SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
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SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – I – BASICS & STATICS OF PARTICLES – SMEA1202

UNIT 1 BASICS & STATICS OF PARTICLES 9 Hrs
Introduction-Units and Dimensions-Laws of Mechanics-Vectors-Vectorial representation of forces and moments - Vector operations-resolution and composition of forces - equilibrium of a particle - Free body diagram - forces in space-equilibrium of a particle in space-equivalent systems of forces-principle of transmissibility–Resultant and Equilibrant.

Unit I BASIC & STATICS OF PARTICLES

INTRODUCTION

Mechanics is that branch of science which deals with the behavior of a body when the body is at root or in motion. Classification of engineering mechanics on a broad view:

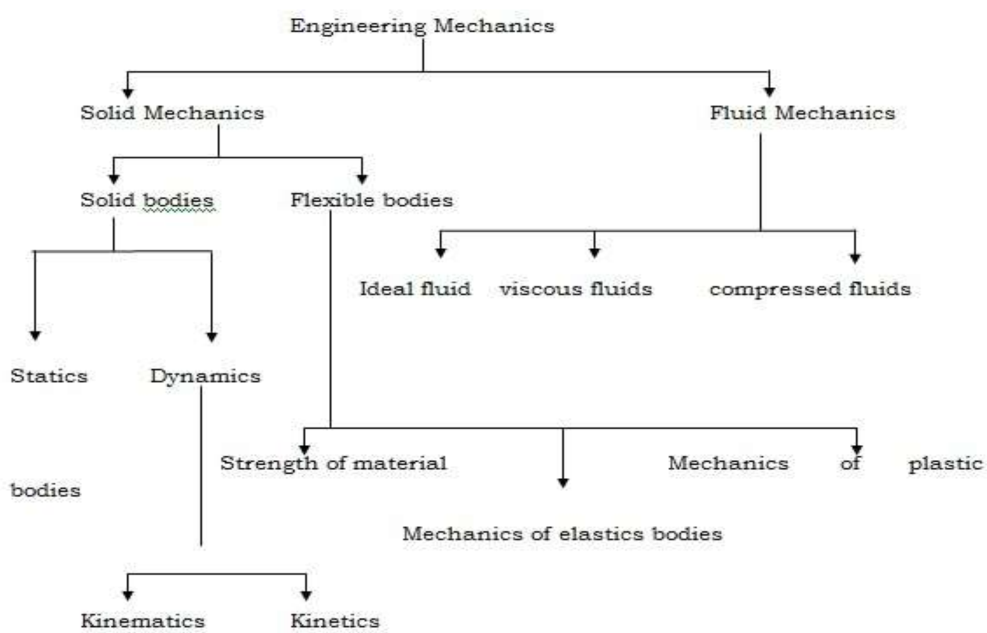


Fig.1. Classification

The engineering mechanics is mainly classified into two branches. They are

1. Statics 2. Dynamics

1. Statics: Statics deals with the forces on a body at rest.

2. Dynamics: Dynamics deals with the forces acting on a body when the body is in motion. Dynamics further subdivided in to two sub branches. They are:

(a) Kinematics: Deals the motion of a body without considering the forces causing the motion.
(b) Kinetics: Deals with the relation between the forces acting on the body and the resulting motion

Rigid body: The rigid body means the body does not deform under the action of force. Engineering Mechanics deals with Rigid body Dynamics.

Particle: It is an object with its mass concentrated at a point

Force: force is defined as an agency which changes or tends to change the body at rest or in motion. Force is a vector quantity. So we have to specify the magnitude, direction and point of action. The unit of force is Newton.

$1\ N = 1\ kgm/s^2$

IMPORTANCE OF MECHANICS TO ENGINEERING:

1) For designing and manufacturing of various mechanical tools and equipments

2) For calculation and estimation of forces of bodies while they are in use.

3) For designing and construing to dams, roads, sheds, structure, building etc.

4) For designing a fabrication of rockets.

Units and dimensions

The following units are used mostly,

1. Centimeter-Gram Second system of unit.
2. Metre-kilogram-second system of units.
3. International system of units.

1. Length is expressed in centimeter, mass in gram and time in second. The unit of force in this system is dyne. Dyne is defined as the force acting on a mass of one gram and producing an acceleration of one centimeter per second square.
2. The length is expressed in metre(m), mass in kilogram and time in second. The unit of force is expressed as kilogram force and is represented as kgf.
3. S.I is abbreviation for "The system International units". It is also called the international system of units.

The length is expressed in metre mass in kilogram and time in second. The unit of force in Newton and is represented N. Newton which is the force acting on a mass of one kilogram and producing as acceleration of one meter per second square. The relation between Newton (N) and dyne is derived as follows,

One Newton = 1 kilogram mass x 1 meter/S2

$= 1000g \times 100\ cm/S^2$

$=1000 \times 100 \times gm \times cm/S^2$

=105 dyne

MKS SYSTEM FORCE Unit is Kgf or kg(wt) or simply Kg. All referring the same.

1 Kgf = 9.81 N

The unit of force, kilo-Newton and mega- Newton is used when the magnitude of forces is very large.

1 kN = 10^3 N

And one Mega- Newton = 10^6 Newton

Kilo(K) = 10^3

Mega(M) = 10^6

Giga(G) = 10^9

Tera(T) = 10^{12}

Basic Units

Physical quantity	Notation or unit	Dimension or symbols
Length	Metre	m
Mass	Kilogram	kg
Time	Second	S
Electric current	Ampere	A
Temperature	Kelvin	K
Luminous Intensity	Candela	cd

Supplementary units

Plane angle	Radian	rad
Solid angle	Steridian	sr

Derived units

Acceleration	metre/second^2	m/s^2
Angular velocity	radian/second	rad/s
Angular acceleration	radian/s^2	rad/s^2
Force	Newton	N
Work, Energy	Joule	J=Nm=kg m^2/s^2
Torque	Newton metre	Nm
Power	Watt	W= J/s
Pressure	Pascal	Pa=N/m2
Frequency	Hertz	Hz=s-1

Laws of Mechanics:

Newton‟s first law of Motion:

Everybody continues in a state of root or uniform motion in a straight line unless it is compelled to change that state by some external force acting on it.

Newton‟s Second Law of Motion:

The net external force acting on a body in a direction is directly proportional to the rate of change of momentum in that direction.

Newton‟s Third law of motion:

To every action there is always equal and opposite reaction. Law of Gravitation:
It states that two bodies will be attracted towards each other along their connecting line with a force which is directly proportional to the product of their masses and inversely proportional to the square of the distance between the centers.

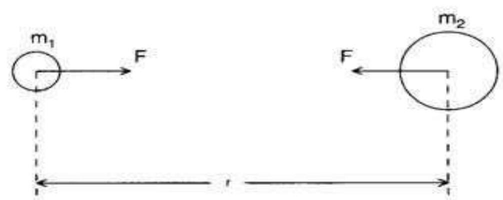


Fig.2 Attraction between bodies

According to law of gravitation

$$F \alpha \frac{m_1 m_2}{r^2}$$

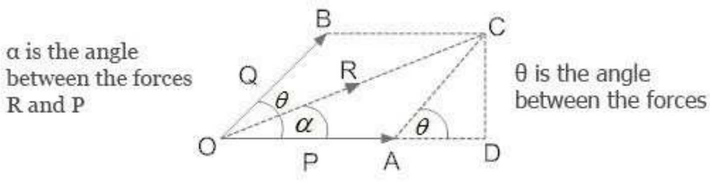
$$F = G\frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant

G = 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}

Parallelogram law of forces:

If two forces acting at a point be represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram ram passing through that point.



P and Q are two forces, meet at a point O

Fig.3 Parallelogram law of forces

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

$$\alpha = \tan^{-1}\left[\frac{Q \sin \theta}{P + Q \cos \theta}\right]$$

Triangular law of forces:

If two forces acting at a point are represented by the two sides of a triangle taken in order then their resultant force is represented by the third side taken in opposite order.

Lame’s Theorem:

If three forces acting at a point are in equilibrium each force will be proportional to the sine of angle between the other two forces.

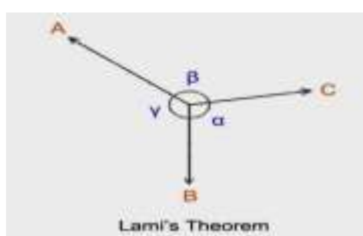


Fig. 4 Lami’s theorem

According to Lami’s theorem, the particle shall be in equilibrium if

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Principle of transmissibility of forces

It states that “ if a force, acting at a point on a rigid body, is shifted to any other point which is on the line of action of the force, the external effect of the force on the body remains unchanged.

For example a force F is acting at point A on a rigid body along the line of action AB. At point B, apply two equal and opposite forces F1 and F2 such that F1 and F2 are collinear and equal in magnitude with F. Now, we can transfer F1from B to A such that F
and F1 are equal and opposite and accordingly they cancel each other. The net result is force F2 at B. This implies that a force acting at any point on a body may also be considered to act at any other point along its line of action without changing the equilibrium of the body.

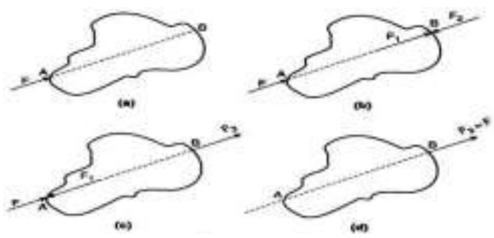


Fig.5 Principle of transmissibility of forces

There is an important observation. If a force is transferred to a different line of action with the force value a couple must be accompanied`

Polygon law of Forces:

If a number of forces acting simultaneously on a particle be represented in magnitude and direction by the sides of a polygon taken in order then the resultant of all three forces may be represented in magnitude and direction by the closing side of the polygon taken in opposite order.

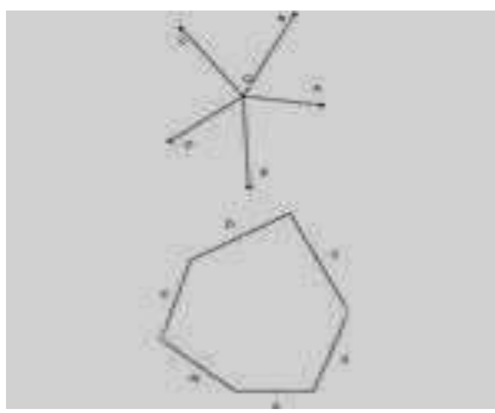


Fig. 6 Polygon law of Forces

Force and Force system:

Force is defined as the agency which changes or tends to change the position of rest or motion of the body. The number of forces acting at a point is called force system.

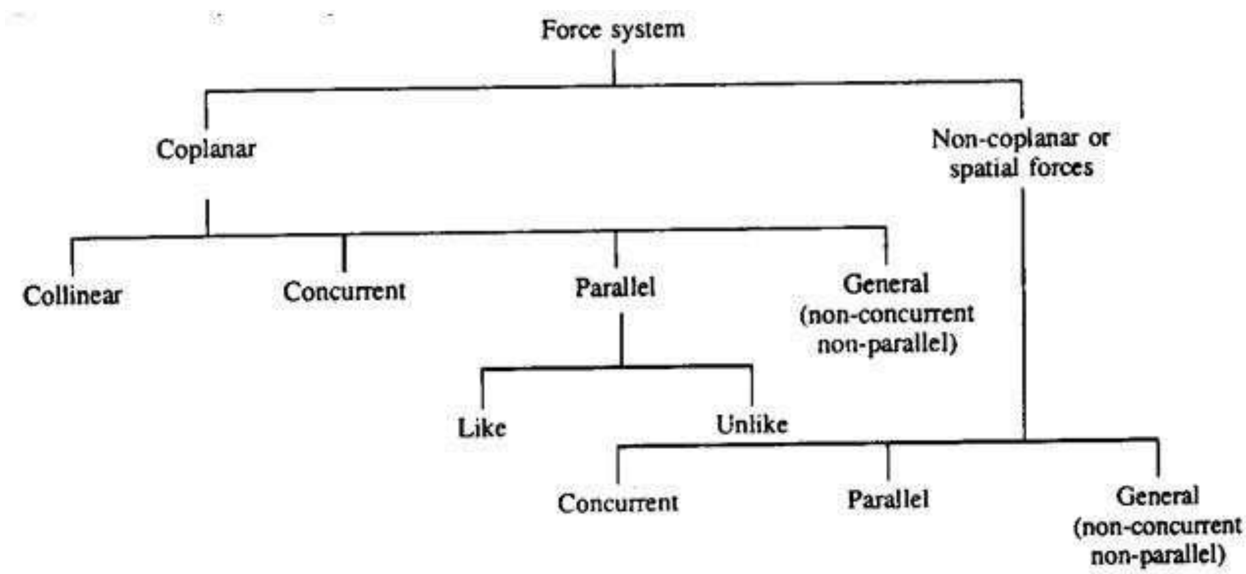


Fig. 7 Force Classification

Coplanar Force system:

Fig. 8 Coplanar force System

Non coplanar force system:- The system in which the forces do not lie on the same plane is called non coplanar force system.

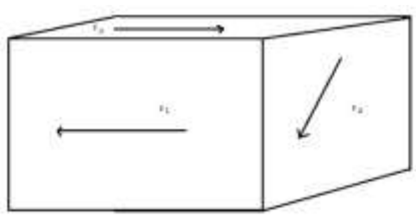


Fig.9 Non coplanar force system

Collinear forces:- The system in which the forces whose line of action lie on the same line and in same plane is called collinear force system.

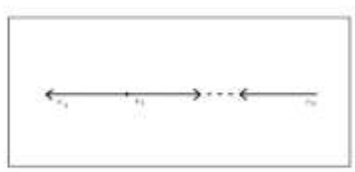


Fig.10 Collinear force system

Concurrent force system:- The system in which the forces meet at one point and lie in the same plane is called concurrent force system.

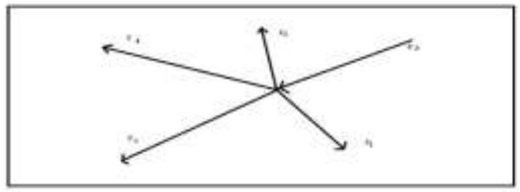


Fig. 11 Concurrent force system

Parallel force system:-

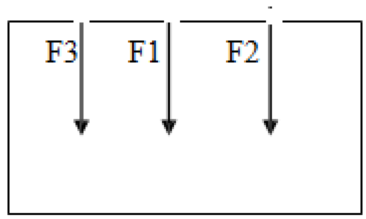


Fig. 12 Parallel force system

In parallel force system the line of action of forces one parallel to each other.

Parallel forces acting in same direction are called like parallel forces and the parallel forces acting in opposite direction are called unlike parallel force system.

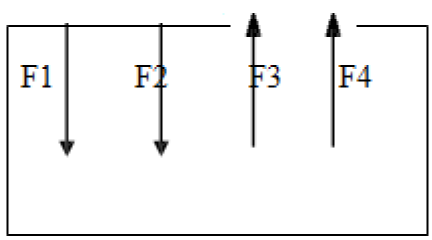


Fig. 13 Parallel force System

Non concurrent force system:- The system in which the forces do not meet at one point but their lines of action lie on same plane is called non concurrent force system.

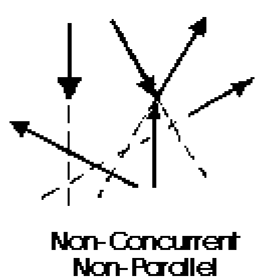


Fig 14 Non coplanar force system:-

NON COPLANNAR NON CONCURRENT FORCE SYSTEM

The forces which do not meet a point and their lines of action do not lie on the same plane , are called non coplanar non con current force system.

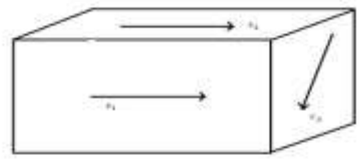


Fig. 15 Non coplanar non con current force system

NON COPLANNAR CONCURRENT FORCE SYSTEM

The forces which meet at a point but their lines of action lies on different planes , are known as non coplanar concurrent force system.

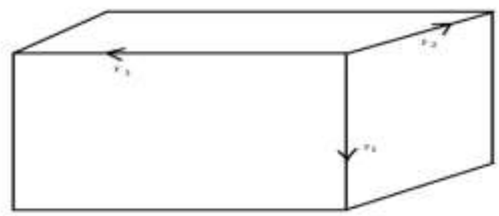


Fig. 16 Non coplanar concurrent force system

Resultant force:

When a number if forces acting on a body are replaced by a single force which has the same effect on the body as that of those number of forces then such a single force is called resultant force.

Composition of forces:

Combining several forces into a single force is called Composition of forces. The single force is called Resultant. The effect by component forces and single force remains the same.

Resolution of a force:

Splitting up of a force into components along the fixed reference axis is called resolution of forces. The effect by single force and component forces remains the same.

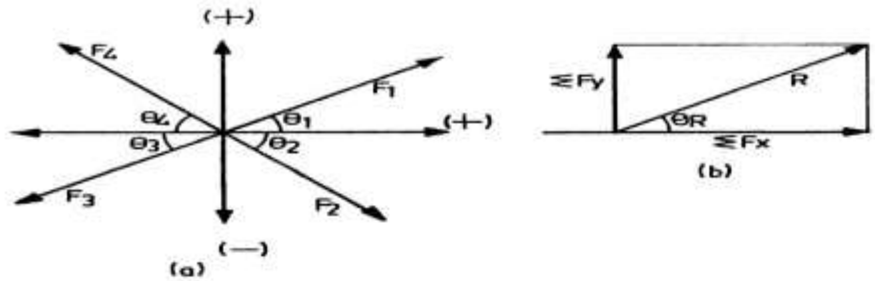


Fig. 17 Resolving of forces

Algebraic sum of horizontal components

$\sum Fx = F1\cos\Theta1 - F2\cos\Theta2 - F3\cos\Theta3 + F4\cos\Theta4$

Algebraic sum of vertical components

$\sum Fy = F1\sin\Theta1 + F2\sin\Theta2 - F3\sin\Theta3 - F4\sin\Theta4$

Resultant R = $\sqrt{(\sum Fx)2 + (\sum Fy)2}$

Angle α mode by the resultant with x axis is given by

$\tan \alpha = \sum Fy / \sum Fx$

A vertical force has no horizontal component

Θ = 900

Fx = FcosΘ   Fy = FSinΘ

= Fcos90   =Fsin90

= 0   = F

A horizontal force has no vertical component

Θ = 00

Fx = FcosΘ   Fy = FSinΘ

= Fcos0   =Fsin0

= F   = 0

NUMERICALS:

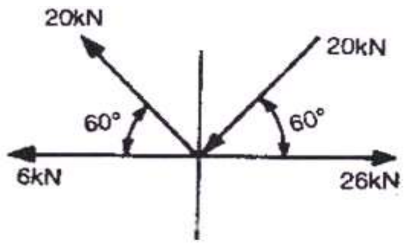
1. Forces R, S, T, U are collinear. Forces R and T act from left to right. Forces S and U act from right to left.

Magnitudes of the forces R, S, T, U are 40 N, 45 N, 50 N and 55 N respectively. Find the resultant of R, S,
T, U.

Given data: R=40 N
S=45 N
T=50 N U=55N

Resultant= -R-U+T=-40-55+45+50=0

2. Find the resultant of the force system shown in Fig



Given data:

$F1=20$ KN ; $\Theta1=60°$ $F2=26$ KN ; $\Theta2=0°$ $F3=6KN$; $\Theta3=00°$ $F4=20KN$; $\Theta4=60°$ Solution:
Resolve the given forces horizontally and calculate the algebraic total of all the horizontal parts or

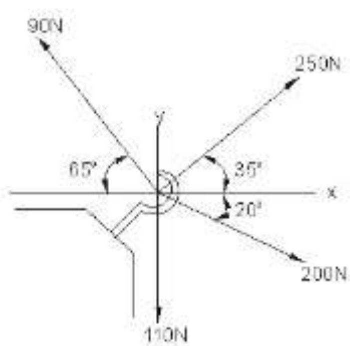
Σ H=-20cos60°+26cos0°-6cos0°-20cos60°=0

Resolve the given forces vertically and calculate the algebraic total of all the vertical parts or Σ V.
Σ V=-20sin60°±26sin0°±6sin0°+20 sin60=0

R = √((∑H)^2+(V)^2)=0

3.Determine the magnitude and direction of the resultant of forces acting on the hook shown

In fig



Given data:

$F1=250$ N ; $\Theta1=35°$ $F2=200$ N ; $\Theta2=20°$ $F3=110$ N $\Theta3=90°$
$F4=90$ N ; $\Theta4=65°$ Solution:
Resolve the given forces horizontally and calculate the algebraic total of all the horizontal parts or

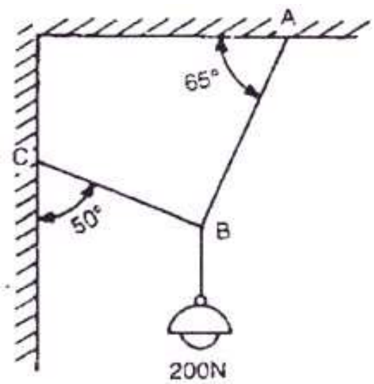
Σ H=250cos35°+200cos30°±110cos90°-90cos65°=170.38N

Resolve the given forces vertically and calculate the algebraic total of all the vertical parts or Σ V.
Σ V=250sin35°-200sin20°-110sin90°+90sin65=46.55N

R = √((∑H)2+(Σ V)2)=176.62N

Θ=tan¯(ΣV/∑H)=15°

4.An electric light fixture weighting 200 N is supported as shown in Fig. Determine the tensile forces in the wires and BA and BC



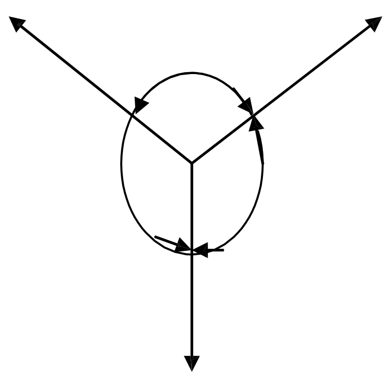
Solution:

Free body diagram(FBD):

TBC TAB

75°

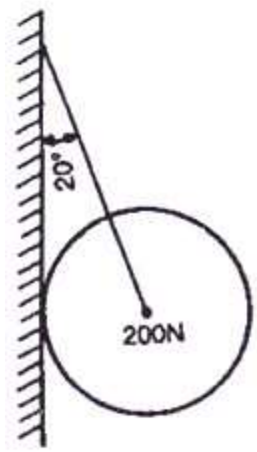
130° 155°



W=200 N By using lami theorem
TAB/sin130°= TBC/sin155°=200/sin75°

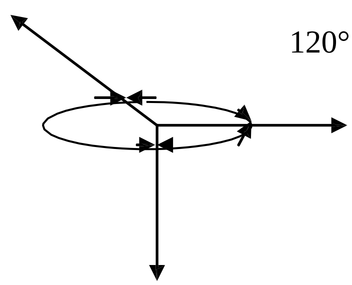
~~TAB=200/sin75~~°*sin130°=158.61N TBC=200/sin75°* sin155°=87.50N

5.A sphere weighing 200 N is tied to a smooth wall by a string as shown in Fig. Find the tension T in the string and reaction R from the wall



Solution: TAC

Free body diagram(FBD): 120°



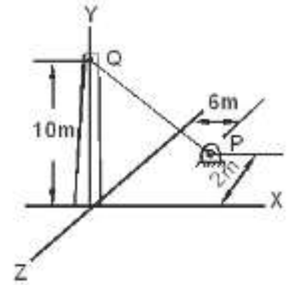
RB

160° 90° W=200N

By using lami theorem

TAC/Sin 90°=RB/Sin160°=200/Sin120° TAC=200/SIN120°* Sin 90°=230.94N RB=200/Sin120°* Sin160°=78.98N

6. A metal guy rope tied to a peg at P shown in Fig.12 keeps an electric post in equilibrium. The force in the guy rope is 1.25 kN. Find the components of the force at P and the angles of inclination of the force with the three rectangular axes



Given data:

Tension in guy wire is 1250 N

1)Components Fx,Fy,Fz

Consider the tension in guy wire, acting at P. the force I directed from P to Q. Let it beTPQ
Co ordinates of P (x1,y1,z1)= (6,0,-2)

Co ordinates of Q (x2,y2,z2)= (0,10,0) Vector TPQ = TPQ *λPQ
λPQ =PQ/PQ= (X2-X1)i+(Y2-Y1)j+(Z2-Z1)k/√((X2-X1)2+(Y2-Y1)2+(Z2-Z1) 2

= (0-6)i+(10-0)j+(0-(-2))k/√((6)2+(10)2+(2) 2

= -6i+10j+2k/11.382

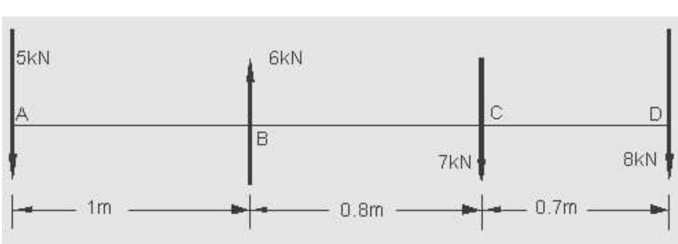
Vector TPQ = TPQ *λPQ

Vector TPQ = 1250 *-6i+10j+2k/11.382=-633i+1056j+211k from above equation
Fx=633i Fy=1056j Fz=211k
2)Angle ΘX, ΘY ΘZ

we know force vector F =-633i+1056j+211k

ΘX =COS¯[Fx/F]=633/1250=59° ΘY =COS¯[FY/F]=1056/1250=32° ΘZ = COS¯[Fz/F]=211/1250=80°

7.Find the resultant of the force system shown in Fig.13 and its position from A. (Force in „kN" and distance in „m")



Solution:

Magnitude of resultant of force R =-5+6-7-8 =-14KN

(-) sign shows that the resultant forces acts in the negative direction i.e., downwards

Location of the resultant force:

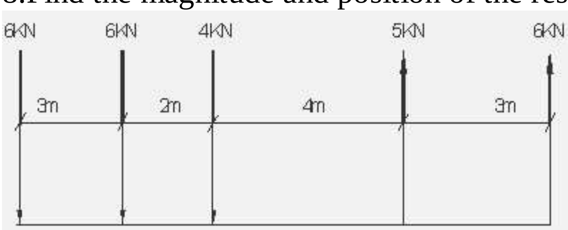
Lt us locate the resulting force with reference to the point A. Hence , taking the moments of given forces and adding,

Algebraic sum of moments about A,

∑MA = -(6*1)+(7*1.8)+(8*2.5)=26.6 KN-m(clockwise)

Hence acting downwards and to have clock moment, resultant force is taken on the right side of A Let resultant force acts at a distance of "x" m from A
∑MA = R*x

26.6= 14*x x =1.9m
8.Find the magnitude and position of the resultant of the system of forces shown in Fig.



Magnitude of resultant of force R =-6-6-4+5+6=-5N

(-) sign how that the resultant forces acts in the negative direction i.e., downwards

Location of the resultant force:

Lt us locate the resulting force with reference to the point A. Hence , taking the moments of given forces and adding,

Algebraic sum of moments about A,

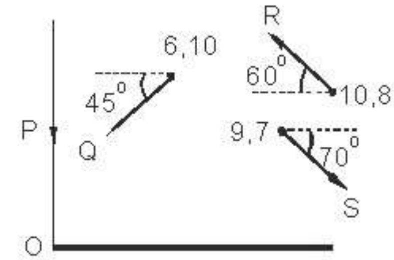
∑MA = (6*3)+(4*5)-(5*9)-(6*12)= 38-117=-79 KN-m(Counter clockwise)

Hence acting downwards and to have counter clock moment, resultant force is taken on the right side of A Let resultant force acts at a distance of "x" m from A
∑MA = R*x

79= 5*x

x =15.9 m

9. A system of four forces P, Q, R and S of magnitude 5 kN, 8 kN, 6 kN and 4 kN respectively acting on a body are shown in rectangular coordinates as shown in Fig.2. Find the moments of the forces about the origin O. Also find the resultant moment of the forces about O. The distances are in metres.

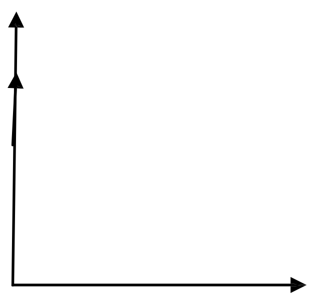


Solution: Moment Of P:

Moment of force P about the origin, MP = Force*perpendicular distance from origin

Y

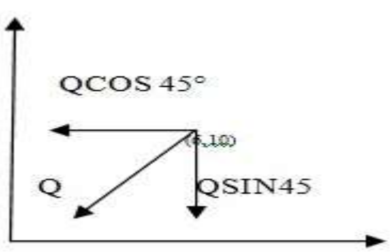
= 5*0=0



P=5KN

X

Moment Of Q :



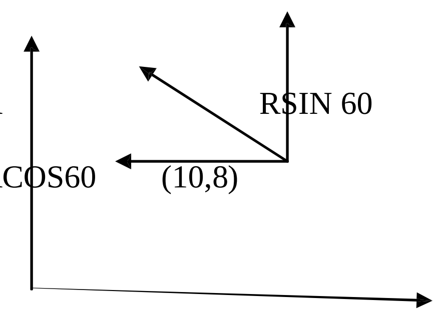
Moment of force Q about the origin, MQ = sum of the moments of components of the force Q about the origin

= -Q COS 45°*10 +Q SIN 45°*6

=-8COS 45°*10 +8 SIN 45°*6

=-42.025N+40.84=-1.185 KN-m(counter clock wise moment)

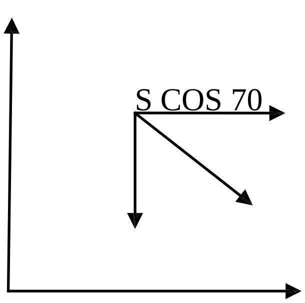
Moment of force R about the origin, MR = sum of the moments of components of the force R about the origin



= -6 COS 60°*10 –6 SIN 60°*8

=--57.14-14.63=-71.77 KN-m(counter clock wise moment)

Moment of force S about the origin, MS = sum of the moments of components of the force R about the origin

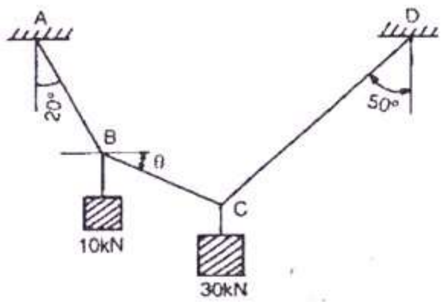


= S COS 70°*7 +S SIN 45°*9

=4COS 70*7 +4 SIN 70°*9

=17.73+27.86=45.59 KN-m(clock wise moment)

10. A wire is fixed at two points A and D as shown in Fig.20.Two weights 10 kN and 30 kN are is 200and that of CD is 500 to the vertical. Determine the tension in the segments AB, BC and CD of the wire and also the inclination of BC to the vertical. Take Θ = 30 0



TBA

Free body diagram of joint B and C

20°

TCB

50°

TCD

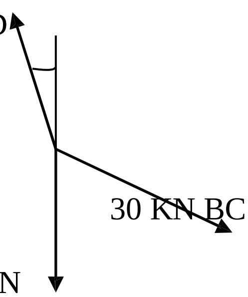
C

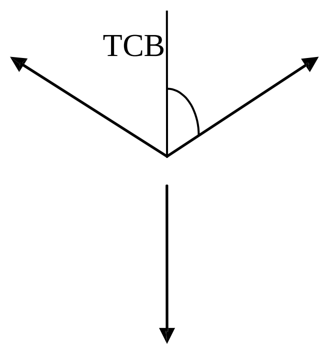
B

T

30 KN BC

10KN





At joint B

$\sum$H = -TBACos70+TBCCos Θ = 0

TBCCos Θ = TBACos70

TBA = TBCCos Θ / Cos70

TBA = 2.92 TBCCos Θ -------------(1)

$\sum$V = -TBASin70 - 10 - TBCSin Θ = 0

TBASin70 - TBCSin Θ = 10 -----------(2) Substitute TBA we get
2.92 TBCCos Θ Sin70 - TBCSin Θ = 10

2.74 Cos Θ TBC - TBCSin Θ =10

TBC(2.74 Cos Θ - Sin Θ) = 10 --------------(3) TBC = 10/2.74 Cos Θ - Sin Θ

At Joint C

$\sum$H = -TCDCos40 - TCBCosΘ = 0

$TCDCos40 = TCBCos\Theta$

$TCD = 1.30\ TCBCos\Theta$ ------------(4)

$\sum V = -\ TCDSin40 + TCBSin\Theta - 30 = 0$

Substitute TCD we get

$1.30\ TCBCos\Theta\ Sin40 + TCBSin\Theta = 30$

$0.835\ \ TCBCos\Theta + TCBSin\Theta =30$

$TCB(0.835\ Cos\Theta +\ Sin\Theta) = 30$ --------------(5) $TCB = \ \ 30/0.835\ Cos\Theta +\ Sin\Theta$
Divide (3) / (5)

$TBC(2.74\ Cos\Theta -\ Sin\Theta) = 10\ /\ TCB(0.835\ Cos\Theta +\ Sin\Theta) = 30\ (2.74\ Cos\Theta -\ Sin\Theta)\ 30 = (0.835\ Cos\Theta +\ Sin\Theta)\ 10$
$82.2\ Cos\Theta -\ 30Sin\Theta = 8.35\ Cos\Theta + 10\ Sin\Theta$

$73.85\ Cos\Theta =40\ Sin\Theta\ Sin\Theta\ /\ Cos\Theta = 1.846\ \tan\Theta = 1.846$
$\Theta = 61.540$

$TBC\ \ = 10/\ (2.74\ Cos\Theta -\ Sin\Theta)\ TBC\ \ \ = 23.44\ N$
$TBA\ \ \ = 2.92\ TBC\ \ Cos\Theta$

$= 32.61\ KN$

$TCD\ \ \ = 1.30\ TCB\ \ Cos\Theta$

$= 14.52\ KN$

SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – I – EQUILIBRIUM OF RIGID BODIES – SMEA1202

UNIT 2 EQUILIBRIUM OF RIGID BODIES 9 Hrs

Types of supports and their reactions - requirements of stable equilibrium – Moments and Couples- Varignon's theorem- Equilibrium of Rigid bodies in two dimensions- Equilibrium of Rigid bodies in three dimensions.

EQUILIBRIUM OF RIGID BODIES

FREE BODY DIAGRAM

Free body diagram is a graphical illustration which shows all the forces acting at a rigid body involving 1.Self weight, 2. Normal reactions, 3.frictional force, 4. Applied force and 5. External moment applied. In a rigid body mechanics, the concept of free body diagram is very useful to solve the problems.

PROCEDURE FOR DRAWING A FREE BODY DIAGRAM:

1. Draw outlined shape

   - Isolate rigid body from its surroundings

2. Show all the forces

   - Show all the external forces and couple moments. This includes Applied Loads, Support reactions, the weight of the body, Identify each force.

   - Known forces should be labeled with proper magnitude and direction.

   - Letters are used to represent magnitude and directions of unknown forces.

Equilibrium of rigid bodies:

Static equilibrium:

A body or part of it which is currently stationary or moving with a constant velocity will remain in its status, if the resultant force and resultant moment are zero for all the forces and couples applied on it.

So:

Thus equations of equilibrium for a rigid body are:

$\Sigma F_x = 0$

$\Sigma F_y = 0$ and

$\Sigma M_{any\ point} = 0$

VARIGNON’S THEOREM (OR PRINCIPAL OF MOMENTS)

Varignon’s Theorem states that the moment of a force about any point is equal to the algebraic sum of the moments of its components about that point.

Principal of moments states that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of all the forces of the system about the same point.

Proof of Varignon’s Theorem

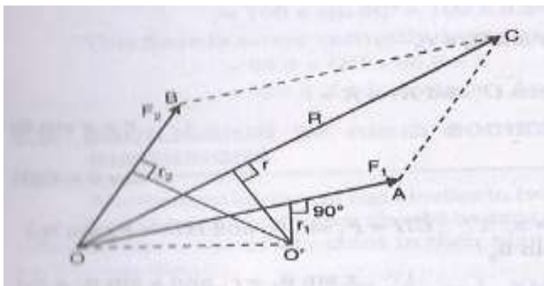


Fig. 1 Varignon’s Theorem

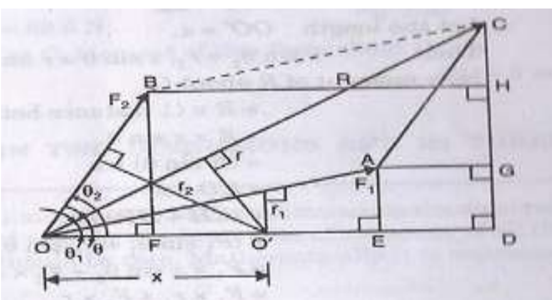


Fig. 2 Varignon’s Theorem explanation

Fig 1 shows two forces Fj and F2 acting at point O. These forces are represented in magnitude and direction by OA and OB. Their resultant R is represented in magnitude and direction by OC which is the diagonal of parallelogram OACB. Let O’ is the point in the plane about which moments of F1, F2 and R are to be determined. From point O’, draw perpendiculars
on OA, OC and OB.

Let r1= Perpendicular distance between F1 and O’.
r2= Perpendicular distance between R and O’. r3= Perpendicular distance between F2 and O’. Then according to Varignon’s principle;
Moment of R about O’ must be equal to algebraic sum of moments of F1 and F2 about O’.
$R \times r = F1 \times r1 + F2 \times r$

Now refer to Fig (b). Join OO' and produce it to D. From points C, A and B draw perpendiculars on OD meeting at D, E and F respectively. From A and B also draw perpendiculars on CD meeting the line CD at G and H respectively.
Let $\Theta1$ = Angle made by F; with OD, Θ = Angle made by R with OD, and $\Theta2$ = Angle made by F2 with OD.
In Fig. (b), OA = BC and also OA parallel to BC, hence the projection of OA and BC on the same vertical line CD will be equal i.e., GD = CH as GD is the projection of OA on CD and CH is the projection of BC on CD.
Then from Fig. 2.(b), we have
P1 sin $\Theta1$ = AE = GD = CH F1 cos $\Theta1$ = OE
F2 sin $\Theta1$ = BF = HD
F2 cos $\Theta2$ = OF = ED

(OB = AC and also OB || AC. Hence projections of OB and AC on the same horizontal line OD will be equal i.e., OF = ED)
R sin Θ =CD R cos Θ =OD
Let the length OO' = x.
Then x sin $\Theta1$ = r, x sin Θ = r and x sin $\Theta2$ = r2

Now moment of R about O'
= R × (distance between O' and R) = R × r
= R × x sin Θ (r = x sin Θ)
=(R sin Θ) × x
= CD × x (R sin Θ = CD)
= (CH +HD)× x
= (F1 sin $\Theta1$ + F2 sin $\Theta2$) × x (CH = F1 sin $\Theta1$ and HD = F2 sin $\Theta2$)
= F1 × x sin $\Theta1$ + F2 × x sin $\Theta2$
= F1 × r1 + F2 × r2 (x sin $\Theta1$ = r1 and x sin $\Theta2$ = r2)
= Moment of F1 about O' + Moment of F2 about O'.
Hence moment of R about any point in the algebraic sum of moments of its components i.e., F1 and F2) about the same point. Hence Varignon's principle is proved. The principle of moments (or Varignon's principle) is not restricted to only two concurrent forces but is also applicable to any coplanar force system, i.e., concurrent or non-concurrent or parallel force system.

Couple

The moment produced by two equal, opposite and non-collinear forces is called a couple.

M = r x F

Resultant of the forces acting on the body
Let o the body on which three types of forces are acting. So the resultant force is

R = F1+F2+F3+……..= ΣF

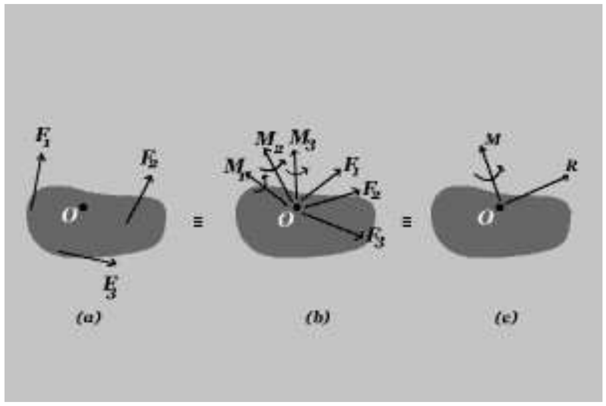


Fig.3 Resultant of the forces acting on a body

For each force let the couple introduced to move the forces to the point O be M1, M2, M3……… respectively. So the resultant couple is

M = M1+M2+M3……….

= (r1 x F1) + (r2 x F2) + (r3 x F3)…….

= Σ(r + F)

The point O selected as the point of concurrency for the forces is arbitrary, and the magnitude and direction of M depend on the particular point O selected. The magnitude and direction of R, however, are the same no matter which point is selected.

In general, any system of forces may be replaced by its resultant force R and the resultant couple M. In dynamics we usually select the mass center as the reference point. The change in the linear motion of the body is determined by the resultant force, and the change in the angular motion of the body is determined by the resultant couple. In statics, the body is in complete equilibrium when the resultant force R is zero and the resultant couple M is also zero. Thus, the determination of resultants is essential in both statics and dynamics.

Resolution of forces

Resolution process is the reverse of addition process or resultant process. A force may be result into several parts, such that addition of these forces provide the same force

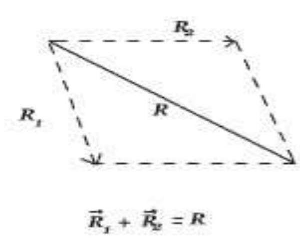


Fig. 4 Resolution of forces

The most common two dimensional resolution of a force vector is into rectangular components. Let i and j be the unit vectors in the direction of x and y,
F = Fxi+Fyj

Hence sum of the resolved components of several forces is equal to the resolved component of the forces.

In general equilibrium means that there is no acceleration i.e., the body is moving with constant velocity but in this special case we take this constant to be zero.

So if we take a point particle and apply a force on it, it will accelerate. Thus if we want its acceleration to be zero, the sum of all forces applied on it must vanish. This is the condition for equilibrium of a point particle. So for a point particle the equilibrium condition is

$$\sum_i \vec{F}_i = 0$$

Where $\vec{F}_i$; i =1, 2, 3…….. are the forces applied on the point particle.
Torque

Torque is defined as the vector product of the displacement vector form the reference point where the force applied. Thus

$$\vec{\tau} = \vec{r}_O \times \vec{F}$$

This is also known as the moment of the force

Equilibrium of rigid bodies

Conditions for Equilibrium

The body should not accelerate (should not move) which, is ensured if $\sum_i \vec{F}_i = 0$

That is the sum of all forces acting on it must be zero no matter at what points on the body they are applied. For example consider the beam in figure given. Let the forces applied by the supports S1 and S2 be F1 and F2, respectively. Then for equilibrium, it is required that the sum of all forces acting on it must be zero no matter at what points on the body they are applied. For example consider the beam in figure given. Let the forces applied by the supports S1 and S2 be F1 and F2, respectively. Then for equilibrium, it is required that

$$\vec{F}_1 + \vec{F}_2 + \vec{W} + \vec{L} = 0$$

Assuming the direction towards the top of the page to be y-direction, this translates to

$$F_1\hat{j} + F_2\hat{j} - W\hat{j} - L\hat{j} = 0 \text{ or } F_1 + F_2 - W - L = 0$$

The condition is sufficient to make sure that the net force on the rod is zero. But as we learned earlier, and also our everyday experience tells us that even a zero net force can give rise to a turning of the rod. So F1 and F2 must be applied at such points that the net torque on the beam is also zero. This is given below as the second rule for equilibrium.

Summation of moment of forces about any point in the body is zero i.e., $\sum_i \vec{\tau}_{iO} = 0$ where $\vec{\tau}_{iO}$ is the torque due to the force $\vec{F}_i$ about point O. One may ask at this point whether

$\sum_i \vec{\tau}_{iO} = 0$

Should be taken about many different points or is it sufficient to take it about any one convenient point. The answer is that any one convenient point is sufficient because if condition (1) above is satisfied, i.e. net force on the body is zero then the torque as is independent of point about which it is taken. These two conditions are both necessary and sufficient condition for equilibrium.

Torque due to a force

Torque about a point due to a for $\vec{F}$ is obtained as the vector product

$$\vec{\tau}_O = \vec{r}_O \times \vec{F}$$
$$= (yF_z - F_y z)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$$

Where $\vec{r}_O$ is a vector from the point O to the point where the force is being applied. Actuall y $\vec{r}_O$ could be a vector from O to any point along the line of action of the force. The magnitude of the torque is given as

$$|\vec{\tau}_O| = |\vec{F}||\vec{r}_O|\sin\theta$$

The unit of a torque is Newton-meter or simply Nm. This is known as Varignon's theorem.

Equilibrium of Rigid bodies in two dimensions

Forces are all in x-y plane

Thus F2 = 0 Mx = My = 0 are automatic satisfied

Equations of equilibrium are reduced, i.e.

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M_A = 0$ (ΣM_z) Where A is any point in the plane.

Equilibrium of Rigid bodies in three dimensions

Equation of equilibrium

$\Sigma F = 0$ $\Sigma M_o = \Sigma(r \times F) = 0$

I.e. $\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$

$\Sigma M_x = 0$ $\Sigma M_y = 0$ $\Sigma M_z = 0$

Equilibrium of rigid bodies

The basics and important condition to be considered for the equilibrium of forces and moments is taken as zero

When a body is acted by upon by some external forces the body starts to rotate or move about any point. If the body does not move or rotate about any point the body said to be in equilibrium.

Moment of force

Moment of a force about a any point is defined as the product of magnitude of force and the perpendicular distance between the forces the point.

The moment (M) of the force (F) about 'o' is given by

M = r*F

F-force acting on the body

r- perpendicular distance from the point O on the line of action of force

If the moment rotates the body in CW direction about 'O' then it is CW moment(+ve). if the moment rotates the body in CCW direction about 'O' then it is CW moment(-ve)

Equilibrium of a particle:

If the resultant of a number of forces acting on a particle is zero then the particle is in equilibrium

Equilibrium and equilibrant:

The force E which brings the particle to equilibrium is called equilibrant(E).

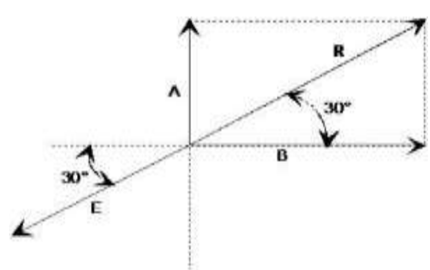


Fig. 5 Equilibrium and equilibrant

Resultant force and equilibrant force both have same magnitude. But if resultant acts at say θ then Equilibrant acts at 180^0+ θ.

Conditions for equilibrium:

1) The algebraic sum of all the external force is zero.$\sum F= 0$ ($.\sum Fx= 0$; $.\sum Fy= 0$; $.\sum Fz= 0$)
2) The algebraic sum of all the Moments about any point in the plane is zero.$\sum M = 0$

Couple:

Two forces F and –F having same magnitude, parallel lines of action and opposite sense are said to form a couple.

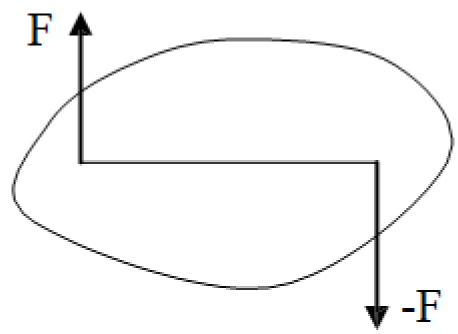


Fig. 6 Couple

When 2 equal and opposite parallel forces act on a body at some distance apart the 2 forces form a couple. Couple has a tendency to rotate the body . The perpendicular distance between the parallel forces is called arm of the couple.

Moment of a couple = forces x Arm of the couple

M = F x a

Equivalent Forces and couples:

The two forces having same magnitude, direction and line of action but acting at different points producing the same external effect on the rigid body are said to be equivalent forces.

If two couples produce the same moment on the rigid body they are called equivalent couples.

Difference between moment and couple:

Couple is a pure turning effect which may be moved anywhere in its own plane or into a parallel plane without change of its effect on the body.

Moment of force must include a description of the reference axis about which the moment is taken.

Resolution of force into a force and a couple at a point.(Force couple system) Figure
Resultant of coplanar non concurrent force system:-

$R = \sqrt{(\sum H)^2+(\Sigma V)^2}$

$\Theta=\tan^{-}(\Sigma V/\sum H)$

Resultant force vector R= formula

Equilibrium of rigid body in two dimensional

The equilibrium state will be achieved when the summation of all the external forces and the moments of all the forces is zero.

Principle of equilibrium

$\sum F=0$ (force law of equilibrium)

$\sum M=0$(Moment law of equilibrium)

Force Law of equilibrium

$\sum F_x=0$ (with respect to horizontal components)

$\sum F_y=0$(with respect to horizontal components)

Equilibrium of particle in space

In three dimension of space if the forces acting on the particle are resolved into their respective i ,j,k components the equilibrium equation is written as,

$\sum F_i+\sum F_j+\sum F_k=0$

The equation for equilibrium of a particle in space is,

$\sum F_x=0$; $\sum F_y=0$; $\sum F_z=0$;

EXAMPLE 1:

An adjustable shelving system consists of rod mounted to a shaft on the left and a frictionless support on the right. Isolate the shelf brace and make a free body diagram of the system

Solution:

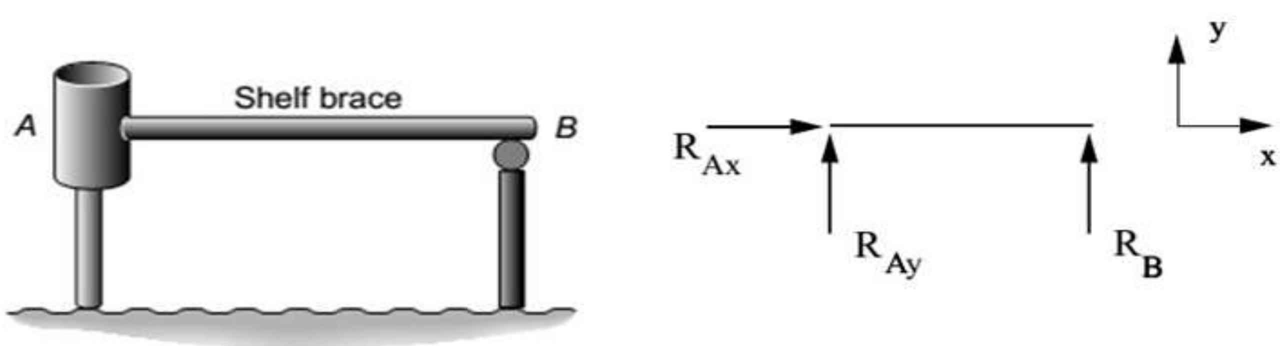


Fig. 7 free body diagram of given situation

Reactions and Support reactions

When a number of forces are acting on a body, and the first body is supported on another body, then the second body exerts a force known as reactions on the first body at the points of contact so that the first body is in equilibrium. The second body is known as support and the force, exerted by the second body on the first body, is known as support reactions.

Types of supports

There are 5 most important supports. They are

- Simple supports or knife edged supports
- Roller support
- Pin-joint or hinged support
- Smooth surface support
- Fixed or built-in support

Simple supports or knife edged support: in this case support will be normal to the surface of the beam. If AB is a beam with knife edges A and B, then R_A and R_B will be the reaction.

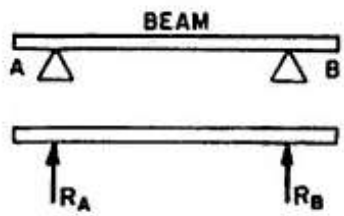


Fig.8 Simply Supported beam

Roller support: here beam AB is supported on the rollers. The reaction will be normal to the surface on which rollers are placed.

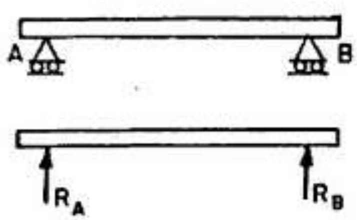


Fig. 9 Roller support

Pin joint (or hinged) support: here the beam AB is hinged at point A. the reaction at the hinged end may be either vertical or inclined depending upon the type of loading. If load is vertical, then the reaction will also be vertical. But if the load is inclined, then the reaction at the hinged end will also be inclined.

Fig.10 Hinged Support

Fixed or built-in support: in this type of support the beam should be fixed. The reaction will be inclined. Also the fixed support will provide a couple.

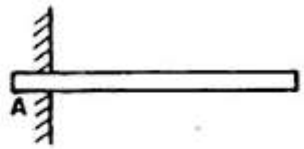


Fig.11 Fixed Support

Types of loading

There are 3 most important type of loading:

Concentrated or point load

Concentrated or point load: here beam AB is simply supported at the ends A and B. A load W is acting at the point C. this load is known as point load or Concentrated load. Hence any load acting at a point on a beam, is known as point load.

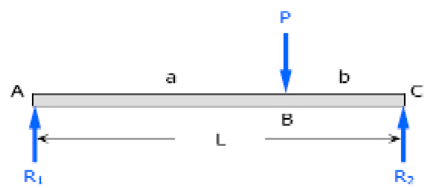


Fig.12 Point load

Uniformly distributed load

A uniformly distributed load (UDL) is a load that is distributed or spread across the whole region of an element such as a beam or slab. In other words, the magnitude of the load remains uniform throughout the whole element. While calculating support reactions the UDL is converted as a point load which is equal to the area of the Rectangle and the load intensity acts a CG point, G as shown.

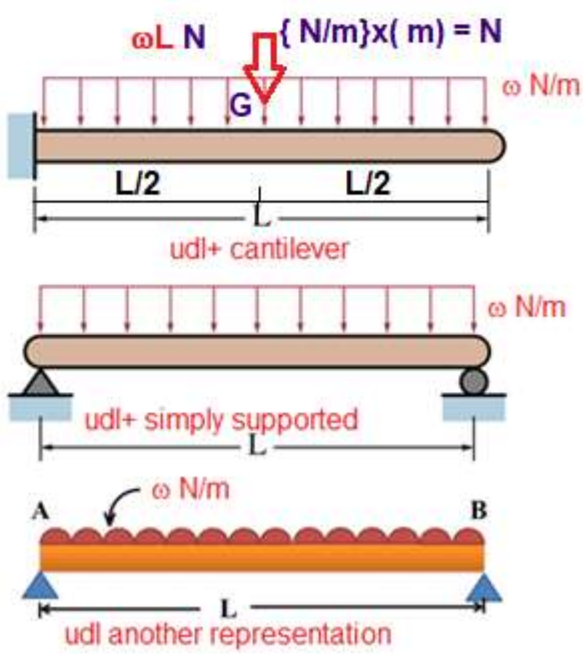


Fig. 13 Uniformly distributed load

Uniformly varying load

A UVL is one which is spread over the beam in such a manner that rate of loading varies from each point along the beam, in which load is zero at one end and increase uniformly to the other end. This type of load is known as triangular load. While calculating support reactions the UVL is converted as a point load which is equal to the area of the triangle and the load intensity acts a CG point, G as shown.

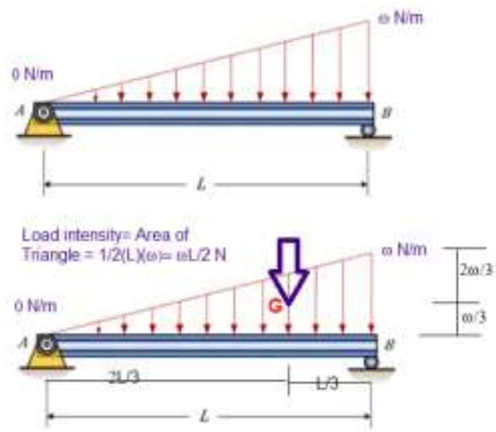


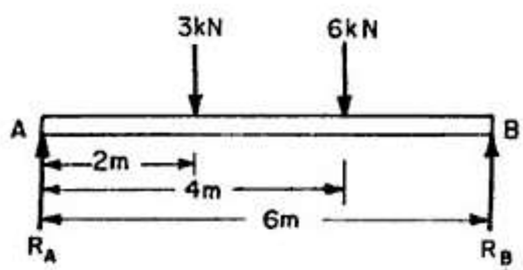
Fig. 14 Uniformly varying load

Numerical Questions:

A simply supported beam AB of span 6m carries point loads of 3kN and 6kN at a distance of 2m and 4m from the left end A as shown in fig. find the reactions at A and B analytically.

Solution

Given, span of beam = 6m



Fig

Let R_A = reaction at A; R_B = reaction at B

As the beam is in equilibrium, the moments of all the forces about any point should be zero. Now taking the moment of all forces about A and equating the resultant moment to zero, we get

$R_B \times 6 - 3 \times 2 - 6 \times 4 = 0$

$6R_B = 6 + 24 = 30$

$R_B = 30/6 = 5$ kN

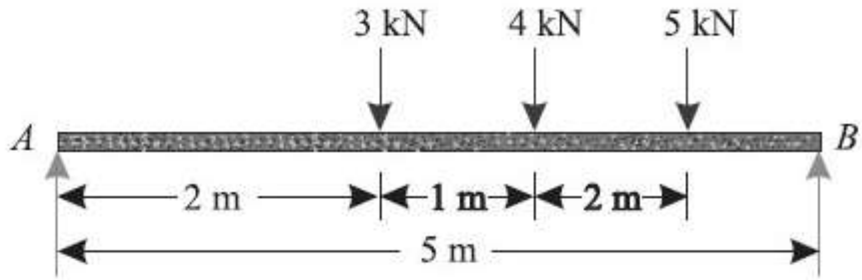
Also for equilibrium, $\Sigma Fy = o$

$\therefore \; R_A + R_B = 3 + 6 = 9$

∴ $R_A = 9 – R_B = 9 – 5 = 4$ kN

EXAMPLE 2:

A simply supported beam AB of span 5 m is loaded as shown in figure. Find the reactions at A and B.



Solution.

Given: Span (l) = 5 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

The example may be solved either analytically or graphically. But we shall solve analytically only.

We know that anticlockwise moment due to RB about A

$= R_B \times l = R_B \times 5 = 5 R_B$ kN-m ...(i)

and sum of the clockwise moments about A,

$= (3 \times 2) + (4 \times 3) + (5 \times 4) = 38$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii),
$5 R_B = 38$

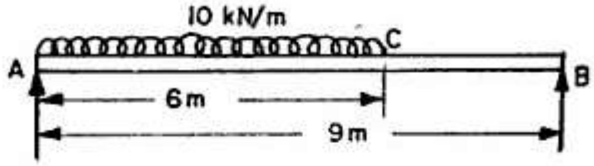
or $R_B = 38/5 = 7.6$ kN

and $R_A = (3+4+5) – 7.6 = 4.4$ kN

EXAMPLE 3:

Problem for Uniformly distributed load

A simply supported beam AB of length 9m, carries a uniformly distributed load of 10 kN/m for distance of 6m from the left end. Calculate the reactions at A and B



Solution

Given,

Length of beem = 9

Rate of U.D.L = 10 kN/m

Length of U.D.L = 6m

Total load due to U.D.L = (Length of U.D.L) x Rate of U.D.L

= 6 x 10 = 60 kN

This load of 60Kn will be acting at the middle point of AC i.e,at a distance of 6/2= 3 m from A. Let R_A = Reaction at A and R_B = reaction at B
Taking the moment of all forces about point A, and equating the resultant moment to zero, we get

R_B x 9 - (6 x 10) x 3 = 0 or $9R_B$ – 180 = 0

∴ R_B = 180/9 =20kN.

Also for equilibrium, $\Sigma F_y = 0$

Or $R_A + R_B$ = 6 x 10 =60

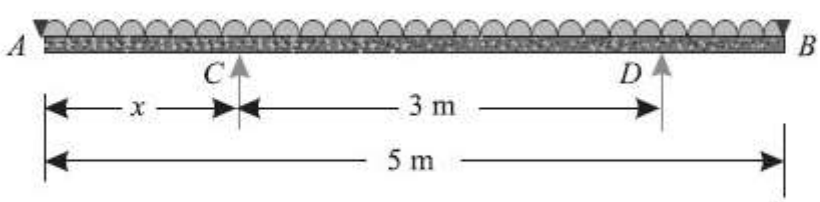
∴ $R_A = 60 - R_B$ = 62 -20 = 40 kN.

EXAMPLE 4:

Problems for overhanging

A beam AB 5 m long, supported on two intermediate supports 3 m apart, carries a uniformly distributed load of 0.6 kN/m. The beam also carries two concentrated loads of 3 kN at left hand end A, and 5 kN at the right hand end B as shown in Figure. Determine the location of the two supports, so that both the reactions are equal.

Solution.



Given:

Length of the beam AB (L) = 5 m and span (l) = 3 m

Let R_C =Reaction at C,

R_D =Reaction at D, and

x =Distance of the support C from the left hand end

We know that total load on the beam

$=3 + (0.6 \times 5) + 5 = 11$ kN

Since the reactions R_C and R_D are equal, therefore reaction at support

$= 11/2 = 5.5$ kN

We know that anticlockwise moment due to R_C and R_D about A

$= 5.5 \times x + 5.5 (x + 3) = 5.5 x + 5.5 x + 16.5$ kN-m

$= 11x + 16.5$ kN-m ...(i)

and sum of clockwise moment due to loads about A

$= (0.6 \times 5) 2.5 + 5 \times 5 = 32.5$ kN-m ...(ii)

Now equating anticlockwise and clockwise moments given in (i) and (ii)

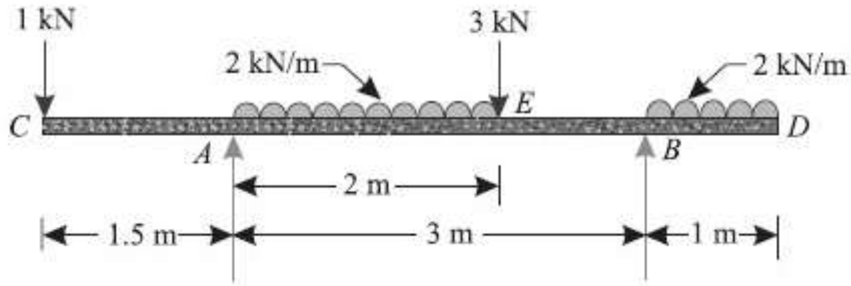
$11 x + 16.5 = 32.5$ or $11 x = 16$

$\therefore$ $11 x = 16 = 1.45$ m

It is thus obvious that the first support will be located at distance of 1.45m from A and second support at a distance of 1.45 + 3 = 4.45 m from A.

EXAMPLE 5:

A beam AB of span 3m, overhanging on both sides is loaded as shown in Figure. Determine the reactions at the support.



Solution.

Given:

Span (l) = 3m

Let RA = Reaction at A, and

R_B = Reaction at B.

We know that anticlockwise moment due to R_B and load at C about A

$=R_B \times l + (1 \times 1.5) = R_B \times 3 + (1 \times 1.5) = 3R_B + 1.5$ kN ...(i)

and sum of clockwise moments due to loads about A

$=(2 \times 2)\ 1 + (3 \times 2) + (1 \times 1)\ 3.5 = 13.5$ kN-m ...(ii) Now equating anticlockwise and clockwise moments given in (i) and (ii),
$3R_B + 1.5 = 13.5$

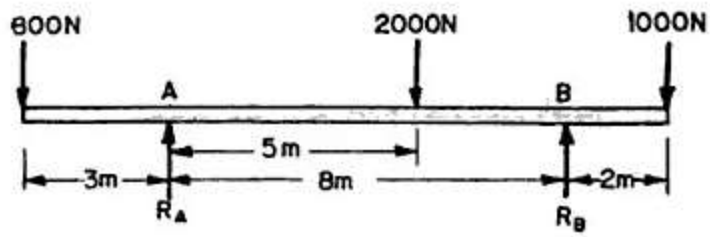
or R_B

= 4 kN

and $R_A = 1 + (2 \times 2) + 3 + (1 \times 1) - 4 = 5$ kN

EXAMPLE 6:

A beam of AB of span 8m, overhanging on both sides is loded as show in figure. Calculate the reactions at both ends.



Solution

Given,

Span of beam = 8m

Let RA = reaction at A RB = reaction at B

Taking the moment of all foreces about point A and equating the resulant moment to zero, we get

$R_B \times 8 + 800 \times 3 - 2000 \times 5 - 1000 \times (8 + 2) = 0$ or

$8R_B + 2400 - 10000 - 10000 = 0$ or

$8R_B = 2000 - 2400 = 17600$

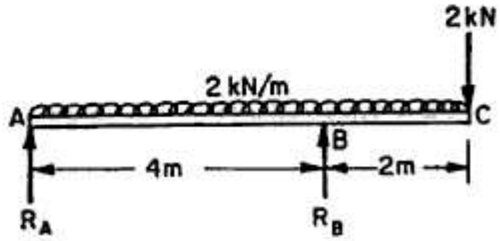
$\therefore R_B = 2200$ N.

$R_A + R_B = 800 + 2000 + 1000 = 3800$

$\therefore R_A = 3800 - R_B = 3800 - 2200 = 1600$

EXAMPLE 7:

A beam AB of span 4m, overhanging on one side upto a length of 2m, carries a uniformaly distributed load of 2kN/m over the entire length of 6m and a point load of 2kN/m as shown in figure. Calculate the reactions at A and B.



Solution

Given,

Span of beam = 4m

Total length = 6m

Rate of U.D.L = 2 kN/m

Total load due to U.D.L = 2 x 6 = 12kN

The toad of 12 kN (i.e., due to U.D.L) will act at the middle point of AC, i.e, at a distance of 3m from A.

Let R_A = reaction at A

and R_B = reaction at B

taking the moment of all forces about point A and equating the resultant moment to zero, we get

$R_B \times 4 - (2 \times 6) \times 3 - 2 \times (4 + 2) = 0$

Or $4R_B = 36 + 12 = 48$

$\therefore \quad R_B = 48/4 = 12kN.$

Also for equilibrium, $\Sigma F_y = 0$ or $R_A + R_B = 12+2 = 14$

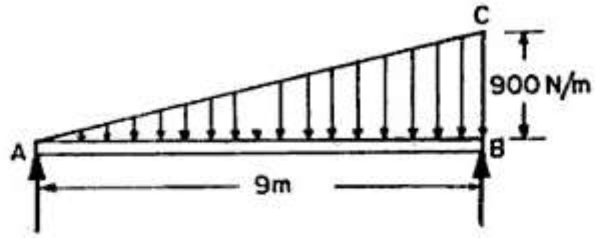
$\therefore R_A = 14 - R_B = 14 - 12 = 2kN$

EXAMPLE 8:

Problem for Uniformly varying load

A simply supported beam of span 9m carries a uniformly varying load from zero at end a to 900 N/m at end B. calculate the reactions at end B. calculate the reaction at the two ends of the support.

Solution



Given,

Span of beam = 9m

Load at end A = 0

Load at end B = 900 N/m

Total load the beam = Area of ABC = (AB x BC)/2 = (9x900)/2

= 4050N

Or $5R_B$ – (5 x 800) x 2.5 – {1/2 x 5 x 800} x {2/3 x 5} = 0

Or $5R_B$ – 1000 – 6666.66 = 0

Or 5RB = 1000 + 6666.66 = 16666.66

Or R_B = 16666.66/5 = 3333.33 N.

Also for the equilibrium of the beam, ΣFY = 0

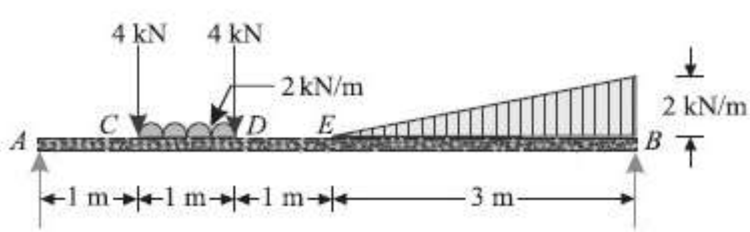
∴ $R_A + R_B$ = total load of the beam

= 6000 N (∵ Total load on beam = 6000 N)

∴ RA = 6000 – RB = 6000 – 3333.33 = 2666.67 N.

EXAMPLE 9:

A simply supported beam AB of 6 m span is subjected to loading as shown in Figure. Find the support reactions at A and B.



Solution.
Given: Span (l) = 6 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that anticlockwise moment due to RB about A

$= R_B \times l = R_B \times 6 = 6\ R_B$ kN-m ...(i)

and sum of clockwise moments due to loads about A

= (4 x 1) + (4 x 2) +[(0 + 2)/2] x 3 x 5 = 30 kN-m

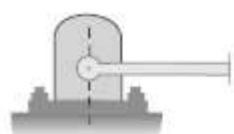
Now equating anticlockwise and clockwise moments given in (i) and (ii),

$6\ R_B = 30$

or $R_B = 30/6 = 5$ Kn

and $R_A = (4 + 2 + 4 + 3) - 5 = 8$ kN

Hinged beams



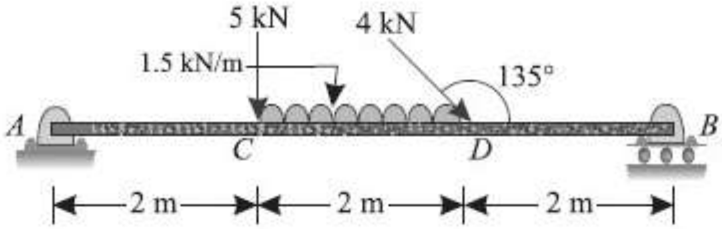
In such a case, the end of a beam is hinged to the support as shown in Figure. The reaction on such an end may be horizontal, vertical or inclined, depending upon the type of loading. All the steel trusses of the bridges have one of their end roller supported, and the other hinged.

The main advantage of such a support is that the beam remains stable. A little consideration will show that the beam cannot be stable, if both of its ends are supported on rollers. It is thus obvious, that one of the supports is made roller supported and the other hinged.

EXAMPLE 10:

Problem for hinged beam

A beam AB of 6 m span is loaded as shown in Figure. Determine the reactions at A and B. Solution.



Given:
Span = 6 m

Let R_A = Reaction at A, and

R_B = Reaction at B.

We know that as the beam is supported on rollers at the right hand support (B), therefore the reaction RB will be vertical (because of horizontal support). Moreover, as the beam is hinged at the left support (A) and it is also carrying inclined load, therefore the reaction at this end will be the resultant of horizontal and vertical forces, and thus will be inclined with the vertical.

Resolving the 4 kN load at D vertically

$= 4 \sin 45° = 4 \times 0.707 = 2.83$ kN

and now resolving it horizontally

$= 4 \cos 45° = 4 \times 0{\cdot}707 = 2{\cdot}83$ kN We know that anticlockwise moment due to R_B about A

$= R_B \times 6 = 6 RB$ kN-m ...(i)

and sum of clockwise moments due to loads about A

$= (5 \times 2) + (1.5 \times 2)\, 3 + 2.83 \times 4 = 30.3$ kN-m ...(ii)

Now equating the anticlockwise and clockwise moments in (i) and (ii),

$6 R_B = 30.3$ kN

$R_B = 5.05$ kN.

We know that vertical component of the reaction RA

$= [5 + (1.5 \times 2) + 2.83] – 5.05 = 5.78$ kN

∴ Reaction at A,

$R_A = \{ (5.78)^2 + (2.83)^2 \}^{1/2} = 6.44$ kN

Let θ = Angle, which the reaction at A makes with vertical.

∴ $\tan \theta = (2.83)/(5.78) = 0.4896$ or $\theta = 26.1°$

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SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – III – PROPERTIES OF SURFACES AND SOLIDS – SMEA1202

UNIT 3 PROPERTIES OF SURFACES AND SOLIDS 9 Hrs

Determination of Areas - First moment of Area and the centroid - simple problems involving composite figures. Second moment of plane area-Parallel axis theorem and perpendicular axis theorem-Polar moment of Inertia – Principal moments of Inertia of plane areas – Principle axes of inertia – relation to area moments of Inertia. Second moment of plane area of simple sections like C,I,T,Z etc. - Basic Concept of Mass moment of Inertia.

PROPERTIES OF SURFACES AND SOLIDS

First moment of area and the centroid

Centroid, Centre of gravity, Centre of mass and moment of inertia are the important properties of a section which are required frequently in the analysis of many engineering problems.

Centroid

It is the point at which the total area of the plane figure (namely rectangle, square, triangle, circle etc.) is assumed to be concentrated.

Centre of gravity

It is a point through which the resultant of the distributed gravity forces (weights) act irrespective of the orientation of the body.

Centre of mass

It is the point where the entire mass of the body may be assumed to be concentrated.

For all practical purposes the centroid and Centre of gravity are assumed to be the same.

Centroid of one dimensional body (Line)

Let us consider a homogeneous wire which is having length 'L', uniform cross sectional area 'a' and density ρ

The weight of the wire = W= ρga X L

The wire is considered to be made up of a number of elemental lengths $L_1, L_2, \dots L_n$

Then to find the centroid, substitute for W in the following equation

$$\bar{x} = \frac{\sum W_1 x_1}{W}$$

$$= \frac{(\rho g a) L_1 X_1 + (\rho g a) L_2 X_2 + (\rho g a) L_3 X_3 \dots\dots\dots (\rho g a) L_n X_n}{(\rho g a) L}$$

$$= \frac{L_1 X_1 + L_2 X_2 + L_3 X_3 + \dots\dots\dots + L_n X_n}{L}$$

$$\bar{X} = \frac{\int X \, dL}{L}$$

Similarly,

$$\bar{Y} = \frac{\int Y \, dL}{L}$$

Centroid of two dimensional body (area)

Let us consider a rectangular plate P,Q,R,S of uniform thickness- t ,density- ρ and area –A

The weight of the plate= W=ρgt x A

This body is considered to be made up of number of imaginary strips or particles of area $A_1, A_2, A_3, \dots \dots \dots \dots A_n,$

$\bar{x} = \frac{\sum W_{1} x_1}{W}$

$= \frac{(\rho g t)A_1 X_1 + (\rho g t)A_2 X_2 + (\rho g t)A_3 X_3 \dots\dots\dots\dots (\rho g t)A_n X_n}{(\rho g t)A}$

$= \bar{X} = \frac{\int X\, dA}{A}$

Similarly,

$\bar{Y} = \frac{\int Y\, dA}{A}$

The integral ∫xdA is known as the first moment of area with respect to the y axis and the integral ∫ydA is known as the first moment of area with respect to the x axis .

Moment of Inertia

the concept which gives a quantitative estimate of the relative distribution of area or mass of a body with respect to some reference axis is termed as the moment of inertia of the body.

The moment of inertia of a body about an axis is defined as the resistance offered by the body to rotation about that axis.it is also defined as the product of the area and the square of the distance of the center of gravity of the area from that axis. Moment of is denoted by I. Hence the moment of inertia about the x axis is represented by I_{xx} and about the y axis is represented by I_{yy}

The moment of inertia of an area is called as the area moment of inertia or the second moment of area .the moment of inertia of the mass of a body is called as the mass moment of inertia

I_{xx}=$\int y^2$dA

I_{yy}=$\int x^2$dA

Parallel axis theorem

Parallel axis theorem states that, the moment of inertia of an area with respect to any axis in its plane is equal to the moment of inertia of the area with to a parallel centroidal axis plus the product of the area and the distance between the two axes

Perpendicular axis theorem (polar moment of inertia)

Perpendicular axis theorem states that the moment of inertia of an area with respect to an axis perpendicular to the x-y plane (z axis) and passing through a pole O is equal to the sum of the moment of inertia of the area about the other two axis (x&y axis) passing through the pole. It's also called as polar moment of inertia and is denoted by the letter J.

J=I_{zz}=$I_{xx} + I_{yy}$

Radius of Gyration

$I_{xx} = k_x^2 A$

$k_x = \sqrt{\frac{I_{xx}}{A}}$

k_x is known as the radius of gyration of the area with respect to the X – axis and has the unit of length (m)

Radius of gyration with respect to the Y – axis

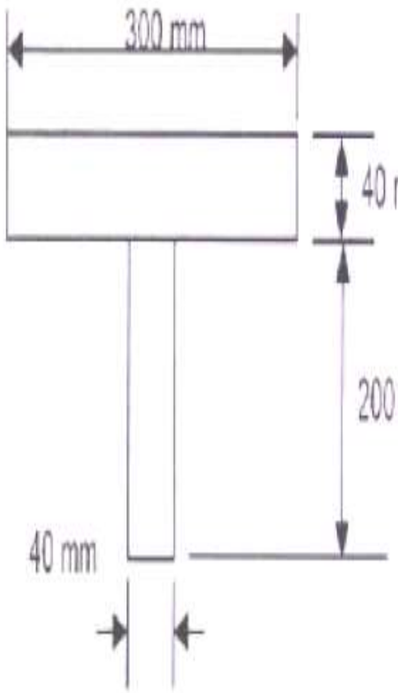
$I_{yy} = k_y^2 A$

$k_x = \sqrt{\frac{I_{yy}}{A}}$

General : The Student is advised to take bottom most line and left most line as reference axes for measuring the CG s of segments.

Problems for finding centroidal axes

1. Locate the centroid of T-section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(12000 \ x \ 150) + (8000 \ x \ 150)}{12000 + 8000}$$

$$= \frac{180000 + 1200000}{20000}$$

$$= \frac{1380000}{20000}$$

= 69 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(12000 \ x \ 220) + (8000 \ x \ 100)}{12000 + 8000}$$

$$= \frac{2640000 + 800000}{20000}$$

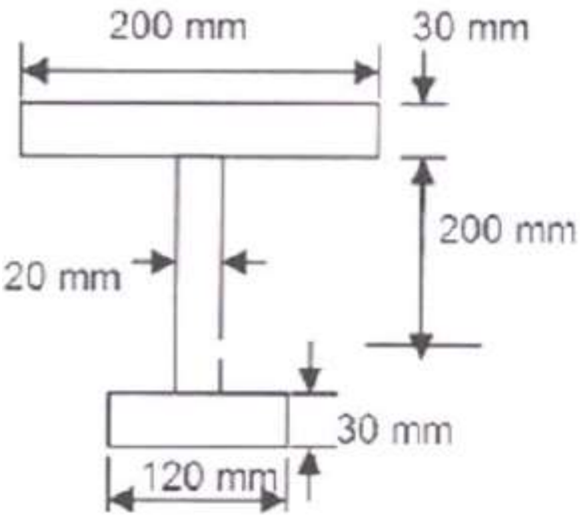
$$= \frac{3440000}{20000}$$

= 172 mm

Result :

The centroid of the given section is (69, 172)

2. Determine the centre of gravity of the I-Section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Middle rectangle section 2

Bottom rectangle section 3

section	Area	X in mm	Y in mm
1	200 x 30 =6000	200/2 =100	30 + 200 + 30/2 =245
2	20 x 200 = 4000	200/2 =100	30 + 200/2 = 130
3	120 x 30 = 3600	200/2 = 100	30/2 =15

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(6000 \ x \ 100)+(4000 \ x \ 100)+(3600 \ x \ 100)}{6000+4000+3600}$$

$$= \frac{600000+400000+360000}{13600}$$

$$= \frac{1360000}{13600}$$

= 100 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(6000 \ x \ 245)+(4000 \ x \ 130)+(3600 \ x \ 15)}{6000+4000+3600}$$

$$= \frac{1470000+520000+54000}{13600}$$

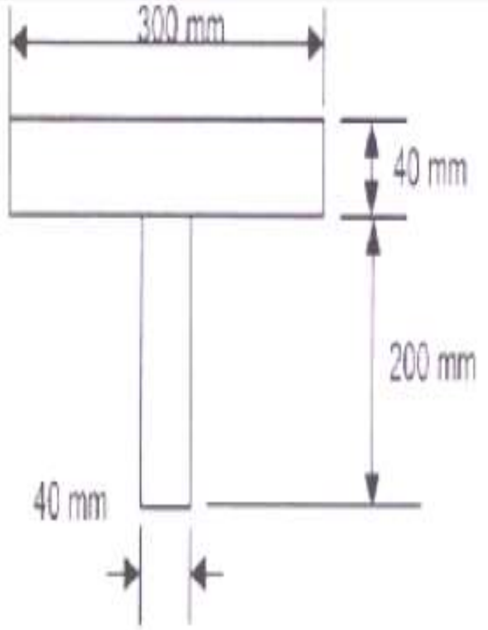
$$= \frac{2044000}{13600}$$

= 150.294 mm

Result :

The Centre of gravity of the given section is (100, 150.294)

3. Locate the centroid of T-section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$

$$= \frac{(12000\ X 150)+(8000\ X\ 150)}{12000 + 8000}$$

$$= \frac{1800000 + 1200000}{20000}$$

$$= \frac{3000000}{20000}$$

= 150 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(12000\ x\ 220)+(8000\ x\ 100)}{12000 + 8000}$$

$$= \frac{2640000 + 800000}{20000}$$

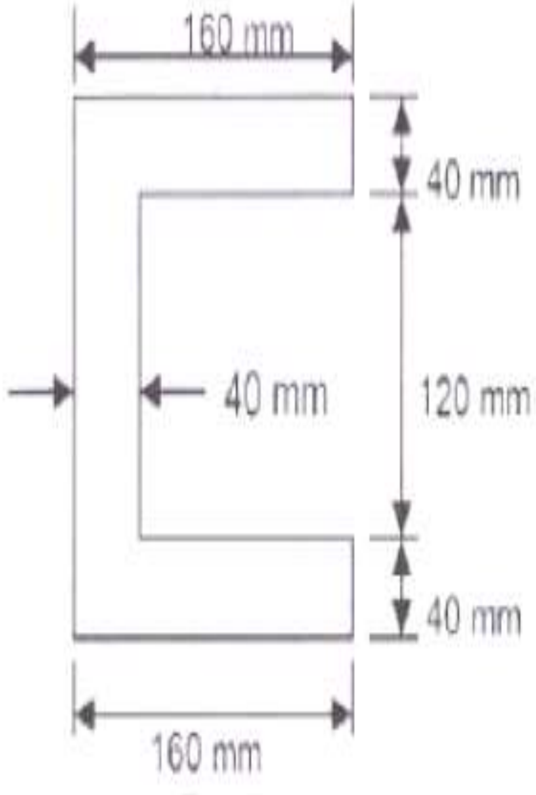
$$= \frac{3440000}{20000}$$

= 172 mm

Result :

The Centre of gravity of the given section is (150, 172)

4. Determine the centre of gravity of the channel section shown in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Middle rectangle section 2

Bottom rectangle section 3

Section	Area	X in mm	Y in mm
1	160 x 40 =6400	160/2 =80	40/2 =20
2	120 x 40 = 4800	40/2 =20	40 + 120/2 = 100
3	160 x 40 = 6400	160/2 = 80	40 +120 +40/2 =180

$$\bar{X} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(6400\ x\ 80)+(4800\ x\ 20)+(6400\ x\ 80)}{6400+4800+6400}$$

$$= \frac{512000+96000+512000}{17600}$$

$$= \frac{1120000}{17600}$$

= 63.636 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(6400 \, x \, 20)+(4800 \, x \, 100)+(6400 \, x \, 180)}{6400+4800+6400}$$

$$= \frac{128000+480000+1152000}{17600}$$

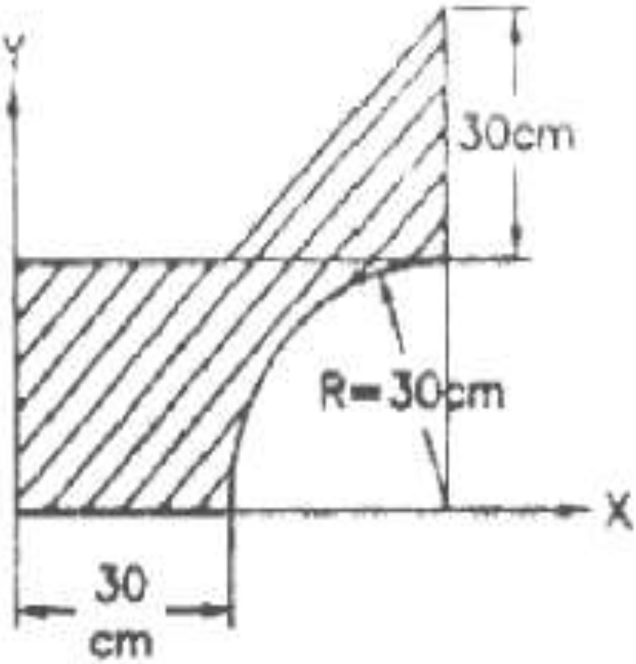
$$= \frac{1760000}{17600}$$

= 100 mm

Result :

The Centre of gravity of the given section is (63.636, 100)

5.Locate the centroid of plane area shaded shown in Fig.



Divide the diagram in to three sections with their individual centroid

Bottom rectangle section 1

Top triangle section 2

Bottom quarter circle section 3

Section	Area in mm^2	X in mm	Y in mm
1	60 x 30 =1800	60/2 =30	30/2 =15
2	½ x 30 x 30 = 450	30 + (2 X 30/3) =50	30 + 1 x 30/3 = 40
3	$\pi X 30^2/4$ = 706.858	60 – (4 x30/3π) = 47.268	(4 x30/3π) =12.732

$$\bar{X} = \frac{A_{1x_1+}A_2x_2 - A_3x_3}{A_{1+}A_2 - A_3}$$

$$= \frac{(1800 X 30)+(450 X 50)-(706.858 X 47.268)}{1800 + 450 - 706.858}$$

$$= \frac{54000 + 22500 - 33411.764}{1543.142}$$

$$= \frac{43088.236}{1543.142}$$

$$= 27.922 \text{ mm}$$

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2 - A_3 y_3}{A_1 + A_2 - A_3}$$

$$= \frac{(1800 X 15) + (4500 X 40) - (706.858 X 27.922)}{1800 + 450 - 706.858}$$

$$= \frac{27000 + 18000 - 19736.889}{1543.142}$$

$$= \frac{25263.111}{1543.142}$$

$$= 16.371 \text{ mm}$$

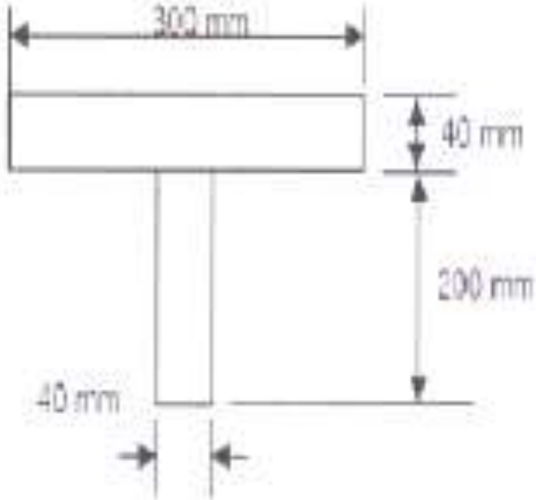
Result :

The Centre of gravity of the given section is (27.922, 16.371)

General : The Student is advised to take bottom most line and left most line as reference axes for measuring the CG s of segments. Finding CG of total fig. is done in accordance to that.

Problems on MI

6.Find the moment of Inertia about the centroidal axes of the section in Fig.



Divide the section in to two rectangles with their individual centroid

Top rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	300 x 40 =12000	300/2 =150	200 + 40/2 =220
2	40 x 200 = 8000	300/2 =150	200/2 = 100

$$\bar{X} = \frac{A_{1x_1+}A_2x_2}{A_{1+A2}}$$

$$= \frac{(12000\ X150)+(8000\ X\ 150)}{12000+8000}$$

$$= \frac{1800000+1200000}{20000}$$

$$= \frac{3000000}{20000}$$

= 150 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(12000 \times 220) + (8000 \times 100)}{12000 + 8000}$$

$$= \frac{2640000 + 800000}{20000}$$

$$= \frac{3440000}{20000}$$

= 172 mm

Result :

The Centre of gravity of the given section is (150, 172)

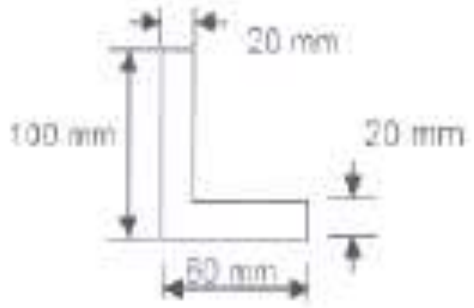
Section	MI about X axis passing through individual centroid I_x	A_1 X $(y_{1-\bar{y}})^2$	MI about X axis passing through $\bar{X}$ I_{XX}	MI about y axis passing through individual centroid I_y	A_1 X $(x_{1-\bar{x}})^2$	MI about y axis passing through $\bar{Y}$ I_{YY}
1	$\frac{bd^3}{12}$ = $\frac{300 \times 40^3}{12}$ = 1600000	A_1 X $(y_{1-\bar{y}})^2$ = 12000 x $(220 - 172)^2$ =27648000	$I_{X_1} + A_1$ X $(y_{1-\bar{y}})^2$ =1600000 + 27648000 =29248000	$\frac{bd^3}{12}$ = $\frac{40 \times 300^3}{12}$ = 90000000	A_1 X $(x_{1-\bar{x}})^2$ = 12000 x $(150 - 150)^2$ =0	$I_{Y_1} + A_1$ X $(x_{1-\bar{x}})^2$ =90000000 + 0 =90000000
2	$\frac{bd^3}{12}$ = $\frac{40 \times 200^3}{12}$ = 26666667	A_2X $(\bar{Y} - Y_2)^2$ = 8000 x $(172 - 100)^2$ =41472000	I_{X_2} + A_2X$(\bar{Y} - Y_2)^2$ =26666667+ 41472000 = 68138667	$\frac{bd^3}{12}$ = $\frac{200 \times 40^3}{12}$ = 10666667	A_2X $(x_{2-\bar{x}})^2$ = 8000 x $(150 - 150)^2$ =0	$I_{Y_2} + A_2$X $(x_{2-\bar{x}})^2$ =10666667 + 0 =10666667
			$\sum I_{XX}$ = 97386667			$\sum I_{YY}$= 100666667

Answer :

Moment of inertia about the centriodal X axis = 97386667 mm^4 = 97.387 X $10^6 mm^4$

Moment of inertia about the centriodal Y axis = 100666667 mm^4 = 100.667 X $10^6 mm^4$

7. .Find the moment of Inertia about the centroidal axes of the section in Fig.



Divide the section in to two rectangles with their individual centroid

left rectangle section 1

Bottom rectangle section 2

Section	Area	X in mm	Y in mm
1	100 x 20 =2000	20/2 =10	100/2 =50
2	40 x 20 = 800	20 + 40/2 =40	20/2 = 10

$$\bar{X} = \frac{A_1x_1 + A_2x_2}{A_1 + A_2}$$

$$= \frac{(2000 X 10)+(800 X 40)}{2000 + 800}$$

$$= \frac{20000 + 32000}{2800}$$

$$= \frac{52000}{2800}$$

= 18.571 mm

$$\bar{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(2000 \times 50) + (800 \times 10)}{2000 + 800}$$

$$= \frac{100000 + 8000}{2800}$$

$$= \frac{108000}{2800}$$

= 38.571 mm

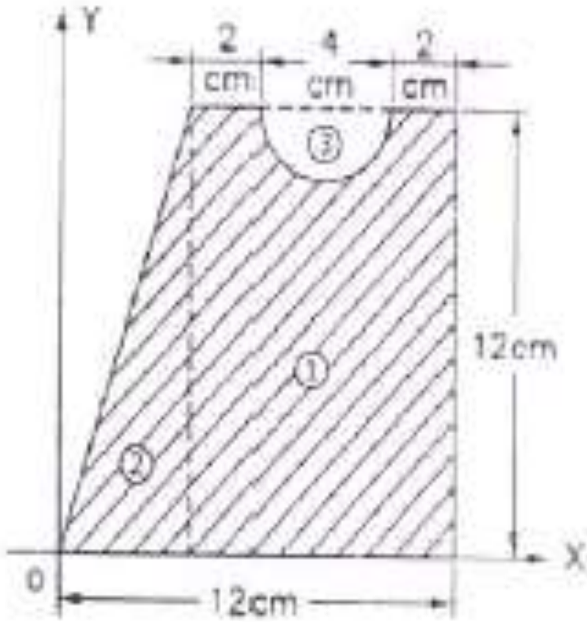
Section	MI about X axis passing through individual centroid I_x	A_1 X $(y_{1-\bar{y}})^2$	MI about X axis passing through $\bar{X}$ I_{XX}	MI about y axis passing through individual centroid I_y	A_1 X $(x_{1-\bar{x}})^2$	MI about y axis passing through $\bar{Y}$ I_{YY}
1	$\frac{bd^3}{12} = \frac{20 X 100^3}{12}$ = 1666667	A_1 X $(y_{1-\bar{y}})^2$ = 2000 x $(50 - 38.571)^2$ =261244	$I_{X_1} + A_1$ X $(y_{1-\bar{y}})^2$ =1666667 + 261244 =1927911	$\frac{bd^3}{12} = \frac{100 X 20^3}{12}$ = 66667	A_1 X $(x_{1-\bar{x}})^2$ = 2000 x $(18.571 - 10)^2$ =146924	$I_{Y_1} + A_1$ X $(x_{1-\bar{x}})^2$ =66667+ 146924 =213951
2	$\frac{bd^3}{12} = \frac{40 X 20^3}{12}$ = 26667	A_2 X $(\bar{Y} - Y_2)^2$ = 800 x $(38.571 - 10)^2$ =653042	I_{X_2}+ A_2 X$(\bar{Y} - Y_2)^2$ =26667+ 653042= 679709	$\frac{bd^3}{12} = \frac{20 X 40^3}{12}$ = 106667	A_2 X $(x_{2-\bar{x}})^2$ = 800 x $(40 - 18.571)^2$ =367362	$I_{Y_2} + A_2$ X $(x_{2-\bar{x}})^2$ =106667 + 367362 =474029
			ΣI_{XX} = 2607620			ΣI_{YY}= 687980

Answer :

Moment of inertia about the centriodal X axis = 2607620mm^4 = 2.608 X $10^6 mm^4$

Moment of inertia about the centriodal Y axis = 687980mm^4 = 6.88 X $10^5 mm^4$

8. Find the MI about the horizontal axes of the section shown in Fig.



Moment of inertia of section 1

$I_{X_1} = \frac{bd^3}{12} = \frac{6 X 12^3}{12}$

= 864

Moment of inertia of section 2

$I_{X_2} = \frac{bh^3}{36} = \frac{6 X 12^3}{36}$

$= 288 cm^4$

Moment of inertia of section 3

= MI of semi-circle about its Centre+ $A_3 X (\bar{Y} - Y_3)^2$

= $0.1097r^4 + (\frac{\pi X r^2}{2}) X (\bar{Y} - Y_3)^2$ (find $\bar{Y}$ and Y_3 like the previous problems)

= $0.1097 X 2^4) + (\frac{\pi X 2^2}{2}) X (11.151 - 4.43)^2$

= 1.7552 + 283.82

= 285.578 cm^4

Moment of inertia of whole section

= $I_{X_1+}I_{X_2} - I_{X_3}$

= 864 +288 – 285.578

= 866.422 cm^4

SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

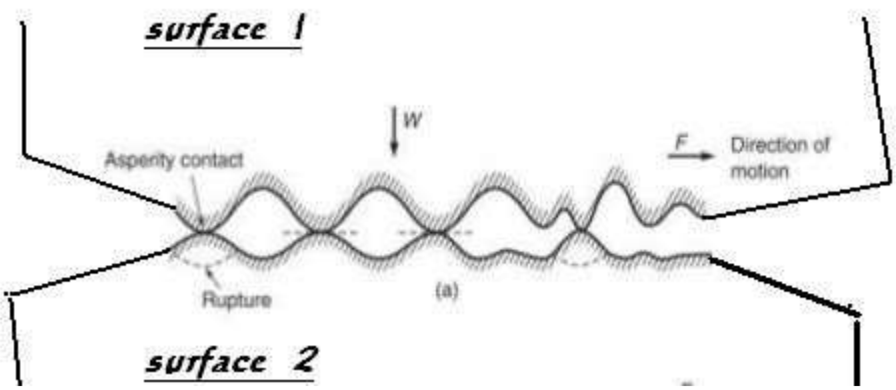
UNIT – IV – FRICTION – SMEA1202

Frictional Force - Laws of Coulomb friction - Cone of friction-Angle of repose-relation between cone of friction and angle of repose- limiting friction-Rolling resistance- Simple contact friction - Screw - Wedge– Ladder- Belt friction.

FRICTION

In the previous units, the surfaces in contact have been assumed to be frictionless. But practically, the surfaces are rough in nature.

Friction gets developed because of surface irregularities between the contact surfaces.



Let us say a block of weight W rests on a table. Let us further say a maximum of 20 N force it can generate against the applied load which will always oppose motion.

If we apply say 1N the generated Frictional force is also 1 N.

If we apply say 5N the generated Frictional force is also 5 N.

If we apply say 10 N the generated Frictional force is also 10 N.

If we apply say 20 N the generated Frictional force is also 20 N.

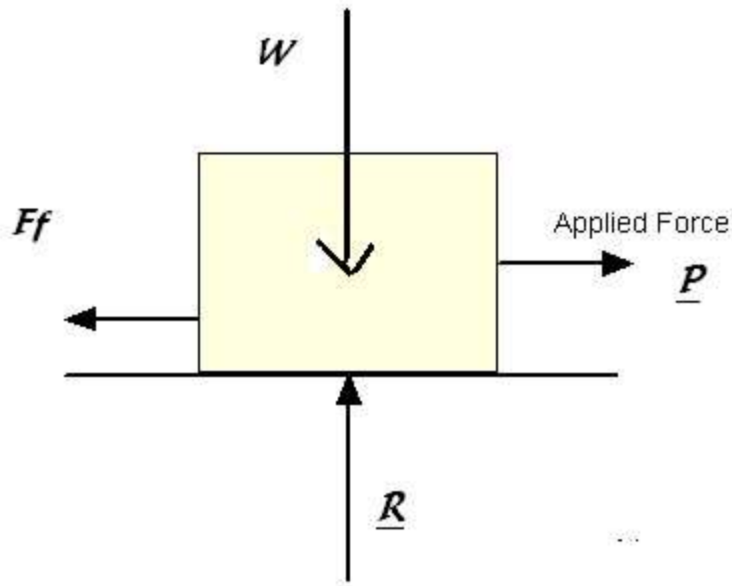
It is sure that it will generate equal frictional force as above. Otherwise because of force imbalance the object will move. It does not happen Hence the applied load= generated Frictional force.

Read the last case discussed above. i.e., **If we apply say 20 N the generated Frictional force is also 20 N. This state is called impending motion state where motion is likely to occur but is in equilibrium(i.e., not moving)**

However if we apply more than to 20 N say 21 N then object can generate only a maximum of 20 N in the opposite direction of applied force. Hence the object will move with 21 N- 20 N= 1N force in the applied force direction.

<u>Evaluation of Frictional force:</u>

Let us consider a block of weight W rests on a table. Let the developed reaction to support the load is R.



Now W=R, Is it not?

Let us assume that an applied load P acts on the block to RHS.

Frictional force F_f will act to the LHS to oppose motion.

The frictional force is directly proportional to the normal reaction

i.e., $F_f \alpha R$

F_f = (a constant) R

The constant is called the coefficient of Friction and is referred as μ.

i.e., $F_f = \mu R$

Coefficient of Friction μ is defined as the ratio of the frictional force to the normal reaction which is dimensionless.

i.e., $\mu = F_f / R$

Types of friction:

1. **Dry or Coulomb Friction:** When friction occurs between two non-lubricated bodies in contact, it is known as dry friction. The two surfaces of bodies may be at rest or one of the bodies is moving and the other is at rest.

2. **Fluid friction:** When adjacent layers in a fluid are moving with different velocities, then the friction is called fluid friction.

Classification of Dry friction

1. Static Friction (F_s): Frictional force acting between two bodies which are in contact but are not sliding with respect to each other is called static friction.

$$F = \mu N$$

   a) Limiting Friction (F_{max}): The maximum frictional force that a body can exert on the other body having contact with it is known as limiting friction.

$$F_{max} = \mu_s N$$

   where, F_{max} is the maximum possible force of static friction, N is normal force and μ_s is a constant known as coefficient of static friction. Always F_s is smaller than $\mu_s N$ and its value depends on other forces acting on the body. The magnitude of frictional force is equal to that required to keep the body at relative rest. Therefore

$$F_s \leq F_{max} = \mu_s N$$

2. Dynamic or Kinetic Friction(F_k): When two bodies are in contact moving with respect to each other experiences some friction and this force is known as dynamic or kinetic friction(f_k).

$$F_k = \mu_k N$$

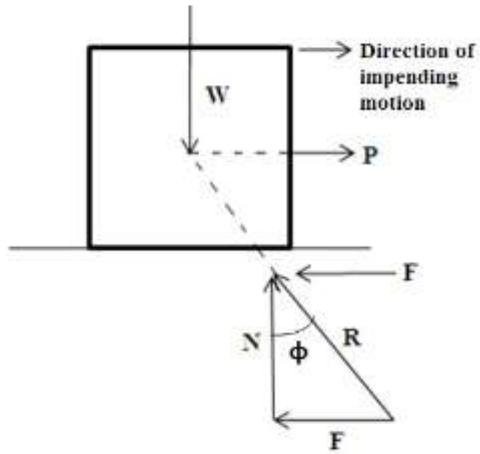
   where, F_k is Kinetic friction, N is normal force and μ_k is a constant known as coefficient of kinetic friction.

   a) Sliding Friction: If a body is moving over or within the other body experiences some frictional force. This force is known as sliding friction.
   b) Rolling Friction: This is the force experienced by a body when it is rolling on the other body.

Laws of Coulomb friction:

1. The frictional force developed is equal to the external force applied to the surface, till the maximum friction.
2. The frictional force is always acting in the opposite direction in which the surface tends to move.
3. The frictional force is independent to the surface area of contact.
4. μ_s and μ_k are not depend upon the area of the surfaces but depends upon the nature of the surfaces which are in contact.
5. μ_s is always greater than μ_k.

Angle of friction (Φ): Angle of friction is defined as the angle made by the resultant and the normal to the surface.

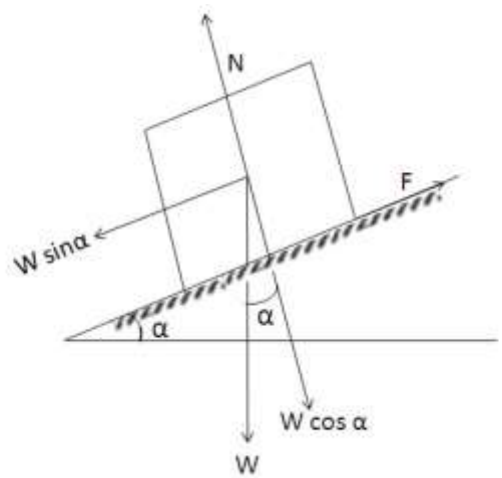


$$\tan \Phi = (F/N) = (\mu N/N) = \mu$$

where, **Φ** is angle of friction and μ is coefficient of friction.

Angle of repose (α): Angle of repose is the maximum angle of inclination of an inclined plane on which a body remains in equilibrium or sleep over the inclined plane by the assistance of friction only.

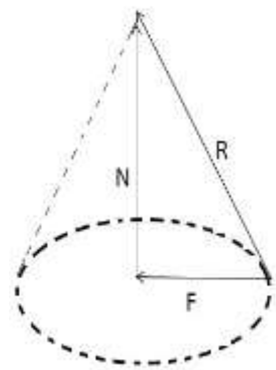
(English Meaning of Repose is Sleep.)



$$\tan \alpha = (W\sin \alpha / W\cos \alpha) = (F_f / R_N) = (\mu R_N / R_N) = \mu = \tan \Phi$$

(OR) $\boxed{\alpha = \Phi}$

Cone of friction: When the direction of external force is changed gradually through 360°, the resultant generates a right circular cone with semi central angle of cone about normal plane is equal to the angle of friction.



Screw friction: It is a device used for lifting or lowering heavy load by applying comparatively smaller efforts at the end of the lever. The thread of the screw jack can be considered as an inclined plane.

$$P \times a = F \times r$$

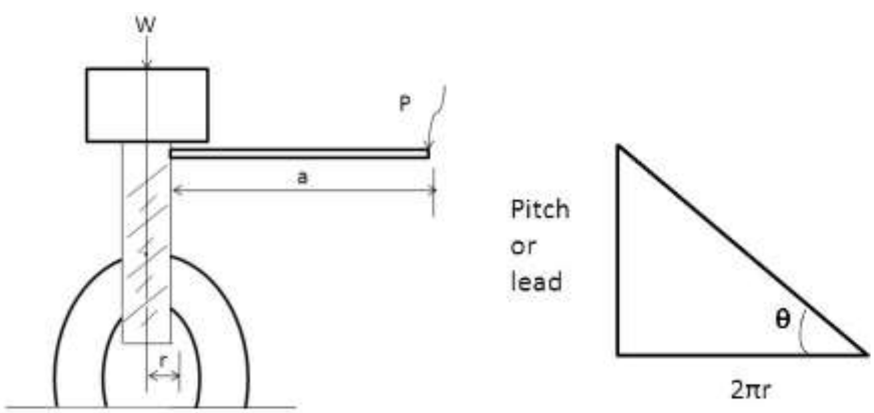
$$F = [(W (\tan\theta + \tan\phi)/ (1 - \tan\theta \tan\phi)]$$

For Lifting $$P = [(W\, r/a) \tan(\theta + \phi)]$$

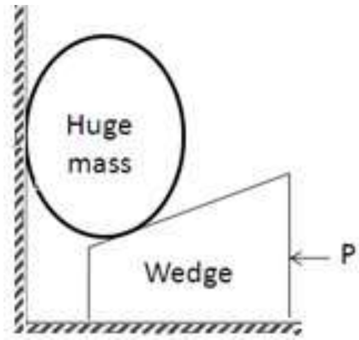
For Lowering $$P = [(W\, r/a) \tan(\phi - \theta)]$$

$$\tan\theta = \{P \text{ (or) } L / 2\pi r\}$$

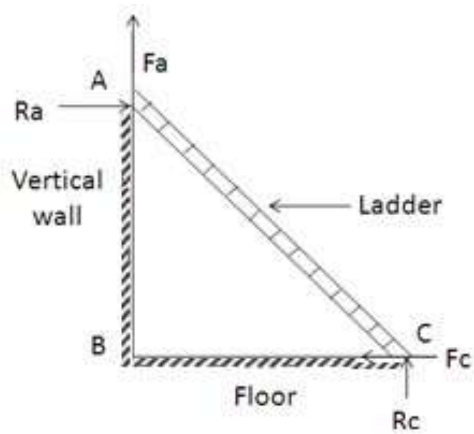
For single start $P = L$ and for multi start $nP = L$.



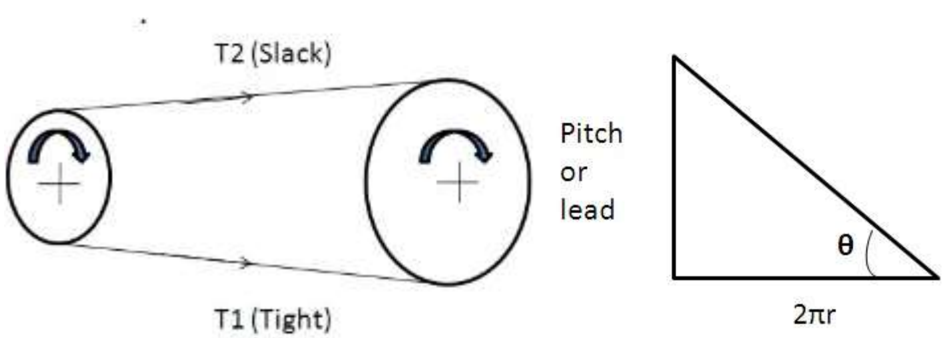
Wedge friction: A wedge is a small wooden or metal piece placed under the huge mass for lifting. This wedge experiences friction at its contact surfaces.



Ladder friction: A ladder placed against a vertical wall and horizontal floor experiences friction at two contact points, one with the wall and the other with the floor. This problem can be solved with equilibrium conditions applicable to non-concurrent and coplanar system of forces *(∑Fx=0, ∑Fy=0* and *∑M=0*).



Belt friction: The friction experiences between the pulley and belt is called belt friction.



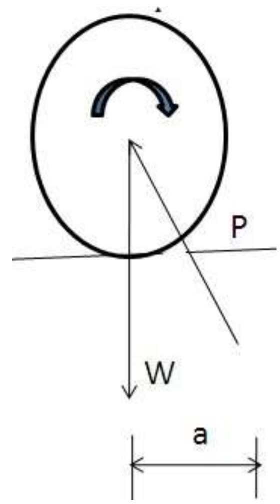
$(T_1/T_2) = e^{\mu\theta}$ Torque

transmitted, $T = (T_1 - T_2)\, r$ Power

transmitted, $P = (T_1 - T_2)\, v$

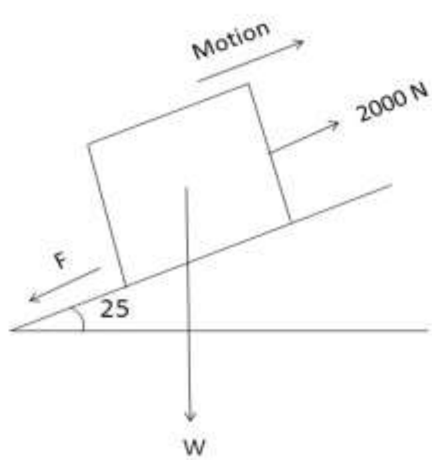
where, T_1 and T_2 are tension in tight and slack sides, r is radius of pulley , v is linear velocity and μ is coefficient of friction.

Rolling Resistance: When a wheel is made to roll freely with constant angular velocity over a horizontal surface, the wheel slows down due to the deformation of the surface which causes the wheel to have surface contact instead of line contact. This contact surface that resists the motion of wheel called rolling resistance.

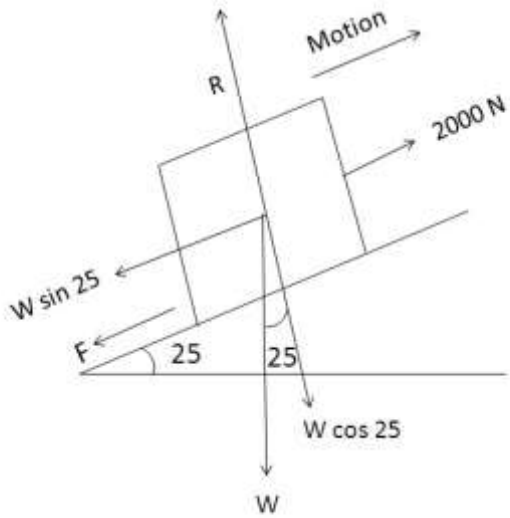


Sample Solved Problems

1. An effort of 2000 N is required to move a certain body up a 25° inclined plane. The force acting parallel to the plane. If the angle of inclination is changed from 25° to 30°, the effort required to move the body increases to 2250 N. Determine the weight of the body and the coefficient of friction.



<u>Free Body Diagram of 2000 N weight:</u>



Case I : When α = 25°

$\sum Fx = 0$:

2000 - F – W sin α = 0

2000 - μ R – W sin α = 0

2000 - μ W cos 25 – W sin 25 = 0

0.908 x μ W + 0.4226 x W = 2000

W (0.908 x μ + 0.4226) = 2000

W = 2000 / (0.908 x μ + 0.4226) -------------- 1

Case II : When α = 30°, P = 2250 N

$\sum Fx = 0$:

2250 - μ R – W sin α = 0

2250 - μ W cos α – W sin α = 0

2250 - μ W cos 30 – W sin 30 = 0

0.866 x μ W + 0.5 x W = 2250

W x (0.866 x μ + 0.5) = 2250

W = 3000 / (0.866 x μ + 0.5) -------------- 2

Equating Eq. 1 and 2

2000 / (0.908 x μ + 0.4226) = 2250 / (0.866 x μ + 0.5)

(0.866 x μ + 0.5) / (0.908 x μ + 0.4226) = 2250 / 2000

(0.866 x μ + 0.5) / (0.908 x μ + 0.4226) = 1.125

(0.866 x μ + 0.5) = 1.125 x (0.908 x μ + 0.4226)

0.866 x μ + 0.5 = 1.0215 μ + 0.475425

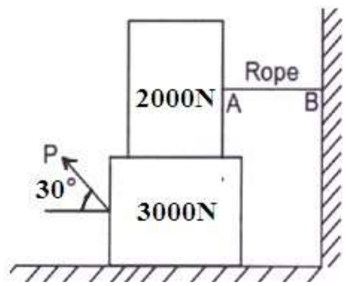
$$1.0215\ \mu - 0.866 \times \mu = 0.5 - 0.475425$$

$$\mu\ (1.0215 - 0.866) = 0.024575$$

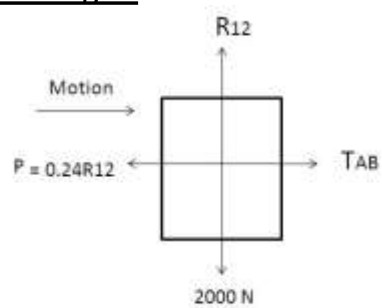
$$\mu\ (0.1555) = 0.024575$$

$$\boldsymbol{\mu = 0.158}$$

2. For the blocks shown in Fig. 1 Find the value of pull 'P'. The coefficient of friction between blocks is 0.24 and the same between block and floor is 0.3.



Free Body Diagram of 2000 N weight:



$\sum Fx = 0$:

$T_{AB} - 0.24\ R_{12} = 0$

$\sum Fy = 0$:

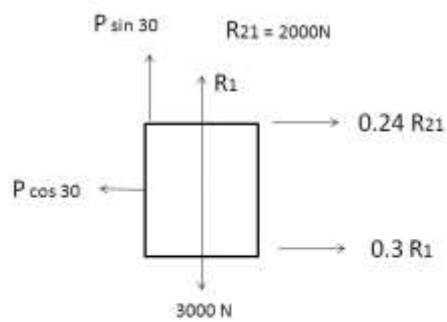
$R_{12} - 2000 = 0$

$\mathbf{R_{12} = 2000\ N}$

Therefore, $T_{AB} = 0.24 \times 2000$

$\mathbf{T_{AB} = 480\ N}$

Free Body Diagram of 3000 N weight:



$\sum Fx = 0$:

$-P \cos 30 + 0.24 \times 2000 + 0.3\ R_1 = 0$

$-P \cos 30 + 0.3\ R_1 = -480$ ------- 1

$\sum Fy = 0$:

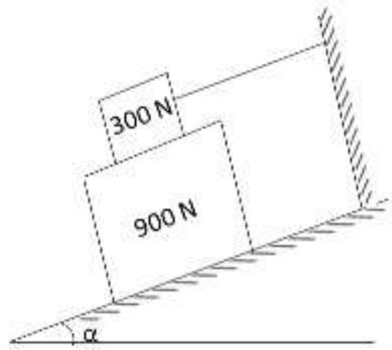
$P \sin 30 + R_1 - 2000 - 3000 = 0$ ------2

Solving equations 1 & 2 we will get

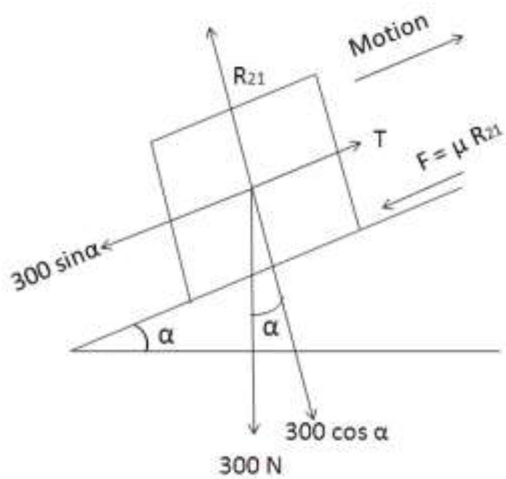
$\mathbf{P = 1948.7\ N}$

$\mathbf{R_1 = 4025.6\ N}$

3. What should be the value of α in Fig. which will make the motion of 900 N blocks down the plane to impend? Take the coefficient of friction for all contact surfaces as 1/3.



Free Body Diagram for 300 N weight:



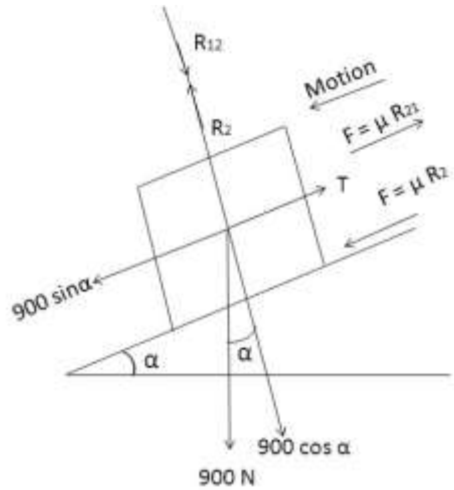
$\sum Fx = 0$:

$T - 0.33\ R_{21} - 300 \sin \alpha = 0$ ------- 1

$\sum Fy = 0$:

$R_{21} = 300 \cos \alpha$ -----------------2

Free Body Diagram For 900N weight:



$\sum Fx = 0$:

$0.33\ R_{21} + 0.33R_2 - 900 \sin\alpha = 0$ ------- 3

$\sum Fy = 0$:

$R_2 - R_{21} - 900 \cos\alpha = 0$

$R_2 - 300 \cos\alpha - 900 \cos\alpha = 0$ [from Eq 2]

$R_2 = 1200 \cos\alpha$ -------------------- 4

Substituting Eq 2 & 4 in Eq 3

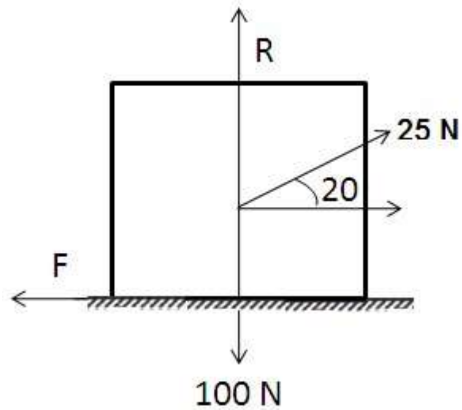
$0.33\ (300 \cos\alpha) + 0.33\ (1200 \cos\alpha) - 900 \sin\alpha = 0$

$0.33 \times 1500 \cos\alpha = 900 \sin\alpha$

or, $\alpha = \tan^{-1}(495/900) = \tan^{-1} 0.55$

Therefore, $\boldsymbol{\alpha = 29.05°}$

4. The force required to pull a block of weight 100 N on a rough plane is 25 N. Find the co-efficient of friction if the force is applied at an angle of 20° with the horizontal.



$\sum Fx = 0$:

$$\mu R = 25 \cos 20° \qquad \text{------- } 1$$

$\sum Fy = 0$:

$$R + 25 \sin 20° = 100$$

$$\mathbf{R = 91.44\ N} \qquad \text{------- } 2$$

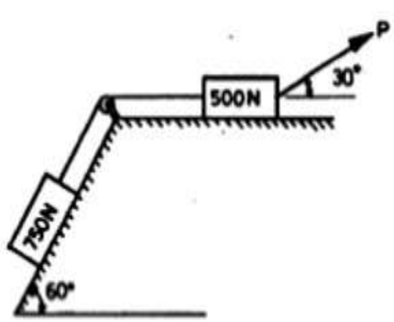
Substituting the value of R in Eq. 1

$$\mu = 25 \cos 20° / R$$

$$\mu = 23.49 / 91.44$$

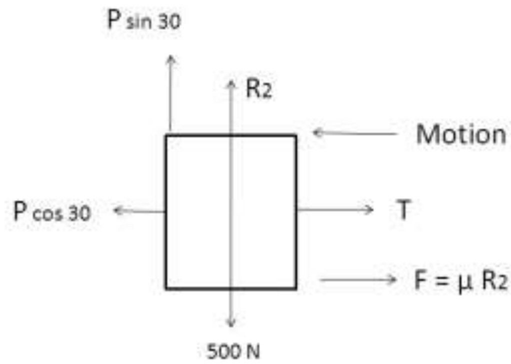
$$\boldsymbol{\mu = 0.256}$$

5. What is the value of P in the system shown in Fig.4 to cause the motion to impend? Assume the pulley is smooth and coefficient of friction between other contact surfaces is 0.22.



Solution:

FBD For 500N weight:



$\sum Fx = 0$:

$T + \mu R_2 - P \cos 30 = 0$

$T + 0.22 R_2 - P \cos 30 = 0$ ------- 1

$\sum Fy = 0$:

$R_2 + P \sin 30 - 500 = 0$

$R_2 = 500 - P \sin 30$ ------- 2

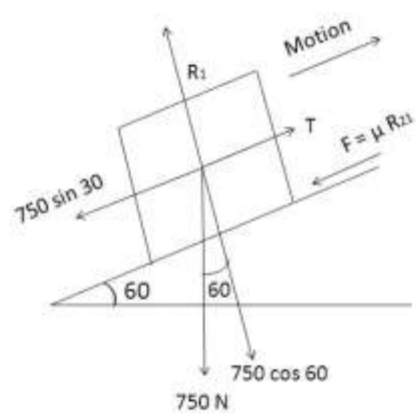
Substituting in Eq. 1

$T + 0.22 (500 - P \sin 30) - P \cos 30 = 0$

$T + 110 - 0.5 P - 0.866 P = 0$

$T + 110 - 1.366 P = 0$ ------- 3

FBD For 750N weight:



$\sum Fx = 0$:

$T - \mu R_1 - 750 \sin 60 = 0$

$T - 0.22 R_1 - 649.52 = 0$ ------- 4

$\sum Fy = 0$:

$R_1 + 750 \cos 60 = 0$

$R_1 = 750 \cos 60$

$\mathbf{R_1 = 375\ N}$

Substituting in Eq. 4

$T = 0.22 \times 375 + 649.52$

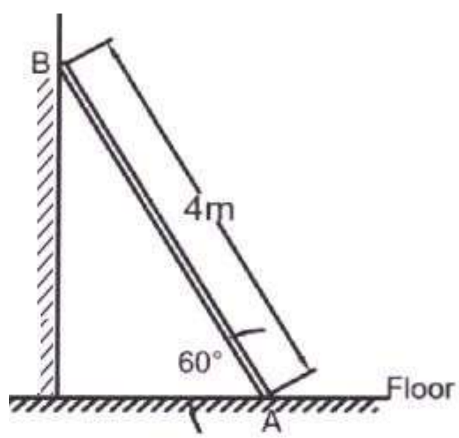
$\mathbf{T = 732\ N}$

Substituting in Eq. 3

$732 + 110 - 1.336 P = 0$

$\mathbf{P = 630.239\ N}$

6. A ladder of weight 1000 N and length 4m rests as shown in Fig.6. If a 750 N weight is acting a distance of 3m from the bottom of ladder, it is at the point of sliding. Determine the co-efficient of friction between ladder and the floor. Assume the co-efficient of friction is same for all the contacting surfaces.



Solution:

$\sum Fx = 0$:

$R_w - \mu_f R_f = 0$

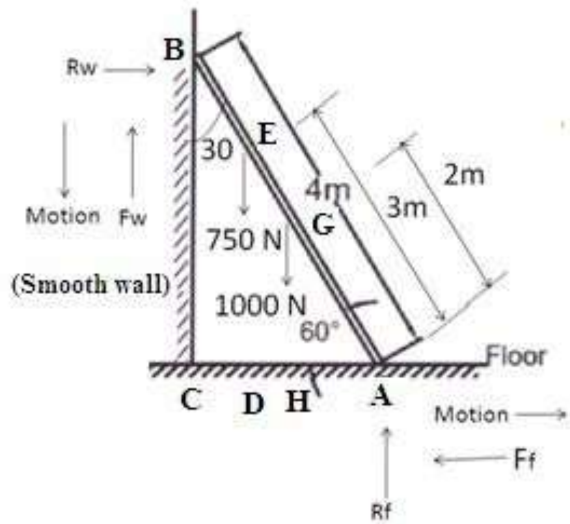
$\mu_f = R_w / R_f$ ------- 1

$\sum Fy = 0$:

$\mu_w R_w - 1000 - 750 + R_f = 0$

$0 - 1000 - 750 + R_f = 0$

R_f = 1750 N ------- 2



$\sum M_A = 0$:

$(1000 \text{ x } 2 \text{ x } \cos 60) + (750 \text{ x } 3 \text{ x } \cos 60) - (R_w \text{ x } 4 \text{ x } \sin 60) - (\mu_w R_w \text{ x } 4 \text{ x } \cos 60) = 0$

$(1000 \text{ x } 2 \text{ x } \cos 60) + (750 \text{ x } 3 \text{ x } \cos 60) - (R_w \text{ x } 4 \text{ x } \sin 60) - (0) = 0$

$(R_w \text{ x } 4 \text{ x } \sin 60) = (1000 \text{ x } 2 \text{ x } \cos 60) + (750 \text{ x } 3 \text{ x } \cos 60)$

$3.464 \text{ x } R_w = 2125$

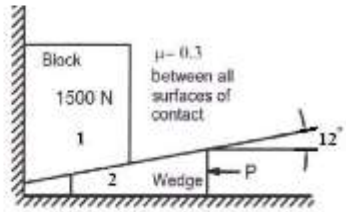
R_w = 613.45 N

From Eq. 1

$\mu_f = R_w / R_f = 613.45 / 1750$

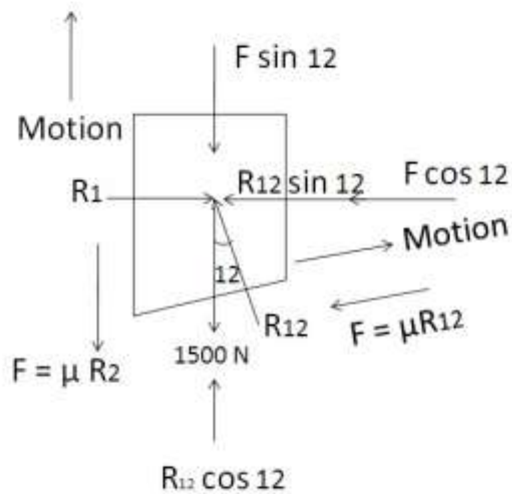
μ_f = 0.35

7. For the block and wedge shown in Fig., determine the value of 'P' required for raising the block. Weight of the wedge is 150N.



Solution:

FBD for Block of weight 1500 N:



$\sum Fx = 0$:

$$R_1 - R_{21}\sin 12° - F\cos 12° = 0$$

$$R_1 - R_{21}\sin 12° - \mu R_{21}\cos 12° = 0$$

$$R_1 - R_{21}\sin 12° - 0.3R_{21}\cos 12° = 0$$

$$R_1 = 0.501\ R_{21} \qquad \text{-------- 1}$$

$\sum Fy = 0$:

$$- F\sin 12° - 1500 + R_{21}\cos 12° - \mu R_2 = 0$$

$$- \mu R_{21}\sin 12° - 1500 + R_{21}\cos 12° - \mu R_2 = 0$$

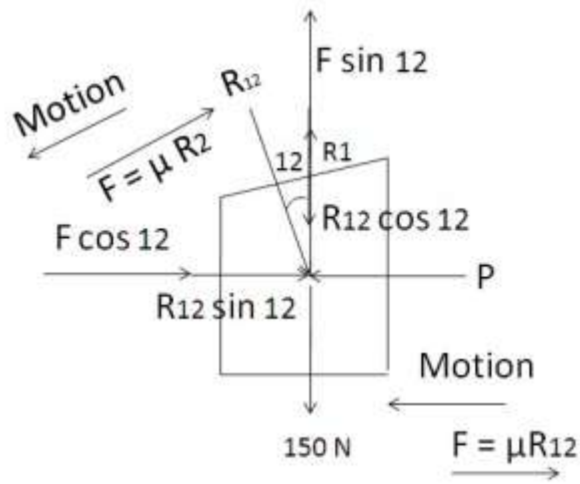
$$- 0.3\ R_{21}\sin 12° + R_{21}\cos 12° - \mu R_2 = 1500 \qquad \text{-------- 2}$$

Substituting R_1 value in Eq. 2 we will get

$$\mathbf{R_{21} = 1959.75\ N}$$

R_1 = 981.8 N

FBD for Wedge of weight 150 N:



$\sum Fx = 0$:

$\mu R_{21} \cos 12° + R_{21} \sin 12° - P + \mu R_2 = 0$

$0.3 \times 1959.75 \cos 12° + 1959.75 \sin 12° - P + 0.3 R_2 = 0$

$P - 0.3 R_2 = 982.5$ -------- 3

$\sum Fy = 0$:

$- R_{21} \cos 12° + R_2 + 0.3 R_{21} \sin 12° - 150 = 0$

$- 1959.75 \cos 12° + R_2 + 0.3 \times 1959.75 \sin 12° - 150 = 0$

R_2 = 1944.68 N

Substituting R_2 value in Eq. 3 we will get

P = 1565.9 N

8. The pitch of a single threaded screw jack is 6 mm and its mean diameter is 60 mm. If μ is 0.1, determine the force required at the end of lever 250 mm long measure from the axis of screw to a) raise a 65 kN load b) lower the same load.

Given:

$P = L = 6$ mm

$D = 60$ mm: $r = 30$ mm

$\mu = 0.1$

$a = 250$ mm

$W = 65$ KN

Solution:

For Raising

$$P = [(W\ r/a) \tan(\theta + \phi)]$$

$$\tan\theta = (P/2\pi r)$$

$$\theta = \tan^{-1}(P/2\pi r)$$

$$\boldsymbol{\theta} = \tan^{-1}(6/2\pi x30)$$

$$\boldsymbol{\theta = 1.82°}$$

WKT, $\phi = \tan^{-1}\mu = \tan^{-1}(0.1)$

$$\boldsymbol{\phi = 5.71°}$$

Substituting in Eq. 1

$$P = \{[(65 \times 30)/250)] \times \tan(1.82 + 5.71)\}$$

$$\mathbf{P = 1.031\ KN}$$

For Lowering

$$P = [(W\ r/a) \tan(\phi - \theta)]$$

$$P = \{[(65 \times 30)/250)] \times \tan(5.71 - 1.82)\}$$

$$\mathbf{P = 0.5302\ KN}$$

9. A belt is running over a pulley of diameter 1 m at 300 rpm. The angle of contact is 160^0 and coefficient of friction is 0.25. If the maximum tension in the belt is 1200 N, determine the power transmitted by it.

Given:

$D = 1$ m

$N = 300$ rpm

$\Theta = 160^0$

$\mu = 0.25$

$T_1 = 1200$ N

Solution:

Power transmitted, $P = (T_1 - T_2)\, v$

$V = (\pi DN/60) = \pi \times 1 \times 300 / 60 = 15.7$ m/s

$(T_1/T_2) = e^{\mu\theta}$

$(1200/T_2) = e^{0.25 \times 160 \times \pi / 180}$

$T_2 = 1200 / e^{0.25 \times 160 \times \pi / 180}$

$T_2 = 334.83$ N

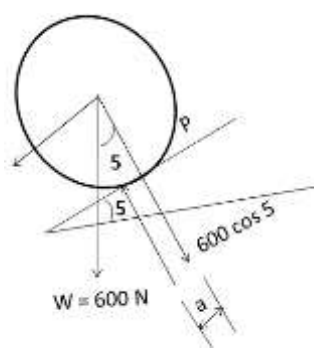
Power transmitted, $P = (T_1 - T_2)\, v$

$P = (1200 - 334.83) \times 15.7$

$\mathbf{P = 13583.16}$ **watts**

10. A wheel of weight 600 N and radius 350 mm rolls down a 5 inclined plane. Find the coefficient of rolling resistance.

Solution:



$\sum Mp = 0$

$-(600 \cos 5 \times a) + (600 \sin 5 \times 0.35) = 0$

$a = (600 \sin 5 \times 0.35) / (600 \cos 5)$

$\mathbf{a = 0.0306\ m}$

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SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIT – V – KINETICS OF RIGID BODIES & DYNAMICS OF PARTICLES – SMEA1202

Unit 5 KINETICS OF RIGID BODIES AND DYNAMICS OF PARTICLES 9 Hrs

Dynamics- Classification- Kinematics- Kinetics- Types of energy-Displacement, Velocity and acceleration their relation- Relative motion - Curvilinear motion - Newton's Law - D'Alembert's Principle, Work Energy Equation- Impulse and Momentum- Impact of elastic bodies- General plane motion.

KINETICS OF RIGID BODIES AND DYNAMICS OF PARTICLES

INTRODUCTION

The dynamics of particles deals with the study of forces acting on a body and its effects, when the body is in motion. It is further divided into Kinematics and kinetics.

Kinematics – The study of motion of body without considering the forces which cause the motion of the body.

Kinetics – The study of motion of body with considering the external forces which cause the motion of the body.

<u>*Plane motion*</u> – If a particle has no size but mass it is considered to have only plane motion, not rotation. In this chapter the study motion of particles with only plane motion is taken without considering force that cause motion i.e., Kinematics.

The plane motion of the body can be sub divided into two types

(i) Rectilinear motion

(ii) Curvilinear motion

1. RECTILINEAR MOTION *(Straight Line Motion)* - It is the motion of the particle along a straight line.

Example: A car moving on a straight road

A stone falls vertically downwards

A ball thrown vertically upwards

This deals with the relationship among displacement, velocity, acceleration and time for a moving particle. The rectilinear motion is of two types as Uniform acceleration and Variable acceleration.

1.1 **Displacement** –The displacement of a moving particle is the change in its position, during which the particle remain in motion. It is the vector quantity, i.e., it has both magnitude and direction. The SI unit for displacement is the metre (m).

1.2 **Velocity** – The rate of change of displacement is velocity. It is the ratio between distances travelled in particular direction to the time taken. It is also a vector quantity, i.e., it has both magnitude and direction. The SI unit for velocity is the metre/second (m/sec) or kilometer/hour (km/h)

1.3 **Acceleration** – The rate of change of velocity is acceleration. It is the ratio between changes in velocity to the time taken. The change in velocity means the difference between final velocity and initial velocity. It is also a vector quantity. The SI unit for acceleration is the metre/second^2 (m/sec^2).

1.4 **Retardation** – The negative acceleration is retardation. It occurs when final velocity is less than initial velocity ($v<u$).

1.5 **Speed** – The distance travelled by a particle or a body along its path per unit time. It is a scalar quantity, i.e., it has only magnitude. The SI unit for speed is the metre/second (m/sec) or kilometer/hour (km/h)

RELATIVE MOTION

A body is said to be in motion if it changes its position with respect to the surroundings, taken as fixed. This type of motion is known as the individual motion of the body. An example of relative motion is how the sun appears to move across the sky, when the earth is actually spinning and causing that apparent motion. Usually, we consider motion with respect to the ground or the Earth. Within the Universe there is no real fixed point. The basis for Einstein's Theory of Relativity is that all motion is relative to what we define as a fixed point.

Relative velocity – Basic concept

Let's consider two motors A and B are moving on a road in same direction moving in uniform speed. Let the uniform velocities of motors A and B be u m/sec and v m/sec respectively (assume $v > u$)

Now, a person standing on the road looks at the motor A and finds that it is going at a speed of u m/sec. Similarly, looks at motor B and finds it is going at a speed of v m/sec separately. But for the driver of motor A, the motor B seems to move faster than him at the rate of only (v - u) m/sec. i.e., the motor A is imagined to be at ret or, the driver of motor A forgets his own motion.

Relative velocity of B with respect to A is (v-u). It is denoted by $V_{B/A}$

$$\therefore V_{B/A} = V_B - V_A = (v - u) \text{ m/sec}$$

Similarly for the driver of motor B, the motor A seems to move slower (assume $u < v$) than him at the rate of only (u - v) m/sec. i.e., the motor B is imagined to be at ret or, the driver of motor B forgets his own motion.

Relative velocity of A with respect to B is (v - u). It is denoted by $V_{A/B}$

$$\therefore V_{A/B} = V_A - V_B = (u - v) \text{ m/sec}$$

PROBLEM

Example1. The car A travels at a speed of 30 m/ sec and car B travels at a speed of 20 m/ sec in the same direction. Determine, i) the velocity of car A relative to car B ii) the velocity of car B, relative to car A

Given data

$V_A = 30$ m/se

$V_B = 20$ m/sec

Same direction

Solution

Let the cars A and B, travels in the same direction, say towards right.

Now, let's use the sign convention, the RHS velocity is taken as positive, and the LHS velocity is taken as negative. Hence, $V_A = 30$ m/sec and $V_B = 20$ m/sec.

Velocity of car A relative to car B

$V_{A/B} = V_A - V_B = 30 - 20 = 10$ m/sec ($\rightarrow$) (since due to positive)

Velocity of car B relative to car A

$V_{B/A} = V_B - V_A = 20 - 30 = -10$ m/sec ($\leftarrow$) (since due to negative)

Example2. The car A travels at a speed of 30 m/ sec and car B travels at a speed of 20 m/ sec in the opposite direction. Determine, i) the velocity of car A relative to car B ii) the velocity of car B, relative to car A

Given data

$V_A = 30$ m/se

$V_B = -20$ m/sec (- due to LHS)

Opposite direction

<u>Solution</u>

Let the cars A and B, travels in the opposite direction, say A towards right and towards left.

Velocity of car A relative to car B

$V_{A/B} = V_A - V_B = 30 - (-20) = 50$ m/sec ($\rightarrow$) (since due to positive)

Velocity of car B relative to car A

$V_{B/A} = V_B - V_A = -20 - 30 = -50$ m/sec ($\leftarrow$) (since due to negative)

MATHEMATICAL EXPRESSION FOR VELOCITY AND ACCELERATION

(i) Velocity, $v = ds/dt$

(ii) Acceleration, $a = d^2s/dt^2$

Where, s – distance travelled by a particle in a straight line.

t – time taken by the particle to travel the distance 's'

Equation of motion in straight line

Let, u – initial velocity (m/sec)

v – Final velocity (m/sec)

s – Distance travelled by a particle (m)

t – Time taken by the particle to change from u to v (second)

a – acceleration of the particle (m/sec^2)

$$\boxed{\begin{gathered} v = u + at \\ s = ut + \tfrac{1}{2}(at^2) \\ v^2 = u^2 + 2as \end{gathered}}$$

Note: 1) If a body starts from rest, its initial velocity is zero i.e., u=0 2)

If a body comes to rest, its final velocity is zero i.e., v=0

PROBLEMS

Example1. A car is moving with a velocity of 20 m/sec. the car is brought to rest by applying brakes in 6 seconds. Find i) retardation ii) distance travelled by the car after applying brakes.

Given data

u = 20 m/s

v = 0 (car is brought to

rest) t = 6 sec

Solution

i) Retardation or negative acceleration

Using equation of motion, v = u+at

$$0 = 20 + (a*6)$$

$$a = -3.33\ m/sec^2$$

$$\boxed{\textbf{Retardation} = 3.33\ m/sec^2}$$

ii) Distance travelled

Using equation of motion, $s = ut + 1/2\,(at^2)$

$$= (20*6) + 1/2(3.33*6^2)$$

$$= 60\ m$$

$$\boxed{\textbf{Distance}, s = 60\ m}$$

Example2. A train starts from rest and attains a velocity of 45 kmph in 2 minutes, with uniform acceleration. Calculate i) acceleration ii) distance travelled and iii) time required to reach a velocity of 36n kmph.

Given data

Initial velocity, $u = 0$ (train starts from rest)

Final velocity, $v = 45$ kmph = 12.5 m/sec

Time taken , $t = 2$ minutes = 120 seconds

Solution

i) Acceleration, a

Using equation of motion, $v = u + at$

$$A = 0.104\ m/sec^2$$

ii) Distance travelled in 2 minutes, s

Using equation of motion, $s = ut + 1/2$ (at^2) $S = 748.8$ m

iii) Time required to attain velocity of 36 kmph $u = 0$

$v = 36$ kmph = 10 m/sec

Using equation of motion, $v = u + at$ t

$$= 96.15\ sec$$

Example3. A thief's car had a start with an acceleration of 2 m/sec^2. A police's car came after 5 seconds and continued to chase the thief's car with a uniform velocity of 20 m/sec. Find the time taken in which the police car will overtake the thief's car?

Given data

Initial velocity of thief's car = 0

Acceleration of thief's car = 2 m/sec^2

Uniform velocity of police van = 20 m/sec

Police's car came after 5 seconds of the start of thief's car.

Solution

Let us consider that the police's car takes 't' seconds to overtake thief's car. Now, the cars are taken separately to solve.

Motion of thief's car

$u = 0$

$a = 2$ m/sec^2

$t = (t+5)$

Using equation of motion, $s = ut+1/2\,(at^2) = (t+5)^2$ -------------------- (1)

Motion of police's car

The police's car is moving with an uniform velocity of 20 m/sec.

Therefore, distance travelled by the police's car, from starting point of thief's car and to overtake it

Take, s = uniform velocity * time taken

$= 20*t = 20t$ -------------------- (2)

The police car overtakes the thief's car. Hence, the distances travelled by both the cars should be equal.

Therefore, equate (1) and (2)

$(t+5)^2 = 20t$

$t^2+25+10t = 20t$

$t^2+25-10t = 0$

The t is found as 5 seconds.

Conclusion - The time taken by police's car to overtake thief's car is 5 seconds.

2. **CURVILINEAR MOTION** - It is the motion of the particle along a curved path. It has two dimensions.

Example: A stone thrown into the air at an angle

Throwing paper airplanes in air

There are two systems involved in curvilinear motion. They are

(i) Cartesian systems (rectangular coordinates)

(ii) Polar system (radial coordinates)

CARTESIAN SYSTEMS

It is a rectangular coordinate system which has the horizontal component in X-axis and vertical component in Y-axis.

Horizontal component of velocity, $V_x = dx/dt$

Vertical component of velocity, $V_y = dy/dt$

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

Angle of inclination of velocity with X-axis, $\alpha=\tan^{-1}(V_y/V_x)$

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)}$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x)$

PROBLEMS

Example1. The portion of a particle along a curved path is given by the equations $x=t^2+8t+4$ and $y=t^3+3t^2+8t+4$. Find the i) initial velocity, u ii) velocity of the particle at t=2 sec iii) acceleration of the particle at t=0 and iv) acceleration of the particle at t= 2 sec.

Given data

$x=t^2+8t+4$

$y=t^3+3t^2+8t+4$

Solution

Horizontal component of velocity , $V_x = dx/dt = d(t^2+8t+4)/dt = 2t+8$ ------------ (1)

Vertical component of velocity, $V_y = dy/dt = d(t^3+3t^2+8t+4)/dt = 3t^2+6t+8$ ------ (2)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = d(2t+8)/dt = 2$ ------------ (3)

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = d(3t^2+6t+8)/dt = 6t+6$ ----- (4)

i) **Initial velocity, u**

Put t = 0 in equation (1) and

(2) $V_x = 2t+8$

Now, $\mathbf{V_x = 8}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 8}$ **m/sec**

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

$= \sqrt{(8^2+8^2)}$

$\mathbf{V = 11.31}$ **m/sec**

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$= \tan^{-1}(8/8)$

$\boldsymbol{\alpha = 45^\circ}$

ii) **Velocity at t= 2 sec**

Put t = 2 seconds in equation (1) and (2)

$V_x = 2t+8$

Now, $\mathbf{V_x = 12}$ **m/sec**

$V_y = 3t^2+6t+8$

Now, $\mathbf{V_y = 32}$ **m/sec**

Therefore, resultant velocity of a particle, $V = \sqrt{(V_x^2+V_y^2)}$

$$= \sqrt{(12^2+32^2)}$$

$$\mathbf{V = 34.17\ m/sec}$$

Angle of inclination of velocity with X-axis, $\alpha = \tan^{-1}(V_y/V_x)$

$$= \tan^{-1}(32/12)$$

$$\boldsymbol{\alpha = 69.4^\circ}$$

iii) **<u>Acceleration at t=0</u>**

Put $t = 0$ in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2\ m/sec^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 6\ m/sec^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)} = \sqrt{(2^2+6^2)} = 6.34\ m/sec^2$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(6/2) = 71.56^\circ$

iv) **<u>Acceleration at t = 2 sec</u>**

Put $t = 2$ sec in equation (3) and (4)

Acceleration of a particle along X-axis, $a_x = d^2x/dt^2 = 2\ m/sec^2$

Acceleration of a particle along Y-axis, $a_y = d^2y/dt^2 = 6t+6 = 18\ m/sec^2$

Resultant acceleration of a particle, $a = \sqrt{(a_x^2+a_y^2)} = \sqrt{(2^2+18^2)} = 18.11\ m/sec^2$

Angle of inclination of acceleration with X-axis, $\varphi = \tan^{-1}(a_y/a_x) = \tan^{-1}(18/2) = 83.66^\circ$

<u>PROJECTILES</u>

The projectile is an example of curvilinear motion of a particle in plane motion. The motion of a particle is neither vertical nor horizontal, but inclined to the horizontal plane.

It is classified under Kinematics since the force which is responsible for motion is left out in the analysis and the rest are considered/

<u>Definitions</u>

<u>Projectile</u> – A particle projected in space at an angle to the horizontal plane.

<u>Angle of projection</u> means the angle to the horizontal at which the projectile is projected. It is denoted by a.

<u>Velocity of projectile</u> means the velocity with which the projectile is thrown into space. It is denoted by u (m/sec)

Trajectory means the path described by the projectile.

Time of flight is the total time taken by the projectile from the instant of projection up to the projectile hits the plane again.

Range is the distance along the plane between the point of projection and the point at which the projectile hits the plane at the end of its journey.

Path of the Projectile

The *horizontal distance* travelled by the projectile in any time t.

X = Velocity * Time taken

Therefore, $X = u \cos \alpha t$

Or

$t = X / u \cos \alpha$

Similarly for *vertical distance*,

$$Y = \tan\alpha\, X - \tfrac{1}{2}\,(g^2 X/u^2 \cos^2\alpha)$$

From the equation of the trajectory, it is clear that the two variables of projectile motion are initial velocity (u) and the angle of projection (a) to arrive standard results of projectile motion.

Time of flight (T) and time taken to reach highest point (t):

$T = 2\,u \sin\alpha / g$

$t = u \sin\alpha / g$

Maximum height attained:

$h_{max} = u^2 \sin^2\alpha / 2g$

Horizontal range:

$R = u^2 \sin 2\alpha / g$

PROBLEMS

Example1. A particle is projected with an initial velocity of 60 m/sec, at an angle of 75° with the horizontal. Determine i) the maximum height attained by the particle ii) horizontal range of the particle iii) time taken by the particle to reach highest point iv) time of flight

Given data

Initial velocity, u = 60 m/sec

Angle of projection, $\alpha = 75°$

Solution

i) the maximum height attained by the particle

$h_{max} = u^2 \sin^2\alpha / 2g = 171.19$ m (take $g = 9.81$ m/sec^2) ii) horizontal range

$R = u^2 \sin 2\alpha / g = 183.48$ m

iii) time taken to reach highest point $t = u \sin\alpha / g = 5.9$ sec

iv) time of flight

$T = 2u \sin\alpha / g = 11.8$ sec

Example2. A particle is projected with an initial velocity of 12 m/sec at an angle α with the horizontal. After sometime the position of the particle is observed by its x and y distances of 6m and 4 m respectively from the point of projection. Find the angle of projection?

Given data

Initial velocity, u = 12 m/sec

Horizontal distance, x = 6 m

Vertical distance, y = 4 m

Solution

If the coordinate points on the projectile path are given, then use equation of trajectory. Equation of path of projectile (trajectory)

$Y = \tan\alpha\, X - ½ (g^2 X/u^2 \cos^2\alpha)$

Put u = 12 m/sec, X = 6 m and Y = 4 m

Take $g = 9.81$ m/sec^2

We get,

$4 = 6 \tan \alpha - (1.226/ \cos^2 \alpha)$

$1.226 \tan^2 \alpha - 6 \tan \alpha + 5.226 = 0$

Therefore, $\tan \alpha = - 6 \pm \sqrt{(6^2 - (4*1.226*5.226)}/(2*1.226)$

$\alpha = 75.1^\circ$ or 53.06°

Important definitions on kinetics

a) Mass - a fundamental measure of the amount of matter in the object. It is denoted by 'm'. The SI unit of mass is Kilograms (Kg). It's a scalar quantity.

b) Weight - The weight of an object is defined as the force of gravity on the object and may

be calculated as the mass times the acceleration of gravity, w = mg. Since the weight is a force, its SI unit is the Newton.

Weight = mass * acceleration due to gravity

$$\boxed{W = mg}$$

c) Momentum - Momentum can be defined as "mass in motion." All objects have mass; so if an object is moving, then it has momentum - it has its mass in motion. It depends upon the variables mass and velocity. In terms of an equation, the momentum of an object is equal to the mass of the object times the velocity of the object. Its SI unit is kg.m/sec^2

Momentum = mass • velocity

$$\boxed{M = mv}$$

LAWS OF MOTION

When a particle / body is at rest, or moving in a straight line (rectilinear motion) or in a curved line (curvilinear motion), the particle / body obeys certain laws of motion. These laws are called Newton's law of motion. These laws are also called the principles of motion, or principles of Dynamics.

First Law

Every body continues to be in its state of rest or of uniform motion in a straight line unless and until it is acted upon some external force to change that state. It is also called ***the law of inertia***, and consists of the following two parts:

1. A body at rest continues in the same state, unless acted upon by some external force. It appears to be self-evident, as a train at rest on a level track will not move unless pulled by an engine. Similarly, a book lying on a table remains at rest, unless it is lifted or pushed.
2. A body moving with a uniform velocity continues its state of uniform motion in a straight line, unless it is compelled by some external force to change its state. It cannot be exemplified because it is, practically, impossible to get rid of the forces acting on a body.

Second Law

The rate of change of momentum of a moving body is directly proportional to the impressed force and takes place in the direction of the force applied.

The change of momentum = final momentum – initial momentum

$$= mv - mu = m(v - u)$$

The rate of change of momentum = change of momentum / time taken

$$= m(v - u) / t = m^*a \text{ (since } (v - u) / t = a)$$

Basically, to increase the velocity of the moving body from u to v, there must be some external force to cause this change. Let that external force be 'F'.

As per the law, the external force 'F' is directly proportional to the rate of change of momentum i.e., $F \propto ma \rightarrow F = k * ma$ where, k is the constant of proportionality.

But for a moving body, k and m are constants, and hence it states that, the force acting on the body is directly proportional to the acceleration of the body. From this we can conclude that,

1. For a given body, greater force produces greater acceleration and the lesser force produces the lesser acceleration.
2. The acceleration is zero, if there is no external force on the body which results in u = v.

To find the value of constant 'k' in equation F = k*ma We know that, $1\ N = 1\ kg * 1\ m/sec^2$

That is, the unit force (N) is a force, which produce unit acceleration (1 m/sec^2) on an unit mass (1 kg) hence, by substituting F = 1; m = 1 and a = 1. We get

$$\boxed{F = ma}$$

Example1. A body of mass 4 kg is moving with a velocity of 2 m/sec and when certain force is applied, it attains a velocity of 8 m/sec in 6 seconds?

Given data

Mass, m = 4 kg

Initial velocity, u = 2

m/sec Final velocity, v = 8

m/sec Time, t = 6 sec

Solution

Acceleration, a = v - u / t = 8-2/6 = 1 m/sec^2

Let, 'P' be the force applied to cause this acceleration.

P = ma = 4*1 = 4 N

Example2. A body of mass 4 kg is at rest. What force should be applied to move it to a distance f 2 m in 4 seconds?

Given data

Mass, m = 4 kg

Distance, s = 12 m

Time taken, t = 4 sec

Initial velocity, u = 0

Solution

Using the equation, $s = ut + \frac{1}{2} at^2$

$12 = 0 + 8a$

Therefore, a = 12/8 m/sec^2

The force required to move, P = m*a = 4*(12/8) = 6 N

Therefore, P = 6 N

4. D'ALEMBERT'S PRINCIPLE

It states, "If a rigid body is acted upon by a system of forces, this system may be reduced to a single resultant force whose magnitude, direction and the line of action may be found out by the methods of graphic statics."

We know that, that force acting on a body.

$$P = ma \text{ ------- (i)}$$

Where, m = mass of the body, and

a = Acceleration of the body.

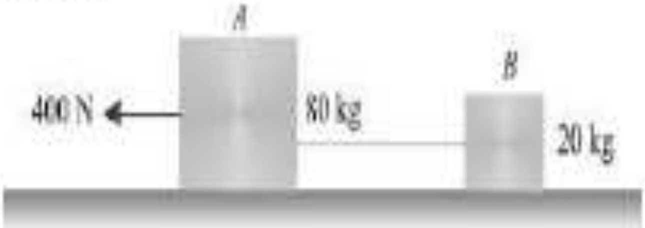
The equation (i) may also be written as:

$$P - ma = 0 \text{ -----(ii)}$$

It may be noted that equation (i) is the equation of dynamics whereas the equation (ii) is the equation of statics. The equation (ii) is also known as the equation of dynamic equilibrium under the action of the real force P. This principle is known as D' Alembert's principle.

PROBLEMS

Example1. Two bodies A and B of mass 80 kg and 20 kg are connected by a thread and move along a rough horizontal plane under the action of a force 400 N applied to the first body of mass 80 kg as shown in Figure. The coefficient of friction between the sliding surfaces of the bodies and the plane is 0.3 Determine the acceleration of the two bodies and the tension in the thread, using D' Alembert's principle.



Given data

Mass of body *A* (m_1) = 80 kg

Mass of the body *B* (m_2) = 20 kg

Force applied on first body (*P*) = 400 N and

Coefficient of friction (μ) = 0.3

Solution

Let a = Acceleration of the bodies, and

T = Tension in the thread.

(*a*) *Body A* (*b*) *Body B*

Consider the body A. The forces acting on it are: 400 N forces (acting towards left)

Mass of the body = 80 kg (acting downwards)

Reaction $R_1 = 80 \times 9.8 = 784$ N (acting upwards)

Force of friction, $F_1 = \mu R_1 = 0.3 \times 784 = 235.2$ N (acting towards right) Tension in the thread = T (acting towards right).

$\therefore$ Resultant horizontal force, $P_1 = 400 - T - F_1 = 400 - T - 235.2 = 164.8 - T$ (acting towards left)

We know that force causing acceleration to the body $A; \rightarrow m_1 a = 80\,a$

And according to D'Alembert's principle $P_1 - m_1 a = 0 \rightarrow 164.8 - T - 80\,a = 0$

$\therefore T = 164.8 - 80a$ ....(i)

Now consider the body B. The forces acting on it are:

Tension in the thread = T (acting towards left)

Mass of the body = 20 kg (acting downwards)

Reaction $R_2 = 20 \times 9.8 = 196$ N (acting upwards)

Force of friction, $F_2 = \mu R_2 = 0.3 \times 196 = 58.8$ N (acting towards right)

$\therefore$ Resultant horizontal force, $P_2 = T - F_2 = T - 58.8$

We know that force causing acceleration to the body $B \rightarrow m_2 a = 20\,a$

And according to D'Alembert's principle $P2 - m2a = 0 \rightarrow (T - 58.8) - 20\,a = 0$

$\therefore T = 58.8 + 20a$ ....(ii)

Now equating the two values of T from equation (i) and (ii),

$$164.8 - 80\,a = 58.8 + 20\,a$$

$$100\,a = 106$$

$$a = 106/100$$

$\therefore a = 1.06\ m/s^2$

Tension in the thread

Substituting the value of a in equation (ii)

$$T = 58.8 + (20 \times 1.06)$$

<u>*Third Law*</u>

To every action, there is always an equal and opposite reaction

This law appears to be self-evident as when a bullet is fired from a gun, the bullet moves out with a great velocity, and the reaction of the bullet, in the opposite direction, gives an unpleasant shock to the man holding the gun. Similarly, when a swimmer tries to swim, he pushes the water backwards and the reaction of the water pushes the swimmer forward.

Example: When a bullet is fired from a gun, the opposite reaction of the bullet is known as the recoil of gun.

Let M = Mass of the gun,

V = Velocity of the gun with which it recoils,

m = mass of the bullet, and

v = Velocity of the bullet after explosion.

∴ Momentum of the bullet after explosion = mv ---- (i)

Momentum of the gun = MV ------------------------ (ii)

Equating the equations (i) and (ii), MV = mv

This relation is popularly known as *Law of Conservation of Momentum.*

PROBLEMS

Example1. A machine gun of mass 25 kg fires a bullet of mass 30 gram with a velocity of 250 m/s. Find the velocity with which the machine gun will recoil?

Given data

Mass of the machine gun (M) = 25 kg

Mass of the bullet (m) = 30 g = 0.03kg and

Velocity of firing (v) = 250 m/s.

Solution

Let V = Velocity with which the machine gun will recoil.

We know that $MV = mv$

$2.5 \times v = 0.03 \times 250 = 7.5 \rightarrow v = 7.5 / 25$

Example2. A bullet of mass 20 g is fired horizontally with a velocity of 300 m/s, from a gun carried in a carriage; which together with the gun has mass of 100 kg. The resistance to sliding of the carriage over the ice on which it rests is 20 N. Find (a) velocity with which the gun will recoil, (b) distance, in which it comes to rest, and (c) time taken to do so.

Given data

Mass of the bullet (m) = 20 g = 0.02 kg

Velocity of bullet (v) = 300 m/s

Mass of the carriage with gun (M) = 100 kg and

Resistance to sliding (F) = 20 N

Solution

(a) Velocity, with which the gun will recoil

Let V = velocity with which the gun will recoil.

We know that $MV = mv$

$100 \times V = 0.02 \times 300 = 6 \rightarrow V = 6 / 100 = 0.06$ m/s *(b) Distance, in which the gun comes to rest*

Now consider motion of the gun. In this case, initial velocity (u) = 0.06 m/s and final velocity, $v = 0$ (because it comes to rest)

Let a = Retardation of the gun, and

s = Distance in which the gun comes to rest.

We know that resisting force to sliding of carriage (F)

$$20 = Ma = 100\,a \rightarrow a = 20 / 100$$

We also know that $v^2 = u^2 - 2as$ (Minus sign due to retardation) $0 = (0.06)2 - 2 \times 0.2\,s$

$$= 0.0036 - 0.4\,s \rightarrow s = 0.0036 / 0.4 = 0.009 \text{ m or } 9 \text{ mm}$$

(c) *Time taken by the gun in coming to rest*

Let t = Time taken by the gun in coming to rest.

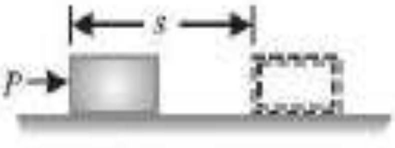
We know that final velocity of the gun (v)

$0 = u + at = 0.06 - 0.2\,t$ (Minus sign due to retardation)
$t = 0.06 / 0.2$

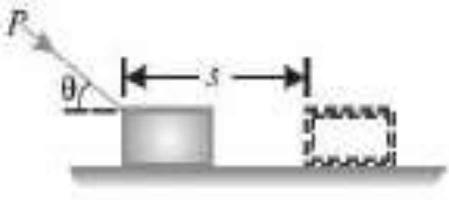
WORK ENERGY EQUATION

Work

Whenever a force acts on a body, and the body undergoes some displacement, then work is said to be done. *e.g.*, if a force P, acting on a body, causes it to move through a distance s as shown in Figure (a).



(a) Body moving in the direction of force



(b) Body not moving in the direction of force

Then work done by the force P = Force × Distance = $P \times s$

Work done by the force = P*S

Sometimes, the force P does not act in the direction of motion of the body, or in other words, the body does not move in the direction of the force as shown in Figure (*b*).

Then work done by the force P = Component of the force in the direction of motion × Distance

$$= P \cos\theta \times s$$

Work done by the force = P cos θ*S

In SI system of units, force is in Newton and the distance is in meters.

PROBLEMS

Example1.A horse pulling a cart exerts a steady horizontal pull of 300 N and walks at the rate of 4·5 kmph. How much work is done by the horse in 5 minutes?

Given data

Pull (*i.e.* force) = 300 N

Velocity (v) = 4·5 kmph. = 75 m/ min

and Time, t = 5 min.

Solution

We know that distance travelled in 5 minutes
$s = 75 \times 5 = 375$ m → ∴ $s = 375$ m

Work done by the horse, W = Force × Distance

$$= 300 \times 375 = 112\,500 \text{ N-m} = 112.5 \text{ kN-m}$$

∴ $W = 112.5$ kJ

Example2. A spring is stretched by 50 mm by the application of a force. Find the work done, if the force required to stretch 1 mm of the spring is 10 N.

Given data

Spring stretched by the application of force (s) = 50 mm Stretching of spring = 1 mm and force = 10 N

Solution

We know that force required stretching the spring by 50 mm = 10 × 50 = 500 N

∴ **Average force = 500 / 2 = 250 N**

Work done = Average **force** × Distance = 250 × 50 = 12 500 N-mm = 12.5 N-m

∴ **Work done = 12.5 J**

Power

The power may be defined as the rate of doing work.

∴ Power = work done / time

= **(Force * Distance) / Time**

Or

∴ Power = Force * (Distance/Time)

= **Force * Velocity**

In SI systems of units, unit of work is Newton metre, and the unit of time is seconds. Unit of power = Nm / Seconds = 1 watt

∴ **In SI systems, unit of power is watt**

Energy

The energy may be defined as the capacity to do work. It exists in many forms i.e., mechanical, electrical chemical, heat, light etc. the energy is the capacity to do work. Since the energy of a body is measured by the work it can do, therefore the units of energy will be the same as those of the work. Therefore, the SI system of unit of work is joule.

In the study of mechanics, we are concerned only with mechanical energy. Mechanical energy is classified into two types.

1. Potential energy. 2. Kinetic energy.

Potential energy

It is the energy possessed by a body, for doing work, by virtue of its position.

Example1. A body, raised to some height above the ground level, possesses some potential energy; because it can do some work by falling on the earth's surface.

Example2. Compressed air also possesses potential energy; because it can do some work in expanding, to the volume it would occupy at atmospheric pressure.

Example3. A compressed spring also possesses potential energy; because it can do some work in recovering to its original shape.

Now consider a body of mass (m) raise through a height (h) above the datum level. We know that work done in raising the body = Weight × Distance = (mg) h = mgh

Potential Energy, P.E = mg *h

PROBLEM

Example1. A man of mass 60 kg dives vertically downwards into a swimming pool from a tower of height 20 m. He was found to go down in water by 2 m and then started rising. Find the average resistance of the water. Neglect the air resistance.

Given data

Mass of the man (m) = 60 kg and

Height of the tower (h) = 20 m

Solution

Let P = Average resistance of the water

We know that potential energy of the man before jumping

P.E = mg*h = 60 × 9.8 × 20 = 11 760 N-m ------------- (i)

Work done by the average resistance of water = Average resistance of water × Depth of water

= P × 2 = 2 P N-m ------ (*ii*)

Since the total potential energy of the man is used in the work done by the water, therefore equating equations (*i*) and (*ii*),

→ 11 760 = 2 P → P = 11760 / 2 ∴ **P = 5880 N**

Kinetic energy

It is the energy, possessed by a body, for doing work by virtue of its mass and velocity of motion. Now consider a body, which has been brought to rest by a uniform retardation due to the applied force.

Let m = Mass of the body

u = Initial velocity of the body

P = Force applied on the body to bring it to rest, a = Constant retardation, and

s = Distance travelled by the body before coming to rest.

Since the body is brought to rest, therefore its final velocity, $v = 0$

and Work done, W = Force × Distance = $P \times s$ ---------- (*i*)

Now substituting value of ($P = m.a$) in equation (*i*),

$W = ma \times s = mas$ ---------- (*ii*)

We know that $v^2 = u^2 - 2\,as$ (Minus sign due to retardation)

Now substituting the value of ($a.s$) in equation (*ii*) and replacing work done with kinetic energy,

$$K.E = mu^2/2$$

In most of the cases, the initial velocity is taken as v (instead of u), therefore kinetic energy,

$$K.E = mv^2/2$$

Kinetic Energy, K.E = ½ (mv²)

PROBLEM

Example1. A truck of mass 15 tones travelling at 1.6 m/s impacts with a buffer spring, which compresses 1.25 mm per kN. Find the maximum compression of the spring?

Given data

Mass of the truck (m) = 15 t

Velocity of the truck (v) = 1.6 m/s and

Buffer spring constant (k) = 1.25 mm/ kN

Solution

Let x = Maximum compression of the spring in mm.

We know that kinetic energy of the truck = $mv^2/2 = (15*1.6^2) / 2 = 19.2 = 19200$ kN - mm

Kinetic Energy, K.E = 19200 kN-mm -------------- (i)

Compressive load = x / 1.25 = 0.8 x kN

Work done in compressing the spring = Average compressive load × Displacement =

$(0.8 x / 2) * x = 0.4 x^2$ ---------------- (ii)

Since the entire kinetic energy of the truck is used to compress the spring therefore equating equations (i) and (ii),

$$19\,200 = 0.4 x^2 \rightarrow x^2 = 19200 / 0.4$$
$$= 48000$$
$$\therefore \mathbf{x = 219\ mm}$$

<u>Work Energy Equation</u>

The equation of motion in one-dimension (taking the variable to be x, and the force to be F) is

$$m\frac{d^2x}{dt^2} = F(x)$$

Let us again eliminate time from the left-hand using the technique used above

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\frac{dx}{dt} = \frac{dx}{dt}\frac{d}{dx}\frac{dx}{dt} = v\frac{dv}{dx}$$

To get

$$mv\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}mv^2\right) = F(x)$$

On integration this equation gives

$$\frac{1}{2}mv_1^2 - \frac{1}{2}mv_0^2 = \int F(x)\,dx$$

where x_i and x_f refer to the initial and final positions, and v_i and v_f to the initial and final velocities, respectively. We now interpret this result. We define the kinetic energy of a particle of mass m and velocity v to be

$$\text{Kinetic energy} = \frac{1}{2}mv^2$$

and the work done in moving from one position to the other as the integral given above

$$\text{Work done} = \int_{x_i}^{x_f} F\,dx$$

With these definitions the equation derived above tells us that work done on a particle changes its kinetic energy by an equal amount; this known as the *work-energy theorem*.

IMPULSE AND MOMENTUM

Impulse

The impulse of a constant force F is defined as the product of the force and the time t for which it acts. The SI unit of linear impulse is ***N.sec***

$$\text{Impulse} = Ft \text{ ----------- (i)}$$

The effect of the impulse on a body can be found using equation (i) where, a is acceleration, u and v are initial and final velocities respectively and t is time.

$$v = u + at$$

So

$$mat = m(v - u)$$
$$F = ma$$
$$Ft = m(v - u) = \text{change in momentum} \text{ ----------- (ii)}$$

So we can say that

$$\text{Impulse of a constant force} = Ft = \text{change in momentum produced}$$

Impulse is a vector quantity and has the sane units as momentum, Ns or kg m/s. The impulse of a variable force can be defined by the integral

$$\text{Impulse} = \int_0^t F\,dt$$

Where, t is the time for which F acts.

By Newton's 2^{nd} law

$$F = ma = m\frac{dv}{dt}$$

So impulse can also be written

$$\text{Impulse} = \int_0^t m\frac{dv}{dt}dt$$

$$= \int_u^v m\,dv$$

$$= \left[mv\right]_u^v$$

Which for a constant mass

$$\text{Impulse} = m(v-u)$$

In summary

$$\text{Impulse} = \int_0^t F\,dt = \text{change in momentum produced} \quad \text{---------- (iii)}$$

Impulsive force

Suppose the force F is very large and acts for a very short time. During this time the distanc moved is very small and under normal analysis would be ignored. Under these condition the onl effect of the force can be measured is the impulse, or change I momentum - the force is called a impulsive force.

In theory this force should be infinitely large and the time of action infinitely small. Som applications where the conditions are approached are collision of snooker balls, a hammer hittin a nail or the impact of a bullet on a target.

PROBLEMS

Example1. A nail of mass 0.02 kg is driven into a fixed wooden block, its initial speed is 30 m/s and it is brought to rest in 5ms. Find a) the impulse b) value of the force (assume this constant) on the nail.

Given data

Mass, m = 0.02 kg

Velocity, v = 30 m/sec

Initial velocity, u = 0

Time, t = 5 minutes

Solution

Using the equation,

$$\begin{aligned} \text{Impulse} &= \text{change in momentum of the nail} \\ &= 0.02(30-0) \\ &= 0.6\ Ns \end{aligned}$$

$$\text{Impulse} = Ft$$

$$F = \frac{\text{Impulse}}{t} = \frac{0.6}{0.005} = 120\ N$$

Momentum

The quantity of motion possessed by the moving body is called momentum. It is the product of mass and velocity.

Momentum = Mass * Velocity

i.e., M = mv

Where, m is mass I kilogram

v = velocity in m/sec

M = Momentum in kg.m/sec

→ mv = (w/g) * v

The SI unit of momentum is also ***N.sec***

Impulse – Momentum equation

The impulse – Momentum equation is also derived from the Newton's secong law,

F = ma = m * (dv/dt) i.e., F dt = m dv

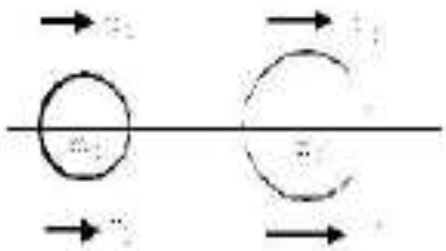
As derived in the impulse, the term $\int_0^t F\,dt$ is called impulse and m (v-u) is called the change of momentum, i.e., Final momentum – Initial momentum.

Impulse = m (v-u) = (W/g) * (v-u)

Impact of elastic bodies

In the last section the bodies were assumed to stay together after impact. An elastic body is one which tends to return to its original shape after impact. When two elastic bodies collide, they rebound after collision. An example is the collision of two snooker balls.

If the bodies are travelling along the same straight line before impact, then the collision is called a direct collision. This is the only type of collision considered here.



Direct collision of two elastic spheres

Consider the two elastic spheres as shown in figure. By the principle of conservation of linear momentum

$$\text{Momentum befor impact} = \text{Momentum after impact}$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Where the *u*'s are the velocities before collision and the *v*'s, the velocities after.

When the spheres are inelastic v_1 and v_2 are equal as we saw in the last section. For elastic bodies v_1 and v_2 depend on the elastic properties of the bodies. A measure of the elasticity is the **coefficient of restitution *e***, for direct collision this is defined as

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$

This equation is the result of experiments performed by Newton. The values of e in practice vary from between 0 and 1. For inelastic bodies $e = 0$, for completely elastic $e = 1$. In this latter case no energy is lost in the collision.

PROBLEMS

Example1. A body of mass 2kg moving with speed 5m/s collides directly with another of mass 3 kg moving in the same direction. The coefficient of restitution is 2/3. Find the velocities after collision.

Solution

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$2\times5 + 3\times4 = 2v_1 + 3v_2$$
$$22 = 2v_1 + 3v_2 \qquad \text{--------- (i)}$$

From equation

$$e = -\left(\frac{v_1 - v_2}{u_1 - u_2}\right)$$
$$\frac{2}{3} = -\left(\frac{v_1 - v_2}{5 - 4}\right)$$
$$-2 = 3v_1 - 3v_2 \qquad \text{-------------- (ii)}$$

Adding [i] and [ii] gives

$$20 = 5v_1$$
$$v_1 = 4\ m/s$$

And by [i]

$$22 = 8 + 3v_2$$
$$v_2 = \frac{14}{3}\ m/s$$

Example2. A railway wagon has mass 15 tones and is moving at 1.0 m/s. It collides with a second wagon of mass 20 tones moving in the opposite direction at 0.5m/s. After the collision the second wagon has changed its speed to 0.4m/s in the opposite direction as before the collision. Find i) the velocity of the 15 tones wagon after the collision ii) the coefficient of restitution and iii) the loss in kinetic energy.

Solution

$$\text{Momentum befor impact} = \text{Momentum after impact}$$
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$15000\times1.0 - 20000\times0.5 = 15000v_1 + 20000\times0.4$$
$$-3000 = 15000v_1$$
$$v_1 = -0.2\,m/s$$

The negative sign means it has change direction of travel.

Coefficient of restitution is

$$e = -\left(\frac{v_2 - v_2}{u_2 - u_2}\right)$$
$$e = -\left(\frac{(-0.2) - 0.4}{1.0 - (-0.5)}\right)$$
$$e = 0.4$$

$$\text{kinetic energy before impact} = \frac{1}{2}15000\times1.0^2 + \frac{1}{2}20000\times0.5^2$$
$$= 10000\,J$$

$$\text{kinetic energy after impact} = \frac{1}{2}15000\times0.2^2 + \frac{1}{2}20000\times0.4^2$$
$$= 1900\,J$$

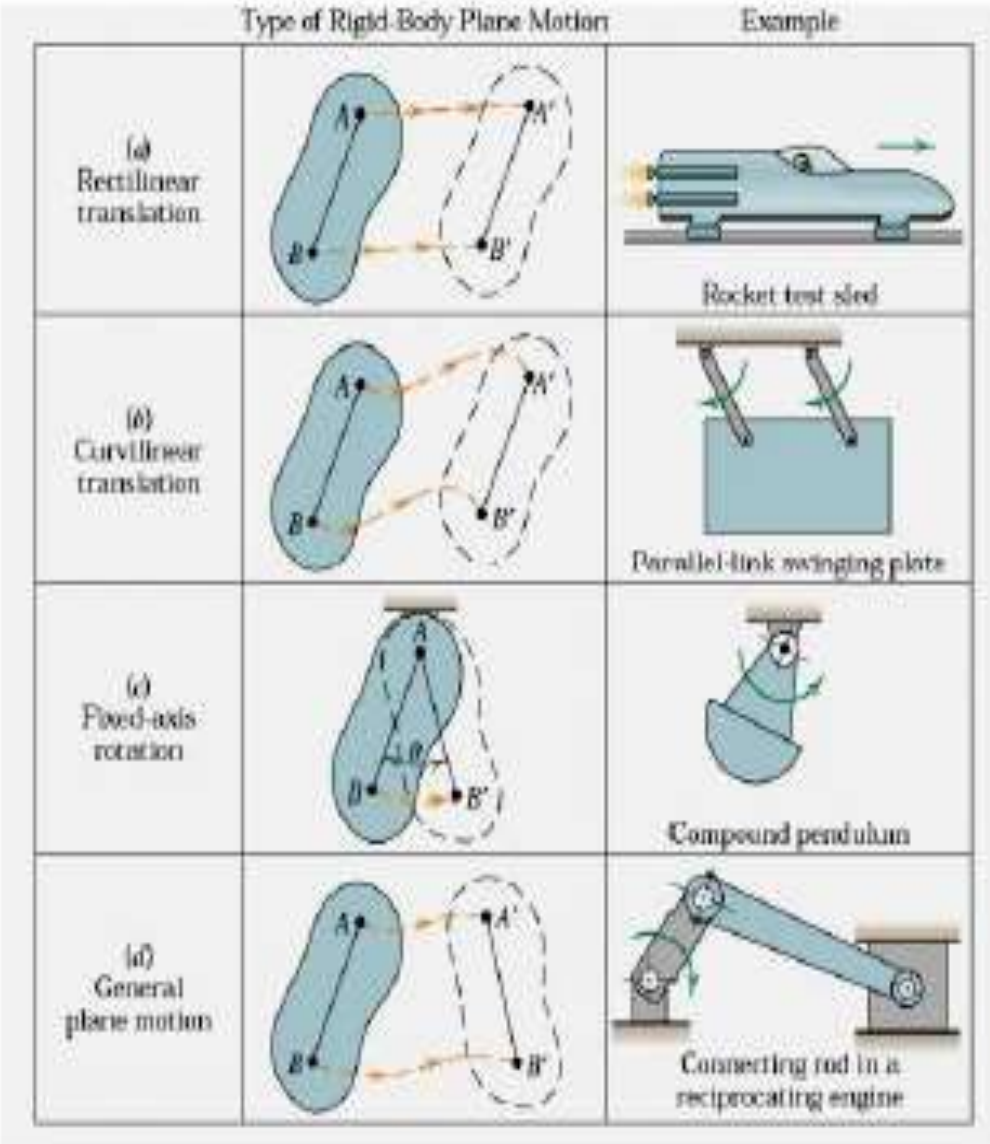
$$\text{loss of kinetic energy} = 10000 - 1900 = 8100\,J$$

Translation, Rotation of rigid bodies and General plane motion

Introduction

Forces acting of rigid bodies can be also separated in two groups: (a) The *external forces* represent the action of other bodies on the rigid body under consideration; (b) The *internal forces* are the forces which hold together the particles forming the rigid body. Only external forces can impart to the rigid body a motion of translation or rotation or both.

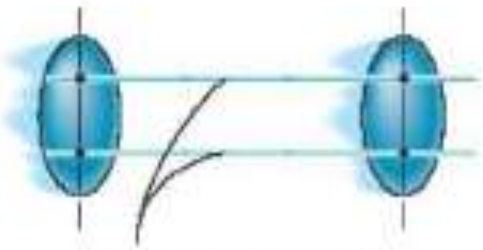
In kinematics the types of motion are *TRANSLATION, ROTATION* about a fixed axis and *GENERAL PLANE MOTION*.



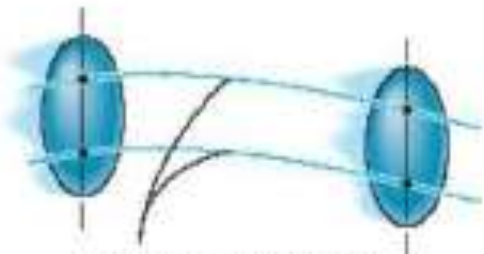
TRANSLATION

A motion is said to be a translation if any straight line inside the body keeps the same direction during the movement. It occurs if every line segment on the body remains parallel to its original direction during the motion

All the particles forming the body move along parallel paths. If these paths are straight lines, the motion is said a ***rectilinear translation***; if the paths are curved lines, the motion is a ***curvilinear motion*** as given below in figure.



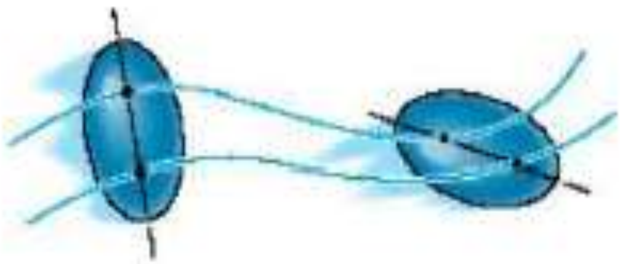
Path of rectilinear translation



Path of curvilinear translation

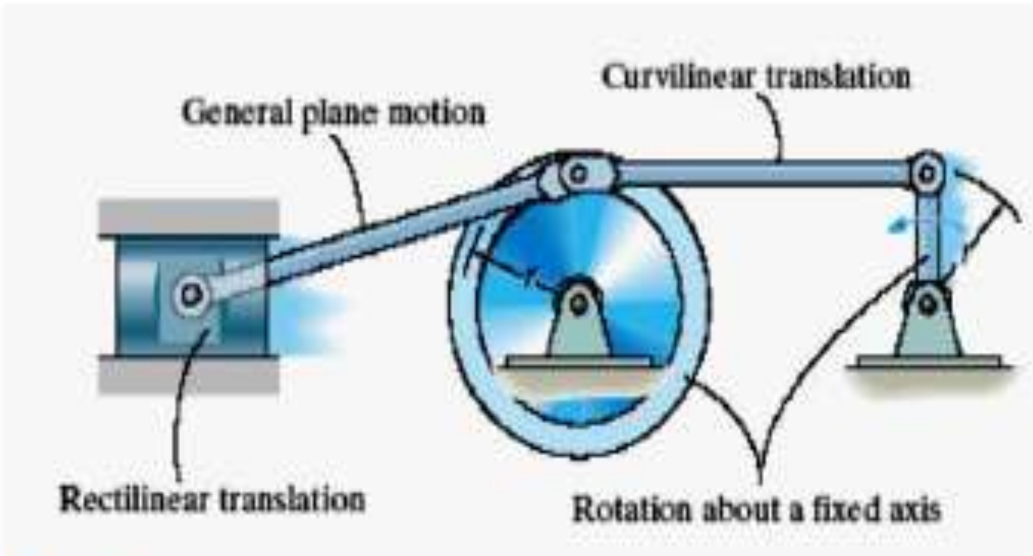
GENERAL PLANE MOTION

Any plane motion which is neither a translation nor a rotation is referred as a general plane motion. Plan motion is that in which all the particles of the body move in parallel planes. Translation occurs within a plane and rotation occurs about an axis perpendicular to this plane.



General plane motion

An example of bodies undergoing the three types of motion is shown in this mechanism. The wheel and crank undergo rotation about a fixed axis. In this case, both axes of rotation are at the location of the pins and perpendicular to the plane of the figure. The piston undergoes rectilinear translation since it is constrained to slide in a straight line. The connecting rod undergoes curvilinear translation, since it will remain horizontal as it moves along a circular path. The connecting rod undergoes general plane motion, as it will both translate and rotate.



ROTATION

Some bodies like pulley, shafts, and flywheels have motion of rotation (*i.e.*, angular motion) which takes place about the geometric axis of the body. The angular velocity of a body is always expressed in terms of revolutions described in one minute, *e.g.*, if at an instant the angular velocity of rotating body in *N* r.p.m. (*i.e.* revolutions per min) the corresponding angular velocity ω (in rad) may be found out as discussed below:

1 revolution/min = 2π rad/min

[illegible]

$$\omega = 2\pi N / 60 \text{ rad/sec}$$

Important Terms

The following terms, which will be frequently used in this chapter, should be clearly understood at this stage:

Angular velocity - It is the rate of change of angular displacement of a body, and is expressed in r.p.m. (revolutions per minute) or in radian per second. It is, usually, denoted by ω (omega).

Angular acceleration - It is the rate of change of angular velocity and is expressed in radian per second per second (rad/s^2) and is usually, denoted by α. It may be constant or variable.

Angular displacement - It is the total angle, through which a body has rotated, and is usually denoted by θ. If a body is rotating with a uniform angular velocity (ω) then in t seconds, the angular displacement is $\theta = \omega * t$

Motion of rotation under constant angular acceleration

Consider a particle, rotating about its axis.

Let ω_0 = Initial angular velocity,

ω = Final angular velocity,

t = Time (in seconds) taken by the particle to change its velocity from ω_0 to ω. α = Constant angular acceleration in rad/s^2, and

θ =Total angular displacement in radians.

Since in t seconds, the angular velocity of the particle has increased steadily from ω_0 to ω at the rate of α rad/s^2, therefore

$$\omega = \omega_0 + \alpha t \qquad ...(i)$$

and average angular velocity $= \dfrac{\omega_0 + \omega}{2}$

We know that the total angular displacement,

$$\theta = \text{Average velocity} \times \text{Time} = \left(\frac{\omega_0 + \omega}{2}\right) \times t \qquad ...(ii)$$

Substituting the value of ω from equation (i),

$$\theta = \frac{\omega_0 + (\omega_0 + \alpha t)}{2} \times t = \frac{2\omega_0 + \alpha t}{2} \times t = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad ...(iii)$$

and from equation (i), we find that

$$t = \frac{\omega - \omega_0}{\alpha}$$

Substituting this value of t in equation (ii),

$$\theta = \left(\frac{\omega_0 + \omega}{2}\right) \times \left(\frac{\omega - \omega_0}{\alpha}\right) = \frac{\omega^2 - \omega_0^2}{2\alpha}$$

$$\therefore \qquad \omega^2 = \omega_0^2 + 2\alpha\theta \qquad ...(iv)$$

Relation between linear motion and angular motion

S. No	Particulars	Linear motion	Angular motion
1.	Initial velocity	u	ω_0
2.	Final velocity	v	ω
3.	Constant acceleration	a	α
4.	Total distance traversed	s	θ
5.	Formula for final velocity	$v = u + at$	$\omega = \omega_0 + \alpha t$
6.	Formula for distance traversed	$s = ut + \frac{1}{2}at^2$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
7.	Formula for final velocity	$v^2 = u^2 + 2as$	$\omega^2 = \omega_0^2 + 2\alpha\theta$
8.	Differential formula for velocity	$v = \frac{ds}{dt}$	$\omega = \frac{d\theta}{dt}$
9.	Differential formula for acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

PROBLEMS

Example1. A flywheel starts from rest and revolves with an acceleration of 0.5 rad/ sec^2. What will be its angular velocity and angular displacement after 10 seconds?

Given data

Initial angular velocity ($\omega 0$) = 0 (becasue it starts from rest)

Angular acceleration (α) = 0.5 rad/sec^2 and

Time (t) = 10 sec.

Solution

Angular velocity of the flywheel

We know that angular velocity of the flywheel,

$\omega = \omega 0 + \alpha t = 0 + (0.5 \times 10) = 5$ rad/sec

Angular displacement of the flywheel

We also know that angular displacement of the flywheel,

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (0 \times 10) + \left[\frac{1}{2} \times 0.5 \times (10)^2\right] = 25 \text{ rad}$$

Example2. A wheel rotates for 5 seconds with a constant angular acceleration and describes during this time 100 radians. It then rotates with a constant angular velocity and during the next five seconds describes 80 radians. Find the initial angular velocity and the angular acceleration.

Given data

Time (t) = 5 sec and

Angular displacement (θ) = 100 rad

Solution

Initial angular velocity

Let ω_0 = Initial angular velocity in rad/s,

α = Angular acceleration in rad/s^2, and

ω = Angular velocity after 5 s in rad/s.

First of all, consider the angular motion of the wheel with constant acceleration for 5 seconds. We know that angular displacement (θ),

$$100 = \omega_0 t + \frac{1}{2}\alpha t^2 = \omega_0 \times 5 + \frac{1}{2} \times \alpha (5)^2 = 5\omega_0 + 12.5\alpha$$

$\therefore$ $\quad 40 = 2\omega_0 + 5\alpha \quad$ ...(*i*)

and final velocity, $\quad \omega = \omega_0 + \alpha t = \omega_0 + \alpha \times 5 = \omega_0 + 5\alpha$

Now consider the angular motion of the wheel with a constant angular velocity of $(\omega_0 + 5\alpha)$ for 5 seconds and describe 80 radians. We know that the angular displacement,

$$80 = 5(\omega_0 + 5\alpha)$$

or $\quad 16 = \omega_0 + 5\alpha \quad$ ...(*ii*)

Subtracting equation (*ii*) from (*i*),

$$24 = \omega_0 \quad \text{or} \quad \omega_0 = 24 \text{ rad/s} \quad \textbf{Ans.}$$

Angular acceleration

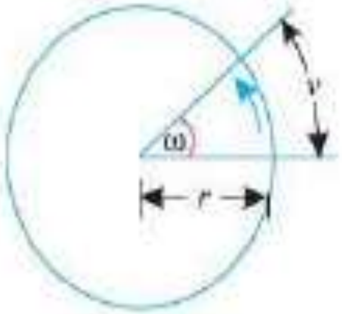
Substituting this value of ω_0 in equation (*ii*),

$$16 = 24 + 5\alpha \quad \text{or} \quad \alpha = \frac{16 - 24}{5} = -1.6 \text{ rad/s}^2 \quad \textbf{Ans.}$$

...(Minus sign means retardation)

Linear (Or Tangential) Velocity of a Rotating Body

Consider a body **rotating about** its **axis** as shown in Figure.



Let ω = Angular velocity of the body in rad/s,

r = Radius of the circular path in meters, and

v = Linear velocity of the particle on the periphery in m/s.

After one second, the particle will move v meters along the circular path and the angular displacement will be ω rad.

We know that length of arc = Radius of arc × Angle subtended in rad.

PROBLEMS

Example1. A wheel of 1.2 m diameter starts from rest and is accelerated at the rate of 0.8 rad/s2 . Find the linear velocity of a point on its periphery after 5 seconds.

Given data

Diameter of wheel = 1.2 m or radius (r) = 0.6 m

Initial angular velocity ($\omega 0$) = 0 (becasue, it starts from rest)

Angular acceleration (α) = 0.8 rad/s2 and

Time (t) = 5 s

Solution

We know that angular velocity of the wheel after 5 seconds,

$\omega = \omega 0 + \alpha t = 0 + (0.8 \times 5) = 4$ rad/s

∴ Linear velocity of the point on the periphery of the wheel, $v = r\omega = 0.6 \times 4 = 2.4$ m/s

Example2. A pulley 2 m in diameter is keyed to a shaft which makes 240 r.p.m. Find the linear velocity of a particle on the periphery of the pulley.

Given data

Diameter of pulley = 2 m or radius (r) = 1 m and

Angular frequency (N) = 240 r.p.m.

Solution

We know that angular velocity of the pulley,

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60} = 25.1 \text{ rad/s}$$

$\therefore$ Linear velocity of the particle on the periphery of the pulley,

$$v = r\omega = 1 \times 25.1 = 25.1 \text{ m/s}$$

Linear (Or Tangential) Acceleration of a Rotating Body

Consider a body rotating about its axis with a constant angular (as well as linear) acceleration. We know that linear acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}(v) \qquad ...(i)$$

We also know that in motion of rotation, the linear velocity,

$$v = r\omega$$

Now substituting the value of v in equation (i),

$$a = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha$$

Where α = Angular acceleration in rad/sec^2 and is equal to $d\omega/dt$.

PROBLEMS

Exapmle1. A car is moving at 72 kmph. If the wheels are 75 cm diameter, find the angular velocity of the tyre about its axis. If the car comes to rest in a distance of 20 meters, under a uniform retardation, find angular retardation of the wheels.

Given data

Linear velocity (v) = 72 kmph = 20 m/s

Diameter of wheel (d) = 75 cm or radius (r) = 0.375 m and

Distance travelled by the car (s) = 20 m.

Solution

Angular retardation of the wheel

We know that the angular velocity of the wheel,

$$\omega = \frac{v}{r} = \frac{20}{0.375} = 53.3 \text{ rad/sec}$$

Let $\quad a$ = Linear retardation of the wheel.

We know that $\quad v^2 = u^2 + 2as$

$\therefore \quad 0 = (20)^2 + 2 \times a \times 20 = 400 + 40a$

or $\quad a = -\frac{400}{40} = -10 \text{ m/sec}^2$ ...(Minus sign indicates retardation)

We also know that the angular retardation of the wheel,

$$\alpha = \frac{a}{r} = \frac{-10}{0.375} = -26.7 \text{ rad/sec}^2$$

...(Minus sign indicates retardation)

Example2. The equation for angular displacement of a body moving on a circular path is given by θ = 2t3 + 0.5 where θ is in rad and t in sec. Find angular velocity, displacement and acceleration after 2 sec.

Given data

Equation for angular displacement θ = 2t3 + 0.5 ---- (i)

Solution

Angular displacement after 2 seconds

Substituting $t = 2$ in equation (*i*),

$$\theta = 2\,(2)^3 + 0.5 = 16.5 \text{ rad}$$

Angular velocity after 2 seconds

Differentiating both sides equation (*i*) with respect to *t*,

$$\frac{d\theta}{dt} = 6\,t^2 \qquad ...(ii)$$

velocity, $\omega = 6\,t^2$ ...(*iii*)

Substituting $t = 2$ in equation (*iii*),

$$\omega = 6\,(2)^2 = 24 \text{ rad/sec}$$

Angular acceleration after 2 seconds

Differentiating both sides of equation (*iii*) with respect to *t*,

$$\frac{d\omega}{dt} = 12t \text{ or Acceleration } \alpha = 12t$$

Now substituting $t = 2$ in above equation,

$$\alpha = 12 \times 2 = 24 \text{ rad/sec}^2$$

Example3. The equation for angular displacement of a particle, moving in a circular path (radius 200 m) is given by $\theta = 18t + 3t^2 - 2t^3$ where θ is the angular displacement at the end of t sec. Find (i) angular velocity and acceleration at start, (ii) time when the particle reaches its maximum angular velocity; and (iii) maximum angular velocity of the particle.

Given data

Equation for angular displacement $\theta = 18t + 3t2 - 2t^3$ ------------ (i)

Solution

(i) Angular velocity and acceleration at start

Differentiating both sides of equation (i) with respect to t, $d\theta / dt = 18 + 6t - 6t$

^{2} i.e. angular velocity, $\omega = 18 + 6t - 6t^2$ -------------(ii)

Substituting t = 0 in equation (ii),

$$\omega = 18 + 0 - 0 = 18 \text{ rad/s}$$

Differentiating both sides of equation (ii) with respect to t, $d\omega / dt = 6 -$

$12t$ i.e. angular acceleration, $\alpha = 6 - 12t$ -------------- (iii)

Now substituting $t = 0$ in equation (iii),

$$\alpha = \mathbf{6\ rad/s^2}$$

(ii) Time when the particle reaches maximum angular velocity

For maximum angular velocity, take equation (iii) and equate it to zero $6 -$
$12t = 0$ or $t = 6 / 12$

$$\mathbf{t = 0.5\ seconds.}$$

(iii) Maximum angular velocity of the particle

The maximum angular velocity of the particle may now be found out by substituting $t = 0.5$ in equation (ii),

$$\omega_{max} = 18 + (6 \times 0.5) - 6\,(0.5)^2$$
$$\omega_{max} = \mathbf{19.5\ rad/s}$$