UNIT – I
MEASUREMENT AND INSTRUMENTATION – SIC 1203
UNIT 1
1. BASIC MEASUREMENTS


1.1 METHODS OF MEASUREMENT

Measurement is the assignment of a number to a physical quantity or characteristic of an object or event, which can be compared with other objects or events.

(i) Direct method of measurement.

In this method the value of a quantity is obtained directly by comparing the unknown with the standard. Direct methods are common for the measurement of physical quantities such as length, mass and time. It involves no mathematical calculations to arrive at the results, for example, measurement of length by a graduated scale. The method is not very accurate because it depends on human insensitiveness in making judgment.

(ii) Indirect method of measurement.

In this method several parameters (to which the quantity to be measured is linked with) are measured directly and then the value is determined by mathematical relationship. For example, measurement of density by measuring mass and geometrical dimensions.

1.2 MEASUREMENT SYSTEM

Measurement system, any of the systems used in the process of associating numbers with physical quantities and phenomena. Although the concept of weights and measures today includes such factors as temperature, luminosity, pressure, and electric current, it once consisted of only four basic measurements: mass (weight), distance or length, area, and
Basic to the whole idea of weights and measures are the concepts of uniformity, units, and standards. Uniformity, the essence of any system of weights and measures, requires accurate, reliable standards of mass and length and agreed-on units. A unit is the name of a quantity, such as kilogram or pound. A standard is the physical embodiment of a unit, such as the platinum-iridium cylinder kept by the International Bureau of Weights and Measures at Paris as the standard kilogram. Two types of measurement systems are distinguished historically: an evolutionary system, such as the British Imperial, which grew more or less haphazardly out of custom, and a planned system, such as the International System of Units (SI), in universal use by the world’s scientific community and by most nations.

The International System of Units (French: Système international d'unités, SI) is the modern form of the metric system, and is the most widely used system of measurement. It comprises a coherent system of units of measurement built on seven base units. It defines twenty-two named units, and includes many more unnamed coherent derived units. The system also establishes a set of twenty prefixes to the unit names and unit symbols that may be used when specifying multiples and fractions of the units. The system was published in 1960 as the result of an initiative that began in 1948. It is based on the metre-kilogram-second system of units (MKS) rather than any variant of the centimetre-gram-second system (CGS).

Basic classification of measuring instruments:

1. Mechanical Instruments:– They are very reliable for static and stable conditions. The disadvantage is they are unable to respond rapidly to measurement of dynamic and transient conditions.
2. Electrical Instruments:– Electrical methods of indicating the output of detectors are more rapid than mechanical methods. The electrical system normally depends upon a mechanical meter movement as indicating device.
3. Electronic Instruments:– These instruments have very fast response.
example a cathode ray oscilloscope (CRO) is capable to follow dynamic and transient changes of the order of few nano seconds (10-9 sec).

**Absolute instruments or Primary Instruments**

These instruments gives the magnitude of quantity under measurement in terms of physical constants of the instrument e.g. Tangent Galvanometer. These instruments do not require comparison with any other standard instrument. These instruments give the value of the electrical quantity in terms of absolute quantities of the instruments and their deflections. In this type of instruments no calibration or comparison with other instruments is necessary. They are generally not used in laboratories and are seldom used in practice by electricians and engineers. They are mostly used as means of standard measurements and are maintained in national laboratories and similar institutions. Examples of absolute instruments are: Tangent galvanometer, Raleigh current balance, Absolute electrometer

**Secondary instruments**

These instruments are so constructed that the quantity being measured can only be determined by the output indicated by the instrument. These instruments are calibrated by comparison with an absolute instrument or another secondary instrument, which has already been calibrated against an absolute instrument. Working with absolute instruments for routine work is time consuming since every time a measurement is made, it takes a lot of time to compute the magnitude of quantity under measurement. Therefore secondary instruments are most commonly used.

- They are direct reading instruments. The quantity to be measured by these instruments can be determined from the deflection of the instruments.
- They are often calibrated by comparing them with either some absolute instruments or with those which have already been calibrated.
- The deflections obtained with secondary instruments will be meaningless until it is not calibrated.
- These instruments are used in general for all laboratory purposes.
- Some of the very widely used secondary instruments are: ammeters, voltmeter, wattmeter, energy meter (watt-hour meter), ampere-hour meters etc.
Classification of Secondary Instruments:

Classification based on the way they present the results of measurements  
**Deflection type:**
Deflection of the instrument provides a basis for determining the quantity under measurement. The measured quantity produces some physical effect which deflects or produces a mechanical displacement of the moving system of the instrument.

**Null Type:** In a null type instrument, a zero or null indication leads to determination of the magnitude of measured quantity.

Classification based on the various effects of electric current (or voltage) upon which their operation depend.

They are:

- **Magnetic effect:** Used in ammeters, voltmeters, watt-meters, integrating meters etc.
- **Heating effect:** Used in ammeters and voltmeters.
- **Chemical effect:** Used in dc ampere hour meters.
- **Electrostatic effect:** Used in voltmeters.
- **Electromagnetic induction effect:** Used in ac ammeters, voltmeters, watt meters and integrating meters.

Generally the magnetic effect and the electromagnetic induction effect are utilized for the construction of the commercial instruments. Some of the instruments are also named based on the above effect such as electrostatic voltmeter, induction instruments, etc.

Classification based on the Nature of their Operations

We have the following instruments.

1. **Indicating instruments**

   Indicating instruments indicate, generally the quantity to be measured by means of a pointer which moves on a scale. Examples are ammeter, voltmeter, wattmeter etc.

2. **Recording instruments**

   These instruments record continuously the variation of any electrical quantity with respect to time. In principle, these are indicating instruments but so arranged that a permanent continuous record
of the indication is made on a chart or dial. The recording is generally made by a pen on a graph paper which is rotated on a dice or drum at a uniform speed. The amount of the quantity at any time (instant) may be read from the traced chart. Any variation in the quantity with time is recorded by these instruments. Any electrical quantity like current, voltage, power etc., (which may be measured lay the indicating instruments) may be arranged to be recorded by a suitable recording mechanism.

3. Integrating instruments:

These instruments record the consumption of the total quantity of electricity, energy etc., during a particular period of time. That is, these instruments totalize events over a specified period of time. No indication of the rate or variation or the amount at a particular instant are available from them. Some widely used integrating instruments are: Ampere-hour meter: kilowatthour (kWh) meter, kilovolt-ampere-hour (kVARh) meter.

Classification based on the Kind of Current that can be Measurand.

Under this heading, we have:

• Direct current (dc) instruments

• Alternating current (ac) instruments

• Both direct current and alternating current instruments (dc/ac instruments).

Classification based on the method used.

Under this category, we have:

Direct measuring instruments: These instruments converts the energy of the measured quantity directly into energy that actuates the instrument and the value of the unknown quantity is measured or displayed or recorded directly. These instruments are most widely used in engineering practice because they are simple and inexpensive. Also, time involved in the measurement is shortest. Examples are Ammeter, Voltmeter, Watt meter etc.

Comparison instruments: These instruments measure the unknown quantity by comparison with a standard. Examples are dc and ac bridges and potentiometers. They are used when a higher accuracy of measurements is desired.
1.4 FUNCTIONAL ELEMENTS OF MEASUREMENT SYSTEM

A systematic organization and analysis are more important for measurement systems. The whole operation system can be described in terms of functional elements. The functional elements of generalized measurement system are shown in figure 1.

Fig 1: Functional elements of generalized measurement system

Most of the measurement system consists of following functional elements.

1. Primary sensing element
2. Variable conversion element
3. Variable manipulation element
4. Data transmission element
5. Data storage and playback element
6. Data presentation element

1. Primary Sensing Element

The quantity under measurement makes its first contact with primary sensing element of measurement system. The quantity is first sensed or detected by primary sensor. Then detected physical quantity signal is converted into an electrical signal by a transducer.

Transducer is defined as a device which converts a physical quantity into an electrical quantity. Sensor is act as primary element of transducer. In many cases the physical quantity
is directly converted into an electrical quantity by a transducer. So the first stage of a measurement system is known as a detector transducer stage.

Example, Pressure transducer with pressure sensor, Temperature sensor etc.,

2. Variable Conversion Element

The output of primary sensing element is electrical signal of any form like a voltage, a frequency or some other electrical parameter. Sometime this output not suitable for next level of system. So it is necessary to convert the output some other suitable form while maintaining the original signal to perform the desired function the system.

For example the output primary sensing element is in analog form of signal and next stage of system accepts only in digital form of signal. So, we have to convert analog signal into digital form using an A/D converter. Here A/D converter is act as variable conversion element.

3. Variable Manipulation Element

The function of variable manipulation element is to manipulate the signal offered but original nature of signal is maintained in same state. Here manipulation means only change in the numerical value of signal.

Examples,
1. Voltage amplifier is act as variable manipulation element. Voltage amplifier accepts a small voltage signal as input and produces the voltage with greater magnitude. Here numerical value of voltage magnitude is increased.
2. Attenuator acts as variable manipulation element. It accepts a high voltage signal and produces the voltage or power with lower magnitude. Here numerical value of voltage magnitude is decreased.
   - Linear process manipulation elements: Amplification, attenuation, integration, differentiation, addition and subtraction etc.,
   - Nonlinear process manipulation elements: Modulation, detection, sampling, filtering, chopping and clipping etc.,

All these elements are performed on the signal to bring it to desired level to be accepted by the next stage of measurement system. This process of conversion is called signal
conditioning. The combination of variable conversion and variable manipulation elements are called as Signal Conditioning Element.

4. **Data Transmission Element**

The elements of measurement system are actually physically separated; it becomes necessary to transmit the data from one to another. The element which is performs this function is called as data transmission element.

Example, Control signals are transmitted from earth station to Space-crafts by a telemetry system using radio signals. Here telemetry system is act as data transmission element.

The combination of Signal conditioning and transmission element is known as Intermediate Stage of measurement system.

5. **Data storage and playback element**

Some applications requires a separate data storage and playback function for easily rebuild the stored data based on the command. The data storage is made in the form of pen/ink and digital recording. Examples, magnetic tape recorder/ reproducer, X-Y recorder, X-t recorder, Optical Disc recording etc.,

6. **Data presentation Element**

The function of this element in the measurement system is to communicate the information about the measured physical quantity to human observer or to present it in an understandable form for monitoring, control and analysis purposes. Visual display devices are required for monitoring of measured data. These devices may be analog or digital instruments like ammeter, voltmeter, camera, CRT, printers, analog and digital computers. Computers are used for control and analysis of measured data of measurement system. This Final stage of measurement system is known as Terminating stage.

1.41 **EXAMPLE OF GENERALIZED MEASUREMENT SYSTEM**

**Bourdon Tube Pressure Gauge:**

The simple pressure measurement system using bourdon tube pressure gauge is shown in figure 2. The detail functional elements of this pressure measurement system is given below.

- Primary sensing element and : Pressure Sensed
- Variable conversion element : Bourdon Tube
In this measurement system, bourdon tube is act as primary sensing and variable conversion element. The input pressure is sensed and converted into small displacement by a bourdon tube. On account of input pressure the closed end of the tube is displaced. Because of this pressure in converted into small displacement. The closed end of bourdon tube is connected through mechanical linkage to a gearing arrangement.

The small displacement signal can be amplified by gearing arrangement and transmitted by mechanical linkages and finally it makes the pointer to rotate on a large angle of scale. If it is calibrated with known input pressure, gives the measurement of the pressure signal applied to the bourdon tube in measurand.

**Fig. 2 Bourdon tube pressure gauge**

![Diagram of Bourdon tube pressure gauge](image)
2. CHARACTERISTICS OF MEASURING INSTRUMENTS

These performance characteristics of an instrument are very important in their selection.

- Static Characteristics: Static characteristics of an instrument are considered for instruments which are used to measure an unvarying process condition. Performance criteria based upon static relations represent the static Characteristics. (The static characteristics are the value or performance given after the steady state condition has reached).

- Dynamic Characteristics: Dynamic characteristics of an instrument are considered for instruments which are used to measure a varying process condition. Performance criteria based upon dynamic relations represent the dynamic Characteristics.

2.1 STATIC CHARACTERISTICS

1) Accuracy

Accuracy is defined as the degree of closeness with which an instrument reading approaches to the true value of the quantity being measured. It determines the closeness to true value of instrument reading. Accuracy is represented by percentage of full scale reading or in terms of inaccuracy or in terms of error value. Example, Accuracy of temperature measuring instrument might be specified by ±3°C. This accuracy means the temperature reading might be within + or -3°C deviation from the true value. Accuracy of an instrument is specified by ±5% for the range of 0 to 200°C in the temperature scale means the reading might be within + or -10°C of the true reading.

2) Precision

Precision is the degree of repeatability of a series of the measurement. Precision is measures of the degree of closeness of agreement within a group of measurements are repeatedly made under the prescribed condition. Precision is used in measurements to describe the stability or reliability or the reproducibility of results.
Table1: Comparison between accuracy and precision

<table>
<thead>
<tr>
<th>Accuracy</th>
<th>Precision</th>
</tr>
</thead>
<tbody>
<tr>
<td>It refers to degree of closeness of the</td>
<td>It refers to the degree of agreement among</td>
</tr>
<tr>
<td>measured value to the true value</td>
<td>group of readings</td>
</tr>
<tr>
<td>Accuracy gives the maximum error that is</td>
<td>Precision of a measuring system gives</td>
</tr>
<tr>
<td>maximum departure of the final result from its</td>
<td>its capability to reproduce a certain</td>
</tr>
<tr>
<td>true value</td>
<td>reading with a given accuracy</td>
</tr>
</tbody>
</table>

3) **Bias**

Bias is a quantitative term describing the difference between the average of measured readings made on the same instrument and its true value (It is a characteristic of measuring instruments to give indications of the value of a measured quantity for which the average value differs from true value).

4) **Sensitivity**

Sensitivity is defined as the ratio of change in output signal (response) to the change in input signal (measurand). It is the relationship indicating how much output changes when input changes.

\[
\text{Sensitivity} = \frac{\text{change in output}}{\text{change in input}}
\]

\[
\text{Sensitivity} = \frac{\Delta q_o}{\Delta q_i}
\]

If the sensitivity is constant then the system is said to be linear system. If the sensitivity is variable then the system is said to be non linear system.
Fig. 3: Definition of sensitivity for (a) Linear and (b) Non linear instrument

When the calibration curve is linear as in figure 3a the sensitivity of the instrument can be defined as in slope of the calibration curve. In this case sensitivity is constant over the entire range of instrument. If the curve is not normally straight line or nonlinear instrument sensitivity varies with the input or varies from on range to another as in figure 3b.

4) Linearity

Linearity is the best characteristics of an instrument or measurement system. Linearity of the instrument refers to the output is linearly or directly proportional to input over the entire range of instrument. So the degree of linear (straight line) relationship between the output to input is called as linearity of an instrument.

Fig.4: Representation of Linearity and Non-Linearity of an Instrument

Nonlinearity: The maximum difference or deviation of output curve from the Specified idealized straight line as shown in figure 4. Independent nonlinearity may be defined as
5) Resolution

Resolution or Discrimination is the smallest change in the input value that is required to cause an appreciable change in the output. (The smallest increment in input or input change which can be detected by an instrument is called as resolution or discrimination)

6) Hysteresis

Hysteresis is Non-coincidence of loading and unloading curves on output. Hysteresis effect shows up in any physical, chemical or electrical phenomenon. When input increases, output also increases and calibration curve can be drawn. If input is decreases from maximum value and output also decreases but does not follow the same curve, then there is a residual output when input is zero. This phenomenon is called Hysteresis. The difference between increasing change and decreasing change of output values is known as hysteresis error as shown in figure 5.(The different outputs from the same value of quantity being measured are reached by a continuously increasing change or a continuously decreasing change)

![Fig. 5: Hysteresis Error of an instrument](image)
7) Dead Zone

Dead zone or dead band is defined as the largest change of input quantity for which there is no output the instrument due the factors such as friction, backlash and hysteresis within the system. (The region upto which the instrument does not respond for an input change is called dead zone). Dead time is the time required by an instrument to begin to respond to change in input quantity.

8) Backlash

The maximum distance through which one part of the instrument is moved without disturbing the other part is called as backlash. (Backlash may be defined as the maximum distance or angle through which any part of the instrument can be moved without causing any motion of next part of the system)

![Fig. 6: Threshold because of backlash](image)

Reasons for the presence of backlash in an instrument include allowing for lubrication, manufacturing errors, deflection under load, and thermal expansion.

9) Drift

Drift is an undesirable change in output over a period of time that is unrelated to change in input, operating conditions. Drift is occurred in instruments due to internal temperature variations, ageing effects and high stress ect.

Zero drift is used for the changes that occur in output when there is zero output. It is expressed as percentage of full range output.

10) Threshold

The minimum value of input which is necessary to activate an instrument to produce an output is termed its threshold as shown in figure 7. (Threshold is the minimum value of the input required to cause the pointer to move from zero position).
11) Input Impedance

The magnitude of the impedance of element connected across the signal source is called Input Impedance. Figure 8 shows a voltage signal source and input device connected across it.

![Fig. 8 voltage source and input device](image)

The magnitude of the input impedance is given by

\[ Z_i = \frac{\hat{e}_i}{\hat{i}_i} \]

Power extracted by the input device from the signal source is

\[ P = e_i i_i = \frac{e_i^2}{Z_i} \]

From above two expressions it is clear that a low input impedance device connected across the voltage signal source draws more current and more power from signal source than high input impedance device.
12) **Loading Effect**

Loading effect is the incapability of the system to faithfully measure, record or control the input signal in accurate form.

13) **Repeatability**

Repeatability is defined as the ability of an instrument to give the same output for repeated applications of same input value under same environmental condition.

14) **Reproducibility**

Reproducibility is defined as the ability of an instrument to reproduce the same output for repeated applications of same input value under different environment condition. In case of perfect reproducibility the instrument satisfies no drift condition.

15) **Static Error**

The difference between the measured value of quantity and true value (Reference Value) of quantity is called as Error.

\[
\text{Error} = \text{Measured value - True Value} \\
\delta A = A_m - A_t \\
\delta A - \text{error}
\]

\[A_m - \text{Measured value of quantity} \]
\[A_t - \text{True value of quantity}\]

16) **Static Correction**

It is the difference between the true value and the measurement value of the quantity

\[
\delta C = A_t - A_m = - \delta A \\
\delta C - \text{Static correction}
\]

17) **Scale Range**

It can be defined as the measure of the instrument between the lowest and highest readings it can measure. A thermometer has a scale from −40°C to 100°C. Thus the range varies from −40°C to 100°C.
18) Scale Span

It can be defined as the range of an instrument from the minimum to maximum scale value. In the case of a thermometer, its scale goes from $-40^\circ$C to $100^\circ$C. Thus its span is $140^\circ$C. As said before accuracy is defined as a percentage of span. It is actually a deviation from true expressed as a percentage of the span.

2.2 DYNAMIC CHARACTERISTICS

The dynamic behaviour of an instrument is determined by applying some standard form of known and predetermined input to its primary element (sensing element) and then studies the output. Generally dynamic behaviour is determined by applying following three types of inputs.

1. **Step Input**: Step change in which the primary element is subjected to an instantaneous and finite change in measured variable.

2. **Linear Input**: Linear change, in which the primary element is, follows a measured variable, changing linearly with time.

3. **Sinusoidal input**: Sinusoidal change, in which the primary element follows a measured variable, the magnitude of which changes in accordance with a sinusoidal function of constant amplitude.

The dynamic characteristics of an instrument are

(i) Speed of response

(ii) Fidelity

(iii) Lag

(iv) Dynamic error

(i) **Speed of Response**

It is the rapidity with which an instrument responds to changes in the measured quantity.

(ii) **Fidelity**

It is the degree to which an instrument indicates the changes in the measured variable without dynamic error (faithful reproduction or fidelity of an instrument is the ability of reproducing an input signal faithfully (truly).
(iii) Lag

It is the retardation or delay in the response of an instrument to changes in the measured variable. The measuring lags are two types:

- **Retardation type:** In this case the response of an instrument begins immediately after a change in measured variable is occurred.

- **Time delay type:** In this case the response of an instrument begins after a dead time after the application of the input quantity.

(iv) Dynamic Error

Error which is caused by dynamic influences acting on the system such as vibration, roll, pitch or linear acceleration. This error may have an amplitude and usually a frequency related to the environmental influences and the parameters of the system itself.

3. CLASSIFICATION OF ERRORS

All measurement can be made without perfect accuracy (degree of error must always be assumed). In reality, no measurement can ever made with 100% accuracy. It is important to find that actual accuracy and different types of errors can be occurred in measuring instruments. Errors may arise from different sources and usually classified as follows, Classification of Error

1. Gross Errors
2. Systematic Errors
   a) Instrumental errors
      i) Inherent shortcomings of instruments
      ii) Misuse of instruments
      iii) Loading effects
   b) Environmental errors
   c) Observational errors
3. Random Errors

1. Gross Errors

The main source of Gross errors is human mistakes in reading or using instruments and in recording and calculating measured quantity. As long as human beings are involved and they may grossly misread the scale reading, then definitely some gross errors will be occurred in measured value.

Example, Due to an oversight, Experimenter may read the temperature as $22.7^\circ C$ while the actual reading may be $32.7^\circ C$ He may transpose the reading while recording. For example, he may read $16.7^\circ C$ and record $27.6^\circ C$ as an alternative.

The complete elimination of gross errors is maybe impossible, one should try to predict and correct them. Some gross errors are easily identified while others may be very difficult to detect. Gross errors can be avoided by using the following two ways.

Great care should be taken in reading and recording the data.

Two, three or even more readings should be taken for the quantity being measured by using different experimenters and different reading point (different environment condition of instrument) to avoid re-reading with same error. So it is suitable to take a large number of readings as a close agreement between readings assures that no gross error has been occurred in measured values.

2. Systematic Errors

Systematic errors are divided into following three categories.

i. Instrumental Errors

ii. Environmental Errors

iii. Observational Errors

i) Instrumental Errors

These errors are arises due to following three reasons (sources of error).

a) Due to inherent shortcoming of instrument

b) Due to misuse of the instruments, and

c) Due to loading effects of instruments
a) Inherent Shortcomings of instruments

These errors are inherent in instruments because of their mechanical structure due to construction, calibration or operation of the instruments or measuring devices. These errors may cause the instrument to read too low or too high. Example, if the spring (used for producing controlling torque) of a permanent magnet instrument has become weak, so the instrument will always read high. Errors may be caused because of friction, hysteresis or even gear backlash.

Elimination or reduction methods of these errors,

  o The instrument may be re-calibrated carefully.

  o The procedure of measurement must be carefully planned. Substitution methods or calibration against standards may be used for the purpose.

  o Correction factors should be applied after determining the instrumental errors.

b) Misuse of Instruments

In some cases the errors are occurred in measurement due to the fault of the operator than that of the instrument. A good instrument used in an unintelligent way may give wrong results.

Examples, Misuse of instruments may be failure to do zero adjustment of instrument, poor initial adjustments, using leads of too high a resistance and ill practices of instrument beyond the manufacturer’s instruction and specifications ect.

c) Loading Effects

The errors committed by loading effects due to improper use of an instrument for measurement work. In measurement system, loading effects are identified and corrections should be made or more suitable instruments can be used.

Example, a well calibrated voltmeter may give a misleading (may be false) voltage reading when connected across a high resistance circuit. The same voltmeter, when connected across a low resistance circuit may give a more reliable reading (dependable or
steady or true value). In this example, voltmeter has a loading effect on the circuit, altering the actual circuit conditions by measurement process. So errors caused by loading effect of the meters can be avoided by using them intelligently.

**ii) Environmental Error**

Environmental error occurs due to external environmental conditions of the instrument, such as effects of temperature, pressure, humidity, dust, vibration or external magnetic or electrostatic fields.

Elimination or reduction methods of these undesirable errors are

- Arrangements should be made to keep the conditions as nearly as constant as possible. Example, temperature can be kept constant by keeping the instrument in the temperature controlled region.
- The device which is used against these environmental effects.

  Example, variations in resistance with temperature can be minimized by using very low resistance temperature co-efficient of resistive material.
  
  Employing techniques which eliminate the effects of these disturbances. For example, the effect of humidity dust etc., can be entirely eliminated by tightly sealing the equipment.
  
  The external or electrostatic effects can be eliminated by using magnetic or electrostatic shield on the instrument.
  
  Applying computed corrections: Efforts are normally made to avoid the use of application of computed corrections, but where these corrections are needed and are necessary, they are incorporated for the computations of the results

**iii) Observational Errors**

There are many sources of observational errors. As an example, the pointer of a voltmeter rests slightly above the surface of the scale. Thus an error on account of PARALLAX will be acquired unless the line of vision of the observer is exactly above the pointer. To minimize parallax errors, highly accurate meters are provided with mirrored scales as shown in figure 3.1.
Fig. 9: Errors due to parallax

When the pointer’s image appears hidden by the pointer, observer’s eye is directly in line with the pointer. Although a mirrored scale minimizes parallax error,

Fig.10: Arrangements showing scale and pointer in the same plane

The observational errors are also occurs due to involvement of human factors. For example, there are observational errors in measurements involving timing of an event Different observer may produce different results, especially when sound and light measurement are involved. The complete elimination of this error can be achieved by using digital display of output.

3. Random Errors

These errors are occurred due to unknown causes and are observed when the magnitude and polarity of a measurement fluctuate in changeable (random) manner. The quantity being measure is affected by many happenings or disturbances and ambient influence about which we are unaware are lumped together and called as Random or Residual. The errors
caused by these disturbances are called Random Errors. Since the errors remain even after the systematic errors have been taken care, those errors are called as Residual (Random) Errors. Random errors cannot normally be predicted or corrected, but they can be minimized by skilled observer and using a well maintained quality instrument.

4.1 SOURCES OF ERRORS

The sources of error, other than the inability of a piece of hardware to provide a true measurement are listed below,

1) Insufficient knowledge of process parameters and design conditions.

2) Poor design

3) Change in process parameters, irregularities, upsets (disturbances) etc.

4) Poor maintenance

5) Errors caused by people who operate the instrument or equipment.

Certain design limitations.

Errors in Measuring Instruments

No measurement is free from error in reality. An intelligent skill in taking measurements is the ability to understand results in terms of possible errors. If the precision of the instrument is sufficient, no matter what its accuracy is, a difference will always be observed between two measured results. So an understanding and careful evaluation of the errors is necessary in measuring instruments. The Accuracy of an instrument is measured in terms of errors.

True value

The true value of quantity being measured is defined as the average of an infinite number of measured values when the average deviation due to the various contributing factors tends to zero. In ideal situation is not possible to determine the True value of a quantity by experimental way. Normally an experimenter would never know that the quantity being measured by experimental way is the True value of the quantity or not. In practice the true value would be determined by a “standard method”, that is a method agreed by experts with sufficient accurate.
Static Error

Static error is defined as a difference between the measured value and the true value of the quantity being measured. It is expressed as follows.

$$\delta A = A_m - A_t$$  \hspace{1cm} (1)

Where, $\delta A$ = Error, $A_m$ = Measured value of quantity and $A_t$ = True value of quantity. $\delta A$ is also called as absolute static error of quantity A and it is expressed as follows.

$$\varepsilon_0 = \delta A$$  \hspace{1cm} (2)

Where, $\varepsilon_0$ = Absolute static error of quantity A under measurement.

The absolute value of $\delta A$ does not specify exactly the accuracy of measurement, so the quality of measurement is provided by relative static error.

Relative static error

Relative static error is defined as the ratio between the absolute static errors and true value of quantity being measured. It is expressed as follows.

$$\varepsilon_r = \frac{\text{Absolute Error}}{\text{True Value}} = \frac{\delta A}{A_t} = \frac{\varepsilon_0}{A_t}$$  \hspace{1cm} (3)

Percentage static error $= \% \varepsilon_r = \varepsilon_r \times 100$

From equation (1),

$$A_t = A_m - \delta A$$
$$A_t = A_m - \varepsilon_0$$
$$A_t = A_m - \varepsilon_r A_t$$  \hspace{1cm} (4)
$$A_t + \varepsilon_r A_t = A_m$$
$$A_t (1 + \varepsilon_r) = A_m$$
$$A_t = \frac{A_m}{1 + \varepsilon_r}$$
ε₀=δA is small, which means that the difference between measured value and true values is very small, \( A_m - A_t = \) Negligible or small. So Almost
\( A_m = A_t \) (that is \( \varepsilon_r<<1 \)).

From equation (4), \( A_t = A_m - \varepsilon_r A_t \)
Substitute \( A_t = A_m \) in equation (4),
\( A_t = A_m - \varepsilon_r A_m \)
\( A_t = A_m (1 - \varepsilon_r) \)

4.2 ERROR CORRECTION OR METHOD OF CORRECTION

It is the difference between the true value and the measured value of quantity. \( \delta C = A_t - A_m \)------------------(5)
Where, \( \delta C = \) Static Error Correction = - \( \delta A \)

* For Detail Error correction (Rectification or Elimination or Reduction) methods of all categories of errors are discussed in the topic of classification of errors.

5.ANALYSIS OF DATA

Analysis of data is a process of inspecting, cleaning, transforming, and modeling data with the goal of discovering useful information, suggesting conclusions, and supporting decision-making. Analysis refers to breaking a whole into its separate components for individual examination. Data analysis is a process for obtaining raw data and converting it into information useful for decision-making by users.
STATISTICAL EVALUATION OF MEASUREMENT DATA

Statistical Evaluation of measured data is obtained in two methods of tests as shown in below.

- **Multi Sample Test:** In multi sample test, repeated measured data have been acquired by different instruments, different methods of measurement and different observer.

- **Single Sample Test:** measured data have been acquired by identical conditions (same instrument, methods and observer) at different times.

Statistical Evaluation methods will give the most probable true value of measured quantity.

The mathematical background statistical evaluation methods are Arithmetic Mean, Deviation Average Deviation, Standard Deviation and variance.

**Arithmetic Mean**

The most probable value of measured reading is the arithmetic mean of the number of reading taken. The best approximation is made when the number of readings of the same quantity is very large. Arithmetic mean or average of measured variables $X$ is calculated by taking the sum of all readings and dividing by the number of reading.

The Average is given by,

$$X = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} = \frac{\sum x}{n}$$

Where, $X =$ Arithmetic mean, $x_1, x_2, \ldots, x_n =$ Readings or variable or samples and $n =$ number of readings.

**Deviation (Deviation from the Average value)**

The Deviation is departure of the observed reading from the arithmetic mean of the group of reading. Let the deviation of reading $x_1$ be $d_1$ and that of $x_2$ be $d_2$ etc.,

**Average Deviation:**

Average deviation defined as the average of the modulus (without respect to its sign) of the individual deviations and is given by,

Where, $D =$ Average Deviation.
The average deviation is used to identify precision of the instruments which is used in making measurements. Highly precise instruments will give a low average deviation between readings.

Standard Deviation

Standard deviation is used to analysis random errors occurred in measurement. The standard Deviation of an infinite number of data is defined as the square root of the sum of individual deviations squared, divided by the number of readings (n).

\[
S.D = \sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n}} = \sqrt{\frac{\sum d_i^2}{n}} ; \text{ for } n > 20
\]

\[
S.D = s = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n-1}} = \sqrt{\frac{\sum d_i^2}{n-1}} ; \text{ for } n < 20
\]

Variance

The variance is the mean square deviation, which is the same as S.D except Square root. Variance is Just the squared standard deviation.

\[
V = (\text{Standard deviation})^2
\]

\[
V = \sigma^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n} = \frac{\sum d_i^2}{n} ; \text{ for } n > 20
\]

\[
V = s^2 = \frac{d_1^2 + d_2^2 + d_3^2 + \ldots + d_n^2}{n-1} = \frac{\sum d_i^2}{n-1} ; \text{ for } n < 20
\]
Histogram:

When a number of Multisample observations are taken experimentally there is a scatter of the data about some central value. For representing this results in the form of a Histogram. A histogram is also called a frequency distribution curve.

Example: Following table 3.1 shows a set of 50 readings of length measurement. The most probable or central value of length is 100mm represented as shown.

<table>
<thead>
<tr>
<th>Length (mm)</th>
<th>Number of observed readings (frequency or occurrence)</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.7</td>
<td>1</td>
</tr>
<tr>
<td>99.8</td>
<td>4</td>
</tr>
<tr>
<td>99.9</td>
<td>12</td>
</tr>
<tr>
<td>100.0</td>
<td>19</td>
</tr>
<tr>
<td>100.1</td>
<td>10</td>
</tr>
<tr>
<td>100.2</td>
<td>3</td>
</tr>
<tr>
<td>100.3</td>
<td>1</td>
</tr>
</tbody>
</table>

Total number of readings =50

Fig. 11: Histogram

This histogram indicates the number of occurrence of particular value. At the central value of 100mm is occurred 19 times and recorded to the nearest 0.1mm as shown in figure 3.3. Here bell shape dotted line curve is called as normal or Gaussian curve.
Measure of Dispersion from the Mean

The property which denotes the extent to which the values are dispersed about the central value is termed as dispersion. The other name of dispersion is spread or scatter.

Measure of dispersion from central value is an indication of the degree of consistency (precision) and regularity of the data. Example: Figure 3.4 shows the two sets of data and curve 1 vary from $x_1$ to $x_2$ and curve 2 vary from $x_3$ to $x_4$. Curve 1 is having smaller dispersion from central value than the curve 2. Therefore curve 1 is having greater precision than the curve 2.

![Curves showing different ranges and precision index](image)

**Fig. 12: Curves showing different ranges and precision index**

**Range**

The simplest possible measure of dispersion is the range which is the difference between greatest and least values of measured data.

Example: In figure 3.4, the range of curve1 is $(x_2 - x_1)$ and range of curve 2 is $(x_4 - x_3)$.

**Limiting Errors (Guarantee Errors or Limits of errors):**

In most of the instruments the accuracy is guaranteed to be within a certain percentage of full scale reading. The manufacturer has to specify the deviations from the nominal value of a particular quantity. The limits of these deviations from the specified value are called as Limiting Errors or Guarantee Errors.

The magnitude of Limiting Error = Accuracy x Full scale reading. In general the actual value of quantity is determined as follows.
Actual Value of Quantity = Nominal value ± Limiting Error

\[ A_a = A_n \pm \delta A \]

Where, \( A_a \) = Actual value of quantity; \( A_n \) = Nominal value of Quantity; \( \pm \delta A \) = Limiting error.

**For Example**, Nominal magnitude of resistor is 1000Ω with a limiting error \( \pm 100Ω \).

Determine the Actual magnitude of the resistance.

Actual value of quantity \( A_a = 1000 \pm 100Ω \) or \( A_a \geq 900Ω \) and \( A_a \leq 1100Ω \).

Therefore the manufacturer guarantees that the value of resistance of resistor lies between 900Ω and 1100Ω.

**Relative (Fractional) Limiting Error**

The relative limiting error is defined as the ratio of the error to the specified (nominal) magnitude of the quantity.

Relative Limiting Error \( \varepsilon_r = \)

Then limiting values calculated as follows,

We know that \( A_a = A_n \pm \delta A = A_n \pm \varepsilon_r A_n = A_n (1 \pm \varepsilon_r) \)

Percentage limiting error \% \( \varepsilon_r = \varepsilon_r \times 100 \)

In limiting errors the nominal value \( A_n \) is taken as the true value or quantity, the quantity which has the maximum deviation from \( A_a \) is taken as the incorrect quantity.

Then \( \delta A = A_a - A_n \)

Therefore Relative Limiting Error \( \varepsilon_r = \frac{A_a - A_n}{A_n} = \frac{\text{Actual value } - \text{nominal value}}{\text{nominal value}} \)
**For Example,** considered $A_n = 100 \Omega$ and $\delta A = \pm 10 \Omega$;

Relative limiting error $\varepsilon_r = \frac{\delta A}{A_n} = \frac{10}{100} = \pm 0.1$

Percentage Limiting error $%\varepsilon_r = 0.1 \times 100 = \pm 10\%$

Limiting values of resistance are:

$$A_\alpha = A_n (1 \pm \varepsilon_r) = 100 (1 \pm 0.1) = 100 \pm 10 \Omega$$

**Probable error**

The most probable or best value of a Gaussian distribution is obtained by taking arithmetic mean of the various values of the variety. A convenient measure of precision is achieved by the quantity $r$. It is called Probable Error of P.E. It is expressed as follows,

$$\text{Probable Error} = P.E = r = \frac{0.4769}{h}$$

Where $r$= probable error and $h$= constant called precision index.

Gaussian distribution and Histogram are used to estimate the probable error of any measurement.

**Normal or Gaussian curve of errors**

The normal or Gaussian law of errors is the basis for the major part of study of random errors. The law of probability states the normal occurrence of deviations from average value of an infinite number of measurements can be expressed by,

$$y = \frac{h}{\sqrt{\pi}} \exp\left(-h^2x^2\right)$$

Where, $x$= magnitude deviation from

mean

$y$=Number of readings at any deviation $x$ (the probability of occurrence of deviation $x$) $h$= A constant called precision index.

The Normal or Gaussian probability curve is shown in figure 3.5. In this curve $r$ is the measure of precision quantity (probable error=$r$). The points $-r$ and $+r$ are locating the area bounded by the Gaussian curve.
Precision index \( x = 0 \) then, \( y = \frac{h}{\sqrt{\pi}} \). The maximum value of \( y \) depends upon \( h \). If \( y \) is larger, then the corresponding curve is having greater precision. Then the probable is determined using following expression.

\[
\text{Probable Error} = r = \frac{0.4769}{h}
\]

**Instrument Error Combination**

When two or more quantities are measurand, then final result is easily generalised for many measurand by using the combination of these quantities. In many measurand each quantity having limiting error, are combined, it is easy to compute the limiting error of combination. The limiting error can be easily found by considering the relative increment of the function if the final result is in the form of an algebraic equation.

1) **Sum of two Quantities (Addition)**

Let us consider \( r_1 \) and \( r_2 \) are measurand quantities and final result is \( X \) Therefore, \( X = r_1 + r_2 \)

The relative increment of the function is given by,

\[
\frac{dX}{X} = \frac{d(r_1 + r_2)}{X} = \frac{dr_1}{X} + \frac{dr_2}{X}
\]

Expressing the result in terms of relative increment of the component quantities,
If the limiting errors of quantities are represented by \( \pm \delta r_1 \) and \( \pm \delta r_2 \), then the corresponding relative limiting error in \( X \) is given by,

\[
\frac{dX}{X} = \frac{r_1 dr_1}{X r_1} + \frac{r_2 dr_2}{X r_2}
\]

The above equation shows that the resultant limiting error is equal to the sum of the products formed by multiplying the individual relative limiting errors by ratio of each term to the function.

2) Difference of two Quantities (Subtraction)

Let us consider \( r_1 \) and \( r_2 \) are measurand quantities and final result is \( X \). Therefore, \( X = r_1 - r_2 \)
4) **Product of two Quantities (Multiplication)**

Let us consider $r_1$ and $r_2$ are measurand quantities and final result is $X$

Therefore, $X = r_1 \times r_2$

$log_e X = log_e r_1 + log_e r_2$

Differentiating the above with respect to $X$,

$$\frac{1}{X} = \frac{1}{r_1} \frac{dr_2}{dX} + \frac{1}{r_2} \frac{dr_2}{dX}$$

$$\frac{1}{X} = \frac{1}{dX} \left[ \frac{dr_1}{r_1} + \frac{dr_2}{r_2} \right]$$

$$\frac{dX}{X} = \frac{dr_1}{r_1} + \frac{dr_2}{r_2}$$

If the limiting errors of quantities are represented by $\pm \delta r_1$ and $\pm \delta r_2$, then the corresponding relative limiting error in $X$ is given by,

$$\frac{\delta X}{X} = \pm \left[ \frac{\delta r_1}{r_1} + \frac{\delta r_2}{r_2} \right]$$

Therefore the relative limiting error of product of terms is equal to the sum of relative limiting errors of terms.

7) **Power of a factor**

Let us consider,

$$X = r_1^n$$

$log_e X = n log_e r_1$

Differentiating above with respect to $X$,

$$\frac{1}{X} = n \frac{1}{r_1} \frac{dr_1}{dX}$$

$$\frac{dX}{X} = n \frac{dr_1}{r_1}$$

Therefore the relative limiting error in $X$ is,
Calibration is the process of checking the accuracy of instrument by comparing the instrument reading with a standard or against a similar meter of known accuracy. So using calibration is used to find the errors and accuracy of the measurement system or an instrument.

Calibration is an essential process to be undertaken for each instrument and measuring system regularly. The instruments which are actually used for measurement work must be calibrated against some reference instruments in which is having higher accuracy. Reference instruments must be calibrated against instrument of still higher accuracy or against primary standard or against other standards of known accuracy.

The calibration is better carried out under the predetermined environmental conditions. All industrial grade instruments can be checked for accuracy in the laboratory.

\[
\frac{\delta X}{X} = \pm n \frac{\delta r_1}{r_1}
\]

It clear from above result under these conditions is magnified n times.

7) Composite factors
Let us consider, \( X = r_1^n r_1^m \)
\( \log_e X = n \log_e r_1 + m \log_e r_2 \)
Differentiating above with respect to \( X \).

\[
\frac{1}{X} = \frac{n}{r_1} \frac{dr_1}{dX} + \frac{m}{r_2} \frac{dr_2}{dX}
\]

\[
\frac{dX}{X} = n \frac{dr_1}{r_1} + m \frac{dr_2}{r_2}
\]

Thus the limiting error in \( X \) is,

\[
\frac{\delta X}{X} = \pm \left[ n \frac{\delta r_1}{r_1} + m \frac{\delta r_2}{r_2} \right]
\]
by using the working standard. Certification of an instrument manufactured by an industry is undertaken by National Physical Laboratory and other authorizes laboratories where the secondary standards and working standards are kept.

**Process of Calibration**

The procedure involved in calibration is called as process of calibration. Calibration procedure involves the comparison of particular instrument with either

- A primary standard,
- A secondary standard with higher accuracy than the instrument to be calibrated
- An instrument of known accuracy.

**Procedure of calibration as follows.**

- Study the construction of the instrument and identify and list all the possible inputs.
- Choose, as best as one can, which of the inputs will be significant in the application for which the instrument is to be calibrated.
- Standard and secure apparatus that will allow all significant inputs to vary over the ranges considered necessary.
- By holding some input constant, varying others and recording the output, develop the desired static input-output relations.

**Theory and Principles of Calibration Methods**

Calibration methods are classified into following two types,

1) Primary or Absolute method of calibration
2) Secondary or Comparison method of calibration
   i. Direct comparison method of calibration
   ii. Indirect comparison method of calibration
1) Primary or Absolute method of calibration

If the particular test instrument (the instrument to be calibrated) is calibrated against primary standard, then the calibration is called as primary or absolute calibration. After the primary calibration, the instrument can be used as a secondary calibration instrument.

2) Secondary or Comparison calibration method

If the instrument is calibrated against secondary standard instrument, then the calibration is called as secondary calibration. This method is used for further calibration of other devices of lesser accuracy. Secondary calibration instruments are used in laboratory practice and also in the industries because they are practical calibration sources.

i) Direct comparison method of Calibration

Direct comparison method of calibration with a known input source with same order of accuracy as primary calibration. So the instrument which is calibrated directly is also used as secondary calibration instruments.

ii) Indirect comparison method of Calibration

The procedure of indirect method of calibration is based on the equivalence of two different devices with same comparison concept.

Standards of measurement:

A standard is a physical representation of a unit of measurement. A known accurate measure of physical quantity is termed as standard. These standards are used to determine the accuracy of other physical quantities by the comparison method.

Example, the fundamental unit of mass in the International System is the Kilogram and defined as the mass of a cubic decimetre of water at its temperature of maximum of density of 4°C.

Different standards are developed for checking the other units of measurements
and all these standards are preserved at the International Bureau of Weight and Measures at Serves, Paris.

Classification of Standards

Standards are classified into four types, based on the functions and applications.

1) International standards
2) Primary standards
3) Secondary standards
4) Working standards

1) International Standard

International standards are defined and established upon internationally. They are maintained at the International Bureau of Weights and measures and are not accessible to ordinary users for measurements and calibration. They are periodically evaluated and checked by absolute measurements in terms of fundamental units of physics.

- **International Ohms** It is defined as the resistance offered by a column of mercury having a mass of 14.4521gms, uniform cross sectional area and length of 106.300cm, to the flow of constant current at the melting point of ice.

2) Primary Standards

- Primary standards are maintained by the National Standards Laboratories (NSL) in different parts of the world.
- The principle function of primary standards is the calibration and verification of secondary standards.
- They are not available outside the National Laboratory for calibration.
- These primary standards are absolute standards of high accuracy that can be used as ultimate reference standards.
Secondary Standards
These standards are basic reference standards used in industrial laboratories for calibration of instruments. Each industry has its own secondary standard and maintained by same industry. Each laboratory periodically sends its secondary standard to the NSL for calibration and comparison against the primary standards. Certification of measuring accuracy is given by NSL in terms of primary standards.

4. Working Standards
The working standards are used for day-to-day use in measurement laboratories. So this standard is the primary tool of a measurement laboratory. These standards may be lower in accuracy in comparison with secondary standard. It is used to check and calibrate laboratory instruments for accuracy and performance.

Example, a standard resistor for checking of resistance value manufactured.

Reference books
UNIT – II
MEASUREMENT AND INSTRUMENTATION – SIC 1203
UNIT 2 ELECTRICAL MEASUREMENTS

Units of voltage and current - principle of operation of D’Arsonval Galvanometer - principle, operation, constructional details and comparison of the following: permanent magnet moving coil, permanent magnet moving iron, Dynamometer, Induction, thermal and rectifier type instruments, Power measurement - Voltmeter ammeter method, Ammeter voltmeter method, Electro-dynamic wattmeter - Low power factor wattmeter

Current
Current is the rate at which electric charge flows past a point in a circuit. Symbol is I and unit is A or amps

Voltage
Voltage is the electrical force that would drive an electric current between two points. And symbol is v units is volts or voltage

1. PRINCIPLE OF D’ARSONVAL GALVANOMETER
An action caused by electromagnetic deflection, using a coil of wire and a magnetized field. When current passes through the coil, a needle is deflected. Whenever electrons flow through a conductor, a magnetic field proportional to the current is created. This effect is useful for measuring current and is employed in many practical meters.

Moving Coil:
It is the current carrying element. It is either rectangular or circular in shape and consists of number of turns of fine wire. This coil is suspended so that it is free to turn about its vertical axis of symmetry. It is arranged in a uniform, radial, horizontal magnetic field in the air gap between pole pieces of a permanent magnet and iron core. The iron core is spherical in shape if the coil is circular but is cylindrical if the coil is rectangular. The iron core is used to provide a flux path of low reluctance and therefore to provide strong magnetic field for the coil to move in. this increases the deflecting torque and hence the sensitivity of the galvanometer. The length of air gap is about
1.5mm. In some galvanometers the iron core is omitted resulting in a decreased value of flux density and the coil is made narrower to decrease the air gap. Such a galvanometer is less sensitive, but its moment of inertia is smaller on account of its reduced radius and consequently a short periodic time.

2) **Damping:**

There is a damping torque present owing to production of eddy currents in the metal former on which the coil is mounted. Damping is also obtained by connecting a low resistance across the galvanometer terminals. Damping torque depends upon the resistance and we can obtain critical damping by adjusting the value of resistance.

3) **Suspension:**

The coil is supported by a flat ribbon suspension which also carries current to the coil. The other current connection in a sensitive galvanometer is a coiled wire. This is called the lower suspension and has a negligible torque effect. This type of galvanometer must be leveled carefully so that the coil hangs straight and centrally without rubbing the
poles or the soft iron cylinder. Some portable galvanometers which do not require exact leveling have "taut suspensions" consisting of straight flat strips kept under tension for at the both top and at the bottom.

The upper suspension consists of gold or copper wire of nearly 0.012-5 or 0.02-5 mm diameter rolled into the form of a ribbon. This is not very strong mechanically; so that the galvanometers must be handled carefully without jerks. Sensitive galvanometers are provided with coil clamps to the strain from suspension, while the galvanometer is being moved.

4) **Indication:**

The suspension carries a small mirror upon which a beam of light is cast. The beam of light is reflected on a scale upon which the deflection is measured. This scale is usually about 1 meter away from the instrument, although ½ meter may be used for greater compactness.

5) **Zero Setting:**

A torsion head is provided for adjusting the position of the coil and also for zero setting.

**Operation**

When a current flows through the coil, the coil generates a magnetic field. This field acts against the permanent magnet. The coil twists, pushing against the spring, and moves the pointer. The hand points at a scale indicating the electric current. Careful design of the pole pieces ensures that the magnetic field is uniform, so that the angular deflection of the pointer is proportional to the current. A useful meter generally contains provision for damping the mechanical resonance of the moving coil and pointer, so that the pointer settles quickly to its position without oscillation.
2. PERMANENT MAGNET MOVING COIL INSTRUMENT

Principle of moving coil instrument

Moving coil instrument depends on the principle that when a current carrying conductor is placed on a magnetic field, mechanical force acts on the conductor. The coil placed on the magnetic field and carrying operating current is attached to the moving system. With the movement of the coil the pointer moves over the scale.

![Fig.2 Permanent magnet moving coil](image)

Construction of PMMC instrument

Moving coil instrument consists of a powerful permanent magnet with soft iron pieces and light rectangular coil of many turns of fine wire wound on aluminum former inside which is an iron core as shown in the figure. As it uses permanent magnets they are called “Permanent magnet moving coil instrument”. The purpose of the coil is to make the field uniform. The coil is mounted on the spindle and acts as the moving element. The current is led into and out of the coil by means of the two control hair
springs, one above and the other below the coil. The springs also provide the controlling torque. Damping torque is provided by eddy current damping.

**Working of PMMC instrument**

When the moving coil instrument is connected in the circuit, operating current flows through the coil. This current-carrying coil is placed in the magnetic field produced by the permanent magnet and therefore, mechanical force acts on the coil. As the coil attached to the moving system, the pointer moves over the scale. It may be noted here that if current direction is reversed the torque will also be reversed since the direction of the field of permanent magnet is same. Hence, the pointer will move in the opposite direction, i.e. it will go on the wrong side of zero. In other words, these instruments work only when current in the circuit is passed in a definite direction i.e. for d.c. only. So it is called permanent magnet moving coil instruments because a coil moves in the field of a permanent magnet.

**Torque Equation for PMMC**

The equation for the developed torque of the PMMC can be obtained from the basic law of electromagnetic torque.

The deflecting torque is given by,

\[ T_d = N B A I \]

Where, \( T_d \) = deflecting torque in N-m
\( B \) = flux density in air gap, Wb/m²
\( N \) = Number of turns of the coils
\( A \) = effective area of coil
\( m^2 \)
\( I \) = current in the moving coil, amperes

Therefore, \( T_d = G I \)

Where, \( G = N B A \) = constant

The controlling torque is provided by the springs and is proportional to the angular deflection of the pointer.

\( T_c = K \Phi \)

Where, \( T_c \) = Controlling Torque
\( K \) = Spring Constant Nm/rad or Nm/deg
\( \Phi \) = angular deflection
For the final steady state position,

\[ T_d = T_c \]

Therefore \( G I = K \Phi \)

So, \( \Phi = \frac{(G/K)I}{I} \) or \( I = \frac{(K/G)}{\Phi} \)

Thus the deflection is directly proportional to the current passing through the coil. The pointer deflection can therefore be used to measure current.

**Advantage of PMMC instrument**

1. Uniform scale.
2. Very effective eddy current damping
3. Power consumption is low.
4. No hysteresis loss.
5. They are not affected by stray field.
6. Require small operating current.
7. Accurate and reliable.

**Disadvantage of PMMC instrument**

1. Only used for D.C measurement.
2. Costlier compared to moving iron instrument.
3. Some errors are caused due to the aging of the control springs and the permanent magnets.

**3. MOVING IRON INSTRUMENT**

There are classified in to two type

1. Attraction type moving iron instrument
2. Repulsion type moving iron instrument

**Attraction type moving iron instrument**

**Principle of attraction type moving iron instrument**

An “attraction type” moving-iron instrument consists of a coil, through which the test current is passed, and a pivoted soft-iron mass attached to the pointer. The resulting magnetic polarity at the end of the coil nearest the iron mass then induces the opposite magnetic polarity into the part of the iron mass nearest the coil, which is then drawn by attraction towards the coil, deflecting the pointer across a scale.

The coil is flat and has a narrow slot like opening. The moving iron is a flat disc or a sector
eccentrically mounted. When the current flows through the coil, a magnetic field is produced and the moving iron moves from the weaker field outside the coil to the stronger field inside it or in other words the moving iron is attracted in. The controlling torque is provided by springs but gravity control can be used for panel type of instruments which are vertically mounted. Damping is provided by air friction with the help of a light aluminium piston (attached to the moving system) which move in a fixed chamber closed at one end as shown in Fig. or with the help of a vane (attached to the moving system) which moves in a fixed sector shaped chamber as shown.

**Operation**

The current to be measured is passed through the fixed coil. As the current is flow through the fixed coil, a magnetic field is produced. By magnetic induction the moving iron gets magnetized. The north pole of moving coil is attracted by the south pole of fixed coil. Thus the deflecting force is produced due to force of attraction. Since the moving iron is attached with the spindle, the spindle rotates and the pointer moves over the calibrated scale. But the force of attraction depends on the current flowing through the coil.

The force $F$, pulling the soft iron piece towards the coil is directly proportional to;

a) Field strength $H$, produced by the coil.

b)pole strength „m” developed in the iron piece.

$$ F \alpha mH $$

Since, $m \alpha$.
H, F α H^2
Instantaneous deflecting torque α H^2
Also, the field strength H = μi
If the permeability(μ) of the iron is assumed constant,
Then, H α i
Where, i® instantaneous coil current, Ampere
Instantaneous deflecting torque α i^2
Average deflecting torque, Td α mean of i^2 over a cycle.
Since the instrument is spring controlled,
Tc α θ
In the steady position of deflection, Td = Tc
θ α mean of i^2 over a cycle
α I^2
Since the deflection is proportional to the square of coil current, the scale of such instruments is non-uniform (being crowded in the beginning and spread out near the finishing end of the scale).

**Moving iron repulsion type instrument**
These instruments have two vanes inside the coil, the one is fixed and other is movable. When the current flows in the coil, both the vanes are magnetized with like polarities induced on the same side. Hence due to repulsion of like polarities, there is a force of repulsion between the two vanes causing the movement of the moving van. The repulsion type instruments are the most commonly used instruments.
The two different designs of repulsion type instruments are:
i) Radial vane type and
ii) Co-axial vane type
Radial van repulsion type instrument

Out of the other moving iron mechanism, this is the most sensitive and has most linear scale. The two vanes are radial strips of iron. The fixed vane is attached to the coil. The movable vane is attached to the spindle and suspended in the induction field of the coil. The needle of the instrument is attached to this vane. Even though the current through the coil is alternating, there is always repulsion between the like poles of the fixed and the movable vane. Hence the deflection of the pointer is always in the same direction. The deflection is effectively proportional to the actual current and hence the scale is calibrated directly to rad amperes or volts. The calibration is accurate only for the frequency for which it is designed because the impedance is different for different frequencies.

Fig.4 Radial vane
**Concentric vane repulsion type instrument**

The instrument has two concentric vanes. One is attached to the coil frame rigidly while the other can rotate coaxially inside the stationary vane. Both the vanes are magnetized to the same polarity due to the current in the coil. Thus the movable vane rotates under the repulsive force. The movable vane is attached to the pivoted shaft, the repulsion results in a rotation of the shaft. The pointer deflection is proportional to the current in the coil. The concentric vane type instrument is moderately sensitive and the deflection is proportional to the square of the current through coil. Thus the instrument said to have square low response. Thus the scale of the instrument

![Fig.5 Vane repulsion type](image)

is non-uniform in nature. Thus whatever may be the direction of the current in the coil, the deflection in the moving iron instruments is in the same direction. Hence moving iron instruments can be used for both a.c. and d.c. measurements. Due to square low response, the scale of the moving iron instrument is non-uniform.

**Torque equation**

The deflecting torque results due to repulsion between the similarly charged soft-iron pieces or vanes. If the two pieces develop pole strength of $m_1$ and $m_2$ respectively, then;

- Instantaneous deflecting torque $\propto m_1 m_2 \propto H^2$

If the permeability of iron is assumed constant, then; $H \propto i$, where, $i$ is the coil current

- Instantaneous deflecting torque $\propto i^2$

Average deflecting torque, $T_d \propto \text{mean of } i^2$ over a cycle.

Since the instrument is spring controlled, $T_c \propto \theta$
In the steady position of deflection, $T_d = T_c$

$\theta \propto \text{mean of } i^2 \text{ over a cycle.}$

$\propto i^2$

Thus, the deflection is proportional to the square of the coil current. The scale of the instrument is non-uniform; being crowded in the beginning and spread out near the finish end of the scale. However, the non-linearity of the scale can be corrected to some extent by the accurate shaping and positioning of the iron vanes in relation to the operating coil.

**Advantages**

1) The instruments can be used for both a.c. and d.c. measurements.
2) As the torque to weight ratio is high, errors due to the friction are very less.
3) A single type of moving element can cover the wide range hence these instruments are cheaper than other types of if instruments.
4) There are no current carrying parts in the moving system hence these meters are extremely rugged and reliable.
5) These are capable of giving good accuracy. Modern moving iron instruments have a d.c. error of 2% or less.
6) These can withstand large loads and are not damaged even under sever overload conditions.
7) The range of instruments can be extended.

**Disadvantages**

1) The scale of moving iron instruments is not uniform and is cramped at the lower end. Hence accurate readings are not possible at this end.
2) There are serious errors due to hysteresis, frequency changes and stray magnetic fields.
3) The increase in temperature increases the resistance of coil, decreases stiffness of the springs, decreases the permeability and hence affect the reading severely.
4) Due to the non linearity of B-H curve, the deflecting torque is not exactly proportional to the square of the current.
5) There is a difference between a.c. and d.c. calibration on account of the effect of inductance of the meter. Hence these meters must always be calibrated at the frequency at which they are to be used. The usual commercial moving iron instrument may be used within its specified accuracy from 25 to 125 HZ frequency range.
6) Power consumption is on higher side.
Errors in moving iron instrument

1) **Hysteresis error:** Due to hysteresis effect, the flux density for the same current while ascending and descending values is different. While descending, the flux density is higher and while ascending it is lesser. So meter reads higher for descending values of current or voltage. So remedy for this is to use smaller iron parts which can demagnetise quickly or to work with lower flux densities.

2) **Temperature error:** The temperature error arises due to the effect of temperature on the temperature coefficient of the spring. This error is of the order of 0.02% change in temperature. Errors can cause due to self-heating of the coil and due to which change in resistance of the coil. So coil and series resistance must have low temperature coefficient. Hence manganin is generally used for the series resistance.

3) **Stray magnetic Field Error:** The operating magnetic field in case of moving iron instruments is very low. Hence effect of external i.e. stray magnetic field can cause error. This effect depends on the direction of the stray magnetic field with respect to the operating field of the instrument.

4) **Frequency Error** : These are related to a.c. operation of the instrument. The change in frequency affects the reactance of the working coil and also affects the magnitude of the eddy currents. This cause error in the instrument.

5) **Eddy Current Error** : When instrument is used for a.c. measurements the eddy currents are produced in the iron parts of the instrument. The eddy current affects the instrument current causing the change in the deflection torque. This produce the error in the meter reading. As eddy current are frequency dependent, frequency changes cause eddy current error.
4. DYNAMAMETER

![Diagram of Dynamometer](image)

**Fig.6 Position of Fixed and Moving Coil**

**Construction**

**Fixed Coils:** The necessary field required for the operation of the instrument is produced by the fixed coils. A uniform field is obtained near the center of coil due to division of coil in two sections. These coils are air cored. Fixed coils are wound with fine wire for using as voltmeter, while for ammeters and wattmeters it is wound with heavy wire. The coils are usually varnished. They are clamped in place against the coil supports. This makes the construction rigid.

Ceramic is usually used for mounting supports. If metal parts would have been used then it would weaken the field of the fixed coil.

**Moving Coil**

The moving coil is wound either as a self-sustaining coil or else on a non-metallic former. If metallic former is used, then it would induce eddy currents in it. The construction of moving coil is made light as well as rigid. It is air cored.

**Controlling**

The controlling torque is provided by springs. These springs act as leads to the moving coil.

**Moving System**

The moving coil is mounted on an aluminium spindle. It consists of counter weights and pointer. Sometimes a suspension may be used, in case a high accuracy is desired.

**Damping**

The damping torque is provided by air friction, by a pair of aluminium vanes which are attached to the spindle at the bottom. They move in sector shaped chambers. As operating field would
be distorted by eddy current damping, it is not employed.

**Shielding**

The field produced by these instruments is very weak. Even earth's magnetic field considerably affects the reading. So shielding is done to protect it from stray magnetic fields. It is done by enclosing in a casing high permeability alloy.

**Cases and Scales**

Laboratory standard instruments are usually contained in polished wooden or metal cases which are rigid. The case is supported by adjustable levelling screws. A spirit level may be provided to ensure proper levelling. For using electrodynamometer instrument as ammeter, fixed and moving coils are connected in series and carry the same current. A suitable shunt is connected to these coils to limit current through them upto desired limit. The electrodynamometer instruments can be used as a voltmeter by connecting the fixed and moving coils in series with a high non-inductive resistance. It is most accurate type of voltmeter.

For using electrodynamometer instrument as a wattmeter to measure the power, the fixed coils acts as a current coil and must be connected in series with the load. The moving coils acts as a voltage coil or pressure oil and must be connected across the supply terminals. The wattmeter indicates the supply power.

**Working**

When current passes through the fixed and moving coils, both coils produce the magnetic fields. The field produced by fixed coil is proportional to the load current while the field produced by the moving coil is proportional to the voltage. As the deflecting torque is produced due to the interaction of these two fields, the deflection is proportional to the power supplied to the load.

**Torque Equation**

Let $i_1 =$ Instantaneous value of current in fixed coil $i_2 =$ Instantaneous value of current in moving coil $L_1 =$ Self-inductance of fixed coil $L_2 =$ self-inductance of moving coil $M =$ Mutual inductance between fixed and moving coils

![Fig.7 Equivalent Circuit](image-url)
The electrodynamometer instrument can be represented by an equivalent circuit,

From the principle of conservation of energy,

Energy input = Energy stored + Mechanical energy

Mechanical energy = energy input - energy stored

Subtraction (2) from equation (1),

\[ \text{Mechanical energy} = \frac{1}{2} i_1^2 dL_1 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM \]

and \( e_2 = \frac{d\phi_2}{dt} \)

Electrical input energy = \( e_1 i_1 dt + e_2 i_2 dt \)

\[ = i_1 d\phi_1 + i_2 d\phi_2 \]

\[ = i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1) \]

\[ = i_1 L_1 d i_1 + i_1^2 d L_1 + i_1 i_2 d M + i_1 M d i_2 + i_2 L_2 d i_2 + i_2^2 d L_2 + i_1 i_2 d M + i_2 M d i_1 \] \( \ldots (1) \)

The energy stored in the magnetic field due to \( L_1, L_2 \) and \( M \) is given by,

Energy stored = \( \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M \)

Change in stored energy = \( d \left[ \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M \right] \)

\[ = i_1 L_1 d i_1 + \frac{1}{2} i_1^2 d L_1 + i_1 i_2 L_2 d i_2 + \frac{1}{2} i_2^2 d L_2 + i_1 M d i_2 + i_2 M d i_1 + i_1 i_2 d M \] \( \ldots (2) \)

The self inductance \( L_1 \) and \( L_2 \) are constant and hence \( dL_1 \) and \( dL_2 \) are zero. Mechanical energy = \( i_1 i_2 dM \)

If \( T_i \) is the instantaneous deflecting torque and \( d\theta \) is the change in the deflection then

Mechanical energy = Mechanical work done

\[ = T_i d\theta \]

\[ i_1 i_2 d M = T_i d\theta \]

\[ T_i = i_1 i_2 \frac{d M}{d\theta} \]
**Advantage of Electrodynamometer instrument**

1) As the coils are air cored, these instruments are free from hysteresis and eddy current losses.
2) They have a precision grade security.
3) These instruments can be used on both a.c. and d.c. They are also used as a transfer instruments.
4) Electrodynamometer voltmeter are very useful where accurate r.m.s values of voltage, irrespective of waveforms, are required.
5) Free from hysteresis errors.
6) Low power Consumption.
7) Light in weight.

**Disadvantage of electrodynamometer instrument**

1) These instruments have a low sensitivity due to a low torque to weight ratio. Also it introduces increased frictional losses. To get accurate results, these errors must be minimized.
2) They are more expensive than other type of instruments.
3) These instruments are sensitive to overload and mechanical impacts. Therefore can must be taken while handling them.
4) They have a nonuniform scale.
5) The operation current of these instruments is large due to the fact that they have weak magnetic field

**Error in electrodynamometer instrument**

The various errors in electrodynamometer instruments are,

1. **Torque to weight ratio** : To have reasonable deflecting torque, mmf of the moving coil must be large enough. Thus m.m.f. = NI hence current through moving coil should be high or number of turns should be large. The current cannot be made very high because it may cause excessive heating of springs. Large number of turns hence is the only option but it increases weight of the coil. This makes the system heavy reducing torque to weight ratio. This can cause frictional errors in the reading

2. **Frequency errors** : The changes in the frequency causes to change self inductances of moving coil and fixed coil. This causes the error in the reading. The frequency error can be
3. **Eddy current errors**: In metal parts of the instrument the eddy current gets produced. The eddy current interacts with the instrument current, to cause change in the deflecting torque, to cause error. Hence metal parts should be kept as minimum as possible. Also the resistivity of the metal parts used must be high, to reduce the eddy currents.

4. **Stray magnetic field error**: Similar to moving iron instruments the operating field in electrodynamometer instrument is very weak. Hence external magnetic field can interact with the operation field to cause change in the deflection, causing the error. To reduce the effect of stray magnetic field, the shields must be used for the instruments.

5. **Temperature error**: The temperature errors are caused due to the self heating of the coil, which causes change in the resistance of the coil. Thus temperature compensating resistors can be used in the precise instrument to eliminate the temperature errors.

5. **INDUCTION TYPE INSTRUMENT**

![Diagram of an induction type wattmeter]

**Fig.8 Induction Type Wattmeter**

Induction type wattmeter consists of two laminate electromagnets known as shunt...
electromagnet and series electromagnet respectively. Shunt magnet is excited by the current proportional to the voltage across load flowing through the pressure coil and series magnet is excited by the current, proportional to the voltage across the load flowing through the pressure coil and series magnet is excited by the load current flowing through the current coil. A thin disc made of Cu or Al, pivoted at its centre, is placed between the shunt and series magnets so that it cuts the flux from both of the magnets. The deflection torque is produced by interaction of eddy current induced in the disc and the inducing flux in order to cause the resultant flux in shunt magnet to lag in phase by exactly 90° behind the applied voltage, one or more copper rings, known as copper shading bond are provided on one limb at the shunt magnet. Correct disappointed between shunt and series magnet fluxes may be attained by adjusting the position of copper shading bonds. The pressure coil circuit of introduction type instrument is made

6. THERMAL TYPE INSTRUMENT

The hot wire instruments are based on the principle that length of wire increases due to heating effect when a current passed through it. It is a square law device with a non-linear relationship because increase in length of a wire is directly proportional to the square of current passing through wire. Note that the increase in the length of a wire is very small percentage of the total length of wire. Hence various mechanical linkages have been devised to expand this effect and convert it into motion of a point of a circular scale.

A hot working wire denoted by W in the Figure is made up of platinum-irridum alloy. The main

![Graduated scale](image)

**Fig.9 Thermal Type Instrument**

A hot working wire denoted by W in the Figure is made up of platinum-irridum alloy. The main
advantages of using platinum-irridum alloy is that it can withstand high temperatures without deterioration of wire material caused by oxidation. The working wire W is very fine and its diameter is of the order of 0.1 mm. The wire W is stretched between two point A and B where point B is fixed point and point A is tension adjustment point at which tension adjustment mechanism is placed. One more wire W1, made up of phosphor-bronze is connected to main hot wire W at point C while other end of wire W1 is connected to fixed point D. A fine thread of silk material represented by G is connected between spring S and point F. The thread G is wound around a pulley denoted by E. Both points F and spring S are fixed points. A point P used for indication and thin aluminium disc L are mounted on the spindle. The pulley system E is also mounted on the spindle.

When a current to be measured is passed through the wire, it gets expanded as a result of heating effect by current flowing through it. Because of heating effect, a sag is produced in the wire W. Now the sag in main working wire W causes sag in other wire W1. This sag is transferred to the spring S through a fine silk thread G. Thus the sag produced gets magnified and the spring activates pulley to rotate and pointer gets deflected indicating value of the current under measurement on a graduated scale.

The expansion of the wire is proportional to the heating effect of the current. As heat produced is in the form of power dissipated give $P = I^2R$, the expansion of the wire in hot wire instrument is proportional to the square of r.m.s. value of the current. A thin aluminium disc L rotates between poles of the permanent magnet M and it provides eddy current damping to the instrument. The base of the main instrument is made up of a material which is having coefficient of expansion same as that of hot wire. As base and hot wire, both have same coefficient of expansion, the errors due to uneven expansion between base of instrument and hot wire are minimized.

In early hot wire instruments, the hot wire used was made up of platinum-silver alloy. But the main drawback of early instruments was the low value of the full-scale operating temperature (about 135-150 °C). Because of very low operating temperature, even small variations in room temperature affects the position of the pointer. Today's hot wire instruments use wire made up of platinum-irridum alloy with which operating temperature range can be extended upto 300 to 500 °C. As the temperature range is increased, the effects due to room temperature variations
are minimized. The hot wire is made very thin so that it can attain steady temperature quickly when current flows through wire. The size of wire is decided such that it can bare normal mechanical stresses developed in the instruments.

The hot wire instrument can be used as ammeter over a range 0 to 1 A without a shunt, while 0 to 5 A with a shunt. It can be used as a voltmeter to measure voltage upto 400 V by using high value non-inductive resistance in series with instruments.

**Advantages of hot wire type instrument**

(i) no stray magnetic field effect
(ii) same calibration for dc as well as for ac
(iii) fair accuracy
(iv) simple construction
(v) low cost
(vi) negligible temperature error if suitably adjusted and
(vii) suitability for measurement of currents at very high frequencies

**Disadvantages of hot wire type instrument**

(i) delicate construction
(ii) relatively higher power consumption
(iii) uneven scale
(iv) incapability of taking over-load
(v) sluggish in action
(vi) need of frequent adjustment of zero position due to temperature variations and
(viii) different deflections for ascending and descending values.

**7. RECTIFIER TYPE INSTRUMENT**

Rectifier type instrument measures the alternating voltage and current with the help of rectifying elements and permanent magnet moving coil type of instrument However the primary function of rectifier type of instruments work as voltmeter

We have used here a bridge rectifier circuit as shown. Again we divide our operation into two parts. In the first we analyze the output by applying the dc voltage and in another we will apply ac voltage to the circuit. A series multiplier resistance is connected in series with the voltage source which has the same function as described above.
Let us consider first case here we applying dc voltage source to the circuit. Now the value of full scale deflection current in this case is again \( \frac{V}{(R+R_1)} \), where \( V \) is the root mean square value of the applied voltage, \( R \) is the resistance of the resistance multiplier and \( R_1 \) which is the electrical resistance of the instrument. The \( R \) and \( R_1 \) are marked in the circuit diagram. Now let us consider second case, in this case we will apply ac sinusoidal voltage to the circuit which is given \( v=V_m \sin(wt) \) where \( V_m \) is the peak value of the applied voltage again if we calculate the value of full scale deflection current in this case by applying the similar procedure then we will get an expression of full scale current as \( 0.9V/(R+R_1) \). Remember in order to obtain the average value of voltage we should integrate the instantaneous expression of voltage from zero to pi. Thus comparing it dc output we conclude that the sensitivity with ac input voltage source is 0.9 times the as in the case of dc input voltage source.

**Advantages of rectifier instruments:**

1. The primary advantage of the rectifier voltmeter is that it is far more sensitive as compared to other types of voltmeter. Suitable for measuring A.C. voltages.

2. Metal rectifiers can be incorporated in universal instruments, such as ammeter, thereby enabling a moving-coil milliammeter to be used in combination with shunt and series
resistances to measure various ranges of D.C. and D.C. voltage, and in combination with a bridge rectifier and suitable resistors to measure various ranges of A.C. and A.C. voltage.

8. POWER MEASUREMENT

VOLTMETER AMMETER METHOD AND AMMETER VOLTMETER METHOD

Electrical Power dissipated by a load \( L \) fed by a dc power supply \( E \) is the product of the voltage across the load \( V_L \) and the current flowing in it \( I_L \)

\[
P = V_L \times I_L
\]

The power measurement in a dc circuit can be carried out using a voltmeter \( V \) and ammeter \( A \) as shown in the figure. In figure a the ammeter measures the current flowing in to the voltmeter, as well as that into the load; whereas in figure b the voltmeter measures the voltage drop across the ammeter in addition to that dropping across the load.

![Fig.11.a circuit diagram](image)

![Fig.11.b.Circuit diagram](image)

\( I \) – Current measured by ammeter
\( V \) – Voltage measured by voltmeter
\( R_V, R_A \) – Internal resistance of voltmeter and ammeter
\( R_L \) – Load resistance
\( I_V \) – Current flowing into the voltmeter
\( V_A \) – Voltage drop across the ammeter
9. THREE VOLTMETER METHOD

\[ P = V_L \times I_L = V \times I \times \left( \frac{R_V - R_A}{R_V} \right) \]

\[ P = V_L \times I_L = V \times I \times \left( \frac{R_L - R_A}{R_L} \right) \]

\( V_1, V_2 \) and \( V_3 \) are the three voltmeters and \( R \) is a non-inductive resistance connected in series with the load as shown in figure.

![Three voltmeter method](image1)

![Phasor diagram](image2)

Fig.12 Three voltmeter method and Phasor diagram
From phasor diagram,

\[ V_1^2 = V_2^2 + V_3^2 + 2V_2V_3\cos\phi \]

\[ = V_2^2 + V_3^2 + 2(IR)V_3\cos\phi \]

\[ = V_2^2 + V_3^2 + 2R(V_3I\cos\phi) \]

\[ = V_2^2 + V_3^2 + 2PR \]

Since power in the inductive circuit, \( P = V_3I\cos\phi \)

Or, power,

\[ P = \frac{V_1^2 - V_2^2 - V_3^2}{2R} \]

Power factor of the circuit is given by:

\[ \cos\phi = \frac{V_1^2 - V_2^2 - V_3^2}{2V_2V_3} \]

The assumptions are made that the current in the resistor \( R \) is same as the load current.

**Disadvantages:**

- Supply voltage higher than normal voltage is required because an additional resistance \( R \) is connected in series with the load \( Z \) (inductive circuit).
- Even small errors in measurement of voltages may cause serious errors in the value of power determined by this method.

**10. THREE AMMETER METHOD**

Following figures shows the circuit diagram and phasor diagram of three ammeter method for measurement of power. The current measured by the ammeter \( A_1 \), is the vector sum of the load current and that taken by the non-inductive resistor \( R \), this latter being in phase with \( V \).
Three ammeter method and Phasor diagram

Fig. 13 Three ammeter method and Phasor diagram
From phasor diagram, we have:

\[ I_1^2 = I_2^2 + I_3^2 + 2I_2I_3\cos\phi \]

But,

\[ I_2 = \frac{V}{R} \]

\[ I_1^2 = I_2^2 + I_3^2 + 2\frac{V}{R}I_3\cos\phi \]

Hence the power, \( P = VI_3\cos\phi \), is given by:

\[ P = \frac{R}{2}(I_1^2 - I_2^2 - I_3^2) \]

Also,

\[ \cos\phi = \frac{I_1^2 - I_2^2 - I_3^2}{2I_2I_3} \]

Advantages:
- The advantage of this method is that the value of determined is independent of supply frequency and waveforms.
- The disadvantages of measurement of power by three voltmeter method are overcome in this method.

**11. INDUCTION TYPE WATTMETER**

These types of watt-meters operate on the same working principle on which the induction type ammeter and voltmeter operates. These instruments can only be used on ac supply while dynamo-meter type watt meters can be used on either ac or dc supply system. Induction type watt-meters are useful only when the supply and frequency remains constant. Since both the coils i.e. current coil and pressure coils are necessary in such instrument, it is not essential to use shaded pole principle. Because for producing a deflecting torque, two fluxes are essential with suitable phase angle and it would be available from these two coils.
A watt-meter has two laminated electromagnet, one of which is excited by load current or definite fraction of it, and is connected in series with the circuit, known as series magnet and the other is excited by the current proportional to the applied voltage or fraction of it and is always connected across the supply, known as shunt magnet. An aluminum disc is so mounted so that it cuts the fluxes produced by both the magnets. As a result of which, two e.m.f are produced which induces two eddy currents in the disc. C - Magnet is used to provide necessary damping torque to the pointer, to damp out the oscillations. Deflecting torque is produced due to interaction of these eddy currents and the inducing flux. Copper shading bands are provided either on central limb or on the outer limb of the shunt magnet, and can be so adjusted as to make the resultant flux in the shunt magnet lag behind the applied voltage by 90. Both the wattmeters are provided with spiral springs A and B, for producing controlling torque to counter balance the deflecting torque. In Fig.a the spiral spring and damping magnet is omitted for simplicity. The scale of such type instruments is quite uniform and extends over an angle of 300. Currents up to 100 A can be handled by these watt-meters directly where as beyond this current transformers are used. Two types of induction type watt meters are available. Line diagrams of both of the types are detailed in Figures a and b.

![Diagram of Induction Type Wattmeter](image)

**Fig.14 Induction type Wattmeter**

In the form of the instrument shown in Fig. a, two pressure coils are connected in series in such a way that both of them send flux through the central limb. The series magnet
also carries two small current coils connected in series and wound so that they magnetized their respective cores in the same direction. Correct phase displacement between the fluxes produced by series and shunt magnet is obtained by the adjustment of copper shading band on the central limb.

Fig.15 Induction type Wattmeter

In Fig. b, there is only one pressure and one current coil. Two projecting poles of shunt magnet are surrounded by a copper shading band whose position can be adjusted for correcting the phase of the flux of this magnet with the applied voltage. The pressure coil circuit of induction type instrument is made as inductive as possible so that the flux of the shunt magnet may lag nearly by 90 behind the applied voltage.

Advantages
The advantages of induction watt meters are the same as those of induction ammeters long scale, freedom from effects of stray field, and have effective damping torque.

Disadvantages
Following are the disadvantage of the induction type instruments:

a) Change in temperature causes variation in the resistance of the moving element, affects the eddy currents therein, and so the operating torque. The error due to this is in part offset by a balancing effect due to change in temperature of the windings.
b) Change in frequency from that of the calibration value causes variations in both the reactance of the voltage coil circuit, which is highly inductive, and also in the amount of compensation from the phase compensating circuit. Within the limits of frequency variation met within practice on the mains, this last error is not important.

**12. ELECTRODYNAMOMETER WATTMETER**

Dynamometer type wattmeter works on very simple principle and this principle can be stated as "when any current carrying conductor is placed inside a magnetic field, it experiences a mechanical force and due to this mechanical force deflection of conductor takes place".

*Construction and Working Principle*

It consists of following parts

![Fig.16 Electrodynamometer](image)

There are two types of coils present in the electrodynamometer. They are:

(a) Moving coil: Moving coil moves the pointer with the help of spring control instrument. A limited amount of current flows through the moving coil so as to avoid heating. So in order to limit the current we have connect the high value resistor in series with the moving coil. The moving is air cored and is mounted on a pivoted spindle and can moves freely. In electrodynamometer type wattmeter, moving coil works as pressure coil. Hence moving coil is connected across the voltage and thus the current flowing through this coil is always proportional to the voltage.

(b) Fixed coil: The fixed coil is divided into two equal parts and these are connected in series with the load, therefore the load current will flow through these coils. Now the reason is very obvious of using two fixed coils instead of one, so that it can be constructed to carry considerable amount of electric current. These coils are called the current coils of electrodynamometer type wattmeter. Earlier these fixed coils are designed to carry the current...
of about 100 amperes but now the modern wattmeter are designed to carry current of about 20 amperes in order to save power.

(c) Control system: Out of two controlling systems i.e.
(1) Gravity control
(2) Spring control, only spring controlled systems are used in these types of wattmeter. Gravity controlled system cannot be employed because they will appreciable amount of errors.

(d) Damping system: Air friction damping is used, as eddy current damping will distort the weak operating magnetic field and thus it may leads to error.

(e) Scale: There is uniform scale is used in these types of instrument as moving coil moves linearly over a range of 40 degrees to 50 degrees on either sides.

Now let us derive the expressions for the controlling torque and deflecting torques. In order to derive these expressions let us consider the circuit diagram given.

We know that instantaneous torque in electrodynamic type instruments is directly proportional to product of instantaneous values of currents flowing through both the coils and the rate of change of flux linked with the circuit.

Let \( I_1 \) and \( I_2 \) be the instantaneous values of currents in pressure and current coils respectively.

So the expression for the torque can be written as:

\[
T = I_1 \times I_2 \times \frac{dM}{dx}
\]

where \( x \) is the angle.

Now let the applied value of voltage across the pressure coil be

\[
= 2V \sin \omega t
\]

Assuming the electrical resistance of the pressure coil be very high hence we can neglect reactance with respect to its resistance. In this the impedance is equal to its electrical resistance therefore it is purely resistive. The expression for instantaneous current can be written as \( I_2 = \frac{V \sin \omega t}{R_p} \) where \( R_p \) is the resistance of pressure coil.

\[
I_2 = 2 \times \frac{V \sin \omega t}{R_p}
\]

If there is phase difference between voltage and electric current, then expression for instantaneous current through current coil can be written as
\[ I_1 = I \ \text{t} = 2I \sin(\omega t - \Phi) \]

As current through the pressure coil in very very small compare to current through current coil hence current through the current coil can be considered as equal to total load current.

Hence the instantaneous value of torque can be written as

\[
\frac{V \sin \omega t \times 2I \sin \omega t - \Phi}{R_p} \ \frac{dM}{dx}
\]
Average value of deflecting torque can be obtained by integrating the instantaneous torque from limit 0 to T, where T is the time period of the cycle. Controlling torque is given by $T_c = Kx$ where $K$ is spring constant and $x$ is final steady state value of deflection.

**Advantages of Electrodynamometer Type Wattmeter**

Following are the advantages of electrodynamometer type wattmeters and they are written as follows:

(a) Scale is uniform upto certain limit.

(b) They can be used for both to measure ac as well dc quantities as scale is calibrated for both.

**Errors in Electrodynamometer Type Wattmeter**

Following are the errors in the electrodynamometer type wattmeters:

(a) Errors in the pressure coil inductance

(b) Errors may be due to pressure coil capacitance

(c) Errors may be due to mutual inductance effects

(d) Errors may be due connections (i.e. pressure coil is connected after current coil)

(e) Error due to Eddy currents

(f) Errors caused by vibration of moving system

(g) Temperature error

(h) Errors due to stray magnetic field

**MEASUREMENT OF THREE PHASE POWER BY TWO WATTMETERS METHOD**

In this method we have two types of connections

(a) Star connection of loads

(b) Delta connection of loads

When the star connected load, the diagram is shown in below- (c) Errors may be due to mutual inductance effects

(d) Errors may be due connections (i.e. pressure coil is connected after current coil)

(e) Error due to Eddy currents

(f) Errors caused by vibration of moving system

(g) Temperature error

(h) Errors due to stray magnetic field
For star connected load clearly the reading of wattmeter one is product phase current and voltage difference \((V_2-V_3)\). Similarly the reading of wattmeter two is the product of phase current and the voltage difference \((V_2-V_3)\). Thus the total power of the circuit is sum of the reading of both the wattmeters. Mathematically we can write

\[
P = P_1 + P_2 = I_1(V_1 + V_2) + I_2(V_2 - V_3)
\]

but we have \(I_1 + I_2 + I_3 = 0\), hence putting the value of \(I_1 + I_2 = -I_3\)

We get total power as \(V_1I_1 + V_2I_2 + V_3I_3\)

When delta connected load, the diagram is shown in below

---

Fig.17 Two Wattmeter method

Fig.18 Three Wattmeter method
The reading of wattmeter one can be written as

\[ P_1 = -V_3 (I_1 - I_3) \]

and reading of wattmeter two is

\[ P_2 = -V_2 (I_2 - I_1) \]

Total power is \[ P = P_1 + P_2 = V_2 I_2 + V_3 I_3 - I_1 (V_2 + V_3) \]

but \( V_1 + V_2 + V_3 = 0 \), hence expression for total power will reduce to \( V_1 I_1 + V_2 I_2 + V_3 I_3 \)

**MEASUREMENT OF THREE PHASE POWER BY ONE WATTMETER METHOD**

Limitation of this method is that it cannot be applied on unbalanced load. So under this condition we have \( I_1 = I_2 = I_3 = I \) and \( V_1 = V_2 = V_3 = V \)

Diagram is shown below

![Diagram](image)

**Fig. 19 Three Phase power measurement**

Two switches are given which are marked as 1-3 and 1-2, by closing the switch 1-3 we get reading of wattmeter as

\[ P_1 = V_{13} I_1 \cos(30 - \phi) = \sqrt{3} \times VI \cos(30 - \phi) \]

Similarly the reading of wattmeter when switch 1-2 is closed is

\[ P_2 = V_{12} I_1 \cos(30 + \phi) = \sqrt{3} \times VI \cos(30 + \phi) \]

Total power is \[ P_1 + P_2 = 3VI \cos \phi \]
THREE PHASE ELECTRODYNAMOMETER WATTMETER

A dynamometer type three-phase wattmeter consists of two separate wattmeter movements mounted together in one case with the two moving coils mounted on the same spindle.

- The arrangement is shown in Fig.
- There are two current coils and two pressure coils.
- A current coil together with its pressure coil is known as an element. Therefore, a three phase wattmeter has two elements.
- The connections of two elements of a 3 phase wattmeter are the same as that for two wattmeter method using two single phase wattmeter.
- The torque on each element is proportional to the power being measured by it.
- The total torque deflecting the moving system is the sum of the deflecting torque of the two elements.
- Hence the total deflecting torque on the moving system is proportional to the total Power.
- In order that a 3 phase wattmeter read correctly, there should not be any mutual interference between the two elements.
- A laminated iron shield may be placed between the two elements to eliminate the mutual effect

![Fig.20 Three Phase Electrodynamometer](image)
LOW POWER FACTOR ELECTRO-DYNAMOMETER TYPE WATTMETERS

- Small deflecting torque on the moving system even when the current and pressure coils are fully excited and
- Introduction of large error due to inductance of pressure coil at low power factor.

The special features incorporated in an electro-dynamometer type wattmeter to make it suitable for measurement of power in low power factor circuits are given below.

1. Pressure coil Circuit. The pressure coil circuit is made of low resistance in order to make the pressure coil current large resulting in increased operating torque. The pressure coil current in a low pf wattmeter may be as much as 10 times the value used for ordinary wattmeters.
2. Compensation For Pressure Coil current
3. Compensation For Inductance of Pressure Coil. The error caused by pressure coil inductance is \((\sin \Theta) / \cos \phi + \sin \Theta\) times the actual reading of wattmeter. Now with low pf, the value of \(\phi\) is large and, therefore, the error is large. The error caused by inductance of pressure coil is compensated by connecting a capacitor across a part of series resistance in the pressure coil circuit, as shown in fig

![Fig.21 low Power factor Electro dynamometer](image)
Reference books


SCHOOL OF ELECTRICAL AND ELECTRONICS
DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – III
MEASUREMENT AND INSTRUMENTATION – SIC 1203
UNIT 3 MEASUREMENT OF RESISTANCE, INDUCTANCE AND CAPACITANCE


CLASSIFICATION OF RESISTANCES

For the purposes of measurements, the resistances are classified into three major groups based on their numerical range of values as under:

- Low resistance (0 to 1 ohm)
- Medium resistance (1 to 100 kilo-ohm) and
- High resistance (>100 kilo-ohm)

Accordingly, the resistances can be measured by various ways, depending on their range of values, as under:

1. Low resistance (0 to 1 ohm): AV Method, Kelvin Double Bridge, potentiometer, doctor ohmmeter, etc.
2. Medium resistance (1 to 100 kilo-ohm): AV method, wheat stone’s bridge, substitution method, etc.
3. High resistance (>100 kilo-ohm): AV method, Fall of potential method, Megger, loss of charge method, substitution method, bridge method, etc.

1. LOW RESISTANCE

1.1 KELVIN DOUBLE BRIDGE

The Kelvin double bridge is one of the best devices available for the precise measurement of low resistances. It is the modification of wheatstone bridge by which the errors due to contact resistance and lead resistances are eliminated. This bridge is named double bridge because it contains a second set of ratio arms. An interesting variation of the Wheatstone bridge is the Kelvin Double bridge, used for measuring very low resistances (typically less than 1/10 of an ohm)

THEORY

Consider the bridge circuit shown in figure below. Here ‘r’ represents the resistance of the lead that connects the unknown resistance ‘R’ to standard resistance ‘S’. Two galvanometer
connections indicated by dotted lines are possible. The connection may be either to point 'm'
or to point 'n'. When the galvanometer is connected to point ‘m’ the resistance ‘r’ of the
connecting leads is added to the standard resistance ‘S’ resulting in indication of too low an
indication for unknown resistance ‘R’. When the connection made to point the resistance ‘r’ is
added to the unknown resistance resulting in indication of too high a value for ‘R’.
Suppose that instead of using point ‘m’ which gives a low result or ‘n’ which makes the result
High, we make the galvanometer connection to any intermediate point ‘d’ as shown by full line.
If at point ‘d’ the resistance ‘r’ is divided into two parts r1, r2 such that r1/r2 = P/Q

\[ \text{Balance condition} \]
\[ E_{45} = E_{513} \]

\[ \frac{a}{b} = \frac{R_1}{R_2} \]
\[ E_{45} = R_2 \frac{E}{R_1 + R_2} \]

Consider the path 5-1-2-6 back to 5 through the battery
\[ E = I \times [R_3 + R_y \frac{1}{(a+b)+R_x}] \]
\[ E = I \left[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \]

Sub eqn. 4 in eqn. 2
\[ E_{45} = \frac{R_2}{R_1 + R_2} \times I \left[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] \]
\[ V_{12} = I \left[ \frac{R_y (a+b)}{R_y + a+b} \right] \]

\[ V_{13} = \frac{b}{a+b} V_{12} \]

\[ V_{13} = \frac{b}{a+b} \left[ \frac{R_y (a+b)}{R_y + a+b} \right] \]

\[ E_{513} = IR_3 + V_{13} \]

\[ E_{513} = IR_3 + I \left[ \frac{b}{a+b} \left[ \frac{R_y (a+b)}{R_y + a+b} \right] \right] \]

\[ E_{45} = E_{513} \]

\[ \frac{IR_2}{R_1 + R_2} \left[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} \right] = I \left[ R_3 + \frac{b}{a+b} \left[ \frac{R_y (a+b)}{a+b+R_y} \right] \right] \]

\[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \frac{R_1 + R_2}{R_2} \left[ R_3 + \frac{b}{a+b} \left[ \frac{R_y (a+b)}{a+b+R_y} \right] \right] \]

\[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = \left[ 1 + \frac{R_1}{R_2} \right] \left[ R_3 + \frac{bR_y}{R_y + a+b} \right] \]

\[ R_3 + R_x + \frac{(a+b)R_y}{a+b+R_y} = R_3 + \frac{R_1 R_3}{R_2} + \frac{bR_y}{R_y + a+b} + \frac{R_1 b R_y}{R_2 (R_y + a+b)} \]

\[ R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{R_y + a+b} + \frac{R_1 b R_y}{R_2 (R_y + a+b)} \]

\[ R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{R_2 (R_y + a+b)} - \frac{aR_y}{(a+b+R_y)} \]

\[ R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{(R_y + a+b)} \left[ \frac{R_1}{R_2} - \frac{a}{b} \right] \]
Then the presence of the resistance of connecting leads causes no error in the result. We have, 1 Therefore we conclude that making the galvanometer connection as at C, the resistance of leads does not affect the result. The process described above is obviously not a practical way of achieving the desired result, as there would certainly be a trouble in determining the correct point for galvanometer connection. It does however suggest the simple modification that two actual resistance units of correct ratio be connected between points ‘m’ and ‘n’ the galvanometer be connected to the junction of the resistors. This is the actual Kelvin bridge arrangement.

\[
\frac{a}{b} = \frac{R_1}{R_2} \quad \text{thus} \quad \frac{R_1}{R_2} \cdot \frac{a}{b} = 0
\]

\[
R_S = \frac{R_1 R_2}{R_S}
\]

Then the presence of the resistance of connecting leads causes no error in the result. We have, 1 Therefore we conclude that making the galvanometer connection as at C, the resistance of leads does not affect the result. The process described above is obviously not a practical way of achieving the desired result, as there would certainly be a trouble in determining the correct point for galvanometer connection. It does however suggest the simple modification that two actual resistance units of correct ratio be connected between points ‘m’ and ‘n’ the galvanometer be connected to the junction of the resistors. This is the actual Kelvin bridge arrangement.

![Fig.2 Kelvin double bridge](image)

The Kelvin double bridge incorporates the idea of a second set of ratio arms, hence the name of double bridge- and the use of four terminal resistors for the low resistance arms. Figure shows the schematic diagram of the Kelvin Bridge. The first of ratio arms is P and Q. The second set of ratio arms, p and q is used to connect the galvanometer to a point ‘d’ at the appropriate potential between points ‘m’ and ‘n’ to eliminate the effect of connecting lead of resistance ‘r’ between the known resistance ‘R’ and the standard resistance ‘S’. The ratio p /q is made equal to P/Q. Under balance conditions there is no current through the galvanometer, which means that the voltage drop between a and b, E is equal to the voltage drop Ed between a and b.
Now if \( p/q = p/q \) becomes \( R = P / Q \) \( *S \) Above equation is the usual working equation for the Kelvin double bridge. It indicates that the resistance of connecting lead ‘r’ has no effect on the measurement provided that the two sets of ratio arms have equal ratios. The above equation is useful however as it shows the error that is introduced in case the ratios are not exactly equal. It is indicated that it is desirable to keep ‘r’ as small as possible in order to minimize the errors in case there is a difference between ratios \( P / Q \) and \( p/q \). In a typical Kelvin bridge, the range of resistance calculated is 0.1S to 1.0S.

2. MEDIUM RESISTANCE

2.1 VOLTMETER AMMETER METHOD

In this method ammeter reads the true value of the current through the resistance but voltmeter does not measure the true voltage across the resistance. Voltmeter reads sum of voltage across ammeter and resistance \( R_a \)=resistance of ammeter \( R_m \)= measured value

\[
R_m = V/I = (V_r+V_a)/I
\]
\[
= I(R+R_a)/I
\]
\[
R_m = (R+R_a)
\]

True Value of Resistance
\[
R = (R_m - R_a)
\]

Relative error \( \varepsilon_r = R_a / R \)

Thus measured value is higher than true value. True value is equal to measured value only if the ammeter resistance \( R_a \) is zero.
\[ R_m = \frac{V}{I} = \frac{V}{I_r + I_v} \]
\[ V / V (R / (1+R/R_v)) \]
\[ R = R_v R_m / (R_v - R_m) \]

Relative error, \( \varepsilon_r = (R_m - R) / R = \frac{Ra}{R} \)

If the value of resistance under measurement is large as compared to internal resistance of ammeter, the error in measurement would be small.

**Advantage**

Easy, simple, rough method.

**Disadvantage**
- At full scale error may be around 0-1%.
- Errors sometimes considerably high.

### 2.2 SUBSTITUTION METHOD

A --- Ammeter

r---regulating resistance

S---standard variable resistance

R--- Unknown resistance

S/W ---switch for putting R and S alternatively into circuit.

![Circuit diagram](image)

**Fig.4:Circuit diagram**

**Procedure**

Case 1: Resistance R in the circuit.

- S/W is set to position 1.
This brings R into the circuit.

- r is adjusted to give ammeter the chosen scale mark.

Case 2: Resistance S in the circuit.
- Change S/W to position 2.
- This brings S in to the circuit.
- Adjust S to give same chosen scale mark by ammeter.
- The substitution for one resistance by another has left current unaltered.
- The value of S gives the value of R.

**Advantage**
- More accurate than ammeter voltmeter method as no error as the case of ammeter voltmeter method.
- Many applications in bridge method.
- Used in high frequency ac measurement.

**Disadvantage**
- Accuracy depends upon constancy of battery emf and resistance of circuit (excluding R & S)
- Also depends upon accuracy of measurement of S.

### 2.3 WHEATSTONE BRIDGE

For measuring accurately any electrical resistance Wheatstone bridge is widely used. There are two known resistors, one variable resistor and one unknown resistor connected in bridge form as shown below. By adjusting the variable resistor the current through the Galvanometer is made zero. When the current through the galvanometer becomes zero, the ratio of two known resistors is exactly equal to the ratio of adjusted value of variable resistance and the value of unknown resistance. In this way the value of unknown electrical resistance can easily be measured by using a Wheatstone Bridge.

**THEORY**

The general arrangement of Wheatstone bridge circuit is shown in the figure below. It is a four arms bridge circuit where arm AB, BC, CD and AD are consisting of electrical resistances P, Q, S and R respectively. Among these resistances P and Q are known fixed electrical resistances and these two arms are referred as ratio arms. An accurate and sensitive
Galvanometer is connected between the terminals B and D through a switch S2. The voltage source of this Wheatstone bridge is connected to the terminals A and C via a switch S1 as shown. A variable resistor S is connected between point C and D.

The potential at point D can be varied by adjusting the value of variable resistor. Suppose current $I_1$ and current $I_2$ are flowing through the paths ABC and ADC respectively. If we vary the electrical resistance value of arm CD the value of current $I_2$ will also be varied as the voltage across A and C is fixed. If we continue to adjust the variable resistance one situation may comes when voltage drop across the resistor S that is $I_2.S$ is becomes exactly equal to voltage drop across resistor Q that is $I_1.Q$. Thus the potential at point B becomes equal to the potential at point D hence potential difference between these two points is zero hence current through galvanometer is nil. Then the deflection in the galvanometer is nil when the switch S2 is closed.

Now, from Wheatstone bridge circuit

$$\text{current } I_1 = \frac{V}{P + Q}$$

And
Now potential of point B in respect of point C is nothing but the voltage drop across the resistor Q and this is

\[ I_{1Q} = \frac{V_Q}{P + Q} \]  

Again potential of point D in respect of point C is nothing but the voltage drop across the resistor S and this is

\[ I_{2S} = \frac{V_S}{R + S} \]  

Equating, equations (i) and (ii) we get,

\[
\frac{V_Q}{P + Q} = \frac{V_S}{R + S} \Rightarrow \frac{Q}{P + Q} = \frac{S}{R + S} \\
\Rightarrow \frac{Q}{S} = \frac{P + Q}{P + S} = \frac{P}{Q} + 1 = \frac{P}{Q} + \frac{R}{S} + 1 = \frac{P}{Q} + \frac{R}{S} \\
\Rightarrow R = S \times \frac{P}{Q}
\]

3. MEASUREMENT OF HIGH RESISTANCE

There are different methods that can be employed for the measurement of high resistances. Some of the important methods are as follows.

i) Direct deflection method

ii) Loss of charge method

iii) Megohm bridge

iv) Megger

3.1 Direct Deflection Method

In this method, a high resistance (more than 1000 Ω) and very sensitive moving coil galvanometer is connected in series with the resistances to be measured along with supply voltage as shown in the Figure 1.
The direct deflection method is used for very high resistances such as insulation resistances cables. A sensitive galvanometer is used in place of micro ammeter.

The Figure 1 shows this method making use of with and without Guard wire at the terminal A. The Galvanometer G measures the current Ig between the conductor and the metal sheath. The leakage current IL is actually carried out by the guard wire which ultimately do not flow through the galvanometer and thereby eliminating the source of error.

Cables without the metal sheaths can be tested in similar way but the cable is immersed in water tank first except its end where connections are made. The cable should be immersed for at least 24 hours in slightly alkaline water at a room temperature (approx. 20°C) which will provide return path for the current as in Figure 1. The readings obtained give the Volume resistance of the conductor. The insulation resistance of the cable is given by,

\[ R = V/I \]

In some cases, the deflection of the galvanometer is observed and its scale is afterwards calibrated by replacing the insulation by a standard high resistance (usually 1MΩ), the galvanometer shunt being varied, as required to give a deflection on the same order as before.

In tests on cable the galvanometer should be short-circuited before applying the voltage. The short-circuiting connection is removed only after sufficient time is elapsed so that charging and absorption currents cases to flow. The galvanometer should be well shunted during the early stages of measurement, and it is normally desirable to influence a protective series resistance (of several megaohm) in the galvanometer circuit. The value of this resistance should be subtracted from the observed resistance value in order to determine the true resistance. A high voltage battery of 500V emf is required and its emf should remain constant throughout the test.
3.2 Loss of charge method

In loss of charge method unknown resistance is connected in parallel with capacitor and electrostatic voltmeter. The capacitor is initially charged to some suitable voltage by means of a battery of voltage $V$ and then allowed to discharge through the resistance. The terminal voltage is observed during discharge and it is given by,

\[ V = v \exp\left(-\frac{t}{CR}\right) \]

Or insulation resistance is given by,

\[ R = \frac{t}{(C \ln \frac{V}{v})} = 0.4343 \frac{t}{(C \log \frac{V}{v})} \]

Fig.8 Circuit Diagram

Fig.9 Voltage vs Time

From above equation it follows that if $V$, $v$, $C$ and $t$ are known the value of $R$ can be computed. If the resistance $R$ is very large the time for an appreciable fall in voltage is very large and thus this process may become time consuming. Also the voltage-time curve will thus be very flat and unless great care is taken in measuring voltages at the beginning and at the end of time $t$, a serious error may be made in the ratio $V/v$ causing the considerable corresponding error in the measured value of $R$. More accurate results may be obtained by change in the
voltage \( V-v \) directly and calling this change as \( e \), the expression for \( R \) becomes:

\[
R = \frac{0.4343 \, t}{C \, \log_{10} \left( \frac{V}{V-e} \right)}
\]

This change in voltage may be measured by a galvanometer. However, from the experimental point of view, it may be advisable to determine the time \( t \) from the discharge curve of the capacitor by plotting curve of \( \log v \) against time \( t \). This curve is linear as shown in second figure and thus determination of time \( t \) from this curve for the voltage to fall from \( V \) to \( v \) yields more accurate results. Loss of charge method is applicable to some high resistances, but it requires a capacitor of very high leakage resistance as high as resistance being measured. The method is very attractive if the resistance being measured is the leakage resistance of a capacitor as in this case auxiliary \( R \) and \( C \) units are not required.

Actually in this method, the true value of resistance is not measured, since it is assumed that the value of resistance of electrostatic voltmeter and the leakage resistance of the capacitor have infinite value. But in practice corrections must be applied to take into consideration the above two resistances. Let \( R_1 \) be the leakage resistance of the capacitor. Also \( R' \) be the equivalent resistance of the parallel resistances \( R \) and \( R_1 \).

Then discharge equation of capacitor gives,

\[
R' = \frac{0.4343 \, t}{C \, \log_{10} \left( \frac{V}{V/v} \right)}
\]

The test is then repeated with the unknown resistance \( R \) disconnected and the capacitor discharging through \( R_1 \). The value of \( R_1 \) obtained from this second test and substituted into the expression,

\[
R' = \frac{(R \, R_1)}{(R+R_1)}
\]

In order to get value of \( R \),

The leakage resistance of the voltmeter, unless very high resistance should also be
taken into consideration.

3.3 Megohm bridge

Megohm bridge is another important method for measurement of high resistances. It has one three terminal high resistance located in one arm of the bridge. Figure shows the very high resistance with terminals A and B, and a guard terminal, which is put on the insulation. So it forms a three terminal resistance.

![Three terminal resistance](image)

**Fig.11 circuit Diagram**

Let us consider take the hypothetical case of a 100 Mohm resistance and assume that this resistance is measured by an ordinary Wheatstone bridge. It is clear that Wheatstone will measure a resistance of $100 \times \frac{200}{100+200} = 67$ Mohm instead of 100 Mohm thus the error is 33 percent.

![Wheatstone Bridge](image)

**Fig.12 Wheatstone Bridge**
However if the same resistance is measured by a modified Wheatstone bridge as shown in fig b) with the guard connection G connected as indicated, the error in measurement will be reduced and this modified Wheatstone bridge is called megohm bridge.

The arrangement of above figure illustrated the operation of Megohm Bridge.

Figure shows the circuit of the completely elf-contained Megohm Bridge which includes power supplies, bridge members, amplifiers, and indicating instrument. It has range from 0.1MΩ to 10^6MΩ. The accuracy is within 3% for the lower part of the range to possible 10% above 10000MΩ.

Sensitivity of balancing against high resistance is obtained by using an adjustable high voltage supplies of 500V or 1000V and the use of a sensitive null indicating arrangement such as a high gain amplifier with an electronic voltmeter or a C.R.O. The dial on Q is calibrated 1-10-100-1000MΩ, with main decade 1-10 occupying greater part of the dial space. Since unknown resistance R=PS/Q, the arm Q is made, tapered, so that the dial calibration is approximately logarithmic in the main decade, 1-10. Arm S give five multipliers, 0.1,1,10,100 and 1000.

3.4 MEGGER
1) **Deflecting & Control coil**: Connected parallel to the generator, mounted at right angle to each other and maintain polarities in such a way to produced torque in opposite direction.
2) **Permanent Magnets**: Produce magnetic field to deflect pointer with North-South pole magnet.
3) **Pointer**: One end of the pointer connected with coil another end deflects on scale from infinity to zero.
4) **Scale**: A scale is provided in front-top of the megger from range ‘zero’ to ‘infinity’, enable us to read the value.
5) **D.C generator or Battery connection**: Testing voltage is produced by hand operated D.C generator for manual operated Megger. Battery / electronic voltage charger is provided for automatic type Megger for same purpose.
6) **Pressure coil resistance and Current coil resistance**: Protect instrument from any damage because of low external electrical resistance under test.
**Working Principle of Megger**

- Voltage for testing produced by hand operated Megger by rotation of crank in case of hand operated type, a battery is used for electronic tester.
- 500 Volt DC is sufficient for performing test on equipment range up to 440 Volts.
- 1000V to 5000V is used for testing for high voltage electrical systems.
- Deflecting coil or current coil connected in series and allows flowing the electric current taken by the circuit being tested.
- The control coil also known as pressure coil is connected across the circuit. Current limiting resistor (CCR & PCR) connected in series with control & deflecting coil to protect damage in case of very low resistance in external circuit.
- In hand operated megger electromagnetic induction effect is used to produce the test voltage i.e. armature安排s to move in permanent magnetic field or vice versa.
- Where as in electronic type megger battery are used to produce the testing voltage.
- As the voltage increases in external circuit the deflection of pointer increases and deflection of pointer decreases with an increase of current.
- Hence, resultant torque is directly proportional to voltage & inversely proportional to current.
- When electrical circuit being tested is open, torque due to voltage coil will be maximum & pointer shows ‘infinity’ means no shorting throughout the circuit and has maximum resistance within the circuit under test.
- If there is short circuit pointer shows ‘zero’, which means ‘NO’ resistance within circuit being tested.

Work philosophy based on ohm-meter or ratio-meter. The deflection torque is produced with
megger tester due to the magnetic field produced by voltage & current, similarly like ‘Ohm's Law’ Torque of the megger varies in ratio with V/I, (Ohm's Law : - V=IR or R=V/I). Electrical resistance to be measured is connected across the generator & in series with deflecting coil. Produced torque shall be in opposite direction if current supplied to the coil.

1. High resistance = No current :- No current shall flow through deflecting coil, if resistance is very high i.e. infinity position of pointer
2. Small resistance = High current :- If circuit measures small resistance allows a high electric current to pass through deflecting coil, i.e. produced torque make the pointer to set at ‘ZERO’.
3. Intermediate resistance = varied current :- If measured resistance is intermediate, produced torque align or set the pointer between the range of ‘ZERO to INFINITY’

**Maxwell’s inductance bridge**

The choke for which R1 and L1 have to measure connected between the points ‘A’ and ‘B’. In this method the unknown inductance is measured by comparing it with the standard inductance.

L2 is adjusted, until the detector indicates zero current.

Let R1=unknown resistance
L1= unknown
L2 is adjusted, until the detector indicates zero current.

Let R1= unknown resistance

L1= unknown inductance of the choke.
L2= known standard inductance
R1,R2,R4= known resistances

![Maxwell's Inductance Bridge](image)

**Fig 14 Circuit Diagram**
Impedance of arm ab, \( Z_1 = (R_1 + j\omega L_1) \)
Impedance of arm ad, \( Z_2 = (R_2 + r_2 + j\omega L_2) \)
Impedance of arm bc, \( Z_3 = R_3 \)
Impedance of arm cd, \( Z_4 = R_4 \)
Hence for balanced bridge,

### Table 1: Maxwell Bridge Parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>IMPEDANCE ((Z))</th>
<th>ADMITTANCE ((Y)) [(Y=1/Z)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance(R)</td>
<td>(R)</td>
<td>(1/R)</td>
</tr>
<tr>
<td>Inductance(L)</td>
<td>(j\omega L)</td>
<td>(1/j\omega L)</td>
</tr>
<tr>
<td>Capacitance(C)</td>
<td>(1/j\omega C)</td>
<td>(j\omega C)</td>
</tr>
</tbody>
</table>

\(Z_1Z_4 = Z_2Z_3\)
\((R_1 + j\omega L_1) R_4 = (R_2 + r_2 + j\omega L_2) R_3\)
\(R_1 R_4 + j\omega L_1 R_4 = R_2 R_3 + r_2 R_3 + j\omega L_2 R_3\)
\(R_1 R_4 + j\omega L_1 R_4 = (R_2 + r_2) R_3 + j\omega L_2 R_3\)

- **Equating real and imaginary parts**
- \(R_1 R_4 = (R_2 + r_2) R_3\)
  - \(R_1 = (R_2 + r_2) (R_3/R_4)\)
- \(L_1 R_4 = L_2 R_3\)
  - \(L_1 = L_2 (R_3/R_4)\)
- **Thus unknown inductance \(L_1\) and its resistance \(R_1\) may be calculated.**
Advantage

Expression for R1 and L1 are simple.
Equations area simple
They do not depend on the frequency (as w is cancelled) R1
and L1 are independent of each other

Disadvantage

Variable inductor is costly.
Variable inductor is bulky

Maxwell's Inductance

Capacitance Bridge

The unknown inductance is measured with the help of the standard variable capacitance.

\[ L_1 = \text{Unknown inductance with resistance } R_1 \]

\[ C_4 = \text{variable standard capacitor} \]

\[ R_2, R_3, R_4 = \text{Known non inductive resistances} \]

Impedance of arm ab, \( Z_1 = (R_1 + j\omega L_1) \)

Impedance of arm ad, \( Z_2 = R_2 \)

Impedance of arm bc, \( Z_3 = R_3 \)

Impedance of arm cd, \( Z_4 = 1/JY_4 \quad (R_4 \text{ parallel to } C_4) \)

\[ Y_4 = (1/R_4) + j\omega C_4 \]

Hence for balanced bridge,

\[ Z_1Z_4 = Z_2Z_3 \]

\[ Z_1(1/JY_4) = Z_2Z_3 \]

\[ Z_1 = Z_2Z_3Y_4 \]

\[ (R_1 + j\omega L_1) = R_2R_3 \left( (1/R_4) + j\omega C_4 \right) \]

\[ (R_1 + j\omega L_1) = (R_2R_3/R_4) + j\omega C_4R_2R_3 \]

Equating real and imaginary parts

\[ R_1 = (R_2R_3/R_4) \]
Thus unknown inductance \( L_1 \) and its resistance \( R_1 \) may be calculated

### Table 2: Maxwell Capacitance Bridge

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Impedance (Z)</th>
<th>Admittance ([Y=1/Z])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (R)</td>
<td>( R )</td>
<td>( 1/R )</td>
</tr>
<tr>
<td>Inductance (L)</td>
<td>( j\omega L )</td>
<td>( 1/j\omega L )</td>
</tr>
<tr>
<td>Capacitance (C)</td>
<td>( 1/j\omega C )</td>
<td>( j\omega C )</td>
</tr>
</tbody>
</table>

**HAY’S BRIDGE**

Hay's bridge is modified Maxwell bridge, now question arises here in our mind that where we need to do modification.

- The **Hay’s bridge** is used for determining the self-inductance of the circuit.
- The bridge is the **advanced form** of Maxwell's bridge.
- The Maxwell’s bridge is only appropriate for measuring the medium quality factor.
- Hence, for measuring the high-quality factor the Hays bridge is used in the circuit.
- In Hay’s bridge, the capacitor is connected in series with the resistance, the voltage drop across the capacitance and resistance are varied.
- And in Maxwell bridge, the capacitance is connected in parallel with the resistance.
- Thus, the magnitude of a voltage pass through the resistance and capacitor is equal.

**Fig 16 Hays Bridge**

The unknown inductor \( L_1 \) is placed in the arm \( ab \) along with the resistance \( R_1 \). This unknown inductor is compared with the standard capacitor \( C_4 \) connected across the arm \( cd \). The resistance \( R_4 \) is connected in series with the capacitor \( C_4 \). The other two non-inductive resistor \( R_2 \) and \( R_3 \) are connected in the arm \( ad \) and \( bc \) respectively. The \( C_4 \) and \( R_4 \) are
adjusted for making the bridge in the balanced condition. When the bridge is in a balanced condition, no current flows through the detector which is connected to point b and c respectively. The potential drops across the arm ad and cd are equal and similarly, the potential across the arm ab and bc are equal.

Let,
- \( L_1 \) = unknown inductance having a resistance \( R_1 \)
- \( R_2, R_3, R_4 \) = known non-inductive resistance.
- \( C_4 \) = standard capacitor

At balance condition,
\[
(R_1 + j\omega L_1)(R_4 - j/\omega C_4) = R_2 R_3
\]
\[
R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3
\]

Separating the real and imaginary term, we obtain
\[
R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \quad \text{and} \quad L_1 = \frac{-R_1}{\omega^2 R_4 C_4}
\]

Solving the above equation, we have
\[
L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2}
\]
\[
R_1 = \frac{\omega^2 C_4^2 R_2 R_3 R_4}{1 + \omega^2 R_4^2 C_4^2}
\]

The equation of the unknown inductance and capacitance consists frequency term. Thus for finding the value of unknown inductance the frequency of the supply must be known.

For the high-quality factor, the frequency does not play an important role.
\[
Q = \frac{1}{\omega^2 C_4 R_4}
\]

Substituting the value of Q in the equation of unknown inductance, we get
\[
L_1 = \frac{R_2 R_3 C_4}{1 + (1/Q)^2}
\]

For greater value of Q the \( 1/Q \) is neglected and hence the equation become
\[
L_1 = R_2 R_3 C_4
\]

The quality factor of the coil is
\[
Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega^2 C_4 R_4}
\]
The phasor diagram of the Hay’s bridge is shown in the figure below. The magnitude and the phase of the $E_3$ and $E_4$ are equal and hence they are overlapping each other and draw on the horizontal axis. The current $I_1$ flow through the purely resistive arm $bd$. The current $I_1$ and the potential $E_3 = I_3 R_3$ are in the same phase and represented on the horizontal axis. The current passes through the arm $ab$ produces a potential drop $I_1 R_1$ which is also in the same phase of $I_1$. The total voltage drop across the arm $ab$ is determined by adding the voltage $I_1 R_1$ and $\omega I_1 L_1$.

The voltage drops across the arm $ab$ and $ad$ are equal. The voltage drop $E_1$ and $E_2$ are equal in magnitude and phase and hence overlap each other. The current $I_2$ and $E_2$ are in the same phase as shown in the figure above. The current $I_2$ flows through the arms $cd$ and produces the $I_2 R_4$ voltage drops across the resistance and $I_2/\omega C_4$ voltage drops across the capacitor $C_4$. The capacitance $C_4$ lags by the currents $90^\circ$.

The voltage drops across the resistance $C_4$ and $R_4$ gives the total voltage drops across the arm $cd$. The sum of the voltage $E_1$ and $E_3$ or $E_2$ and $E_4$ gives the voltage drops $E$.

Comparing imaginary part

**Advantage**

- Fixed capacitor is cheaper than variable capacitor.

- This bridge is best suitable for measuring high value of Q-factor

**Disadvantage**

- Equations of $L_1$ and $R_1$ are complicated.

- Measurement of $R_1$ and $L_1$ require the value of frequency

- This bridge cannot be used for measuring low Q-factor
ANDERSON’S BRIDGE

- The Anderson’s bridge gives the accurate measurement of self-inductance of the circuit.
- The bridge is the advanced form of Maxwell’s inductance capacitance bridge.
- In Anderson bridge, the unknown inductance is compared with the standard fixed capacitance which is connected between the two arms of the bridge.
- The bridge has fours arms ab, bc, cd, and ad.
- The arm ab consists unknown inductance along with the resistance.
- And the other three arms consist the purely resistive arms connected in series with the circuit.
- The static capacitor and the variable resistor are connected in series and placed in parallel with the cd arm.
- The voltage source is applied to the terminal a and c.

**Fig.18 Anderson bridge**

- The phasor diagram of the Anderson bridge is shown in the figure below. The current I1 and the E3 are in phase and represented on the horizontal axis. When the bridge is in balance condition the voltage across the arm bc and ec are equal. The current enters into the bridge is divided into the two parts I1 and I2. The I1 is entered into the arm ab and causes the voltage drop I1(R1+R) which is in phase with the I1. As the bridge is in the balanced condition, the same current is passed through the arms bc and ec. The voltage drop E4 is equal to the sum of the IC/ωC and the IC r. The current I4 and the voltage E4 are in the same phase and representing on the same line of the phasor diagram.
The sum of the current IC and I4 will give rise to the current I2 in the arm ad. When the bridge is at balance condition the emf across the arm ab and the point a, d and e are equal. The phasor sum of the voltage across the arms ac and de will give rise the voltage drops across the arm ab. The V1 is also obtained by adding the I1(R1+r1) with the voltage drop of I1L1 in the arm AB. The phasor sum of the E1 and E3 or E2 and E4 will give the supply voltage. Let, Li: unknown inductance having a resistance .R1,R2,R3,R3-know non inductive resistance

At balance Condition, \[ I_1 = I_3 \text{ and } I_2 = I_C + I_4 \]

Now, \[ I_1 R_3 = I_C \times \frac{1}{j \omega C} \]
\[ I_C = I_1 \omega CR_3 \]

The other balance condition equation is expressed as
\[ I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_C r \]
\[ I_C \left( r + \frac{1}{j \omega C} \right) = (I_2 - I_C) R_4 \]

By substituting the value of I_C in the above equation we get,
\[ I_1(r_1 + R_1 + j\omega L_1) = I_2 R_2 + I_1 j\omega CR_3 r \]
\[ I_1(r_1 + R_1 + j\omega L_1 - j\omega CR_3 r) = I_2 R_2 \]

and
\[ I_1(R_3 + j\omega R_3 R_4 + j\omega CR_3 r) = I_2 R_4 \]

on equating the equation, we get
\[ I_1(r_1 + R_1 + j\omega L_1 - j\omega CR_3 r) = I_1 \left( \frac{R_1 R_2}{R_3} + \frac{j\omega CR_3 r R_2}{R_4} + j\omega CR_3 R_2 \right) \]

Equating the real and the imaginary part, we get
\[ R_1 = \frac{R_1 R_3}{R_4} - r_1 \]
\[ L_1 = C \frac{R_3}{R_4} [4(R_4 + R_2) + R_2 R_4] \]

The above equation obtained is more complex that we have obtained in Maxwell bridge. On observing the above equations we can easily say that to obtain convergence of balance more easily, one should make alternate adjustments of r1 and r in Anderson’s bridge. Now let us look how we can obtain the value of unknown inductor experimentally. At first set the signal generator frequency at audible range. Now adjust r1 and r such that phones gives a minimum sound. Measure the values of r1 and r (obtained after these adjustments) with the help of multimeter. Use the formula that we have derived above in order to find out the value of
unknown inductance. The experiment can be repeated with the different value of standard capacitor.

**Phasor Diagram of Anderson's Bridge**

Let us mark the voltage drops across ab, bc, cd and ad as e1, e2, e3 and e4 as shown

![Phasor Diagram](image)

**Phasor Diagram for Andersons**

Fig.19 Phasor Diagram

Here in the phasor diagram of Anderson's bridge, we have taken i1 as reference axis. Now ic is perpendicular to i1 as capacitive load is connected at ec, i4 and i2 are lead by some angle as shown in figure. Now the sum of all the resultant voltage drops i.e. e1, e2, e3 and e4 is equal to e, which is shown in phasor diagram. As shown in the phasor diagram of Anderson's bridge the resultant of voltages drop i1 (R1 + r1) and i1.ω.l1 (which is shown perpendicular to i1) is e1. e2 is given by i2.r2 which makes angle 'A' with the reference axis. Similarly, e4 can be obtained by voltage drop i4.r4 which is making angle 'B' with reference axis.

**Advantages of Anderson's Bridge**

1. It is very easy to obtain the balance point in Anderson's bridge as compared to Maxwell bridge in case of low quality factor coils.
2. There is no need of variable standard capacitor is required instead of thin a fixed value capacitor is used.
3. This bridge also gives accurate result for determination of capacitance in terms of inductance.

**Disadvantages of Anderson's Bridge**

1. The equations obtained for inductor in this bridge is more complex as complex as compared to Maxwell's bridge.
2. The addition of capacitor junction increases complexity as well as difficulty of shielding the bridge.
Measurement of Mutual Inductance Heaviside Bridge

Let us consider two coils connected in series as shown in figure given below.

Such that the magnetic fields are additive, the resultant inductor of these two can be calculated as

$$ L_x = L_1 + L_2 + 2M $$

Where, $L_1$ is the self inductor of first coil, $L_2$ is the self inductor of second coil, $M$ is the mutual inductor of these two coils. Now if the connections of any one of the coils is reversed then we have

$$ L_y = L_1 + L_2 - 2M $$

On solving these two equations we have

$$ M = \frac{L_x - L_y}{4} $$

Thus the mutual inductor of the two coils connected in series is given by one-fourth of the difference between the measured value of self inductor when taking the direction of field in the same direction and value of self inductor when the direction of field is reversed. However, one needs to have the two series coils on the same axis in order to get most accurate result. Let us consider the circuit of Heaviside mutual inductor bridge, given below.
Main application of this bridge in industries is to measure the mutual inductor in terms of self inductance. Circuit of this bridge consists of four non inductive resistors $r_1$, $r_2$, $r_3$ and $r_4$ connected on arms 1-2, 2-3, 3-4 and 4-1 respectively. In series of this bridge circuit an unknown mutual inductor is connected. A voltage is applied across terminals 1 and 3. At balance point electric current flows through 2-4 is zero hence the voltage drop across 2-3 is equal to voltage drop across 4-3. So by equating the voltage drops of 2-4 and 4-3 we have,

$$i_1 r_3 = i_2 r_4$$

Also we have,

$$(i_1 + i_2)(j \omega M) + i_1(r_1 + r_3 + j \omega l_1) = i_2(r_2 + r_4 + j \omega l_2)$$

Therefore,

$$i_2 \frac{r_4}{r_3 + 1} j \omega M + i_2 \frac{r_2}{r_3} (r_1 + r_3 + j \omega l_1) = i_2(r_2 + r_4 + j \omega l_2)$$

or,

$$j \omega M \left( \frac{r_4}{r_3 + 1} + 1 \right) + \frac{r_4}{r_3} r_1 + r_4 + j \omega l_1 \frac{r_4}{r_3} = r_2 + r_4 + j \omega l_2$$

Thus,

$$r_1 = r_2 \frac{r_3}{r_4}$$

and mutual inductor is given by,

$$\frac{r_3 l_2 - r_4 l_1}{r_3 + r_4}$$
Measurement of Capacitance Schering Bridge

This bridge is used to measure the capacitance of the capacitor, dissipation factor and measurement of relative permittivity. Let us consider the circuit of Schering bridge

![Schering Bridge Diagram]

Fig.22 Schering Bridge

Here, \( c_1 \) is the unknown capacitance whose value is to be determined with series electrical resistance \( r_1 \). \( c_2 \) is a standard capacitor. \( c_4 \) is a variable capacitor. \( r_3 \) is a pure resistor (i.e. non inductive in nature). And \( r_4 \) is a variable non inductive resistor connected in parallel with variable capacitor \( c_4 \). Now the supply is given to the bridge between the points a and c.

The detector is connected between b and d. From the theory of ac bridges we have at balance condition,

Substituting the values of \( z_1, z_2, z_3 \) and \( z_4 \) in the above equation, we get

\[
\left( r_1 + \frac{1}{j\omega c_1} \right) \left( \frac{r_4}{1 + j\omega c_4 r_4} \right) = \frac{r_3}{j\omega c_2}
\]

\[
(r_1 + \frac{1}{j\omega c_1})r_4 = \frac{r_3}{j\omega c_2}(1 + j\omega c_4 r_4)
\]

\[
r_1 r_4 - \frac{jr_4}{\omega c_1} = -\frac{jr_3}{\omega c_2} + \frac{r_3 r_4 c_4}{c_2}
\]

Equating the real and imaginary parts and the separating we get,
consider the phasor diagram of the above Schering bridge circuit and mark the voltage drops across ab, bc, cd and ad as e₁, e₃, e₄ and e₂ respectively. From the above Schering bridge phasor diagram, we can calculate the value of tanδ which is also called the dissipation factor.

\[
tan\delta = \omega c_1 r_1 = \omega \frac{c_2 r_4}{r_3} \times \frac{r_3 c_4}{c_2} = \omega c_4 r_4
\]

**DE SAUTY’S BRIDGE**

This bridge provides us the most suitable method for comparing the two values of capacitor if we neglect dielectric losses in the bridge circuit. The circuit of De Sauty’s bridge is shown.

Battery is applied between terminals marked as 1 and 4. The arm 1-2 consists of capacitor c₁.
(whose value is unknown) which carries current $i_1$ as shown, arm 2 - 4 consists of pure resistor (here pure resistor means we assuming it non inductive in nature), arm 3 - 4 also consists of pure resistor and arm 4 - 1 consists of standard capacitor whose value is already known to us. Let us derive the expression for capacitor $c_1$ in terms of standard capacitor and resistors.

At balance condition we have,

$$\frac{1}{j\omega c_1} \times r_4 = \frac{1}{j\omega c_2} \times r_3$$

It implies that the value of capacitor is given by the expression

$$c_1 = c_2 \times \frac{r_4}{r_3}$$

In order to obtain the balance point we must adjust the values of either $r_3$ or $r_4$ without disturbing any other element of the bridge. This is the most efficient method of comparing the two values of capacitor if all the dielectric losses are neglected from the circuit.

![Phasor diagram of De Sauty bridge](image)

**Fig. .24 Phasor diagram of De Sauty bridge**

Now instead of some advantages like bridge is quite simple and provides easy calculations, there are some disadvantages of this bridge because this bridge give inaccurate results for imperfect capacitor (here imperfect means capacitors which not free from dielectric losses). Hence we can use this bridge only for comparing perfect capacitors. Here we interested in modify the De Sauty's bridge, we want to have such a kind of bridge that will gives us accurate results for imperfect capacitors also.
This modification is done by Grover. The modified circuit diagram is shown below:

![De Sauty bridge diagram](image)

**Fig. 25 De Sauty bridge**

Here Grover has introduced electrical resistances $r_1$ and $r_2$ as shown in above on arms 1 - 2 and 4 - 1 respectively, in order to include the dielectric losses. Also he has connected resistances $R_1$ and $R_2$ respectively in the arms 1 - 2 and 4 - 1. Let us derive the expression capacitor $c_1$ whose value is unknown to us. Again we connected standard capacitor on the same arm 1 - 4 as we have done in De Sauty's bridge. At balance point on equating the voltage drops we have:

$$\left( R_1 + r_1 + \frac{1}{j \omega c_1}\right) r_4 = \left( R_2 + r_2 + \frac{1}{j \omega c_2}\right) r_3 \cdots \cdots \cdots (1)$$

On solving above equation we get:

$$\frac{c_1}{c_2} = \frac{R_2 + r_2}{R_1 + r_1} = r_4 r_3$$

This the required equation. By making the phasor diagram we can calculate dissipation factor. Phasor diagram for the above circuit is shown below.
Let us mark $\delta_1$ and $\delta_2$ be phase angles of the capacitors $c_1$ and $c_2$ capacitors respectively. From the phasor diagram we have $\tan(\delta_1) = $ dissipation factor $= \omega c_1 r_1$ and similarly we have $\tan(\delta_2) = \omega c_2 r_2$. From equation (1) we have

$$c_2 r_2 - c_1 r_1 = c_1 R_1 - c_2 R_2$$

$$= \omega c_2 r_2. \text{From equation (1) we have}$$

on multiplying $\omega$ both sides we have

$$\omega c_2 r_2 - \omega c_1 r_1 = \omega(c_1 R_1 - c_2 R_2)$$

Therefore the final expression for the dissipation factor is written as

$$B u t \quad \frac{c_1}{c_2} = \frac{r_4}{r_3}$$

$$\tan(\delta_1) - \tan(\delta_2) = \omega c_2 \left( \frac{r_4}{r_3} - R_2 \right)$$

Hence if dissipation factor for one capacitor is known. However this method is gives quite inaccurate results for dissipation factor.
Wien's Bridge

Circuit and derives the expression for the unknown element at balance, Wien Bridge has a series RC combination in one and a parallel combination in the adjoining arm. Wien's bridge is shown in fig. Its basic form is designed to measure frequency. It can also be used for the instrument of an unknown capacitor with great accuracy. The impedance of one arm is

![Fig.27 circuit Diagram](Image)

The admittance of the parallel arm is

\[ Y_3 = \frac{1}{R_3} + j\omega C_3 \]

Using the bridge balance equation, we have

\[
\begin{align*}
Z_1 Z_4 &= Z_2 Z_3 \\
Z_1 Z_4 &= Z_2 Y_3, \text{ i.e. } Z_2 &= Z_1 Z_4 Y_3
\end{align*}
\]

Therefore

\[
R_2 = R_4 \left( \frac{R_1 - j}{\omega C_1} \right) \left( \frac{1}{R_3} + j\omega C_3 \right)
\]

\[
R_2 = \frac{R_1 R_4}{R_3} - \frac{j R_4}{\omega C_1 R_3} + j\omega C_3 R_1 R_4 + \frac{C_3 R_4}{C_1}
\]

Equating the real and imaginary terms we have as,

\[
R_2 = \left( \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1} \right) - j \left( \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 \right)
\]

\[
R_2 = \frac{R_1 R_4}{R_3} + \frac{C_3 R_4}{C_1}\quad \text{and}\quad \frac{R_4}{\omega C_1 R_3} - \omega C_3 R_1 R_4 = 0
\]
The bridge is used for measuring frequency in the audio range. Resistances R1 and R3 can be ganged together to have identical values. Capacitors C1 and C3 are normally of fixed values. The audio range is normally divided into 20 - 200 - 2 k - 20 kHz range. In this case, the resistances can be used for range changing and capacitors, and C3 for fine frequency control within the range. The bridge can also be used for measuring capacitance. In that case, the frequency of operation must be known.

The bridge is also used in a harmonic distortion analyzer, as a Notch filter, as an in audio frequency and radio frequency oscillators as a frequency determine element. An accuracy of 0.5% - 1% can be readily obtained using this bridge. Because it is frequency sensitive, it is difficult to balance unless the waveform of the applied voltage is purely sinusoidal.

Reference books

SCHOOL OF ELECTRICAL AND ELECTRONICS
DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION

UNIT – IV
MEASUREMENT AND INSTRUMENTATION – SIC 1203
UNIT-4

ELECTRONIC MEASUREMENTS


1. INTRODUCTION- CATHODE RAY TUBE (CRT)

In studying the various electronic, electrical networks and systems, signals which are functions of time, are often encountered. Such signals may be periodic or non-periodic in nature. The device which allows the amplitude of such signals, to be displayed primarily as a function of time, is called cathode ray oscilloscope, commonly known as C.R.O. The C.R.O. gives the visual representation of the time varying signals. The oscilloscope has become an universal instrument and is probably most versatile tool for the development of electronic circuits and systems. It is an integral part of electronic laboratories.

The oscilloscope is, in fact, a voltmeter. Instead of the mechanical deflection of a metallic pointer as used in the normal voltmeters, the oscilloscope uses the movement of an electron beam against a fluorescent screen, which produces the movement of a visible spot. The movement of such spot on the screen is proportional to the varying magnitude of the signal, which is under measurement.

The electron beam can be deflected in two directions: the horizontal or x-direction and the vertical or y-direction. Thus an electron beam producing a spot can be used to produce two dimensional displays. Thus C.R.O. can be regarded as a fast x-y plotter. The X-axis and y-axis can be used to study the variation of one voltage as a function of another. Typically the x-axis of the oscilloscope represents the time while the y-axis represents variation of the input voltage signal. Thus if the input voltage signal applied to the y-axis of C.R.O. is sinusoidal varying and if x-axis represents the time axis, then the spot moves sinusoidal, and the familiar sinusoidal waveform can be seen on the screen of the oscilloscope. The oscilloscope is so fast device that it can display the periodic signals whose time period is as small as microseconds and even nanoseconds. The C.R.O. basically operates on voltages, but it is possible to convert current, pressure, strain, acceleration and other physical quantities into the voltage using transducers and obtain their visual representations on the C.R.O.
The Cathode Ray Tube

The cathode ray tube (CRT) is the heart of the C.R.O. The CRT generates the electron beam, accelerates the beam, deflects the beam and also has a screen where beam becomes visible as a spot. The main parts of the CRT are:

i) Electron gun ii) Deflection system iii) Fluorescent screen iv) Glass tube or envelope v) Base

A schematic diagram of CRT, showing its structure and main components is shown in the figure below

![Block diagram of cathode ray tube (CRT)](image)

Electron Gun

The electron gun section of the cathode ray tube provides a sharply focused electron beam directed towards the fluorescent-coated screen. This section starts from thermally heated cathode, emitting the electrons. The control grid is given negative (-) potential with respect to cathode. This grid controls the number of electrons in the beam, going to the screen.

The momentum of the electrons (their number x their speed) determines the intensity, or brightness, of the light emitted from the fluorescent screen due to the electron bombardment. The light emitted is usually of the green colour. Because the electrons are negatively charged, a repulsive force is created by applying a negative voltage to the control grid in CRT, voltages applied to various grids are stated with respect to cathode, which is taken as common point. This negative control voltage can be made variable. Since the electron beam consists of many electrons, the beam tends to diverge. This is because the similar (negative) charges on the electron repel each other. To compensate for such repulsion forces, an adjustable electrostatic field is created between two cylindrical anodes, called the focusing anodes. The high positive potential is also given to the pre-accelerating anodes and accelerating anodes, which results into the required acceleration of the electrons.

Both focusing and accelerating anodes are cylindrical in shape having small openings located in the centre of each electrode, co-axial with the tube axis. The pre-accelerating and
Accelerating anodes are connected to a common positive high voltage which varies between 2 kV to 10 kV. The focusing anode is connected to a lower positive voltage of about 400 V to 500 V.

**Deflection system**

When the electron beam is accelerated it passes through the deflection system, with which beam can be positioned anywhere on the screen. The deflection system of the cathode-ray-tube consists of two pairs of parallel plates, referred to as the vertical and horizontal deflection plates. One of the plates in each set is connected to ground (0 V). To the other plate of each set, the external deflection voltage is applied through an internal adjustable gain amplifier stage. To apply the deflection voltage externally, an external terminal, called the Y input or the X input, is available.

As shown in the figure above, the electron beam passes through these plates. A positive voltage applied to the Y input terminal \(V_y\) causes the beam to deflect vertically upward due to the attraction forces, while a negative voltage applied to the Y input terminal will cause the electron beam to deflect vertically downward, due to the repulsion forces.

Similarly, a positive voltage applied to X-input terminal \(V_x\) will cause the electron beam to deflect horizontally towards the right; while a negative voltage applied to the X-input terminal will cause the electron beam to deflect horizontally towards the left of the screen. The amount of vertical or horizontal deflection is directly proportional to the correspondingly applied voltage.

When the voltages are applied simultaneously to vertical and horizontal deflecting plates, the electron beam is deflected due to the resultant of these two voltages. The face of the screen can be considered as an x-y plane. The \((x,y)\) position of the beam spot is thus directly influenced by the horizontal and the vertical voltages applied to the deflection plates \(V_x\) and \(V_y\), respectively.

The horizontal deflection \((x)\) produced will be proportional to the horizontal deflecting voltage, \(V_x\), applied to X-input.

\[
x \propto V \\
x = K_x V_x
\]

where \(K_x\) is constant of proportionality. The deflection produced is usually measured in cm or as number of divisions, on the scale, in the horizontal direction.

Then \(K_x = x/V_x\) where \(K_x\) is expressed as cm/volt or division/volt, is called **horizontal sensitivity** of the oscilloscope.

Similarly, the vertical deflection \((y)\) produced will be proportional to the vertical deflecting voltage, \(V_y\), applied to the y-input.

\[
y \propto V \\
y = K_y V_y
\]

Then \(K_y = y/V_y\) and \(K_y\), the **vertical sensitivity**, will be expressed as cm/volt, or division/volt.

The values of vertical and horizontal sensitivities are selectable and adjustable through multipositional switches on the front panel that controls the gain of the corresponding internal amplifier stage. The bright spot of the electron beam can thus trace (or plot) the x-y relationship between the two voltages, \(V_x\) and \(V_y\).

The schematic arrangement of the vertical and the horizontal plates, controlling the position of the spot on the screen is shown in the figure below:
Fluorescent screen

The light produced by the screen does not disappear immediately when bombardment by electrons ceases, i.e., when the signal becomes zero. The time period for which the trace remains on the screen after the signal becomes zero is known as "persistence". The persistence may be as short as a few microsecond, or as long as tens of seconds or even minutes. Medium persistence traces are mostly used for general purpose applications. Long persistence traces are used in the study of transients. Long persistence helps in the study of transients since the trace is still seen on the screen after the transient has disappeared. Short persistence is needed for extremely high speed phenomena.

The screen is coated with a fluorescent material called phosphor which emits light when bombarded by electrons. There are various phosphors available which differ in colour, persistence, and efficiency.

One of the common phosphor is Willemite, which is zinc, orthosilicate, ZnO+SiO2, with traces of manganese. This produces the familiar greenish trace. Other useful screen materials include compounds of zinc, cadmium, magnesium and silicon. The kinetic energy of the electron beam is converted into both light and heat energy when it hits the screen. The heat so produced gives rise to "phosphor burn" which is damaging and sometimes destructive. This degrades the light output of phosphor and sometimes may cause complete phosphor destruction. Thus the phosphor must have high burn resistance to avoid accidental damage.

Many phosphor materials having different excitation times and colours as well as different phosphorescence times are available. The type P1, P2, P_{11} or P_{31} are the short persistence phosphors and are used for the general purpose oscilloscopes.
Table 1: The Types of Short Persistence of Phosphor

<table>
<thead>
<tr>
<th>Phosphor</th>
<th>Colour Under excitation</th>
<th>Colour After glow</th>
<th>Persistence</th>
<th>Relative luminance</th>
<th>Relative writing speed</th>
<th>Applications</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>yellow-green</td>
<td>yellow-green</td>
<td>medium</td>
<td>45</td>
<td>35</td>
<td>General purpose</td>
</tr>
<tr>
<td>P2</td>
<td>blue-green</td>
<td>green</td>
<td>medium</td>
<td>60</td>
<td>70</td>
<td>General purpose</td>
</tr>
<tr>
<td>P4</td>
<td>white</td>
<td>white</td>
<td>medium to short</td>
<td>50</td>
<td>75</td>
<td>Black and white T.V.</td>
</tr>
<tr>
<td>P7</td>
<td>blue-white</td>
<td>yellow-green</td>
<td>medium-short</td>
<td>45</td>
<td>95</td>
<td>Radar</td>
</tr>
<tr>
<td>P11</td>
<td>blue-violet</td>
<td>blue</td>
<td>medium-short</td>
<td>25</td>
<td>100</td>
<td>Photographic recording</td>
</tr>
<tr>
<td>P15</td>
<td>blue-green</td>
<td>blue-green</td>
<td>visible-short</td>
<td>15</td>
<td>25</td>
<td>Flying spot scanners for T.V.</td>
</tr>
<tr>
<td>P19</td>
<td>orange</td>
<td>orange</td>
<td>long</td>
<td>25</td>
<td>3</td>
<td>Radar</td>
</tr>
<tr>
<td>P31</td>
<td>green</td>
<td>green</td>
<td>medium-short</td>
<td>100</td>
<td>75</td>
<td>General purpose</td>
</tr>
<tr>
<td>P33</td>
<td>orange</td>
<td>orange</td>
<td>very long</td>
<td>20</td>
<td>7</td>
<td>Radar</td>
</tr>
<tr>
<td>P39</td>
<td>green</td>
<td>green</td>
<td>medium-long</td>
<td>50</td>
<td>40</td>
<td>Computer graphics</td>
</tr>
</tbody>
</table>

**Glass Tube:** All the components of a CRT are enclosed in an evacuated glass tube called envelope. This allows the emitted electrons to move about freely from one end of the tube to the other end.

**Base:** The base is provided to the CRT through which the connections are made to the various parts.

1.2. **CATHODE RAY OSCILLOSCOPE**

1.2.1 **CATHODE RAY OSCILLOSCOPE (CRO)-BLOCK DIAGRAM**

The block diagram of CRO is shown below:

![Block diagram of Cathode ray Oscilloscope (CRO)](image-url)

Fig.3 Block diagram of Cathode ray Oscilloscope (CRO)
**Cathode Ray Tube (CRT):** This is the cathode ray tube which is the heart of C.R.O. It is used to emit the electrons required to strike the phosphor screen to produce the spot for the visual display of the signals.

**Vertical Amplifier:** The input signals are generally not strong to provide the measurable deflection on the screen. Hence the vertical amplifier stage is used to amplify the input signals. The amplifier stages used are generally wide band amplifiers so as to pass faithfully the entire band of frequencies to be measured. Similarly it contains the attenuator stages as well. The attenuators are used when very high voltage signals are to be examined, to bring the signals within the proper range of operation.

The block diagram of a vertical amplifier is shown in the figure below.

![Fig.4 Vertical amplifier](image)

It consists of several stages with overall fixed sensitivity. The amplifier can be designed for stability and required bandwidth very easily due to the fixed gain. The input stage consists of an attenuator followed by FET source follower. It has very high input impedance required to isolate the amplifier from the attenuator. It is followed by BJT emitter follower to match the output impedance of FET output with input of phase inverter. The phase inverter provides two anti-phase output signals which are required to operate the push pull output amplifier.

The push pull operation has advantages like better hum voltage cancellation, even harmonic suppression especially large and harmonic, greater power output per tube and reduced number of defocusing and nonlinear effects.

**Delay Line:** The delay line is used to delay the signal for some time in the vertical sections. When the delay line is not used, the part of the signal gets lost. Thus the input signal is not applied directly to the vertical plates but is delayed by some time using a delay line circuit as shown in the figure below:
If the trigger pulse is picked off at a time \( t = t_0 \) after the signal has passed through the main amplifier then signal is delayed by \( x_1 \) nanoseconds while sweep takes \( y_1 \) nanoseconds to reach. The design of delay line is such that the delay time \( x_1 \) is higher than the time \( y_1 \). Generally \( x_1 \) is 200 nanoseconds while the \( y_1 \) is 80 nanoseconds, thus the sweep starts well in time and no part of the signal is lost.

There are two types of delay lines used in C.R.O. which are:

i) Lumped parameter delay line

ii) Distributed parameter delay line

They are discussed below:

I) **Lumped parameter delay line:**

Lumped parameter delay line consists of number of cascaded symmetrical LC networks called T sections. Each section is capable of delaying the signal by 3 to 6 nanoseconds. Such a T filter section is shown in the figure below.

The T-section filter acts as low pass filter having cut-off frequency given as

\[
f_c = \frac{1}{\pi \sqrt{L/C}}
\]

If \( V_i \) consists of frequencies much less than the cut-off frequency, output signal \( V_o \) will be a faithful reproduction of \( V_i \) but delayed by the time,

\[
t_s = \frac{1}{\pi f_c} = \sqrt{L/C}
\]
where

tₙ - Delay for a single T network

where, \( t_d = n t_s \)

here \( t_d = \) Total delay

\( n = \) Number of T sections

A practical delay line circuit in C.R.O. is driven by push-pull amplifier and is shown in the figure.

![Practical delay line circuit](image)

**Fig. 7 Practical delay line**

II) Distributed parameter delay line:

It is basically a transmission line constructed with a wound helical coil on a mandrel and extruded insulation between it. It is specially manufactured co-axial cable with high inductance per unit length. The construction of such line is shown in the figure below.

The inductance can be increased by winding the helical inner conductor on ferromagnetic core. This increases the characteristics impedance \( Z_o \) and delay time. Typical parameters for helical, distributed parameter delay line are \( Z_o = 1000 \, \Omega \) and \( t_d = 180 \, \text{nsec/m} \). The co-axial delay line is advantageous as:

i) It does not require careful adjustment as lumped parameter.  
ii) It requires less space.

![Distributed delay line](image)

**Fig. 8 Distributed delay line**

**Trigger Circuit**: It is necessary that horizontal deflection starts at the same point of the input vertical signal, each time it sweeps. Hence to synchronize horizontal deflection with vertical
deflection a synchronizing or triggering circuit is used. It converts the incoming signal into the triggering pulses, which are used for the synchronization.

**Time Base Generator:** The time base generator is used to generate the sawtooth voltage, required to deflect the beam in the horizontal section. This voltage deflects the spot at a constant time dependent rate. Thus the x-axis on the screen can be represented as time, which helps to display and analyse the time varying signals.

**Horizontal Amplifier:** The sawtooth voltage produced by the time base generator may not be of sufficient strength. Hence before giving it to the horizontal deflection plates, it is amplified using the horizontal amplifier.

**Power Supply:** The power supply block provides the voltages required by CRT to generate and accelerate an electron beam and voltages required by other circuits of the oscilloscope like horizontal amplifier, vertical amplifier etc.

The negative High Voltage (HV) supply has following advantages:

i) The accelerating anodes and the deflection plates are close to ground potential. This ground potential protects the operator from shocks.

ii) The deflection voltages are measured with respect to ground hence blocking or coupling capacitors are not necessary.

iii) Insulation required between controls and chasis is less.

There are two sections of a power supply block. The High Voltage (HV) section and Low Voltage (LV) section. The high voltages of the order of 1000V to 1500 V are required by CRT. Such high negative voltages are used for CRT. The low voltage is required for the heater of the electron gun, which emits the electrons. This is a positive voltage of the order of few hundred volts. This voltage is also used for other circuits of C.R.O. This is the discussion of basic block diagram of a simple C.R.O.
The CRT needs an anode to cathode supply voltage of 1000V or more to give a bright and sharp display. The deflection plates which move the dot around the screen must be at about the same voltage as the anode. So if the cathode is at or near ground potential as is common in vacuum tube circuits, then the deflection plates are 1KV above ground. If the deflection plates are to be driven by a DC coupled transistor amplifier, and the bases are to be near ground so that they may in turn be driven by low voltage circuits then the output transistors must have a $V_{CEO}$ rating of over 1KV. Transistors with that kind of collector breakdown voltage that are also fast enough to handle megahertz signals are rare indeed.

If, on the contrary, the anode voltage is at ground, then we would need an amplifier with an output stage that could swing 60 or so volts above and below ground. While this is possible, it would be more complex than necessary. A better solution is to put the CRT anode 70 to 80 volts above (more positive than) ground and the cathode about 950 volts below ground. In this way, the drivers for the deflection plates could work from a 150 Volt supply and swing ±60 Volts from a quiescent value of about 75 Volts.

In the circuit above, the two 27KΩ resistors set the anode potential to 75V. The second and fourth grids are internally connected to the anode and along with the third or focus grid form a lens that focuses the electron beam from the cathode to a small sharp dot on the screen. The control grid is held near -1000V by circuitry not shown here. The voltage divider on the left of the drawing provides proper bias voltage for the focus grid and the cathode. The top pot adjusts focus and the lower one sets the brightness of the dot on the screen.
**Vertical deflection system**

The main function of this amplifier is to amplify the weak signal so that the amplified signal can produce the desired signal. To examine the input signals are penetrated to the vertical deflection plates through the input attenuator and number of amplifier stages.

The schematic arrangement of the vertical plates, controlling the position of the spot on the screen is shown in the figure below:

![Arrangements of Vertical plates in CRT](image)

**Fig.10 Arrangements of Vertical plates in CRT**

The input signals are generally not strong to provide the measurable deflection on the screen. Hence the vertical amplifier stage is used to amplify the input signals. The amplifier stages used are generally wide band amplifiers so as to pass faithfully the entire band of frequencies to be measured. Similarly it contains the attenuator stages as well. The attenuators are used when very high voltage signals are to be examined, to bring the signals within the proper range of operation.

The function of vertical deflection system is to provide an amplified signal of the proper level to drive the vertical deflection plates without introducing any appreciable distortion into the system. The input sensitivity of many CROs is of the order of a few milli-volts per division and the voltage required for deflecting the electron beam varies from approximately 100 V (peak to peak) to 500 V depending on the accelerating voltage and the construction of the tube. Thus the vertical amplifier is required to provide this desired gain from milli-volt input to several hundred volt (peak to peak) output. Also the vertical amplifier should not distort the input waveform and should have good response for entire band of frequencies to be measured. The deflection plates of CRO act as plates of a capacitor and when the input signal frequency exceeds over 1 MHz, the current required for charging and discharging of the capacitor formed by the deflection plates increases. So the vertical amplifier should be capable of supplying current enough to charge and discharge the deflection plate capacitor.

As we know that electrical signal is delayed by a certain amount of time when transmitted through an electronic circuitry. In CRO, output signal voltage of the vertical amplifier is fed to the vertical plates of CRT and some of its portion is used for triggering the time base generator circuit, whose output is supplied to the horizontal deflection plates through horizontal amplifier. The whole process, which includes generating and shaping of a trigger pulse and starting of a time-base generator and then its amplification, takes time of the order of 100 ns or so. So the input signal of the vertical deflection plates of a CRT is to be delayed by at least the same or little more amount
of time to allow the operator to see the leading edge of the signal waveform under study on the screen.

The block diagram of a vertical amplifier is shown in the figure below.

![Block Diagram of Vertical Amplifier](image)

**Fig.11 Vertical amplifier**

It consists of several stages with overall fixed sensitivity. The amplifier can be designed for stability and required bandwidth very easily due to the fixed gain. The input stage consists of an attenuator followed by FET source follower. It has very high input impedance required to isolate the amplifier from the attenuator. It is followed by BJT emitter follower to match the output impedance of FET output with input of phase inverter. The phase inverter provides two anti-phase output signals which are required to operate the push pull output amplifier.

The push pull operation has advantages like better hum voltage cancellation, even harmonic suppression especially large and harmonic, greater power output per tube and reduced number of defocusing and nonlinear effects.

**Triggered sweep**

![Triggered Sweep Circuit](image)

**Fig.12 Triggered sweep and its output waveform**

The continuous sweep is of limited use in displaying periodic signals of constant frequency and amplitude. When attempting to display voice or music signals, the pattern falls in and out of sync as the frequency and amplitude of the music varies resulting in an unstable display.
A triggered sweep can display such signals, and those of short duration, e.g. narrow pulses. In triggered mode, the input signal is used to generate substantial pulses that trigger the sweep. Thus ensuring that the sweep is always in step with the signal that drives it.

As shown in figure above, resistance $R_3$ and $R_4$ form a voltage divider such that the voltage $V_D$ at the cathode of the diode is below the peak voltage $V_P$ for UJT conduction. When the circuit is switched on, the UJT is in the non-conducting stage, and $C_T$ charges exponentially through $R_T$ towards $V_{BB}$ until the diode becomes forward biased and conducts, the capacitor voltage never reaches the peak voltage required for UJT conduction but is clamped at $V_D$. If now a -ve pulse of sufficient amplitude is applied to the base and the peak voltage $V_P$, is momentarily lowered, the UJT fires. As a result, capacitor $C_T$, discharges rapidly through the UJT until the maintaining voltage of the UJT is reached; at this point the UJT switches off and capacitor $C_T$ charges towards $V_{BB}$, until it is clamped again at $V_D$. Above figure shows the output waveform.

1.4 CRO SPECIFICATIONS

Oscilloscopes are a very common form of test equipment - possibly the most important type of test equipment. As a result it is often necessary to be able to choose one either from the test equipment store or as a rental or when buying an oscilloscope. When selecting an oscilloscope, there are many different specifications and parameters to consider, each one related to the performance. When selecting an oscilloscope, what are the most important specifications and parameters and which ones will affect the performance of the scope in the particular application.

1) **Types of oscilloscope**: One of the major specifications associated with buying an oscilloscope is the actual type of oscilloscope that is required. Some types of scope will be able to perform other measurements better than others; some use current technology whereas others are older; and there may be cost implications as well. Analogue, analogue storage, digital, digital storage, digital sampling, USB scopes and many more types are available.

2) **Oscilloscope bandwidth specification**: One important oscilloscope specification is related to the frequency or speed of the waveforms that can be measured. This is determined by the bandwidth of the oscilloscope and it is found that the capability of the oscilloscope to accurately display the waveform falls off with increasing frequency. The oscilloscope specification for bandwidth will typically be quoted in the format: Bandwidth = -3dB at 1500 MHz.

3) **Vertical DC gain accuracy**: It is important when measuring the amplitude of signals, to know the accuracy of the measurement that is being made. As oscilloscopes are not intended to be used instead of digital multimeters, it is not anticipated that the voltage elements of the oscilloscope specification will be as accurate.

4) **Vertical channel Resolution**: Digital oscilloscopes need to convert the incoming analogue signal into a digital signal. The vertical channel resolution determines the "granularity" of the signal. Most digital oscilloscopes have 8-bit resolution.

5) **Rise Time Specification**: Another important oscilloscope specification which needs to be accommodated is the rise time of the oscilloscope. This is a particularly important specification for any digital circuits where the edges on square waves and pulses are often of great importance. The oscilloscope must have a sufficiently fast rise time to capture the rapid transitions accurately, otherwise important information may not be displayed and the results could be misleading. The
rise time of the oscilloscope is defined as the time it takes for the image to rise from 10% to 90% of the final value.

6) **Oscilloscope sample rate**: the sample rate oscilloscope specification is becoming a more widespread and important specification. The sample rate is specified in samples per second (S/s). The faster the oscilloscope samples the waveform, the greater the resolution of the detail on the waveform and with greater sample rates the less the likelihood that any critical information will be lost.

   Oscilloscope sample rate = 2.5 x Highest frequency

7) **Memory Depth**: This is the memory for storing signals. The greater the memory depth the more signal it is possible to capture at the highest sample rate.

1.5 **CRO Controls**

The various front panel controls of a simple C.R.O. are described in this section. These are divided into four groups,

1. Basic controls
2. Vertical section
3. Horizontal section
4. Z-axis Intensity control

1. **Basic Controls**:
   1. **ON-OFF**: The on-off switch turns on or off the C.R.O.
   2. **Intensity**: This controls the intensity or brightness of the light produced by beam spot. It actually controls the number of electrons per second that are bombarding the screen which determines the brightness of the spot.
   3. **Focus**: This controls the sharpness of the spot. A sharper spot is always preferred. Focusing or the spot is obtained by varying the voltage applied to the focusing anodes of the cathode ray tube.
   4. **Astigmatism**: This is another focus control. With the help of focus control and astigmatism control, a very sharp spot can be obtained both in the centre and also at the edges of the screen. With the astigmatism control the voltage to accelerating anodes is varied.
   5. **Scale Illumination**: Most C.R.O. s have some sort of plastic screen in front of the cathode ray tube. This screen has a grid engraved on it, giving it an appearance similar to that of graph paper. This is called graticule. This scale facilitates the measurement on the oscilloscope. The scale illumination control, illuminates the scale and hence the lines on the scale can be seen very easily.

2. **Vertical Section**: Most oscilloscopes have two vertical inputs. These are usually called inputs 1 and 2 or A and B. Two input signals can be applied to these two inputs and thereby both the signals can be observed on the screen simultaneously. This is very useful for comparing two signals. The following controls serve for each vertical input.
   a) **volts/division**:

      This control sets the vertical scale; that is, it determines how much the spot will be deflected by an input signals applied to vertical input terminals. The usual units are either volts per
centimeter or volts per division, where division refers to the grid marks on the screen. The actual input voltage can be found by measuring the deflection and multiplying it by the scale factor. Thus, if the scale control was set to 5 V/cm and deflection is 1.3 cm, the input would be 6.5 V. The control is shown in the figure below.

![Scale Control Diagram](image)

**Fig. 13 Volts/division selection**

Suppose the alternating voltage signal of amplitude 10 V is to be displayed. Then if volts/division are selected as 10 then it will be displayed as shown in the figure (a) while if volts/division = 5 is selected, it will be displayed as shown in the figure (b).

![Voltage Display Diagrams](image)

**Fig. 14 Effects of volts/division**

b) **Invert:**

This control inverts the input signal; that is, it multiplies it by -1. Then positive input voltages become negative and cause downward deflections. The effect of invert is shown in the figure below.
c) Position:

With the help of this control, the pattern obtained on the screen can be shifted, as a whole, vertically upwards or downwards. This is achieved by adding a d.c. offset voltage to the input signal.

d) X 10 (Multiplied by 10):

This control makes the gain of the vertical amplifier 10 times as great as normal, it changes the scale factor by factor of 10. Thus if the X 10 switch is turned 'ON' and the scope is set on 0.05 V/cm, if the actual scale factor is 0.005 V/cm or 5 mV/cm.

e) Vertical Coupling:

This switch controls the coupling to the vertical amplifier. The usual choices are A.C., D.C., or ground. The meaning of these various positions are as follows:

i. A.C.: The vertical amplifier is a.c. coupled to the input. Thus the d.c. component of the input is blocked, and only the a.c. components of the input signal deflect the beam vertically. This allows to observe small a.c. signals or large d.c. background.

ii. D.C.: The vertical amplifier has d.c. coupling throughout, so that the deflection corresponds to both the a.c. and the d.c. components of the input.

iii. GROUND: The input to the amplifier is grounded. There will be no vertical deflection. If no voltage is applied to horizontal plates, the spot will be at the position corresponding to ground. It is useful for measuring voltage with respect to ground.

f) Vertical Mode Control: The control serves for the vertical section of the scope as a whole. Assume that two input signals are simultaneously applied to the two vertical inputs of the scope. Then this switch determines what is displayed on the screen. Thus usual choices are:

i) 1 only: Only the signal at input 1 is displayed.

ii) 2 only: Only the signal at input 2 is displayed.

iii) 1 + 2: Sum: The sum of the inputs 1 and 2 is displayed.

iv) 1-2: Sum: Difference: The difference between input 1 and input 2 is displayed.

v) Alternate: Input 1 is displayed first, then input 2 is displayed, then input 1 again and so on. By using the vertical position control, the two traces can be separated vertically, and thus, relations between the two signals can be studied.
vi) Chop: In this mode first input 1 is displayed for a fraction of a microsecond, then input 2 for a fraction of microsecond, then input 1 again, and so on. In this way, plots of both inputs can be drawn at the same time. The chop mode is useful with low frequency signals, while the alternate mode is useful for high frequency signals.

3. Horizontal section:

a) Time Base Control: Very often the oscilloscope is used to observe the waveform of time varying signals. Most of the horizontal section of the scope is devoted to generating a time base for such signals. The time base control is calibrated in terms of time per centimeter or time per division. A typical unit might be 0.10 msec/cm, meaning that horizontal deflection of the spot will be 1 cm in 0.1 msec. The usual range on a scope is from about 0.1 sec/cm to 20 to 50 nsec/cm. This is shown in the figure.

![Fig.16 Times/division selection](image)

If signal has time period of 20 msec then with two different time base control selections, it can be displayed as shown in the figure below (a) and (b).

![Fig.17 Effect of Time base control.](image)

b) Position: This knob can be used to shift the display, as a whole to left or right.

c) Synchronization: It has been mentioned earlier that to obtain the stationary pattern on the screen, the synchronization is must. It is used to operate the time base generator such that the frequency of saw tooth voltage is an integral multiple of input signal frequency. There are various signals which can be applied to the trigger circuit. The signals can be selected using a synchronous selector switch. The types of signals which can be selected are:
i) **Internal**: The trigger is obtained from signal being measured through the vertical amplifier.

ii) **Line**: The input to the trigger circuit is from a.c mains (say 230 V, 50 Hz) supply. This is useful when observing the signals which are synchronized with power line, such as ripple in a power line.

iii) **External**: The input to the trigger circuit is from the external trigger circuit.

**d) Sweep Selector**: When the sweep selector switch $S$, is in linear position, the horizontal amplifier receives an input from the saw tooth sweep generator which is triggered by the synchronous amplifier. The external signal also can be applied to the horizontal deflecting plates, by putting a selector switch $S$, to the external position.

4. **Z-axis Intensity control:**

It is used for brightening the display. Periodic positive pulses are applied to the grid and alternatively negative pulses are applied to cathode, to brighten the beam during its sweep period. This control is obtained by inserting a signal between the ground and the control grid or ground and the cathode.

### 1.6 CRO PROBES

The CRO probe performs the very important function of connecting the test circuit to the oscilloscope without altering, loading or otherwise disturbing the test circuit.

There are three different probes:

a) Direct reading probe, b)circuit isolation probe, c)detector probe.

They are discussed below:

**a) Direct reading probe**

This probe is the simplest of all probes and it uses a shielded coaxial cable.

It avoids stray pickups which may lead to problems when low level signals are being measured. It is used usually for low frequency and low impedance circuits. However in using the shielded probe, the shunt capacitance of the probe is added to the input impedance and capacity of the scope and acts to lower the response of the oscilloscope to high impedance and high frequency circuits.

**b) Isolation probe**

Isolation probe is used in order to avoid the undesirable circuit loading effects of shielded probe. The isolation probe which is used along with the capacitive voltage divider, decreases the input capacitance and increases the input resistance of the oscilloscope. This way the loading effects are drastically reduced.

**c) Detector probe**

When analyzing the response to modulated signals in communication equipment like AM, FM and TV receivers, the detector probe functions to separate the lower frequency modulation component from the higher frequency carrier. The amplitude of the modulator carrier (which is proportional to the response of the receiver to the much high frequency carrier signal) is displayed on the oscilloscope by rectifying and bypassing action. This
permits an oscilloscope capable of audio-frequency response to perform signal tracing tests on communication signals in the range of hundreds of Mhz, a range which is beyond the capabilities of all oscilloscopes except the highly specialized ones.

Fig.18: A CRO Probe

1.7 MEASUREMENTS ON CRO

The various characteristics of an input signal and the properties of the signal can be measured using C.R.O. The various parameters which can be measured using C.R.O. are voltage, current, period, frequency, phase, amplitude, peak to peak value, duty cycle etc. Let us discuss the amplitude, frequency and phase measurements using C.R.O.

(1) Voltage measurement:

The C.R.O. includes the amplitude measurement facilities such as constant gain amplifiers and the calibrated shift controls. The waveform can be adjusted on the screen by using shift controls so that the measurement of divisions corresponding to the amplitude becomes easy. Generally to reduce the error, peak to peak value of the signal is measured and then its amplitude and r.m.s. value is calculated.

To measure the amplitude use the following steps:

1. Note down the selection in volts/ division from the front panel, selected for measurement
2. Adjust shift control to adjust signal on screen so that it becomes easy to count number of divisions corresponding to peak to peak value of the signal.
3. Note down peak to peak value in terms of the number of divisions on screen.
4. Use the following relation to obtain peak to peak value in volts.
   \[
   V_{\text{P-P}} = \text{(Number of divisions or units noted)} \times \left( \frac{\text{volts}}{\text{divisions}} \right)
   \]
5. The amplitude can then be calculated as:
   \[
   V_m = \text{Amplitude} = \frac{V_{\text{P-P}}}{2}
   \]

while the rms value of sinusoidal wave can be given as:

\[
V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} = \frac{V_{\text{P-P}}}{2\sqrt{2}} \text{ only for sinusoidal signals}
\]
(2) Current measurement:

The CRO is basically voltage indicating device. Hence to measure the current, the current is passed through a known standard resistance. The voltage across resistance is displayed on CRO and is measured. This measured voltage divided by the known resistance gives the value of the unknown current. The arrangement is shown in the figure above. Current is obtained using formula,

$$I = \frac{V_{\text{measured on C.R.O.}}}{R}$$

(3) Period & Frequency measurement:

In such measurement, the waveform is displayed on the screen such that one complete cycle is visible on the screen. Thus accuracy increases if a single cycle occupies as much as the horizontal distance on the screen.

Note the time/ division selected on the front panel. Then the period of the waveform can be obtained as,

$$T = \text{(Number of divisions occupied by 1 cycle)} \times \left( \frac{\text{time}}{\text{division}} \right) = \text{time period}$$

The frequency is the reciprocal of the period. That is given by:

$$f = \frac{1}{T}$$

This is the method of frequency measurement without Lissajous pattern.

1.8 MEASUREMENT OF PHASE AND FREQUENCY USING LISSAJOUS PATTERNS:

It is interesting to consider the characteristics of patterns that appear on the screen of a CRT when sinusoidal voltages are simultaneously applied to horizontal and vertical plates. These patterns are called 'Lissajous Patterns'. When two sinusoidal voltages of equal frequency which are in phase with each other are applied to the horizontal and vertical deflection plates, the pattern appearing on the screen is a straight line as its clear from figure below.
Thus when two equal voltages of equal frequency but with 90° phase displacement are applied to a CRO, the trace on the screen is a circle. This is shown in figure below.

When two equal voltages of equal frequency but with a phase shift \( \phi \) (not equal to 0° or 90°) are applied to a CRO we obtain an ellipse as shown in figure below. An ellipse is also obtained when unequal voltages of same frequency are applied to the CRO.
A number of conclusions can be drawn from the above discussions. When two sinusoidal voltages of same frequency are applied:

(i) A straight line results when the two voltages are equal and are either in phase with each other or 180° out of phase with each other. The angle formed with the horizontal is 45° when the magnitudes of voltages are equal. An increase in the vertical detection voltage causes the line to have an angle greater than 45° with the horizontal. On the other hand a greater horizontal voltages makes the angle less than 45° with the horizontal.

(ii) Two sinusoidal waveforms of the same frequency produce a Lissajous pattern, which may be a straight line, a circle or an ellipse depending upon the phase and magnitude of the voltages.

A circle can be formed only when the magnitude of the two signals are equal and the phase difference between them is either 90° or 270°. However, if the two voltages are not equal and/or out of phase an ellipse is formed. If the Y voltage is larger, an ellipse with vertical major axis is formed while if the X plate voltage has a greater magnitude, the major axis of the ellipse lies along horizontal axis.

**Fig.22** Lissajous pattern with two equal voltages of same frequency and phase shift of \( \phi \)
Fig. 23 Lissajous patterns with different phase shifts

Fig. 24 Determination of angle of Phase shift

The sine of phase angle between the voltages is given by

$$\sin \phi = \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$$

For convenience, the gains of the vertical and horizontal amplifiers are adjusted so that the ellipse fits exactly into a square marked by the lines on the graticule. If the major axis of the ellipse lies in the first and third quadrants (i.e., its slope is positive) as in figure (a), the phase angle is either between 0° to 90° or between 270° to 360°. When the major axis of ellipse lies in second and fourth quadrants i.e., when its slope is negative as in figure (b), the phase angle is either between 90° and 180° or between 180° and 270°.
Frequency Measurements: Lissajous patterns may be used for accurate measurement of frequency. The signal, whose frequency is to be measured, is applied to the Y plates. An accurately calibrated standard variable frequency source is used to supply voltage to the X plates, with the internal sweep generator switched off. The standard frequency is adjusted until the pattern appears as a circle or an ellipse, indicating that both signals are of the same frequency. Where it is not possible to adjust the standard signal frequency to the exact frequency of the unknown signal, the standard is adjusted to a multiple or a submultiple of the frequency of the unknown source so that the pattern appears stationary.

Let us consider an example Suppose sine waves are applied to X and Y plates as shown in figure below

![Fig.25 Lissajous patterns with frequency ratio 2:1](image)

Let the frequency of wave applied to Y plates is twice that of the voltage applied to X plates. This means that the CRT spot travels two complete cycles in the vertical direction against one in the horizontal direction. The two waves start at the same instant. Lissajous pattern may be constructed in the usual way and a 8 shaped pattern with two loops is obtained. If the two waves do not start at the same instant we get different patterns for the same frequency ratio. The Lissajous patterns for other frequency ratios can be similarly drawn. Some of these patterns are shown in figure below.

![Fig.26 Lissajous patterns with different frequency ratios](image)
It can be shown that for all the above cases, the ratios of the two frequencies is:

\[
\frac{f_V}{f_X} = \frac{\text{number of times tangent touches top or bottom}}{\text{number of times tangent touches either side}} = \frac{\text{number of horizontal tangencies}}{\text{number of vertical tangencies}}
\]

where, \( f_V \) = frequency of signal applied to Y-plates
\( f_X \) = frequency of signal applied to X-plates

### 1.9 TYPES OF OSCILLOSCOPES

There are a number of oscilloscopes which are used for special applications. Some of the oscilloscopes are described below:

**a) Multiple beam oscilloscopes**

In many cases it becomes necessary to compare one signal with that of the other. In such cases Multiple beam oscilloscopes are used. They enclose in a single tube several beam producing systems each with its vertical pair of plates, but mostly with a common time-base. Each Y-channel has its own amplifier. The synchronization or triggering is done from the input of a desired Y-channel or from an external input voltage. Double beam oscilloscopes use two electron guns within the same cathode ray tube. The electron beam of the two channels are completely independent of each other. The same effect may be produced by a single electron gun, the output from it being split into two.

**b) Multiple Trace Oscilloscopes**

This oscilloscope uses single electron guns and produces multiple traces by switching the Y-deflection plates from one input signal to another (this means that the Y-channel is time shared by many signals). The eyes interpret this is a continuous simultaneous display of the input signals although it is a sampled display. This method reduces the cost of manufacturing multi-channel oscilloscopes.

**c) Sampling oscilloscopes**

The oscilloscopes presently can be used for continuous display for frequencies in the 50-300 Mhz range depending upon the design of the oscilloscopes. The display may have upto 1000 dots of luminescence. The vertical deflection for each dot is obtained from progressively later points in each successive cycle of input waveform as shown below:
The horizontal deflection of the electron beam is obtained by applying staircase waveform to X-deflection plates. The sampling oscilloscope is able to respond and store rapid bits of information and present them in a continuous display. The sampling techniques immediately the input signals into lower frequency domain, where conventional low frequency circuitry is then capable of producing a highly effective display. This type of oscilloscopes can be used beyond 50MHz into the UHF range around 500MHz and beyond upto 10GHz. It should be noted that the sampling techniques cannot be used to display of transient waveforms.

**Sampling Oscilloscope**

An ordinary oscilloscope has a Bandwidth of 10 MHz. The high frequency (HF) performance can be improved by means of sampling the input waveform and reconstructing its shape from the sample, i.e. the signal to be observed is sampled and after a few cycles the sampling point is advanced and another sample is taken. The shape of the waveform is reconstructed by joining the sample levels together. The sampling frequency may be as low as 1/10th of the input signal frequency (if the input signal frequency is 100 MHz, the bandwidth of the CRO vertical amplifier can be as low as 10 MHz). As many as 1000 samples are used to reconstruct the original waveform. Figure below shows a block diagram of a sampling oscilloscope.
The input waveform is applied to the sampling gate. The input waveform is sampled whenever a sampling pulse opens the sampling gate. The sampling must be synchronized with the input signal frequency. The signal is delayed in the vertical amplifier, allowing the horizontal sweep to be initiated by the input signal. The corresponding waveforms are also shown in below figure

![Various waveforms at each block of a sampling oscilloscope.](image)

At the beginning of each sampling cycle, the trigger pulse activates an oscillator and a linear ramp voltage is generated. This ramp voltage is applied to a voltage comparator which compares the ramp voltage to a staircase generator. When the two voltages are equal in amplitude, the staircase advances one step and a sampling pulse is generated, which opens the sampling gate for a sample of input voltage.

The resolution of the final image depends upon the size of the steps of the staircase generator. The smaller the size of the steps the larger the number of samples and higher the resolution of the image.

d) Scanning Oscilloscopes

These oscilloscopes use television tubes. The data to be measured are applied through intensity modulation on the standard screen. Several phenomena can be observed simultaneously on a single screen by using this technique. As a result of large number of factors influencing the quality of recording, experience with the particular camera CRO combination is usually the best guide.

e) Storage type Oscilloscopes

- They are rapidly becoming one of the most useful tools in the presentation of very slowly swept signals and finds many application in mechanical and biomedical fields.
- Usually in conventional CRTs, the persistence of phosphor ranges from microseconds to seconds. In applications where the persistence of the screen is smaller than the rate at which the signal sweeps across the screen, the start of screen will have disappeared before the end of the display is written.
- In storage oscilloscopes, the persistence times are much greater than a few
seconds or even hours are available, making it possible to store events on the CRT screen.

- The special CRT of storage oscilloscope contains electron gun, deflection plates, phosphor bronze screen but also it holds many number of special electrodes. The CRT used here is called as storage tube.
- The schematic diagram of Storage CRT below:

![Schematic diagram of a storage type CRT](image)

Fig. 29 Schematic diagram of a storage type CRT

- The storage mesh or the storage target is mounted just behind the phosphor screen is a conductive mesh covered with a highly resistive coating of magnesium fluoride.
- The write gun is a high-energy electron gun, similar to the conventional gun giving a narrow focussed beam which can be deflected and used to write the information to be stored.
- Because of the excellent insulating properties of the magnesium fluoride coating, the positively charged pattern remains exactly in the same position on the storage target which it was first deposited.
- The stored pattern may be made available for viewing at a later time by the use of two special electron guns called flood guns. The flood guns are placed inside the CRT in a position between the deflection plates and the storage target and they emit low-velocity electrons over a large area towards the entire screen.
- When the flood guns are switched for viewing mode low energy electrons are sprayed towards the screen. The electron trajectories are adjusted by the collimating electrodes which constitute a low-voltage electrostatic lens system, so that the flood electrons cover the entire screen area.
- To erase the pattern which is etched on the storage mesh, a negative voltage is applied to the storage target, neutralizing the stored positive charge.
- To get variable persistence, the erase voltage is applied in the form of pulses instead of a steady dc voltage. By varying the width of these pulses, the rate of erase is verified.
f) Impulse waveform oscilloscopes

These oscilloscopes are used for the investigation of transient non-period phenomena which occur at high voltages. These oscilloscopes use special types of CRT wherein the plates are mounted on the sides. The voltage to be measured is applied to these plates either directly or through capacitive potential dividers. Simultaneously, an impulse is suddenly applied to the cathode voltage. A very bright display is obtained on account of the high voltage and the high beam current which exist for a very short duration. Therefore, photographic records of the display can be obtained even at very high speeds of up to $50 \times 10^6$ m/s.

1.9 STORAGE CRO, DIGITAL STORAGE OSCILLOSCOPE

The storage type CRO is rapidly becoming one of the most useful tools in the presentation of very slowly swept signals and finds many applications in the mechanical and biomedical fields. In the conventional CRT the persistence of the phosphor ranges from microseconds to perhaps seconds. In applications where the persistence of the screen is smaller than the rate at which the signal sweeps across the screen, the start of the display will have disappeared before the end of the display is written.

With the variable-persistence or storage CRO, the slowly swept trace can be kept on display continuously by adjusting the persistence of the CRT screen to match the sweep time. Persistence times much greater than a few seconds or even hours, are available, making it possible to store events on a CRT screen. The storage CRO uses a special CRT, called the storage tube. This special CRT contains all the elements of a conventional CRT, such as the electron gun, the deflection plates, and a phosphor screen, but in addition holds a number of special electrodes. A schematic representation of one type storage tube is given in figure below.

The storage mesh or storage target, mounted just behind the phosphor screen, is a conductive mesh covered with a highly resistive coating of magnesium fluoride. The write gun is a high-energy electron gun, similar to the conventional gun, giving a narrow focused beam which can be deflected and used to write the information to be stored. The write gun etches a positively charged pattern on the storage target by knocking off secondary-emission electrons. Because of the excellent insulating properties of the magnesium fluoride coating, this positively charged pattern remains exactly in the same position on the storage target where it was first deposited. The electron beam, which is deflected in the conventional manner both in the horizontal and the vertical directions, therefore traces out the waveform pattern on the storage target.

![Schematic diagram of Storage type CRT](image-url)
The stored pattern may be made available for viewing at a later time by the use of two special electron guns, called flood guns. The flood guns are placed inside the CRT in a position between the deflection plates and the storage target and they emit low-velocity electrons over a large area towards the entire screen. When the flood guns are switched on (the viewing mode), low energy electrons are sprayed towards the screen. The electron trajectories are adjusted by the collimation electrodes which constitute a low voltage electrostatic lens system, so that the flood-electrons cover the entire screen area. Most of the flood electrons are collected by the collector mesh and therefore never reach the phosphor screen. In the area near the stored positive charge on the storage target, the positive field pulls some of the flood-electrons through the storage mesh and these electrons continue to hit the phosphor. The CRT display therefore will be an exact copy of the pattern which was initially stored on the target and the display will be visible as long as the flood guns continue emission of low-energy electrons. To erase the pattern which is etched on the storage mesh, a negative voltage is applied to the storage target, neutralizing the stored positive charge.

To obtain variable persistence, the erase voltage is applied in the form of pulses instead of a steady d.c. voltage. By varying the width of these pulses the rate of erasing is varied. The variable-persistence control on the front panel of the scope is then the width control of the erase-pulse generator.

2. INDUSTRIAL APPLICATIONS OF CRO

Because the oscilloscope is an extremely flexible and versatile instrument, it can be used to measure a number of parameters associated with DC and AC signals. Using a single channel oscilloscope, it is capable of making measurements of voltage, current, time, frequency and rise/fall time. If a dual trace oscilloscope is used the phase shift between two synchronous signals can be measured. Other major applications of CRO are listed below:

In Radio Work

1. To trace and measure a signal throughout the RF, IF and AF channels of radio and television receivers.
2. It provides the only effective way of adjusting FM receivers, broadband high-frequency RF amplifiers and automatic frequency control circuits;
3. to test AF circuits for different types of distortions and other spurious oscillations;
4. To give visual display of wave-shapes such as sine waves, square waves and their many different combinations;
5. To trace transistor curves
6. To visually show the composite synchronized TV signal
7. To display the response of tuned circuits etc.

Scientific and Engineering applications:

1. Measurement of ac/dc voltages,
2. Finding B/H curves for hysteresis loop,
3. for engine pressure analysis,
4. for study of stress, strain, torque, acceleration etc.
5. Frequency and phase determination by using Lissajous figures,
6. Radiation patterns of antenna,
7. Amplifier gain,
8. Modulation percentage,
9. Complex waveform as a short-cut for Fourier analysis,
10. Standing waves in transmission lines etc.

2 AMMETER AND VOLTOMETER

2.1 DC AMMETER

A PMMC galvanometer constitutes the basic movement of a dc ammeter. Since the coil winding of a basic movement is small and light it can carry only very small currents. When large currents are to be measured, it is needed to bypass major part of the current via a resistance called as shunt as shown below:

![Fig. 31 Basic DC ammeter.]

Since the shunt resistance is in parallel with the meter movement, the voltage drop across the shunt and movement must be the same.

\[
\text{Therefore, } V_{sh} = V_m
\]

\[
i.e., I_{sh}R_{sh} = I_mR_m,
\]

\[
R_{sh} = \frac{I_mR_m}{I_{sh}}
\]

For each required value of full scale meter current, we can determine the value of shunt resistance.
2.1.1 Multirange Ammeters:

![Multirange Ammeters diagram]

The current range of the dc ammeter may be further extended by a number of shunts selected by a range switch. Such a meter is called multirange ammeter. It is shown above.

The circuit has four shunts $R_1$, $R_2$, $R_3$, $R_4$ which can be placed in parallel with the movement to give four different current ranges. Switch 'S' is a multiposition switch having low contact resistance and high current carrying capacity since its contacts are in series with low resistance shunts.

Shunts are used for the extension of range of Ammeters. So a good shunt should have the following properties:
1- The temperature coefficient of shunt should be low
2- Resistance of shunt should not vary with time
3- They should carry current without excessive temperature rise
4- They should have thermal electromotive force with copper

‘Manganin’ is used for DC shunt and ‘Constantan’ as AC shunt.

2.1.2 Extending Ammeter ranges:

![Extending of Ammeters diagram]

The range of the ammeter can be extended to measure high current values by using external shunts connected to the basic meter movement (usually the lowest current range) as given in the figure above. Note that the range of the basic meter movement cannot be lowered. (for example if a 100µA movement with 100 scale division is used to measure 1µA, then the
meter will deflect by only one division. Hence ranges lower than the basic range are not practically possible)

2.2 AC Ammeter

An ideal ammeter senses the current flowing through it while maintaining zero voltage across its terminals. This implies that the meter must have zero internal resistance and behaves like a short circuit when connected to the circuit under test.

![Fig.34 Ideal ammeter has zero internal resistance](image)

A more realistic model of a real ammeter includes the internal resistance of the meter. If the internal resistance of the circuit is less compared with the circuit under test, then there will be minimal effect on the circuit being measured and the meter will approximate the ideal case.

An AC ammeter is similar to DC ammeter except that it measures the current of AC (not DC) waveforms. The same ideal and real (non-zero resistance) ammeter applies to this case of AC ammeter. All of the considerations discussed under AC voltmeters apply to AC ammeters as well.

![Fig.35 A realistic ammeter has a non-zero internal resistance](image)

2.3 DC Voltmeter

The basic PMMC meter can be converted into a DC voltmeter by adding a series resistor known as multiplier as shown below:
The function of the series resistor is to limit the current through the movement so that the current does not exceed the full scale deflection value. A dc voltmeter measures the potential difference between two points in a dc circuit or a circuit component. To measure the potential difference between two points in a dc circuit or a circuit component, dc voltmeter is always connected across them with proper polarity.

The value of the multiplier required is calculated as follows.

\[ I_m = \text{full scale deflection current of the movement (I_{fsd})} \]

\[ R_m = \text{Internal resistance of the movement} \]

\[ R_s = \text{multiplier resistance} \]

\[ V = \text{full range voltage of the instrument.} \]

from the circuit, \[ V = I_m(R_s + R_m) \]

\[ R_s = \frac{V}{I_m} - R_m. \]

The multiplier limits the currents through the movement, so as to not exceed the value of the full scale deflection \( I_{fsd} \). The above equation is used to further extend the range of DC voltmeter.

2.3.1 Multirange Voltmeter:

As in case of an ammeter, to obtain a multirange ammeter, a number of shunts were connected across the movement with a multi-position switch. Likewise a dc voltmeter can be converted into a multirange voltmeter by joining a number of resistors (multipliers) along with a range switch to provide a greater number of workable ranges.

Figure shows the multirange voltmeter using a four position switch.
2.3.2 Extending Voltmeter ranges:

The range of a voltmeter can be extended to measure high voltages by using a high voltage probe or by using an external multiplier resistor as shown below:

In most meters the basic movement is used on the lowest current range.

2.4 AC VOLTMETER

In electronic ac voltmeters input signal is firstly rectified and then supplied to the dc amplifier, as shown in figure. Sometimes signal is firstly amplified by ac amplifier and then rectified before supplying it to dc meter, as shown in figure.
Normally ac voltmeters are average responding type and the meter is calibrated in terms of the rms values for a sine wave. Since most of the voltage measurements involve sinusoidal waveform so this method of measuring rms value of ac voltages works satisfactorily and is less expensive than true rms responding voltmeters. However, in case of measurement of non-sinusoidal waveform voltage, this meter will give high or low reading depending on the form factor of the waveform of the voltage to be measured.

Circuit diagram of an average reading ac voltmeter using semi-conductor diode is shown in figure. The diode conducts during the positive half cycle and does not conduct during the -ve half cycle, as shown in figure.

![Circuit diagram of an average reading AC Voltmeter using semi-conductor diode](image)

**Fig.40 Average reading AC Voltmeter**

The average current through the meter will be given by the expression

\[ I_{av} = \frac{V_{av}}{2R} = 0.45 \times \frac{V_{rms}}{R} \]

\( V_{rms} \) is the effective or rms value of applied voltage and 1.11 is the form factor of sinusoidal wave. R is multiplied by 2 because the voltmeter operates on half-wave rectification. It is to be worth noting that this instrument can be used to indicate dc voltages but in such a case the instrument readings will have to be multiplied by 2 x 1.11, that is, as the diode conducts all the time. Main advantages associated with these voltmeters are that they are simple in construction, have high input impedance, low power consumption and uniform scale. Main disadvantage of these voltmeters is that these operate in audio-frequency range. In radio-frequency range, distributed capacitance of the high resistance R introduces error in the readings.

Another disadvantage of such a voltmeter is that due to non-linear volt-ampere characteristic for lower voltage the readings of the voltmeter at lower voltage are not correct.
2.4.1 Multirange AC Voltmeter:

![Multirange AC Voltmeters](image)

Resistances $R_1$, $R_2$, $R_3$, $R_4$ form a chain of multipliers for voltage ranges of 1000V, 250V, 50V and 10V respectively.

On the 2.5V range, resistance $R_5$ acts as a multiplier and corresponds to the multiplier $R_s$ as shown in the figure. $R_{sh}$ is the meter shunt and acts to improve the rectifier operation.

2.5 Ohmmeter

The ohmmeter means that it is an instrument which measures resistance of a quantity. Resistance in the electrical sense means the opposition offered by a substance to the current flow in the device. Every device has a resistance, it may be large or small and it increases with temperature for conductors, however for semiconducting devices the reverse is true. There are many types of ohmmeters available such as

1. Series ohmmeter.
2. Shunt ohmmeter.
3. Multi range ohmmeter.

![Ohmmeter](image)

The instrument is connected with a battery, a series adjustable resistor and an instrument which gives the reading. The resistance to be measured is connected at terminal ab. When the circuit is completed by connecting output resistance, the circuit current flows and so the deflection is measured. When the resistance to be measured is very high then current in the circuit will be very small and the reading of that instrument is assumed to be maximum resistance to be measured.
When resistance to be measured is zero then the instrument reading is set to zero position which gives zero resistance.

2.5.1 Series type Ohmmeter:

The series type ohmmeter consists of a current limiting resistor \( R_1 \), Zero adjusting resistor \( R_2 \), EMF source E, Internal resistance of D’Arsonval movement \( R_m \) and the resistance to be measured R. When there is no resistance to be measured, current drawn by the circuit will be maximum and the meter will show a deflection. By adjusting \( R_2 \) the meter is adjusted to a full scale current value since the resistance will be zero at that time. The corresponding pointer indication is marked as zero. Again when the terminal AB is opened it provides very high resistance and hence almost zero current will flow through the circuit. In that case the pointer deflection is zero which is marked at very high value for resistance measurement.

![Fig.43 Series type ohmmeter.](image)

So a resistance between zeros to a very high value is marked and hence can be measured. So, when a resistance is to be measured, the current value will be somewhat less than the maximum and the deflection is recorded and accordingly resistance is measured. This method is good but it posses certain limitations such as the decrease in potential of the battery with its use so adjustment must be made for every use. The meter may not read zero when terminals are shorted, these types of problem may arise which is counteracted by the adjustable resistance connected in series with the battery.

2.5.2 Shunt type Ohmmeter:

In this type of meters we have a battery source and an adjustable resistor is connected in series with the source. We have connected the meter in parallel to the resistance which is to be measured. There is a switch by the use of which we can on or off the circuit.

![Fig.44 Shunt type Ohmmeter.](image)
The switch is opened when it is not in use. When the resistance to be measured is zero, the terminals A and F are shorted so the current through the meter will be zero. The zero position of the meter denotes the resistance to be zero. When the resistance connected is very high, then a small current will flow through the terminal AF and hence full scale current is allowed to flow through the meter by adjusting the series resistance connected with the battery. So, full scale deflection measures very high resistance. When the resistance to be measured is connected between A and F, The pointer shows a deflection by which we can measure the resistance values. In this case also, the battery problem may arise which can be counteracted by adjusting the resistance. The meter may have some error due to its repeated use also.

2.5.3 Multirange Ohmmeter:

This instrument provides the reading up to a very wide range. In this case we have to select the range switch according to our requirement. An adjuster is provided so that we can adjust the initial reading to be zero. The resistance to be measured is connected in parallel to the meter.

![Multirange Ohmmeter](image)

**Fig. 45 Multirange Ohmmeter.**

The meter is adjusted so that it shows full scale deflection when the terminals in which the resistance connected is full scale range connected through the range switch. When the resistance is zero or short circuit, there is no current flow through the meter and hence no deflection. Suppose we have to measure a resistance under 1 ohm, then the range switch is selected at 1 ohm range at first. Then that resistance is connected in parallel and the corresponding meter deflection is noted. For 1 ohm resistance it shows full scale deflection but for the resistance other that 1 ohm it shows a deflection which is less than the full load value and hence resistance can be measured. This is the most suitable method of all the ohmmeters as we can get accurate reading in this type of meters. So this meter is most widely used now days.
2.3 ELECTRONIC MULTIMETERS

A multimeter is basically a PMMC meter. To measure dc current the meter acts as an ammeter with a low series resistance. Range changing is accomplished by shunts in such a way that the current passing through the meter does not exceed the maximum rated value. A multimeter consists of an ammeter, voltmeter and ohmmeter combined with a function switch to connect the appropriate circuit to the D'Arsonval movement.

Fig. 46 Diagram of a multimeter

The above figure shows the circuit diagram of a multimeter used as a microammeter, ac voltmeter, dc voltmeter, dc milliammeter and an ohmmeter.

2.3.1 Microammeter & DC ammeter

Following figure shows the circuit of a multimeter used as a microammeter as well as its use as a dc ammeter.

Fig. 47 Multimeter as a microammeter and as a dc ammeter.
2.3.2 DC Voltmeter:

Following figure shows the use of multimeter as a dc voltmeter.

![DC Voltmeter Diagram](image)

**Fig. 20 DC voltmeter section of the multimeter.**

2.3.3 AC Voltmeter:

![AC Voltmeter Diagram](image)

**Fig.48 AC voltmeter section of a multimeter.**

Figure shows the ac voltmeter section of a multimeter. To measure ac voltage, the output voltage is rectified by a half wave rectifier before the current passes through the meter. Across the meter, the other diode serves as protection. The diode conducts when a reverse voltage appears across the diodes, so that current bypasses the meter in the reverse direction.

2.3.4 Ohmmeter:

Referring to the following figure, which shows the ohmmeter section of a multimeter, in the 10k range the 102 Ω resistance is connected in parallel with the total circuit resistance and in the 1M Ω range the 102 Ω resistance is totally disconnected from the circuit.
Therefore on the 1M Ω range, the half scale deflection is 10k. Since on the 10k range, the 102 Ω resistance is joined across the total resistance, therefore in this range the half scale deflection is 100 Ω. The measurement of resistance is done by applying a small voltage installed within the meter. For the 1M Ω range, the internal resistance is 10k Ω. i.e., value at the mid scale.

The range of the ohmmeter can be changed by connecting the switch to a suitable shunt resistance. By using different values of shunt resistance, different ranges can be obtained. By increasing the battery voltage and using a suitable shunt, the maximum values which the ohmmeter reads can be changed.
TYPES OF VOLTMETER

2.4 DIFFERENTIAL TYPE VOLTMETER

The differential voltmeter technique is one of the most common and accurate methods of measuring unknown voltages. In this technique, the voltmeter is used to indicate the difference between known and unknown voltages. i.e., an unknown voltage is compared with a known voltage.

Following figure shows the basic circuit of a differential voltmeter based on the potentiometric method, hence the name potentiometric voltmeter.

Fig. 50 Basic Differential voltmeter.

In this method, the potentiometer is varied until the voltage across it equals the unknown voltage, which is indicated by the null indicator reading zero. Under null conditions, the meter draws current from neither the reference source nor the unknown voltage source and hence the differential voltmeter represents an infinite impedance to the unknown source (The null meter serves as an indicator only). To detect small differences the meter movement must be sensitive but it need not be calibrated, since only zero has to be indicated. The reference source used is usually a 1V dc standard source or a zener controlled precision supply. A high voltage reference supply is used for measuring high voltages.

The usual practice, however is to employ voltage dividers or attenuators across an unknown source to reduce the voltage. The input voltage divider has a relatively low input impedance, especially for unknown voltages much higher than the reference standard. The attenuation will have a loading effect and the input resistance of voltmeter is not infinity when an attenuator is used.

In order to measure ac voltages, the ac voltages must be converted into dc by incorporating a precision rectifier circuit. A block diagram of such ac differential voltmeter is shown below:

Fig. 51 Block diagram of an ac differential voltmeter.
2.4.1 True RMS Voltmeter

RMS value of the sinusoidal waveform is measured by the average reading voltmeter of which scale is calibrated in terms of rms value. This method is quite simple and less expensive. But sometimes rms value of the non-sinusoidal waveform is required to be measured. For such a measurement a true rms reading voltmeter is required. True rms reading voltmeter gives a meter indication by sensing heating power of waveform which is proportional to the square of the rms value of the voltage.

![Fig. 52 True RMS Voltmeter](image)

Thermo-couple is used to measure the heating power of the input waveform of which heater is supplied by the amplified version of the input waveform. Output voltage of the thermocouple is proportional to the square of the rms value of the input waveform. One more thermo-couple, called the balancing thermo-couple, is used in the same thermal environment in order to overcome the difficulty arising out of non-linear behaviour of the thermo-couple. Non-linearity of the input circuit thermo-couple is cancelled by the similar non-linear effects of the balancing thermo-couple. These thermo-couples form part of a bridge in the input circuit of a dc amplifier, as shown in block diagram.

AC waveform to be measured is applied to the heating element of the main thermo-couple through an ac amplifier. Under absence of any input waveform, output of both thermo-couples are equal so error signal, which is input to dc amplifier, is zero and therefore indicating meter connected to the output of dc amplifier reads zero. But on the application of input waveform, output of main thermo-couple upsets the balance and an error signal is produced, which gets amplified by the dc amplifier and feedback to the heating element of the balancing thermo-couple. This feedback current reduces the value of error signal and ultimately makes it zero to obtain the balanced bridge condition. In this balanced condition, feedback current supplied by the dc amplifier to the heating element of the balance thermo-couple is equal to the ac current flowing in the heating element of main thermo-couple. Hence this direct current is directly proportional to the rms value of the input ac voltage and is indicated by the meter connected in the output of the dc amplifier. The PMMC meter may be calibrated to read the rms voltage directly.
By this method, rms value of any voltage waveform can be measured provided that the peak excursions of the waveform do not exceed the dynamic range of the ac amplifier.

2.4.2 Vector Voltmeter

The vector voltmeter is basically a new type of amplitude and phase measuring device. It uses two samplers to sample the two waves whose amplitude and relative phase are to be measured. It measures the voltages at two different points in the circuit and also measures the phase difference between these voltages at these two points. In this voltmeter, two RF signals of the same fundamental frequency (1MHz to GHz) are converted to two IF signals. The amplitudes, waveforms and the phase relations of IF signals are same as that of RF signals. Thus, the fundamental components of the IF signal have the same amplitude and phase relationships as the fundamental components RF signals. These fundamental components are filtered from the IF signals and are measured by a voltmeter and phase meter.

![Block diagram of Vector Voltmeter](image)

**Fig. 53 Block diagram of Vector Voltmeter**

The vector voltmeter is useful in VHF applications and are used as:

1) Insertion losses
2) Complex impedance of mixers
3) S parameters of transistors
4) Radio frequency distortion
5) Amplitude modulation index
6) Amplifier gain and phase shift
7) Filter transfer functions
8) Two port network parameters.

The block diagram shows the instrument consists of five major sections. Two IF-RF converters, an automatic phase control section, a phase meter circuit, a voltmeter circuit. The RF-IF converters and the phase control sections produce two 20KHz sine waves with the same amplitudes and phase relationships as the fundamental components of the RF signals applied to channels A and B. The phase meter section continuously monitor these two 20KHz sine waves and indicates the phase angle between them. The voltmeter section can be switched to channel A or channel B to provide a meter display of the amplitude.

Each RF-IF converter consists of a sampler and a tuned amplifier. The sampler produces a 20 KHz replica of the input RF waveform, and the tuned amplifier extracts the 20KHz fundamental component from this waveform replica. The phase control unit is rather a sophisticated circuit that generates the sampling pulses for both RF-IF converters and automatically controls the pulse rate to produce 20KHz IF signals. The sampling pulse rate is controlled by a voltage tuned oscillator (VTO) for which the tuning voltage is supplied by an automatic phase control section. This section locks the IF signal of channel A to a 20KHz reference oscillator. To get initial locking, the phase control section applies a ramp voltage to the VTO. The ramp voltage sweeps the sampling rate until channel A IF is 20KHz and in phase with the reference oscillator.

To determine the phase difference between the two IF signals, the tuned amplifiers are followed by the phase meter circuit. Each channel is first amplified and then limited resulting in square wave signals at the inputs to the IF phase shifting circuits. The circuit in channel A shifts the phase of the square wave signal by +60°, the circuit in channel B shifts the phase of its signal by -120°. Both phase shifts are accomplished by a combination of capacitive networks and inverting and non-inverting amplifiers whose vector-sum outputs provide the desired phase shift. The outputs of the phase shift circuits are amplified and clipped, producing square waveforms and applied to trigger amplifiers. These circuits convert the square wave input signals to positive spikes with very fast rise times. The bistable multivibrator is triggered by the pulses from both the channels. Channel A is joined to the set input of the MV, channel B is joined to the reset input of the MV. If the initial phase shift between the RF signals at the probes was 0°, the trigger pulses into the multivibrator are 180° out of phase owing to the action of the phase shift circuits. The MV then produces a square wave output voltage which is symmetrical about zero.

The instrument contains a power supply section, which is not shown in the block diagram. The power supply generates all the needed supply voltages for the various sections of the instrument.
3. WAVE ANALYSER:

![Basic Wave Analyzer](image)

**Fig.54 Basic Wave Analyzer.**

It consists of a primary detector, which is a simple LC circuit. This LC circuit is adjusted for resonance at the frequency of the particular harmonic component to be measured. The intermediate stage is a full wave rectifier to obtain the average value of the input signal. The indicating device is a simple DC voltmeter that is calibrated to read the peak value of the sinusoidal input voltage. Since the LC circuit is tuned to a single frequency it passes only the frequency to which it is tuned and rejects all other frequencies. A number of tuned filters, connected to the indicating device through a selector switch would be required for a useful Wave Analyzer.

3.1.1 Frequency Selective Wave Analyzer:

The wave analyzer consists of a very narrow pass-band filter section which can be tuned to a particular frequency within audible frequency range (20Hz-20KHz). The block diagram of wave analyzer is shown below:

![Frequency Selective Wave Analyzer](image)

**Fig.55 Frequency Selective Wave Analyzer.**

The complex wave to be analyzed is passed through an adjustable attenuator which serves as a range multiplier and permits a large range of signal amplitudes to be analyzed without loading the amplifier.

The output of the attenuator is then fed to a selective amplifier which amplifies the selected frequency. The driver amplifier applies the attenuated input signal to a high Q-active filter. This high Q-filter is a low pass filter which allows the frequency which is selected to pass and reject all others. The magnitude of this selected frequency is indicated by the meter and the filter section identifies the frequency of the component. The filter circuit consists of a
cascaded RC resonant circuit and amplifiers. For selecting the frequency range, the capacitors generally used are of the closed tolerance polystyrene type and the resistances used are precision potentiometers. The capacitors are used for range changing and the potentiometer is used to change the frequency within the selected pass-band. Hence this wave analyser is also called frequency selective voltmeter.

The entire AF range is covered in decade steps by switching capacitors in the RC section. Then selected signal output from the final amplifier stage is applied to the meter circuit and to an untuned amplifier. The main function of the buffer amplifier is to drive output devices such as recorders or electronic counters. The meter has several voltage ranges as well as decibel scales marked on it. It is driven by an average reading rectifier type decoder.

The wave analyser must have extremely low input distortion, undetectable by the analyser itself. The bandwidth of the instrument is very narrow, typically about 1% of selective band given by the following response characteristics.

![Fig.56 Relative response in Decibels (Db)](image_url)

3.1.2 Heterodyne Wave Analyzer:

Wave analyzers are useful for measurement in the audio frequency range only. For measurements in the RF range and above (MHz range), an ordinary wave analyzer cannot be used. Hence, special types of wave analyzers working on the principle of heterodyning (mixing) are used. These wave analyzers are known as Heterodyne Wave Analyzer. In this wave analyzer, the input signal to be analyzed is heterodyned with the signal from the internal tunable local oscillator in the mixer stage to produce a higher IF frequency. By tuning the local oscillator frequency, various signal frequency components can be shifted within the pass-band of the IF amplifier. The output of the IF amplifier is rectified and applied to the meter circuit.

An instrument that involves the principle of heterodyning is the Heterodyning tuned voltmeter, shown in figure below:
The input signal is heterodyned to the known IF by means of a tunable local oscillator. The amplitude of the unknown component is indicated by the VTVM (vacuum tube voltmeter) or output meter. The VTVM is calibrated by means of signals of known amplitude. The frequency of the component is identified by the local oscillator frequency, i.e. the local oscillator frequency is varied so that all the components can be identified. The local oscillator can also be calibrated using input signals of known frequency. The fixed frequency amplifier is a multistage amplifier which can be designed conveniently because of its frequency characteristics. This analyzer has good frequency resolution and can measure the entire AF frequency range. With the use of a suitable attenuator, a wide range of voltage amplitudes can be covered. Their disadvantage is the occurrence of spurious cross-modulation products, setting a lower limit to the amplitude that can be measured.

Two types of selective amplifiers find use in Heterodyne wave analyzers. The first type employs a crystal filter, typically having a centre frequency of 50 kHz. By employing two crystals in a band-pass arrangement, it is possible to obtain a relatively flat pass-band over a 4 cycle range. Another type uses a resonant circuit in which the effective Q has been made high and is controlled by negative feedback. The resultant signal is passed through a highly selective 3-section quartz crystal filter and its amplitude measured on a Q-meter.

When a knowledge of the individual amplitudes of the component frequency is desired, a heterodyne wave analyzer is used.

A modified heterodyne wave analyzer is shown in figure below:
In this analyzer, the attenuator provides the required input signal for heterodyning in the first mixer stage, with the signal from a local oscillator having a frequency of 30—48 MHz.

The first mixer stage produces an output which is the difference of the local oscillator frequency and the input signal, to produce an IF signal of 30 MHz. This IF frequency is uniformly amplified by the IF amplifier. This amplified IF signal is fed to the second mixer stage, where it is again heterodyned to produce a difference frequency or IF of zero frequency.

The selected component is then passed to the meter amplifier and detector circuit through an active filter having a controlled band-width. The meter detector output can then be read off on a db-calibrated scale, or may be applied to a secondary device such as a recorder.

This wave analyzer is operated in the RF range of 10 kHz — 18 MHz, with 18 overlapping bands selected by the frequency range control of the local oscillator. The bandwidth, which is controlled by the active filter, can be selected at 200 Hz, 1 kHz and 3 kHz.

3.2. SPECTRUM ANALYSER

The most common way of observing signals is to display them on an oscilloscope, with time as the X-axis (i.e. amplitude of the signal versus time). This is the time domain. It is also useful to display signals in the frequency domain. The instrument providing this frequency domain view is the spectrum analyzer.

A Spectrum Analyzer Block Diagram provides a calibrated graphical display on its CRT, with frequency on the horizontal axis and amplitude (voltage) on the vertical axis. Displayed as vertical lines against these coordinates are sinusoidal components of which the input signal is composed. The height represents the absolute magnitude, and the horizontal location represents the frequency. These instruments provide a display of the frequency spectrum over a given frequency band. Spectrum analyzers use either a parallel filter bank or a swept frequency technique. In a parallel filter bank analyzer, the frequency range is covered by a series of filters whose central frequencies and bandwidth are so selected that they overlap each other, as shown in figure below.

![Spectrum Analyser (parallel filter bank analyser)](Fig.59)

Typically, an audio analyzer will have 32 of these filters, each covering one third of an octave. For wide band narrow resolution analysis, particularly at RF or microwave signals, the swept technique is preferred.
Basic Spectrum Analyser using Swept Receiver Design:

Referring to above block diagram, the sawtooth generator provides the sawtooth voltage which drives the horizontal axis element of the scope and this sawtooth voltage is frequency controlled element of the voltage tuned oscillator. As the oscillator sweeps from $f_{\text{min}}$ to $f_{\text{max}}$ of its frequency band at a linear recurring rate, it beats with the frequency component of the input signal and produce an IF, whenever a frequency component is met during its sweep. The frequency component and voltage tuned oscillator frequency beats together to produce a difference frequency, i.e. IF. The IF corresponding to the component is amplified and detected if necessary, and then applied to the vertical plates of the CRO, producing a display of amplitude versus frequency.

The spectrum produced if the input wave is a single tuned A.M. is given in figure below:

![Spectrum Analyser](image)

**Fig.60 Spectrum Analyser**

![Test wave seen on ordinary CRO](image)

**Fig.61 Test wave seen on ordinary CRO**

![Display on the spectrum CRO](image)

**Fig.62 Display on the spectrum CRO**
Fig. 9 Test waveform as seen on X-axis (time) and Z-axis (frequency)

One of the principal applications of spectrum analyzers has been in the study of the RF spectrum produced in microwave instruments. In a microwave instrument, the horizontal axis can display as a wide a range as 2 — 3 GHz for a broad survey and as narrow as 30 kHz, for a highly magnified view of any small portion of the spectrum. Signals at microwave frequency separated by only a few kHz can be seen individually.

The frequency range covered by this instrument is from 1 MHz to 40 GHz. The basic block diagram is of a spectrum analyzer covering the range 500 kHz to 1 GHz, which is representative of a superheterodyne type.

The input signal is fed into a mixer which is driven by a local oscillator. This oscillator is linearly tunable electrically over the range 2 — 3 GHz. The mixer provides two signals at its output that are proportional in amplitude to the input signal but of frequencies which are the sum and difference of the input signal and local oscillator frequency.

The IF amplifier is tuned to a narrow band around 2 GHz, since the local oscillator is tuned over the range of 2 — 3 GHz, only inputs that are separated from the local oscillator frequency by 2 GHz will be converted to IF frequency band, pass through the IF frequency amplifier, get rectified and produce a vertical deflection on the CRT.

From this, it is observed that as the saw tooth signal sweeps, the local oscillator also sweeps linearly from 2 — 3 GHz. The tuning of the spectrum analyzer is a swept receiver, which sweeps linearly from 0 to 1 GHz. The saw tooth scanning signal is also applied to the horizontal plates of the CRT to form the frequency axis. (The Spectrum Analyzer Block Diagram is also sensitive to signals from 4 — 5 GHz referred to as the image frequency of the super heterodyne. A low pass filter with a cutoff frequency above 1 GHz at the input suppresses these spurious signals.) Spectrum analyzers are widely used in radars, oceanography, and biomedical fields.

3.3. DISTORTION ANALYSER

Harmonic Distortion Analyzer-Fundamental Suppression Type:

A Harmonic Distortion Analyzer measures the total harmonic power present in the test wave rather than the distortion caused by each component. The simplest method is to suppress the fundamental frequency by means of a high pass filter whose cut off frequency is a little above the fundamental frequency. This high pass allows only the harmonics to pass and the total harmonic distortion can then be measured. Other types of Harmonic Distortion Analyzer based on fundamental suppression are as follows.
a) Employing a Resonance Bridge type:

The bridge shown in Figure below. This bridge is balanced for the fundamental frequency, i.e. L and C are tuned to the fundamental frequency. The bridge is unbalanced for the harmonics, i.e. only harmonic power will be available at the output terminal and can be measured. If the fundamental frequency is changed, the bridge must be balanced again. If L and C are fixed components, then this method is suitable only when the test wave has a fixed frequency. Indicators can be thermocouples or square law VTVMs (vacuum tube voltmeters). This indicates the rms value of all harmonics. When a continuous adjustment of the fundamental frequency is desired, a Wien bridge system is employed.

![Resonance Bridge](image)

Fig.64 Resonance Bridge

b) Wien's Bridge method:

The bridge is balanced for the fundamental frequency. The fundamental energy is dissipated in the bridge circuit elements. Only the harmonic components reach the output terminals. The harmonic distortion output can then be measured with a meter. For balance at the fundamental frequency, \( C_1, C_2, C, R_1 = R_2 = R, R_3 = 2R_4 \).

![Wien's Bridge method](image)

Fig.65 Wien's Bridge method.

c) Bridged T-Network:
Referring to figure below, the L and C’s are tuned to the fundamental frequency, and R is adjusted to bypass fundamental frequency. The tank circuit being tuned to the fundamental frequency, the fundamental energy will circulate in the tank and is bypassed by the resistance. Only harmonic components will reach the output terminals and the distorted output can be measured by the meter. The Q of the resonant circuit must be at least 3-5.

One way of using a bridge T-network is given in figure below. The switch S is first connected to point A so that the attenuator is excluded and the bridge T-network is adjusted for full suppression of the fundamental frequency, i.e. minimum output. Minimum output indicates that the bridged T-network is tuned to the fundamental frequency and that the fundamental frequency is fully suppressed.

The switch is next connected to terminal B, i.e. the bridged T-network is excluded. Attenuation is adjusted until the same reading is obtained on the meter. The attenuator reading indicates the total rms distortion. Distortion measurement can also be obtained by means of a wave analyzer, knowing the amplitude and the frequency of each component, the Harmonic Distortion Analyzer can be calculated. However, distortion meters based on fundamental suppression are simpler to design and less expensive than wave analyzers. The disadvantage is that they give only the total distortion and not the amplitude of individual distortion components.

Reference books

SCHOOL OF ELECTRICAL AND ELECTRONICS
DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – V
MEASUREMENT AND INSTRUMENTATION – SIC 1203
UNIT :5

Introduction to ADC / DAC - Specifications, ADC Quantization Error, Types of ADC - Flash, Counter, Successive Approximation, Dual-Slope types and Introduction to Delta-Sigma, Types of DAC - Weighted-Resistor, 2R ladder and PWM.

I. DATA CONVERTERS AND CONNECTORS

1.1 ADC AND DAC SPECIFICATIONS

Resolution

The resolution of a DAC is the smallest change in the output of the DAC for any change in digital input. i.e. if a input to DAC changes one bit, how much analog output has changed in full scale deflection.

\[ \% \text{resolution} = \left( \frac{\text{Step size}}{\text{Full scale output (FSO)}} \right) \times 100 \]

In other way the resolution is the number of states into which the full scale output is divided. i.e if a 8 bit DAC can resolve the FSO up to 255 levels. Each level of output is called step size and for higher number of bits the resolution will be better.

\[ \% \text{resolution} = \left( \frac{1}{2^N - 1} \right) \times 100 \]

Normally the resolution will be in milli volts.

Accuracy

The Accuracy of a DAC is the difference between output practical analog output to the ideal expected output for a given digital input. The DAC is contains electronic components where the gain plays a major role which can introduce gain error in the output. Due to the the full scale output may differ compared to ideal one. For an example if a DAC of 10 V is said to have an accuracy of 0.01% there will be 10mv output deviation. The another factor which implicates the accuracy is the zero offset error i.e for a zero input the output of DAC reflects some offset value.

Conversion Speed

The conversion speed of the DAC is output analog value settling time period for a change in the digital input. This is also called settling time period of DAC. Normally it will be micro seconds and in some advanced micro controller DAC it may be nano seconds.

Monotonicity

The Digital to Analog Converter is said to be monotonic if its analog value is either increasing or equal to previous value for an LSB change in input digital signal.

Offset/ Zero scale error:

An input code of zero may be expected to give 0V output. A small offset may be present and the transfer characteristic does not pass through the origin.
Linearity

The input-output characteristic of a D/A converter. zero offset and gain develop in the characteristic which passes through the origin and full scale points. But it is not sure that the intermediate points will always lie in the straight line. A very small error in the weighting factor for a fraction LSB will cause non-linearity. Linearity can be expressed by deviation from the ideal line as a percentage or fraction of LSB. It is specified as ±LSB or ±1/2 LSB.

![Graph showing linearity error for a 3-bit ADC](image)

**Fig.1: Linearity error for a 3-bit ADC**

Settling time:

This is usually expressed as the time taken to settle within half LSB. Generally settling time will be 500ns.

**Stability:**

The ability of a DAC to produce a stable output all the time is called as Stability. The performance of a converter changes with drift in temperature, aging and power supply variations. So all the parameters such as offset, gain, linearity error & monotonicity may change from the values specified in the datasheet. Temperature sensitivity defines the stability of a D/A converter.

**1.2 Quantization error**

Quantization is representing the sampled values of the amplitude by a finite set of levels, which means converting a continuous-amplitude sample into a discrete-time signal.

For any system, during its functioning, there is always a difference in the values of its input and output. The processing of the system results in an error, which is the difference of those values.
The difference between an input value and its quantized value is called a Quantization Error. A Quantizer is a logarithmic function that performs Quantization (rounding off the value). An analog-to-digital converter (ADC) works as a quantizer.

The following figure illustrates an example for a quantization error, indicating the difference between the original signal and the quantized signal.

![Quantization Error](image)

**Fig.2 Quantization error.**

### 2. TYPES OF ADC

#### 2.1 FLASH TYPE ADC:

This is the simplest possible A/D converter. It is at the same time, the fastest and most expensive technique. Figure shows a 3 bit A/D converter. The circuit consists of a resistive divider network, 8 op-amp comparators and a 8-line to 3-line encoder (3-bit priority encoder). The Comparator and its truth table are shown in Figure below.
Fig. 3 A comparator and its truth table

<table>
<thead>
<tr>
<th>Voltage input</th>
<th>Logic output $X$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a &gt; V_d$</td>
<td>$X = 1$</td>
</tr>
<tr>
<td>$V_a &lt; V_d$</td>
<td>$X = 0$</td>
</tr>
<tr>
<td>$V_a = V_d$</td>
<td>Previous value</td>
</tr>
</tbody>
</table>

Fig. 4 Flash type A-D converter
A small amount of hysteresis is built into the comparator to resolve any problems that might occur if both inputs were of equal voltage as shown in the truth table. From the Figure, at each node of the resistive divider, a comparison voltage is available. Since all the resistors are of equal value, the voltage levels available at the nodes are equally divided between the reference voltage $V_R$ and the ground. The purpose of the circuit is to compare the analog input voltage $V_a$ with each of the node voltages. The truth table for the flash type A/D converter is shown above.

### Advantages of flash type A/D converter

- High speed simultaneous conversion
- Typical conversion time is 100 ns or less.

### Disadvantages of flash type A/D converter

- The number of comparators required almost doubles for each added bit.
- Larger the value of $n$ (number of bits), the more complex is the priority encoder.

#### 2.2 COUNTER TYPE ADC

The counter type ADC is the basic form of ADC which is also called as ramp type ADC or stair case approximation ADC. This circuit consists of N-bit counter, DAC and comparator.

![Fig.5 Counter type ADC](image-url)
The N bit counter generates an n bit digital output which is applied as an input to the DAC. The analog output corresponding to the digital input from DAC is compared with the input analog voltage using an op-amp comparator. The op-amp compares the two voltages and if the generated DAC voltage is less, it generates a high pulse to the N-bit counter as a clock pulse to increment the counter. The same process will be repeated until the DAC output equals to the input analog voltage.

If the DAC output voltage is equal to the input analog voltage, then it generates low clock pulse and it also generates a clear signal to the counter and load signal to the storage resistor to store the corresponding digital bits. These digital values are closely matched with the input analog values with small quantization error. For every sampling interval the DAC output follows a ramp fashion so that it is called as Digital ramp type ADC. And this ramp looks like stair cases for every sampling time so that it is also called as staircase approximation type ADC.

![Fig.6 Digital output and analog input for a counter type ADC](image)

Conversion time of ADC is the time taken by the ADC to convert the input sampled analog value to digital value. Here the maximum conversion of high input voltage for a N bit ADC is the clock pulses required to the counter to count its maximum count value. So

The maximum conversion of Counter type ADC is \( (2^N-1) \times T \)

Where, \( T \) is the time period of clock pulse.

If \( N=2 \) bit then \( T_{max} = 3T \).

By observing the above conversion time of Counter type ADC it is illustrated that the sampling period of Counter type ADC should be as shown below.

\( T_s \geq (2^{N-1}) \times T \)

**Advantages of Counter type ADC:**

- Simple to understand and operate.
- Cost is less because of less complexity in design.
Disadvantages or limitations of Counter type of ADC:

- Speed is less because every time the counter has to start from ZERO.
- There may be clash or aliasing effect if the next input is sampled before completion of one operation.

### 2.3 SUCCESSIVE APPROXIMATION TYPE ADC

- A Successive Approximation Register (SAR) is added to the circuit
- Instead of counting up in binary sequence, this register counts by trying all values of bits starting with the MSB and finishing at the LSB.
- The register monitors the comparators output to see if the binary count is greater or less than the analog signal input and adjusts the bits accordingly

![Fig.7 Successive Approximation ADC Circuit](image)

**Elements:**

- DAC = Digital to Analog Converter
- EOC = End of Conversion
- SAR = Successive Approximation Register
- S/H = Sample and Hold Circuit
- \( V_{in} \) = Input Voltage
- Comparator
- \( V_{ref} \) = Reference Voltage

**Algorithm**
- Uses an \( n \)-bit DAC and original analog results
- Performs a binary comparison of \( V_{DAC} \) and \( V_{in} \)
- MSB is initialized at 1 for DAC
- If \( V_{in} < V_{DAC} \) (\( V_{REF}/2^{n-1} \)) then MSB is reset to 0
- If \( V_{in} > V_{DAC} \) (\( V_{REF}/2^{n} \)) Successive Bits set to 1 otherwise 0
- Algorithm is repeated up to LSB
- At end DAC in = ADC out
- \( N \)-bit conversion requires \( N \) comparison cycles

**Example 1:**

5-bit ADC, \( V_{in}=0.6V, V_{ref}=1V \)

**Cycle 1** => MSB=1

\[
\text{SAR} = 1 0 0 0 0 \\
V_{DAC} = V_{ref}/2^1 = .5 \\
V_{in} > V_{DAC} \quad \text{SAR unchanged} = 1 0 0 0 0
\]

- Cycle 2

\[
\text{SAR} = 1 1 0 0 0 \\
V_{DAC} = .5 + .25 = .75 \\
V_{in} < V_{DAC} \quad \text{SAR bit3 reset to 0} = 1 0 0 0 0
\]

- Cycle 3

\[
\text{SAR} = 1 0 1 0 0 \\
V_{DAC} = .5 + .125 = .625 \\
V_{in} < V_{DAC} \quad \text{SAR bit2 reset to 0} = 1 0 0 0 0
\]

- Cycle 4

\[
\text{SAR} = 1 0 0 1 0 \\
V_{DAC} = .5 + .125 = .625 \\
V_{in} < V_{DAC} \quad \text{SAR bit2 reset to 0} = 1 0 0 0 0
\]
\[ V_{DAC} = 0.5 + 0.0625 = 0.5625 \quad V_{in} > V_{DAC} \quad \text{SAR unchanged} = 10010 \]

- Cycle 5

SAR = 10011

\[ V_{DAC} = 0.5 + 0.0625 + 0.03125 = 0.59375 \]

**Table 2: Input vs Voltage**

<table>
<thead>
<tr>
<th>Bit</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>0.5</td>
<td>0.25</td>
<td>0.125</td>
<td>0.0625</td>
<td>0.03125</td>
</tr>
</tbody>
</table>

**Advantages**

- Capable of high speed and reliable
- Medium accuracy compared to other ADC types
- Good tradeoff between speed and cost
- Capable of output the binary number in serial (one bit at a time) format.
- High resolution
- No precision external components needed

**Disadvantages**

- Higher resolution successive approximation ADC’s will be slower
- Speed limited.

**2.4 DUAL SLOPE ADC**

- An unknown input voltage is applied to the input of the integrator and allowed to ramp for a fixed time period (\( t_u \))
- Then, a known reference voltage of opposite polarity is applied to the integrator and is allowed to ramp until the integrator output returns to zero (\( t_d \))
- The input voltage is computed as a function of the reference voltage, the constant run-up time period, and the measured run-down time period.
- The run-down time measurement is usually made in units of the converter's clock, so longer integration times allow for higher resolutions.
The speed of the converter can be improved by sacrificing resolution.

$$V_{in} = -V_{ref} \frac{t_d}{t_u}$$

**Fig. 8 Dual Slope ADC**

$$V(t) = V(0) + \frac{V_{in}}{RC} t$$

**Fig. 9 Graph**
### Table 3: Comparison

<table>
<thead>
<tr>
<th>Type</th>
<th>Speed (relative)</th>
<th>Cost (relative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual Slope</td>
<td>Slow</td>
<td>Med</td>
</tr>
<tr>
<td>Flash</td>
<td>Very Fast</td>
<td>High</td>
</tr>
<tr>
<td>Successive Appox</td>
<td>Medium – Fast</td>
<td>Low</td>
</tr>
<tr>
<td>Sigma-Delta</td>
<td>Slow</td>
<td>Low</td>
</tr>
</tbody>
</table>

#### 2.5 INTRODUCTION TO DELTA-SIGMA

A Delta Sigma (SD-ADC) has a modulator and a digital filter (also known as decimation filter) as shown in figure below. A modulator converts the input analog signal into digital bit streams (1s and 0s). One can observe a bit, either 1'b1 or 1'b0 coming at every clock edge of the modulator.

The decimation filter receives the input bit streams and, depending on the over sampling ratio (OSR) value, it gives one N-bit digital output per OSR clock edge. For example, if we consider OSR to be 64, then the Filter gives one N-bit output for every 64 clock edges (64 data outputs of the modulator). Here N is the resolution of the SD ADC.

![Fig.10 Delta Sigma ADC](image-url)
In a conventional ADC, an analog signal is integrated, or sampled, with a sampling frequency and subsequently quantized in a multi-level quantizer into a digital signal. This process introduces quantization error noise. The first step in a delta-sigma modulation is delta modulation. In delta modulation the change in the signal (its delta) is encoded, rather than the absolute value. The result is a stream of pulses, as opposed to a stream of numbers as is the case with PCM. In delta-sigma modulation, the accuracy of the modulation is improved by passing the digital output through a 1-bit DAC and adding (sigma) the resulting analog signal to the input signal, thereby reducing the error introduced by the delta-modulation. Primarily because of its cost efficiency and reduced circuit complexity, this technique has found increasing use in modern electronic components such as DACs, ADCs, frequency synthesizers, switched-mode power supplies and motor controllers. Both ADCs and DACs can employ delta-sigma modulation. A delta-sigma ADC first encodes an analog signal using high-frequency delta-sigma modulation, and then applies a digital filter to form a higher-resolution but lower sample-frequency digital output. On the other hand, a delta-sigma DAC encodes a high-resolution digital input signal into a lower-resolution but higher sample-frequency signal that is mapped to voltages, and then smoothed with an analog filter. In both cases, the temporary use of a lower-resolution signal simplifies circuit design and improves efficiency.

In brief, because it is very easy to regenerate pulses at the receiver into the ideal form transmitted. The only part of the transmitted waveform required at the receiver is the time at which the pulse occurred. Given the timing information the transmitted waveform can be reconstructed electronically with great precision. In contrast, without conversion to a pulse stream but simply transmitting the analog signal directly, all noise in the system is added to the analog signal, permanently reducing its quality. Each pulse is made up of a step up followed after a short interval by a step down. It is possible, even in the presence of electronic noise, to recover the timing of these steps and from that regenerate the transmitted pulse stream almost noiselessly. Then the accuracy of the transmission process reduces to the accuracy with which the transmitted pulse stream represents the input waveform.

Delta-sigma modulation converts the analog voltage into a pulse frequency and is alternatively known as Pulse Density modulation or Pulse Frequency modulation. In general, frequency may vary smoothly in infinitesimal steps, as may voltage, and both may serve as an analog of an infinitesimally varying physical variable such as acoustic pressure, light intensity, etc. The substitution of frequency for voltage is thus entirely natural and carries in its train the transmission advantages of a pulse stream. The different names for the modulation method are the result of pulse frequency modulation by different electronic implementations, which all produce similar transmitted waveforms.

The ADC converts the mean of an analog voltage into the mean of an analog pulse frequency and counts the pulses in a known interval so that the pulse count divided by the interval gives an accurate digital representation of the mean analog voltage during the interval. This
interval can be chosen to give any desired resolution or accuracy. The method is cheaply produced by modern methods; and it is widely used.

3. TYPES OF DAC

![Fig 11: DAC](image)

The input in the digital to analog converter is an n-bit binary word D and is combined with a reference voltage $V_R$ to give an analog output signal. The output of a DAC can be either a voltage or current. For a voltage output DAC, the D/A converter is mathematically described as

$$V_o = K \cdot V_{FS} \left( d_1 \cdot 2^{-1} + d_2 \cdot 2^{-2} + \ldots + d_n \cdot 2^{-n} \right) \quad \text{----- (1)}$$

Where,

- $V_o$ = output voltage
- $V_{FS}$ = full scale output voltage
- $K$ = scaling factor usually adjusted to unity
- $d_1, d_2, \ldots, d_n$ = n-bit binary fractional word with the decimal point located at the left
  - $d_1$ = most significant bit (MSB) with a weight of $V_{FS}/2$
  - $d_n$ = most significant bit (MSB) with a weight of $V_{FS}/2^n$

There are various ways of implementing DAC

- Weighted-Resistor DAC
- 2R ladder DAC
- PWM type DAC

3.1 WEIGHTED RESISTOR DAC

One of the simplest circuits is shown in Figure uses a summing amplifier with a binary weighted resistor network. It has n- electronic switches $d_1, d_2, \ldots, d_n$ controlled by binary input word. These switches are single pole double throw (SPDT) type. If the binary input to a particular switch is 1, it connects the resistance to the reference voltage ($-V_R$). And if the input bit is 0, the
switch connects the resistor to the ground. From Figure (a) the output current \( I_o \) for an ideal op-amp can be written as

\[
I_o = I_1 + I_2 + \ldots + I_n
\]

\[
= \frac{V_R}{2R} d_1 + \frac{V_R}{2^2R} d_2 + \ldots + \frac{V_R}{2^nR} d_n
\]

\[
= \frac{V_R}{R} (d_1 2^{-1} + d_2 2^{-2} + \ldots + d_n 2^{-n})
\]

The output voltage

\[
V_o = I_o R_f = V_R \frac{R_f}{R} (d_1 2^{-1} + d_2 2^{-2} + \ldots + d_n 2^{-n}) \quad \text{----- (2)}
\]

Comparing equation (1) with (2) it can be seen that if \( R_f = R \) then \( K = 1 \) and \( V_{FS} = V_R \).

The circuit shown in Figure uses a negative reference voltage. The analog output voltage is therefore positive staircase as shown in Figure for a 3-bit weighted resistor DAC. It may be noted that

- Although the op-amp in Figure is connected in inverting mode, it can also be connected in non-inverting mode.
- The op-amp is simply working as a current to voltage converter.
- The polarity of the reference voltage is chosen in accordance with the type of the switch used. For example, for TTL compatible switches, the reference voltage should be \( = 5 \) V and the output will be negative.

The accuracy and stability of a DAC depends upon the accuracy of the resistors and the tracking of each other with temperature. There are however a number of problems associated with this type of DAC. One of the disadvantages of binary weighted type DAC is the wide range of resistor values required. It may be observed that for better resolution, the input binary word length has to be increased. Thus, as the number of bit increases, the range of resistance value increases. For 8-bit DAC, the resistors required are \( 2^0 R, 2^1 R, 2^2 R \ldots 2^7 R \). The largest resistor is 128 times the smallest one for only 8-bit DAC. For a 12-bit DAC, the largest resistance required is 5.12 MΩ if the smallest is 2.5 kΩ. The fabrication of such a large resistance is IC not practical. Also the voltage drop across such a large resistor due to the bias current would also affect the accuracy. The choice of smallest resistor value as 2.5 kΩ is reasonable; otherwise loading effect will be there. The difficulty of achieving and maintaining accurate ratios over such a wide range especially in monolithic form restricts the use of weighted resistor DACs to below 8-bits.
3.2 R-2R LADDER DAC

Wide range of resistors are required in binary weighted resistor type DAC. This can be avoided by using R-2R ladder type DAC where only two values of resistors are required. It is well suited for integrated circuit realization. The typical value of resistor ranges from 2.5kΩ to 10kΩ.

For simplicity, consider a 3-bit DAC as shown in Figure, where the switch position \( d_1 \), \( d_2 \), \( d_3 \) corresponds to the binary word 100. The circuit can be simplified to the equivalent form of Figure (b) and finally to Figure (c). then, voltage at node C can be easily calculated by the set procedure of network analysis as
The output voltage

\[ V_o = \frac{-2R}{R} \left( -\frac{V_R}{4} \right) = \frac{V_R}{2} = \frac{V_{FS}}{2} \]

The switch position corresponding to the binary word 001 in 3 bit DAC is shown in Figure (a). The circuit can be simplified to the equivalent form of Fig(b). The voltages at the nodes (A,B,C) formed by resistor branches are easily calculated in a similar fashion and the output voltage becomes
\[ V_o = \left( -\frac{2R}{R} \right) \left( -\frac{V_R}{16} \right) = \frac{V_R}{8} = \frac{V_{FS}}{8} \]

In a similar fashion, the output voltage for R-2R ladder type DAC corresponding to other 3-bit binary words can be calculated.

### 3.3 PWM TYPE DAC

The PWM signal outputs on a device are variable duty cycle square-waves with 3.3 volt amplitude. These signals can each be decomposed into a D.C. component plus a new square-wave of identical duty-cycle but with a time-average amplitude of zero. Figure below depicts this graphically.

![Decomposition of PWM signal](image)

**Fig.15 Decomposition of PWM signal**
The idea behind realizing digital-to-analog (D/A) output from a PWM signal is to analog low-pass filter the PWM output to remove most of the high frequency components, ideally leaving only the D.C. component. This is depicted in Figure below. The bandwidth of the low-pass filter will essentially determine the bandwidth of the digital-to-analog converter. A frequency analysis of the PWM signal is given in the next section in order to provide a theoretical basis for the filtering strategy.

![Fig.16 Analog filtering of PWM signal](image)

The PWM/DAC approach is not new, but performance limitations have historically confined its use to low-resolution, low-bandwidth applications. The performance of the method relates directly to the ability of the low-pass filter to remove the high-frequency components of the PWM signal. Use a filter with too low a cut-off frequency, and DAC bandwidth suffers. Use a filter with too high a cut-off frequency or with slow stop-band roll-off, and DAC resolution suffers, but one way to alleviate both of these problems is to increase the frequency of the PWM. However, as PWM frequency increases on conventional microprocessor generated PWM, digital resolution problems begin to manifest.

**Reference books**
