SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE
www.sathyabama.ac.in

SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
GAS DYNAMICS AND JET PROPULSION (SME1303)

UNIT – I
COMPRESSIBLE FLUID FLOW – FUNDAMENTALS

Lecture 1 / Unit 1

Introduction

Fluid is a substance that continuously deforms when shearing forces are applied. E.g., Liquids, gases, vapors and their mixtures.

Mechanics – study of forces acting on bodies. This can be divided into statics, dynamics and kinematics.

Fluid Mechanics - study of the mechanical properties of fluids, when it is subjected to internal and external forces.

Fluid Dynamics - study concerned with the movement of gases and liquids.

Types of fluids – Ideal (non viscous / inviscid), Real (viscous) and Perfect (non viscous and incompressible / Perfect fluid do not stick to the surface with which they are in contact).

Incompressible fluid – density does not change with time. i.e., $\frac{\partial \rho}{\partial t} = 0$ or $\nabla \rho = 0$. Flow of gases at low Mach number (< 0.3) can be assumed as incompressible.

Compressible fluid – density varies with pressure. Flow at Mach numbers higher than 0.3 is considered as compressible.

Gas dynamics – compressible fluid dynamics. Gas dynamics is a branch of fluid dynamics concerned with studying the motion of gases and its consequent effects. Gas dynamics combines the principles of fluid mechanics and thermodynamics.

System, surrounding, boundary, closed system, open system, isolated system.

Ideal gas – which obeys Boyle's law (pV = k, if temperature is kept constant within a closed system) and Charles's law (V α T, if pressure is constant / T is abs. temp.).

Semi perfect gas – is an ideal gas whose specific heats are functions of temperature. i.e., $C_p = f(T)$ and $C_v = f(T)$.

Perfect gas – whose specific heats remain constant at all temperatures.

<u>Types of energy forms in TD system:-</u>

1. **Potential energy** – energy possessed by the fluid by virtue of its height = gZ.
2. **Kinetic energy** – energy possessed by the fluid by virtue of its motion = $\frac{C^2}{2}$.
3. **Internal energy(u)** – Energy stored in the gas by virtue of its molecular motion. At temperature 'T', it is given by $u = C_v T$. In compressible flow, 'u' appears with the quantity *pv*.

   Hence, U + *pv* = h = U + $\frac{p}{\rho}$.

   For a perfect gas, h = $C_v T$ + RT = (C_v + R) T = C_p T.

4. Flow energy or displacement energy(*pv*) – The energy required to push the fluid into or out of the control volume is called the flow work or flow energy. Flow energy is necessary for maintaining a continuous flow through a control volume.

1 – Dimensional flow: Flow properties, such as pressure and velocity, at a given instant of time vary only in the direction of flow and not across the cross section.

Steady flow: Fluid properties, such as pressure, temperature and velocity, in the control volume do not change with time.
Unsteady flow: When one or more fluid property in the control volume change with time.
General form of energy equation:

$$Q = W + (U_2 - U_1) + mg(Z_2 - Z_1) + \frac{1}{2}m(c_2^2 - c_1^2)$$

Energy equation for a non flow process: (expansion and compression of gases in a cylinder with piston). The potential and kinetic energy terms are negligible compared to other quantities and the work term W includes only shaft work.

$$Q = W_s + (U_2 - U_1)$$

Energy equation for a flow process: (expansion of steam and gas in turbines and compression of air and gases in turbo compressors). In this, W includes the flow work also.

$$W = W_s + (p_2V_2 - p_1V_1)$$

and hence the energy equation can be written as

$$h_1 + gZ_1 + \frac{1}{2}c_1^2 + q = h_2 + gZ_2 + \frac{1}{2}c_2^2 + w_s \text{ --- } \textbf{SFEE}$$

Generally in flow problems of gases and vapors the magnitude of $g(Z_2 - Z_1)$ is negligible compared to other quantities. Therefore,

$$h_1 + \frac{1}{2}c_1^2 + q = h_2 + \frac{1}{2}c_2^2 + w_s \text{ --- } \textbf{SFEE}$$

Adiabatic Energy Equation (AEE): In some engineering problems, the heat transfer q during the process is negligibly small and can be ignored. Expansion of gases and vapors in turbines are examples of such processes.

Hence, $h_1 + \frac{1}{2}c_1^2 = h_2 + \frac{1}{2}c_2^2 + w_s$

AEE is involved for processes involving both energy transfer and energy transformation. Some adiabatic processes involve only energy transformation, e.g., expansion of gases in nozzles and compression of gases in diffusers. In these, shaft work is absent

$$h_1 + \frac{1}{2}c_1^2 = h_2 + \frac{1}{2}c_2^2 \text{ (when change in elevation is ignored)}$$

Stagnation or Total enthalpy (h_0): Stagnation enthalpy of a gas or a vapor is its enthalpy when it is adiabatically decelerated to zero velocity at zero elevation. Putting $h_1 = h$ and $c_1 = c$ for the initial state; $h_2 = h_0$ and $c_2 = 0$ for the final state, then

$$h_0 = h + \frac{1}{2}c^2$$

Consider a steady flow of a fluid through a duct such as nozzle or diffuser, where the flow takes place adiabatically with no shaft work and negligible potential energy, we get

$$h_1 + \frac{1}{2}c_1^2 = h_2 + \frac{1}{2}c_2^2 + w_s$$

$$h_{01} = h_{02} \Rightarrow \text{Stagnation enthalpy remains constant.}$$

Any increase in fluid velocity in these devices will create an equivalent decrease in the static enthalpy 'h' of the fluid.

If the fluid were brought to a complete stop, then the velocity at state 2 would be zero, then $h_1 + \frac{1}{2}c_1^2 = h_2 = h_{02} = h_{01}$.

Thus the stagnation enthalpy represents the enthalpy of a fluid when the fluid is brought to rest adiabatically. During a stagnation process, the KE of the fluid is converted to enthalpy, which results in the increase in the fluid temperature and pressure. The properties of the fluid at the stagnation state are called as stagnation properties. It is represented by the subscript 0.

Stagnation temperature(T_0): It is the temperature of the gas when the gas is adiabatically decelerated to zero velocity at zero elevation.

WKT, $h_0 = h + \frac{1}{2}c^2$

The above equation for a perfect gas can be written as

$$c_p T_0 = c_p T + \frac{1}{2}c^2$$

$T_0 = T + \frac{c^2}{2c_p}$. The quantity $\frac{c^2}{2c_p}$ is known as the **velocity temperature (T_c)** corresponding to the velocity c. It is also called as dynamic temperature.

$$T_0 = T + T_c$$

Also, $\frac{T_0}{T} = 1 + \frac{c^2}{2c_p T} = 1 + \frac{c^2}{2\gamma RT/(\gamma - 1)}$

But $\gamma RT = a^2$ and $\frac{c^2}{a^2} = M^2$, Where a = velocity of sound and M = Mach number.

Therefore, $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\frac{c^2}{a^2} = 1 + \frac{\gamma - 1}{2}M^2$

Note: For low speed flow, stagnation and static temperatures are practically identical. But for high speed flows $T_0 \rangle T$.

Velocity of sound (a): The velocity of sound in a medium at a temperature T is given by $a = \sqrt{\gamma RT}$; where $\gamma =$ ratio of specific heats; R = gas constant; T= absolute temperature in K.

Also: $a = \left[\sqrt{\frac{\partial p}{\partial \rho}}\right]_s = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT} = \sqrt{\gamma \frac{\overline{R}}{W} T}$; where K = Bulk Modulus, W = Molar mass, $\overline{R}$ = Gas constant = 8.314 $\frac{J}{molK}$

From the above equation, the following things are observed:

1. In a given fluid, a is higher at higher temperatures.

2. Fluids with higher values of K (or lower values of coefficient of compressibility) have higher velocities of sound.
3. With little change in the values of γ for commonly used gases, a at a given temperature is higher for lower molecular weight gases and vice-versa. Hydrogen, with a very low molecular weight has a high a of about 1400 m/s, while the a in some freons which have higher molecular weights is only about 150 m/s.

Mach Number (M): The ratio of the fluid velocity (c) to the local velocity of sound (a).

$$M = \frac{c}{a} = \frac{c}{\sqrt{\gamma RT}}$$

Also, $M^2 = \dfrac{inertiaforce}{elasticforce} = \dfrac{\rho A c^2}{KA} = \dfrac{\rho c^2}{K}$

Fluid flow regions are often described in terms of flow Mach number.
If $M \cong 0$, then flow is said to be incompressible flow.(M is very low).
Fluid flow regions can be roughly classified in six categories:

Regime	**Subsonic**	**Transonic**	**Sonic**	**Supersonic**	**Hypersonic**	**High-hypersonic**
Mach	<1.0	0.8–1.2	1.0	1.2–5.0	5.0–10.0	>10.0

Stagnation velocity of sound (a_0): For a given value of stagnation temperature, the velocity of sound for a perfect gas is given as $a_0 = \sqrt{\gamma R T_0}$. By substituting, $R = \dfrac{\gamma - 1}{\gamma} c_p$, we get

$$a_0 = \sqrt{(\gamma - 1) c_p T_0} = \sqrt{(\gamma - 1) h_0} \ .$$

For an isentropic process:

$$\left(\frac{T_2}{T_1}\right)_{s=c} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} and \left(\frac{T_2}{T_1}\right)_{s=c} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} and \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma}$$

Stagnation pressure (p_0): The pressure a fluid attains when it is decelerated to zero velocity at zero elevation in a reversible adiabatic process. For a given value of static p and T, its value can be derived from the stagnation temperature.

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

Stagnation density (ρ_0): For given values of stagnation pressure and temperature of an ideal gas, the stagnation density is given by, $\rho_0 = \dfrac{p_o}{RT_0}$ **.**

From isentropic relations: $\dfrac{\rho_0}{\rho} = \left(\dfrac{p_0}{p}\right)^{1/\gamma}$ and $\dfrac{\rho_0}{\rho} = \left(\dfrac{T_0}{T}\right)^{1/(\gamma-1)} = \left(1 + \dfrac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}$

Various regions of flow:
Here AEE is expressed in terms of *c* and *a*.

WKT: $h_0 = h + \frac{1}{2}c^2 = constant$ ----- Eqn.1

$$h = c_p T = \frac{\gamma}{\gamma - 1} RT = \frac{a^2}{\gamma - 1} = \frac{\gamma}{\gamma - 1}\frac{p}{\rho} \text{ ----- Eqn. 2}$$

Using Eqn. 2 in Eqn. 1, we get

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = constant. \text{ ----- Eqn. 3}$$

To solve constants: -

Case 1:- If T = 0, then h = 0 and $c = c_{max}$ and hence $h_0 = \frac{c_{max}^2}{2}$ = constant. ----- Eqn. 4

Case 2:- If c = 0, then a = a_0 and hence $h_0 = \frac{a_0^2}{\gamma - 1}$ = constant. ----- Eqn. 5

Equations 3,4 and 5 gives

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = \frac{c_{max}^2}{2} = \frac{a_0^2}{\gamma - 1} = h_0 \text{ ----- Eqn. 6}$$

Eqn. 6 is yet another form of AEE. Eqn. 6 shows the Total Energy of a fluid.
By plotting Eqn. 6 on *c* and *a* coordinates, a steady flow ellipse is obtained. Five regions of flow are quite distinct on this ellipse.

Reference velocities: Fluid velocity is expressed in non-dimensional forms. This is done by dividing the flow velocity by some reference velocity. The reference velocities which can be used are:
1. local velocity of sound, *a*. 2. stagnation velocity of sound, a_0. 3. maximum velocity of fluid, c_{max}. 4. critical velocity of fluid/sound, $c^* = a^*$.

Maximum velocity of fluid (c_{max}): The AEE has 2 components namely enthalpy and kinetic energy. If KE is absent, the total energy is entirely represented by the stagnation enthalpy. The other extreme condition which can be conceived is when the entire energy is made up of KE only, i.e. h = 0 and c = c_{max}. c_{max} is the velocity that would be achieved by the fluid when it is accelerated to absolute zero temperature(T = 0, h = 0) in an imaginary adiabatic expansion process. For this condition, from Eqn. 4, we get $c_{max} = \sqrt{2h_0} = \sqrt{2c_pT_0} = \sqrt{\frac{2\gamma}{\gamma - 1}RT_0}$ and therefore, $\frac{c_{max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}} = 2.24 (for \gamma = 1.4)$.

Critical velocity of sound: The critical velocity of a fluid is its velocity at a Mach number of unity.

$$M_{critical} = \frac{c^*}{a^*} = 1 \text{ and hence } c^* = a^* = \sqrt{\gamma R T^*}$$

For adiabatic expansion from the stagnation temperature to the critical temperature, the AEE is given as

$$h_0 = h^* + \frac{c^{*2}}{2}$$

For a perfect gas,

$$T_0 = T^* + \frac{c^{*2}}{2c_p} \Rightarrow c^* = \sqrt{2c_p\left(T_0 - T^*\right)} \text{ ------ Eqn. 7}$$

WKT:- $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$

For the critical Mach number (M = 1, T = T^*), we get

$$\frac{T_0}{T^*} = \frac{\gamma + 1}{2} = 1.2 (for \gamma = 1.4) \text{ ------ Eqn. 8}$$

From Eqn. 7,

$$c^* = a^* = \sqrt{2c_p T_0\left(1 - \frac{T^*}{T_0}\right)} = \sqrt{2c_p T_0\left(1 - \frac{2}{\gamma + 1}\right)} = \sqrt{2c_p T_0\left(\frac{\gamma - 1}{\gamma + 1}\right)} = \sqrt{2\frac{\gamma R}{\gamma - 1} T_0\left(\frac{\gamma - 1}{\gamma + 1}\right)}$$

$$c^* = a^* = \sqrt{\frac{2\gamma R T_0}{\gamma + 1}} = a_0\sqrt{\frac{2}{\gamma + 1}} \Rightarrow \frac{c^*}{a_0} = \frac{a^*}{a_0} = \sqrt{\frac{2}{\gamma + 1}} = 0.913$$

WKT:-

$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$ and $\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$

and hence $\frac{T^*}{T} = \frac{T^*}{T_0}\frac{T_0}{T} = \frac{2}{\gamma + 1}\left(1 + \frac{\gamma - 1}{2} M^2\right) = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2$

WKT:-

$$\frac{c_{\max}}{a_0} = \sqrt{\frac{2}{\gamma - 1}} \text{ and } \frac{c^*}{a_0} = \sqrt{\frac{2}{\gamma + 1}} \text{ ,and hence } \frac{c_{\max}}{c^*} = \frac{c_{\max}}{a^*} = \sqrt{\frac{\gamma + 1}{\gamma - 1}} = 2.45$$

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = \frac{c_{\max}^2}{2} = \frac{1}{2} a^{*2} \frac{\gamma + 1}{\gamma - 1} \text{ ----- Eqn. 9}$$

Mach number (M^*): M^* can be defined by non-dimensionalizing the fluid velocity by critical fluid velocity or the critical sound velocity.

$$M^* = \frac{c}{c^*} = \frac{c}{a^*}; \text{ Also, } M^{*2} = \frac{c^2}{a^{*2}} = \frac{c^2}{a^2} \cdot \frac{a^2}{a^{*2}} = \frac{a^2}{a^{*2}} . M^2$$

Sometimes it is more convenient to use M^* instead of M because (i) at high fluid velocities M approaches infinity. (ii) M is not proportional to the fluid velocity alone.

Note: M^* does not mean M = 1.

Crocco Number (C_r): It is a non-dimensional number and it is defined as the ratio of fluid velocity to the maximum fluid velocity, $c_{\max}$.

$$C_r = \frac{c}{c_{\max}} = \frac{c}{c^*}\frac{c^*}{c_{\max}} = M^*\sqrt{\frac{\gamma-1}{\gamma+1}} = \sqrt{\frac{\frac{1}{2}(\gamma+1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}}\sqrt{\frac{\gamma-1}{\gamma+1}} = \sqrt{\frac{\frac{1}{2}(\gamma-1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}}$$

$$C_r^2 = \frac{\frac{1}{2}(\gamma-1)M^2}{1+\frac{1}{2}(\gamma-1)M^2} \Rightarrow 2C_r^2 = \frac{(\gamma-1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}$$

$$2C_r^2 + (\gamma-1)M^2 C_r^2 = (\gamma-1)M^2$$

$$2C_r^2 = (\gamma-1)M^2\left[1-C_r^2\right] \Rightarrow M = \sqrt{\frac{2C_r^2}{(\gamma-1)\left(1-C_r^2\right)}}$$

WKT:- $\frac{T_0}{T} = 1+\frac{\gamma-1}{2}M^2$

$$\frac{T_0}{T} = 1+\frac{\gamma-1}{2}\left[\frac{2C_r^2}{(\gamma-1)\left(1-C_r^2\right)}\right] = 1+\frac{C_r^2}{1-C_r^2} = \frac{1-C_r^2+C_r^2}{1-C_r^2} = \frac{1}{1-C_r^2}$$

Relation between M and M^*:

WKT:-

$$\frac{a^2}{\gamma-1}+\frac{c^2}{2} = \frac{1}{2}a^{*2}\frac{\gamma+1}{\gamma-1}$$

Multiplying by 2 throughout, we get

$$\frac{2a^2}{\gamma-1}+c^2 = a^{*2}\frac{\gamma+1}{\gamma-1}$$

$$\frac{2}{\gamma-1}\frac{a^2}{a^{*2}}+\frac{c^2}{a^{*2}} = \frac{\gamma+1}{\gamma-1}$$

$$\frac{2}{\gamma-1}\frac{M^{*2}}{M^2}+M^{*2} = \frac{\gamma+1}{\gamma-1} \quad \text{----- Eqn. A}$$

$$M^{*2}\left[1+\frac{2}{\gamma-1}.\frac{1}{M^2}\right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2}\left(\frac{2}{\gamma-1}\right)\left[\frac{\gamma-1}{2}+\frac{1}{M^2}\right] = \frac{\gamma+1}{\gamma-1}$$

$$M^{*2} = \frac{(1/2)(\gamma+1)}{\frac{1}{M^2}+\frac{\gamma-1}{2}}$$

$$M^{*2} = \frac{(1/2)(\gamma+1)}{\frac{1}{M^2}\left[1+\frac{\gamma-1}{2}M^2\right]}$$

$$M^{*2} = \frac{(1/2)(\gamma+1)M^2}{1+\frac{1}{2}(\gamma-1)M^2}$$

At M = 0, M^* = 0 ; At M = 1, M^* = 1

At M < 1, M^* < 1 ; At M > 1, M^* > 1

At M = ∞, What is M^* ?

Eqn. A $\Rightarrow$ $\frac{2}{\gamma-1}\frac{M^{*2}}{M^2} = \frac{\gamma+1}{\gamma-1} - M^{*2}$

$$\frac{M^{*2}}{M^2} = \frac{\gamma+1}{2} - \frac{\gamma-1}{2}M^{*2}$$

$$\frac{M^{*2}}{M^2} = \frac{\gamma+1}{2}\left[1-\frac{\gamma-1}{\gamma+1}M^{*2}\right]$$

$$M^{*2}\left(\frac{2}{\gamma+1}\right) = M^2\left[1-\frac{\gamma-1}{\gamma+1}M^{*2}\right]$$

$$M^2 = \frac{\left(\frac{2}{\gamma+1}\right)M^{*2}}{1-\left(\frac{\gamma-1}{\gamma+1}\right)M^{*2}}$$ ----- Eqn. B

At M = ∞, $1-\frac{\gamma-1}{\gamma+1}M^{*2} = 0$

$$M^{*2}_{\max} = \frac{\gamma+1}{\gamma-1}$$

$$M^{*}_{\max} = \frac{c_{\max}}{c^*} = \sqrt{\frac{\gamma+1}{\gamma-1}} = 2.45(for\gamma = 1.4)$$

Compressibility Factor or Pressure Coefficient: -

$$\frac{p_0 - p}{(1/2)\rho c^2} = 1$$

AEE:- $h_0 = h + \frac{1}{2}c^2 = constant$

Differentiating the above, we get

dh + cdc = 0 ----- Eqn. 1

WKT:- $Tds = dh - vdp$

For an isentropic flow:- Tds = 0 and hence

$$dh = vdp = \frac{dp}{\rho}$$ ----- Eqn. 2

Eqn. 2 in Eqn. 1 gives, $\frac{dp}{\rho} + cdc = 0$

If the flow is assumed to

incompressible, $\rho \cong constant$.

Integrating the above equation, we get

$$\frac{p}{\rho}+\frac{c^2}{2}=constant \text{ ----- Eqn. 3}$$

To solve constant:- When the flow is isentropically decelerated to zero velocity, the resultant pressure is stagnation pressure and the corresponding density is the stagnation density.

When c = 0, $p = p_0$ & $\rho = \rho_0$ and hence

$$constant = \frac{p_0}{\rho_0} \text{ and } \therefore Eqn.3 \Rightarrow$$

$$\frac{p}{\rho}+\frac{c^2}{2}=\frac{p_0}{\rho_0} \text{ ------ Eqn. 4}$$

For an incompressible flow:- $\rho \cong \rho_0$

Therefore, Eqn. 4 can be written as

$$p+\frac{1}{2}\rho c^2 = p_0 \text{ ----- Eqn. 5}$$

Eqn. 5 is **Bernoulli's Equation**. This eqn. is applicable for steady, frictionless, incompressible, irrotational flow. (Irrotational – laminar, streamline flow).

$\frac{p_0-p}{(1/2)\rho c^2}=1$ is the **pressure coefficient or compressibility factor.**

Note:- The above eqn. is valid only if the flow is isentropic and incompressible.

Effect of Mach no on Compressibility:-

If the flow is assumed incompressible, the value of pressure coefficient obtained by Bernoulli Equation is unity.

For compressible flow, the value of pressure coefficient deviates from unity. The magnitude of deviation increases with Mach number of flow.

For isentropic compressible flow, the ratio of the p_0 and p is given by

$$\frac{p_0}{p}=\left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}=\left(1+\frac{\gamma-1}{2}M^2\right)^{\gamma/(\gamma-1)}$$

This can be expanded in the following series:-

$$(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+.........$$

Here, $x=\frac{\gamma-1}{2}M^2$ & $n=\frac{\gamma}{\gamma-1}$

$$\therefore \frac{p_0}{p}=1+\frac{\gamma}{2}M^2+\frac{\frac{\gamma}{\gamma-1}\left(\frac{\gamma}{\gamma-1}-1\right)}{2}\left(\frac{\gamma-1}{2}M^2\right)^2+..$$

$$\frac{p_0}{p}=1+\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+......$$

$$\frac{p_0}{p}-1=\frac{\gamma}{2}M^2+\frac{\gamma}{8}M^4+\frac{\gamma(2-\gamma)}{48}M^6+......$$

$$\frac{p_0-p}{p}=\frac{\gamma}{2}M^2\left[1+\frac{M^2}{4}+\frac{(2-\gamma)}{24}M^4+.....\right]$$

$$\frac{p_0-p}{p\left(\frac{\gamma}{2}M^2\right)}=\left[1+\frac{M^2}{4}+\frac{(2-\gamma)}{24}M^4+.....\right]$$

Now, $p\left(\frac{\gamma}{2}M^2\right)=\rho RT\frac{\gamma}{2}M^2=\rho RT\frac{\gamma}{2}\frac{c^2}{a^2}$

$p\left(\frac{\gamma}{2}M^2\right)=\rho RT\frac{\gamma}{2}\frac{c^2}{\gamma RT}=\frac{1}{2}\rho c^2$ and hence the above eqn becomes

$$\frac{p_0-p}{\frac{1}{2}\rho c^2}=\left[1+\frac{M^2}{4}+\frac{(2-\gamma)}{24}M^4+.....\right]$$

If $\gamma=1.4,$ then

$\frac{p_0-p}{\frac{1}{2}\rho c^2}=1+\frac{M^2}{4}+\frac{M^4}{40}+...$ ----- Eqn. A

The above Eqn. A gives the % deviation of the pressure coefficient from its incompressible flow with M. **Effect of compressibility table** (shown below) gives the error or deviation due to assumption of incompressibility.

M	**% deviation**
0.1	0.3
0.2	1.0
0.5	6.4
0.9	22.0
1.0	27.5

Derivation of velocity of sound (a) (or) acoustic velocity (or) velocity of propagation of a small pressure wave.

Velocity of sound is the speed at which sound waves (sound waves – are very small pressure waves) are propagated through a compressible fluid. Each fluid particle present in the sound wave undergoes a nearly **isentropic process** as the disturbances produced in the fluid by a sound wave are very small.

Assumption: 1. Isentropic process 2. Constant area duct.

Assume a wave has been initiated by the slight motion of the piston in a constant area duct. The wave front is propagating steadily to the right at a velocity ‘a’.

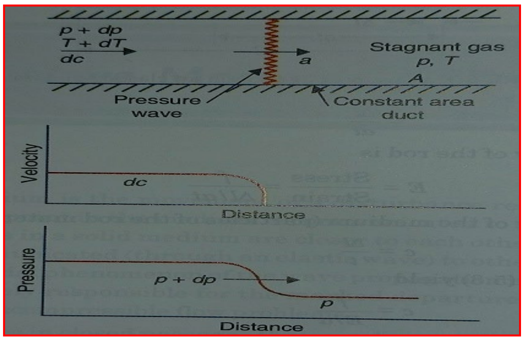


Fig. Propagation of infinitesimal pr. wave (for a stationary observer)

The fluid through which the wave has passed has undergone change in pressure $p+dp, \rho+d\rho, h+dh, T+dT$ and the fluid moves right with a velocity *dc* and the condition of the fluid to the right into which the wave front is just moving but has not passed through has pressure *p*, density ρ, temperature *T* and enthalpy *h* and is *stationary*.

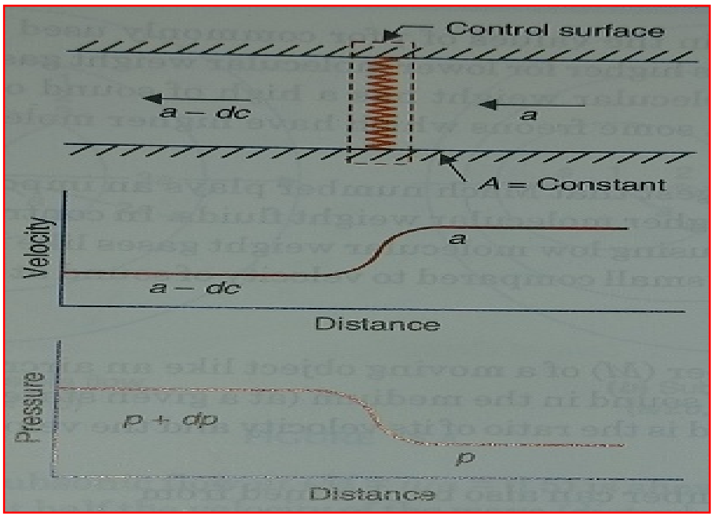


Fig. Propagation of infinitesimal pr. wave (for an observer moving with the wave front)

For an observer travelling with the wave front to the right at the same velocity *'a'*, **it will appear** that the fluid enters with higher velocity *'a'* into the control surface and leaves with a lower velocity *'a-dc'*, but with higher values of pressure and density.

Momentum eqn. for this open system: (sum of the forces in *x* direction is equal to the change in momentum in the *x* direction.)

Considering a stationary control volume surrounding the stationary wave, we get,

$$A[p-(p+dp)] = \dot{m}[(a-dc)-a]$$

$$Adp = \dot{m}\,dc \text{ ----- Eqn. 1}$$

From continuity eqn: $\dot{m} = \rho A a$ ----- Eqn. 2

Eqn.2 in Eqn.1 gives, $Adp = \rho A a dc$ and hence, $dp = \rho a dc$ ----- Eqn. 3

Applying continuity equation for both sides of the wave, $\rho A a = (\rho + d\rho)A(a - dc)$

$\rho a = \rho a - \rho dc + a d\rho - d\rho dc (negligible)$

$\rho dc = a d\rho$ ----- Eqn. 4

$(4) in (3) \Rightarrow \quad dp = a^2 d\rho$

$\therefore a^2 = \frac{dp}{d\rho}$ or $a = \left(\sqrt{\frac{dp}{d\rho}}\right)_s$ ----- Eqn. A

For a perfect gas undergoing an isentropic process, $pv^\gamma = C$ (or) $\frac{p}{\rho^\gamma} = C$

$p\rho^{-\gamma} = constant$

Taking log on both the sides,

$\ln p - \gamma \ln \rho = constant$

On differentiating the above,

$$\frac{dp}{p} - \gamma \frac{d\rho}{\rho} = 0$$

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho}$$

$\left(\frac{dp}{d\rho}\right)_s = \gamma \frac{p}{\rho} = \gamma RT$ ----- Eqn. B

(B) in (A) $\Rightarrow \; a^2 = \gamma RT$

i.e., $a = \sqrt{\gamma RT}$ is the required equation.

2.6 (a) What is velocity temperature? Determine the velocity of air ($\gamma = 1.4$, $c_p = 1.005$ kJ/kg-K) corresponding to a velocity-temperature of 1°C.

(b) Determine the Mach number of an aircraft at which the velocity temperature of air at the entry of the engine equals the static temperature.

Soln:

a). $\frac{c^2}{2c_p}$ is known as the **velocity temperature** or **dynamic temperature(T_c)** corresponding to the velocity c.

Given: T_c = 1 K; C_p = 1.005 kJ/kg-K and hence

$$\frac{C^2}{2x1005} = 1 \rightarrow C = \sqrt{2x1005} = 44.8\frac{m}{s}$$

Note: In the above formula, the value of C_p should be 1005 J/kg-K and you should not substitute 1.005 as it will give you wrong answer. Most of the students do this mistake.

b) Given: $T_c = T$

To find: Mach number, M.

Wkt., $T_0 = T + T_c$

and hence, $T_0 = 2T$

and therefore, $T_0/T=2$.

But, $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}\frac{c^2}{a^2} = 1 + \frac{\gamma - 1}{2}M^2$

$$\frac{T_0}{T} = 2 = 1 + \frac{\gamma - 1}{2}M^2$$

and hence, **M = 2.236 (Ans)**

Answers:

(a) c = 44.8 m/s

(b) M = 2.236

2.10 The jet of a gas at 593 K ($\gamma = 1.3$, $R = 469$ J/kg-K) has a Mach number of 1.2. Determine for local and stagnation conditions velocity of sound and enthalpy.
What is the maximum attainable velocity of this jet?

Soln:

Given: T = 593 K; γ = 1.3; R = 469 J/kg-K; M = 1.2.

To find: a, h, a_0, h_0, C_{max}.

Wkt:

$$a = \sqrt{\gamma RT} = \sqrt{1.3x469x593} = 601.29 m/\sec$$

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{1.3 - 1}{2} x1.2^2 = 1.216$$

and hence, T_0 = 1.216x593 = 721.09 K

$$a_0 = \sqrt{\gamma RT_0} = \sqrt{1.3x469x721.09} = 663.1 m/\sec$$

h = C_pT = (γR/γ-1)xT = (1.3x469/(1.3-1))x593 = 1205.173 kJ/kg.

h_0 = C_pT_0 = (γR/γ-1)xT_0 = (1.3x469/(1.3-1))x721.09 = 1465.49 kJ/kg.

$$c_{\max} = \sqrt{2h_0} = \sqrt{2x1465.49x1000} = 1712.01 m/\sec$$

Note: In the above formula, the value of h_0 should be in J/kg and not in kJ/kg as it will give you wrong answer. Most of the students do this mistake.

Answers:

(a) a = 601.29 m/s

(b) a_0 = 663.1 m/s

(c) h = 1205.173 kJ/kg

(d) h_0 = 1465.49 kJ/kg

(e) C_{max} = 1712.01 m/sec

2.11. In a **settling chamber** air is at pressure of 5 bar and temperature is at 500 K. Determine the values of stagnation enthalpy, stagnation velocity of sound, maximum velocity of fluid, critical temperature, critical velocity of fluid and critical velocity of sound. Which type of flow has been assumed in the calculations?

Soln:

✓ **Note: In a settling chamber, air is not moving and hence the values given should be taken as stagnation values.**

✓ Given: T_0 = 500 K; p_0 = 5 bar.

✓ To find: a_0, h_0, C_{max}, T^*, c^* and a^*.

✓ Type of flow assumed: **Isentropic**.

Wkt: C_p = 1.005 kJ/kg-K

✓ $h_0 = C_pT_0 = 1.005xT_0 = 1.005x500 = 502.5$ kJ/kg.

$$a_0 = \sqrt{\gamma RT_0} = \sqrt{1.4x287x500} = 448.218 m/\sec$$

Note: In the above formula, the value of R should be 287 J/kg-K and you should not substitute 0.287 kJ/kg-K as it will give you wrong answer. Most of the students do this mistake.

$$c_{\max} = \sqrt{2h_0} = \sqrt{2x502.5x1000} = 1002.49 m/\sec$$

Wkt: $\frac{T_0}{T^*} = \frac{\gamma+1}{2}$

and hence, $\mathbf{T^* = T_0/1.2 = 416.67\ K}$

Wkt: $c^* = a^* = \sqrt{\gamma RT^*}$

and hence, $\mathbf{c^* = a^* = 409.16\ m/sec}$

Answers:

(a) a_0 = 448.218 m/s (b) h_0 = 502.5 kJ/kg (c) C_{max} = 1002.49 m/sec (d) T^* = 416.67 K (e) c^* = a^* = 409.16 m/sec.

2.9 An aircraft is flying at an altitude of 12,000 meters ($T =$, $p =$) at a Mach number of 0.82. The cross-sectional area of the inlet diffuser before the L.P. compressor stage is 0.5 m^2. Determine:

(a) the mass of air entering the compressor per second,

(b) the speed of the aircraft and

(c) the stagnation pressure and temperature of air at the diffuser entry.

Soln:

✓ Given: M = 0.82; A = 0.5 m^2; Z = 12,000 m;

✓ To find: (a) m (b) speed of the aircraft (c) T_0 and p_0

✓ Type of flow assumed: **Isentropic**.

Go to Table 2, ppts of standard atmosphere, pg. 20

Z (m)	t (°C)	T (K)	a (m/s)	p (bar)	ρ (kg/m³)	μ ($\frac{Ns}{m^2} \times 10^6$)
12000	-56.50	216.65	295.2	0.193	0.311	14.17

Wkt: From the above table, ρ = 0.311 kg/m^3; T = 216.65 K and p = 0.193 bar

$$\dot{m} = \rho AC = \rho A(M.a) = \rho AM\sqrt{\gamma RT} = 0.311x0.5x0.82x\sqrt{1.4x287x216.65} = 37.62\frac{kg}{sec}$$

(b) Speed of the aircraft

Wkt: M = c/a and hence c = M.a = 0.82x√(γRT) = 0.82x√(1.4x287x216.65) = 241.93 m/sec

To find: speed of the aircraft in km/hr, multiply the above value by 3600/1000 and hence c = 241.93 x (3600/1000) = 870.96 km/hr.

(c) From gas tables: for M = 0.82, γ = 1.4; Table 3.2, pg. 30, we get

0.82	0.843	0.881	0.643	1.031	1.015	0.663

p/p_0 = 0.643 and hence p_0 = p/0.643 = 0.193/0.643 = 0.30 bar;

T/T_0 = 0.881 and hence T_0 = T/0.881 = 216.65/0.881 = 245.91 K;

Answers:

(a) m = 37.62 kg/sec

(b) Speed of the aircraft = 870.96 km/hr

(c) p_0 = 0.30 bar

T_0 = 245.91 K

2.8 The conditions of an air stream at entry to a duct are

$p_1 = 1.0$ bar, $T_1 = 300$ K, $M_1 = 1.3$

If the Mach number at exit of the duct is 0.6 determine for adiabatic flow the temperature and velocity of air at the duct exit.

Soln:

✓ Given: p_1 = 1 bar; T_1 = 300 K; M_1 = 1.3; M_2 = 0.6 → Diffuser.

✓ To find: (a) T_2 (b) c_2

✓ Type of flow assumed: **Isentropic**.

From gas tables: for M_1 = 1.3, γ = 1.4; Table 3.2, pg. 32, we get

1.30	1.231	0.747	0.361	1.066	1.022	0.385

p_1/p_{01} = 0.361 and hence $\mathbf{p_{01}} = p_1/0.361 = 1/0.361 = 2.77 \text{ bar} = \mathbf{p_{02}}$

T_1/T_{01} = 0.747 and hence $\mathbf{T_{01}} = T_1/0.0.747 = 300/0.747 = 401.61 \text{ K} = \mathbf{T_{02}}$

From gas tables: for M_2 = 0.6, γ = 1.4; Table 3.2, pg. 29, we get

0.60	0.635	0.933	0.784	1.188	1.105	0.932

p_2/p_{02} = 0.784 and hence $\mathbf{p_2} = p_{02} \times 0.784 = 2.77 \times 0.784 = \mathbf{2.17 \text{ bar}} \rightarrow \mathbf{p_2 > p_1}$

T_2/T_{02} = 0.933 and hence $T_2 = T_{02} \times 0.933 = 401.61 \times 0.933 = \mathbf{374.7 \text{ K}} \rightarrow \mathbf{T_2 > T_1}$

$c_2 = M_2 \times a_2 = 0.6 \times \sqrt{(\gamma R T_2)} = 232.8$ m/sec.

Answers:
(a) T_2 = 374.7 K
(b) c_2 = 232.8 m/sec.

2.7 Steam at a section of a pipe has pressure = 10 bar, temperature = 600 K, velocity = 120 m/s, datum head = 10 m.
(a) Taking $c_p = 2.150$ kJ/kg K, $c_v = 1.615$ kJ/kg K, determine Mach number, stagnation pressure and temperature.
(b) Compare the stagnation pressure value with that obtained from the Bernoulli Equation and comment on the difference.
(c) Are the effects of Mach number and datum significant in this case?

Soln:

✓ Given: p = 10 bar; T = 600 K; c = 120 m/s; Z = 10 m; C_p = 2.150 kJ/kg-K; C_v= 1.615 kJ/kg-K

✓ To find: (a) M, T_o and p_o (b) Compare this p_o with $p_{o,Bernoulli}$

✓ Type of flow assumed: **Isentropic**.

(a) Wkt: $\gamma = C_p/C_v$= 2.150/1.615 = 1.3;

Also, $C_p = \gamma R/(\gamma-1) \rightarrow R = (\gamma-1)C_p/\gamma$ = 0.496 kJ/kg-K = 496 J/kg-K

and hence M = c/a = 120/√(1.3x496x600) = 0.19

Wkt., $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 = 1 + \frac{1.3 - 1}{2} x 0.19^2 = 1.0054$

and hence, T_o = 1.0054x600 = 603.24 K; and $\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$ and hence, p_o = 10.24 bar;

(b) Bernoulli Equation is given by: $p+\frac{1}{2}\rho c^2 = p_0$

Wkt: ρ = p/RT = $10x10^5/(496x600)$ = 3.36 kg/m^3 and hence $p_0 = (10x10^5)+(1/2)(3.36)(120)^2 = 10.24$ bar.

$p_{0,Bernoulli} = 10.24$ bar.

Comment: At low Mach number(0.19), there is no significant difference in stagnation pressure considering compressible flow and using Bernoulli equation.

(c) Effect of Mach number and datum head are not significant.

Answers:

(a) M = 0.19; T_0 = 603.24 K; p_0 = 10.24 bar.

(b) $p_{0,Bernoulli}$ = 10.24 bar.

(c) Effect of Mach number and datum head are insignificant.

SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
GAS DYNAMICS AND JET PROPULSION (SME1303)

UNIT – 2
FLOW THROUGH VARIABLE AREA DUCTS

Introduction

In the case of high speed flow (compressible flow), the change of state in flow properties is achieved by the following three ways:-

1. Isentropic flow – With area change, treating the fluid to be non-viscous and passage to be frictionless.

2. Fanno flow or frictional flow in a constant area duct - With friction, considering the heat transfer between the surroundings and system to be negligible.

3. Rayleigh flow in a constant area frictionless duct – With heat transfer, assuming the fluid to be non-viscous.

MACH NUMBER VARIATION

WKT:- $h+\frac{c^2}{2}=h_0=C$ ------ Eqn. 1

on differentiation, we get, $dh+cdc=0$

WKT:- $Tds=dh-vdp$ and for an isentropic flow, $ds=0$ and hence, $dh=vdp=\frac{dp}{\rho}$

$$Eqn.1 \Rightarrow \frac{dp}{\rho}+cdc=0 \Rightarrow dp=-\rho cdc \Rightarrow dc=-\frac{dp}{\rho c} \text{ ----- Eqn. 2}$$

From continuity equation, $\dot{m}=\rho Ac=constant$

Taking log and differentiating we get, $\ln\rho+\ln A+\ln c=constant$

$$\frac{d\rho}{\rho}+\frac{dA}{A}+\frac{dc}{c}=0 \Rightarrow dc=-c\left(\frac{d\rho}{\rho}+\frac{dA}{A}\right) \text{ ----- Eqn. 3}$$

Eqn. 2 in Eqn. 3, we get, $-\frac{dp}{\rho c}=-c\left(\frac{d\rho}{\rho}+\frac{dA}{A}\right) \Rightarrow dp=\rho c^2\left(\frac{d\rho}{\rho}+\frac{dA}{A}\right)$

$$\frac{dA}{A}=\frac{dp}{\rho c^2}-\frac{d\rho}{\rho}=\frac{dp}{\rho c^2}\left(1-\frac{d\rho}{dp}c^2\right)$$

$$\frac{dA}{A}=\frac{dp}{\rho c^2}\left(1-\frac{c^2}{a^2}\right) \text{ since } \left[\because\left(\frac{dp}{d\rho}\right)_s=a^2\right]$$

$$\frac{dA}{A}=\frac{dp}{\rho c^2}\left(1-M^2\right) \text{ ----- Eqn. A}$$

Eqn. A is considered for both accelerating and decelerating passages for various values of Mach number, M.

Eqn. A $\Rightarrow$ Shape of the passage depends on local Mach number.

Purpose of nozzle: Gases and vapors are **expanded** in nozzles by providing a pressure ratio across them. Purpose of a nozzle is to **accelerate** the flow by providing a **pressure drop**. Hence *dp* in Eqn. A is always **negative.**

Purpose of diffuser: Gases and vapors are **compressed** in diffusers by providing a pressure ratio across them. Purpose of a diffuser is to **decelerate** the flow by obtain **pressure rise.** Hence *dp* in Eqn. A is always **positive**. Diffusers are used to obtain **static pressure rise** at the cost of the deceleration of flow in the diffuser passage.

In Eqn. A, If $\frac{dA}{A}$ is $\frac{+ve, then \quad areais \uparrow ing}{-ve, then \quad areais \downarrow ing}$

For nozzle, *dp* is negative, $\frac{dA}{A} = \frac{dp}{\rho c^2}\left(1 - M^2\right)$

Flow M at inlet	$1 - M^2$	*dp*	$\frac{dA}{A}$	Area	Passage	Diagram
M < 1 (c < a) Subsonic	+ ve	- ve	- ve$\Rightarrow$	$\downarrow$	Converging	
M =1 Sonic	0	- ve	0$\Rightarrow$	No change in area	There is no change in passage area where M=1.	This section is referred to as the throat of the passage.
M > 1 (c > a) Supersonic	- ve	- ve	+ ve $\Rightarrow$	$\uparrow$	Diverging	

For diffuser, *dp* is positive, $\frac{dA}{A} = \frac{dp}{\rho c^2}\left(1 - M^2\right)$

Flow M at inlet	$1 - M^2$	*dp*	$\frac{dA}{A}$	Area	Passage	Diagram
M < 1 (c < a) Subsonic	+ ve	+ ve	+ ve $\Rightarrow$	$\uparrow$	Diverging	
M =1 Sonic	0	+ ve	0$\Rightarrow$	No change in area	There is no change in passage area where M=1.	This section is referred to as the throat of the passage.
M > 1 (c > a) Supersonic	- ve	+ ve	- ve $\Rightarrow$	$\downarrow$	Converging	

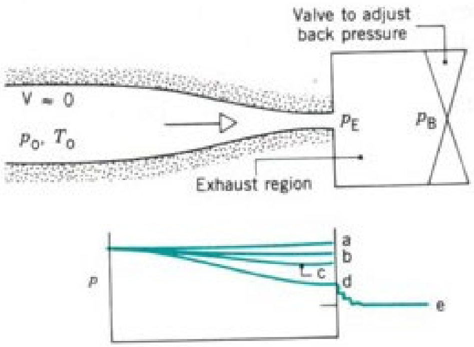
It may be noted that while a continuous acceleration of a real flow from subsonic to supersonic Mach numbers is possible whereas the converse of this is impossible, i.e., continuous deceleration of a real flow from supersonic to subsonic Mach number is impossible.

***Note:- A real supersonic flow decelerates to a subsonic flow only through a* SHOCK WAVE.**

Effect of back pressure

The back pressure is the pressure in the exhaust region outside the nozzle exit.

In a converging nozzle

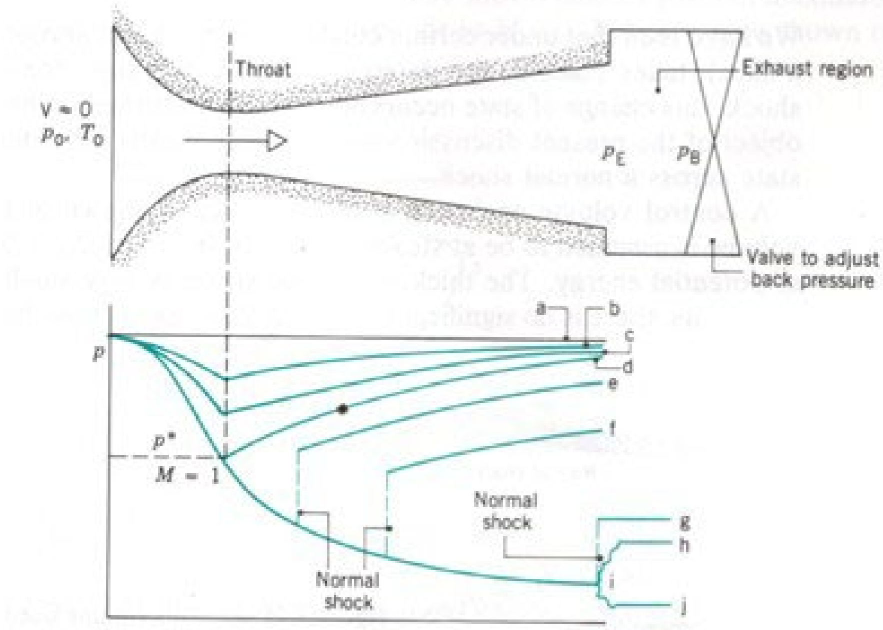


In a converging diverging nozzle

The highest velocity to which a fluid can be accelerated in a converging nozzle is limited to sonic velocity (Mach number, M=1), which occurs at the exit plane (throat) of the nozzle. Accelerating a fluid to supersonic velocities (M>1) can be accomplished only by attaching a diverging flow section to the subsonic nozzle at the throat. The resulting combined flow section is a converging-diverging nozzle.

Forcing a fluid through a converging-diverging nozzle is no guarantee that the fluid will be accelerated to supersonic velocity. For given inlet conditions, the flow through a converging-diverging nozzle is governed by the back pressure.

The effect of variation of back pressure in a convergent-divergent nozzle is shown next:



Effect of back pressure on the operation of a converging–diverging nozzle.

Stagnation and Critical States:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2 \ ; \ \ \frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}; \ \frac{T^*}{T_0} = \frac{2}{\gamma + 1} = 0.833$$

$$\frac{p^*}{p_0} = \left[\frac{2}{\gamma + 1}\right]^{\gamma/(\gamma-1)} = 0.528 \quad \frac{\rho^*}{\rho_0} = \left[\frac{2}{\gamma + 1}\right]^{1/(\gamma-1)} = 0.634 \, ; \ \frac{T^*}{T} = \frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2$$

$$\frac{p^*}{p} = \left[\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2\right]^{\gamma/(\gamma-1)} \quad \frac{\rho^*}{\rho} = \left[\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2\right]^{1/(\gamma-1)}$$

Area ratio $\left(\frac{A}{A^*}\right)$ as function of Mach number:

Here $\frac{A}{A^*}$ is a function of local M.

From continuity equation, $\rho A c = \rho^* A^* c^*$

$$\frac{A}{A^*} = \frac{\rho^*}{\rho} \frac{c^*}{c} \text{ ----- Eqn. 1}$$

WKT:- $\frac{\rho^*}{\rho} = \left[\frac{2}{\gamma + 1} + \frac{\gamma - 1}{\gamma + 1} M^2\right]^{1/(\gamma-1)}$ ----- Eqn. 2

$$M^{*2} = \frac{(1/2)(\gamma + 1) M^2}{1 + \frac{1}{2}(\gamma - 1) M^2} \text{ where } M^* = \frac{c}{c^*}$$

$$M^* = \frac{c}{c^*} = \left(\frac{\frac{1}{2}(\gamma + 1) M^2}{1 + \frac{1}{2}(\gamma - 1) M^2}\right)^{\frac{1}{2}}$$

$$\frac{1}{M^*} = \frac{c^*}{c} = \left(\frac{1 + \frac{1}{2}(\gamma - 1) M^2}{\frac{1}{2}(\gamma + 1) M^2}\right)^{\frac{1}{2}}$$

$$\frac{1}{M^*} = \frac{c^*}{c} = \frac{1}{M}\left(\frac{1+\frac{1}{2}(\gamma-1)M^2}{\frac{1}{2}(\gamma+1)}\right)^{\frac{1}{2}}$$

$$\frac{1}{M^*} = \frac{c^*}{c} = \frac{1}{M}\left(\frac{1}{\frac{1}{2}(\gamma+1)} + \frac{\frac{1}{2}(\gamma-1)M^2}{\frac{1}{2}(\gamma+1)}\right)^{\frac{1}{2}}$$

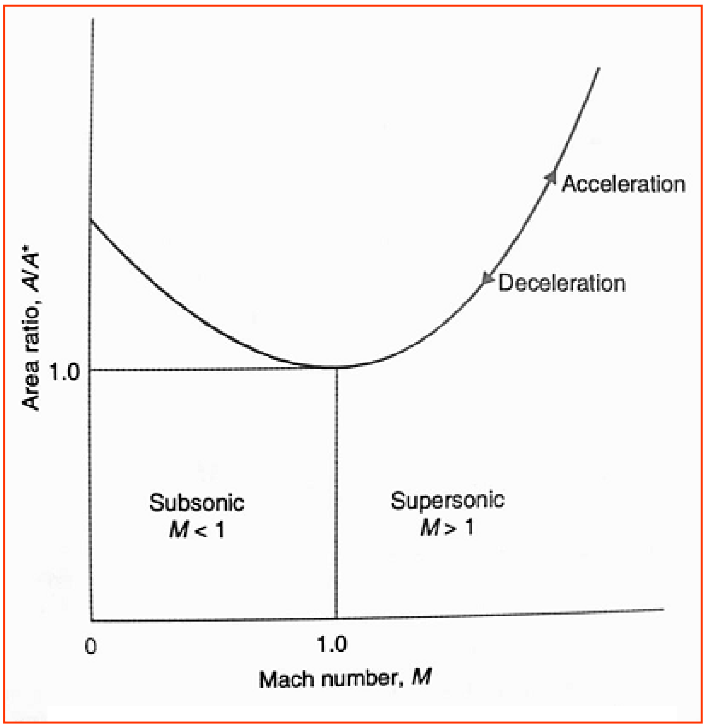
$$\frac{1}{M^*} = \frac{c^*}{c} = \frac{1}{M}\left(\frac{2}{(\gamma+1)} + \frac{(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{1}{2}} \text{---- Eqn. 3}$$

Eqns. 2 and 3 in 1 gives,

$$\frac{A}{A^*} = \left[\frac{2}{\gamma+1} + \frac{\gamma-1}{\gamma+1}M^2\right]^{1/(\gamma-1)} \text{ x } \frac{1}{M}\left(\frac{2}{(\gamma+1)} + \frac{(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{1}{2}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{2}{(\gamma+1)} + \frac{(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

The variation of area ratio for subsonic and supersonic isentropic acceleration and deceleration is shown below.



In many compressible flow calculations, the function $\frac{A}{A^*}\frac{p}{p_0}$ occurs frequently and this will be a function of Mach number.

$$\frac{A}{A^*}\frac{p}{p_0}=\frac{1}{M}\left(\frac{2}{(\gamma+1)}+\frac{(\gamma-1)M^2}{(\gamma+1)}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \text{x}$$

$$\left(1+\frac{\gamma-1}{2}M^2\right)^{-\gamma/(\gamma-1)}$$

$$\frac{A}{A^*}\frac{p}{p_0}=\frac{1}{M}\left(\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}\right)M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} \text{x}$$

$$\left(1+\frac{\gamma-1}{2}M^2\right)^{-\gamma/(\gamma-1)}$$

$$\frac{A}{A^*}\frac{p}{p_0}=\frac{1}{M}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-1/2}$$

$$\frac{A}{A^*}\frac{p}{p_0}=\frac{\frac{1}{M}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}}$$ is the required solution.

Derivation of *equivalent mass flow parameter*:- $\dot{m}=\rho Ac$

$$\Rightarrow \frac{\dot{m}}{A}=\rho c=\frac{p}{RT}c=\frac{p}{\sqrt{RT}}\frac{c}{\sqrt{\gamma RT}}\sqrt{\gamma}$$

$$\frac{\dot{m}}{A}=\sqrt{\frac{\gamma}{R}}\frac{p}{\sqrt{T}}.M=\sqrt{\frac{\gamma}{R}}Mp\sqrt{\frac{T_0}{T}}\frac{1}{\sqrt{T_0}}$$

$$\frac{\dot{m}\sqrt{T_0}}{A}=\sqrt{\frac{\gamma}{R}}Mp\sqrt{1+\frac{\gamma-1}{2}M^2}$$

$$\frac{\dot{m}\sqrt{T_0}}{A}=\sqrt{\frac{\gamma}{R}}M\frac{p_0}{p_0/p}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}$$

$$\frac{\dot{m}\sqrt{T_0}}{Ap_0} = \sqrt{\frac{\gamma}{R}}M \frac{1}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}}\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{1}{2}}$$

$$\frac{\dot{m}\sqrt{T_0}}{Ap_0}\sqrt{\frac{R}{\gamma}} = \frac{M}{\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

The above equation shows that for a given gas, the equivalent mass flow parameter is a fn. of M only.

Non-dimensional mass flow parameter in terms of pressure ratio:

WKT:- $\dot{m} = \rho A c$ ------ Eqn. A

For an isentropic flow, $\frac{p_0}{\rho_0^\gamma} = \frac{p}{\rho^\gamma}$

$$\rho^\gamma = \left(\frac{p}{p_0}\right)\rho_0^\gamma \Rightarrow \rho = \left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\rho_0 \text{ ----- Eqn. B}$$

$$T_0 = T + \frac{c^2}{2c_p} \Rightarrow c^2 = 2c_p\left(T_0 - T\right)$$

$$c^2 = 2\frac{\gamma R}{\gamma-1}\left(T_0 - T\right) = 2\frac{\gamma R}{\gamma-1}T_0\left(1-\frac{T}{T_0}\right)$$

$$\therefore c = \sqrt{2\frac{\gamma R}{\gamma-1}T_0\left\{1-\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}} \text{ ----- Eqn. C}$$

Eqns. B and C in A gives,

$$\dot{m} = A\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\rho_0\sqrt{2\frac{\gamma R}{\gamma-1}T_0\left\{1-\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}}$$

$$\dot{m} = A\left(\frac{p}{p_0}\right)^{\frac{1}{\gamma}}\frac{p_0}{RT_0}\sqrt{2\frac{\gamma R}{\gamma-1}T_0\left\{1-\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right\}}$$

$$\frac{\dot{m}}{A}=\frac{p_0\sqrt{RT_0}}{RT_0}\sqrt{\frac{2\gamma}{\gamma-1}\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}}\left\{1-\frac{p}{p_0}\right\}^{\frac{\gamma-1}{\gamma}}}$$

$$\frac{\dot{m}}{A}=\frac{p_0\sqrt{\gamma}}{\sqrt{RT_0}}\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}}-\left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

$$\frac{\dot{m}\sqrt{T_0}}{Ap_0}\sqrt{\frac{R}{\gamma}}=\sqrt{\frac{2}{\gamma-1}\left[\left(\frac{p}{p_0}\right)^{\frac{2}{\gamma}}-\left(\frac{p}{p_0}\right)^{\frac{\gamma+1}{\gamma}}\right]}$$

To find the **maximum mass flow rate** *for a given pressure ratio,* **differentiate** *the variable quantity* $\left(\frac{p}{p_0}\right)$ *and equate it to* **zero.**

Let $q=\frac{p}{p_0}$

$$\frac{d}{dq}\left[q^{\frac{2}{\gamma}}-q^{\frac{\gamma+1}{\gamma}}\right]=0$$

$$\frac{2}{\gamma}q^{\frac{2}{\gamma}-1}-\left(\frac{\gamma+1}{\gamma}\right)q^{\frac{\gamma+1}{\gamma}-1}=0$$

$$\frac{2}{\gamma}q^{\frac{2-\gamma}{\gamma}}=\left(\frac{\gamma+1}{\gamma}\right)q^{\frac{1}{\gamma}}$$

$$\frac{2}{\gamma+1}=q^{\frac{1}{\gamma}}q^{\frac{-(2-\gamma)}{\gamma}}=q^{\frac{1}{\gamma}}q^{\frac{\gamma-2}{\gamma}}$$

$$\frac{2}{\gamma+1}=q^{\frac{\gamma-1}{\gamma}}$$

$$q=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

But, $\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}=\frac{p^*}{p_0}$

Hence, $q=\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}=\frac{p^*}{p_0}$

Therefore, the pressure ratio which gives maximum $\frac{\dot{m}\sqrt{T_0}}{Ap_0}$ is also the ***CRITICAL PRESSURE RATIO.*** The critical conditions occur when M = 1, i.e., at the minimum area of cross section – Throat section.

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma}\frac{\gamma}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma}\frac{\gamma}{\gamma-1}}\right]}$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{2}{\gamma+1}\right)^{\frac{2}{\gamma-1}} - \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\right]} \quad \textbf{- I}$$

Eqn. **I** can be expressed in a different form.

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\left(\frac{2}{\gamma+1}\right)^{-1} - 1\right]}$$

$$\left[\begin{array}{l} \dfrac{\gamma+1}{\gamma-1} + x = \dfrac{2}{\gamma-1} \Rightarrow x = \dfrac{2}{\gamma-1} - \dfrac{\gamma+1}{\gamma-1} = \dfrac{2-\gamma-1}{\gamma-1} \\ x = \dfrac{1-\gamma}{\gamma-1} = \dfrac{-(\gamma-1)}{(\gamma-1)} = -1 \end{array}\right]$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{\gamma+1}{2} - 1\right]}$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{\gamma+1-2}{2}\right]}$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \sqrt{\frac{2}{\gamma-1}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\frac{\gamma-1}{2}\right]}$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0}\sqrt{\frac{R}{\gamma}} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad \textbf{------ II}$$

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0} = \sqrt{\frac{\gamma}{R}}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$

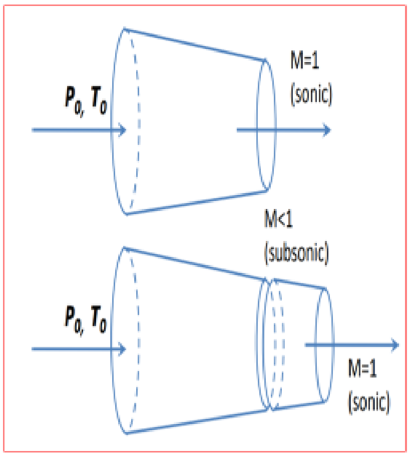
For $\gamma = 1.4$ and R = 287 J / kg-K

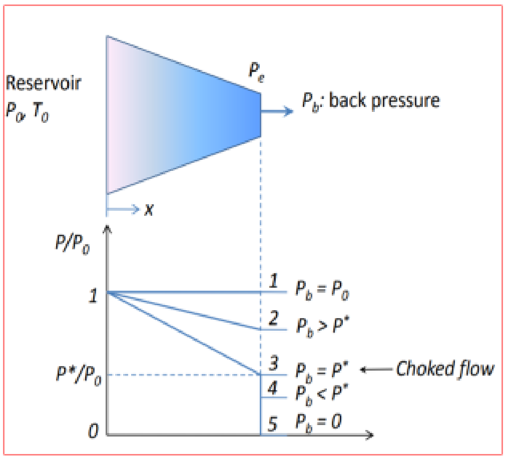
$$\frac{\dot{m}_{max}\sqrt{T_0}}{A^* p_0} = 0.0404 \text{ ----- } \mathbf{III}$$

Eqn. **III** is known as ***Fliegner's formula.***

ISENTROPIC FLOW THROUGH CONVERGENT NOZZLE

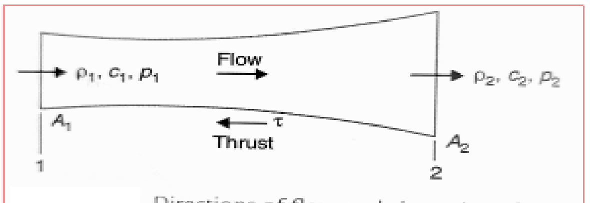
- ✓ Convergent nozzles are used for **subsonic** and **sonic** flows.
- ✓ Used as **flow measuring** and **flow regulating devices**.
- ✓ CD nozzles are used for supersonic flows.





- ✓ **Choking:** Mass flow rate is maximum when M=1, i.e., when the flow is sonic.
- ✓ The maximum mass flow condition is reached when the throat pressure ratio achieves critical value (p_t=p*); there is no further increase in mass flow with decrease in back pressure after this point. This condition is called "**choking**".
- ✓ The phenomenon of choking exists only in compressible flow.

- ✓ **Impulse Function or Wall Force Function, F:**
- ✓ The quantities pA and ρAC² occur frequently in some compressible flow problem.
- ✓ Both are expressed together as an important gas dynamic parameter as the 'wall force function' or the impulse function (F).
- ✓ Both has unit of 'Force".



Directions of flow and thrust in a duct

- ✓ Unit of ρAC² = (kg/m³)(m²)(m/s)² = kg.m/s² = mass x acceleration = Force in Newton. (1 N = 1 kg.m/s²).

- ✓ One-dimensional flow through a control surface is shown.
- ✓ The **thrust** or **wall force** experienced by the duct is a result of change in pressure and Mach number between the cross sections 1 and 2.
- ✓ By momentum equation, the thrust (τ) is given by

$$\tau = (p_2 A_2 + \rho_2 A_2 C_2^2) - (p_1 A_1 + \rho_1 A_1 c_1^2) \quad \text{....Eqn. A}$$

- ✓ $F^* = p^* A^* (1+\gamma)$

$$\tau = F_2 - F_1$$

- ✓ Thrust developed is equal to change in impulse function.

For a perfect gas, $\rho C^2 = \frac{p}{RT} C^2 = \gamma p \frac{C^2}{\gamma RT} = \gamma p M^2$ ....Eqn. 1

The impulse function is **F = pA + ρAC²** ....Eqn. 2

Eqn. 1 and Eqn. 2 give

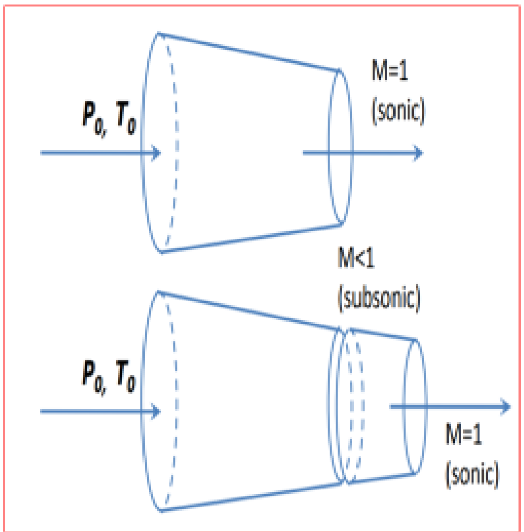
$F = pA + A\gamma p M^2 = pA(1+\gamma M^2)$ ....Eqn. 3

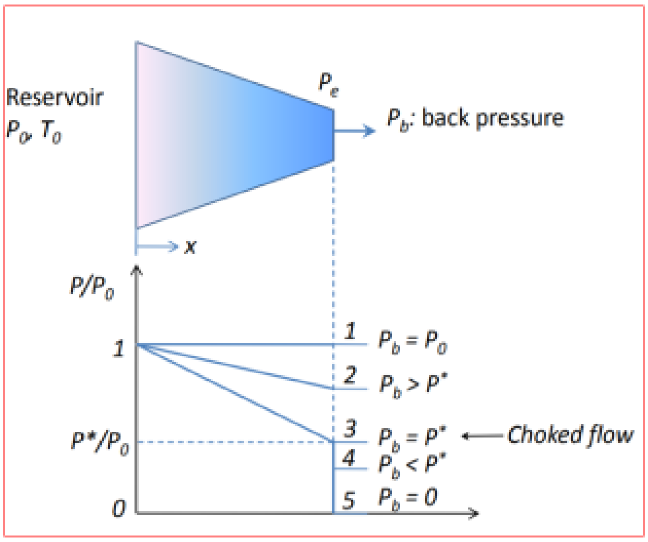
Substituting Eqn. 3 in Eqn. A yields, $\tau = p_2 A_2 (1 + \gamma M_2^2) - p_1 A_1 (1 + \gamma M_1^2)$ ...Eqn. B

Eqn. B demonstrates that the use of impulse function is very convenient in obtaining the thrust exerted by the flowing fluid.

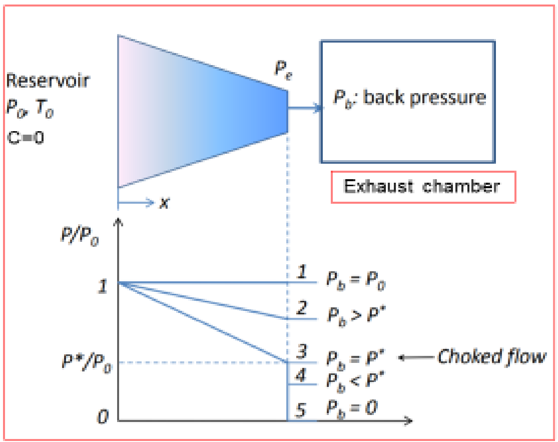
ISENTROPIC FLOW THROUGH CONVERGENT NOZZLE

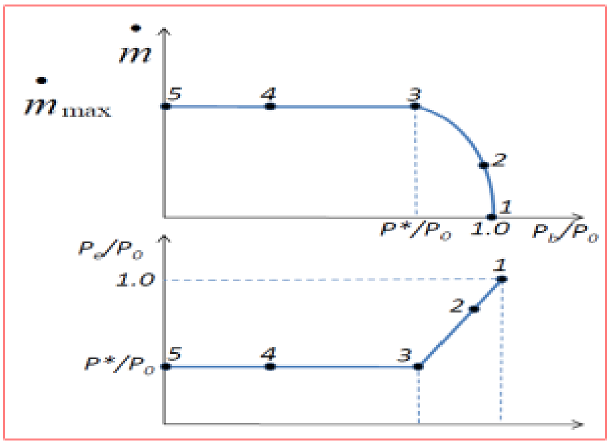
- ✓ Convergent nozzles are used for **subsonic** and **sonic** flows.
- ✓ Used as **flow measuring** and **flow regulating devices**.
- ✓ CD nozzles are used for supersonic flows.



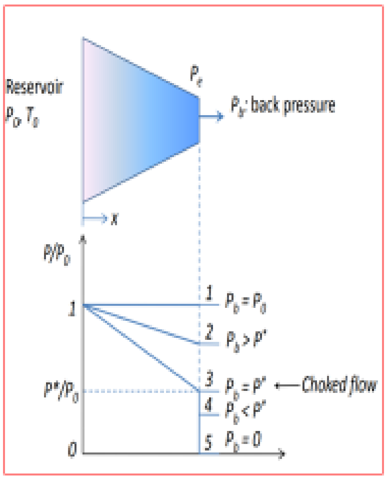


- **Back Pressure, p_b**: The flow in a **nozzle** is caused by a variation in **pressure** between two points.
- Here, the **pressure** at the exit is referred to as the **back-pressure**, and the **pressure** at the entry is the stagnation **pressure**.
- The ratio between them is the **back-pressure** ratio, which can be used to control flow velocity.

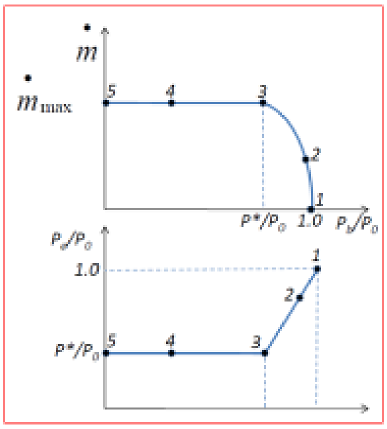


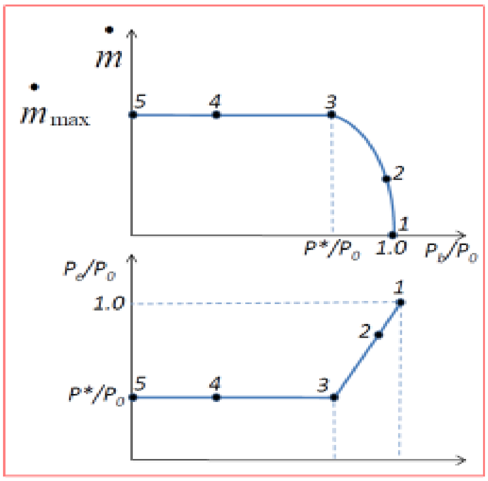


- Fig. shows the flow from an **infinite reservoir** to an exhaust chamber through a convergent nozzle.
- Stagnation conditions in the reservoir are kept constant while the back pressure, i.e., the exhaust chamber pressure can be varied.
- The pressure distribution along the nozzle for various values of the pressure ratio, p_e/p_0, across the nozzle are shown in curves as 1,2,3,4 and 5.



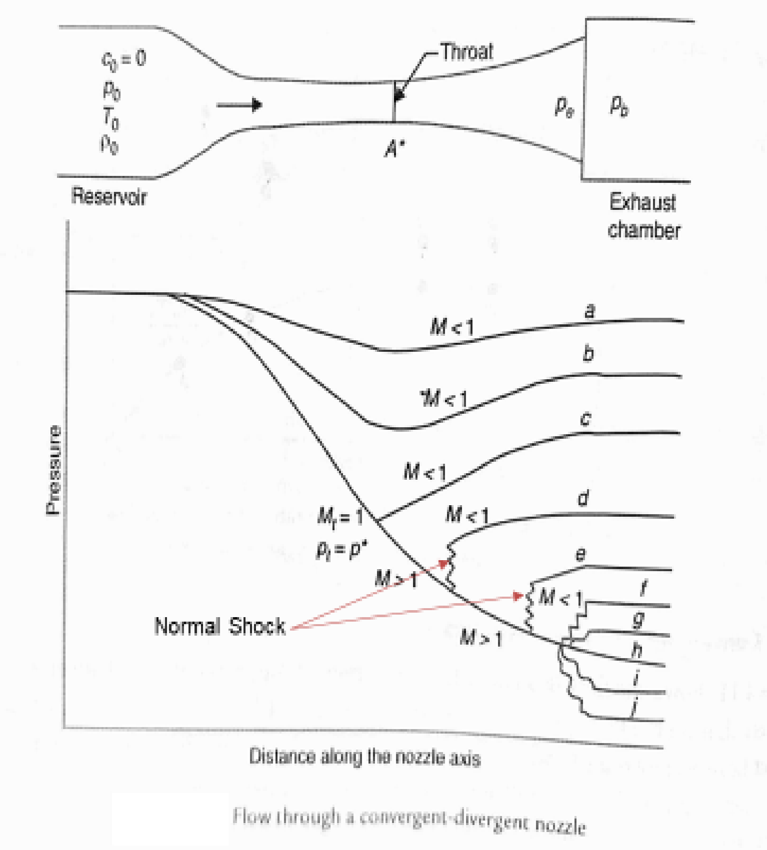
- Curves 1 and 2 correspond to the values of the pressure ratio more than the critical.
- Curve 3 corresponds to the critical pressure ratio, p_e/p_0 = 0.528(for γ=1.4).
- For curves 1, 2 and 3, $p_e = p_b$.
- The nozzle exit pressure, p_e does not decrease when the exhaust chamber pressure, p_b is further reduced below the critical value, as shown by curves 4 and 5.





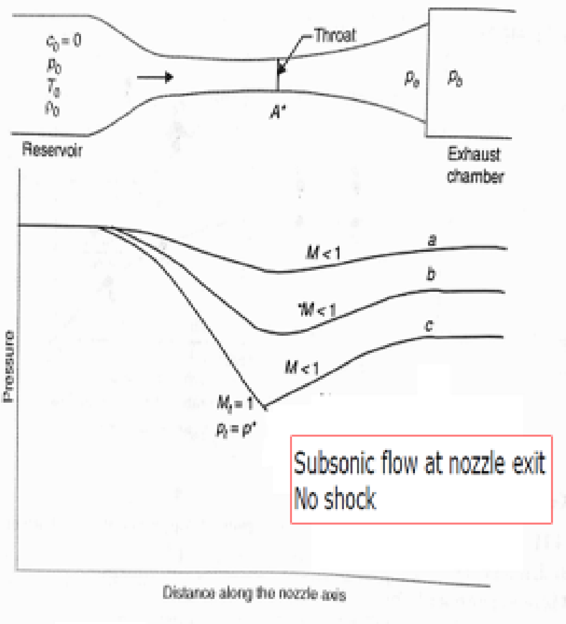
- ✓ Curves 1 and 2 correspond to the values of the pressure ratio more than the critical.
- ✓ Curve 3 corresponds to the critical pressure ratio, p_e/p_0 = 0.528(for γ=1.4).
- ✓ The maximum mass flow occurs at point 3 after which both the mass flow parameter and the nozzle pressure ratio **cease to vary.**
- ✓ A back pressure lower than the critical pressure cannot be sensed in the nozzle upstream flow and does not affect the flow rate.

ISENTROPIC FLOW THROUGH CONVERGENT DIVERGENT NOZZLE



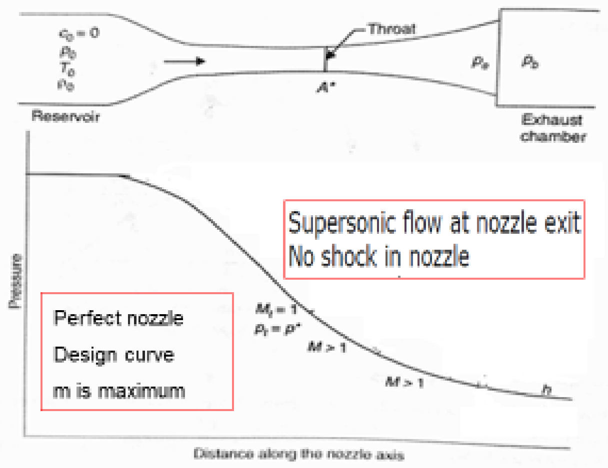
Flow through a convergent-divergent nozzle

- ✓ **Maximum Mach number** attainable in a **converging nozzle** is **unity**.
- ✓ For supersonic Mach numbers, a diverging section after the throat is required.
- ✓ The Mach number at the exit of the converging-diverging nozzle depends upon the back pressure (exhaust chamber pressure).
- ✓ Fig. shows the flow from an infinite reservoir through a CD nozzle to an exhaust chamber.
- ✓ The static pressure distributions for various values of the pressure ratio p_b/p_0 are shown in curves a, b, c, d, e, f, g, h, i and j.



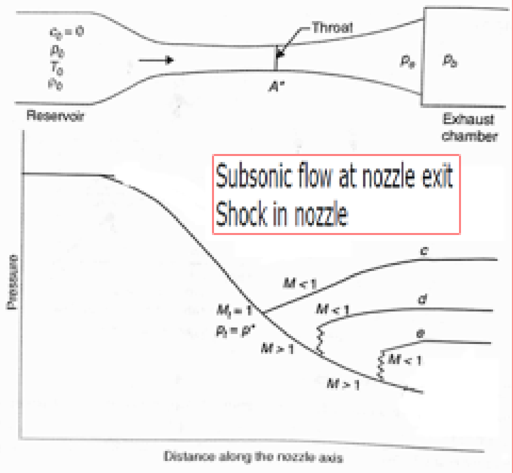
Flow through a convergent-divergent nozzle

- ✓ In 'a' and 'b', the pressure ratio p_e/p_0 across the nozzle is such that the flow is accelerating only up to the throat.
- ✓ In 'a' and 'b', the **diverging part** acts as a **diffuser** through which the pressure rises to the back pressure, p_b.
- ✓ In 'c', the critical conditions are reached at the throat(p_t=p^*; M_t=1) but the back pressure is such that the **diverging part** still acts as a **diffuser**.



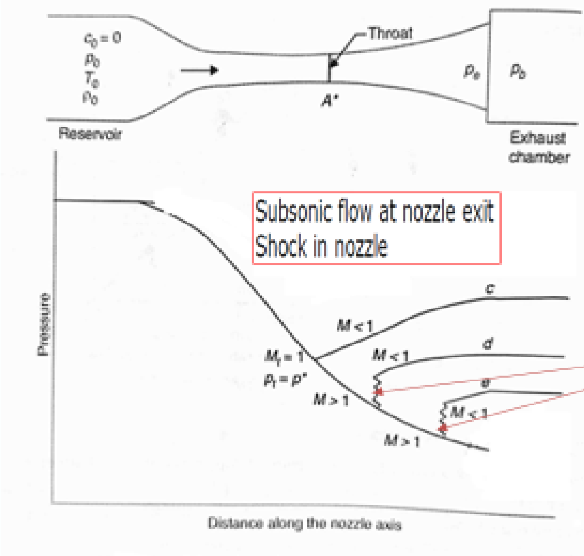
Flow through a convergent-divergent nozzle

- ✓ Curve **'h'** corresponds to the **design value of the back pressure**.
- ✓ Flow is supersonic in the entire divergent part of the nozzle.
- ✓ Flow is **isentropic** in curves a, b, c, h, i and j.



Flow through a convergent-divergent nozzle

- ✓ When the nozzle is operated under **off-design value of the back pressure(curves 'd' & 'e')** there is a discontinuity in the flow in the supersonic velocity and there is a **steep rise in static pressure, i.e., normal shock occurs in the divergent part of the nozzle.**



Flow through a convergent-divergent nozzle

- ✓ The pressure, temperature and density suddenly rises and velocity drops from supersonic to subsonic through a plane of discontinuity (wavy plane).
- ✓ This plane of discontinuity is called a **shock wave.**
- ✓ Flow through shock wave is **no longer isentropic.**
- ✓ Flow is irreversible in shock.

- ✓ Off-design curves – a,b,c – venturi – pressure ratio higher than design pressure ratio.
- ✓ Design curve – h – nozzle operated at design pressure ratio.
- ✓ Off-design curves – d, e – pressure ratio lower than design pressure ratio – shock curves.
- ✓ Off-design curves – f,g, i, j – pressure ratio lower than design pressure ratio.

Describe the behavior of flow in a CD nozzle when it is operated at (a) pressure ratio higher than the design value (b) pressure ratio lower than a design value (c) design pressure/value.

Pressure ratio higher than the design value

(a) off design curves – curves 'a' and 'b'(venturi)

1. Flow remains subsonic throughout the nozzle (M<1).
2. Pr. ratio is not critical at the throat.
3. Acceleration takes place in the converging part and upto the throat.
4. The diverging part still acts as a diffuser, in which the pressure rises to chamber pressure p_b at the expense of the velocity.
5. Flow does not reach the maximum level (not choked) at the throat.
6. Curves '*a*' and '*b*' acts as '*venturi*'.

(b) off-design curve – curve 'c' (venturi)

1. Flow is subsonic in the converging part of the nozzle and reaches sonic at the throat of the nozzle.
2. Pr. ratio at the throat is critical.
3. Acceleration takes place in the converging part and upto the throat.
4. The diverging part still acts as a diffuser, in which the pressure rises at the expense of velocity.
5. Flow at its maximum (choked flow).
6. Lowering the back pressure further will have no influence on the fluid flow in the converging part of the nozzle. But lowering back pressure p_b, will influence the character of flow in the diverging section.
7. Curve '*c*' acts as ***'venturi'***.

Nozzle operated at design pressure ratio

curve 'h' ***– design curve***

1. Flow is subsonic in the convergent part.
2. Flow is sonic at the throat.
3. Flow is supersonic in the divergent part, with ***no normal shock*** within the nozzle. Thus the flow throughout the nozzle can be approximated as ***isentropic***.
4. Mass flow rate is at its ***maximum (choked flow)***.

Pressure ratio lower than the design value

(d) off design curves – curves 'd' ***and*** 'e'.

1. Flow remains subsonic in the converging part & reaches sonic at the throat.
2. Flow continues accelerating to supersonic velocity in the diverging part.
3. This acceleration comes to a sudden stop; as a ***normal shock*** develops between the throat and exit plane of the nozzle, which causes a ***sudden drop in velocity*** to subsonic level and a ***steep or sudden increase in pressure.***
4. The fluid continues to decelerate further in the remaining part of the CD nozzle.
5. Flow through the shock is ***irreversible*** and it cannot be approximated as isentropic.

(e) off design curves – curves 'i' ***and*** 'j'

- lowering the back pressure further moves the shock downstream till it reaches the exit.

4.5 Following quantities are given at the entry and exit of a passage:
Entry: $p_1 = 2.07$ bar, $T_1 = 300$ K, $M_1 = 1.4$
Exit: $M_2 = 2.5$
Assuming isentropic flow of an ideal gas ($\gamma = 1.3$, $R = 0.52$ kJ/kg-K) determine:
(a) velocity of sound (a_0) at stagnation conditions.
(b) the maximum velocity (c_{max}).
(c) the Mach numbers M_1^* and M_2^*.
(d) temperature and pressure at exit.

Soln:

✓ Given: $\gamma = 1.3$; $R = 0.52$ kJ/kg-K; $p_1 = 2.07$ bar; $T_1 = 300$ K; $M_1 = 1.4$; $M_2 = 2.5 \rightarrow$ Nozzle

✓ To find: (a) a_0 (b) C_{max} (c) M_1^* and M_2^* (d) p_2 and T_2

✓ Type of flow assumed: **Isentropic**.

From gas tables: for $M_1 = 1.4$, $\gamma = 1.3$; Table 3.1, pg. 23, we get

1.40	1.320	0.773	0.327	1.123	1.039	0.367

$p_1/p_{01} = 0.327$ and hence $\mathbf{p_{01}} = p_1/0.327 = 2.07/0.327 = 6.33$ bar $= \mathbf{p_{02}}$

$T_1/T_{01} = 0.773$ and hence $\mathbf{T_{01}} = T_1/0.0.773 = 300/0.773 = 388.09$ K $= \mathbf{T_{02}}$

$a_0 = \sqrt{(\gamma R T_0)} = \sqrt{(1.3x0.52x1000x388.09)} = 512.2$ m/sec.

$C_p = \gamma R/(\gamma-1) \rightarrow (1.3x0.52)/(1.3-1) = 2.253$ kJ/kg-K

(b) $c_{max} = \sqrt{(2xh_0)} = \sqrt{(2xc_pT_0)} = 1322.4$ m/sec.

(c) $M_1^* = 1.320$

From gas tables: for $M_2 = 2.5$, $\gamma = 1.3$; Table 3.1, pg. 25, we get

2.50	1.926	0.516	0.057	2.954	1.223	0.168

(c) $M_2^* = 1.926 \approx 1.93$

(d) $p_2/p_{02} = 0.057$ and hence $p_2 = 0.057xp_{02} = 0.057x6.33 = 0.361$ bar.

$T_2/T_{02} = 0.516$ and hence $T_2 = 0.516xT_{02} = = 200.25$ K.

Answers:

(a) $a_0 = 512.2$ m/s

(b) (b) $C_{max} = 1322.4$ m/sec

(c) (c) $M_1^* = 1.320$ and $M_2^* \approx 1.93$

(d) (d) $p_2 = 0.361$ bar; $T_2 = 200.25$ K.

4.7 The Mach number and pressure at the entry of a subsonic diffuser are 0.9 and 4.165 bar. Determine the area ratio required and the pressure rise if the Mach number at the exit of diffuser is 0.20. Assume isentropic diffusion of air.

Soln:

✓ Given: $\gamma = 1.4$; R = 0.287 kJ/kg-K; $p_1 = 4.165$ bar; $M_1 = 0.9$; $M_2 = 0.2 \rightarrow$ Diffuser

✓ To find: (a) A_2/A_1 (b) $p_2 - p_1$

✓ Type of flow assumed: **Isentropic Diffusion**.

From gas tables: for $M_1 = 0.9$, $\gamma = 1.4$; Table 3.2, pg. 30, we get

0.90	0.915	0.861	0.591	1.009	1.004	0.596

$p_1/p_{01} = 0.591$ and hence $\mathbf{p_{01}} = \mathbf{p_{02}} = p_1/0.591 = 4.165/0.591 = 7.05$ bar

From gas tables: for $M_2 = 0.2$, $\gamma = 1.4$; Table 3.2, pg. 28, we get

0.20	0.218	0.992	0.973	2.964	2.400	2.882

$p_2/p_{02} = 0.973$ and hence $p_2 = 6.86$ bar

Pressure rise = $p_2 - p_1 = 6.86 - 4.165 = 2.695$ bar.

$$\frac{A_2}{A_1} = \frac{A_2}{A_2^*} x \frac{A_1^*}{A_1} = \frac{2.964}{1.009} = 2.94$$

Note: $\mathbf{A_1^* = A_2^* = A^*}$

Answers:

(a) $A_2/A_1 = 2.94$.

(b) $p_2 - p_1 = 2.695$ bar.

4.8 The entry conditions of air for a nozzle are $p_1 = 1.392$ bar, $T_1 = 333$ K, $M_1 = 0.40$. If the exit pressure is 1.039 bar determine the exit Mach number and the maximum specific mass flow m_{max}/A^*.

Soln:

✓ Given: $\gamma = 1.4$; R = 0.287 kJ/kg-K; $p_1 = 1.392$ bar; $M_1 = 0.4$; $T_1 = 333$ K; $p_2 = 1.039$ bar $\rightarrow$ Nozzle, as $p_2 < p_1$(pressure drop; expansion occurs)

✓ To find: (a) M_2 (b) m_{max}/A^*.

✓ Type of flow assumed: **Isentropic Acceleration**

From gas tables: for $M_1 = 0.4$, $\gamma = 1.4$; Table 3.2, pg. 29, we get

0.40	0.431	0.969	0.895	1.590	1.375	1.424

$p_1/p_{01} = 0.895$ and hence $\mathbf{p_{01}} = \mathbf{p_{02}} = p_1/0.895 = 1.392/0.895 = 1.555$ bar

✓ Now, $p_2/p_{02} = 1.039/1.555 = 0.668$ and for this pressure ratio, refer gas tables and get M_2 by interpolation.

0.78	0.807	0.892	0.669	1.047	1.023	0.700
0.79	0.816	0.889	0.663	1.043	1.021	0.691

✓ M_2 (by interpolation) = 0.781

From gas tables: for $M_1 = 0.4$, $\gamma = 1.4$; Table 3.2, pg. 29, we get, $T_1/T_{01} = 0.969$ and hence $T_{01} = T_{02} = 343.65$ K.

$$\frac{\dot{m}_{\max}\sqrt{T_o}}{A^* p_o} = 0.0404$$

and hence $m_{max}/A^* = 338.88$ kg/m²-sec.

Answers:

(a) $M_2 = 0.781$

(b) $m_{max}/A^* = 338.88$ kg/m²-sec.

4.9 The pressure, velocity and temperature of air ($\gamma = 1.4$, $c_p = 1.0$ kJ/kg K) at the entry of a nozzle are 2 bar, 145 m/s and 330 K; the exit pressure is 1.5 bar.

(*a*) what is the shape of the nozzle?

(*b*) Determine for isentropic flow

(*i*) the Mach numbers at entry and exit

(*ii*) the flow rate and maximum possible flow rate

Soln:

✓ Given: $\gamma = 1.4$; $C_p = 1$ kJ/kg-K; $p_1 = 2$ bar; $T_1 = 330$ K; $c_1 = 145$ m/s; $p_2 = 1.5$ bar

✓ To find: (a) Shape of the nozzle (b) M_1 and M_2 (c) m/A_1 and m_{max}/A^*.

✓ Type of flow assumed: **Isentropic Acceleration**.

✓ $C_p = \gamma R/(\gamma-1) \rightarrow R = (\gamma-1)C_p/\gamma = 285.7$ J/kg-K

✓ $M_1 = c_1/\sqrt{(\gamma RT_1)} = 145/\sqrt{(1.4 \times 285.7 \times 330)} = 0.399 \approx 0.4$

From gas tables: for $M_1 = 0.4$, $\gamma = 1.4$; Table 3.2, pg. 29, we get

0.40	0.431	0.969	0.895	1.590	1.375	1.424

$p_1/p_{01} = 0.895$ and hence $\mathbf{p_{01}} = \mathbf{p_{02}} = p_1/0.895 = 2/0.895 = 2.235$ bar

$T_1/T_{01} = 0.969$ and hence $T_{01} = T_{02} = T_1/0.969 = 330/0.969 = 340.56$ K

Now, $p_2/p_{02} = 1.5/2.235 = 0.671$ and for this pressure ratio, refer gas tables for $\gamma = 1.4$

Now, p_2/p_{02} = 1.5/2.235 = 0.671 and for this pressure ratio, refer gas tables for γ = 1.4

0.77	0.797	0.894	0.676	1.052	1.026	0.711
0.78	0.807	0.892	0.669	1.047	1.023	0.700

By interpolation @0.671, we get M_2 and T_2/T_{02}

M_2 = 0.777

T_2/T_{02} = 0.8926 → T_2 = 0.8926x340.56 = 303.98 K

✓ m = ρ_2 A_2 c_2 and hence, **m/A_2 =** ρ_2 c_2 = (p_2/RT_2) c_2 = $(1.5x10^5/(285.7x303.98))x0.777x(\sqrt{}\ (1.4x285.7x303.98)$ = **467.95 kg/m²-sec.**

$$\frac{\dot{m}_{\max}\sqrt{T_0}}{A^* p_0} = \sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\frac{\dot{m}_{max}}{A^*} = \frac{p_0}{\sqrt{T_0}} x \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma+1)}}$$

$$\frac{\dot{m}_{max}}{A^*} = \frac{p_0}{\sqrt{T_0}} x \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma+1)}} = \frac{2.235x10^5}{\sqrt{340.56}} x \sqrt{\frac{1.4}{285.7}} x \left(\frac{2}{2.4}\right)^{\frac{2.4}{2x0.4}}$$

m_{max}/A^*= 490.6 kg/m²-sec

Answers:

(a) $M_1 < M_2$ (0.4 < 0.777) → convergent nozzle as M_2 is still subsonic.

(b) M_1 = 0.4; M_2 = 0.777;

(c) m/A_2 = 467.95 kg/m²-sec.

(d) m_{max}/A^* = 490.6 kg/m²-sec.

4.10 A gas is isentropically expanded from $p = 10$ bar and $t = 525$°C in a nozzle to a pressure of 7.6 bar. If the rate of flow of the gas is 1.5 kg/s determine: (*a*) pressure, temperature and velocity at the nozzle throat and exit; (*b*) maximum possible velocity attainable by the gas and (*c*) the type of the nozzle and its throat area. Take $\gamma = 1.3$ and $R = 0.464$ kJ/kg K.

Soln:

✓ Given: $\gamma = 1.3$; R = 0.464 kJ/kg-K; $p_0 = 10$ bar $= p_{01} = p_{02}$; $T_0 = 525$ °C = 798 K $= T_{01} = T_{02}$; $p_2 = 7.6$ bar; m = 1.5 kg/sec;

✓ **Note: *isentropically expanded from*** means ***isentropically expanded from reservoir***

✓ To find: (a) p*, T*, c* and p_2, T_2, c_2 (b) C_{max} (c) Type of nozzle and A*.

✓ $C_p = \gamma R/(\gamma-1) \rightarrow (1.3x0.464)/(1.3-1) = 2.01$ kJ/kg-K

Now, $p_2/p_{02} = 7.6/10 = 0.76$ and for this pressure ratio, refer gas tables for $\gamma = 1.3$ and get M_2 and T_2/T_{02}.

0.66	0.686	0.938	0.760	1.130	1.072	0.859

We get: $M_2 = 0.66$;

$T_2/T_{02} = 0.938$ and hence $T_2 = 748.52$ K;

$A_2/A_2^* = 1.130$

$c_2 = M_2 \times a_2 = 0.66 \times \sqrt{(\gamma R T_2)} = 443.48$ m/sec.

$$c_{max} = \sqrt{2xh_0} = \sqrt{2xc_p xT_0} = \sqrt{2x\frac{\gamma R}{\gamma - 1}xT_0} = \sqrt{2x\frac{1.3x464}{0.3}x798} = 1791.37 \text{ m/sec}$$

Wkt., $\rho_2 = p_2/RT_2 = 7.6x10^5/(464x748.52) = 2.188$ kg/m³.

✓ $m = \rho_2 A_2 c_2$ and hence,

✓ A_2 = Throat area $= m/\rho_2 c_2 = 1.5/(2.188x443.48) = 0.001546$ m² = 15.46 cm².

✓ As $M_2 = 0.66 \rightarrow$ the given nozzle is of convergent type as M_2 is still subsonic.

Answers:

(a) $p_2 = 7.6$ bar; $T_2 = 748.52$ K; $c_2 = 443.48$ m/sec.

(b) $c_{max} = 1791.37$ m/sec.

(c) Type of nozzle: convergent and $A_2 = 15.46$ cm².

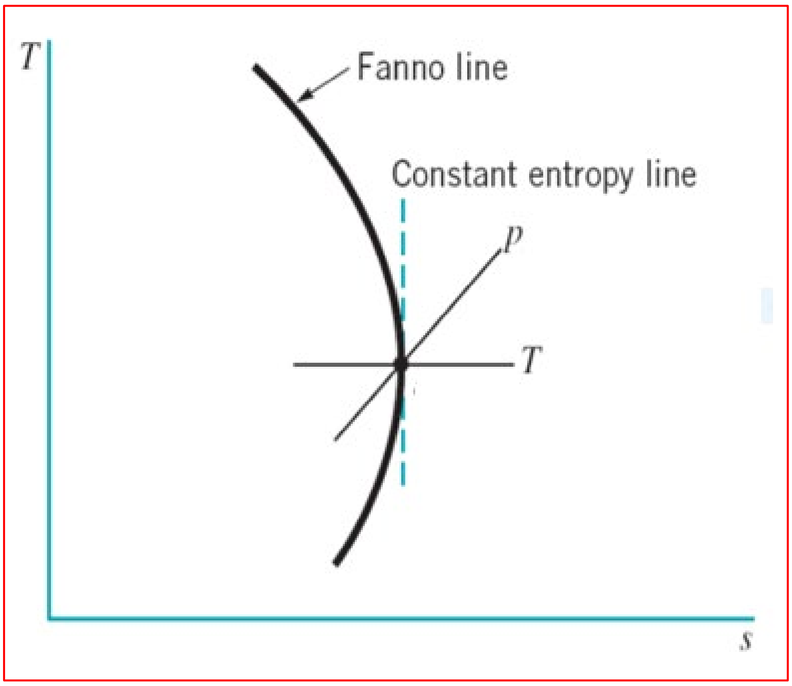
SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE
www.sathyabama.ac.in

SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
GAS DYNAMICS AND JET PROPULSION (SME1303)

UNIT 3
FLOW THROUGH CONSTANT AREA DUCTS

FANNO FLOW OR FRICTIONAL FLOW

Adiabatic flow process in a constant area duct with friction in the absence of work transfer. Flow properties vary due to duct wall friction.



Note: In Fanno flow/Fanno line/ Fanno curve, at the point of maximum entropy F, M = 1 or the flow is sonic. i.e., flow is choked.

Applications of Fanno flow

- ✓ Flow processes occurring in gas ducts of air craft engines.
- ✓ Air conditioning systems.
- ✓ Transport of fluids in a chemical process plants.
- ✓ Thermal and nuclear power plants.
- ✓ Petrochemical and gas industries.
- ✓ High vacuum technology.
- ✓ Transport of natural gas in long pipe lines.
- ✓ Exhaust system of an internal combustion engine.

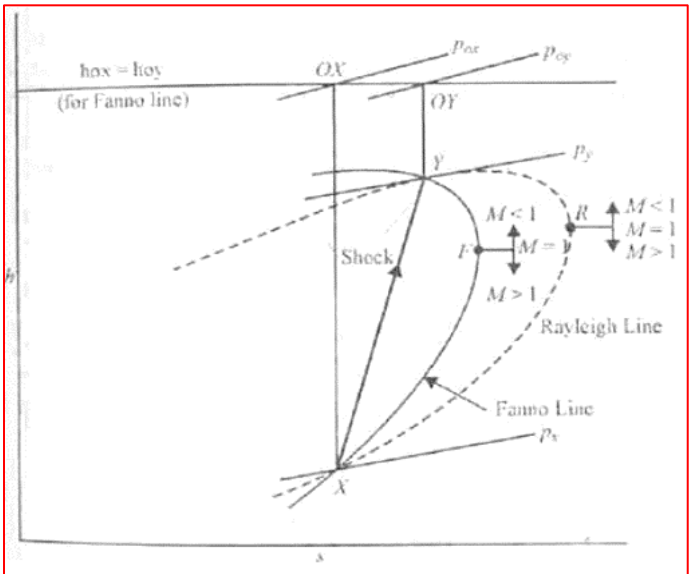
Equations derived for Fanno flow is derived under the following assumptions:

- ✓ Perfect gas.
- ✓ Constant area duct.
- ✓ One dimensional steady frictional flow.
- ✓ Absence of heat and work transfer.
- ✓ Absence of body force (gravitational effects).

GOVERNING EQUATIONS FOR FANNO FLOW

1. Continuity equation
2. Energy equation
3. Momentum equation
4. Equation of state

Using the above equations, a curve in h-s plane is obtained, which is called the Fanno line/Fanno curve.



Normal shock on Fanno and Rayleigh curves

Upper Branch, AF – Subsonic

- ✓ Process occur in the direction AF.
- ✓ Friction in the passage leads to increase in entropy and the flow reaches sonic. i.e., M = 1.
- ✓ Process in the direction FA is not possible because it leads to decrease in entropy, which violates II law of thermodynamics.
- ✓ Friction causes **irreversible acceleration** of the flow with **pressure drop**.
- ✓ Maximum velocity obtainable in subsonic flow is **sonic (F)**.
- ✓ Further acceleration of flow towards B cannot occur because of the impossibility of decrease in entropy.

Lower Branch, BF – Supersonic

- ✓ Process occur in the direction BF.
- ✓ Friction in the passage leads to increase in entropy and the flow reaches sonic. i.e., M = 1.
- ✓ Process in the direction FB is not possible because it leads to decrease in entropy, which violates II law of thermodynamics.
- ✓ Friction causes **irreversible deceleration** of the flow with **pressure rise**.
- ✓ **Minimum velocity obtainable in supersonic flow is sonic (F).**
- ✓ Further deceleration of flow towards A cannot occur because of the impossibility of decrease in entropy.

✓ Fanning friction factor or coefficient of skin friction, f

- It is a measure of frictional resistance to flow.
- A dimensionless number characterizing the frictional force at the boundary between fluid and a wall.
- It is defined by the identity: a) in the case of flow in pipes. b) in the case of external flow past a body.
- f = wall shear stress / dynamic head.

$$f = \frac{\tau_w}{\left(\frac{1}{2}\right)\rho c^2}$$

VARIATION OF PROPERTIES IN FANNO FLOW

Properties	Subsonic Region	Supersonic Region
M	increases	decreases
c	increases	decreases
p	decreases	increases
T	decreases	increases
T_0	constant	constant
p_0	decreases	decreases
ρ	decreases	increases
F	decreases	decreases
$4f(L_{max}/D)$	decreases	decreases

VARIATION OF FLOW PARAMETERS

$$\frac{c_2}{c_1} = \frac{M_2}{M_1}\left(\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right)^{1/2}$$

$$\frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\left\{\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right\}^{1/2}$$

$$\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$$

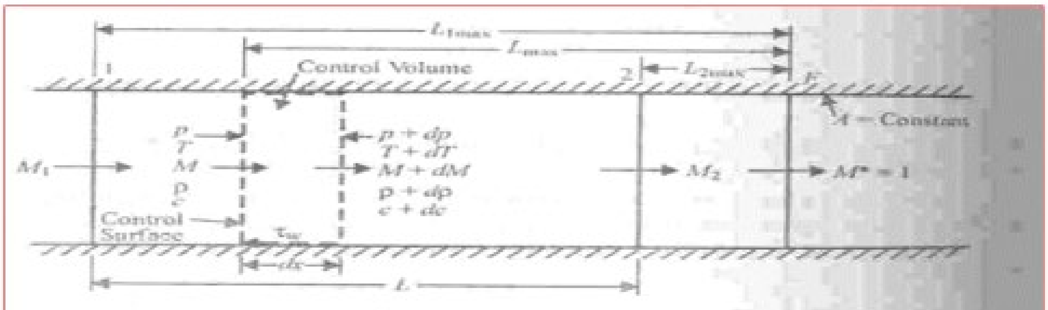
$$\frac{p_2}{p_1} = \frac{M_1}{M_2}\left\{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right\}^{1/2}$$

$$\frac{p_{02}}{p_{01}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{(\gamma+1)/2(\gamma-1)}$$

$$\frac{F_2}{F_1} = \frac{1+\gamma M_2^2}{1+\gamma M_1^2}\,\frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{1/2}$$

$$\frac{s_2-s_1}{R} = \ln\frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{(\gamma+1)/2(\gamma-1)}$$

VARIATION OF MACH NUMBER WITH DUCT LENGTH



Variation of flow properties in Fanno flow

- ✓ The distance between the two sections of the duct where the Mach numbers are M_1 and M_2 is given by

$$4\bar{f}\frac{L}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_1} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{M_2}$$

$$4\bar{f}\frac{L}{D} = \frac{M_2^2 - M_1^2}{\gamma M_1^2 M_2^2} + \frac{\gamma+1}{2\gamma}\ln\frac{M_1^2}{M_2^2}\,\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}$$

Prove that at the point of maximum entropy (F) in Fanno line, the flow is sonic. i.e., Mach number is equal to one.

Continuity equation:

$$\dot{m} = \rho_x A_x c_x = \rho_y A_y c_y$$

Since,

$$A_x = A_y = A = constant$$

$$\frac{\dot{m}}{A} = \rho_x c_x = \rho_y c_y \quad \text{----- Eqn. 1}$$

Energy equation:

$$h_{ox} = h_{oy} = h_o = constant$$

$$T_{ox} = T_{oy} = T_o = constant$$

$$h_x + \frac{1}{2}c_x^2 = h_y + \frac{1}{2}c_y^2 \quad --- Eqn.2$$

Momentum equation:

$$(p_x - p_y)A = \dot{m}(c_y - c_x)$$

$$p_x - p_y = \frac{\dot{m}}{A}(c_y - c_x) \quad \text{-- Eqn. 3}$$

Equation 1 and 3 gives

$$p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2 \quad \text{-- Eqn. 4}$$

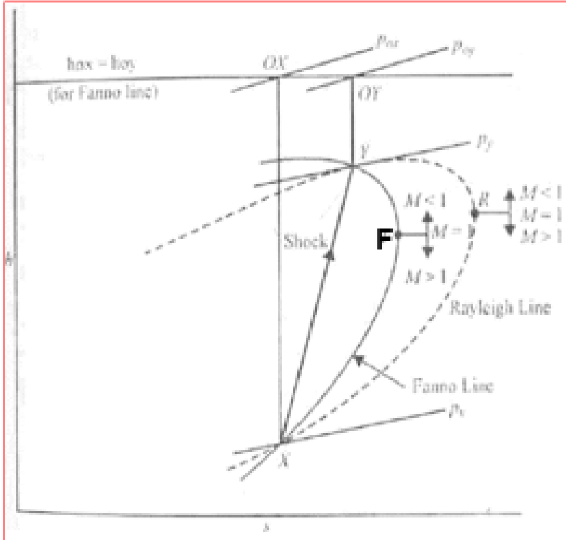
$$F_x = F_y = constant \quad \text{-- Eqn. 5}$$

Equation of state: Equation of state gives

$$h = f(s, \rho)$$

$$s = f(p, \rho) \quad \text{-- Eqn. 6}$$

Combine Eqns. 1 & 2 into a single relation and plot it on the h-s diagram using property relations, the resultant curve is called the Fanno line or Fanno curve.



Normal shock on Fanno and Rayleigh curves

- ✓ F – Point of maximum entropy on Fanno line
- ✓ At point F, M = 1 (flow is sonic) (flow is choked)
- ✓ Upper branch – Subsonic flow
- ✓ Lower branch – Supersonic flow
- ✓ Shock: X to Y
- ✓ $p_y > p_x$; $p_{0x} > p_{0y}$; $S_y > S_x$

- ✓ Consider an infinitesimal process in the vicinity of maximum entropy point, F.
- ✓ **At this point F**, the changes being small, the process may be considered as reversible ($\Delta s = 0$).
- ✓ For this reversible process, we have,

$h + (c^2/2) = \text{constant}$

On differentiation, we get,

$dh + cdc = 0$

$dh = -cdc$ -- Eqn. 7

Also,

$\rho c = \text{constant}$

On differentiation, we get

$\rho dc + cd\rho = 0$ s

$dc = -(c/\rho)\, d\rho$ -- Eqn. 8

Wkt., $Tds = dh - vdp$

For an isentropic process, ds = 0 and hence the above equation becomes,

$dh = vdp$ (or) $dh = dp/\rho$ -- Eqn. 9

Eqns. 8 and 9 in Eqn. 7 gives,

$dp/\rho = -c[-(c/\rho)\, d\rho] \longrightarrow dp = c^2 \times d\rho$

$$\left(\frac{dp}{d\rho}\right)_s = c^2$$

But,

$$\left(\frac{dp}{d\rho}\right)_s = a^2$$

Hence,

- ✓ $c^2 = a^2$
- ✓ $c = a$
- ✓ i.e., $M = 1$
- ✓ i.e., Velocity of gas at maximum entropy point, F, is sonic.
- ✓ Upper branch represents subsonic flow.
- ✓ Lower branch represents supersonic flow.

8.5 The friction factor for a 25 mm diameter 11.5 m long pipe is 0.004. The conditions of air at entry are

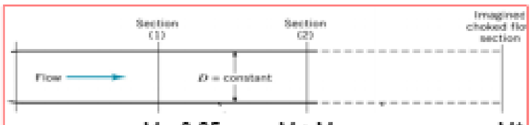
$$p_1 = 2.0 \text{ bar}, \ T_1 = 301 \text{ K}, \ M_1 = 0.25$$

Determine the mass flow rate, and the pressure, temperature and the Mach number at exit.

Given: D = 25 mm = 0.025 m; L = 11.5 m; f = 0.004; p_1 = 2 bar; T_1 = 301 K; M_1 = 0.25;

To find: (a) m (b) p_2 (c) T_2 (d) M_2

Solution:



M_1=0.25 M_2>M_1 M*

- ✓ $m = \rho_1 A_1 c_1 = (p_1/RT_1)A_1 c_1$
- ✓ $c_1 = M_1 \times a_1 = 0.25 \times \sqrt{(1.4\times287\times301)} = 86.94$ m/sec

Now,

$m = (2\times10^5/(287\times301))\times(\pi/4)(0.025)^2\times86.94 = 0.098$ kg/sec

T / T* = 1.185

p / p* = 4.362

$$\boxed{4f\frac{L}{D} = \left[4f\frac{L_{Max}}{D}\right]_{M_1} - \left[4f\frac{L_{max}}{D}\right]_{M_2}}$$

$$4\bar{f}\frac{L}{D} = 8.537$$

- ✓ At M_1 =0.25, refer Fanno table for γ = 1.4, Table 6.2, pg. 81, by interpolation, we get

0.24	4.538	0.261	1.186	2.496	2.043	9.387
0.26	4.185	0.283	1.184	2.317	1.909	7.687
	p / p*		T / T*			

$$4\bar{f}\frac{L}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_1} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{M_2}$$

- ✓ (4x0.004x11.5)/0.025 = 8.537 - $\left(4\bar{f}\frac{L}{D}\right)_{M_2}$

$$\left(4\bar{f}\frac{L}{D}\right)_{M_2} = 1.177$$

- ✓ At 1.177, refer Fanno table for γ = 1.4, Table 6.2, pg. 81, by interpolation, we get

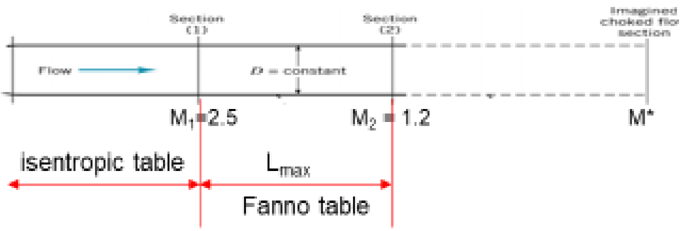
0.48	2.231	0.514	1.147	1.380	1.230	1.245
0.50	2.138	0.534	1.143	1.340	1.203	1.069

- ✓ $M_2 = 0.49$
- ✓ $p_2 / p_2^* = 2.195 \longrightarrow p_2 = \frac{(p/p^*)_2}{(p/p^*)_1} x p_1 = 1.006$ bar or $p_2 = \frac{(p/p^*)_{M_2}}{(p/p^*)_{M_1}} x p_1$
- ✓ $T_2 / T_2^* = 1.145 \longrightarrow T_2 = \frac{(T/T^*)_2}{(T/T^*)_1} x T_1 = 290.8$ K or $T_2 = \frac{(T/T^*)_{M_2}}{(T/T^*)_{M_1}} x T_1$

Answers: $M_2 = 0.49$; $p_2 = 1.006$ bar; $T_2 = 290.8$ K; m = 0.098 kg/sec.

8.7 A long pipe of 25.4 mm diameter has a mean coefficient of friction of 0.003. Air enters the pipe at a Mach number of 2.5, stagnation temperature 310 K and static pressure 0.507 bar. Determine for a section at which the Mach number reaches 1.2,

(*a*) static pressure and temperature,
(*b*) stagnation pressure and temperature,
(*c*) velocity of air,
(*d*) distance of this section from the inlet, and
(*e*) mass flow rate of air.



Given: D = 25.4 mm = 0.0254 m; f = 0.003; $p_1 = 0.507$ bar; $T_{01} = 310$ K; $M_1 = 2.5$; $M_2 = 1.2$;

To find: (a) p_2 and T_2 (b) p_{02} and T_{02} (c) c_2 (d) L_{max} (e) m

- ✓ From isentropic tables, for $M_1 = 2.5$, γ = 1.4; Table 3.2, pg. 36, we get

2.50	1.826	0.444	0.0585	2.637	1.187	0.154

- ✓ $p_1 / p_{01} = 0.0585 \longrightarrow p_{01} = 8.667$ bar
- ✓ $T_1 / T_{01} = 0.444 \longrightarrow T_1 = 137.64$ K

✓ From Fanno tables, for $M_1 = 2.5$, $\gamma = 1.4$; Table 6.2, pg. 85, we get

2.50	0.292	1.826	0.533	2.637	1.187	0.432

✓ From Fanno tables, for $M_2 = 1.2$, $\gamma = 1.4$; Table 6.2, pg. 82, we get

1.20	0.804	1.158	0.932	1.030	1.011	0.034

$$p_2 = \frac{(p/p^*)_{M_2}}{(p/p^*)_{M_1}} x p_1 = 1.396 \text{ bar}$$

$$T_2 = \frac{(T/T^*)_{M_2}}{(T/T^*)_{M_1}} x T_1 = 240.676 \text{ K}$$

$T_{01} = T_{02} = 310$ K

$$p_{02} = \frac{(p_0/p_0{}^*)_{M_2}}{(p_0/p_0{}^*)_{M_1}} x p_{01} = 3.385 \text{ bar}$$

$c_2 = M_2 \text{ x } a_2 = M_2 \text{ x } \sqrt{(1.4x287x240.676)} = 373.16$ m/sec

$$4\bar{f}\frac{L}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_1} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{M_2} \rightarrow$$

$(4x0.003xL_{max})/0.0254 = 0.432 - 0.034 \rightarrow$

$L_{max} = 0.842$ m

✓ $m = \rho_1 A c_1 = (p_1/RT_1)Ac_1 = 0.382$ kg/sec.

Answers:

(a) $p_2 = 1.396$ bar; $T_2 = 240.676$ K;

(b) $p_{02} = 3.385$ bar; $T_{01} = T_{02} = 310$ K;

(c) $c_2 = 373.16$ m/sec;

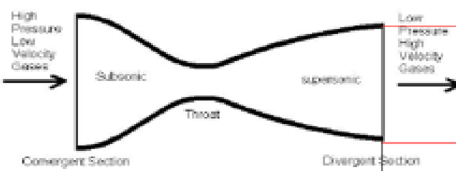
(d) $L_{max} = 0.842$ m;

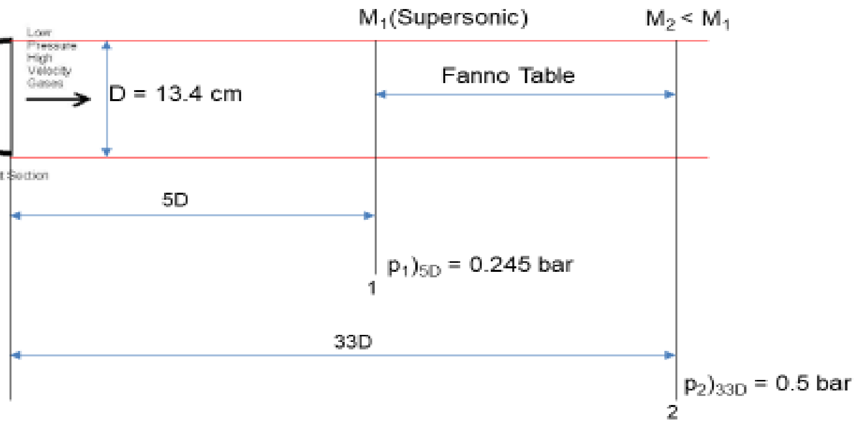
(e) $m = 0.382$ kg/sec;

8.8 A circular duct of 13.4 cm diameter is fed with air by a supersonic nozzle. The stagnation pressure at the nozzle entry and static pressures at sections 5 D and 33 D downstream are 7.00, 0.245 and 0.50 bar respectively. The nozzle throat diameter is 6.46 cm. Determine (a) Mach numbers at the two sections downstream of the nozzle (b) the mean value of the skin friction between the two sections. Assume isentropic flow upto the nozzle throat and adiabatic in the rest.

Given: D = 13.4 cm = 0.134 m; $p_0 = 7$ bar; $p_1)_{5D} = 0.245$ bar; $p_2)_{33D} = 0.5$ bar; $D_t = 6.46$ cm = 0.0646 m; isentropic flow up to throat and adiabatic in the rest.

To find: (a) M_1 and M_2 (b) f





At station 1

$$\frac{A_1}{A^*} = \left(\frac{D_1}{D^*}\right)^2 = \left(\frac{13.4}{6.46}\right)^2 = 4.303$$

$p_1/p_0 = 0.245/7 = 0.035$

At station 1,

$$\frac{A_1}{A^*} x \frac{p_1}{p_0} = 4.303x0.035 = 0.150$$

From Isentropic tables, for this ratio of 0.150, γ = 1.4; Table 3.2, pg. 36, we get

2.54	1.838	0.436	0.0550	2.737	1.191	0.150

- ✓ $M_1 = M_{5D} = 2.54$

At station 2

$p_2/p_0 = 0.5/7 = 0.071$

$$\frac{A_2}{A^*} x \frac{p_2}{p_0} = 4.303x0.071 = 0.306$$

From Isentropic tables, for this ratio of 0.306, γ = 1.4; Table 3.2, pg. 33, by interpolation, we get

1.55	1.395	0.675	0.253	1.211	1.056	0.307
1.56	1.402	0.673	0.249	1.219	1.057	0.304

- ✓ $M_2 = M_{33D} = 1.55$

Wkt.,

$$4\bar{f}\frac{L}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{M_1} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{M_2}$$

$$4\bar{f}\frac{L}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{2.54} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{1.55}$$

From Fanno table, M_2 = 1.55, γ = 1.4; Table 6.2, pg. 83., by interpolation, we get

1.54	0.586	1.389	0.814	1.204	1.055	0.151
1.56	0.576	1.401	0.807	1.219	1.057	0.158

From Fanno table, M_1 = 2.54, γ = 1.4; Table 6.2, pg. 85, we get

2.54	0.285	1.838	0.524	2.737	1.191	0.440

$$4\bar{f}\frac{28D}{D} = 0.440 - 0.155 \longrightarrow \boxed{f = 0.0025 \approx 0.003}$$

Answers:

(a) $M_1 = M_{5D} = 2.54$

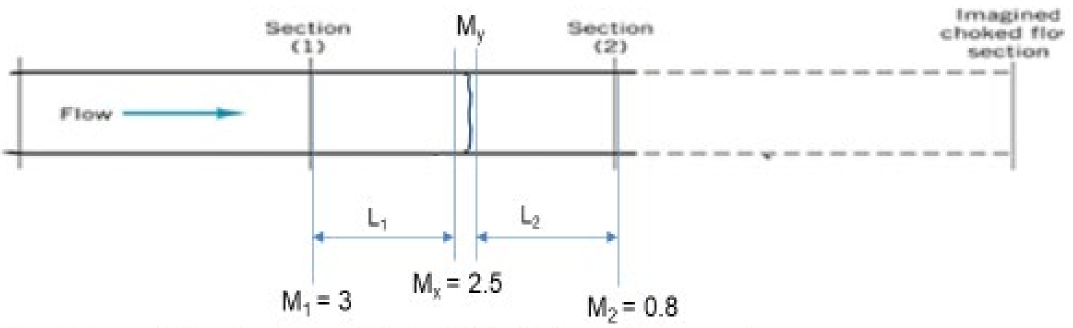
$M_2 = M_{33D} = 1.55$

(b) f = 0.003

8.9 Air at a pressure of 685 mbar and temperature 310 K enters a 60 cm diameter duct at a Mach number of 3.0. The flow passes through a normal shock wave at a section L_1 metres downstream of the entry where the Mach number is 2.5. The Mach number at the exit (at a distance L_2 metres downstream of the shock) is 0.8. The mean coefficient of skin friction is 0.005. Determine:
(a) the length L_1 and L_2,
(b) state of air at exit, and
(c) mass flow rate through the duct.

Given: D = 60 cm = 0.6 m; p_1 = 685 mbar; T_1 = 310 K; M_1 = 3.0; M_x = 2.5; M_2 = 0.8; f = 0.005;

To find: (a) L_1 and L_2 (b) p_2, T_2, c_2 (c) m



At M_1 = 3, γ = 1.4; refer Fanno table, Table 6.2, pg. 85, we get

3.00	0.218	1.964	0.428	4.235	1.237	0.522

At M_x = 2.5, γ = 1.4; refer Fanno table, Table 6.2, pg. 85, we get

2.50	0.292	1.826	0.533	2.637	1.187	0.432

Now,

$$\frac{p_x}{p_1} = \frac{p_x/p^*}{p_1/p^*}$$

$$\frac{p_x}{p_1} = \frac{0.292}{0.218} = 1.3395$$

✓ p_x = 1.3395 x 685 = 917.56 mbar

$$\frac{T_x}{T_1} = \frac{T_x/T^*}{T_1/T^*}$$

$$\frac{T_x}{T_1} = \frac{0.533}{0.428} = 1.2453$$

✓ T_x = 1.2453 x 310 = 386.04 K

At M_x = 2.5, γ = 1.4; refer shock table, Table 4.2, pg. 54, we get

2.50	0.513	7.125	2.138	0.499	8.526

✓ M_y = 0.513

✓ p_y/p_x = 7.125 ⟶ p_y = 7.125 x 917.56 = 6537.62 mbar

✓ T_y/T_x = 2.138 ⟶ T_y = 2.138 x 386.04 = 825.35 K

Between M_1 and M_x:

$$4\bar{f}\frac{L_1}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_3 - \left[4\bar{f}\frac{L_{max}}{D}\right]_{2.5}$$

✓ From Fanno table, M_1 = 3, γ = 1.4; Table 6.2, pg. 85., we get

$$4x0.005x\frac{L_1}{D} = 0.522 - 0.432 \longrightarrow L_1 = 2.7 \text{ meters}$$

Between M_y and M_2:

✓ From Fanno table at M_y = 0.513, γ = 1.4; Table 6.2, pg. 85, by interpolation, we get

0.50	2.138	0.534	1.143	1.340	1.203	1.069
0.52	2.052	0.555	1.138	1.303	1.179	0.917

✓ From Fanno table at M_2 = 0.8, γ = 1.4; Table 6.2, pg. 85, by interpolation, we get

0.80	1.289	0.825	1.064	1.038	1.018	0.073

$$4\bar{f}\frac{L_2}{D} = \left[4\bar{f}\frac{L_{Max}}{D}\right]_{0.513} - \left[4\bar{f}\frac{L_{max}}{D}\right]_{0.8}$$

$$4x0.005x\frac{L_2}{D} = 0.970 - 0.073 \longrightarrow L_2 = 26.9 \text{ meters}$$

Total pipe length = 2.7 + 26.9 = 29.6 meters

✓ p_y/p^* = 2.082 ⟶ p^* = 6537.62/2.082 = 3140.07 mbar

✓ T_y/T^* = 1.139 ⟶ T^* = 825.35/1.139 = 724.63 K

✓ p_2/p^* = 1.289 ⟶ p_2 = 1.289 x 3140.07 = 4047.55 mbar = 4.05 bar

✓ T_2/T^* = 1.064 ⟶ T_2 = 1.064 x 724.63 = 771.01 K

✓ m = $\rho_1 A c_1$ = $(p_1/RT_1)Ac_1$ = ((685x10^{-3}x10^5)/(287x310)) **x** ((3.14/4)x0.6^2) **x** (M_1xa_1) = 230.5 kg/sec.

Answers:

(a) L_1 = 2.7 meters

L_2 = 26.9 meters

(b) p_2 = 4.05 bar

T_2 = 771.01 K

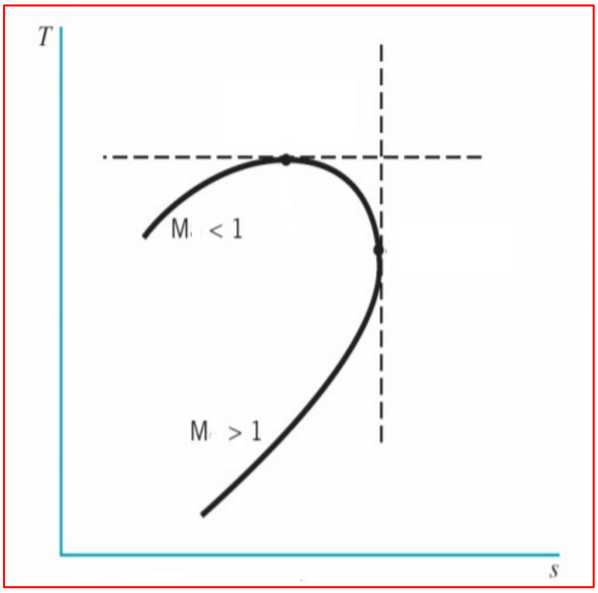
(c) m = 230.5 kg/sec

- ✓ **Rayleigh flow or Flow with heat transfer**

  - **Frictionless** flow process in a constant area duct with heat transfer (diabatic flow).

Applications of Rayleigh flow:

1. Heat exchangers
2. Combustion chamber inside turbojet engines
3. Gas coolers



Note:

- In Rayleigh curve, at the point of maximum entropy R, M = 1 or the flow is sonic. i.e., flow is choked.
- At the point of maximum enthalpy E, $M = 1/\sqrt{\gamma}$

Equations derived for Rayleigh flow is derived under the following assumptions:

- ✓ Perfect gas with constant specific heats and molecular weight.
- ✓ Constant area frictionless duct.
- ✓ One dimensional flow with heat transfer.

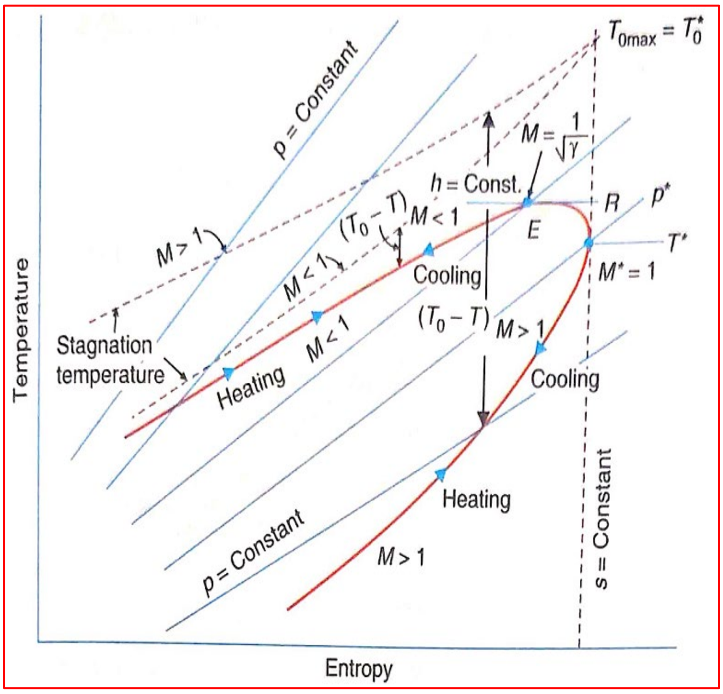
Absence of body forces (gravitational effects).

GOVERNING EQUATIONS FOR RAYLEIGH FLOW

1. Continuity equation
2. Energy equation
3. Momentum equation
4. Equation of state
5. Mach Number

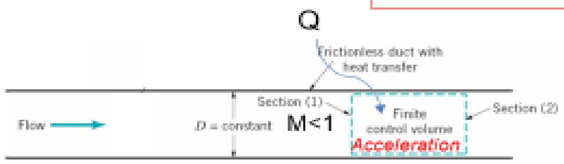
- Using the above equations, a curve in h-s plane is obtained, which is called the Rayleigh curve.

RAYLEIGH FLOW



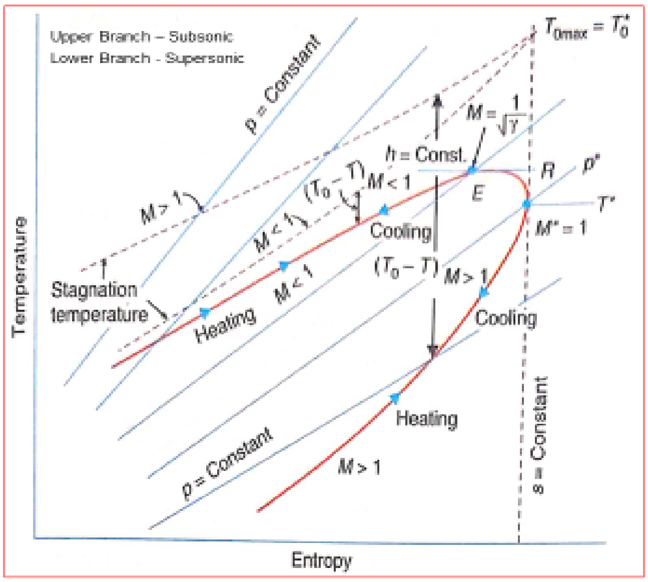
- ✓ The heat transfer causing heating or cooling of the gas changes its stagnation temperature.
- ✓ During heating, entropy must increase.
- ✓ Hence, the state of the gas moves towards the right approaching the maximum entropy point R.
- ✓ Heating process beyond R is not possible as it leads to decrease in entropy.
- ✓ *Limit of the heating process on both subsonic and supersonic branches of the Rayleigh line is the maximum entropy point R.*
- ✓ The direction of cooling processes is away from the limiting point R.
- ✓ The stagnation temperature lines for the subsonic and supersonic branches meet at M = M* = 1.
- ✓ Since no further heating beyond this (M = 1) is possible, the stagnation temperature at this point has the maximum value, i.e., at M = M* = 1: $T_0 = T_0^* = T_{0\,max}$

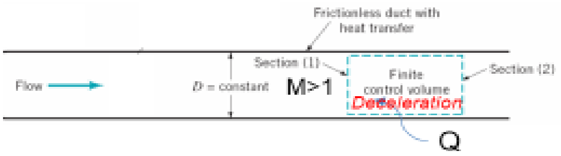
RAYLEIGH FLOW



- ✓ Subsonic flow is accelerated by heat addition.
- ✓ The maximum obtainable velocity is sonic at point R.
- ✓ The static enthalpy or static temperature rise up to the maximum enthalpy point E beyond which further heat addition leads to decrease in enthalpy or temperature.

Reason for this: Pressure drop is greater than the temp. increase by heat addition.





- ✓ Supersonic flow is decelerated by heat addition.
- ✓ The minimum obtainable velocity is sonic at point R.

VARIATION OF PROPERTIES IN RAYLEIGH FLOW - HEATING

Properties	Subsonic Region	Supersonic Region
M	increases	decreases
c	increases	decreases
p	decreases	increases
T	increases for $M<1/\sqrt{(\gamma)}$ decreases for $M>1/\sqrt{(\gamma)}$	increases
T_0	increase	increase
p_0	decreases	decreases
ρ	decreases	increases
s	increases	increase

VARIATION OF PROPERTIES IN RAYLEIGH FLOW - COOLING

Properties	Subsonic Region	Supersonic Region
M	decreases	increases
c	decreases	increases
p	increases	decreases
T	decreases for M<1/√(γ) increases for M>1/√(γ)	decreases
T_0	decreases	decreases
p_0	increases	increases
ρ	increases	decreases
s	decreases	decreases

Prove that at the point of maximum entropy (R) in Rayleigh line, the flow is sonic. i.e., Mach number is equal to one.

Continuity equation:

$$\dot{m} = \rho_x A_x c_x = \rho_y A_y c_y$$

Since,

$$A_x = A_y = A = constant$$

$$\frac{\dot{m}}{A} = \rho_x c_x = \rho_y c_y \quad \text{----- Eqn. 1}$$

Energy equation:

$$h_{ox} = h_{oy} = h_o = constant$$

$$T_{ox} = T_{oy} = T_o = constant$$

$$h_x + \frac{1}{2}c_x^2 = h_y + \frac{1}{2}c_y^2 \; --- Eqn.2$$

Momentum equation:

$$(p_x - p_y)A = \dot{m}(c_y - c_x)$$

$$p_x - p_y = \frac{\dot{m}}{A}(c_y - c_x) \quad \text{-- Eqn. 3}$$

Equation 1 and 3 gives

$$p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2 \quad \text{-- Eqn. 4}$$

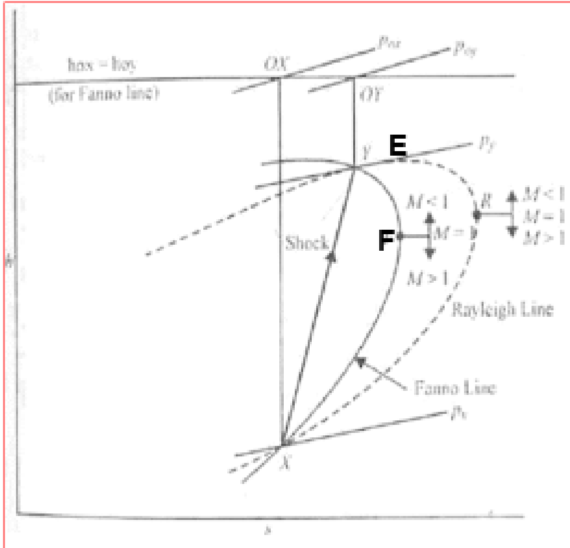
$$F_x = F_y = constant \quad \text{-- Eqn. 5}$$

Equation of state: Equation of state gives

$$h = f(s, \rho)$$

$$s = f(p, \rho) \quad \text{-- Eqn. 6}$$

Combine the governing equations of Rayleigh flow into a single relation and plot it on the h-s diagram using property relations, the resultant curve is called the Rayleigh line or Rayleigh curve.



Normal shock on Fanno and Rayleigh curves

- R – Point of maximum entropy on Rayleigh line
- At point R, M = 1 (flow is sonic) (flow is choked)
- At point E, M = 1/√(γ) and T = T_{max}
- Upper branch – Subsonic flow
- Lower branch – Supersonic flow
- Shock: X to Y

- Consider an infinitesimal process in the vicinity of maximum entropy point, R.
- **At this point R**, the changes being small, the process may be considered as reversible (Δs = 0).
- From **momentum** equation,

  $p + \rho c^2 = \text{constant}$

  On differentiation, we get,

  $dp + \rho.2cdc + c^2.dp = 0$ -- Eqn. 1

Also, from **continuity** equation

  $\rho c = \text{constant}$

On differentiation, we get

  $\rho dc + cd\rho = 0$

  $dc = -(c/\rho)\, d\rho$ -- Eqn. 2 s

Eqn. 2 in Eqn. 1 gives

$dp + \rho.2c.\,(-(c/\rho)d\rho + c^2.d\rho = 0$

$dp - c^2.d\rho = 0 \longrightarrow \left(\frac{dp}{d\rho}\right)_s = c^2$

But,

$$\left(\frac{dp}{d\rho}\right)_s = a^2$$

Hence,

- $c^2 = a^2$
- $c = a$
- i.e., M = 1
- i.e., Velocity of gas at maximum entropy point, R, is sonic.
- Upper branch represents subsonic flow.
- Lower branch represents supersonic flow.

ENERGY EQUATION WITH HEAT TRANSFER

$$h_1 + \frac{1}{2}c_1^2 + q = h_2 + \frac{1}{2}c_2^2$$

Solving the above equation for 'q' with $h = c_pT$

Wkt., $h + c^2/2 = h_0 = c_pT_0$

Therefore,

$q = c_pT_{02} - c_pT_{01}$

$\mathbf{q = c_p\,(T_{02} - T_{01})}$

- If heat is added, T_0 increases.
- If heat is extracted, T_0 decreases.

- The above equation clearly indicates that the effect of heat addition is to directly change the total temperature of the fluid.

SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE
www.sathyabama.ac.in

SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
GAS DYNAMICS AND JET PROPULSION (SME1303)

UNIT – 4
NORMAL SHOCK AND OBLIQUE SHOCKS

NORMAL SHOCK

- Shock wave – is a steep finite pressure wave.
- Changes in the flow properties across shock are abrupt.

Normal shock wave Shocks are classified as:

Normal shock – is $\perp$r to the direction of flow.

Oblique shock – is inclined to the direction of flow.

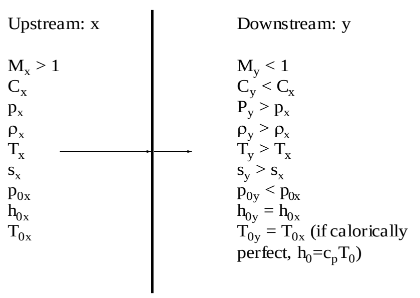
Note: -

1. Normal shock wave is a special case of oblique shock wave in which the ***wave angle*** ***(*** σ ***)*** is **equal to 90**°.
2. Oblique shock is also called as a 2 dimensional plane shock wave.

Normal shock:-

- is $\perp$r to the 1-D flow.
- is adiabatic and not isentropic.
- across the normal shock, flow is irreversible.
- an irreversible process is always accompanied by an $\uparrow$se in entropy, with a consequent $\downarrow$se in available energy which is a source of loss.
- fluid should attain **supersonic velocity** before it can enter a normal shock region.
- after the shock, the fluid velocity falls to subsonic value with $\uparrow$ in static pressure and entropy.
- the loss in KE due to this irreversible process results in the sudden $\uparrow$se in static temperature of the fluid.
- across a shock wave, there is a loss in stagnation pressure.
- shock is always a compression shock and occurs in supersonic region. Compression shock is always accompanied by $\uparrow$se in entropy.
- rarefaction shocks are not possible.
- Mach number after the shock (M_y) is always less than 1.
- larger the supersonic Mach number before the shock, the smaller the subsonic Mach number after the shock.

NORMAL SHOCK WAVE



Typical shock wave thickness 1/1,000 mm

Expansion wave and Compression wave:

- A wave which is at a lower pressure than the fluid into which it is moving is called as **rarefaction** or **expansion wave.**
- A wave which is at a higher pressure than the fluid into which it is moving is called as **compression wave.**

Shocks in compression waves are possible whereas shocks in expansion waves are not possible.

Effects of shock wave:-

1. Shock may cause boundary layer separation.
2. Deviation of flow from its designed direction.
3. Efficiency of the machine experiencing shock waves are considerably low.
4. Creates ***sonic boom,*** which results in blast waves. A **sonic boom** is the sound associated with the shock waves created by an object traveling through the air faster than the speed of sound. Sonic booms generate enormous amounts of sound energy, sounding much like an explosion. ***These wave have a damaging effect on human life and buildings.***

Useful applications of shock wave:-

1. Shock tubes - A shock wave inside a shock tube is generated by a small explosion (blast-driven) or by the buildup of high pressures which cause diaphragm to burst and a shock wave to propagate down the shock tube

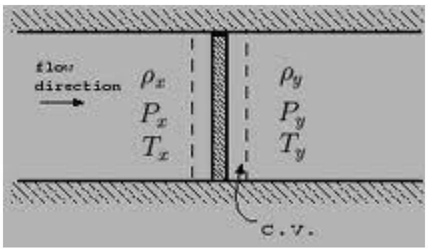
The shock tube is an instrument used to replicate and direct blast waves at a sensor or a model in order to simulate actual explosions and their effects, usually on a smaller scale.

A strong moving shock wave is utilized to accelerate the flow to a high Mach number in a shock tube where flow behavior at high Mach numbers can be studied.

Shock tubes are also used in biomedical research to find out how biological tissues are affected by blast waves.

2. Supersonic compressors – on account of abrupt changes of pressure, density etc., across shock waves, they are profitably used in supersonic compressors to obtain considerably high-pressure ratios in one stage; in such compressors the pressure ratio developed per stage may be as high as 10.0.

Governing Equations:



Frictionless duct with constant A

Properties upstream of the shock – *x*

Properties downstream of the shock – *y*

Fig. shows a normal shock in a constant area frictionless duct. The shock wave is considered to be contained in a control volume.

Continuity, momentum, energy and the state equations govern the flow through the shock.

Continuity equation: $\dot{m} = \rho_x A_x c_x = \rho_y A_y c_y$

Since $A_x = A_y = A = constant$

$$\frac{\dot{m}}{A} = \rho_x c_x = \rho_y c_y \text{ ----- Eqn. 1}$$

Energy equation: If heat transfer is assumed negligible, in the absence of shaft work, the AEE for the control volume containing the shock gives

$$h_{ox} = h_{oy} = h_o = constant$$

$$T_{ox} = T_{oy} = T_o = constant$$

$$h_x + \frac{1}{2}c_x^2 = h_y + \frac{1}{2}c_y^2 --- Eqn.2$$

Momentum equation: $(p_x - p_y)A = \dot{m}(c_y - c_x)$

$$p_x - p_y = \frac{\dot{m}}{A}(c_y - c_x) \text{ -- Eqn. 3}$$

Equation 1 and 3 give

$$p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2 \text{ ----- Eqn. 4}$$

$$F_x = F_y = constant \text{ ----- Eqn. 5}$$

Equation of state: Equation of state gives

$$\begin{aligned} h &= f(s, \rho) \\ s &= f(p, \rho) \end{aligned} \text{ ----- Eqn. 6}$$

These above mentioned equations are used to define two important curves namely ***Fanno line*** and ***Rayleigh line.***

Prandtl-Meyer Relation:

PM relation gives the relation between gas velocities before and after the normal shock and critical velocity of sound. This equation is the basis of other equations for shock waves.

$$c_x c_y = a^{*2}$$

Proof:

WKT:- Adiabatic Energy Equation is given as

$$\frac{a^2}{\gamma - 1} + \frac{c^2}{2} = \frac{1}{2}a^{*2}\,\frac{\gamma + 1}{\gamma - 1}$$

Applying AEE before and after the shock, we get

$$\frac{a_x^2}{\gamma - 1} + \frac{c_x^2}{2} = \frac{a_y^2}{\gamma - 1} + \frac{c_y^2}{2} = \frac{1}{2}a^{*2}\,\frac{\gamma + 1}{\gamma - 1}$$

$$\frac{a_x^2}{\gamma - 1} = \frac{1}{2}a^{*2}\,\frac{\gamma + 1}{\gamma - 1} - \frac{c_x^2}{2}$$

Multiplying by $(\gamma - 1)$ on both the sides,

$$a_x^2 = \frac{\gamma+1}{2} a^{*2} - \frac{\gamma-1}{2} c_x^2$$

Dividing by c_x on both the sides,

$$\frac{a_x^2}{c_x} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2} c_x \text{----- Eqn. 1}$$

Similarly

$$\frac{a_y^2}{c_y} = \frac{\gamma+1}{2}\frac{a^{*2}}{c_y} - \frac{\gamma-1}{2} c_y \text{ ----- Eqn. 2}$$

WKT:- $\left(p_x - p_y\right)A = \dot{m}\left(c_y - c_x\right)$

$$\left(p_x - p_y\right)\frac{A}{\dot{m}} = \left(c_y - c_x\right) \text{ ----- Eqn. 3}$$

Also, $\dfrac{\dot{m}}{A} = \rho_x c_x = \rho_y c_y$

Using this in the Eqn. 3, we get,

$$\frac{p_x}{\rho_x c_x} - \frac{p_y}{\rho_y c_y} = c_y - c_x$$

Multiplying by γ on both the sides, we get

$$\frac{1}{c_x}\frac{\gamma p_x}{\rho_x} - \frac{1}{c_y}\frac{\gamma p_y}{\rho_y} = \gamma\left(c_y - c_x\right) \text{ ----- Eqn. 4}$$

But, $\dfrac{\gamma p_x}{\rho_x} = a_x^2$ and $\dfrac{\gamma p_y}{\rho_y} = a_y^2$

Hence, Eqn. 4 becomes:

$$\frac{a_x^2}{c_x} - \frac{a_y^2}{c_y} = \gamma\left(c_y - c_x\right) \text{ ----- Eqn. 5}$$

Eqn. 1 and Eqn. 2 in Eqn. 5 gives,

$$\frac{\gamma+1}{2}\frac{a^{*2}}{c_x} - \frac{\gamma-1}{2} c_x - \frac{\gamma+1}{2}\frac{a^{*2}}{c_y} + \frac{\gamma-1}{2} c_y = \gamma\left(c_y - c_x\right)$$

$$\frac{\gamma+1}{2} a^{*2}\left(\frac{1}{c_x} - \frac{1}{c_y}\right) + \frac{\gamma-1}{2}\left(c_y - c_x\right) = \gamma\left(c_y - c_x\right)$$

$$\frac{\gamma+1}{2} a^{*2}\left(\frac{c_y - c_x}{c_x c_y}\right) + \frac{\gamma-1}{2}\left(c_y - c_x\right) = \gamma\left(c_y - c_x\right)$$

$$\frac{\gamma+1}{2} a^{*2}\left(\frac{1}{c_x c_y}\right) + \frac{\gamma-1}{2} = \gamma$$

$$\frac{\gamma+1}{2}a^{*2}\left(\frac{1}{c_x c_y}\right)=\gamma-\frac{\gamma-1}{2}$$

$$\frac{\gamma+1}{2}a^{*2}\left(\frac{1}{c_x c_y}\right)=\frac{2\gamma-\gamma+1}{2}$$

$$\frac{\gamma+1}{2}a^{*2}\left(\frac{1}{c_x c_y}\right)=\frac{\gamma+1}{2}$$

$c_x c_y = a^{*2}$ ----- **Prandtl-Meyer Relation**

Another form of Prandtl-Meyer relation:

$$\frac{c_x}{a^*}\frac{c_y}{a^*}=1$$

Now, $a^* = a_x^* = a_y^*$ and hence,

$$\frac{c_x}{a_x^*}\frac{c_y}{a_y^*}=1;\ \text{But}\ \frac{c_x}{a_x^*}=M_x^*\ \text{and}\ \frac{c_y}{a_y^*}=M_y^*$$

$\therefore M_x^* M_y^*$ **= 1 ----- Prandtl-Meyer Relation**

Impossibility of a shock in subsonic flow:

Equation $\frac{\Delta s}{R}$ for a given value of γ is plotted.

Perfect gases of engineering applications have γ between 1.0 and 1.67; for this range $\frac{\Delta s}{R}$ always has +ve values for values of the upstream Mach no. M_x greater than unity.

At M_x =1, $\frac{\Delta s}{R}$ = 0.

For the sake of explanation, equation $\frac{\Delta s}{R}$ has also been plotted for values of M_x < 1; this branch of curve represents a decrease in entropy, which is by II law of TDs is impossible. Therefore, shock waves cannot develop in a subsonic flow.

Strength of a shock wave: It is given by

$$\xi=\frac{pr.increaseduetoshock}{upstreampressure}=\frac{p_y-p_x}{p_x}=\frac{p_y}{p_x}-1$$

Substituting for p_y/p_x, we get,

$$\xi=\frac{2\gamma}{\gamma+1}M_x^2-\frac{\gamma-1}{\gamma+1}-1$$

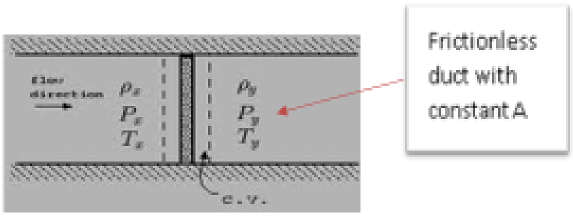
$$\xi=\frac{2\gamma M_x^2-\gamma+1-\gamma-1}{\gamma+1}=\frac{2\gamma M_x^2-2\gamma}{\gamma+1}$$

$$\xi = \frac{2\gamma}{\gamma+1}\left(M_x^2 - 1\right)$$

Thus the shock strength is proportional to $\left(M_x^2 - 1\right)$ and high values of upstream Mach number M_x results in strong shock.

Shocks of vanishing strength: Shock wave for which ξ is almost zero is referred to as shocks of vanishing strength. For such shocks, $M_x \approx 1$; $\frac{\rho_y}{\rho_x} \approx 1$; $\frac{p_y}{p_x} \approx 1$; $\frac{T_y}{T_x} \approx 1$; $\frac{p_{oy}}{p_{ox}} \approx 1$; $\frac{\Delta s}{R} \approx 0$.

GOVERNING EQUATIONS FOR NORMAL SHOCK WAVE



- ✓ The shock wave is considered to be contained in a control volume.
- ✓ Equations that govern the flow through the shock are:
  1. Continuity equation
  2. Energy equation
  3. Momentum equation
  4. Equation of state

Properties upstream of the shock: *x*

Properties downstream of the shock: *y*

Continuity equation:

$$\dot{m} = \rho_x A_x c_x = \rho_y A_y c_y$$

Since, $A_x = A_y = A = constant$

$$\frac{\dot{m}}{A} = \rho_x c_x = \rho_y c_y \text{ ----- Eqn. 1}$$

Energy equation: If heat transfer is assumed negligible, in the absence of shaft work, the AEE for the control volume containing the shock gives

$$h_{ox} = h_{oy} = h_o = constant$$

$$T_{ox} = T_{oy} = T_o = constant$$

$$h_x + \frac{1}{2}c_x^2 = h_y + \frac{1}{2}c_y^2 \; --- Eqn.2$$

Momentum equation:

$$(p_x - p_y)A = \dot{m}(c_y - c_x)$$

$$p_x - p_y = \frac{\dot{m}}{A}(c_y - c_x) \quad \text{-- Eqn. 3}$$

Equation 1 and 3 gives

$$p_x + \rho_x c_x^2 = p_y + \rho_y c_y^2 \quad \text{-- Eqn. 4}$$

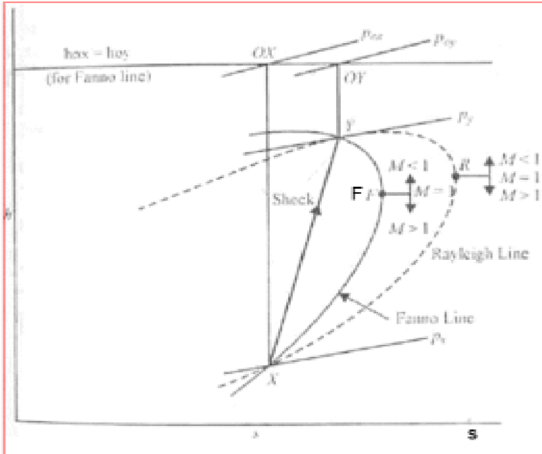
$$F_x = F_y = constant \quad \text{-- Eqn. 5}$$

Equation of state: Equation of state gives

$$h = f(s, \rho)$$

$$s = f(p, \rho) \quad \text{-- Eqn. 6}$$

The equations shown above are used to define two important curves namely ***Fanno line*** and ***Rayleigh line.***



Normal shock on Fanno and Rayleigh curves

- ✓ F – Point of maximum entropy on Fanno line
- ✓ R – Point of maximum entropy on Rayleigh line
- ✓ At point F and R, M = 1 (flow is sonic)
- ✓ Upper branch – Subsonic flow
- ✓ Lower branch – Supersonic flow
- ✓ Shock: X to Y
- ✓ $p_y > p_x$; $p_{0x} > p_{0y}$; $S_y > S_x$

<u>Mach Number downstream of the shock (M_y)</u>

Generally the upstream Mach No. (M_x) is given and we have to find the downstream Mach No. (M_y)

WKT:- $c^* = a^* = \sqrt{\frac{2\gamma}{\gamma+1} \cdot RT_0}$

$$\therefore a^{*2} = \frac{2\gamma}{\gamma+1} \cdot RT_0 \quad \text{--- (1)}$$

Also, WKT: $c_x \cdot c_y = a^{*2}$ --- (2)

Eqn. (1) in (2) gives

$$c_x \cdot c_y = \left(\frac{2\gamma}{\gamma+1}\right) \cdot RT_0 \quad \text{--- (3)}$$

But, $c_x = M_x \cdot a_x = M_x \cdot \sqrt{\gamma R T_x}$

$c_y = M_y \cdot a_y = M_y \cdot \sqrt{\gamma R T_y}$

Substituting these in eqn (3), we get

$$M_x \sqrt{\gamma R T_x} \cdot M_y \cdot \sqrt{\gamma R T_y} = \left(\frac{2\gamma}{\gamma+1}\right) \cdot RT_0$$

$$M_x \cdot M_y \cdot \cancel{\gamma}\cancel{R} \cdot \sqrt{T_x T_y} = \left(\frac{2}{\gamma+1}\right)\cancel{\gamma}\cancel{R}T_0$$

$$M_x \cdot M_y = \left(\frac{2}{\gamma+1}\right)\frac{T_0}{\sqrt{T_x T_y}}$$

<u>On squaring both sides,</u> we get

$$M_x^2 \cdot M_y^2 = \left(\frac{2}{\gamma+1}\right)^2 \cdot \frac{T_0}{T_x} \cdot \frac{T_0}{T_y}$$

Substituting the values of $\frac{T_0}{T_x}$ & $\frac{T_0}{T_y}$ for adiabatic flow, we get,

$$M_x^2 \cdot M_y^2 = \left(\frac{2}{\gamma+1}\right)^2\left(1+\frac{\gamma-1}{2}M_x^2\right)\left(1+\frac{\gamma-1}{2}M_y^2\right)$$

r_devaraj67@yahoo.com (1)

$$M_x^2 \cdot M_y^2 = \left[\frac{4}{(\gamma+1)^2} + \frac{\overset{2}{\cancel{4}}}{(\gamma+1)^2}\frac{(\gamma-1)}{\cancel{2}}M_x^2\right]\left(1+\frac{\gamma-1}{2}M_y^2\right)$$

$$= \frac{4}{(\gamma+1)^2} + \frac{\overset{2}{\cancel{4}}}{(\gamma+1)^2}\left(\frac{\gamma-1}{\cancel{2}}\right)M_y^2 + \frac{2(\gamma-1)}{(\gamma+1)^2}M_x^2 + \frac{\cancel{2}(\gamma-1)^2}{\cancel{2}(\gamma+1)^2}M_x^2 \cdot M_y^2$$

$$= \frac{4}{(\gamma+1)^2} + \frac{2(\gamma-1)}{(\gamma+1)^2}M_y^2 + \frac{2(\gamma-1)}{(\gamma+1)^2}M_x^2 + \frac{(\gamma-1)^2}{(\gamma+1)^2} \cdot M_x^2 \cdot M_y^2$$

$$M_x^2 \cdot M_y^2 - \frac{2(\gamma-1)}{(\gamma+1)^2}M_y^2 - \frac{(\gamma-1)^2}{(\gamma+1)^2} \cdot M_x^2 M_y^2 = \frac{4}{(\gamma+1)^2} + \frac{2(\gamma-1)}{(\gamma+1)^2}M_x^2$$

$$\frac{M_x^2 \cdot M_y^2(\gamma+1)^2 - 2(\gamma-1)M_y^2 - (\gamma-1)^2 M_x^2 M_y^2}{\cancel{(\gamma+1)^2}} = \frac{4 + 2(\gamma-1)M_x^2}{\cancel{(\gamma+1)^2}}$$

$$M_y^2\left[M_x^2(\gamma+1)^2 - 2(\gamma-1) - (\gamma-1)^2 M_x^2\right] = 4 + 2(\gamma-1)M_x^2$$

$$M_y^2 = \frac{4 + 2(\gamma-1)M_x^2}{M_x^2(\gamma+1)^2 - 2(\gamma-1) - (\gamma-1)^2 M_x^2}$$

$$M_y^2 = \frac{2(\gamma-1)\left[\frac{4}{2(\gamma-1)} + M_x^2\right]}{(\gamma+1)^2 M_x^2 - 2(\gamma-1)\left[1+\frac{\gamma-1}{2}M_x^2\right]}$$

$$M_y^2 = \frac{\frac{\cancel{4}\,2}{\cancel{2}(\gamma-1)} + M_x^2}{\frac{(\gamma+1)^2 M_x^2 - 2(\gamma-1)\left[1+\frac{\gamma-1}{2}M_x^2\right]}{2(\gamma-1)}}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{\frac{(\gamma+1)^2 M_x^2}{2(\gamma-1)} - \frac{\cancel{2}(\cancel{\gamma-1})\left[1+\frac{\gamma-1}{2}M_x^2\right]}{\cancel{2}(\cancel{\gamma-1})}}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{\frac{(\gamma+1)^2 M_x^2}{2(\gamma-1)} - 1 - \frac{(\gamma-1)}{2}M_x^2}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{M_x^2\left[\frac{(\gamma+1)^2}{2(\gamma-1)} - \frac{(\gamma-1)}{2}\right] - 1}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{M_x^2\left[\frac{(\gamma+1)^2 - (\gamma-1)^2}{2(\gamma-1)}\right] - 1}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{M_x^2\left[\frac{\cancel{\gamma^2}+\cancel{1}+2\gamma-\cancel{\gamma^2}-\cancel{1}+2\gamma}{2(\gamma-1)}\right] - 1}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{M_x^2\left[\frac{\cancel{4}\,2\gamma}{\cancel{2}(\gamma-1)}\right] - 1}$$

$$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{M_x^2\left[\frac{2\gamma}{\gamma-1}\right] - 1}$$

$$\boxed{M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}} \quad \text{— (A)}$$

Eqn (A) shows, $M_y = f(M_x)$.

Static pressure ratio across the shock (P_y/P_x)

WKT: $F_x = F_y$ (F = Impulse fn).

$$P_x + \rho_x c_x^2 = P_y + \rho_y c_y^2 \quad \text{— (1)}$$

$$\left[\rho c^2 = \frac{P}{RT}c^2 = \frac{\gamma P c^2}{\gamma RT} = \frac{\gamma P c^2}{a^2} = \gamma P M^2\right]$$

$\because a = \sqrt{\gamma RT}$

$\therefore \rho_x c_x^2 = \gamma P_x M_x^2$

$\rho_y c_y^2 = \gamma P_y \cdot M_y^2$

Eqn (1) becomes,

$$P_x + \gamma P_x M_x^2 = P_y + \gamma P_y M_y^2$$

$$P_x(1+\gamma M_x^2) = P_y(1+\gamma M_y^2)$$

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma M_y^2} \quad \text{— (2)}$$

WKT: $$M_y^2 = \frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}$$

Substituting for M_y^2 in Eqn (2), we get

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\gamma\left[\frac{\left(\frac{2}{\gamma-1}\right)+M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2-1}\right]}$$

$$\frac{P_y}{P_x} = \frac{1+\gamma M_x^2}{1+\left[\frac{\left(\frac{2\gamma}{\gamma-1}\right)+\gamma M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2-1}\right]}$$

Y_devaraj67@yahoo.com (2)

$$\frac{P_y}{P_x} = \frac{\left\{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1\right\}(1+\gamma M_x^2)}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1 + \left(\frac{2\gamma}{\gamma-1}\right) + \gamma M_x^2}$$

$$\frac{P_y}{P_x} = \frac{\left\{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1\right\}(1+\gamma M_x^2)}{\gamma M_x^2\left(\frac{2}{\gamma-1}+1\right) - 1 + \left(\frac{2\gamma}{\gamma-1}\right)}$$

$$\frac{P_y}{P_x} = \frac{\left(\frac{2\gamma}{\gamma-1}M_x^2 - 1\right)(1+\gamma M_x^2)}{\gamma M_x^2\left(\frac{2+\gamma-1}{\gamma-1}\right) + \frac{-(\gamma-1)+2\gamma}{(\gamma-1)}}$$

$$\frac{P_y}{P_x} = \frac{\left(\frac{2\gamma}{\gamma-1}M_x^2 - 1\right)(1+\gamma M_x^2)}{\gamma M_x^2\left(\frac{\gamma+1}{\gamma-1}\right) + \frac{(\gamma+1)}{(\gamma-1)}}$$

$$\frac{P_y}{P_x} = \frac{\left(\frac{2\gamma}{\gamma-1}M_x^2 - 1\right)\cancel{(1+\gamma M_x^2)}}{\left(\frac{\gamma+1}{\gamma-1}\right)\cancel{(1+\gamma M_x^2)}}$$

$$\frac{P_y}{P_x} = \frac{\frac{2\gamma}{\gamma-1}M_x^2}{\frac{\gamma+1}{\gamma-1}} - \frac{1}{\frac{\gamma+1}{\gamma-1}}$$

$$\boxed{\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)} \quad \text{— (B)}$$

From (B), At $M_x = 1$, $\frac{P_y}{P_x} = 1$

At $M_x = \infty$, $\frac{P_y}{P_x} = \infty$.

Note:- (i) for $M_x > 1$, there is always pressure rise across it.

(ii) values of pressure ratio $\left(\frac{P_y}{P_x}\right)$ corresponding to $M_x < 1$ are absurd.

Static temperature ratio across the shock (T_y/T_x)

At upstream of the shock,

$$\frac{T_{0x}}{T_x} = 1 + \frac{\gamma-1}{2} M_x^2$$

At downstream of the shock,

$$\frac{T_{0y}}{T_y} = 1 + \frac{\gamma-1}{2} M_y^2$$

AEE for a perfect gas is given as, $T_{0x} = T_{0y} = T_0$ & hence

$$\frac{T_y}{T_x} = \frac{T_y}{T_{0y}} \cdot \frac{T_{0x}}{T_x} = \frac{1 + \frac{\gamma-1}{2} M_x^2}{1 + \frac{\gamma-1}{2} M_y^2} \quad (1)$$

WKT:- $$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$$

Substituting for M_y^2 in Eqn (1), we get.

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{\gamma-1}{2}\right) M_x^2}{1 + \left(\frac{\gamma-1}{2}\right)\left[\frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]}$$

$$\frac{T_y}{T_x} = \frac{1 + \left(\frac{\gamma-1}{2}\right) M_x^2}{1 + \left[\frac{1 + \left(\frac{\gamma-1}{2}\right) M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}\right]}$$

$$\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{\frac{2\gamma}{\gamma-1} M_x^2 - \cancel{1} + \cancel{1} + \left(\frac{\gamma-1}{2}\right) M_x^2}$$

$$\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{M_x^2\left[\frac{2\gamma}{\gamma-1} + \frac{\gamma-1}{2}\right]}$$

$$\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{M_x^2\left[\frac{4\gamma + (\gamma-1)^2}{2(\gamma-1)}\right]}$$

$$\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{M_x^2\left[\frac{4\gamma + \gamma^2 + 1 - 2\gamma}{2(\gamma-1)}\right]}$$

$$\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{M_x^2\left[\frac{\gamma^2 + 1 + 2\gamma}{2(\gamma-1)}\right]}$$

$$\boxed{\frac{T_y}{T_x} = \frac{\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}{\frac{1}{2} \cdot M_x^2 \cdot \frac{(\gamma+1)^2}{(\gamma-1)}}} \quad (C)$$

$$\therefore \frac{a_y}{a_x} = \frac{\sqrt{\gamma R T_y}}{\sqrt{\gamma R T_x}} = \sqrt{\frac{T_y}{T_x}}$$

$$\boxed{\frac{a_y}{a_x} = \frac{\sqrt{2(\gamma-1)\left[\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1\right]\left[1 + \left(\frac{\gamma-1}{2}\right) M_x^2\right]}}{(\gamma+1) M_x}}$$

(C) $\Rightarrow \frac{T_y}{T_x} = f(M_x)$

Stagnation pressure ratio across the shock (P_{0y}/P_{0x}).

* Shock wave is an irreversibility across which there is a stagnation pressure loss and increase in entropy. The stagnation pr. ratio is a fn. of upstream mach no. (M_x)

r_devaraj67@yahoo.com (3)

$$\frac{P_{0y}}{P_{0x}} = \frac{P_{0y}}{P_y} \cdot \frac{P_y}{P_x} \cdot \frac{P_x}{P_{0x}} \quad (A)$$

Let $$\frac{P_{0y}}{P_y} = \left(1 + \frac{\gamma-1}{2} M_y^2\right)^{\gamma/\gamma-1}$$

WKT:- $$M_y^2 = \frac{\frac{2}{\gamma-1} + M_x^2}{\frac{2\gamma}{\gamma-1} M_x^2 - 1}$$

substituting for M_y^2 in Eqn (1)

$$\frac{P_{0y}}{P_y} = \left[1 + \left(\frac{\gamma-1}{2}\right)\frac{\left(\frac{2}{\gamma-1}\right) + M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{0y}}{P_y} = \left[1 + \frac{1 + \left(\frac{\gamma-1}{2}\right) M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1}\right]^{\frac{\gamma}{\gamma-1}}$$

$$\frac{P_{0y}}{P_y} = \left[\frac{\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - \cancel{1} + \cancel{1} + \left(\frac{\gamma-1}{2}\right) M_x^2}{\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{0y}}{P_y} = \left[\frac{M_x^2\left[\frac{2\gamma}{\gamma-1} + \frac{\gamma-1}{2}\right]}{\left(\frac{2\gamma}{\gamma-1}\right) M_x^2 - 1}\right]^{\gamma/\gamma-1}.$$

$$\frac{P_{oy}}{P_y} = \left[\frac{M_x^2\left(\frac{4\gamma + (\gamma-1)^2}{2(\gamma-1)}\right)}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_y} = \left[\frac{M_x^2\left(\frac{4\gamma+\gamma^2+1-2\gamma}{2(\gamma-1)}\right)}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_y} = \left[\frac{M_x^2\left(\frac{(\gamma+1)^2}{2(\gamma-1)}\right)}{\left(\frac{2\gamma}{\gamma-1}\right)M_x^2 - 1}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_y} = \left[\frac{(\gamma+1)(\gamma+1)\,M_x^2}{2(\gamma-1)\left[\frac{2\gamma}{\gamma-1}M_x^2 - 1\right]}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_y} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M_x^2}{\left(\frac{\gamma-1}{\gamma+1}\right)\left[\frac{2\gamma}{\gamma-1}M_x^2 - 1\right]}\right]^{\gamma/\gamma-1}$$

$$\frac{P_{oy}}{P_y} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M_x^2}{\left(\frac{2\gamma}{\gamma+1}\right)M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}\right]^{\gamma/\gamma-1} \quad \text{— (2)}$$

Also, WKT:

$$\frac{P_x}{P_{ox}} = \left(1 + \frac{\gamma-1}{2}M_x^2\right)^{-\frac{\gamma}{\gamma-1}} \quad \text{— (3)}$$

WKT:- $$\frac{P_y}{P_x} = \frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right) \quad \text{— (4)}$$

Eqn. (2), (3) & (4) in Eqn (A) gives

$$\frac{P_{oy}}{P_{ox}} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M_x^2}{\left(\frac{2\gamma}{\gamma+1}\right)M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)}\right]^{\gamma/\gamma-1}\left(1+\frac{\gamma-1}{2}M_x^2\right)^{-\frac{\gamma}{\gamma-1}} \times \left[\frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right]$$

$$\boxed{\frac{P_{oy}}{P_{ox}} = \left[\frac{\left(\frac{\gamma+1}{2}\right)M_x^2}{1+\left(\frac{\gamma-1}{2}\right)M_x^2}\right]^{\gamma/\gamma-1}\left(\frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right)^{-\frac{1}{\gamma-1}}} \quad \text{— (B)}$$

Eqn (B) shows that $\frac{P_{oy}}{P_{ox}} = f(M_x)$

$$\boxed{\because 1 - \frac{\gamma}{\gamma-1} = \frac{\gamma-1-\gamma}{\gamma-1} = -\frac{1}{\gamma-1}.}$$

Change in entropy across the shock ($\Delta S = S_y - S_x$)

For a perfect gas; WKT.

$$S_2 - S_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$

Now, change of entropy across a shock is given as

$$\Delta S = S_y - S_x = C_p\ln\left(\frac{T_y}{T_x}\right) - R\ln\left(\frac{P_y}{P_x}\right)$$

$$S_y - S_x = C_p\ln\left(\frac{T_y}{T_x}\right) - \frac{C_p(\gamma-1)}{\gamma}\ln\left(\frac{P_y}{P_x}\right) \qquad \left[\because C_p = \frac{\gamma R}{\gamma-1}\right]$$

$$\Delta S = C_p \ln\frac{\left(T_y/T_x\right)}{\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}}} \quad \text{— (A)} \qquad \left[\because \ln a - \ln b = \ln\frac{a}{b}\right]$$

WKT:-

$$\frac{T_y}{T_x} = \frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}\cdot M_y^2}\cdot\frac{T_{oy}}{T_{ox}} \quad \text{— (1)}$$

$$\left[\because \frac{T_{oy}}{T_y} = 1+\frac{\gamma-1}{2}M_y^2;\ \frac{T_{ox}}{T_x} = 1+\frac{\gamma-1}{2}M_x^2\right]$$

$$\frac{P_y}{P_x} = \left[\frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2}\right]^{\gamma/\gamma-1} \times \frac{P_{oy}}{P_{ox}} \qquad \left[\because \frac{P_y}{P_x} = \left(\frac{T_y}{T_x}\right)^{\frac{\gamma}{\gamma-1}}\right]$$

$$\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1+\frac{\gamma-1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_y^2}\right)\left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{— (2)}$$

combining (1) & (2), we get.

$$\frac{\frac{T_y}{T_x}}{\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}}} = \frac{\frac{T_{oy}}{T_{ox}}}{\left(\frac{P_{oy}}{P_{ox}}\right)^{\frac{\gamma-1}{\gamma}}}$$

r_devaraj67@yahoo.com (4)

But, $\frac{T_{oy}}{T_{ox}} = 1$, we get

$$\frac{\frac{T_y}{T_x}}{\left(\frac{P_y}{P_x}\right)^{\frac{\gamma-1}{\gamma}}} = \left(\frac{P_{oy}}{P_{ox}}\right)^{-\frac{(\gamma-1)}{\gamma}} \quad \text{— (3)}$$

Eqn (3) in (A) gives

$$\Delta S = C_p\ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-\frac{(\gamma-1)}{\gamma}}$$

$$\Delta S = C_p\left(\frac{\gamma-1}{\gamma}\right)\ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-1}$$

$$\therefore \Delta S = S_y - S_x = R\ln\frac{P_{ox}}{P_{oy}}$$

$$\boxed{\frac{\Delta S}{R} = \ln\frac{P_{ox}}{P_{oy}}} \quad \text{— (B)}$$

$$\boxed{\frac{P_{ox}}{P_{oy}} = e^{\frac{\Delta S}{R}}}$$

(B) $\Rightarrow \frac{\Delta S}{R} = \ln\left(\frac{P_{oy}}{P_{ox}}\right)^{-1} = -\ln\left(\frac{P_{oy}}{P_{ox}}\right)$

Substituting for $\frac{P_{oy}}{P_{ox}}$ in the above equation

$$\frac{\Delta S}{R} = -\left[\ln\left[\left(\frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2}\right)^{\gamma/\gamma-1} \times \left(\frac{2\gamma}{\gamma+1}M_x^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right)^{-\frac{1}{\gamma-1}}\right]\right]$$

$$\frac{\Delta S}{R} = -\frac{\gamma}{\gamma-1}\ln\frac{\frac{\gamma+1}{2}M_x^2}{1+\frac{\gamma-1}{2}M_x^2} + \frac{1}{\gamma-1}\ln\left(\frac{2\gamma}{\gamma+1}M_x^2 - \frac{\gamma-1}{\gamma+1}\right)$$

r_devaraj67@yahoo.com

$$= \frac{\left[1+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x}+\frac{(\gamma-1)^2}{4\gamma}\right]\left[\frac{\gamma+1}{\gamma-1}\frac{\cancel{2\gamma}}{\cancel{2\gamma}}\frac{P_y}{P_x}+\frac{\cancel{2\gamma}}{\cancel{\gamma-1}}\frac{\cancel{\gamma-1}}{\cancel{2\gamma}}-1\right]}{D_r}$$

$$= \frac{\left[1+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x}+\frac{(\gamma-1)^2}{4\gamma}\right]\left[\frac{\gamma+1}{\gamma-1}\frac{P_y}{P_x}+\cancel{1}-\cancel{1}\right]}{D_r}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\cdot\frac{P_y}{P_x}+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x}\frac{(\gamma+1)}{(\gamma-1)}\frac{P_y}{P_x}+\frac{(\gamma-1)^2(\gamma+1)}{4\gamma\cdot(\gamma-1)}\frac{P_y}{P_x}}{D_r}$$

$$= \frac{\frac{\gamma+1}{\gamma-1}\frac{P_y}{P_x}+\frac{(\gamma+1)^2}{4\gamma}\frac{P_y}{P_x}\cdot\frac{P_y}{P_x}+\frac{(\gamma-1)(\gamma+1)}{4\gamma}\frac{P_y}{P_x}}{D_r}$$

$$= \frac{\frac{(\gamma+1)^2}{4\gamma}\frac{P_y}{P_x}\frac{P_y}{P_x}+\left(\frac{\gamma+1}{\gamma-1}\right)\frac{P_y}{P_x}+\frac{(\gamma^2-1)}{4\gamma}\cdot\frac{P_y}{P_x}}{D_r}$$

$$= \frac{\frac{(\gamma+1)^2}{4\gamma}\cdot\frac{P_y}{P_x}\frac{P_y}{P_x}+\frac{P_y}{P_x}\left[\frac{\gamma+1}{\gamma-1}+\frac{\gamma^2-1}{4\gamma}\right]}{D_r}$$

$$= \frac{\frac{(\gamma+1)^2}{4\gamma}\cdot\frac{P_y}{P_x}\frac{P_y}{P_x}+\frac{P_y}{P_x}\left[\frac{4\gamma^2+4\gamma+\gamma^3-\gamma-\gamma^2+1}{4\gamma(\gamma-1)}\right]}{D_r}$$

$$= \frac{\frac{(\gamma+1)^2}{4\gamma}\cdot\frac{P_y}{P_x}\frac{P_y}{P_x}+\frac{P_y}{P_x}\left[\frac{\gamma^3+3\gamma^2+3\gamma+1}{4\gamma(\gamma-1)}\right]}{D_r}$$

$$= \frac{\frac{(\gamma+1)^2}{4\gamma}\cdot\frac{P_y}{P_x}\frac{P_y}{P_x}+\frac{P_y}{P_x}\left[\frac{(\gamma+1)^3}{4\gamma(\gamma-1)}\right]}{D_r}$$

$$= \frac{\frac{P_y}{P_x}\frac{(\gamma+1)^3}{4\gamma\cdot(\gamma-1)}\left[\frac{(\gamma-1)}{(\gamma+1)}\frac{P_y}{P_x}+1\right]}{D_r}$$

$$= \frac{\frac{P_y}{P_x}\frac{(\gamma+1)^3}{4\gamma\cdot(\gamma-1)}\left[\frac{\gamma-1}{\gamma+1}\cdot\frac{P_y}{P_x}+1\right]}{\frac{(\gamma+1)^2}{4\gamma(\gamma-1)}\left[(\gamma+1)\cdot\frac{P_y}{P_x}+(\gamma-1)\right]}$$

$$= \frac{\frac{P_y}{P_x}\frac{\cancel{(\gamma+1)^3}}{\cancel{4\gamma\cdot(\gamma-1)}}\left[\frac{\gamma-1}{\gamma+1}\cdot\frac{P_y}{P_x}+1\right]}{\frac{\cancel{(\gamma+1)^3}}{\cancel{4\gamma(\gamma-1)}}\left[\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}\right]}$$

$$\therefore\ \frac{T_y}{T_x} = \frac{\frac{P_y}{P_x}\left[1+\frac{\gamma-1}{\gamma+1}\frac{P_y}{P_x}\right]}{\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}}$$

$$\frac{P_y}{P_x}\cdot\frac{T_x}{T_y} = \frac{\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\cdot\frac{P_y}{P_x}}$$

But $\frac{\rho_y}{\rho_x} = \frac{P_y}{P_x}\cdot\frac{T_x}{T_y}$

$$\therefore\ \frac{\rho_y}{\rho_x} = \frac{\frac{P_y}{P_x}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\cdot\frac{P_y}{P_x}}$$

$$\frac{\rho_y}{\rho_x} = \frac{\cancel{\left(\frac{\gamma-1}{\gamma+1}\right)}\left[\frac{\gamma+1}{\gamma-1}\cdot\frac{P_y}{P_x}+1\right]}{\cancel{\left(\frac{\gamma-1}{\gamma+1}\right)}\left[\frac{\gamma+1}{\gamma-1}+\frac{P_y}{P_x}\right]}$$

$$\boxed{\frac{\rho_y}{\rho_x} = \frac{\frac{\gamma+1}{\gamma-1}\cdot\frac{P_y}{P_x}+1}{\frac{\gamma+1}{\gamma-1}+\frac{P_y}{P_x}}} \quad \text{(I)}$$

$$\frac{\gamma+1}{\gamma-1}\cdot\frac{\rho_y}{\rho_x}+\frac{P_y}{P_x}\cdot\frac{\rho_y}{\rho_x} = \frac{\gamma+1}{\gamma-1}\cdot\frac{P_y}{P_x}+1$$

$$\frac{P_y}{P_x}\left(\frac{\rho_y}{\rho_x}-\frac{\gamma+1}{\gamma-1}\right) = 1-\frac{\gamma+1}{\gamma-1}\cdot\frac{\rho_y}{\rho_x}$$

$$\frac{P_y}{P_x} = \frac{-\left[\frac{\gamma+1}{\gamma-1}\cdot\frac{\rho_y}{\rho_x}-1\right]}{-\left[\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}\right]}$$

$$\boxed{\frac{P_y}{P_x} = \frac{\frac{\gamma+1}{\gamma-1}\cdot\frac{\rho_y}{\rho_x}-1}{\frac{\gamma+1}{\gamma-1}-\frac{\rho_y}{\rho_x}}} \quad \text{(II)}$$

(I) – Density ratio in terms of pr. ratio

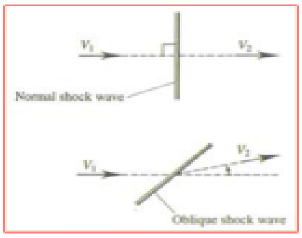
(II) – Pressure ratio in terms of density ratio

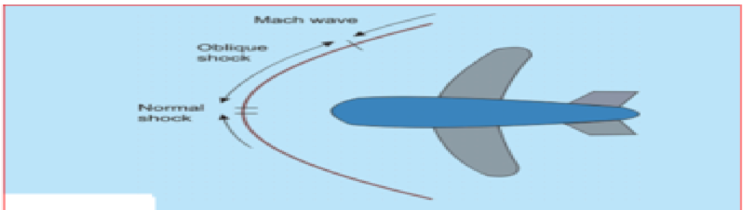
(I) and (II) are called

Rankine – Hugoniot equations.

FLOW WITH OBLIQUE SHOCK

- ✓ Oblique or 2-Dimensional plane shock wave
  - When the direction of flow is inclined at an oblique angle to the shock wave.

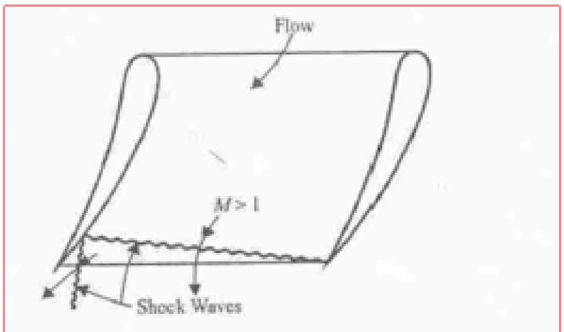




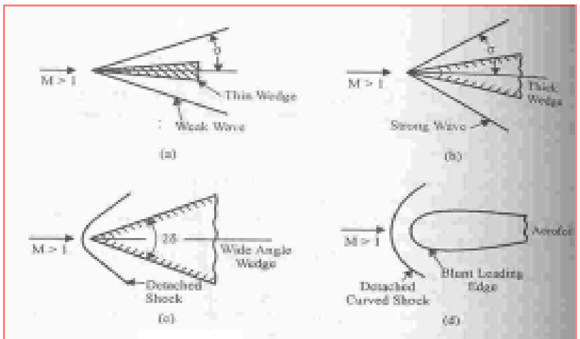
Normal and oblique Shock in front of an Obstacle

- ✓ Normal shock wave is a special case of oblique shock wave in which the ***wave angle,*** $\boldsymbol{\sigma_{\text{normal shock}} = 90^o}$.
- ✓ A stronger oblique shock wave is nearer to a normal shock wave and has a larger wave angle (approaching 90º).
- ✓ Weak oblique shock waves have smaller wave angles and are closer to the Mach waves.
- ✓ There are larger changes in flow parameters across strong oblique shocks.

- ✓ Oblique shocks are found to occur at the exit of the turbine blade passage with supersonic flow.
- ✓ Their presence alters the direction of the jet at the exit of the blades.



Oblique shock waves in a turbine blade passage



Examples of oblique shock waves

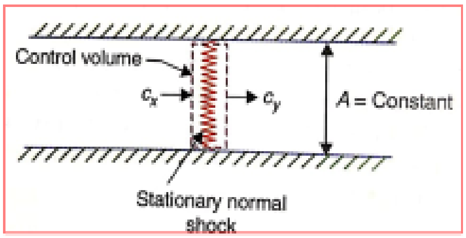
- ✓ Interferograms of supersonic flow fields over wedges of small and larger angles show the presence of attached shocks on thinner wedges and detached shocks upstream of thick wedges and blunt bodies (aero foils with thick leading edges).
- ✓ Note: $\delta_{\text{normal shock}} = 0$; δ = Deflection angle.

6.6 A jet of air at 275 K and 0.69 bar has an initial Mach number of 2.0. If it passes through a normal shock wave determine (*a*) Mach number (*b*) pressure (*c*) temperature (*d*) density (*e*) speed of sound and (*f*) jet velocity downstream of the shock.

Soln:

✓ Given: p_x = 0.69 bar; T_x = 275 K; M_x = 2.0;

✓ To find: (a) M_y (b) p_y (c) T_y (d) ρ_y (e) a_y (f) c_y



From gas tables: refer shock tables, for M_x = 2.0, γ = 1.4; Table 4.2, pg. 53, we get

2.00	0.577	4.500	1.687	0.721	5.641

✓ M_y = 0.577

✓ $p_y / p_x = 4.500 \longrightarrow p_y$ = 3.105 bar

✓ $T_y / T_x = 1.687 \longrightarrow T_y$ = 463.93 K

✓ $\rho_y = p_y / RT_y = 3.105 \times 10^5 / (287 \times 463.93) = 2.332$ kg/m^3

$$a_y = \sqrt{\gamma R T_y} = \sqrt{1.4x287x463.93} = 431.75\,\frac{m}{sec}$$

$C_y = M_y$ x a_y = 249.1 m/sec

Answers:

(a) M_y = 0.577; (e) a_y = 431.75 m/sec

(b) p_y = 3.105 bar; (f) C_y = 249.1 m/sec

(c) T_y = 463.93 K;

(d) ρ_y = 2.332 kg/m^3;

6.7 An aircraft flies at a Mach number of 1.2 at an altitude of 16000 metres. The compression in its engine is partly achieved by a normal shock wave standing at the entry of its diffuser. Determine immediately downstream of the shock:

(*a*) Mach number,

(*b*) temperature of the air,

(*c*) pressure of the air, and

(*d*) stagnation pressure loss across the shock.

Soln:

✓ Given: M_x = 1.2; z = 16000 m;

✓ At this altitude, refer gas tables, Table 2, Page no. 20 and get p_x = 0.103 bar; T_x = 216.65 K;

✓ To find: (a) M_y (b) p_y (c) T_y (d) p_{0x} - p_{0y}

Z m	t °C	T K	a m/s	p bar	ρ kg/m³	μ $\frac{Ns}{m^2} \times 10^6$
16000	-56.50	216.65	295.2	0.103	0.165	14.17

From normal shock tables: for M_x = 1.2, γ = 1.4; Table 4.2, pg. 52, we get

1.20	0.842	1.513	1.128	0.993	2.407

✓ From normal shock tables: for $M_x = 1.2$, $\gamma = 1.4$; Table 4.2, pg. 52, we get

1.20	0.842	1.513	1.128	0.993	2.407

✓ $M_y = 0.842$

✓ $p_y / p_x = 1.513 \longrightarrow p_y = 0.15584$ bar = 155.84 mbar

✓ $T_y / T_x = 1.128 \longrightarrow T_y = 244.38$ K

✓ $p_{oy} / p_x = 2.407 \longrightarrow p_{oy} = 0.2479$ bar and hence

✓ $p_{oy} / p_{0x} = 0.993 \longrightarrow p_{ox} = 0.2496$ bar and hence

✓ $P_{ox} - p_{0y} = 0.0017$ bar = 1.7 mbar

Answers:

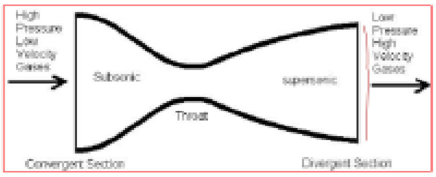
(a) $M_y = 0.842$;

(b) $p_y = 155.84$ mbar;

(c) $T_y = 244.38$ K;

(d) $P_{ox} - p_{0y} = 1.7$ mbar;

6.11 The stagnation pressure and temperature of air at the entry of a nozzle are 5 bar and 500 K respectively. The exit Mach number is 2.0 where a normal shock occurs. Calculate the following quantities before and after the shock: Static and stagnation pressures and temperatures, air velocities and Mach numbers. What are the values of stagnation pressure loss and increase in entropy across the shock?



To find: (a) T_x and T_y (b) T_{0x} and T_{0y} (c) p_x and p_y (d) p_{0x} and p_{0y} (e) c_x and c_y (f) M_x and M_y (g) p_{0x}-p_{0y} (h) s_y-s_x

$p_0 = 5$ bar $\quad M_x = 2.0$

$T_0 = 500$ K

From the given data, we found: $T_{0x} = T_{0y} = 500$ K;

$M_x = 2.0$

$p_{0x} = 5$ bar

✓ From isentropic tables: for $M_x = 2.0$, $\gamma = 1.4$; Table 3.2, pg. 34, we get

2.00	1.633	0.555	0.128	1.687	1.123	0.216

✓ $T_x / T_{0x} = 0.555 \longrightarrow T_x = 277.5$ K and

✓ $M_x = c_x / a_x \longrightarrow c_x = M_x \cdot a_x = 667.83$ m/sec

✓ $p_x / p_{0x} = 0.128 \longrightarrow p_x = 0.64$ bar

From normal shock tables, for $M_x = 2.0$, $\gamma = 1.4$; Table 4.2, pg. 53, we get

2.00	0.577	4.500	1.687	0.721	5.641

$M_y = 0.577$

$p_y / p_x = 4.500 \longrightarrow p_y = 2.88$ bar

$T_y / T_x = 1.687 \longrightarrow T_y = 468.14$ K

$p_{0y} / p_{0x} = 0.721 \longrightarrow p_{0y} = 3.605$ bar

$c_y = M_y \cdot a_y = 250.24$ m/sec

$\Delta p_0 = p_{0x} - p_{0y} = 5 - 3.605 = 1.395$ bar

$$\frac{\Delta S}{R} = \frac{S_y - S_x}{R} = ln\frac{p_{0x}}{p_{0y}} \longrightarrow \Delta S = 93.88 \text{ J/kg-K}$$

Answers:

(a) $M_x = 2.0$; $M_y = 0.577$;

(b) $p_x = 0.64$ bar; $p_y = 2.88$bar;

(c) $T_x = 277.5$ K; $T_y = 468.14$ K;

(d) $c_x = 667.83$ m/sec; $c_y = 250.24$ m/sec

(e) $p_{0x} = 5$ bar; $p_{0y} = 3.605$ bar;

(f) $T_{0x} = T_{0y} = 500$ K;

(g) $\Delta p_0 = 1.395$ bar;

(h) $\Delta S = +93.88$ J/kg-K;

A compression shock occurs in a divergent flow passage. On the upstream side of the shock, the velocity of air is 400 m/s and is at 2 bar and 35º C. Determine (a) Mach number downstream of the shock (b) air velocity on the downstream of the shock (c) stagnation pressure before and after the shock (d) Change in entropy per unit mass of air as a result of shock. Take $\gamma = 1.3$ and R = 287 J/kg-K.

Soln:

✓ Given: $c_x = 400$ m/s; $p_x = 2$ bar; $T_x = 35º\text{ C} + 273 = 308$ K; $\gamma = 1.3$ and R = 287 J/kg-K

✓ To find: (a) M_y (b) c_y (c) p_{0x} and p_{0y} (d) s_y-s_x

✓ $M_x = c_x/a_x = 400/\sqrt{(1.3x287x308)} = 1.179 \approx 1.18$

From normal shock tables, for $M_x = 1.18$, $\gamma = 1.3$; Table 4.1, pg. 47, we get

1.18	0.853	1.444	1.089	0.994	2.262

✓ $M_y = 0.853$

✓ $T_y / T_x = 1.089 \longrightarrow T_y = 1.089x308 = 335.412$ K

✓ $C_y = M_y \times a_y = 0.853 \times \sqrt{(1.3x287x335.412)} = 301.75$ m/sec

✓ $p_{oy} / p_x = 2.262 \longrightarrow p_{oy} = 4.524$ bar

✓ $p_{oy} / p_{0x} = 0.994 \longrightarrow p_{ox} = 4.551$ bar

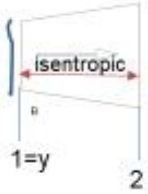
✓ s_y-s_x = R x $\ln(p_{0x}/p_{0y})$ = 287 x $\ln(1/0.994)$ = 1.727 J/kg-K.

6.12 A Mach-2 aircraft engine employes a subsonic inlet diffuser of area ratio 3. A normal shock is formed just upstream of the diffuser inlet. The free-stream conditions upstream of the diffuser are: $p = 0.10$ bar, $T = 300$ K. Determine:

(*a*) Mach number, pressure and temperature at the diffuser exit.

(*b*) Diffuser efficiency including the shock.

Assume Isentropic flow in the diffuser downstream of the shock.



Soln: Given: $M_x = 2.0$; $p_x = 0.10$ bar; $T_x = 300$ K; $A_2 / A_1 = 3$

✓ To find: (a) M_2, p_2, T_2 (b) η_D

From normal shock tables, for $M_x = 2.0$, $\gamma = 1.4$; Table 4.2, pg. 53, we get

2.00	0.577	4.500	1.687	0.721	5.641

✓ $M_y = 0.577 = M_1$

✓ $T_y / T_x = 1.687 \longrightarrow T_y = 506.1\text{ K} = T_1$

✓ $p_y / p_x = 4.500 \longrightarrow p_y = 0.45\text{ bar} = p_1$

✓ From isentropic tables, for $M_1 = 0.577$, $\gamma = 1.4$; Table 3.2, pg. 29, by interpolation, we get

0.57	0.605	0.939	0.802	1.226	1.129	0.984
0.58	0.615	0.937	0.796	1.213	1.121	0.966

$T_1/T_{01} = 0.9376$ and hence $T_{01} = T_{02} = T_1/0.9376 = 506.1/0.969 = 539.78$ K

0.57	0.605	0.939	0.802	1.226	1.129	0.984
0.58	0.615	0.937	0.796	1.213	1.121	0.966

$p_1/p_{01} = 0.7978$ and hence $p_{01} = p_{02} = p_1/0.7978 = 0.45/0.7978 = 0.5641$ bar

$A_1/A_1^* = 1.2169$

$$\frac{A_2}{A_1} = 3 = \frac{A_2}{A_2^*} X \frac{A_1^*}{A_1} \quad [Note\!:\ A^* = A_1^* = A_2^*]$$

2.84	1.925	0.382	0.0347	3.636	1.222	0.126
2.85	1.927	0.381	0.0342	3.670	1.223	0.125

Now, $A_2/A_2^* = 3 \times 1.2169 = 3.651$

For this area ratio of $A_2/A_2^* =$ **3.651**, refer isentropic table and by interpolation, we get,

0.16	0.1750	0.9949	0.982	3.673	2.947	3.607
0.17	0.186	0.9943	0.980	3.464	2.786	3.394

$M_2 = 0.161$;

$p_2 / p_{02} = 0.9818 \longrightarrow p_2 = 0.5538$ bar

$T_2 / T_{02} = 0.9948 \longrightarrow T_2 = 536.97$ K

This equation shows that the efficiency of diffuser with normal shock is a function of entry Mach number.

✓ The diffuser efficiency with shock is given by: $$\eta_D = \frac{\frac{T_{01}}{T_1}\left(\frac{p_{0y}}{p_{0x}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{\gamma-1}{2} M_1^2}$$

At $M_x = 2.0$, refer isentropic table and normal shock tables, for $\gamma = 1.4$;

2.00	1.633	0.555	0.128	1.687	1.123	0.216

2.00	0.577	4.500	1.687	0.721	5.641

$$\eta_D = \frac{\dfrac{T_{01}}{T_1}\left(\dfrac{p_{0y}}{p_{0x}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\dfrac{\gamma-1}{2}M_1^2}$$

Here, $M_1 = 2.0$

✓ Hence, $\eta_D = 80.1\%$

Answers:

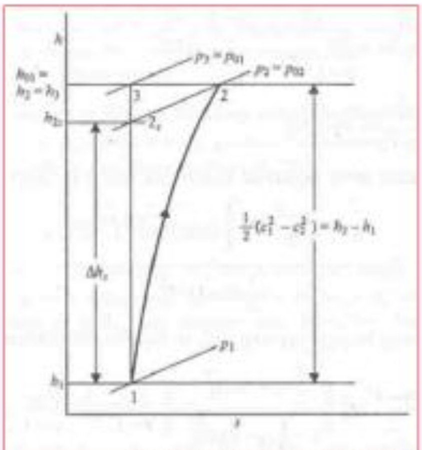
$M_2 = 0.161$;

$p_2 = 0.5538$ bar

$T_2 = 536.97$ K

$\eta_D = 80.1\%$

$$Efficiency\ of\ diffuser = \frac{Change\ of\ enthalpy\ in\ reversible\ diffusion}{Change\ of\ enthalpy\ in\ actual\ diffusion} = \frac{h_{2s} - h_1}{h_2 - h_1}$$



Diffusion process in a supersonic diffuser

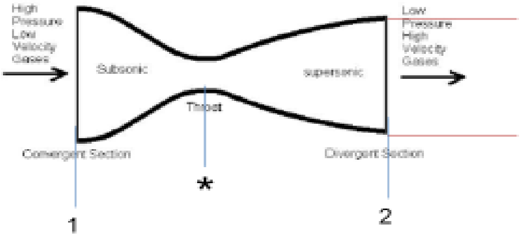
$$\eta_D = \frac{\dfrac{\gamma+1}{\gamma-1}}{\left(\dfrac{2\gamma}{\gamma+1}M_1^2 - \dfrac{\gamma-1}{\gamma+1}\right)^{1/\gamma}} - \left(\frac{2}{\gamma-1}\right)\frac{1}{M_1^2}$$

6.14 A supersonic nozzle is provided with a constant diameter circular duct at its exit. The duct diameter is same as the nozzle exit diameter. Nozzle exit cross-section is three times that of its throat. The entry conditions of the gas ($\gamma = 1.4$, $R = 0.287$ kJ/kg K) are $p_0 = 10$ bar, $T_0 = 600$ K. Calculate the static pressure, Mach number and the velocity of the gas in the duct:

(a) when the nozzle operates at its design condition,

(b) when a normal shock occurs at its exit, and

(c) when a normal shock occurs at a section in the diverging part where the area ratio $A/A^* = 2.0$



- ✓ Diagram for case 'a'
- ✓ $p_0 = p_{01} = p_{02} = 10$ bar
- ✓ $T_0 = T_{01} = T_{02} = 600$ K
- ✓ $A_2/A_2^* = 3$

Soln: Given: Supersonic nozzle; $A_2/A^*=3$; $\gamma=1.4$; R = 0.287 kJ/kg-K; p_0 = 10 bar; T_0 = 600 K;

✓ To find: **In the duct;**

(a) M_2, p_2, c_2 when the nozzle operates at design condition.

(b) M_y, p_y, c_y when a normal shock occurs at its exit.

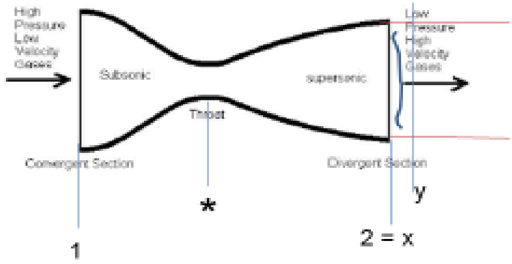
(c) M_2, p_2, c_2 when a normal shock occurs at a section in the divergent part where the area ratio $A/A^* = 2.0$

✓ At $A_2/A_2^* = 3$, refer isentropic table for $\gamma = 1.4$, Table 3.2, pg 36, by interpolation, we get

2.63	1.866	0.419	0.0478	2.980	1.201	0.1425
2.64	1.869	0.417	0.0471	3.007	1.202	0.142

- ✓ $M_2 = 2.637 \approx 2.64$
- ✓ $p_2 / p_{02} = 0.0473 \longrightarrow p_2 = 0.473$ bar
- ✓ $T_2 / T_{02} = 0.418 \longrightarrow T_2 = 250.8$ K
- ✓ $c_2 = M_2 a_2 = 2.64 \times \sqrt{(1.4 \times 287 \times 250.8)} = 838.06$ m/sec

- Diagram for case ‘b’
- $p_0 = p_{01} = p_{02} = 10$ bar
- $T_0 = T_{01} = T_{02} = 600$ K
- $A_2/A_2^* = 3$

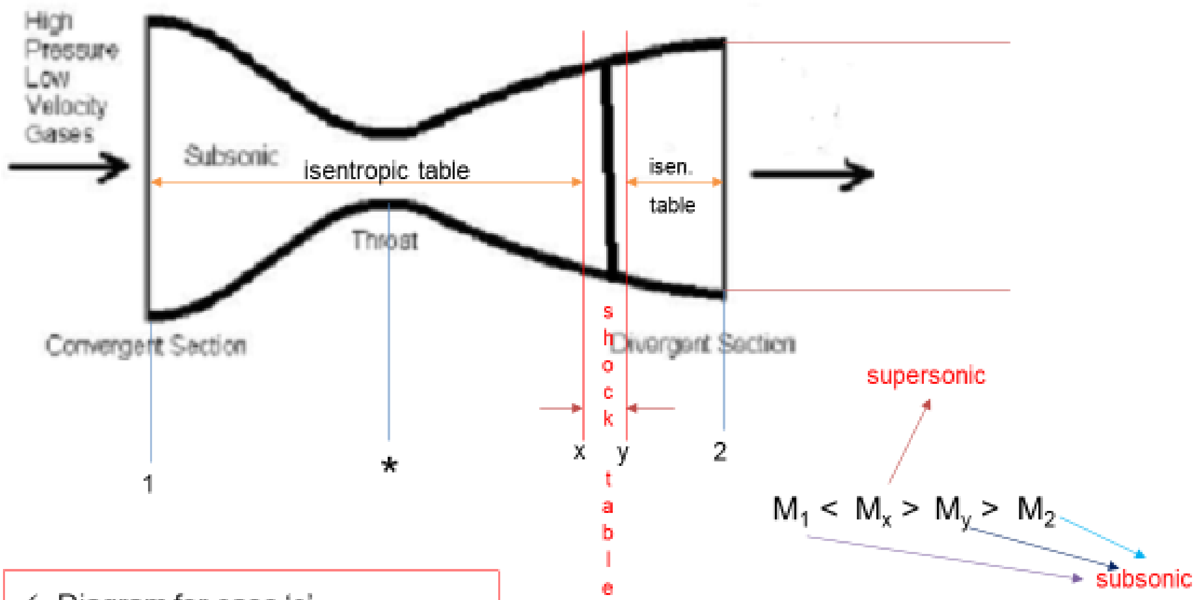


- $M_2 = 2.64 = M_x$
- $p_2 = 0.473$ bar $= p_x$
- $T_2 = 250.8$ K $= T_x$
- $c_2 = 838.06$ K $= c_x$

- At $M_x = 2.64$, refer normal shock table for $\gamma = 1.4$, Table 4.2, pg 54, we get

2.64	0.500	7.965	2.279	0.445	9.451

- $M_y = 0.5$
- $T_y / T_x = 2.279 \longrightarrow T_y = 571.57$ K
- $p_y / p_x = 7.965 \longrightarrow p_y = 3.767$ bar
- $c_y = M_y a_y = 0.5 \times \sqrt{(1.4 \times 287 \times 571.57)} = 239.61$ m/sec



- Diagram for case ‘c’
- $p_0 = p_{01} = p_{0x} = 10$ bar
- $T_0 = T_{01} = T_{02} = T_{0x} = T_{0y} = 600$ K
- $A_2/A_2^* = 3$
- $A_x/A_x^* = 2$

Following relations may be noted:

- $A_1^* = A_x^* = A_{throat} = A_t^*$
- $A_2^* = A_y^*$
- $A_x = A_y$

✓ At $A_x/A_x^* = 2$, refer isentropic table for γ = 1.4, Table 3.2, pg 35, by interpolation, we get

2.19 2.20	1.714 1.718	0.510 0.508	0.095 0.0935	1.987 2.005	1.148 1.150	0.188 0.187

✓ $M_x = 2.197 \approx 2.2$

✓ $p_x / p_{0x} = 0.0939 \longrightarrow p_x = 0.0939 \times 10 = 0.939$ bar

✓ $T_x / T_{0x} = 0.509 \longrightarrow T_x = 0.509 \times 600 = 305.4$ K

✓ At $M_x = 2.2$, refer normal shock table for γ = 1.4, Table 4.2, pg 53, we get

2.20	0.547	5.480	1.857	0.628	6.716

✓ $M_y = 0.547$

✓ $p_{0y} / p_{0x} = 0.628 \longrightarrow p_{0y} = 6.28$ bar $= p_{02}$

✓ At $M_y = 0.547$, refer isentropic table for γ = 1.4, Table 3.2, pg 29, by interpolation, we get

0.54 0.55	0.575 0.585	0.945 0.943	0.820 0.814	1.270 1.255	1.157 1.147	1.042 1.022

✓ $A_y/A_y^* = 1.2595$

$$\frac{A_2}{A_2^*} = \frac{A_2}{A_Y} x \frac{A_Y}{A_Y^*} = \frac{A_2}{A_X^*} x \frac{A_X^*}{A_X} x \frac{A_Y}{A_Y^*} = 3x\frac{1}{2}x1.2595 = 1.889$$

✓ At $A_2/A_2^* = 1.889$, refer isentropic table for γ = 1.4, Table 3.2, pg 29, by interpolation, we get

0.32 0.33	0.347 0.357	0.979 0.978	0.932 0.927	1.922 1.871	1.614 1.577	1.790 1.735

✓ $M_2 = 0.326$

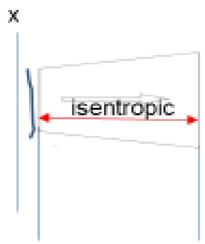
✓ $p_2 / p_{02} = 0.929 \longrightarrow p_2 = 0.929 \times 6.28 = 5.834$ bar

✓ $T_2 / T_{02} = 0.9783 \longrightarrow T_2 = 0.9783 \times 600 = 586.98$ K

✓ $c_2 = M_2 a_2 = 0.326 \times \sqrt{(1.4 \times 287 \times 586.98)} = 158.3$ m/sec

CONDITION	M_2	p_2(bar)	c_2 (m/sec)
When the nozzle operates at design condition (no shock, isentropic flow)	2.64	0.473	838.06
When a normal shock occurs at its exit	0.5	3.767	239.61
when a normal shock occurs at a section in the divergent part where the area ratio A/A* = 2.0	0.326	5.834	158.3

UQ: An air plane having a diffuser designed for subsonic flight has a normal shock attached to the edge(entry) of the diffuser when the plane is flying at a certain Mach number. If at the exit of the diffuser, the Mach number is 0.3. What must be the flight Mach number assuming isentropic diffusion behind the shock? The area at inlet is 0.29 m² and that at exit is 0.44 m².



Y=1

$A_y = A_1 =$ 0.29 m²

$M_2 = 0.3$

$A_2 = 0.44$ m²

✓ To find: M_x

✓ Note: $A_2^* = A_y^*$

✓ At $M_2 = 0.3$, refer isentropic table for γ = 1.4, Table 3.2, pg 28, we get

0.30	0.325	0.982	0.939	2.035	1.698	1.912

✓ $A_2/A_2^* = 2.035 = A_2/A_y^*$

✓ $A_y^* = 0.44/2.035 = 0.216$ m²

✓ Now, $A_y/A_y^* = 0.29/0.216 = 1.343$

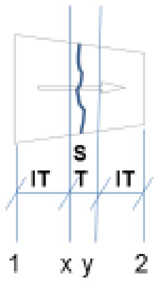
✓ At $A_y/A_y^* = 1.343$, refer isentropic table for γ = 1.4, Table 3.2, pg 29, by interpolation, we get

0.49	0.524	0.954	0.848	1.360	1.216	1.154
0.50	0.534	0.952	0.843	1.340	1.203	1.129

✓ $M_y = 0.498$; ✓ For $M_y = 0.49$, refer shock table for γ = 1.4, Table 4.2, pg 54, we get, $\mathbf{M_x = 2.66}$

2.66	0.498	8.088	2.300	0.438	9.587

UQ: Air with Mach number 2.5 enters a convergent duct with an area ratio of $A_2/A_1 = 0.5$. Under certain conditions, normal shock occurs at a point where $A/A_1 = 0.6$. For this condition, find exit Mach number and pressure ratio across the duct.



$M_1 > M_x > M_y < M_2$

Given: $M_1 = 2.5$; $A_2/A_1 = 0.5$; $A_x/A_1 = 0.6$;

To find: (a) M_2 (b) p_2 / p_1

Following relations may be noted:

✓ $A_1^* = A_x^*$; $A_2^* = A_y^*$; $A_x = A_y$

✓ $p_{01} = p_{0x}$; $p_{0y} = p_{02}$

✓ At $M_1 = 2.5$, refer isentropic table for γ = 1.4, Table 3.2, pg 36, we get

2.50	1.826	0.444	0.0585	2.637	1.187	0.154

✓ $A_1/A_1^* = 2.637 = A_1/A_x^*$

✓ $p_1 / p_{01} = 0.0585$

✓ $A_x/A_x^* = (A_x/A_1)(A_1/A_x^*) = 0.6\text{x}2.637 = 1.582$

✓ At $A_x/A_x^* = 1.582$, refer isentropic table for γ = 1.4, Table 3.2, pg 34, by interpolation, we get

1.92	1.596	0.575	0.145	1.580	1.111	0.228
1.93	1.600	0.573	0.142	1.593	1.113	0.227

✓ $M_x = 1.92$

✓ $p_x / p_{0x} = 0.1445$

✓ At M_x = 1.92, refer normal shock table for γ = 1.4, Table 3.2, pg 53, by interpolation, we get

1.92	0.592	4.134	1.624	0.758	5.239

✓ M_y = 0.592

✓ p_{0y} / p_{0x} = 0.758

✓ At M_y = 0.592, refer isentropic table for γ = 1.4, Table 3.2, pg 29, by interpolation, we get

0.59	0.625	0.935	0.790	1.200	1.113	0.948
0.60	0.635	0.933	0.784	1.188	1.105	0.932

✓ A_y/A_y^* = 1.198

$$\frac{A_2}{A_2^*} = \frac{A_2}{A_Y} x \frac{A_Y}{A_Y^*} = \frac{A_2}{A_X} x \frac{A_Y}{A_Y^*} = \frac{A_2}{A_1} x \frac{A_1}{A_X} x \frac{A_Y}{A_Y^*} = 0.5x \frac{1}{0.6} x1.198 = 0.998{\sim}1.0$$

✓ For this area ratio, refer isentropic table for γ = 1.4, Table 3.2, pg 31, we get

✓ M_2 = 1 and $p^* / p_{02} = 0.528 = p_2 / p_{02}$

$$\boxed{\frac{p_2}{p_1} = \frac{p_2}{p_{02}} x \frac{p_{0y}}{p_{0x}} x \frac{p_{01}}{p_1} = 1x0.758x \frac{1}{0.0585} = 12.9 \sim 13}$$

Answers:

M_2 = 1.0;

p_2 / p_1 = 13

SATHYABAMA
INSTITUTE OF SCIENCE AND TECHNOLOGY
(DEEMED TO BE UNIVERSITY)
Accredited "A" Grade by NAAC | 12B Status by UGC | Approved by AICTE
www.sathyabama.ac.in

SCHOOL OF MECHANICAL ENGINEERING
DEPARTMENT OF MECHANICAL ENGINEERING
GAS DYNAMICS AND JET PROPULSION (SME1303)

UNIT – 5
PROPULSION

JET PROPULSION

INTRODUCTION: Gas turbine cycles for jet propulsion differ from shaft power cycles because of the fact that the useful power output for jet propulsion is produced, *wholly* or *partially*, as a result of expansion of gas in a propelling nozzle; **wholly** in **turbojet** engines and **partially** in **turboprop** engines. A second distinguishing feature is the need to consider the effect of **forward speed** and **altitude** on the performance of the propulsion engines. The beneficial effect of those parameters, together with an inherently **high power/weight** ratio have enabled the gas turbine to replace the reciprocating engine for aircraft propulsion except for the very low power levels.

The principle of jet propulsion is obtained from the application of Newton's laws of motion (Newton's second law: **rate of change of momentum in any direction is proportional to the force acting in that direction.** Newton's third law: **for every action there is equal and opposite reaction**)

We know that when a fluid is to be accelerated, a force is required to produce this acceleration in the fluid. At the same time there is an equal and opposite reaction force of the fluid on the engine which is known as the **thrust**. Hence, it may be stated that the working of the jet propulsion is based on the reaction principle. Thus all devices that move through fluid must follow this basic principle.

In principle, any fluid can be used to achieve the jet propulsion. Thus water, steam or combustion gases can be used to propel a body in a fluid. But there are limitations in the choice of the fluid when the bodies are to be propelled in the atmosphere. Experience shows that only two types of fluids are particularly suitable for jet propulsion.

(i) A heated and compressed atmospheric air –admixed with the products of combustion produced by burning fuel in that air can be used for jet propulsion. The thermo chemical energy of the fuel is utilized for increasing the temperature of the air to the desired value. The jet of this character is called a thermal jet and the jet propulsion engine using atmospheric air is called **air breathing engines.**

(ii) Another class of jet propulsion engines use a jet of gas produced by the chemical reactions of the fuel and oxidizer. Each of them is carried with the system itself. The fuel-oxidant mixture is called the **propellant**. No atmospheric used for the formation of the jet. But the oxidant in the propellant is used for generating the thermal jet. A jet produced in this way is known as **rocked jet** and the equipment wherein the chemical reaction takes place is called a **rocket motor.** The complete unit including the propellant is called a **rocket engine.**

From the above discussion it is clear that the jet propulsion engines may be classified broadly into two groups.

(i) Air breathing engines and

(ii) Rocket engines.

Air breathing engines can be further classified as follows:

(i) Reciprocating or propeller engines and

(ii) Gas turbine engines.

GAS TURBINE ENGINES:

All modern aircrafts are fitted with gas turbines. Gas turbine engines are classified as :

(i) Ram jet engine

(ii) Pulse jet engine

(iii) Turbo jet engine

(iv) Turbo prop engine

Taken in the above order they provide propulsive jets of **increasing mass flow** and **decreasing jet velocity**. Therefore, in that order, it will be seen that ramjet can be used for **highest cruising speed** whereas the turbo prop engine will be useful for the **lower cruising speed at low altitudes.**

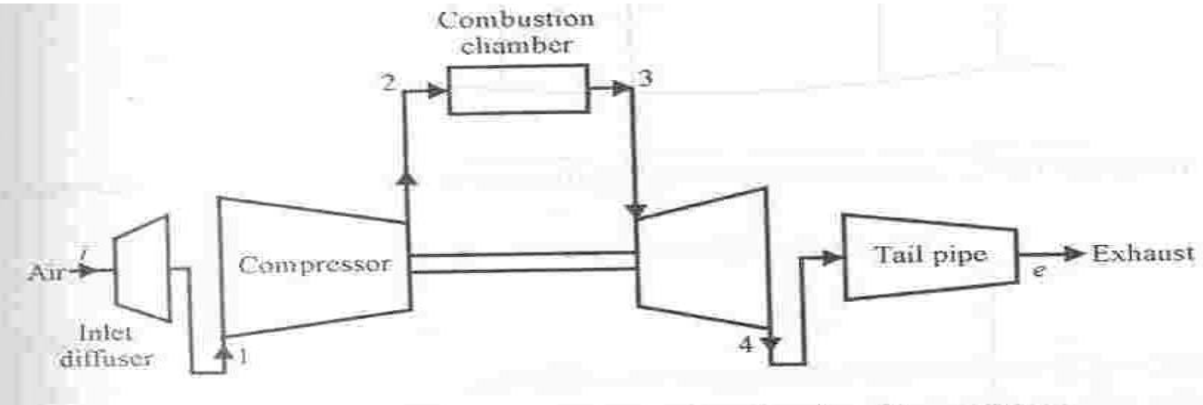
In practice, the choice of the power plant will depend on the required cruising speed, desired range of the air craft and maximum rate of the climb.

The details of various gas turbine engines mentioned above are discussed under two categories.

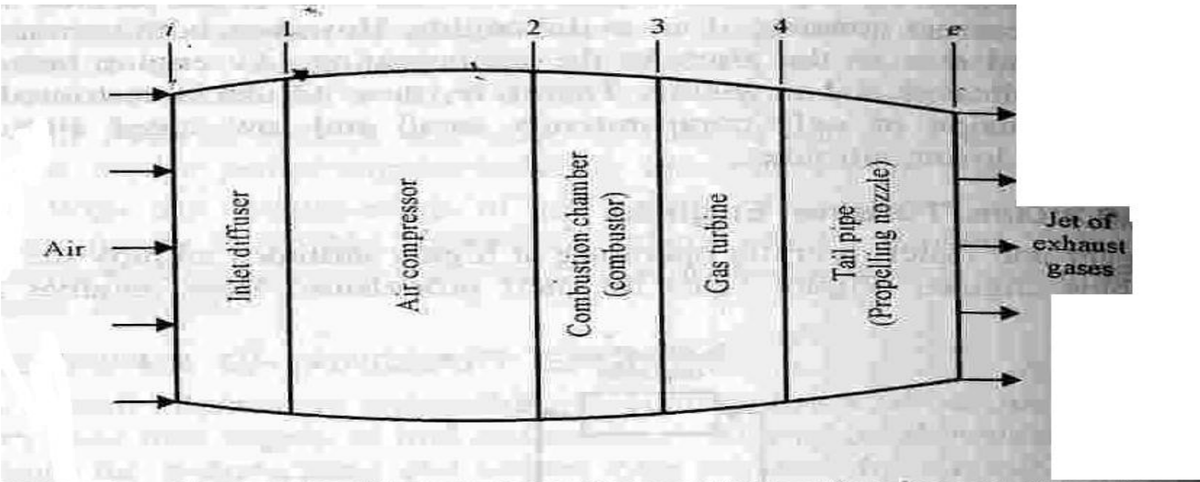
(i) pilot less operation and

(ii) piloted operation.

The *ram jet and pulse jet engines* come under the category of **pilot less operation** whereas the *turbo jet and turbo prop engines* are used for **piloted operation.**

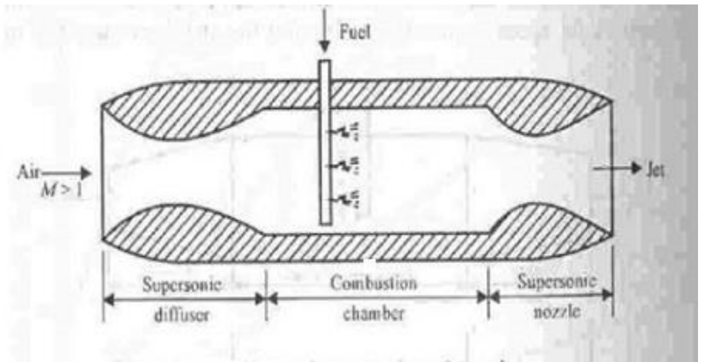


Gas turbine power plant for aircraft propulsion

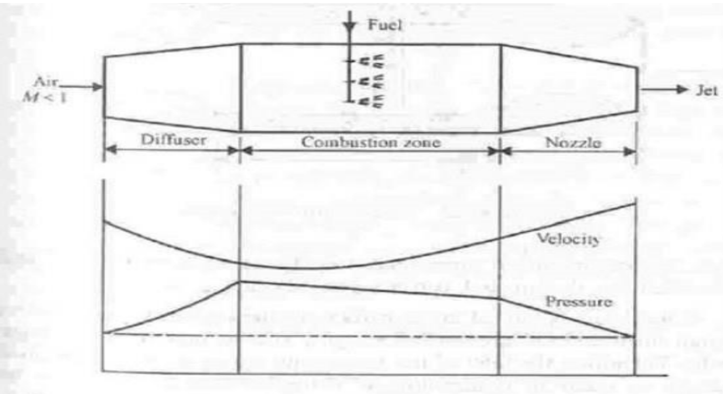


Layout of the principal components in an aircraft gas turbine engine

THE RAM JET ENGINE: The fact of obtaining very high pressure ratios of about 8 to 10 by a ram compression has made it possible to design a jet engine **without a mechanical compressor.** A deceleration of the air from Mach number 3 at a diffuser inlet to Mach number 0.3 in combustion chamber would cause pressure ratio of more than 30. Due to shock and other losses inevitable at such velocities, all of this pressure rise is not available; still whatever we get is more than sufficient for raising the air pressure to the required combustion pressure. This principle of ram pressure rise is used in the ram jet engines. The **ram pressure rise** can be achieved in the **diffusers.** The simplest type of **air breathing engine** is **ram jet engine.**



A supersonic ramjet engine



A subsonic ramjet engine

PARTS OF A RAM JET ENGINE:

1. SUPERSONIC DIFFUSER (1-2)
2. SUB SONIC DIFFUSER (2-3)
3. COMBUSTION CHAMBER (3-4)
4. DISCHARGE NOZZLE (4-5)

RAM EFFECT: Both supersonic and subsonic diffusers convert the kinetic energy of the entering air into pressure rise. This energy transformation is called *ram effect* and the pressure rise is called the *ram pressure.* The principle of operation is as follows:

Air from the atmosphere enters the engine at a very high speed and its velocity gets reduced first in the supersonic diffuser, thereby its static pressure increases. The air then enters the subsonic diffuser wherein it is compressed further. Afterwards the air flows into the combustion chamber, the fuel is injected by the suitable injectors and mixed with the unburnt air. The air is heated to a temperature of the order of 1500 – 2000 K by the continuous combustion of the fuel. The fresh supply of air to the diffuser builds up pressure at the diffuser end so that these gases cannot expand towards the diffuser. Instead, the gases are made to expand in the combustion chamber towards the tail pipe. Further, they are allowed to expand in the exhaust nozzle section. The products will leave the engine with a speed exceeding that of the entering air. Because of the rate of increase in the momentum of the working fluid, a thrust, F, is developed in the direction of flight.

The cycle pressure ratio of a ramjet engine depend upon its flight velocity. The higher the flight velocity the larger is the ram pressure, and consequently larger will be the thrust. This is true until a condition is reached where the discharge nozzle becomes choked. Thereafter, the nozzle operates with the constant Mach NO. of 1 at its throat. Therefore, a ramjet having fixed geometry is designed for a specific mach no. and altitude, and at the design point, will give the best performance.

Since the ram jet engine cannot operate under static conditions, as there will be no pressure rise in the diffuser, it is not self propelling at zero flight velocity. To initiate its operation, the ramjet must either launched from an air plane in flight or be given an initial velocity by some auxiliary means, such as launching rockets. Since the ramjet is an air breathing engine, its maximum altitude is limited. Its field of operation is inherently in speed ranges above those of the other air breathing engines. However, it has a limited use in a high subsonic speed range. Its best performance capabilities, however, are in supersonic speed range of mach no. between 2 and 5. The upper speed is limited by the problem of cooling of the outer skin of the engine body at high flight mach numbers.

Ramjets have higher thrust per unit weight amongst air breathing engines and is only next to rockets in this respect. Though a ram jet can operate at subsonic velocities just below the sonic velocity, it is most efficient at high velocity of about 2400 – 6000 km/hr and at very high altitudes.

<u>EFFECT OF FLIGHT VELOCITY</u>: The cycle pressure ratio of a ram jet engine depends upon its flight velocity. The **higher the flight velocity the larger is the ram pressure**, and consequently **larger will be thrust.**

Advantages and disadvantages of ramjet engine

Advantages	Disadvantages
1. Ram jet is very simple and does not have any moving part. It is very cheap to produce and requires almost no maintenance. 2. Due to the fact that a turbine is not used to drive the mechanical compressor, the maximum temperature which can be allowed in ram jet is very high, about 2000 deg. Celsius as compared to about 900 deg. Celsius in turbojets. This allows a greater thrust to be obtained by burning fuel at air-fuel ratio of about 13:1, which gives higher temperatures. 3. The specific fuel consumption is better than other gas turbine power plants at high speed and high altitudes. 4. There seems to be no upper limit to the flight speed of the ramjet.	1. Since the compression of air is obtained by virtue of its speed relative to the engine, the take-off thrust is zero and it is not possible to start the ramjet without an external launching device. 2. The engine heavily relies on the diffuser and it is very difficult to design a diffuser which will give good pressure recovery over a wide range of speeds. 3. Due to a high air speed the combustion chamber requires flame holder to stabilize the combustion. 4. At very high temperatures of about 2000 deg.Celsius dissociation of products of combustion occurs which will reduce the efficiency of the plant if not recovered in nozzle during expansion

<u>Applications</u>: Due to its high thrust at high operational speed it is widely used in high speed air craft and missiles. Subsonic ramjets are used in target weapons, in conjunction with turbojets or rockets for getting the starting torque.

Advantages and disadvantages of pulse jet engines

Advantages	Disadvantages
1. This is a very simple device next to ramjet and is light in weight. It requires very small and occasional maintenance. 2. Unlike ramjet, it has static thrust because of the compressed air starting ; thus it does not need a device for initial propulsion. The static thrust is even more than the cruise thrust. 3. It can run on almost any types of liquid fuels with out much effect on the performance. It can also operate on gaseous fuel with the little modifications. 4. Pulse jet engine is relatively cheap.	1. The biggest disadvantage is very short life of flapper valves and high rates of fuel consumption. The s.f.c is as high as that of ramjet. 2. The speed of the pulse jet is limited to a very narrow range of about 650- 800 km/hr, because of the limitations in the aero dynamic design of an efficient diffuser suitable for wide speed range. 3. The operational pulse jet is also limited in altitude range. 4. The high degree of vibrations due to intermittent nature of the cycle and the

	buzzing noise has made it suitable for pilot less crafts only. 5. It has lower propulsive efficiency than turbojet engines

Applications: Pulse jet is highly suited for bombers like the German V-1. It has also been used in some helicopters, target aircrafts and missiles.

Advantages and disadvantages of turbo prop engine

Advantages	Disadvantages
1. It has a higher thrust at take-off and better fuel economy. 2. The frontal area is less than the propeller engines so that the drag is reduced 3. The turbo prop can operate economically over a wide range of speeds ranging from low speeds where pure jet engine is uneconomical to high speeds of about 800 km/hr where the propeller engine efficiency is low. 4. It is easy to maintain and has lower vibrations and noise. 5. The power output is not limited as in the case of propeller engines. 6. The multi shaft arrangement allows a great flexibility of operation over a wide range of speeds.	1. The main disadvantage is that at high speeds, due to shocks and flow separation, the propeller efficiency decreases rapidly, thereby, putting up a maximum speed limit on the engine 2. It requires a reduction gear which increases the cost and also consumes certain energy developed by the turbine in addition to requiring more space. **Application**: It is widely used in commercial and military air craft due to its high flexibility of operation and good fuel economy. It is likely to eliminate propeller engines from moderate power and speed air craft.

Advantages and disadvantages of turbo jet engine

Advantages	Disadvantages
1. The power to weight ratio of a turbojet is about 4 times of a propeller system having reciprocating engine. 2. It is simple, easy to maintain and requires lower lubricating oil consumption. Furthermore, complete absence of liquid cooling results in reduced frontal area. 3. There is no limit to the power output which can be obtained from a turbojet while the piston engines have reached almost their peak power and further increase will be the cost of complexity and greater engine weight and frontal area of the air craft.	1. The fuel economy at low operational speeds is extremely poor. 2. It has low take-off thrust and hence poor starting characteristics.

4. The speed of the turbojet is not limited by the propeller and it can attain higher flight speeds than engine propeller aircrafts.	

Ram efficiency or inlet diffuser: In all jet propulsion plants, except turbo prop and rocket engines, a diffuser is used at the inlet to convert kinetic energy into static pressure rise. This compression process is essentially adiabatic but it cannot be considered as reversible since fluid friction is present. The most widely used efficiency of the inlet jet process is based upon the pressure rise that actually takes place; compared to the pressure rise which would have taken place had the process been isentropic. By definition, this efficiency is

$$\eta_{ram} = \frac{P_{01} - P_a}{P_{01}' - P_a}$$

Thrust augmentation:

The poor take-off characteristic of the turbojet engine can be improved by augmenting the thrust. The thrust from a turbojet is given by

$$F = m_a[(1 + f)\, c_j - c_i]$$

In which the exhaust c_j , is the function of the maximum temperature in the cycle. Higher the maximum temperature higher is the value of c_j . Another method of increasing thrust is to increase the mass flow rate. Improved thrust results in shorter take off distances , high climb rate and good maneuverability at high altitudes. The thrust augmentation can be effected by the following methods :

1. Burning of additional fuel in the tail pipe between the turbine exhaust section and entrance section of the exhaust nozzle. This method of thrust augmentation increases the jet velocity and is known as **after burning.** The device used is called the **after burner.**
2. Injecting refrigerants, water or water alcohol mixture at some point between inlet and exit sections of the air compressor. This method of thrust augmentation increases the mass flow rate and decreases the work of compression.
3. Bleeding off air in excess of that required for stoichiometric combustion in the main combustion chamber – at the entrance section of the combustion chamber, and burning it with the stoichiometric fuel-air ratio in a separate one. The combustion products from the latter combustor are expanded in a separate auxiliary exhaust nozzle. The bled air is replaced by water which is injected in the main combustors. This method of thrust augmentation is known as **bleed burn cycle.**

<u>PULSE JET ENGINE</u>

Pulse jet engine is very similar to ramjet engine in construction except that in addition to the diffuser at intake, combustion chamber and exhaust nozzle, it has mechanically operated flapper valve grids which can allow or stop air flow in the combustion chamber. Thus pulse jet is an intermittent flow, **compressor less type of device with minimum no. of moving parts.**

It consists essentially of the following parts:

1. a diffuser
2. a valve grid which contains springs that close on their own spring pressure
3. a combustion chamber
4. a spark plug
5. a tail pipe or discharge nozzle

<u>operation of the pulse jet</u>:

During starting compressed air is forced into the inlet which opens the spring loaded flapper valve grid; the air enters the combustion chamber into which the fuel is injected and burnt with the help of the spark plug. Combustion occurs with the sudden explosion process, i.e., the **combustion is at constant volume** *instead* of at constant pressure as in other propulsive devices. The pulse jet cycle is ***more near*** to **Otto cycle. Ram action** can also be used to increase the pressure of the cycle.

The function of the diffuser is to convert the kinetic energy of the entering air into static pressure rise by slowing down the air velocity. When a certain pressure difference builds across the valve grid, the valves will open. This makes the fresh air to enter the combustion chamber, where fuel is mixed with the air and combustion starts. To start the combustion initially, the spark plug is used. Once the combustion starts, it proceeds at constant volume. Thereby, there is a rapid increase in pressure, which causes the valve to close rapidly. The products of combustion surges towards the nozzle. They expand in the nozzle and escape into the atmosphere with a higher velocity so that the exit velocity is much higher than the inlet velocity.

Thus, the rate of momentum of the working fluid is changed so as to cause a propulsive thrust. Since, the combustion process causes the pressure to increase, the **engine can operate** even at **static conditions** once it gets started. When the combustion products accelerate from the chamber, they leave a slight vacuum in the combustion chamber. This, in turn, produces sufficient pressure drop across the valve grid allowing the valves to open again.

A new charge of air enters the combustion chamber which is mixed with the fuel that flows continuously. The fresh fuel-air mixture is ignited by the charge leaving and /or by residual charge. New charge need not be ignited with the spark plug again. Proper design allows the duct to fire at the given pulse rate when the fuel flows continuously. The frequency of pulsation is determined by the duct shape and working temperatures and may be as high as 500 cycles/sec in very small units. The **thrust of the pulse jet engine** is **proportional** to the **average mass flow rate of gases through the engine multiplied by its increase in velocity.**

Like ramjet engines, the maximum operating altitude of the pulse jet is also limited by the air density consideration. Unlike the ramjet, the **pulse jet engine** develops **thrust at zero speed.** A high initial launching velocity, however, improves its performance. The thrust of the engine, of course, decreases with altitude and does not continue to increase with increasing flight speeds up to supersonic range as is true of ram jet. The pulse jet engine is simple and cheap for subsonic flight and well adopted to **pilot less air craft.** The use of the pulse jet engine is restricted to pilot less air craft due to its severe **vibration** and **high intensity noise.**

The pulse jet has low thermal efficiency and limited speed range. The maximum operating speed of the pulse jet is seriously limited by two factors:

1. It is not possible to design a good diffuser at high speeds.
2. The flapper valves, the only mechanical part in the pulse jet, also have certain natural frequency and if it coincides with the cycle frequency, resonance occurs and the valves may remain open and no compression will take place.

Also, as the speed increases it is difficult for the air to flow back. this reduces the total compression pressure as well as the mass flow of air which results in inefficient combustion and lower thrust. The reduction in thrust and efficiency is quite sharp as the speed increases. At subsonic speed it might not operate as the speed is not sufficient to raise the air pressure to the required combustion pressure.

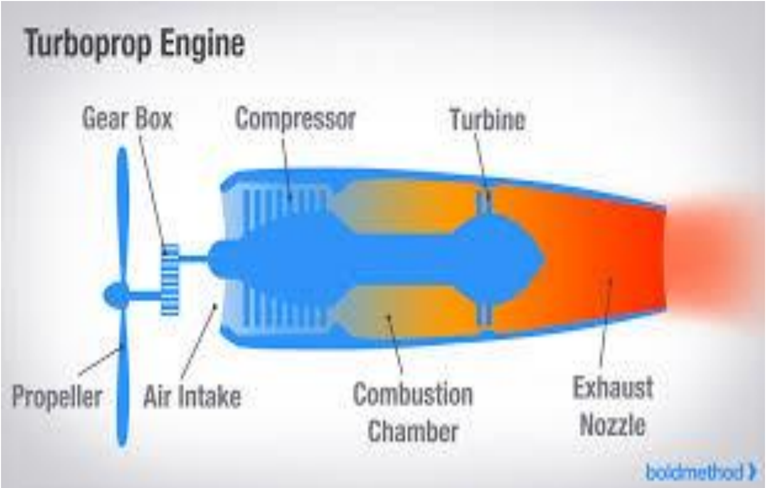
TURBOPROP ENGINE

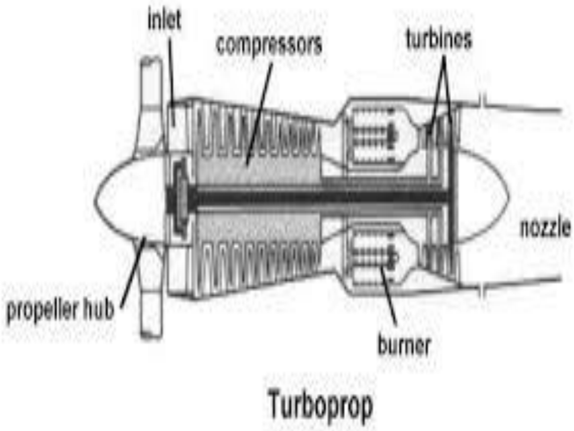
It is a known fact that an higher thrust per unit mass flow of fuel can be obtained by increasing the mass flow of air which results in better fuel economy. This fact is utilized in a turbo prop engine which is an intermediate between a pure jet engine and propeller engine. Turboprop engine attempts to increase the air flow by using a propeller driven by the turbine in addition to a small thrust produced by the exhaust nozzle.(Refer Yahya/Pandian for fig)

In this engine the turbine is designed so as to develop enough shaft power for driving the propeller which provides most of the propulsive thrust. It may be noted that the thrust produced due to jet action is quite small.

The engine consists of the following components:

1. a diffuser
2. a compressor
3. a combustion chamber(c.c)
4. a turbine
5. an exhaust nozzle
6. a reduction gear and
7. a propeller.





In the turbo prop engine, the turbine extracts much more power because the turbine is to provide power for both the compressor and the propeller. When all of this energy is extracted from the high temperature gases, there is still a little energy left for production jet thrust. Thus the turbo prop engine derives only a small portion(10 to 20 % depending upon the flight velocity) from the exhaust nozzle. Since the shaft speed of gas turbine engine is very much higher than that of a propeller, a reduction gear must be placed between the turbine shaft and the propeller, to enable the propeller to operate efficiently. Still, as the flight speed increases, the ratio of nozzle power to propeller power for maximum thrust tends to increase. It may be noted that the propulsive thrust developed is due to the following:

1. the propeller increases the air momentum, and
2. the overall engine – from diffuser to nozzle - provides an internal momentum increase.

The sum of these two thrusts is the total thrust developed by the engine.

The propeller produces its own thrust and thus the turboprop engine is essentially a two fluid stream engine. Turbo prop engines combine in them the high take-off thrust and good propeller efficiency of the propeller engines at speed lower than 800 km/hr and the small weight, lower frontal area, and reduced vibration and noise of the pure jet engine. Its operational range is between that of propeller engines and turbojets though it can operate in any speed up to 800 km/hr.

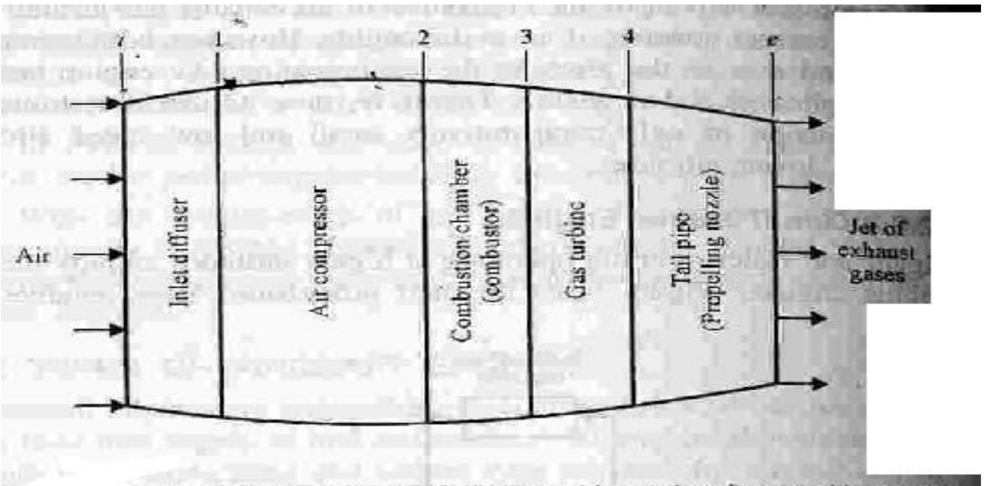
The power developed by the turbo prop remains almost same at high altitudes and high speeds as that under sea-level and take-off conditions because as speed increases ram effect also increases. The specific fuel consumption(sfc) increases with increase in speed and altitude. The thrust developed is high at take-off and reduces at increased speed.

TURBO JET ENGINE

The two pilot less air breathing engines described, viz., ram jet and pulse jet are simple in construction. However, their application is limited, and, to date, they have not been used very extensively. The most common type of air breathing engine apart from turboprop is the turbojet engine

This engine consists of the following components:

1. a diffuser
2. a mechanical compressor
3. a combustion chamber
4. a mechanical turbine and
5. an exhaust nozzle.



Layout of the principal components in an aircraft gas turbine engine

The function of the diffuser is to convert the kinetic energy of the entering air into a static pressure rise which is achieved by the ram effect. After this air enters the mechanical compressor.

The compressor used in a turbojet can be either **centrifugal type or axial flow type.** The use of a particular type of compressor gives the turbojet typical characteristics. The centrifugal compressor produces a high pressure ratio of about 4:1 to 5:1 in a single stage and usually a double-sided rotor is used to reduce the engine diameter. The turbojet using a centrifugal compressor are high durability, ease of manufacture and low cost, and good operation under adverse circumstances such as icing and when sand and small foreign particles are inhaled in the duct.

After the compressor air enters to the combustion chamber, the fuel nozzles feed fuel continuously and continuous combustion takes place at constant pressure. The high pressure, high temperature gases then enter the turbine, where they expand to provide enough power output from the turbine.

The turbine is directly connected to the compressor and all the power developed by the turbine is absorbed by the compressor and the auxiliaries. The main function of the turbine is to provide power, to drive the compressor. After the gases leave the turbine they expand further in the exhaust nozzle, and are ejected into the atmosphere with a velocity greater than the flight velocity thereby producing thrust for propulsion.

Current turbojet engines operate with compressor pressure ratios between 6 and 16, and with turbine inlet temperatures of the order of 1200 K. The corresponding speed of the exhaust jet when propelling an aircraft at 900 km/hr (250 m/s) is of the order of 500 m/s.

Like the ramjet engine, the turbo jet engine is a continuous flow engine. Here a compressor run by a turbine is used to provide additional pressure rise which is not available in a ramjet engine. Since this engine has a separate mechanical compressor, it is capable of operating even under static conditions. However, increase in flight velocity improves its performance because of the benefit of ram pressure achieved by the diffuser.

Because of the turbine material limitations, only a limited amount of fuel can be burnt in the combustion chamber. The exhaust products downstream of the turbine still contain a considerable amount of excess oxygen. Additional thrust augmentation can be achieved from the turbojet engine by providing an **afterburner** in which additional fuel can be burnt. Properly designed afterburner can greatly increase the temperature and hence the velocity of exhaust gases – providing **thrust augmentation.**

Of the air breathing engines, discussed so far only the turbojet and turboprop engines have been applied as the propulsion device for ***piloted aircraft***. Turbojet engine is eminently suited for propelling aircraft at speeds above 800 km/hr. As the flight speed is increased, the ram pressure increases rapidly and at supersonic speeds(mach no. = 3) the characteristics of the turbojet engine tend to merge with those of ramjet engine.

Basic thermodynamic cycle of a turbojet engine is **Joule or Brayton cycle**. The turbojet engine is almost a ***constant thrust engine***. The turbojet is most efficient when flown at high altitudes and at relatively high speeds. The operational range of turbojet engine is about 800 to 1100 km/hour and the s.f.c is about 1 – 1.5 kg/thrust hr at cruising speeds and are still greater at lower speeds. The altitude limit is about 10000 meters.

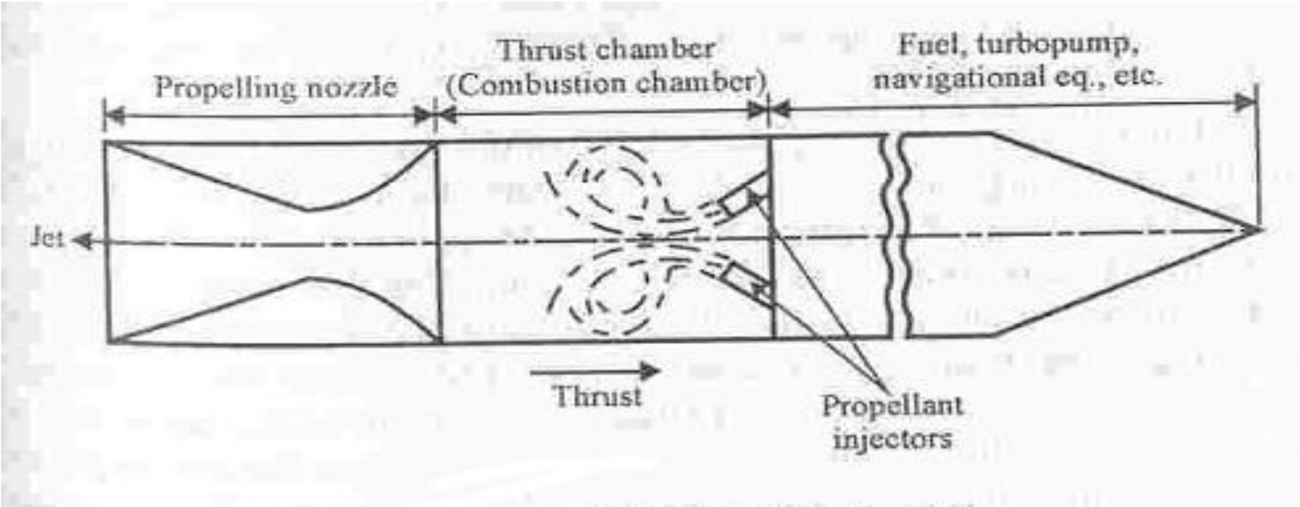
ROCKET PROPULSION

- The functioning of the air breathing propulsive devices is dependant of ambient air; they cannot operate in vacuum. Therefore, their operation and performance depend on the altitude. The thrust of air breathing engines is also dependant on flight speed since the ambient air is admitted at the front of the engine at this speed.
- Turbojet and ramjet engines can operate at supersonic speeds, the latter achieving a maximum speed of about Mach 4.
- For very high speeds in the **hypersonic** range, **rockets are employed** which **carry their own fuel and oxidizer** supply; therefore, their operation and performance do not depend on the ambient air and altitude. They can operate in vacuum and achieve any altitude. The **thrust of the rocket engines** is **independent of the flight speed.**

LIQUID PROPELLANT ROCKET

- A rocket engine mainly consists of a container or containers for propellants, combustion chamber (frequently called as a thrust chamber) and a propulsion or exhaust nozzle besides control and navigational equipment, payload etc., (**Payload**: For a **rocket**, the **payload** can be a satellite, space probe, or spacecraft carrying humans, animals, or cargo. For a ballistic missile, the **payload** is one or more warheads and related systems; the total weight of these systems is referred to as the throw-weight).
- The fuel and oxidizer (known as propellants) are introduced into the thrust chamber for chemical reaction (combustion) at high pressure and temperature.
- The products of combustion expand through the propelling nozzle to very high velocities. High flow rate of exhaust gases at high velocities gives very high value of thrust; this is due to a large change in the momentum flux as well as the difference between the nozzle exhaust pressure and the ambient pressure.

- Both liquid and solid propellants can be used in rocket engines. The larger and long range rockets are generally liquid propellant engines whereas the smaller ones are solid propellant type.
- In a hybrid type, a solid fuel is used with a liquid oxidizer or vice-versa.
- A monopropellant engine uses only one chemical (propellant) which dissociates after ignition.
- Conventionally, the **liquid propellant** rockets are known as **"engines"** whereas the **solid propellant type** are known as **"motors"**.
- On account of limited quantity of the propellants stored in the rockets, the exhaust jet velocities must be very high (about 3000 m/s) for given operation time and altitude. This requires very high pressures and temperatures of gases in the thrust chamber, their maximum values are over 200 bar and 4000 K respectively.



A liquid propellant rocket

Rocket engines or simply rockets can be classified in the following manner:

- ✓ **On the basis of source of energy employed:** chemical rockets, solar rockets, nuclear rockets, electrical rockets.
- ✓ **On the basis of propellants used:** liquid propellant rockets, solid propellant rockets, hybrid propellant rockets.
- ✓ **On the basis of application:** space rockets, military rockets (missiles), weather or sounding rockets, aircraft propulsion, turbo-rocket engines, ramjet rocket engines, booster rockets, sustainer rockets, retro rockets.
- ✓ **On the basis of number of stages:** single stage rockets, multi-stage rockets,
- ✓ **On the basis of size and range:** short range small rockets, long range large rockets.

You must also refer: GDJP UNIT 5 - 2 Marks Q & A - Word File for the definition of thrust, thrust power, propulsive and overall efficiencies, thrust augmentations in turbo jet engines, ram jet and pulse jet engines.

LIQUID PROPELLANT ROCKETS

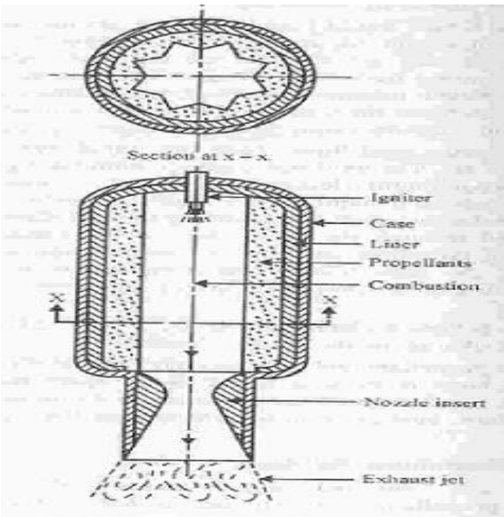
- ◎ Rocket engines in which liquid fuels and oxidizers are used are known as liquid propellant rocket engines; Refer the above figure.
- ◎ The propellants are fed into the thrust chamber (combustor or combustion chamber) from their containers for combustion or chemical reaction.
- ◎ For better mixing and efficient combustion, the fuel and the oxidizer are atomized through the injectors. The feed system regulates the optimum mixture ratio for a given set of propellants.
- ◎ **Commonly used liquid fuels:** liquid hydrogen, RFNA (Red Fuming Nitric Acid), liquid fluorine, WFNA (White Fuming Nitric Acid).
- ◎ Most of the liquid propellants are toxic and require very high combustion temperatures; this demands special materials and handling system.

- One of the propellants is circulated through the outer shell of the exhaust nozzle and the inner walls of the thrust chamber before injection from combustion.
- Besides cooling the high temperature components, it also achieves regenerative heating of the propellant giving an increase in the thermal efficiency.
- Ignition is achieved electrically or by a pilot flame obtained by burning a small charge in the thrust chamber.
- Besides the propellant tanks, thrust chamber and the exhaust nozzle which occupy a large proportion of the rocket space, there are several other things that a rocket engine carries such as the propellant pumping and control system, navigational equipment, auxiliary power unit and the payload.

SOLID PROPELLANT ROCKETS

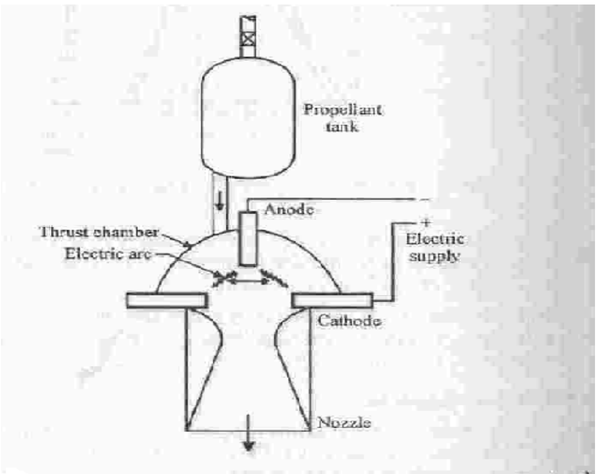
- Rockets which use solid fuels and solid propellants (oxidizers) are known as solid propellant rocket engines or motors.

- Solid fuel (plastic or resin material) and oxidizers (nitrates, perchlorates etc.) are mixed in a single propellant grain and packed into the steel shell or case of the rocket. The method of casting the fuel and the oxidizer mixture in fluid state into the rocket shell is widely employed.
- A cylindrical star-shaped cavity is provided for combustion in the centre along the axis of the shell.
- A liner provided between the case and the propellant protects the case from high temperatures developing inside the propellant layers.
- The igniter located at the top starts the combustion or the chemical reaction between the fuel and oxidizer. Once the flame front is established, combustion is self sustaining. The rate of burning for a given fuel-oxidizer mixture depends on the internal shape of the cast propellant shell besides the other factors.
- Excessive pressures which are a serious problem in solid propellant rockets can be kept lower by suitably choosing the fuel and oxidizer combinations.
- Solid propellant rockets are comparatively simpler and lighter, which are used in small sizes; however, recent developments have made it possible to design large size solid propellant rockets for space vehicles.



A solid propellant rocket

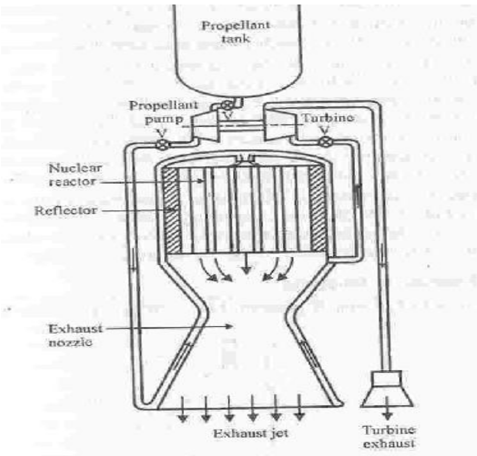
ELECTRICAL ROCKETS



Main components of an electric rocket engine (arc-heating type)

- Rockets in which thrust is produced by employing the electrical energy are called 'electrical rockets'.
- In nuclear-electric rocket engines, nuclear energy is converted into electrical energy.
- In the solar-electric rocket engines, electrical energy is obtained from solar radiation.
- Electrical energy is used to generate thrust in the rockets by different methods. In an electro-thermal rocket (shown in the figure), the propellant is heated by an electric arc struck between central anode and an annular cathode housed in the thrust chamber. Very high temperatures of the propellant in the thrust chamber can be obtained in this type.
- In MHD (Magneto Hydro Dynamic) rocket propulsion, the propellant at very high temperatures is accelerated by a magnetic field.
- In an electrostatic or ion propulsion rocket engine, the propellant is ionized. The ions are then accelerated by an electric field. Heavy metals such as mercury and caesium are suitable for generating an "ion beam" which gives maximum specific impulse of the order of 20,000 secs.

NUCLEAR ROCKETS



Main components of a nuclear rocket engine

- In a nuclear rocket, nuclear energy is used to heat the propellant or the working fluid, to obtain high stagnation temperature in the thrust chamber.
- Liquid hydrogen is widely used in such rockets.
- A nuclear reactor takes the place of the combustor in the thrust chamber.
- The reactor with nuclear fission of the uranium fuel consists of the conventional components such as the fuel, moderator, reflector, shield, control rods etc.
- Liquid hydrogen is pumped from its tank to the reactor through the shell jacketing the exhaust nozzle.
- A small portion of the heated gas (propellant) is tapped out for driving the turbine which in turn drives the propellant pump.
- The turbine exhaust is also expanded in a small nozzle providing a fraction of the total thrust.
- A solid-core nuclear fission reactor employing hydrogen as propellant can give a specific impulse of about 1000 seconds at thrust chamber temperature of little over 2000℃.
- Nuclear rockets can employ a Rankine power cycle for generating electricity which in turn can be employed to heat the propellant.

ROCKET APPLICATIONS: Rockets have several applications in military and peace time operations such as space exploration, weather prediction, communication, aircraft propulsion and scientific investigations.

SPACE FLIGHTS: Operation of the modified V-2 missile by the US in 1949 can be regarded as the first attempt towards space flights; the missile entered the 'ionosphere' reaching an altitude of 400 km.

On account of very long distances involved in, space travel rockets have several stages (upto ten) which are successively dropped after their operations. This eliminates the problem of carrying the structures of the "spent up" rockets over long distances. Besides this each individual rocket must have large proportion of the propellant weight compared to the total weight.

Besides putting weather and communication satellites into various orbits, rockets have been taking spacecrafts to orbits around other planets for data collection and handling, if possible. The historic moon landing took place in September 1959.

ORBITAL VELOCITY OF THE EARTH-SATELLITE: (**Earth-Satellite:** A space vehicle orbiting around earth; **Orbital Velocity:** velocity of the satellite to maintain it in an orbit) Rockets are used to launch space stations or satellites into earth-orbit; they revolve round the earth at an altitude beyond the earth's atmosphere where the drag force is absent. Therefore, such satellites can remain in the orbit forever without expenditure of energy.

If the orbital velocity of an earth-satellite is equal to the angular velocity of the earth, it has zero velocity relative to the earth. Such a satellite is known as the '**stationary satellite**' which can be usefully employed for several purposes.

Earth satellites are used for several scientific and technological programs such as observation of space phenomena, study of weather, military missions and communication.

The orbital velocity of the satellite can be found by equating the centrifugal force acting on it to the gravitational pull. The velocity of the satellite u_{orb} at a radius R from the centre of the earth.

Therefore,

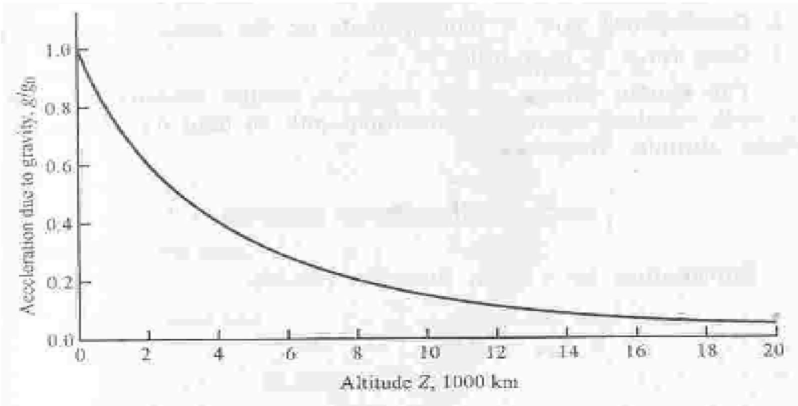
$$m.\frac{u_{orb}^2}{R} = \text{mg}$$

$$u_{orb} = \sqrt{Rg} \text{ ----------------------- Eqn.1}$$

The variation of the gravitational force along the radius $R = R_o + Z$ is given by

$g = \left(\frac{R_o}{R}\right)^2 . g_o = \left(\frac{R_o}{R_0+Z}\right)^2 . g_o$------------------- Eqn. 2

Below figure shows the plot of the acceleration due to gravity (in the dimensionless form g/g_o) against the altitude. It is seen that there is only little decrease in the gravitational pull with altitude even in the upper regions of the atmosphere such as the ionosphere and the exosphere.



Variation of acceleration due to gravity with altitude

Substituting from Equation 2 into Equation 1, yields the expression for the orbital velocity in terms of the earth's radius at mean sea level, and altitude.

$$u_{orb} = \sqrt{\frac{g_o R_o^2}{R}} = R_o \sqrt{\frac{g_o}{R_o + Z}}$$

ESCAPE VELOCITY: A rocket vehicle system destined to travel in the outer space to other planets or their orbits must escape from the earth's gravitational field. The thrust should be able to accelerate the rocket to a velocity at which its kinetic energy equals the work required to overcome gravitational force.

The escape velocity at the earth's surface is about 11.2 km/sec. If the rocket-vehicle systems are accelerated to this velocity within the earth's atmosphere, the frictional heating will be excessive leading to unsurmountable problems. Therefore, a practical and safer method is first to accelerate the rocket at lower velocities through the atmosphere; the rocket is finally accelerated to the escape velocity at a much higher altitude where the atmospheric drag and heating are absent.

An expression for the escape velocity of the rocket at altitude Z is derived under the following assumptions:

1. Vertical flight of the rocket 2. Gravitational pull of the other planets on the rocket is negligible 3. Drag force is negligible.

The kinetic energy of the rocket at escape velocity is equal to the work required against the gravitational pull to take the rocket to an infinite altitude. Therefore,

$$u_{esc} = \sqrt{2}.u_{orb}$$

TWO MARKS

1. What is the difference between shaft propulsion and jet propulsion?
Performance of the air craft propulsion cycle depends upon

- Forward speed of the air craft
- Altitude at which it is flying.

The above two variables do not enter into performance calculations of shaft power cycles. Also in jet propulsion, power output is produced, wholly or partially, as a result of expansion of gas in a propelling nozzle; wholly in a turbojet engine and partially in a turboprop engine.

2. List the different types of jet engines.
a. Reciprocating or propeller engines.
b. Gas turbine engines

- Ram jet engine
- Pulse jet engine
- Turbo prop engine
- Turbo prop engine

3. What is thrust augmentation?
The poor take-off characteristic of the turbojet engine can be improved by augmenting the thrust.

The thrust from a turbojet is given by $F = m_a [(1+f) c_j - c_i]$
Where m_a = mass of air
$f = m_f / m_a$ (fuel-air ratio)
c_i= velocity of air at inlet
c_j= velocity of air at exit

- Also, c_j is a function of maximum temperature in the cycle.
- Higher the maximum temperature, higher is the value of c_j.
- Another method of improving the trust is to increase the mass flow rate.
- Improved thrust results in shorter take-off distance, high climb rate and good maneuverability at high altitudes.

<u>The thrust augmentation can be effected by the following methods:</u>
1. by using after burner
2. by injecting refrigerants, between inlet & exit sections of air compressor and thereby increasing the mass flow rate.

3. by bleed burn cycle - i.e., by bleeding off excess air.
4. What is specific impulse? or specific thrust?

The thrust per kg of air flow is known as specific thrust or specific impulse.

$$I_s = \frac{F}{\dot{w}}$$

5. Define thrust and propulsive efficiency for a jet propulsion system.
The force which propels the aircraft at a given speed is called the thrust or propulsive force.

$$Propulsive\ efficiency = \frac{Propulisive\ or\ thrust\ power}{Power\ output\ of\ the\ engine} = \frac{2\sigma}{1+\sigma}$$

Where,

$$\sigma = \frac{effective\ jet\ velocity}{flight\ speed} = \frac{c_j}{u}$$

6. Give the difference between ram jet & pulse jet engine.

Ram Jet	Pulse Jet
Unsuitable for Subsonic speeds.	Can work at any flight speed.
Thrust is steady.	Thrust is not steady; it is produced in short pulses.
No upper limit to the flight speed of the ramjet.	Limited altitude and flight speed. Maximum flight speed is 1000 kmph.
It is not possible to start a ram jet without an external launching device.	Does not need a device for initial propulsion.

7. What is mono propellant? Give examples.
A liquid propellant which contains both fuel and oxidizer in a single chemical is known as 'mono propellant'.
(Eg.) Hydrogen Peroxide (H_2O_2), Hydrazine (N_2H_4), Nitro Glycerin [$C_3H_5(ONO_2)_3$], Nitro Methane [CH_3NO_2].

8. Define specific impulse for a rocket engine.
Specific impulse of a rocket engine is the thrust per unit weight flow rate of the propellant.

$I_S = \frac{F}{\dot{W}_p}$ [units: $\frac{N}{N/S}$ = seconds]

Unit for Specific Impulse: seconds.

9. What is bipropellant? Give examples.
A number of fuel and oxidizer combinations can be found to fit a given application.

Oxidizer	Fuel
H_2O_2	Gasoline
Liquid Oxygen	Ethanol

Nitrogen Tetroxide Nitric Acid	Hydrazine UDMH(Unsymmetical Dimethyl Hydrazine)

10. Define propulsive power and overall efficiency for jet propulsion system. η

Propulsive Power: It is defined as the power required to propel the air craft.

$$Overall\ Efficiency = \eta_0 = \frac{Propulsive\ Power}{Power\ input\ to\ the\ engine\ through\ fuel}$$

Also,

$$\eta_0 = \eta_p . \eta_{th}$$

11. Define thrust for a rocket engine.

The force that propels the rocket at a given velocity is known as the thrust. This is produced due to the change in the momentum flux of the outgoing gases as well as the difference between the nozzle exit pressure and the ambient pressure.

12. Give the different between turbojet and ramjet engine.

Turbo Jet	Ram Jet
Turbine & compressor are present.	Turbine & compressor are not provided.
Capable of operating under static conditions since the engine has a separate mechanical compression	Cannot operate under static conditions.
Because of turbine material limitations, only a limited amount of fuel can be burnt in the combustion chamber.	No such limitation as there is no turbine.
Requires Maintenance.	Maintenance not required.
It has moving parts like compressor and turbine.	Very simple in construction and does not have any moving part.
Maximum temperature is about 900 degree Celsius and thrust produced is lesser compared to ram jet engine.	Maximum temperature allowed is about 2000 degree Celsius. This temperature allows a greater thrust to be obtained by burning the fuel at A/F ratio of about 13:1.

13. Define the characteristic velocity of a rocket (v*).

Rocket performance is also frequently expressed in terms of characteristic velocity which is defined as

$$v^* = \frac{Effective\ jet\ velocity}{Thrust\ coefficient}$$

14. What are the liquid propellants commonly used in rockets?
Mono and bi propellants are the liquid propellants that are commonly used in rockets.

15. What is the difference between turbojet and turboprop engine?

Turbojet	Turbo prop
Reduction gear & propeller are not present.	Reduction & propeller are provided propulsive thrust.
Higher flight speeds than engine propeller aircrafts.	Flight speed is comparatively lesser.
Cost is less.	Cost is more because it requires a reduction gear.
Low take-off thrust & hence poor starting characteristics.	Have higher thrust at take off and better fuel economy.

16. What is turbo prop unit?
The turbo prop engine consists of the following components: Diffuser, compressor, combustion chamber, turbine, an exhaust nozzle, reduction gear and a propeller.

17. What is the A/F ratio in a gas turbine and a reciprocating engine?

Gas turbine – 50:1 to 150:1

Reciprocating Engine – 10:1 to 15:1

18. What is the type of compressor used in turbo jet? Why?
Centrifugal type - produces high per ratio of about.4:1 to 5:1 in a single stage.
Turbojet using centrifugal compressor has short & sturdy appearance.
High durability, ease of manufacture and low cost.
Axial flow compressor - More efficient than the centrifugal type.
Multistage axial flow compressor can develop pressure ratio as high as 6:1 and above.
The air handled by this is more than that handled by a centrifugal computer of same diameter.
6-8% less SFC (Specific Fuel Consumption) than the centrifugal type.

19. What is the difference between jet propulsion and rocket propulsion?

Jet propulsion engine uses atmospheric air for its combustions.
In rocket propulsion, no atmospheric air is used. Fuel and oxidizer is carried with the system itself. The fuel-oxidant mixture is called the propellant. The oxidant in the propellant is used for generating the thermal jet.

20. Give the important requirements of rocket engine Fuels.

- Energy released during contribution should be high.
- High density propellants as they require smaller tanks.

- Lower freezing point rules out any possibility of freezing of the propellants at high altitudes.
- Must be non-corrosive.
- Chemically stable. Properties should not deteriorate with time. They should not absorb moisture.
- Low vapor pressure & viscosity reduces the power required for pumping.
- Should not be poisonous and hazardous.
- Cheap and abundantly available.

21. Define thrust power and propulsive power.

Propulsive power is the net increase in the kinetic energy of the workout fluid between exit.

Propulsive efficiency is the measure of the effectiveness with which the kinetic energy imparted to the fluid is transferred into useful work.

The useful work done by the system is the product of the thrust and flight velocity i.e., F.C_i, which is called the thrust power.

Leaving losses = Difference between the thrust and the propulsive power.

22. What is after burning in turbo jet engines?

Burning of the additional fuel in tail pipe between turbine exhaust section and entrance of the exhaust section nozzle. This method of thrust augmentation increases the jet velocity and is known as after burning. The device used is called the after burner.

23. Name three early rockets.

a. JATO - Jet Assisted Take Off

b. RATO - Rocket Assisted Take Off.

c. Sputniks

d. Satellite Explorer I

24. What are cryogenic propellants?

Cryogenic propellants are in the gaseous state at normal temperatures and require extremely low temperature to maintain them in liquid state; they release much higher heat energy in the thrust chamber. Some commonly used cryogenic propellants are liquid oxygen, hydrogen, fluorine and ammonia.

25. What are the different methods of ignition employed in liquid propellant rockets?

- Parallel stream type
- Impinging stream type
- Spray injection type

26. Give important applications of rocket propulsion.

1. Military operation
2. Space exploration
3. Weather prediction
4. Communications

5. Air craft propulsion
6. Scientific investigation

27. ICBM - Inter Continental Ballistic Missile
IRBM - Intermediate Range
Ballistic Missile
GAM - Guided Air Missile
SAM - Surface to Air Missile
SSM - Surface to Surface Missile
AAM - Air to Air Missile
RFNA - Red Fuming Nitric acid
Booster Rocket Stage - High thrust first stage powerful rocket.

28. What are the main components of a gas turbine engine used for turbojets aircraft?
Diffuser, mechanical compressor, a combustion chamber, mechanical turbine & exhaust nozzle.

29. Compare solid and liquid propellant rockets.

Solid Propellant Rockets	Liquid Propellant Rockets
Solid fuels and oxidizers are used.	Liquid fuels oxidizers are used.
Comparatively simple and lighter.	Heavier and complicated compared to solid propellant rockets.
Space required is less due to high density of solid propellants.	Larger storage space is required due to the lower density of the liquid propellants.
Can be mass produced in a short time.	Cannot be mass produced in a short time for military applications.
Does not require feed system.	Require feed system to feed liquid propellants.
Less frequent servicing problem.	Maintenance required is more.
Vibration problems due to turbo pump liquid flow system are absent.	Vibrations problems are present as turbo pump feed system is involved.
Difficult to control combustion.	Combustion can be controlled.
Lower values of specific impulse.	High values of specific impair.

30. What are the various propulsive devices for aircraft?
1. I.C Engines
2. Gas Turbine Engines
    a. Ram Jet Engine
    b. Pulse Jet Engine
a & b are of Pilotless operation.
    c. Turbo Jet Engine
    d. Turbo Prop Engine
c & d are of Piloted operation.

31. Why is the rocket called as a non-breathing engine?
It does not take atmosphere air for combustions. It carries its own oxidizer for combustion. Hence it is called non-breathing engine.

32. How does aircraft lift off the ground?
The aerodynamic resistance to a body moving through the air is called the drag D. The force perpendicular to the motion is called the lift L. An aerofoil (the wings, tail plan and fin of an aircraft) is a body so shaped that the lift may be greater than the drag and because of this lift force, aircraft lifts off the ground.

33. Define the principle of ram jet engine.
A deceleration of the air from Mach 3 (supersonic) at diffuses inlet of a ram jet engine to Mach 0.3 (subsonic) in cornbustion chamber would cause a pressure ratio of more than 30. Due to shock and other losses unavoidable at such velocities, all of this pressure rise (of pressure ratio 30) is not available; still whatever we get is more than sufficient for raising the air pressure to the required combustion pressure. This principle of ram pressure rise is used in the ram jet engines. The ram pressure rise can be achieved in diffusers.

Both supersonic and subsonic diffusers (present at the entry of the ram jet engine) convert the kinetic energy of the entering air into pressure rise. This energy transformation is called **ram effect** and the pressure rise is called the **ram pressure**.

34. State the effect of forward speed of jet engine on its propulsive efficiency.
Propulsive efficiency is maximum when the flight speed is close to the jet velocity.

35. Name the different stages in PSLV rocket engine.
PSLV - Polar Satellite Launch Vehicle.
Totally there are four stages using solid and liquid propellant systems alternately.

I stage	-	Solid propellant booster stage.
II stage	-	Liquid propellant stage generates a maximum thrust of 725 KN.
III stage	-	With high performance solid motor.
IV stage	-	Use twin pressure fed liquid fuelled engine.

36. Name the propellants used in PSLV.
1.HTPB based propellants (HTPB- Hydroxyl Terminated Poly Butadiene (solid propellant).
2.UDMH (Unsymmetrical Di Methyl Hydrazine) as fuel and N204 (Nitrogen Tetroxide) as oxidizer (liquid propellant).
3. Mono Methyl Hydrazine (MMH) as fuel and mixed oxides of Nitrogen as oxidizer (liquid propellant).

37. Brief description about turbo fan engine (By pass engine).
Turbo fan is a combination of the turbo prop and the turbo jet engines combining the advantages of both. A turbo fan is quieter and more fuel efficient then a turbo jet and provides greater thrust. Turbo fan engine is also called as a By-pass engine.

38. Define by-pass ratio for a turbofan engine.
By-pass ratio, B:

$$B = \frac{Mass\ flow\ rate\ of\ cold\ air\ stream}{Mass\ flow\ rate\ of\ hot\ air\ stream}$$

For a modern turbo fan, $B \geq 8.0$

39. Which part of the rocket engine is having high heat intensity?
Combustion chamber

40. What are the effects of forward speed and altitude in the case of jet propulsion?
Ram effect or ram: The increase of pressure and temperature due to aircraft speed is known as ram effect or ram. Because of aircraft forward speed, the net specific thrust reduces.
The effect of altitude on a turbo jet is by virtue of reduction of ambient pressure and temperature.