$$

= 11169 Nm

4. A two cylinders uncoupled locomotive has inside cylinders 0.6m apart. The radius of each crank is 300mm and is at right angles. The revolving masses per cylinder are 250kg and the reciprocating mass per cylinder is 300kg. The whole of the revolving and 2/3rd of the reciprocating masses to be balanced and the balancing masses are to be placed in the plane of rotation of the driving wheels at a radius of 0.8m. The driving wheels are 2m in diameter and 1.5m apart. If the speed of the engine is 80kmph, find the hammer blow, max variation in tractive effort and max swaying couple.

Solution

Balancing masses are m_A and m_B

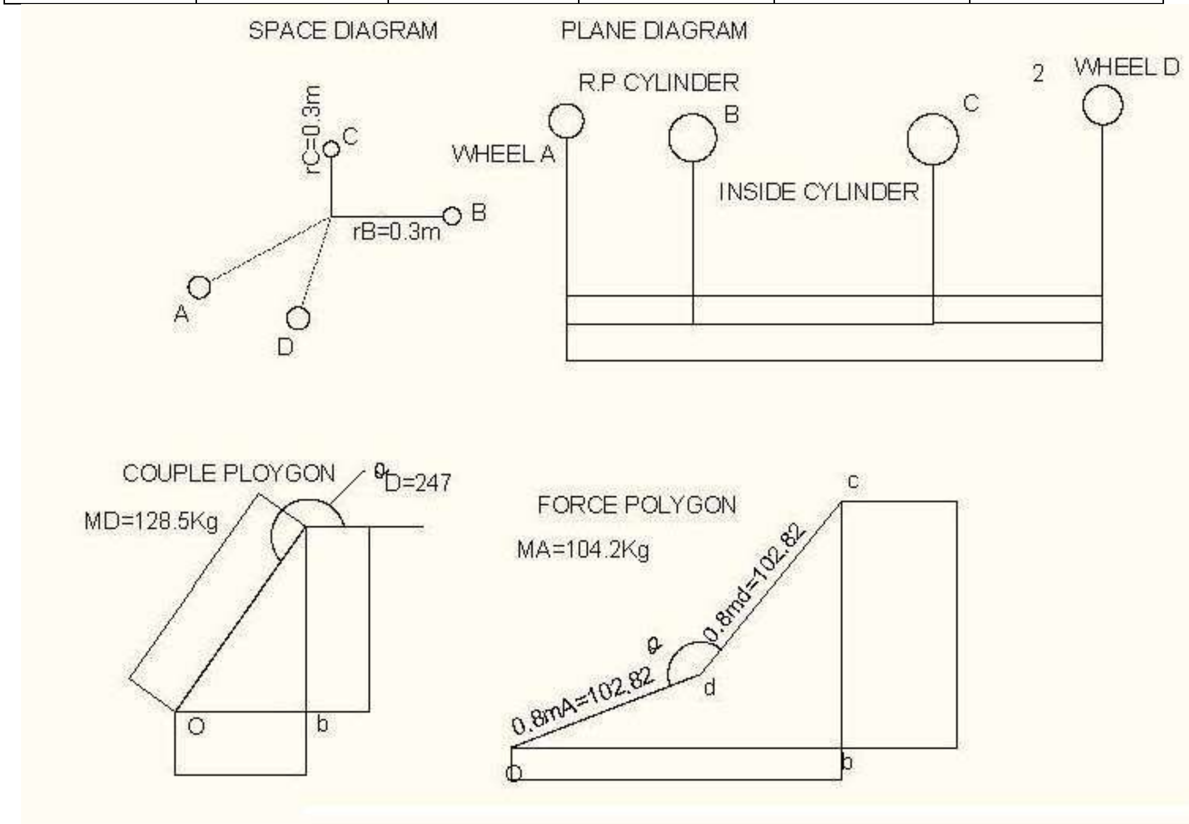
$m_A = m_B$ =Whole revolving mass + 2/3rd of reciprocating mass

= 250 + 2/3 x300 = 450 kg

Take A as reference plane

Plane	Mass in kg	r in m	Force/ω^2 = mr kg m	Distance from wheel A l m	Couple /ω^2 = mrl kg m^2
A	m_A	0.8	0.8m_A	0	0
B	450	0.3	135	0.45	60.75

C	450	0.3	135	1.05	141.75
D	m_D	0.3	$0.8m_D$	1.5	$1.2m_D$



Draw couple polygon first to find m_D and Θ_D. Then draw force polygon to find m_A and Θ_A.

From couple polygon,

$1.2m_D = 154.22$

$m_D = 128.52$ kg

$\Theta_D = 247$®

Substitute $m_D = 128.52$ kg in force column and draw force polygon.

From force

polygon, $0.8m_A =$

102.83 kgm mA =

128.52 kg

$\Theta_A = 203$®

To find hammer blow

450kg of mass contains 2/3 x 300 = 200kg of reciprocating mass alone.

128.52 kg mass contain = 200/450 x 128.52

= 57.12 kg

This balancing mass 57.12 kg is the cause for the hammer blow.

ꝏ, B = 57.12 kg and b = 0.8

m Speed of engine, V = 80

km/hr

= 22.22 m/s

Angular velocity (ω) = V /radius of wheel

= 22.22/1.5

= 14.815 rad/s

Hammer blow = B ω^2b

= 57.12 x 14.85^2x 0.8

= 10.029 N

To find max variation in tractive effort

$= \sqrt{2}(1 - C)\, m\, \omega^2 r$

$= \sqrt{2}(1 - \frac{2}{3})300 x 14.815^2 x 0.8$

= 9312 N

To find max swaying couple,

$= \frac{q}{\sqrt{2}}(1 - C)\, m\, \omega^2 r$

2

= 3951 Nm

5. The 3 cranks of a 3 cylinder locomotive are all on the same shaft and are set at 120 degrees. The distance between cylinders is 1m and the radius of the crank of the cylinder is 0.4m. The reciprocating masses are 300 kg for inside cylinder and 250 kg for outside cylinder and the plane of rotation of the balancing masses are 0.75m from the inside crank.

If 50 % of the reciprocating masses are balanced, determine the magnitude and the positioning of the balancing masses required at a radius of 0.6m. The hammer blow per wheel when the shaft makes 360 rpm. The speed in kmph at which the wheel will lift off the rails, when the load on each driving wheel is 100 KN and the diameter of the tread of wheel is 1m.

Solution

C = 0.5

Here we have to balance only reciprocating masses. Since 50 % of the reciprocating masses are to be balanced.

The reciprocating masses to be balanced for each outside cylinder

= $m_A = m_C$ = 0.5 X 250 = 125 kg

The reciprocating masses are to be balanced for inside cylinder

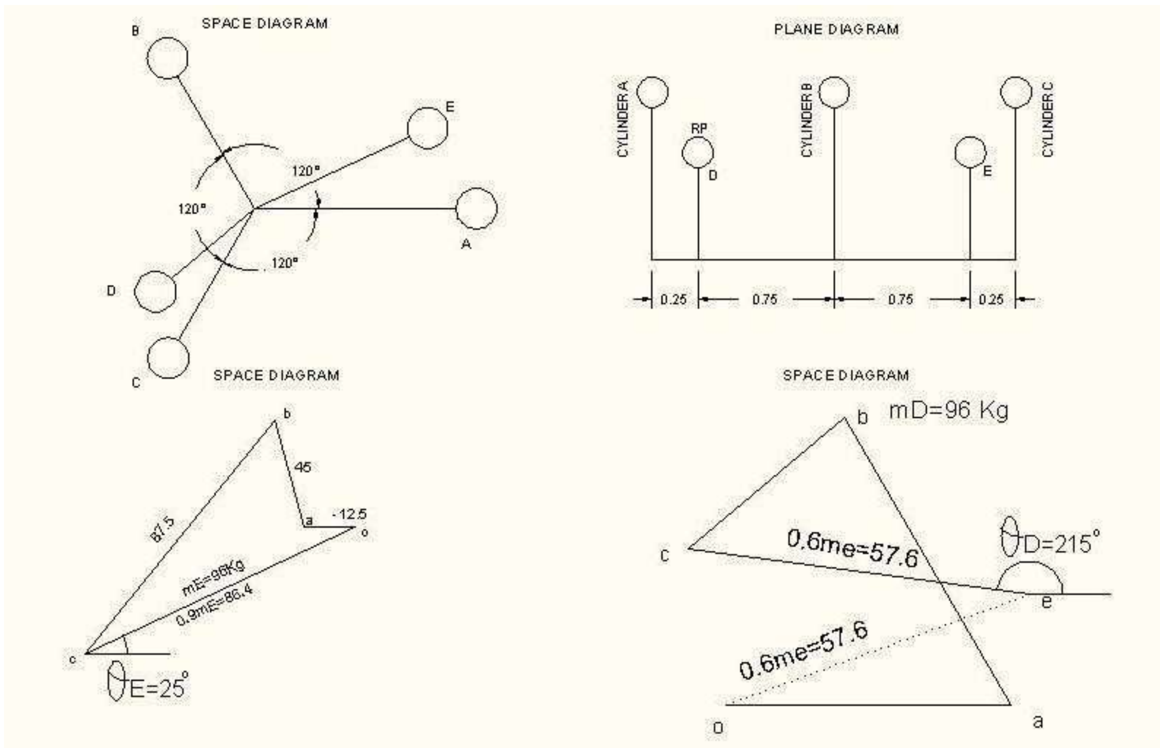
= m_B

= 0.5 X 300

= 150 kg

Take D as reference plane (R.P)

Plane	Mass in kg	r in m	Force/ω^2 = mr kg m	Distance from wheel A l m	Couple /ω^2 = mrl kg m^2
Outside cylinder A	125	0.4	50	-0.25	-12.5
D (R. P)	m_D	0.6	0.6m_D	0	0
Inside cylinder B	150	0.4	60	0.75	45
E	m_E	0.6	0.6m_E (57.6)	1.5	0.9mE
Outside cylinder C	125	0.4	50	1.75	87.5



Draw couple polygon first and the force polygon to find m_E and

m_D . By measurement m_E = 96 kg :m_D = 96 kg

Hammer blow (or) Fluctuation in rail pressure

In this problem rotating masses are not given. We have considered only the masses.

So the balancing masses D and E are used to balance only reciprocating masses

Here

$m_D = m_E$ balancing masses for reciprocating mass alone

= B = 96 kg also b = 0.6m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi x 360}{60} = 37.7 rad/s$$

Hammer blow per wheel = B ω^2b

= 96 x 37.7^2 x 0.6

= 81866.3 N

peed in km/hr at which the wheel will lift off the rails when the load on each driving wheel is 100 KN and the diameter of the tread of driving wheel is 1m.

$$\omega_{limiting} = \sqrt{\frac{W}{Bb}} = \sqrt{\frac{100x10^3}{96x0.6}} = 41.67 rad/s$$

$$V_{limiting} = \omega R = 41.67 X1 = 41.67 m/s$$

= 150 km/hr

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DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – III - FREE VIBRATION – SMEA1402

UNIT III

FREE VIBRATION

BASIC FEATURES OF VIBRATORY SYSTEMS

Periodic Motion

The motion which repeats after a regular interval of time is called periodic motion.

Frequency

The number of cycles completed in a unit time is called frequency. Its unit is cycles per second (cps) or Hertz (Hz).

Time Period

Time taken to complete one cycle is called periodic time. It is represented in seconds/cycle.

Amplitude

The maximum displacement of a vibrating system or body from the mean equilibrium position is called amplitude.

Free Vibrations

When a system is disturbed, it starts vibrating and keeps on vibrating thereafter without the action of external force. Such vibrations are called free vibrations.

Natural Frequency

When a system executes free vibrations which are undamped, the frequency of such a system is called natural frequency.

Forced Vibrations

The vibrations of the system under the influence of an external force are called forced vibrations. The frequency of forced vibrations is equal to the forcing frequency.

Resonance

When frequency of the exciting force is equal to the natural frequency of the system it is called resonance. Under such conditions the amplitude of vibration builds up dangerously.

Degree of Freedom

The degree of freedom of a vibrating body or system implies the number of independent coordinates which are required to define the motion of the body or system at given instant.

LUMPED MASS PARAMETER SYSTEMS

Instead of considering distributed mass, a lumped mass is easier to analyse, whose dynamic behaviour can be determined by one independent principal coordinate, in a single degree freedom system. It is important to study the single degree freedom system for a clear understanding of basic features of a vibration problem.

Elements of Lumped Parameter Vibratory System

The elements constituting a lumped parameter vibratory system are :

The Mass

The mass is assumed to be rigid and concentrated at the centre of gravity.

The Spring

It is assumed that the elasticity is represented by a helical spring. When deformed it stores energy. The energy stored in the spring is given by

$$PE = \frac{1}{2} k x^2$$

where*k* is stiffness of the spring. The force at the spring is given by

F= k x

The springs work as energy restoring element. They are treated massless.

The Damper

In a vibratory system the damper is an element which is responsible for loss of energy in the system. It converts energy into heat due to friction which may be either sliding friction or viscous friction. A vibratory system stops vibration because of energy conversion by damper. There are two types of dampers.

Viscous Damper

A viscous damper consists of viscous friction which converts energy into heat due to this. For this damper, force is proportional to the relative velocity.

F_d α relative velocity (v)

$F_d = cv$

where*c* is constant of proportionality and it is called coefficient of damping.

The coefficient of viscous damping is defined as the force in '*N*' when velocity is 1 m/s.

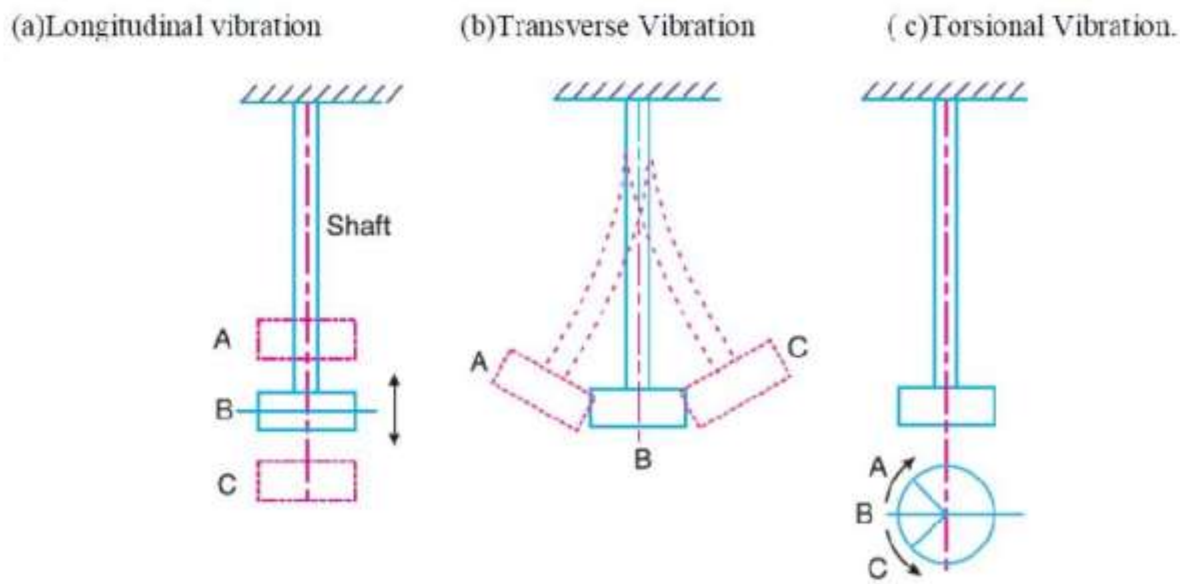
Coulumb's Damper

The dry sliding friction acts as a damper. It is almost a constant force but direction is always opposite to the sliding velocity. Therefore, direction of friction changes due to change in direction of velocity.

DEGREES OF FREEDOM

The number of independent coordinates required to completely define the motion of a system is known as degree of freedom of the system.

FREE VIBRATION OF LONGITUDINAL, TRANSVERSE AND TORSIONAL SYSTEMS OF SINGLE DEGREE OF FREEDOM



B = Mean position ; A and C = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

Longitudinal Vibration: When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations known as longitudinal vibrations.

Transverse Vibration: When the particles of the shaft or disc moves approximately perpendicular to the axis of the shaft, then the vibrations known as transverse vibrations.

Torsional Vibration: When the particles of the shaft or disc move in a circle about the axis of the shaft, then the vibrations known as torsional vibration

Natural frequency of free undamped longitudinal vibration:

Equilibrium method or Newton's method

Consider a constraint (i.e.Spring) of negligible mass in an unstrainded.

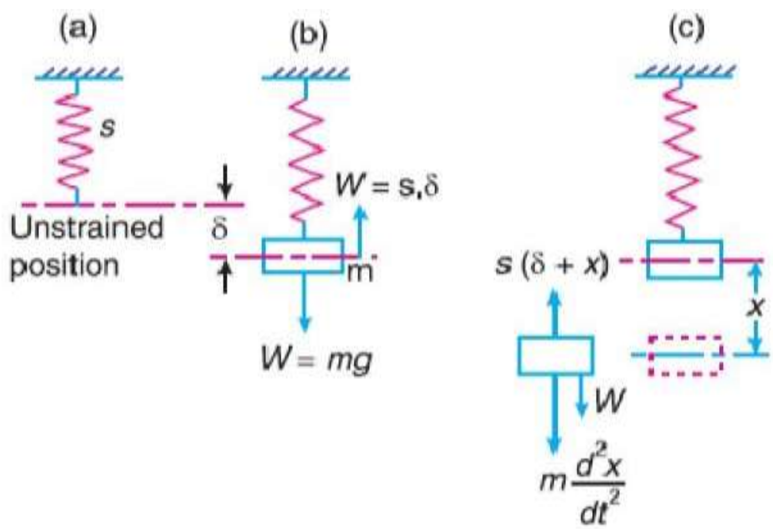
Let S = Stiffness of the constraint. It is the force required to produce unit displacement in the direction of vibration. It is usually expressed in N/ M.

m = Mass of the body suspended from the constraint in kg,

W= Weight of the body in newtons = m.g,

δ = Static deflection of the spring in metres due to weight W newtons, and

x = Displacement given to the body by the external force, in metres.



Natural frequency of free longitudinal vibrations.

In the equilibrium position, as shown in Fig, The gravitational pull W = m.g, is balanced by a force of spring , such that W = s.δ.

Since the mass is now displaced from its equilibrium position by a distance x, as shown in Fig .(c), and is then released, therefore after time t,

$$\text{Restoring force} = W - s(\delta + x) = W - s.\delta - s.x$$

$$= s.\delta - s.\delta - s.x = -s.x \quad \ldots (\because W = s.\delta) \quad \ldots (i)$$

and Accelerating force = Mass × Acceleration

$$= m \times \frac{d^2x}{dt^2} \quad \text{. . . (Taking downward force as positive) . . . } (ii)$$

Equating equations *(i)* and *(ii)*, the equation of motion of the body of mass m after time t is

$$m \times \frac{d^2x}{dt^2} = -s.x \quad \text{or} \quad m \times \frac{d^2x}{dt^2} + s.x = 0$$

$$\therefore \quad \frac{d^2x}{dt^2} + \frac{s}{m} \times x = 0 \quad \text{. . . } (iii)$$

We know that the fundamental equation of simple harmonic motion is

$$\frac{d^2x}{dt^2} + \omega^2.x = 0 \quad \text{. . . } (iv)$$

Comparing equation (iii) and (iv), we have

$$\omega = \sqrt{\frac{s}{m}}$$

$$\text{Time period,} \quad t_p = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{s}}$$

$$\text{and natural frequency,} \quad f_n = \frac{1}{t_p} = \frac{1}{2\pi}\sqrt{\frac{s}{m}} = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \quad \text{. . . } (\because \ m.g = s.\delta)$$

Taking the value of g as 9.81 m/s^2 and δ in metres,

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{Hz}$$

ii) ENERGY METHODS :

In Free vibrations, no energy is transferred in to the system or from the system. Therefore, the total energy (sum of KE and PE) is constant and is same all the times.

$$\frac{d}{dt}(K.E. + P.E.) = 0$$

We know that K.E = $\frac{1}{2}$ m^2

$= \frac{1}{2}$ m(dx / dt)^2 i

And P.E = Mean force X Displacement

$= \frac{\text{force at A+force at B}}{2}$ X Displacement

$= \frac{0+S.x}{2}$ x

$= \frac{1}{2}$ s x^2 ii

Substituting equations (ii) and (iii) in equation (i), we get

$$\frac{d}{dt}\left[\frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}\cdot s\,x^2\right] = 0$$

$$\text{or}\left[\frac{1}{2}\times m\times 2\times\frac{dx}{dt}\times\frac{d^2x}{dt^2}\right] + \left[\frac{1}{2}\times s\times 2\,x\times\frac{dx}{dt}\right] = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{s}{m}\cdot x = 0}$$

This is the same differential equation as obtained by newton's

Problem :

A car having a mass of 100 kg deflects its spring 4 cm under its load. Determine the natural frequency of the car in the vertical direction.

Solution :

m = 1000 kg; δ = 4 cm = 0.04m.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{1}{2\pi}\sqrt{\frac{9.81}{0.04}} = 2.492Hz$$

The damping force per unit velocity is known as <u>damping coefficient.</u>

WHIRLING OF SHAFTS AND CRITICAL SPEED

The speed at which resonance occurs is called critical speed of the shaft. In other words, the speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical speed.

Critical speed occurs when the speed of the rotation of shaft is equal to the natural frequency of the lateral vibration of shafts, At this speed shaft starts to vibrate violently in the transverse direction. Whirling is defined as the rotation of the plane created by bent shaft and the line of centre of bearings.

The excessive vibrations associated with critical speeds may cause permanent deformation resulting in structural damage. Example: The rotor blades of a turbine may come in contact with stator blades. Larger shaft deflections produce larger bearing reactions, which may lead to bearing failure. The amplitude build up is a time dependent phenomenon and therefore, it is very dangerous to continue to run the shaft at it critical speed.

The critical speed essentially depends on

a) The eccentricity of the C.G of the rotating masses from the axis of rotation of the shaft.
b) Diameter of the disc
c) Span (length) of the shaft, and
d) Type of supports connections at its ends.

In a shaft whirling situation, let y=additional displacement of C.G from axis of rotation due to centrifugal force. Discuss as to how 'y' varies with respect to the operating speed and critical speed.

Solution : $y = \dfrac{\pm e}{\left(\dfrac{N_c}{N}\right)^2 - 1}$

If the operating sped is equal to critical speed, then

$\left(\dfrac{N_c}{N}\right)^2$ - 1 = 0 ; hence y = 0 ; deflection is zero.

and $N_c = \dfrac{0.4985}{\sqrt{\delta}} rps$

Critical speed of shaft is the same as the natural frequency of transverse vibration – Justify.

We know that critical or whirling speed $\omega_c = \omega_n$

$$\omega_c = \omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\delta}} Hz$$

If N_c is the critical speed in rps, then

$$2\pi N_c = \sqrt{\frac{g}{\delta}} \Rightarrow N_c = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} = \frac{0.4985}{\sqrt{\delta}}\ rps$$ Hence proved.

PROBLEM

A shaft of diameter 10 mm carries at its centre a mass of 12 kg. It is supported by two short bearings, the centre distance of which is 400 mm. Find the whirling speed: (i) Neglecting the mass of the shaft, and (ii) considering the mass of the shaft. The density of shaft material is 7500 kg/m^3. Take E = 200 GN/m^2.

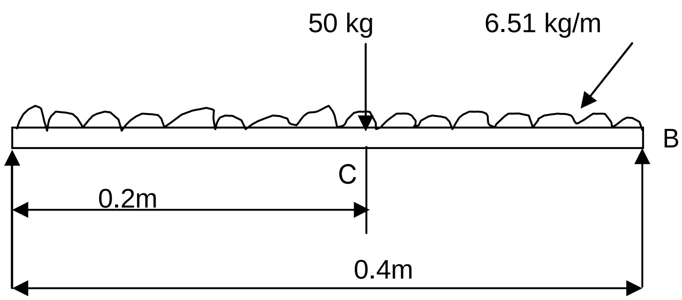
Given Data: **d = 10mm = 0.01m;**

m = 12 kg;

ρ = 7500 kg/m^3;

E = 200 GN/m^2

= 200 x 10^9 N/m^2



To find Whirling speed by (i) neglecting the mass of the shaft, and (ii) considering the mass of the shaft

Solution:

A shaft supported in short bearings is assumed to be a simply supported beam. The given shaft is shown in figure.

Since the density of shaft material is given as 7500 kg/m^3, therefore mass of the shaft per metre length,

m_s = Area × Length × Density

$= \frac{\pi}{4}(0.01)^2 \times 1 \times 7500 = 0.589 kg/m$

Moment of inertia, $I = \frac{\pi}{64} d^4 = \frac{\pi}{64}(0.01)^4 = 4.91 \times 10^{-10} m^4$

We known the static deflection due to a mass of 12 kg at C,

$$\delta_1 = \frac{Wa^2b^2}{3EI.l}$$

$$= \frac{(12 \times 9.81)(0.2)^2(0.2)^2}{3 \times 200 \times 10^9 \times 4.91 \times 10^{-10} \times 0.4}$$

$$= 1.598 \times 10^{-3} \text{m}$$

......[Here a = 0.2m, and b=0.2m]

and the static deflection due to mass of the shaft,

$$\delta_s = \frac{5}{384} \times \frac{wl^4}{EI} = \frac{5}{384} \times \frac{(0.589 \times 9.81)(0.4)^4}{200 \times 10^9 \times 4.91 \times 10^{-10}}$$

$$= 1.961 \times 10^{-5} \text{m}$$

(i) Neglecting the mass of the shaft:

The natural frequency of transverse vibrations is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1}} = \frac{0.4985}{\sqrt{1.598 \times 10^{-3}}} = 12.47 \text{Hz}$$

We know that whirling speed (N_{cr}) of the shaft in r.p.s is equal to the frequency of transverse vibration in Hz.

N_{cr} = 12.47 r.p.s. = 12.47 x 60 = **748.22 r.p.m**

(ii) Considering the mass of the shaft:

The natural frequency of transverse vibrations is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \frac{\delta_s}{1.27}}}$$

$$= \frac{0.4985}{\sqrt{1.598 \times 10^{-3} + \left(\frac{1.961 \times 10^{-5}}{1.27}\right)}}$$

$$= 12.41 \text{Hz}$$

Therefore, whirling speed,

N_{cr} = 12.41 r.p.s. = 12.41 x 60 = **744.63 r.p.m**

Dunkerley’s Method

theDunkerley’s method used in natural transverse vibration for a shaft carrying a number of point loads and uniformly distributed load is obtained by Dunkerley’s formula.

Dunkerley’sFormula :

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Dunkerley’sFormula :

$$\frac{1}{f_n^2} = \frac{1}{f_{n1}^2} + \frac{1}{f_{n2}^2} + \frac{1}{f_{n3}^2} + \ldots\ldots + \frac{1}{f_{ns}^2}$$

f_{n1}, f_{n2}, f_{n3}etc : Natural frequency of transverse vibration at each point loads, and

f_{ns} = Natural frequency of transverse vibration of the UDL.

PROBLEM

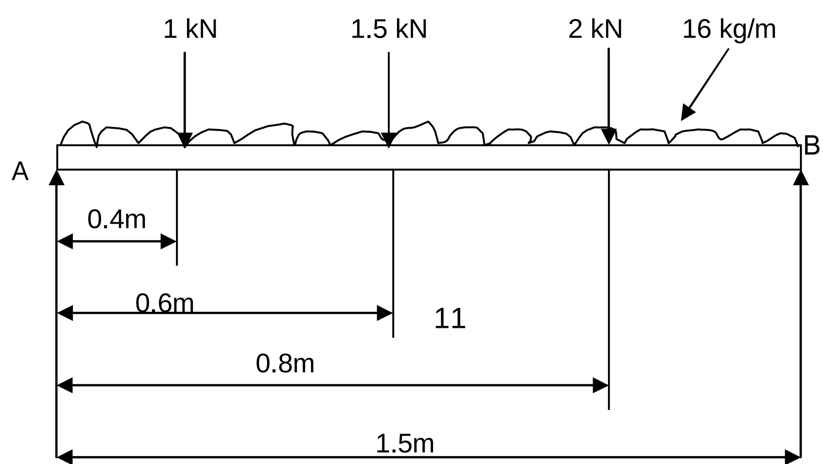
A shaft 30mm diameter and 1.5 long has a mass of 16 kg per metre length. It is simply supported at the ends and carries three isolated loads 1 kN, 1.5 kN and 2 kN at 0.4 m, 0.6 m and 0.8 m respectively from the left support. Find the frequency of the transverse vibrations: 1. Neglecting the mass of the shaft, and 2. considering the mass of the shaft. Take E = 200 GPa.

Given Data:

d = 30 mm = 0.03m; l = 1.5m;

m = 16 kg/m; E=200 GPa = 200 x 10^9N/m^2

To Find: Frequency of the transverse vibrations by (a) neglecting the mass of the shaft, and (b) considering the mass of the shaft.



Solution:

The shaft carrying the loads is shown in figure

Moment of inertia of the shaft,

$$I = \frac{\pi}{64} d^4$$

$$= \frac{\pi}{64} (0.03)^4 = 3.976 \times 10^{-8} m^4$$

Static deflection due to a load of 1 kN,

$$\delta_1 = \frac{Wa^2b^2}{3EI.l} = \frac{1000(0.4)^2 (1.1)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$

$$= 5.41 \times 10^{-3} m$$

....[Here a=0.4m, and b=1.1m]

Similarly, static deflection due to a load of 1.5 kN,

$$\delta_2 = \frac{Wa^2b^2}{3EI.l} = \frac{1500(0.6)^2 (0.9)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$

$$= 0.01222 m$$

....[Here a=0.6m, and b=0.9m]

Static deflection due to a load of 2 kN,

$$\delta_3 = \frac{Wa^2b^2}{3EI.l} = \frac{2000(0.8)^2 (0.7)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-8} \times 1.5}$$

$$= 0.01752 m$$

....[Here a=0.8m, and b=0.7m]

and static deflection due to the mass of the shaft (i.e., a udl)

$$\delta_s = \frac{5}{384} \times \frac{wl^4}{EI} = \frac{5}{384} \times \frac{(16 \times 9.81)(1.5)^4}{200 \times 10^9 \times 3.976 \times 10^{-8}}$$

$$= 1.301 \times 10^{-3} m$$

(a) Neglecting the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}}$$

$$= \frac{0.4985}{\sqrt{5.41 \times 10^{-3} + 0.01222 + 0.01752}}$$

$$\mathbf{= 2.659 Hz}$$

(b) Considering the mass of the shaft:

The natural frequency of transverse vibrations, according to the Dunkerley's equation is given by

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \frac{\delta_s}{1.27}}}$$

$$= \frac{0.4985}{\sqrt{5.41 \times 10^{-3} + 0.01222 + 0.01752 + \left(\frac{1.301 \times 10^{-3}}{1.27}\right)}}$$

$$= \mathbf{2.62 Hz}$$

TORSIONAL VIBRATION OF TWO AND THREE ROTOR SYSTEM

Torsional vibrations.

When the particles of a shaft or disc move in a circle about the axis of the shaft, then the vibrations are known as torsional vibrations.

Differentiate between transverse and torsional vibrations.

1. In transverse vibrations, the particles of the shaft move approximately perpendicular to the axis of the shaft. But in torsional vibrations, the particles of the shaft move in a circle about the axis of the shaft.
2. Due to transverse vibrations, tensile and compressive stresses are induced. Due to torsional shear stresses are induced in the shaft.

The expression for natural frequency of free torsional vibration (a) without considering the effect of inertia of the constraint, and (b) considering the effects of inertia of the constraints :

a) without considering the effect of the inertia of constraint

natural frequency of torsional vibration $f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{q}{I}}$

b) with the effect of the inertia of constraint

$$f_x = \frac{1}{2\pi}\sqrt{\frac{q}{1+\left(\frac{I_c}{3}\right)}}$$

where q = Torsional stiffness of shaft in N-m

I = Mass moment of inertia of disc in kg-m^2 = m k^2, and

I_c = Mass moment of inertia of constraint (i.e) shaft of spring etc.

Expression for the frequency of vibration of a two rotor system.

Natural frequency of torsional vibration, $f_x = \dfrac{1}{2\pi}\sqrt{\dfrac{CJ}{Il}}$

Where C = rigidity modulus of shaft,
I = Mass M.I of rotor
J = Polar M.I of shaft, and
l = Length of node from rotor.

Two equations for two rotor system are :

1) $l_A I_A = l_B I_B$ and (2) $l = l_A + l_B$
2) From equation (1), if I_B value is large, $\left(\text{then } l_B = \frac{l_A \cdot I_A}{I_B}\right)$; then l_B value will be lesser that the value of l_A. It means that the rotor having larger mass moment of inertia will have node closer to it.

The equation used to obtain torsionally equivalent shaft of a stepped shaft.

The total angle of twist of the shaft is equal to the sum of the angle of twists of different lengths.

So $\theta = \theta_1+\theta_2+\theta_3$

Or

$$\frac{Tl}{CJ} = \frac{Tl_1}{CJ_1} + \frac{Tl_2}{CJ_2} + \frac{Tl_3}{CJ_3}$$

$$\frac{l}{J} = \frac{l_1}{J_1} + \frac{l_2}{J_2} + \frac{l_3}{J_3}$$

$$\frac{l}{\frac{\pi}{32} d^4} = \frac{l_1}{\frac{\pi}{32} d_1^4} + \frac{l_2}{\frac{\pi}{32} d_2^2} + \frac{l_3}{\frac{\pi}{32} d_3^2}$$

→ Let d = d1, then

$$l = l_1 + l_2\left(\frac{d_1}{d_2}\right)^4 + l_3\left(\frac{d_1}{d_3}\right)^4$$

where l = Length of a torsionally equivalent shaft.

PROBLEM

A shaft of 100mm diameter and 1 metre long is fixed at one end and the other end carries a flywheel of mass 1 tonne. The radius of gyration of the fly wheel is 0.5m. Find the frequency of torsional vibrations. If the modulus of rigidity for the shaft material is 80 GN/m^2

Given data:

D=100mm=0.1m;l=1m;m=1tonne=1000kg;

K=0.5m;C=80GN/m^2 =80x10^9N/m^2.

To find:

Frequency of torsional vibrations (f_n).

Solution:

We know that polar moment of inertia of the shaft.

$$j = \frac{\pi}{32} \times d^4 = \frac{\pi}{32}(0.1)^4 = 9.82 \times 10^{-6} m^4$$

$\therefore$ Torsional stiffness of the shaft is given by

$$q = \frac{C.J}{l} = \frac{80 \times 10^9 (9.82 \times 10^{-6})}{1} = 785.6 \times 10^3 \ N - m.$$

We also know that mass moment of inertia of the shaft.

1=m.k^2=1000$(0.5)^2$=250kg-m^2.

$\therefore$ Frequency of torsional vibrations.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{q}{l}} = \frac{1}{2\pi}\sqrt{\frac{785.6 \times 10^3}{250}} = 8.922 Hz.$$

Damped free vibration

It is the resistance to the motion of a vibrating body. The vibrations associated with this resistance are known as damped vibrations.

Types of damping:

(1) Viscous damping
(2) Dry friction or coulomb damping
(3) Solid damping or structural damping
(4) Slip or interfacial damping.

Damping ratio (ζ).

It is defined as the ratio of actual damping coefficient (c) to the critical damping coefficient (C_C).

Mathematically, Damping ratio $\varsigma = \dfrac{c}{c_c} = \dfrac{c}{2m\omega_n}$

logarithmic decrement.

Logarithmic decrement is defines as the natural logarithm of the amplitude reduction factor. The amplitude reduction factor is the ratio of any two successive amplitudes on the same side of the mean position.

$$\therefore = \delta = \log_e\left(\frac{x_1}{x_2}\right) \quad \log_e\left(\frac{x_n}{x_{n+1}}\right)$$

It is a convenient way to measure the amount of damping is to measure the rate of decay of oscillation. This is best expressed by the term L.D. it is defined as the natural logarithm of the ratio of any two successive amplitudes.

A vibrating system consist of a mass of 7 kg and a spring stiffness 50 N/cm and damper of damping coefficient 0.36 Ncm^{-1} sec. find damping factor.

Solution :

m= 7 kg; s=50 N/cm = 5000 N/m ; c = 0.36 N/cm/sec = 36 N/m/sec

$$\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{5000}{7}} = 26.72 \text{ rad/sec}$$

$c_c = 2\ m\ \omega_n = 2 \text{ X } 7 \text{ X } 26.72 = 374.16$ N/m/s

Damping factor = $\frac{c}{c_c} = 0.0962$

PROBLEM

A vibrating system consists of a mass of 8 kg, spring of stiffness 5.6 N/mm and a dashpot of damping coefficient of 40 N/m/s. Find:

- **(a) the critical damping coefficient,**
- **(b) the damping factor,**
- **(c) the natural frequency of damped vibration,**
- **(d) the logarithmic decrement,**
- **(e) the ratio of two consecutive amplitudes, and**
- **(f) the number of cycles after which the original amplitude is reduced to 20 percent**

Given data:

m = 8 kg; s = 5.6 N/mm = 5.6 x 10^3 N/m; C = 40 N/ m/ s

Solution:

(a) Critical damping coefficient (c_c):

We know that

$$c_c = 2m\omega_n = 2m\sqrt{\frac{s}{m}} = 2\sqrt{s.m}$$

$$c_c = 2\sqrt{5.6\times10^3\times8} = \mathbf{422.32 N/m/s}$$

(b) Damping factor (ζ):

We know that,

Damping factor, $\zeta = \frac{c}{c_c} = \frac{40}{422.32} = \mathbf{0.0945}$

(c) Natural frequency of damped vibration (f_d):

We know that the circular frequency of damped vibrations,

$$\omega_d = \sqrt{1-\zeta^2}\,.\omega_n$$

where $\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{5.6\times10^3}{8}} = 26.34\,\text{rad/s}$

$$\therefore \omega_d = \sqrt{1-(0.0945)^2} \times 26.34 = 26.22\,\text{rad/s}$$

$\therefore$ Natural frequency of damped vibration of the system

$$f_d = \frac{\omega_d}{2\pi} = \frac{26.22}{2\pi} = \mathbf{4.173Hz}$$

(d) Logarithmic decrement (δ):

We know that logarithmic decrement,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.0945)}{\sqrt{1-(0.0945)^2}} = \mathbf{0.596}$$

(e) Ratio of two consecutive amplitudes $\left(\frac{x_n}{x_{n+1}}\right)$**:**

Let x_n and x_{n+1} = Magnitudes of two consecutive amplitudes

The logarithmic decrement can also be given by

$$\delta = \ln\left[\frac{x_n}{x_n+1}\right] \text{ or } \frac{x_n}{x_n+1} = e^{\delta}$$

$$\frac{x_n}{x_n+1} = e^{0.596} = \mathbf{1.8156}$$

(f) Number of cycles after which the amplitude is reduced to 20% (n):

Let x_1 = Amplitude at the starting position

x_n = Amplitude after n cycle = 20% x_1 = $0.2x_1$

The logarithmic decrement in terms of number of cycles (n) is given by

$$\delta = \frac{1}{n}.\ln\left(\frac{x_1}{x_n}\right)$$

or

$$0.596 = \frac{1}{n}.\ln\left(\frac{x_1}{0.2x_1}\right)$$

or $\mathbf{n = 2.7 cycles}$

PROBLEM

A vibrating system consists of a mass of 20 kg, a spring of stiffness 20 kN/m and a damper. The damping provided is only 30% of the critical value. Determine:

- **(i)** **the damping factor,**
- **(ii)** **the critical damping coefficient,**
- **(iii)** **the natural frequency of damped vibrations,**
- **(iv)** **the logarithmic decrement, and**
- **(v)** **the ratio of the consecutive amplitudes**

Given Data:

m = 20kg; s = 20kN/m = 20 x 10^3N/m; c = 30%; c = 0.3c_c

Solution:

(i) Damping factor (ζ):

We know that,

Damping factor, $\zeta = \frac{c}{c_c} = \frac{0.3c_c}{c_c} = \mathbf{0.3}$

(ii) Critical damping coefficient (c_c):

The critical damping coefficient is given by

$$c_c = 2\sqrt{s.m} = 2\sqrt{20\times10^3\times20}$$
$$= \mathbf{1264.91 N/m/s}$$

(iii) Natural frequency of damped vibrations (f_d):

We know that the circular frequency of damped vibrations,

$$\omega_d = \sqrt{1-\zeta^2}\,.\omega_n$$

where $\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{20\times10^3}{20}} = 31.622\text{rad/ s}$

$$\therefore \omega_d = \sqrt{1-(0.3)^2}.31.622 = 30.165\text{rad/s}$$

Therefore the natural frequency of damped vibrations is given by

$$f_d = \frac{\omega_d}{2\pi} = \frac{30.165}{2\pi} = \mathbf{4.8Hz}$$

(iv) Logarithmic decrement (δ):

We know that logarithmic decrement,

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.3)}{\sqrt{1-(0.3)^2}} = \mathbf{1.976}$$

(v) Ratio of two consecutive amplitudes:

Let x_n and x_{n+1} = Magnitudes of two consecutive amplitudes

The logarithmic decrement can also be given by

$$\delta = \ln\left[\frac{x_n}{x_n+1}\right] \text{or } \frac{x_n}{x_n+1} = e^{\delta}$$

$$\frac{x_n}{x_n+1} = e^{1.976} = \mathbf{7.213}$$

PROBLEM

An instrument vibrates with a frequency of 1.24 Hz when there is no damping. When the damping is provided, the frequency of damped vibrations was observed to be 1.03 Hz. Find:

- **(i) the damping factor, and**
- **(ii) the logarithmic decrement**

Given data: f_n = 1.24 Hz; f_d = 1.03 Hz

Solution:

(i) Damping factor (ζ):

We know that the natural circular frequency of undamped vibrations,

$$\omega_n = 2\pi \times f_n$$
$$= 2\pi \times 1.24 = 7.791 \text{ rad/s}$$

and circular frequency of damped vibrations,

$$\omega_d = 2\pi \times f_d$$
$$= 2\pi \times 1.03 = 6.472 \text{ rad/s}$$

We also know that circular frequency of damped vibrations (ω_d);

$$\omega_d = \sqrt{1-\zeta^2} \times \omega_n$$

or $$6.472 = \sqrt{1-\zeta^2} \times 7.791$$

$\therefore$ Damping factor, ζ = **0.556**

(ii) Logarithmic decrement (δ):

We know that the logarithmic decrement (δ) in terms of damping factor (ζ),

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} = \frac{2\pi(0.556)}{\sqrt{1-(0.556)^2}} = \mathbf{4.21}$$

SCHOOL OF MECHANICAL ENGINEERING

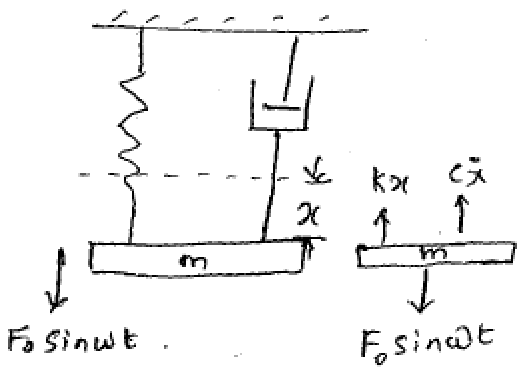
DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – IV - FORCED VIBRATION – SMEA1402

UNIT - IV

Force Vibration with Harmonic Excitation

Consider a spring mass system with viscous damping and subjected to a harmonic excitation $F_o \sin \omega t$, in which F_o is constant. Consider the mass to be displaced by a distance 'x' downwards with respect to static equilibrium position as the reference line. Selection of static equilibrium position as the reference line eliminates the need to consider the weight of the mass [which is nullified by spring force due to deflection] in the free body diagram.



The type of vibration which occurs under the influence of external force, is called "FORCED VIBRATION". The external force is called External excitation.

The excitation may be periodic, impulsive or random in nature.

Sources of Excitation

- Thermal effect, (un even expansion of embers give rise to unbalance,
- Resonance (large amplitudes), loose or defective mating part, bent shaft (because of critical speeds)

From Neωton's 2^{nd} law of motion.

Rate of change of momentum = - $F_s = F_d + F_o$ Sinωt

i.e, $m\ddot{x} = -kx - c\dot{x} + F_o \sin\omega t$

$\Rightarrow \boxed{m\ddot{x} + c\dot{x} + kx = F_o \sin\omega t}$ Governing differential equation … (1)

⇑ Governing Differential Equation Eqn (1) is Non-homogeneous, 2^{nd} order Diff. eqn of motion. The complete solution consists of two parts (i) complementary function part [x_c] (ii) particular integral part [x_p]

We know, that

$X_c = x_e X_c = X_e^{-} \zeta w_n t \quad \sin[\omega_d + \psi]$

where $\omega_d = \omega_n \sqrt{1-\zeta^2}$

Due to exponential decay, x_c dies out eventually. This part is called **'transient motion'** particular integral part [x_p]

$x_p = X \sin[\omega t + \psi]$ …(2)

Where $\boxed{X = \frac{F_o}{\sqrt{[k-m\omega^2]^2+[c\omega]^2}}}$ …(2A)

The amplitude of x of the particular Internal Part Replace ω by does not depend on time. In other words, the amplitude of vibration represented by X_p does not change with time and therefore it is called steady state motion.

Also $\tan\psi = \frac{2\zeta r}{1-r^2}$ …(3)

Where r = frequency ration = $\frac{\omega}{\omega_n}$ & ψ = Phase angle

(Note $\omega < \omega_n$ the $r < 1$
$\omega < \omega_n$ the $r > 1$

Equation (2A) can be written as,

$$\boxed{X = \frac{x_o}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}} \qquad \ldots(4)$$

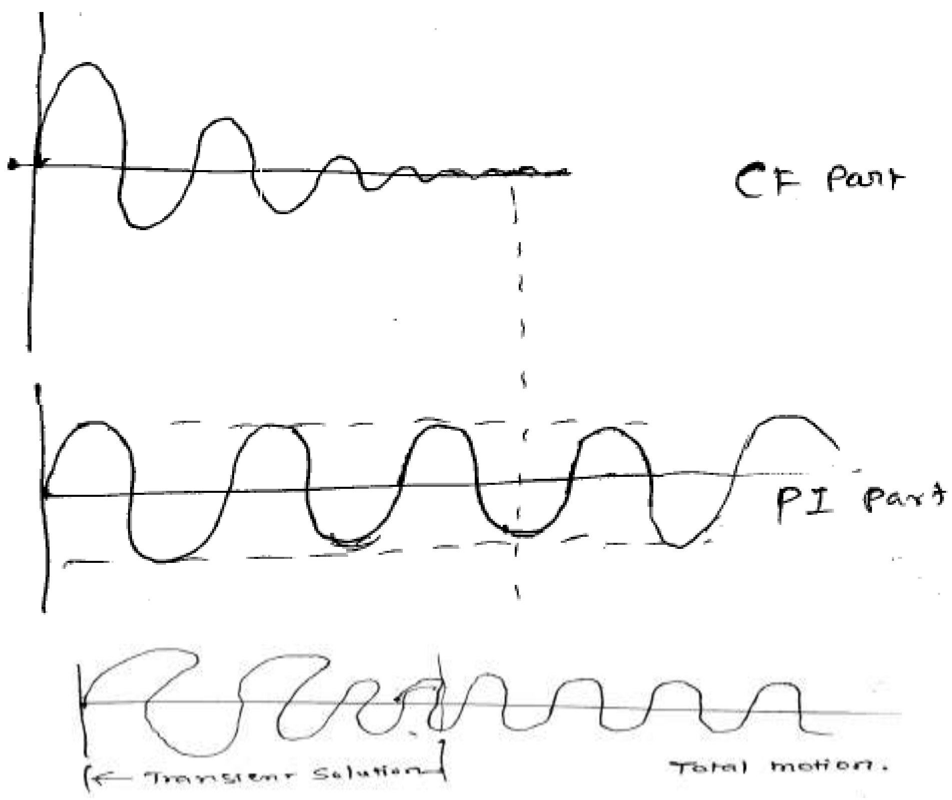
Let $M = \frac{x}{x_o} \rightarrow$ Magnification Factor

Then $M = \frac{x}{x_o} = \frac{1}{(1-r^2)^2+(2\zeta r)^2} \qquad \ldots(5)$

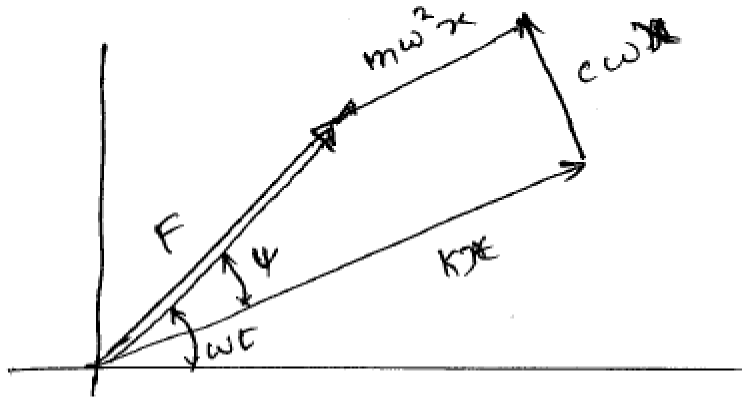
Note 1 : Eqn. (5) has been got from (2A), as follows. In Eqn. 2A ÷ by k (spring stiffness) in both Nr & Dr and be simplified.

Note 2 : Magnification factor : (m) : It is the ratio of maximum displacement of forced vibration to the static deflection due to static force.

Note 3 :



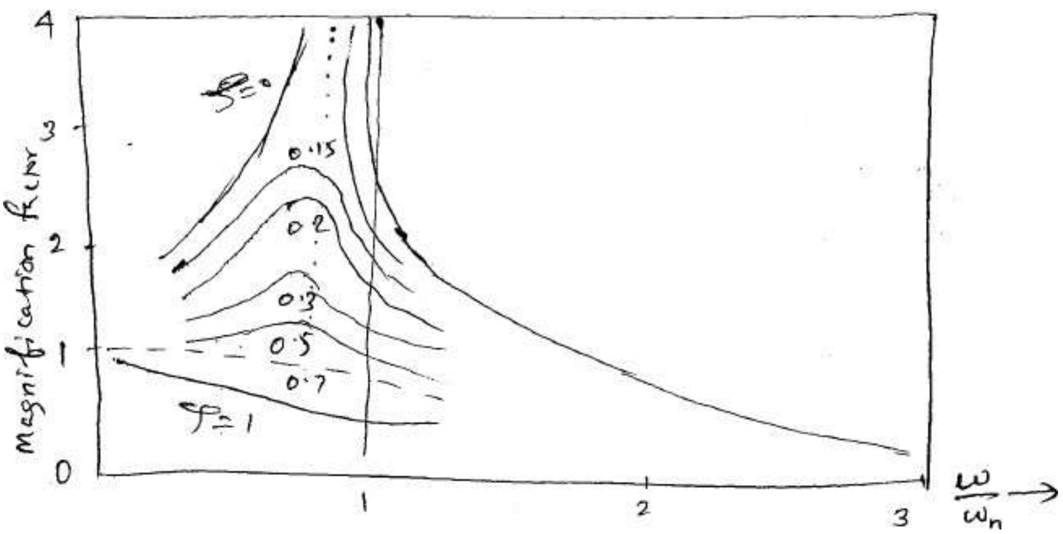
Note 4 :		Vector representation of forced vibration with damping.

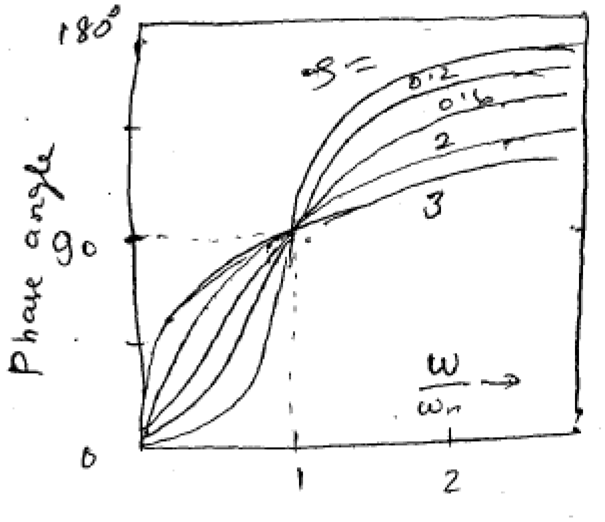


Note 5 :		The frequency at which max amplitude occurs is given by

$\frac{\omega_{max}}{\omega_n} = \sqrt{1 - 2\zeta^2}$ where ω_{max} – fore corresponding do max amplitude.

Characteristic curves : A curve between frequency ratio and magnification factor is known as **frequency response curve**. Similarly a curve between phase angle and frequency ratio is known as phase – frequency response curve.





The following points are noted:

1. At zero frequency magnification is unity & damping does not have any effect on it.
2. Damping reduces the magnification factor for all values of frequency.
3. The maximum value of amplitude occurs, a little towards left at resonant frequency.
4. At resonant frequency the phase angle is 90°.
5. The phase angle increases for decreasing value of damping above resonance.
6. The amplitude of vibration is infinite at resonant freq. and zero damping factor.
7. The amplitude ratio is below unity for all values of damping which was more than 0.70.
8. The variation in phase angle is because of damping without damping it is either 180° or ‘0°’)

Variation of frequency Ratio ω/ω_n

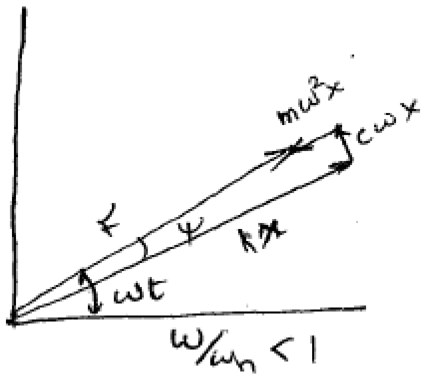
- Three possibilities of ω variation i.e., $\omega<\omega_n$, $\omega = \omega_n$ & $\omega > \omega_n$

Case : i $\quad \frac{\omega}{\omega_n} <<1$

$\therefore$ ω is very small

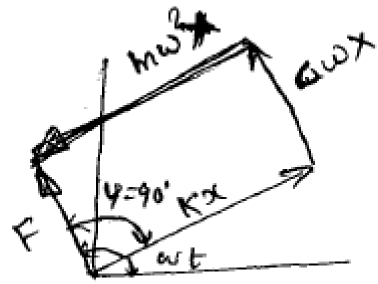
$\therefore \underset{\substack{\uparrow \\ \text{Inertia} \\ \text{force}}}{\underline{m\omega^2 x}}$ & $\underset{\substack{\uparrow \\ \text{damping} \\ \text{force}}}{\underline{c\omega x}}$ get reduced greatly

This results in small value of ψ



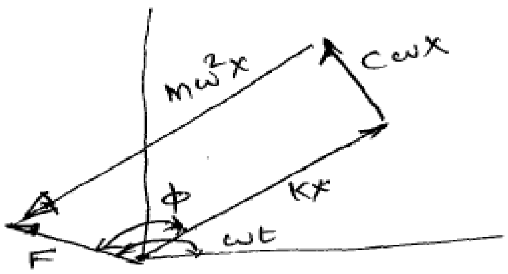
Case : ii

$\frac{\omega}{\omega_n}$ = 1 when $\omega = \omega_n$ i.e, Excitation ω = natural Frequency



Case : iii $\frac{\omega}{\omega_n}$ >> 1

At very high frequencies, of ω inertia force increases very rapidly. Damping & spring forces are small in magnitude for high values of $\frac{\omega}{\omega_n}$ phase angle ψ is close to 180°.



Here, inertia force = spring force

Excitation force balances the damping force here $x = \frac{F_o}{c\omega_n}$

List of Formulae :

1. $\omega = \frac{2\pi N}{60}$ rad/s

2. K = load/deflection N/m

3. Static deflection $X_o = F_{o/k}$ $\frac{N}{N/m} = \frac{N}{1} \times \frac{m}{N}$ =m

4. $r = \frac{\omega}{\omega_n}$ → frequency ratio & $\omega = \frac{2\pi N}{60}$ rps

5. $\zeta = \frac{C}{C_c}$;

6. $C_c = 2\sqrt{km} = 2m\omega_n$

7. δ = log, decrement = $\frac{1}{n}\ln\left(\frac{x_o}{x_n}\right)$; $= \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$

8. When damper is not present (ζ =0)

$\frac{X_{\max}}{X_o} = \frac{1}{r^2-1}$ if r > 1

$= \frac{1}{1-r^2}$ if r < 1

9. When damper is present

$\frac{X_{\max}}{X_o} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$ and xo = $\frac{F_o}{K}$;

$\omega_n = \sqrt{\frac{k}{m}}$ Therefore , $\omega_n^2 = \frac{k}{m}$ and k = $m\omega_n^2$

10. Amplitude at resonance [X_{max}] resonance = $\frac{x_o k}{c\omega_n} = \frac{\text{Fo}}{c\omega_n}$

11. Force transmitted to the foundation, F_T,

$\frac{F_T}{F_o} = \frac{\sqrt{1+(2\,\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

A reciprocating pump 200 kg is driven through a belt by an electric motor at 3000 rpm. The pump is mounted on isolators with total stiffness 5 MN/m and damping 3.125 KN.S Determine (i) the vibratory amplitude of the pump at the running speed due to fundamental harmonic force of excitation 1 KN (ii) Also find the max vibratory amplitude when the pump is switched on and the motor speed passes through resonant condition.

Q. A 300 kg Single cylinder vertical petrol engine is mounted upon chasis frame & cause defection = 1.5 mm. Mass of reciprocating parts of engine = 25 kg and it has a stroke of 145mm. A dashpot is provided whose damping resistance = 1.5 KN/(m/s) Determine (1) Amplitude of forced vibration when driving shaft rotates at 480 rpm (2) speed of the driving shaft at which resonance occurs.

m = 300 kg deflection $\Delta = 1.5\text{mm} = 1.5 \times 10^{-3}\text{m}$ m = 25 kg

$L = 145 \text{ x } 10^{-3}\text{m}$ $\quad \therefore$ radius of crank $= \frac{L}{2} = 0.0725$ m

$C = 1.5 \text{ x } 10^{3} \frac{N}{(\frac{m}{S})}$ $\quad$ Forcing speed N = 480 rpm

Forcing angular speed $\omega = \frac{2\pi N}{60} = 50.3$ rad/s

$$K = \frac{load}{\text{deflection}} = \frac{300\,(9.81)}{1.5\, x\, 10^{-3}} = 1.96 \text{ x } 10^{6} \text{ N/m}$$

$$C = 1.5 \text{ x } 10^{3} \frac{N}{(m/S)}$$

$$C_c = 2\sqrt{\text{km}} = 2\left\{\sqrt{1.96\, x\, 10^{6}(300)}\right\} = 48497.4 \frac{N}{(m/S)}$$

$$C_\zeta = \frac{C}{C_c} = 0.0309$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.96\, x\, 10^{6}}{300}} = 80.83 \text{ rad/s}$$

$$r = \frac{\omega}{\omega_n} = \frac{50.3}{80.829} = 0.622$$

Let due to reciprocating parts centrifugal force gets developed,

$F_o = m_1$, (radius) ω^2

$= 25\,(0.0725)\,(50.3)^2 = 4585.78\text{N}$

$$X = \frac{X_o}{\sqrt{1-r^2+(2\zeta r)^2}} = \frac{(4585.78/\, 1.96\, x\, 10^{6}}{\sqrt{[1-0.622^2]^2+[2\,(0.0309)(0.622)]^2}}$$

$$= 3.8 \text{ x } 10^{-3} \text{ m} = 3.8\text{mm}$$

Speed of driving shaft at which resonance occurs, when $\omega=\omega_n$ resonance occurs.

$\omega = \omega_n = \sqrt{\frac{K}{m}} = 80.83$ rps

$\frac{2\pi N}{60} = 80.83 \Rightarrow$ N = 771.86 rpm

Note:1 For $0 < \zeta < \frac{1}{\sqrt{2}} = 0.707$, the max value of M occurs when $\sqrt{1 - 2\zeta^2} = \frac{\omega}{\omega_n}$

2. The maximum value of x when $r = \sqrt{1 - 2\zeta^2}$ is given by $\frac{x}{x_0} = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$

and the value of x at $\omega = \omega_n$ $\left[\frac{x}{x_0}\right]_{\omega=\omega_n} = \frac{1}{2\zeta}$

3. For small values of damping ($\zeta < 0.05$), we

can take $\left[\frac{x}{x_0}\right]_{max} \cong \left[\frac{x}{x_0}\right]_{\omega=\omega_n} = \frac{1}{2\zeta} = Q$

Q factor or quality factor of the system

4. $[X_{max}]_{resonance} = \cong \frac{x_o K}{C\omega_n} = \cong \frac{F_o}{c\omega_n}$

Q. A machine part having a mass of 2.5kg executes vibration in a viscous damping medium. A harmonic exciting force of 30N acts on the part and causes a resonant amplitude of 14mm, with a period of 0.22S. Find the damping co eff. when the freq. of exciting force is changed to 4 H_2. Determine the increase of forced vibn upon the removal of damper

F=30N, m = 2.5kg, $t_p = 0.22$s, $f_n = \frac{1}{t_p} = 4.545$Hz

$[X_{max}]_{resonance} = 14\text{mm} = 14 \times 10^{-3}$m. c = ? when f = 4 H_2

$\omega_n = 2\pi f_n = 2\pi\,(4.545) = 28.56$ rad/s

$\omega = 2\pi f = 2\pi\,(4) = 25.13$ rps

$r = \frac{\omega}{\omega_n} = \frac{25.13}{28.56} = 0.8817$

Critical damping co eff. $C_c = 2_m\,\omega_n = 2\sqrt{km}$

$= 2\,(2.5)\;(28.56) = 142.8\;\frac{N}{(m/s)}$

$[X_{max}]_{resonance} = \frac{x_o k}{c\omega_n} = \frac{F_o}{c\omega_n}$

i.e., $14 \times 10^{-3} = \frac{30}{c(28.56)}$

$\Rightarrow\; c = 75.03\;\frac{N}{(\mathrm{m/s})}$

Damping factor, $\zeta = \frac{c}{c_c}$

$= \frac{75.03}{142.8}$

$= 0.525$

When damper is not removed, $\frac{X}{X_o} = \frac{1}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$

$\frac{X}{X_o} = \frac{1}{\sqrt{[1-0.8817^2]^2+[2(0.525)(0.8817)]^2}} = 1.0502$

$X_{\max} = \frac{(F_o/k)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = \frac{30}{m\omega_n^2}\;[1.0502]$

$= \frac{30}{(2.5)\;(28.56)^2}\,(1.0502)$

$= 0.01545^{m}$

= 15.45mm

When damper is removed

$\frac{X_{max}}{X_0} = \frac{1}{1-r^2}$ r < 1

$X_{max} = \frac{F_o/k}{1-r^2}$ $\left(\because X_o = \frac{F_0}{k}\right)$

$= \frac{30}{2.5(28.56)^2} [\frac{1}{1-(0.8817)^2}]^{\& \, k=m\omega_n^2}$

= 0.06609m

= 66.09mm

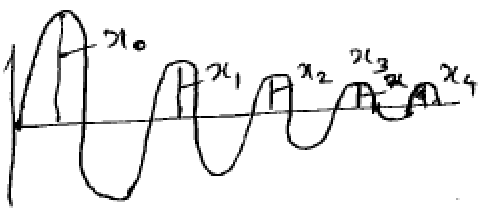
% increase in amplitude

$= \frac{[X_{max}]_{Nodamping} - [X_{max}]_{damping}}{[X_{max}]_{damping}}$

= [66.09 ~15.45]/15.45

= 321.92%

A 12 Kg mass is suspended from end of helical spring, other end is fixed, spring stiffness =15 N/mm. Due to viscous damping, amplitude decreases to 1/10th of initial value in 4 oscillations. If a periodic force 150 cos 50t N is applied at mass in vertical direction, find amplitude of forced vibration. What is its value of resonance?



m = 12 kg   k = $15\frac{N}{\text{mm}}$ x $\frac{1000\text{mm}}{1\text{m}} = 15\ x\ 10^3\ \frac{\text{N}}{\text{m}}$

$x_4 = 0.1\ x_o$

$\frac{x_o}{x_4} = 10$

∴ Logarithmic decrement, $\delta = \frac{1}{4}\ln[10] = 0.5756$

$\zeta = \frac{\delta}{\sqrt{(2\pi)^2+\delta^2}} = 0.09122$

$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{15\ x\ 10^3}{12}} = 35.3$ rps

$F_x = F_o \cos \omega t$

$= 50 \cos 50t$

∴ $F_o = 150$N & $\omega = 50$ rad/s

∴ freq. ratio $r = \frac{\omega}{\omega_n} = \frac{50}{35.3} = 1.416$

$Cc = \sqrt[2]{\text{km}} = \sqrt[2]{15\ x\ 10^3\ (12)} = 848.52\ \frac{N}{(m/s)}$

$\zeta = \frac{c}{C_c}$

$0.09122 = \frac{c}{848.52} \Rightarrow c = 77.40 = \frac{N}{(m/s)}$

$\frac{X}{Fo/k} = \frac{1}{\sqrt{[1-1.416^2]^2+[2(0.09122)(1.416)]^2}}$

$\frac{X}{[150/15x10^3} = 0.09637$

$$X = \left[\frac{150}{15\, x 10^3}\right] 0.9637$$

$= 9.637 \times 10^{-3}$m

$= 9.637$mm

Amplitude at resonance,

$$[X_{max}]_{resonance} = \frac{x_0 k}{c\omega_n} = \frac{F_o}{c\omega_n}$$

$$= \frac{150}{(77.40)(35.3)}$$

$= 0.05490$m

$= 54.9$mm

Force transmitted to the foundation, F_T

$$\frac{F_T}{F_o} = \frac{\sqrt{1 + (2\zeta r)^2}}{\sqrt{[1-r^2]^2 + [2\zeta r]}}$$

$$\text{Also } \frac{X}{X_0} = M \frac{1}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

A machine of mass one tonne is acted upon by an external force of 2450 N at a frequency of 1500 rpm. To reduce the effects of vibration. Isolator of rubber having a static deflection of 2mm under the m/c load and an estimated damping $\zeta = 0.2$ are used. Determine (1) the force transmitted to the foundation (2) the amplitude of vibration of m/c (3) The phase lag.

Given $\Delta = 2 \times 10^{-3}$m m= 1000kg F = 2450 N

Forcing frequency $\omega = \frac{2\pi N}{60} = \frac{2\pi(1500)}{60} = 157$ rad/s

$\zeta = 0.2$

$$K = \frac{\text{Force or load}}{Static\ deflection} = \frac{mg}{\Delta} = \frac{1000\ (9.81)}{2\ x\ 10^{-3}} = 49 \text{ x } 10^5 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{49 \text{ x } 10^5}{1000}} = 70 \text{ rad/s}$$

Freq ratio 'r' $= \frac{\omega}{\omega_n} = \frac{157}{70} = 2.2428$

Let force transmitted to the foundation, $= F_T$

$$\frac{F_T}{F_o} = \frac{\sqrt{1+[2\zeta r]^2}}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$$

$$\frac{F_T}{F_o} = \frac{\sqrt{1+[2(0.2)(2.24280]^2}}{\sqrt{[1-2.2428^2)^2+[2(0.2)(2.2428)]^2}}$$

$$\frac{F_T}{2450} = \frac{1.3434}{4.128}$$

$= 0.3254$

Also

$$\frac{X}{X_o} = \frac{1}{\sqrt{(1-r)^2+(2\zeta r)^2}}$$

$$X = \frac{F_o/k}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{[2450\ /49\text{x}10^5]}{4.128}$$

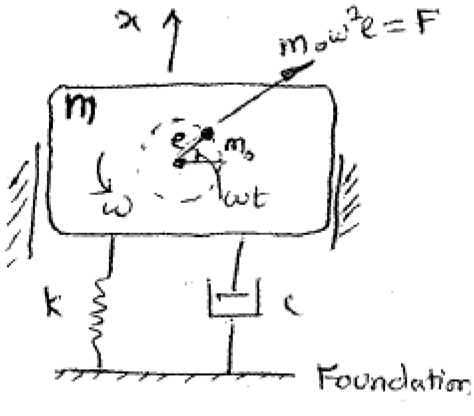
$= 1.211 \text{ x } 10^{-4}$m

$= 0.121$mm

Let phase lay = ψ

$$\text{Tan } \psi = \frac{2\varsigma r}{1-r^2} = \frac{2(0.2)(2.24)}{1-2.24^2} = -0.223 \qquad \therefore \psi = \tan^{-1}(\psi) = -12.57^o$$

RESPONSE OF A ROTATING AND RECI PRORATING UNBALANCE SYSTEM



Rotating Un balance

- A machine having rotor as one of its components is called a rotating machine eg. Turbines and I.C. Engines.
- When C.G. of rotor does not coincide with axis of rotation then unbalance occurs.

  Let : e = Distance betweem Being axis of rotation & C.G.

  m_o = mass acting at distance 'e' (i.e, Eccentric mass)

  Centrifugal force = $m_o e\omega^2$

  At any moment vertical displacement = x + esinωt

  The centrifugal force $m_o e\omega^2$ has two components vertical and horizontal. The vertical component has the significance and is given by $m_o e\omega^2$ sinωt

  When we consider single degree Problem (motion in vertical) the excitation is available in vertical direction ie., F_o sinωt = $m_o e\omega^2$ sinωt.

$$[m-m_o]\ \frac{d^2x}{dt^2} + m_o\frac{d^2}{dt^2}\ (x + e\sin\omega t) + kx + c\frac{dx}{dt} = 0$$

$$(m-m_o)\ \ddot{x} + m_o\ \ddot{x} - m_o e\omega^2\ e\sin\omega t + kx + c\dot{x} = 0$$

$$\Rightarrow\ m\ddot{x} + c\ \dot{x} + kx = m_o e\omega^2\ \sin\omega t$$

Where $m_o\omega^2 e = F_o$

It represent forced vibration.

Note -1 :

$$\frac{X}{X_o} = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}\ \text{holds good}$$

$$\text{Where}\quad X_o = \frac{F_0}{k} = \frac{m_o e\omega^2}{k}$$

$$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{\sqrt{[1-r^2]^2+[2\zeta r]^2}}$$

At resonance, $\omega = \omega_n$ & $r = 1$ $\quad\therefore = 1$

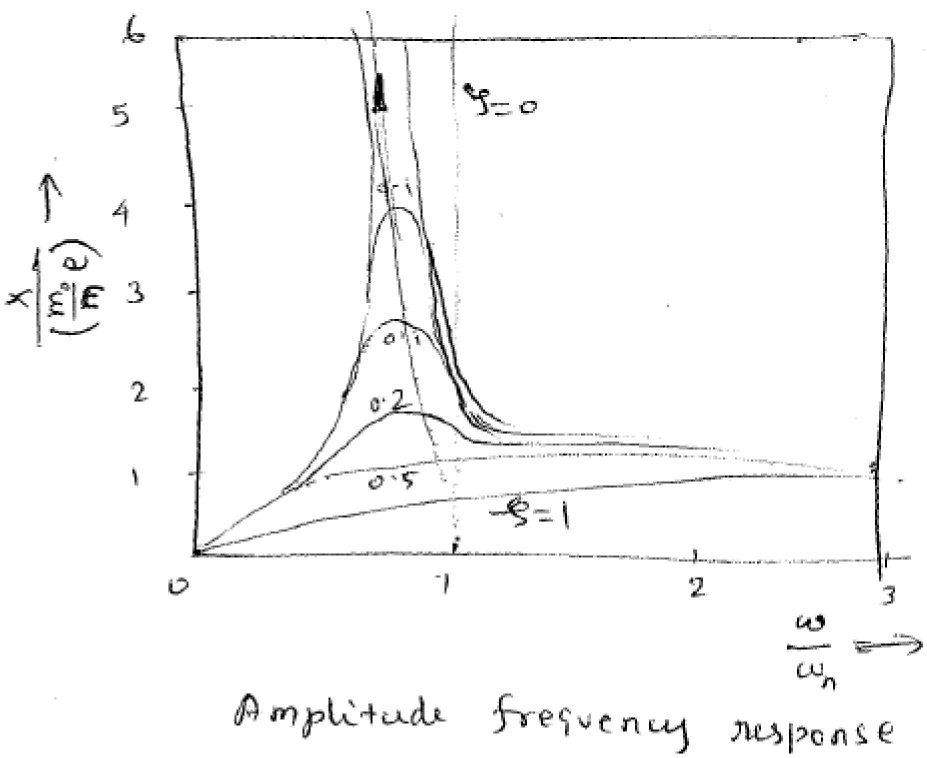
Then $\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{2\zeta}$ and share angle $\tan\psi = \frac{2\zeta r}{1-r^2}$

The complete solution $x = x_c + x_p$

$$x = x_e^{-\zeta w_n t}\ [\cos(\omega_d\, t + \psi)] + \frac{(m_o e\ \omega^2/k)}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

Characteristic Curve:

1. Damping plays very important role during resonance.
2. When $\omega \ll \omega_n$ then the system is known as low speed system for a low speed system $\frac{X}{\frac{m_o}{m}e} \to 0$



Amplitude frequency response curve

3. When $\omega >> \omega_n$, then the system is called high speed system.

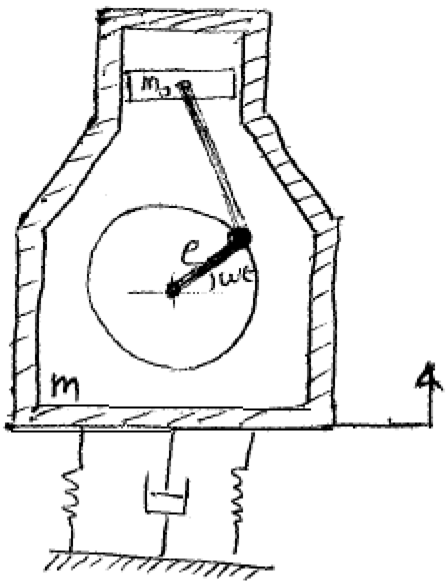
   Here $\frac{X}{\left(\frac{m_o}{m}e\right)} \to 1$

4. At very high speed effect of damping is negligible.
5. The peak occurs at the left (in comparison to the previous characteristic ‘c’
6. At resonance $\omega = \omega_n$ &

$$\frac{X}{\left(\frac{m_o}{m}e\right)} = \frac{1}{2\varsigma} \quad X = \frac{1}{2\varsigma}, \frac{m_o}{m}\ e \ \ \varsigma = \ = \ \frac{1}{2}, \frac{m_o}{m}, \frac{e}{x_{reso}}$$

Inertia force due to reciprocating mass approximately is equal to

$F_o = m_o\, e\omega^2\, [\sin\omega t + \frac{e}{l} \sin 2\omega t]$



Reciprocating Unbalance

If e is very small as compared to 'l', second term (2nd Harmonic) can be neglected & $F_o = m_o e\omega^2 \sin\omega t$

Determine, (i) ζ = ?

(ii) x & ψ = ?

(iii) Resonant speed, ω_n = ?

$X_{resonance}$ = ?

(iv) Resuttant force = ?

Now

$$C_c = 2\sqrt{km} = 2\sqrt{(6400 \ x \ 20)} = 715.54 \ \frac{\text{N}}{\text{m/s}}$$

(1) Damping ratio, $\zeta = \frac{C}{C_c} = \frac{125}{715.54} = 0.175$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{6400}{20}} = 17.88 \text{ rad/s}$$

Frequency ratio $r = \frac{\omega}{\omega_n} = \frac{41.866}{17.88} = 2.342$

(2) $$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\varsigma r)^2}}$$

$$\frac{X}{\left[\frac{0.5}{20}(0.05)\right]} = \frac{(2.342)^2}{(1-2.342^2)^2+[2(0.175)(2.342)]^2}$$

$X = 1.5 \times 10^{-3}$m

And phase angle = ψ

Tan $\psi = \frac{2\zeta r}{1-r^2} = \frac{2(0.175)(2.342)}{1-2.342^2}$

$\Rightarrow \psi = -10.36^o$ or $180 - 10.36^o = 169.63^o$

(3) Resonant speed = ω_n = 17.88 rps

$\omega_n = \frac{2\pi N}{60} \Rightarrow 17.88 = \frac{2\pi N}{60}$

$\therefore$ N = 170.74 rpm

In a vibrating system the total mass of the system is 25kg. At speed of 1000 rpm., the system and eccentric mass have a phase difference of 90^o and the corresponding amplitude is 1.5cm. The eccentric unbalanced mass of 1 kg has a radius of rotation 4cm. Determine (i) the natural frequency of the system (ii) the damping factor (iii) the amplitude at 1500 rpm and (iv) the phase angle at 1500 rpm.

$m = 25$ kg $x = 1.5 \times 10^{-2}$m $m_o = 1$ kg $e = 4 \times 10^{-2}$m

At phase angle 90^o, the condition of resonance occurs [$\omega = \omega_n$]

$\omega = \frac{2\pi N}{60}$ (i) $f_n \Rightarrow$ rps

$f_n = \frac{N}{60} = \frac{1000}{60}$ = 16.67 cycles/s

$= \frac{2\pi(1000)}{60}$ = 104.72 rps

At resonance

ii. $$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{1}{2\zeta}$$

$$\Rightarrow \zeta = \frac{1}{x}\left(\frac{m_o e}{m}\right)\frac{1}{x} = \frac{m_o e}{2mx}$$

$$= \frac{(1)(4\,x\,10^{-2})}{2(25)(1.5\,x\,10^{-2})}$$

$$= 0.053$$

r = $\frac{\omega}{\omega_n} = \frac{f}{f_n} = \frac{N}{N_n} = \frac{1500}{1000} = 1.5$ Amplitude at 1500 rpm, is x

$$\frac{X}{\left(\frac{m_o e}{m}\right)} = \frac{r^2}{\sqrt{(1-r^2)^2+(2\varsigma r)^2}}$$

$$\frac{X}{\left(\frac{1}{25}\right)(4x10^{-2})} = \frac{(1.5)^2}{\sqrt{[1-1.5^2]^2+[2(0.053)(1.5)]^2}} \Rightarrow x = 0.226 \text{ x } 10^{-2}\text{m}$$

The phase angle to 1500 rpm.

Now r $= \frac{1500}{1000} = 1.5$

$$\tan\ \psi = \frac{2\zeta r}{1-r^2}$$

$$= \frac{2(0.053)(1.5)}{1-1.5^2}$$

$\Rightarrow\ \psi = -7.24900^o = 180^o - 7.249^o = 172.75$

An electric motor is supported on a spring and a dashpot. The spring has the stiffness 6400 N/m and the dashpot offers resistance of 500N at 4 m/s. The unbalanced mass 0.5kg rotates at 5cm rad and the total mass of vibratory system is 20 kg. The motor runs at 400 rpm. Determine 1) Damping factor, 2) amplitude of vibration & phase angle, 3) resonant speed & resonant amplitude 4) Force exerted by the spring and dashpot on the motor.

K = 6400 N/m

Damping force = 500 N

Velocity = 4 M/s.

$$\therefore\ c = \frac{\text{Damping force}}{\text{velocity}} = \frac{500}{4} = 125\ \frac{N}{(m/s)}$$

m_o = 0.5 kg

Eccentricity = 0.05m

m = 20kg

N = 400 rpm $\therefore \omega = \frac{2\pi N}{60}$ =41.866 rad/s

Amplitude at resonance :

$$\frac{X_{reso}}{\left(\frac{m_o e}{m}\right)} = \frac{1}{\sqrt{2\varsigma}}$$

$\therefore X_{reso} = \left[\frac{m_o e}{m}\right] \frac{1}{2\varsigma} = \frac{m_o e}{2m\varsigma}$ when e = eccentricity

$= \frac{(0.5)(0.05)}{2(0.175)(20)}$ = 3.57 x $10^{-3} m$ = 3.57mm

4) Force because of dashpot on the motor F_d = cωx

= 125 [41.866] [1.5 x 10^{-3}]

= 7.85 N

Force because of spring, F_s = Kx

= (6400) (1.5 x 10^{-3})

= 22.4

Resultant force, F = N $= \sqrt{F_d^2 + F_s^2}$

$= \sqrt{7.85^2 + 9.6^2}$

= 12.4N

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SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – V - GOVERNORS AND GYROSCOPES – SMEA1402

UNIT V GOVERNORS AND GYROSCOPES

Governors - Types - Centrifugal governors – Porter, Proel and Hartnell Governors – Characteristics –Sensitivity– Stability – Hunting – Isochronisms – equilibrium speed - Effect of friction - Controlling Force

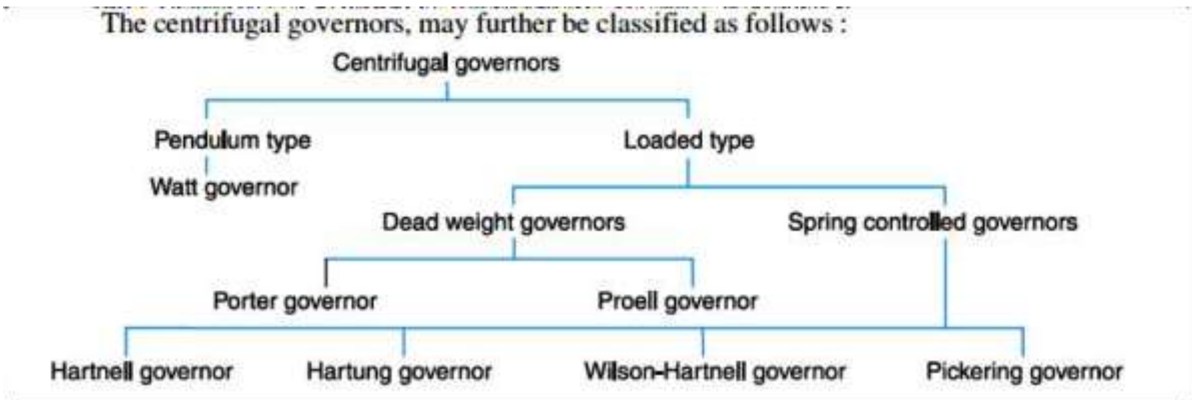
Introduction

The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g. when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits. A little consideration will show, that when the load increases, the configuration of the governor changes and a valve is moved to increase the supply of the working fluid ; **conversely,** when the load decreases, the engine speed increases and the governor decreases the supply of working fluid.

Types of Governors

The governors may, broadly, be classified as

1. Centrifugal governors,
2. Inertia governors.



Centrifugal Governors

The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the ***controlling force****.It consists of two balls of equal mass, which are attached to the arms as shown in Fig. 1. These balls are known as ***governor balls*** **or** ***fly balls*.** The balls revolve with a spindle, which is driven by the engine through bevel gears. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the links to a sleeve, which is keyed to the spindle. This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rises when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops *S*, *S* are provided on the spindle. The sleeve is connected by a bell crank lever to a throttle valve. The supply of the working fluid decreases when the sleeve rises and increases when it falls. When the load on the engine increases, the engine and the governor speed decreases. This results in the decrease of centrifugal force on the balls. Hence the balls move inwards and the sleeve moves downwards. The downward movement of the sleeve operates a throttle valve at the other end of the bell crank lever to increase the supply of working fluid and thus the engine speed is increased. In this case, the extra power output is provided to balance the increased load. When the load on the engine decreases, the engine and the governor speed increases, which results in the increase of centrifugal force on the balls. Thus the balls move outwards and the sleeve rises upwards. This upward movement of the sleeve reduces the supply of the working fluid and hence the speed is decreased. In this case, the power output is reduced.

Note : When the balls rotate at uniform speed, controlling force is equal to the centrifugal force and they balance each other.

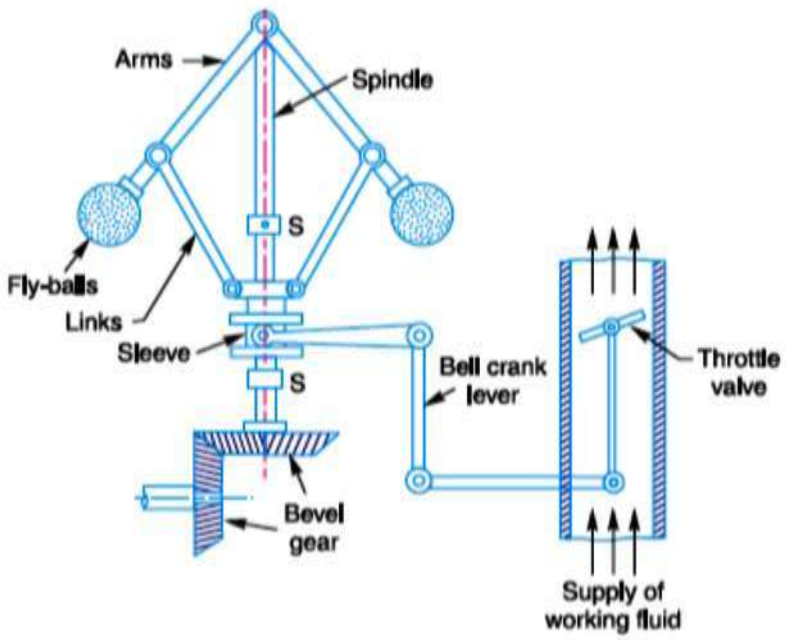


Fig. 1. Centrifugal governor

Terms Used in Governors

The following terms used in governors are important from the subject point of view ;

1. Height of a governor. It is the vertical distance from the centre of the ball to a point where the axes of the arms (or arms produced) intersect on the spindle axis. It is usually denoted by *h*.

2. ***Equilibrium speed.*** It is the speed at which the governor balls, arms etc., are in complete equilibrium and the sleeve does not tend to move upwards or downwards.

3. ***Mean equilibrium speed.*** It is the speed at the mean position of the balls or the sleeve.

4. ***Maximum and minimum equilibrium speeds.*** The speeds at the maximum and minimum radius of rotation of the balls, without tending to move either way are known as maximum and minimum equilibrium speeds respectively.

Note : There can be many equilibrium speeds between the mean and the maximum and the mean and the minimum equilibrium speeds.

5. ***Sleeve lift.*** It is the vertical distance which the sleeve travels due to change in equilibrium speed.

Porter Governor

The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve as shown in Fig. 2 (*a*). The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Consider the forces acting on one-half of the governor as shown in Fig. 2 (*b*).

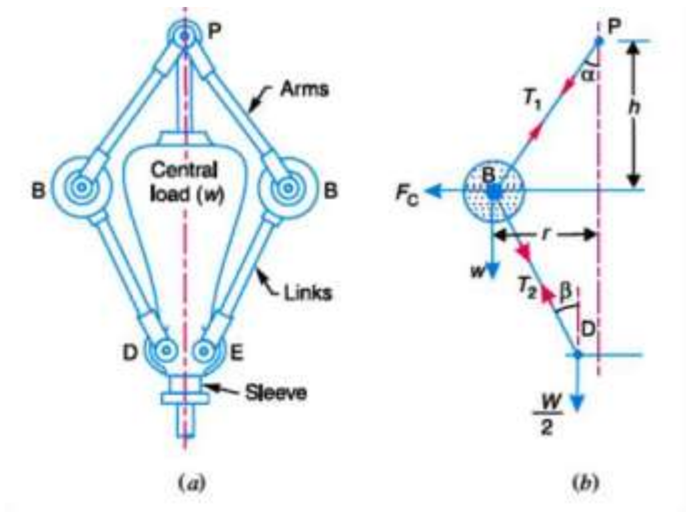


Fig. 2. Porter governor.

Let m = Mass of each ball in kg,

w = Weight of each ball in newtons = $m.g$,

M = Mass of the central load in kg,

W = Weight of the central load in newtons = $M.g$,

r = Radius of rotation in metres,

h = Height of governor in metres ,

N = Speed of the balls in r.p.m .,

ω= Angular speed of the balls in rad/s= $2\pi N/60$ rad/s,

FC = Centrifugal force acting on the ball in newtons = $m.\omega^2.r$,

T1 = Force in the arm in newtons,

T2 = Force in the link in newtons,

α= Angle of inclination of the arm (or upper link) to the vertical, and

β= Angle of inclination of the link (or lower link) to the vertical.

Though there are several ways of determining the relation between the height of the governor (h) and the angular speed of the balls (ω), yet the following two methods are important from the subject point of view :

1. Method of resolution of forces; and

2. Instantaneous centre method.

1. Method of resolution of forces

Considering the equilibrium of the forces acting at *D*, we have

$$T_2 \cos\beta = \frac{W}{2} = \frac{M.g}{2}$$

$$T_2 = \frac{M.g}{2\cos\beta}$$

Again, considering the equilibrium of the forces acting on *B*. The point *B* is in equilibrium under the action of the following forces, as shown in Fig. 2 (*b*).

(i) The weight of ball ($w = m.g$),

(ii) The centrifugal force (FC),

(iii) The tension in the arm (T1), and

(iv) The tension in the link (T2).

Resolving the forces vertically,

$$T_1 \cos\alpha = T_2 \cos\beta + w = \frac{M.g}{2} + m.g \qquad \ldots (ii)$$

$$\ldots \left(\because\ T_2 \cos\beta = \frac{M.g}{2}\right)$$

Resolving the forces horizontally,

$$T_1 \sin\alpha + T_2 \sin\beta = F_C$$

$$T_1 \sin\alpha + \frac{M.g}{2\cos\beta} \times \sin\beta = F_C \qquad \ldots \left(\because\ T_2 = \frac{M.g}{2\cos\beta}\right)$$

$$T_1 \sin\alpha + \frac{M.g}{2} \times \tan\beta = F_C$$

$$T_1 \sin\alpha = F_C - \frac{M.g}{2} \times \tan\beta \qquad \ldots (iii)$$

Dividing equation *(iii)* by equation *(ii)*,

$$\frac{T_1 \sin\alpha}{T_1 \cos\alpha} = \frac{F_C - \frac{M.g}{2} \times \tan\beta}{\frac{M.g}{2} + m.g}$$

or $$\left(\frac{M.g}{2} + m.g\right)\tan\alpha = F_C - \frac{M.g}{2} \times \tan\beta$$

$$\frac{M.g}{2} + m.g = \frac{F_C}{\tan\alpha} - \frac{M.g}{2} \times \frac{\tan\beta}{\tan\alpha}$$

Substituting $\frac{\tan\beta}{\tan\alpha} = q$, and $\tan\alpha = \frac{r}{h}$, we have

$$\frac{M.g}{2} + m.g = m.\omega^2.r \times \frac{h}{r} - \frac{M.g}{2} \times q \qquad \ldots (\because F_C = m.\omega^2.r)$$

or $$m.\omega^2.h = m.g + \frac{M.g}{2}(1+q)$$

$\therefore$ $$h = \left[m.g + \frac{M.g}{2}(1+q)\right]\frac{1}{m.\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2} \qquad \ldots (iv)$$

or $$\omega^2 = \left[m.g + \frac{Mg}{2}(1+q)\right]\frac{1}{m.h} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

or $$\left(\frac{2\pi N}{60}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}$$

$\therefore$ $$N^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{h}\left(\frac{60}{2\pi}\right)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h} \qquad \ldots (v)$$

. . . (Taking $g = 9.81$ m/s²)

Notes : 1. When the length of arms are equal to the length of links and the points *P* and *D* lie on the same vertical line, then

tan a = tan b or *q* = tan a / tan b = 1

Therefore, the equation **(*v*)** becomes

$$N^2 = \frac{(m+M)}{m} \times \frac{895}{h}$$

2. When the loaded sleeve moves up and down the spindle, the frictional force acts on it in a direction opposite to that of the motion of sleeve.

If *F* = Frictional force acting on the sleeve in newtons, then the equations **(*v*)** and **(*vi*)** may be written as

$$N^2 = \frac{m.g + \left(\frac{M.g \pm F}{2}\right)(1+q)}{m.g} \times \frac{895}{h} \qquad \ldots (vii)$$

$$= \frac{m.g + (M.g \pm F)}{m.g} \times \frac{895}{h} \qquad \ldots (\text{When } q = 1) \ldots (viii)$$

The + sign is used when the sleeve moves upwards or the governor speed increases and negative sign is used when the sleeve moves downwards or the governor speed decreases.

3. Mass of the central load (*M*) increases the height of governor in the ratio (m+M)/m

2. *Instantaneous centre method*

In this method, equilibrium of the forces acting on the link *BD* are considered. The instantaneous centre *I* lies at the point of intersection of *PB* produced and a line through *D* perpendicular to the spindle axis, as shown in Fig. 3. Taking moments about the point *I*,

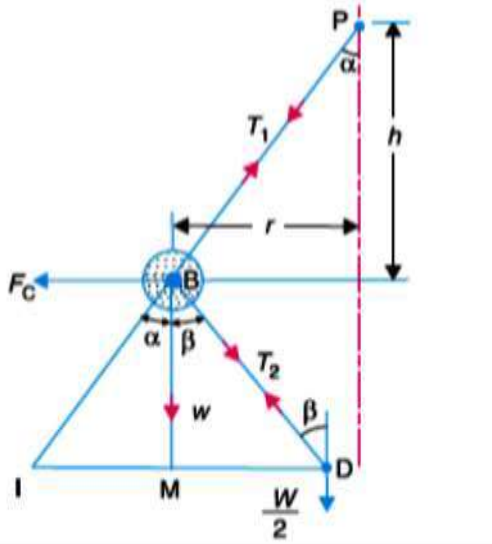


Fig. 3. Instantaneous centre method

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID$$

$$= m.g \times IM + \frac{M.g}{2} \times ID$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2} \times \frac{ID}{BM}$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right)$$

$$= m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM}{BM} + \frac{MD}{BM}\right)$$

$$= m.g \tan \alpha + \frac{M.g}{2}(\tan \alpha + \tan \beta)$$

$$\ldots \left(\because \frac{IM}{BM} = \tan \alpha, \text{ and } \frac{MD}{BM} = \tan \beta\right)$$

Dividing throughout by $\tan \alpha$,

$$\frac{F_C}{\tan \alpha} = m.g + \frac{M.g}{2}\left(1 + \frac{\tan \beta}{\tan \alpha}\right) = m.g + \frac{M.g}{2}(1+q) \qquad \ldots \left(\because q = \frac{\tan \beta}{\tan \alpha}\right)$$

We know that $F_C = m.\omega^2.r$, and $\tan \alpha = \frac{r}{h}$

$$\therefore \; m\omega^2.r \times \frac{h}{r} = m.g + \frac{M.g}{2}(1+q)$$

or

$$h = \frac{m.g + \frac{M.g}{2}(1+q)}{m} \times \frac{1}{\omega^2} = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{g}{\omega^2}$$

. . . (Same as before)

When $\tan \alpha = \tan \beta$ or $q = 1$, then

$$h = \frac{m + M}{m} \times \frac{g}{\omega^2}$$

Proell Governor

The Proell governor has the balls fixed at *B* and *C* to the extension of the links *DF* and *EG*, as shown in Fig. 4 (*a*). The arms *FP* and *GQ* are pivoted at *P* and *Q* respectively. Consider the equilibrium of the forces on one-half of the governor as shown in Fig. 4 (*b*). The instantaneous centre (*I*) lies on the intersection of the line *PF* produced and the line from *D* drawn perpendicualr to the spindle axis. The prependicular *BM* is drawn on *ID*.

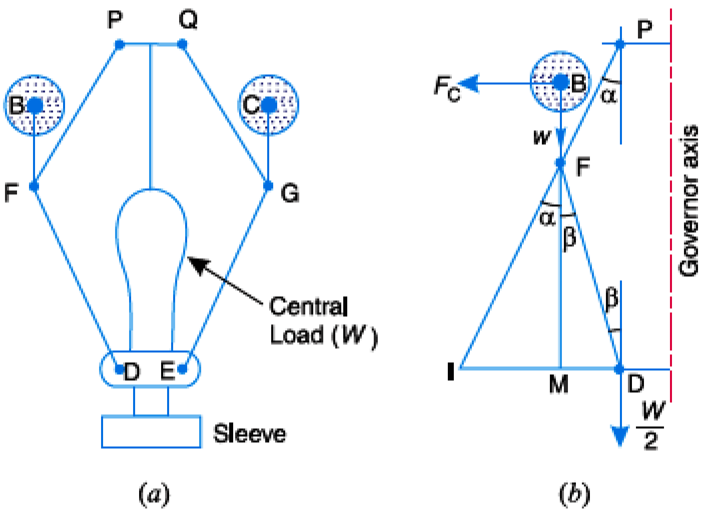


Fig. 4. Proell governor.

Taking moments about *I*,

$$F_C \times BM = w \times IM + \frac{W}{2} \times ID = m.g \times IM + \frac{M.g}{2} \times ID \qquad \dots (i)$$

$$\therefore \qquad F_C = m.g \times \frac{IM}{BM} + \frac{M.g}{2}\left(\frac{IM + MD}{BM}\right) \qquad \dots (\because \; ID = IM + MD)$$

Multiplying and dividing by FM, we have

$$F_C = \frac{FM}{BM}\left[m.g \times \frac{IM}{FM} + \frac{M.g}{2}\left(\frac{IM}{FM} + \frac{MD}{FM}\right)\right]$$

$$= \frac{FM}{BM}\left[m.g \times \tan\alpha + \frac{M.g}{2}(\tan\alpha + \tan\beta)\right]$$

$$= \frac{FM}{BM} \times \tan\alpha\left[m.g + \frac{M.g}{2}\left(1 + \frac{\tan\beta}{\tan\alpha}\right)\right]$$

We know that $F_C = m.\omega^2 r$; $\tan\alpha = \frac{r}{h}$ and $q = \frac{\tan\beta}{\tan\alpha}$

$$\therefore \quad m.\omega^2.r = \frac{FM}{BM} \times \frac{r}{h}\left[m.g + \frac{M.g}{2}(1+q)\right]$$

and

$$\omega^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{g}{h} \qquad \ldots (ii)$$

Substituting $\omega = 2\pi N/60$, and $g = 9.81$ m/s², we get

$$N^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h} \qquad \ldots (iii)$$

Notes : 1. The equation *(i)* may be applied to any given configuration of the governor.

2. Comparing equation *(iii)* with the equation *(v)* of the Porter governor (Art. 18.6), we see that the equilibrium speed reduces for the given values of m, M and h. Hence in order to have the same equilibrium speed for the given values of m, M and h, balls of smaller masses are used in the Proell governor than in the Porter governor.

3. When $\alpha = \beta$, then $q = 1$. Therefore equation *(iii)* may be written as

$$N^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h} \qquad (h \text{ being in metres}) \ldots (iv)$$

Hartnell Governor

A Hartnell governor is a spring loaded governor as shown in Fig. 5. It consists of two bell crank levers pivoted at the points *O*, *O* to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical arm *OB* and a roller at the end of the horizontal arm *OR*. A helical spring in compression provides equal downward forces on the two rollers through a collar on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

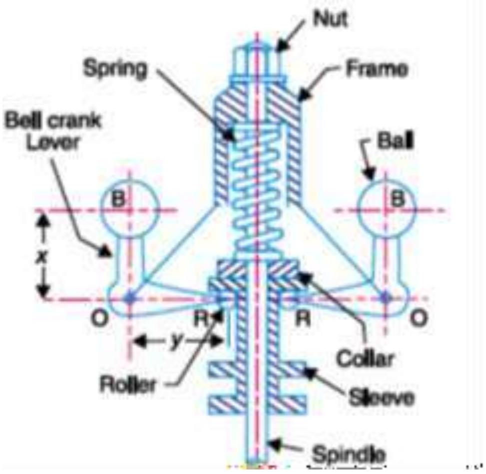


Fig. 5. Hartnell governor.

Let m = Mass of each ball in kg,

M = Mass of sleeve in kg,

$r1$ = Minimum radius of rotation in metres,

$r2$ = Maximum radius of rotation in metres,

w_1 = Angular speed of the governor at minimum radius in rad/s,

w_2 = Angular speed of the governor at maximum radius in rad/s,

S_1 = Spring force exerted on the sleeve at w_1 in newtons,

S_2 = Spring force exerted on the sleeve at w_2 in newtons,

$FC1$ = Centrifugal force at w_1 in newtons = $m\ (\omega_1)^2\ r1$,

$FC2$ = Centrifugal force at w_2 in newtons = $m\ (\omega_2)^2\ r2$,

s = Stiffness of the spring or the force required to compress the spring by one mm,

x = Length of the vertical or ball arm of the lever in metres,

y = Length of the horizontal or sleeve arm of the lever in metres, and

r = Distance of fulcrum O from the governor axis or the radius of rotation when the governor is in mid-position, in metres.

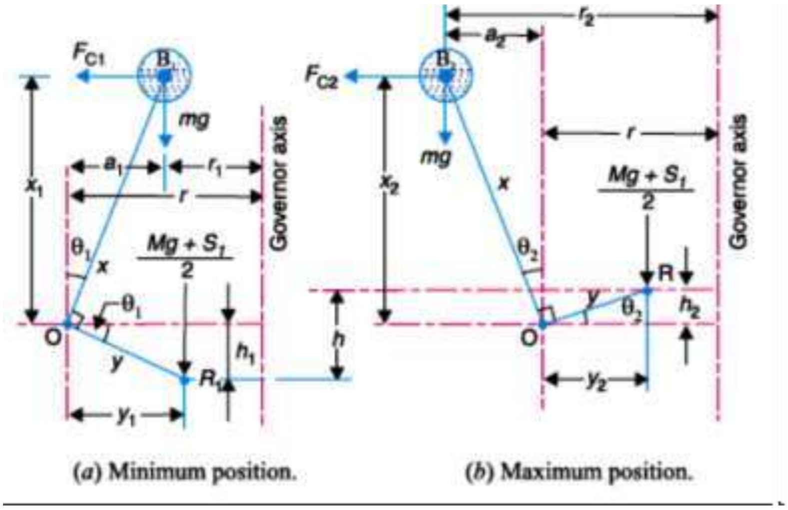


Fig 6

Consider the forces acting at one bell crank lever. The minimum and maximum position is shown in Fig. 6. Let *h* be the compression of the spring when the radius of rotation changes from *r*1 to *r*2.

$$\frac{h_1}{y} = \frac{a_1}{x} = \frac{r - r_1}{x} \qquad \ldots (i)$$

For the minimum position *i.e.* when the radius of rotation changes from *r* to *r*1, as shown in Fig. 6 (*a*), the compression of the spring or the lift of sleeve *h*1 is given by

Similarly, for the maximum position *i.e.* when the radius of rotation changes from *r* to *r*2, as shown in Fig. 6 (*b*), the compression of the spring or lift of sleeve *h*2 is given by

$$\frac{h_2}{y} = \frac{a_2}{x} = \frac{r_2 - r}{x} \qquad \ldots (ii)$$

Adding equations (*i*) and (*ii*),

$$\frac{h_1 + h_2}{y} = \frac{r_2 - r_1}{x} \quad \text{or} \quad \frac{h}{y} = \frac{r_2 - r_1}{x} \qquad \ldots (\because h = h_1 + h_2)$$

$$\therefore \qquad h = (r_2 - r_1)\frac{y}{x} \qquad \ldots (iii)$$

Now for minimum position, taking moments about point O, we get

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

or
$$M.g + S_1 = \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1) \qquad \ldots(iv)$$

Again for maximum position, taking moments about point O, we get

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

or
$$M.g + S_2 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) \qquad \ldots(v)$$

Subtracting equation (*iv*) from equation (*v*),

$$S_2 - S_1 = \frac{2}{y_2}(F_{C2} \times x_2 + m.g \times a_2) - \frac{2}{y_1}(F_{C1} \times x_1 - m.g \times a_1)$$

We know that

$$S_2 - S_1 = h.s, \qquad \text{and} \qquad h = (r_2 - r_1)\frac{y}{x}$$

$\therefore$
$$s = \frac{S_2 - S_1}{h} = \left(\frac{S_2 - S_1}{r_2 - r_1}\right)\frac{x}{y}$$

Neglecting the obliquity effect of the arms (*i.e.* $x_1 = x_2 = x$, and $y_1 = y_2 = y$) and the moment due to weight of the balls (*i.e.* $m.g$), we have for minimum position,

$$\frac{M.g + S_1}{2} \times y = F_{C1} \times x \qquad \text{or} \qquad M.g + S_1 = 2F_{C1} \times \frac{x}{y} \qquad \ldots(vi)$$

Similarly for maximum position,

$$\frac{M.g + S_2}{2} \times y = F_{C2} \times x \quad \text{or} \quad M.g + S_2 = 2F_{C2} \times \frac{x}{y} \qquad \ldots (vii)$$

Subtracting equation (*vi*) from equation (*vii*),

$$S_2 - S_1 = 2\,(F_{C2} - F_{C1})\,\frac{x}{y} \qquad \ldots (viii)$$

We know that

$$S_2 - S_1 = h.s, \quad \text{and} \quad h = (r_2 - r_1)\,\frac{y}{x}$$

$$\therefore \quad s = \frac{S_2 - S_1}{h} = 2\left(\frac{F_{C2} - F_{C1}}{r_2 - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (ix)$$

Notes : 1. Unless otherwise stated, the obliquity effect of the arms and the moment due to the weight of the balls is neglected, in actual practice.

2. When friction is taken into account, the weight of the sleeve ($M.g$) may be replaced by ($M.g. \pm F$).

3. The centrifugal force (F_C) for any intermediate position (*i.e.* between the minimum and maximum position) at a radius of rotation (r) may be obtained as discussed below :

Since the stiffness for a given spring is constant for all positions, therefore for minimum and intermediate position,

$$s = 2\left(\frac{F_C - F_{C1}}{r - r_1}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (x)$$

and for intermediate and maximum position,

$$s = 2\left(\frac{F_{C2} - F_C}{r_2 - r}\right)\left(\frac{x}{y}\right)^2 \qquad \ldots (xi)$$

$\therefore$ From equations (*ix*), (*x*) and (*xi*),

$$\frac{F_{C2} - F_{C1}}{r_2 - r_1} = \frac{F_C - F_{C1}}{r - r_1} = \frac{F_{C2} - F_C}{r_2 - r}$$

or

$$F_C = F_{C1} + (F_{C2} - F_{C1})\left(\frac{r - r_1}{r_2 - r_1}\right) = F_{C2} - (F_{C2} - F_{C1})\left(\frac{r_2 - r}{r_2 - r_1}\right)$$

Sensitiveness of Governors

Consider two governors *A* and *B* running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor *A* is greater than the lift of the sleeve of governor *B*. It is then said that the governor *A* is more sensitive than the governor *B*. In general, the greater the lift of the sleeve corresponding to a given fractional change in speed, the greater is the sensitiveness of the governor. It may also be stated in another way that for a given lift of the sleeve, the sensitiveness of the governor increases as the speed range decreases. This definition of sensitiveness may be quite satisfactory when the governor is considered as an independent mechanism. But when the governor is fitted to an engine, the practical requirement is simply that the change of equilibrium speed from the full load to the no load position of the sleeve should be as small a fraction as possible of the mean equilibrium speed. The actual displacement of the sleeve is immaterial, provided that it is sufficient to change the energy supplied to the engine by the required amount. For this reason, the sensitiveness is defined as the ***ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.***

Let N_1 = Minimum equilibrium speed,

N_2 = Maximum equilibrium speed, and

N = Mean equilibrium speed $= \dfrac{N_1 + N_2}{2}$.

$\therefore$ Sensitiveness of the governor

$$= \frac{N_2 - N_1}{N} = \frac{2(N_2 - N_1)}{N_1 + N_2}$$

$$= \frac{2(\omega_2 - \omega_1)}{\omega_1 + \omega_2} \qquad \text{. . . (In terms of angular speeds)}$$

Stability of Governors

A governor is said to be ***stable*** when for every speed within the working range there is a definite configuration *i.e*. there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

Note : A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Isochronous Governors

A governor is said to be ***isochronous*** when the equilibrium speed is constant (*i.e*. range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

Let us consider the case of a Porter governor running at speeds $N1$ and $N2$ r.p.m.

$$(N_1)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_1} \qquad \ldots (i)$$

$$(N_2)^2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h_2} \qquad \ldots (ii)$$

For isochronism, range of speed should be zero *i.e.* $N2 - N1 = 0$ or $N2 = N1$. Therefore from equations ***(i)*** and ***(ii)***, $h1 = h2$, which is impossible in case of a Porter governor. Hence a ***Porter governor cannot be isochronous.***

Now consider the case of a Hartnell governor running at speeds $N1$ and $N2$ r.p.m.

$$M.g + S_1 = 2\,F_{C1} \times \frac{x}{y} = 2 \times m\left(\frac{2\pi N_1}{60}\right)^2 r_1 \times \frac{x}{y} \qquad \ldots (iii)$$

$$M.g + S_2 = 2\,F_{C2} \times \frac{x}{y} = 2 \times m\left(\frac{2\pi N_2}{60}\right)^2 r_2 \times \frac{x}{y} \qquad \ldots (iv)$$

For isochronism, $N2 = N1$. Therefore from equations ***(iii)*** and ***(iv)***,

$$\frac{M.g + S_1}{M.g + S_2} = \frac{r_1}{r_2}$$

Note : The isochronous governor is not of practical use because the sleeve will move to one of its extreme positions immediately the speed deviates from the isochronous speed.

Hunting

A governor is said to be ***hunt*** if the speed of the engine fluctuates continuously above and below the mean speed. This is caused by a too sensitive governor which changes the fuel supply by a large amount when a small change in the speed of rotation takes place. For example, when the load on the engine increases, the engine speed decreases and, if the governor is very sensitive, the governor sleeve immediately falls to its lowest position. This will result in the opening of the control valve wide which will supply the fuel to the engine in excess of its requirement so that the engine speed rapidly increases again and the governor sleeve rises to its highest position. Due to this movement of the sleeve, the control valve will cut off the fuel supply to the engine and thus the engine speed begins to fall once again. This cycle is repeated indefinitely. Such a governor may admit either the maximum or the minimum amount of fuel. The effect of this will be to cause wide fluctuations in the engine speed or in other words, the engine will hunt.

Effort and Power of a Governor

The ***effort of a governor*** is the mean force exerted at the sleeve for a given percentage change of speed* (or lift of the sleeve). It may be noted that when the governor is running steadily, there is no force at the sleeve. But, when the speed changes, there is a resistance at the sleeve which opposes its motion. It is assumed that this resistance which is equal to the effort varies uniformly from a maximum value to zero while the governor moves into its new position of equilibrium.

The ***power of a governor*** is the work done at the sleeve for a given percentage change of speed. It is the product of the mean value of the effort and the distance through which the sleeve moves. Mathematically, Power = Mean effort × lift of sleeve.

Effort and Power of a Porter Governor

The effort and power of a Porter governor may be determined as discussed below.

Let N = Equilibrium speed corresponding to the configuration as shown in Fig. 7 (a), and

c = Percentage increase in speed.

Increase in speed = $c.N$

and increased speed = $N + c.N = N(1 + c)$

The equilibrium position of the governor at the increased speed is shown in Fig. 7 (b).

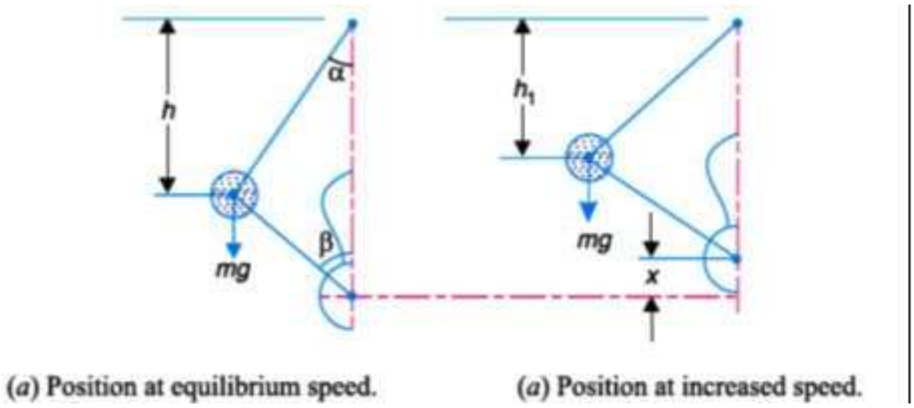


Fig. 7

When the speed is N r.p.m., the sleeve load is $M.g$. Assuming that the angles a and b are equal, so that $q = 1$, then the height of the governor,

$$h = \frac{m + M}{m} \times \frac{895}{N^2} \text{ (in metres)} \qquad \ldots (i)$$

When the increase of speed takes place, a downward force P will have to be exerted on the sleeve in order to prevent the sleeve from rising. If the speed increases to $(1 + c)\,N$ r.p.m. and the height of the governor remains the same, the load on the sleeve increases to $M1.g$. Therefore

$$h = \frac{m + M_1}{m} \times \frac{895}{(1 + c)^2 N^2} \text{ (in metres)} \qquad \ldots (ii)$$

Equating equations (*i*) and (*ii*), we have

$$m + M = \frac{m + M_1}{(1 + c)^2} \quad \text{or} \quad M_1 = (m + M)(1 + c^2) - m$$

and

$$M_1 - M = (m + M)(1 + c)^2 - m - M = (m + M)[(1 + c)^2 - 1] \qquad \ldots (iii)$$

A little consideration will show that $(M1 - M)g$ is the downward force which must be applied in order to prevent the sleeve from rising as the speed increases. It is the same force which acts on the governor sleeve immediately after the increase of speed has taken place and before the sleeve begins to move. When the sleeve takes the new position as shown in Fig. 7 (*b*), this force gradually diminishes to zero.

Let $\quad P$ = Mean force exerted on the sleeve during the increase in speed or the effort of the governor.

$$\therefore \quad P = \frac{(M_1 - M)\,g}{2} = \frac{(m + M)\,[(1 + c)^2 - 1]\,g}{2}$$

$$= \frac{(m + M)\,[1 + c^2 + 2c - 1]\,g}{2} = c\,(m + M)\,g \qquad \ldots (iv)$$

. . . (Neglecting c^2, being very small)

If F is the frictional force (in newtons) at the sleeve, then

$$P = c\,(m.g + M.g \pm F)$$

We have already discussed that the power of a governor is the product of the governor effort and the lift of the sleeve.

Let $\quad x$ = Lift of the sleeve.

$\therefore$ Governor power $= P \times x \qquad \ldots (v)$

If the height of the governor at speed N is h and at an increased speed $(1 + c)\,N$ is h_1, then

$$x = 2\,(h - h_1)$$

As there is no resultant force at the sleeve in the two equilibrium positions, therefore

$$h = \frac{m + M}{m} \times \frac{895}{N^2}, \quad \text{and} \quad h_1 = \frac{m + M}{m} \times \frac{895}{(1 + c)^2\,N^2},$$

$$\therefore \quad \frac{h_1}{h} = \frac{1}{(1 + c)^2} \quad \text{or} \quad h_1 = \frac{h}{(1 + c)^2}$$

We know that

$$x = 2\,(h - h_1) = 2\left[h - \frac{h}{(1 + c)^2}\right] = 2\,h\left[1 - \frac{1}{(1 + c)^2}\right]$$

$$= 2\,h\left[\frac{1 + c^2 + 2c - 1}{1 + c^2 + 2c}\right] = 2\,h\left(\frac{2c}{1 + 2c}\right) \qquad \ldots (vi)$$

. . . (Neglecting c^2, being very small)

Substituting the values of P and x in equation (v), we have

$$\text{Governor power} = c\,(m + M)\,g \times 2\,h\left(\frac{2c}{1 + 2c}\right) = \frac{4c^2}{1 + 2c}\,(m + M)\,g.h \qquad \ldots (vii)$$

Notes : 1. If α is not equal to β, *i.e.* $\tan\beta / \tan\alpha = q$, then the equations ($i$) and ($ii$) may be written as

$$h = \frac{m + \frac{M}{2}(1 + q)}{m} \times \frac{895}{N^2} \qquad \ldots (viii)$$

When speed increases to $(1 + c)\,N$ and height of the governor remains the same, then

$$h = \frac{m + \frac{M_1}{2}(1 + q)}{m} \times \frac{895}{(1 + c)\,N^2} \qquad \ldots (ix)$$

From equations (*viii*) and (*ix*), we have

$$m+\frac{M}{2}(1+q)=\frac{m+\frac{M_1}{2}(1+q)}{(1+c)^2}$$

or
$$\frac{M_1}{2}(1+q)=\left[m+\frac{M}{2}(1+q)\right](1+c)^2-m$$

$\therefore$
$$\frac{M_1}{2}=\frac{m(1+c)^2}{1+q}+\frac{M}{2}(1+c)^2-\frac{m}{1+q}$$

or
$$\frac{M_1}{2}-\frac{M}{2}=\frac{m(1+c)^2}{1+q}+\frac{M}{2}(1+c)^2-\frac{m}{1+q}-\frac{M}{2}$$

$$=\frac{m}{1+q}[(1+c)^2-1]+\frac{M}{2}[(1+c)^2-1]$$

$$=\left[\frac{m}{1+q}+\frac{M}{2}\right][(1+c)^2-1]$$

$\therefore$ Governor effort, $P=\left(\frac{M_1-M}{2}\right)g=\left[\frac{m}{1+q}+\frac{M}{2}\right][1+c^2+2c-1]\,g$

$$=\left(\frac{m}{1+q}+\frac{M}{2}\right)(2c)\,g=\left(\frac{2m}{1+q}+M\right)c.g \qquad \text{. . . (Neglecting } c^2\text{)}$$

The equation (*vi*) for the lift of the sleeve becomes,

$$x=(1+q)\,h\left(\frac{2c}{1+2c}\right)$$

$\therefore$ Governor power $=P\times x=\left(\frac{2m}{1+q}+M\right)c.g\,(1+q)\,h\left(\frac{2c}{1+2c}\right)$

$$=\frac{2c^2}{1+2c}[2m+M(1+q)]g.h=\frac{4c^2}{1+2c}\left[m+\frac{M}{2}(1+q)\right]g.h$$

2. The above method of determining the effort and power of a Porter governor may be followed for any other type of the governor.

Controlling Force

We have seen earlier that when a body rotates in a circular path, there is an inward radial force or centripetal force acting on it. In case of a governor running at a steady speed, the inward force acting on the rotating balls is known as ***controlling force.*** It is equal and opposite to the centrifugal reaction.

Controlling force, $FC = m.\text{w}^2.r$

The controlling force is provided by the weight of the sleeve and balls as in Porter governor and by the spring and weight as in Hartnell governor (or spring controlled governor).

When the graph between the controlling force (FC) as ordinate and radius of rotation of the balls (r) as abscissa is drawn, then the graph obtained is known as ***controlling force diagram.*** This diagram enables the stability and sensitiveness of the governor to be examined and also shows clearly the effect of friction.

Controlling Force Diagram for Porter Governor

The controlling force diagram for a Porter governor is a curve as shown in Fig. 8. We know that controlling force,

$$F_C = m.\omega^2.r = m\left(\frac{2\pi N}{60}\right)^2 r$$

$$N^2 = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2\left(\frac{F_C}{r}\right) = \frac{1}{m}\left(\frac{60}{2\pi}\right)^2(\tan\phi) \qquad \ldots\left[\because \frac{F_C}{r} = \tan\phi\right]$$

$$N = \frac{60}{2\pi}\left(\frac{\tan\phi}{m}\right)^{1/2} \qquad \ldots(i)$$

where f is the angle between the axis of radius of rotation and a line joining a given point (say A) on the curve to the origin O.

Notes: 1. in case the governor satisfies the condition for stability, the angle f must increase with radius of rotation of the governor balls. In other words, the equilibrium speed must increase with the increase of radius of rotation of the governor balls.

2. For the governor to be more sensitive, the change in the value of f over the change of radius of rotation should be as small as possible.

3. For the isochronous governor, the controlling force curve is a straight line passing through the origin. The angle f will be constant for all values of the radius of rotation of the governor. From equation ***(i)***

$$\tan\phi = \frac{F_C}{r} = \frac{m.\omega^2.r}{r} = m.\omega^2 = m\left(\frac{2\pi N}{60}\right)^2 = C.N^2$$

where $$C = m\left(\frac{2\pi}{60}\right)^2 = \text{constant}$$

Using the above relation, the angle f may be determined for different values of *N* and the lines are drawn from the origin. These lines enable the equilibrium speed corresponding to a given radius of rotation to be determined. Alternatively, the same results may be obtained more simply by setting-off a speed scale along any arbitrarily chosen ordinate. The controlling force is calculated for one constant radius of rotation and for different arbitrarily chosen values of speed. The values thus obtained are set-off along the ordinate that corresponds to the chosen radius and marked with the appropriate speeds.

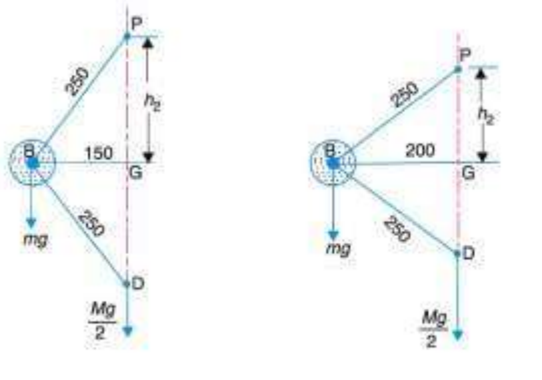
1) The arms of a Porter governor are each 250 mm long and pivoted on the governor axis. The mass of each ball is 5 kg and the mass of the central sleeve is 30 kg. The radius of rotation of the balls is 150 mm when the sleeve begins to rise and reaches a value of 200 mm for maximum speed. Determine the speed range of the governor. If the friction at the sleeve is equivalent of 20 N of load at the sleeve, determine how the speed range is modified.

Given : *BP* = *BD* = 250 mm ; *m* = 5 kg ; *M* = 30 kg ; *r*1 = 150 mm ; *r*2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig. (a) and (b) respectively.

Let N1 = Minimum speed when r1 = BG = 150 mm, and

N2 = Maximum speed when r2 = BG = 200 mm.



(*a*) Minimum position. (*b*) Maximum position.

Speed range of the governor

From Fig. (*a*), we find that height of the governor,

$$h_1 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$$

$$(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+30}{5} \times \frac{895}{0.2} = 31\,325$$
$$N_1 = 177 \text{ r.p.m.}$$

Height of the governor,

$$h_2 = PG = \sqrt{(PB)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

$$(N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+30}{5} \times \frac{895}{0.15} = 41\,767$$
$$N_2 = 204.4 \text{ r.p.m.}$$

speed range of the governor= $N2 - N1 = 204.4 - 177 = 27.4$ r.p.m.

Speed range when friction at the sleeve is equivalent of 20N of load (i.e. when F = 20 N)

We know that when the sleeve moves downwards, the friction force (F) acts upwards and the minimum speed is given by

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$
$$= \frac{5 \times 9.81 + (30 \times 9.81 - 20)}{5 \times 9.81} \times \frac{895}{0.2} = 29500$$
$$\therefore \quad N_1 = 172 \text{ r.p.m.}$$

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum speed is given by

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$
$$= \frac{5 \times 9.81 + (30 \times 9.81 + 20)}{5 \times 9.81} \times \frac{895}{0.15} = 44200$$
$$\therefore \quad N_2 = 210 \text{ r.p.m.}$$

Speed range of the governor = $N2 - N1 = 210 - 172 = 38$ r.p.m.

2) The arms of a Porter governor are 300 mm long. The upper arms are pivoted on the axis of rotation. The lower arms are attached to a sleeve at a distance of 40 mm from the axis of rotation. The mass of the load on the sleeve is 70 kg and the mass of each ball is 10 kg. Determine the equilibrium speed when the radius of rotation of the balls is 200 mm. If the

friction is equivalent to a load of 20 N at the sleeve, what will be the range of speed for this position ?

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Solution. Given : BP = BD = 300 mm ; DH = 40 mm ; M = 70 kg ; m = 10 kg ; r = BG = 200 mm

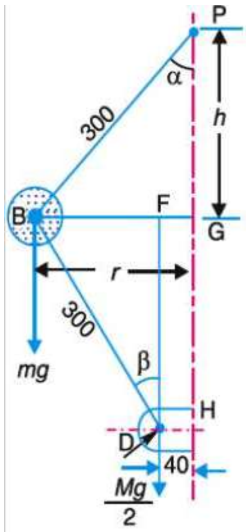
Equilibrium speed when the radius of rotation r = BG = 200 mm

Let N = Equilibrium speed.

The equilibrium position of the governor is shown in Fig. From the figure, we find that height of the governor,

$$h = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(300)^2 - (200)^2} = 224 \text{ mm}$$

$$= 0.224\text{m}$$



All dimensions in mm.

BF = BG – FG = 200 – 40 = 160 . . . (FG = DH)

and $$DF = \sqrt{(DB)^2 - (BF)^2} = \sqrt{(300)^2 - (160)^2} = 254 \text{ mm}$$

$\therefore \tan\alpha = BG/PG = 200 / 224 = 0.893$

and $\tan\beta = BF/DF = 160 / 254 = 0.63$

$$\therefore \quad q = \frac{\tan\beta}{\tan\alpha} = \frac{0.63}{0.893} = 0.705$$

We know that

$$N_2 = \frac{m + \frac{M}{2}(1+q)}{m} \times \frac{895}{h}$$

$$= \frac{10 + \frac{70}{2}(1+0.705)}{10} \times \frac{895}{0.224} = 27\,840$$

$\therefore \quad N_2 = 167$ r.p.m. **Ans.**

Range of speed when friction is equivalent to load of 20 N at the sleeve (i.e. when F = 20 N)

Let N1 = Minimum equilibrium speed, and
N2 = Maximum equilibrium speed.

We know that when the sleeve moves downwards, the frictional force (F) acts upwards and the minimum equilibrium speed is given by

$$(N_1)^2 = \frac{m.g + \left(\frac{M.g - F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 - 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 27\,144$$

N1 = 164.8 r.p.m.

We also know that when the sleeve moves upwards, the frictional force (F) acts downwards and the maximum equilibrium speed is given by

$$(N_2)^2 = \frac{m.g + \left(\frac{M.g + F}{2}\right)(1+q)}{m.g} \times \frac{895}{h}$$

$$= \frac{10 \times 9.81 + \left(\frac{70 \times 9.81 + 20}{2}\right)(1+0.705)}{10 \times 9.81} \times \frac{895}{0.224} = 28\,533$$

$\therefore \quad N_2 = 169$ r.p.m.

We know that range of speed = $N_2 - N_1$ = – 164.8 = 4.2 r.p.m.

3) All the arms of a Porter governor are 178 mm long and are hinged at a distance of 38 mm from the axis of rotation. The mass of each ball is 1.15 kg and mass of the sleeve is 20 kg. The governor sleeve begins to rise at 280 r.p.m. when the links are at an angle of 30° to the vertical. Assuming the friction force to be constant, determine the minimum and maximum speed of rotation when the inclination of the arms to the vertical is 45°.

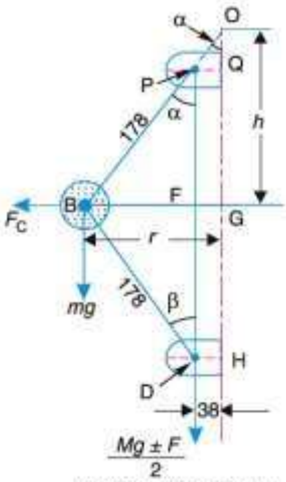
Given: *BP* = *BD* = 178 mm; *PQ* = *DH* = 38 mm; *m* = 1.15 kg; *M* = 20 kg; *N* = 280 r.p.m. ; $\alpha = \beta = 30°$

First of all, let us find the friction force (*F*). The equilibrium position of the governor when the lines are at 30° to vertical, is shown in Fig.. From the figure, we find that radius of rotation,

$r = BG = BF + FG = BP \times \sin\alpha + FG = 178 \sin 30° + 38 = 127$ mm

and height of the governor,

$h = BG / \tan\alpha = 127 / \tan 30° = 220$ mm $= 0.22$ m



All dimensions in mm.

$$N^2 = \frac{m.g + (Mg \pm F)}{m.g} \times \frac{895}{h}$$

$$\ldots (\because \tan\alpha = \tan\beta \text{ or } q = 1)$$

$$(280)^2 = \frac{1.15 \times 9.81 + 20 \times 9.81 \pm F}{1.15 \times 9.81} \times \frac{895}{0.22}$$

or

$$\pm F = \frac{(280)^2 \times 1.15 \times 9.81 \times 0.22}{895} - 1.15 \times 9.81 - 20 \times 9.81$$

$$= 217.5 - 11.3 - 196.2 = 10 \text{ N}$$

We know that radius of rotation when inclination of the arms to the vertical is 45 (*i.e.* when $\alpha = \beta = 45°$),

$r = BG = BF + FG = BP \times \sin\alpha + FG = 178 \sin 45° + 38 = 164$ mm

and height of the governor,

$h = BG / \tan\alpha = 164 / \tan 45° = 164$ mm $= 0.164$ m

Let $N1$ = Minimum speed of rotation, and

$N2$ = Maximum speed of rotation.

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 - 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 95\ 382$$

$\therefore \quad N_1 = 309$ r.p.m. **Ans.**

and
$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h}$$

$$= \frac{1.15 \times 9.81 + (20 \times 9.81 + 10)}{1.15 \times 9.81} \times \frac{895}{0.164} = 105\ 040$$

$N_2 = 324$ r.p.m. **Ans.**

4) A Proell governor has equal arms of length 300 mm. The upper and lower ends of the arms are pivoted on the axis of the governor. The extension arms of the lower links are each 80 mm long and parallel to the axis when the radii of rotation of the balls are 150 mm and 200 mm. The mass of each ball is 10 kg and the mass of the central load is 100 kg. Determine the range of speed of the governor.

Given :
PF = DF = 300 mm ; BF = 80 mm ; m = 10 kg ; M = 100 kg ; r_1 = 150 mm; r_2 = 200 mm

First of all, let us find the minimum and maximum speed of the governor. The minimum and maximum position of the governor is shown in Fig

Let N_1 = Minimum speed when radius of rotation, r_1 = FG = 150 mm ; and
N_2 = Maximum speed when radius of rotation , r_2 = FG = 200 mm.

From Fig. (a), we find that height of the governor,

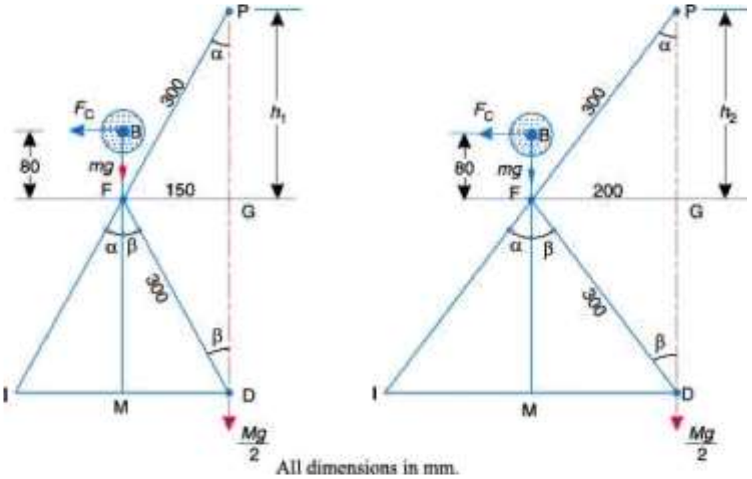
$$h_1 = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(300)^2 - (150)^2} = 260 \text{ mm} = 0.26 \text{ m}$$

FM = *GD* = *PG* = 260 mm = 0.26 m
BM = *BF* + *FM* = 80 + 260 = 340 mm = 0.34 m

$$(N_1)^2 = \frac{FM}{BM}\left(\frac{m+M}{m}\right)\frac{895}{h_1} \qquad \ldots (\because \alpha = \beta \text{ or } q = 1)$$

$$= \frac{0.26}{0.34}\left(\frac{10+100}{10}\right)\frac{895}{0.26} = 28\ 956 \quad \text{or} \quad N_1 = 170 \text{ r.p.m.}$$

Now from Fig. *b*, we find that height of the governor,



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From Fig. 18.13 (*a*), we find that

$$BM = BF + FM = 80 + 260 = 340 \text{ mm} = 0.34 \text{ m}$$

$$ID = IM + MD = 0.15 + 0.15 = 0.3 \text{ m}$$

$$F_C = m(\omega_1)^2 r_1 = 10 \left(\frac{2\pi N_1}{60}\right)^2 0.15 = 0.0165 (N_1)^2$$

$$F_C \times BM = m.g \times IM + \frac{}{2}$$

a.oias‹/v/›' as= ‹oxsai xo.is + $\frac{}{2'}$ or

0 tSG f/¥' = t4.7i3 + l47.t3 = t6t.86S

Similarly N_2 may be calculated.

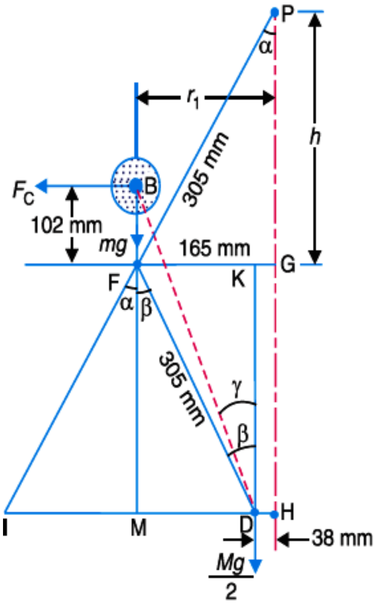
A Proell governor has all four arms of length 305 mm. The upper arms are pivoted on the axis of rotation and the lower arms are attached to a sleeve at a distance of 38 mm from the axis. The mass of each ball is 4.8 kg and are attached to the extension of the lower arms which are 102 mm long. The mass on the sleeve is 45 kg. The minimum and maximum radii of governor are 165 mm and 216 mm. Assuming that the extensions of the lower arms are parallel to the governor axis at the minimum radius, find the corresponding equilibrium speeds.

Given : *PF* = *DF* = 305 mm ; *DH* = 38 mm ; *BF* = 102 mm ; *m* = 4.8 kg ; *M* = 54 kg

Equilibrium speed at the minimum radius of governor

The radius of the governor is the distance of the point of intersection of the upper and lower arms from the governor axis. When the extensions of the lower arms are parallel to the governor axis, then the radius of the governor (*FG*) is equal to the radius of rotation (r_1).

The governor configuration at the minimum radius (*i.e.* when *FG* = 165 mm) is shown in Fig.



N_1 = Equilibrium speed at the minimum radius *i.e.* when $FG = r_1$ = 165 mm.

$$\sin\alpha = \frac{FG}{FP} = \frac{165}{305} = 0.541$$

$$\alpha = 32.75°$$

$$\tan\alpha = \tan 32.75° = 0.6432$$

$$\sin\beta = \frac{FK}{DF} = \frac{FG - KG}{DF}$$

$$= \frac{165 - 38}{305} = 0.4164$$

$$\beta = 24.6°$$

$$\tan\beta = \tan 24.6° = 0.4578$$

$$q = \frac{\tan\beta}{\tan\alpha} = \frac{0.4578}{0.6432} = 0.712$$

From Fig, we find that height of the governor

$$h = PG = \sqrt{(PF)^2 - (FG)^2} = \sqrt{(305)^2 - (165)^2} = 256.5 \text{ mm} = 0.2565 \text{ m}$$

$$MD = FK = FG - KG = 165 - 38 = 127 \text{ mm}$$

$$FM = \sqrt{(DF)^2 - (MD)^2} = \sqrt{(305)^2 - (127)^2} = 277 \text{ mm} = 0.277 \text{ m}$$

$$BM = BF + FM = 102 + 277 = 379 \text{ mm} = 0.379 \text{ m}$$

$$(N_1)^2 = \frac{FM}{BM}\left[\frac{m + \frac{M}{2}(1+q)}{m}\right]\frac{895}{h}$$

$$= \frac{0.277}{0.379}\left[\frac{4.8 + \frac{54}{2}(1+0.712)}{4.8}\right]\frac{895}{0.2565} = 27\,109$$

$$N_1 = 165 \text{ r.p.m. } \textbf{Ans.}$$

Note: The valve of $N1$ may also be obtained by drawing the governor configuration to some suitable scale and measuring the distances BM, IM and ID. Now taking moments about point I,

$$F_C \times BM = m.g \times IM + \frac{M.g}{2} \times ID,$$

$$F_C = \text{Centrifugal force} = m(\omega_1)^2 r_1 = m\left(\frac{2\pi N_1}{60}\right)^2 r_1$$

Equilibrium speed at the maximum radius of governor

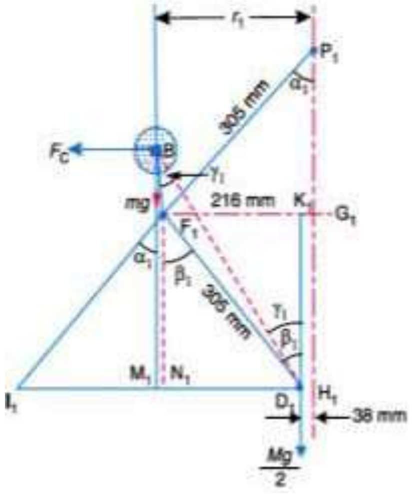
Let $N2$ = Equilibrium speed at the maximum radius of governor, *i.e.* when $F1G1 = r_2 = 216$ mm.

First of all, let us find the values of BD and g in Fig. We know that

$$BD = \sqrt{(BM)^2 + (MD)^2} = \sqrt{(397)^2 + (}$$

$$\tan\gamma = MD/BM = 127/379 = 0.335 \quad \text{or}$$

The governor configuration at the maximum radius of $F1G1$ = 216 mm is shown in Fig. From the geometry of the figure,



$$\sin \alpha_1 = \frac{F_1 G_1}{P_1 F_1} = \frac{216}{305} = 0.7082$$

$$\alpha_1 = 45.1°$$

$$\sin \beta_1 = \frac{F_1 K_1}{F_1 D_1} = \frac{F_1 G_1 - K_1 G_1}{F_1 D_1}$$

$$= \frac{216 - 38}{305} = 0.5836$$

$$\beta_1 = 35.7°$$

Since the extension is rigidly connected to the lower arm (*i.e*. *DFB* or *D*1*F*1*B*1 is one continuous link) therefore *B*1*D*1 and angle *B*1*D*1*F*1 do not change. In other words,

*B*1*D*1 = *BD* = 400 mm

$\gamma - \beta = \gamma_1 - \beta_1$ ορ $\gamma_1 = \gamma - \beta + \beta_1 = 18.5° - 24.6° + 35.7° = 29.6°$

$\therefore$ Radius of rotation,

$$r_2 = M_1 D_1 + D_1 H_1 = B_1 D_1 \times \sin \gamma_1 + 38 \text{ mm}$$
$$= 400 \sin 29.6° + 38 = 235.6 \text{ mm} = 0.2356 \text{ m}$$

From Fig. 18.17, we find that

$$B_1 M_1 = B_1 D_1 \times \cos \gamma_1 = 400 \times \cos 29.6° = 348 \text{ mm} = 0.348 \text{ m}$$
$$F_1 N_1 = F_1 D_1 \times \cos \beta_1 = 305 \times \cos 35.7° = 248 \text{ mm} = 0.248 \text{ m}$$
$$I_1 N_1 = F_1 N_1 \times \tan \alpha_1 = 0.248 \times \tan 45.1° = 0.249 \text{ m}$$
$$N_1 D_1 = F_1 D_1 \times \sin \beta_1 = 305 \times \sin 35.7 = 178 \text{ mm} = 0.178 \text{ m}$$
$$\therefore \quad I_1 D_1 = I_1 N_1 + N_1 D_1 = 0.249 + 0.178 = 0.427 \text{ m}$$
$$M_1 D_1 = B_1 D_1 \sin \gamma_1 = 400 \sin 29.6° = 198 \text{ mm} = 0.198 \text{ m}$$
$$\therefore \quad I_1 M_1 = I_1 D_1 - M_1 D_1 = 0.427 - 0.198 = 0.229 \text{ m}$$

We know that centrifugal force,

$$F_C = m(\omega_2)^2 \, r_2 = 4.8 \left(\frac{2 \pi N_2}{60} \right)^2 0.2356 = 0.0124 \, (N_2)^2$$

Now taking moments about point I_1,

$$F_C \times B_1 M_1 = m.g \times I_1 M_1 + \frac{M.g}{2} \times I_1 D_1$$

$$0.0124 \, (N_2)^2 \times 0.348 = 4.8 \times 9.81 \times 0.229 + \frac{54 \times 9.81}{2} \times 0.427$$

$$0.0043 \, (N_2)^2 = 10.873 + 113.1 = 123.883$$

$$(N_2)^2 = \frac{123.883}{0.0043} = 28\,810 \qquad \text{or} \qquad N_2 = 170 \text{ r.p.m.}$$

A Hartnell governor having a central sleeve spring and two right-angled bell crank levers moves between 290 r.p.m. and 310 r.p.m. for a sleeve lift of 15 mm. The sleeve arms and the ball arms are 80 mm and 120 mm respectively. The levers are pivoted at 120 mm from the governor axis and mass of each ball is 2.5 kg. The ball arms are parallel to the governor axis at the lowest equilibrium speed. Determine : **1.** loads on the spring at the lowest and the highest equilibrium speeds, and **2.** stiffness of the spring.

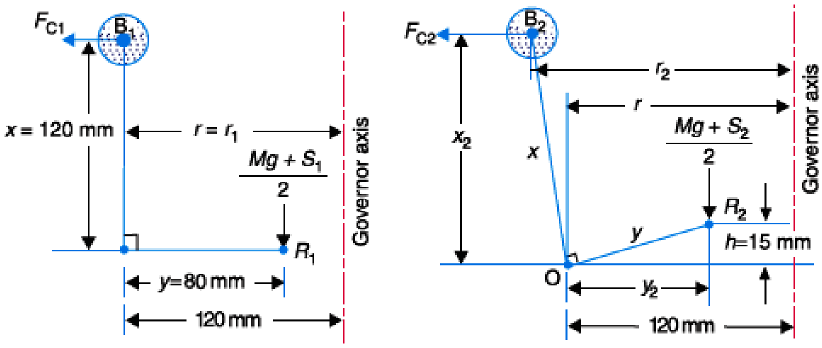
Solution. Given : N1 = 290 r.p.m. or ω_1 = 2 _ × 290/60 = 30.4 rad/s ; N2 = 310 r.p.m. or ω_2 = 2 _ × 310/60 = 32.5 rad/s ; h = 15 mm = 0.015 m ; y = 80 mm = 0.08 m ; x = 120 mm = 0.12 m ; r = 120 mm = 0.12 m ; m = 2.5 kg

1. Loads on the spring at the lowest and highest equilibrium speeds

Let S = Spring load at lowest equilibrium speed, and

S2 = Spring load at highest equilibrium speed.

Since the ball arms are parallel to governor axis at the lowest equilibrium speed (i.e. at N1 = 290 r.p.m.), as shown in Fig. (a), therefore r = r1 = 120 mm = 0.12 m



(*a*) Lowest position. (*b*) Highest position.

We know that centrifugal force at the minimum speed,

$FC1 = m\,(\omega_1)^2\,r1 = 2.5\,(30.4)^2\,0.12 = 277$ N

Now let us find the radius of rotation at the highest equilibrium speed, *i.e.* at *N*2 = 310 r.p.m. The position of ball arm and sleeve arm at the highest equilibrium speed is shown in Fig. (*b*).

Let *r*2 = Radius of rotation at *N*2 = 310 r.p.m.

$$h = (r_2 - r_1)\frac{y}{x}$$

$$r_2 = r_1 + h\left(\frac{x}{y}\right) = 0.12 + 0.015\left(\frac{0.12}{0.08}\right) = 0.1425 \text{ m}$$

Centrifugal force at the maximum speed,

$$FC_2 = m\,(\omega_2)^2\,r_2 = 2.5 \times (32.5)2 \times 0.1425 = 376 \text{ N}$$

Neglecting the obliquity effect of arms and the moment due to the weight of the balls, we have for lowest position,

$$M.g + S_1 = 2F_{C1} \times \frac{x}{y} = 2 \times 277 \times \frac{0.12}{0.08} = 831 \text{ N}$$

$$\therefore \quad S_2 = 831 \text{ N} \qquad (\because M = 0)$$

and for highest position,

$$M.g + S_2 = 2F_{C2} \times \frac{x}{y} = 2 \times 376 \times \frac{0.12}{0.08} = 1128 \text{ N}$$

$$\therefore \quad S_1 = 1128 \text{ N} \qquad (\because M = 0)$$

2. Stiffness of the spring

We know that stiffness of the spring,

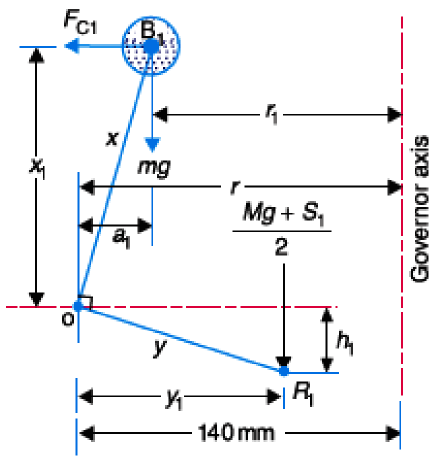
$$s = \frac{S_2 - S_1}{h} = \frac{1128 - 831}{15} = 19.8 \text{ N/mm}$$

In a spring loaded governor of the Hartnell type, the mass of each ball is 5 kg and the lift of the sleeve is 50 mm. The speed at which the governor begins to float is 240 r.p.m., and at this speed the radius of the ball path is 110 mm. The mean working speed of the governor is 20 times the range of speed when friction is neglected. If the lengths of ball and roller arm of the bell crank lever are 120 mm and 100 mm respectively and if the distance between the centre of pivot of bell crank lever and axis of governor spindle is 140 mm, determine the initial compression of the spring taking into account the obliquity of arms. If friction is equivalent to a force of 30 N at the sleeve, find the total alteration in speed before the sleeve begins to move from mid-position.

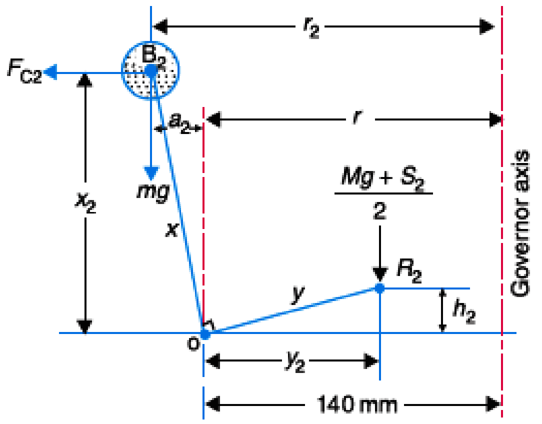
Solution. Given : m = 5 kg ; h = 50 mm = 0.05 m ; $N1$ = 240 r.p.m. or $\omega_1 = 2\pi \times 240/60$ = 25.14 rad/s ; r_1 = 110 mm = 0.11 m ; x = 120 mm = 0.12 m ; y = 100 mm = 0.1 m ; r = 140 mm = 0.14 m ; F = 30 N

Initial compression of the spring taking into account the obliquity of arms

First of all, let us find out the maximum speed of rotation (ω_2) in rad/s.
We know that mean working speed,



(*b*) Minimum position



(*b*) Maximum position

$$\omega = \frac{\omega_1 + \omega_2}{2}$$

and range of speed, neglecting friction

$$= \omega_2 - \omega_1$$

Since the mean working speed is 20 times the range of speed, therefore

$$\omega = 20\,(\omega_2 - \omega_1)$$

or
$$\frac{\omega_1 + \omega_2}{2} = 20\,(\omega_2 - \omega_1)$$

$$25.14 + \omega_2 = 40\,(\omega_2 - 25.14) = 40\,\omega_2 - 1005.6$$

$\therefore$ $40\,\omega_2 - \omega_2 = 25.14 + 1005.6 = 1030.74$ or $\omega_2 = 26.43$ rad/s

The minimum and maximum position of the governor balls is shown in Fig. (*a*) and (*b*) respectively.

Let r_2 = Maximum radius of rotation.

We know that lift of the sleeve,

$$h = (r_2 - r_1)\,\frac{y}{x}$$

or
$$r_2 = r_1 + h \times \frac{x}{y} = 0.11 + 0.05 \times \frac{0.12}{0.1} = 0.17 \text{ m}$$

We know that centrifugal force at the minimum speed,

$$F_{C1} = m\,(\omega_1)^2\,r_1 = 5\,(25.14)^2\,0.11 = 347.6 \text{ N}$$

and centrifugal force at the maximum speed,

$$F_{C2} = m\,(\omega_2)^2\,r_2 = 5\,(26.43)^2\,0.17 = 593.8 \text{ N}$$

Since the obliquity of arms is to be taken into account, therefore from the minimum position as shown in Fig. (*a*),

$a_1 = r - r_1 = 0.14 - 0.11 = 0.03$ m

$$x_1 = \sqrt{x^2 - (a_1)^2} = \sqrt{(0.12)^2 - (0.03)^2} = 0.1162 \text{ m}$$

and
$$y_1 = \sqrt{y^2 - (h_1)^2} = \sqrt{(0.1)^2 - (0.025)^2} = 0.0986 \text{ m}$$

. . . ($\because$ $h_1 = h/2 = 0.025$ m)

Similarly, for the maximum position, as shown in Fig. 18.21 (*b*),

$$a_2 = r_2 - r = 0.17 - 0.14 = 0.03 \text{ m}$$

$\therefore$ $x_2 = x_1 = 0.1162$ m . . . ($\because$ $a_2 = a_1$)

and $y_2 = y_1 = 0.0986$ m . . . ($\because$ $h_2 = h_1$)

Now taking moments about point *O* for the minimum position as shown in Fig. (*a*),

$$\frac{M.g + S_1}{2} \times y_1 = F_{C1} \times x_1 - m.g \times a_1$$

$$\frac{S_1}{2} \times 0.0968 = 347.6 \times 0.1162 - 5 \times 9.81 \times 0.03 = 38.9 \text{ N} \quad \ldots (\because M = 0)$$

$\therefore$ $S_1 = 2 \times 38.9/0.0968 = 804$ N

Similarly, taking moments about point *O* for the maximum position as shown in Fig. (*b*),

$$\frac{M.g + S_2}{2} \times y_2 = F_{C2} \times x_2 + m.g \times a_2$$

$$\frac{S_2}{2} \times 0.0968 = 593.8 \times 0.1162 + 5 \times 9.81 \times 0.03 = 70.47 \text{ N} \quad \ldots (\because M = 0)$$

$\therefore$ $S_2 = 2 \times 70.47/0.0968 = 1456$ N

We know that stiffness of the spring

$$s = \frac{S_2 - S_1}{h} = \frac{1456 - 804}{50} = 13.04 \text{ N/mm}$$

$\therefore$ Initial compression of the spring

$$= \frac{S_1}{s} = \frac{804}{13.04} = 61.66 \text{ mm}$$

Total alternation in speed when friction is taken into account

We know that spring force for the mid-position,

$$S = S_1 + h_1.s = 8.4 + 25 \times 13.04 = 1130 \text{ N} \quad \ldots(\because h_1 = h/2 = 25 \text{ mm})$$

and mean angular speed, $\omega = \frac{\omega_1 + \omega_2}{2} = \frac{25.14 + 26.43}{2} = 25.785 \text{ rad/s}$

or $N = \omega \times 60 / 2\pi = 25.785 \times 60 / 2\pi = 246.2 \text{ r.p.m.}$

$\therefore$ Speed when the sleeve begins to move downwards from the mid-position,

$$N' = N\sqrt{\frac{S - F}{S}} = 246.2\sqrt{\frac{1130 - 30}{1130}} = 243 \text{ r.p.m.}$$

and speed when the sleeve begins to move upwards from the mid-position,

$$N'' = N\sqrt{\frac{S + F}{S}} = 246.2\sqrt{\frac{1130 + 30}{1130}} = 249 \text{ r.p.m.}$$

$\therefore$ Alteration in speed $= N'' - N' = 249 - 243 = 6$ r.p.m. **Ans.**

A Porter governor has equal arms each 250 mm long and pivoted on the axis of rotation. Each ball has a mass of 5 kg and the mass of the central load on the sleeve is 25 kg. The radius of rotation of the ball is 150 mm when the governor begins to lift and 200 mm when the governor is at maximum speed. Find the range of speed, sleeve lift, governor effort and power of the governor in the following cases :

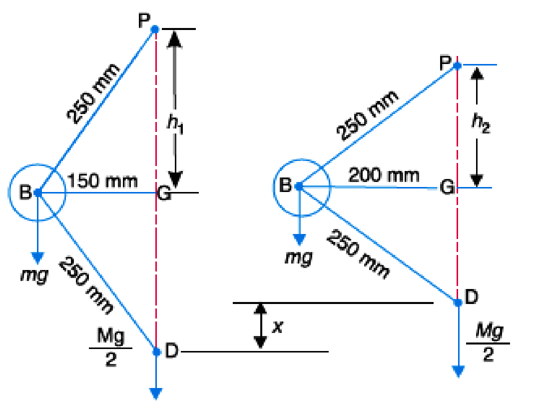
1. When the friction at the sleeve is neglected, and
2. When the friction at the sleeve is equivalent to 10 N.

Given : $BP = BD = 250$ mm ; $m = 5$ kg ; $M = 25$ kg ; $r1 = 150$ mm ; $r2 = 200$ mm ; $F = 10$ N

1. When the friction at the sleeve is neglected

First of all, let us find the minimum and maximum speed of rotation. The minimum and maximum position of the governor is shown in Fig. 18.34 (*a*) and (*b*) respectively.

Let $N1$ = Minimum speed, and
N2 = Maximum speed.



(*a*) Minimum position. (*b*) Maximum position.

From Fig a

` $h_1 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (150)^2} = 200 \text{ mm} = 0.2 \text{ m}$

From Fig. (*b*),

$$h_2 = PG = \sqrt{(BP)^2 - (BG)^2} = \sqrt{(250)^2 - (200)^2} = 150 \text{ mm} = 0.15 \text{ m}$$

We know that $(N_1)^2 = \frac{m+M}{m} \times \frac{895}{h_1} = \frac{5+25}{5} \times \frac{895}{0.2} = 26\ 850$

$\therefore \quad N_1 = 164$ r.p.m.

l $\quad (N_2)^2 = \frac{m+M}{m} \times \frac{895}{h_2} = \frac{5+25}{5} \times \frac{895}{0.15} = 35\ 800$

$\because \quad N_2 = 189$ r.p.m.

Range of speed

We know that range of speed = N2 – N1 = 189 – 164 = 25 r.p.m.

Sleeve lift

We know that sleeve lift, x = 2 (h_1 – h_2) = 2 (200 – 150) = 100 mm = 0.1 m

Governor effort

Let c = Percentage increase in speed.

We know that increase in speed or range of speed,

c.N1 = N_2 – N_1 = 25 r.p.m.

c = 25/N_1 = 25/164 = 0.152

We know that governor effort

P = c (m + M) g = 0.152 (5 + 25) 9.81 = 44.7 N

Power of the governor

We know that power of the governor = P.x = 44.7 × 0.1 = 4.47 N-m

2. When the friction at the sleeve is taken into account

$$(N_1)^2 = \frac{m.g + (M.g - F)}{m.g} \times \frac{895}{h_1}$$

$$= \frac{5 \times 9.81 + (25 \times 9.81 - 10)}{5 \times 9.81} \times \frac{895}{0.2} = 25\ 938$$

$N_1 = 161$ r.p.m.

$$(N_2)^2 = \frac{m.g + (M.g + F)}{m.g} \times \frac{895}{h_2}$$

$$= \frac{5 \times 9.81 + (25 \times 9.81 + 10)}{5 \times 9.81} \times \frac{895}{0.15} = 37\ 016$$

$N_2 = 192.4$ r.p.m.

Range of speed

We know that range of speed = N2 – N1 = 192.4 – 161 = 31.4 r.p.m.

Sleeve lift

The sleeve lift (x) will be same as calculated above.

Sleeve lift, x = 100 mm = 0.1 m

Governor effort

Let c = Percentage increase in speed.

We know that increase in speed or range of speed,

c.N1 = N2 – N1 = 31.4 r.p.m.

c = 31.4/N1 = 31.4/161 = 0.195

We know that governor effort,

P = c (m.g + M.g + F) = 0.195 (5 × 9.81 + 25 × 9.81 + 10) N= 57.4 N

Power of the governor

We know that power of the governor= P.x = 57.4 × 0.1 = 5.74 N-m

Exercise Problems

1) A loaded governor of the Porter type has equal arms and links each 250 mm long. The mass of each ball is 2 kg and the central mass is 12 kg. When the ball radius is 150 mm, the valve is fully open and when the radius is 185 mm, the valve is closed. Find the maximum speed and the range of speed. If the maximum speed is to be increased 20% by an addition of mass to the central load, find what additional mass is required.

[Ans. 193 r.p.m. ; 16 r.p.m.; 6.14 kg]

2) In a Porter governor, the upper and lower arms are each 250 mm long and are pivoted on the axis of rotation. The mass of each rotating ball is 3 kg and the mass of the sleeve is 20 kg. The sleeve is in its lowest position when the arms are inclined at 30° to the governor axis. The lift of the sleeve is 36 mm. Find the force of friction at the sleeve, if the speed at the moment it rises from the lowest position is equal to the speed at the moment it falls from the highest position. Also, find the range of speed of the governor.

[Ans. 9.8 N ; 16 r.p.m.]

3) A Proell governor has all the four arms of length 250 mm. The upper and lower ends of the arms are pivoted on the axis of rotation of the governor. The extension arms of the lower links are each 100 mm long and parallel to the axis when the radius of the ball path is 150 mm. The mass of each ball is 4.5 kg and the mass of the central load is 36 kg. Determine the equilibrium speed of the governor.

[Ans. 164 r.p.m.]

4) A Proell governor has arms of 300 mm length. The upper arms are hinged on the axis of rotation, whereas the lower arms are pivoted at a distance of 35 mm from the axis of rotation. The extension of lower arms to which the balls are attached are 100 mm long. The mass of each ball is 8 kg and the mass on the sleeve is 60 kg. At the minimum radius of rotation of 200 mm, the extensions are parallel to the governor axis. Determine the equilibrium speed of

the governor for the given configuration. What will be the equilibrium speed for the maximum radius of 250 mm?

[Ans. 144.5 r.p.m. ; 158.2 r.p.m.]

5) A spring controlled governor of the Hartnell type with a central spring under compression has balls each of mass 2 kg. The ball and sleeve arms of the bell crank levers are respectively 100 mm and 60 mm long and are at right angles. In the lowest position of the governor sleeve, the radius of rotation of the balls is 80 mm and the ball arms are parallel to the governor axis. Find the initial load on the spring in order that the sleeve may begin to lift at 300 r.p.m. If the stiffness of the spring is 30 kN/m, what is the equilibrium speed corresponding to a sleeve lift of 10 mm?

[Ans. 527 N ; 342 r.p.m.]

6) In a governor of the Hartnell type, the mass of each ball is 1.5 kg and the lengths of the vertical and horizontal arms of the bell crank lever are 100 mm and 50 mm respectively. The fulcrum of the bell crank lever is at a distance of 90 mm from the axis of rotation. The maximum and minimum radii of rotation of balls are 120 mm and 80 mm and the corresponding equilibrium speeds are 325 and 300 r.p.m. Find the stiffness of the spring and the equilibrium speed when the radius of rotation is 100 mm.

[Ans. 18 kN/m, 315 r.p.m.]

7) A Porter governor has all four arms 200 mm long. The upper arms are pivoted on the axis of rotation and the lower arms are attached to a sleeve at a distance of 25 mm from the axis. Each ball has a mass of 2 kg and the mass of the load on the sleeve is 20 kg. If the radius of rotation of the balls at a speed of 250 r.p.m. is 100 mm, find the speed of the governor after the sleeve has lifted 50 mm. Also determine the effort and power of the governor.

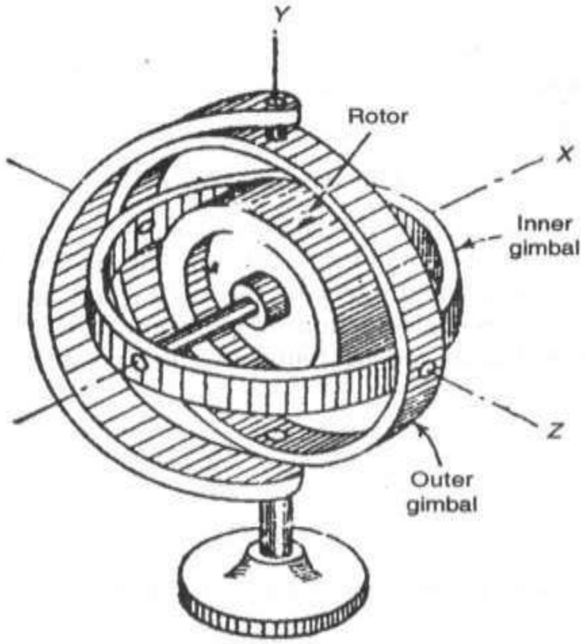
[Ans. 275.6 r.p.m.; 22.4 N; 1.12 N-m]

8) A Porter governor has arms 250 mm each and four rotating flyballs of mass 0.8 kg each. The sleeve movement is restricted to ± 20 mm from the height when the mean speed is 100 r.p.m. Calculate the central dead load and sensitivity of the governor neglecting friction when the flyball exerts a centrifugal force of 9.81 N. Determine also the effort and power of the governor for 1 percent speed change.

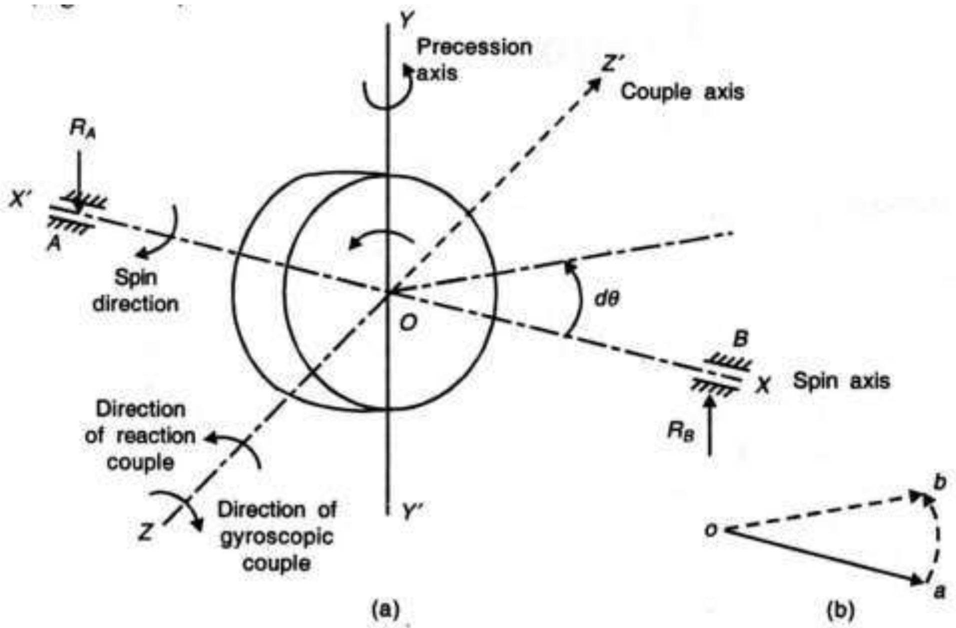
[Ans. 11.76 N; 11.12; 0.196 N; 7.7 N-mm]

Gyroscopes:

- A gyroscope is a spatial mechanism which is generally employed for the study of precessional motion of a rotary body. Gyroscope finds applications in gyrocompass, used in aircraft, naval ship, control system of missiles and space shuttle. The gyroscopic effect is also felt on the automotive vehicles while negotiating a turn.
- A gyroscope consists of a rotor mounted in the inner gimbal. The inner gimbal is mounted in the outer gimbal which itself is mounted on a fixed frame as shown in Fig.1. When the rotor spins about X-axis with angular velocity ω rad/s and the inner gimbal precesses (rotates) about Y-axis, the spatial mechanism is forced to turn about Z-axis other than its own axis of rotation, and the gyroscopic effect is thus setup. The resistance resistance to change in the direction of rotational axis is called gyroscopic effect.



Gyroscopic couple:
Consider a rotary body of mass *m* having radius of gyration *k* mounted on the shaft supported at two bearings. Let the rotor spins (rotates) about X-axis with constant angular velocity ω rad/s. The X-axis is, therefore, called spin axis, Y-axis, precession axis and Z-axis, the couple or torque axis



The angular momentum of the rotating mass is given by,

$$H = mK^2\omega = I\omega$$

Now, suppose the shaft axis (X-axis) precesses through a small angle δθ about Y-axis in the plane *XOZ,* then the angular momentum varies from *H* to *H* + δ*H,* where δ*H* is the change in the angular momentum, represented by vector ab.For the small value of angle of rotation 50, we can write,

$$ab = oa \times \delta\theta$$

$$\delta H = H \times \delta\theta$$

$$= I\omega\delta\theta$$

However, the rate of change of angular momentum is:

$$C = \frac{dH}{dt} = \lim_{\delta t \to 0}\left(\frac{I\omega\delta\theta}{\delta t}\right)$$

$$= I\omega\frac{d\theta}{dt}$$

$$C = I\omega\omega_p$$

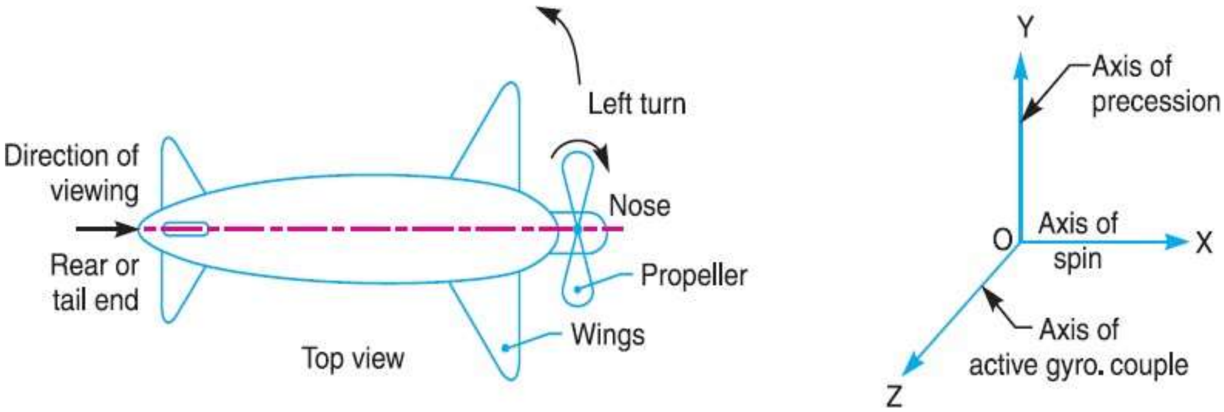
Where,

C = gyroscopic couple (N-m)
ω = angular velocity of rotary body (rad/s)
ωp = angular velocity of precession (rad/s)

The couple *I*.ω.ωp, in the direction of the vector *xx′* (representing the change in angular momentum) is the *active gyroscopic couple,* which has to be applied over the disc when the axis of spin is made to rotate with angular velocity ω_p about the axis of precession. When the axis of spin itself moves with angular velocity ω_p, the disc is subjected to *reactive couple* whose magnitude is same (*i.e. I.* $\omega.\omega_p$) but opposite in direction to that of active couple. This reactive couple to which the disc is subjected when the axis of spin rotates about the axis of precession is known as *reactive gyroscopic couple.*

Effect of the Gyroscopic Couple on an Aeroplane:

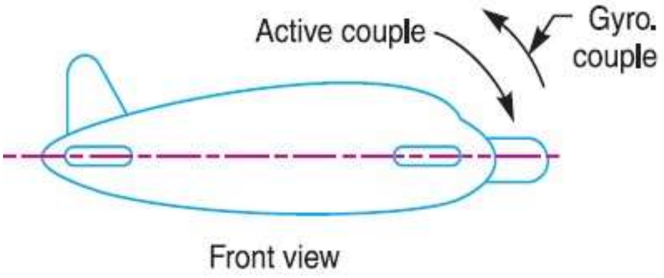
The top and front view of an aeroplane is shown in Fig Let engine or propeller rotates in the clockwise direction when seen from the rear or tail end and the aeroplane takes a turn to the left.

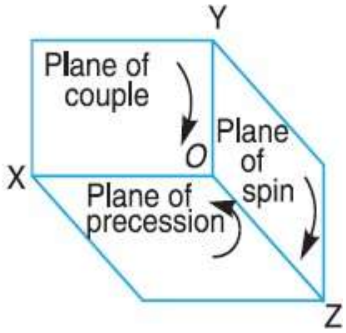


ω = Angular velocity of the engine in rad/s,
m = Mass of the engine and the propeller in kg,
k = Its radius of gyration in metres,
I = Mass moment of inertia of the engine and the propeller in kg-m2 $= m.k^2$,
v = Linear velocity of the aeroplane in m/s,
R = Radius of curvature in metres, and
ω_p = Angular velocity of precession= *v/R* rad/s

∴ Gyroscopic couple acting on the aeroplane,

$C = I.\omega.\omega_p$





1. When the aeroplane takes a right turn under similar conditions as discussed above, the effect of the reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
2. When the engine or propeller rotates in anticlockwise direction when viewed from the rear or tail end and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to dip the nose and raise the tail of the aeroplane.
3. When the aeroplane takes a right turn under similar conditions as mentioned in note 2 above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.
4. When the engine or propeller rotates in clockwise direction when viewed from the front and the aeroplane takes a left turn, then the effect of reactive gyroscopic couple will be to raise the tail and dip the nose of the aeroplane.
5. When the aeroplane takes a right turn under similar conditions as mentioned in note 4-above, the effect of reactive gyroscopic couple will be to raise the nose and dip the tail of the aeroplane.

GYROSCOPIC EFFECT ON SHIP

Gyroscope is used for stabilization and directional control of a ship sailing in the rough sea. A ship, while navigating in the rough sea, may experience the following three different types of motion

(i) Steering—The turning of ship in a curve while moving forward

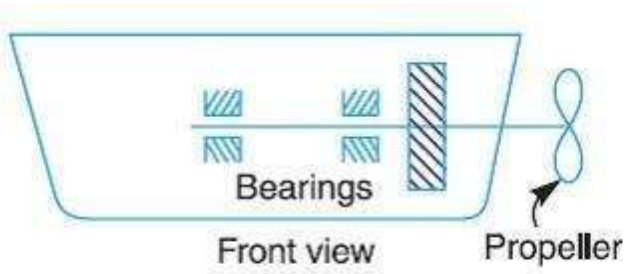
(ii) Pitching—The movement of the ship up and down from horizontal position in a vertical plane about transverse axis

(iii)Rolling—Sideway motion of the ship about longitudinal axis.

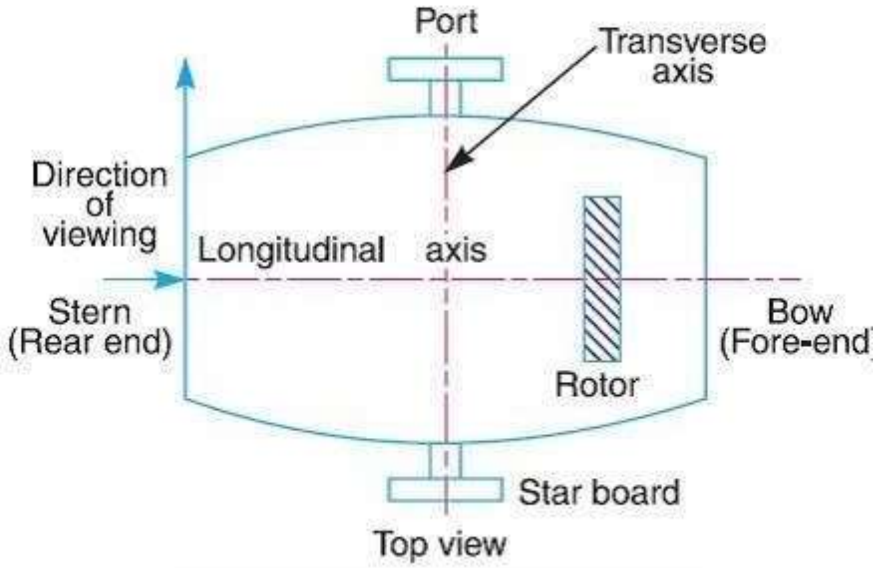
For stabilization of a ship against any of the above motion, the major requirement is that the gyroscope shall be made to precess in such a way that reaction couple exerted by the rotor opposes the disturbing couple which may act on the frame.

The top and front views of a naval ship are shown in Fig. The fore end of the ship is called bow and the rear end is known as stern or aft. The left hand and right hand sides of the ship, when viewed from the stern are called port and star-board respectively.

1. Steering, 2. Pitching, and 3. Rolling.

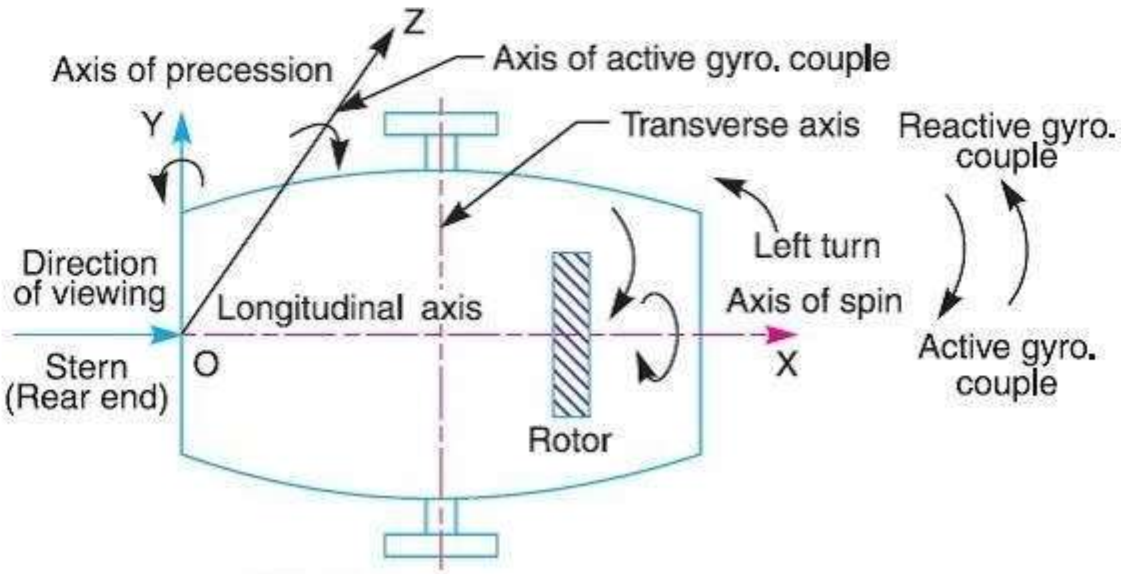


Terms used in a naval ship.



Effect of Gyroscopic Couple on a Naval Ship during Steering

Steering is the turning of a complete ship in a curve towards left or right, while it moves forward. Consider the ship taking a left turn, and rotor rotates in the clockwise direction when viewed from the stern, as shown in Fig. 14.8. The effect of gyroscopic couple on a naval ship during steering taking left or right turn may be obtained in the similar way as for an aeroplane



Naval ship taking a left turn.

When the rotor of the ship rotates in the clockwise direction when viewed from the stern, it will have its angular momentum vector in the direction *ox* as shown in Fig. As the ship steers to the left, the active gyroscopic couple will change the angular momentum vector from *ox* to *ox*′. The vector *xx*′ now represents the active gyroscopic couple and is perpendicular to *ox*. Thus the plane of active gyroscopic couple is perpendicular to *xx*′ and its direction in the axis *OZ* for left hand turn is clockwise as shown. The reactive gyroscopic couple of the same magnitude will act in the opposite direction (*i.e.* in anticlockwise direction). The *effect of this reactive gyroscopic couple is to raise the bow and lower the stern.*

Notes: 1. When the ship steers to the right under similar conditions as discussed above, the effect of the reactive gyroscopic couple, will be to raise the stern and lower the bow.

2. When the rotor rates in the anticlockwise direction, when viewed from the stern and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to lower the bow and raise the stern.

3. When the ship is steering to the right under similar conditions as discussed in note 2 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.

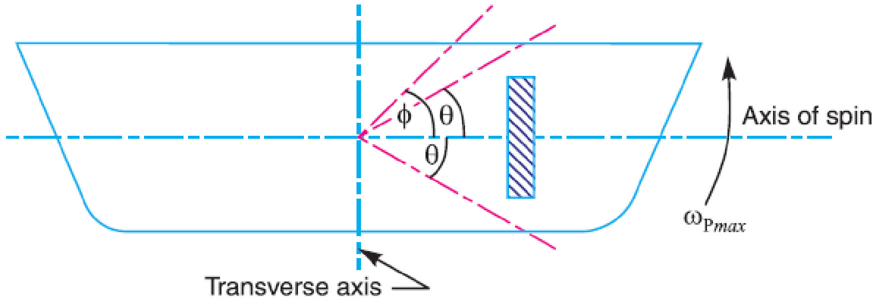
4. When the rotor rotates in the clockwise direction when viewed from the bow or fore end and the ship is steering to the left, then the effect of reactive gyroscopic couple will be to raise the stern and lower the bow.

5. When the ship is steering to the right under similar conditions as discussed in note 4 above, then the effect of reactive gyroscopic couple will be to raise the bow and lower the stern.

6. The effect of the reactive gyroscopic couple on a boat propelled by a turbine taking left or right turn is similar as discussed above.

Effect of Gyroscopic Couple on a Naval Ship during Pitching

Pitching is the movement of a complete ship up and down in a vertical plane about transverse axis. In this case, the transverse axis is the axis of precession. The pitching of the ship is assumed to take place with simple harmonic motion *i.e.* the motion of the axis of spin about transverse axis is simple harmonic.



(*a*) Pitching of a naval ship

Notes:

- The effect of the gyroscopic couple is always given on specific position of the axis of spin *i.e.* whether it is pitching downwards or upwards.
- The pitching of a ship produces forces on the bearings which act horizontally and perpendicular to the motion of the ship.
- The maximum gyroscopic couple tends to shear the holding-down bolts.

Effect of Gyroscopic Couple on a Naval Ship during Rolling

- We know that, for the effect of gyroscopic couple to occur, the axis of precession should always be perpendicular to the axis of spin. If, however, the axis of precession becomes parallel to the axis of spin, there will be no effect of the gyroscopic couple acting on the body of the ship.

- In case of rolling of a ship, the axis of precession (*i.e.* longitudinal axis) is always parallel to the axis of spin for all positions. Hence, there is no effect of the gyroscopic couple acting on the body of a ship.

Problems:

1. *A uniform disc of diameter 300 mm and of mass 5 kg is mounted on one end of an arm of length 600 mm. The other end of the arm is free to rotate in a universal bearing. If the disc rotates about the arm with a speed of 300 r.p.m. clockwise, looking from the front, with what speed will it precess about the vertical axis*?

   Solution. Given: d = 300 mm or r = 150 mm = 0.15 m ; m = 5 kg ; l = 600 mm = 0.6 m ; N = 300 r.p.m. or $\omega = 2\pi \times 300/60 = 31.42$ rad/s

   We know that the mass moment of inertia of the disc, about an axis through its centre of

   gravity and perpendicular to the plane of disc,

   $$I = m.r2/2 = 5(0.15)2/2 = 0.056 \text{ kg-m2}$$

   and couple due to mass of disc,

   $$C = m.g.l = 5 \times 9.81 \times 0.6 = 29.43 \text{ N-m}$$

   Let wP = Speed of precession.

   We know that couple (C),

   $$29.43 = I.\omega\ \omega_p = 0.056 \times 31.42 \times \omega P = 1.76\ \omega P$$

   $$\omega P = 29.43/1.76 = 16.7 \text{ rad/s } \textbf{Ans.}$$

2. *An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hr. The rotary engine and the propeller of the plane has a mass of 400 kg and a radius of gyration of 0.3 m. The engine rotates at 2400 r.p.m. clockwise when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it.*

   Solution:

   Given : R = 50 m ; v = 200 km/hr = 55.6 m/s ; m = 400 kg ; k = 0.3 m ;

   N = 2400 r.p.m. or $\omega = 2\pi \times 2400/60 = 251$ rad/s

   We know that mass moment of inertia of the engine and the propeller,

   $$I = m.k2 = 400(0.3)2 = 36 \text{ kg-m2}$$

   and angular velocity of precession,

   $$\omega P = v/R = 55.6/50 = 1.11 \text{ rad/s}$$

   We know that gyroscopic couple acting on the aircraft,

   $$C = I.\ \omega.\ \omega_p = 36 \times 251.4 \times 1.11 = 100\ 46 \text{ N-m}$$

   $$= 10.046 \text{ kN-m Ans.}$$

   when the aeroplane turns towards left, the effect of the gyroscopic couple is to lift the nose upwards and tail downwards.

3. The turbine rotor of a ship has a mass of 8 tonnes and a radius of gyration 0.6 m. It rotates at 1800 r.p.m. clockwise, when looking from the stern. Determine the gyroscopic couple, if the ship travels at 100 km/hr and steer to the left in a curve of 75 m radius.

Solution. Given: $m = 8$ t $= 8000$ kg ; $k = 0.6$ m ; $N = 1800$ r.p.m. or $\omega = 2\pi \times 1800/60 = 188.5$ rad/s ; $v = 100$ km/h $= 27.8$ m/s ; $R = 75$ m

We know that mass moment of inertia of the rotor,

$$I = m.k2 = 8000 (0.6)2 = 2880 \text{ kg-m2}$$

and angular velocity of precession,

$$\omega P = v / R = 27.8 / 75 = 0.37 \text{ rad/s}$$

We know that gyroscopic couple,

$$C = I.\omega.\ \omega_p = 2880 \times 188.5 \times 0.37 = 200\,866 \text{ N-m}$$
$$= 200.866 \text{ kN-m Ans.}$$

When the rotor rotates in clockwise direction when looking from the stern and the ship steers to the left, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

4. The heavy turbine rotor of a sea vessel rotates at 1500 r.p.m. clockwise looking from the stern, its mass being 750 kg. The vessel pitches with an angular velocity of 1 rad/s. Determine the gyroscopic couple transmitted to the hull when bow is rising, if the radius of gyration for the rotor is 250 mm. Also show in what direction the couple acts on the hull?

Solution. Given: $N = 1500$ r.p.m. or $\omega = 2\pi \times 1500/60 = 157.1$ rad/s; $m = 750$ kg; $\omega P = 1$ rad/s; $k = 250$ mm $= 0.25$ m

We know that mass moment of inertia of the rotor,

$$I = m.k2 = 750 (0.25)2 = 46.875 \text{ kg-m2}$$

$\therefore$ Gyroscopic couple transmitted to the hull (i.e. body of the sea vessel),

$$C = I.\omega.\omega_p = 46.875 \times 157.1 \times 1 = 7364 \text{ N-m} = 7.364 \text{ kN-m}$$

When the bow is rising i.e. when the pitching is upward, the reactive gyroscopic couple acts in the clockwise direction which moves the sea vessel towards star-board.

5. The turbine rotor of a ship has a mass of 3500 kg. It has a radius of gyration of 0.45 m and a speed of 3000 r.p.m. clockwise when looking from stern. Determine the gyroscopic couple and its effect upon the ship:

   1. when the ship is steering to the left on a curve of 100 m radius at a speed of 36 km/h.

   2. when the ship is pitching in a simple harmonic motion, the bow falling with its maximum
   velocity. The period of pitching is 40 seconds and the total angular displacement between the two extreme positions of pitching is 12 degrees.

Solution

Given : $m = 3500$ kg ; $k = 0.45$ m; $N = 3000$ r.p.m. or $\omega = 2\pi \times 3000/60 = 314.2$ rad/s

1. When the ship is steering to the left Given: $R = 100$ m ; $v =$ km/h $= 10$ m/s

We know that mass moment of inertia of the rotor,

$$I = m.k2 = 3500 (0.45)2 = 708.75 \text{ kg-m2}$$

and angular velocity of precession,

$$\omega_p = v/R = 10/100 = 0.1 \text{ rad/s}$$

$\therefore$ Gyroscopic couple,

$$C = I.\omega.\ \omega_p = 708.75 \times 314.2 \times 0.1 = 22\,270 \text{ N-m}$$
$$= 22.27 \text{ kN-m Ans.}$$

When the rotor rotates clockwise when looking from the stern and the ship takes a left turn, the effect of the reactive gyroscopic couple is to raise the bow and lower the stern.

2. When the ship is pitching with the bow falling
Given: tp = 40 s
Since the total angular displacement between the two extreme positions of pitching is 12°
(i.e. $2\varphi = 12°$), therefore amplitude of swing,

$$\varphi = 12 / 2 = 6° = 6 \times \pi/180 = 0.105 \text{ rad}$$

and angular velocity of the simple harmonic motion,

$$\omega 1 = 2\pi / tp = 2\pi / 40 = 0.157 \text{ rad/s}$$

We know that maximum angular velocity of precession,

$$\omega_p = \varphi.\omega 1 = 0.105 \times 0.157 = 0.0165 \text{ rad/s}$$

∴ Gyroscopic couple,

$$C = I.\omega.\ \omega_p = 708.75 \times 314.2 \times 0.0165 = 3675 \text{ N-m}$$

$$= 3.675 \text{ kN-m Ans.}$$

When the bow is falling (i.e. when the pitching is downward), the effect of the reactive gyroscopic couple is to move the ship towards port side.

EXERCISES

1. A flywheel of mass 10 kg and radius of gyration 200 mm is spinning about its axis, which is horizontal and is suspended at a point distant 150 mm from the plane of rotation of the flywheel. Determine the angular velocity of precession of the flywheel. The spin speed of flywheel is 900 r.p.m. [Ans. 0.39 rad/s]

2. A horizontal axle AB, 1 m long, is pivoted at the mid point C. It carries a weight of 20 N at A and a wheel weighing 50 N at B. The wheel is made to spin at a speed of 600 r.p.m in a clockwise direction looking from its front. Assuming that the weight of the flywheel is uniformly distributed around the rim whose mean diameter is 0.6 m, calculate the angular velocity of precession of the system around the vertical axis through C.
[Ans. 0.52 rad/s]

3. Each paddle wheel of a steamer have a mass of 1600 kg and a radius of gyration of 1.2 m. The steamer turns to port in a circle of 160 m radius at 24 km / h, the speed of the paddles being 90 r.p.m. Find the magnitude and effect of the gyroscopic couple acting on the steamer. [Ans. 905.6 N-m]

4. An aeroplane makes a complete half circle of 50 metres radius, towards left, when flying at 200 km per hour. The rotary engine and the propeller of the plane has a mass of 400 kg with a radius of gyration of 300 mm. The engine runs at 2400 r.p.m. clockwise, when viewed from the rear. Find the gyroscopic couple on the aircraft and state its effect on it. What will be the effect, if the aeroplane turns to its right instead of to the left?
[Ans. 10 kN-m]
5. The rotor of the turbine of a yacht makes 1200 r.p.m. clockwise when viewed from stern. The rotor has a mass of 750 kg and its radius of gyration is 250 mm. Find the maximum gyroscopic couple transmitted to the hull (body of the yacht) when yacht pitches with maximum angular velocity of 1 rad /s. What is the effect of this couple? [Ans. 5892 N-m]