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SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING

DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

DEPARTMENT OF ELECTRONICS AND INSTRUMENTATION ENGINEERING

UNIT -I

Electrical and Electronics Engineering – SEEA1103

DC CIRCUITS

Electrical Quantities, Ohm's law, Kirchoff's laws, Resistors in series and parallel combinations, Current and Voltage division rules, Node and Mesh Analysis.

ELECTRICAL QUANTITIES – DEFINITIONS, SYMBOLS AND UNITS

- Charge:

A body is said to be charged positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge on 6.28×10^{18} electrons.

- **Atom:**

To understand the basic concepts of electric current, we should know the Modern Electron Theory. Consider the matter which is in the form of solid, liquid (or) gas. Smallest particle of matter is molecule. Minute Particles are called molecules, which are themselves made up of still minute particles known as Atoms.

Atom: Minute tiny Particles with the central Part Nucleus.

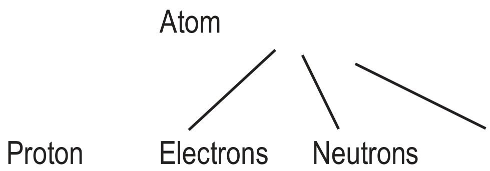


Figure 1.1

These are the types of tiny Particles in an Atom.

Protons: It is charged with positive charge.

Neutron: It is uncharged and hence it is neural.

Electron: It is revolving around nucleus. It is charged with small and constant amount of negative charge.

In an Atom, No of electrons = No of Protons

- **Electric Potential:**

When a body is charged, either electrons are supplied on it (or) removed from it. In both cases the work is done. The ability of the charged body to do work is called electric potential. The charged body has the capacity to do, by moving the other charges by either attraction (or) repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential. And the work done, to charge a body to 1 Colomb is the measure of electric potential.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

W = Work done per unit charge.
Q = Charge measured in Coulombs.

Unit of electric potential is **Joules / Coulomb (or) Volt**. If W = 1 joule; Q = 1 Coulomb, then V = 1/1 = 1 Volt.

A body is said to have an electric potential of 1 Volt, if one Joule of work is done to charge a body to one Coulomb. Hence greater the Joules / Coulomb on a charged body, greater is electric potential.

- **Potential Difference:**

The difference in the potentials of two charged bodies is called potential difference.

Consider two charged bodies A and B having Potentials of 5 Volts and 3 Volts respectively.

Potential Difference is +2v.
Unit of potential difference is Volts.
Potential difference is sometimes called Voltage.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is Ampere (or) Coulomb / sec.

$$\text{Current, } (I) = \frac{\text{Charge(q)}}{\text{Sec Time(t)}} = \frac{q}{t} \text{ Coulombs /}$$

In differential form, $i = \frac{dq}{dt}$ Coulombs / Sec

Consider a conducting material like metal, say Copper. A large number of free electrons are available. They move from one Atom to the other at random, before an electric force is applied. When an electric potential difference is applied across the metallic conductors, free electrons start moving towards the positive terminal of the cell. This continuous flow of electrons forms electric current. According to modern electronic theory, the direction of conventional current is form positive terminal to negative terminal through the external circuit.

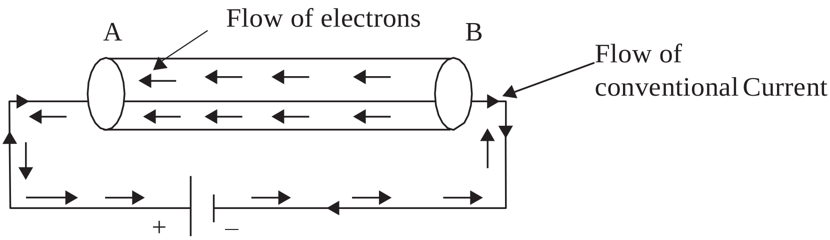


Figure 1.2

Thus, a wire is said to carry a current of 1 Ampere when charge flows through it at the rate of one Coulomb per second.

- **Resistance:**

Consider a conductor which is provided some potential difference. The free electrons start moving in a particular direction. While moving, the free electrons may collide with some Atoms (or) Molecules. They oppose the flow of electrons. Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical property of the substance due to which it opposes (or) restricts the flow of electricity (i.e. electrons) through it. Its unit is Ohms.

A wire is said to have a resistance of 1 ohm if a potential difference of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 Joule, when a current of 1A flows through it.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is inversely proportional to the cross sectional area of the Conductor, ie, $R \alpha \frac{l}{A}$

(iii) It depends upon the nature of the material of the conductor.

(iv) It depends upon the temperature of the conductor.

From the First three points and assuming the temperature to remain constant, we get,

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{1}{A}$$

ρ (‘Rho’) is a constant of proportionality called **Resistivity** (or) Specific Resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

- **Specific Resistance (or) Resistivity:**

Resistance of a wire is given by $R = \rho \frac{1}{A}$

If l = 1metre, A =1m^2 then, R = ρ . The resistance offered by a wire of length 1 metre and across sectional area of Cross-section of 1m^2 is called the Resistivity of the material of the wire.

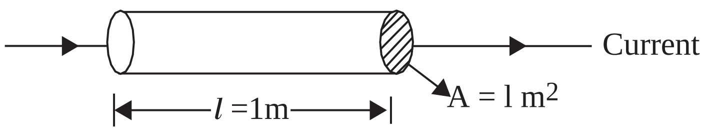


Figure 1.3

If a cube of one meter side is taken instead of wire, ρ is defined as below., Let l = 1 metre, A = 1 m^2, then R = ρ. “Hence, the resistance between the opposite faces of 1 metre cube of the given material is called the resistivity of that material”. The unit of resistivity is ohm-metre

$$\left[\ \rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m(\text{ohm-metre})\ \right]$$

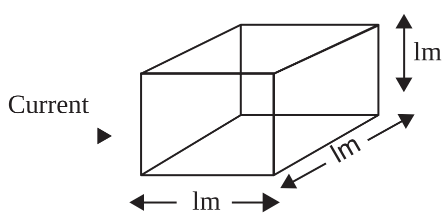


Figure 1.4

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho Symbol for its unit is ($\mho$)

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the Conductivity (or) Specific Conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows flow of current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G \frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{\text{m}^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in an electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done inmoving the electrons in a unit time is called Electric Power. The unit of Electric Power is Joules/sec (or) Watt. $\{P = VI = I^2 R = V^2 / R\}$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = (Electrical Power)*(Time)

$$\text{Electrical Energy} = I^2Rt = \frac{V^2}{R} t$$

Electrical Energy is measured in Kilowatt hour (kwh)

Problem 1.1 The resistance of a conductor 1 mm^2 in cross section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material.

Given Data

Area of cross-section A = 1 mm^2
Length, l = 20 m
Resistance, R = 0.346 Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A}$

$$\Rightarrow \rho = \frac{RA}{l}$$

Solution: Area of Cross-section, $A = 1mm^2$

$$= 1 * 10^{-6} m^2$$

$$\rho = \frac{1*10^{-6} * 0.346}{20} = 1.738*10^{-8}\ \Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\,\Omega m$.

Problem 1.2 A Coil consists of 2000 turns of copper wire having a cross-sectional area of 1 mm^2. The mean length per turn is 80 cm and resistivity of copper is 0.02 μΩm at normal working temperature. Calculate the resistance of the coil.

Given data:

No of turns = 2000
Length / turn = 80 cm =0.8 m

Resistivity, = 0.02 μΩm = 0.02*10^{-6} Ωm = 2*10^{-8} Ωm
Cross sectional area of the wire, A= 1mm^2 = 1*10^{-6}m^2

Solution:
Mean length of the wire, l = 2000*0.8 =1600 m.

We know that, $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2 * 10^{-8} * 1600}{1*10^{-6}} = 32\Omega$

Resistance of the coil $= 32\Omega$

Problem 1.3 A wire of length 1m has a resistance of 2Ω. What is the resistance of the second wire, whose specific resistance is double that of first, if the length of wire is 3m and the diameter is double that of first?

Given Data:

For the first wire: $l_1 = 1m$, $R_1 = 2\ \Omega$, $\rho_1 = \rho$ (say)

$$d_1 = d\ \ (say)$$

For the Second wire: $l_2 = 3m$, $d_2 = 2d$, $\rho_2 = 2\rho$

Solution:

$$R_1 = \rho_1 \frac{l_1}{A_1} = \frac{\rho * 1}{\frac{\pi d^2}{4}} \quad \text{[Radius of the wire } = \pi r^2 \text{, where } r = \frac{d}{2}\text{]}$$

$$\text{ie, } R_1 = \frac{4\rho}{\pi d^2} = \frac{\rho_1 * 1}{\pi d^2 / 4} \ \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} = \frac{2\rho * 3}{\frac{\pi (2d)^2}{4}} = \frac{6\rho}{\pi d^2} \qquad (2)$$

Dividing equation (1) by (2),

$$\frac{4\rho}{\pi d^2} * \frac{\pi d^2}{6\rho} \Rightarrow \frac{4}{6} = \frac{R_1}{R_2}$$

$$R_2 = \frac{6R_1}{4} = \frac{6 * 2}{4} = 3\ \Omega$$

$$R_2 = 3\ \Omega$$

The Resistance of the second wire, $R_2 = 3\ \Omega$

Problem 1.4 A Rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (i) opposite ends and (ii) opposite sides. The resistivity of copper is $1.7 * 10^{-6}\ \Omega$cm.

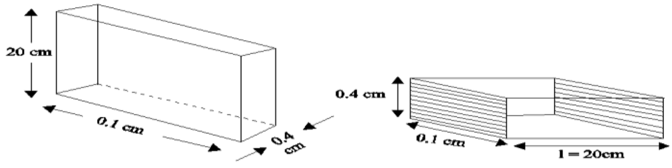


Figure 1.5 **Figure 1.6**

(i) Opposite Ends

Wide, w = 0.1cm

Thickness, t = 0.4cm

Length, l = 20cm

(ii) Opposite Sides:

Wide, w = 0.1cm

Thickness, t = 20 cm

Length, l = 0.4 cm

(a) Area = $w*t = 0.1*0.4 = 0.04cm^2$

$$R_1 = \frac{\rho l}{A} = \frac{1.7*10^{-6}*20}{0.04} = 0.85*10^{-3}\,\Omega$$

$R_1 = 0.85\,m\wedge$ [Opposite ends, referring to Figure 1.5]

Area, $A = w*t = 0.1*20 = 20cm^2$

$R_2 = \frac{1.7*10^{-6}*0.4}{2} = 0.34*10^{-6}\,\Omega$ [Opposite Sidesi referring to Figure 1.6]

$R_2 = 0.34\mu\,\Omega$

Problem 1.5 A silver wire of length 12m has a resistance of 0.2Ω. Find the specific resistivity of the material. The cross-sectional area of the wire is 0.01cm^2.

$R = \frac{\rho l}{A}$ $\Rightarrow$ length, l = 12m

Resistance, R= 0.2Ω

A = 0.01cm^2

$$\rho = \frac{RA}{l} = \frac{0.2*0.01*10^{-4}}{12}$$

$$\rho = 1.688*10^{-8}\,\Omega\,m$$

OHM'S LAW AND ITS LIMITATIONS

The relationship between DC potential difference (V) current (I) and Resistance (R) in a DC circuit was first discovered by the scientist George Simon Ohm, is called Ohm's law.

- **Statement:**

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition (eg. Temperature, etc.) do not change.

$$\text{ie, } \frac{V}{I} = \text{Constant}$$

(or)

$$\frac{V}{I} = R$$

$$\Rightarrow V = I * R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

$$\text{Power, } P = V * I = I^2 R = \frac{V^2}{R}$$

- **Illustration:**

Let the potential difference between points A and B be V volts and current flowing be I Amp. Then, $\frac{V}{I} = \text{Constant}$,

$$\frac{V}{I} = R \text{ (say)}$$

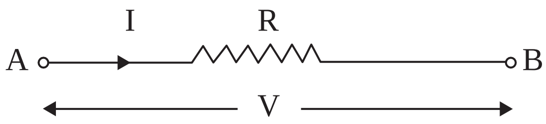


Figure 1.7

We know that, if the voltage is doubled (2V), the current flowing will also be doubled (2I). So, the ratio $\frac{V}{I}$ remains the same (ie, R). Also when voltage is measured in volts, current in ampere, then resistance will be in ohms.

- **Graphical representation of Ohm's law**

[Slope line of the graph represents the resistance]

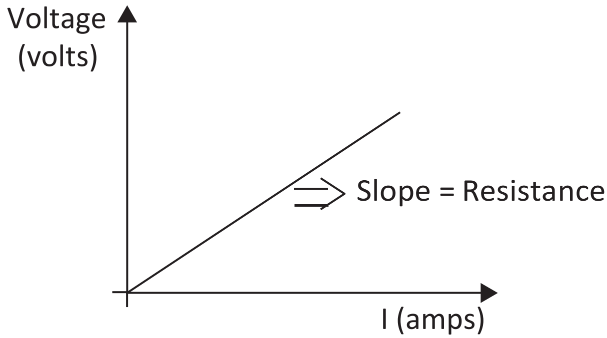


Figure 1.8

- **Limitations in ohm's law**:

  (i) Ohm's law does not apply to all non-metallic conductors. For eg. Silico Carbide.
  (ii) It also does not apply to non-linear devices such as Zener diode, etc.
  (iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

- **Problems based on ohm's law**:

Problem 1.6. An electric heater draws 8A from 250V supply. What is the power rating? Also find the resistance of the heater element.

Given data:

Current, $I = 8A$
Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8 * 250 = 2000 Watt$

Resistance (R) = $\frac{V}{I} = \frac{250}{8} = 31.25\ \Omega$

Problem 1.7 What will be the current drawn by a lamp rated at 250V, 40W, connected to a 230 V supply.

Given Data:

Rated Power = 40 W

Rated Voltage = 250 V

Supply Voltage = 230 V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\ \Omega$$

$$\text{Current, } I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472\ A$$

Problem 1.8 A Battery has an emf of 12.8 volts and supplies a current of 3.24 A. What is the resistance of the circuit? How many Coulombs leave the battery in 5 minutes?

Solution:

$$\text{Circuit Resistance, R} = \frac{V}{I} = \frac{12.8}{3.24} = 4\ \Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.24×5×60 = 960 Coulomb

Problem 1.9 If a resistor is to dissipate energy at the rate of 250W, find the resistance for a terminal voltage of 100V.

Given data:

Power = 250W

Voltage = 100V

Solution:

$$\text{Resistance, } R = \frac{V^2}{\rho} = \frac{100^2}{250} = 40\ \Omega$$

$$R = 40\ \Omega\ .$$

Problem 1.10 A voltmeter has a resistance of, 20,200 Ω. When connected in series with an external resistance across a 230 V supply, the instrument reads 160 V. What is the value of external resistance?

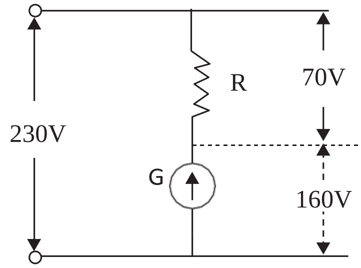


Figure 1.9

The voltage drop across external resistance, R

$$V_R = 230 - 160 = 70V$$

$$\text{Circuit current, } I = \frac{160}{20{,}000} = \frac{1}{125} A$$

We know that, $V = IR$

$$70 = IR$$

$$70 = \frac{1}{125} \times R$$

$$R = 8750\ \Omega$$

COMBINATION OF RESISTORS

- **Introduction:**

The closed path followed by direct Current (DC) is called a DC Circuit A d.c circuit essentially consist of a source of DC power (eg. Battery, DC generator, etc.) the conductors used to carry current and the load. The load for a DC circuit is usually a resistance. In a DC circuit, loads (i.e, resistances) may be connected in series, parallel, series – parallel. Hence the resistor has to be connected in the desired way for getting the desired resistance.

Resistances in series (or) series combination

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends. [A and B]

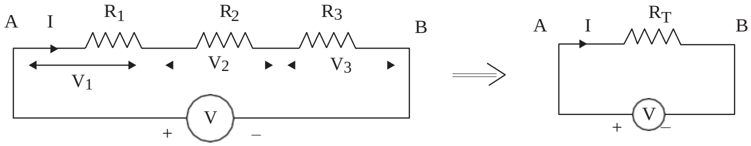


Figure 1.10

In the above circuit, there is only one closed path, so only one current flows through all the elements. In other words, if the Current is same through all the resistors, the combination is called series combination.

- **To find equivalent Resistance**:

Let, V = Applied voltage
I = Source current = Current through each element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By Ohms law, $V_1 = IR_1$

$V_2 = IR_2$ and $V_3 = IR_3$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\left(R_1 + R_2 + R_3\right)$

$$V = I\left(R_1 + R_2 + R_3\right)$$

$$V = IR_T$$

$$\frac{V}{I} = R_T$$

The ratio of $\left(V/I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\dfrac{1}{G_T} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$ (In terms of conductance)

$\therefore$ Equivalent resistance (R_T) is the sum of all individual resistances.

- **Concepts of series circuit**:

i. The current is same through all elements.
ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.
iii. The equivalent resistance (R_T) is greater than the greatest individual resistance of that combination.
iv. Voltage drops are additive.
v. Powers are additive.
vi. The applied voltage is equals to the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 interms of V and R_1, R_2, R_3:

Equivalent Resistance, $R_T = R_1 + R_2 + R_3$

By ohm's low, $I = \dfrac{V}{R_T} = \dfrac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

$\therefore$ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms in series, then the total Resistance is given by,

$$R_T = n * R$$

- **Applications:**
  - ∗ When variable voltage is given to the load, a variable resistance (Rheostat) is connected in series with the load. Example: Fan regulator is connected in series with the fan.
  - ∗ The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
  - ∗ When a load of low voltage is to be operated on a high voltage supply, a fixed value of resistance is connected in series with the load.
- **Disadvantage of Series Circuit:**
  - ∗ If a break occurs at any point in the circuit, no current will flow and the entire circuit becomes useless.
  - ∗ If 5 numbers of lamps, each rated 230 volts are to be connected in series circuit, then the supply voltage should be 5 x 230 = 1150 volts. But voltage available for lighting circuit in each and every house is only 230 V. Hence, series circuit is not practicable for lighting circuits.
  - ∗ Since electrical devices have different current ratings, they cannot be connected in series for efficient operation.
- **Problems based on series combination:**

Problem 1.11 Three resistors 30 Ω, 25 Ω, 45 Ω are connected in series across 200V. Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.

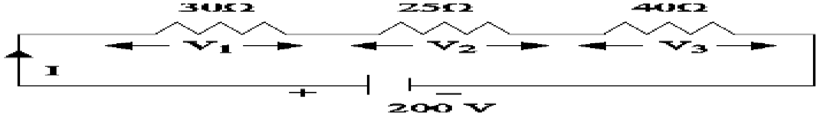


Figure 1.11

(i) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3$$
$$R_T = 30 + 25 + 45 = 100\ \Omega$$

(ii) Current, $I = \dfrac{V}{R_T} = \dfrac{200}{100} = 2\ A$

(iii) Potential difference across each element,

$$V_1 = IR_1 = 2 * 30 = 60\ V$$
$$V_2 = IR_2 = 2 * 25 = 50\ V$$
$$V_3 = IR_3 = 2 * 45 = 90\ V$$

Problem 1.12 Find the value of 'R' in the circuit diagram, given below.

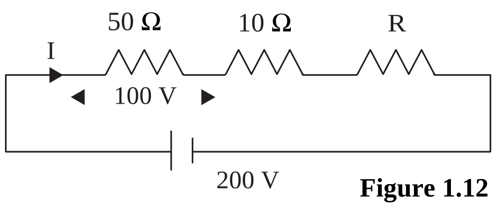


Figure 1.12

We know that, $V_1 = IR_1$

$$I = V_1 / R_1 = 100/50 = 2\ A$$

Similarly, $V_2 = IR_2 = 2 * 10 = 20\ V$

Total voltage drop, $V = V_1 + V_2 + V_3$

$$V_3 = V - (V_1 + V_2) = 200 - (100 + 20)$$
$$V_3 = 80\ V$$
$$V_3 = IR_3,\ R_3 = V_3 / I = 80/2 = 40\ \Omega$$
$$\therefore R_3 = 40\ \Omega$$

Problem 1.13 A 100W, 200V bulb is put in series with a 60W bulb across a supply. What will be the current drawn? What will be the voltage across the 60W bulb? What will be the supply voltage?

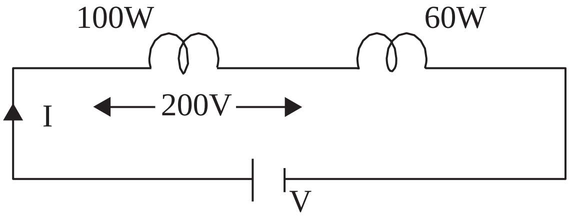


Figure 1.13

Power dissipated in the first bulb, $P_1 = V_1 I$
Current, I = P_1 / V_1 = 100/200 = 0.5 A
Power dissipated in the second bulb, $P_2 = V_2 I$
Voltage across the 60 W bulb,

$$V_2 = \frac{P_2}{I} = \frac{60}{0.5} = 120V$$

The supply voltage, $V = V_1 + V_2 = 200 + 120$

$$V = 320V$$

The supply voltage, V = 320 V.

Problem 1.14 An incandescent lamp is rated for 110V, 100W. Using suitable resistor how can you operate this lamp on 220V mains.

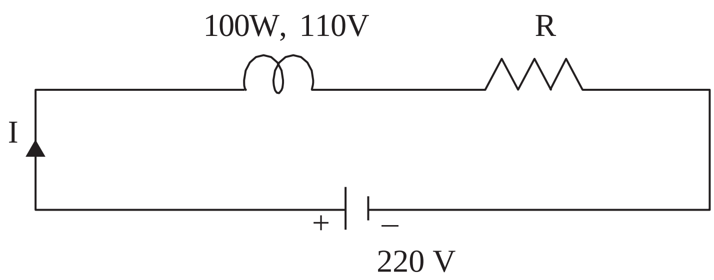


Figure 1.14

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110}$ = 0.909A, I = 0.909A

For satisfactory operation of the lamp, Current of 0.909A should flow. When the voltage across the lamp is 110V, then the remaining voltage must be across R

$$\text{Supply voltage} = V = 220 \text{ Volts}$$
$$\text{Voltage across } R = V - 110 \text{ Volts}$$
$$\text{ie, } V_R = 220 - 110 = 110V$$
$$\text{By ohm's law, } V_R = IR$$
$$110 = 0.909 \text{ R}$$
$$R = 121\ \Omega$$

Problem 1.15 The lamps in a set of decoration lights are connected in series. If there are 20 lamps and each lamp has resistance of 25Ω, calculate the total resistance of the set of lamp and hence calculate the current taken from a supply of 230 volts.

Given Data: Supply voltage, $V = 230\ volts$

Resistance of each lamp, $R = 25\ \Omega$

No of lamps in series, $n = 20$

Solution: Total Resistance, $R_T = n * R = 20 * 25$

$$R_T = 500\ \Omega$$

Current from supply. $I = \dfrac{V}{R_T} = \dfrac{230}{500} = 0.46\ A$

Problem 1.16 The field coil of a d.c generator has a resistance of 250Ω and is supplied from a 220 V source. If the current in the field coil is to be limited to 0.44 A. Calculate the resistance to be connected in series with the coil.

Given Data: Source voltage, $V = 220$ volts, $I = 0.44\ A$

Field coil resistance, $R_f = 250\ \Omega$

Solution: Let the resistance in series with R_f be R in Ohms.

Total resistance, $R_T = R_f + R = 250 + R$

Current, $I = 0.44\ A$

By ohm's law, $R_T = \dfrac{V}{I} = \dfrac{220}{0.44} = 500\ \Omega$

$$250 + R = 500\ \Omega$$

$$R = 500 - 250 = 250\ \Omega$$

$$R = 250\ \Omega$$

Resistance in Parallel (or) Parallel Combination

If one end of all the resistors are joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and is given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B, as shown in the Figure 1.15(a)..

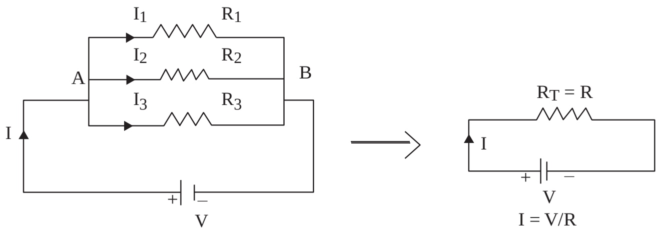


Figure 1.15

$$I = \frac{V}{R} \qquad (1)$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \qquad (2)$$

Total current is the sum of three individual currents,

$$I_T = I = I_1 + I_2 + I_3 \qquad (3)$$

Substituting the above expression for the current in equation (3),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Referring to Figure (1.15(b)), $R_T = R$

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (4)$$

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie, Power dissipated by R = Power dissipated by R_1 + Power dissipated by R_2 + Power dissipated by R_3

We know that reciprocal of Resistance is called as conductance.

Conductance = 1 / Resistance

[G = 1/R]

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

- **Concepts of Parallel Circuit:**
  - Voltage is same across all the elements.
  - All elements will have individual currents, depends upon the resistance of element.
  - The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
  - If n resistance each of R are connected in parallel then,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots n\,terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

(or)

$$R_T = \frac{R}{n}$$

  - Powers are additive.
  - Conductance are additive.
  - Branch currents are additive.

- **Current Division Technique:**

Case (i) When two resistances are in parallel:

Two resistance R_1 and R_2 ohms are connected in parallel across a battery of V (volts) Current through R_2 is I_2 and through R_2 is I_2 The total current is I.

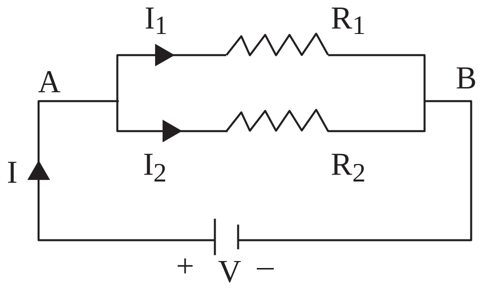


Figure 1.16

To express I_1 and I_2 interms of I, R_1 and R_2 (or) to find branch currents I_1, I_2:

$$I_2 R_2 = I_1 R_1$$

$$I_2 = \frac{I_1 R_1}{R_2} \qquad (1)$$

Also, the total current, $I = I_1 + I_2$ (2)

Substituting (1) in (2), $I_1 + \dfrac{I_1 R_1}{R_2} = I$

$$\frac{I_1 R_2 + I_1 R_1}{R_2} = I$$

$$I_1 (R_1 + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \dfrac{IR_1}{(R_1 + R_2)}$

To find the equivalent Resistance, (R_T) :

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, the total value of two resistances connected parallel is equal to their product divided by their sum i.e.,

$$\text{Equivalent Resistance} = \frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel. Let I be the supply current (or) total current. I_1, I_2, and I_3 are the currents through the resistors R_1, R_2 and R_3.

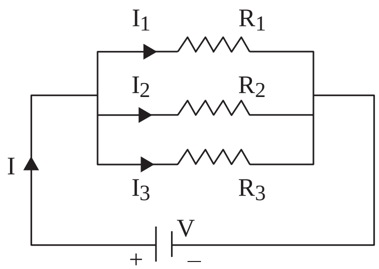


Figure 1.17

To find the equivalent Resistance (R_T):

$$\frac{1}{R}=\frac{1}{R_T}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$$

$$\frac{1}{R_T}=\frac{R_1R_2+R_2R_3+R_3R_1}{R_1R_2R_3}$$

$$R_T=\frac{R_1R_2R_3}{R_1R_2+R_2R_3+R_3R_1}$$

To find the branch currents I_1, I_2 and I_3:

We know that, $I_1+I_2+I_3=I$ (1)

Also, $I_3R_3=I_1R_1=I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (1)

$$I_2=\frac{I_1R_1}{R_2}\,;\,I_3=\frac{I_1R_1}{R_3}$$

$$I_1+\frac{I_1R_1}{R_2}+\frac{I_1R_1}{R_3}=I$$

$$I_1(1+\frac{R_1}{R_2}+\frac{R_1}{R_3})=I$$

$$\frac{I_1(R_2R_3+R_3R_1+R_1R_2)}{R_2R_3}=I$$

$$I_1=\frac{I(R_2R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

$$I_3=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

- **Advantages of parallel circuits:**
  - The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other's performance.
  - If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

- **Applications of parallel circuits:**
  * All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switches.
  * Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided, inversely proportional to resistance.
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal to sum of individual resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	Reciprocal of the equivalent resistance is equal to sum of reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest resistances in the circuit.
There is only one path for the flow of current.	There are more than one path for the flow of current.

- **Problems based on parallel combinations:**

Problem 1.17 What is the value of the unknown resistor R shown in Figure 1.18. If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

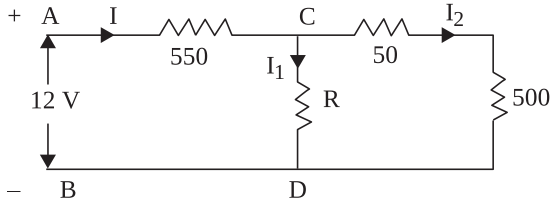


Figure 1.18

Given Data:

$$V_{500} = 2.5V$$

$$I_2 = \frac{V_{500}}{R} = \frac{2.5}{500} = 0.005A$$

V_{50} = Voltage across 50 Ω

$V_{50} = I_2 R = 0.005*50 = 0.25$ V

$V_{CD} = V_{50} + V_{500} = 0.25 + 2.5 = 2.75$ V

V_{550} = Drop across 550Ω = 12 – 2.75 = 9.25 V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 232.69\ \Omega$$

R = 232.69 Ω

Problem 1.18 Three resistors 2 Ω, 3 Ω and 4 Ω are in parallel. How will be a total current of 8A is divided.

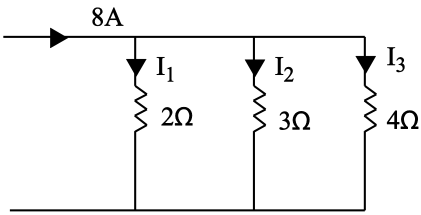


Figure 1.19

This given circuit can be reduced as, 3 Ω and 4 Ω are connected in parallel.

Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\Omega$

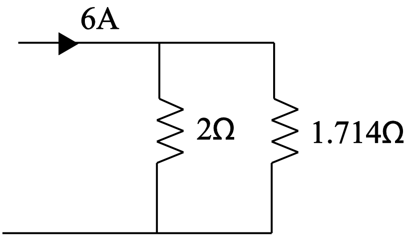


Figure 1.20

1.714 Ω and 2 Ω are connected in parallel, its equivalent resistance is 0.923 Ω

$$\frac{1.714 * 2}{2 + 1.714} = 0.923$$

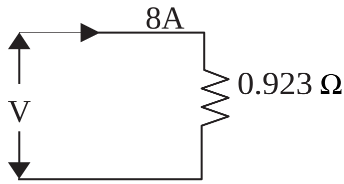


Figure 1.21

$$V = IR = 8 * 0.923$$
$$V = 7.385V$$

Branch currents, $I_1 = \dfrac{V}{R_1} = \dfrac{7.385}{2} = 3.69$ A

$$I_2 = \frac{V}{R_2} = \frac{7.385}{3} = 2.46\,A$$
$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Problem 1.19 What resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω

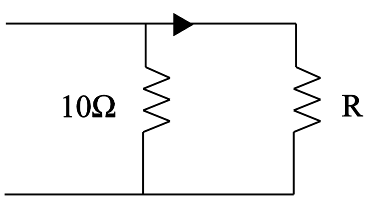


Figure 1.22

R is connected in parallel with 10 Ω Resistor to given an equivalent resistance of 6 Ω.

$$\frac{10 * R}{10 + R} = 6$$
$$10R = (10 + R)6$$
$$10R = 60 + 6R$$
$$10R - 6R = 60$$
$$R = \frac{60}{4} = 15\ \Omega$$
$$R = 15\ \Omega$$

Problem 1.20 Two resistors R_1 and R_2 are connected in Parallel and a Voltage of 200V DC is applied to the terminals. The total current drawn is 20A, R_1=30 Ω. Find R_2 and power dissipated in each resistor, for the figure 1.23.

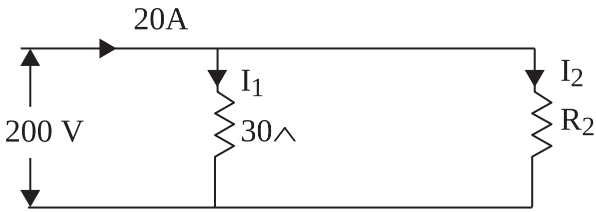


Figure 1.23

Given Data:

V = 200V, I = 20A, R_1 = 30 Ω

Solution: $I_1 = \frac{V}{R_1} = \frac{200}{30} = 6.667\ A$

$$I_1 + I_2 = I$$

$$I_2 = I - I_1$$

$$= 20 - 6.667 = 13.33 \text{ A}$$

$$I_2 = \frac{IR_1}{R_1 + R_2}$$

$$13.33 = \frac{20*30}{30 + R_2}$$

$$(30 + R_2)13.33 = 600$$

$$13.33R_2 = 600 - 400$$

$$13.33R_2 = 200$$

$$R_2 = \frac{200}{13.33} = 15\ \Omega$$

$$R_2 = 15\ \Omega$$

Power dissipated in 30 Ω, $P_1 = VI_1$ = 200*6.667

P_1 = 1333 W

Power dissipated in 15 Ω, $P_2 = VI_2$

P_2 = 200*13.33 = 2667

P_2 = 2667 W

Problem 1.21 Calculate the current supplied by the battery in the given circuit as shown in the figure 1.24.

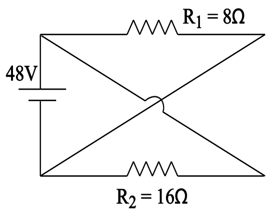


Figure 1.24

Solution: The above given circuit can be redrawn as,

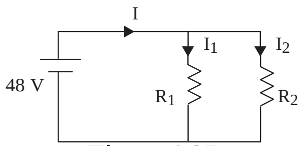


Figure 1.25

R_1 and R_2 are in parallel across the voltage of 48 volts.

Equivalent Resistance, $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{8*16}{8+16} = \dfrac{16\ \Omega}{3}$

$$R_T = 5.33\wedge$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9\text{A}$$

Problem 1.22 Calculate the total resistance and battery current in the given circuit

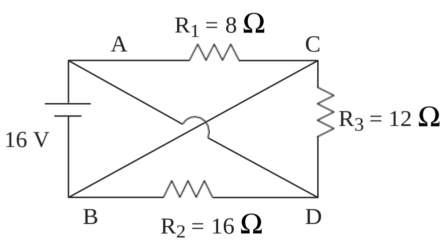


Figure 1.26

The given above circuit can be re-drawn as,

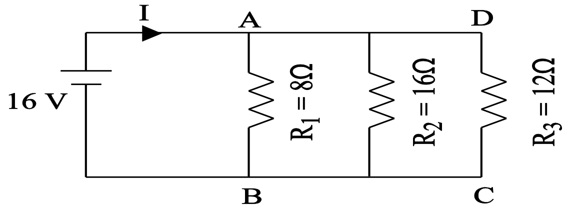


Figure 1.27

8 Ω, 16 Ω, 12 Ω are connected in parallel. Its equivalent resistance,

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

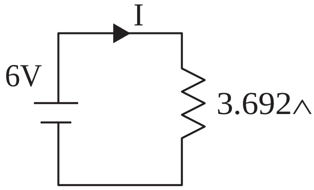


Figure 1.28

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\ \Omega$$

$$R_T = 3.692\ \Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Problem 1.23 In the Circuit shown in the figure 1.29, calculate

(i) The current in all resistors.
(ii) The value of unknown resistance ‘x’
(iii) The equivalent resistance between A and B.

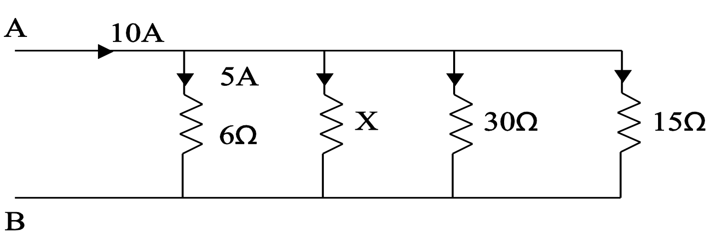


Figure 1.29

Solution: As all the resistors are in parallel, the voltage across each one is same. Give that current through 6 Ω , ie, $I_{6\,\Omega} = 5A$

Voltage across 6 Ω = 5×6 = 30volts.

Hence, current through 30 Ω, $I_{30} = \frac{V_{30}}{R} = \frac{30}{30} = 1A$

Similarly, current through 15 Ω, $I_{15} = \frac{V_{15}}{R} = \frac{30}{15} = 2A$

Total Current, $I = I_6 + I_x + I_{30} + I_{15}$

$$10 = 5 + I_x + 1 + 2$$

$$I_X = 2\,A$$

Hence, the current flowing through the 'X' Resistor is, $I_X = 2\,A$

Value of the Resistor 'X' is given by,

$$X = \frac{30}{I_x} = \frac{30}{2} = 15\ \Omega$$

Let, the equivalent resistance across AB = R_T

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{x} + \frac{1}{30} + \frac{1}{15}$$

$$\frac{1}{R_T} = \frac{5+2+1+2}{30} = \frac{1}{3}$$

$$R_T = 3\ \Omega$$

Series — Parallel Combination

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Figure 1.30. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with R_1.

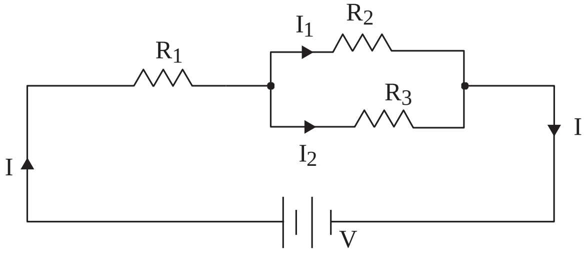


Figure 1.30

Equivalent Resistance: R_T for parallel combination.

$$R_P = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel combination = $I * \frac{R_2 R_3}{R_2 + R_3}$.

- **Problems based on Series – Parallel Combination:**

Problem 1.24 In the circuit, find the current in all the resistors. Also calculate the supply voltage.

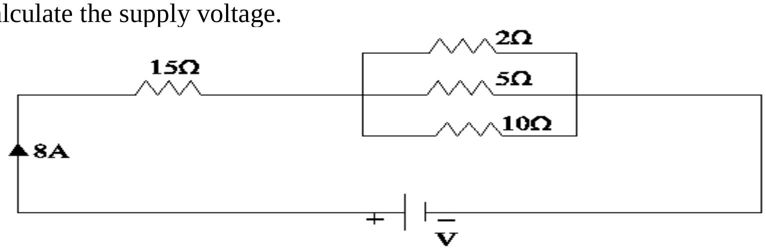


Figure 1.31

Voltage across 15∧, $V_{15} = I_{15} \times R = 8 \times 15 = 120V$

Resistors 2 Ω, 5 Ω, 10 Ω are connected in parallel, it equivalent resistance is given by,

$$R_P = \frac{2 * 5 * 10}{2 \times 5 + 5 \times 10 + 10 \times 2} = 1.25\ \Omega$$

Voltage across the parallel Combination is given by

$$V_p = V_2 = V_5 = V_{10} = I \times R_P = 8 \times 1.25 = 10V$$

Total supply Voltage, $V = V_{15} + V_p$

$$V = 120 + 10 = 130\text{V}$$

$$V = 130\text{V}$$

Hence, the Current through the parallel combination of the resistors are given by,

Current through 2 Ω resistor, $I_2 = \frac{V_2}{R} = \frac{10}{2} = 5A$

Current through 5 Ω Resistor, $I_5 = \frac{V_5}{R} = \frac{10}{5} = 2\ A$

Current through 10 Ω Resistor, $I_{10} = \frac{V_{10}}{R} = \frac{10}{10} = 1A$

The Current of 8A across the parallel combination is divided as 5A, 2A, and 1A.

Problem 1.25 Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current

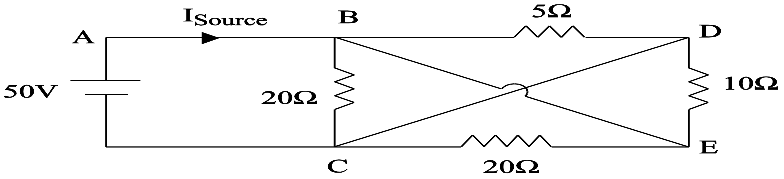


Figure 1.32

Solution: The given above circuit can be re-drawn as

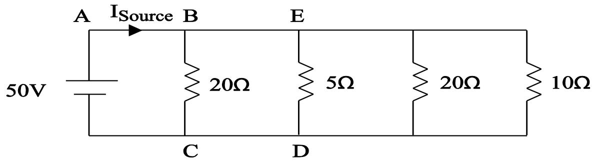


Figure 1.33

20 Ω and 10 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{20 * 10}{20 + 10} = 6.667\ \Omega$

The given circuit is reduced as,

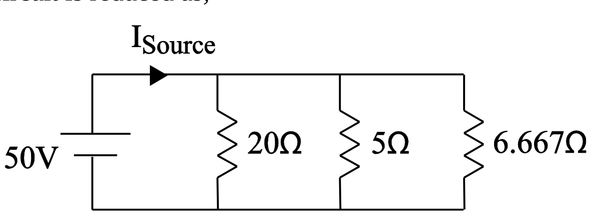


Figure 1.34

6.667 Ω and 5 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{6.667 * 5}{6.667 + 5} = 2.857\ \Omega$

The circuit is reduced as,

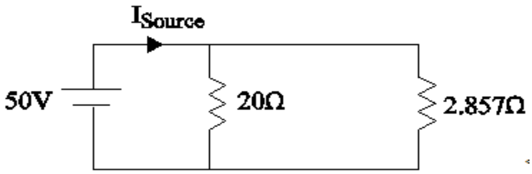


Figure 1.35

20 Ω and 2.857 Ω are connected in parallel. It equivalent resistance is,

$\frac{20 * 2.857}{20 + 2.857} = 2.497\ \Omega$

The Circuit is re-drawn as,

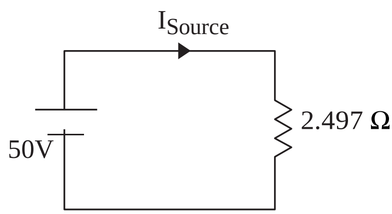


Figure 1.36

Hence the equivalent resistance of the Circuit is $R_T = 2.497\ \Omega = 2.5\ \Omega$

Source Current of the Circuit is given by,

$$I_{source} = \frac{V}{R} = \frac{50}{2.5} = 20A$$

Problem 1.26 Find the equivalent resistance between the terminals A and B.

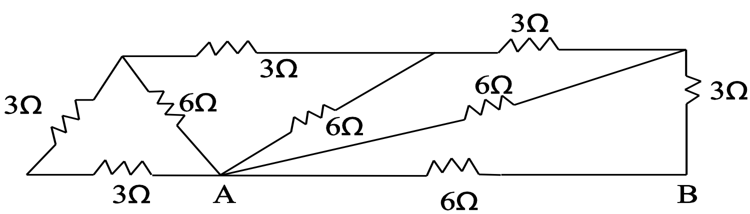


Figure 1.37

Solution:

3 Ω and 3 Ω are connected in Series, it equivalent resistance is, (3 + 3) = 6 Ω. The Circuit gets reduced as

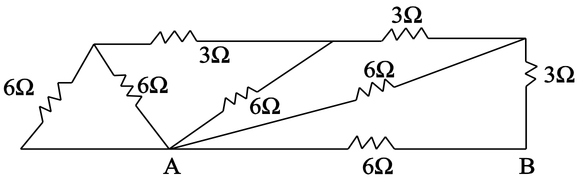


Figure 1.38

6 Ω and 6 Ω are connected in parallel. The circuit gets reduced as,

$$\frac{6*6}{6+6} = 3\text{ohms.}$$

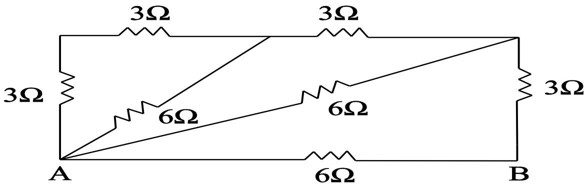


Figure 1.39

3 Ω and 3 Ω are connected in series (3 + 3 = 6 Ω).

The reduced Circuit is,

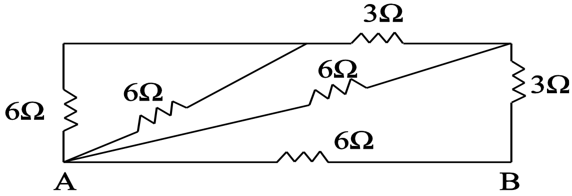


Figure 1.40

6 Ω and 6 Ω are connected in parallel. Its equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

The circuit can be reduced as,

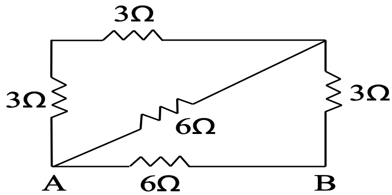


Figure 1.41

3 Ω and 3 Ω are connected in series. (3 + 3 = 6 Ω).

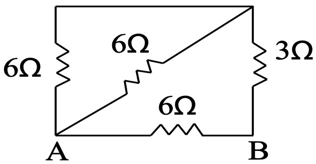


Figure 1.42

6 Ω and 6 Ω are connected in parallel. It equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

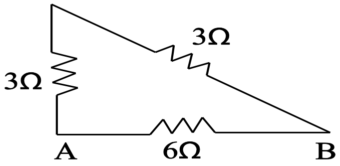


Figure 1.43

3 Ω and 3 Ω are connected in series, the reduced Circuit is 3 + 3 = 6 Ω

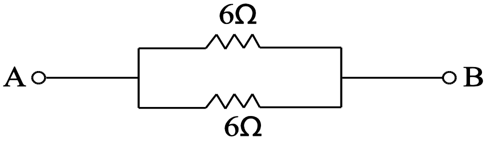


Figure 1.44

6 Ω and 6 Ω are connected in parallel.

$\frac{6*6}{6+6}$ = 3 Ω . The equivalent resistance between the terminals A and B given by R_{AB} = 3 Ω.

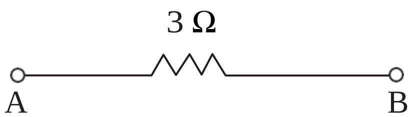


Figure 1.45

$$\therefore R_{AB} = 3\ \Omega$$

Problem 1.27 Determine the value of R if the power dissipated in 10 Ω resistor is 90 W.

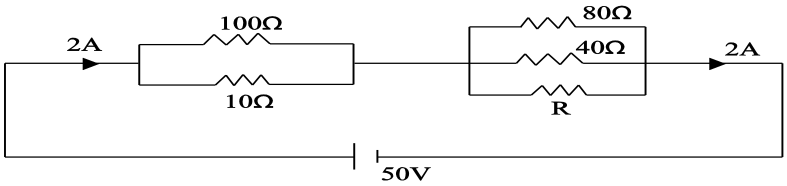


Figure 1.46

Solution:

100 Ω and 10 Ω are connected in parallel.

Its equivalent resistance is, $\frac{100 * 10}{100 + 10}$ = 9.09 Ω

The circuit is reduced as,

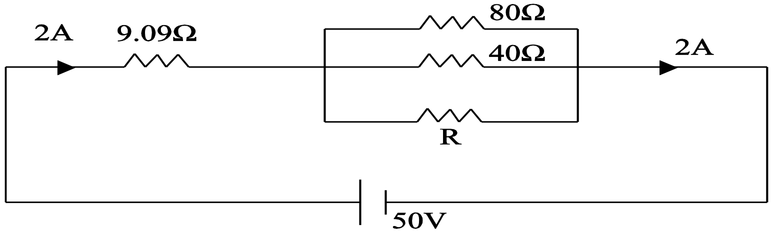


Figure 1.47

Current of 2A flows through the 9.09 Ω resistor. Voltage across 9.09 Ω is given by,

$$V_{9.09} = I_{9.09} \times R$$
$$V_{9.09} = 2 \times 9.09 = 18.18V$$

Similarly voltage across the unknown resistor V_R,

$$V_R = V - V_{9.09} = 50 - 18.18 = 31.818V$$

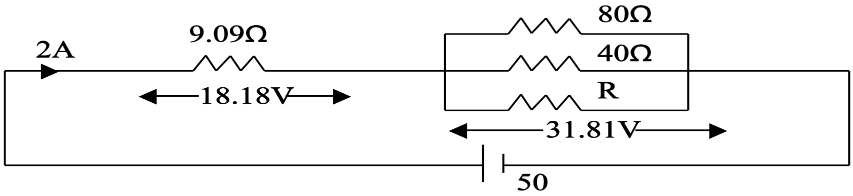


Figure 1.48

Hence the Current through 40 Ω, 80 Ω resistors can be found out with the voltage drop of 31.818V across it.

$$I_{80} = \frac{V_R}{80} = \frac{31.818}{80} = 0.397\,A$$
$$I_{40} = \frac{V_R}{40} = \frac{31.818}{40} = 0.7954\,A$$

Hence current through the unknown resistor R is I_R,

$$I_R = I - [I_{80} + I_{40}]$$
$$I_R = 2 - (0.397 + 0.7954) = 0.8075A$$

Hence, the value of the unknown Resistor R is given by

$$R = \frac{V_R}{I_R} = \frac{31.818}{0.8075} = 39.4\ \Omega$$

The value of the unknown resistor R is given by, R = 39.4 Ω.

Problem 1.28 Calculate the following for the circuits given,

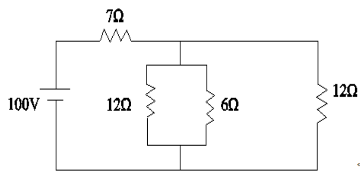


Figure 1.49

(i) Total resistance offered to the Source.
(ii) Total Current from the Source.
(iii) Power Supplied by the Source.

Solution: 12 Ω and 6 Ω are connected in Parallel.

Its equivalent resistance, $\frac{12*6}{12+6} = 4\ \Omega$. The reduced circuit is given as,

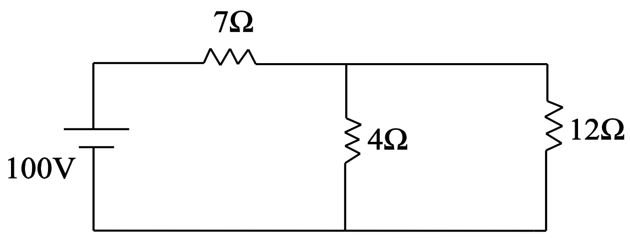


Figure 1.50

4 Ω and 12 Ω are connected in parallel. $\frac{4*12}{12+4} = 3\ \Omega$

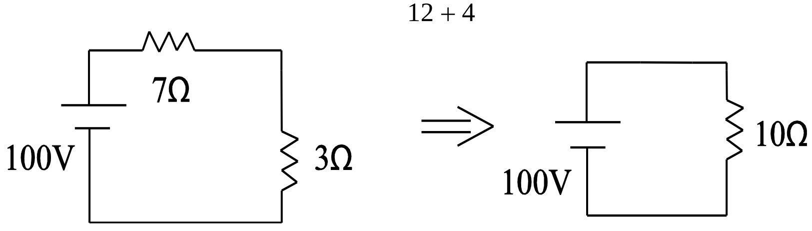


Figure 1.51

7 Ω and 3 Ω are connected in series, $7 + 3 = 10\Omega$

Total resistance offered to the Source, $R = 10\ \Omega$

Total Current from the Source, $I = \frac{100}{10} = 10$ A

$$I = 10A$$

Power supplied by the Source, $P = I^2R = 10^2 \times 10 = 1000W$

P = 1000W.

Problem 1.29 A letter A is Constructed of an uniform wire of 1 Ω resistance per cm. The signs of the letter are 60cm long and the cross piece is 30cm long, Apex angle 60°. Find the resistance of the letter between two ends of the legs.

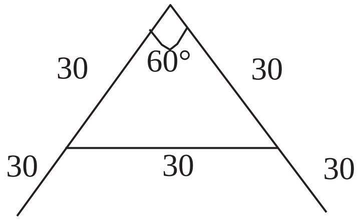


Figure 1.52

Solution:

The given circuit can be redrawn as,

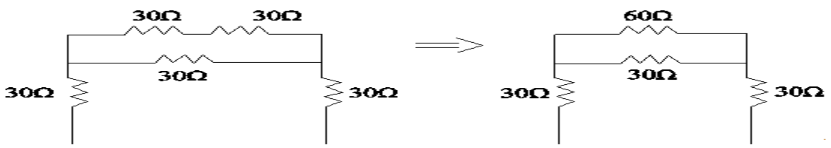


Figure 1.53

60 Ω and 30 Ω are connected in parallel

$$\frac{60 * 30}{60 + 30} = 2O \text{ ohms}$$

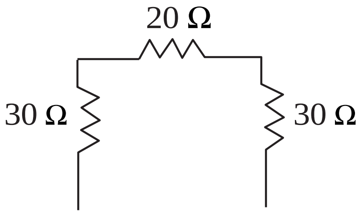


Figure 1.54

Equivalent Resistance = 80 Ω.

Problem 1.30 Find the current supplied by the battery.

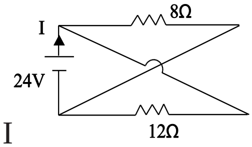


Figure 1.55

Solution:

The given circuit can be re-drawn as,

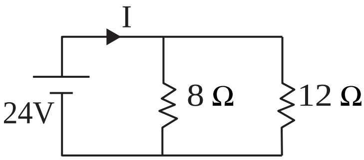


Figure 1.56

8 Ω and 12 Ω connected in parallel.

$$\frac{8*12}{8+12} = 4.8\ \Omega$$

Reduced circuit is,

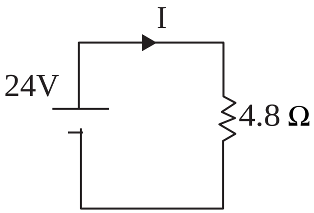


Figure 1.57

Current, $I = \frac{V}{R} = \frac{24}{4.8} = 5A$

$I = 5A$

Problem 1.31 Find the current supplied by the battery for the figure shown below.

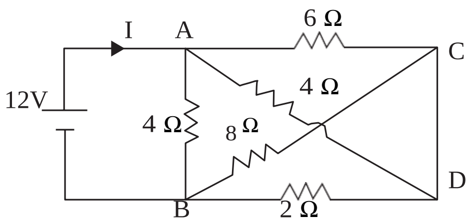


Figure 1.58

Solution:

The given above circuit can be redrawn as,

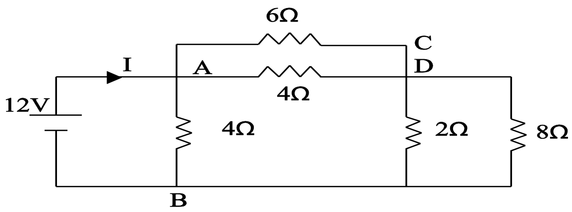


Figure 1.59

4 Ω and 6 Ω are connected in parallel. $\dfrac{6*4}{6+4} = 2.4\ \Omega$

Similarly, 2 Ω and 8 Ω are connected in parallel.

$$\frac{2*8}{8+2} = 1.6\ \Omega$$

The reduced circuit can be re drawn as,

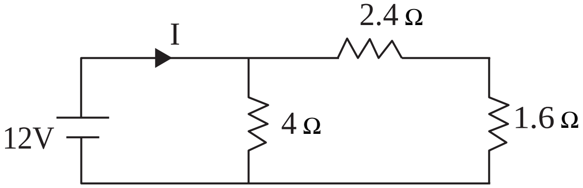


Figure 1.60

2.4 Ω and 1.6 Ω are connected in series. 2.4 +1.6 = 4 Ω

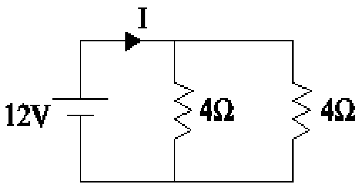


Figure 1.61

4 Ω and 4 Ω are connected in parallel $\frac{4*4}{4+4} = 2\ \Omega$

The reduced circuit is,

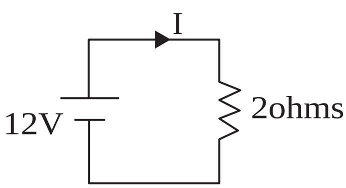


Figure 1.62

$$I = \frac{V}{R} = \frac{12}{2} = 6A$$

Current I, supplied by the battery = 6A.

Problem 1.32 Two Resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100v supply. The voltage drop across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50,000 Ω. Find the sum of the Reading.

Solution:

Case (i) A voltmeter is connected across 2500 Ω.

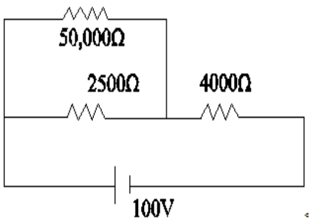


Figure 1.63

2500 Ω and 50,000 Ω are connected is parallel.

$$\frac{2500 * 50000}{2500 + 50000} = 2381\text{ohms}$$

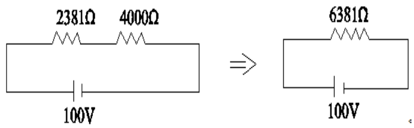


Figure 1.64

2381 Ω and 4000 Ω are connected in series.

$$2381 + 4000 = 6381\ \Omega$$

Current $I = \frac{V}{R} = \frac{100}{6381} = 0.01567\text{A}$

Voltage drop across, the Resister R_1 is measured by connecting a voltmeter having resistance of 50,000 across $R_{1.}$ Hence V_A be voltage drop across R_1

$$V_A = IR = 0.01567 * 2381$$

$$V_A = 37.31\text{V}$$

Case (ii) Voltmeter is connected across 4000 Ω.

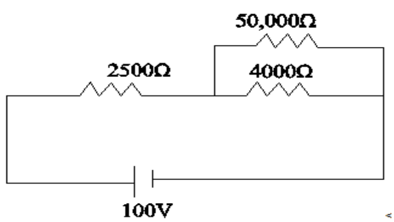


Figure 1.65

4000 Ω and 50,000 Ω are connected in parallel.

$$\frac{4000 * 50000}{4000 + 50000} = 3703.7\text{ohms}$$

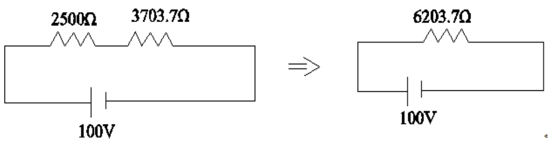


Figure 1.66

Current, $I = \frac{V}{R} = \frac{100}{6203.7} = 0.0161\ A$

Voltage drop across the resistor R_2 is measured by connecting a voltmeter having resistance of 50000 across R_2.Hence, V_B be the voltage drop across R_2.

$$V_B = IR = 0.6161 * 3703.7$$
$$V_B = 59.7V$$

The total voltage drop = $V_A + V_B$

$$V = 37.31 + 59.7$$
$$V = 97\ V$$

Problem 1.33 Find the value of 'R' and the total current when the total power dissipated in the network is 16W as shown in the figure.

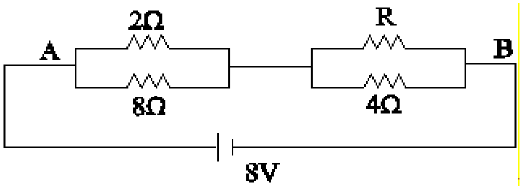


Figure 1.67

Solution:

Total Power (P) = 16w

Total Current, $I = \frac{P}{V} = \frac{16}{8} = 2A$

Total Resistance, $(R_{AB}) = \frac{P}{I^2} = \frac{16}{4} = 4\ \Omega$

Total Resistance between A and B is given by,

$$R_{AB} = \frac{2*8}{2+8} + \frac{4*R}{4+R}$$

$$4 = 1.6 + \frac{4R}{4+R}$$

$$4(4+R) = 1.6(4+R) + 4R \quad R = 6\ \Omega.$$

KIRCHOFF'S LAWS

Kirchhoff's current law

The kirchoff's current law states that the algebraic sum of currents in a node is zero.

It can also be stated that "sum of incoming currents is equal to sum of outgoing currents."

Kirchhoff's current law is applied at nodes of the circuit. A node is defined as two or more electrical elements joined together. The electrical elements may be resistors, inductors capacitors, voltage sources, current sources etc.

Consider a electrical network as shown below.

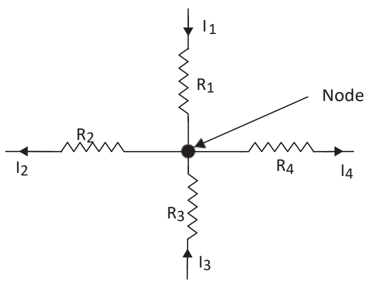


Figure 1.68

Four resistors are joined together to form a node. Each resistor carries different currents and they are indicated in the diagram.

$I_1 \rightarrow$ Flows towards the node and it is considered as positive current. $(+\ I_1)$

$I_2 \rightarrow$ Flows away from the node and it is considered as negative current. $(-\ I_2)$

$I_3 \rightarrow$ Flows towards the node and it is considered as positive current. $(+I_3)$

$I_4 \rightarrow$ Flows away from the node and hence it is considered as negative current $(-I_4)$

Applying KCL at the node, by diffinition-1 algebraic sum of currents in a node is zero.

$$+I_1 - I_2 + I_3 - I_4 = 0 \qquad (1)$$

taking the I_2 & I_4 to other side

$$I_1 + I_3 = I_2 + I_4 \qquad (2)$$

From equation (2) we get the definition – 2. Where I_1 & I_3 are positive currents (Flowing towards the node) I_2 & I_4 are negative currents. (Flowing away from the node).

Kirchoff's voltage Law: (KVL)

Kirchhoff's voltage law states that "sum of the voltages in a closed path (loop) is zero".

In electric circuit there will be closed path called as loops will be present.

The KVL is applied to the closed path only the loop will consists of voltage sources, resistors, inductors etc.

In the loop there will be voltage rise and voltage drop. This voltage rise and voltage drop depends on the direction traced in the loop. So it is important to understand the sign convention and the direction in which KVL is applied (Clock wise Anti clock wise).

- **Sign Conventions**

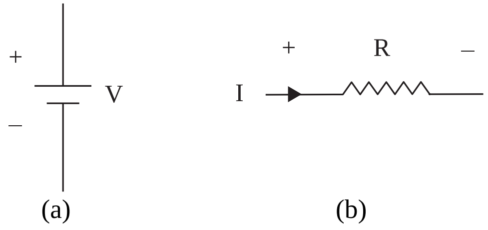
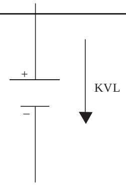


Figure 1.69

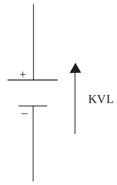
Consider a battery source V as shown in the figure 1.69(a). Here positive of the battery is marked with + sign and negative of the battery is marked with – sign.

When we move from + sign to – sign, it is called voltage drop.

When we move from – sign to + sign, it is called as voltage rise.

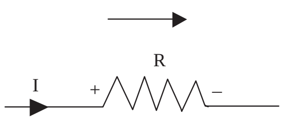


When KVL is applied in Anti clockwise direction as shown above it is called as voltage drop. A voltage drop is indicated in a loop with "—" sign (–V)

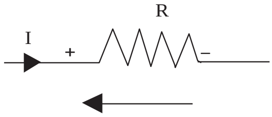


For the same battery source if the KVL is applied in clock wise direction we move from – sign to + sign. Hence it is called as Voltage Rise. A Voltage rise indicated in the loop with + sign. (+V).

Similarly in the resistor the current entry point is marked as positive (+ sign) and current leaving point is marked as negative sign. (– sign).



For the resistor shown in the diagram above, if KVL is applied in clock wise direction then it is called as voltage drop. Voltage drop in KVL equation must be indicated with negative sign (–). $\therefore$ –IR.



For the resistor shown in the diagram above, if KVL is applied in anti clockwise direction then it is called as voltage rise. A voltage rise is indicated in the KVL equation as positive. i.e. + IR.

In short the above explanation is summarized below in a Table.

S.No.	Element	KVL in clockwise	KVL in anticlockwise
1.	+ –	+ – Drop –V	+ – Rise +V
2.	I + R –	I + R – Drop –IR	I + R – Drop +IR

- **Procedure for KVL:**
  * Identify the loops and Name them.
  * Mark the branch currents and name them.
  * Apply the sign convention.
  * Select a loop & apply KVL either in clockwise or Anticlockwise and frame the equation.
  * Solve all the equations of the loop.

- **Problems based on Kirchhoff's laws**

Problem 1.34 For the given circuit find the branch currents and voltages by applying KVL.

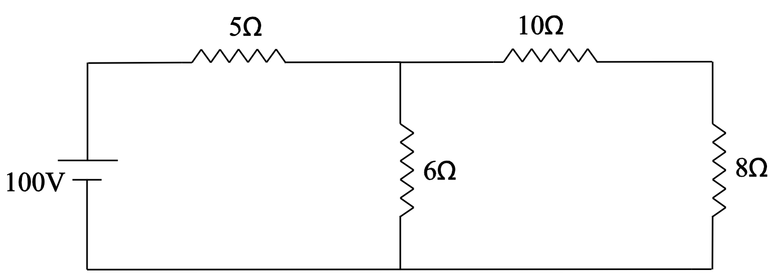


Figure 1.70

Solution:

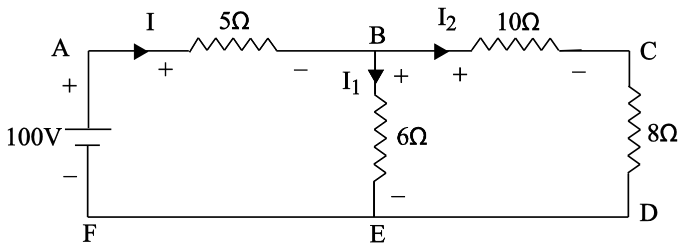


Figure 1.71

Consider loop ABEF & Apply KVL in CLK wise direction

$$100 - 5I - 6I_1 = 0$$

But $I = I_1 + I_2$

$$100 - 5\left(I_1 + I_2\right) - 6I_1 = 0$$
$$100 - 5I_1 - 5I_2 - 6I_1 = 0$$
$$-11I_1 - 5I_2 + 100 = 0$$
$$11I_1 + 5I_2 = 100 \qquad (1)$$

Consider loop BCDEB & Apply KVL in CLK wise direction

$$-10I_2 - 8I_2 + 6I_1 = 0$$
$$-18I_2 + 6I_1 = 0$$
$$6I_1 = 18I_2$$
$$I_1 = 3I_2 \qquad (2)$$

Sub I_1 in equ (1)

$$11\left(3I_2\right) + 5I_2 = 100$$
$$33I_2 + 5I_2 = 100$$
$$38I_2 = 100$$
$$I_2 = \frac{100}{38} = 2.63\ Amps.$$
$$I_2 = 2.63\ Amps$$

Sub I_2 in equ (2)

$$I_1 = 3(2.63) = 7.89\text{Amps}$$
$$I_1 = 7.89\text{Amps}$$
$$I = I_1 + I_2 = 10.52$$
$$I = 10.52\text{Amps.}$$

Voltage Across 5∧ = $5 \times I = 5 \times 10.52$
$= 52.6\ volts$

Voltage Across 6∧ = $6 \times I_1 = 6 \times 7.89$
$= 47.34\ volts$

Voltage Across 10∧ = $10 \times I_2 = 10 \times 2.63$
$= 26.3\ volts$

Voltage Across 8∧ = $8 \times I_2 = 8 \times 2.63$
$= 21.04\ volts$

(Or)

The above problem can be solved by applying KVL in Anti clock wise directions.

Consider loop ABEF & Apply KVL in anti clock wise direction

$$6I_1 + 5I - 100 = 0$$

But $I = I_1 + I_2$

$$6I_1 + 5(I_1 + I_2) - 100 = 0$$
$$6I_1 + 5I_1 + 5I_2 = 100$$
$$11I_1 + 5I_2 = 100 \qquad (3)$$

Consider loop BCDEB & Apply KVL in anti clockwise direction

$$8I_2 + 10I_2 - 6I_1 = 0$$
$$18I_2 = 6I_1$$
$$I_1 = 3I_2 \qquad (4)$$

equations (3) & (1) are identical

equations (2) & (4) are identical

Hence we get the same answer irrespective of directions of applying KVL.

Problem 1.35 Calculate the branch current in 15 Ω resistor by Applying kirchhoff's law

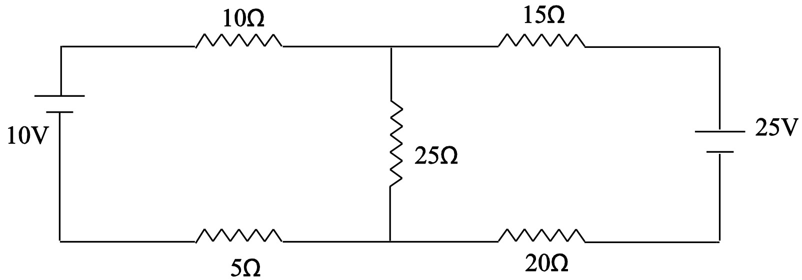


Figure 1.72

Figure 72 battery voltage value 25 volt missing

Solution:

Name the loop and Mark the current directions

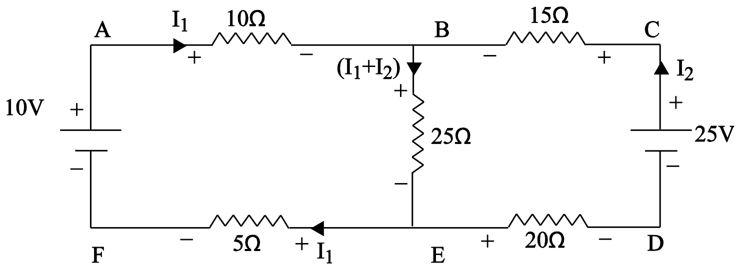


Figure 1.73

Consider the loop ABEFA & apply KVL in CLK wise

$$10 - 10I_1 - 25(I_1 + I_2) - 5I_1 = 0$$

$$10 - 10I_1 - 25I_1 - 25I_2 - 5I_1 = 0$$

$$-40I_1 - 25I_2 + 10 = 0$$

$$40I_1 + 25I_2 = 10 \qquad (1)$$

Consider the loop BCDEB and Apply KVL in CLK wise direction

Consider the loop BCDEB and Apply KVL in CLK wise direction

$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$
$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$
$$15I_2 - 25 + 20I_2 + 25I_1 + 25I_2 = 0$$

$$25I_1 + 60I_2 - 25 = 0$$

$$25I_1 + 60I_2 = 25 \text{ ……………..(2)}$$

Solve (1) & (2) & find I_2 alone

$(1) \times 25 \Rightarrow 1000\, I_1 + 625 I_2 = 25$

$(2) \times 40 \Rightarrow 1000\, I_1 + 2400 I_2$

$(A) - (B) \Rightarrow -1775\, I_2 = -750$

$I_2 = 0.42$ Amps.

Current in 15 Ω resistor is 0.42Amps.

Problem 1.36 For the given network find the branch current in 8 Ω and voltage across the 3 Ω by applying KVL

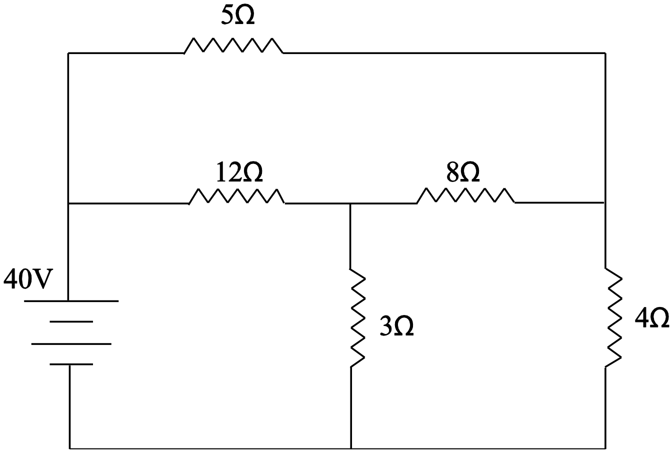


Figure 1.74

Solution:

Name the loop and mark the current directions and apply sign convention.

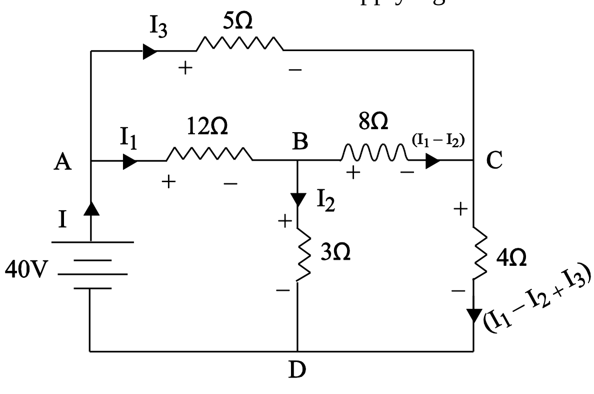


Figure 1.75

Consider loop ABDA and apply KVL $-8I_1 + 8$

$$-12I_1 - 3I_2 + 40 = 0$$

$$12I_1 + 3I_2 = 40$$

Consider loop BCDB and apply KVL

$$-8(I_1 - I_2) - 4(I_1 - I_2 + I_3) + 3I_2 = 0$$

$$I_2 - 4I_1 + 4I_2 - 4I_3 + 3I_2 = 0 \qquad (1)$$

$$-12I_1 + 15I_2 - 4I_3 = 0 \qquad (2).$$

Consider loop ABCA and apply KVL

$$-12I_1 - 8(I_1 - I_2) + 5I_3 = 0$$

$$-12I_1 - 8I_1 + 8I_2 + 5I_3 = 0$$

$$-20I_1 + 8I_2 + 5I_3 = 0 \qquad (3)$$

Solve equ (2) & (3) and cancel out I_3

$$(2) \times 5 \Rightarrow -60I_1 + 75I_2 - 20I_3 = 0$$

$$(3) \times 4 \Rightarrow -80I_1 + 32I_2 + 20I_3 = 0$$

Add the above two equations $\Rightarrow -140I_1 + 107I_2 = 0$ (4)

Solve equ (4) & (1) and find I_1 & I_2

$$12I_1 + 3I_2 = 40 \qquad (1)$$

$$-140I_1 + 107I_2 = 0 \qquad (4)$$

$$(1) \times 107 \Rightarrow 1284I_1 + 321I_2 = 4280$$

$$(4) \times 3 \Rightarrow -420I_1 + 321I_2 = 0$$

Subtract the above two $1704I_1 = 4280$

$$I_1 = 2.51 \text{ Amps}$$

Sub I_1 in (4)

$$-140 \times 2.51 + 107I_2 = 0$$

$$-351.4 + 107I_2 = 0$$

$$107I_2 = 351.4$$

$$I_2 = 3.28 \text{ Amps}$$

Current in 8 Ω resistor $= I_1 - I_2$

$= 2.51 - 3.28$

$= -0.77$ Amps.

Negative sign indicates that current flows in the opposite direction of our assumption.

Voltage in 3 Ω resistor = $3I_2$
$$= 3 \times 3.28 = 9.84 \text{ volts}$$

Note: Since there are 3 loops three unknown currents I_1, I_2 and I_3 should be named in the loop.

Problem 1.37 For the given network shown below find the branch currents by applying KVL and also find the voltage across 5 Ω resistor.

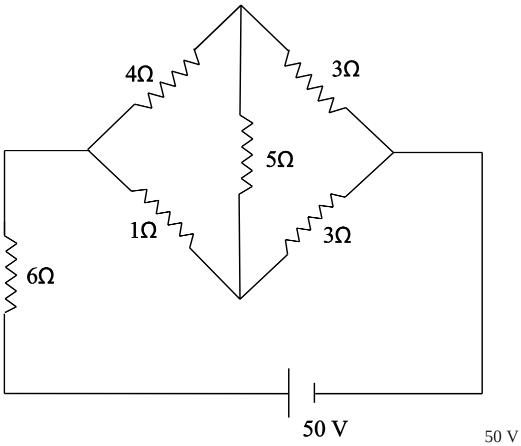


Figure 1.76

Solution:
Name the loop and assume the branch currents.

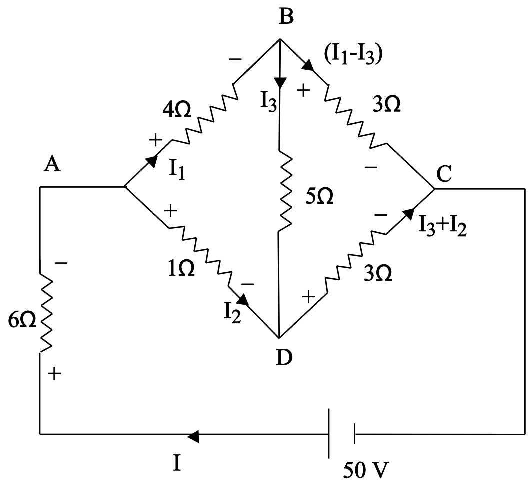


Figure 1.77

Consider the loop ABDA and apply KVL.

$$-4I_1 - 5\,I_3 + I_2 = 0$$

$$-4\,I_1 + I_2 - 5\,I_3 = 0 \qquad (1)$$

Consider the loop BCDB and apply KVL.

$$-3(I_1 - I_3) + 3(I_3 + I_2) + 5I_3 = 0$$

$$-3I_1 + 3I_3 + 3I_3 + 3I_2 + 5I_3 = 0$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (2)$$

Consider the loop ADCA and apply KVL.

$$-6(I_1 + I_2) - I_2 - 3(I_3 + I_2) + 50 = 0$$

$$-6I_1 - 6I_2 - I_2 - 3I_3 - 3I_2 = -50$$

$$-6I_1 - 10I_2 - 3I_3 = -50$$

$$6I_1 + 10I_2 + 3I_3 = 50 \qquad (3)$$

From eqn is (1) & (2) Cancel I_3

$$-4I_1 + I_2 - 5I_3 = 0 \qquad (4)$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (5)$$

$$(4) \text{ x } 3 \Rightarrow -12I_1 + 3I_2 - 15I_3 = 0$$

$$(5) \text{ x } 4 \Rightarrow -12I_1 + 12I_2 - 44I_3 = 0$$

By subtracting the above two equations $-9I_2 - 59I_3 = 0$

$$9I_2 = -59I_3$$

$$I_2 = -6.56I_3 \quad (6)$$
$$-3I_1 + 3I_2 + 11I_3 = 0 \quad (7)$$
$$6I_1 + 10I_2 + 3I_3 = 50 \quad (8)$$

$$(7) \times 2 \Rightarrow -6I_1 + 6I_2 + 22I_3 = 0$$
$$(8) \Rightarrow 6I_1 + 10I_2 + 3I_3 = 50$$

By adding the above two equations $16I_2 + 28I_3 = 50$ (9)

Sub eqn (6) in (9)

$$16(-6.56\, I_3) + 25\, I_3 = 50$$
$$-104.96 + 25\, I_3 = 50$$
$$-79.96\, I_3 = 50$$
$$I_3 = -0.625 \text{ Amps} \quad (10)$$

Sub eqn (10) in (6)

$$I_2 = -6.56 \times (-0.625)$$
$$I_2 = 4.1 \text{ Amps} \quad (7)$$

Sub (10) & (11) in eqn (8)

$$6I_1 + 10I_2 + 3I_3 = 50$$
$$6I_1 + 10(4.1) + 3(-0.625) = 50$$
$$6I_1 + 41 - 1.875 = 50$$
$$6I_1 = 10.875$$
$$I_1 = 1.81 \text{ Amps}$$

Current in 6 Ω resistor = $(I_1 + I_2) = (1.81 + 4.1) = 5.91$Amps
Current in 4 Ω resistor = $I_1 = 1.81$ Amps

Current in 5 Ω resistor = $I_3 = -0.625$ Amps
Current in 3 Ω resistor = $(I_1 - I_3) = 1.81 + 0.625 = 2.44$ Amps
Current in 3 Ω resistor = $(I_3 + I_2) = 3.475$ Amps
Current in 1 Ω resistor = $I_2 = 4.1$ Amps.

Voltage Across 5 Ω resistor= $5 \times 0.625 = 3.13$ volts.

Problem 1.38 For the Circuit shown below determine voltages (i) V_{df} and (ii)V_{ag}

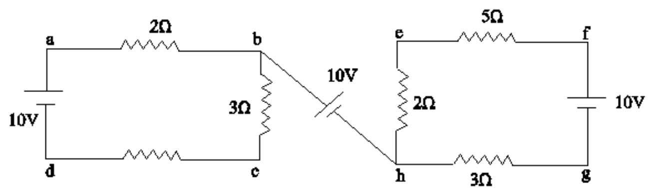


Figure 1.78

Solution:
Mark the current directions and mark the polarity

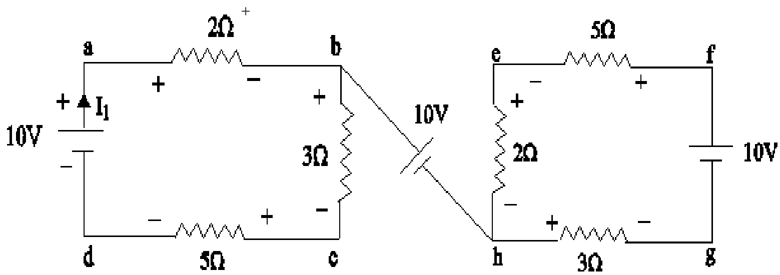


Figure 1.79

Apply KVL to loop abcda
$10 - 2I_1 - 3I_1 - 5I_1 = 0$
$-10I_1 = -10$
$I_1 = 1$ Amps

Apply KVL to loop efghe
$5I_2 - 10 + 3I_2 + 2I_2 = 0$
$10I_2 = 10$
$I_2 = 1$ Amps

To find V_{df}:
Trace the path V_{df}

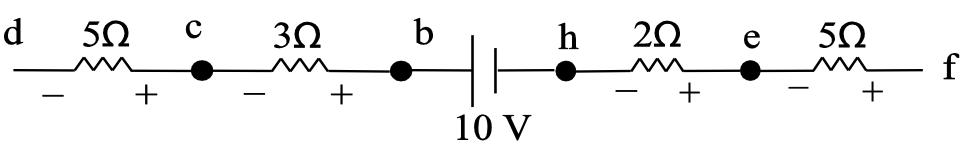


Figure 1.80

$V_{df} = -5(I_1 - 3I_1 + 10 + 2I_2 + 5I_2)$
$V_{df} = -5 - 3 + 10 + 2 + 5$
$V_{df} = 9$ Volts.
$V_{df} = -9$ Volts [$\because$ because - sign on d side + on f side]

To find V_{ag}:

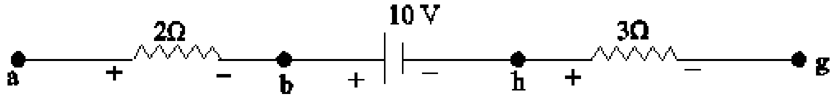


Figure 1.81

Apply KVL to the above Trace
$-2I_1 - 10 - 3I_2 = V_{ag}$
$V_{ag} = -2 - 10 - 3$
$V_{ag} = -15$

$V_{ag} = 15$ Volts. (With a side + w.r.t g)

Problem 1.39 Find the currents through R_2, R_3, R_4, R_5 and R_6 of the network.

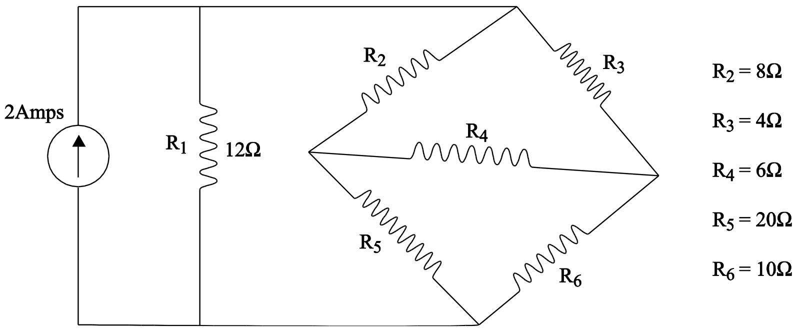


Figure 1.82

$R_2 = 8\,\Omega$
$R_3 = 4\,\Omega$
$R_4 = 6\,\Omega$
$R_5 = 20\,\Omega$
$R_6 = 10\,\Omega$

Solution:
Name the circuit and mark the current directions and polarity as shown below

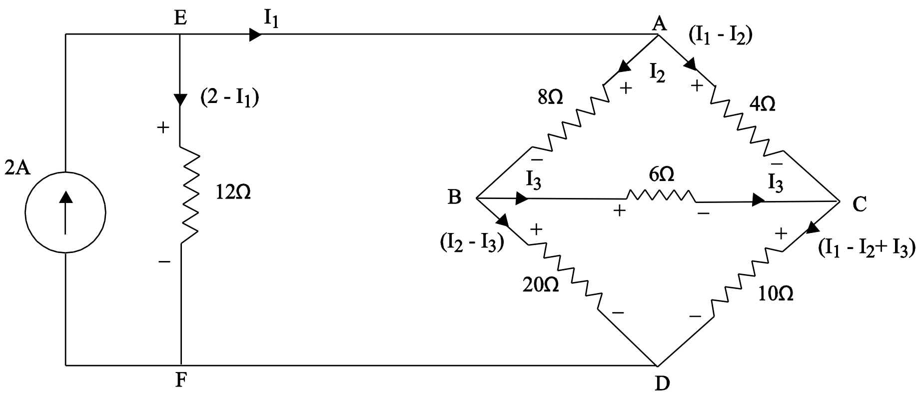


Figure 1.83

Apply KVL to the loop ACBA.

$$-4(I_1 - I_2) + 6I_3 + 8I_2 = 0.$$
$$-4I_1 + 4I_2 + 6I_3 + 8I_2 = 0$$
$$-4I_1 + 12I_2 + 6I_3 = 0 \qquad (1)$$

Apply KVL to the loop BCDB

$$-6I_3 - 10(I_1 - I_2 + I_3) + 20(I_2 - I_3) = 0$$
$$-6I_3 - 10I_1 + 10I_2 - 10I_3 + 20I_2 - 20I_3 = 0$$
$$-10I_1 + 30I_2 - 36I_3 = 0 \qquad (2)$$

Apply KVL to loop EABDFE

$$-8I_2 - 20(I_2 - I_3) + 12(2 - I_1) = 0$$
$$-8I_2 - 20I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-28I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-12I_1 - 28I_2 + 20I_3 = -24$$
$$12I_1 + 28I_2 - 20I_3 = 24 \qquad (3)$$

Solving equ. (1) (2) & (3). We get

$I_1 = 1.125$ Amps
$I_2 = 0.375$ Amps
$I_3 = 0$ Amps

$\therefore$ Current in $R_2 = 0.375$ Amps
$R_3 = 0.75$ Amps
$R_4 = 0$ Amps
$R_5 = 0.375$ Amps
$R_6 = 0.75$ Amps

NODAL ANALYSIS

- In nodal analysis, node equations relating node voltages are obtained for a multi node network.
- These node voltages are derived from kirchoff's current law (KCL)

- In this method the number of equations required to be solved is N-1, where N is the number of nodes.
- A node is a junction in a network where three or more branches meet. One of the nodes in a network is regarded as reference (datum) node and the potential of the other nodes are defined with reference to the datum node.

Case I.

Consider figure 1 Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node 'a' we get

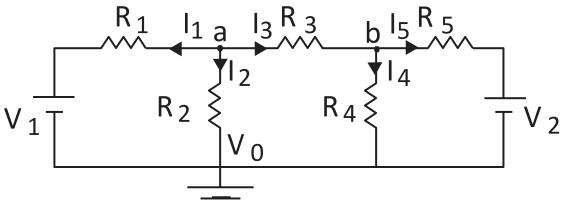


Figure 1.84

$$I_1 + I_2 + I_3 = 0 \quad (1)$$

Where

$$I_1 = \frac{V_a - V_1}{R_1};\ I_2 = \frac{V_a - V_0}{R_2};\ I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ.(1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [$V_0 = 0$]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \quad \text{.................. (2)}$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \quad \text{.................. (3)}$$

$$I_4 = \frac{V_b - V_0}{R_4};\ I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

$V_o = 0$ [reference node]

$$V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]-V_a\left[\frac{1}{R_3}\right]=\frac{V_2}{R_5} \text{ .... (4)}$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Sathyabama University

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\begin{pmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2,

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{pmatrix}$$

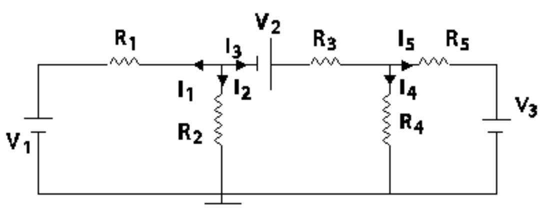
$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$$V_a = \frac{\Delta_1}{\Delta}; \qquad V_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

CASE II:



Consider fig 2

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = 0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1}-\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a+V_2-V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b-V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b}{R_5}-\frac{V_3}{R_5}$$

$$\frac{V_a}{R_3}-V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=-\frac{V_3}{R_5}-\frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3}+V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_3}{R_5}+\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1=\begin{pmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)$$

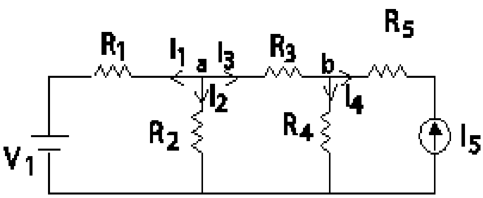
$$\Delta_2 = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

(case iii)



Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1}+\frac{V_a}{R_2}+\frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3} + \frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \text{.................}(8)$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

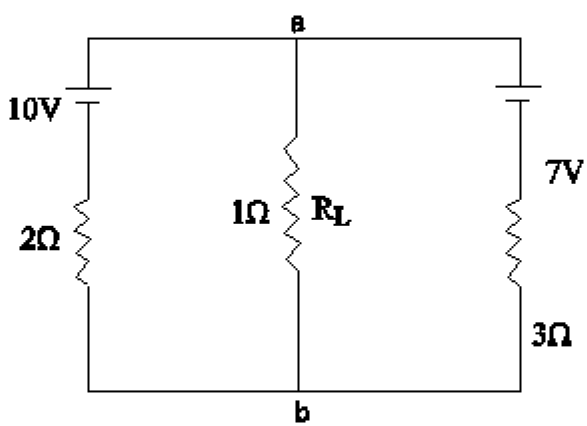
1) Two batteries having emf of 10V and 7V and internal resistances of 2Ω and 3Ω respectively, are connected in parallel across a load of resistance 1Ω. Calculate

(i) The individual battery currents

(ii) The current through the load

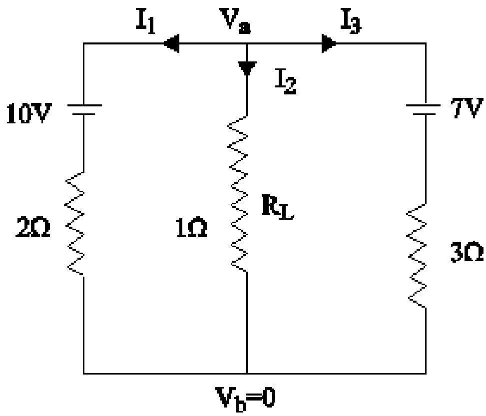
(iii) The Voltage across the load

Solution:



Step 1) Select the nodes and mark the nodes

Step 2) Select the datum or reference node.



<fig 84>

b is the ground node $V_b = 0$

Step 3: Mark the currents I_1, I_2 & I_3

Step 4: Write the node equations for node a and solve for V_a.

$$I_1 + I_2 + I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_1 = \frac{V_a - 10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$I_2 = \frac{V_a}{1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting (2), (3) & (4) in (1)

$$\frac{V_a - 10}{2} + V_a + \frac{V_a - 7}{3} = 0$$

$$V_a\left[\frac{1}{2} + 1 + \frac{1}{3}\right] = \frac{10}{2} + \frac{7}{3}$$

$$1.83\ V_a = 7.33$$

$$V_a = 4\ V$$

(i) Individual battery currents

$$I_1 = \frac{V_a - 10}{2} = \frac{4-10}{2}$$

$$= -3A$$

Ans: I_1=3 A

$$I_3 = \frac{V_a - 7}{3} = \frac{4-7}{3} = -1$$

Ans: I_3 = 1A

(ii) Current through the load

$$I_L = I_{1-2} = \frac{V_a}{1} = 4A$$

(iii)Voltage across the load

$$V_L = V_a - V_b$$
$$= 4 - 0$$
$$V_L = 4\ V$$

2) Write the node voltage equation and calculate the currents in each branch for the network.

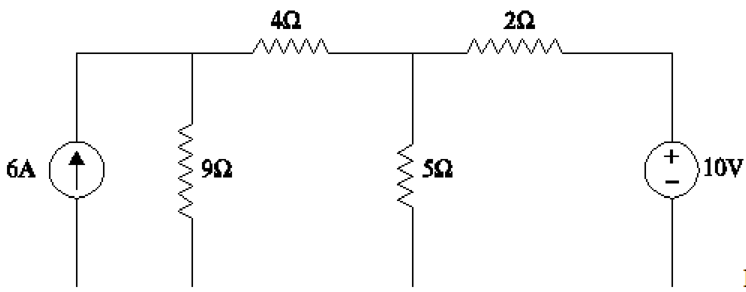


FIG85

Step 1: To assign voltages at each node

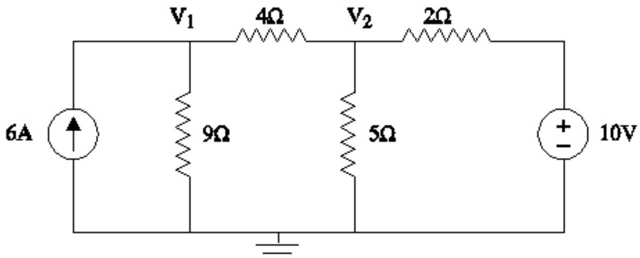


FIG86

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

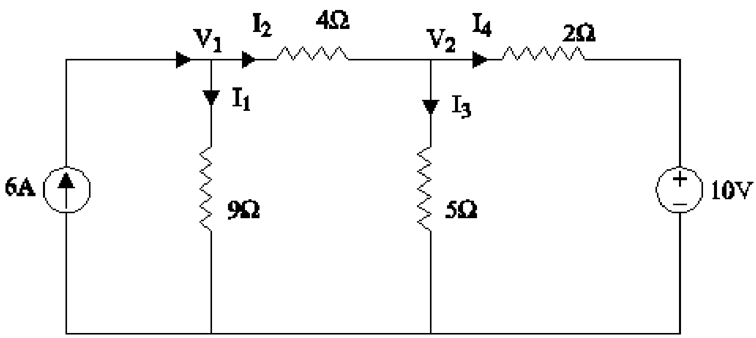


FIG87

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \text{................} \quad (1)$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \text{....................} \quad (2)$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule.

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86v$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ -.25 & 5 \end{bmatrix} = 3.3$$

$$V_{22}\frac{3.3}{.2795} = 11.8V$$

$$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$$

$$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$$

$$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$$

$$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$$

Hence currents in all the branches are found.

Problem 1.42 Use the Nodal Method to find V_{ba} and current through 30 Ω resistor in the circuit shown

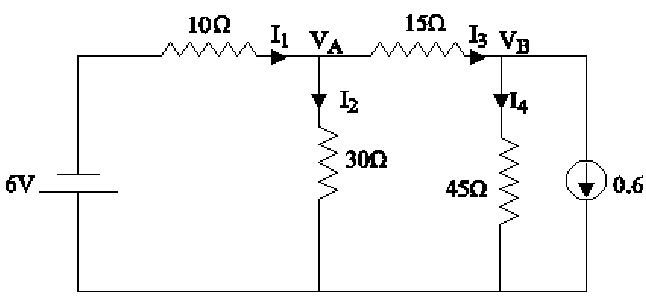


FIG88

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A\left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15}\right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} \quad +0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35 \times 10^{-3}\right]$$

$$\Delta = [0.01324]$$
$$\Delta = 0.01324$$

$$\Delta_1 = \begin{pmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15} + \frac{1}{45} \end{pmatrix} = -0.093$$

$$\Delta_1 = [-0.053 - 0.04] = \ -0.093$$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$$\Delta_2 = [-0.12 - 0.0396]$$

$$\Delta_2 = -0.1596$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$$V_{ba} = V_A - V_B = \ -7 + 12 \ = \ 5V$$

$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$$I_2 = -0.233A$$

Maxwell's Mesh method (Loop method).

This method was first proposed by Maxwell simplifies the solution of several networks. In this method, KVL is used. In any network, the number of independent loop equations will be

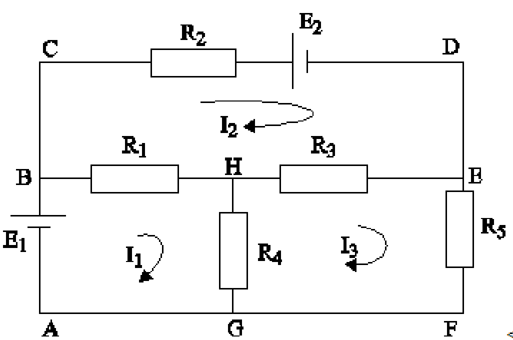
$$m = l - (j - 1)$$

Where l is the number of branches and j is the number of junctions.

Let us consider the circuit shown in fig(). for writing the mesh equations. It has

Number of junctions = 4 (B, H, E, G).

Number of branches = 6 (AB, BC, CD, DE, EF, HG).



<fig89>

In the above figure we shall name the three loop currents I_1, I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is $(I_1 - I_3)$ in a downward direction and R_1 is $(I_1 - I_2)$ from left to Right

Apply KVL for the first loop ABHGA,

$$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$$

$$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$$

$$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Apply KVL for the loop BEDC,

$$-R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$$

$$\therefore -R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2 \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop i.

$$R_{ij} = \begin{cases} -\sum(\text{Resistance common to the loop i and loop j}, \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances}) \\ +\sum(\text{Resistance common to the loop i and loop j}, \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances}) \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{bmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{bmatrix};$$

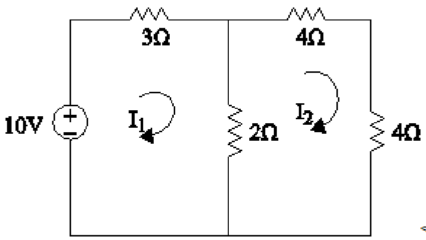
$$\Delta_3 = \begin{bmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{bmatrix}; \quad \Delta = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig () using Mesh current method

Solution:



<fig90>

Method 1:

Apply KVL for the first loop,

$$10 - 3I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Apply KVL for the second loop,

$$-4I_2 - 4I_2 - 2(I_2 - I_1) = 0$$

$$-2I_1 + 10I_2 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Solve eqn (1) & (2), we get

$$(1) \times 5 \Rightarrow 25I_1 - 10I_2 = 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$(2) \quad \Rightarrow -2I_1 + 10I_2 = 0$$

$$(3) + (2) \Rightarrow \quad 23\, I_1 = 50$$

$$I_1 = \frac{50}{23} = 2.174 \text{ A}$$

Sub I_1 in (2)

$$I_2 = \frac{2 \times 2.174}{10} = 0.435 \text{ A}$$

$$I_3\Omega = 2.174 \text{ A}$$

$$I_2\Omega = I_1 - I_2 = 1.739 \text{ A}$$

$$I_4\Omega = 0.435 \text{ A}$$

Method 2:

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

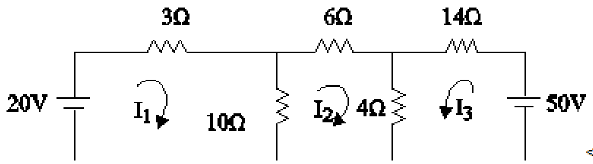
$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the loop currents for the network shown in figure below by using Loop Analysis.



<fig91>

Solution

For loop 1,

$$3I_1 + 10(I_1 - I_2) = 20$$

$13I_1 - 10I_2 = 20$ .... (1)

For loop 2,

$$10(I_2 - I_1) + 6I_2 + 4(I_2 + I_3) = 0$$

$$10I_2 - 10I_1 + 6I_2 + 4I_2 + 4I_3 = 0$$

$\div 2 \Rightarrow -5I_1 + 10I_2 + 2I_3 = 0$ .... (2)

For loop 3,

$$4(I_3 + I_2) + 14I_3 = 50$$

$$4I_2 + 18I_3 = 50$$

$\div 2 \Rightarrow 2I_2 + 9I_3 = 25$ .... (3)

$$\therefore \begin{bmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{vmatrix}$$

$$\Delta = 13(90 - 4) + 10(-45 - 0)$$

$$= 668$$

For loop 1,

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$$

$$18I_1 + 5I_2 - 3I_3 = 50 \text{ .... (1)}$$

For loop 2,

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$$

$$5I_1 + 8I_2 + I_3 = 10 \text{ .... (2)}$$

For loop 3,

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5$$

$$-3I_1 + I_2 + 4I_3 = -5 \text{ .... (3)}$$

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$= 13\ (250)\ + 10\ (-125)\ + 20\ (-10)$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$= 18\ (32-1)\ -5\ (20+3)\ -3(5+24)$$
$$= 356$$

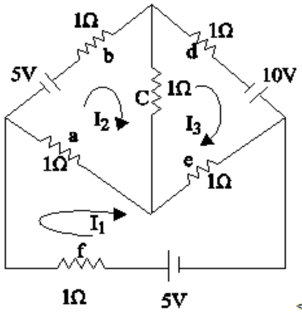
$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= 18\ (-40-10)\ -5\ (-25+30)\ + 50(5+24)$$
$$= -900 - 25 + 1450$$
$$= 525$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

Solution

4) Determine the currents in various elements of the bridge circuit as shown below.



<fig92>

Solution

For loop 1,

$$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$$

$$3I_1 - I_2 - I_3 = 5 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

For loop 2,

$$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$$

$$-I_1 + 3I_2 - I_3 = 5 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

For loop 3,

$$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$$

$$-I_1 - I_2 + 3I_3 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$= 3\,(9-1) + 1\,(-3-1)\ -1(1+3)$

$= 16$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

$= 3\ (15 + 10)\ -5\ (-3\ -\ 1)\ -1\ (-10 + 5)$

$= 100$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

$= 3\ (30 + 5) + 1\ (-10 + 5) + 5\ (1 + 3)$

$= 120.$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2 = 6.25 - 6.25 = 0$ A

$I_b = I_2 = 6.25$ A.

$I_c = I_2 - I_3 = 6.25 - 7.5 = -1.25$ A

$I_d = I_3 = 7.5$ A

$I_e = I_1 - I_3 = 6.25 - 7.5 = -1.25$ A.

$I_f = I_1 = 6.25$ A.

TEXT / REFERENCE BOOKS

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2. B.L.Theraja, Fundamentals of Electrical Engineering and Electronics, 1st edition, S.Chand & Co., 2009.

3. Smarajit Ghosh, Fundamentals of Electrical and Electronics Engineering, 2nd edition, PHI Learning Private Ltd, 2010.

INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

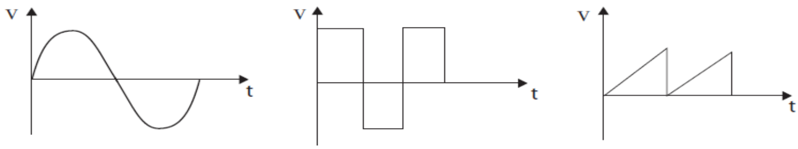

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurring interval.
3. Cycle

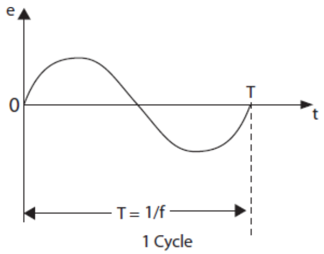

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period
   The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.
5. Frequency
   The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz

$$f = 1/T$$

6. Amplitude
   The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.
7. R.M.S value [Root Mean Square]
   The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$RMS\ Value = \sqrt{\frac{Area\ Under\ the\ square\ curve\ for}{one\ complete\ cycle\ /\ Period}}$$

8. Average Value of AC
   The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

   Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)
   It is the ratio of RMS value to average value

   Form Factor = RMS value/Average Value

10. Peak Factor (Ka)
    It is the ratio of Peak (or) maximum value to RMS value.

    Peak Factor Ka=Peak Value/RMS value

Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

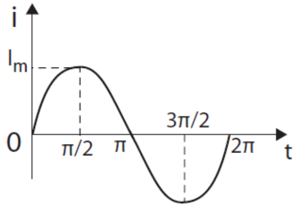

Figure 2.3

$$i = I_m \sin\omega t \;\; ; \omega t=\theta$$

Mean square of AC $I^2_{RMS} = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \quad \text{[since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\frac{1-\cos 2\theta}{2}d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi}\frac{i d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi}\sin\theta \, d\theta$$

$$= -\frac{I_m}{\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$= \frac{I_m}{\pi}(-1-1)$$

$$= -\frac{2I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

Avg

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

Expression for RMS, Average, Form Factor, Peak factor for Half wave rectifier

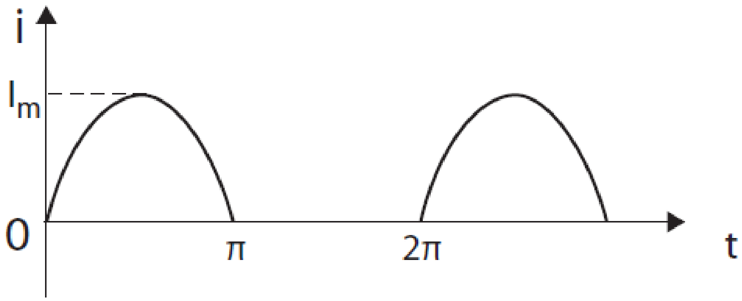

Figure 2.4

1) RMS value

$i = I_m \sin\theta \; ; 0 < \theta < \pi$

$i = 0 \qquad \pi < \theta < 2\pi$

Mean square of AC $I^2_{RMS} = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{Im^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{id\theta}{2\pi}$$

$$= \frac{1}{\ }\left[\int^{\pi} id\theta + 0\right]$$

$2\pi \lfloor {}_{0} \rfloor$

$$= \frac{1}{2\pi}\int_0^\pi I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\int_0^\pi I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_0^\pi$$

$$= \frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{2\pi}\left(-1-1\right)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{I_m} = \frac{2}{2}$$

Examples:

The equation of an alternating current is given by

i = 40sin 314 t

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

i = 40sin 314 t

We know that i = I_m sin ωt

S_o $I_m = 40$

$\omega = 314\ rad / \sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50 Hz$

(v) $Form\ Factor\ \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(vi) $Peak\ \ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

what is the equation of a 50Hz voltage sin wave having an rms value of 50 volt

Solution:

f = 50Hz

$V_{rms} = 50V$

$v = V_m \sin\omega t$

$\omega = 2\pi f = 2\pi\times 50 = 314$ rad/sec

$V_m = V_{rms}\sqrt{2} = 50\times\sqrt{2} = 70.7$ volt

$\therefore v = 70.7 \sin 314t$

PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

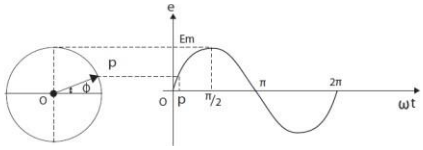

Figure 2.5

$OP = E_m$

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = I_{m1} \sin\theta$

$i_2 = I_{m2} \sin\theta$

can be represented graphically as shown in Fig 2.6(a).

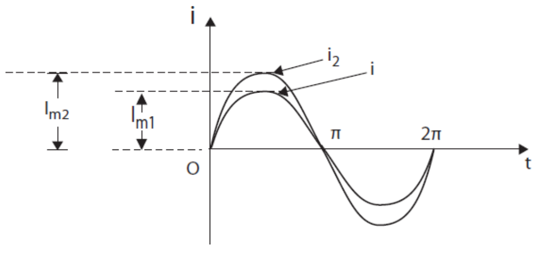

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

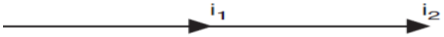

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag**' and '**lead**' and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = I_{m1} \sin(\omega t - \Phi)$
$i_2 = I_{m2} \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

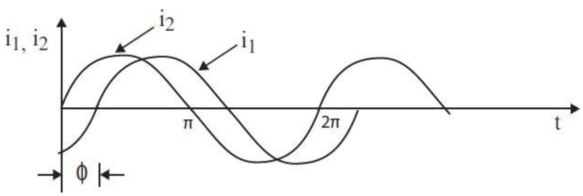

Figure 2.7a

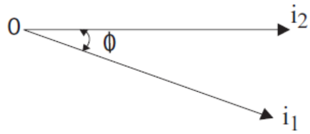

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of 'Φ' electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = I_{m1} \sin(\omega t + \Phi)$
$i_2 = I_{m2} \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

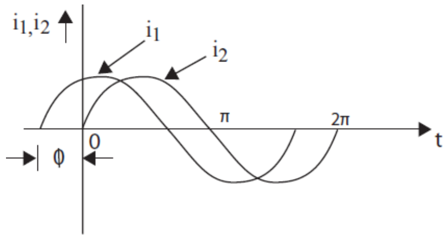

Figure 2.8(a)

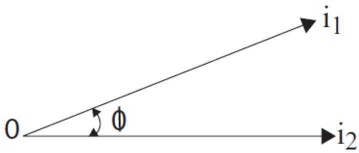

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference between them is 90°.
2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

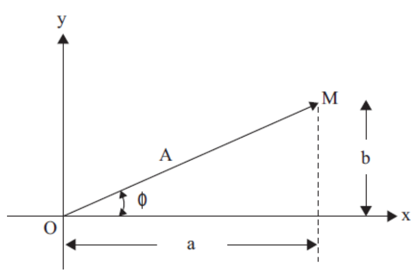

Figure 2.9

In Fig 2.9, OM represents vector A
Φ represents the phase angle of vector A

A = a + jb
a – Horizontal component or active component or in phase component
b – Vertical component or reactive component or quadrature component

The magnitude of vector $'A' = \sqrt{a^2 + b^2}$
Phase angle of Vector 'A' = α = $\tan^{-1}$ (b/a)
Features of j – Operator

1. $j = \sqrt{-1}$
   It indicates anticlockwise rotation of Vector through 90°.
2. $j^2 = j \cdot j = -1$
   It indicates anticlockwise rotation of vector through 180°.
3. $j^3 = j \cdot j \cdot j = -j$
   It indicates anticlockwise rotation of vector through 270°.
4. $j^4 = j \cdot j \cdot j \cdot j = 1$
   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.
A = 2 + j3

This representation is known as rectangular form.
Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606
Phase angle of A = α = $\tan^{-1}$ (3/2) = 56°.31
A=|A| ∠α°
A=3.606∠56°.31

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

$\therefore$ Magnitude = | A + B | = $\sqrt{6^2+8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.
Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

$\therefore$ Magnitude = | A – B | = $\sqrt{-2^2+7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.
Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 $\angle$ 56°.310

B = 4 – j5

|B| = $\sqrt{4^2+-5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340

B= 6.403$\angle$ -51°.340

A X B = 3.606 $\angle$ 56°.310 X 6.403$\angle$ -51°.340

= 3.606 X 6.403$\angle$ (56°.310 + (-51°.340))

= 23.089 $\angle$4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

|A| = $\sqrt{4^2+-2^2}$ = 4.472

$\alpha = \tan^{-1}(-2/4) = -26°.565$

$A = 4.472 \angle -26°.565$

$B = 2 + j3$

$|B| = \sqrt{2^2 + 3^2} = 3.606$

$\alpha = \tan^{-1}(3/2) = 56°.310$

$B = 3.606\angle 56°.310$

$$\frac{A}{B} = \frac{4.472\angle -26^o.565}{3.606\angle 56^o.310} = \frac{4.472}{3.606}\angle -26^o.565 - 56^o.310 = 1.240\ \angle -82.875$$

ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

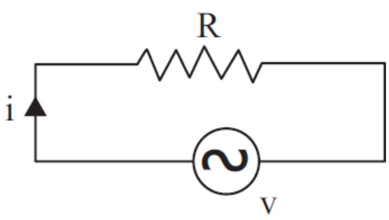

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

From Ohms law,

$$v = i R$$

$$I = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

$$\because I_m = \frac{V_m}{R}$$

$$= I_m \sin\omega t$$

where,

$V_m \rightarrow$ Maximum value of voltage (V)

$I_m \rightarrow$ Maximum value of current (A)

$\omega \rightarrow$ Angular frequency (rad/sec)

$t \rightarrow$ Time period (sec)

Phasor Representation:

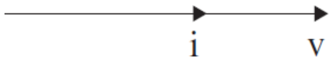

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

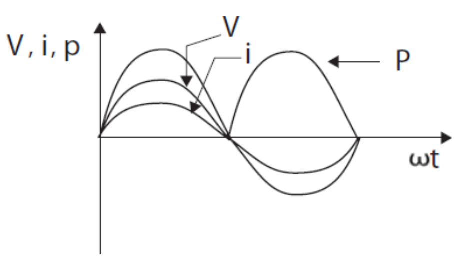

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:

In an AC circuit, impedance is the ratio of the maximum value of voltage to the maximum value of current.

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\frac{V_m}{R}} = R$$

$$\therefore Z = R$$

Power:

(i) Instantaneous power:

It is defined as the product of instantaneous voltage and instantaneous current.

p= v i

$$= V_m \sin\omega t \, I_m \sin\omega t = V_m I_m \sin^2\omega t$$

$[\because \omega t = \theta]$

$$p = V_m I_m \sin^2\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta \, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2\pi}[\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS} = V.I$$

$$P = V\,I$$

Power Factor:

It is defined as the cosine of the phase angle between voltage and current.

$$\cos\phi = \cos 0 = 1(unity)$$

Problems:

A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: v = 240 sin 377t

V_m = 240 V

ω = 377 rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin \omega t}{R}$$

$$= \frac{240}{6} \sin 377t$$

$$= 40 \sin 377tA$$

I. Phase angle:

$\phi = 0$

II. Impedance:

$Z = R = 6\Omega$

III. Instantaneous power:

IV. $p = V_m I_m \sin^2 \omega t$

$= 240.40.\sin^2 377t$

$= 9600\sin^2 377t$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800\text{watts}$$

VI. Power factor:

$\cos\Phi = \cos 0 = 1$

A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: $e = 200\sin\omega t$

$V_m = 200$

$P = 100w$

Average power, $P = \frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1\text{ A}$

Also, $V_m = I_m.R$

$R = 200\Omega$

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t$ A

A voltage e = 250sinωt when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:

Given: $e = 250\sin\omega t$

$V_m = 250$

$P = 100W$

I. $P = \dfrac{V_m I_m}{2}$

$100 = \dfrac{250 I_m}{2}$

$I_m = 0.8\ A$

II. $I_m = \dfrac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8\sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

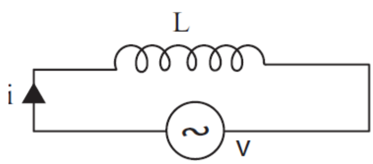

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

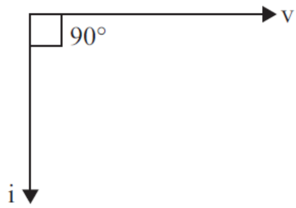

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

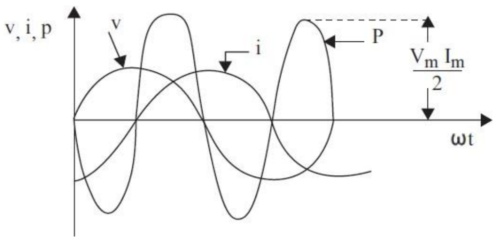

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m / \omega L} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i) Instantaneous power:

$$p = v\,i$$
$$= V_m \sin\omega t \; I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= V_m I_m \sin\omega t\,(-\cos\omega t)$$
$$= -V_m I_m \sin\omega t \cos\omega t = -V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta\cos\theta\, d\theta$$
$$= -\frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2}\, d\theta$$
$$\left[\because \sin 2\theta = 2\sin\theta\cos\theta\right]$$
$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}\left[\cos 2\pi - \cos 0\right]$$
$$= \frac{V_m I_m}{4\pi}\left[1-1\right] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor

(iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: L = 0.225 H

V_{RMS} = V = 120 V

f = 50Hz

I. Inductive reactance, XL = 2πfL =2π x 50 x 0.225 = 70.68Ω

II. Instantaneous current, i =-I_m cosωt

$\because I_m = \frac{V_m}{\omega L}$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$$V_m = V_{RMS}\sqrt{2} = 169.71V$$

$$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$$

Maximum power, $P_m = \frac{V_m I_m}{2}$ = 203.74 W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 344t volts

Instantaneous current, i = -2.4 cosωt A

A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: L = 0.01 H

I = 10cos1500t

I_m = 10A

ω = 1500 rad/sec

I. Inductive reactance, X_L = ωL = 1500 x 0.01 = 15Ω

II. The voltage across the inductor, e= $L\frac{di}{dt}$

$$= 0.01\frac{d(10\cos 1500t)}{dt} = 0.01 \times 10[-\sin 1500t.1500]$$

$$= -150 \sin 1500t \text{ V}$$

III. X_L = 40Ω; 2πfL = 40

$$f = \frac{40}{2\pi \times 0.01} = 637\text{Hz}$$

In the circuit, source voltage is v=200 sin $\left(314t + \frac{\pi}{6}\right)$ and the current is

$i = 20 \sin\left(314t - \frac{\pi}{3}\right)$ Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$

$I_m = 20A$

$\omega = 314$ rad/sec

I. $\omega = 2\pi f$

$f = 50Hz$

II. $V_m = 200V$ and $I_m = 20A$

III. $V_{RMS} = \frac{V_m}{\sqrt{2}} = 141.42V$

$I_{RMS} = \frac{I_m}{\sqrt{2}} = 14.142A$

IV. For a sinusoidal wave, Average value of current, $I_{av} = \frac{2I_m}{\pi} = 12.732A$

Average value of voltage, $V_{av} = \frac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

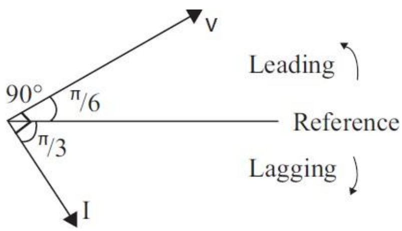

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle (90^0). So the circuit is purely inductive.

$I_m = \frac{V_m}{\omega L}$

$L = \frac{200}{314 \times 20} = 31.85mH$

Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

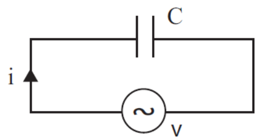

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, $Q = C.v$

$$= C.\ V_m \sin\omega t$$

$$\text{Current}, i = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}(CV_m \sin \omega t) = \omega CV_m \cos\omega t$$

$$= \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$[\because I_m = \omega CV_m]$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

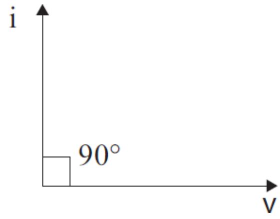

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

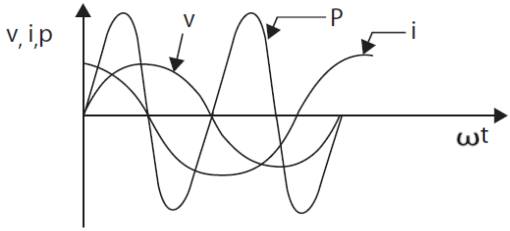

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$

$$= V_m \sin\omega t\, I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\,(\cos\omega t)$$

$$= V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta\cos\theta\, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2}\, d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:
In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

A 135µF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:
Given: V_{RMS}= V= 150V
C = 135µF
f = 50Hz

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \because I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150 X \sqrt{2} = 212.13V$
$I_m = 314X135X10^{-6}X212.13 = 8.99A$
$i = 8.99 \sin\left(314t + \frac{\pi}{2}\right)$ A

III. $p = V_m I_m \sin\omega t (\cos \omega t) = 212.13X8.99 \sin314t.\cos314t$
$= 66642.6 \sin 314t.\cos314t = 66642.6 \frac{\sin 628t}{2}$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$
$= 33321.3 \sin628t$ W

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}} = 6.36A$

VI. $P_m = \frac{V_m I_m}{2} = 953.52$ W

A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: $V_{RMS} = V = 100V$

$C = 12\mu F$

$I = 0.5A$

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$V_m = 100X\sqrt{2} = 141.42V$

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

$I_m = 0.5X\sqrt{2} = 0.707A$

II. $I_m = \omega CV_m = 2\pi fCV_m$

$f = 66.3Hz$

RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

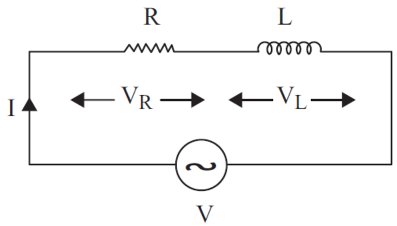

Figure 2.20

Let $V = V_m$ Sin ωt be the applied voltage.

i = Circuit current at any constant.

I = Effective Value of Circuit Current.

V_R= Potential difference across inductor.

V_L= Potential difference across inductor.

F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

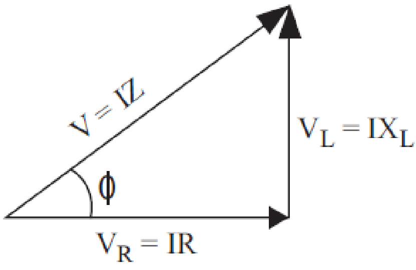

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$

$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

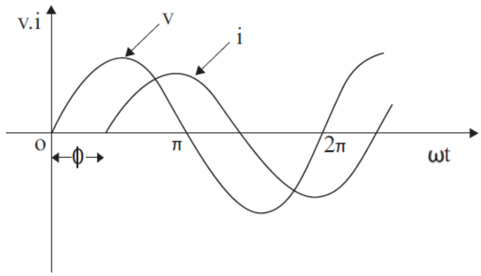

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual Power $P = VI\cos\phi$ – Current component is phase with voltage

Reactive or Quadrature Power

$Q = VI\sin\phi$ – Current component is quadrature with voltage

Complex or Apparent Power

$S = VI$ – Product of voltage and current

$S = P + jQ$

Problem

A series RL Circuit has

$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$ and $V(t) = 20\sin\left(314t + \frac{5\pi}{3}\right)$

Determine
(a) the impedence of the circuit
(b) the values of R_1L and power factor
(c) average power of the circuit

Solution:

$i(t) = 5\sin(314t + \frac{2\pi}{3})$

$V(t) = 20\sin(314t + \frac{5\pi}{3})$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2\times 180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5\times 180}{} = 150°$

3 3

Phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$= 150 - 120$

$\theta = 30^0$

Power factor $= \cos\theta$

$= \cos 30$

$= 0.866$ (lagging)

Impedence of the circuit $Z = \dfrac{V_m}{I_m}$

$= \dfrac{20}{5}$

$Z = 4\Omega$

(i) But $\cos\phi = \dfrac{R}{Z}$

$0.866 = \dfrac{R}{4}$

$\therefore R = 4 \times 0.866$

$R = 3.46\Omega$

$|Z| = \sqrt{R^2 + X_L^2}$

$X_L = \sqrt{Z^2 + R^2}$

$= \sqrt{(4)^2 - (3.46)^2}$

$X_L = 2\Omega$

$\omega L = 2\Omega$

$L = \dfrac{2}{\omega}$

$= \dfrac{2}{314}$

$L = 6.37 \times 10^{-3}$ H

(ii) Average power = VI cos ϕ

$= \dfrac{20}{\sqrt{2}} \cdot \dfrac{5}{\sqrt{2}} (0.866)$

$= 43.3$ watts

A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current
(ii) The phase angle between the current and the voltage
(iii) Power factor

(iv) Power

Solution:

$R = 6\Omega$
$L = 0.03$ H
$X_L = 2\pi fL$
$X_L = 2\pi\times50\times0.03$
$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$
$$= \sqrt{(6)^2 + (9.42)^2}$$
$$|Z| = 11.17\Omega$$

(i) $I = \dfrac{V}{Z} = \dfrac{100}{11.17} = 8.95$ amps

(ii) $\phi = tan^{-1}\left(\dfrac{X_L}{R}\right)$

$= tan^{-1}\left(\dfrac{9.42}{6}\right)$

$\Phi = 57.5$ (lagging)

(iii) Power factor = cos ϕ
= cos 57.5
= 0.537 (lagging)

(iv) Power = Average power
= VI cos Φ
= 100 ×8.95×0.537

Power = 480.6 Watts

A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

$R = 10\Omega$
$L = 20\text{ mH} = 20\times10^{-3}$H
$X_L = 2\pi fL$
$= 2\pi\times60\times20\times10^{-3}$
$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R^2 + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \dfrac{V}{Z} = \dfrac{250}{12.5} = 20\,amps$

$V_R = IR = 20\times10 = 200$ volts

(iii) $V_L = I\,X_L = 20\times7.54 = 150.8$ volts

(iv) Apparent power S = VI

$$= 250 \times 20$$

$$S = 5000VA$$

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \text{ (lagging)}$$

Active power = VI cos ϕ

=250×20×0.8

P = 4000 Watts

$$\sin\phi = \sqrt{1 - \cos^2\Phi} = \sqrt{1-(0.8)^2} = 0.6$$

Reactive Power Q = VI sin ϕ

= 250×20×0.6

Q= 3000 KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$

$Z_2 = 10 - j7$

V = 200 volts

$Z_{Total} = Z_1 + Z_2$

$= 5 + j7 + 10 - j7$

$Z_{Total} = 15 < 0^{\cdot}$

$\therefore \phi = 0^{\cdot}$

Taking V as referenve,

$V = 200 < 0^{\circ\cdot}$ Volts

(i) $I = \frac{V}{Z} = \frac{200\angle 0^\circ}{15\angle 0^\circ} = 13.33\angle 0^\circ\ amps$

(ii) ϕ = 0

PF = cos ϕ = cos 0 = 1

(iii) Power = VI cos ϕ

= 200 ×13.33×1

Power = 2666 watts

RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.
I = Circuit current of any instant
I = Effective value of circuit current
V_R = Potential Difference across Resistor
V_c = Potential Difference across Capacitor
f = Frequency of applied voltage
The same Current I flows through R and C
Voltage across R = V_R = IR in phase with I
Voltage across C = V_c = IX_c lagging I by 90^0
Applied voltage $V = IR - jIX_c$

$$= I(R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$

Figure 3.27

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF
R = 120Ω
V = 100V
F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\Omega$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$= 340\ \Omega$$

(a) $$|I| = \frac{|V|}{|Z|}$$

$$= \frac{100}{340}$$

$$= 0.294\ amps$$

(b) $$Phase Difference\ \phi = \tan^{-1}\left(\frac{X_c}{R}\right)$$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3^\circ\ (Leading)$$

$$\cos\phi = \cos(69.3)^\circ$$

$$= 0.353\ (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38\ Watts$$

The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$

$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$

$$= \sqrt{(240)^2 - (100)^2}$$

$$|V_C| = 218.2 \; volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$

$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3 amps$$

$$|X_C| = \frac{V_C}{|I|} \; (Apply\ ohm's\ law\ for\ C)$$

$$= \frac{218.2}{3} = 72.73\Omega$$

$$\frac{1}{2\pi fc} = 72.73$$

$$C = \frac{1}{2\pi \times 50 \times 72.73} = 43.77 \times 10^{-6}\ F$$

$$C = 43.77\mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum V_c = $\sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$$|V| = 60 volts$$

$$F = 50\ Hz$$

$$|I| = 2\ amps$$

Power Dissipated = P = 80 watts

(a) $|Z| = \dfrac{|V|}{|I|} = \dfrac{60}{2} = 30\Omega$

(b) $As\ P = I^2 R$

$$R = \frac{P}{I^2} = \frac{80}{4}$$

$$= 20\Omega$$

(c) $Since, |Z|^2 = R^2 + X_c^2$

$$X_c = \sqrt{(z)^2 - R^2}$$

$$= \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$\frac{1}{2\pi fc} = 22.36$$

$$c = \frac{1}{2\pi f (22.36)}$$

$$= \frac{1}{2\pi \times 50 \times 22.36}$$

$$= 142 \times 10^{-6}\ F$$

C = 142 μF

(or) Power factor = $\cos\phi = \dfrac{R}{|Z|}$

$$= \frac{20}{30}$$

= 0.67(Leading)

It is capacitive circuit.

A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

$V_R = 100$ volts

$W = I^2R = V_RI$

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\ amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$

$$V_C = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(230)^2 - (100)^2}$$

$$= 207 volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$

$$= 27.6\Omega$$

$$\frac{1}{2\pi fc} = 27.6$$

$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$

$$= 96.19\ \mu F$$

$$Power\ factor = \cos\phi = \frac{R}{|Z|}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$

$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$

$$= 13.33\Omega$$

$$Powerfactor = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{13.33}{30.7}$$

= 0.434(Leading)

Solution

RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$

$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$

$$= |I|^2 [R^2 + (X_L - X_C)^2]$$

$$|V| = |I| \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \frac{|V|}{|I|} \qquad \because X = (X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{R^2 + X^2}$$

Three cases of Z

<u>Case 1</u> If $X_L > X_C$
The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

<u>Case 2</u> If $X_L < X_C$
The circuit behaves like RC circuit current leads applied voltage power factor is leading.

<u>Case 3</u> When $X_L = X_C$, the circuit behaves like pure resistance.
Current is in phase with the applied voltage power factor is unity.
Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage
   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by
   1) $i = I_m \sin(\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin(\omega t + \theta)$, for $X_L > X_C$

3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j\,(X_L - X_C)$$

Problems

In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V$ $\quad V_R = 3V$ $\quad V_L = 1V$

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$(V_L - V_C)^2 = V^2 - V_R^2$$

$$= 25 - 9 = 16$$

$$V_L - V_C = \pm 4$$

$$V_C = V_L \pm 4 = 1 + 4$$

$$V_C = 5V$$

A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1$ H

$C = 150\ \mu F \qquad = 150 \times 10^{-6}$ F

$V = 200V \qquad f = 50$ Hz

a) $X_L = 2\pi f L = 2\pi\,(50)\,0.1$

$= 31.4\ \Omega$

b) $X_C = \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi(50)(150\times10^{-6})}$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $|Z| = \sqrt{R^2 + X^2}$

$= \sqrt{10^2 + (10.2)^2}$

$= 14.28\Omega(\mathit{Inductive})$

$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

$= 0.7$ (lagging) (I lags behind V)

Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi f c_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi f L_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^2 + X_{C1}^2}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^2 + X_{L2}^2}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g
Susceptance = b
Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \ (positive)$$

$$|Y_1| = \sqrt{g_1^2 + b_1^2}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2} \ \text{(Negative)}$$

$$|Y_2| = \sqrt{g_2^{\,2} + b_2^{\,2}}$$

Total conductance $G = g_1 + g_2$
Total Suceptance $B = b_1 - b_2$
Total admittance $|Y| = \sqrt{G^2 + B^2}$
Branch current $|I_1| = |V||Y_1|$

$$|I_2| = |V||Y_2|$$

$$|I| = |V||Y|$$

Phase angle = $tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$
$Z_2 = 10 - j60\ \Omega$
$F = 50$ Hz
$Z_1 = R_1 + jX_L$
$Z_2 = R_2 - jX_C$

J term positive for in inductive
J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\ (50)\ (L)$
$X_L = 15$
$2\pi\ (50)\ L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$
$R_2 = 10$
$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$$C = \frac{1}{2\pi(50)60}$$

$C = 53\ \mu F$.

2.3.27 Two circuits, the impedances of which are $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$
$$Z2 = (6 - j8)\Omega = 10\angle -53.13$$
$$I = 15\text{ A}$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \quad \text{(Current divider rule)}$$
$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$
$$(Z_1 + Z_2 = 10 + j15 + 6 - j8)$$
$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$
$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$
$$= 15 - (1.97 - j8.37)$$
$$= 15.5 - 32.7\text{A}$$

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$
$$= 739.6 \text{ watts}$$

Power taken by branch 2

$$= |I_2|^2 R_2$$
$$= (15.5)^2 \times 6$$
$$= 1442 \text{ watts}$$

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$Z_1 = R = 100\Omega$

$Z_2 = j X_L = j2\pi fL$

$= j\,(2\pi\times 50\times 0.6)$

$= j\ 188.5\Omega$

$= 188.5 \angle 90$

$Z_T = Z_1 * Z_2$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

$Total\ impedance\ |Z_T| = 88.33\Omega$

$R = 78\Omega$, $X_L = 41.46\Omega$

$X_L = 2\pi f L_{eq}$

$41.46 = 2\pi \times 50 \times Leq$

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

= 30 – j40 + 24 +j32

= 54 – j8

= $54.6 \angle -8.43$ A

Comparing ‘V’ and ‘I_T’ current I_T lag voltage ‘V’

$\therefore \phi = 8.43°$ lag

Power factor = cos ϕ= cos 8.43

= 0.99 lag

True Power = $W = |V||I|\cos\phi$

=200 × 54.6 × cos 8.43

= 10802 watts

= 10.802 KW

Apparent Power $= |V|I$

= 200× 54.6

= 10920 VA = 10.920 KVA

Reactive Power $= |V| I \sin\phi$

= 200 × 54.6 × sin 8.43

= 1601 VAR

= 1.601 KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

$= 3.663 \angle 8.43$

= 3.623 + j0.54

$= R + j X_L$

$R = 3.623\Omega \qquad X_L = 0.54\ \Omega$

(or)

$$Z_{Total} = Z1*Z2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2.4 + j3.2)(3 - j4)}{2.4 + j3.2 + 3 - j4}$$

$$= \frac{4\angle 53.13 \times 5\angle -53.13}{5.46\angle -8.43}$$

$$= \frac{20\angle 0^0}{5.46\angle - 8.43}$$

$$= 3.663 \angle 8.43$$

$$= 3.623 + j\,0.54\ \Omega$$

THREE PHASE A.C. CIRCUITS

Three Phase Connection

We have seen above only about single phase systems. Generally generation transmission and distribution of electrical energy are of three phase in nature. Three phase system is a very common poly phase system. It could be viewed combination of three single phase system with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system where as pulsating torque is produced in the case of single phase system. Because of these advantages the overall generation, transmission and distribution of electrical power is usually of three phase.

There are two possible connections in 3-phase system. One is star connection and the other one is delta or mesh connection. Each type of connection is governed by characteristics equations relating the currents and the voltages.

Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called star point or the neutral point. The free ends of the three phase coils will be operating at specific potential with respect to the zero potential of star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system and three phase three wire system.

Analysis

Let us analyze the relationship between currents and voltages. In a three phase circuit, the voltage across the individual coil is known as phase voltage and the voltage between two lines is called line voltage. Similarly the current flowing through the coil is called phase current and the current flowing through the line is called line current.

Notations Defined

E_R , E_Y , E_B	: Phase voltages of R, Y and B phases.
I_R , I_Y, I_B	: Phase currents
V_{RY}, V_{YB}, V_{BR}	: Line voltages
I_{L1}, I_{L2}, I_{L3}	: Line currents

Figure 3.32

A balanced system is one in which the currents in all phases are equal in magnitude and are displaced from one another by equal angles. Also the voltages in all the phases are equal in magnitude and are displaced from one another by equal angles. Thus,

$$E_R=E_Y=E_B=E_P \qquad V_{RY}=V_{YB}=V_{BR}=V_L$$

$$I_R=I_Y=I_B=I_P \qquad I_{L1}=I_{L2}=I_{L3}=I_L$$

Figure 3.33

Current Relationship:

Apply Kirchhoff's current law at nodes R_1, Y_1, B_1 We get

$$I_R = I_{LI};\ I_Y = I_{L1}\ ;\ I_B=I_{L3}$$

This means that in a balanced star connected system, phase current equals the line current

$$I_P = I_L$$
Phase current = Line current

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_{R}; V_{Ry} and E_y.

$$\vec{E}_R - \vec{E}_Y = \vec{V}_{RY}$$

Using law of parallelogram

$$\left|\vec{V}_{RY}\right| = V_{RY} = \sqrt{E_R^2 + E_R^2 + 2E_R E_Y \cos 60}$$

$$= \sqrt{E_P^2 + E_P^2 + 2E_P E_P \cos 60} = E_P\sqrt{3}$$

Similarly,

$$\vec{E}_Y - \vec{E}_B = \vec{V}_{YB} \text{ and } \vec{E}_B - \vec{E}_R = \vec{V}_{BR}$$
$$V_{RY} = E_P\sqrt{3} \text{ and } V_{BR} = E_P\sqrt{3}$$

Hence $V_L = \sqrt{3}\, E_P$

Line Voltage $= \sqrt{3}$ phase voltage

Power relationship:

Let cosϕ be the power factor of the system.

Power consumed in one phase=$E_P I_P \cos\phi$

Power consumed in three phase $= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi \text{ watts}$$

Reactive power in one phase $= E_P I_P \sin\phi$

Total Reactive power $= 3E_P I_P \sin\phi$

$= \sqrt{}$ Apparent Power=$3E_P I_P =$

Apparent power per phase=$E_P I_P \sqrt{}$ Total

$3V_L I_L \sin\phi$ VAR

$3V_L I_L$ Volt

Delta Connection:

The dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Figure 3.33

Let us analyze the relationship between currents and voltages. The system is balanced one. Notation used in the star connection are used here.

E_R, E_Y, E_B : Phase voltages of R, Y and B phases.
I_R, l_Y, I_B : Phase currents
V_{RY}, V_{YB}, V_{BR}: Line voltages
I_{L1}, I_{L2}, I_{L3} : Line currents

Voltage relationship:
Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R,V_{RY}

We Have $E_R = V_{RY}$
Similarly $E_Y = V_{YB}$ *and* $E_B = V_{BR}$

Thus $E_P = V_L$
Phase voltage= line voltage

Current Relationship:
Apply Kirchhoff's current law at node A (i.e.) R_1 , B_2 We get

$$\vec{I}_R - \vec{I}_B = \vec{I}_{1,1}$$

Referring to the phasor diagram and applying the law of parallelogram,
We get

$$I_{L1} = \sqrt{I_R^2 + I_Y^2 + 2I_RI_Y\cos 60}$$
$$= \sqrt{I_P^2 + I_P^2 + 2I_PI_P\cos 60}$$

Similarly,

$$\vec{I}_Y - \vec{I}_R = \vec{I}_{1,2} \text{ and } \vec{I}_B - \vec{I}_Y = \vec{I}_{1,3}$$

Hence $I_{L2} = I_P\sqrt{3}$ and $I_{L3} = I_P\sqrt{3}$

Thus Line current $= \sqrt{3}$ Phase current

$$I_L = I_P\sqrt{3}$$

Power relationship:
Let cosϕ be the power factor of the system.

Power consumed in one phase $= E_P l_P cos\phi$

Power consumed in three phase $= 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$

$$= \sqrt{3}V_L I_L \cos\phi \; watts$$

Reactive power in one phase $= E_p I_p sin\phi$

Total Reactive power $= 3E_p I_p cos\phi$

$$= \sqrt{3}V_L I_L \sin\phi \; VAR$$

Apparent power per phase = E_pI_p

Total Apparent Power $= 3E_p I_p = \sqrt{3}\, V_L I_L$ volt

MEASUREMENT OF POWER IN THREE PHASE CIRCUITS:

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phase are not of same value. As a result, the current flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The three phase power can be measured by using three watt maters in each phases. The algebraic sum of the reading gives the total three phase power consumed. However three phase power can also measured using two watt meter.

Case I Star Connected load

In this section we analyse the measurement of three phase power using two wattmeter, when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Diagram 4

Figure 3.35

For Wattmeter 1

$$\text{Current measured} = \vec{I}_{L1} = \vec{I}_R$$

$$\text{Voltage measured} = \vec{V}_{RY}$$

$$\text{Phase angle between them} = 30 + \phi$$

$$\text{Power measured} = P1 = V_{RY} I_R \cos(30 + \phi)$$

For Wattmeter 2

$$\text{Current measured} = \vec{I}_{L3} = \vec{I}_B$$

$$\text{Voltage measured} = \vec{V}_{BY}$$

$$\text{Phase angle between them} = 30 - \phi$$

$$\text{Power measured} = P1 = V_{BY} I_B \cos(30 - \phi)$$

$$= V_L I_L \cos(30 - \phi)$$

$$\text{Now,} \quad P1 + P2 = V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi = \text{Total power in a three phase circuit}$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$
$$= V_L I_L \times 2 \times \sin 30 s \text{ in} \phi$$
$$= V_L I_L \sin\phi$$

$$\frac{P2 - P1}{P2 + P1} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]$$

$$\tan\phi = \sqrt{3}\ (P\ 2 - P1/P\ 2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Thus, two wattmeters connected appropriately in a three phase circuit can measure the total power consumed in the circuit.

Case II Delta Connected load

In this section we analyse the measurement of three phase power using two wattmeter, power when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Figure 3.36

For Wattmeter 1

$$\textit{Current measured} = I_{1,1} = \vec{I}_R - \vec{I}_B$$

$$\textit{Voltage measured} = \vec{V}_{RY} = \vec{E}_R$$

$$\textit{Phase angle between them} = 30 + \phi$$

$$\textit{Power measured} = P1 = V_{RY} I_{L1} \cos(30 + \phi)$$

$$=V_L I_L \cos(30 + \phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{1,3} = \vec{I}_B - \vec{I}_Y$$

$$Voltage\ measured = \vec{V}_{BY} = -\vec{E}_Y$$

$$Phase angle between them = 30 - \phi$$

$$Power\ measured = P1 = V_{BY} I_{1,3} \cos(30 - \phi)$$

$$= \mathrm{V_L I_L} \cos(30 - \phi)$$

$$Now, \quad P1 + P2 = V_L I_L \cos(30 + \phi) + \mathrm{V_L I_L} \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\,\mathrm{V_L I_L} \cos\phi = Total\ power\ in\ a three\ phase circuit$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$= V_L I_L \times 2 \times \sin 30 s\ in\phi$$

$$= \mathrm{V_L I_L} \sin\phi$$

$$\mathrm{Tan}\,\phi = \sqrt{3}(\mathrm{P}\,2 - P1/\mathrm{P}\,2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Problems 3.30

Three similar coils of Resistance of 10Ω and inductance 0.15 Henry are connected in star across a 3Φ, 440V, 50Hz supply. Find the line and phase values of current. Also find the above values when they are connected in Delta.

Solution:

Given Data

$$V_L = 440V,\ R_{ph} = 10\Omega,\ L_{ph} = 0.15H,\ f = 50Hz$$

$$X_{L\,ph} = 2\pi f L_{ph} = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$|Z_{ph}| = \sqrt{R_{ph}^2 + X_{L\,ph}^2} = \sqrt{10^2 + (47.12)^2}$$

$$= 48.17\Omega$$

In star Connection

$$I_L = I_{ph} \qquad V_L = \sqrt{3}V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 230.95 \text{ Volt}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.95}{48.17} = 4.794\,A$$

$$I_L = I_{ph} = 4.794\,A$$

Active power = $3V_{ph}I_{ph}\cos\Phi$

$$\cos\Phi = \frac{R_{ph}}{Z_{ph}} = 0.2075$$

$$\text{Activepower} = 3*230.95*4.794*0.2075$$
$$= 689.54W$$

Reactive power = $3V_{ph}I_{ph}\sin\Phi$

$$\sin\Phi = \sqrt{1-\cos^2\Phi} = 0.9782$$

$$\text{Reactive power} = 3*230.95*4.794*0.9782$$
$$= 3249.23VAR$$

$$\text{Apparent power} = 3V_{ph}I_{ph} = 3*230.95*4.794$$
$$= 3321.52V$$

If it is Delta connected coils, then

$$VL = V_{ph} \;\&\; IL = \sqrt{3}\,I_{ph}$$

$$VL = V_{ph} = 440V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} \frac{440}{48.17} = 9.134\,A$$

$$IL = \sqrt{3}\,I_{ph} = \sqrt{3}*9.134 = 15.82\,A$$

$$\text{Active power} = 3\,V_{ph}I_{ph}\cos\Phi$$
$$= 3*440*9.134*0.2075$$
$$= 2501.80\,watt$$

$$\text{Reacive power} = 3\, V_{ph} I_{ph} \sin\Phi$$

$$= 3 * 440 * 9.134 * 0.9782$$

$$\text{Apparent power} = 3 V_{ph} I_{ph} = 3 * 440 * 9.134$$

$$= 12056.88\ VA$$

Problem 3.31

Two wattmeters connected to measure the 3Φ power indicate 1000 watts and 500 watts respectively. Calculate the power factor of the ckt.

Solution:

Given data

$$p_1 = 500 \text{ watts}, p_2 = 1000 \text{ watts},$$
$$p_1 + p_2 = 1000 + 500 = 1500 \; watts$$
$$p_2 - p_1 = 1000 - 500 = 500 \; watts$$
$$p_1 = VL\,IL \cos(30 + \Phi)$$
$$p_2 = VL\,IL \cos(30 - \Phi)$$
$$p_1 + p_2 = \sqrt{3} VL\ IL \cos \Phi$$

$$p_2 - p_1 = \sqrt{3} * \frac{(p_2 - p_1)}{(p_1 + p_2)} = \frac{\sqrt{3} * 500}{1500}$$
$$= 0.5773$$
$$\Phi = 29.99°$$

Power factor cos Φ = 0.866

Problem 3.32

A balanced star connected load of (3+j4) Ω impedance is connected to 400V, three phase supply. What is the real power consumed by the load?

Solution:

Given data

$$V_L = 400 \; volt$$
$$\text{Impedence / phase} = Z_{ph} = 3 + j4 = 5\angle 53°$$
In starconnection
$$I_L = I_{ph} \; \& \; V_L = \sqrt{3} V_{ph}$$
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 volt$$
$$\text{Current in each } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5\angle 53°}$$
$$= 46.02\angle - 53° \; A$$
$$\text{Linecurrent } I_L = 46.02 \; A$$

$$\text{Total power consumed in the load} = \sqrt{3}\, V_L I_L \cos\Phi$$
$$= \sqrt{3} * 400 * 46.02 * \cos(-53°)$$
$$= 19188 \; watt$$

PART A – QUESTIONS

1. Define Form factor and Peak factor
2. What is meant by average value?
3. Give the relation between line voltage and phase voltage, line current and phase current for star and delta connection.
4. What are the advantages of polyphase system ?
5. Define power factor?
6. What is phase sequence?
7. Define inductance and write its unit.
8. What is meant by balanced system?
9. Write down the expression for power factor in two wattmeter method.

PART B – QUESTIONS

1. Explain with neat figures the power measurement in three phase circuits using two-wattmeter method.
2. A given load consisting of a resistor R & a capacitor C, takes a power of 4800W from 200V, 60HZ supply mains, Given that the voltage drop across the resistor is 120V, Calculate the (a) impedance (b) current (c) power factor (d) resistance (e) capacitance. Write down the equations for the current and voltage.
3. A coil of 10 ohms and inductance of 0.1H in series with a 150μF capacitor across 200V,250HZ supply. Calculate (i) inductive reactance, capacitive reactance and impedance of the circuit (ii) current (iii)power factor(iv)voltage across the coil and capacitor respectively.
4. An impedance z_1= (2.4+j3.2) ohms is in parallel with another impedance z_2= (3-j4) ohms. The combination is given a supply of 200 V. Calculate (i) total impedance (ii) individual & total currents (iii) power factor (iv) power in the circuit.
5. A balanced three phase load consists of 6 ohms resistor & 8 ohms reactor (inductive) in each phase. The supply is 230V, 3 phases, 50HZ. Find (a) phase current (b) line current (c) total power. Assume the load to be connected in star & delta.
6. A 3phase, 4 wire 208 V, ABC system supplies a star connected load in which $Z_A=10\angle 0$, $Z_B=15\angle 30$, $Z_C=10\angle -30$. Find the line currents, the neutral current and the load power.
7. A coil having R = 10Ω and L = 0.2H is connected to a 100V, 50 Hz supply. Calculate (i) the impedance of the coil (ii) the current (iii) the phase difference between the current and voltage and (iv) the power.
8. Three similar coils of resistance of 10Ω and inductance 0.15H are connected in star across a 3 phase 440V, 50 Hz supply. Find the line

and phase values of current. Also find the above values when they are connected in delta.

9. Each phase of a delta connected load comprises a resistor of Ohm and a capacitor of μF in series. Calculate the line current for a 3 - ϕ voltages of 400V at 50 Hz. Also evaluate the power factor and the total 3 - ϕ power absorbed by the load.

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UNIT-2

Electrical Circuits and Network Analysis – SEEA1304

NETWORK THEOREMS

Superposition Theorem - Reciprocity Theorem - Thevenin's Theorem - Norton's Theorem - Maximum power transfer Theorem..

2.1 SUPERPOSITION THEOREM

The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals. This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the circuit which contains two sources as shown in Fig. 1.

Now let us find the current passing through the 3 V resistor in the circuit. According to the superposition theorem, the current I_2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A

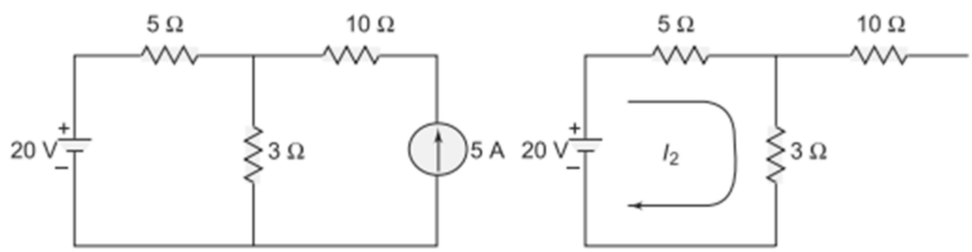


Figure 1 : Superposition theorem

The current I_5 due to the 5 A source with the 20 V source short circuited is

$$I_5 = 5 \times \frac{5}{(3+5)} = 3.125\,\text{A}$$

The total current passing through the 3 V resistor is

(2.5 + 3.125) = 5.625 A

Let us verify the above result by applying nodal analysis.

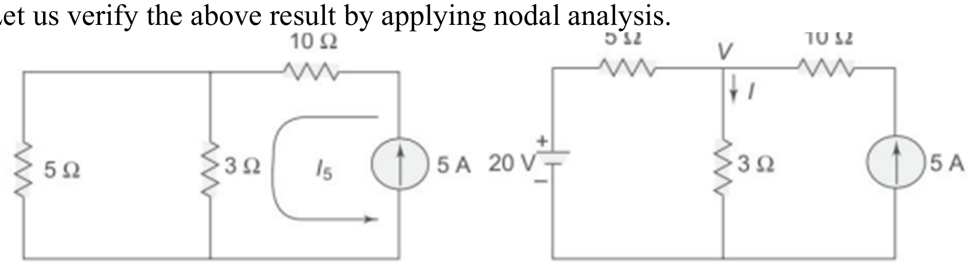


Figure 2: Superposition theorem

The current passing in the 3 V resistor due to both sources should be 5.625 A.
Applying nodal analysis to Fig. 2, we have

$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$V\left[\frac{1}{5}+\frac{1}{3}\right]=5+4$$

$$V=9\times\frac{15}{8}=16.875\ \text{V}$$

The current passing through the 3 V resistor is equal to V/3,

i.e. I = 16.875/3=5.625 A

So the superposition theorem is verified.
Let us now examine the power responses.
Power dissipated in the 3 V resistor due to the voltage source acting alone

$$P_{20}=(I_2)^2R=(2.5)^2\,3=18.75\text{ W}$$

Power dissipated in the 3 V resistor due to the current source acting alone

$$P_5=(I_5)^2R=(3.125)^2\,3=29.29\text{ W}$$

Power dissipated in the 3V resistor when both the sources are acting simultaneously is given by

$$P=(5.625)^2\times 3=94.92\text{ W}$$

From the above results, the superposition of P20 and P5 gives

$$P_{20}+P_5=48.04\text{ W}$$

which is not equal to P = 94.92 W
We can, therefore, state that the superposition theorem is not valid for power responses. It is applicable only for computing voltage and current responses.

Example 1: Find the voltage across the 2 V resistor in Fig. 3 by using the superposition theorem.

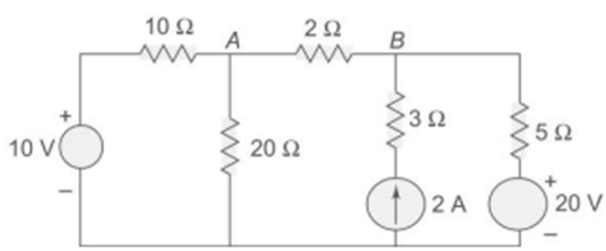


Figure 3

Solution
Let us find the voltage across the 2 V resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2 V resistor.

Our first step is to find the voltage across the 2 V resistor due to the 10 V source, while other sources are set equal to zero.

The circuit is redrawn as shown in Fig. 4

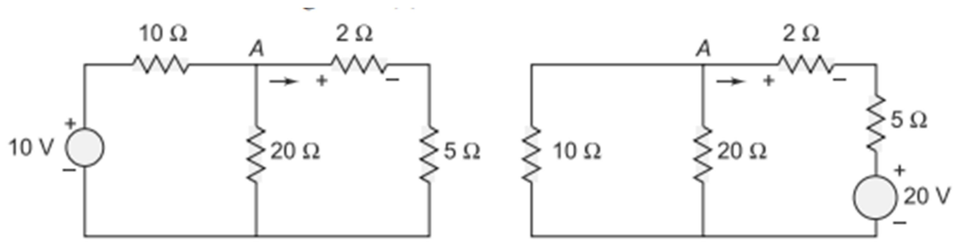


Figure 4

Assuming a voltage V at the node 'A' as shown in Fig. 4, the current equation is

$$\frac{V-10}{10}+\frac{V}{20}+\frac{V}{7}=0$$

$$V\,[0.1 + 0.05 + 0.143] = 1$$

$$\text{or}\quad V = 3.41\text{ V}$$

The voltage across the 2 V resistor due to the 10 V source is

$$V_2=\frac{V}{7}\times 2=0.97\text{ V}$$

Our second step is to find out the voltage across the 2 V resistor due to the 20 V source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 4.

Assuming voltage V at the node A as shown in Fig. 4, the current equation is

$$\frac{V-20}{7}+\frac{V}{20}+\frac{V}{10}=0$$

$$V\,[0.143 + 0.05 + 0.1] = 2.86$$

$$\text{or}\quad V=\frac{2.86}{0.293}=9.76\text{ V}$$

The voltage across the 2 V resistor due to the 20 V source is

$$V_2=\left(\frac{V-20}{7}\right)\times 2=-2.92\text{ V}$$

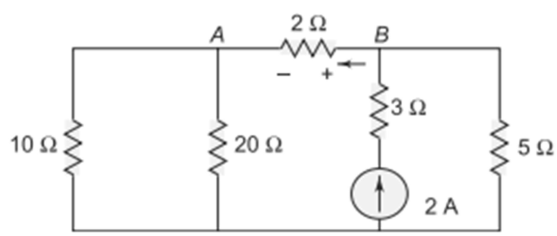


Figure 5

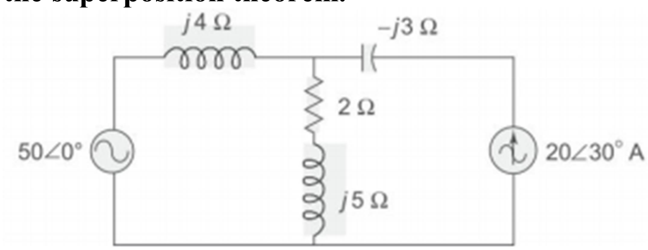
The last step is to find the voltage across the 2 V resistor due to the 2 A current source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 5

$$\text{The current in the } 2\ \Omega \text{ resistor} = 2 \times \frac{5}{5+8.67}$$

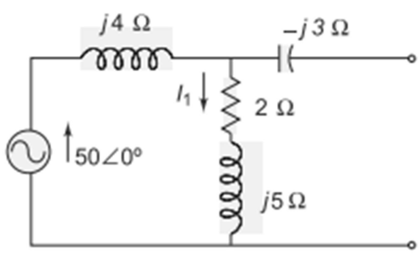
$$= \frac{10}{13.67} = 0.73\,\text{A}$$

The voltage across the 2 V resistor = 0.73 x 2 = 1.46 V
The algebraic sum of these voltages gives the total voltage across the 2 V resistor in the network
V = 0.97 – 2.92 – 1.46 5 = - 3.41 V
The negative sign of the voltage indicates that the voltage at ‘A’ is negative

Example2:
Determine the voltage across the (2 + j5) V impedance as shown in Fig below by using the superposition theorem.



Solution According to the superposition theorem, the current due to the 50 ∠0° V voltage source is I1 as shown in Fig below with current source 20 ∠30° A open-circuited.



$$\text{Current } I_1 = \frac{50\angle 0^\circ}{2+j4+j5} = \frac{50\angle 0^\circ}{(2+j9)}$$

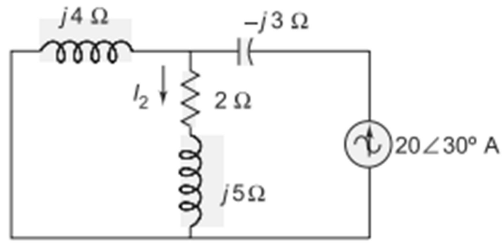
$$= \frac{50\angle 0^\circ}{9.22\angle 77.47} = 5.42\angle -77.47^\circ\ \text{A}$$

Voltage across $(2 + j5)\ \Omega$ due to the current I_1 is

$$V_1 = 5.42 \angle -77.47^\circ\,(2 + j5)$$

$$= (5.38)\,(5.42) \angle -77.47^\circ + 68.19^\circ$$

$$= 29.16 \angle -9.28^\circ$$



The current due to the 20 ∠30° A current source is I_2 as shown in Fig. 7.18, with the voltage source 50 ∠0° V short-circuited.

$$\text{Current } I_2 = 20\angle 30^\circ \times \frac{(j4)\,\Omega}{(2+j9)\,\Omega}$$

$$= \frac{20\angle 30^\circ \times 4\angle 90^\circ}{9.22\angle 77.47^\circ}$$

$$\therefore \quad I_2 = 8.68\angle 120^\circ - 77.47^\circ = 8.68\angle 42.53^\circ$$

Voltage across $(2 + j5)\,\Omega$ due to the current I_2 is

$$V_2 = 8.68\angle 42.53^\circ\,(2 + j5)$$
$$= (8.68)\,(5.38)\,\angle 42.53^\circ + 68.19^\circ$$
$$= 46.69\angle 110.72^\circ$$

Voltage across $(2 + j5)\,\Omega$ due to both sources is

$$V = V_1 + V_2$$
$$= 29.16\angle -9.28^\circ + 46.69\angle 110.72^\circ$$
$$= 28.78 - j4.7 - 16.52 + j43.67$$
$$= (12.26 + j38.97)\text{ V}$$

Voltage across $(2 + j5)\,\Omega$ is $V = 40.85\angle 72.53^\circ$.

2.2 THEVENIN'S THEOREM

In many practical applications, it is always not necessary to analyse the complete circuit; it requires that the voltage, current, or power in only one resistance of a circuit be found. The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network. Thevenin's theorem states that any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the open-circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances. According to Thevenin's theorem, an equivalent circuit can be found to replace the circuit in Fig. 6

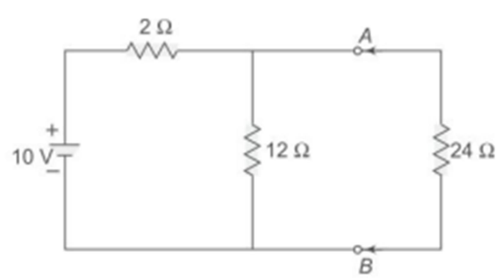


Figure 6

In the circuit, if the 24 V load resistance is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 V resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

$$\text{where} \quad I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1 \text{ A}$$

$$\therefore \quad I_{24} = 1 \times \frac{12}{12+24} = 0.33 \text{ A}$$

The voltage across the 24 V resistor = 0.33 x 24 = 7.92 V.
Now let us find Thevenin's equivalent circuit.
The Thevenin voltage is equal to the open-circuit voltage across the terminals 'AB', i.e. the voltage across the 12 V resistor. When the load resistance is disconnected from the circuit, the Thevenin voltage

$$V_{Th} = 10 \times \frac{12}{14} = 8.57 \text{ V}$$

The resistance into the open-circuit terminals is equal to the Thevenin resistance

$$R_{Th} = \frac{12 \times 2}{14} = 1.71 \ \Omega$$

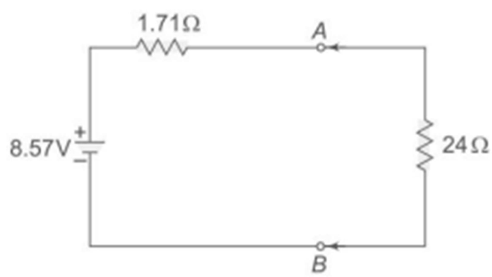


Figure 7

Thevenin's equivalent circuit is shown in Fig. 7. Now let us find the current passing through the 24 V resistance and voltage across it due to Thevenin's equivalent circuit. Fig. 7

$$I_{24} = \frac{8.57}{24+1.71} = 0.33 \text{A}$$

The voltage across the 24 V resistance is equal to 7.92 V. Thus, it is proved that RL (5 24 V) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Example : Determine the Thevenin's equivalent circuit across 'AB' for the given circuit shown in Fig. 8

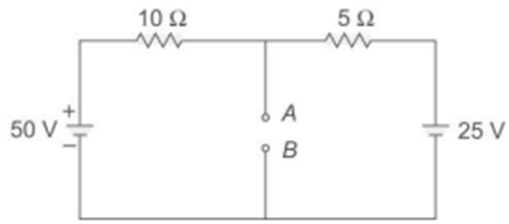


Figure 8

Solution The complete circuit can be replaced by a voltage source in series with a resistance as shown in Fig. 9

where VTh is the voltage across terminals AB, and
RTh is the resistance seen into the terminals AB.

To solve for VTh, we have to find the voltage drops around the closed path as shown in Fig. 9

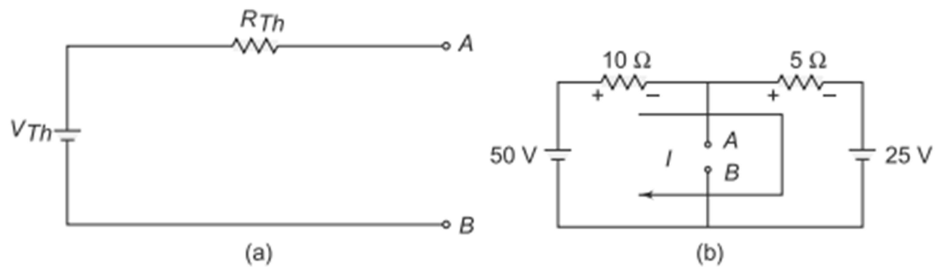


Figure 9

We have $50 - 25 = 10I + 5I$

or $15I = 25$

$$\therefore \quad I = \frac{25}{15} = 1.67\ \text{A}$$

Voltage across 10 Ω = 16.7 V

Voltage drop across 5 Ω = 8.35 V

or $V_{Th} = V_{AB} = 50 - V_{10}$

$= 50 - 16.7 = 33.3$ V

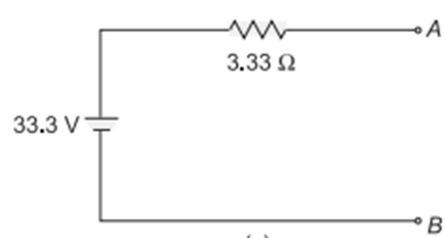


Figure 10

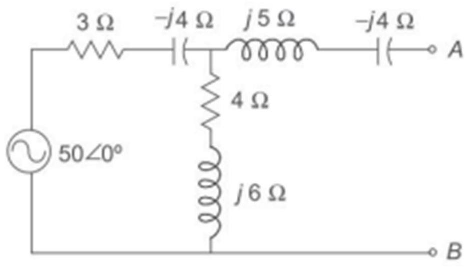
To find RTh, the two voltage sources are removed and replaced with short circuit. The resistance at terminals AB then is the parallel combination of the 10 V resistor and 5 V resistor; or

$$R_{Th} = \frac{10 \times 5}{15} = 3.33\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 9

Example 2: For the circuit shown in Fig. 7.22, determine Thèvenin's equivalent between the output terminals.

Solution The Thèvenin voltage, VTh, is equal to the voltage across the (4 + j6) V impedance. The voltage across (4 + j6) V is



$$V = 50\angle 0^\circ \times \frac{(4+j6)}{(4+j6)+(3-j4)}$$

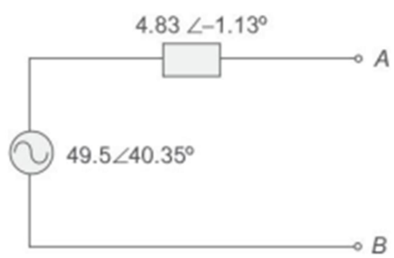
$$= 50\angle 0^\circ \times \frac{4+j6}{7+j2}$$

$$= 50\angle 0^\circ \times \frac{7.21\angle 56.3^\circ}{7.28\angle 15.95^\circ}$$

$$= 50\ \angle 0^\circ \times 0.99\ \angle 40.35^\circ$$

$$= 49.5\ \angle 40.35^\circ \text{ V}$$

The impedance seen from terminals A and B is



$$Z_{\text{Th}} = (j5 - j4) + \frac{(3-j4)(4+j6)}{3-j4+4+j6}$$

$$= j1 + \frac{5\angle 53.13^\circ \times 7.21\angle 56.3^\circ}{7.28\angle 15.95^\circ}$$

$$= j1 + 4.95\ \angle -12.78^\circ = j1 + 4.83 - j1.095$$

$$= 4.83 - j0.095$$

$$\therefore\ \ Z_{\text{Th}} = 4.83\ \angle -1.13^\circ\ \Omega$$

The Thèvenin equivalent circuit is shown in Fig above.

2.3 NORTON'S THEOREM

Another method of analysing the circuit is given by Norton's theorem, which states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short-circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.

According to Norton's theorem, an equivalent circuit can be found to replace the circuit in Fig. 11

In the circuit, if the load resistance of 6 V is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6 V resistor due to the original circuit

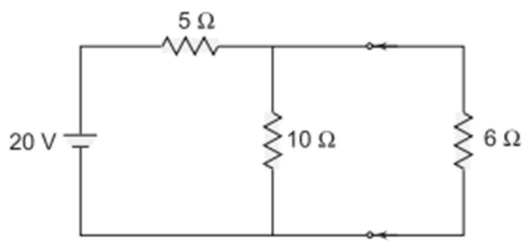


Figure 11

$$I_6 = I_T \times \frac{10}{10+6}$$

$$\text{where} \quad I_T = \frac{20}{5+(10\|6)} = 2.285\,\text{A}$$

$$\therefore \quad I_6 = 2.285 \times \frac{10}{16} = 1.43\,\text{A}$$

i.e. the voltage across the 6 V resistor is 8.58 V. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short-circuited terminals as shown in Fig. 12

$$\text{Here,} \quad I_N = \frac{20}{5} = 4\,\text{A}$$

Norton's resistance is equal to the parallel combination of both the 5 V and 10 V resistors

$$R_N = \frac{5 \times 10}{15} = 3.33\,\Omega$$

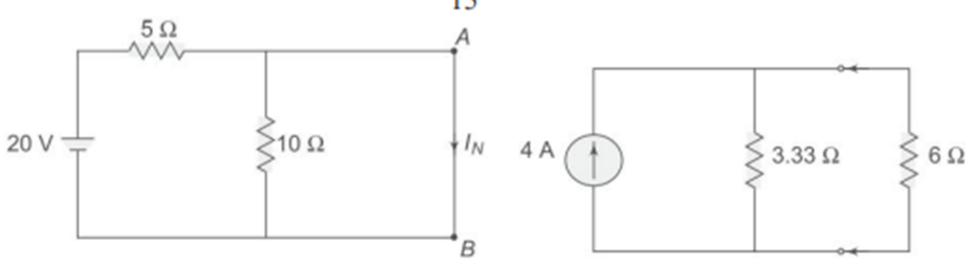


Figure 12

The Norton's equivalent source is shown in Fig. 12.
Now let us find the current passing through the 6 V resistor and the voltage across it due to Norton's equivalent circuit

$$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43 \text{ A}$$

The voltage across the 6 Ω resistor = 1.43 x 6 = 8.58 V
Thus, it is proved that RL (= 6 Ω) has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

Example Determine Norton's equivalent circuit at terminals AB for the circuit shown in Fig. 13

Solution The complete circuit can be replaced by a current source in parallel with a single resistor as shown in Fig. 14, where IN is the current passing through the short circuited output terminals AB and RN is the resistance as seen into the output terminals.

To solve for IN, we have to find the current passing through the terminals AB as shown in Fig. 14

From Fig. 14, the current passing through the terminals AB is 4 A. The resistance at terminals AB is the parallel combination of the 10 V resistor and the 5 V resistor
Norton's equivalent circuit is shown in Fig. 14

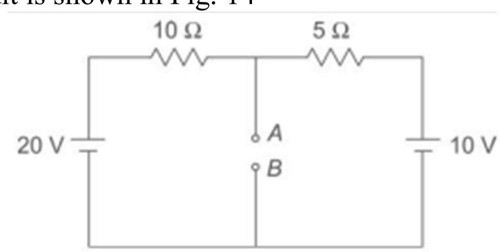


Figure 13

$$\text{or} \quad R_N = \frac{10 \times 5}{10+5} = 3.33 \ \Omega$$

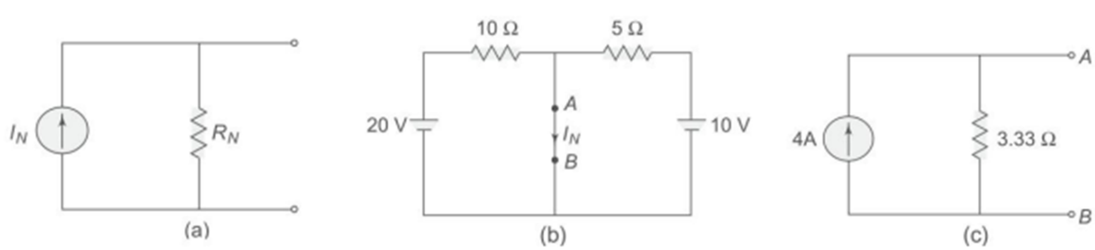
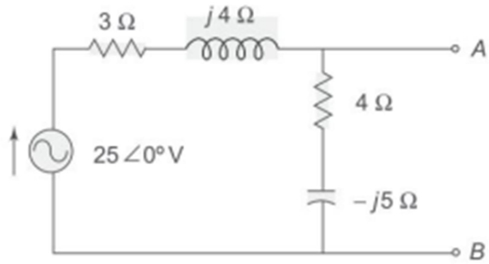
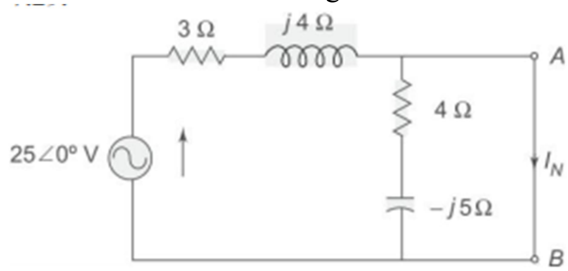


Figure 14

Example 2: For the circuit shown in Fig below, determine Norton's equivalent circuit between the output terminals, AB



Solution Norton's current IN is equal to the current passing through the short-circuited terminals AB as shown in Fig. below

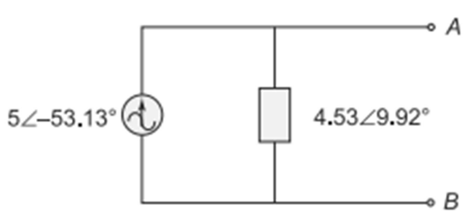


The current through terminals AB is

$$I_N = \frac{25\angle 0^\circ}{3+j4} = \frac{25\angle 0^\circ}{5\angle 53.13^\circ}$$
$$= 5\angle -53.13^\circ$$

The impedance seen from terminals AB is

$$Z_N = \frac{(3+j4)(4-j5)}{(3+j4)+(4-j5)}$$
$$= \frac{5\angle 53.13^\circ \times 6.4\angle -51.34^\circ}{7.07\angle -8.13^\circ}$$
$$= 4.53\angle 9.92^\circ$$



Norton's equivalent circuit is shown in Fig above.

2.4 RECIPROCITY THEOREM

In any linear bilateral network, if a single voltage source Va in branch 'a' produces a current Ib in branch 'b', then if the voltage source Va is removed and inserted in branch 'b' will produce a current Ib in branch 'a'. The ratio of response to excitation is same for the two conditions mentioned above. This is called the reciprocity theorem.

Consider the network shown in Fig. 15. AA’ denotes input terminals and BB9 denotes output terminals.

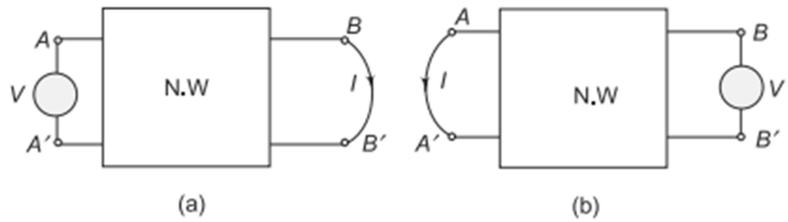


Figure 15

The application of voltage V across AA’ produces current I at BB’. Now if the positions of the source and responses are interchanged, by connecting the voltage source across BB9, the resultant current I will be at terminals AA’. According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

Example Verify the reciprocity theorem for the network shown in Fig. 16

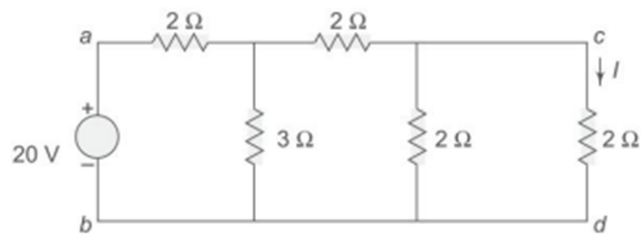


Figure 16

Solution Total resistance in the circuit = 2 + [3 || (2 + 2 || 2)] = 3.5 Ω

The current drawn by the circuit (See Fig. 17 (a))

$$I_T = \frac{20}{3.5} = 5.71\ \Omega$$

The current in the 2 V branch cd is I = 1.43 A.

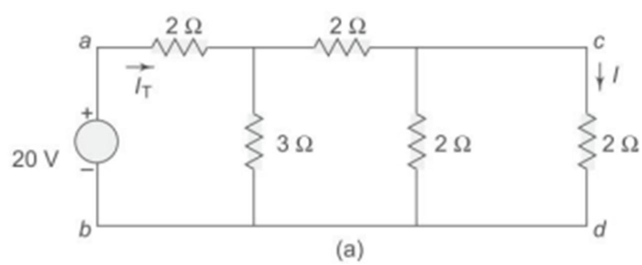


Figure 17 (a)

Applying the reciprocity theorem, by interchanging the source and response, we get Fig. 17 (b).

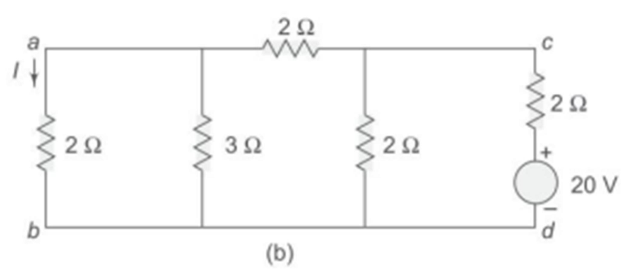


Figure 17 (b)

Total resistance in the circuit = 3.23 V.
Total current drawn by the circuit= = 20/3.23 = 6.19 A
The current in the branch ab is I = 1.43 A
If we compare the results in both cases, the ratio of input to response is the same, i.e. (20/1.43) = 13.99

2.5 MAXIMUM POWER TRANSFER THEOREM

Many circuits basically consist of sources, supplying voltage, current, or power to the load; for example, a radio speaker system, or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from the source to the load. In the simple resistive circuit shown in Fig. 18, Rs is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to the load is maximum

It is a fact that more voltage is delivered to the load when the load resistance is high as compared to the resistance of the source. On the other hand, maximum current is transferred to the load when the load resistance is small compared to the source resistance.

For many applications, an important consideration is the maximum power transfer to the load; for example, maximum power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum power transfer theorem states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance. In Fig. 18, assume that the load resistance is variable.

Current in the circuit is I = VS /(RS + RL)

Power delivered to the load RL is $P = I^2R_L = V^2$ S R_L / (RS + RL)^2

To determine the value of RL for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to RL, i.e. when dP/dR_L equals zero.

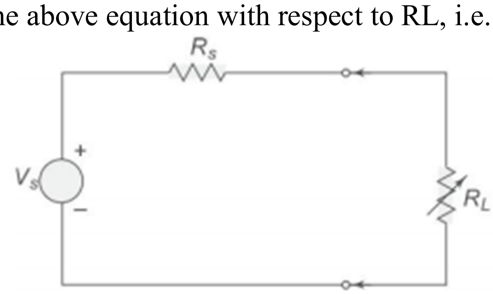


Figure 18

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2}{(R_S+R_L)^2} R_L\right]$$

$$= \frac{V_S^2\left\{(R_S+R_L)^2-(2R_L)(R_S+R_L)\right\}}{(R_S+R_L)^4}$$

$$\therefore \quad (R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$$

$$R^2_S + R^2_L + 2R_S R_L - 2R^2_L - 2R_S R_L = 0$$

$$\therefore \quad R_S = R_L$$

So, maximum power will be transferred to the load when load resistance is equal to the source resistance

Example In the circuit shown in Fig. 19, determine the value of load resistance when the load resistance draws maximum power. Also find the value of the maximum power

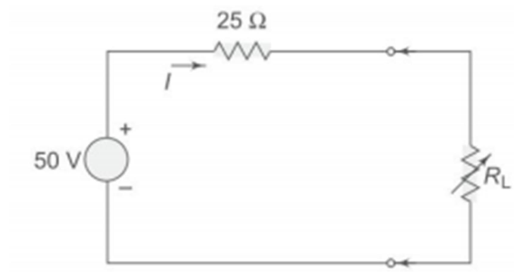
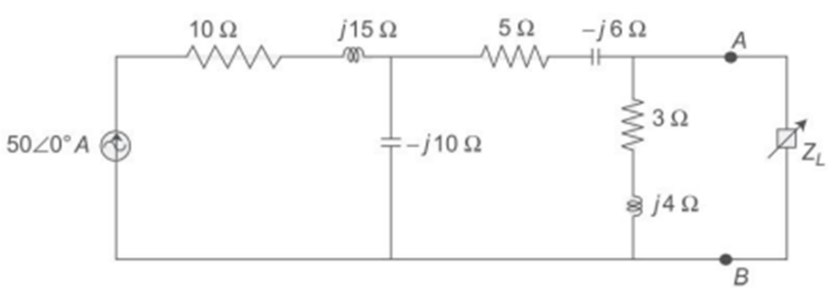


Figure 19

Solution In Fig. 19, the source delivers the maximum power when load resistance is equal to the source resistance.
RL = 25 V
The current I = 50/(25 + RL) = 50/50 = 1 A
The maximum power delivered to the load P = I^2R_L= 1 x 25= 25 W

Example 2: Determine the maximum power delivered to the load in the circuit shown in Fig below



Solution The circuit is replaced by Thèvenin's equivalent circuit in series with ZL as shown in Fig. below

where $V_{AB} = I_{(3+j4)\Omega} \times (3 + j4)$ volts

$$I_{(3+j4)\Omega} = \frac{50\angle 0^\circ \times (-j10)}{5 - j6 + 3 + j4 - j10} = 34.67\angle -33.7^\circ \text{ A}$$

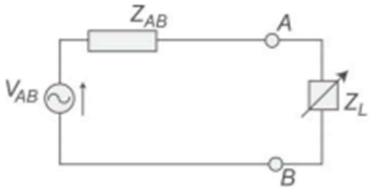
Voltage across AB is $V_{AB} = 34.67\angle{-33.7^\circ} \times 5\angle{53.13^\circ}$

$$= 173.35\angle{19.43^\circ}\ \text{V}$$

Impedance across terminals AB is

$$Z_{AB} = \{[(10 + j15) \parallel (-j10)] + (5 - j6)\} \parallel (3 + j4$$

$$Z_{AB} = 5.27\angle{41.16^\circ} = 4 + j3.5$$



To get the maximum power delivered to the load impedance, the load impedance must be equal to complex conjugate of source impedance. Therefore, the total impedance in the circuit shown in Fig. 7.44 is 8 Ω. The current in the circuit is

$$I_2 = \frac{V_{AB}}{8} = \frac{173.35}{8} = 21.66\ \text{A}$$

The maximum power transferred to the load is

$$P = I_L^2 R_L = (21.66)^2 \times 4 = 1874.9\ \text{watts}$$

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UNIT –III

Electrical Circuits and Network Analysis – SEEA1304

NETWORK TOPOLOGY

1. Basic definitions:

Network Topology:

- Is another method of solving electric circuits
- Is generalized approach

Network:

A combination of two or more network elements is called a network.

Topology:

Topology is a branch of geometry which is concerned with the properties of a geometrical figure, which are not changed when the figure is physically distorted, provided that, no parts of the figure are cut open or joined together.

The geometrical properties of a network are independent of the types of elements and their values.

Every element of the network is represented by a line segment with dots at the ends irrespective of its nature and value.

Circuit:

If the network has at least one closed path it is a circuit.

Note that every circuit is a network but every network is not a circuit.

Branch:

Representation of each element (component) of a electric network by a line segment is a branch.

Node:

A point at which two or more elements are joined is a node. End points of the branches are called nodes.

Graph:

It is collection of branches and nodes in which each branch connects two nodes.

Graph of a Network:

The diagram that gives network geometry and uses lines with dots at the ends to represent network element is usually called a graph of a given network. For example,

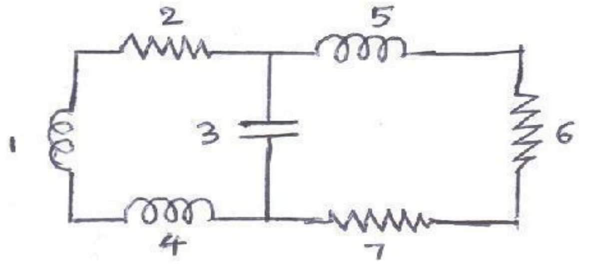


Fig.3.1 Network

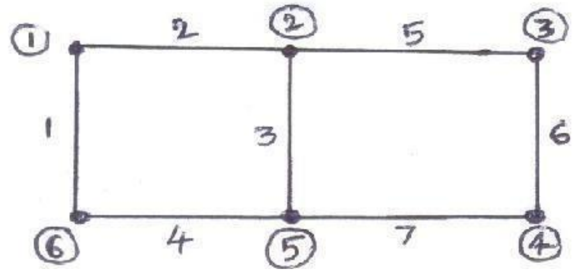


Fig.3.2 Graph

SUB GRAPH

A sub-graph is a subset of branches and nodes of a graph for example branches 1, 2, 3 & 4 forms a sub-graph. The sub-graph may be connected or unconnected. The sub- graph of graph shown in figure 2 is shown in figure 3.

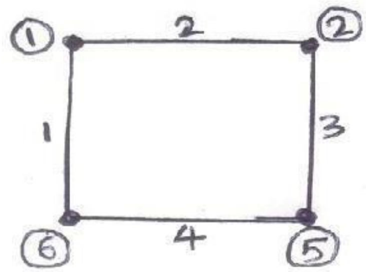


Fig.3.3 Sub-graph

Connected Graph:

If there exists at least one path from each node to every other node, then graph is said to be connected. Example,

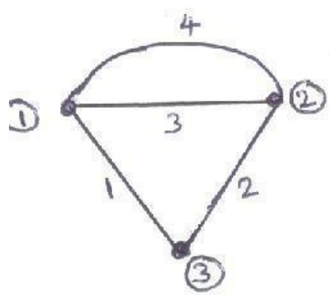


Fig.3.4 Connected Graph

Un-connected Graph:

If there exists no path from each node to every other node, the graph is said to be un-connected graph. For example, the network containing a transformer (inductively coupled parts) its graph could be un-connected.

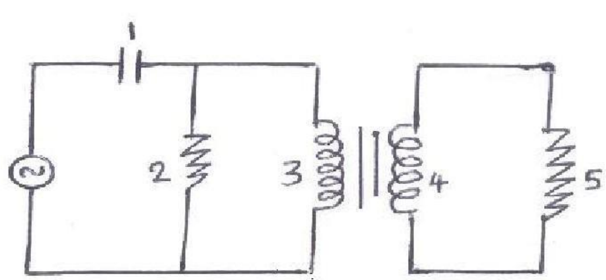


Fig.3.5

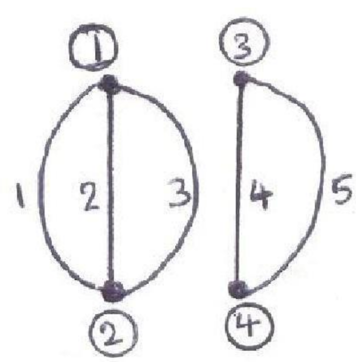


Fig.3.6 Un-connected Graph

A sequence of branches going from one node to other is called path. The node once considered should not be again considered the same node.

Loop (Closed Path):

Loop may be defined as a connected sub-graph of a graph, which has exactly two branches of the sub-graph connected to each of its node.

For example, the
branches1, 2 & 3 in figure 7 constitute a loop.

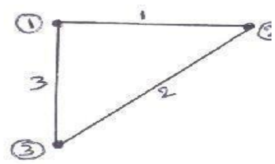


Fig 3.7 Graph

Planar and Non-planar Graphs:

A planar graph is one where the branches do not cross each other while drawn on a plain sheet of paper. If they cross, they are non-planar.

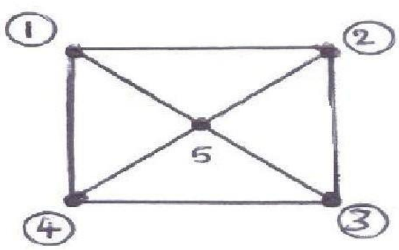


Fig.3.8 Planar Graph

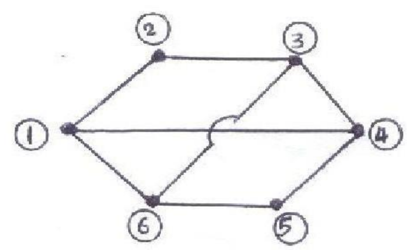


Fig.3.9 Non-planar graph

Oriented Graph:

The graph whose branches carry an orientation is called an oriented graph

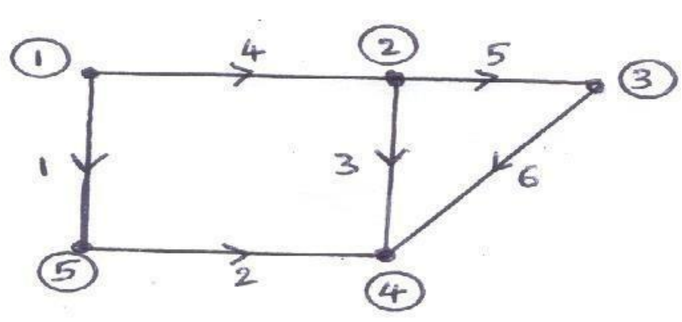


Fig.3.10 Oriented Graph

The current and voltage references for a given branches are selected with a +ve sign at tail side and –ve sign at head

Tree:

Tree of a connected graph is defined as any set of branches, which together Connect all the nodes of the graph without forming any loops. The branches of a tree are called **Twigs.**

Co-tree:

Remaining branches of a graph, which are not in the tree, form a co-tree. The branches of a co-tree are called **links** or **chords.**

The tree and co-tree for a given oriented graph shown in figure5.11 is shown in Figure 5.12 and figure5.13.

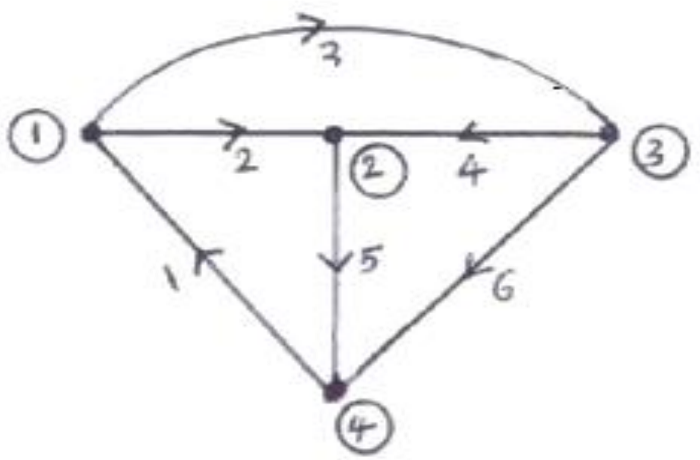


Fig. 3.11 Oriented Graph

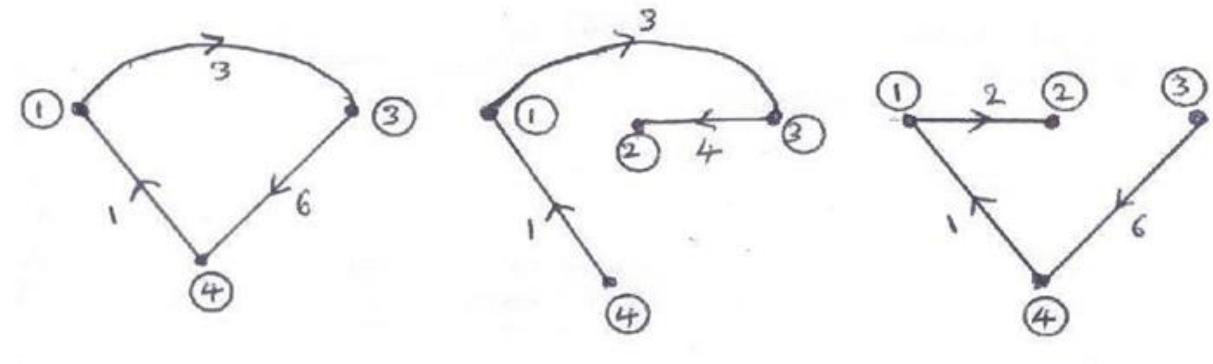


Fig. 3.12 Trees

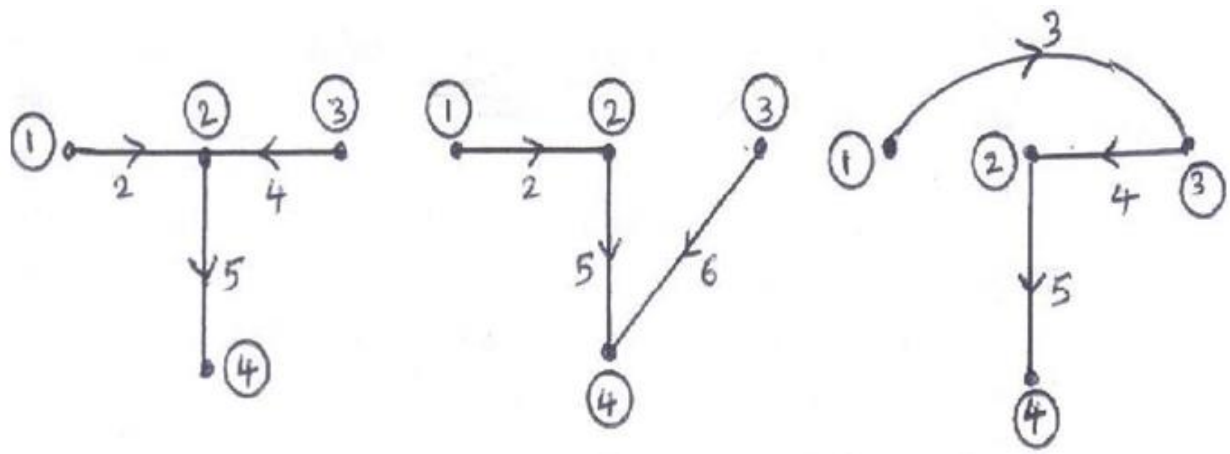


Fig.3.13 Co-trees

Tree	Twigs	Links (Chords)
1	2, 4 & 5	1, 3 & 6
2	3, 4 & 5	1, 2 & 6
3	2, 5 & 6	1, 3 & 4

Properties of Tree:

i) It contains all the nodes of the graph.

ii) It contains (n_t-1) branches. Where 'n_t' is total number of nodes in the given graph.

iii) There are no closed paths.

Total number of tree branches, n = (n_t-1)

Where n_t = Total number of nodes Total number of links, l = (b-n)

Where b = Total number of branches in the graph.

Degree of Node:

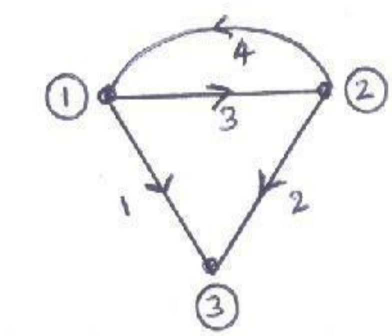
The number of branches attached to the node is degree of node.

II. Complete Incidence Matrix (A_a):

Incidence matrix gives us the information about the branches, which are joined to the nodes and the orientation of the branch, which may be towards a node or away from it.

Nodes of the graph form the rows and branches form the columns. If the branch is not connected to node, corresponding element in the matrix is given the value **'0'**. If a branch is joined, it has two possible orientations. If the orientation is away from the node, the corresponding matrix element is written as **'+1'**. If it is towards the node, the corresponding matrix element is written as **'-1'**.

Example: 1) Obtain complete incidence matrix for the graph shown



Solution: Aa =

Nodes	Branches			
	1	2	3	4
1	1	0	1	-1
2	0	1	-1	1
3	-1	-1	0	0

$$A_a = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

Properties of Incidence Matrix:

i) Each column has only two non-zero elements and all other elements are zero.

ii) If all the rows of **'A_a'** are added, the sum will be a row whose elements equal zero.

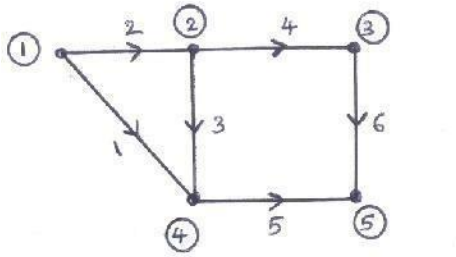
If the graph has **'b'** branches and **'n_t'** nodes, the complete incidence matrix is of the order (n_t x b).

III. Reduced Incidence Matrix (A):

When one row is eliminated from the complete incidence matrix, the remaining matrix is called **reduced incidence matrix**

If the graph has **'b'** branches and **'n_t'** nodes, the reduced incidence matrix is of the order (n_t-1) x b.

Example: 2) write the complete and reduced incidence matrix for the given graph shown



Solution:

Aa =

Nodes	Branches					
	1	2	3	4	5	6
1	1	1	0	0	0	0
2	0	-1	1	1	0	0
3	0	0	0	-1	0	1
4	-1	0	-1	0	1	0
5	0	0	0	0	-1	-1

Complete Incidence Matrix, $A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

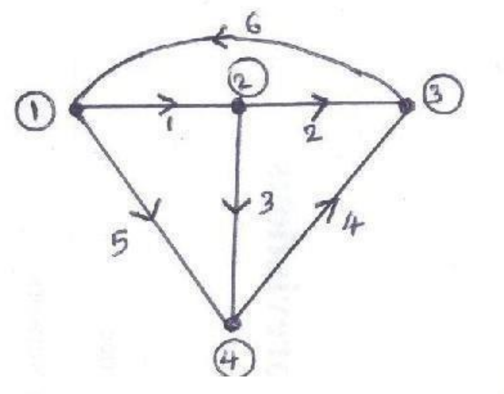
Reduced Incidence Matrix, $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$

Example: 3) Draw the oriented graph of incidence matrix shown below

$$\mathbf{A_a} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Solution:

Total number of nodes = n_t = 4 Total number of branches = b = 6



Oriented Graph

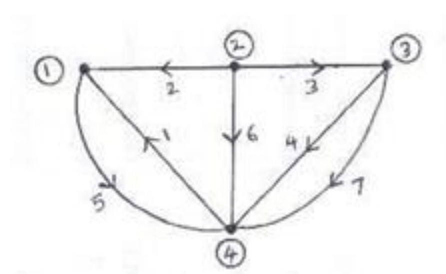
Example: 4) Draw the oriented graph of incidence matrix shown below

$$\mathbf{A} = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$\mathbf{A_a} = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix}$$

Total number of nodes = n_t = 4
Total number of branches = b = 7



Oriented Graph

TIESET

A tie-set is a set of branches contained in a loop such that each loop contains one link or chord and remainder are tree branches.

Or

The set of branches forming the closed loop in which link or loop current circulate is called a Tie-set. The tie-set consists of only one link and remaining are Twigs.

- The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link. This loop is also called f-loop or a tie set.

- The orientation of the cut-set is same as orientation of link.

TIE-SET SCHEDULE

For a given network tree, a systematic way of indicating the links through the use of a schedule is called tie-set schedule

To write the tie-set for network graph,

(i) Consider an oriented network graph

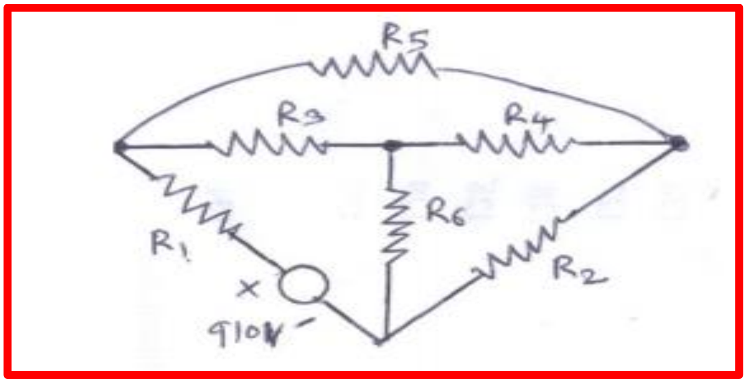
(ii) Write any one possible tree of the network graph

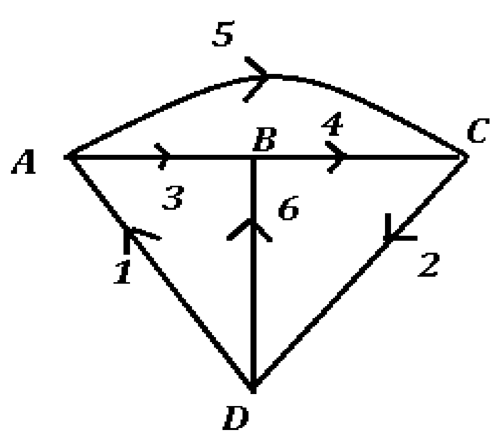
(iii)Connect a link to the tree branches to form a loop. In the same way form all Fundamental loops.

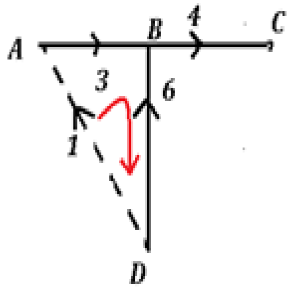
(iv)The loop current direction is same as that of the link.

(v) Form the Matrix the rows denotes the loop and columns denotes the branches

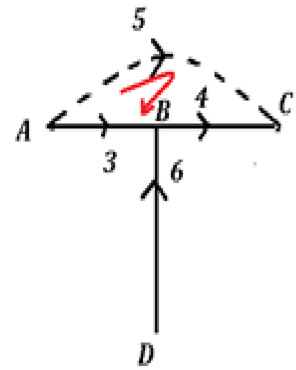
Problem 1:For the Given Network, Write a tie-set Schedule.



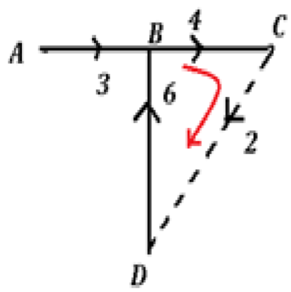




Loop	1	2	3	4	5	6
ABD	1	0	1	0	0	-1



Loop	1	2	3	4	5	6
ABC	0	0	-1	-1	1	0

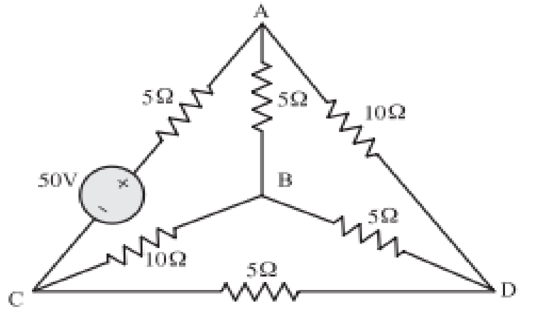


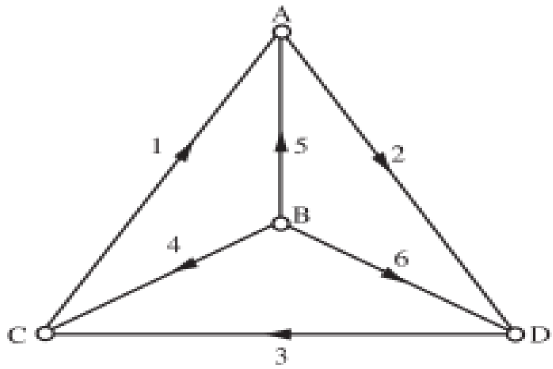
Loop	1	2	3	4	5	6
BCD	0	1	0	1	0	1

LOOPS	1	2	3	4	5	6
ABD	1	0	1	0	0	-1
ABC	0	0	-1	-1	1	0
BCD	0	1	0	1	0	1

$$M = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Problem 2: For the Given Network, Write a tie-set Schedule.

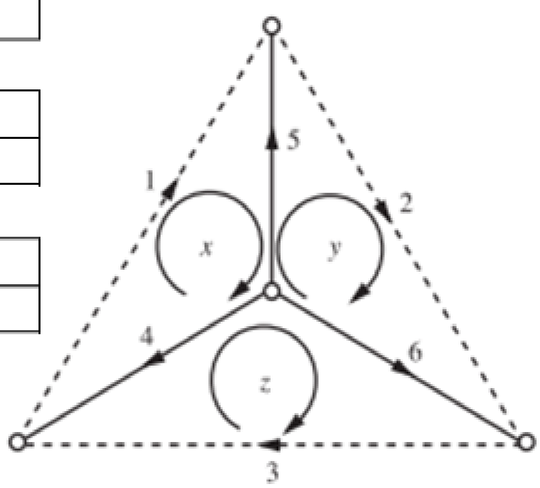




Loop	1	2	3	4	5	6
x	1	0	0	1	-1	0

Loop	1	2	3	4	5	6
y	0	1	0	0	1	-1

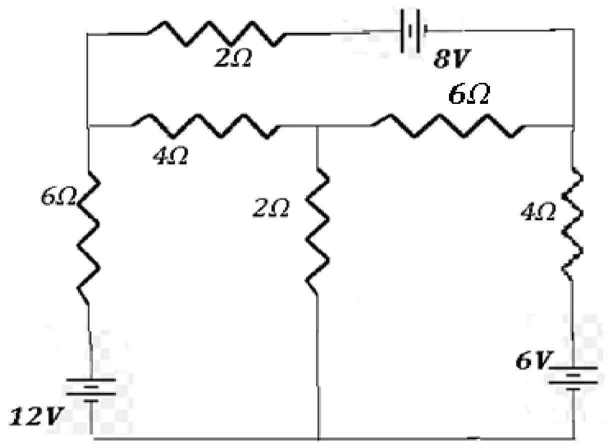
Loop	1	2	3	4	5	6
z	0	0	1	-1	0	1

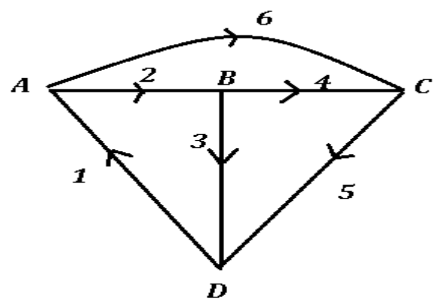


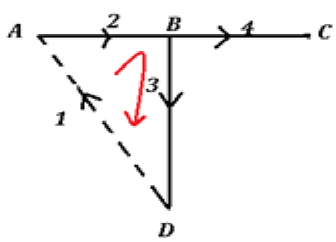
LOOPS	1	2	3	4	5	6
x	1	0	0	1	-1	0
y	0	1	0	0	1	-1
z	0	0	1	-1	0	1

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

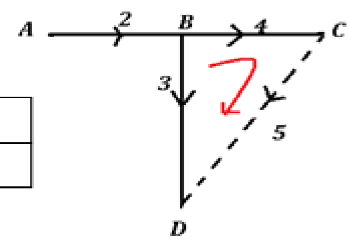
Problem 3: Find the Tieset Matrix and the Branch Voltages.



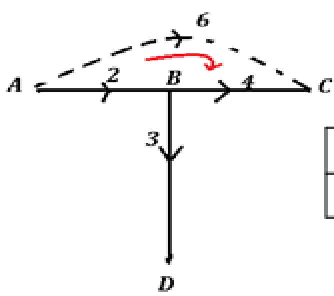




Loop	1	2	3	4	5	6
ABD	1	1	1	0	0	0



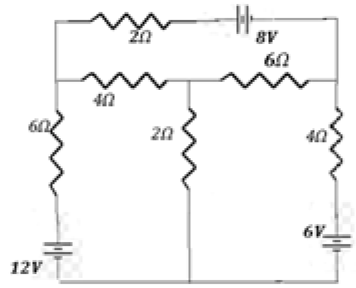
Loop	1	2	3	4	5	6
BCD	0	0	-1	1	1	0



Loop	1	2	3	4	5	6
AC	0	-1	0	-1	0	1

LOOPS	1	2	3	4	5	6
ABD	1	1	1	0	0	0
BCD	0	0	-1	1	1	0
AC	0	-1	0	-1	0	1

$$M = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$



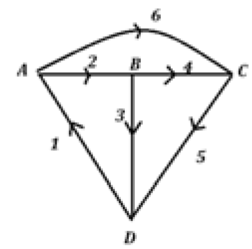
Rowwise

$$\begin{aligned} e_1 + e_2 + e_3 &= 0 \\ -e_3 + e_4 + e_5 &= 0 \\ -e_2 - e_4 + e_6 &= 0 \end{aligned} \quad \text{(I)}$$

Columnwise

$$\begin{aligned} i_1 &= I_1 \\ i_2 &= I_1 - I_3 \\ i_3 &= I_1 - I_2 \\ i_4 &= I_2 - I_3 \\ i_5 &= I_2 \\ i_6 &= I_3 \end{aligned} \quad \text{(II)}$$

$$\begin{aligned} e_1 + 12 &= 6i_1 \\ e_2 &= 4i_2 \\ e_3 &= 2i_3 \\ e_4 &= 6i_4 \\ e_5 - 6 &= 4i_5 \\ e_6 - 8 &= 2i_6 \end{aligned} \quad \text{(III)}$$



Comparing (II) and (III),

$$\begin{aligned} e_1 + 12 &= 6I_1 \\ e_2 &= 4(I_1 - I_3) \\ e_3 &= 2(I_1 - I_2) \\ e_4 &= 6(I_2 - I_3) \\ e_5 - 6 &= 4I_2 \\ e_6 - 8 &= 2I_3 \end{aligned} \quad \text{(IV)}$$

Substituting (IV) in (I),

$$\begin{aligned} 6I_1 - 12 + 4I_1 - 4I_3 + 2I_1 - 2I_2 &= 0 \\ \mathbf{12I_1 - 2I_2 - 4I_3} &\mathbf{= 12} \\ -2I_1 + 2I_2 + 6I_2 - 6I_3 + 6 + 4I_2 &= 0 \\ \mathbf{-2I_1 + 12I_2 - 6I_3} &\mathbf{= -6} \\ -4I_1 + 4I_3 - 6I_2 + 6I_3 + 8 + 2I_3 &= 0 \\ \mathbf{-4I_1 - 6I_2 + 12I_3} &\mathbf{= -8} \end{aligned}$$

(V)

$$\begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{vmatrix} = 960$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -4 \\ -6 & 12 & -6 \\ -8 & -6 & 12 \end{vmatrix} = 528$$

$$\Delta_2 = \begin{vmatrix} 12 & 12 & -4 \\ -2 & -6 & -6 \\ -4 & -8 & 12 \end{vmatrix} = -832$$

$$\Delta_3 = \begin{vmatrix} 12 & -2 & 12 \\ -2 & 12 & -6 \\ -4 & -6 & -8 \end{vmatrix} = -880$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{528}{960} = 0.55\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-832}{960} = -0.866\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-880}{960} = -0.916\text{A}$$

(IV)

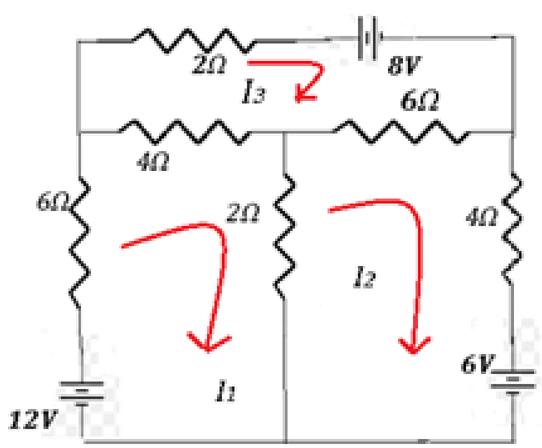
$$e_1 + 12 = 6I_1$$
$$e_2 = 4(I_1 - I_3)$$
$$e_3 = 2(I_1 - I_2$$
$$e_4 = 6(I_2 - I_3)$$
$$e_5 - 6 = 4I_2$$
$$e_6 - 8 = 2I_3$$

$$e_1 + e_2 + e_3 = 0$$
$$-e_3 + e_4 + e_5 = 0$$
$$-e_2 - e_4 + e_6 = 0$$

(I)

$$6I_1 - 12 + 4I_1 - 4I_3 + 2I_1 - 2I_2 = 0$$
$$12I_1 - 2I_2 - 4I_3 = 12$$
$$-2I_1 + 2I_2 + 6I_2 - 6I_3 + 6 + 4I_2 = 0$$
$$-2I_1 + 12I_2 - 6I_3 = -6$$
$$-4I_1 + 4I_3 - 6I_2 + 6I_3 + 8 + 2I_3 = 0$$
$$-4I_1 - 6I_2 + 12I_3 = -8$$

Verification by Mesh Analysis



$$\begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} = 960$$

$$\Delta_1 = \begin{bmatrix} 12 & -2 & -4 \\ -6 & 12 & -6 \\ -8 & -6 & 12 \end{bmatrix} = 528$$

$$\Delta_2 = \begin{bmatrix} 12 & 12 & -4 \\ -2 & -6 & -6 \\ -4 & -8 & 12 \end{bmatrix} = -832$$

$$\Delta_3 = \begin{bmatrix} 12 & -2 & 12 \\ -2 & 12 & -6 \\ -4 & -6 & -8 \end{bmatrix} = -880$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{528}{960} = 0.55\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-832}{960} = -0.866\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-880}{960} = -0.916\text{A}$$

CUTSET

The cut set is a minimal set of branches of the graph, removal of which cuts the graph into two parts. It separates the nodes of the graph into two groups.

- The cut-set consists of only one tree branch and remainders are links.
- Each branch of the cut-set has one of its terminal incident at a node in one group and its other end at a node in the other group and its other end at a node in the other group.
- The orientation of the cut-set is same as orientation of tree branch.

CUT-SET SCHEDULE

For a given network tree, a systematic way of indicating the tree branch voltage through use of a schedule called cut-set schedule

To write the cut-set schedule for network graph,

(i) Consider an oriented network graph

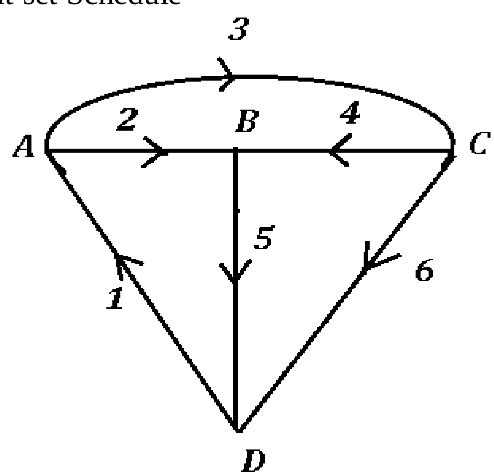
(ii) Write any one possible tree of the network graph

(iii)Assume tree branch voltages as (e1, e2…en) independent variables.

(iv)Assume the independent voltage variable is same direction as that of a tree branch voltage

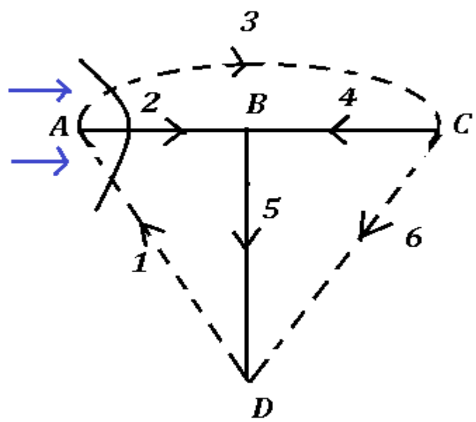
(v) Mark the cut-sets (recognize) in the network graph.

PROBLEM 5: Determine the Cut set Schedule



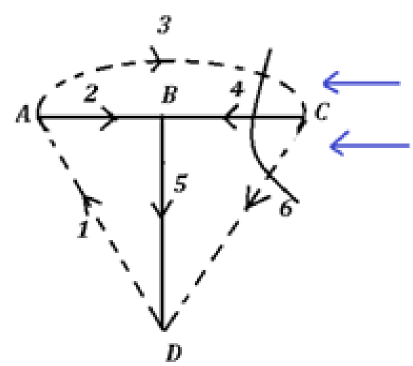
Number of Nodes= n =4

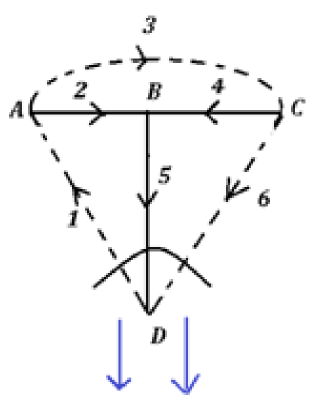
Number of Cutsets = n-1= 3



BRANCHES	1	2	3	4	5	6
NODE A	-1	1	1	0	0	0

BRANCHES	1	2	3	4	5	6
NODE C	0	0	-1	1	0	1



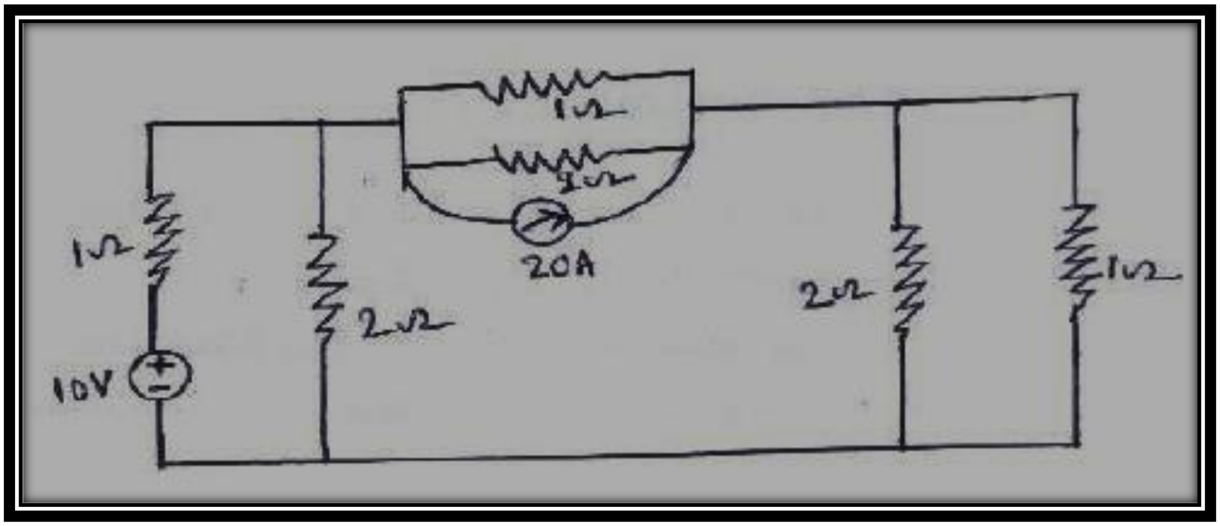


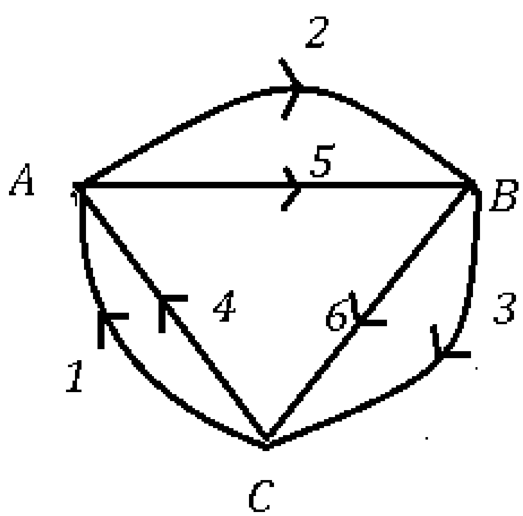
BRANCHES	1	2	3	4	5	6
NODE D	-1	0	0	0	1	1

CUTSET SCHEDULE

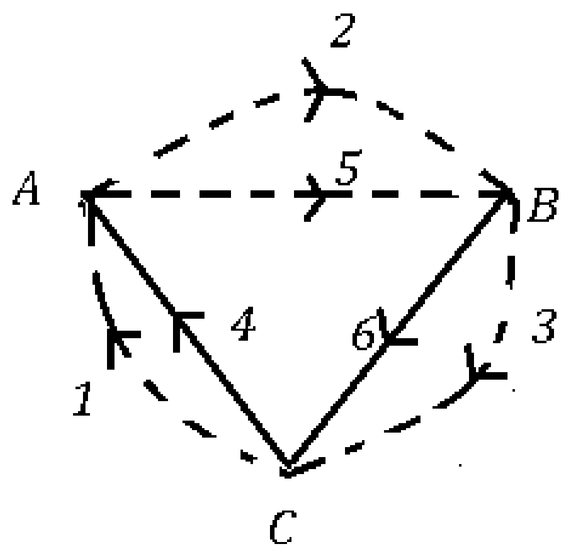
BRANCHES	1	2	3	4	5	6
NODE A	-1	1	1	0	0	0
NODE C	0	0	-1	1	0	1
NODE D	-1	0	0	0	1	1

Problem 6: Find the Cutset Matrix and the Branch Voltages

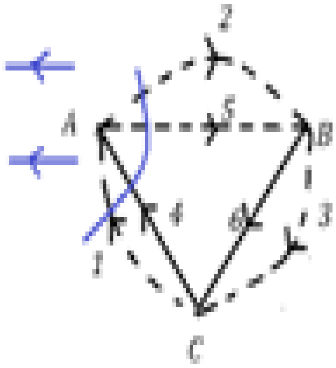




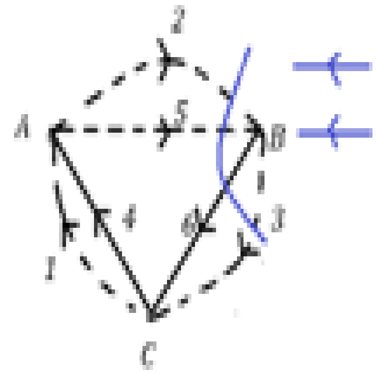
Oriented Graph



TREE & Co-TREE



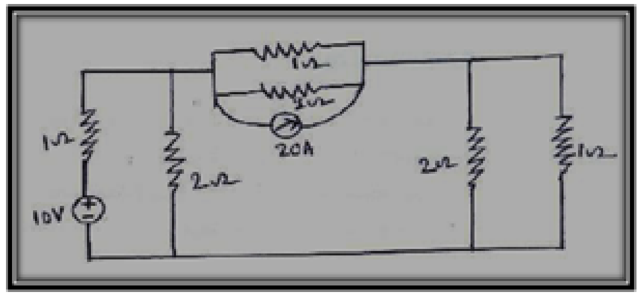
BRANCHES	1	2	3	4	5	6
NODE A	1	-1	0	1	-1	0



BRANCHES	1	2	3	4	5	6
NODE B	0	-1	1	0	-1	1

BRANCHES	1	2	3	4	5	6
NODE A	1	-1	0	1	-1	0
NODE B	0	-1	1	0	-1	1

Column----Branch Voltage(e)
Row -----Branch Current(i)



Rows of Cutset

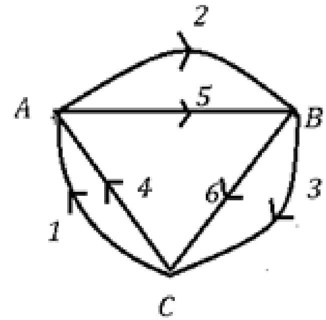
$$\begin{aligned} i_1 - i_2 + i_4 - i_5 &= 0 \\ -i_2 + i_3 - i_5 + i_6 &= 0 \end{aligned} \quad \text{(I)}$$

Columns of Cutset

$$\begin{aligned} e_1 &= E_1 \\ e_2 &= -E_1 - E_2 \\ e_3 &= E_2 \\ e_4 &= E_1 \\ e_5 &= -E_1 - E_2 \\ e_6 &= E_2 \end{aligned} \quad \text{(II)}$$

From Circuit,

$$\begin{aligned} e_1 + 10 &= i_1 \\ e_2 &= i_2 \\ e_3 &= i_3 \\ e_4 &= 2i_4 \\ e_5 + 40 &= 2i_5 \\ e_6 &= 2i_6 \end{aligned} \quad \text{(III)}$$



Substituting (II) in (III),

$$\begin{aligned} E_1 + 10 &= i_1 \\ -E_1 - E_2 &= i_2 \\ E_2 &= i_3 \\ E_1 &= 2i_4 \\ -E_1 - E_2 + 40 &= 2i_5 \\ E_2 &= 2i_6 \end{aligned} \quad \text{(IV)}$$

Substituting (IV) in (I),

$$E_1 + 10 + E_2 + E_1 + 0.5E_1 + 0.5E_1 - 20 + 0.5E_2 = 0$$
$$\mathbf{3E_1 + 1.5E_2 = 10}$$
$$2E_2 + E_1 + 0.5E_1 + 0.5E_2 + 0.5E_2 - 20 = 0$$
$$\mathbf{1.5E_1 + 3E_2 = 20}$$

(V)

$$\begin{bmatrix} 3 & 1.5 \\ 1.5 & 3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 3 & 1.5 \\ 1.5 & 3 \end{bmatrix} = 6.75$$

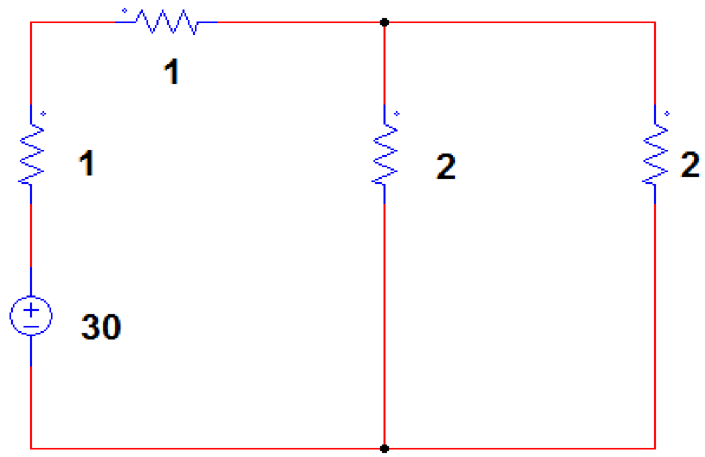
$$\Delta_1 = \begin{bmatrix} 10 & -1.5 \\ 20 & 3 \end{bmatrix} = 0$$

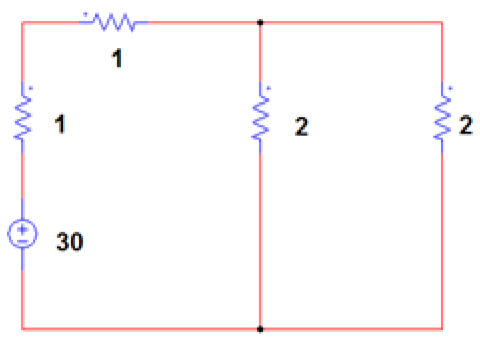
$$\Delta_2 = \begin{bmatrix} 3 & 10 \\ 1.5 & -20 \end{bmatrix} = 45$$

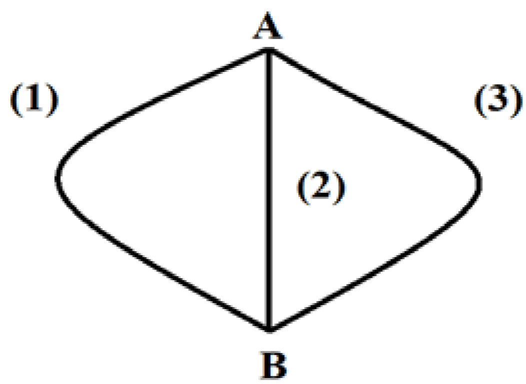
$$E_1 = \frac{\Delta_1}{\Delta} = 0$$

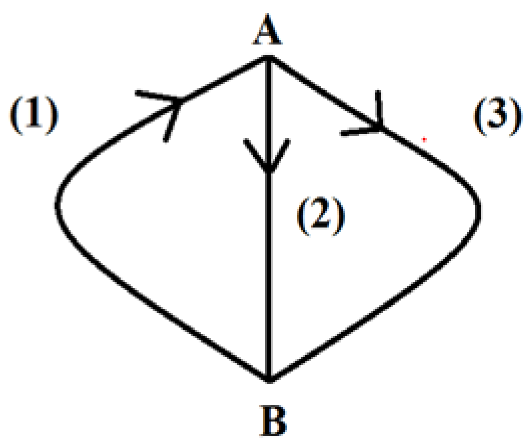
$$E_2 = \frac{\Delta_2}{\Delta} = 6.667$$

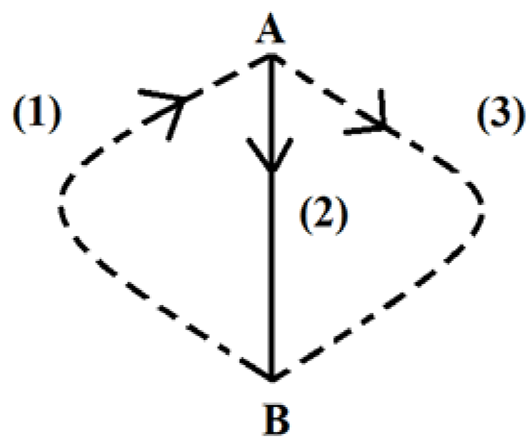
Problem 8:For the circuit shown frame the Cutset schedule and find the branch currents

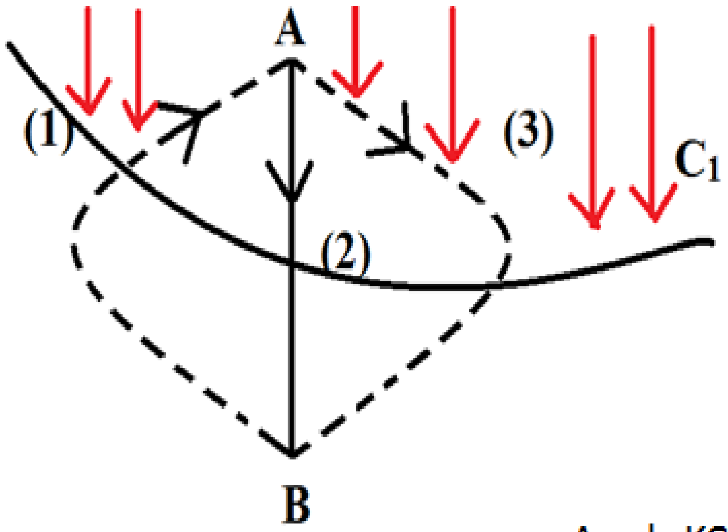












No. of cut set	Branches		
	1	2	3
C_1	-1	1	1

Apply KCL in rows

$-i_1 + i_2 + i_3 = 0$ (I)

APPLY KVL in column

Let the branch voltages be e_1, e_2, & e_3

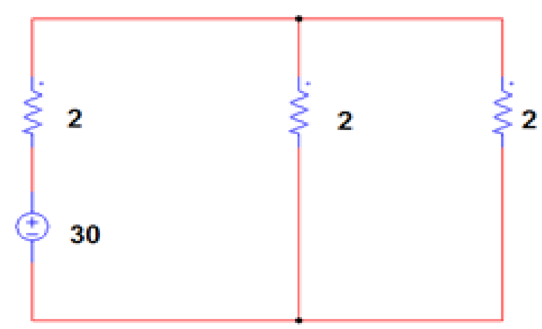
E_1 be the tree branch voltages

$$e_1 = -E_1$$
$$e_2 = E_1 \qquad \text{(II)}$$
$$e_3 = E_1$$

from the circuit

$$e_1 = -E_1$$
$$e_2 = E_1 \qquad \text{(II)}$$
$$e_3 = E_1$$

$$e_1 + 30 = 2i_1$$
$$e_2 = 2i_2 \qquad \text{(III)}$$
$$e_3 = 2i_3$$



Comparing (II) and (III)

$$-E_1 + 30 = 2i_1$$
$$E_1 = 2i_2 \qquad \text{(IV)}$$
$$E_1 = 2i_3$$

Simplifying,

$$i_1 = \frac{-E_1 + 30}{2} \qquad i_2 = \frac{E_1}{2} \qquad i_3 = \frac{E_1}{2}$$

$$-i_1 + i_2 + i_3 = 0 \qquad \text{(I)}$$

$$0.5E_1 - 15 + 0.5E_1 + 0.5E_1 = 0$$

$$-15 = -1.5E_1 \qquad E_1 = 10V$$

Substituting in Previous equation to obtain the branch current

$$i_1 = \frac{-E_1 + 30}{2} = 10A$$

$$i_2 = \frac{E_1}{2} = 5A$$

. . . .

$$i_3 = \frac{E_1}{2} = 5A$$

DUALITY AND DUAL NETWORK:

The network is said to be dual network of each other if the mesh equations of given network are the node equations of other network. The property of duality is a mutual property. If network A is dual network B, then the network B is also dual of network A.

Some of the dual pairs are given in the following table:

	Element	Dual Element
1	Resistance	Conductance
2	Capacitance	Inductance
3	Inductance	Capacitance
4	Series Branch	Parallel Branch
5	Voltage Source	Current Source
6	Current Source	Voltage Source
7	Switch Closed (at t = 0)	Switch Opened (at t = 0)
8	Charge	Flux linkage
9	Mesh	Node
10	Link	Twig

Methods of drawing the dual of a network

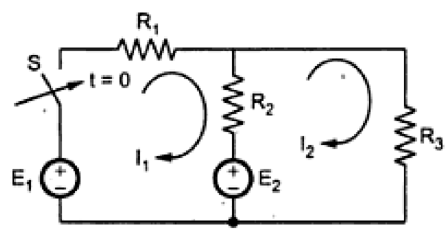
The following steps are followed to draw the dual of given electrical network:

1. A dot is placed in each independent loop of the original network. These dots placed inside the loops correspond to the independent nodes in dual network.
2. A dot is placed outside the given network. This corresponds to the reference node of the dual network.
3. All the dots are connected by dotted lines crossing all the branches. The dotted lines should cross only one branch at a time. The dual elements will form the branches connecting the corresponding nodes in the dual network.

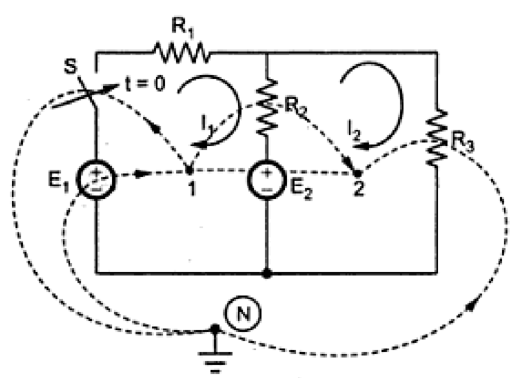
   Note A: The voltage rise in the clockwise direction corresponds to a current flowing towards the independent network.

   Note B: A clockwise current in a loop corresponds to positive polarity for the at the dual independent node.

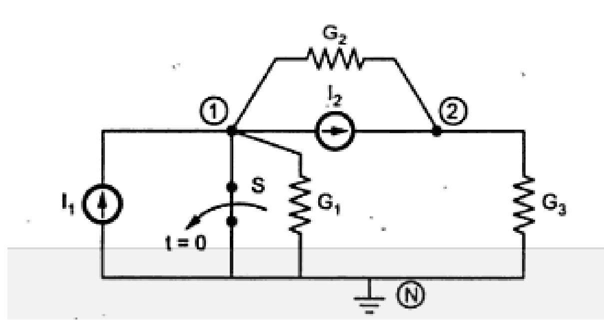
Example Draw a dual network for the given network below.



The procedure for drawing the dual network is given below:



The dual network is given below:



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Pvt. Ltd, 2015.

2. M.E.Van Valkenburg, “Network Analysis”, 4th edition, Pearson Education Pvt. Ltd, 2015.

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UNIT-IV

Electrical Circuits and Network Analysis- SEEA1304

UNIT 4

Two-Port Networks

4.1 Terminals and Ports

In a two-terminal network, the terminal voltage is related to the terminal current by the impedance $Z = V/I$. In a four-terminal network, if each terminal pair (or port) is connected separately to another circuit as in Fig. 4-1, the four variables i , i , u , and u are related by two equations called the terminal characteristics. These two equations, plus the terminal characteristics of the connected circuits, provide the necessary and sufficient number of equations to solve for the four variables.

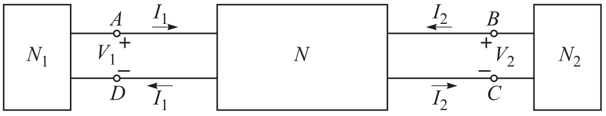

Fig. 4-1

4.2 Z-Parameters

The terminal characteristics of a two-port network, having linear elements and dependent sources, may be written in the **s**-domain as

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2 \end{aligned} \tag{1}$$

The coefficients $\mathbf{Z}_{ij}$ have the dimension of impedance and are called the **Z**-*parameters* of the network. The **Z**-parameters are also called *open-circuit impedance parameters* since they may be measured at one terminal while the other terminal is open. They are

$$\begin{aligned} \mathbf{Z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{Z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{Z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{Z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{aligned} \tag{2}$$

EXAMPLE 4.1 Find the Z-parameters of the two-port circuit in Fig. 4-2.

Apply KVL around the two loops in Fig. 4-2 with loop currents I and I to obtain

$$\begin{aligned} \mathbf{V}_1 &= 2\mathbf{I}_1 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = (2 + \mathbf{s})\mathbf{I}_1 + \mathbf{s}\mathbf{I}_2 \\ \mathbf{V}_2 &= 3\mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{s}\mathbf{I}_1 + (3 + \mathbf{s})\mathbf{I}_2 \end{aligned} \tag{3}$$

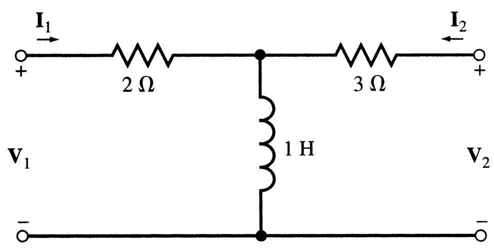

Fig. 4-2

By comparing (1) and (3), the **Z**-parameters of the circuit are found to be

$$\begin{aligned} \mathbf{Z}_{11} &= \mathbf{s} + 2 \\ \mathbf{Z}_{12} &= \mathbf{Z}_{21} = \mathbf{s} \\ \mathbf{Z}_{22} &= \mathbf{s} + 3 \end{aligned} \tag{4}$$

Note that in this example $\mathbf{Z}_{12} = \mathbf{Z}_{21}$.

Reciprocal and Nonreciprocal Networks

A two-port network is called *reciprocal* if the open-circuit transfer impedances are equal: $\mathbf{Z}_{12} = \mathbf{Z}_{21}$. Consequently, in a reciprocal two-port network with current **I** feeding one port, the open-circuit voltage measured at the other port is the same, irrespective of the ports. The voltage is equal to $\mathbf{V} = \mathbf{Z}_{12}\mathbf{I} = \mathbf{Z}_{21}\mathbf{I}$. Networks containing resistors, inductors, and capacitors are generally reciprocal. Networks that additionally have 4dependent sources are generally nonreciprocal (see Example 4.2).

EXAMPLE 4.2 The two-port circuit shown in Fig. 4-3 contains a current-dependent voltage source. Find its **Z**-parameters.

As in Example 4.1, we apply Kirchhoff's Voltage Law (KVL) around the two loops:

$$\begin{aligned} \mathbf{V}_1 &= 2\mathbf{I}_1 - \mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = (2 + \mathbf{s})\mathbf{I}_1 + (\mathbf{s} - 1)\mathbf{I}_2 \\ \mathbf{V}_2 &= 3\mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{s}\mathbf{I}_1 + (3 + \mathbf{s})\mathbf{I}_2 \end{aligned}$$

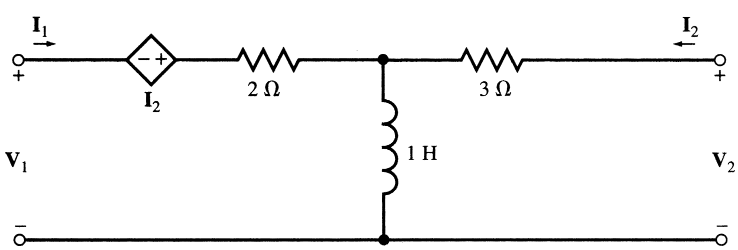

Fig. 4-3

The **Z**-parameters are

$$
\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{s} + 2 \\
\mathbf{Z}_{12} &= \mathbf{s} - 1 \\
\mathbf{Z}_{21} &= \mathbf{s} \\
\mathbf{Z}_{22} &= \mathbf{s} + 3
\end{aligned} \tag{5}
$$

With the dependent source in the circuit, $\mathbf{Z}_{12} \neq \mathbf{Z}_{21}$ and so the two-port circuit is nonreciprocal.

4.3 T-Equivalent of Reciprocal Networks

4A reciprocal network may be modeled by its T-equivalent as shown in the circuit of Fig. 4-4. Z , Z , and Z are obtained from the **Z**-parameters as follows.

$$
\begin{aligned}
\mathbf{Z}_a &= \mathbf{Z}_{11} - \mathbf{Z}_{12} \\
\mathbf{Z}_b &= \mathbf{Z}_{22} - \mathbf{Z}_{21} \\
\mathbf{Z}_c &= \mathbf{Z}_{12} = \mathbf{Z}_{21}
\end{aligned} \tag{6}
$$

The T-equivalent network is not necessarily realizable.

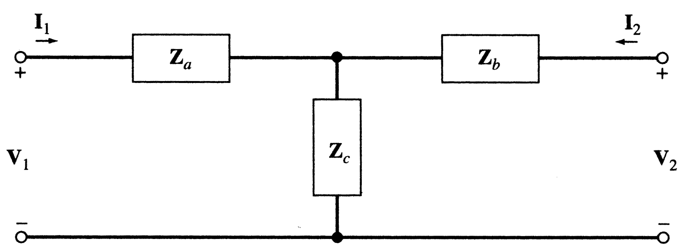

Fig. 4-4

EXAMPLE 4.3 Find the Z-parameters of Fig. 4-4.

Again we apply KVL to obtain

$$
\begin{aligned}
\mathbf{V}_1 &= \mathbf{Z}_a\mathbf{I}_1 + \mathbf{Z}_c(\mathbf{I}_1 + \mathbf{I}_2) = (\mathbf{Z}_a + \mathbf{Z}_c)\mathbf{I}_1 + \mathbf{Z}_c\mathbf{I}_2 \\
\mathbf{V}_2 &= \mathbf{Z}_b\mathbf{I}_2 + \mathbf{Z}_c(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{Z}_c\mathbf{I}_1 + (\mathbf{Z}_b + \mathbf{Z}_c)\mathbf{I}_2
\end{aligned} \tag{7}
$$

By comparing (1) and (7) the **Z**-parameters are found to be

$$
\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{Z}_a + \mathbf{Z}_c \\
\mathbf{Z}_{12} &= \mathbf{Z}_{21} = \mathbf{Z}_c \\
\mathbf{Z}_{22} &= \mathbf{Z}_b + \mathbf{Z}_c
\end{aligned} \tag{8}
$$

4.4 Y-Parameters

The terminal characteristics may also be written as in (9), where $\mathbf{I}_1$ and $\mathbf{I}_2$ are expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$.

$$
\begin{aligned}
\mathbf{I}_1 &= \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2 \\
\mathbf{I}_2 &= \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2
\end{aligned} \tag{9}
$$

The coefficients $\mathbf{Y}_{ij}$ have the dimension of admittance and are called the **Y**-*parameters* or *short-circuit admittance parameters* because they may be measured at one port while the other port is short-circuited. The **Y**-parameters are

$$
\begin{aligned}
\mathbf{Y}_{11} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \\
\mathbf{Y}_{12} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0} \\
\mathbf{Y}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0} \\
\mathbf{Y}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}
\end{aligned}
\tag{10}
$$

EXAMPLE 4.4 Find the Y-parameters of the circuit in Fig. 4-5.

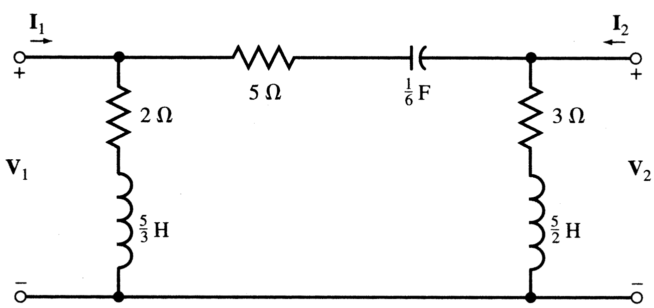

Fig. 4-5

We apply Kirchhoff's Current Law (KCL) to the input and output nodes (for convenience, we designate the admittances of the three branches of the circuit by Y , Y , and Y as shown in Fig. 4-6). Thus,

$$
\begin{aligned}
\mathbf{Y}_a &= \frac{1}{2 + 5\mathbf{s}/3} = \frac{3}{5\mathbf{s} + 6} \\
\mathbf{Y}_b &= \frac{1}{3 + 5\mathbf{s}/2} = \frac{2}{5\mathbf{s} + 6} \\
\mathbf{Y}_c &= \frac{1}{5 + 6/\mathbf{s}} = \frac{\mathbf{s}}{5\mathbf{s} + 6}
\end{aligned}
\tag{11}
$$

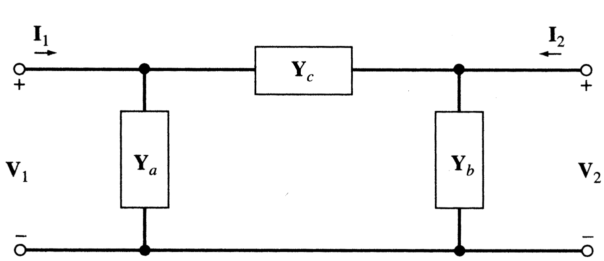

Fig. 4-6

The node equations are

$$
\begin{aligned}
\mathbf{I}_1 &= \mathbf{V}_1\mathbf{Y}_a + (\mathbf{V}_1 - \mathbf{V}_2)\,\mathbf{Y}_c = (\mathbf{Y}_a + \mathbf{Y}_c)\mathbf{V}_1 - \mathbf{Y}_c\mathbf{V}_2 \\
\mathbf{I}_2 &= \mathbf{V}_2\mathbf{Y}_b + (\mathbf{V}_2 - \mathbf{V}_1)\,\mathbf{Y}_c = -\mathbf{Y}_c\mathbf{V}_1 + (\mathbf{Y}_b + \mathbf{Y}_c)\mathbf{V}_2
\end{aligned}
\tag{12}
$$

By comparing (9) with (12), we get

$$\mathbf{Y}_{11} = \mathbf{Y}_a + \mathbf{Y}_c$$
$$\mathbf{Y}_{12} = \mathbf{Y}_{21} = -\mathbf{Y}_c \tag{13}$$
$$\mathbf{Y}_{22} = \mathbf{Y}_b + \mathbf{Y}_c$$

Substituting $\mathbf{Y}_a$, $\mathbf{Y}_b$, and $\mathbf{Y}_c$ from (11) into (13), we find

$$\mathbf{Y}_{11} = \frac{\mathbf{s}+3}{5\mathbf{s}+6}$$
$$\mathbf{Y}_{12} = \mathbf{Y}_{21} = \frac{-\mathbf{s}}{5\mathbf{s}+6} \tag{14}$$
$$\mathbf{Y}_{22} = \frac{\mathbf{s}+2}{5\mathbf{s}+6}$$

Since $\mathbf{Y}_{12} = \mathbf{Y}_{21}$, the two-port circuit is reciprocal.

4.5 Pi-Equivalent of Reciprocal Networks

A reciprocal network may be modeled by its Pi-equivalent as shown in Fig. 4-6. The three elements of the Pi-equivalent network can be found by reverse solution. We first find the Y-parameters of Fig. 4-6. From (10) we have

$$\begin{aligned} \mathbf{Y}_{11} &= \mathbf{Y}_a + \mathbf{Y}_c && \text{[Fig. 4-7-(a)]} \\ \mathbf{Y}_{12} &= -\mathbf{Y}_c && \text{[Fig. 4- 7-(b)]} \\ \mathbf{Y}_{21} &= -\mathbf{Y}_c && \text{[Fig. 4- 7-(a)]} \\ \mathbf{Y}_{22} &= \mathbf{Y}_b + \mathbf{Y}_c && \text{[Fig. 4-7-(b)]} \end{aligned} \tag{15}$$

from which

$$\mathbf{Y}_a = \mathbf{Y}_{11} + \mathbf{Y}_{12} \qquad \mathbf{Y}_b = \mathbf{Y}_{22} + \mathbf{Y}_{12} \qquad \mathbf{Y}_c = -\mathbf{Y}_{12} = -\mathbf{Y}_{21} \tag{16}$$

The Pi-equivalent network is not necessarily realizable.

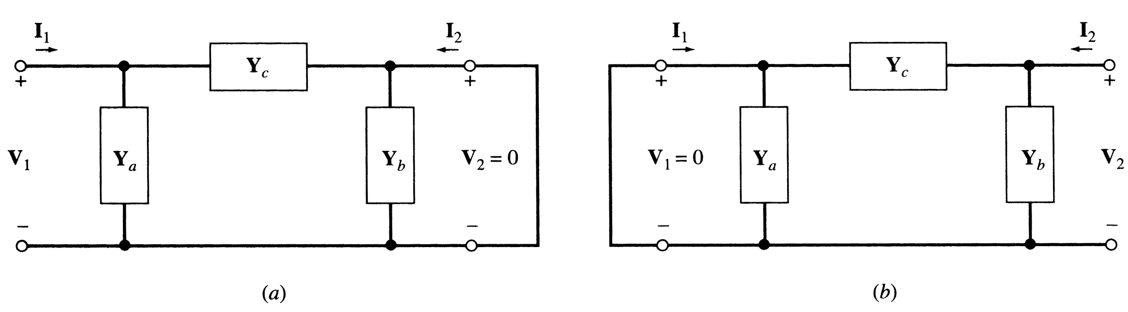

Fig. 13-7

4.6 Application of Terminal Characteristics

The four terminal variables $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in a two-port network are related by the two equations (1) or (9). By connecting the two-port circuit to the outside as shown in Fig. 4-1, two additional equations are obtained. The four equations then can determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ without any knowledge of the inside structure of the circuit.

EXAMPLE 4.5 The Z-parameters of a two-port network are given by

$$\mathbf{Z}_{11} = 2\mathbf{s} + 1/\mathbf{s} \qquad \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s} \qquad \mathbf{Z}_{22} = 2\mathbf{s} + 4$$

4The network is connected to a source and a load as shown in Fig. 4-8. Find I , I , V , and V .

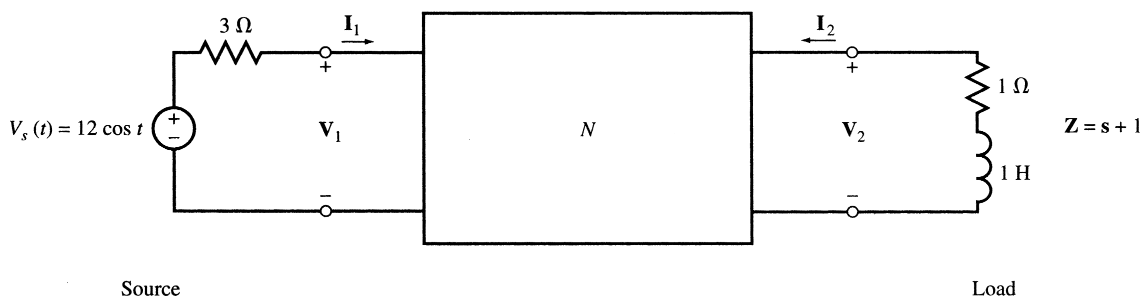

Fig. 4-8

The terminal characteristics are given by

$$\begin{aligned} \mathbf{V}_1 &= (2\mathbf{s} + 1/\mathbf{s})\mathbf{I}_1 + 2\mathbf{s}\mathbf{I}_2 \\ \mathbf{V}_2 &= 2\mathbf{s}\mathbf{I}_1 + (2\mathbf{s} + 4)\mathbf{I}_2 \end{aligned} \tag{17}$$

The phasor representation of voltage $v_s(t)$ is $\mathbf{V}_s = 12$ V with $\mathbf{s} = j$. From KVL around the input and output loops we obtain the two additional equations

$$\begin{aligned} \mathbf{V}_s &= 3\mathbf{I}_1 + \mathbf{V}_1 \\ 0 &= (1 + \mathbf{s})\mathbf{I}_2 + \mathbf{V}_2 \end{aligned} \tag{18}$$

Substituting $\mathbf{s} = j$ and $\mathbf{V}_s = 12$ in (17) and in (18) we get

$$\begin{aligned} \mathbf{V}_1 &= j\mathbf{I}_1 + 2j\mathbf{I}_2 \\ \mathbf{V}_2 &= 2j\mathbf{I}_1 + (4 + 2j)\mathbf{I}_2 \\ 12 &= 3\mathbf{I}_1 + \mathbf{V}_1 \\ 0 &= (1 + j)\mathbf{I}_2 + \mathbf{V}_2 \end{aligned}$$

from which

$$\mathbf{I}_1 = 3.29 \underline{/-10.2^\circ} \qquad \mathbf{I}_2 = 1.13 \underline{/-131.2^\circ}$$

$$\mathbf{V}_1 = 2.88 \underline{/37.5^\circ} \qquad \mathbf{V}_2 = 1.6 \underline{/93.8^\circ}$$

4.7 Conversion between Z- and Y-Parameters

The **Y**-parameters may be obtained from the **Z**-parameters by solving (1) for $\mathbf{I}_1$ and $\mathbf{I}_2$. Applying Cramer's rule to (1), we get

$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{Z}_{22}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_1 - \frac{\mathbf{Z}_{12}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_2 \\ \mathbf{I}_2 &= \frac{-\mathbf{Z}_{21}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_1 + \frac{\mathbf{Z}_{11}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_2 \end{aligned} \tag{19}$$

where $\mathbf{D_{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21}$ is the determinant of the coefficient matrix in (1). By comparing (19) with (9) we have

$$\begin{aligned}
\mathbf{Y}_{11} &= \frac{\mathbf{Z}_{22}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{12} &= \frac{-\mathbf{Z}_{12}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{21} &= \frac{-\mathbf{Z}_{21}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{22} &= \frac{\mathbf{Z}_{11}}{\mathbf{D_{ZZ}}}
\end{aligned} \tag{20}$$

Given the **Z**-parameters, for the **Y**-parameters to exist, the determinant $\mathbf{D}_{ZZ}$ must be nonzero. Conversely, given the **Y**-parameters, the **Z**-parameters are

$$\begin{aligned}
\mathbf{Z}_{11} &= \frac{\mathbf{Y}_{22}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{12} &= \frac{-\mathbf{Y}_{12}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{21} &= \frac{-\mathbf{Y}_{21}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{22} &= \frac{\mathbf{Y}_{11}}{\mathbf{D_{YY}}}
\end{aligned} \tag{21}$$

where $\mathbf{D}_{YY} = \mathbf{Y}_{11}\mathbf{Y}_{22} - \mathbf{Y}_{12}\mathbf{Y}_{21}$ is the determinant of the coefficient matrix in (9). For the **Z**-parameters of a two-port circuit to be derived from its **Y**-parameters, $\mathbf{D}_{YY}$ should be nonzero.

EXAMPLE 4.6 Referring to Example 4.4, find the Z-parameters of the circuit of Fig. 4-5 from its Y-parameters.

The **Y**-parameters of the circuit were found to be [see (14)]

$$\mathbf{Y}_{11} = \frac{\mathbf{s}+3}{5\mathbf{s}+6} \qquad \mathbf{Y}_{12} = \mathbf{Y}_{21} = \frac{-\mathbf{s}}{5\mathbf{s}+6} \qquad \mathbf{Y}_{22} = \frac{\mathbf{s}+2}{5\mathbf{s}+6}$$

Substituting into (21), where $\mathbf{D_{YY}} = 1/(5\mathbf{s}+6)$, we obtain

$$\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{s} + 2 \\
\mathbf{Z}_{12} &= \mathbf{Z}_{21} = \mathbf{s} \\
\mathbf{Z}_{22} &= \mathbf{s} + 3
\end{aligned} \tag{22}$$

The Z-parameters in (22) are identical to the Z-parameters of the circuit of Fig. 4-2. The two circuits are equivalent as far as the terminals are concerned. This was by design. Figure 4-2 is the T-equivalent of Fig. 4-5. The equivalence between Fig. 4-2 and Fig. 4-5 may be verified directly by applying (6) to the **Z**-parameters given in (22) to obtain the T-equivalent network.

4.8 h-Parameters

Some two-port circuits or electronic devices are best characterized by the following terminal equations:

$$\begin{aligned}
\mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\
\mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2
\end{aligned} \tag{23}$$

where the $\mathbf{h}_{ij}$ coefficients are called the *hybrid* or **h**-*parameters*.

EXAMPLE 4.7 Find the h-parameters of Fig. 4-9.

This is the simple model of a bipolar junction transistor in its linear region of operation. By inspection, the terminal characteristics of Fig. 4-9 are

$$\mathbf{V}_1 = 50\mathbf{I}_1 \qquad \text{and} \qquad \mathbf{I}_2 = 300\mathbf{I}_1 \tag{24}$$

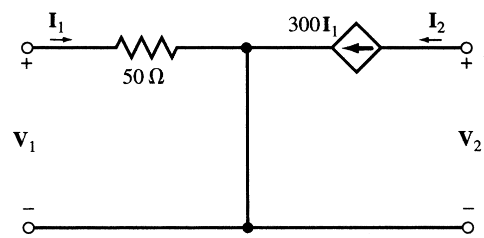

Fig. 4-9

By comparing (24) and (23) we get

$$\mathbf{h}_{11} = 50 \qquad \mathbf{h}_{12} = 0 \qquad \mathbf{h}_{21} = 300 \qquad \mathbf{h}_{22} = 0 \tag{25}$$

4.9 g-Parameters

The terminal characteristics of a two-port circuit may also be described by still another set of hybrid parameters as given in (26).

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{aligned} \tag{26}$$

where the coefficients $\mathbf{g}_{ij}$ are called *inverse hybrid* or **g**-*parameters.*

EXAMPLE 4.8 Find the g-parameters in the circuit shown in Fig. 4-10.

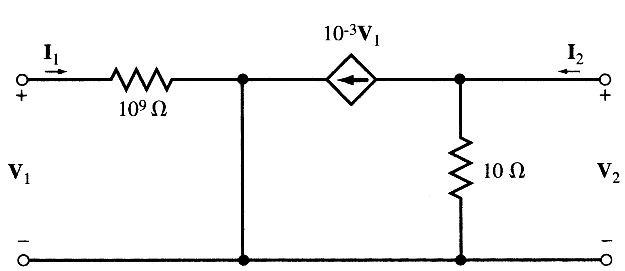

Fig. 4-10

This is the simple model of a field effect transistor in its linear region of operation. To find the **g**-parameters, we first derive the terminal equations by applying Kirchhoff's laws at the terminals:

At the input terminal: $\mathbf{V}_1 = 10^9\,\mathbf{I}_1$

At the output terminal: $\mathbf{V}_2 = 10(\mathbf{I}_2 - 10^{-3}\,\mathbf{V}_1)$

or

$$\mathbf{I}_1 = 10^{-9}\,\mathbf{V}_1 \qquad \text{and} \qquad \mathbf{V}_2 = 10\mathbf{I}_2 - 10^{-2}\,\mathbf{V}_1 \tag{27}$$

By comparing (27) and (26) we get

$$\mathbf{g}_{11} = 10^{-9} \qquad \mathbf{g}_{12} = 0 \qquad \mathbf{g}_{21} = -10^{-2} \qquad \mathbf{g}_{22} = 10 \tag{28}$$

4.10 Transmission Parameters

The transmission parameters **A**, **B**, **C**, and **D** express the required source variables $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of the existing destination variables $\mathbf{V}_2$ and $\mathbf{I}_2$. They are called **ABCD** or T-parameters and are defined by

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{aligned} \tag{29}$$

EXAMPLE 4.9 Find the T-parameters of Fig. 4-11 where Z and Z are nonzero.

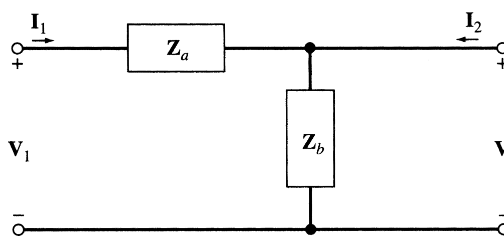

Fig. 4-11

This is the simple lumped model of an incremental segment of a transmission line. From (29) we have

$$\begin{aligned}
\mathbf{A} &= \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{\mathbf{Z}_a + \mathbf{Z}_b}{\mathbf{Z}_b} = 1 + \mathbf{Z}_a\mathbf{Y}_b \\
\mathbf{B} &= -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \mathbf{Z}_a \\
\mathbf{C} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \mathbf{Y}_b \\
\mathbf{D} &= -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = 1
\end{aligned} \tag{30}$$

4.11 Interconnecting Two-Port Networks

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connections, resulting in new two-port networks. For each configuration, a certain set of parameters may be more useful than others to describe the network.

Series Connection

Figure 4-12 shows a series connection of two two-port networks a and b with open-circuit impedance parameters $\mathbf{Z}_a$ and $\mathbf{Z}_b$, respectively. In this configuration, we use the **Z**-parameters since they are combined as a series connection of two impedances. The Z-parameters of the series connection are (see Problem 4.10):

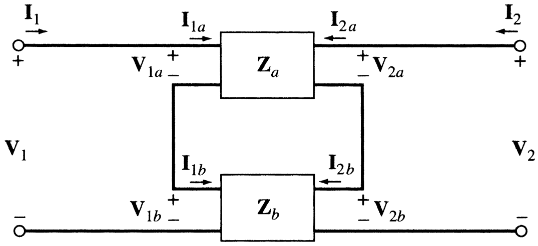

Fig. 4-12

$$
\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{Z}_{11,a} + \mathbf{Z}_{11,b} \\
\mathbf{Z}_{12} &= \mathbf{Z}_{12,a} + \mathbf{Z}_{12,b} \\
\mathbf{Z}_{21} &= \mathbf{Z}_{21,a} + \mathbf{Z}_{21,b} \\
\mathbf{Z}_{22} &= \mathbf{Z}_{22,a} + \mathbf{Z}_{22,b}
\end{aligned} \tag{31a}
$$

or, in the matrix form,

$$[\mathbf{Z}] = [\mathbf{Z}_a] + [\mathbf{Z}_b] \tag{31b}$$

Parallel Connection

Figure 4-13 shows a parallel connection of the two-port networks a and b with short-circuit admittance parameters $\mathbf{Y}_a$ and $\mathbf{Y}_b$, respectively. In this case, the $\mathbf{Y}$-parameters are convenient to work with. The $\mathbf{Y}$-parameters of the parallel connection are (see Problem 4.11):

$$
\begin{aligned}
\mathbf{Y}_{11} &= \mathbf{Y}_{11,a} + \mathbf{Y}_{11,b} \\
\mathbf{Y}_{12} &= \mathbf{Y}_{12,a} + \mathbf{Y}_{12,b} \\
\mathbf{Y}_{21} &= \mathbf{Y}_{21,a} + \mathbf{Y}_{21,b} \\
\mathbf{Y}_{22} &= \mathbf{Y}_{22,a} + \mathbf{Y}_{22,b}
\end{aligned} \tag{32a}
$$

or, in matrix form,

$$[\mathbf{Y}] = [\mathbf{Y}_a] + [\mathbf{Y}_b] \tag{32b}$$

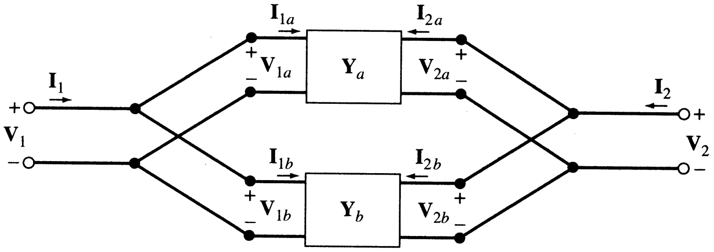

Fig. 4-13

Cascade Connection

The cascade connection of the two-port networks a and b is shown in Fig. 4-14. In this case the T-parameters are particularly convenient. The T-parameters of the cascade combination are

$$
\begin{aligned}
\mathbf{A} &= \mathbf{A}_a\mathbf{A}_b + \mathbf{B}_a\mathbf{C}_b \\
\mathbf{B} &= \mathbf{A}_a\mathbf{B}_b + \mathbf{B}_a\mathbf{D}_b \\
\mathbf{C} &= \mathbf{C}_a\mathbf{A}_b + \mathbf{D}_a\mathbf{C}_b \\
\mathbf{D} &= \mathbf{C}_a\mathbf{B}_b + \mathbf{D}_a\mathbf{D}_b
\end{aligned} \tag{33a}
$$

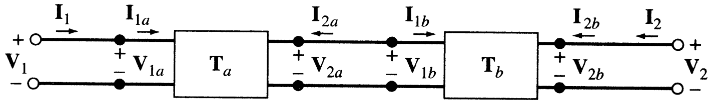

Fig. 4-14

or, in matrix form,

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] \tag{33b}$$

4.12 Choice of Parameter Type

What types of parameters are appropriate for and can best describe a given two-port network or device? Several factors influence the choice of parameters. (1) It is possible that some types of parameters do not exist as they may not be defined at all (see Example 4.10). (2) Some parameters are more convenient to work with when the network is connected to other networks, as shown in Section 4.11. In this regard, by converting the two-port network to its T- and Pi-equivalents and then applying the familiar analysis techniques, such as element reduction and current division, we can greatly reduce and simplify the overall circuit. (3) For some networks or devices, a certain type of parameter produces better computational accuracy and better sensitivity when used within the interconnected circuit.

EXAMPLE 4.10 Find the Z- and Y-parameters of Fig. 4-15.

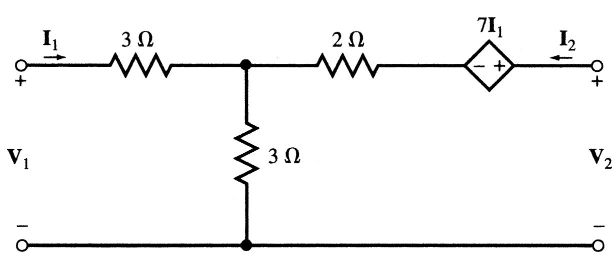

Fig. 4-15

We apply KVL to the input and output loops. Thus,

Input loop: $\mathbf{V}_1 = 3\mathbf{I}_1 + 3(\mathbf{I}_1 + \mathbf{I}_2)$

Output loop: $\mathbf{V}_2 = 7\mathbf{I}_1 + 2\mathbf{I}_2 + 3(\mathbf{I}_1 + \mathbf{I}_2)$

or

$$\mathbf{V}_1 = 6\mathbf{I}_1 + 3\mathbf{I}_2 \qquad \text{and} \qquad \mathbf{V}_2 = 10\mathbf{I}_1 + 5\mathbf{I}_2 \tag{34}$$

By comparing (34) and (2) we get

$$\mathbf{Z}_{11} = 6 \qquad \mathbf{Z}_{12} = 3 \qquad \mathbf{Z}_{21} = 10 \qquad \mathbf{Z}_{22} = 5$$

The **Y**-parameters are, however, not defined, since the application of the direct method of (10) or the conversion from **Z**-parameters (19) produces $\mathbf{D}_{zz} = 6(5) - 3(10) = 0$.

4.13 Summary of Terminal Parameters and Conversion

The various terminal parameters are defined by the following equations:

Z-parameters
$$\mathbf{V}_1 = \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2$$
$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{I}]$$

h-parameters
$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

T-parameters
$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

Y-parameters
$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2$$
$$[\mathbf{I}] = [\mathbf{Y}][\mathbf{V}]$$

g-parameters
$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

Table 4-1 summarizes the conversion between the Z-, Y-, h-, g-, and T-parameters. For the conversion to be possible, the determinant of the source parameters must be nonzero.

Table 4-1

		Z		**Y**		**h**		**g**		**T**	
Z		$\mathbf{Z}_{11}$	$\mathbf{Z}_{12}$	$\frac{\mathbf{Y}_{22}}{\mathbf{D}_{\mathbf{YY}}}$	$\frac{-\mathbf{Y}_{12}}{\mathbf{D}_{\mathbf{YY}}}$	$\frac{\mathbf{D}_{\mathbf{hh}}}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{g}_{11}}$	$\frac{-\mathbf{g}_{12}}{\mathbf{g}_{11}}$	$\frac{\mathbf{A}}{\mathbf{C}}$	$\frac{\mathbf{D}_{\mathrm{TT}}}{\mathbf{C}}$
		$\mathbf{Z}_{21}$	$\mathbf{Z}_{22}$	$\frac{-\mathbf{Y}_{21}}{\mathbf{D}_{\mathbf{YY}}}$	$\frac{\mathbf{Y}_{11}}{\mathbf{D}_{\mathbf{YY}}}$	$\frac{-\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$	$\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$	$\frac{\mathbf{D}_{\mathbf{gg}}}{\mathbf{g}_{11}}$	$\frac{1}{\mathbf{C}}$	$\frac{\mathbf{D}}{\mathbf{C}}$
Y		$\frac{\mathbf{Z}_{22}}{\mathbf{D}_{\mathbf{zz}}}$	$\frac{-\mathbf{Z}_{12}}{\mathbf{D}_{\mathbf{zz}}}$	$\mathbf{Y}_{11}$	$\mathbf{Y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$\frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}}$	$\frac{\mathbf{D}_{\mathbf{gg}}}{\mathbf{g}_{22}}$	$\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$	$\frac{\mathbf{D}}{\mathbf{B}}$	$\frac{-\mathbf{D}_{\mathrm{TT}}}{\mathbf{B}}$
		$\frac{-\mathbf{Z}_{21}}{\mathbf{D}_{\mathbf{zz}}}$	$\frac{\mathbf{Z}_{11}}{\mathbf{D}_{\mathbf{zz}}}$	$\mathbf{Y}_{21}$	$\mathbf{Y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{-\mathbf{D}_{\mathbf{nn}}}{\mathbf{h}_{11}}$	$\frac{-\mathbf{g}_{21}}{\mathbf{g}_{22}}$	$\frac{1}{\mathbf{g}_{22}}$	$\frac{-1}{\mathbf{B}}$	$\frac{\mathbf{A}}{\mathbf{B}}$
h		$\frac{\mathbf{D}_{\mathbf{ZZ}}}{\mathbf{Z}_{22}}$	$\frac{\mathbf{Z}_{12}}{\mathbf{Z}_{22}}$	$\frac{1}{\mathbf{Y}_{11}}$	$\frac{-\mathbf{Y}_{12}}{\mathbf{Y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$	$\frac{\mathbf{g}_{22}}{\mathbf{D}_{\mathrm{gg}}}$	$\frac{\mathbf{g}_{12}}{\mathbf{D}_{\mathrm{gg}}}$	$\frac{\mathbf{B}}{\mathbf{D}}$	$\frac{\mathbf{D}_{\mathrm{TT}}}{\mathbf{D}}$
		$\frac{-\mathbf{Z}_{21}}{\mathbf{Z}_{22}}$	$\frac{1}{\mathbf{Z}_{22}}$	$\frac{\mathbf{Y}_{21}}{\mathbf{Y}_{11}}$	$\frac{\mathbf{D}_{\mathrm{yy}}}{\mathbf{Y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$	$\frac{\mathbf{g}_{21}}{\mathbf{D}_{\mathrm{gg}}}$	$\frac{\mathbf{g}_{11}}{\mathbf{D}_{\mathrm{gg}}}$	$\frac{-1}{\mathbf{D}}$	$\frac{\mathbf{C}}{\mathbf{D}}$
g		$\frac{1}{\mathbf{Z}_{11}}$	$\frac{-\mathbf{Z}_{12}}{\mathbf{Z}_{11}}$	$\frac{\mathbf{D}_{\mathbf{YY}}}{\mathbf{Y}_{22}}$	$\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{22}}$	$\frac{\mathbf{h}_{22}}{\mathbf{D}_{\mathbf{hh}}}$	$\frac{-\mathbf{h}_{12}}{\mathbf{D}_{\mathbf{hh}}}$	$\mathbf{g}_{11}$	$\mathbf{g}_{12}$	$\frac{\mathbf{C}}{\mathbf{A}}$	$\frac{-\mathbf{D}_{\mathrm{TT}}}{\mathbf{A}}$
		$\frac{\mathbf{Z}_{21}}{\mathbf{Z}_{11}}$	$\frac{\mathbf{D}_{\mathbf{ZZ}}}{\mathbf{Z}_{11}}$	$\frac{-\mathbf{Y}_{21}}{\mathbf{Y}_{22}}$	$\frac{1}{\mathbf{Y}_{22}}$	$\frac{-\mathbf{h}_{21}}{\mathbf{D}_{\mathbf{hh}}}$	$\frac{\mathbf{h}_{11}}{\mathbf{D}_{\mathbf{hh}}}$	$\mathbf{g}_{21}$	$\mathbf{g}_{22}$	$\frac{1}{\mathbf{A}}$	$\frac{\mathbf{B}}{\mathbf{A}}$
T		$\frac{\mathbf{Z}_{11}}{\mathbf{Z}_{21}}$	$\frac{\mathbf{D}_{\mathbf{ZZ}}}{\mathbf{Z}_{21}}$	$\frac{-\mathbf{Y}_{22}}{\mathbf{Y}_{21}}$	$\frac{-1}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{D}_{\mathbf{hh}}}{\mathbf{h}_{21}}$	$\frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}}$	$\frac{1}{\mathbf{g}_{21}}$	$\frac{\mathbf{g}_{22}}{\mathbf{g}_{21}}$	$\mathbf{A}$	$\mathbf{B}$
		$\frac{1}{\mathbf{Z}_{21}}$	$\frac{\mathbf{Z}_{22}}{\mathbf{Z}_{21}}$	$\frac{-\mathbf{D}_{\mathbf{YY}}}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{Y}_{11}}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}}$	$\frac{-1}{\mathbf{h}_{21}}$	$\frac{\mathbf{g}_{11}}{\mathbf{g}_{21}}$	$\frac{\mathbf{D}_{\mathbf{gg}}}{\mathbf{g}_{21}}$	$\mathbf{C}$	$\mathbf{D}$

$\mathbf{D}_{\mathbf{PP}} = \mathbf{P}_{11}\mathbf{P}_{22} - \mathbf{P}_{12}\mathbf{P}_{21}$ is the determinant of the coefficient matrix for the **Z**–, **Y**–, **h**–, **g**–, or T-parameters.

SOLVED PROBLEMS

4.1. Find the Z-parameters of the circuit in Fig. 4-16(a).

Z and Z are obtained by connecting a source to port #1 and leaving port #2 open [Fig. 4-16(b)]. The parallel and series combination of resistors produces

$$\mathbf{Z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}\bigg|_{\mathbf{I}_{22}=0} = 8 \qquad \text{and} \qquad \mathbf{Z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}\bigg|_{\mathbf{I}\ =0} = \frac{1}{3}$$

Similarly, $\mathbf{Z}_{22}$ and $\mathbf{Z}_{12}$ are obtained by connecting a source to port #2 and leaving port #1 open [Fig. 4-16(c)].

$$\mathbf{Z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0} = \frac{8}{9} \qquad \mathbf{Z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0} = \frac{1}{3}$$

The circuit is reciprocal, since $\mathbf{Z}_{12} = \mathbf{Z}_{21}$.

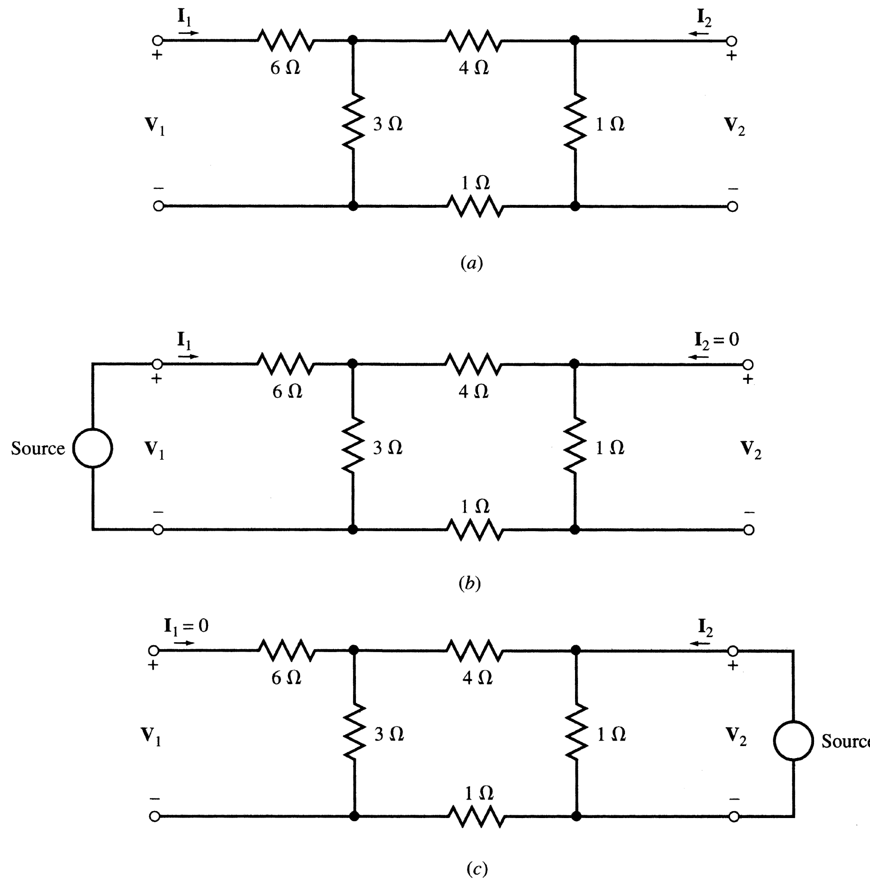

Fig. 4-16

4.2. The Z-parameters of a two-port network N are given by

$$\mathbf{Z}_{11} = 2\mathbf{s} + 1/\mathbf{s} \qquad \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s} \qquad \mathbf{Z}_{22} = 2\mathbf{s} + 4$$

(*a*) Find the T-equivalent of *N*. (*b*) The network *N* is connected to a source and a load as shown in the circuit of Fig. 4-8. Replace N by its T-equivalent and then solve for i , i , u , and u .

(a) The three branches of the T-equivalent network (Fig. 4-4) are

$$\mathbf{Z}_a = \mathbf{Z}_{11} - \mathbf{Z}_{12} = 2\mathbf{s} + \frac{1}{\mathbf{s}} - 2\mathbf{s} = \frac{1}{\mathbf{s}}$$

$$\mathbf{Z}_b = \mathbf{Z}_{22} - \mathbf{Z}_{12} = 2\mathbf{s} + 4 - 2\mathbf{s} = 4$$

$$\mathbf{Z}_c = \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s}$$

(*b*) The T-equivalent of *N*, along with its input and output connections, is shown in the phasor domain in Fig. 4-17.

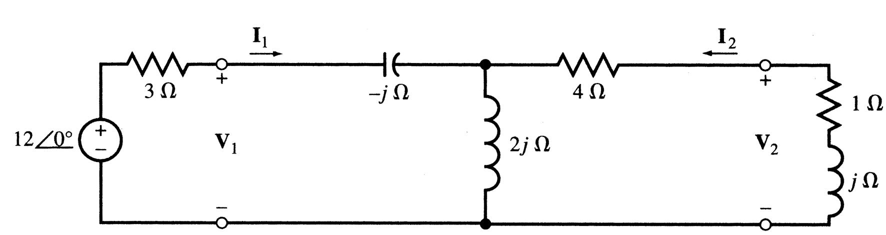

Fig. 4-17

By applying the familiar analysis techniques, including element reduction and current division, to Fig. 4-17 we find i , i , u , and u .

In the phasor domain	In the time domain:
$\mathbf{I}_1 = 3.29 \angle{-10.2^\circ}$	$i_1 = 3.29 \cos(t - 10.2^\circ)$
$\mathbf{I}_2 = 1.13 \angle{-131.2^\circ}$	$i_2 = 1.13 \cos(t - 131.2^\circ)$
$\mathbf{V}_1 = 2.88 \angle{37.5^\circ}$	$\upsilon_1 = 2.88 \cos(t + 37.5^\circ)$
$\mathbf{V}_2 = 1.6 \angle{93.8^\circ}$	$\upsilon_2 = 1.6 \cos(t + 93.8^\circ)$

4.3. Find the Z-parameters of the two-port network in Fig. 4-18.

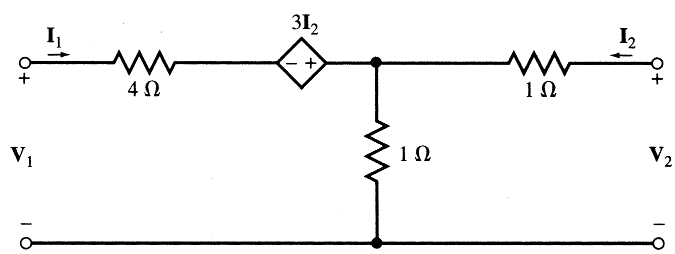

Fig. 4-18

KVL applied to the input and output ports results in the following:

Input port: $\mathbf{V}_1 = 4\mathbf{I}_1 - 3\mathbf{I}_2 + (\mathbf{I}_1 + \mathbf{I}_2) = 5\mathbf{I}_1 - 2\mathbf{I}_2$

Output port: $\mathbf{V}_2 = \mathbf{I}_2 + (\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{I}_1 + 2\mathbf{I}_2$

By applying (1) to the above, $\mathbf{Z}_{11} = 5$, $\mathbf{Z}_{12} = -2$, $\mathbf{Z}_{21} = 1$, and $\mathbf{Z}_{22} = 2$.

4.4. Find the Z-parameters of the two-port network in Fig. 4-19 and compare the results with those of Problem 4.3.

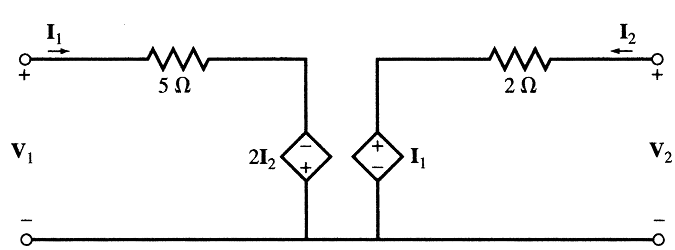

Fig. 4-19

KVL gives

$$\mathbf{V}_1 = 5\mathbf{I}_1 - 2\mathbf{I}_2 \qquad \text{and} \qquad \mathbf{V}_2 = \mathbf{I}_1 + 2\mathbf{I}_2$$

The above equations are identical with the terminal characteristics obtained for the network of Fig. 13-18. Thus, the two networks are equivalent.

4.5. Find the Y-parameters of Fig. 4-19 using its Z-parameters.

From Problem 4.4,

$$\mathbf{Z}_{11} = 5, \mathbf{Z}_{12} = -2, \mathbf{Z}_{21} = 1, \mathbf{Z}_{22} = 2$$

Since $\mathbf{D_{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21} = (5)(2) - (-2)(1) = 12$,

$$\mathbf{Y}_{11} = \frac{\mathbf{Z}_{22}}{\mathbf{D}_{zz}} = \frac{2}{12} = \frac{1}{6} \qquad \mathbf{Y}_{12} = \frac{-\mathbf{Z}_{12}}{\mathbf{D}_{zz}} = \frac{2}{12} = \frac{1}{6} \qquad \mathbf{Y}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{D}_{zz}} = \frac{-1}{12} \qquad \mathbf{Y}_{22} = \frac{\mathbf{Z}_{11}}{\mathbf{D}_{zz}} = \frac{5}{12}$$

4.6. Find the Y-parameters of the two-port network in Fig. 4-20 and thus show that the networks of Figs. 4-19 and 4-20 are equivalent.

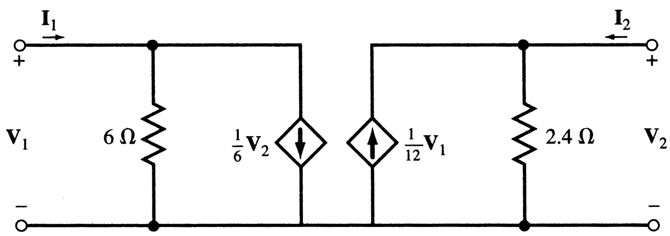

Fig. 4-20

Apply KCL at the ports to obtain the terminal characteristics and **Y**-parameters. Thus,

Input port: $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{6} + \frac{\mathbf{V}_2}{6}$$

Output port: $$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2.4} - \frac{\mathbf{V}_1}{12}$$

and $$\mathbf{Y}_{11} = \frac{1}{6} \qquad \mathbf{Y}_{12} = \frac{1}{6} \qquad \mathbf{Y}_{21} = \frac{-1}{12} \qquad \mathbf{Y}_{22} = \frac{1}{2.4} = \frac{5}{12}$$

which are identical with the **Y**-parameters obtained in Problem 13.5 for Fig. 13-19. Thus, the two networks are equivalent.

4.7. Apply the short-circuit equations (10) to find the Y-parameters of the two-port network in Fig. 4-21.

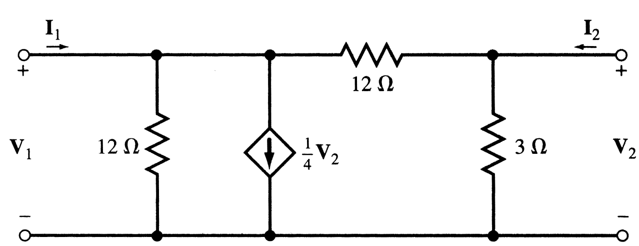

Fig. 4-21

$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1\big|_{\mathbf{V}_2=0} = \left(\frac{1}{12} + \frac{1}{12}\right)\mathbf{V}_1 \qquad \text{or} \qquad \mathbf{Y}_{11} = \frac{1}{6}$$

$$\mathbf{I}_1 = \mathbf{Y}_{12}\mathbf{V}_2\big|_{\mathbf{V}_1=0} = \frac{\mathbf{V}_2}{4} - \frac{\mathbf{V}_2}{12} = \left(\frac{1}{4} - \frac{1}{12}\right)\mathbf{V}_2 \qquad \text{or} \qquad \mathbf{Y}_{12} = \frac{1}{6}$$

$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1\big|_{\mathbf{V}_2=0} = -\frac{\mathbf{V}_1}{12} \qquad \text{or} \qquad \mathbf{Y}_{21} = -\frac{1}{12}$$

$$\mathbf{I}_2 = \mathbf{Y}_{22}\mathbf{V}_2\big|_{\mathbf{V}_1=0} = \frac{\mathbf{V}_2}{3} + \frac{\mathbf{V}_2}{12} = \left(\frac{1}{3} + \frac{1}{12}\right)\mathbf{V}_2 \qquad \text{or} \qquad \mathbf{Y}_{22} = \frac{5}{12}$$

4.8. Apply KCL at the nodes of the network in Fig. 4.21 to obtain its terminal characteristics and Y-parameters. Show that the two-port networks of Figs. 4.18 to 4.21 are all equivalent.

Input node: $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{12} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{12} + \frac{\mathbf{V}_2}{4}$$

Output node: $$\mathbf{I}_2 = \frac{\mathbf{V}_2}{3} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{12}$$

$$\mathbf{I}_1 = \frac{1}{6}\mathbf{V}_1 + \frac{1}{6}\mathbf{V}_2 \qquad \mathbf{I}_2 = -\frac{1}{12}\mathbf{V}_1 + \frac{5}{12}\mathbf{V}_2$$

The **Y**-parameters observed from the above characteristic equations are identical with the **Y**-parameters of the circuits in Figs. 4.18, 4.19, and 4.20. Therefore, the four circuits are equivalent.

4.9. Z-parameters of the two-port network N in Fig. 4.22(a) are Z = 4s, Z =Z = 3s, and Z = 9s. (*a*) Replace *N* by its T-equivalent. (*b*) Use part (*a*) to find input current i_1 for $\upsilon_s = \cos 1000t$ (V).

(*a*) The network is reciprocal. Therefore, its T-equivalent exists. Its elements are found from (6) and shown in the circuit of Fig. 4.22(b).

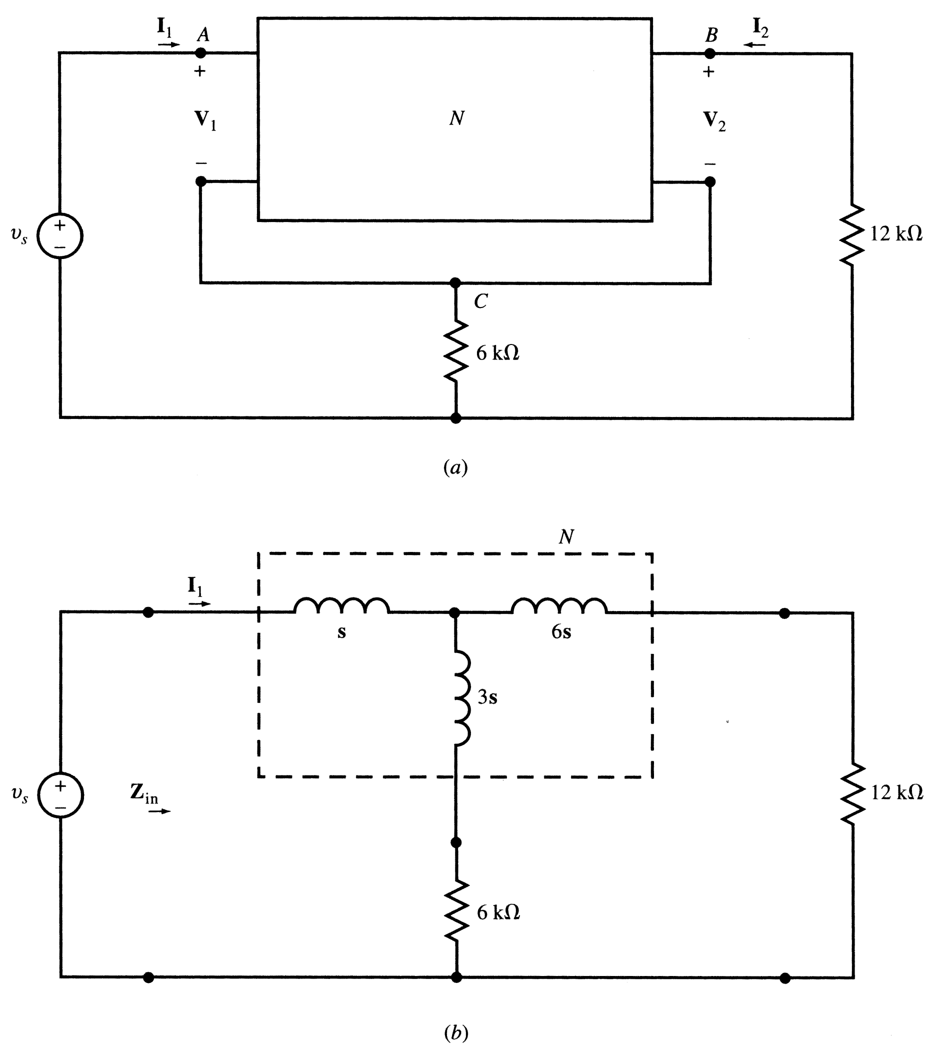

Fig. 4.22

$$\mathbf{Z}_a = \mathbf{Z}_{11} - \mathbf{Z}_{12} = 4\mathbf{s} - 3\mathbf{s} = \mathbf{s}$$

$$\mathbf{Z}_b = \mathbf{Z}_{22} - \mathbf{Z}_{21} = 9\mathbf{s} - 3\mathbf{s} = 6\mathbf{s}$$

$$\mathbf{Z}_c = \mathbf{Z}_{12} = \mathbf{Z}_{21} = 3\mathbf{s}$$

(b) We repeatedly combine the series and parallel elements of Fig. 4.22(b), with resistors being in kΩ and **s** in krad/s, to find $\mathbf{Z}_{\text{in}}$ in kΩ:

$$\mathbf{Z}_{\text{in}}(\mathbf{s}) = \mathbf{V}_s/\mathbf{I}_1 = \mathbf{s} + \frac{(3\mathbf{s}+6)(6\mathbf{s}+12)}{9\mathbf{s}+18} = 3\mathbf{s} + 4 \qquad \text{or} \qquad \mathbf{Z}_{\text{in}}(j) = 3j + 4 = 5\underline{/36.9^\circ} \text{ k}\Omega$$

and $i_1 = 0.2\cos(1000t - 36.9^\circ)$ (mA).

4.10. Two two-port networks a and b, with open-circuit impedances Z and Z , respectively, are connected in series (see Fig. 4.12). Derive the Z-parameter equations (31a).

From network **a** we have

$$\mathbf{V}_{1a} = \mathbf{Z}_{11,a}\mathbf{I}_{1a} + \mathbf{Z}_{12,a}\mathbf{I}_{2a}$$

$$\mathbf{V}_{2a} = \mathbf{Z}_{21,a}\mathbf{I}_{1a} + \mathbf{Z}_{22,a}\mathbf{I}_{2a}$$

From network **b** we have

$$\mathbf{V}_{1b} = \mathbf{Z}_{11,b}\mathbf{I}_{1b} + \mathbf{Z}_{12,b}\mathbf{I}_{2b}$$

$$\mathbf{V}_{2b} = \mathbf{Z}_{21,b}\mathbf{I}_{1b} + \mathbf{Z}_{22,b}\mathbf{I}_{2b}$$

From the connection between **a** and **b** we have

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b} \qquad \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b} \qquad \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b}$$

Therefore,

$$\mathbf{V}_1 = (\mathbf{Z}_{11,a} + \mathbf{Z}_{11,b})\mathbf{I}_1 + (\mathbf{Z}_{12,a} + \mathbf{Z}_{12,b})\mathbf{I}_2$$

$$\mathbf{V}_2 = (\mathbf{Z}_{21,a} + \mathbf{Z}_{21,b})\mathbf{I}_1 + (\mathbf{Z}_{22,a} + \mathbf{Z}_{22,b})\mathbf{I}_2$$

from which the **Z**-parameters (31*a*) are derived.

13.11. Two two-port networks **a** and **b**, with short-circuit admittances $\mathbf{Y}_a$ and $\mathbf{Y}_b$, respectively, are connected in parallel (see Fig. 4.13). Derive the Y-parameter equations (32a).

From network **a** we have

$$\mathbf{I}_{1a} = \mathbf{Y}_{11,a}\mathbf{V}_{1a} + \mathbf{Y}_{12,a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{2a} = \mathbf{Y}_{21,a}\mathbf{V}_{1a} + \mathbf{Y}_{22,a}\mathbf{V}_{2a}$$

and from network **b** we have

$$\mathbf{I}_{1b} = \mathbf{Y}_{11,b}\mathbf{V}_{1b} + \mathbf{Y}_{12,b}\mathbf{V}_{2b}$$

$$\mathbf{I}_{2b} = \mathbf{Y}_{21,b}\mathbf{V}_{1b} + \mathbf{Y}_{22,b}\mathbf{V}_{2b}$$

From the connection between **a** and **b** we have

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b} \qquad \mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b} \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

Therefore,

$$\mathbf{I}_1 = (\mathbf{Y}_{11,a} + \mathbf{Y}_{11,b})\mathbf{V}_1 + (\mathbf{Y}_{12,a} + \mathbf{Y}_{12,b})\mathbf{V}_2$$

$$\mathbf{I}_2 = (\mathbf{Y}_{21,a} + \mathbf{Y}_{21,b})\mathbf{V}_1 + (\mathbf{Y}_{22,a} + \mathbf{Y}_{22,b})\mathbf{V}_2$$

from which the **Y**-parameters of (32*a*) result.

13.12. Find (a) the Z-parameters of the circuit of Fig. 4.23(a) and (b) an equivalent model which uses three positive-valued resistors and one dependent voltage source.

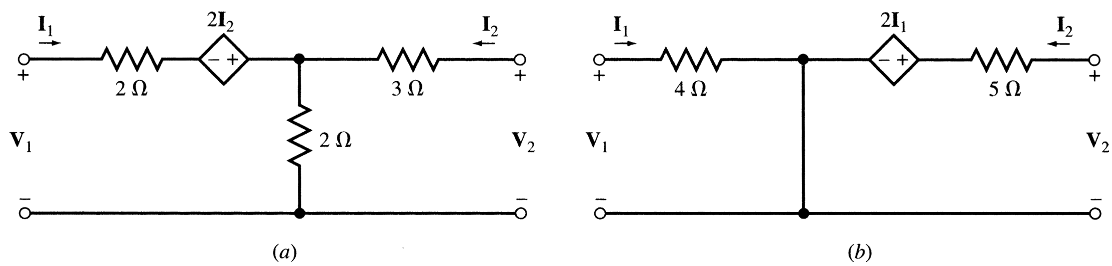

Fig. 4.23

(a) From application of KVL around the input and output loops we find, respectively,

$$\mathbf{V}_1 = 2\mathbf{I}_1 - 2\mathbf{I}_2 + 2(\mathbf{I}_1 + \mathbf{I}_2) = 4\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_2 + 2(\mathbf{I}_1 + \mathbf{I}_2) = 2\mathbf{I}_1 + 5\mathbf{I}_2$$

The **Z**-parameters are $\mathbf{Z}_{11} = 4$, $\mathbf{Z}_{12} = 0$, $\mathbf{Z}_{21} = 2$, and $\mathbf{Z}_{22} = 5$.

(b) The circuit of Fig. 4.23(b), with two resistors and a voltage source, has the same Z-parameters as the circuit of Fig. 4.23(a). This can be verified by applying KVL around its input and output loops.

4.13. (a) Obtain the Y-parameters of the circuit in Fig. 13-23(a) from its Z-parameters. (b) Find an equivalent model which uses two positive-valued resistors and one dependent current source.

(a) From Problem 13.12, $\mathbf{Z}_{11} = 4$, $\mathbf{Z}_{12} = 0$, $\mathbf{Z}_{21} = 2$, $\mathbf{Z}_{22} = 5$, and so $\mathbf{D_{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21} = 20$. Hence,

$$\mathbf{Y}_{11} = \frac{\mathbf{Z}_{22}}{\mathbf{D_{ZZ}}} = \frac{5}{20} = \frac{1}{4} \qquad \mathbf{Y}_{12} = \frac{-\mathbf{Z}_{12}}{\mathbf{D_{ZZ}}} = 0 \qquad \mathbf{Y}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{D_{ZZ}}} = \frac{-2}{20} = -\frac{1}{10} \qquad \mathbf{Y}_{22} = \frac{\mathbf{Z}_{11}}{\mathbf{D_{ZZ}}} = \frac{4}{20} = \frac{1}{5}$$

(b) Figure 13-24, with two resistors and a current source, has the same **Y**-parameters as the circuit in Fig.13-23(a). This can be verified by applying KCL to the input and output nodes.

4.14. Referring to the network of Fig. 4.23(b), convert the voltage source and its series resistor to its Norton equivalent and show that the resulting network is identical to that in Fig. 4.24.

The Norton equivalent current source is $\mathbf{I}_N = 2I_1/5 = 0.4I_1$. But $I_1 = V_1/4$. Therefore, $\mathbf{I}_N = 0.4I_1 = 0.1V_1$. The 5-Ω resistor is then placed in parallel with $\mathbf{I}_N$. The circuit is shown in Fig. 13-25 which is the same as the circuit in Fig. 13-24.

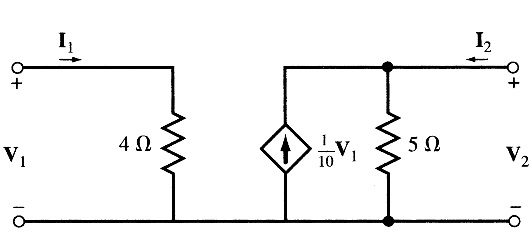

Fig. 4.24

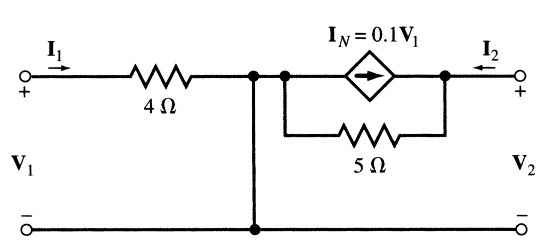

Fig. 4.25

4.15. The h-parameters of a two-port network are given. Show that the network may be modeled by the network in Fig. 13-26 where $\mathbf{h}_{11}$ is an impedance, $\mathbf{h}_{12}$ is a voltage gain, $\mathbf{h}_{21}$ is a current gain, and $\mathbf{h}_{22}$ is an admittance.

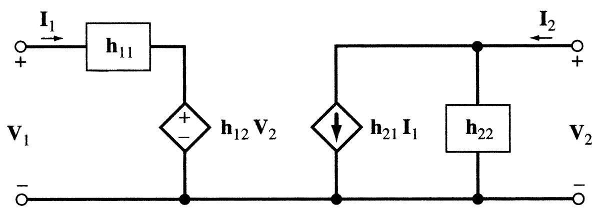

Fig. 4.26

Apply KVL around the input loop to get

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

Apply KCL at the output node to get

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

These results agree with the definition of **h**-parameters given in (23).

4.16. Find the h-parameters of the circuit in Fig. 4.25.

By comparing the circuit in Fig. 4.25 with that in Fig. 4.26, we find

$$\mathbf{h}_{11} = 4\,\Omega, \qquad \mathbf{h}_{12} = 0, \qquad \mathbf{h}_{21} = -0.4, \qquad \mathbf{h}_{22} = 1/5 = 0.2\ \Omega^{-1}$$

4.17. Find the h-parameters of the circuit in Fig. 4.25 from its Z-parameters and compare with the results of Problem 4.16.

Refer to Problem 4.13 for the values of the Z-parameters and D . Use Table 4.1 for the conversion of the **Z**-parameters to the **h**-parameters of the circuit. Thus,

$$\mathbf{h}_{11} = \frac{\mathbf{D}_{ZZ}}{\mathbf{Z}_{22}} = \frac{20}{5} = 4 \qquad \mathbf{h}_{12} = \frac{\mathbf{Z}_{12}}{\mathbf{Z}_{22}} = 0 \qquad \mathbf{h}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{Z}_{22}} = \frac{-2}{5} = -0.4 \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{Z}_{22}} = \frac{1}{5} = 0.2$$

The above results agree with the results of Problem 4.16.

4.18. The simplified model of a bipolar junction transistor for small signals is shown in Fig. 4.27. Find its **h**-parameters.

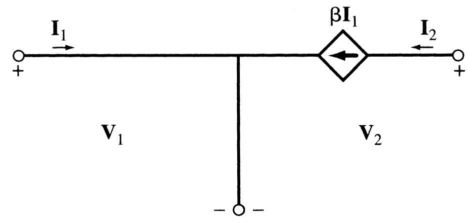

Fig. 4.27

The terminal equations are $\mathbf{V}_1 = 0$ and $\mathbf{I}_2 = \beta\mathbf{I}_1$. By comparing these equations with (23), we conclude that $\mathbf{h}_{11} = \mathbf{h}_{12} = \mathbf{h}_{22} = 0$ and $\mathbf{h}_{21} = \beta$.

4.19. The h-parameters of a two-port device H are given by

$$\mathbf{h}_{11} = 500\ \Omega \qquad \mathbf{h}_{12} = 10^{-4} \qquad \mathbf{h}_{21} = 100 \qquad \mathbf{h}_{22} = 2(10^{-6})\ \Omega^{-1}$$

Draw a circuit model of the device made of two resistors and two dependent sources. Include the values of each element.

From a comparison with Fig. 4.26, we draw the model of Fig. 4.28.

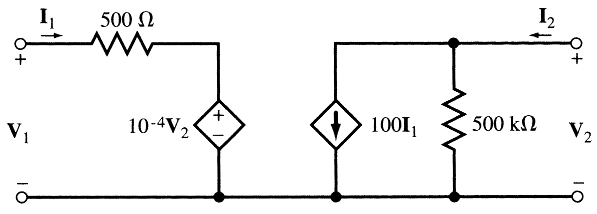

Fig. 4.28

4.20. The device H of Problem 4.19 is placed in the circuit of Fig. 4.29(a). Replace H by its model of Fig. 4.28 and find V /V.

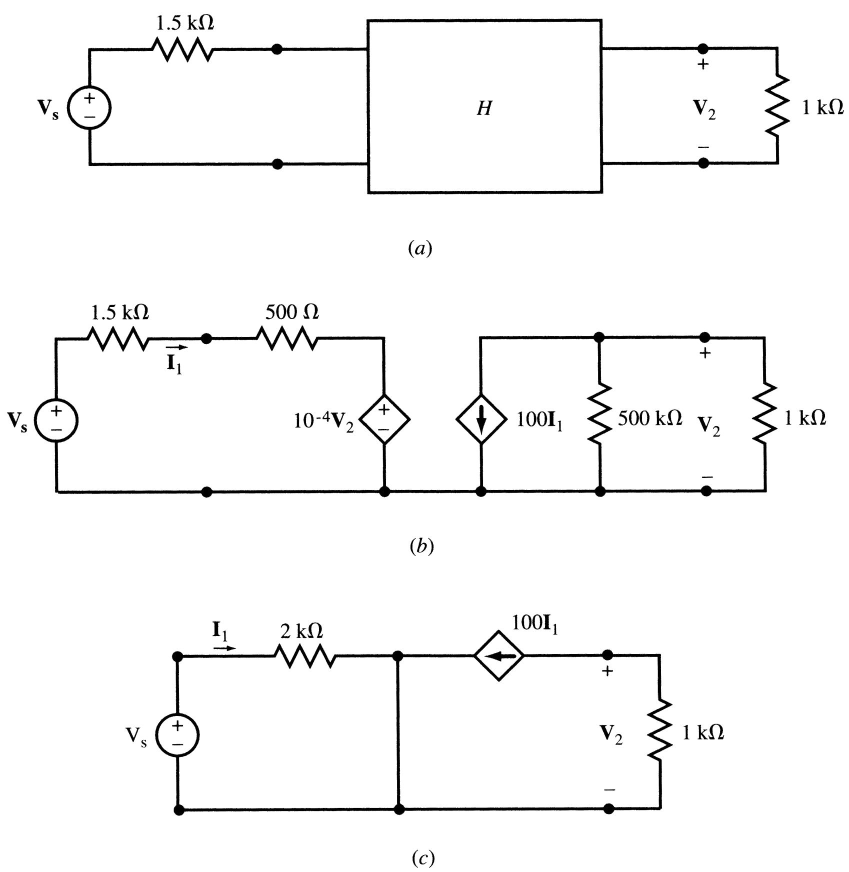

Fig. 4.29

The circuit of Fig. 4.29(b) contains the model. With good approximation, we can reduce it to Fig. 4.29(c) from which

$$\mathbf{I}_1 = \mathbf{V}_s/2000 \qquad \mathbf{V}_2 = -1000(100\mathbf{I}_1) = -1000(100\mathbf{V}_s/2000) = -50\ \mathbf{V}_s$$

Thus, $\mathbf{V}_2/\mathbf{V}_s = -50$.

4.21. A load Z is connected to the output of a two-port device N (Fig. 13-30) whose terminal characteristics are given by $\mathbf{V}_1 = (1/N)\mathbf{V}_2$ and $\mathbf{I}_1 = -N\mathbf{I}_2$. Find (*a*) the T-parameters of *N* and (*b*) the input impedance $\mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1$.

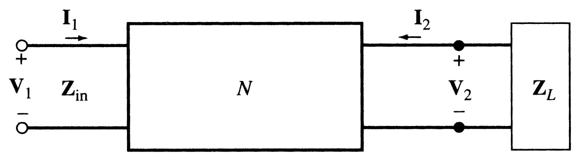

Fig. 4.30

(*a*) The T-parameters are defined by [see (29)]

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

The terminal characteristics of the device are

$$\mathbf{V}_1 = (1/N)\mathbf{V}_2$$
$$\mathbf{I}_1 = -N\mathbf{I}_2$$

By comparing the two pairs of equations we get $\mathbf{A} = 1/N$, $\mathbf{B} = 0$, $\mathbf{C} = 0$, and $\mathbf{D} = N$.

(*b*) Three equations relating $\mathbf{V}_1$, $\mathbf{I}_1$, $\mathbf{V}_2$, and $\mathbf{I}_2$ are available: two equations are given by the terminal characteristics of the device and the third equation comes from the connection to the load,

$$\mathbf{V}_2 = -\mathbf{Z}_L\mathbf{I}_2$$

By eliminating $\mathbf{V}_2$ and $\mathbf{I}_2$ in these three equations, we get

$$\mathbf{V}_1 = \mathbf{Z}_L\mathbf{I}_1/N^2 \qquad \text{from which} \qquad \mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1 = \mathbf{Z}_L/N^2$$

SUPPLEMENTARY PROBLEMS

4.22. The Z-parameters of the two-port network N in Fig. 4.22(a) are Z = 4s, Z =Z = 3s, and Z = 9s. Find the input current i_1 for $\upsilon_s = \cos 1000t$ (V) by using the open circuit impedance terminal characteristic equations of *N*, together with KCL equations at nodes *A*, *B*, and *C*.

Ans. $i_1 = 0.2 \cos(1000t - 36.9°)$ (A)

4.23. Express the reciprocity criteria in terms of h-, g-, and T-parameters.

Ans. $\mathbf{h}_{12} + \mathbf{h}_{21} = 0$, $\mathbf{g}_{12} + \mathbf{g}_{21} = 0$, and $\mathbf{AD} - \mathbf{BC} = 1$

4.24. Find the T-parameters of a two-port device whose Z-parameters are Z =s, Z =Z = 10s, and Z = 100s.

Ans. $\mathbf{A} = 0.1$, $\mathbf{B} = 0$, $\mathbf{C} = 10^{-1}/\mathbf{s}$, and $\mathbf{D} = 10$

4.25. Find the T-parameters of a two-port device whose Z-parameters are Z = 10 s, Z =Z = 10 s, and Z = 10 s. Compare with the results of Problem 4.21.

Ans. $\mathbf{A} = 0.1$, $\mathbf{B} = 0$, $\mathbf{C} = 10^{-7}/\mathbf{s}$ and $\mathbf{D} = 10$. For high frequencies, the device is similar to the device of Problem 13.21, with $N = 10$.

4.26. The Z-parameters of a two-port device N are Z = $k\mathbf{s}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = 10k\mathbf{s}$, and $\mathbf{Z}_{22} = 100k\mathbf{s}$. A 1-Ω resistor is connected across the output port (see Fig. 4.30). (a) Find the input impedance Z =V /I and construct its equivalent circuit. (*b*) Give the values of the elements for $k = 1$ and 10 .

Ans. (*a*) $\mathbf{Z}_{\text{in}} = \dfrac{k\mathbf{s}}{1 + 100k\mathbf{s}} = \dfrac{1}{100 + 1/k\mathbf{s}}$

The equivalent circuit is a parallel RL circuit with $R = 10^{-2}\ \Omega$ and $L = 1$ kH.

(*b*) For $k = 1$, $R = \dfrac{1}{100}\ \Omega$ and $L = 1$ H. For $k = 10^6$, $R = \dfrac{1}{100}\ \Omega$ and $L = 10^6$ H.

4.27. The device N in Fig. 4.30 is specified by the following Z-parameters: $Z_{22} = N Z_{11}$ and $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \sqrt{\mathbf{Z}_{11}\mathbf{Z}_{22}} = N\mathbf{Z}_{11}$. Find $\mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1$ when a load $\mathbf{Z}_L$ is connected to the output terminal. Show that if $\mathbf{Z}_{11} \gg \mathbf{Z}_L/N^2$ we have impedance scaling such that $\mathbf{Z}_{\text{in}} = \mathbf{Z}_L/N^2$.

Ans. $\mathbf{Z}_{\text{in}} = \dfrac{\mathbf{Z}_L}{N^2 + \mathbf{Z}_L/\mathbf{Z}_{11}}$. For $\mathbf{Z}_L \ll N^2\mathbf{Z}_{11}$, $\mathbf{Z}_{\text{in}} = \mathbf{Z}_L/N^2$.

4.28. Find the Z-parameters in the circuit of Fig. 4.31. Hint: Use the series connection rule.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \mathbf{s} + 3 + 1/\mathbf{s}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \mathbf{s} + 1$

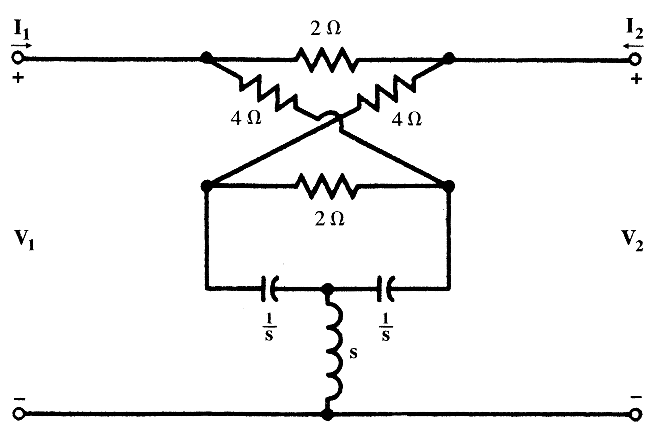

Fig. 4.31

4.29. Find the Y-parameters in the circuit of Fig. 4.32. Hint: Use the parallel connection rule.

Ans. $\mathbf{Y}_{11} = \mathbf{Y}_{22} = 9(\mathbf{s} + 2)/16$, $\mathbf{Y}_{12} = \mathbf{Y}_{21} = -3(\mathbf{s} + 2)/16$

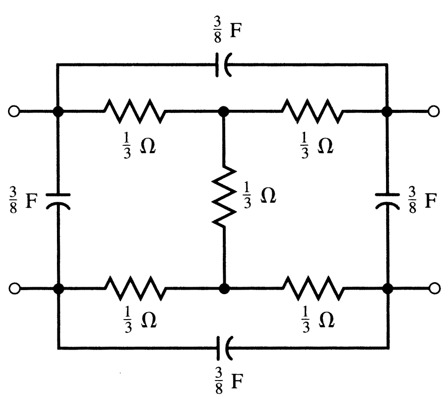

Fig. 4.32

4.30. Two two-port networks a and b with transmission parameters T and T are connected in cascade (see Fig. 13-14). Given $I_{2a} = -I_{1b}$ and $V_{2a} = V_{1b}$, find the T-parameters of the resulting two-port network.

Ans. $\mathbf{A} = \mathbf{A}_a\mathbf{A}_b + \mathbf{B}_a\mathbf{C}_b$, $\mathbf{B} = \mathbf{A}_a\mathbf{B}_b + \mathbf{B}_a\mathbf{D}_b$, $\mathbf{C} = \mathbf{C}_a\mathbf{A}_b + \mathbf{D}_a\mathbf{C}_b$, $\mathbf{D} = \mathbf{C}_a\mathbf{B}_b + \mathbf{D}_a\mathbf{D}_b$

4.31. Find the T- and Z-parameters of the network in Fig. 4.33. The impedances of the capacitors are given. Hint: Use the cascade connection rule.

Ans. $\mathbf{A} = 5j - 4$, $\mathbf{B} = 4j + 2$, $\mathbf{C} = 2j - 4$, and $\mathbf{D} = j3$, $\mathbf{Z}_{11} = 1.3 - 0.6j$, $\mathbf{Z}_{22} = 0.3 - 0.6j$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = -0.2 - 0.1j$

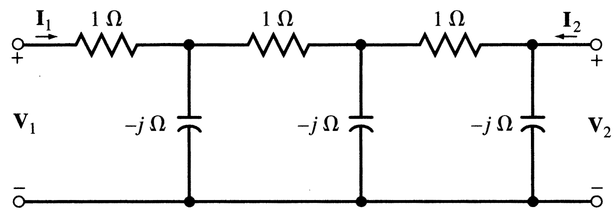

Fig. 4.33

4.32. Find the Z-parameters of the two-port circuit of Fig. 4.34.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \frac{1}{2}(\mathbf{Z}_b + \mathbf{Z}_a)$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \frac{1}{2}(\mathbf{Z}_b - \mathbf{Z}_a)$

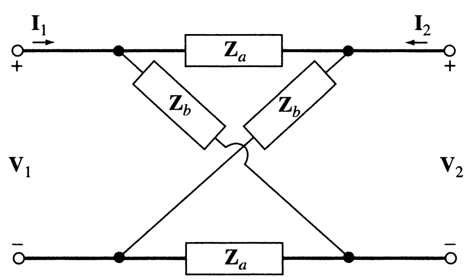

Fig. 4.34

4.33. Find the Z-parameters of the two-port circuit of Fig. 4.35.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \dfrac{1}{2}\dfrac{\mathbf{Z}_b(2\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \dfrac{1}{2}\dfrac{\mathbf{Z}_b^2}{\mathbf{Z}_a + \mathbf{Z}_b}$

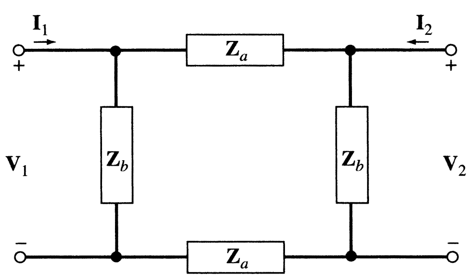

Fig. 4.35

4.34. Referring to the two-port circuit of Fig. 4.36, find the T-parameters as a function of w and specify their values at $\omega = 1$, 10^3, and 10^6 rad/s.

Ans. $\mathbf{A} = 1 - 10^{-9}\,\omega^2 + j10^{-9}\,\omega$, $\mathbf{B} = 10^{-3}\,(1 + j\omega)$, $\mathbf{C} = 10^{-6}\,j\omega$, and $\mathbf{D} = 1$. At $\omega = 1$ rad/s, $\mathbf{A} = 1$, $\mathbf{B} = 10^{-3}\,(1 + j)$, $\mathbf{C} = 10^{-6}\,j$, and $\mathbf{D} = 1$. At $\omega = 10^3$ rad/s, $\mathbf{A} \approx 1$, $\mathbf{B} \approx j$, $\mathbf{C} = 10^{-3}j$, and $D = 1$. At $\omega = 10^6$ rad/s, $\mathbf{A} \approx -10^3$, $\mathbf{B} \approx 10^3 j$, $\mathbf{C} = j$, and $\mathbf{D} = 1$.

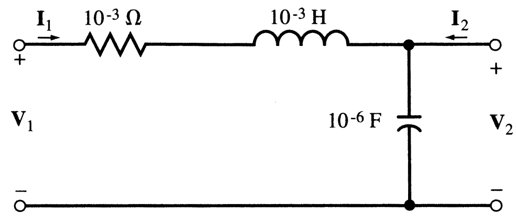

Fig. 4.36

4.35. A two-port network contains resistors, capacitors, and inductors only. With port #2 open [Fig. 4.37(a)], a unit step voltage $v_1 = u(t)$ produces $i_1 = e^{-t}u(t)$ (μA) and $v_2 = (1 - e^{-t})\,u(t)$ (V). With port #2 short-circuited [Fig. 13-37(*b*)], a unit step voltage $v_1 = u(t)$ delivers a current $i_1 = 0.5(1 + e^{-2t})u(t)$ (μA). Find i_2 and the T-equivalent network. *Ans.* $i_2 = 0.5(-1 + e\,)u(t)$ *[see Fig. 4.37(c)]*

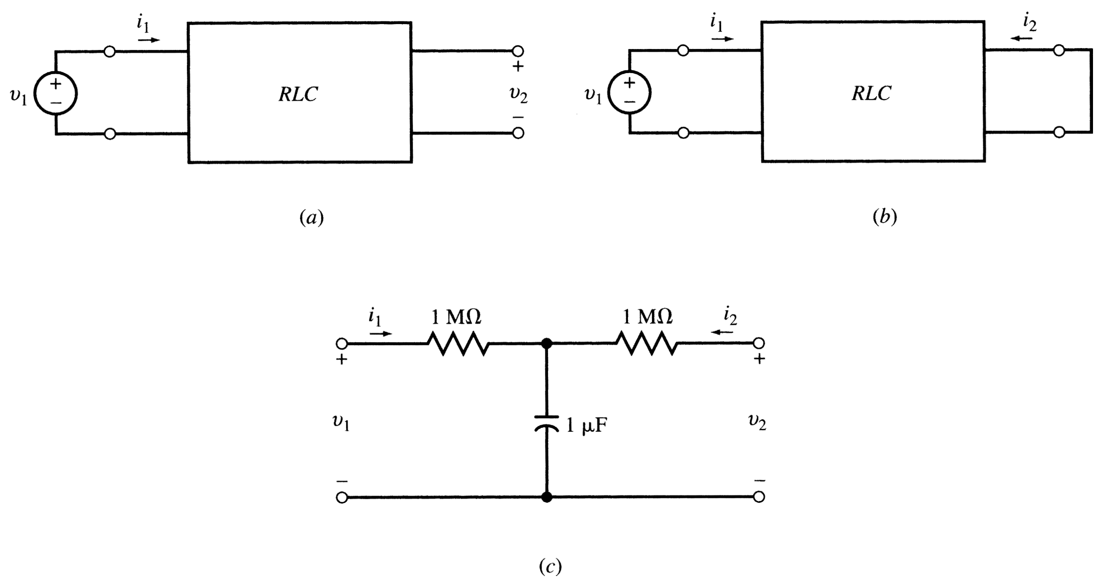

Fig. 4.37

4.36. The two-port network N in Fig. 4.38 is specified by $Z_{11} = 2$, $Z_{12} = Z_{21} = 1$, and $Z_{22} = 4$. Find I_1, I_2, and I_3.

Ans. $I_1 = 24$ A, $I_2 = 1.5$ A, and $I_3 = 6.5$ A

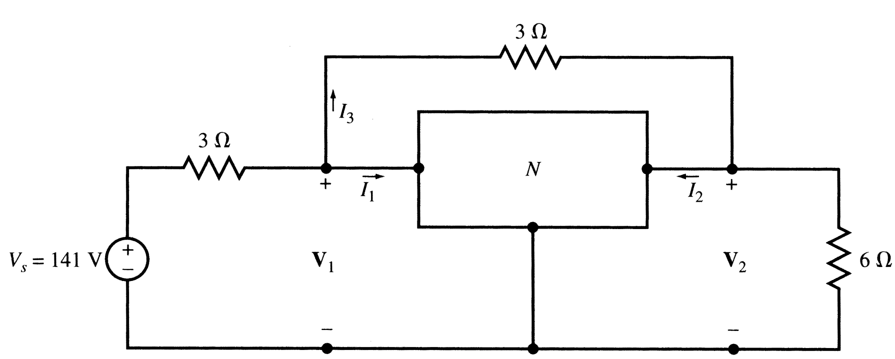

Fig. 4.38

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UNIT - V

Electrical Circuits and Network Analysis – SEEA1304

V. TRANSIENTS AND RESONANCE CIRCUITS

Introduction:

In this chapter we shall study transient response of the RL, RC series and RLC circuits with external DC excitations. Transients are generated in Electrical circuits due to abrupt changes in the operating conditions when energy storage elements like Inductors or capacitors are present. Transient response is the dynamic response during the initial phase before the steady state response is achieved when such abrupt changes are applied. To obtain the transient response of such circuits we have to solve the differential equations which are the governing equations representing the electrical behavior of the circuit. A circuit having a single energy storage element i.e. either a capacitor or an Inductor is called a Single order circuit and it's governing equation is called a First order Differential Equation. A circuit having both Inductor and a Capacitor is called a Second order Circuit and it's governing equation is called a Second order Differential Equation. The variables in these Differential Equations are currents and voltages in the circuit as a function of time.

A solution is said to be obtained to these equations when we have found an expression for the dependent variable that satisfies both the differential equation and the prescribed initial conditions. The solution of the differential equation represents the Response of the circuit. Now we will find out the response of the basic RL and RC circuits with DC Excitation.

RL CIRCUIT with external DC excitation:

Let us take a simple RL network subjected to external DC excitation as shown in the figure 5.1. The circuit consists of a battery whose voltage is V in series with a switch, a resistor R, and an inductor L. The switch is closed at t = 0.

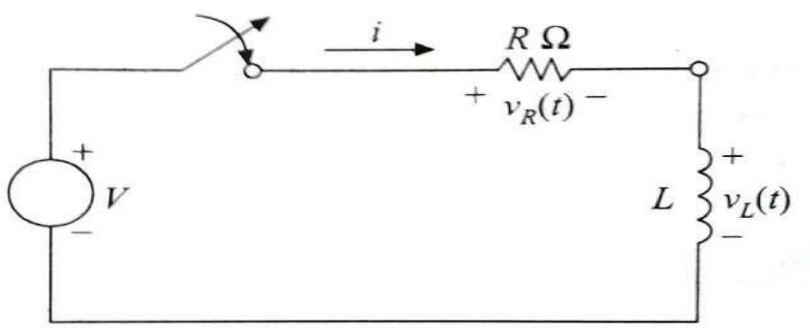


Figure 5.1 RL Circuit with external DC excitation

When the switch is closed current tries to change in the inductor and hence a voltage V_L (t) is induced across the terminals of the Inductor in opposition to the applied voltage. The rate of change of current decreases with time which allows current to build up to it's maximum value.

It is evident that the current i(t) is zero before t = 0.and we have to find out current i(t)for time t >0. We will find i(t)for time t >0 by writing the appropriate circuit equation and then solving it by separation of the variables and integration.

Applying Kirchhoff's voltage law to the above circuit we get:

$$V = V_R(t) + V_L(t)$$

$$i\,(t) = 0 \text{ fort} < 0 \text{ and}$$

Using the standard relationships of Voltage and Current for the Resistors and Inductors we can rewrite the above equations as

$$V = R\,i + L\,di\,/\,dt \text{ for } t > 0$$

One direct method of solving such a differential equation consists of writing the equation in such a way that the variables are separated, and then integrating each side of the equation. The variables in the above equation are i and t. This equation is multiplied by dt and arranged with the variables separated as shown below:

$$Ri.\,dt + Ldi = V.\,dt$$

$$\text{i.e } Ldi = (V - Ri)dt$$

$$\text{i.e} \quad Ldi\,/\,(V - Ri) \quad = \quad dt$$

Next each side is integrated directly to get :

$$-\,(L/R\,)\,\ln(V - Ri) = t + k$$

Where k is the integration constant. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, i (t)is zero, and thus i (0−) = 0. Since the current in an inductor cannot change by a finite amount in zero time without being associated with an infinite voltage, we have i (0+) = 0. Setting i = 0 at t = 0, in the above equation we obtain and, hence,

$$-(L/R)\ln(V) = k$$

$$-L/R[\ln(V-Ri)-\ln V]=t$$

Rearranging we get

$$\ln[(V-Ri)/V]=-(R/L)t$$

Taking antilogarithm on both sides we get

$$(V-Ri)/V=e^{-Rt/L}$$

From which we can see that

$$i(t)=(V/R)-(V/R)e^{-Rt/L} \text{ for } t>0$$

Thus, an expression for the response valid for all time t would be

$$i(t)=V/R\,[1-e^{-Rt/L}]$$

This is normally written as:

$$i(t)=V/R\,[1-e^{-t/\tau}]$$

where 'τ' is called the *time constant* of the circuitand it's unit is seconds.

The voltage across the resistance and the Inductor for t >0 can be written as:

$$V_R(t) = i(t).R = V\,[1-e^{-t/\tau}]$$

$$V_L(t) = V - V_R(t) = V - V\,[1-e^{-t/\tau}] = V\,(e^{-t/\tau})$$

A plot of the current i(t) and the voltages $V_R(t)$ & $V_L(t)$ is shown in the figure 5.2.

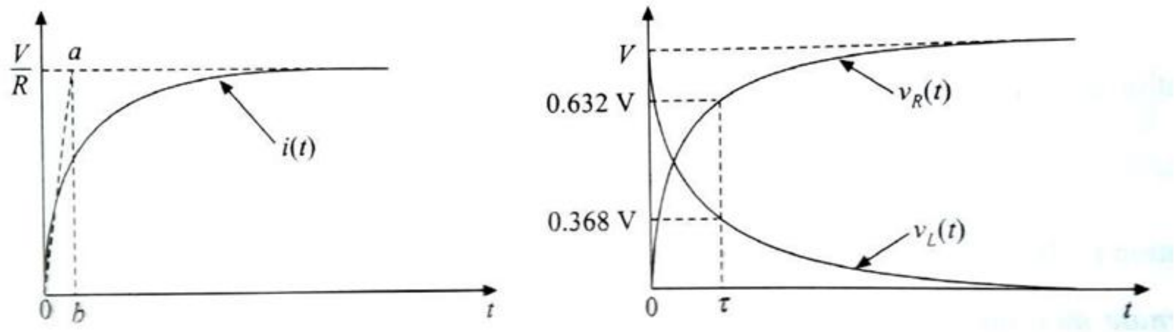


Figure 5.2 Transient current and voltages in the Series RL circuit.

At t = 'τ' the voltage across the inductor will be

$$v_L(\tau)=V\,(e^{-\tau/\tau}) = V/e = 0.36788\,V$$

and the voltage across the Resistor will be

$$v_R(\tau) \;=\; V\,[1- e^{-\tau/\tau}\,] = 0.63212\ V$$

The plots of current i(t) and the voltage across the Resistor $v_R(t)$ are called exponential growth curves and the voltage across the inductor $v_L(t)$is called exponential decay curve.

RC CIRCUIT with external DC excitation:

A series RC circuit with external DC excitation V volts connected through a switch is shown in the figure 5.3. If the capacitor is not charged initially i.e. it's voltage is zero ,then after the switch S is closed at time t=0, the capacitor voltage builds up gradually and reaches it's steady state value of V volts after a finite time. The charging current will be maximum initially (since initially capacitor voltage is zero and voltage across a capacitor cannot change instantaneously) and then it will gradually comedown as the capacitor voltage starts building up. The current and the voltage during such charging periods are called Transient Current and Transient Voltage.

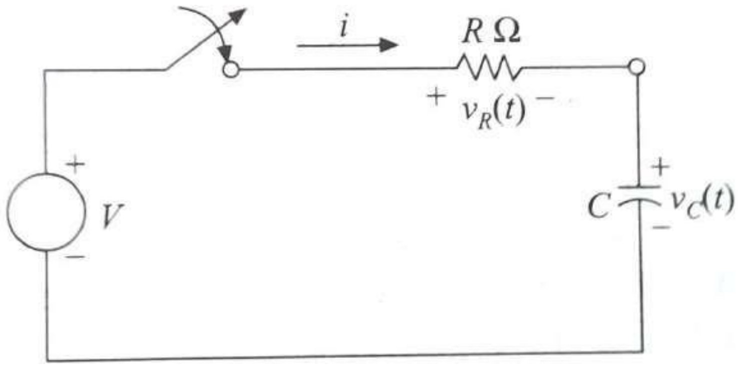


Figure 5.3 RC Circuit with external DC excitation

Applying KVL around the loop in the above circuit we can write

$$V \;=\; vR(t) + vC(t)$$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$$vC(t) = (1/C\,)\int \boldsymbol{i(t)dt} \text{ or } i(t) = C.[dvC(t)/dt]$$

and using this relation, vR(t) can be written asvR(t) = Ri(t) = R. C.[dvC(t)/dt]

Using the above two expressions for $\mathbf{v_R(t)}$ **and** $\mathbf{v_C(t)}$the above expression for **V** can be rewritten as **:**

$$\mathbf{V = R.\ C.[dv_C(t)/dt] + v_C(t)}$$

Or finally $\mathbf{dv_C(t)/dt + (1/RC).\ v_C(t) = V/RC}$

The inverse coefficient of $\mathbf{v_C(t)}$ is known as the time constant of the circuit $\boldsymbol{\tau}$ and is given by $\boldsymbol{\tau} = \mathbf{RC}$ and it's units are seconds**.**

The above equation is a first order differential equation and can be solved by using the same method of

separation of variables as we adopted for the LC circuit.

Multiplying the above equation $\mathbf{dv_C(t)/dt + (1/RC).\ v_C(t) = V/RC}$

both sides by 'dt' and rearranging the terms so as to separate the variables $v_C(t)$ and t we get:

$$dv_C(t) + (1/RC).\ v_C(t)\ .\ dt = (V/RC).dt$$

$$dv_C(t) \;=\; [(V/RC) - (1/RC).\ v_C(t)].\ dt$$

$$dv_C(t)\ /\ [(V/RC) - (1/RC).\ v_C(t)] \;=\; dt$$

$$R.\ C\ .\ dv_C(t)\ /\ [(V - v_C(t)] = dt$$

Now integrating both sides w.r.t their variables i.e. '$v_C(t)$' on the LHS and't' on the RHS we get

$$-RC \ln [V - v_C(t)] = t + k$$

where 'k'is the constant of integration.In order to evaluate k, an initial condition must be invoked. Prior to t = 0, vC(t)is zero, and thus vC(t)(0−) = 0. Since the voltage across a capacitor cannot change by a finite amount in zero time, we have vC(t)(0+) = 0. Setting vC(t)= 0 att = 0, in the above equation we obtain:

$$-RC \ln [V] = k$$

and substituting this value of $k = -RC \ln [V]$ in the above simplified equation

$-RC \ln [V - vC(t)] = t + k$

we get :

$-RC \ln [V - v_C(t)] = t - RC \ln [V]$

i.e. $-RC \ln [V - v_C(t)] + RC \ln [V] = t$ i.e. $-RC [\ln \{V - v_C(t)\} - \ln (V)] = t$

i.e. $[\ln \{V - v_C(t)\}] - \ln [V]\} = -t/RC$

i.e. $\ln [\{V - v_C(t)\}/(V)] = -t/RC$

Taking anti logarithm we get $[\{V - v_C(t)\}/(V)] = e^{-t/RC}$

i.e $v_C(t) = V(1 - e^{-t/RC})$

which is the voltage across the capacitor as a function of time .

The voltage across the Resistor is given by :

$v_R(t) = V - v_C(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$

And the current through the circuit is given by:

$i(t) = C.[dv_C(t)/dt] = (CV/CR)e^{-t/RC} = (V/R)e^{-t/RC}$

Or the other way: $i(t) = v_R(t)/R = (V.e^{-t/RC})/R = (V/R)e^{-t/RC}$

In terms of the time constant τ the expressions for $v_C(t)$, $v_R(t)$and $i(t)$ are given by :

$v_C(t) = V(1 - e^{-t/RC})$

$v_R(t) = V.e^{-t/RC}$

$i(t) \quad = \ (V/R\,)e^{-t/RC}$

The plots of current i(t) and the voltages across the resistor v_R(t)and capacitor v_C(t)are shown in the figure 5.4

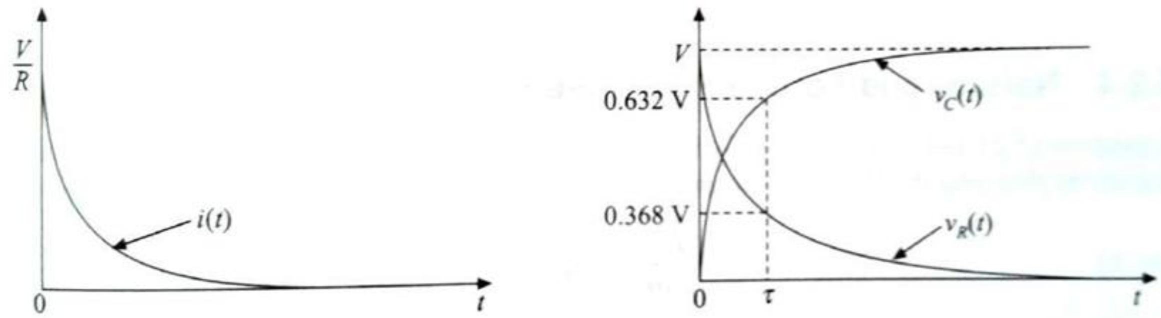


Figure 5.4 Transient current and voltages in RC circuit with DC excitation.

At **t** = **'τ'** the voltage across the capacitor will be**:**

$$\mathbf{v_C(\tau) \quad = V\,[1 - e^{-\tau/\tau}\,] \ = \ 0.63212\ V}$$

the voltage across the Resistor will be:

$$\mathbf{v_R(\tau) \ = \ V\,(e^{-\tau\,/\tau}) \ = \ V/e \quad = \quad 0.36788\,V}$$

and the current through the circuit will be:

$$\mathbf{i(\tau) \ = \ (V/R)\,(e^{-\tau\,/\tau}) \ = \ V/R.\ e \qquad = \ 0.36788\ (V/R)}$$

Thus it can be seen that after one time constant the charging current has decayed to approximately

% of it's value at **t=0** . At **t= 5 τ**charging current will be

$$\mathbf{i(5\tau) \ = \ (V/R)\,(e^{-5\tau\,/\tau}) \ = \ V/R.\ e^5 \quad = \ 0.0067(V/R)}$$

This value is very small compared to the maximum value of **(V/R)** at **t=0** .Thus it can be assumed that the capacitor is fully charged after 5 time constants.

The following similarities may be noted between the equations for the transients in the **LC** and **RC** circuits:

- The transient voltage across the Inductor in a **LC** circuit and the transient current in the **RC** circuit have the same form $\mathbf{k.(e^{-t\,/\tau})}$

- The transient current in a **LC** circuit and the transient voltage across the capacitor in the **RC** circuit have the same form **k.(1−$e^{-t/\tau}$)**

But the main difference between the **RC and RL** circuits is the effect of resistance on the duration of the transients.

- In a **RL** circuit a large resistance shortens the transient since the time constant τ **=L/R** become small.
- Where as in a **RC** circuit a large resistance prolongs the transient since the time constant τ **= RC** become large.

Discharge transients: Consider the circuit shown in the figure 5.5 where the switch allows both charging and discharging the capacitor. When the switch is position 1 the capacitor gets charged to the applied voltage V. When the switch is brought to position 2, the current discharges from the positive terminal of the capacitor to the negative terminal through the resistor R as shown in the figure 5.5 (b). The circuit in position 2 is also called ***source free circuit*** since there is no any applied voltage.

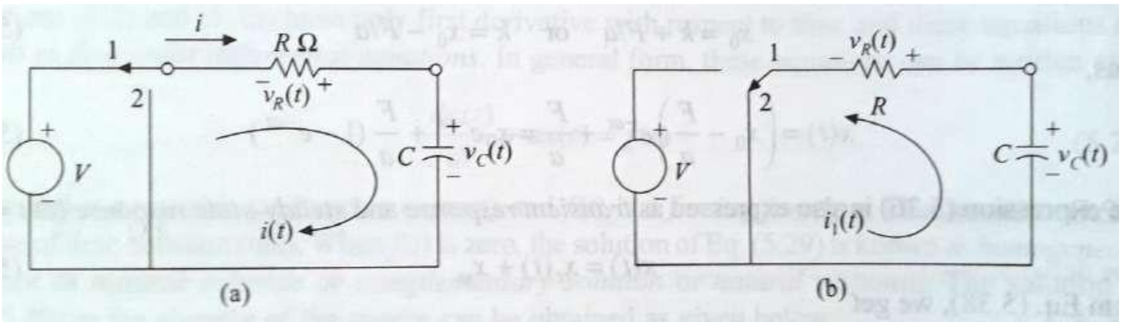


Figure 5.5: RC circuit (a) During Charging (b) During Discharging

The current i1 flow is in opposite direction as compared to the flow of the original charging current i. This process is called the *discharging of the capacitor*. The decaying voltage and the current are called the *discharge transients*. The resistor ,during the discharge will oppose the flow of current with the polarity of voltage as shown. Since there is no any external voltage source ,the algebraic sum of the voltages across the Resistance and the capacitor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$v_R(t)+v_C(t) = 0$ or $v_R(t) = - v_C(t)$

We know that vR(t) = R.i(t) = R. C.$dv_C(t)$ /dt. Substituting this in the first loop equation we get R. C.$dv_C(t)$/dt + $v_C(t)$ = 0

The solution for this equation is given by $v_C(t) = Ke^{-t/\tau}$ where K is a constant decided by the initial conditions and τ =RC is the time constant of the RC circuit

The value of K is found out by invoking the initial condition $v_C(t) = V$ @t = 0

Then we get K = V and hence **$v_C(t) = Ve^{-t/\tau}$; $v_R(t) = -Ve^{-t/\tau}$ and**

$i(t) = vR(t)/R = (-V/R)e^{-t/\tau}$

The plots of the voltages across the Resistor and the Capacitor are shown in the figure 5.6.

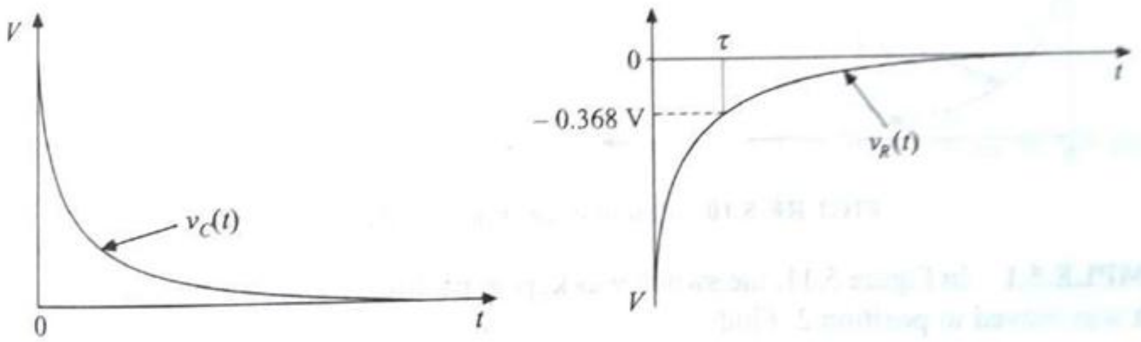


Figure5.6. Plot of Discharge transients in RC circuit

Decay transients: Consider the circuit shown in the figure 5.7 where the switch allows both growing and decaying of current through the Inductance . When the switch is position 1 the current through the Inductance builds up to the steady state value of V/R. When the switch is brought to position 2, the current decays gradually from V/R to zero. The circuit in position 2 is also called a ***source free circuit*** since there is no any applied voltage.

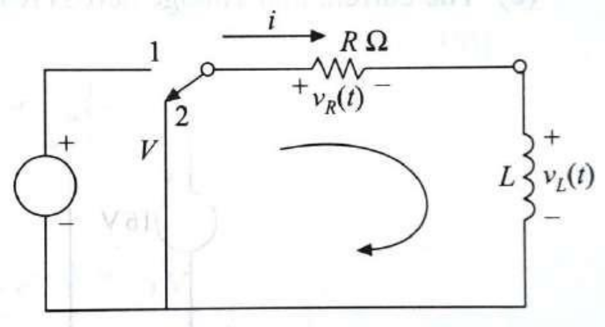


Figure 5.7: Decay Transient In RL circuit

The current flow during decay is in the same direction as compared to the flow of the original growing /build up current. The decaying voltage across the Resistor and the current are called the decay transients. Since there is no any external voltage source, the algebraic sum of the voltages across the Resistance and the Inductor will be zero (applying KVL) .The resulting loop equation during the discharge can be written as

$$v_R(t)+v_L(t) = R.i(t) + L.di(t)/dt = 0 \text{and } v_R(t) = - v_L(t)$$

The solution for this equation is given by $i(t) = Ke^{-t/\tau}$ where K is a constant decided by the initial conditions and τ =L/R is the time constant of the RL circuit.
The value of the constant K is found out by invoking the initial condition i(t) = V/R @t = 0
Then we get K = V/R and hence $i(t) = (V/R) . e^{-t/\tau}$;

$v_R(t) = R.i(t)= Ve^{-t/\tau}$ and $v_L(t) = - Ve^{-t/\tau}$

The plots of the voltages across the Resistor and the Inductor and the decaying current through the circuit are shown in the figure 5.8

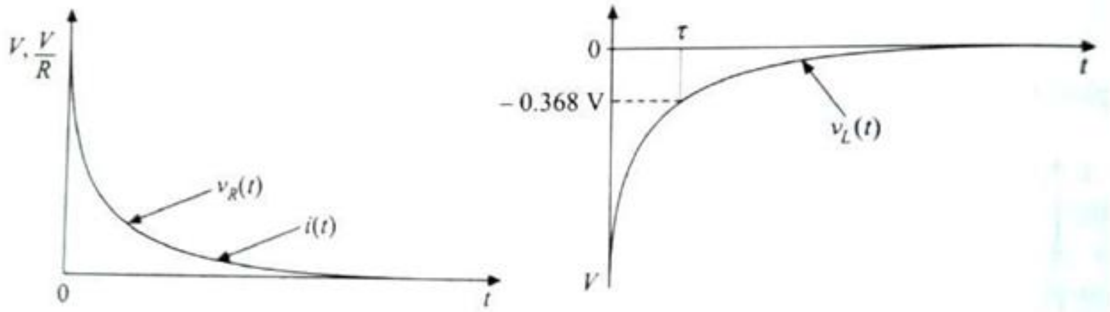


Figure 5.8 Plot of Decay transients in RL circuit

The Concept of Natural Response and forced response:

The **RL** and **RC** circuits we have studied are with external DC excitation. These circuits without the external DC excitation are called **source free circuits** and their Response obtained by solving the corresponding differential equations is known by many names. Since this response depends on the general **nature** of the circuit (type of elements, their size, their interconnection method etc.,) it is often called a **Natural response**. However any real circuit we construct cannot store energy forever. The resistances intrinsically associated with Inductances and Capacitors will eventually dissipate the stored energy into heat. The response eventually dies down,. Hence it is also called **Transient response**. As per the mathematician's nomenclature the solution of such a homogeneous linear differential equation is called **Complementary function**.

When we consider independent sources acting on a circuit, part of the response will resemble the nature of the particular source. (Or forcing function) This part of the response is called **particular solution. , the steady state response** or **forced response**. This will be complemented by the complementary function produced in the source free circuit. The complete response of the circuit is given by the sum of the **complementary function** and the **particular solution**. In other words:

The Complete response = Natural response + Forced response

There is also an excellent mathematical reason for considering the complete response to be composed of two parts—the **forced response** and the **natural response**. The reason is based on the fact that the solution of any linear differential equation may be expressed as the sum of two parts: the **complementary solution**(natural response) and **the particular solution**(forced response).

Determination of the Complete Response:

Let us use the same **RL** series circuit with external DC excitation to illustrate how to determine the complete response by the addition of the natural and forced responses. The circuit shown in the figure 5.9

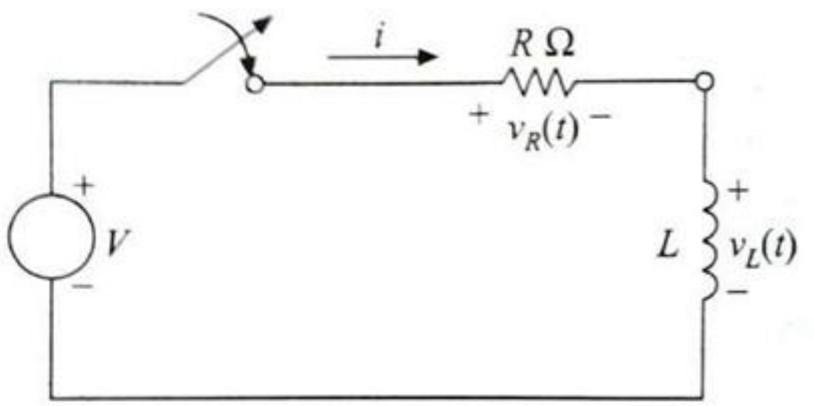


Figure 5.9 RL circuit with external DC excitation

was analyzed earlier, but by a different method. The desired response is the current *i (t)*, and now we first express this current as the sum of the natural and the forced current,

i = in+ i f

The functional form of the natural response must be the same as that obtained without any sources. We therefore replace the step-voltage source by a short circuit and call it the *RL source free* series loop. And in can be shown to be :

in= *Ae*−*Rt*/L

where the amplitude A is yet to be determined; since the initial condition applies to the complete response, we cannot simply assume A = i (0).We next consider the forced response. In this particular problem the forced response is constant, because the source is a constant V for all positive values of time. After the natural response has died out, there can be no voltage across the inductor; hence the all ythe applied voltage V appears across R, and the forced response is simply

i f = V/R

Note that the forced response is determined completely. There is no unknown amplitude. We next combine the two responses to obtain :

$i = Ae^{-Rt/L} + V/R$

And now we have to apply the initial condition to evaluate A. The current is zero prior to t = 0,and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after t = 0, and

$$A + V/R = 0$$

So that

$$A = -V/R$$

And $i = (V/R)(1 - e^{-Rt/L})$

Note carefully that *A* is not the initial value of i, since A = −V/R, while i (0) = 0.

But In source-free circuits, A would be the initial value of the response given by

in= I0e−Rt/L (where I0 =A is the current at time t=0). When forcing functions are present, however, we must first find the initial value of the complete response and then substitute this in the equation for the complete response to find A. Then this value of A is substituted in the expression for the total response i

Note that the forced response is determined completely. There is no unknown amplitude. We next combine the two responses to obtain :

$$i = Ae^{-Rt/L} + V/R$$

And now we have to apply the initial condition to evaluate *A*. The current is zero prior to *t* = 0,and it cannot change value instantaneously since it is the current flowing through an inductor. Thus, the current is zero immediately after *t* = 0, and

$$A + V/R = 0$$

So that

$$A = -V/R$$

And $i = (V/R)(1 - e^{-Rt/L})$

Note carefully that *A* is not the initial value of i, since A = −V/R, while i (0) = 0.

But In source-free circuits, A would be the initial value of the response given by $i_n = I_0 e^{-Rt/L}$ (where I_0 =A is the current at time t=0). When forcing functions are present, however,we must first find the initial value of the complete response and then substitute this in the equation for the complete response to find A.Then this value of A is substituted in the expression for the total response i

RLC CIRCUITS:

Earlier, we studied circuits which contained only one energy storage element, combined with a passive network which partly determined how long it took either the capacitor or the inductor to charge/discharge. The differential equations which resulted from analysis were always first-order. In this chapter, we consider more complex circuits which contain both an inductor and acapacitor. The result is a second-order differential equation for any voltage or current of interest. What we learned earlier is easily extended to the study of these so-called RLC circuits, although now we need two initial conditions to solve each differential equation. There are two types of RLC circuits: Parallel RLC circuits and Series circuits .Such circuits occur routinely in a wide variety of applications and are very important and hence we will study both these circuits.

Parallel RLC circuit:

Let us first consider the simple parallel RLC circuit with DC excitation as shown in the figure 5.10

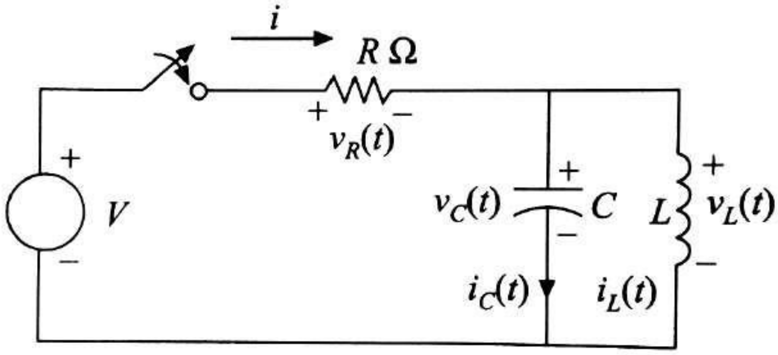


Figure 5.10:Parallel RLC circuit with DC excitation.

For the sake of simplifying the process of finding the response we shall also assume that the initial current in the inductor and the voltage across the capacitor are zero. Then applying theKirchhoff's current law **(KCL)($i = i_C + i_L$)**to the common node we get the following integrodifferential equation:

$$(V-v)/R = 1/L\int_{to}^{t} v\,dt' + C.dv/dt$$

$$V/R = v/R+1/L\int_{to} v\,dt' + C.dv/dt$$

Where $v = v_C(t) = v_L(t)$ is the variable whose value is to be obtained .

When we differentiate both sides of the above equation once with respect to time we get thestandard Linear second-order homogeneous differential equation

$C.(d^2v/dt^2)+ (1/R).(dv/dt)+ (1/L).v = 0$ $(d^2v/dt^2)+ (1/RC).(dv/dt)+ (1/LC).v = 0$

whose solution v(t) is the desired response. This can be written in the form:

$$[s^2 + (1/RC)s + (1/LC)].v(t) = 0$$

where 's' is an operator equivalent to (d/dt) and the corresponding characteristic equation(as explained earlier as a direct route to obtain the solution) is then given by :

$[s^2 + (1/RC)s + (1/LC)] = 0$

This equation is usually called the auxiliary equationor the characteristicequation, as we discussed earlier .If it can be satisfied, then our assumedsolution is correct. This is a quadratic equation and the roots s1 and s2are given as :

$s1= - 1/2RC+\sqrt{[(1/2RC)^2- 1/LC]}$ $s2= - 1/2RC-\sqrt{[(1/2RC)^2- 1/LC]}$

And we have the general form of the response as :

$v(t) = A1e^{s1t}+ A2e^{s2t}$

where s1 and s2 are given by the above equations and A1 and A2 are two arbitrary constants which are to be selected to satisfy the two specified initial conditions.

Definition of Frequency Terms:

The form of the natural response as given above gives very little insight into the nature of the curve we might obtain if v(t)were plotted as a function of time. The relative amplitudes of A1 and A2, for example, will certainly be important in determining the shape of the response curve. Further the constants s1 and s2 can be real numbers or conjugate complex numbers, depending upon the values of R, L, and Cin the given network. These two cases will produce fundamentally different response forms. Therefore, it will be helpful to make some simplifying substitutions in the equations for s1 and s2.Since the exponents s1tand s2t must be dimensionless, s1 and s2 must have the unit of some dimensionless quantity "per second." Hence in the equations for s1 and s2 we see that the units of 1/2RC and $1/\sqrt{LC}$ must also be s^{-1}(i.e., $seconds^{-1}$). Units of this type are called frequencies.

Now two new terms are defined as below :

$$\omega_0 = 1/\sqrt{LC}$$

which is termed as **resonant frequency** and

$$\alpha = 1/2RC$$

which is termed as the **exponential damping coefficient**

α the exponential damping coefficient is a measure of how rapidly the natural response decays or damps out to its steady, final value(usually zero). And **s, s_1, and s_2,** are called **complex frequencies.**
We should note that **s_1, s_2, α,** and **ω_0** are merely symbols used to simplify the discussion of **RLC** circuits. They are not mysterious new parameters of any kind. It is easier, for example, to say **"alpha"** than it is to say **"the reciprocal of 2RC."**

Now we can summarize these results.

The response of the parallel **RLC** circuit is given by :

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \cdots [1]$$

where

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \cdots [2]$$

$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2} \cdots [3]$$

and

$$\alpha = 1/2RC \text{ ....} [4]$$

$$\omega_0 = 1/\sqrt{LC} \text{......} [5]$$

A_1 and **A_2** must be found by applying the given initial conditions.
We note three basic scenarios possible with the equations for **s_1** and **s_2** depending on the relative values of α and **ω_0** (which are in turn dictated by the values of **R, L,** and **C**).

CaseA:

$\alpha > \omega_0$, i.e when **$(1/2RC)^2 > 1/LC$ s_1** and **s_2** will both be negative real numbers, leading to what is referred to as an **over damped response** given by :

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

Since **s_1** and **s_2** are both negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Since **s_2** is a larger negative number it decays faster and then the response is dictated by the first term **$A_1 e^{s_1 t}$.**

CaseB :

$\alpha = \omega_0$, ,i.e when **$(1/2RC)^2 = 1/LC$** , **s_1** and **s_2** are equal which leads to what is called a **critically damped response** given by **:**

$$v(t) = e^{-\alpha t}(A_1 t + A_2)$$

Case C :

$\alpha < \omega_0$,i.e when $(1/2RC)^2 < 1/LC$ both s_1 and s_2 will have nonzero imaginary components, leading to what is known as an under damped response given by :

$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where ω_d is called natural resonant frequency and is given by:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

We should also note that the general response given by the above equations [1] through [5] describe not only the voltage but all three branch currents in the parallel RLC circuit; the constants A_1 and A_2 will be different for each, of course.

Transient response of a series RLC circuit:

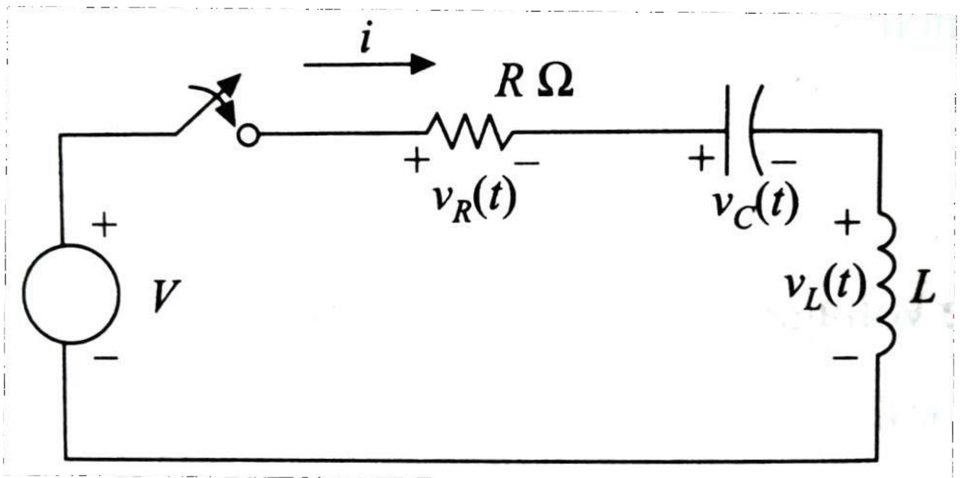


Figure 5.11: Series RLC circuit with external DC Excitation

Applying **KVL** to the series **RLC** circuit shown in the figure 5.11 at **t= 0** gives the following basic relation:

$$\mathbf{V = v_R(t) + v_C(t) + v_L(t)}$$

Representing the above voltages in terms of the current **i** in the circuit we get the following integrodifferential equation:

$$\mathbf{Ri + 1/C\int idt + L.\ (di/dt) = V}$$

To convert it into a differential equation it is differentiated on both sides with respect to time and we get

$L(d^2i/dt^2) + R(di/dt) + (1/C)i = 0$

This can be written in the form

$[S^2 + (R/L)s + (1/LC)].i = 0$ where 's' is an operator equivalent to (d/dt)

And the corresponding characteristic equation is then given by

$[s^2 + (R/L)s + (1/LC)] = 0$

This is in the standard quadratic equation form and the roots s1 and s2 are given by

$s1,s2 = -R/2L \pm \sqrt{[(R/2L)^2 - (1/LC)]} = -\alpha \pm \sqrt{(\alpha^2 - \omega 02)}$

Where α is known as the same exponential damping coefficient and ω0is known as the same Resonant frequency as explained in the case of Parallel RLC circuit and are given by :

$\alpha = R/2L$ and $\omega 0 = 1/\sqrt{LC}$

and A1 and A2must be found by applying the given initial conditions.

Here also we note three basic scenarios with the equations for s1 and s2 depending on the relative sizes of α and ω0 (dictated by the values of R, L, and C).

Case A:

$\alpha > \omega 0$,i.e when $(R/2L)^2 > 1/LC$, s1 and s2 will both be negative real numbers, leading to what is referred to as an over damped response given by :

$i(t) = A1e^{s1t} + A2e^{s2t}$

Since s1 and s2are both be negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Since s2 is a larger negative number it decays faster and then the response is dictated by the first term $A1e^{s1t}$.

Case B :

$\alpha = \omega 0$, ,i.e when $(R/2L)^2=1/LC$s1 and s2are equal which leads to what is called a critically damped responsegiven by :

$i(t) = e^{-\alpha t}(A1t + A2)$

Case C :

$\alpha < \omega 0$,i.e when $(R/2L)^2<1/LC$both s1 and s2 will have nonzero imaginary components, leading to what is known as an under damped responsegiven by :

$i(t) = e^{-\alpha t}(A1 \cos\omega d\, t + A2 \sin\omega d\ \ t)$

where ωd is called natural resonant frequency and is given given by:

$\omega d = \sqrt{\omega 02} - \alpha^2$

Here the constants A1 and A2 have to be calculated out based on the initial conditions case by case.

Series RL circuit with DC excitation:

Let us take the series RL circuit with external DC excitation shown in the figure 5.12.

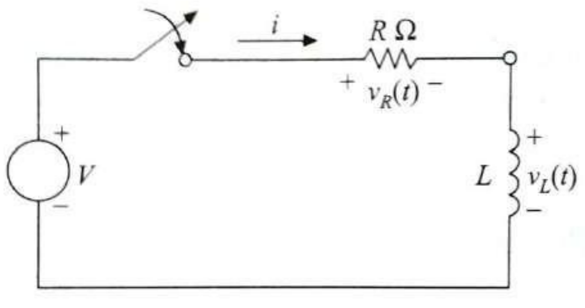


Figure 5.12 RL Circuit with external DC excitation

The governing equation is same as what we obtained earlier.

$$V = Ri + L\,di/dt \text{ for } t > 0$$

Taking Laplace transform of the above equation using the standard transform functions we get

$$V/s = R.I(s) + L[sI(s) - i(0)]$$

It may be noted here that i(0)is the initial value of the current at t=0 and since in our case at t=0 just when the switch is closed it is zero , the above equation becomes:

$$V/s = R.I(s) + L[sI(s)] = I(s)[R + L.s]$$

Taking inverse transform of the above expression for I(s)using the standard transform pairs we get the solution for i(t) as

$$i(t) = (V/R) - (V/R).e^{-(R/L)t} = (V/R)(1 - e^{-(R/L)t})$$

Which is the same as what we got earlier by solving the governing differential equation directly.

RC Circuit with external DC excitation:

Let us now take the series RC circuit with external DC excitation shown in the figure 5.13.

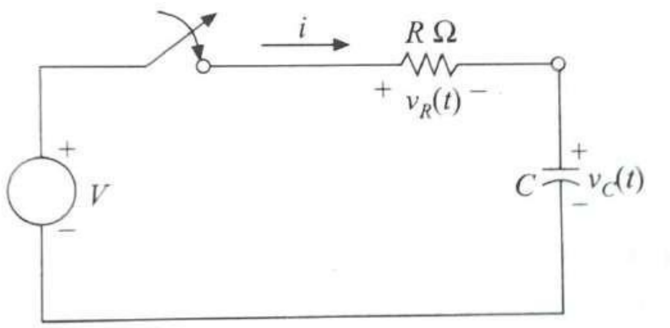


Figure 5.13 RC Circuit with external DC excitation

The governing equation is same as what we obtained earlier and is worked out again for easy understanding:

Applying KVL around the loop in the above circuit we can write:

$V = vR(t) + vC(t)$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$vC(t) = (1/C)\int \boldsymbol{i(t)dt}$ or $i(t) = C.[dvC(t)/dt]$

(Assuming that the initial voltage across the capacitor $vc(0) = 0$) and using this relation, $vR(t)$ can be written as $vR(t) = Ri(t) = R.\ C.[d\ vC(t)/dt]$

Using the above two expressions for vR(t) and vC(t)the above expression for V can be rewritten as :

$V = R.\ C.[d\ vC(t)/dt] + vC(t)$

Now we will take Laplace transform of the above equation using the standard Transform pairs and rules:

$V/s = R.C.s.vC(s) + vC(s)$

$V/s = vC(s)\ (R.C.s.+ 1)$

$vC(s) = (V/s)/\ (R.C.s + 1)$

$vC(s) = (V/RC)/\ [s.\ (s + 1/RC)]$

Now expanding this equation into partial fractions we get

$vC(s) = (V/RC)/\ [s.\ (s + 1/RC)] = A/s + B/(s + 1/RC) \quad (1)$

Where $A = (V/RC)/\ (1/RC)] = V$ and $B = (V/RC)/ -(1/RC)] = -V$

Substituting these values of A and B into the above equation (1) for vC(s) we get

$vC(s) = (V/s) - [V/(s + 1/RC)] = V[(1/s) - \{1/(s + 1/RC)\}]$

And now taking the inverse Laplace transform of the above equation we get

$vC(t) = V(1 - e^{-t/RC})$

which is the voltage across the capacitor as a function of time and is the same as what we obtained earlier by directly solving the differential equation. And the voltage across the Resistor is given by

$vR(t) = V - vC(t) = V - V(1 - e^{-t/RC}) = V.e^{-t/RC}$

And the current through the circuit is given by $i(t) = C.[dvC(t)/dt] = (CV/RC)e^{-t/RC} = (V/R)e^{-t/RC}$

Series RLC circuit with DC excitation:

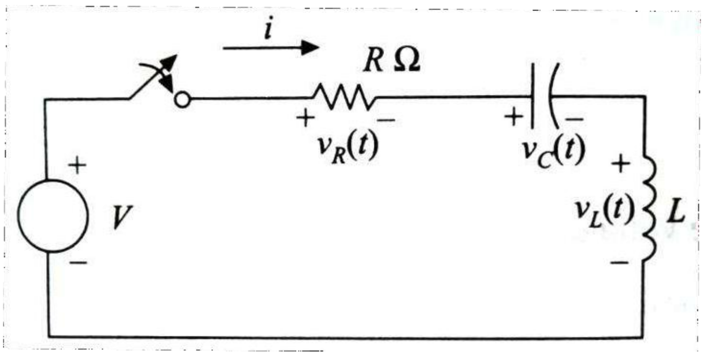


Figure 5.13 Series RLC circuit with DC excitation

The current through the circuit in the Laplace domain is given by :

$$I(s) = \frac{(V/s)}{(R + Ls + 1/Cs)}$$

[since **L [V] = V/s** and the Laplace equivalent of the series circuit is given by **Z(s) = (R + Ls + 1/Cs)]**

$$= V/ (Rs + Ls^2 + 1/C) = (V/L) / [s^2 + (R/L) s + 1/LC] = \frac{(V/L)}{(s+a)(s+b)}$$

Where the roots **'a'** and **'b'** are given by

$$a = -R/2L + \sqrt{(R/2L)^2 - 1/LC} \text{ and}$$

$$b = -R/2L - \sqrt{(R/2L)^2 - 1/LC}$$

It may be noted that there are three possible solutions for for **I(s)** and we will consider them.

Case A: Both **a** and **b** are real and not equal i.e. **(R/2L) > 1/√ LC**

Then **I(s)** can be expressed as $I(s) = \frac{(V/L)}{(s+a)(s+b)} = \frac{K1}{(s+a)} + \frac{K2}{(s+b)}$

Where $K1 = \left[\frac{(V/L)}{(s+b)} \right]_{s=-a} = \frac{(V/L)}{(b-a)}$

Where $K2 = \left[\frac{(V/L)}{(s+a)} \right]_{s=-b} = \frac{(V/L)}{(a-b)}$

Substituting these values of **K1** and **K2** in the expression for **I(s)** we get :

$$I(s) = \frac{(V/L)}{(s+a)(s+b)} = \frac{(V/L)}{(b-a)}\frac{1}{(s+a)} + \frac{(V/L)}{(a-b)}\frac{1}{(s+b)} \text{ and}$$

$$i(t) = \frac{(V/L)}{(b-a)} e^{-at} + \frac{(V/L)}{(a-b)} e^{-bt}$$

Case B : Both **a** and **b** are real and equal i.e. **(a=b=c) i.e. (R/2L) = 1/√ LC**

I(s) =(V/L)/ (s+c)² when **a = b = c** and

$$i(t) = (V/L).\, t.\, e^{-ct}$$

Case C : Both **a** and **b** are complex conjugates i.e. **a = b*** when **(R/2L) < 1/√ LC**

Adopting our standard definitions of $\alpha = R/2L$ $\omega_0 = 1/\sqrt{LC}$ and $\omega_d = \sqrt{(\omega_0^2 - \alpha^2)}$

The roots **a** and **b** are given by $\mathbf{a = \alpha + j\omega_d}$ and $\mathbf{b = \alpha - j\omega_d}$

Then **I(s)** can be expressed as $\mathbf{I(s)} = \dfrac{(V/L)}{(s+\alpha-j\,\omega d)\,(s+\alpha+j\,\omega d)} = \dfrac{K3}{(s+\alpha-j\,\omega d)} + \dfrac{K3*}{(s+\alpha+j\,\omega d)}$

Here $\mathbf{K3} = (s + \alpha - j\,\omega d).\,\mathbf{I(s)}\big|_{\,s = -\alpha + j\,\omega d} = \dfrac{(V/L)}{(s+\alpha+j\,\omega d)}\Big|_{\,s = -\alpha + j\,\omega d} = \dfrac{(V/L)}{2j\,\omega d}$

Therefore: $\mathbf{K3} = \dfrac{(V/L)}{2j\,\omega d}$ and $\mathbf{K3*} = -\dfrac{(V/L)}{2j\,\omega d}$

Now substituting these values **K3** and **K3*** in the above expanded equation for **I(s)** we get

$$I(s) = \frac{(V/L)}{2j\omega d}\,\frac{1}{(s+\alpha-j\omega d)} - \frac{(V/L)}{2j\omega d}\,\frac{1}{(s+\alpha+j\omega d)}$$

And now taking inverse transform of **I(s)** we get

$$i(t) = \frac{(V/L)}{2j\omega d}\,e^{-\alpha t}.\,e^{j\omega d\,t} - \frac{(V/L)}{2j\omega d}\,e^{-\alpha t}.\,e^{-j\omega d\,t}$$

$$\mathbf{i(t)} = \frac{(V/L)}{\omega d}\,e^{-\alpha t}\,\mathrm{Sin}\,\omega_d t$$

Example 1: Find the current in a series RL circuit having R =2Ω and L = 10H when a DC voltage V of 100V is applied. Find the value of the current 5 secs. after the application of the DC voltage.

Solution: This is a straightforward problem which can be solved by applying the formula. First let us find out the Time constant τ of the series LR circuit which is given by τ = L/R secs.

∴ $\tau = 10/2 = 5$ secs

The current in a series LR circuit after the sudden application of a DC voltage is given by :

$i(t) = V/R\,(1 - e^{-t/\tau})$

∴ i(t)at 5 secs = $100/2\,(1 - e^{-5/5}) = 5\,(1 - e^{-1}) = 50\,(1 - 1/e) = 31.48$

∴ **i(t)at 5 secs = 31. 48 Amps**

Example 2: A series RL circuit has R= 25 Ω and L = 5 Henry. A dc voltage V of 100 V is applied to this circuit at t = 0 secs. Find :

(a) The equations for the charging current , and voltage across R & L

(b) The current in the circuit 0.5 secs after the voltage is applied.

(c) The time at which the drops across R and L are equal.

Solution: The solutions for (a) and (b) are straightforward as in the earlier problem. (a)Time constant τ of the series LR circuit which is given by $\tau = L/R$ secs

$\therefore$ $\tau = 5/25 = 1/5$ secs

- The charging current is given by $i(t) = V/R\ (1 - e^{-t/\tau})$

It is also given by $i(t) = I\ (1 - e^{-t/\tau})$ where I is the final steady state current and is equal to $V/R = 100/25\ (1 - e^{-t/(1/5)}) = 4\ (1 - e^{-5t})$ Amps

$i(t) = 4\ (1 - e^{-5t})$ Amps

- The voltage across R is given by $v_R = i(t).R = V/R\ (1 - e^{-t/\tau}).R = V\ (1 - e^{-t/\tau})$

$v_R = 100\ (1 - e^{-5t})$

- The voltage drop across L can be found in two ways.

1. Voltage across Inductor $v_L = L\ di/dt$

2. Voltage across Inductor $v_L = V - v_R$

But it is easier to find using the second method. $\therefore$ $v_L = 100 - 100\ (1 - e^{-5t})$

$v_L = 100.\ e^{-5t}$

(b) At time t= 0.5 secs $i(t) = 4\ (1 - e^{5t}) = 4\ (1 - e^{-2.5}) = 3.67$ Amps

(c) To find out the time at which the voltages across the Inductor and the Resistor are equal we can equate the expressions for $v_R = 100\ (1 - e^{-5t})$ and $v_L = 100.e^{-5t}$ and solve for t. But the simpler method is, we know that since the applied voltage is 100 V the condition $v_R = v_L$ will also be satisfied when $v_R = v_L = 50$ V. i.e $v_R = 100\ (1 - e^{-5t}) = 50$ volts and $v_L = 100.e^{-5t} = 50$ V. We will solve the second equation [$v_L = 100.\ e^{-5t} = 50$ V] to get t which is easier.

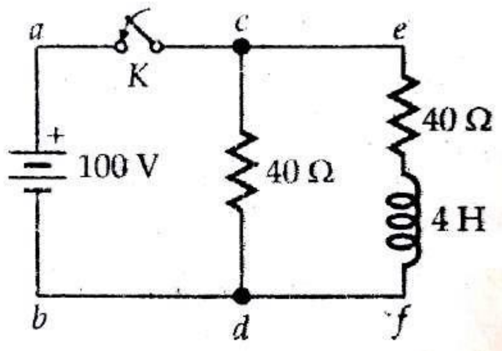
$e^{-5t} = 50/100 = 0.5$.

Taking natural logarithm on both sides we get:

$-5t\ .\ln(e) = \ln 0.5$ i.e $-5t\ .1 = -0.693$ i.e $t = 0.693/5 = 0.139$ secs

$\therefore$ The voltages across the resistance and the Inductance are equal at time t = 0.139 secs

Example 3: In the figure shown below after the steady state condition is reached , at time t=0 the switch K is suddenly opened. Find the value of the current through the inductor at time t = 0.5 seconds.



Solution: The current in the path acdb (through the resistance of 40 Ω alone) is 100/40 = 2.5Amps.(Both steady state and transient are same)
The steady state current through the path aefb (through the resistance of 40 Ω and inductance of 4H) is also = 100/40 = 2.5 Amps.

Now when the switch K is suddenly opened, the current through the path acdb(through the resistance of 40 Ω alone) immediately becomes zero because this path contains only resistance. But the current through the inductor decays gradually but now through the different path efdce
The decay current through a closed RL circuit is given by $I.e^{-t/\tau}$ where I is the earlier steady state current of 2.5 amps through L and $\tau = L/R$ of the decay circuit. It is to be noted carefully here that in the decay path both resistors are there and hence R =40+40 = 80Ω

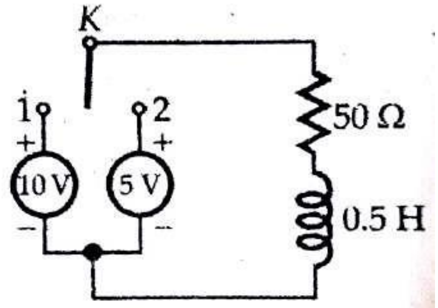
Hence $\tau = L/R = 4/80 = 0.05$ secs
Hence the current through the inductor at time 0.5 secs is given by i(t) @0.5secs $=2.5.e^{-0.5/0.05}$

i.e i(t) @0.5secs $= 2.5.e^{-10}$

i.e i(t) @0.5secs= $1.14\text{x}10^{-4}$ Amps

Example 4: In the circuit shown below the switch is closed to position 1 at time t = 0 secs. Then at time t = 0.5 secs the switch is moved to position 2. Find the expressions for the current through the circuit from 0 to 0. 5 msecs and beyond 0. 5 msecs.

Solution:The time constant $\tau$of the circuit in both the conditions is same and is given by $\tau = L/R$ $= 0.5/50 = 0.01$ secs



1. During the time t=0 to 0.5 msecs. i(t) is given by the standard expression for growing current through a L R circuit: i(t)during 0 to 0.5 msecs = V/R (1—e $^{-t/\tau}$)

i(t)during 0 to 0.5 msecs = V/R (1—e –t / 0.01) Amps

And the current i(t) @ t= 0.5 msecs = 10/50 (1-- e $^{-0.5x10-3 / 0.01}$) = 0.2 (1 – e$^{—0.05}$) = 9.75 mA

i(t) @ t = 0.5 msecs = 9.75 mA and this would be the initial current when the switch is moved to position 2

2. During the time beyond 0.5 msecs (switch is in position 2): The initial current is 9.75 mA . The standard expression for the growing currenti(t) = V/R (1—e –t / τ) is not applicable now since it has been derived with initial condition of i(t) =0 at t=0 where as the initial condition for the current i(t) now in position 2 is 9.75 mA . Now an expression for i(t) in position 2 is to be derived from first principles taking fresh t=0 and initial current i(0) as 9.75mA.

The governing equation in position 2 is given by :

50i+0.5di/dt = 5

We will use the same separation of variables method to solve this differential equation. Dividing the above equation by 0.5, then multiplying by dt and separating the terms containing the two variables i and t we get:

100i + di/dt = 10 i.e 100i.dt +di = 10.dt i.e di = dt (10 – 100i) i.e di/ (10 – 100i) = dt Now integrating on both sides we get --1/100 ln (10 -- 100i) = t + K (1)

The constant K is now to be evaluated by invoking the new initial condition i(t) = 9.75 mAat t =0 --1/100 ln (10 – 100x9.75X10--3) = K = --1/100 ln (10 – 0.975) = --1/100 ln (9.025)

Substituting this value of K in the above equation (1) we get

--1/100 ln (10 -- 100i) = t --1/100 ln (9.025)

--1/100 ln (10 -- 100i) + 1/100 ln (9.025) = t

--1/100 [ln (10 -- 100i) -- ln (9.025)] = t

--1/100 . ln [(10 -- 100i) / (9.025)] = t

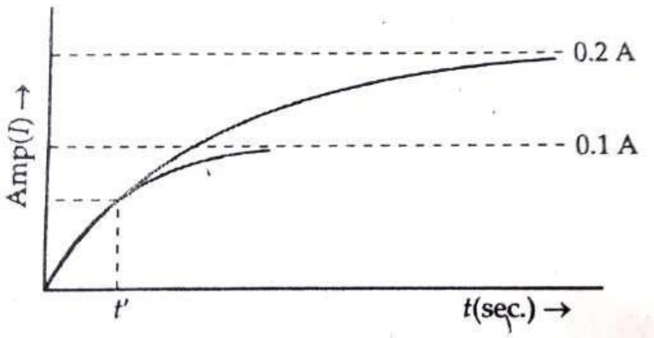
ln [(10 -- 100i) / (9.025)] = --100t

Taking antilogarithm to base **e** on both sides we get:

(10 -- 100i) / (9.025)] = e--100t (10 -- 100i) = 9.025 x e--100t (10 --9.025 x e--100t) = 100i

i = (10 --9.025 x e--100t)/ 100 = 10/100 -- 9.025 x e--100t /100

And finally i = 0.1 –0.09. e--100t



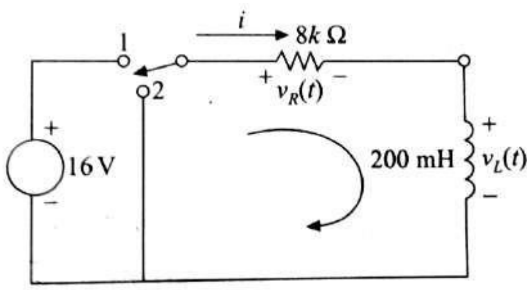
The currents during the periods t = o to 0.5 msec and beyond t = 0.5msec are shown in the figure below. Had the switch been in position 1 all through, the current would have reached the steady state value of 0.2 amps corresponding to source voltage of 10 volts as shown in the top curve. But since the switch is changed to position 2 the current changed it's path towards the new steady state current of 0.1 Amps corresponding the new source voltage of 5 Volts from 0.5 msecs onwards.

Example 5: In the circuit shown below the switch is kept in position 1 upto 250 μsecs and then moved to position 2. Find

(a) The current and voltage across the resistor at t = 100 μsecs

(b) The current and voltage across the resistor at t = 350 μsecs

Solution : The time constant τ of the circuit is given by τ = L/R = 200mH/8KΩ = 25 μsec and is same in both the switch positions.



(a) The current in the circuit upto 250 μsec (till switch is in position 1) is given by : i(t) growing = V/R (1 – e –t / τ) = (16/8)X10--3 (1 – e –t / 25 x10--6) = 2x(1 – e –t / 25 x10 –6) mA

- The current in the circuit @100µsec is given by

i(t) @100 µsec = 2x (1 – e $^{-100\ \mu sec / 25\ \mu sec}$) mA = 2x(1 – e $^{-4}$) mA = 1.9633 mA

i(t) @100 µsec = 1.9633 m

- The Voltage across the resistor is given by vR@100 µsec = R x i(t) @100 µsec

vR@100 µsec = 8 KΩ x1.9633 mA = 15.707 V vR@100 µsec = 15.707 V

(b)(b)

- The current in the circuit @350 µsec is the decaying current and is given by:

i(t)Decaying= I(0).e $^{-t/\tau}$ where I(0) is the initial current and in this case it is the growing current @250µsec. (Since the switch is changed @250µsec) The time t is to be reckoned from this time of 250 µsec. Hence t = (350—250) = 100µsec. So we have to calculate first i(t)growing(@250 µsec)which is given by:

i(t) growing(@250 µsec) = V/R (1 – e $^{-t/\tau}$) = (16/8)X10--3 (1 – e $^{-t / 25\ \mu sec}$) = 2x(1 – e $^{-250/ 25\ \mu sec}$) mA

=2x(1 – e $^{-10}$) mA = 1.999 mA

i(t)growing(@250 µsec)= 1.999 mA = I(0)

Hence i(t)@350 µsec =I(0).e $^{-t/\tau}$ = 1.99x e $^{-100\ \mu sec /25\ \mu sec}$mA = 1.99x e $^{-4}$mA = 0.03663 mA

i(t)@350 µsec = 0.03663 mA

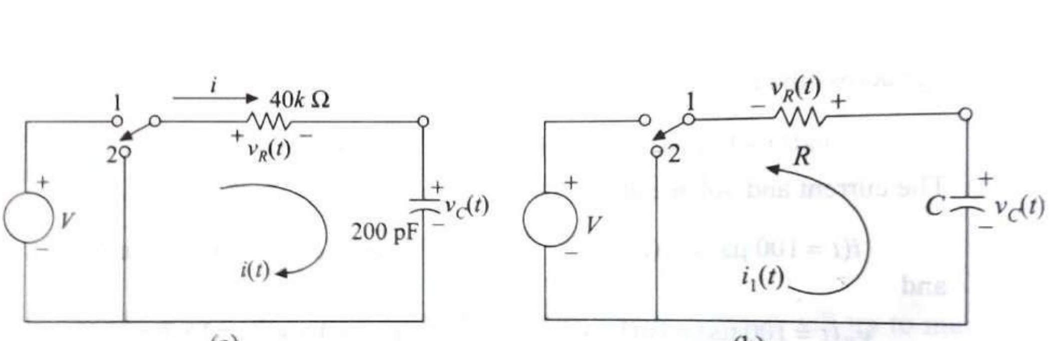
- The voltage across the resistor vR @350 µsec = Rxi(t@350 µsec) = 8KΩx0.03663 mA

vR @350 µsec= 0.293V

Example 6: In the circuit shown below the switch is kept in position 1 up to 100 µ secs and then it is moved to position 2 . Supply voltage is 5V DC . Find

a) The current and voltage across the capacitor at t = 40 µ secs

b) The current and voltage across the resistor at t = 150 µ secs



(a) (b)

Solution:The time constant τ of the circuit is same in both conditions and is given by τ = RC = 40x103x200x10x-12 = 8 µsec

a) The time t = 40 µsec corresponds to the switch in position 1 and in that condition the current i(t) is given by the standard expression for charging current

i(t) = (V/R) [e-t/τ]

i(t) @40 µsec = 5v/40KΩ [$e^{-40/8}$] Amps = 0.125x[e^{-5}] mA = 0.84224 µA

i(t) @40 µsec = 0.84224 µA

The voltage across the capacitor during the charging period is given by V [1- e-t/τ]. vC(t) @40 µsec = 5[1 -- $e^{-40/8}$] = 5[1 -- e^{-5}] = 4.9663 Volts

vC(t) @40 µsec = 4.9663 Volts

b) The time t = 150 µsec corresponds to the switch in position 2 and the current i(t) is given by the discharge voltage expression i(t) = [vC(t)0/R]. $e^{-t/\tau}$

Where vC(t)0 is the initial capacitor voltage when the switch was changed to position 2 and it is the voltage that has built up by 100 µsec during the charging time (switch in position 1) and hence is given by

vC(t)@100µsec = 5[1- $e^{-100/8}$] volts = 5x[1- $e^{-12.5}$] Volts = 4.999 Volts

And now t=150 µsec from beginning is equal to t = (150-100) = 50 µsec from the time switch is changed to position 2.

Therefore the current through the resistor at 150 µsec from the beginning = i(t)150µsec= (4.999/40KΩ). e-t/τ

i(t)150µsec = 0.1249 x $e^{-50/8}$ = 0.241 µA

i(t)150µsec = 0.241 µA

And the voltage across the resistor = R x i(t) = 40KΩ x 0.241 µA = 0.00964v

Series and Parallel Resonance

Series Resonance

Resonance occurs in electric circuits due to the presence of energy storing elements like inductor and capacitor. It is the fundamental concept based on which, the radio and TV receivers are designed in such a way that they should be able to select only the desired station frequency.

There are two types of resonances, namely series resonance and parallel resonance. These are classified based on the network elements that are connected in series or parallel. In this chapter, let us discuss about series resonance.

Series Resonance Circuit Diagram

If the resonance occurs in series RLC circuit, then it is called as Series Resonance. Consider the following series RLC circuit, which is represented in phasor domain.

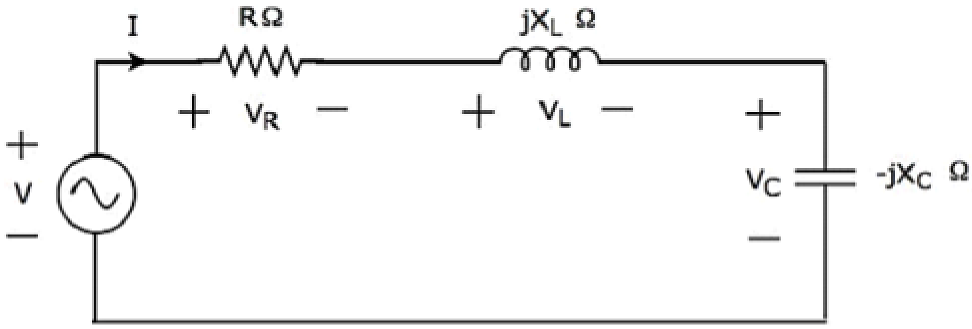


Figure 5.14 Series Resonance

Here, the passive elements such as resistor, inductor and capacitor are connected in series. This entire combination is in series with the input sinusoidal voltage source.

Apply KVL around the loop.

$$V-VR-VL-VC=0V-VR-VL-VC=0$$

$$\Rightarrow V-IR-I(jXL)-I(-jXC)=0\Rightarrow V-IR-I(jXL)-I(-jXC)=0$$

$$\Rightarrow V=IR+I(jXL)+I(-jXC)\Rightarrow V=IR+I(jXL)+I(-jXC)$$

$\Rightarrow V=I[R+j(XL-XC)]\Rightarrow V=I[R+j(XL-XC)]$Equation 1

The above equation is in the form of $V = IZ$.

Therefore, the impedance Z of series RLC circuit will be

$$Z=R+j(XL-XC)Z=R+j(XL-XC)$$

Parameters & Electrical Quantities at Resonance

Now, let us derive the values of parameters and electrical quantities at resonance of series RLC circuit one by one.

Resonant Frequency

The frequency at which resonance occurs is called as resonant frequency f_r. In series RLC circuit resonance occurs, when the imaginary term of impedance *Z* is zero, i.e., the value of XL−XCXL−XC should be equal to zero.

⇒XL=XC⇒XL=XC

Substitute XL=2πfLXL=2πfL and XC=12πfCXC=12πfC in the above equation.

2πfL=12πfC2πfL=12πfC

⇒f2=1(2π)2LC⇒f2=1(2π)2LC

⇒f=1(2π)LC−−−√⇒f=1(2π)LC

Therefore, the **resonant frequency** $\boldsymbol{f_r}$ of series RLC circuit is

fr=1(2π)LC−−−√fr=1(2π)LC

Where, ***L*** is the inductance of an inductor and ***C*** is the capacitance of a capacitor.

The **resonant frequency** $\boldsymbol{f_r}$ of series RLC circuit depends only on the inductance ***L*** and capacitance ***C***. But, it is independent of resistance ***R***.

Impedance

We got the **impedance Z** of series RLC circuit as

Z=R+j(XL−XC)Z=R+j(XL−XC)

Substitute XL=XCXL=XC in the above equation.

Z=R+j(XC−XC)Z=R+j(XC−XC)

⇒Z=R+j(0)⇒Z=R+j(0)

⇒Z=R⇒Z=R

At resonance, the **impedance *Z*** of series RLC circuit is equal to the value of resistance ***R***, i.e., ***Z = R***.

Current flowing through the Circuit

Substitute XL−XC=0XL−XC=0 in Equation 1.

V=I[R+j(0)]V=I[R+j(0)]

⇒V=IR⇒V=IR

⇒I=VR⇒I=VR

Therefore, current flowing through series RLC circuit at resonance is I=VRI=VR.

At resonance, the impedance of series RLC circuit reaches to minimum value. Hence, the maximum current flows through this circuit at resonance.

Voltage across Resistor

The voltage across resistor is

VR=IRVR=IR

Substitute the value of ***I*** in the above equation.

VR=VRRVR=VRR

⇒VR=V⇒VR=V

Therefore, the voltage across resistor at resonance is $V_R = V$.

Voltage across Inductor

The voltage across inductor is

VL=I(jXL)VL=I(jXL)

Substitute the value of ***I*** in the above equation.

VL=VR(jXL)VL=VR(jXL)

⇒VL=jXLRV⇒VL=jXLRV

⇒VL=jQV⇒VL=jQV

Therefore, the **voltage across inductor** at resonance is VL=jQVVL=jQV.

So, the **magnitude** of voltage across inductor at resonance will be

|VL|=QV|VL|=QV

Where ***Q*** is the **Quality factor** and its value is equal to XLRXLR

Voltage across Capacitor

The voltage across capacitor is

VC=I(−jXC)VC=I(−jXC)

Substitute the value of *I* in the above equation.

VC=VR(−jXC)VC=VR(−jXC)

⇒VC=−jXCRV⇒VC=−jXCRV

⇒VC=−jQV⇒VC=−jQV

Therefore, the voltage across capacitor at resonance is VC=−jQVVC=−jQV.

So, the magnitude of voltage across capacitor at resonance will be

|VC|=QV|VC|=QV

Where ***Q*** is the **Quality factor** and its value is equal to XCRXCR

Parallel Resonance

If the resonance occurs in parallel RLC circuit, then it is called as Parallel Resonance. Consider the following parallel RLC circuit, which is represented in phasor domain.

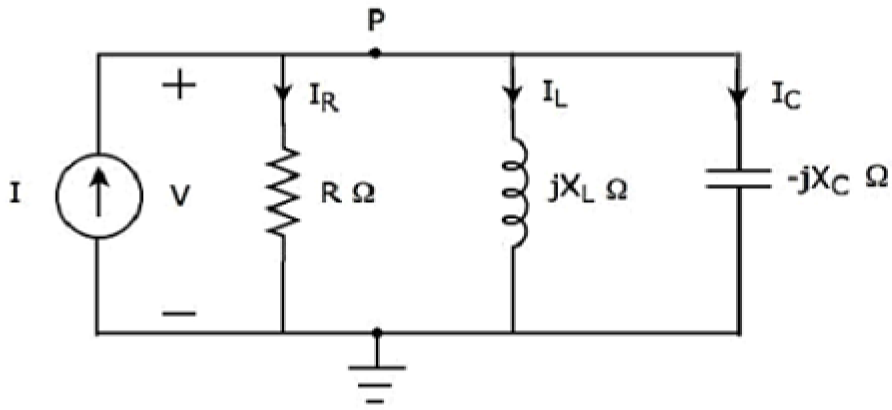


Figure 5.15 Parallel Resonance

Here, the passive elements such as resistor, inductor and capacitor are connected in parallel. This entire combination is in parallel with the input sinusoidal current source.

Write nodal equation at node P.

−I+IR+IL+IC=0−I+IR+IL+IC=0

⇒−I+VR+VjXL+V−jXC=0⇒−I+VR+VjXL+V−jXC=0

⇒I=VR−jVXL+jVXC⇒I=VR−jVXL+jVXC

⇒I=V[1R+j□1XC−1XL□]⇒I=V[1R+j□1XC−1XL□]Equation 1

The above equation is in the form of *I = VY*.

Therefore, the admittance Y of parallel RLC circuit will be

Y=1R+j□1XC−1XL□Y=1R+j□1XC−1XL□

Parameters & Electrical Quantities at Resonance

Now, let us derive the values of parameters and electrical quantities at resonance of parallel RLC circuit one by one.

Resonant Frequency

We know that the resonant frequency, f_r is the frequency at which, resonance occurs. In parallel RLC circuit resonance occurs, when the imaginary term of admittance, Y is zero. i.e., the value of 1XC−1XL1XC−1XL should be equal to zero

⇒1XC=1XL⇒1XC=1XL

⇒XL=XC⇒XL=XC

The above resonance condition is same as that of series RLC circuit. So, the resonant frequency, f_r will be same in both series RLC circuit and parallel RLC circuit.

Therefore, the resonant frequency, f_r of parallel RLC circuit is

fr=12πLC−−−√fr=12πLC

Where,

- L is the inductance of an inductor.
- C is the capacitance of a capacitor.

The resonant frequency, f_r of parallel RLC circuit depends only on the inductance *L* and capacitance *C*. But, it is independent of resistance *R*.

Admittance

We got the admittance Y of parallel RLC circuit as

Y=1R+j□1XC−1XL□Y=1R+j□1XC−1XL□

Substitute, XL=XCXL=XC in the above equation.

Y=1R+j□1XC−1XC□Y=1R+j□1XC−1XC□

⇒Y=1R+j(0)⇒Y=1R+j(0)

⇒Y=1R⇒Y=1R

At resonance, the admittance, Y of parallel RLC circuit is equal to the reciprocal of the resistance, R. i.e., Y=1RY=1R

Voltage across each Element

Substitute, 1XC−1XL=01XC−1XL=0 in Equation 1

I=V[1R+j(0)]I=V[1R+j(0)]

⇒I=VR⇒I=VR

⇒V=IR⇒V=IR

Therefore, the voltage across all the elements of parallel RLC circuit at resonance is $V = IR$.

At resonance, the admittance of parallel RLC circuit reaches to minimum value. Hence, maximum voltage is present across each element of this circuit at resonance.

Current flowing through Resistor

The current flowing through resistor is

IR=VRIR=VR

Substitute the value of *V* in the above equation.

IR=IRRIR=IRR

⇒IR=I⇒IR=I

Therefore, the current flowing through resistor at resonance is IR=IIR=I.

Current flowing through Inductor

The current flowing through inductor is

$$IL=VjXLIL=VjXL$$

Substitute the value of *V* in the above equation.

$$IL=IRjXLIL=IRjXL$$

$$\Rightarrow IL=-j□RXL□I\Rightarrow IL=-j□RXL□I$$

$$\Rightarrow IL=-jQI\Rightarrow IL=-jQI$$

Therefore, the current flowing through inductor at resonance is $IL=-jQIIL=-jQI$.

So, the magnitude of current flowing through inductor at resonance will be

$$|IL|=QI|IL|=QI$$

Where, Q is the Quality factor and its value is equal to RXLRXL

Current flowing through Capacitor

The current flowing through capacitor is

$$IC=V-jXCIC=V-jXC$$

Substitute the value of *V* in the above equation.

$$IC=IR-jXCIC=IR-jXC$$

$$\Rightarrow IC=j□RXC□I\Rightarrow IC=j□RXC□I$$

$$\Rightarrow IC=jQI\Rightarrow IC=jQI$$

Therefore, the current flowing through capacitor at resonance is $IC=jQIIC=jQI$

So, the magnitude of current flowing through capacitor at resonance will be

$$|IC|=QI|IC|=QI$$

Where, Q is the Quality factor and its value is equal to RXCRXC

TEXT / REFERENCE BOOKS

1. Kuo, "Network Analysis and Synthesis", John Wiley and Sons Inc., 2014.

2. Umesh Sinha, "Network Analysis and Synthesis", 5th Edition, Sathya Prakashan Publishers, 2010.