SCHOOL OF MECHANICAL ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

COURSE CODE: SME1307

COURSE NAME: DESIGN OF TRANSMISSION SYSTEMS

SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
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Unit 1 Design of Flexible Drives – SME1307

DESIGN OF FLAT BELT DRIVE

STEPS IN FLAT BELT DRIVE

1. Calculation of velocity ratio
2. Find the velocity of the belt
3. Take load correction factor(Ks) from DDB 7.53 based on given application
4. Calculation of Arc of contact from DDB 7.54 and take the value of correction factor(K) for arc of contact from the table DDB 7.54
5. Calculation of corrected power
6. Calculation of belt rating from DDB 7.54
7. Calculation of width of belt
8. Calculation of length of belt from DDB 7.53

1. **Design a flat belt drive to transmit 10kW at 1500 rpm to a line shaft to run at 500 rpm. Approximate centre distance is m. the diameter of the larger pulley is around 750mm.**

Solution

P-10kW **N1= 1500rpm** **N2= 500rpm** **C=2m** **D=750 mm**

$$\text{Velocity ratio} = \frac{N_2}{N_1} = \frac{d}{D}$$

$$\frac{500}{1500} = \frac{d}{750} \therefore d = \frac{750 \times 500}{1500} = 225 \text{ mm}$$

$$\text{Velocity of belt} = V = \frac{\pi d N_1}{60} = \frac{\pi \times 225 \times 1500}{60 \times 1000} = 17.67 \text{ m/sec}$$

Load correction factor (K_s)

According to load classification, refer data book, Page No.7.53 and take t value of K_S.

$K_s = 1.3$.... (for line shafts)

Correction arc of contact: K_α, refer data book, Page No.7.54.

$$\text{Arc of contact} = 180 - \frac{D-d}{C} \times 60 = 180 - \frac{750-225}{2000} \times 60$$

$= 164.25° = 165°$

From the databook, page No.7.54

Correction factor for Arc of contact, (at $\theta - 165°$)

$K_\alpha = 1.06$

Interpolation of K_α

$170° - 104$

for $5° - ? - 0.04 \times \frac{5}{10} = -0.02$

$160° - 1.08$

$5° - 0.02$

$165 = 1.06$

Calculation of Corrected Power

$$\text{corrected power} = \frac{K_s \cdot (\text{Given power in kW})}{K_\alpha}$$

$$= \frac{1.3 \times 10}{1.06} = 12.26 \text{ kW}$$

Refer databook, page No. 7.52,

According to the minimum pulley diameter and the maximum belt speed assume the no. of plies, from table at $v = 17.67$ m/sec and $d = 225$ mm;

Take, n = no. of plies = 5

Calculation of Load rating:

Select high speed belt,

The load rating per mm width per ply at 10 m/sec = 0.023 kw/mm/ply

$$\text{Load rating at belt speed } (at V = 17.67 \text{ m/sec}) = \frac{0.023 \times 17.67}{10}$$

$$= 0.0406 \text{ kW/mm/ply}$$

Calculation of width of the belt

Refer databook, page No.7.54

$$\text{Millie meter plies of belt} = \frac{\text{Corrected load (or) Corrected power}}{\text{Load rating /mm/ply}}$$

$$\text{Width} \times \text{no. of plies} = \frac{12.26}{0.0406} = 301.97$$

$\therefore$ Width of the belt $= \frac{301.97}{5} = 60.39$ mm Since $n =$ no. of plies $= 5$

Refer databook, Page No.7.52,

For 5 ply belt, the standard width of belt $= 76$ mm

Calculation of length of the belt (L)

Refer databook, page No.7.53

$$L = \frac{\pi}{2}(D+d) + 2C + \frac{(D-d)^2}{4C}$$

$$L = \frac{\pi}{2}(750+225) + 2 \times 2000 + \frac{(750-225)^2}{4 \times 2000}$$

$= 5565.97$ mm

Width of Pulley

Refer databook, Page No. 7.54

up to including 125 mm belt width, pulley than the belt width by 13 mm

$\therefore$width of pulley $= 76 + 13 = 89$ mm

Refer databook, page No.7.54

The recommended pulley nominal diameter $= 90$ mm; with tolerance of nominal diameter as ± 1.2

2. **Design a fabric belt to transmit 10 kW at 450 rpm from an engine to a line shaft as 1200 rpm. The diameter of the engine pulley is 6—mm and the distance of the shaft from the engine is 2m.**
   Solution:
   P=10 kW N1=450 rpm N2= 1200 rpm
   D=600 mm C=2m

$$\frac{N_2}{N_1} = \frac{D}{d} \quad \frac{\text{(dia. of Driver pulley)}}{\text{(dia. of Driven pulley)}}$$

$$\frac{1200}{450} = \frac{600}{d}$$

$$d = \frac{600 \times 450}{1200} = 225 \text{ mm}$$

$\therefore$ Dia. of line shaft pulley $= d = 225$ mm.

Velocity of the belt $= v = \dfrac{\pi D N_1}{60} = \dfrac{\pi \times 600 \times 450}{60 \times 1000} = 14.137$ m/sec

Load correction factor (K_s)

According to Load classification, refer databook Page No.7.53

Take the value of (K_s)

$K_S = 1.3$ (for line shafts)

Correction factor for Arc of contact (K_α): Refer databook page No. 7.54.

$$\text{Arc of contact} = 180 - \frac{D-d}{C} \times 60°$$

$$= 180 - \frac{600-225}{2000} \times 60$$

$$= 168.75 \text{ take approximately } \theta = 170°$$

From the table, (refer page No.7.54)

Correction factor (at $\theta = 170°$) $= K_\alpha = 1.04$ for ARc of contact

Calculation of Corrected Power

$$\text{Corrected power} = \frac{K_S \times (\text{Given power in kW})}{K_a}$$

$$= \frac{1.3 \times 10}{1.04}$$

$$= 12.5 \text{ kW}$$

Refer Page No. 7.52

According to the minimum pulley diameter and the maximum belt speed, assume the no. of plies

From the table, at $v = 14.137$ m/sec and $d = 225$ mm, $n = n_p$ = no.of plies = 5

Calculation of Load rating

Select high speed belt.

The load rating per mm width per ply at 10 m/sec = 0.023 kW/mm/ply

$$\text{Load rating at belt spread (at } v = 14.137 \text{ m/sec)} = \frac{0.023 \times 14.137}{10}$$

$$= 0.0325 \text{ kw/mm/ply}$$

Calculation of width of the belt

Refer page No. 7.54

$$\text{Millimeter plies of belt} = \frac{\text{Corrected load or Corrected power}}{\text{Loaded rating /mm/ply}}$$

$$\text{Width} \times \text{no.of plies} = \frac{12.5}{0.0325}$$

$$= 384.43$$

Since no. of plies $n = 5$

$$\therefore \text{Width of the belt} = \frac{384.43}{5}$$

$$= 76.88 \text{ mm}$$

Refer P.No. 7.52

For 5 ply belt the standard width of belt = 90 mm

Calculation of Length of the belt (L): Refer Databook Page No. 7.53.

$$L = \frac{\pi}{2}(D + d) + 2C + \frac{(D - d)^2}{4C}$$

$$= \frac{\pi}{2}(600 + 225) + 2 \times 2000 + \frac{(600 - 225)^2}{4 \times 2000}$$

$L = 5313.48$ mm

Width of Pulley

Refer Databook Page No. 7.54

Upto and including 125 mm belt width pulleys to be wider than the belt width by 13 mm

$\therefore$ Width of the Pulley $= 90 + 13 = 103$ mm

Refer Databook, Page No. 7.54.

The recommended pulley nominal diameter $= 112$ mm

With tolerance on nominal diameter as ± 1.2.

DESIGN OF V-BELT DRIVE

STEPS IN FLAT BELT DRIVE

1. Select the cross selection of the belt depending on the power to be transmitted.
2. Calculate the speed ratio.
3. Calculate the length of the belt from DDB 7.53.
4. Calculate the design power from DDB 7.64.
5. Find number of belts from DDB 7.70
6. Calculate new centre distance from DDB 7.61

1. A 30 kW 1440 rpm motor is to drive a compressor by means of V-belts. The Diameter of the pulleys are 220 mm and 750 mm. the centre distance between the compressor and the motor is 1440 mm. design a suitable drive.

Solution:

P=30 kw N1=1440 rpm d = 220mm

D=750 mm C=1440 mm

Step 1

From databook, P. No. 7.58.

Select Cross-section of the belt.

Select either (C), (D) or (E)

Select (C) type Belt (Since min. pulley pitch dia = 220 mm given)

Load of drive $= P =$ ~~22 KW~~ $-$ 150 KW   7.5-75 KW

Min. pulley pitch diameter $= 200$ mm

Nominal top width $= W = 22$ mm

Nominal thickness $= T = 14$ mm;

Weight/meter $= 0.343$ Kg_f/m length

C.S. area of belt $= \frac{1}{2}(W + b)\,T$

$$= \frac{1}{2}[22 + 11.808] \times 14$$

$$= 236.66 \text{ mm}^2$$

$$x = T \cdot \tan 20^\circ$$

$$= 14 \cdot \tan 20^\circ$$

$$= 5.095 \text{ mm}$$

$$b = W - 2x$$

$$= 22 - 2\,(5.095)$$

$$= 11.808 \text{ mm}$$

Step 2

Nominal pitch length $= L = \frac{\pi}{2}(D + d) + \frac{(D - d)^2}{4c} + 2c$

$$= \frac{\pi}{2}(750 + 220) + \frac{(750 - 220)^2}{4 \times 1440} + 2 \times 1440$$

$$= 4452.439 \text{ mm}$$

$$= 4.452 \text{ m}$$

Take the nearest nominal pitch length from databook.

Refer P.No. 7.60,

Take standard nominal pitch length $= 4450$ mm; (Nearest)

The corresponding nominal inside length $= 4394$ mm.

The Designation of V-belt C 4394 – IS2494

Step 3

Calculation of Design Power

$d = 220$ mm

$N_1 = 1440$ rpm

Refer Page No. 7.62

$$\text{kW} = (1.47S^{-0.09} - \frac{142.7}{de} - 2.34 \times 10^{-4} S^2)\, S$$

$$S = \frac{\pi d N_1}{60} = \frac{\pi \times 0.22 \times 1440}{60} = 16.587 \text{ m/sec}$$

$de =$ Equivalent pitch diameter

$= d_p \times F_b$

$= 220 \times 1.14$

$= 250.8$ m.m

$d_p =$ pitch dia. of smaller pulley

$= 220$ mm

$\frac{D}{d} = \frac{750}{220}$

$= 3.4 \rightarrow F_b = 1.14$ (P. No. 7.62)

$$\text{kW} = \left[1.47\,(16.587)^{-0.09} - \frac{142.7}{250.8} - 2.34 \times 10^{-4} \times 16.587^2 \right] 16.587$$

$$= 8.4312 \text{ kW}$$

Step 4

To find the no. of belts (n)

Refer page No. (7.70)

$$\text{No. of belts } = n = \frac{P \times F_a}{\text{kW} \times F_c \times F_d}$$

where $P =$ given power KW

$F_a =$ correction factor (from Page No. 7.69)

Let the time period is upto 10 hr.

$\therefore F_a = 1$ (for compressor)

kW = Power at the corresponding C.S. (ie at (C) – Cross section)

$F_c =$ Correction factor for length (P.No.7.60) = 1.04

$F_d =$ Correction factor for arc of contact

$$\theta = 180 - \left(\frac{D-d}{c}\right)60^\circ$$
$$= 180 - \left(\frac{750-220}{1440}\right)60 = 157.9^\circ$$
$$\approx 158^\circ$$

Refer page No. (7.68)

$\therefore$ for 158°, $F_d = 0.94$

$160^\circ - 0.95$

$\frac{157^\circ}{3^\circ} - \frac{0.94}{0.01}$

$1^\circ - ?\ \frac{0.01 \times 1}{3}$

$= 0.0033$

$\therefore 158^\circ = 0.94 + 0.0033$

$= 0.9433$

$$n = \text{no. of belts} = \frac{30 \times 1}{8.4312 \times 1.04 \times 0.94}$$
$$= 3.63$$
$$\approx 4 \text{ belts}$$

$\therefore$ No. of belts required $= n = 4$

CALCULATION OF NEW CENTRE DISTANCE

$$C = A + \sqrt{A^2 - B} \quad \text{(from P.No. 7.61)}$$

$$A = \frac{L}{4} - \pi\left(\frac{D+d}{8}\right)$$

L = nominal pitch length

$= 4450$ mm

$$B = \frac{(D-d)^2}{8}$$

$$A = \frac{4450}{4} - \pi\left(\frac{750+220}{8}\right) = 731.58$$

$$B = \frac{(750-220)^2}{8} = 35112.5$$

$$C = 731.58 + \sqrt{731.58^2 - 35112.5}$$

New centre distance $C = 1438.75$ mm

Calculation of T_1 and T_2

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\,\theta/\sin\beta} \quad \dots (1)$$

$2\beta = 40^\circ$

Semi-groove angle $\beta = 20^\circ$

Let $\mu = 0.25$

$\theta = 158^\circ \times \frac{\pi}{180}$

$= 2.75$ radians

$T_c = mv^2 = 0.343 \times (16.587)^2$

$= 94.36$ N

$m = 0.343$ Kg/m length

$v = S = 16.587$ m/sec

$$\text{Power/belt} = [T_1 - T_c]\left\{1 - \frac{1}{e^{\mu\theta/\sin\beta}}\right\} v$$

$$8.3412 \times 10^3 = (T_1 - 94.36)\left[1 - \frac{1}{e^{0.25 \times 2.75/\sin 20^\circ}}\right] \times 16.587$$

$$8.4312 \times 10^3 = (T_1 - 94.36)\,(14.348)$$

$$\boldsymbol{T_1 = 681.9819\ \text{N}}$$

$$\frac{T_1 - T_c}{T_2 - T_c} = e^{\mu\theta/\sin\beta}$$

$$\frac{681.9819 - 94.36}{T_2 - 94.36} = e^{0.25 \times 2.75/\sin 20^\circ}$$

$$\frac{587.8219}{(T_2 - 94.36)} = 7.464$$

$$T_2 = 78.754 + 94.36$$

$$\boldsymbol{T_2 = 173.114\ \text{N}}$$

To Find Stress (f_b)

$T = (T_1 + T_c) = f_b \times$ area of belt

$681.9819 + 94.36 = f_b \times 236.66$

Permissible stress in the belt material

$$f_b = 3.28\ \text{N/mm}^2$$

2. **Design a V belt drive to the following specification**
   Power to be transmitted -75 kW, speed of the driving wheel (N1)=1440 rpm, speed of the driving wheel (N2)=440 rpm, diameter of the driving wheel (d) =300 mm, Centre distance=2500mm, service =16hrs/ day.

   Solution :
   P=75kw N1=1440 rpm N2=440 rpm d =300mm
   C=2500mm service =16hrs/ day

To Find Stress (f_b)

$T = (T_1 + T_c) = f_b \times$ area of belt

$681.9819 + 94.36 = f_b \times 236.66$

Permissible stress in the belt material

$f_b = 3.28$ N/mm^2

Problem 2: ***Design a V-belt drive to the following specifications***

Power to be transmitted **= 75 kW**

Speed of the driving wheel **= 1440 rpm; (N_1)**

Speed of the driven wheel **= 400 rpm; (N_2)**

Diameter of the driving wheel **= 300 mm; (d)**

Centre distance **= 2500 mm**

Service **= 16 hours/day**

Assume any other relevant data if necessary. (Oct. '97)

Given Data:

$P = 75 \times 10^3$ Watts

Driving Wheel Speed $N_1 = 1440$ rpm;

Driven Wheel Speed $= N_2 = 400$ rpm

Driving wheel diameter $= d = 300$ mm

Driven wheel diameter $= D = ?$

Centre distance $= C = 2500$ mm;

Service $= 16$ hr/day.

Step 1

Selection of Cross section;

From databook, Page No. 7.58

Since minimum pulley dia. is 300 mm

Select (C) cross section

Nominal Width $= W = 22$ mm

Nominal thickness$= T = 14$ mm

Mass/kg = m = 0.343 kg/m length

$b = W - 2x$

$= 22 - 10.19$

$= 11.808$ mm

$x = T \cdot \tan 20° = 14 \times \tan 20°$

$= 5.095$ mm

$$\text{C.S. area} = \frac{1}{2}(W+b)\,T$$

$$= \frac{1}{2}[22 + 11.808] \times 14 = 236.656\ \text{mm}^2$$

Step 2

$$\text{Nominal Pitch length } = L = \frac{\pi}{2}(D+d) + \frac{(D-d)^2}{4c} + 2c$$

$$\frac{N_1}{N_2} = \frac{D}{d} \qquad = \frac{\pi}{2}(1080+300) + \frac{(1080-300)^2}{4 \times 2500} + 2 \times 2500$$

$$D = \frac{N_1}{N_2} \cdot d \qquad = 7228.53 \text{ mm}$$

$$= \frac{1440}{400} \times 300 \qquad \mathbf{L = 7.228\ m}$$

$$= 1080 \text{ mm}$$

Take the nearest value of nominal pitch length (from databook) 7.60 $\mathbf{L = 6863\ mm}$

*The Corresponding nominal inside length = 6807 mm

Step 3

Calculation of Design Power

From page No. 7.62 : for (C) C.S. of the belt

$$\text{kW} = \left[1.47S^{-0.09} - \frac{142.7}{de} - 2.34 \times 10^{-4} S^2\right] S$$

$$S = \frac{\pi d N_1}{60} = \frac{\pi \times 0.3 \times 1440}{60} = 22.61 \text{ m/sec}$$

de = equivalent pitch diameter where

d_p = pitch dia. of smaller pulley

$= d_p \times F_b$

$= 300 \times 1.14$

$= 342$ mm

refer page 7.62

at $\frac{D}{d} = 3.6$ the value of $F_b = 1.14$

But the max. value of (d_e) in the formula is 300 mm for (C) cross-section.

$\therefore$ Take $d_e = 300$ mm

$$kW = \left[1.47\,(22.61)^{-0.09} - \frac{142.7}{300} - 2.34 \times 10^{-4}(22.61)^2 \right] 22.61$$

Max. power transmitted by belt $= 11.643$ kW

Step 4

To find the no. of belts

(Refer page No. 7.70)

$$\text{No. of belts } = n = \frac{P \times F_a}{kW \times F_c \times F_d}$$

$P = 75$ kW

$F_a = 16$ hrs/day Page No. 7.6

Select medium duty [Since specific application is not given]

$\therefore F_a = 1.2$

kW = 11.643 kW

$F_c = 1.14$ refer Page No. 7.60 at Cross-section C

Refer databook, Page No. 7.68.

$F_d = 0.95$

$$\theta = 180^\circ - \left[\frac{D-d}{c} \right] \times 60^\circ$$

$$= 180^\circ - \left(\frac{1080 - 300}{2500} \right) 60^\circ$$

$$= 161.28^\circ$$

Let $\theta = 160^\circ$;

$$\therefore n = \text{no. of belts required} = \frac{75 \times 1.2}{11.643 \times 1.14 \times 0.95}$$

$$= 7.137$$

$n \approx 8$ belts

8 belts are required.

DESIGN OF CHAIN DRIVE

STEPS IN CHAIN DRIVE

1. Calculate transmission ratio
2. Select number of teeth on sprocket pinion Z_1 from DDB7.74. calculate the number of teeth on sprocket wheel Z_2.
3. Calculate pitch from DDB 7.74 and select the standard pitch from DDB 7.71-7.73. and select the std pitch with in P_{max} and P_{min}
4. Select chain number from DDB 7.71-7.73.
5. Check for breaking load from DDB 7.77
6. Check for actual factor of safety from DDB 7.78.
7. Find the length of chain from DDB.7.75
8. Find exact centre distance from DDB 7.76
9. Check for bearing stress from DDB 7.77
10. Calculate number of teeth on pinion and wheel (d_1 and d_2) from DDB 7.78

Problem

Design a chain drive to operate a compressor from a 15kW electric motor at 900 rpm. The compressor is to run at a speed of 300 rpm, the minimum centre distance should be 550mm.

Solution:

P=15Kw n_1=900rpm n_2 = 300 rpm C=550 mm

Step 1: Calculation of transmission ratio

$$\text{Transmission ratio} = \frac{z_2}{z_1} = \frac{n_1}{n_2}$$

$$i = \text{Transmission ratio} = \frac{n_1}{n_2} = \frac{900}{300} = 3$$

Step 2: From Design data book, refer Page No. 7.74.

For $i = 2$ to 3; $z_1 = 25$ to 27

$i = 3$ to 4; $z_1 = 23$ to 25

Take $z_1 = 23$ *to* 27 (select any odd no. of teeth)

Select $z_1 = 27$ teeth (no. of teeth on sprocket pinion)

$z_2 = i\, z_1 = 3 \times 27 = 81$

$= 82$ (no. of teeth on sprocket wheel)

Step 3: From design data book, P.No.7.74

Optimum centre distance $= a = (30 \text{ to } 50)\, p$

where $a =$ approximate centre distance

$$(\text{pitch})_{max} = p_{max} = \frac{550}{30} = 18.33 \text{ mm}$$

$$(\text{pitch})_{min} = p_{min} = \frac{550}{50} = 11 \text{ mm}$$

Select standard pitch from databook,

Take any standard pitch between 11 to 18.33 mm

$\therefore$ Select Pitch $= p = 15.875$ mm

ep 4: Selection of Chain No.

Select roller chain from Page No. 7.72

The available chain No. are 10A and 10B

Select 10A-2 Duplex Chain.

Pitch $= p = 15.875$ mm

Corresponding to chain No. selected, take the values of

$a =$ Bearing area $= 1.4 \text{ cm}^2$

$w =$ Weight per m length $= 1.78 \text{ kg}_f$

$Q =$ Breaking load $= 4440 \text{ kg}_f$

5: Calculate Power transmitted based on breaking load:

From data book, P. No. 7.77

$$N = \frac{Q \cdot v}{102 n \cdot K_s} K_w$$

From the above equation calculate 'Q' breaking load by considering

$N =$ given power

$N = 15$ KW

$$V = \frac{z_1 p n_1}{60 \times 1000} = \frac{27 \times 15.875 \times 900}{60 \times 1000}$$

$$= 6.429 \text{ m/sec}$$

$n =$ Minimum value of factor of safety

$(K_s = 1 ; z_1 = 15 \text{ to } 30)$ from data book, P. No. 7.77,

Select $n = 11$ for a pitch 15.875 mm and $n_1 < 1000$ rpm

Since the specific conditions are not given, in the problem, assume $K_1 = K_2 = K_3 = K_4 = K_5 = K_6 = 1$

$\therefore K_s = 1$

$$15 = \frac{Q \times 6.429}{102 \times 11 \times 1}$$

Breaking load $Q = 2617.825 \text{ Kg}_f$ which is less than the selected chain Breaking load (4440 Kg_f)

The selection of chain no. is satisfactory based on breaking load.

Step 6

(a) Calculation of Length of chain

(b) Final centre distance.

(a) Length of continuous chain in multiples of pitches

$$l_p = 2a_p + \frac{z_1 + z_2}{2} = \frac{\left(\frac{z_2 - z_1}{2\pi}\right)^2}{a_p}$$

where $a_p = \dfrac{a_o}{p}$ $\quad$ $a_o =$ initially assumed centre distance in mm

$$= \frac{550}{15.875} = 34.64 \qquad = 550 \text{ mm}$$

$$= 34.64 \qquad p = \text{pitch} = 15.875 \text{ mm}$$

No. of teeth on sprocket wheel $z_2 = 82$; $\left(\frac{z_2 - z_1}{2\pi}\right)^2 = m$

No. of teeth on sprocket pinion $z_1 = 27$ Read the value of 'm' directly fr databook at $(z_2 - z_1) = 81 - 27 = 54$;

Refer databook Page No. 7.76, the value of $m = 76.6$

$$l_p = 2(34.64) + \frac{27+82}{2} + \frac{76.6}{34.64}$$

$$= 125.99$$

$$= 126 \text{ (approximated to 126)}$$

Length of chain $= l = l_p \cdot p$

$$= 126 \times 15.875 = 2000.25 \text{ mm}$$

Take, $l = 2000$ mm

(b) Final centre distance

$$a = \frac{e + \sqrt{e^2 - 8m}}{4} \cdot p$$

$$e = l_p - \frac{z_1 + z_2}{2}$$

$m = 76.6$ (from distance)

$$= 126 - \frac{27+82}{2}$$

$$= 126 - \frac{109}{2}$$

$$= 71.5 \text{ m.m}$$

Final centre distance $= a = \frac{71.5 + \sqrt{71.5^2 - 8 \times 76.6}}{4} \times 15.875$

$$a = 549.98$$

$$a = 550 \text{ mm}$$

Step 7: Check the actual factor of safety.

From databook, Refer P.No. 7.78

Actual factor of safety $= [n] = \frac{Q}{\Sigma P}$

$Q =$ Breaking load of the chain $= 4440$ Kg

$\Sigma P = P_t + P_s + P_c$

$P_t =$ Tangential force due to power Transmission $= \dfrac{102\,N}{v}$

$P_t = \dfrac{102 \times 15}{6.429}$ $\quad\quad N = 15$ kW

$= 237.98$ kg$_f$ $\quad\quad V = 6.429$ m/sec

$P_c =$ Centrifugal fusion $= \dfrac{W \cdot v^2}{g}$

$= \dfrac{1.78 \times 6.429^2}{9.81}$ $\quad\quad W = 1.78$ kg$_f$/m length

$p = 15.875$ mm

$= 7.499$ kg$_f$ $\quad\quad z_1 = 27$

$P_s =$ tension due to sagging $=$ kWa metres $\quad\quad k =$ Coeff of Sag databook 7.78

$= 6 \times 1.78 \times \dfrac{550}{1000}$ $\quad\quad = 6$ (Horizontal)

$= 5.874$ Kg $\quad\quad a = 550$ mm $= 0.55$ m

$\Sigma p = 237.98 + 7.499 + 5.874$ $\quad\quad z_2 = 82$

$= 251.353$ Kg$_f$

$[n] = \dfrac{Q}{\Sigma P} = \dfrac{4440}{251.353}$

$= 17.66 > 11$

which is greater than allowable factor safety

$\therefore$ The design is safe.

Step 8: Checking of Allowable Bearing Stress

From databook, (P.No. 7.77)

The allowable bearing stress $= \sigma = 2.24$ Kg$_f$/mm^2

(for a pitch of 15.875 and speed < 1000 rpm)

Refer databook, from Pg. no. 7.77

Power transmitted on the basis of allowable bearing stress

$$N = \frac{(\sigma)\, av}{102\, K_s} \text{ kW}$$

$$15 = \frac{(\sigma) \times 1.4 \times 6.429}{102 \times 1} \qquad N = 15 \text{ kW}$$

$$(\sigma) = 169.98 \text{ Kgf/cm}^2 \quad \text{(or)} \qquad a = 1.4 \text{ cm}^2$$

$$(\sigma) = 1.69 \text{ Kgf/mm}^2 \qquad v = 6.429 \text{ m/sec}$$

$$(\sigma) < [\sigma] \qquad K_s = 1$$

$$< 2.24 \text{ Kgf/mm}^2$$

Therefore the design is safe.

Step 9: Pitch dia. of small sprocket $= d_1 = \dfrac{p}{\sin \dfrac{180°}{z_1}}$

$$= \frac{15.875}{\sin \frac{180°}{27}}$$

$$= 136.74 \text{ mm}$$

Pitch dia. of large sprocket $= d_2 = \dfrac{p}{\sin \dfrac{180°}{z_2}}$

$$= \frac{15.875}{\sin \frac{180°}{82}}$$

$$= 363.94 \text{ mm}$$

UNIT 2 – DESIGN OF SPUR AND PARALLEL AXIS HELICAL GEAR

UNIT 2

SPUR GEARS AND PARALLEL AXIS HELICAL GEARS

DESIGN PROCEDURE FOR SPUR GEARS

1. Selection of materials from DDB 8.4.
2. Calculation of life N in cycles
3. Calculation of equivalent youngs modulus From DDB 8.14
4. Calculation of design bending stress [σ_b] from DDB 8.18
5. Calculation of design contact stress [σ_c] from DDB 8.18
6. Calculation of design twisting moment [M_t] from DDB 8.15
7. Calculation of approximate centre distance (a) from DDB8.13
8. Calculation of Z1 and Z2.
9. Calculation of normal module (m_n) from DDB 8.22, take nearest value fr DDM8.2
10. Re Calculation of centre distance from DDB 8.23
11. Calculation of face width b from DDB 8.14, table 10
12. Calculation of velocity
13. To find the constant K and K_d
14. Recalculation of design twisting moment [M_t] from DDB 8.15
15. Check for bending stress σ_b
16. Check for contact stress σc

1. Calculation of gear ratio (i):

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, $N_A$and N_B=speed of the drive rand driven respectively, and
Z_A and Z_B=Number of teeth on driver and driven respectively.

2. Selection of material

Consulting Table8.4, knowing the gear ratio i, choose the suitable material.

3. If not given, assume gear life(say20000 hrs)

Life in cycles – life in hrs x 60 x RPM

4. Calculation of initial design torque:

$[M_t]=M_t.K.\ K_d$

where, $[M_t]$=transmission torque
K =Load factor, Table5.11
K_d =Dynamic load factor, Table5.12

Assume K. K_d=1.3 (ifnot given)

5. **Calculation of E_{eq},[σ_b]and[σ_c]:**
   - ✓From table8.14,T.NO.9 Calculate E_{eq}
   - ✓Calculate Design bending stress [σ_b]
   - ✓Calculate Design contact stress [σ_c]by

$[\sigma_c]=C_B.\ HB.\ K_{cl}$ (or)
$[\sigma_c]=C_R.\ HRC.\ K_{cl}$

where, $C_B C_R$=Coefficient of surface hardness from table8.16,T.No.16

HBHRC =Hardness number

6. **Calculation of centre distance (a):**

$$a \geq (i+1) \sqrt{\left(\frac{.74}{[c]}\right)^2 \times \frac{Eeq\,[Mt}{i\varphi}}$$

φ =b/a from table5.21

7. **Select number of teeth on gear and pinion:**
   - ➢On pinion, Z_1=Assume18
   - ➢On gear, Z_2=i x Z_1

8. **Calculation of module:**

$$m = \frac{2a}{z(1+z2)}$$

Choose standard module from table5.8

9. **Revision of centre distance (m):**

$$a = \frac{(r\ \ z1 - z2)}{2}$$

10. **Calculate b, d_1, v and ψ_p:**

- ✓ Calculate face width, $b = \psi . a$
- ✓ Calculatepitch dia, $d = m . z_1$
- ✓ Calculatepitch linevelocity, $v = (\pi d_1 N_1)/60$
- ✓ Calculatevalueof $\psi_p = b/d_1$

11. Selection of quality of gear:

Knowing the pitch linevelocityandconsultingtable5.22, select a suitable quality Of gear.

12. Revision of design torque [M$_t$]:

Revise K:

Using the calculated value of ψ_p revise the K value by using table5.11

$$[M_t] = M_t . K . K_d$$

13. Check for bending:

Calculate induced bending stress.

$$\sigma_b = \frac{(i \pm 1)}{a m \, b . Y} [M_t]$$

✓Compare σ_b and $[\sigma_b]$.

✓If $\sigma_b \leq [\sigma_b]$,then design is safe.

14. Check for wear strength:

Calculate induced contact stress.

$$\sigma_c = 0.74 \left(\frac{i \pm 1}{a} \right) \sqrt{\left(\frac{i \pm 1}{ib} \right) E_{eq} [M_t]}$$

✓Compare σ_c and $[\sigma_c]$.

✓If $\sigma_c \leq [\sigma_c]$,then design is safe.

15. If the design is not satisfactory($\sigma_b > [\sigma_b]$ and/or $\sigma_c > [\sigma_c]$), then increase the module of face width value of the gear material

PROBLEM

Design a spur gear to transmit 22.5 kW at 900 rpm, speed reduction is 2.5, material for pinion and wheel are C15 and Cast iron grade 30. Take pressure angle of 20^o and working life of the gears are 10,000 hrs.

Material Given: PINION– C15 - Steel

Wheel - Cast iron grade 30

Speed reduction $= i = 2.5$

ie, Gear ratio $= i = \frac{z_2}{z_1} = 2.5$

Power $= P = \mathbf{22.5\ kW}$

Pinion **Speed** $= n = N_1 = 900$ rpm;

Life $= N = 10{,}000$ **hrs**

Step 1: Material properties

Refer P.No. 8.16; **Table 16**

Pinion: C15 (Given)

Assume C15 - Case hardened

$HRC = 55$ to 63 assume $HRC = 60$

ie, Surface hardness > 350 BHN

$\therefore$ core hardness < 350 BHN

Wheel: Cast Iron - GRADE 30

$HB = 200$ to 260

Assume surface hardness $HB = 250$ (Brinell hardness)

$\therefore$ Core hardness < 350

.

Step 2: Life required $= 10,000$ hrs

$= 10000 \times 60 \times 900$ cycles

Life required $= N = 54 \times 10^7$ cycles.

Step 3: Calculation of Equivalent Young's Modulus

Refer P.No. 8.14; Table 9

Pinion material - Steel - $E_1 = 2.15 \times 10^6$ Kg_f/cm^2

Gear (or) Wheel material - C.I. Grade 30 - $E_2 = 1.4 \times 10^6$ Kg_f/cm^2

Therefore, equivalent young's modulus $= E = 1.7 \times 10^6$ Kg_f/cm^2

Since $\sigma_u > 28$ Kg_f/mm^2

For CI Grade 30, the ultimate stress $\sigma_u = 30\ Kg_f/mm^2$ $\qquad \frac{Kg_f/cm^2}{10} = \frac{N}{mm^2}$

(Convert into N/mm^2)

$E = 1.7 \times 10^5$ N/mm^2

Step 4: Calculation of design bending stress $[\sigma_b]$

(Refer P.No. 8.18)

Design bending stress $[\sigma_b] = \frac{1.4\, K_{bl}}{n\, K_\sigma}\sigma_{-1}$

(assume rotation in one direction)

$K_{bl} =$ Life factor for bending $= 1$

Table (22); P.No. 8.20

(Consider core hardness)

$= 1$ (for $N > 10^7$ cycles and BHN < 350)

$n =$ Factor of safety

Table (20) P.No. 8.18

$= 2$

(for case hardend)

$k_\sigma =$ fillet stress con ntration factor

Refer Table (21) P.No. 8.19 for case hardened, $(0 \le x < 0.1)$

$= 1.2$

X = Addendum modification coefficient.

σ_{-1} = Endurance limit stress in bending for complete reversal of stresses

Refer P.No. 8.19; table 19

Assume C15 as forged steel, for forged steel $= \sigma_{-1} = 0.25\,(\sigma_u + \sigma_y) + 500$

From P.No. 1.9 for C15 material.

$\sigma_u = 37 \text{ to } 49 \ \text{Kg}_f/\text{mm}^2$

$\sigma_y = 24 \ \text{Kg}_f/\text{mm}^2$

select $\sigma_u = 40 \ \text{Kg}_f/\text{mm}^2$

$= 40 \times 100 \ \text{Kg}_f/\text{cm}^2$

$\sigma_y = 24 \times 100 \ \text{Kg}_f/\text{cm}^2$

where:

σ_y = yield stress Kg_f/cm^2

σ_u = ultimate stress Kg_f/cm^2

$\therefore \sigma_{-1} = 0.25\,(40 \times 100 + 24 \times 100) + 500$

$= 2100 \ \text{Kg}_f/\text{cm}^2$

$$\therefore \text{Design bending stress } = [\sigma_b] = \frac{1.4\,K_{bl}}{n \cdot K_\sigma}\,\sigma_{-1}$$

$$= \frac{1.4 \times 1}{2 \times 1.2} \times 2100 = 1225 \ \text{Kg}_f/\text{cm}^2$$

(Convert into N/mm^2)

$[\sigma_b] = 122.5 \ \text{N/mm}^2$

Step 5: Calculation of Design surface (Contact compressive stress) $[\sigma_c]$

P.No. 8.16

Design surface (contact compressive stress)

$$= [\sigma_c] = C_R \cdot \text{HRC} \cdot K_{Cl} \cdot \text{Kg}_f/\text{cm}^2$$

C_R = Coeff. depending on surface hardness

Table 16 P.No. 8.16

$C_R = 220$ (for C15, Case hardened)

$HRC = 60$ (assumed)

K_{cl} = Life factor (Consider surface hardness)

Table 17: P.No. 8.17

$= 0.585$ for $N > 10^7$ cycles and surface hardness > 350 BHN

$[\sigma_c] = 220 \times 60 \times 0.585$

$[\sigma_c] = 7722\ Kg_f/cm^2$ (Convert into N/mm^2)

Design surface (Contact compressive) stress $= [\sigma_c] = 772.2\ N/mm^2$

Step 6: Calculation of design torque $[M_t]$

P. No. 8.15

$kW = 22.5\ kW$

$n = 900$ rpm

$[M_t] = M_t \cdot K_d \cdot K$

$$M_t = 97420 \frac{kW}{n}$$

$$= \frac{97420 \times 22.5}{900}$$

$$= 2435.5\ Kg_f - cm$$

Initially assume, for symmetry scheme

$K_d \cdot K = 1.3$

$[M_t] = 2435.5 \times 1.3 = 3166.15\ Kg_f\text{-cm}$ (Convert into N-mm)

$[M_t] = 316615$ N-mm

Step 7: Calculation of Approximate centre distance 'a'.

P.No.8.13

$$\text{Centre distance} = a \geq (i+1) \sqrt[3]{\left\{\frac{0.74}{[\sigma_c]}\right\}^2 \frac{E[M_t]}{i\,\psi}}$$

$i = 2.5$

$[\sigma_c] = 772.2\ N/mm^2$ from Step (5)

$E = 1.7 \times 10^5\ N/mm^2$ from Step (3)

$[M_t] = 316615$ N–mm from Step (6)

$$\psi = \frac{b}{a}$$

where b = face width -mm

a = centre distance -mm

Assume $z_1 =$ no. of teeth on pinion $= 25$ teeth.

(Refer P.No. 8.1)

Speed reduction $i = \frac{z_2}{z_1}$

$z_2 =$ No. of teeth on wheel $= i \cdot z_1$

$$= 2.5 \times 25$$

$$= 62.5 \text{ teeth}$$

Take $z_2 = 63$ **teeth**

Step 9: Calculation of module (P.No. 8.22)

Take, Module $= m = \frac{2a}{z_1 + z_2}$

$$= \frac{\mathbf{2 \times 142.74}}{\mathbf{(25+63)}} = \frac{285.48}{88}$$

$$= 3.24 \text{ mm}$$

Take Standard module, from P.No. 8.2 Table (1)

* module = 4 mm

Step 10: Recalculate **centre** distance 'a' and rounded **to R10** series.

$$a = \frac{m(z_1 + z_2)}{2}$$

$$= \frac{4\,(25+63)}{2}$$

Centre distance $= a = 176 \text{ mm} \approx$ Standard R10 series $a = 200$ mm

Step 11: Calculation of face width (b)

ψ value already assumed as 0.3 but $\psi = \frac{b}{a} = 0.3$

face width $= b = 0.3 \times 200 = 60$ mm

Step 12: Calculation of PCD (d_1) and PCD (d_2)

PCD of pinion $= (PCD)_1 = d_1 = m \cdot z_1$

$= 4 \times 25 = 100$ mm

PCD of wheel $= (PCD)_2 = d_2 = m \cdot z_2$

$= 4 \times 64 = 252$ mm

Step 13: Cal. of pitch line velocity: (v)

$$v = \frac{\pi d_1 N_1}{60} \text{ m/sec}$$

$$= \pi \times 0.1 \times \frac{900}{60} = 4.712 \text{ m/sec}$$

$d_1 = 100$ mm

$N_1 = 900$ rpm

Step 14: Selection of Quality of Gear.

From table (2); P.No. 8.3

at $v =$ upto 8 m/sec

Select the Quality of Gear - IS Quality 8

Step 15: Find load concentration factor (k) for IS Quality 8 Gear,

P.No. 8.15; table (14)

k depends on $\psi_p = \frac{b}{d_1}$ value

$b_1 = 52.8$ mm

$d_1 = 100$ rpm

$$= \frac{60}{100} = 0.6$$

at $\psi_p = 0.6$; the value of $k = 1.03$

Step 16: Find Dynamic load factor k_d

Refer P.No. 8.16; Table (15)

for pitch line vel up to 8 m/sec, cylindrical gears,

surface hardness > 350 BHN

$K_d = 1.4$

Step 17: Recalculate, $[M_t]$ design twisting moment,

$M_t = 2435.5$ Kg$_f$-cm

$$[M_t] = M_t\, k_d\, K$$

$$= 2435.5 \times 1.4 \times 1.03$$

$$[M_t] = 3511.99 \text{ Kg}_f \text{ cm}$$

$$[M_t] = 351199 \text{ N-mm.}$$

Step 18: Check σ_b induced bending stress

P.No. 8.13; table (8)

$$\sigma_b = \frac{i+1}{a \cdot m \cdot b \cdot y}[M_t] \le [\sigma_b]$$

$[M_t] = 351199$ N-mm

$i = 2.5$

$a = 200$ mm

$m = 4$ mm

$b = 60$ mm

$y =$ form factor

(P.No. 8.18) table (18)

(y value depends X and z value)

for $z_1 = 25; X = 0;$

$= 0.4205$

take average value of y

$24 \rightarrow 0.414$

$26 \rightarrow 0.427$

$25 \rightarrow 0.4205$

$$\sigma_b = \frac{2.5+1}{200 \times 60 \times 0.4205} 351199 = 60.89 \text{ N/mm}^2 < [\sigma_b] = 1225 \text{ N/mm}^2$$

Step 19: Check σ_c induced surface (contact compressive stress)

P.No. 8.13, Table (8)

$$\sigma_c = 0.74\,\frac{i+1}{i \cdot b}\sqrt{\frac{i+1}{a}\,E\,[M_t]}$$

$$= 0.74\ \frac{2.5+1}{200}\ \sqrt{\frac{2.5+1}{2.5 \times 60} \times 1.7 \times 10^5 \times 351199}$$

$$= 483.34 \text{ N/mm}^2 < [\sigma_c] = 772.2 \text{ N/mm}^2$$

$\therefore$ The design of pinion is ssatisfactory.

Step 20: Check the stresses in the wheel:

(a) Gear Material

Given - cast iron - Grade (30)

Since contact area is same,

$\sigma_{c_{wheel}} = \sigma_{c_{pinion}} = 483.34$ N/mm^2

(Already calculated)

Life of Wheel $= (N)_{Wheel} = \dfrac{N_{pinion}}{2.5} = \dfrac{54 \times 10^7}{2.5} = 21.6 \times 10^7$ cycles

From P.No. 8.16,

$[\sigma_c]$ = design surface contact compressive stress

$= C_B \;\; HB \cdot K_{CL}$

for CI - Grade (30) HB = 200 to 260 BHN take HB = 260 BHN $C_B = 23$;

$HB = 260$

K_{CL} = from table **(17)**; P.No. 8.17

$$= \sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{21.6 \times 10^7}} = 0.5992$$

$[\sigma_c] = 23 \times 260 \times 0.5992 = $ **3583.216 Kg$_f$**/cm^2

Convert into (N/mm^2)

$= 358.3216$ N/mm^2

$= \sigma_c > [\sigma_c]$

The design of wheel is not satisfactory.

Therefore, change the material of wheel and recalculate $[\sigma_c]$ and $\sigma_c < [\sigma_c]$

Select another material,

From P.No. 8.16; Table 16

Select either C40 or C45 Steel;

Heat treatment - Surface hardened.

K_{CL} = Surface hardness > 350

$= N <$ **25×10^7**

$= \sqrt{\dfrac{10^7}{N}}$

= 0.5992

Select C40 - Surface hardened

$HRC = 40$ to 55

Assume $HRC = 50$; $C_R = 230$; $K_{Cl} = 0.5992$

Design surface $[\sigma_c] = C_R \, HRC \, K_{Cl}$

Contact Compressive stress $= 230 \times 50 \times 0.5992 = 6890.8 \text{ Kg}_f/\text{cm}^2$

$[\sigma_c] = 689.08 \text{ N/mm}^2$

$$\therefore \sigma_c < [\sigma_c]$$

Therefore the selection of material C40, Surface hardened is satisfacto

* Checkup of σ_b induced bending stress;

$\sigma_{b_1} y_1 = \sigma_{b_2} y_2$

σ_{b_1} - (induced bending stress)

$[\sigma_{b_2}]_2$ - design bending stress of wheel

σ_{b_2} (induced bending stress)$_{w}$

$$= \frac{1.4\, K_{bl}}{K_n \cdot n} \sigma_{-1}$$

$y_1 = (\text{form factor})_{\text{pinion}} = 0.42$
$y_2 = (\text{form factor})_{\text{wheel}} = 0.4$

$\sigma_{b_2} < [\sigma_b]_{2_{\text{wheel}}}$

at $z = 63$; $x = 0$ by interpo

Find $[\sigma_b]_{2_{(\text{wheel})}}$

Design bending stress

$\sigma_{-1} = 0.25\,(\sigma_u + \sigma_y) + 500$

$$= [\sigma_b]_{\text{wheel}} = \frac{1.4\, K_{bl}}{K_\sigma\, n} \sigma_{-1}$$

for C 40 material, **P.No. 1**

$\sigma_u = 58$ to 68 $= 1054.66 \text{ Kg}_f$

$k_{bl} = 1$ table (22)

Assume $\sigma_u = 60 \text{ Kg}_f/\text{mm}^2$

$< 350\, BHN$ core hardness; $N > 10^7$

$= 60 \times 10^2 \text{ Kg}_f$

$n = 2.5$ table (20)

Pinion:	Wheel
$\sigma b_1 = 77.49\ \text{N/mm}^2$	$\sigma b_2 =$ Induced bending stress (to be cal.)
$y_1 = 0.4205$	$y_2 = 0.4927$

$$\sigma_{b_1} y_1 = \sigma_{b_2} y_2$$

$$\sigma_{b_2} = \frac{77.49 \times 0.4205}{0.4927}$$

$$= 66.13\ \text{N/mm}^2$$

$$\sigma_{b_2} < [\sigma_b]_2 = 105.46\ \text{N/mm}^2$$

Gear design is satisfactory.

DESIGN PROCEDURE FOR HELICAL GEARS

1. Selection of materials from DDB 8.4.
2. Calculation of life N in cycles
3. Calculation of equivalent youngs modulus From DDB 8.14
4. Calculation of design bending stress [σ_b] from DDB 8.18
5. Calculation of design contact stress [σ_c] from DDB 8.18
6. Calculation of design twisting moment [M_t] from DDB 8.15
7. Calculation of approximate centre distance (a) from DDB8.13
8. Calculation of Z1 and Z2.
9. Calculation of normal module (m_n) from DDB 8.22, take nearest value fr DDM8.2
10. Re Calculation of centre distance from DDB 8.23
11. Calculation of face width b from DDB 8.14, table 10
12. Calculation of velocity
13. To find the constant K and K_d
14. Recalculation of design twisting moment [M_t] from DDB 8.15
15. Check for bending stress σ_b
16. Check for contact stress σc

1. Calculation of gear ratio(i):

$$i = \frac{N_A}{N_B} = \frac{Z_B}{Z_A}$$

where, $N_A$and N_B=speed of the driver and driven respectively, and
$Z_A$and Z_B=Number of teeth on driver and driven respectively.

2. **Selection of material**

   Consulting Table8.3, knowing the gear ratio i, choose the suitable material.

3. If not given, assume gear life(say20000 hrs)

   Life in cycles – life in hrs x 60 x RPM

4. **Calculation of initial design torque:**

   $[M_t]=M_t.K. K_d$

where, $[M_t]$=transmission torque
K =Load factor, Table
K_d =Dynamic load factor, Table

Assume K. K_d=1.3 (if not given)

5. **Calculation of E_{eq},[σ_b]and[σ_c]:**

✓From table8.16 Calculate E_{eq}

✓From table8.16 Calculate Design bending stress [σ_b]

✓Calculate Design contact stress [σ_c]by

$$[\sigma_c]=C_B.\ HB.\ K_{cl} \quad \text{(or)}$$

$$[\sigma_c]=C_R.\ HRC.\ K_{cl}$$

$C_B C_R$=Coefficient of surface hardness from table8.16

HBHRC =Hardness number

6. **Calculation of centre distance (a):**

$$a \geq (i+1)\sqrt{\left(\frac{.74}{[c]}\right)^2 \times \frac{Eq][Mt}{i\varphi}}$$

φ =b/a from table5.21

7. **Select number of teeth on gear and pinion:**
   - On pinion, Z_1=Assume≥ 17
   - On gear, Z_2=i x Z_1

8. **Calculation of module:**

$$m = \frac{2a}{z1+z2)} \times \cos\ \beta$$

Choose standard module from table5.8

9. **Revision of centre distance (m):**

$$a= \frac{m}{\cos\beta}\frac{(\ z1+z2)}{2}$$

10. Calculate b, d_1, v and ψ_p:

- ✓ Calculate face width, $b = \psi . a$
- ✓ Calculate pitch dia, $d = (m_n . z_1)/ \cos \beta$
- ✓ Calculate pitch line velocity, $v = (\pi d_1 N_1)/60$
- ✓ Calculate value of $\psi_p = b/d_1$

11. Selection of quality of gear:

Knowing the pitch line velocity and a consulting table, select a suitable quality Of gear.

12. Revision of design torque$[M_t]$:

$$[M_t] = M_t . K . K_d$$

Check for bending:

Calculate induced beding stress,

$$\sigma_b = \frac{0.7\,(i \pm 1)}{a m\, b . Y}\,[M_t]$$

✓Compare σ_b and $[\sigma_b]$.
✓If $\sigma_b \leq [\sigma_b]$, then design is safe.

14. Check for wear strength:

Calculate induced contact stress

$$\sigma_c = 0.7\left(\frac{i \pm 1}{a}\right)\sqrt{\left(\frac{i \pm 1}{ib}\right) E_{eq}\,[M_t]}$$

✓Compare σ_c and $[\sigma_c]$.
✓If $\sigma_c \leq [\sigma_c]$, then design is safe.

15. If the design is not satisfactory($\sigma_b > [\sigma_b]$ and/or $\sigma_c > [\sigma_c]$), then increasethe module of face width value oft heg earmaterial.

16. Check for gear:

c. Check for bending:

$$\sigma_{b1} . y_1 . \frac{b1 . y1}{y2}$$

Using $\sigma_{b1}.y_1$ and

$\sigma_{b2}=$.

✓Compare σ_{b2} and $[\sigma_{b2}]$.
✓If $\sigma_{b2} \leq [\sigma_{b2}]$, then design is safe.

d. Check for wear strength:

Calculate induced contact stress will be same for pinion and gear, So,

$\sigma_{c2} = \sigma_c$

✓Compare σ_c and $[\sigma_c]$
✓If $\sigma_c \leq [\sigma_c]$, then design is safe

PROBLEM

Design a pair of helical gears for the following data:
Power- 7.5 kW, speed of pinion -1440 rpm, speed reduction -3, pressure angle-20^o, Helix angle – 10^o.
Select the materials and heat treatment.

Solution:

Step:1

Selection of material and heat treatment fromDDB.8.16, assume same material for both pinion and wheel

Select Alloy steel with case hardened neat treatment (Refer P. No. 1.14) .
40 Cr 1 MO 28 - Material, $\sigma_u = 70 - 85 \text{ Kg}_f/\text{mm}^2$

Take $\sigma_u = 80 \text{ Kg}_f/\text{mm}^2$

$HRC = 55$ to 63 and $C_R = 280$

Take $HRC = 60$ $\left[\begin{array}{l}\text{Surface } \textbf{hardness} > 350\, BHN \\ \text{Core hardness} < 350\, BHN\end{array}\right]$

Step 2: Life in cycles.

Assume life of the gear drive $= N = 10{,}000$ hrs

$\therefore N = 10{,}000 \times 60 \times RPM = 10{,}000 \times 60 \times 1400$

$= 84 \times 10^7$

Step 3: Equivalent young's modulus (refer databook P.No. 8.14) Since th material for the pinion and wheel is same,

$E_{eq} = 2.15 \times 10^6 \text{ Kg}_f/\text{cm}^2$

Step 4: Calculation of design bending stress $[\sigma_b]$

(Refer databook P.No. 8.18)

Design bending stress

$= [\sigma_b] = \dfrac{1.4\, K_{bl}}{n\, K_\sigma}(\sigma_{-1})$

$K_{bl} =$ Life factor for bending (P.No. 8.20)

(Consider core hardness)

$= 1$

$\sigma_{-1} =$ Endurance limit stress (P.No. 8.19)

$= (0.35\, \sigma_u + 1200)$

$0.35\,(8000) + 1200$

$= 4000 \text{ Kg}_f/\text{cm}^2$

$n =$ factor of safety (P. No. 8.19)

$= 2$

$K_\sigma =$ Fillet stress concentration (P. No. 8.19)

$= 1.5$

$\therefore [\sigma_b] = \dfrac{1.4 \times 1}{2 \times 1.5} \times (4000)$

$= 1866.66 \text{ Kg}_f/\text{cm}^2$

Take $[\sigma_b] = 1867 \text{ Kg}_f/\text{cm}^2$

Step 5: Calculation of design surface (contact compressive) stress. (Refer databook, P.No. 8.16)

Design surface (contact compressive stress) $= [\sigma_c] = C_R \, HRC \, K_{Cl}$

$[\sigma_c] = 280 \times 60 \times 0.585$

$[\sigma_c] = 9828 \text{ Kgf/cm}^2$

$C_R = 280$

$HRC = 60$

$K_{Cl} =$ Life factor (P. No. 8.17)

(Consider surface hardness)

$= 0.585$

Step 6: Calculation of Design twisting moment $[M_t]$ (Refer databook, P. No. 8.15)

Design twisting moment

$= [M_t] = M_t \cdot K_d \cdot K$

$= 521.89 \times 1.3$

$[M_t] = 678.46 \text{ Kgf} - \text{cm}$

Initially assume

$M_t = 97420 \dfrac{kW}{n}$

$= \dfrac{97420 \times 7.5}{1400}$

$= 521.89 \text{ Kgf} - \text{cm}$

Step 7: Calculation of approximate centre distance (a)

(Refer databook, Page No. 8.13)

Centre distance $a \geq (i+1) \sqrt[3]{\left\{\dfrac{0.7}{[\sigma_c]}\right\}^2 \dfrac{E\,[M_t]}{i\,\psi}}$

$$a \geq (3+1) \sqrt[3]{\left\{\frac{0.7}{9828}\right\}^2 \frac{2.15 \times 10^6 \times 678.46}{3 \times 0.3}}$$

$a \geq 8.07$ cm

assume $\psi = \dfrac{b}{a}$

$= 0.3$

(Open type gearing)

(Refer P.No. 8.14)

$i = 3$

(Centre distance should be rounded to R10 series)

Refer databook, Page No. 7.20

Take, Centre distance $= a = 80$ mm

Step 8: Calculation of z_1 and z_2

Assume, $z_1 =$ No. of teeth on pinion $= 20$

No. of teeth on wheel $= z_2 = i \cdot z_2 = 3 \times 20 = 60$ teeth

Step 9: Calculation of Normal module (m_n)

(refer page No. 8.22)

$$m_n = \text{Normal module} = \frac{2a \cos \beta}{z_1 + z_2}$$

$$m_n = \frac{2 \times 80 \times \cos 10^\circ}{20 + 60} = 1.9696 \text{ mm}$$

Select recommended value of module, (from P.No. 8.2)

Select, module $= m_n = 2.5$ mm

(For safe design, always select m_n slightly greater than required)

Step 10: Recalculate the centre distance (a)

$$a = \frac{m_n}{\cos \beta} \cdot \frac{(z_1 + z_2)}{2}$$

$$= \frac{2.5}{\cos 10} \frac{(20 + 60)}{2}$$

$a = 101.54$ mm

$\therefore$ Take, $a =$ centre distance $= 100$ mm (nearest value)

(from R10 series)

Step 11: Calculation of face width 'b'.

$$\psi = \frac{b}{a} = 0.3$$

$\therefore b = 0.3 \times a = 0.3 \times 100 = 30$ mm

Face width $= b = 30$ mm

Step 12: Calculation of pitch line velocity, v in m/sec.

Refer P. No. 8.22

Pitch line velocity $v = \frac{\pi d_1 n_1}{60}$ m/sec

$= 3.7216$ m/sec

$d_1 =$ pitch circle dia of pinion

$$d_1 = \frac{m_n}{\cos\beta} z_1$$

$$= \frac{2.5}{\cos 10} \times 20$$

$$= 50.77 \text{ mm}$$

Selection of Quality of gears:

(Refer page No. 8.3)

Select IS Quality - Medium

Preferred Quality - 8

Step 13: To find the constants K and K_d.

$K =$ Load correction factor

$= 1.03$ at $\psi = 0.6$

$$\psi = \frac{b}{d_1} = \frac{30}{50.77} = 0.59$$

$\psi \approx 0.6$

$K_d =$ Dynamic load factor

$= 1.3$ (For IS-8 Quality, and velocity upto 3 m/sec, $> 350\,BHN$ surface hardness)

Step 14: Recalculation of $[M_t]$

$[M_t] = M_t \cdot K_d \cdot K$

$= 521.89 \times 1.03 \times 1.3 = 698.81$ Kg_f-cm

Step 15: Checking of bending stress.

Refer Page No. 8.13

$y_v =$ form factor

Refer P.No. 8.18

$$\sigma_b = 0.7 \frac{i \pm 1}{a \cdot b \cdot m_n \cdot y_v} [M_t] \leq [\sigma_b]$$

$$= 0.7 \frac{(3+1)}{10 \times 3 \times 0.25 \times 0.3955} (698.81$$

$$= 659.64 \text{ Kg}_f/\text{cm}^2 < [\sigma_b]$$

$$z_v = \frac{z_1}{\cos^3 \beta} \quad \text{(P. No. 8.22)}$$

$$= \frac{20}{\cos^3 10}$$

$$= 20.93$$

$$= 21 \text{ teeth}$$

$\therefore$ Design is satisfactory based on $y_v = 0.3955$ at $z_v = 21$ teeth bending stress

Step 16: Checking of Surface (contact compressive) stress

refer databook, P. No. 8.13

$$\sigma_c = 0.7 \frac{i \pm 1}{a} \sqrt{\frac{i+1}{ib} E\,[M_t]} \leq [\sigma_c]$$

$$= 0.7 \frac{3+1}{10} \sqrt{\frac{3+1}{3 \times 3} \times 2.15 \times 10^6 \times 698.81}$$

$$= 7235.45 \text{ Kg}_f/\text{cm}^2 < [\sigma_c]$$

Therefore, the design is satisfactory based on surface (contact con stress.

UNIT 3 – DESIGN OF BEVEL AND WORM AND WORM WHEEL GEAR

Bevel gears are gears where the axes of the two shafts intersect and the tooth-bearing faces of the gears themselves are conically shaped. Bevel gears are most often mounted on shafts that are 90 degrees apart, but can be designed to work at other angles as well. The pitch surface of bevel gears is a cone.

TYPES

- Straight bevel gears have conical pitch surface and teeth are straight and tapering towards apex.
- Spiral bevel gears have curved teeth at an angle allowing tooth contact to be gradual and smooth.
- Zerol bevel gears are very similar to a bevel gear, but the teeth are curved: the ends of each tooth are coplanar with the axis, but the middle of each tooth is swept circumferentially around the gear. Zerol bevel gears can be thought of as spiral bevel gears, which also have curved teeth, but with a spiral angle of zero, so the ends of the teeth align with the axis.
- Hypoid bevel gears are similar to spiral bevel, but the pitch surfaces are hyperbolic and not conical. The pinion can be offset above or below the gear center, thus allowing larger pinion diameter, longer life, and smoother mesh. If the beveled surface is made parallel with the axis of rotation, this configuration resembles a worm drive. Hypoid gears were widely used in automobile rear axles

MITER GEAR

Miter gears are a special case of bevel gears that have equal numbers of teeth. The shafts are positioned at right angles from each other, and the gears have matching pitch surfaces and angles, with a conically-shaped pitch surface. Miter gears are useful for transmitting rotational motion at a 90-degree angle with a 1:1 ratio

MITER GEAR

APPLICATIONS

The bevel gear has many diverse applications such as locomotives, marine applications, automobiles, printing presses, cooling towers, power plants, steel plants, railway track inspection machines, etc.

For examples, see the following articles on:

- Bevel gears are used in differential drives, which can transmit power to two axles spinning at different speeds, such as those on a cornering automobile.
- Bevel gears are used as the main mechanism for a hand drill. As the handle of the drill is turned in a vertical direction, the bevel gears change the rotation of the chuck to a horizontal rotation. The bevel gears in a hand drill have the added advantage of increasing the speed of rotation of the chuck and this makes it possible to drill a range of materials.
- The gears in a bevel gear planer permit minor adjustment during assembly and allow for some displacement due to deflection under operating loads without concentrating the load on the end of the tooth.
- Spiral bevel gears are important components on rotorcraft drive systems. These components are required to operate at high speeds, high loads, and for a large number of load cycles. In this application, spiral bevel gears are used to redirect the shaft from the horizontal gas turbine engine to the vertical rotor. Bevel gears are also used as speed reducers

ADVANTAGES

- This gear makes it possible to change the operating angle.
- Differing of the number of teeth (effectively diameter) on each wheel allows mechanical advantage to be changed. By increasing or decreasing the ratio of teeth between the drive and driven wheels one may change the ratio of rotations between the two, meaning that the rotational drive and torque of the second wheel can be changed in relation to the first, with speed increasing and torque decreasing, or speed decreasing and torque increasing.

DISADVANTAGES

- One wheel of such gear is designed to work with its complementary wheel and no other.
- Must be precisely mounted.
- The shafts' bearings must be capable of supporting significant forces.

DESIGN PROCEDURES

1. Calculation of gear ratio from psg data book pg 8.6
2. Selection of materials. from psg data book pg 1.40
3. Selection of gear life based on given data.
4. Calculation of initial design torque from psg data book pg 8.15. table no13
5. Determination of equivalent young's modulus from psg data book pg8.14. table no.9
6. Calculation of design contact stress from psg data book pg 8.16. table no.15
7. Calculation of design of bending stress from psg data book pg 8.18.
8. Calculation of cone distance from psg data book pg8.13. table no.8

9. Selection of Z1 and Z2 assume Z1 initially as ≥17.
10. Calculation for transverse module psg data book pg8.38 table no31.
11. Revision of cone distance psg data book pg8.38 table no.31.
12. Calculation of face width from psg data book pg8.15 table no. 13
13. Calculation of reference diameter from psg data book pg8.38 table no. 31.
14. Selection of quality of gear from psg data book pg8.3 table no. 2
15. Revision of design torque from psg data book pg8.15
16. Check for contact stress from psg data book pg8.13
17. Check for bending stress from psg data book pg8.13 A.

Problem:

A pair of 20° full depth involute teeth bevel gear connects two shafts at right angles having a velocity ratio of 3.2: 1. The gear is made of cast steel with an allowable static stress as72 N/mm^2, and the pinion is made of steel having a static of 100 N/mm^2. The pinion transmits 40kW and at 840 rpm. Find the module, face width, and pitch diameter from the stand point of the beam strength, and check the design from the stand point of wear.

Given data:

$\Theta = 90°$, $\acute{\alpha} = 20°$, $i = 3.2$, $[\sigma b2] = 72\ N/mm^2$, $[\sigma b1] = 100\ N/mm^2$, $P = 40$ kW ,
N1= 840 rpm

To find:

Module, face width, and pitch diameter of the gears.

Solution:

Step: 1: Selection of materials
Material for gear : Cast Iron
Material for pinion: Steel

Step: 2: Assume $Z_1 = 20$, then $Z_2 = i \times Z1$

$Z_2 = 3.2 \times 20 = 64$

$Z_2 = 64$

Step : 3: Calculation of pitch angle [(ie) δ1 and δ2] and the virtual number of teeth (ie) Zp_1 and Zp_2 using the following relations ,

$\tan \delta_2 = i = 3.2$ or $\delta_2 = \tan^{-1}(3.2) = 72.64°$

then $\delta_1 = 90° - \delta_2$

$\delta_1 = 90° - 72.64° = 17.36°$

The virtual number of the teeth on the gears is given by

$$Zv1 = \frac{Z1}{\cos \delta 1} = \frac{20}{\cos 17.36°}$$

$Zv1 = 21$

$$Zv2 = \frac{Z2}{\cos \delta 2} = \frac{64}{\cos 72.64°}$$

$Zv2 = 215$

Then form factors based on virtual number of teeth are given by

$$\acute{Y}1 = 0.154 - \frac{0.912}{Zv1}$$

$\acute{Y}1 = 0.1106$

$$\acute{Y}2 = 0.154 - \frac{0.912}{Zv2}$$

$\acute{Y}2 = 0.1497$

For pinion: $[\sigma b1] \times \acute{y}1 = 100 \times .0116 = 11.06$ **N/mm^2**

For gear: $[\sigma b2] \times \acute{y}2 = 72 \times 1497 = 10.78$ **N/mm^2**

Hence the value of gear is less than pinion. Thus we have to design for gear only.

Step : 4: Calculating the tangential load using the relation We know that ,

$$Ft = \frac{P}{v} \times Ko$$

where $v = \frac{\pi d_1 N_1}{60} = \frac{\pi \times N_1}{60} \times \frac{(mt \times Z_1)}{1000}$

$= \frac{\pi \times 840}{60} \times \frac{(mt \times 20)}{1000} = .879\ mt$

Ko = 1.25 , assuming medium shock

$Ft = \frac{40 \times 10^3}{0.879\ mt} \times 1.25 = \frac{56841}{mt}$

Step :5:

$Fd = \frac{F t}{Cv}$ where $Cv = \frac{5.6}{5.6 + \sqrt{V}}$ where V = 5 **m/s**

Cv = 0.715

$\frac{Fd}{mt} = \frac{56841}{} \times \frac{1}{0.715}$

$Fd = \frac{79497.9}{mt}$

Step: 6: Calculating the preliminary value of dynamic load using the relation

$$F_s = \pi \times m_t \times b \times [\sigma_b] \times y' \times \frac{[R - b]}{R}$$

Where $b = 10\ m_t$; $Y'_2 = 0.1497$

$[\sigma_{b2}] = 72$ **N/mm²** ; $R = 0.5\ m_t \times \sqrt{(Z_1^2 + Z_2^2)} = 33.53\ m_t$

$$F_s = \pi \times m_t \times 10\ m_t \times 72 \times 0.1497 \times \frac{[33.53\ m_t - 10\ m_t]}{33.53\ m_t}$$

$$F_s = 237.62\ m_t^2$$

Step :7:

Calculation of transverse module mt

We know that $F_s \geq F_d$

$$237.62\ m_t^2 \geq \frac{79497.9}{m_t}$$

$m_t \geq 6.94$ **mm**

Step : 8:

Calculate the values of b , d1 and v

Face width $b = 10\ m_t = 10 \times 7 = 70$ **mm**

Pitch circle diameter $d_1 = m_t \times Z_1 = 7 \times 20 = 140$ **mm**

Pitch line velocity $V_2 = V_1 = \frac{\pi d N}{60} = 6.61$**m/s**

Step : 9:

Recalculation of the beam strength

$$F_s = 237.62 \times m_t^2 = 11643.38\ \mathbf{N}$$

Step: 10:

Calculation of the dynamic load , using Buckingham's equation ,

$$F_d = F_t + \frac{21\ V\ (bc + F_t)}{21\ V + \sqrt{(bc + F_t)}}$$

$$F_t = \frac{P}{v} = \frac{40 \times 10^3}{6.16} = 6493.55\ \mathbf{N}$$

$c = 11860 \times 0.017 = 201.62$ **N/mm**

$$F_d = 6493.5 + \frac{21 \times 6.16 \times 10^3 \times (70 \times 201.62 + 6493.5)}{21 \times 6.16 \times 10^3 + \sqrt{(70 \times 201.62 + 6493.5)}}$$

$F_d = 27077.55$ **N**

Step :11:

Check for the beam strength or tooth breakage, but Fd >> Fs

Taking module as 14

Face width $b = 10\ m_t = 10 \times 14 = 140$ **mm**

Pitch circle diameter $d_1 = m_t \times Z_1 = 14 \times 20 = 280$ **mm**

Pitch line velocity $V_2 = V_1 = \underline{\pi d N} = 12.315$**m/s**

$Fs = 237.62 \times mt^2 = 237.62 \times 14^2 = 46573.52$ **N**

$Ft = \dfrac{P}{v} = \dfrac{40 \times 10^3}{12.315} = 3248$ **N**

$c = 11860 \times .025 = 296.62$ **N/mm**

$$Fd = 3248 + \frac{21 \times 12.315 \times 10^3 \times (140 \times 296.5 + 3248)}{21 \times 12.315 \times 10^3 + \sqrt{(140 \times 296.5 + 3248)}}$$

$Fd = 47969.4$ **N**

We find Fs > Fd , now the design is safe and satisfactory against the tooth failure.

Step: 12:

Calculation of wear load (Fw)

$$Fs = \frac{0.75 \times d1 \times b \times Q \times Kw}{\cos \delta 1}$$

$Q = \text{Ratio Factor} = \dfrac{2 \times Zv2}{Zv1 \pm Zv2} = \dfrac{2 \times 215}{21 + 215} = 1.822$, and

$Kw = 0.919$ **N/mm²** , for steel gears hardened to 250 **BHN** ,

$Fs = \dfrac{0.75 \times 280 \times 140 \times 1,822 \times 0.919}{\cos 17.36°} = 51578.25$ **N**

Cos 17.36°

Step : 13:

Checking for wear , we found that Fw > Fd , it means the gear tooth has adequate wear capacity and will not wear out. Thus the design is safe against wear failure also.

Step : 14

Module $mt = 14$ **mm**

Face width $b = 10 \times mt = 140$ **mm**

Pitch diameter $d1 = mt \times Z1 = 14 \times 20 = 280$ **mm**

$d2 = mt \times Z2 = 14 \times 64 = 896$ **mm**

Problem on bevel gear:

Design a cast iron bevel gear drive for a pillar drilling machine to transmit 1875 W, at 800 Rpm, to a spindle at 400 rpm. The gear is to work for 40 hours per week for 3 years. Pressure angle is 20°.

Given Data:

P= 1875 W, N1= 800 rpm, N2= 400 rpm, α= 20°

To find:

Design a bevel gear dive

Solution:

Since the materials of pinion and gear are same we have to design only the pinion

1. Gear ratio:

$$Mt = \left(\frac{60 \, x \, 1875}{2 \, \pi \, x \, 800} \right) = 22.38 \text{ N-m and}$$

$K.K_d$ = 1.3 (as per assumption)

[Mt] = 22.38 x 1.3 = 29.095 N-m

5. **Calculation of Eeq , [бb],** [бc]:

To find Eeq: Eeq = 1.4 x 10^5 N/ mm^2 for cast iron, бu > 280 N/ mm^2 PSG 8.14

To find [бb] = $\frac{1.4 \, K_{bl}}{n \, . \, K_\sigma}$x σ_{-1} , for rotation in one direction PSG 8.20

$$K_{bl} = \sqrt[9]{\frac{10^7}{N}} = 0.8852, \text{ for CI}$$

Kб = 1.2 ; PSG 8.19
n = 2, PSG 8.19
б-1 = 0.45 бu PSG 8.19

Бu = 350 N / mm^2 PSG 1.40
Б-1 = 0.45 x 350 = 157.5 N/ mm^2

i = N1 / N2 = 800 / 400 = 2

pitch angle : for right angle bevel gear, tan δ2 = I = 2
or δ2= $\tan^{-1}$ (2) = 63.43°
and δ1 = 90 – δ2 = 26.57°

2. Material for pinion and gear:

Cast iron grade 35 heat treated

Бu = 350 N/ mm^2, from PSG 1.40

Then [бb] = (1.4 x 0.8852 x 157.2 / (2x 1.2)) = 81.33 N/ mm^2

To find [бc] :

[бc] = Cb x HB x Kcl

Cb = 2.3 PSG 8.16

Hb = 200 to 260 PSG 8.16

Kcl = $\sqrt[6]{\frac{10^7}{N}} = \sqrt[6]{\frac{10^7}{29.952 \times 10^7}}$

= 0.833, for C I

[бc] = 2.3 x 260 x 0.833 = 498.08 N/ mm^2

6. **Calculation of cone distance (R):**

We know that $R \geq \psi y \left(\sqrt{(i^2+1)}\right)\left\{\sqrt[3]{\left[\frac{0.72}{\psi y - 0.5[бc]}\right]^2 x\, Eeq \frac{[Mt]}{i}}\right\}$

$$\psi y = \frac{R}{b} = 3$$

R ≥ 50.2

R= 51 mm

7. **Assume** Z1= 20, Then Z2 = I x Z1= 2 x 20 = 40

Virtual number of teeth $Zv_1 = Z_1 / \cos \delta_1$ = 20 / (cos 26.57°) = 23

And $Zv_2 = Z_2 / \cos\delta_2$ = 40 / (cos 63.43) = 90

8. **Calculating the transverse module (mt):**

$Mt = \frac{R}{(0.5\sqrt{Z1^2 + Z2^2})}$ = 2.28 mm take as 2.5 PSG 8.2

9. Revision of cone distance R:

we know that ,

$R = \left(0.5\, Mt\ \sqrt{Z1^2 + Z2^2}\right) = 0.5\, x\, 2.5\, \sqrt{(20^2 + 40^2)}$= 55.9 mm

10. Calculation of b, Mav, d_{1av}, v, and ψy:

Face width (b) ; b = R / ψy = 55.9 / 3 = 18.63 mm

Average module (m_{av}) : $m_t - (b \sin \delta_1 / Z1)$ = 2.0863 mm

Average pcd of pinion ($d_{1\,av}$) = d1av = $m_{av} \times Z1$ = 2.083 x 20 = 41.66 m

Pitch line velocity v : $\frac{\pi \times d1av \times N1}{60}$= 1.745 m /s

Ψy = b / d_{1av} = 18.63 / 41.66 = 0.477

14. Check for wear strength :We know that the induced contact stress,

$$Бc = \left(\frac{0.72}{R - 0.5b}\right)\left(\left(\frac{\sqrt{(i^2+1)^3}}{(i\,x\,b)}\right) x\ Eeq\ [Mt]\right]^{\frac{1}{2}}$$

K= 1.1 for b / d_{1av} ≤ 1 , PSG 8.15

Kd = 1.35 P SG 8.16

[Mt] = 22.38 x 1.1 x 1.35 = 33.24 N- m

13. Check for bending stress

We know that the induced bending stress

$$Б\,b = \left\{\frac{R\sqrt{(i^2+1)}[Mt]}{((R - 0.5b)^2 x\ b\ x\ mt\ x\ Y_{v1})}\right)$$

Where Yv_1= 0.408 for Zv_1 = 23 .,, PSG 8.18

Б b = 100.75 N/ mm²

Which is not satisfactory

Recalculate with various b, d_{av} ,v, ψ_y, m_{av} ,,

$= 439.33 \text{ N / mm}^2$

We find that бc < [бc], thus the design is safety

15. **Calculation of basic dimensions of pinion and gear :**

Transverse module : mt = 3 mm

Number of teeth : Z1 = 20, Z2 = 40

Pitch circle diameter : d1= mt x Z1 = 3 x20 = 60 mm and

D2 = Mt x Z2 = 3 x 40 = 120 mm

Cone distance R = 67.08 mm

Face width b = 22.36 mm

Pitch angle = $\delta_1 = 26.57°$, and $\delta_2 = 63.43°$

Height factor : fo = 1

Clearance : c = 0.2

Virtual number of teeth : Zv_1= 23, and Zv_2= 90

1. DESIGN OF WORM AND WORMWHEEL

When the shafts are non-parallel and non-intersecting worm and worm wheel drive is used. It can be treated as screw and nut pair. Since the sliding occurs the materials used should have low coefficient of friction.

A worm gear is a species of helical gear, but its helix angle is usually somewhat large (close to 90 degrees) and its body is usually fairly long in the axial direction. These attributes give it screw like qualities. The distinction between a worm and a helical gear is that at least one tooth persists for a full rotation around the helix.

Worm : It is made of steel. The threads are grounded and polished to reduce the surface hardness as low as possible.

Worm wheel: They are made of bronze and cast iron.

DESIGN PROCEDURE OF WORM GEARS

STEP-1: Selection of Material PSG. 8.5

STEP-2: Calculation of Initial Design Torque

[Mt]=Mt x K x Kd.
Initially, Assume K x Kd = 1.
Mt=60 x p / 2ΠN.

STEP-3: Selection of Z_1&Z_2.

Select Z1 FOR VARIOUS EFFICIENCIES PSG. 8.46
$Z_2 = i \times Z_1$.

STEP-4: Selection of [σb] & [σc] PSG.8.45

STEP-5: Calculation of Centre Distance PSG.8.44

a = [(z/q)|+1] x 3√[540/(Z/q) x [σc]]2 x [Mt] / 10

STEP-6: Calculation of Axial Module PSG.8.43
m= 2a / (q+z)

STEP-7: Calculation of Revised Centre Distance PSG.8.43
$a=0.5m\,(q+Z_2)$

STEP-8: Calculation of d, v, γ, Vs.
d = q x m
v = πdn / 60
$\gamma = \tan^{-1}\{Z/q\}$
Vs = v/cos γ

STEP-9: Recalculation of Design Contact Stress Using Vs. PSG.8.45

STEP-10: Revise K, d, Mt Values.

STEP-11: CHECK FOR BENDING STRESS PSG.8.44

[σb] = 1.9[Mt] / m3 x q x z x y

STEP-12: Check for Wear σc PSG.8.44

STEP-13: Check for Efficiency

η=0.95 x tan γ/tan (γ+ρ)

ρ=TAN-1(μ)

STEP-14: Calculation of Cooling Area Required

(1-η) x INPUT POWER = Kt x A (to-ta)

STEP-15: Calculation of Basic Dimensions PSG.8.43

PROBLEM ON WORM GEARS

A steel worm running at 240 rp , receives 1.5 kw from its shaft. The speed reduction is 10:1, design the drive so as to have an efficiency of 80 %, also determine the cooling area required, if the temperature rise is restricted to 45° C, and take overall heat transfer co efficient as 10 W / m² ° C.

Given data:

N1= 240 rpm, P= 1.5 kW, I= 10, η desired = 80%, to – ta = 45°C, Kt = 10 W/ m² °C.

To find:

1. Design the worm gear drive
2. The cooling area required

Solution:

STEP-1: Selection of Material

Worm – Steel

Wheel – Bronze (sand cast), selected from Table PSG. 8.5

STEP-2: Calculation of Initial Design Torque

[Mt]=Mt x K x Kd.

Initially, Assume K x Kd = 1.

Mt=60 x p / 2ΠN.

Mt= (60 x 1.5 x 10^3 / 2πN2) = 596.83 N-m

K.Kd = 1

[Mt]= 596.83 N-m

STEP-3: Selection of Z_1&Z_2.

Select Z1, η desired = 80%,, Z1 = 3 PSG. 8.46

Z_2 = i x Z_1.= 10 x 3 = 30

STEP-4: Selection of [σb] & [σc] PSG.8.45

For bronze wheel бu < 390 N / mm², [бb]= 50 N/ mm² is selected in one rotation in one direction

[бc]= 159 N / mm² is selected

STEP-5: Calculation of Centre Distance PSG.8.44

a = [(z/q)|+1] x 3√[540/(Z/q) x [σc]]2 x [Mt] / 10

a = [(30/11)|+1] x 3√[540/(30/11) x [σc]]2 x [596.83x 10^3] / 10

a = 168.6 mm

STEP-6: Calculation of Axial Module PSG.8.43

m= 2a / (q+z)

m= 2168.6/ (11+30) = 8.22 mm

STEP-7: Calculation of Revised Centre Distance PSG.8.43

a=0.5m ($q+Z_2$)

a=0.5x 10 (11+30) = 205 mm

STEP-8: Calculation of d, v, γ, Vs.

d = q x m

d 1= q x m= 11 x 10 = 110 mm

d 2= Z2 x m= 30 x 10 = 300 mm

$v_1 = \pi d n_1 / 60$= 1.382 m / s

$v_2 = \pi d n_2 / 60$= 0.377 m / s

$\gamma = \tan^{-1}\{Z/q\}$ = 15.25 °

Vs = v/cos γ = 1.432 m/s

STEP-9: Recalculation of Design Contact Stress Using Vs.

For vs = 1.432 m /s , [бc] = 172 N/ mm² PSG.8.45

STEP-10: Revise K, d, Mt Values.

[Mt]=Mt x K x Kd.

= 596.83 x 1 x1 = 596.83 N-m

STEP-11: CHECK FOR BENDING STRESS PSG.8.44

[σb] = 1.9[Mt] / m3 x q x z x y

[σb] = 1.9 x 596.863 x 10^3 / 10^3 x 11 x 30 x 0.432

= 7.6 N / mm^2

STEP-12: Check for Wear σc

Бc = 540 / (Z2/q) √((Z2/ q) +1) / a $)^3$ x (Mt / 10)

= 118.59 N / mm^2

STEP-13: Check for Efficiency

η=0.95 x tan γ/tan (γ+ρ)

ρ=TAN-1(μ) = 2.862 °

η=0.95 x tan γ/tan (γ+ρ) = 80%

STEP-14: Calculation of Cooling Area Required

(1-η) x INPUT POWER = Kt x A (to-ta)

(1 – 0.8) x 1.5 x 10^3 = 10 x A x 45°

= 0.666 m^2

STEP-15: Calculation of Basic Dimensions PSG.8.43

Axial module = Mx = 10 mm

Number of starts = Z1 = 3

Number of teeth = Z2 = 30

Length of worm 152 mm (L≥ (12.5 + 0.09 x Z2) mx = 152 mm

Center distance = a = 205 mm

Height factor = 1

ASSIGNMENT PROBLEMS

1. Design a pair of straight bevel gears to transmit 10 kW at 1000 rpm between two perpendicular shafts. A speed ratio desire is 3.5; Select suitablematerials.

2. Design a suitable worm and wonn wheel drive to transmit 10 kW at 1000 rpm of worm. Select suitable materials. The speed of wheel is to be around 40rpm.

3. Design a bevel gear drive to transmit 7.4 kW at 1440 rpm for the following data: Gear ratio=3 ,Material for pinion and gear :C45 steel ,Life=10,000hrs.7

UNIT 4 – DESIGN OF GEAR BOX

Geometric progression - Standard step ratio - Ray diagram, kinematics layout -Design of sliding mesh gearbox -Constant mesh gear box. - Design of multi speed gear box.

Gear boxes are employed to convert input from a high speed power sources to low speed (Eg. Lift, Cranes and Crushing Machine) or into a many of speeds (Lathe, Milling Machine and Automobiles).A gearbox that converts a high speed input into a single output it is called a single stage gearbox. It usually has two gears and shafts.

A gearbox that converts a high speed input into a number of different speed output it is called a multi-speed gear box. Multi speed gear box has more than two gears and shafts.

Requirements of Gear Box:

- Provides the designed series of spindle speeds
- Transmits the required amount of power to the spindle
- Provides smooth silent operation of the transmission
- Simple in construction
- Mechanism of speed gear boxes should be easily accessible so that it is easier to carry out preventive maintenance

Law of speed in Multispeed Gear box

- ARITHMATIC PROGRESSION
- GEOMETRIC PROGRESSION
- HARMONIC PROGRESSION

ARITHMATIC PROGRESSION: The difference between and two successive spindle speed is constant.
GEOMETRIC PROGRESSION: The ratio of any successive spindle speed is constant.
HARMONIC PROGRESSION: The difference between reciprocal of any two successive speed is constant.

Advantages of Geometric Progression

- In order to get a series of output speeds from a gearbox.
- The speed is reduced uniformly in different stages.
- The speed loss is minimum i.e., Speed loss = Desired optimum speed – Available speed
- The number of gears to be employed is minimum
- G.P. provides a more even range of spindle speeds at each step
- The layout is comparatively very compact.
- Productivity of a machining operation, i.e., surface area of the metal removed in unit time, is constant in the whole speed range.
- G.P. machine tool spindle speeds can be selected easily from preferred numbers. Because **PREFERRED NUMBERS** are in geometric progression.
- Geometric progression, also known as a geometric sequence

Standard progression

When the spindle speed are arranged in geometric progression then the ratio between thetwo adjacentspeeds is known as step ratio or progressionratio

$$\frac{N_2}{N_1} = \frac{N_3}{N_2} = \frac{N_4}{N_3} = \ldots\ldots\ldots\ldots\ldots\ldots = \frac{N_n}{N_{(n-1)}} = \text{constant} = \phi$$

Step ratio

- The ratio between the two adjacent speeds is known as step ratio or progression ratio. It is denoted by ϕ.

Basic series	Step ratio (φ)
R5	1.58
R10	1.26
R20	1.12
R40	1.06
R80	1.03

Preferred basic series (or) Preferred Numbers:

- Preferred numbers are the conventionally rounded off values derived from geometric series.
- There are five basic series, denoted as R 5, R 10, R 20, R 40 and R 80 series.
- Each series has its own step ratio i.e., series factor.
- The series of preferred numbers is obtained by multiplying a step ratio with the first number to get the second number. The third number is obtained by multiplying a step ratio with the second number. Similarly the procedure is continued until the series is completed
-

Basic series	Preferred number
R5 (φ=1.6)	1.00, 1.60, 2.50, 4.00, 6.30,10.00
R10 (φ=1.26)	1.00, 1.25, 1.60, 2.00, 2.50, 3.15, 4.00, 5.00, 6.30, 8.00, 10.00
R20 (φ=1.12)	1.00, 1.06, 1.25, 1.18, 1.60, 1.25, 2.00, 2.24, 2.50, 2.80, 3.15, 3.55, 4.00, 4.50, 5.00, 5.60, 6.30, 7.10, 8.00, 9.00, 10.00
R40 (φ=1.06)	1.00, 1.06, 1.18, 1.25, 1.32, 1.18, 1.40,1.60, 1.70, 1.25,1.80, 1.90, 2.00, 2.10, 2.24, 2.36, 2.50, 2.65, 2.80, 3.00, 3.15, 3.35, 3.55, 4.00, 4.25, 4.50, 4.75, 5.00, 5.30, 5.60, 6.00, 6.30, 6.70, 7.10, 7.50, 8.00, 8.50, 9.00, 9.50, 10.00

Design of gear box

1. Selection of SpindleSpeed:
Find thestandardstep by using therelations,= ϕn-1

2. Structuralformula

It can be selected based on the number of speed:

Number of speed	Structural formula
6	3(1) 2(3) 2(1) 3(2)
8	2(1) 2(2) 2(4) 4(1) 2(4)
9	3(1) 3(3)
12	3(1) 2(3)2(6) 2(1) 3(2)2(6) 2(1) 2(2)3(4)
14	3(1) 3(3)2(5) 4(1) 2(4)2(6)
15	3(1) 3(3) 2(6)
16	4(1) 2(4)2(8) 2(1) 4(2)2(8) 2(1) 2(2)4(4)
18	3(1) 3(3)2(9) 3(1) 2(3)3(6) 2(1) 3(2)3(6)

3. Ray Diagram

The ray diagram is the graphical representation of the drive arrangement in general from. In other words, The ray diagram is the graphical representation of the structural formula.

The basic rules to be followed while designing the gear box as

- ✓ Transmissi n ration (i):

$\frac{1}{4}$ ≤ i ≤ 2

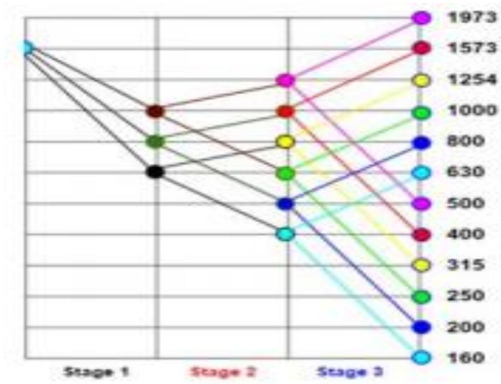
(or) $\frac{1}{4}$

$N_{min} / N_{input} \geq$

$i_{max} = N_{max} / N_{input} \leq 2$

For stable operation the speed ratio at any stage should not be greaterthan 8.

$N_{max} / N_{min} \leq 8$

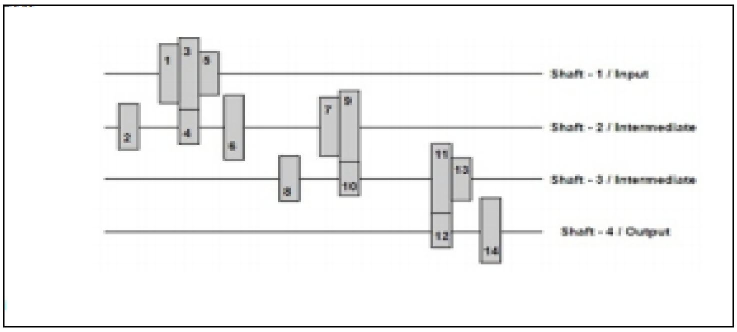


4. KinematicLayout:

The kinematic arrangement shows the arrangement of gears in a gear box.

Formula for kinematic arrangement,

$$n = p_1(X_1) \,.\, p_2(X_2)$$



5. Calculation of number of teeth.

In each stage first pair,

Assume, driver $Z_{min} \geq 17$,

Assume Z = 20 (driver)

SOLVEDPROBLEMS:

1. Design a 12 speed gear box with a minimum and maximum speed of 100 rpm and 355 rpm. Draw the ray and kinematic diagram and calculate number of teeth in each gear.

Given Data:

Z = 12, N1 = 100 rpm, N2 = 355 rpm

To Find: Design a Gear Box

Solution:

Step 1: Calculation of Step Ratio (ϕ) :

$$\frac{N_{max}}{N_{min}} = \phi^{z-1}$$

$$\frac{355}{100} = \phi^{11}$$

$$\phi = 1.12.$$

STEP 2: Selection of Intermediate speed from DB: 7.2

Based on the Step ratio 1.12 select the values

$N_1 = 100$ rpm.
$N_2 = 112$ rpm.
$N_3 = 125$ rpm.
$N_4 = 140$ rpm.
$N_5 = 160$ rpm.
$N_6 = 180$ rpm.
$N_7 = 200$ rpm
$N_8 = 224$ rpm
$N_9 = 250$ rpm
$N_{10} = 280$ rpm
$N_{11} = 315$ rpm
$N_{12} = 355$ rpm.

Step 3: Formation Of Structural Formula

For 12 speed 3 x 2 x 2

	Speed	Space.
12 = 3x2x2	$P_1 = 3$	$x_1 = 1$
	$P_2 = 2$	$x_2 = P_1 = 3$
	$P_3 = 2$	$x_3 = P_1 \times P_2 = 6$

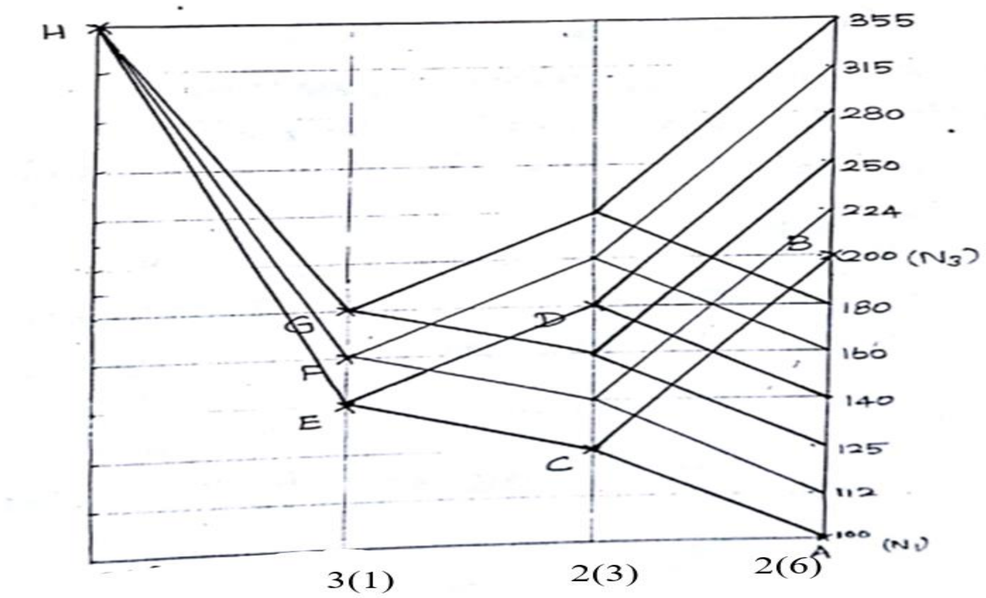
No. of Speed, Z = P1 (X1) . P2 (X2) . P3(X3)

Z = 3 (1) . 2 (3) . 2 (6)

I II III STAGE

Step 4: FORMATION OF RAY DIAGRAM

U – No. of Stages + 1 = 3 +1



4 vertical lines to be drawn and 12 Horizontal lines to be drawn

I II III

Location of C

$\frac{N_{max}}{N_{I/P}} \leq 2.$

$N_{max} = 200.$

$N_{I/P} = 100, 112, 125, 140, 160, 180$ rpm.

$\frac{N_{max}}{N_{I/P}} = \frac{200}{100} = 2.$

$\frac{200}{112} = 1.785$

$\frac{200}{125} = 1.6$

$\frac{200}{140} = 1.428$

$\frac{200}{160} = 1.25.$

$\frac{200}{180} = 1.11$

Avg. = 1.52.

The Average closer to the 125 rpm.

Optimum input Speed. = 125 rpm.

Location of E :

$\frac{N_{max}}{N_{I/P}} \leq 2$

$N_{max} = 180$ rpm.

$N_{I/P} = 125, 140, 160$ rpm.

* $\frac{N_{max}}{N_{I/P}} = \frac{180}{125} = 1.44$

$\frac{180}{140} = 1.28$

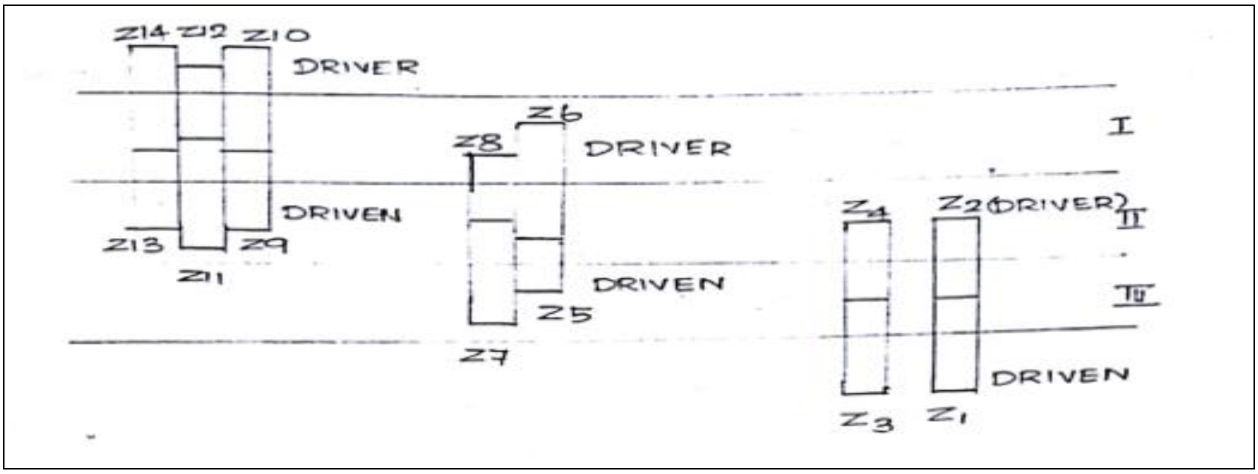
$\frac{180}{160} = 1.125$

Avg. = 1.28

The average is closer to 140 rpm.

Optimum I/P Speed is 140 rpm.

STEP 5: CONSTRUCTION OF KINEMATIC DIAGRAM



STEP 6: Calculation of No. of Teeth:

(Note: In every Stage Driver Speed should be constant)

IIIrd Stage:

Driver Speed (C) = 125 rpm

N2 = N4 = 125 rpm

Driven Speed (A, B) : N1 (A) = 100 rpm, N2 (B) = 200 rpm

Case(i) : [1st Pair 'AC']

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1}$$

Assume, $Z_2 = 20$.

NOTE : { Always Each pair 1st driver Gear = 20

$$* \frac{100}{125} = \frac{20}{Z_1}$$

$$Z_1 = 25.$$

Case (ii) : [2nd Pair 'BC']

$$\frac{N_3}{N_4} = \frac{Z_4}{Z_3}$$

$$\frac{200}{125} = \frac{Z_4}{Z_3}$$

$$Z_4 = 1.6 Z_3 \quad —(1)$$

We know that,

$$Z_1 + Z_2 = Z_3 + Z_4.$$

$$20 + 25 = Z_3 + 1.6 Z_3$$

$$2.6 Z_3 = 45$$

$$Z_3 = 17.$$

$$\therefore Z_4 = 1.6 \times 17. \Rightarrow 28.$$

$Z_3 = 17.$

$\therefore \quad Z_4 = 1.6 \times 17. \Rightarrow 28.$

2nd stage. [ED, EC Pair]

Case(i): Reducing pair.

* Driver Speed (E) -

$N_6 = N_8 = 140$ rpm.

* Driven Speed (C, D)

$N_5 = 125$ rpm.

$N_7 = 180$ rpm.

$$* \frac{N_5}{N_6} = \frac{Z_6}{Z_5}$$

Assume $Z_6 = 20.$

$$\frac{125}{140} = \frac{20}{Z_5}.$$

$Z_5 = .$

Case(ii) Increasing pair (CD)

$$\frac{N_7}{N_8} = \frac{Z_8}{Z_7}.$$

$$\frac{180}{140} = \frac{Z_8}{Z_7}.$$

We know that,

$$Z_5 + Z_6 = Z_7 + Z_8.$$

$$43 = 2.28 Z_7.$$

$$Z_7 = 19.$$

$$Z_8 = 1.27 \times 19.$$

$$Z_8 = 24.$$

1^{st} stage,

* HE, HF, HG Pair.

* Driver Speed (H)

$$N_{10} = N_{12} = N_{14} = 355 \text{ rpm}.$$

* Driven Speed (E, F, G).

$$N_9 \text{ (E)} = 140 \text{ rpm}.$$

$$N_{11} \text{ (F)} = 160 \text{ rpm}.$$

$$N_{13} \text{ (G)} = 180 \text{ rpm}.$$

Case (i) : Reducing pair (EH).

$$\frac{N_9}{N_{10}} = \frac{Z_{10}}{Z_9}.$$

$$\frac{140}{355} = \frac{Z_{10}}{Z_9} \Rightarrow \frac{20}{Z_9}$$

$$Z_9 = 51.$$

Case (ii) : Intermediate pair.

$$\frac{N_{11}}{N_{12}} = \frac{Z_{12}}{Z_{11}}.$$

$$\frac{160}{355} = \frac{Z_{12}}{Z_{11}}$$

$$Z_{12} = 0.45 Z_{11}$$

We know that,

$$Z_9 + Z_{10} = Z_{11} + Z_{12}.$$

$$71 = Z_{11} + 0.45 Z_{11}$$

$$\rightarrow Z_{11} = 49.$$

$$\rightarrow Z_{12} = 22.$$

Case (iii): Increasing pair (HG).

$$\frac{N_{13}}{N_{14}} = \frac{Z_{14}}{Z_{13}}$$

$$\frac{180}{355} = \frac{Z_{14}}{Z_{13}}$$

$$Z_{14} = 0.507 Z_{13} \quad \text{—— (3).}$$

We know that,

$$Z_{13} + Z_{14} = Z_{11} + Z_{12}.$$

$$71 = Z_{13} + 0.507 Z_{13}$$

$$\rightarrow Z_{13} = 47.$$

$$\rightarrow Z_{14} = 24.$$

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UNIT 5 – INTRODUCTIO N TO FINITE ELEMENT ANALYSIS

Objectives of Analysis:

Engineering analysisis adopted for machineries and buildingstructures beforeandafter assemblingtheirparts inordertodetermine

i. Thetypeandqualityofload ii.
Locationofloading
iii. Developedstress
iv. ermissibledeflection v.
Vibrationproperties
vi. Pressureandtemperaturevariation

Methods ofEngineeringAnalysis:

Thethreemethods adoptedforanalyzingengineeringproductstoevaluatetheir mechanicalandotherproperties are:

1) Experimental methods
2) Analyticalmethods
3) Numericalmethods orapproximationmethods

Experimentalmethods:

1) Inthesemethods,theactualproductortheirprototype modelarereallytested byusingtestingequipment.
2) Ifthereis aneedtochangethedimensions oftheprototype, theentireprototypeis tobedisassembledandtobe reassembledandthentesting shouldbecarriedout.
3) Itneedsmanpowerandmaterials.

AnalyticalMethods:

1) Thesemethodsaretheoreticallyanalyzing methods.

2) Onlysimpleandregularshapedproducts likebeams,columns,shafts,plates can beanalyzedbythesemethods

3) Theproducts andtheirloadings specifiedbymathematicalexpressions andthey areanalyzed.

NumericalMethods:

1) For the products of complicated sizes, shapes with complicated material propertiesandboundary conditions,gettingsolutionsusinganalyticalmethodis highlydifficult.Insuchsituation engineerprefersnumericalmethodsthatgives approximatebutacceptablesolutions.

2) Bythismethod, theapproximatebutacceptablesolutions willbeobtained.

Threemethods inNumericalmethods

i. FunctionalApproximatingMethod
ii. FiniteDifferenceMethod(FDM). iii. FiniteElementMethod(FEM).

FunctionalApproximation:

1. Inthismethod, thephysicalproblems arefirstwritteninterms ofdifferentialequationor anypossiblemathematicalexpressions.
2. Thentheapproximatesolutioncanbeobtainedbyintegrationandbyapplyingboundary condition.
3. Thevariationmethodspecificallyknownas Rayleigh-Ritzmethods andweighted residualmethods aresomeofthefunctionalapproximatingmethods.

FiniteDifferenceMethod(FDM):

1. Thefinitedifferencemethodapproximates thederivativesinthegoverningdifferential equationusingdifferenceequations.

2. Finitedifferencemethodis usefulforsolvingheattransferandfluidmechanics problems.
3. Thismethodcannotbeeffectivelyusedforregionshavingorirregularboundaries.

FiniteElement Method(FEM):

1. Inthismethod, thecomplexregiondefiningthedomainis dividedintosmaller elements calledfiniteelements.
2. Thephysicalpropertieslikeshape,dimensions andotherboundaryconditions are imposedontheelements.
3. Thentheseelements areassembledinaproperwayandthesolutionfortheentiresystem canbeobtained.

Steps inFEA

1. **Discretizationofstructure-**Dividingthewholecomplexstructureintofiniteelements bylines orsurfaces.
2. **Numberingofnodes andelements-**InFEM,physicalproblems aresolvedusing matricesandthesizeofthematrixdepends onthenumberofnodes oftheelement.
3. **Selectionofdisplacementfunction-**Linear,quadraticandcubicpolynomialsareusedto evaluatethevalueofthefieldvariableatanypartoftheelement.
4. **Formationof elementstiffness matrixandloadvector-**Basedonequilibrium conditionsorvariationalprinciples stiffnessmatrixisformulated.
5. **Formationofglobal stiffness matrixandloadvector-**Theelementstiffnessmatrices areassembledusingthefollowingformulaetogettheglobalstiffnessmatrix

   $[K][\delta] = [F]$

   where $[K]$ – Global stiffness matrix, $[\delta]$ – Nodal displacement vector and $[F]$ – Nodal force vector

6. **Incorporationofboundaryconditions**
7. **Computeelementstresses andstrain**

8. Analysis andinterpretationofresults

ClassificationofFunctionalApproximationMethods:

1. Variational Methods
2. Weightedresidualmethods

VariationalMethod: Rayleigh- Ritzmethod:

1. Itisatypicalvariationalmethodinwhichtheprincipleofintegralapproachis adopted forsolvingmostlythecomplexstructuralproblems.
2. Inthismethod, thepotentialenergy$\pi$is consideredas thefunctionofRitzparameters whichareonetoinfinity.
3. Inpractice,thedisplacementfunctiony(x)canbeexpressedinterms ofpolynomialseries ortrigonometricseries suchas,

$$y(x)= a_1+a_2x+a_3x^2+a_4x^3+\ldots \qquad (1.1)$$

wherea$_1$,a$_2$, a$_3$ …areknownas RitzparametersorRitzcoefficients.

4. Selectinganyoneoftheabovetwofunctions appropriatelyas atrialfunction,thetotal potentialenergycanbeformulated.
5. Thetotalpotentialenergyis thealgebraicsumof "Integralstrainenergyandexternal workdone".

   Mathematically,Totalpotentialenergy,π=U–W

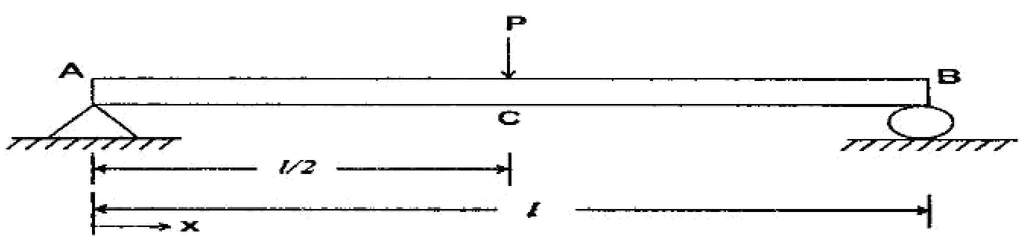
   WhereU–InternalstrainenergyandW–Workdonebyexternalforce
6. Bymakingthetotalpotentialenergytoreachminimumvalue(i.e.,stationarycondition),

theapproximatesolutioncanbedetermined.

7. Theaccuracyofthesolutiondepends onthenumberofRitzcoefficients.

Problem1

Findthedeflectionatthecenterofasimply supportedbeamofspanlengthl,subjectedtoa concentratedloadPatitsmid-point.



Thetotalpotentialenergyforabeamis givenby,π=U–W

Strain energy for a beam, $U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$

WhereEis themodulus ofelasticity,Iis theareamomentofinertiaofthebeam sectionandyis thedeflectionwhichcanbeexpressedas,

$$y=a_1+a_2x+a_3x^2+a_4x^3+\ldots. \tag{1}$$

tosimplifytheproblem,considerthefirstthreeterms suchas,

$$y=a_1+a_2x+a_3x^2 \tag{2}$$

Theboundaryconditions are y=0atx=0andx=l

Henceequations (2)becomes $0=a_1$and$0=a_2l+a_3l^2$whichgives $a_2=-a_3l$

Thenycanbeexpressedas,

$y=-a_3lx+a_3x^2=a_3(x^2-lx)$ (3)

Differentiatingtwotimes weget,

$\frac{dy}{dx} = a_3(2x - l) and \frac{d^2y}{dx^2} = 2a_3$

Thenstrainenergyis givenby,

$U = \frac{EI}{2}\int_0^l (2a_3)^2 dx = \frac{EI}{2} 4a_3{}^2 l = 2EIa_3{}^2 l$

Workdone,W=P* $y_{atx=l/2}$

$=Pa_3(x^2-lx)_{atx=l/2}$(fromequation(3))

$= Pa_3\left(\frac{l^2}{4} - \frac{l*l}{2}\right) \quad = -Pa_3\frac{l^2}{4}$

Thetotalpotentialenergyis givenby, $\pi=U-W$

$= 2EIa_3{}^2l - \left(-Pa_3\frac{l^2}{4}\right) = 2EIa_3{}^2l + Pa_3\frac{l^2}{4}$

Forminimumpotentialenergycondition,

$\frac{\partial \pi}{\partial a_3} = 0$

Substituting hevalueof$a_3$inEquation(3) weget,

$y = a_3(x^2 - lx) \quad = -\frac{Pl}{16EI}(x^2 - lx)$

Maximumdeflectionoccurs atx=l/2

Hence,

$$y_{max} = -\frac{Pl}{16EI}\left(\frac{l}{4}^{2} - l\,\frac{l}{2}\right) = -\frac{Pl}{16EI}\left(-\frac{l}{4}^{2}\right)$$

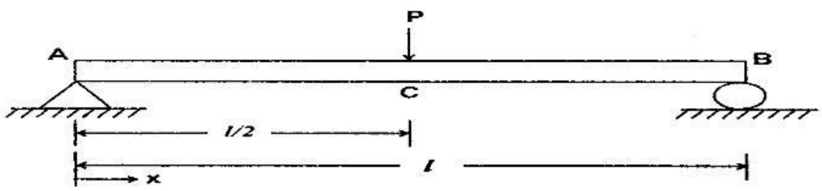
Therefore

$y_{max} = -\frac{Pl^3}{64EI}$

is theappaccuratesolution,thedisplacementfunctionshouldcontainmorenumberofRitz parameters.

Problem2

Findthedeflectionatthecenterofasimply supportedbeamofspanlengthlsubjectedtoa concentratedloadPatitsmid-pointusingtrailfunctionfromtrigonometricseries.



y=a1 — +a2 3— +a3 5— +…

Tosimplytheproblem,selectonetermfunctionas,

y=a1 —=a — (1)

Nowconsiderthepotentialenergyasπ=U–W

Strain energy for a beam, $U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$

Differentiatingthedisplacementfunctiontwotimes weget,

$$\frac{d^2y}{dx^2} = -a\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}$$

Then strain energy, $U = \frac{EI}{2}\int_0^l \left(\frac{d^2y}{dx^2}\right)^2 dx$

$$= \frac{EI}{2}\int_0^l \left(-a\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}\right)^2 dx \qquad (2)$$

$$= \frac{EI}{2}\left(-a\frac{\pi^2}{l^2}\right)^2 \int_0^l \sin^2\frac{\pi x}{l}\, dx$$

Now, $\int_0^l \sin^2\frac{\pi x}{l}\, dx = \int_0^l \frac{1}{2}\left(1-\cos\frac{2\pi x}{l}\right) dx$ (Since $\sin^2 A = \frac{1-\cos 2A}{2}$)

$$= \frac{1}{2}\left[x - \left\{\frac{\sin\frac{2\pi x}{l}}{\frac{2\pi}{l}}\right\}\right]_0^l = \frac{1}{2}\left[(l-0) - \frac{l}{2\pi}\{\sin 2\pi - \sin 0\}\right]$$

Nowtheequation(2)implies

Strain energy, $U = \frac{EI}{2}\int_0^l \left(-a\frac{\pi^2}{l^2}\sin\frac{\pi x}{l}\right)^2 dx = \frac{EI}{2}\left(-a\frac{\pi^2}{l^2}\right)^2 \frac{l}{2} = \frac{a^2\pi^4 EI}{4l^3}$

WorkDone,W=P*y_{max}

=P* $y_{atx=l/2}$ $= P\left(a\sin\frac{\pi x}{l}\right)_{at\ x=l/2}$ From Equation (1)

$$= Pa\sin\frac{\pi}{2} = Pa \quad (\text{Since } \sin\frac{\pi}{2} = 1)$$

The totalpotentialenergy, π= U– W

$$\pi = \frac{a^2\pi^4 EI}{4l^3} - Pa$$

$$\frac{\partial\pi}{\partial a} = 0 \rightarrow \frac{2a\pi^4 EI}{4l^3} = P \quad \text{Therefore, } a = \frac{2pl^3}{\pi^4 EI}$$

Maximumdeflectionoccurs
atx=l/2

$$\text{Hence } y_{max} = \left(a\sin\frac{\pi x}{l}\right)_{at\ x=l/2} \quad y_{max} = \left(\frac{2pl^3}{\pi^4 EI}\sin\frac{\pi x}{l}\right)_{at\ x=l/2} \quad = \frac{2pl^3}{\pi^4 EI}\sin\frac{\pi}{2}$$

$$\text{Therefore, } y_{max} = \frac{pl^3}{48.7EI}$$

**WeightedResidualMe
thod**

Theweightedresidual methodisemployed to obtain approximatesolutionstolinearandnon linearnonstructuralproblemswhosecharacteristicsareexpressedintermsof differential equations.Therequiredsimultaneousequationstofindthesolutioncanbederivedfrom the governingdifferentialequation, withoutknowingthefunctional.Themethods are

1. PointCollocationmethod
2. Subdomaincollocationmethod
3. Leastsquaremethod
4. Galerkin'smethod.

S

1. Lety_e(x)is theexactsolutionofthedifferentialequation
2. Anapproximatefunctioncalledthetrialfunctionisconsideredy(x)=f(x,a_i),i=1,2…. Andissubstitutedinthedifferentialequationtofindtheresidual R.Thetrialfunction shouldsatisfytheboundaryconditions.
3. Thisresidualisfurthertreatedtoevaluatetherequiredsolution.Itisessentialthatthe residualmultipliedbyaweighingfunction andthedomainintegral oftheproductshould bezero.
4. Thenumberofweighingfunctionsis equaltothenumberofunknowncoefficients inthe approximatefunction.

Point Collocation method:

1. In the collocation method, also called point collocation, the residual $R(x, a_i)$ is set equal to zero at n specific points $x_1, x_2, x_3, \ldots x_n$.
2. The weighting function Wi can be expressed as
   $$W_i = \int_D \delta(x - x_i) R(x, a_i) dx = 0$$
3. At point $x=x_i$, $W_i = 1$ and hence $R(x, a_i) = 0$.
4. At other points in the domain, $W_i = 0$.

Sub-domain Collocation method:

1. In this method, the domain is subdivided into n subdomains and the integral of the residual over each sub-domain is then required to be zero.
2. That is the weighting function selected is unity (W_i =1) over the domain

3. Here $\int_D R(x, a_i)dx = 0$

Least squares method:

1. In this method, the integral of the weighted square of the residual over the domain is required to be minimum.
2. That is $I = \int_D [R(x, a_i)]^2 dx = minimum$
3. The minimization of the integral is with respect to the unknown coefficients in the approximate solution.
4. That is $\frac{\partial I}{\partial a} = 0$

Galerkin's method:

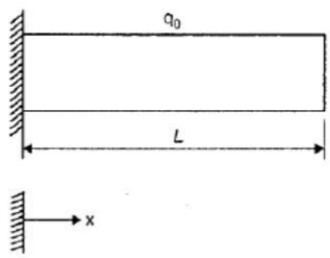
1. In this method, the trial function y(x) itself considered as the weighting function.
2. The domain integral of the product of trial function with the residual is then set equal to zero.

3. That is $\int_D W_i R(x, a_i)\, dx = \int_D y(x) R(x, a_i)\, dx = 0$

Problem

Consider a uniform rod subjected to a uniform axial load as illustrated in Figure. It can be readily shown that the deformation of bar is governed by the differential equation

$$AE\frac{d^2u}{dx^2} + q_0 = 0 \qquad (1)$$



$$u(0) = 0, \left|\frac{du}{dx}\right|_{x=L} = 0$$

Find an approximate solution of the above differential equation by using (i) Point Collocation method (ii) Sub domain Collocation method (iii) Least Square method and (iv) Galerkin's method

Solution:

The given differential equation is $AE\frac{d^2u}{dx^2} + q_0 = 0$

The boundary conditions are, $u(0) = 0, \left|\frac{du}{dx}\right|_{x=L} = 0$

Let us assume an approximate solution, $u(x) = a_1 + a_2 x + a_3 x^2$ (2)

(Order of approximate solution is equal to order of differential equation)

From Boundary equation, $u(0) = 0$

Hence, $u(0) = a_1 + a_2(0) + a_3(0)^2$ Therefore $a_1 = 0$

This method requires $\int_0^l R_d \, dx = 0$ (6)

Substituting (4) in (6) we get,

$\int_0^l (AE2a_3 + q_0) \, dx = 0$

The approximate solution becomes, $u(x) = a_2 x + a_3 x^2$

From $\left|\frac{du}{dx}\right|_{x=L} = 0$ $\left|\frac{du}{dx}\right|_{x=L} = a_2 + 2a_3 L = 0$ Hence, $a_2 = -2a_3 L$

Now the approximate solution becomes, $u(x) = -2a_3 Lx + a_3 x^2$

$u(x) = a_3(x^2 - 2Lx)$ (3)

To get residual equation, by substituting (3) in (1)

We get, $AE \frac{d^2}{dx^2}[a_3(x^2 - 2Lx)] + q_0 = R_d$

$AE \frac{d}{dx}[a_3(2x - 2L)] + q_0 = R_d$

$AE2a_3 + q_0 = R_d$ (4)

Point Collocation method:

In Collocation method, residual are set to zero. $(R_d = 0)$

Therefore, $AE2a_3 + q_0 = 0$

$AE2a_3 = -q_0$

Therefore, $a_3 = \frac{-q_0}{2AE}$ (5)

This method requires $\int_0^l R_d \, dx = 0$ (6)

Substituting (4) in (6) we get,

$$\int_0^l (AE2a_3 + q_0) \, dx = 0$$

By substituting (5) in (3), the final solution is, $u(x) = \frac{-q_0}{2AE}(x^2 - 2Lx)$

Sub domain collocation method:

Upon integration,

$$[AE2a_3x + q_0x]_0^l = 0$$

By substituting the limits we get,

$$[AE2a_3l + q_0l] = 0$$

Hence $a_3 = \frac{-q_0}{2AE}$ (7)

By substituting (7) in (3) we get,

$$u(x) = \frac{-q_0}{2AE}(x^2 - 2Lx)$$

(or) $u(x) = \frac{q_0}{2AE}\left(2Lx - x^2\right)$

Least Squares Method:

The functional $I = \int_0^l R_d^2 dx = minimum$ (8)

It can also be written as, $\frac{\partial I}{\partial a_3} = \int_0^l R_d \frac{\partial R_d}{\partial a_3} dx$

$\frac{\partial R_d}{\partial a_3} = 2AE$ (From (4))

$\frac{\partial I}{\partial a_3} = \int_0^l (AE2a_3 + q_0)(2AE) \, dx \quad \frac{\partial I}{\partial a_3} = \int_0^l (A^2E^2 4a_3 + 2AEq_0) dx$

Upon integration,

$$\frac{\partial I}{\partial a_3} = [A^2E^2 4a_3 x + 2AEq_0 x]_0^l$$

By substituting the limits we get,

$$\frac{\partial I}{\partial a_3} = [A^2E^2 4a_3 l + 2AEq_0 l]$$

The requirement is $\frac{\partial I}{\partial a_3} = 0$ Therefore, $[A^2E^2 4a_3 l + 2AEq_0 l] = 0$

$$A^2E^2 4a_3 l = -2AEq_0 l$$

Hence $a_3 = \frac{-q_0}{2AE}$ (10)

By substituting (10) in (3) we get,

$u(x) = \frac{-q_0}{2AE}(x^2 - 2Lx)$ (or) $u(x) = \frac{q_0}{2AE}(2Lx - x^2)$

(iv) Galerkin's Method:

In this method, the trial function itself is considered as the weighting function and this method requires,

$\int_0^l W_i R_d dx = 0$ where i = 1 to n (11)

Hence $u(x) = W_i = a_3(x^2 - 2Lx)$ (12)

Substituting (4) and (12) in (11) we get,

$$\int_0^l a_3(x^2 - 2Lx)(AE2a_3 + q_0)dx = 0$$

$$\int_0^l [2AEa_3{}^2x^2 + a_3x^2q_0 - 4AELxa_3{}^2 - 2Lxa_3q_0]dx = 0$$

Upon integration,

$$\left[\frac{2AEa_3{}^2x^3}{3} + \frac{a_3x^3q_0}{3} - \frac{4AELx^2a_3{}^2}{2} - \frac{2Lx^2a_3q_0}{2}\right]_0^l = 0$$

$$\left[\frac{2AEa_3{}^2x^3}{3} + \frac{a_3x^3q_0}{3} - 2AELx^2a_3{}^2 - Lx^2a_3q_0\right]_0^l = 0$$

By substituting the limits we get,

$$\left[\frac{2AEa_3{}^2L^3}{3} + \frac{a_3L^3q_0}{3} - 2AELL^2a_3{}^2 - LL^2a_3q_0\right] - [0] = 0$$

$$\left[\frac{2AEa_3{}^2L^3}{3} + \frac{a_3L^3q_0}{3} - 2AEL^3a_3{}^2 - L^3a_3q_0\right] = 0$$

Dividing by a_3 and L^3, we get,

$$\left[\frac{2AEa_3}{3} + \frac{q_0}{3} - 2AEa_3 - q_0\right] = 0$$

$$\frac{2AEa_3}{3} - 2AEa_3 = q_0 - \frac{q_0}{3} \qquad -\frac{4AEa_3}{3} = \frac{2q_0}{3}$$

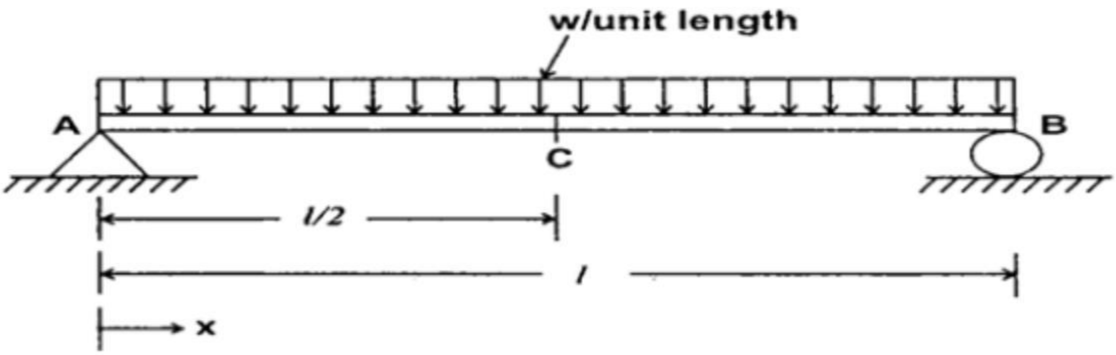
Hence $a_3 = \frac{-q_0}{2AE}$ (13)

By substituting (13) in (3) we get,

$$u(x) = \frac{-q_0}{2AE}(x^2 - 2Lx) \qquad \text{(or) } u(x) = \frac{q_0}{2AE}\left(2Lx - x^2\right)$$

Substituting the eqn. (3) in the governing differential equation, we get the

Find the deflection at the centre of a simply supported beam of span length l subjected to uniformly distributed load throughout its length as shown in figure using (a) Point Collocation method, (b) Sub-domain collocation method (c) Least squares method, (d) Galerkin's method.



The differential equation governing the deflection of beam subjected to uniformly distributed load is given by

$$EI\frac{d^4y}{dx^4} - w = 0, \qquad 0 \le x \le l$$

Now, let us select the trial function for deflection as

$$y = a \sin\frac{\pi x}{l} \qquad \text{.... (1)}$$

The boundary conditions to be satisfied are $y = 0$ at $x = 0$ and $x = l$ where y is the deflection and $EI\frac{d^2y}{dx^2} = 0$ at $x = 0$ and $x = l$ where $EI\frac{d^2y}{dx^2} = M$ (Bending moment) and E = Young's modulus, I = Moment of inertia of the beam.

$$\frac{dy}{dx} = a\frac{\pi}{l}\cos\frac{\pi x}{l}$$

$$\frac{d^2y}{dx^2} = -a\frac{\pi^2}{l^2}\sin\frac{\pi x}{l} \qquad \text{.... (2)}$$

Eqn. (1) satisfies the boundary conditions as $y = 0$ at $x = 0$ and $x = l$

Similarly the eqn. (2) satisfies the boundary conditions as $EI\frac{d^2y}{dx^2} = 0$ at $x = 0$ and $x = l$

$$\frac{d^3y}{dx^3} = -a\frac{\pi^3}{l^3}\cos\frac{\pi x}{l} \quad \text{and} \quad \frac{d^4y}{dx^4} = a\frac{\pi^4}{l^4}\sin\frac{\pi x}{l} \qquad \text{.... (3)}$$

Substituting the eqn. (3) in the governing differential equation, we get the residual as

$$R = EI\, a \frac{\pi^4}{l^4} \sin\frac{\pi x}{l} - w \quad \text{.... (4)}$$

(a) Point Collocation Method:

Inthismethodtheresidual is set to zero.

i.e., $EI\, a \frac{\pi^4}{l^4} \sin\frac{\pi x}{l} - w = 0$

To get maximum deflection, take $x = \frac{l}{2}$ (i.e., at the centre of beam)

Then, $EI\, a \frac{\pi^4}{l^4} \sin\frac{\pi}{l}\left(\frac{l}{2}\right) = w$ $\qquad a = \frac{w\, l^4}{\pi^4 EI}$

The trial function $y = \frac{w\, l^4}{\pi^4 EI} \sin\frac{\pi x}{l}$ $\qquad$ At $x = \frac{l}{2}$, $y_{max} = \frac{w\, l^4}{\pi^4 EI} \sin\frac{\pi}{2}$

(b) Sub-domain Collocation Method:

In this method, the integral of the residual over the sub-domain is set to zero.

i.e., $\int_0^l R\, dx = 0$ $\quad$ i.e., $\int_0^l \left(a\, EI \frac{\pi^4}{l^4} \sin\frac{\pi x}{l} - w \right) dx = 0$

i.e., $\left[a\, EI \frac{\pi^4}{l^4} \left(-\cos\frac{\pi x}{l} \right)\left(\frac{l}{\pi}\right) - w\, x \right]_0^l = 0$

$\therefore a = \frac{w\, l^4}{2\, \pi^3 EI}$ $\qquad$ At $x = \frac{l}{2}$, $y_{max} = \frac{wl^4}{62EI} \sin\frac{\pi}{2}$

(c) Least Squares Method:

In this method the functional

$$I = \int_0^l R^2 \, dx \text{ is minimum}$$

$$\text{Now } I = \int_0^l \left(a\,EI \frac{\pi^4}{l^4} \sin \frac{\pi x}{l} - w \right)^2 dx$$

$$= \int_0^l \left(a^2 E^2 I^2 \frac{\pi^8}{l^8} \sin^2 \frac{\pi x}{l} + w^2 - 2a\,EI\,w \frac{\pi^4}{l^4} \sin \frac{\pi x}{l} \right) dx$$

$$= \int_0^l \left(a^2 E^2 I^2 \frac{\pi^8}{l^8} \left\{ \frac{1 - \cos\left(\frac{2\pi x}{l}\right)}{2} \right\} + w^2 - 2a\,EI\,w \frac{\pi^4}{l^4} \sin \frac{\pi x}{l} \right) dx$$

$$= \frac{a^2 E^2 I^2 \pi^8}{2\,l^7} + w^2 l - 4\,a\,EI\,w \frac{\pi^3}{l^3}$$

Now,

$$\frac{\partial I}{\partial a} = 0 \Rightarrow a\,E^2 I^2 \frac{\pi^8}{l^7} = 4\,EI\,w \frac{\pi^3}{l^3} \qquad \therefore\ a = \frac{4\,w\,l^4}{\pi^5\,EI}$$

$$\text{At } x = \frac{l}{2}, \text{ Maximum deflection, } y_{max} = \frac{4\,w\,l^4}{\pi^5\,EI} \cdot \sin \frac{\pi}{2}$$

Galerkin's Method:

$$\text{In this method, } \int_0^l (y\,R)\,dx = 0$$

$$\text{i.e., } \int_0^l \left\{ \left(a \sin \frac{\pi x}{l} \right) \left(a\,EI \frac{\pi^4}{l^4} \sin \frac{\pi x}{l} - w \right) \right\} dx = 0$$

Solving weget

$$a = \frac{2\,w\,l}{\pi} \frac{2\,l^3}{EI\,\pi^4} = \frac{4\,w\,l^4}{\pi^5\,EI} \qquad \text{At } x = \frac{l}{2}, y_{max} = \frac{4w\,l^4}{\pi^5\,EI}$$

Problems forpractice

1. Derivetheexpressionfordeflectionandbendingmomentinasimplysupportedbeamof spanoflengthl,subjectedtoUDLoverentirespanusingtwoterm trigonometrictrial functionusingRayleighRitzmethod.

   Consider the differential equation for a problem such as $\frac{d^2y}{dx^2} + 300x^2 = 0; 0 \le x \le 1$ with the boundary conditions y(0) = y(1) = 0. The functional corresponding to this problem to be extremized is given by, $I = \int_0^1 \left\{-\frac{1}{2}\left(\frac{dy}{dx}\right)^2 + 300x^2 y\right\} dx$ Find the solution of the problem using Rayleigh-Ritz method using a one term solution as $y = ax(1 - x^3)$

2. **Consider the differential equation for a problem as**

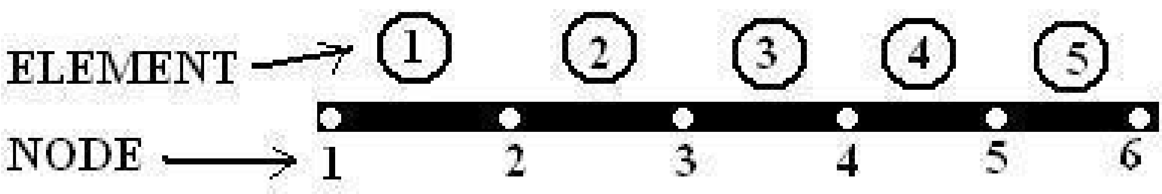
$$\frac{d^2 y}{dx^2} + 300\, x^2 = 0, \qquad 0 \le x \le 1.$$

   with the boundary conditions $y(0) = 0$, $y(1) = 0$. Find the solution of the problem using a one coefficient trial function as $y = a_1 x(1 - x^3)$. Use (i) Point collocation method, (ii) Sub-domain collocation method, (iii) Least square method and (iv) Galerkin's method.

3. Solve thefollowingequationusingatwoparametertrial solutionby(a)Point Collocationmethodand(b)Galerkin's method.

ONE DIMENSIONAL ELEMENT

Barandbeam elementsareconsideredasOneDimensional elements.Theseelementsare oftenusedtomodeltrusses andframestructures. Baris amember whichresists onlyaxialloads. Abeamcanresistaxial,lateralandtwistingloads.Atrussisanassemblageofbarswithpin joints andaframeis anassemblageofbeamelements.



As showninthefigure,aonedimensional structureis dividedintoseveralelements andtheeach elementhas 2nodes.

Shapefunction

N1N2N3areusuallydenotedas shapefunction.Inonedimensionalproblem,thedisplacementu =∑Niui=N1u1

Fortwonodedbar element, thedisplacementatanypointwithintheelementis givenby,

u=N1u1+N2u2

Forthreenodedtriangular element, thedisplacementatanypointwithintheelementis givenby,

$u=N_1u_1+N_2u_2+N_3u_3$

$v=N_1v_1+N_2v_2+N_3v_3$

Shapefunctionneedtosatisfythefollowing

4) Firstderivatives shouldbefinitewithinanelement;

5) Displacementshouldbecontinuous acrosstheelementboundary

PROBLEMS

Consider a bar as shown in Fig.(i). An axial load of 200 kN is applied at point p. Take $A_1 = 2400\ mm^2$, $E_1 = 70 \times 10^9\ N/m^2$, $A_2 = 600\ mm^2$, $E_2 = 200 \times 10^9\ N/m^2$.

Calculate the following:

(a) The nodal displacement at point p.

(b) Stress in each material.

(c) Reaction force.

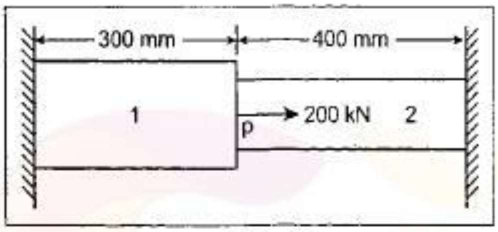


Fig. (i)

Given:

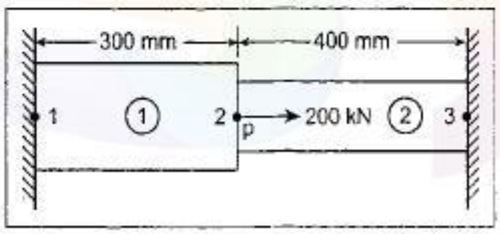
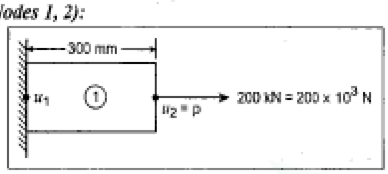


Fig. (ii)

$$\text{Area of element (1), } A_1 = 2400\ mm^2$$
$$\text{Area of element (2), } A_2 = 600\ mm^2$$
$$\text{Length of element (1), } l_1 = 300\ mm$$
$$\text{Length of element (2), } l_2 = 400\ mm$$
$$\text{Young's modulus of element (1), } E_1 = 70 \times 10^9\ N/m^2$$
$$= 70 \times 10^3\ N/mm^2$$

For element 1: (Nodes 1, 2):



Finite element equation is,

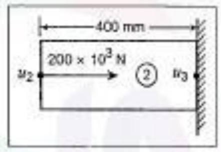
$$\frac{A_1 E_1}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \quad \frac{2400 \times 70 \times 10^3}{300}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \quad 1 \times 10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad \text{... (1)}$$

For element 2: (Nodes 2, 3): Finite element equation is,

$$\frac{A_2 E_2}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$



$$\Rightarrow \quad \frac{600 \times 200 \times 10^3}{400}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \quad 1 \times 10^5 \begin{bmatrix} \overset{a_{22}}{3} & \overset{a_{23}}{-3} \\ \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} \qquad \ldots (2)$$

Assemble the finite elements. *i.e.*, assemble the finite element equations (1) and (2).

$$\Rightarrow \quad 1 \times 10^5 \begin{bmatrix} \overset{a_{11}}{5.6} & \overset{a_{12}}{-5.6} & \overset{a_{13}}{0} \\ \overset{a_{21}}{-5.6} & \overset{a_{22}}{5.6+3} & \overset{a_{23}}{-3} \\ \overset{a_{31}}{0} & \overset{a_{32}}{-3} & \overset{a_{33}}{3} \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \quad 1 \times 10^5 \underbrace{\begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix}}_{[\,K\,]}\begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \qquad \ldots (3)$$

[*Note:* The bar has 3 nodes. Each node has single degree of freedom. So, the global stiffness matrix [K] size is 3 × 3. The properties of the stiffness matrix are also satisfied.

(*i*) [K] matrix is symmetric.

(*ii*) The sum of elements in any column is equal to zero.]

Applying boundary conditions:

Displacements at node 1 and node 3 are zero. So, $u_1 = u_3 = 0$. A load of 200×10^3 N is acting at node 2. So, $F_2 = 200 \times 10^3$ N. Self-weight is neglected. *i.e.*, $F_1 = F_3 = 0$. Substitute u_1, u_3 and F_1, F_2 and F_3 values in equation (3).

In the above equation, $u_1 = 0$. So, neglect first row and first column of [K] matrix. $u_3 = 0$, so, neglect third row and third column of [K] matrix. The final reduced equation is,

$$1 \times 10^5\,[\,8.6\,]\,\{\,u_2\,\} = \{\,2 \times 10^5\,\}$$

$$\Rightarrow \quad 8.6 \times 10^5\, u_2 = 2 \times 10^5$$

$$8.6\, u_2 = 2$$

$$\boxed{u_2 = 0.2325 \text{ mm}}$$

Stress in each element:

We know that, $\quad$ Stress, $\sigma = E\,\dfrac{du}{dx}$

For element (1), $\quad$ Stress, $\sigma_1 = E_1 \times \dfrac{u_2 - u_1}{l_1} = 70 \times 10^3 \times \dfrac{(0.2325 - 0)}{300}$

http://Easyengineering.net N/mm²

For element (2), $\quad$ Stress, $\sigma_2 = E_2 \times \dfrac{u_3 - u_2}{l_2}$

$$= 200 \times 10^3 \times \frac{(0 - 0.2325)}{400}$$

$$\Rightarrow \quad \sigma_2 = -116.25 \text{ N/mm}^2 \text{ (Compressive stress is acting)}$$

Reaction force: We know that,

Reaction force, $\{R\} = [K]\{u^*\} - \{F\}$

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} - \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = 1 \times 10^5 \begin{bmatrix} 5.6 & -5.6 & 0 \\ -5.6 & 8.6 & -3 \\ 0 & -3 & 3 \end{bmatrix} \begin{Bmatrix} 0 \\ 0.2325 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{bmatrix} 0 - 5.6\,(0.2325) + 0 \\ 0 + 8.6\,(0.2325) + 0 \\ 0 - 3\,(0.2325) + 0 \end{bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= 1 \times 10^5 \begin{Bmatrix} -1.302 \\ 2 \\ -0.6975 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} -1.302 \times 10^5 \\ 2 \times 10^5 \\ -0.6975 \times 10^5 \end{Bmatrix} - \begin{Bmatrix} 0 \\ 2 \times 10^5 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = \begin{Bmatrix} -1.302 \times 10^5 \\ 0 \\ -0.6975 \times 10^5 \end{Bmatrix}$$

$$\Rightarrow \quad R_1 = -1.302 \times 10^5 \text{ N}$$

$$R_2 = 0 \text{ N}$$

$$R_3 = -0.6975 \times 10^5 \text{ N}$$

We know that, Reaction force is equivalent and opposite to the applied force.

Verification: $R_1 + R_2 + R_3 = -1.302 \times 10^5 + 0 - 0.6975 \times 10^5$

$= -200 \times 10^3$ N (Applied force)

Result: (*i*) Nodal displacement at point p, *i.e.*, $u_2 = 0.2325$ mm

(*ii*) Stress in each material, $\sigma_1 = 54.25$ N/mm^2 (tensile)

$\sigma_2 = -116.25$ N/mm^2 (compressive)

(*iii*) Reaction forces, $R_1 = -1.302 \times 10^5$ N; $R_2 = 0$

$R_3 = -0.6975 \times 10^5$ N

Problem2.

A thin steel plate of uniform thickness 25 mm is subjected to a point load of 420 N at mid depth as shown in Fig.(i). The plate is also subjected to self-weight. If Young's modulus, $E = 2 \times 10^5$ N/mm² and unit weight density, $\rho = 0.8 \times 10^{-4}$ N/mm³, calculate the following:

(i) Displacement at each nodal point.

(ii) Stresses in each element.

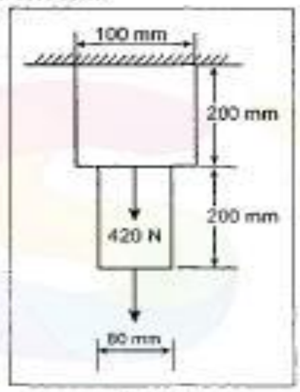


Fig. (i)

Given:

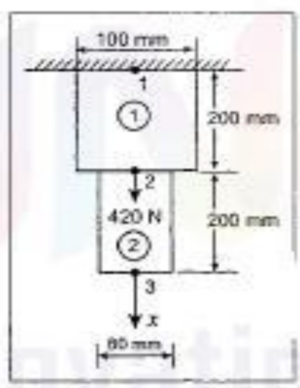


Fig. (ii)

Thickness, $t = 25$ mm

For element (1): Area, $A_1 = 100 \times 25 = 2500$ mm²

For element (2): Area, $A_2 = 80 \times 25 = 2000$ mm²

Point load, $p = 420$ N

Young's modulus, $E = 2 \times 10^5$ N/mm²

Unit weight density, $\rho = 0.8 \times 10^{-4}$ N/mm³

To find: (*i*) Displacement at each nodal points, u_1, u_2 and u_3.

(*ii*) Stress in each element, σ_1 and σ_2.

☺ ***Solution:*** The steel plate is subjected to self-weight. So, we have to find the body force acting at nodal point 1, 2 and 3.

We know that, Body force vector, $\{F\} = \frac{\rho A l}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$ [From equation no.(2.44)]

For element (1): Force vector, $\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \frac{\rho_1 A_1 l_1}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

$$= \frac{0.8\times10^{-4}\times2500\times200}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = 20\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$\begin{Bmatrix}F_1\\F_2\end{Bmatrix} = \begin{Bmatrix}20\\20\end{Bmatrix} \quad \text{... (1)}$$

For element (2): Force vector, $\begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \frac{\rho_2 A_2 l_2}{2}\begin{Bmatrix}1\\1\end{Bmatrix}$

$$= \frac{0.8\times10^{-4}\times2000\times200}{2}\begin{Bmatrix}1\\1\end{Bmatrix} = 16\begin{Bmatrix}1\\1\end{Bmatrix}$$

$$\begin{Bmatrix}F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}16\\16\end{Bmatrix} \quad \text{... (2)}$$

Assembling the force vector, *i.e.*, assemble the equation (1) and (2).

$$\Rightarrow \begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}20\\20+16\\16\end{Bmatrix} = \begin{Bmatrix}20\\36\\16\end{Bmatrix}$$

A point load of 420 N is acting at mid depth *i.e.*, at nodal point 2, as shown in Fig.(*ii*) So, add 420 N in F_2 vector.

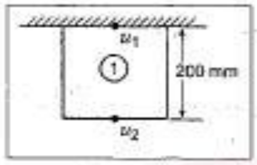
$$\Rightarrow \begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}20\\36+420\\16\end{Bmatrix}$$

$$\boxed{\text{Global force vector} \begin{Bmatrix}F_1\\F_2\\F_3\end{Bmatrix} = \begin{Bmatrix}20\\456\\16\end{Bmatrix}} \quad \text{... (3)}$$

Finite element equation for one dimensional plate element is given by,

$$\begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} = \frac{AE}{l}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$ [From equation no.(2.36)]

For element 1: (Nodes 1, 2):



Finite element equation is,

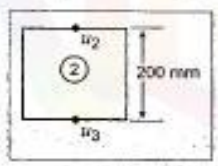
$$\frac{A_1 E}{l_1}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \quad \frac{2500 \times 2 \times 10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix}$$

$$\Rightarrow \quad 2 \times 10^5 \begin{array}{c} \begin{array}{cc} \mathbf{1} & \mathbf{2} \end{array} \\ \begin{bmatrix} 12.5 & -12.5 \\ -12.5 & 12.5 \end{bmatrix} \end{array} \begin{array}{c} \\ \mathbf{1} \\ \mathbf{2} \end{array} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad \ldots (4)$$

For element 2: (Nodes 2, 3): Finite element equation is,

$$\frac{A_2 E}{l_2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$



$$\Rightarrow \quad \frac{2000 \times 2 \times 10^5}{200}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \quad 2 \times 10^5 \begin{array}{c} \begin{array}{cc} \mathbf{2} & \mathbf{3} \end{array} \\ \begin{bmatrix} 10 & -10 \\ -10 & 10 \end{bmatrix} \end{array} \begin{array}{c} \\ \mathbf{2} \\ \mathbf{3} \end{array} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_2 \\ F_3 \end{Bmatrix} \qquad \ldots (5)$$

Assemble the finite elements *i.e.*, assemble the finite element equations (4) and (5).

$$\Rightarrow \quad 2 \times 10^5 \begin{array}{c} \begin{array}{ccc} \mathbf{1} & \mathbf{2} & \mathbf{3} \end{array} \\ \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 12.5+10 & -10 \\ 0 & -10 & 10 \end{bmatrix} \end{array} \begin{array}{c} \\ \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix}$$

$$\Rightarrow \quad 2 \times 10^5 \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 22.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \end{Bmatrix} \qquad \ldots (6)$$

Apply the boundary conditions *i.e.*, at node 1, displacement $u_1 = 0$. Substitute u_1, F_1, F_2 and F_3 values in equation (6).

$$\Rightarrow \quad 2 \times 10^5 \begin{bmatrix} 12.5 & -12.5 & 0 \\ -12.5 & 22.5 & -10 \\ 0 & -10 & 10 \end{bmatrix} \begin{Bmatrix} 0 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 456 \\ 16 \end{Bmatrix}$$

In the above equation, $u_1 = 0$. So, neglect first row and first column of [K] matrix. The reduced equation is,

$$\Rightarrow \quad 2 \times 10^5 \begin{bmatrix} 22.5 & -10 \\ -10 & 10 \end{bmatrix} \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 456 \\ 16 \end{Bmatrix}$$

$$\Rightarrow \quad 2 \times 10^5 (22.5\, u_2 - 10\, u_3) = 456 \qquad \ldots (7)$$

$$2 \times 10^5 (-10\, u_2 + 10\, u_3) = 16 \qquad \ldots (8)$$

$$\text{Solving,} \quad 2 \times 10^5 (12.5\, u_2) = 472$$

$$\Rightarrow \boxed{u_2 = 1.888 \times 10^{-4} \text{ mm}}$$

Substitute u_2 value in equation (7),

$$\Rightarrow \quad 2 \times 10^5 [-10 (1.888 \times 10^{-4}) + 10\, u_3] = 16$$

$$\Rightarrow \quad -10 (1.888 \times 10^{-4}) + 10\, u_3 = 8 \times 10^{-5}$$

$$\Rightarrow \quad -10\, u_3 = 1.968 \times 10^{-3}$$

$$\Rightarrow \quad \boxed{u_3 = 1.968 \times 10^{-4} \text{ mm}}$$

We know that, Stress, $\sigma = E \dfrac{du}{dx}$

For element (1): $\sigma_1 = E \times \dfrac{u_2 - u_1}{l_1} = 2 \times 10^5 \times \dfrac{1.888 \times 10^{-4} - 0}{200}$

$$\boxed{\sigma_1 = 0.188 \text{ N/mm}^2}$$

For element (2): $\sigma_2 = E \times \dfrac{u_3 - u_2}{l_2}$

$$= 2 \times 10^5 \times \frac{(1.968 \times 10^{-4} - 1.888 \times 10^{-4})}{200}$$

$$\boxed{\sigma_2 = 0.008 \text{ N/mm}^2}$$

Result: (*i*) Displacement at each nodal points:

$$u_1 = 0$$
$$u_2 = 1.888 \times 10^{-4} \text{ mm}$$
$$u_3 = 1.968 \times 10^{-4} \text{ mm}$$

(*ii*) Stresses in each element:

$$\sigma_1 = 0.188 \text{ N/mm}^2$$
$$\sigma_2 = 0.008 \text{ N/mm}^2$$