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SCHOOL OF ELECTRICAL AND ELECTRONICS ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT - I - Electromagnetic Theory - SEEA1202

I. Electric Field

COORDINATE SYSTEMS:

The dimension of space comes from nature. The measurement of space comes from us. The laws of electromagnetic are independent of a particular coordinate system. However application of the laws to the solution of a particular problem imposes the need to use a suitable coordinate system. It is the shape of the boundary that determines the most suitable coordinate system to use in its solution. To represent points in space we need a coordinate system. The co-ordinate system may be orthogonal or non-orthogonal. We discuss only right handed, orthogonal coordinate systems. The coordinate systems discussed are three dimensional coordinate systems. The coordinate systems are dened by a set of planes and/or surfaces. Coordinate system denes a set of three reference directions at each and every point in space. The origin of the coordinate system is the reference point relative to which we locate every other point in space. A position vector denes the position of a point in space relative to the origin. The three reference directions are referred to as coordinate directions. Unit vectors along the coordinate directions are called base vectors. In any three dimensional coordinate system , an arbitrary vector can be expressed in terms of a superposition of the three base vectors.

CARTESIAN COORDINATE SYSTEM

A Cartesian coordinate system in two dimensions (also called a **rectangular coordinate system** or an **orthogonal coordinate system)** is defined by an ordered pair of perpendicular lines (axes), a single unit of length for both axes, and an orientation for each axis. The point where the axes meet is taken as the origin for both, thus turning each axis into a number line. For any point *P*, a line is drawn through *P* perpendicular to each axis, and the position where it meets the axis is interpreted as a number. The two numbers, in that chosen order, are the *Cartesian coordinates* of *P*. The reverse construction allows one to determine the point *P* given its coordinates.

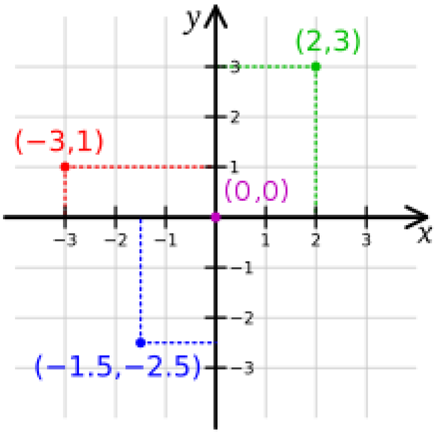
The first and second coordinates are called the *abscissa* and the *ordinate* of *P*, respectively; and the point where the axes meet is called the *origin* of the coordinate system. The coordinates are usually written as two numbers in parentheses, in that order, separated by a comma, as in (3, −10.5). Thus the origin has coordinates (0, 0), and the points on the positive half-axes, one unit away from the origin, have coordinates (1, 0) and (0, 1).

In mathematics, physics, and engineering, the first axis is usually defined or depicted as horizontal and oriented to the right, and the second axis is vertical and oriented upwards. (However, in some computer graphics contexts, the ordinate axis may be oriented downwards.) The origin is often labeled *O*, and the two coordinates are often denoted by the letters *X* and *Y*, or *x* and *y*. The axes may then be referred to as the *X*-axis and *Y*-axis. The choices of letters come from the original convention, which is to use the latter part of the alphabet to indicate unknown values. The first part of the alphabet was used to designate known values.

A Euclidean plane with a chosen Cartesian coordinate system is called a *Cartesian plane*. In a Cartesian plane one can define canonical representatives of certain geometric figures, such as the unit circle (with radius equal to the length unit, and center at the origin), the unit square (whose diagonal has endpoints at (0, 0) and (1, 1)), the unit hyperbola, and so on.

The two axes divide the plane into four right angles, called *quadrants*. The quadrants may be named or numbered in various ways, but the quadrant where all coordinates are positive is usually called the *first quadrant*.

Illustration of a Cartesian coordinate plane. Four points are marked and labeled with their coordinates: (2, 3) in green, (−3, 1) in red, (−1.5, −2.5) in blue, and the origin (0, 0) in purple.



Cylindrical coordinate system

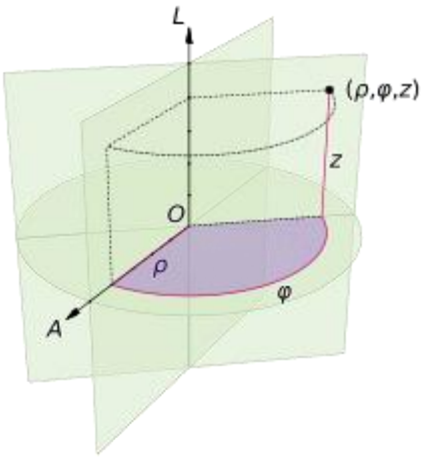
A **cylindrical coordinate system** is a three-dimensional coordinate system that specifies point positions by the distance from a chosen reference axis, the direction from the axis relative to a chosen reference direction, and the distance from a chosen reference plane perpendicular to the axis. The latter distance is given as a positive or negative number depending on which side of the reference plane faces the point.

The *origin* of the system is the point where all three coordinates can be given as zero. This is the intersection between the reference plane and the axis. The axis is variously called the *cylindrical* or *longitudinal* axis, to differentiate it from the *polar axis*, which is the ray that lies in the reference plane, starting at the origin and pointing in the reference direction. Other directions perpendicular to the longitudinal axis are called *radial lines*.

The distance from the axis may be called the *radial distance* or *radius*, while the angular coordinate is sometimes referred to as the *angular position* or as the *azimuth*. The radius and the azimuth are together called the *polar coordinates*, as they correspond to a two-dimensional polar coordinate system in the plane through the point, parallel to the reference plane. The third coordinate may be called the *height* or *altitude* (if the reference plane is considered horizontal), *longitudinal position*, or *axial position*.

Cylindrical coordinates are useful in connection with objects and phenomena that have some rotational symmetry about the longitudinal axis, such as water flow in a straight pipe with round cross-section, heat distribution in a metal cylinder, electromagnetic fields produced by an electric current in a long, straight wire, accretion disks in astronomy, and so on.

A cylindrical coordinate system with origin *O*, polar axis *A*, and longitudinal axis *L*. The dot is the point with radial distance $\rho = 4$, angular coordinate $\varphi = 130°$, and height $z = 4$.



Spherical coordinate system

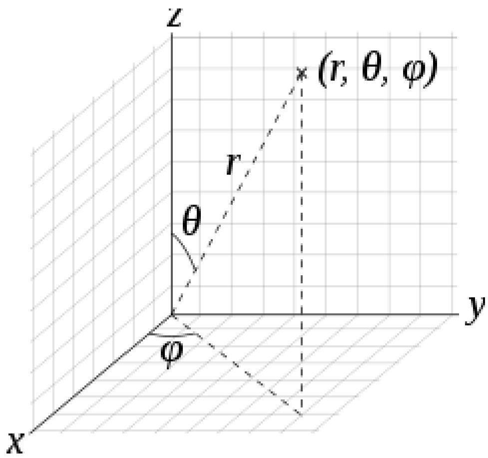
A **spherical coordinate system** is a coordinate system for three-dimensional space where the position of a point is specified by three numbers: the *radial distance* of that point from a fixed origin, its *polar angle* measured from a fixed zenith direction, and the *azimuthal angle* of its orthogonal projection on a reference plane that passes through the origin and is orthogonal to the zenith, measured from a fixed reference direction on that plane. It can be seen as the three-dimensional version of the polar coordinate system.

The radial distance is also called the *radius* or *radial coordinate*. The polar angle may be called *colatitude*, *zenith angle*, *normal angle*, or *inclination angle*.

According to the conventions of geographical coordinate systems, positions are measured by latitude, longitude, and height (altitude). There are a number of celestial coordinate systems based on different fundamental planes and with different terms for the various coordinates. The spherical coordinate systems used in mathematics normally use radians rather than degrees and measure the azimuthal angle counterclockwise from the *x*-axis to the *y*-axis rather than clockwise from north (0°) to east (+90°) like the horizontal coordinate system. The polar angle is often replaced by the *elevation angle* measured from the reference plane, so that the elevation angle of zero is at the horizon.

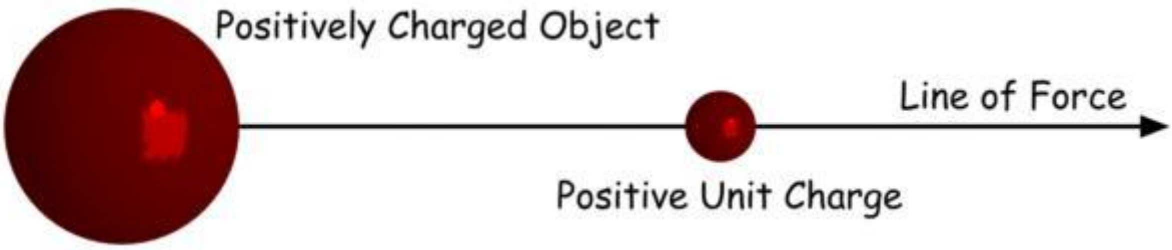
The spherical coordinate system generalizes the two-dimensional polar coordinate system. It can also be extended to higher-dimensional spaces and is then referred to as a hyperspherical coordinate system.

Spherical coordinates (r, θ, φ) as commonly used in ***physics*** (ISO convention): radial distance r, polar angle θ (theta), and azimuthal angle φ (phi). The symbol ρ (rho) is often used instead of r.

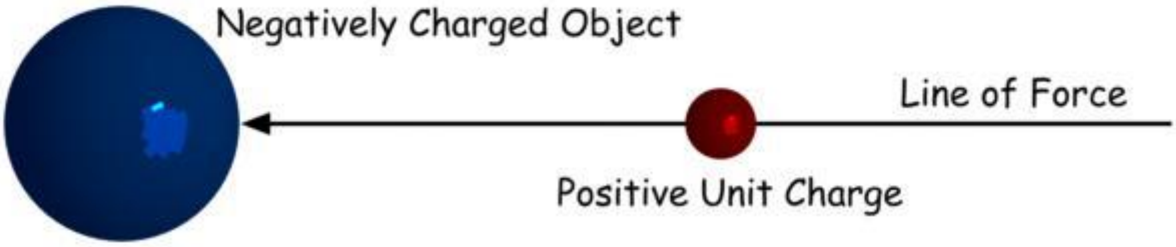


Electric Field

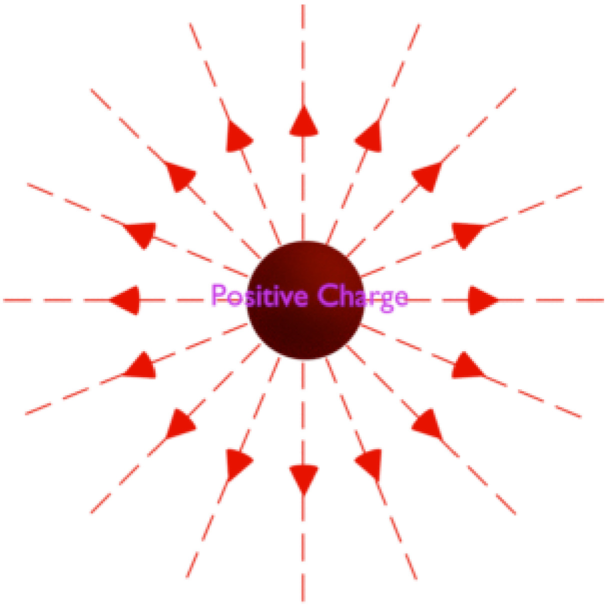
An electric field is measured by a term known as electric field intensity. If we place a positive unit charge near a positively charged object, the positive unit charge will experience a repulsive force. Due to this force, the positive unit charge will move away from the said charged object. The imaginary line through which the unit positive charge moves, is known as line of force.



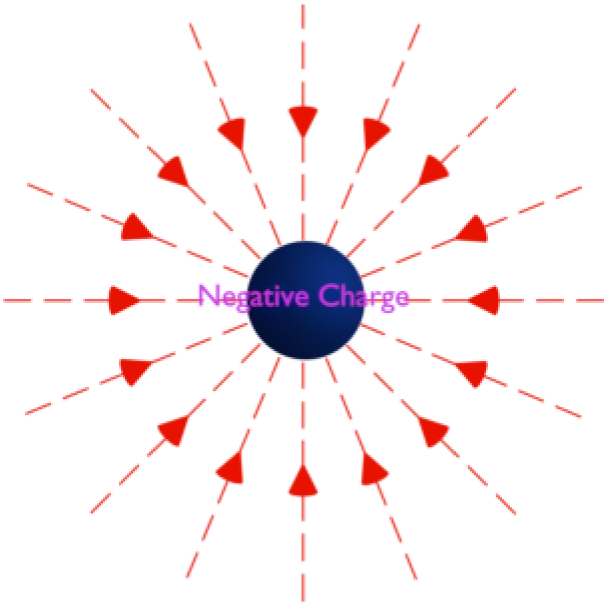
Similarly, if we place a positive unit in the field of a negatively charged object, the unit positive charge will experience an attractive force. Due to this force, the unit positive charge will come closer to the said negatively charged object. In that case, line through which the positive unit charge moves, is called line of force.



We can place a unit positive anywhere surround the positively charged object and each position where we place it, the unit positive charge follows a separate line force to move. Hence, we can say, the lines of force get radiated or come out from this charged object.



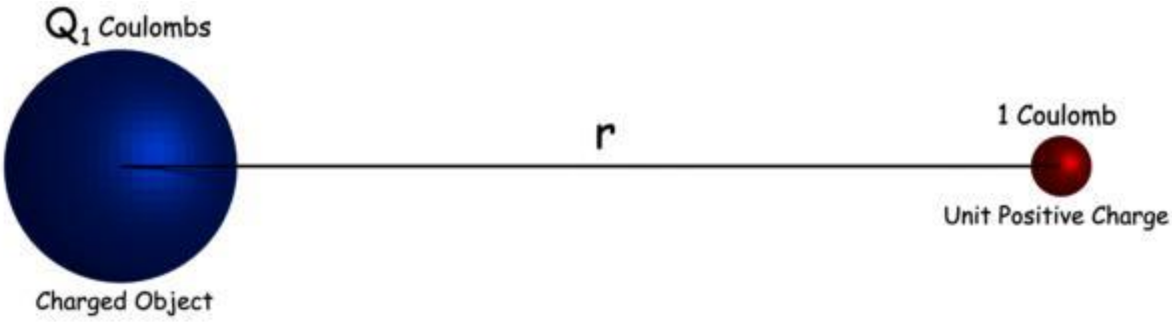
But for a negatively charged object, these lines of force come into this negatively charged object.



VALUE OF ELECTRIC FIELD

Electric field at a point in the space is measured as the force acting on the unit positive charge at that point. When a charged object enters the **electric field** of another charged object, it experiences a force as per Coulomb's law.

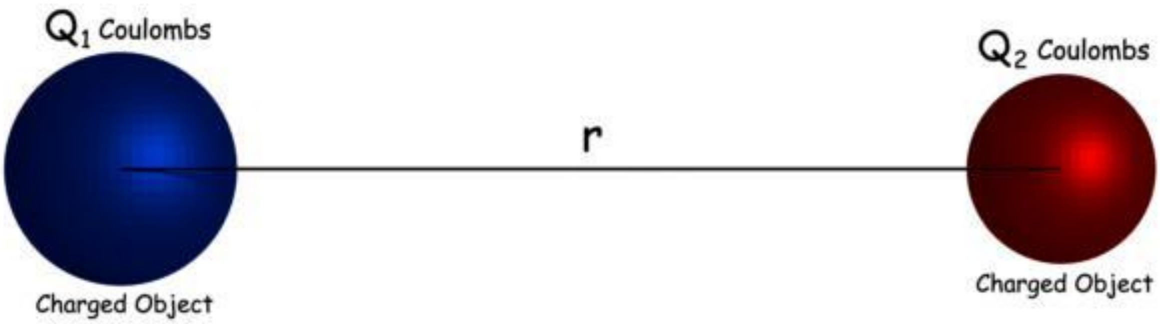
Let us take a charged object, of charge Q_1 Coulomb. Let us place one unit positive charge at r meter away from the center of Q_1.



So, as per Coulomb's law, the force experienced by the unit positive charge is,

$$\frac{Q_1.1}{4\pi\epsilon_0 r^2} = \frac{Q_1}{4\pi\epsilon_0 r^2} = \vec{E}$$

Here, we have considered that the medium is air or vacuum in which we placed both charge Q_1 and unit positive charge. The force experienced by the unit positive charge is the measurement of the electric field of Q_1 at the point where we have placed the unit positive charge. This force vector is denoted by $\overrightarrow{E}$. The term $\overrightarrow{E}$ is known as electric field intensity or electric field strength. Now, we place an object of charge Q_2 coulombs at same point where, unit positive charge was placed.



Two positively charged objects repel each other, two negatively charged objects repel each other and two oppositely charged objects attract each other with force $\overrightarrow{F}$.

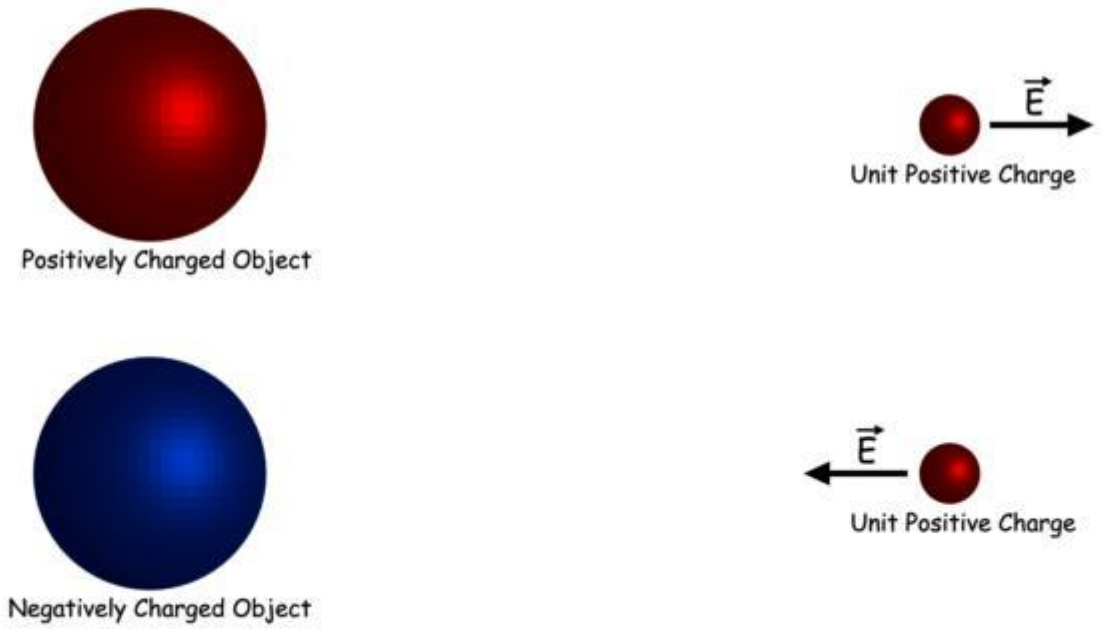
This attraction or repulsion force $\overrightarrow{F}$ can be written as.

$$\vec{F} = \frac{Q_1 . Q_2}{4\pi\epsilon_0 r^2} = \left(\frac{Q_1}{4\pi\epsilon_0 r^2}\right) . Q_2 = \vec{E} . Q_2$$

So, the vector of electric field $\overrightarrow{E}$, determines how strongly an electric charge is repulsed or attracted by the charge which has created the electric field.

DIRECTION OF ELECTRIC FIELD

When we place the imaginary unit positive charge in an electric field, the unit positive charge starts moving due to electrostatic force of the field. The unit charge either comes closer or goes far, depending upon the nature of the charge, by which the field is created. The direction of the movement of the unit positive charge in the field is considered as the direction of the electric field. So, an electric field is a vector quantity.



Static Electric Field

We know there are two types of charge present in nature (i) positive and (ii) negative charge. In positive charge, there is mainly deficiency of electrons and in a negative charge, there are excess of electrons. Now, we can simply understand the concepts of charge from a very basic example. Take a dry comb, comb your hair (which should be dry) two to three times, now take that comb near tiny pieces of paper, you will see that the paper pieces are getting attracted to the comb. This is the very basic example of electric charge and **static electric field**. Due to friction, there is a movement of electrons between comb and hair, so one of them gets positively charged and another one gets negatively charged and as the paper is neutral (i.e. not charged) they get attracted to the comb.

So, we can see that there is an attraction force works between a charged particle and neutral particle, it has been seen further that there is repulsion between two same charged particles and attraction between two oppositely charged particles. This happens due to the field created by a particle. This can be understood if we imagine a glowing bulb, the bulb can be taken as the charge and the visible light can be compared to **static electric field**, the characteristic of the field is similar to the light in the sense that the intensity of the field is greater near the source and it fades as we move further from the source. Now from another point of view, we can say that static electric field is nothing but an intense space, in terms of power where work is done or needed to be done upon in presence of an electrically charged particle depending on the nature of the charged particle.

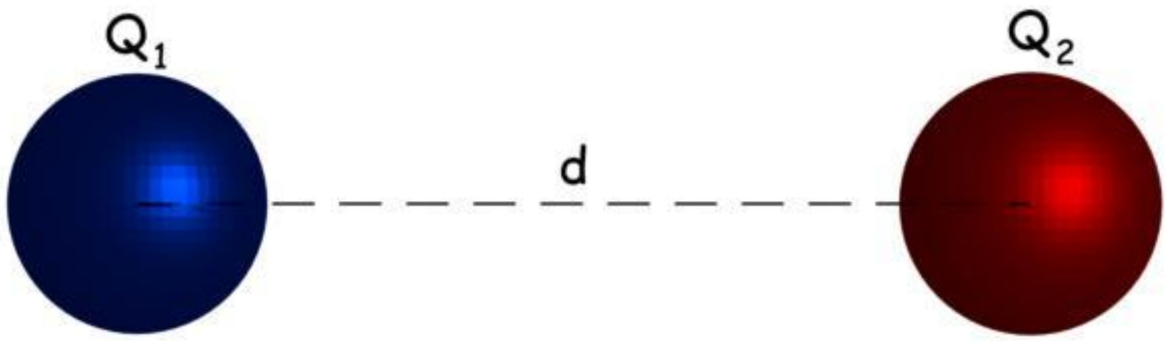
Coulomb's Law: Definition, Formula, & Constant

If two electrically charged bodies are placed nearby each other there will be an attraction or a repulsion force acting on them depending upon the nature of the charge of the bodies. The formula for the force acting between two electrically charged bodies was first developed by Charles-Augustin de Coulomb and the formula he established for determining the value of force acting to nearby charged objects is known as Coulomb's law.

In his law, he stated that to similarly charged (either positive or negative) bodies will repeal each other and two dissimilarly charged bodies (one is positively charged and other is negatively charged) will attract each other. He had also stated that the force acting between the electrically charged bodies is proportional to the product of the charge of the charged bodies and inversely proportional to the square of the distance between the center of the charged bodies.

Coulomb's Law Formula

Let us imagine, Q_1 and Q_2 are the electrical charges of two objects.
d is the distance between the center of the objects.



The charged objects are placed in a medium of permittivity $\varepsilon_o\varepsilon_r$

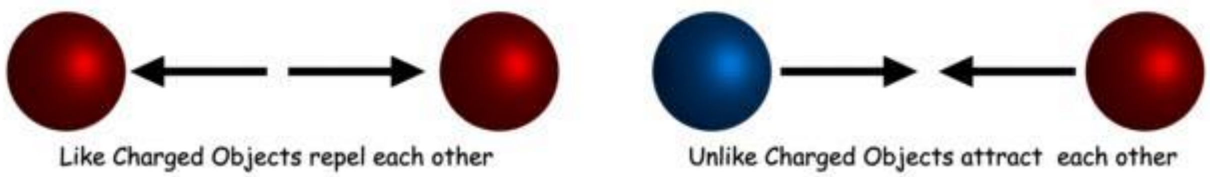
Then we can write the force 'F' as:

$$F = \frac{Q_1\ Q_2}{4\pi\epsilon_o\epsilon_r\ d^2}$$

Statement of Coulomb's Law

If you would prefer a video explanation, you can watch the video below:

First Law



Like charged objects (bodies or particles) repel each other and unlike charged objects (bodies or particles) attract each other.

Second Law

The force of attraction or repulsion between two electrically charged objects is directly proportional to the magnitude of their charge and inversely proportional to the square of the distance between them. Hence, according to the Coulomb's second law,

$$F \propto Q_1\, Q_2 \;\; \& \;\; F \propto \frac{1}{d^2}$$

$$\Rightarrow F \propto \frac{Q_1\, Q_2}{d^2} \Rightarrow F = k\frac{Q_1\, Q_2}{d^2}$$

Where,

1. 'F' is the repulsion or attraction force between two charged objects.
2. 'Q_1' and 'Q_2' are the electrical charged of the objects.
3. 'd' is distance between center of the two charged objects.
4. 'k' is a constant that depends on the medium in which charged objects are placed. In S.I. system, as well as in M.K.S. system $k=1/4\pi\varepsilon_o\varepsilon_r$. Hence, the above equation becomes.

$$F = \frac{Q_1\, Q_2}{4\pi\epsilon_o\epsilon_r\, d^2}$$

The value of $\varepsilon_o = 8.854 \times 10^{-12}\ C^2/Nm^2$.

$$Now,\ F = \frac{Q_1\, Q_2}{4\pi \times\ 8.854\ 10^{-12} \times \epsilon_r\, d^2}$$

$$= 8.9878 \times 10^9 \left[\frac{Q_1\, Q_2}{\epsilon_r\, d^2}\right] \approx 9 \times 10^9 \left[\frac{Q_1\, Q_2}{\epsilon_r\, d^2}\right]$$

Hence, Coulomb's law can be written for medium as,

$$F_{medium} = 9 \times 10^9 \left[\frac{Q_1\, Q_2}{\epsilon_r\, d^2}\right]$$

Then, in air or vacuum $\varepsilon_r = 1$. Hence, **Coulomb's law** can be written for air medium as,

$$F_{air} = 9 \times 10^9 \left[\frac{Q_1 Q_2}{d^2}\right]$$

The value of ε_r would change depends on the medium. The expression for relative permittivity ε_r is as follows;

$$\epsilon_r = \frac{F_{air}}{F_{medium}} = \frac{Force\ developed\ between\ charged\ bodies\ in\ the\ air}{Force\ developed\ between\ same\ charged\ bodies\ in\ the\ medium}$$

Principle of Coulomb's Law

Suppose if we have two charged bodies one is positively charged and one is negatively charged, then they will attract each other if they are kept at a certain distance from each other. Now if we increase the charge of one body keeping other unchanged, the attraction force is obviously increased. Similarly, if we increase the charge of the second body keeping the first one unchanged, the attraction force between them is again increased. Hence, the force between the charge bodies is proportional to the charge of either bodies or both.

$$F \propto Q_1 \ \ \& \ \ F \propto Q_2 \ \Rightarrow \ F \propto Q_1 Q_2$$

Now, by keeping their charge fixed at Q_1 and Q_2 if you bring them nearer to each other the force between them increases and if you take them away from each other the force acting between them decreases. If the distance between the two charge bodies is d, it can be proved that the force acting on them is inversely proportional to d^2.

$$F \propto \frac{1}{d^2}$$

This development of force between two same charged bodies is not the same in all mediums. As we discussed in the above formulas, ε_r would change for various medium. So, depends on the medium, creation of force can be varied.

$$F \propto \frac{1}{\epsilon_r}$$

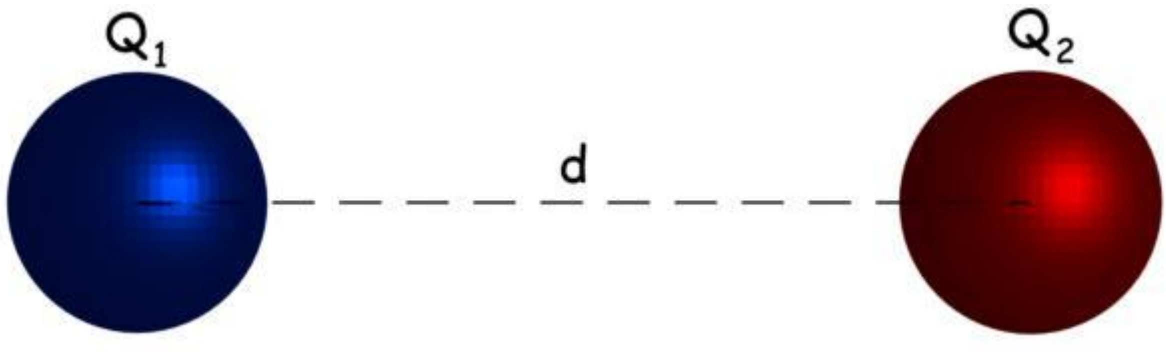
Limitation of Coulomb's Law

1. **Coulomb's law** is valid, if the average number of solvent molecules between the two interesting charge particles should be large.
2. Coulomb's law is valid, if the point charges are at rest.
3. It is difficult to apply the Coulomb's law when the charges are in arbitrary shape. Hence, we cannot determine the value of distance 'd' between the charges when they are in arbitrary shape.

Electric Field Strength or Electric Field Intensity

The force acting on a unit positive charge inside an electric field is termed as electric field strength or electric field intensity.

Electric field strength or **electric field intensity** is the synonym of electric field. **Electric field strength** can be determined by Coulomb's law. According to this law, the force 'F' between two point charges having charge Q_1 and Q_2 Coulombs and placed at a distance d meter from each other is given by,



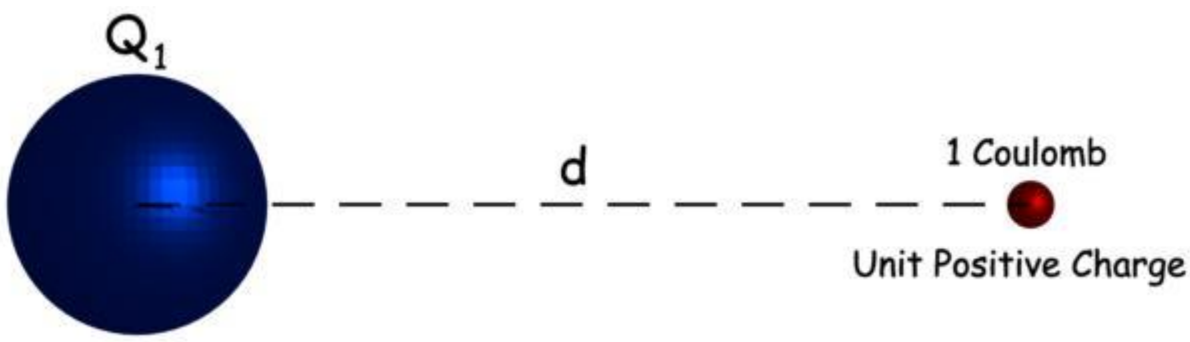
$$\overrightarrow{F} = \frac{Q_1 \, Q_2}{4\pi\epsilon_o\epsilon_r \, d^2} \cdots\cdots(1)$$

Here, ε_o is the permittivity of vacuum = 8.854×10^{-12} F/m and ε_r is the relative permittivity of the surrounding medium.

Now, let us put $Q_2 = +1$ Coulomb and let us denote force F by E in the equation (1), and by doing these we get,

$$\overrightarrow{E} = \frac{Q_1}{4\pi\epsilon_o\epsilon_r \, d^2}$$

This equation shows the force acting the a unit positive charge placed at a distance d from charge Q_1.



As per definition, this is nothing but of **electric field strength** of charge Q_1 at a distance d from that charge. Now, we got the expression of electric field strength or intensity. Now, by combining this expression with equation (1), we get,

$$\overrightarrow{F} = \frac{Q_1\ Q_2}{4\pi\epsilon_o\epsilon_r\ d^2} = \frac{Q_1}{4\pi\epsilon_o\epsilon_r\ d^2} \cdot Q_2 = \overrightarrow{E} \cdot Q_2$$

The above expression shows that, if we place a charge at any point in an electric field, the product of the electric field strength at that point and the charge of the body gives the force acting on the body at that point in the field. The above expression can also be rewritten as,

$$\overrightarrow{E} = \frac{\overrightarrow{F}}{Q_2} \quad Newton/Coulomb$$

Depending on this expression, the **electric field strength** can be expressed in Newton / Coulomb. That is unit of electric field strength is Newton / Coulomb. The electric field strength has direction and hence it is a vector quantity. Intensity means the magnitude or amount. Now field intensity similarly means the magnitude of the strength of the field. Finally **electric field intensity or strength** can be written as,

$$\overrightarrow{E} = \frac{Q_1}{4\pi\epsilon_o\epsilon_r\ d^2} = 9 \times 10^9\ \frac{Q_1}{\epsilon_r\ d^2}$$
$$\left[\because\ \frac{1}{4\pi\epsilon_o} = 9 \times 10^9\right]$$

So far we have discussed the electric field intensity at a point due to the influence of a single charge, but there may be a case where the point is under the filed of many charged bodies. In that case, we first have to calculate, the electric field strength at that point for individual charges and then we have to vectorially add up all the field strengths to get resultant field strength at that point.

$$\overrightarrow{E} = \overrightarrow{E_1} + \overrightarrow{E_2} + \overrightarrow{E_3} + \cdot\ \cdot\ \cdot\ \cdot\ \cdot$$

Gauss Theorem

We know that there is always a static electric field around a positive or negative electrical charge and in that static electric field there is a flow of energy tube or flux. Actually this flux is radiated/emanated from the electric charge. Now amount of this flow of flux depends upon the quantity of charge it is emanating from. To find out this relation, the **Gauss's theorem** was introduced. This theorem can be considered as one of the most powerful and most useful theorem in the field of electrical science. We can find out the amount of flux radiated through the surface area surrounding the charge from this theorem.

This theorem states that the total electric flux through any closed surface surrounding a charge, is equal to the net positive charge enclosed by that surface.

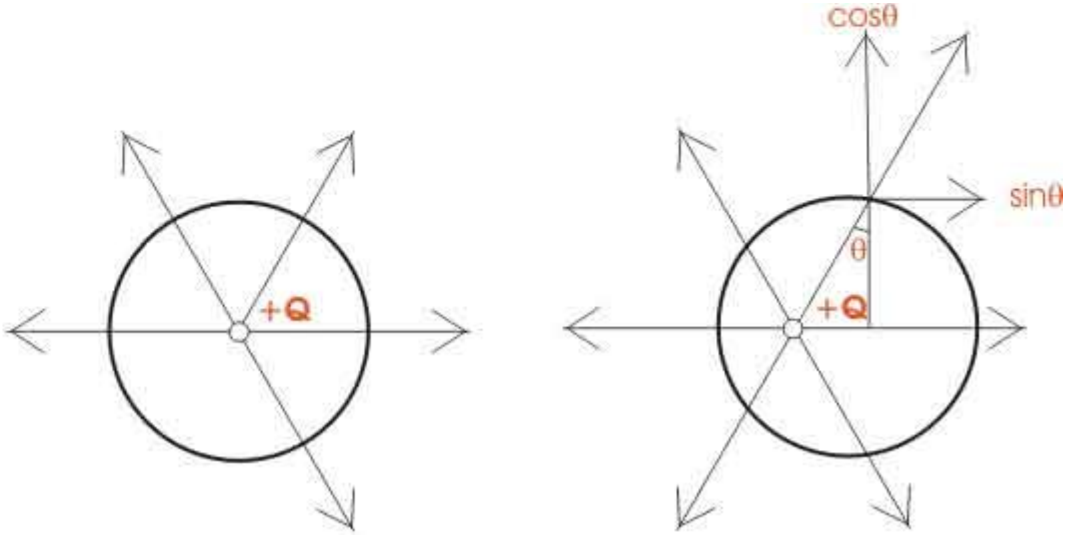
Suppose the charges Q_1, Q_2_ _ _ _Q_i, _ _ _ Q_n are enclosed by a surface, then the theorem may be expressed mathematically by surface integral as

$$\psi = \int\int D.dS = \sum_{i=1}^{n} Qi$$

Where, D is the flux density in coulombs/m^2 and dS is the outwardly directed vector.

Explanation of Gauss's Theorem

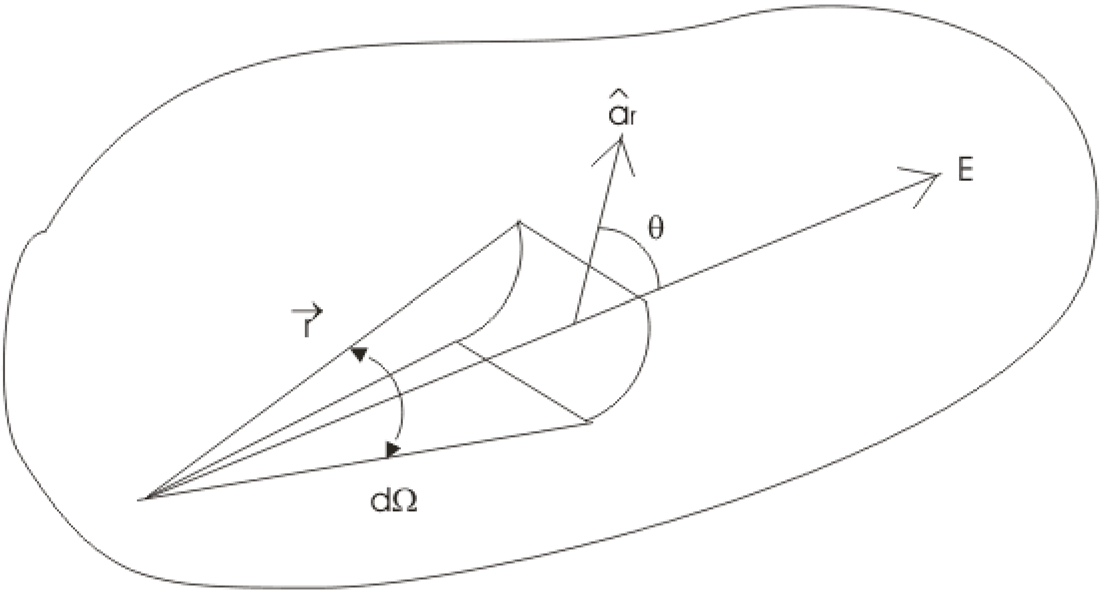
For explaining the **Gauss's theorem**, it is better to go through an example for proper understanding. Let Q be the charge at the center of a sphere and the flux emanated from the charge is normal to the surface. Now, this theorem states that the total flux emanated from the charge will be equal to Q coulombs and this can be proved mathematically also. But what about when the charge is not placed at the center but at any point other than the center (as shown in the figure).



At that time, the flux lines are not normal to the surface surrounding the charge, then this flux is resolved into two components which are perpendicular to each other, the horizontal one is the sinθ component and the vertical one is the cosθ component. Now when the sum of these components is taken for all the charges, then the net result is equal to the total charge of the system which proves **Gauss's theorem**.

Proof of Gauss's Theorem

Let us consider a point charge Q located in a homogeneous isotropic medium of permittivity ε.



The electric field intensity at any point at a distance r from the charge is

$$\vec{E} = \frac{Q}{4\pi\epsilon\, r^2}\,\hat{a_r}$$

The flux density is given as,

$$\vec{D} = \epsilon\vec{E} = \frac{Q}{4\pi\; r^2}\,\hat{a_r}$$

Now from the figure the flux through area dS

$d\psi = D.dS.\cos\theta$

Where, θ is the angle between D and the normal to dS.

Now, dScosθ is the projection of dS is normal to the radius vector. By definition of a solid angle

$dS.\cos\theta = r^2.d\Omega$

Where, dΩ is the solid angle subtended at Q by the elementary surface are dS. So the total displacement of flux through the entire surface area is

$$\psi = \oint_S d\psi = \oint_S D.dS.cos\theta = \oint_S D^2 d = \frac{Q}{4\pi}\oint d\Omega$$

Now, we know that the solid angle subtended by any closed surface is 4π steradians, so the total electric flux through the entire surface is

$$\psi = \oint_S \vec{D}.d\vec{s} = Q$$

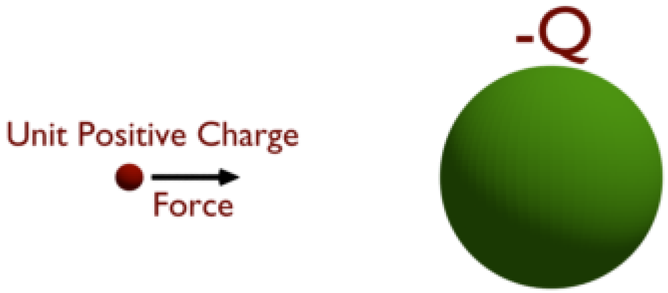
This is the integral form of **Gauss's theorem**. And hence this theorem is proved.

Electric Lines of Force

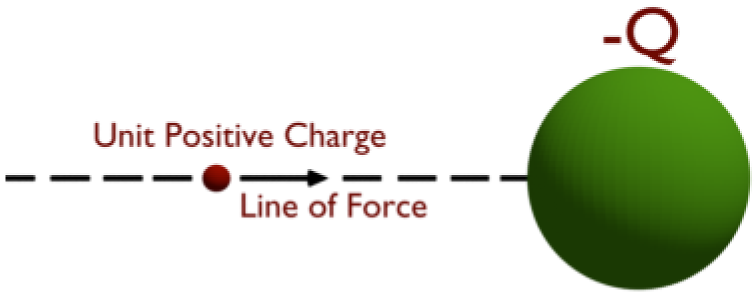
If a free to move unit positive charge is placed in an electric field, it will experience a force due to which the unit positive charge will move along a particular path. The unit positive charge can be placed at different positions in the field and the unit positive charge will follow different paths when it is placed in different positions. The paths, along which the unit positive charge will move due to electrostatic force in the field are called **electric lines of force**.

We consider the direction of electric lines of force as the direction of the movement of unit positive charge in the field. We represent the direction of lines of force with arrowheads.

Let us take a negative charge of – Q coulomb. Also, let us place a unit positive charge at a point in the field of charge – Q.



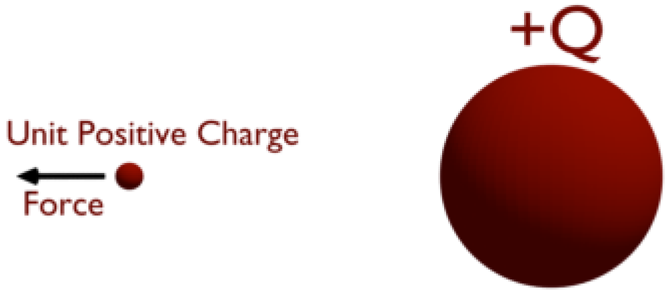
According to Coulomb's law, the unit positive charge will experience an attraction force, towards the center of the said negative charge.



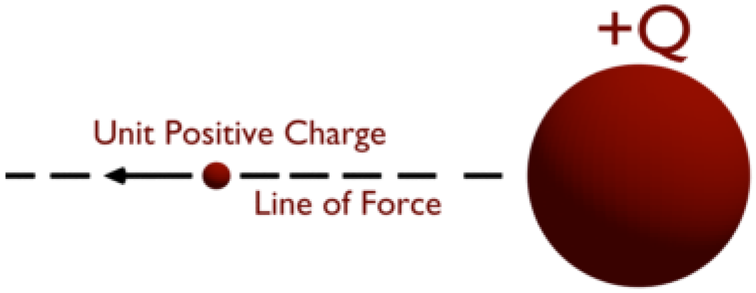
If the unit positive charge is free to move, that will move towards the charge – Q, along the straight line joining the center of unit positive charge and the center of said negative charge.

The line along which the unit positive charge moves towards the said negative charge, is defined as an electric line of force. If we placed the unit positive charge anywhere else in the field, the line of force was created along that new point.

Now, let us take a positive charge of + Q coulomb. We place a unit positive charge at a point in the field.

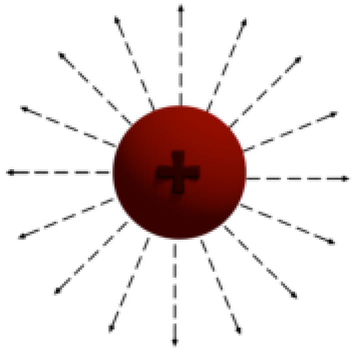


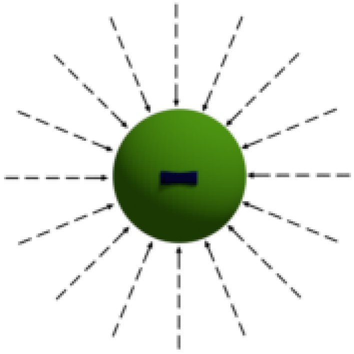
According to Coulomb's law, the unit positive charge will experience a repulsive force.



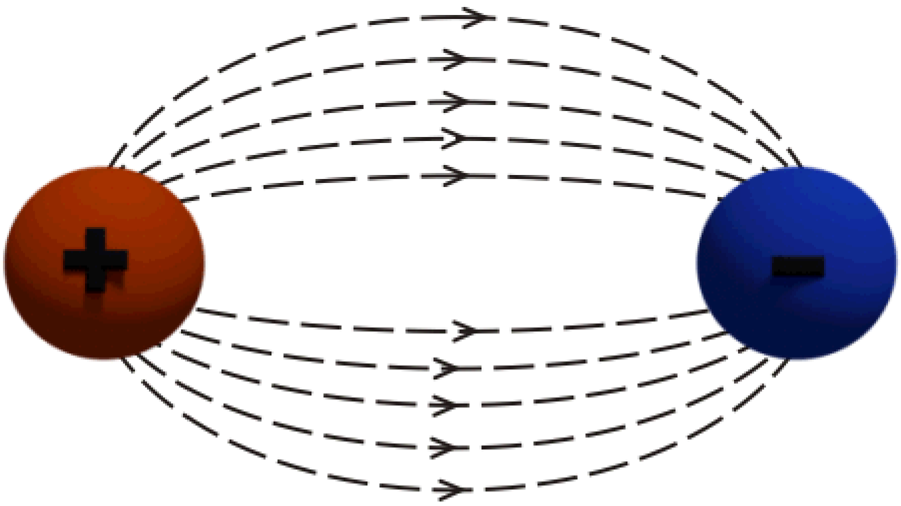
As the unit positive charge is free to move, that will move away along the straight line joining the center of unit positive charge and the said positive charge. The line along which the unit positive charge moves away from the said positive charge is an electric line of force. Similarly, if we placed the unit positive charge anywhere else in the field, the line of force was created along that new point.

Since, we can place a unit positive charge, at infinite number of points in the electric field, theoretically there will be infinite number of lines of force created in the field of an electric charge. But mathematically, to represent the influence of electric charge in the field, the total number of lines of force emanated from an electric charge is taken as equal to the value of electric charge measured in coulomb. That means a Q Coulomb charge emanates Q number of lines of force surrounding it.

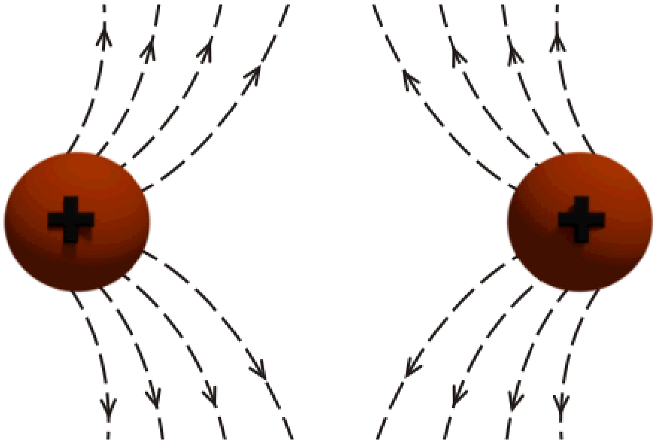




Now, let us place one positively charged object of + Q Coulomb and one negatively charged object of – Q Coulomb side by side as shown, below. Let us place a unit positive charge at any point in between these two charged objects. The unit positive charge will experience an attraction force by the negatively charged object and a repulsion force by the positively charged object. Due to influence attraction and repulsion force, the unit positive charge will move along a curved line from the center of the + Q charged object to the center of – Q charged object. Considering other lines of forces, we can draw the electric field like below.



Now, let us place two positively charged objects of + Q Coulomb side by side as shown, as shown below. Let us place a unit positive charge at any point on any of the charged objects. The unit positive charge will experience a repulsion force by + Q charge on which the unit positive charge is placed. The unit positive charge also experiences another repulsive force by the nearby + Q charge. Due to the influence of these two repulsion forces from both sides, the unit positive charge will move away from both + Q charges, as shown, as shown below. Considering other lines of forces, we can draw the electric field like below.



Properties of Electric Lines of Force

1. Each **electric lines of force** is conceptually imagined it does not have any physical existence.
2. Total number of lines of force emanated from a charge body is equal to the charge of the body measured in Coulomb.
3. Each electric lines of force is originated from positive charge and terminated to the negative charge.
4. A tangent drawn at any point on an electric line of force, indicates the direction electric field at that point in the field.
5. Each electric line of force emanates normally from the surface of the charge body.
6. Electric lines of force can only contract longitudinally.
7. Electric lines of force can expand laterally.
8. No two electric lines of force cross each other.
9. The electric lines of force in same direction repel each other.
10. The electric lines of force in opposite direction attract each other.
11. No lines of force can exists inside the conductor.

Electric Flux and Electric Flux Density

Electric flux is defined as the total number of electric lines of force emanating from a charged body. An electric field is represented by **electric flux**.

Like magnetic flux, **electric flux** lines are not always closed loop. This is because, an isolated magnetic north pole or an isolated magnetic south pole do not exist practically, but an isolated positively charged body and an isolated negatively charged body can exist. We generally denote electric flux with Ψ. We take the measuring unit of flux as the amount flux emanated from one coulomb positive electric charge. We know that number of lines of force emanated from a positive charge body is numerically equal to the charge of the body measured in coulomb. As the flux is total number of lines of force emanated from the charge body, the unit of flux is also taken as Coulomb. So, if Q is the charge of a body and Ψ is the electric flux emanated from the body, then we can write, Q = Ψ

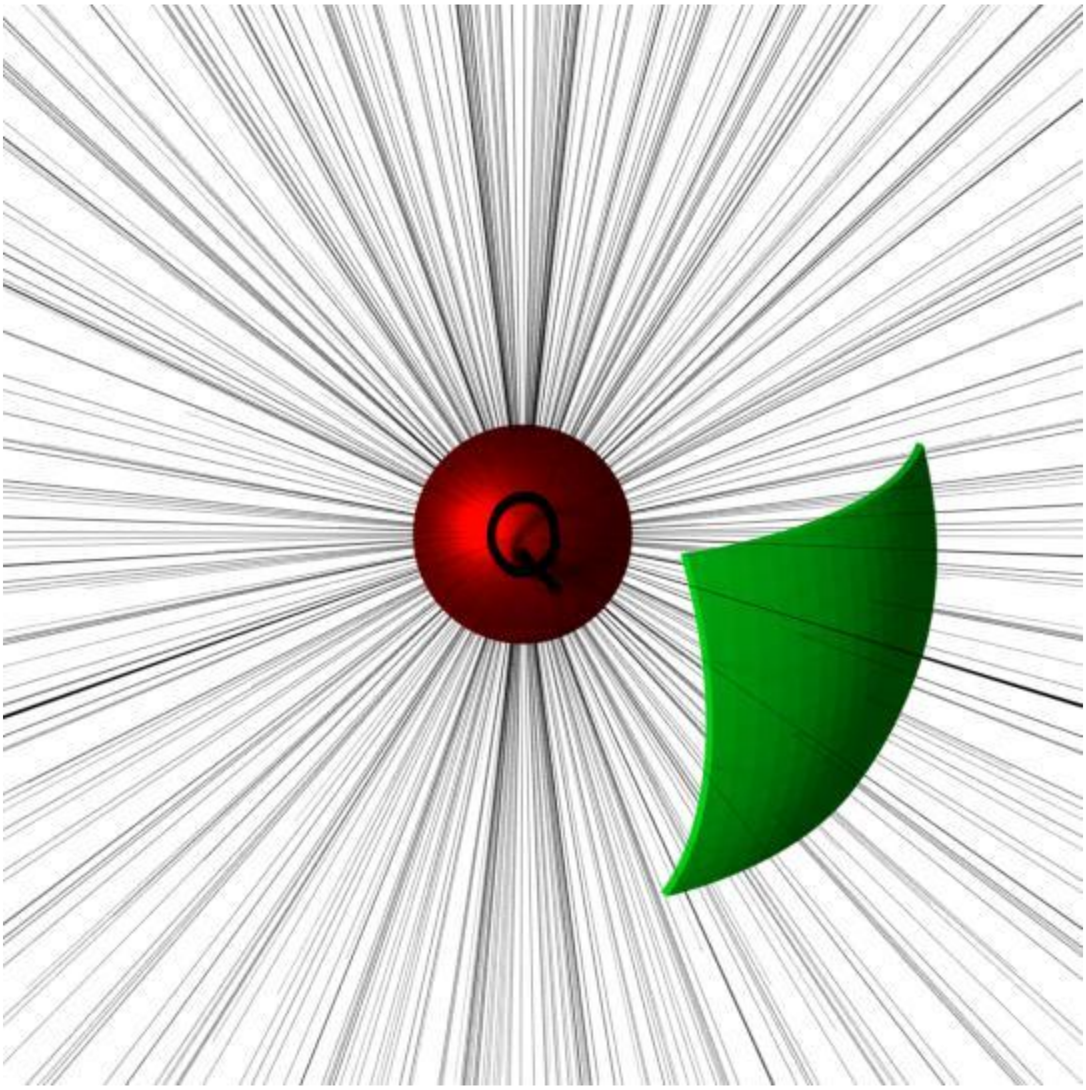
Electric Flux Density

Electric flux density is defined as the amount of flux passes through unit surface area in the space imagined at right angle to the direction of electric field. The expression of electric field at a point is given by

$$E = \frac{Q}{4\pi\epsilon_0\epsilon_r r^2}$$

Where, Q is the charge of the body by which the field is created. R is the distance of the point from the center of the charged body.

As, we know, Q = Ψ



The above equation can be rewritten as,

$$E = \frac{\psi}{4\pi\epsilon_0\epsilon_r r^2} \Rightarrow \epsilon_0\epsilon_r E = \frac{\psi}{4\pi r^2}$$

This is the expression of flux per unit area since, $4\pi r^2$ is the surface area of the imaginary spare of radius r. This is the flux passing through per unit area at a distance r from the center of the charge. This is called **electric flux density** at the said point. We generally denote it with English letter D. Therefore,

$$D = \epsilon_0\epsilon_r E$$

From, the above expression of D, it is clear that electric field intensity and **electric field density** are in same phasor.

As the number of electric lines of force emanated from a charged body is equal to the quantity of charge of the body measured in coulombs, we can also define the **electric flux density** at any point in the electric field of the body as the number of lines of force passing through a unit surface area at that point.

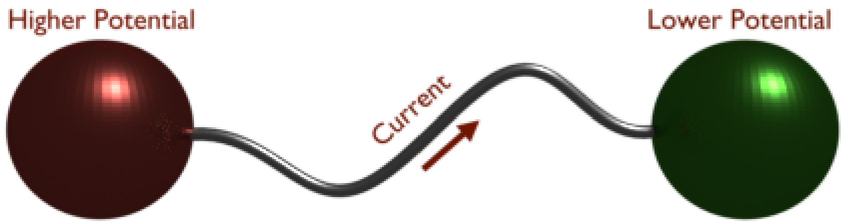
Electric Potential

Electric potential at a point in an electric field is defined as the amount of work to be done to bring a unit positive electric charge from infinity to that point.

Similarly, the potential difference between two points is defined as the work required to be done for bringing a unit positive charge from one point to other point.

When a body is charged, it can attract an oppositely charged body and can repulse a similar charged body. That means, the charged body has ability of doing work. That ability of doing work of a charged body is defined as **electrical potential** of that body.

If two electrically charged bodies are connected by a conductor, the electrons starts flowing from lower potential body to higher potential body, that means current starts flowing from higher potential body to lower potential body depending upon the potential difference of the bodies and resistance of the connecting conductor.



So, **electric potential** of a body is its charged condition which determines whether it will take from or give up electric charge to other body.

Electric potential is graded as electrical level, and difference of two such levels, causes current to flow between them. This level must be measured from a reference zero level. The earth potential is taken as zero level. Electric potential above the earth potential is taken as positive potential and the electric potential below the earth potential is negative.

The unit of electric potential is volt. To bring a unit charge from one point to another, if one joule work is done, then the potential difference between the points is said to be one volt. So, we can say,

$$volt = \frac{joules}{coulomb}$$

If one point has electric potential 5 volt, then we can say to bring one coulomb charge from infinity to that point, 5 joule work has to be done.

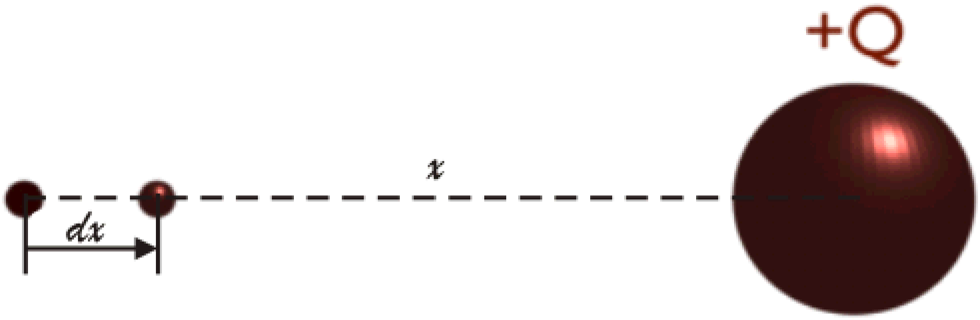
If one point has potential 5 volt and another point has potential 8 volt, then 8 – 5 or 3 joules work to be done to move one coulomb from first point to second.

Potential at a Point due to Point Charge

Let us take a positive charge + Q in the space. Let us imagine a point at a distance x from the said charge + Q. Now we place a unit positive charge at that point. As per Coulomb's law, the unit positive charge will experience a force,

$$F = \frac{Q}{4\pi\varepsilon_0\varepsilon_r x^2}$$

Now, let us move this unit positive charge, by a small distance dx towards charge Q.



During this movement the work done against the field is,

$$dw = -F.\,dx = -\frac{Q}{4\pi\epsilon_0\epsilon_r x^2}$$

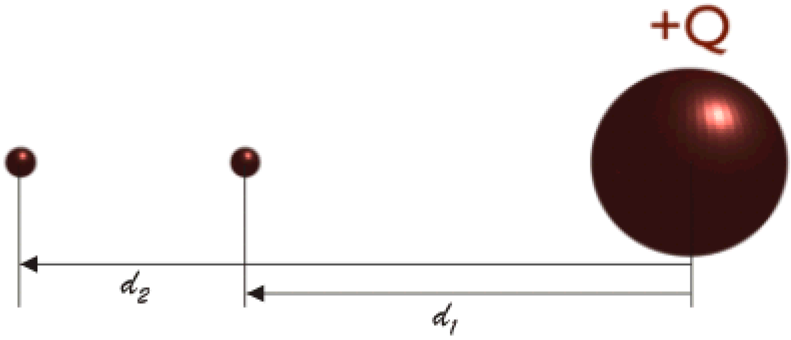
So, total work to be done for bringing the positive unit charge from infinity to distance x, is given by,

$$-\int_{\infty}^{x} dw = -\int_{\infty}^{x} \frac{Q}{4\pi\epsilon_0\epsilon_r x^2}.\,dx = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{x}\right]_{\infty}^{x} = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{x} - \frac{1}{\infty}\right] = \frac{Q}{4\pi\epsilon_0\epsilon_r x}$$

As per definition, this is the electric potential of the point due to charge + Q. So, we can write,

$$V = \frac{Q}{4\pi\epsilon_0\epsilon_r x}$$

Potential Difference between Two Points



Let us consider two points at distance $d_1$meter and d_2 meter from a charge +Q.

We can express the electric potential at the point d_1 meter away from +Q, as,

$$V_{d_1} = \frac{Q}{4\pi\epsilon_0\epsilon_r d_1}$$

We can express the electric potential at the point d_2 meter away from +Q, as,

$$V_{d_2} = \frac{Q}{4\pi\epsilon_0\epsilon_r d_2}$$

Thus, the potential difference between these two points is

$$V_{d_1} - V_{d_2} = \frac{Q}{4\pi\epsilon_0\epsilon_r d_1} - \frac{Q}{4\pi\epsilon_0\epsilon_r d_2} = \frac{Q}{4\pi\epsilon_0\epsilon_r}\left[\frac{1}{d_1} - \frac{1}{d_2}\right]$$

Permittivity and Relative Permittivity or Dielectric Constant

When we place two electrically charged bodies nearby they will experience a force among them. This force may either be attractive or be repulsive depending on the nature of the charge of two bodies. Two like charged bodies repel each other and two unlike charged bodies attract each other. The value of force acting on these two nearby bodies can be formulated by Coulomb's Law.

$$F = \frac{Q_1\ Q_2}{4\pi\epsilon_o\epsilon_r\ d^2}$$

The force acting between nearby electrically charged bodies depends on mainly three factors.

1. Charge of the bodies Q_1 and Q_2 coulombs that is the product of charges of the bodies.
2. Distance between the center of the charges i.e. d meters. The force is inversely proportional to the square of the distance d.
3. The medium in which the bodies are placed.

The role of **permittivity** comes in this third point. It is found that force acting between nearby same charged bodies separated by the same distance is different in different mediums. From the equation of Coulomb's law, we find that the force acting between nearby electrically charged bodies is inversely proportional to the term $\varepsilon_o \varepsilon_r$. This term is called **permittivity** of the medium. Here, ε_o is known as the **absolute permittivity** of the vacuum and ε_r is the **relative permittivity** of the medium in which the bodies are placed.

Relative Permittivity or Dielectric Constant

Relative Permittivity is defined as the ratio of the actual or absolute permittivity of a medium to the absolute permittivity of vacuum. If the permittivity of a medium is ε then $\varepsilon_r = \frac{\varepsilon}{\varepsilon_o}$

This ratio is 1.0006 for air. That means relative permittivity of air is 1.0006.

$$\varepsilon_r(air) = \frac{\varepsilon_{air}}{\varepsilon_o} = 1.0006$$

The electrostatic force acting between nearby electrically charged bodies is inversely proportional to the permittivity of the medium. Hence, the relative permittivity of any medium is defined as the ratio of force acting between nearby electrically charged bodies in the vacuum to the force acting between the same

$$\varepsilon_r = \frac{F_{vacuum}}{F_{medium}}$$

bodies separated by the same distance in the medium.

$$= \frac{Force\ acting\ between\ nearby\ electrically\ charged\ bodies\ in\ vacuum}{Force\ acting\ between\ the\ same\ bodies\ separated\ by\ the\ same\ distance\ in\ the\ medium}$$

We know that electric field intensity at any point in a field is defined as,

$$E = \frac{Q}{4\pi\varepsilon_o\varepsilon_r d^2}$$

From that relationship, we can establish the expression of electrical flux density (D) at that point as,

$$E = \frac{Q}{4\pi\varepsilon_o\varepsilon_r d^2} = \frac{1}{\varepsilon_o\varepsilon_r} \cdot \frac{Q}{4\pi d^2} = \frac{1}{\varepsilon_o\varepsilon_r} \cdot D$$

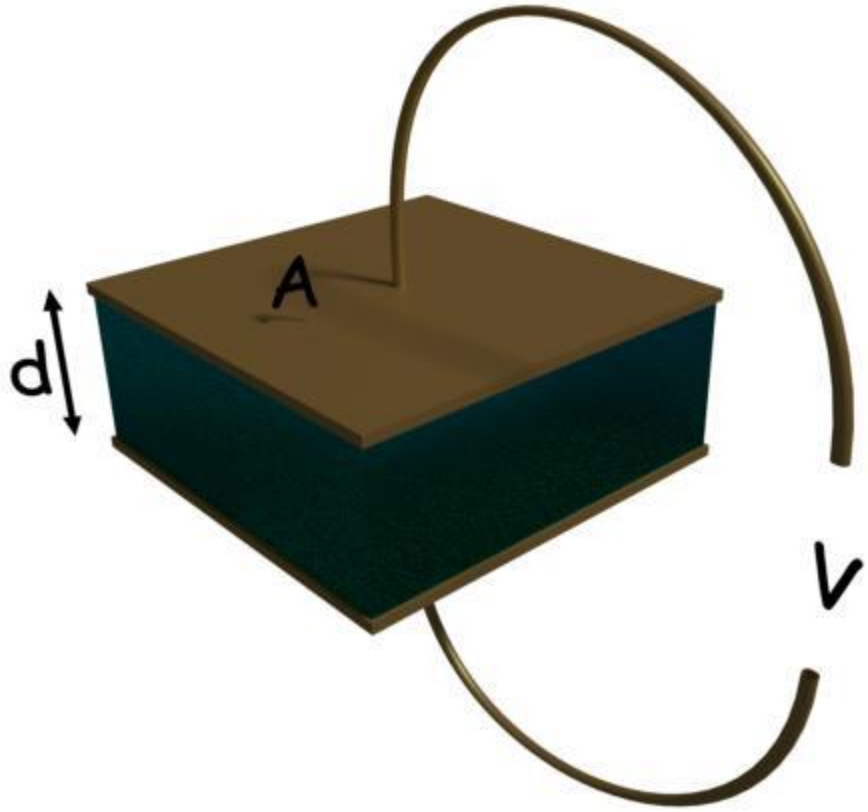
$$D = \varepsilon_o\varepsilon_r E$$

From the expression of electrical field intensity and flux density we can write,

$$\frac{D}{E} = \varepsilon_o\varepsilon_r$$

It is clear that the ratio of the electric flux density to the electric field intensity at a point in the field can be defined as the permittivity of the medium at that point.

Let us take a parallel plates capacitor with effective plate area A and distance between the plates is d and the dielectric between the plates has permittivity ε. The charge accumulated in the capacitor is Q due to an applied voltage across the capacitor is V.



The electric field intensity is $E = \frac{V}{d}$

The flux density is $D = \frac{Q}{A}$

The relation between flux density and electric field intensity is

$$D = \varepsilon_o \varepsilon_r E \Rightarrow \frac{Q}{A} = \varepsilon_o \varepsilon_r \frac{V}{d}$$

Here, in the expression of capacitance, it is found that capacitance of a capacitor is directly proportional to the electric permittivity of the dielectric medium between the plates.

$$\Rightarrow \frac{Q}{V} = \varepsilon_o \varepsilon_r \frac{A}{d} \Rightarrow C = \varepsilon_o \varepsilon_r \frac{A}{d}$$

Again, the expression of energy stored in the capacitor is

$$Energy = \frac{1}{2}CV^2$$

From that expression, it can be concluded that the energy stored in the capacitor is directly proportional to permittivity of the medium in between the plates.

$$Energy = \frac{1}{2}\epsilon_o\epsilon_r\frac{A}{d}V^2$$
$$\Rightarrow Energy \propto \epsilon_o\epsilon_r$$

So capacitance and the energy stored in the capacitor both are directly proportional to permittivity of the dielectric medium.

Hence permittivity of the medium used for making a capacitor is an essential parameter which determines the dimensions of the capacitor during designing of the capacitor.

Absolute Permittivity

Let us define absolute permittivity. The absolute permittivity or simply permittivity of a medium can be defined as the property of the medium which determines certain electric field intensity at a point in the field creates how much flux density at that point. We will show it later in this article. The absolute permittivity of any medium can be represented by the product of two terms one is absolute permittivity of vacuum and relative permittivity of the medium. This is for simplifying the calculations based on permittivity. The absolute permittivity of vacuum is taken as the base value of the permittivity. The relative permittivity of a medium is how many times the permittivity of the medium greater than the absolute permittivity of vacuum.

Unit of Permittivity

Unit of Absolute Permittivity

$$\epsilon = \frac{Q_1Q_2}{F\ 4\pi d^2}$$

From the equation of Coulomb's law, we can write the expression of permittivity as, From the expression of permittivity, as shown above, we can determine the unit of permittivity as

$$\frac{Coulomb^2}{Newton - meter^2} = \frac{C^2}{N - m^2}$$

Another Unit of Absolute Permittivity

The relationship between capacitance and permittivity of dielectric medium of a capacitor can be expressed as

$$C = \epsilon \frac{A}{d} \Rightarrow \epsilon = \frac{Cd}{A}$$

$$Where,\ A\ area\ of\ capacitor\ plate$$
$$and\ d\ is\ the\ distance\ between\ the\ plates$$

From this expression, the unit of permittivity comes as

$$\frac{Farad - meter}{meter^2} \Rightarrow \frac{Farad}{meter} \Rightarrow F/m$$

Unit of Relative Permittivity

As relative permittivity is the ratio of absolute permittivity of a medium to the absolute permittivity of vacuum this is unit less quantity.

Permittivity of Free Space

The permittivity of free space is also called vacuum permittivity. The value of permittivity of free space is

$$\varepsilon_o \approx 8.854187817620 \times 10^{-12}\ \ F/m$$

Divergence

The divergence of a vector field $\mathbf{F}(\mathbf{x})$ at a point $\mathbf{x}_0$ is defined as the limit of the ratio of the surface integral of $\mathbf{F}$ out of the surface of a closed volume V enclosing $\mathbf{x}_0$ to the volume of V, as V shrinks to zero

$$\operatorname{div} \mathbf{F}\big|_{\mathbf{x_0}} = \lim_{V \to 0} \frac{1}{|V|} \oiint_{S(V)} \mathbf{F} \cdot \hat{\mathbf{n}}\, dS$$

where $|V|$ is the volume of V, $S(V)$ is the boundary of V, and $\hat{\mathbf{n}}$ is the outward unit normal to that surface. It can be shown that the above limit always converges to the same value for any sequence of volumes that contain $\mathbf{x}_0$ and approach zero volume. The result, div $\mathbf{F}$, is a scalar function of $\mathbf{x}$.

Divergence Theorem

The flux of a differentiable vector field $\mathbf{F} : \mathbb{R}^3 \to \mathbb{R}^3$ across a closed oriented surface $S \subset \mathbb{R}^3$ in the direction of the surface outward unit normal vector $\mathbf{n}$ satisfies the equation

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma = \iiint_V (\nabla \cdot \mathbf{F})\, dV,$$

where $V \subset \mathbb{R}^3$ is the region enclosed by the surface S.

LaPlace's and Poisson's Equations

A useful approach to the calculation of electric potentials is to relate that potential to the charge density which gives rise to it. The electric field is related to the charge density by the divergence relationship

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

E = electric field

ρ = charge density

ε_0 = permittivity

and the electric field is related to the electric potential by a gradient relationship

$$E = -\nabla V$$

Therefore the potential is related to the charge density by Poisson's equation

$$\nabla \cdot \nabla V = \nabla^2 V = \frac{-\rho}{\varepsilon_0}$$

In a charge-free region of space, this becomes LaPlace's equation

$$\nabla^2 V = 0$$

QUESTIONS

PART A

1. Define electric flux and electric flux density.
2. State Gauss's law.
3. State Coulomb's law.
4. Name a few application of Gauss's law in electrostatics.
5. Define Divergence theorem.
6. Define electric field intensity or electric field.
7. Define potential and potential difference.
8. A potential distribution is given by V=4x+3.Whatis the potential gradient and electric field.
9. What is the electric field intensity at a distance of 20cm from a charge of 2µc in vacuum?
10. Write the Poisson's equation and Laplace equation.

PART B

1. State and explain Gauss law.
2. State and explain divergence theorem.

3. Derive the expression for the field due to line charge distribution with charge Density.

4. Derive the expression for the potential due to surface charge distribution with density of σ.

5. Point charges of $3*10^{+3}$μμcoulombs are situated at each of the three corners of a square of side 0.2m.Find electric field both in magnitude and direction.

6. The point charges of $q1=+10^{-6}C$, $Q2=-10^{-6}C$,$q3=+0.5X10^{-6}C$ are located at the Corners of an equilateral triangle of 50cms side .Determine the magnitude and force on q3.

TEXT / REFERENCE BOOKS

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SCHOOL OF ELECTRICAL AND ELECTRONICS ENGINEERING

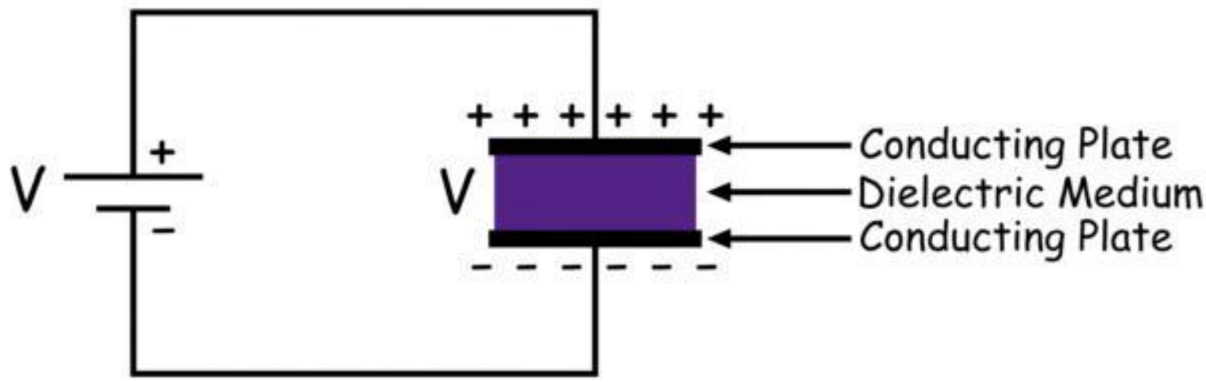
DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT – II - Electromagnetic Theory - SEEA1202

I. Capacitance and Dielectric Material

CAPACITOR AND CAPACITANCE

There are three fundamental electronic components that form the foundation of a circuit – resistors, inductors, and **capacitors**. A **capacitor** in an electrical circuit behaves as a charge storage device. It holds the electric charge when we apply a voltage across it, and it gives up the stored charge to the circuit as when required. The most basic construction of a capacitor consists of two parallel conductors (usually metallic plates) separated by a dielectric material. When we connect a voltage source across the capacitor, the conductor (capacitor plate) attached to the positive terminal of the source becomes positively charged, and the conductor (capacitor plate) connected to the negative terminal of the source becomes negatively charged. Because of the presence of dielectric in between the conductors, ideally, no charge can migrate from one plate to other.



So, there will be a difference in charging level between these two conductors (plates). Therefore an electric potential difference appears across the plates. The charge accumulation in the capacitor plates is not instantaneous rather it is gradually changing. The voltage appears across the capacitor exponentially rises untill it becomes equal to that of the connected voltage source.

CAPACITANCE

Now we understand that the charge accumulation in the conductors (plates) causes the voltage or potential difference across the capacitor. The quantity of charge accumulated in the capacitor for developing a particular voltage across the capacitor is referred to as the charge holding capacity of the capacitor. We measure this charge accumulation capability of a capacitor in a unit called capacitance. The capacitance is the charge gets stored in a capacitor for developing 1 volt potential difference across it. Hence, there is a direct relationship between the charge and voltage of a capacitor. The charge accumulated in the capacitor is directly proportional to the voltage developed across the capacitor.

$$Q \propto V$$

Where Q is the charge and V is the voltage.

$$Q = CV$$

Here C is the constant of proportionality, and this is capacitance,

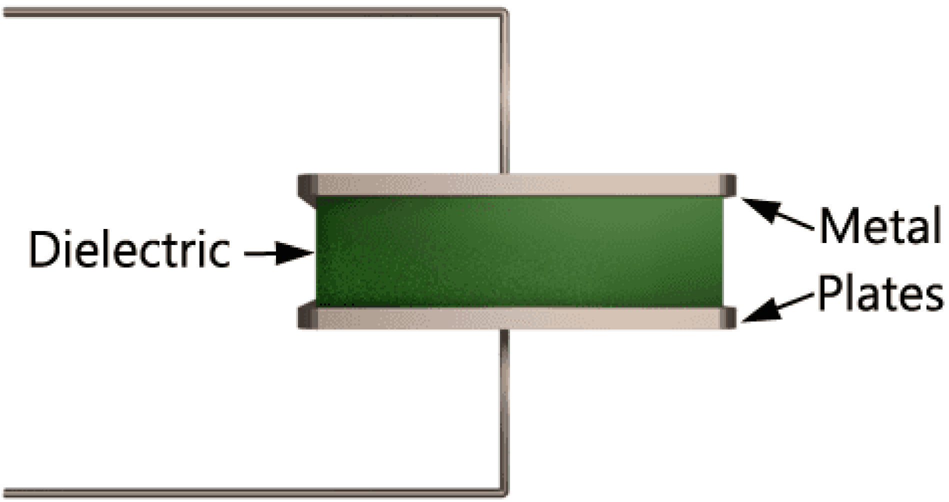
The capacitance depends upon three physical factors, and these are the active area of the capacitor conductor (plates), the distance between the conductors (plates) and permittivity of the dielectric medium.

$$C = \frac{\varepsilon A}{d}$$

Here, ε is permittivity of the dielectric medium, A is the active area of the plate and d is the perpendicular distance between the plates.

Parallel Plate Capacitor

A **parallel plate capacitor** is an arrangement of two metal plates connected in parallel separated from each other by some distance. A dielectric medium occupies the gap between the plates. The dielectric medium can be air, vacuum or some other non conducting material like mica, glass, paper wool, electrolytic gel and many others.



The dielectric does not allow the flow of electric current through it due to its non-conductive property. However, the atoms of the dielectric material get polarised under the effect of electric field of the applied voltage source, and thus there are dipoles formed due to polarisation due to which, a negative and positive charge get deposited on the plates of a **parallel plate capacitor**.

The accumulation of charges on the plates takes place due to which, a charging current flows through the capacitor until the potential difference between the plates equalises the source potential. We define a parallel plate capacitor as a device which is capable of storing electrostatic energy in the form of charge in the dielectric medium between the plates, and thus it can be visualised as equivalent to a rechargeable

DC battery. If the working voltage of the capacitor increases beyond the threshold voltage limit, then a short circuit occurs between the plates due to dielectric breakdown, this breakdown occurs due to excessive heating of the dielectric medium caused by an increase in applied voltage beyond the limit results in rupturing of the capacitor. We should choose the working voltage of capacitor appropriately within the maximum threshold voltage to protect the capacitor from such situations.

We use a various dielectric medium like porcelain, mica, oxides of various metals or other similar materials having high permittivity to increase the threshold voltage level as well as charge storing capability of the capacitor. The capacitance is the function of the overlapping area of the plates, the permittivity of the medium and the distance separation between the plates.

$$Mathematically,\ \ C = \frac{\epsilon\ A}{d}$$

Where, ε = permittivity of the medium, A = overlapping plate area and d = distance separation between the plates.

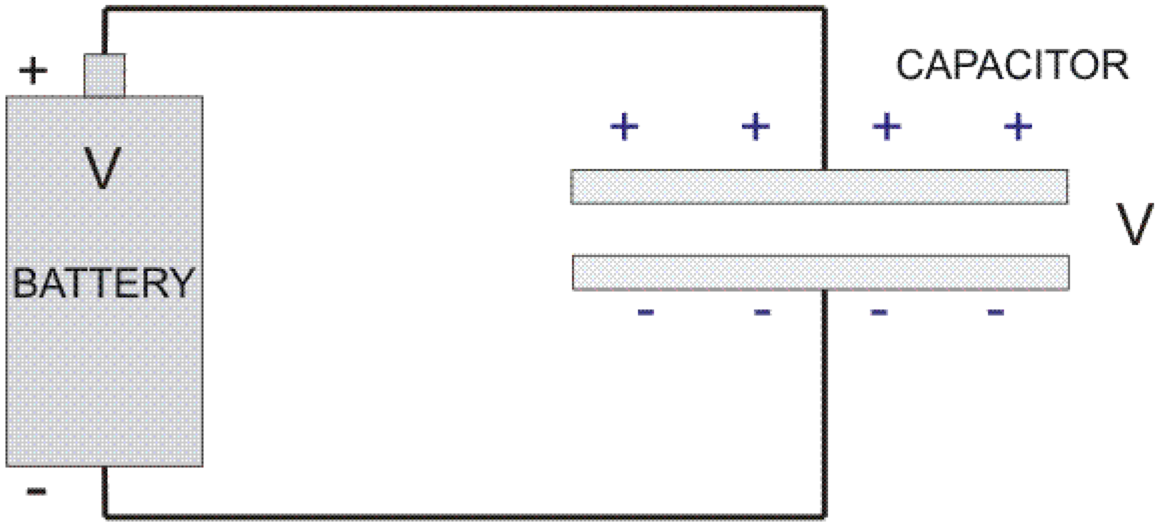
Hence, we can vary the value of capacitance of parallel plate capacitor either by changing the overlapping area or by varying the distance between the plates or by introducing a dielectric medium of different permittivity value.

A **parallel plate capacitor** behaves as open circuited when we connect a DC source across it, while it acts as a short circuit when we connect an AC source to it. The said property of a parallel plate capacitor makes it suitable for filtering of harmonics from AC supply. We can also use a parallel plate capacitor for tunning purpose in electronic circuits for various applications. We also use it in various transducers applications. A capacitor can act as a source of capacitive reactive power, and thus it serves as an essential element in power system auxiliaries for improving the power factor of the system thereby, enhancing the stability of a system. The energy storing capacity of a magnetic field is higher as compared with an electric field, because of this reason we do not usually use a **parallel plate capacitor** as energy storage. The dielectric used in it also suffers from a disadvantage, i.e. leakage of charge, due to which a capacitor cannot hold the charge for a long time and thus we cannot utilize it as an ideal charge storage device.

ENERGY STORED IN CAPACITOR

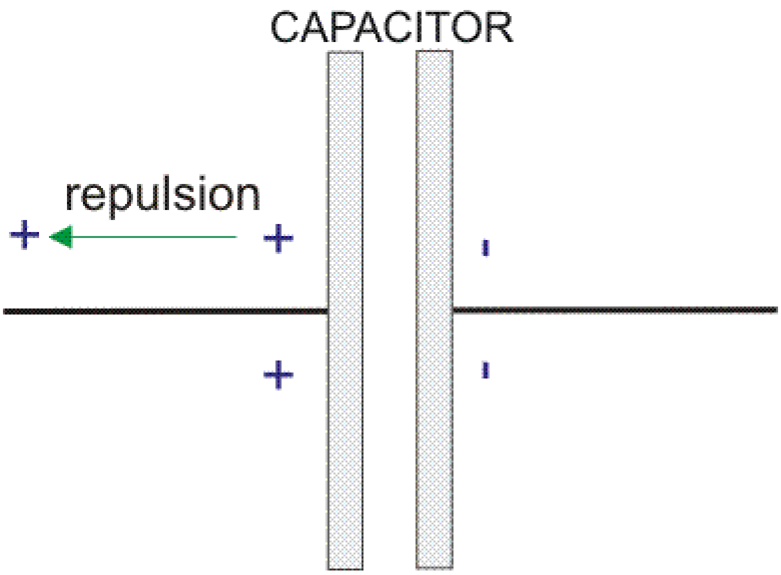
While capacitor is connected across a battery, charges come from the battery and get stored in the capacitor plates. But this process of energy storing is step by step only.

At the very beginning, capacitor does not have any charge or potential. i.e. V = 0 volts and q = 0 C.



Now at the time of switching, full battery voltage will fall across the capacitor. A positive charge (q) will come to the positive plate of the capacitor, but there is no work done for this first charge (q) to come to the positive plate of the capacitor from the battery. It is because of the capacitor does not have own voltage across its plates, rather the initial voltage is due to the battery. First charge grows little amount of voltage across the capacitor plates, and then second positive charge will come to the positive plate of the capacitor, but gets repealed by the first charge. As the battery voltage is more than the capacitor voltage then this second charge will be stored in the positive plate.

At that condition a little amount of work is to be done to store second charge in the capacitor. Again for the third charge, same phenomenon will appear. Gradually charges will come to be stored in the capacitor against pre-stored charges and their little amount of work done grows up.



It can't be said that the capacitor voltage is fixed. It is because of the capacitor voltage is not fixed from the very beginning. It will be at its maximum limit when potency of capacitor will be equal to that of the battery.

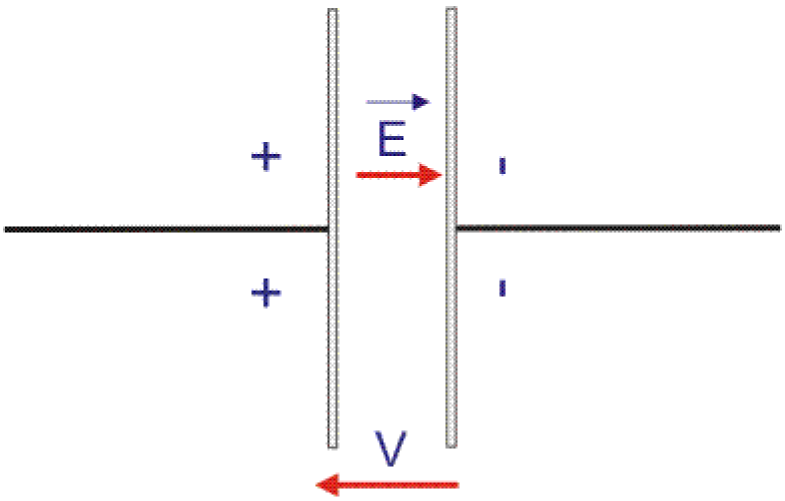
As storage of charges increases, the voltage of the capacitor increases and also energy of the capacitor increases.

So at that point of discussion the energy equation for the capacitor can't be written as energy (E) = V.q

As the voltage increases the electric field (E) inside the capacitor dielectric increases gradually but in opposite direction i.e. from positive plate to negative plate.

$$E = -\frac{dV}{dx}$$

Here dx is the distance between two plates of the capacitor.



Charge will flow from battery to the capacitor plate until the capacitor gains as same potency as the battery.

So, we have to calculate the energy of the capacitor from the very begging to the last moment of charge getting full.

Suppose, a small charge q is stored in the positive plate of the capacitor with respect to the battery voltage V and a small work done is dW.

Then considering the total charging time, we can write that,

$$\int_0^W dW = \int_0^Q V.dQ$$

$$W = \int_0^Q \frac{q}{C}.dq, \ \ [as\ C = \frac{q}{V}] \ \ Or,\ W = \frac{1}{2}.\frac{Q^2}{C} \ \ Or,\ W = \frac{1}{2}.CV^2$$

Now we go for the energy loss during the charging time of a capacitor by a battery.

As the battery is in the fixed voltage the energy loss by the battery always follows the equation, W = V.q, this equation is not applicable for the capacitor as it does not have the fixed voltage from the very beginning of charging by the battery.

Now, the charge collected by the capacitor from the battery is

$$W_{cap} = \frac{1}{2}.CV^2 = \frac{1}{2}.Q.V.$$

Now charge lost by the battery is

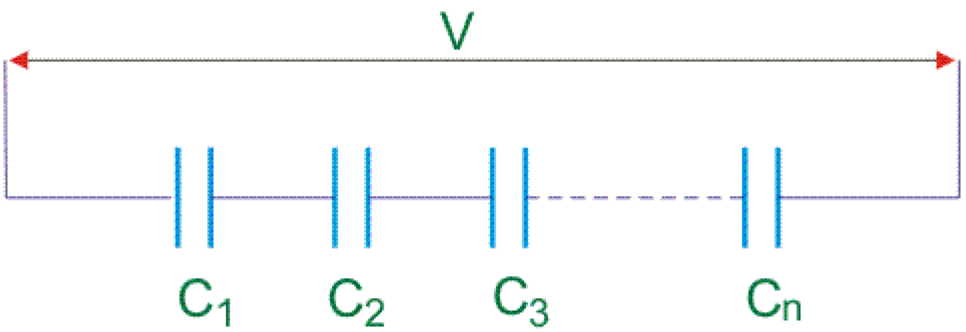
$$W_{loss} = V.Q - \frac{1}{2}.Q.V = \frac{1}{2}.Q.V$$

This half energy from total amount of energy goes to the capacitor and rest half of energy automatically gets lost from the battery and it should be kept in mind always.

CAPACITORS IN SERIES AND PARALLEL

Capacitor in Series

Let us connect n number of **capacitors in series**. V volt is applied across this series combination of capacitors.



Let us consider capacitance of capacitors are C_1, C_2, C_3……C_n respectively, and equivalent capacitance of series combination of the capacitors is C. The voltage drops across capacitors are considered to be V_1, V_2, V_3……V_n, respectively.

$$Therefore,\quad V = V_1 + V_2 + V_3.....V_n....(i)$$

Now, if Q coulomb be the charge transferred from the source through these capacitors, then,

$$V = \frac{Q}{C} \ and \ V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3}.....and \ \ V_n = \frac{Q}{C_n}$$

Since the charge accumulated in each capacitor and I entire series combination of capacitors will be same and it is considered as Q.

Now, equation (i) can be written as,

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + ... + \frac{Q}{C_n} \ \Rightarrow \frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}$$

$$\Rightarrow C = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ... + \frac{1}{C_n}\right]^{-1}$$

Capacitors in Parallel

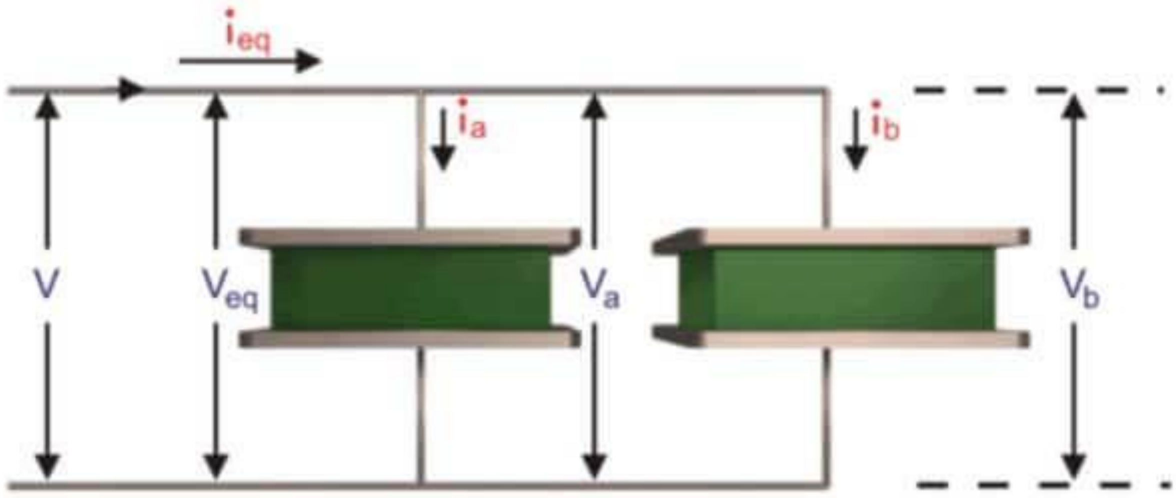
A capacitor is designed to store the energy in the form of its electric field, i.e. electrostatic energy. Whenever there is a necessity to increase more electrostatic energy storing capacity, a suitable capacitor of increased capacitance is required. A capacitor is made up of two metal plates connected in parallel and separated by a dielectric medium like glass, mica, ceramics etc. The dielectric provides a non-conducting medium between the plates and has a unique ability to hold the charge, and the ability of the capacitor to store charge is defined as the capacitance of the capacitor. When a voltage source is connected across the plates of the capacitor a positive charge on one plate, and negative charge on the other plate get deposited. The total amount of charge (q) accumulated is directly proportional to the voltage source (V) such that,

$$q = CV \cdot \cdot \cdot \cdot \cdot (i)$$

Where, C is proportionality constant i.e. capacitance. Its value depends upon physical dimensions of the capacitor.

$$Mathematically, \ \ C = \frac{\epsilon \ A}{d}$$

Where ε = dielectric constant, A = effective plate area and d = space between plates.



To increase the capacitance value of a capacitor, two or more **capacitors are connected in parallel** as two similar plates joined together joined together, then their effective overlapping area is added with constant spacing between them and hence their equivalent capacitance value becomes double ($C \propto A$) of individual capacitance. The capacitor bank is utilized in various manufacturing and processing industries incorporates capacitor in parallel, so to provide a capacitance of desired value as required by regulating the connection of capacitors connected in parallel and thus it is utilized efficiently as a static compensator for the reactive power balance in power system compensation. When two capacitors are connected in parallel then the voltage (V) across each capacitor is same i.e. ($V_{eq} = V_a = V_b$) and current(i_{eq}) is divided into two parts i_a and i_b. As it is known that $i = \frac{\mathrm{d}q}{\mathrm{d}t}$

Putting the value of q from equation (1) in the above equation,

$$i = \frac{d(CV)}{dt} \Rightarrow i = C\frac{dV}{dt} + V\frac{dC}{dt}$$

The later term becomes zero (as capacitor' capacitance is constant). Therefore,

$$i = C\frac{dV}{dt}$$

Applying Kirchhoff's Current Law at the incoming node of the parallel connection

$$i_{eq} = i_a + i_b$$
$$\Rightarrow i_{eq} = C_a\frac{\mathrm{d}V_a}{\mathrm{d}t} + C_b\frac{\mathrm{d}V_b}{\mathrm{d}t}$$

$$\& \;\; V_{eq} = V_a = V_b$$
$$\therefore \; i_{eq} = C_a\frac{\mathrm{d}V_{eq}}{\mathrm{d}t} + C_b\frac{\mathrm{d}V_{eq}}{\mathrm{d}t} \Rightarrow i_{eq} = (C_a + C_b)\frac{\mathrm{d}V_{eq}}{\mathrm{d}t}$$

Finally we get,

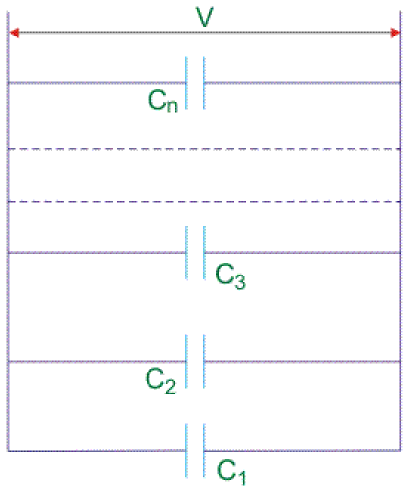
$$i_{eq} = C_{eq}\,\frac{\mathrm{d}V_{eq}}{\mathrm{d}t}$$
$$Where,\; C_{eq} = C_a + C_b$$

Hence, whenever n capacitors are connected in parallel the equivalent capacitance of the whole connection is given by following equation which resembles similar to the equivalent resistance of resistors when connected in series.

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_n$$

Method of Finding Expression of Equivalent Capacitance of Parallel Capacitor

Let us connect n number of **capacitors in parallel**, across a voltage source of V volt.



Let us consider the capacitance of the capacitors are C_1, C_2, C_3…..C_n, respectively and equivalent capacitance of the combination of the capacitor is C. As the **capacitors are connected in parallel**, like current charge in each capacitor will be same. Total charge of the parallel combination, will be divided in each capacitor according to it's capacitance value but voltage across each capacitor will be same and at steady state condition it is exactly equal to the applied voltage.

$$Therefore,\ \ Q = Q_1 + Q_2 + Q_3 + \ldots\ldots Q_n \ldots\ldots\ldots (ii)$$

Where,Q_1, Q_2, Q_3,…….Q_n are the charge of capacitor C_1, C_2, C_3….. C_n respectively.

$$Now,\ \ Q = CV\ and\ Q_1 = C_1V, Q_2 = C_2V, Q_3 = C_3V\ and\ \ldots\ldots Q_n = C_nV$$

Now equation (2) can be written as,

$$CV = C_1V + C_2V + C_3V + \ldots\ldots + C_nV$$
$$\Rightarrow C = C_1 + C_2 + C_3 + \ldots\ldots + C_n$$

BOUNDARY CONDITIONS AT DIELECTRIC AND CONDUCTOR SURFACES

Application of the integral form of Faraday's law to an infinitesimally small loop $\mathcal{C}$ in the plane of the incident and transmitted electric field vectors about the point $\mathbf{r} \in \mathcal{S}$, the upper side tangent to $\mathcal{S}$ in medium 1 and the lower side tangent to $\mathcal{S}$ in medium 2, gives

$$\oint_{\mathcal{C}} \mathbf{E} \cdot d\vec{\ell} = \int_a^b \mathbf{E}_2 \cdot d\vec{\ell} + \int_c^d \mathbf{E}_1 \cdot d\vec{\ell} = 0$$

where the contributions from the sides vanish as $\Delta h \to 0$ about $\mathcal{S}$. In addition,

$$\mathbf{E}_2 \cdot d\vec{\ell} = E_{t2}\Delta\ell, \quad \& \quad \mathbf{E}_1 \cdot d\vec{\ell} = -E_{t1}\Delta\ell$$

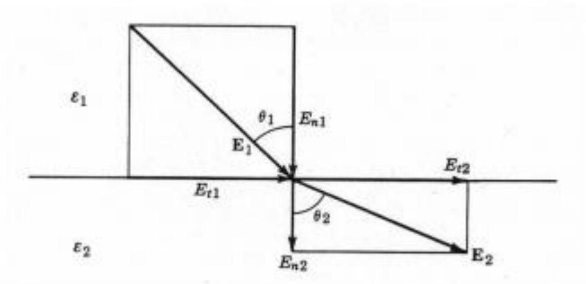
in the limit as $\Delta\ell \to 0$ on $\mathcal{S}$. Hence, in this limit, Faraday's law gives $E_{t2}\Delta\ell - E_{t1}\Delta\ell = 0$, or

$$\boxed{E_{t1}(\mathbf{r}) = E_{t2}(\mathbf{r}), \quad \mathbf{r} \in \mathcal{S}} \tag{3}$$

The tangential component of **E** is continuous across the interface $\mathcal{S}$.

At the interface between two dielectrics with $\varrho_s = 0$, the boundary conditions are

$$\epsilon_1 E_{n1} = \epsilon_2 E_{n2},$$
$$E_{t1} = E_{t2}.$$



Let $\mathbf{E}_1$ be at the angle θ_1 with respect to the surface normal $\hat{\mathbf{n}}$ and $\mathbf{E}_2$ be at the angle θ_2 with respect to the surface normal $-\hat{\mathbf{n}}$, where

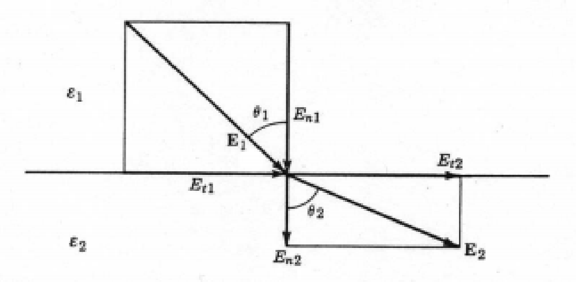
$$\theta_1 = \arctan\left(\frac{E_{t1}}{E_{n1}}\right), \quad \theta_2 = \arctan\left(\frac{E_{t2}}{E_{n2}}\right).$$

Then

$$\tan\theta_2 = \frac{E_{t2}}{E_{n2}} = \frac{\epsilon_2}{\epsilon_1}\frac{E_{t1}}{E_{n1}} = \frac{\epsilon_2}{\epsilon_1}\tan\theta_1,$$

$$\boxed{\epsilon_1 \tan\theta_2 = \epsilon_2 \tan\theta_1} \qquad (6)$$

so that $\theta_2 = Tan^{-1}\left(\frac{\epsilon_2}{\epsilon_1}\tan\theta_1\right)$.



Notice that

$$\epsilon_1 > \epsilon_2 \implies \tan\theta_1 > \tan\theta_2$$
$$\epsilon_2 > \epsilon_1 \implies \tan\theta_2 > \tan\theta_1$$

CAPACITANCE OF UNDERGROUND CABLES

As we saw earlier in the construction of Underground cables, a cable is basically a set of one (or three) conductors surrounded by a metallic sheath. This arrangement can be considered as a set of two long, coaxial, cylinders, separated by insulation. The current carrying conductor forms the inner cylinder while the metallic sheath acts as the outer cylinder. The sheath is grounded, and hence voltage difference appears across the cylinders. The dielectric fills the space between the charged plates (cylinders), making it a capacitor. Hence, **capacitance of the cable** becomes a very important aspect, and must be calculated. We can broadly classify cables as single-cored and three-cored. And the **calculation of capacitance** is different for both.

Capacitance of single core cable

A single core cable can be represented as shown below.

Let,

r = radius of the inner conductor and d = 2r

R = radius of the sheath and D = 2R

ε_0 = permittivity of free space = 8.854 x 10^{-12}

ε_r = relative permittivity of the medium

Consider a cylinder of radius x meters and axial length 1 meter. x be such that, r < x < R.

Now, electric intensity E_x at any point P on the considered cylinder is given as shown in the following equations.

Then, the potential difference between the conductor and sheath is V, as calculated in equations below.

After that, **capacitance of the cable can be calculated as C= Q/V**

$$E_x = \frac{Q}{2\pi\epsilon_0\epsilon_r x} \qquad v/m$$

$$V = \int_r^R E_x . d_x = \int_r^R \frac{Q}{2\pi\epsilon_0\epsilon_r x} . d_x = \frac{Q}{2\pi\epsilon_0\epsilon_r} \ln\frac{R}{r}$$

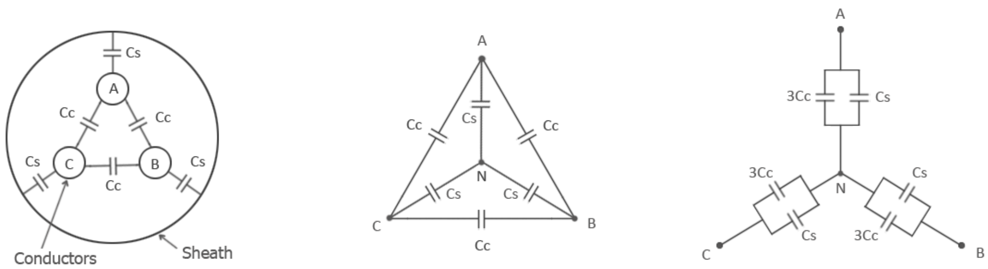
$$C = \frac{Q}{V} = \frac{Q}{\frac{Q}{2\pi\epsilon_0\epsilon_r}\ln\frac{R}{r}} \qquad F/m$$

$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln\frac{R}{r}} \qquad F/m$$

When the capacitance of a cable is known, then its capacitive reactance is given by $X_c = 1/(2\pi fC)\ \Omega$.
Then the charging current of the cable can be given as,
$I_c = V_{ph} / X_c$ A

CAPACITANCE OF THREE CORE CABLE

Consider a three cored symmetric underground cable as shown in the following figure (i). Let Cs be the capacitance between any core and the sheath and Cc be the core to core capacitance (i.e. capacitance between any two conductors).



(i) (ii) (iii)

In the above figure (ii), the three Cc (core to core capacitance) are delta connected and the core to sheath capacitance Cs are star connected due to the sheath forming a single point N. The circuit in figure (ii) can be simplified as shown in figure (iii). Outer points A, B and C represent cable cores and the point N represents the sheath (shown at the middle for simplification of the circuit).

Therefore, the whole three core cable is equivalent to three star connected capacitors each of capacitance Cs + 3Cc as shown in fig. (iii).

The charging current can be given as,

$I_c = 2\pi f(Cs+3Cc)V_{ph}$ A

MEASUREMENT OF CS AND CC

In order to calculate Cs and Cc we perform various experiments like:

1. First, the three cores are connected together and capacitance between the shorted cores and the sheath is measured. Shorting the three cores eliminates all the three Cc capacitors, leaving the three Cs capacitors in parallel. Therefore, if C_1 is the now measured capacitance, Cs can be calculated as, Cs = $C_1/3$.
2. In the second measurement, any two cores and the sheath are connected together and the capacitance between them and the remaining core is measured. If C_2 is the measured capacitance, then C_2 = 2Cc+Cs (imagine the above figure (iii) in which points A, B and N are short circuited). Now, as the value of Cs is known from the first measurement, Cc can be calculated.

EFFECTS OF CAPACITANCE IN UNDERGROUND CABLES

We know that capacitance is inversely proportional to separation between plates. Hence, if the separation between the plates is large, capacitance will be less. This is the case in Overhead Lines where two conductors are separated by several meters. The converse, of course, is also true. If the separation is small, the capacitance is more. In Underground cables, obviously, the separation is relatively smaller. Hence **capacitance of underground cables** is much more than that of Overhead lines.

The most important factor that is affected by this is the Ferranti effect. It is more pronounced in cables than in lines. This induces several limitations.

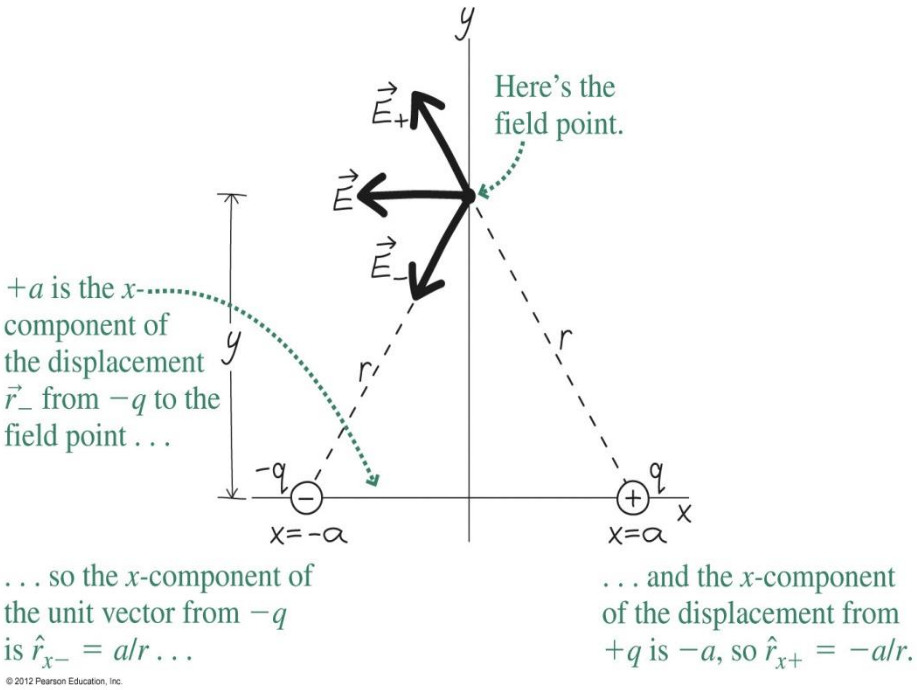
Also, with increased capacitance, the charging current drawn is also increased. Underground cables have 20 to 75 times the line charging current compared to Overhead lines.

Due to these two conditions, the length of Underground cables is limited.

Electric Field due to a Dipole

An electric dipole consists of two point charges of equal magnitude, but opposite sign, separated by a short distance.

- The dipole is electrically neutral, but due to the separation of its charges gives rise to an electric field in its vicinity.



- The electric field at the "field point" is given by $\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q}$. Note that in adding the two electric fields the y-component cancels leaving only an x-component given by,

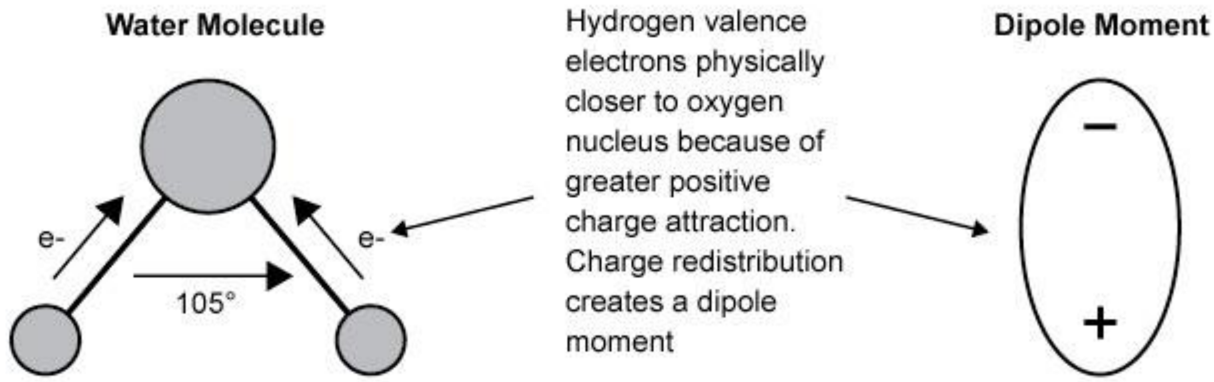
$$|E_x| = \frac{2aq}{4\pi\varepsilon_0 r^3} \approx \frac{2aq}{4\pi\varepsilon_0 R^3} = \frac{p}{4\pi\varepsilon_0 R^3}$$

where R is the distance from the centre of the dipole to the field point and the approximation is valid when r and R are almost equal. In this case the dimension of the dipole (a) is small compared to the field point distance.

- p (=2aq) is called the electric dipole moment. It's actually a vector pointing from the negative to the positive charge in the dipole so that,

$$\vec{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\vec{p}}{R^3}$$

- Many molecules have charge distributions which can be approximated as an electric dipole, water being one of the most common.



QUESTIONS

PART A

1) Define capacitor.
2) Define Capacitance.
3) Express the value of capacitance for a co-axial cable
4) Write the expression for the energy density in electro static field.
5) Estimate the energy stored in a parallel plate capacitor of 0.4m by 1m has a separation of 3cm and a voltage difference of 20V.
6) Write down the expression for the capacitance between two co-axial cylinders.
7) A parallel plate capacitor has a charge of 10-3 C on each plate while the potential difference between the plates is 1020V.Calaculate the value of capacitance.

PART – B

1) What is a dipole. Develop the expression for the potential and field due

to a dipole.

2) Develop the expression for the capacitance for a an isolated sphere and between two concentric spherical shells.
3) Develop the expression for the capacitance between two parallel wires.
4) Develop the expression for the energy stored and energy density in an electrostatic field.

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UNIT – III - Electromagnetic Theory - SEEA1202

I. MAGNETIC FIELD

Magnetic Circuit

A **magnetic circuit** is considered as the path in space through which magnetic flux passes. The figure below shows an iron-cored solenoid. When we supply DC through the solenoid, it will produce flux that's pattern is shown in that figure. Each flux line, as considered, initiated from N pole, passing through the air surrounding the magnet and finally reached to S pole and then from S pole it will come to the N pole through the iron core as shown. As each of the flux lines pass through the air as well as iron, this is called composite magnetic circuit. The lines of force inside the iron core are represented by numbers of uniformly spaced, paralleled lines.

As a result the magnetic field within the iron core is uniform. These lines of force or flux lines through the airspace, are not equally placed at all points, hence, field outside the core is not uniform. In order to make the design and analysis of a **magnetic circuit** easier, it is desired to produce a uniform field. Instead of using a straight iron core, if we have used an uniform cross-sectioned iron toroid, ideally, there would not be any scope for magnetic flux lines to pass through air. As a result the magnetic field within the toroid core is uniform. This is referred as completely enclosed magnetic circuit.

MAGNETIC FLUX

We define magnetic flux as the total number of magnetic lines of force in a magnetic field.

If we place an imaginary isolated unit north pole in a magnetic field it will experience a repulsive force from north pole and an attractive force from south pole of the magnet which has created the field. Due to these both forces, acting on the isolated unit north pole, the north pole will move along a particular path in the field if the pole is free to do so. If we place the same isolated unit north pole at different distance from the magnet in the field, it may follow a different path of travelling.

We call these paths of travelling of the unit north pole in the field, as lines of force. As we can place this imaginary isolated unit north pole at infinite number of points in the field, there may be infinite numbers of lines of force in the field. But visualize a magnetic field with infinite number of lines of force is useless for any scientific calculation. So we have to develop some unique concept, so that we can represent a magnetic field according to its entire strength. We take the unit of magnetic flux as weber. If a field has φ weber flux, it means the field has total φ number of lines of force. Like isolated north pole, the concept of lines of force in a magnetic field is also imaginary. It does not has any physical existence. This is only used for different magnetic calculation and for explaining different magnetic properties.

PROPERTIES OF MAGNETIC FLUX

1. **Magnetic flux** of a filed is considered as the total number of magnetic lines of force in the field. These are also called magnetic flux lines.
2. Each magnetic flux line is closed loop.
3. Each magnetic flux line starts from north pole of a magnet and comes to the south pole through the field and continues from south pole to north pole in the body of the magnet.
4. No two flux lines cross each other.
5. Two similar lines of force travel side by side but repeal each other.
6. The lines of force are stretched like elastic cord.

MAGNETIC FLUX DENSITY

The number of magnetic lines of force passing through a unit area surface perpendicular to the magnetic field is called **magnetic flux density**. If total φ Weber flux perpendicularly through a surface of area A

m^2, Magnetic flux density of the field would be,
We generally represent magnetic flux density by capital letter B.

MAGNETIC PERMEABILITY

Definition of Magnetic Permeability

Magnetic permeability is the ability of a material to respond to how much electromagnetic flux it can support to pass through itself within an applied electromagnetic field.
In other word magnetic permeability of a material is the degree of magnetization capability.

Notation of Magnetic Permeability

Magnetic permeability is expressed in μ that is a Greek Letter. In 1885, Mathematician Oliver Heaviside had termed magnetic permeability as μ.

How to find out Permeability of any Material or Medium

In electromagnetism, H is known as the magnetizing force that signifies the ability of magnetic dipole organizations in any material or medium by magnetic field density B. The relation between B and H is directly proportional, i.e. B α H.

Or, B = μH, where, μ is the proportional constant of that material or medium and it is termed as magnetic permeability.

Hence we can write,

$$\mu = \frac{B}{H}$$

So, in other word magnetic permeability is defined as the ratio of magnetic flux density (B) of a material to its electro-magnetizing force (H).

Unit of Electromagnetic Permeability

The unit of Electromagnetic Permeability is Henry/meter or Newton/sq-ampere.

Permeability in Free Space

Permeability in free pace is denoted as μ_0. Its value is $4л \times 10^{-7}$ H/m. This value of permeability is taken as standard value that is treated as permeability constant.

Permeability of another Medium or Substance

Permeability of another medium or substance is denoted as μ only. Relative permeability is the ratio of permeability of any substance to that of free space and it is denoted as μ_r, i.e.

$$\mu_r = \frac{\mu}{\mu_0}$$

So, permeability of any medium or material is

$$\mu = \mu_r \mu_0$$

Factors Effecting on Permeability

As permeability of any material depends on several factors:

1. Humidity
2. Temperature
3. Position in the medium
4. Frequency of the applied field

Classification of Materials as per Permeability

Diamagnetic: μ_r is less than 1, this material has opposition to external magnetic field. Paramagnetic: μ_r is near about 1 but not exactly 1, this material is weakly attracted by external magnetic field.

Ferromagnetic: μ_r is greater than 1, this material is strongly attracted by external magnetic field.

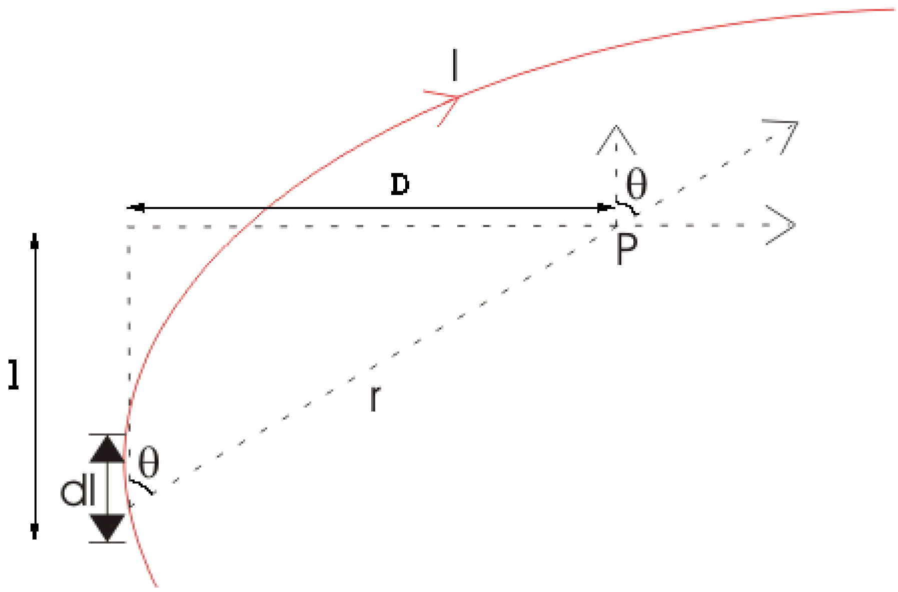
Complex form of Permeability?

Complex permeability comes into account for high frequency magnetic field effects. A phase lag is created between H and B.

Biot Savart Law: Statement, Derivation & Applications

Biot Savart Law

The **Biot Savart Law** is an equation describing the magnetic field generated by a constant electric current. It relates the magnetic field to the magnitude, direction, length, and proximity of the electric current. Biot–Savart law is consistent with both Ampere's circuital law and Gauss's theorem. The Biot Savart law is fundamental to magnetostatics, playing a role similar to that of Coulomb's law in electrostatics.



Biot Savart law was created by two French physicists, Jean Baptiste Biot and Felix Savart derived the mathematical expression for magnetic flux density at a point due a nearby current carrying conductor, in 1820. Viewing the deflection of a magantic compass needle, these two scientists concluded that any current element projects a magnetic field into the space around it.

Through observations and calculations they had derived a mathematical expression, which shows, the magnetic flux density of which dB, is directly proportional to the length of the element dl, the current I, the sine of the angle and θ between direction of the current and the vector joining a given point of the field and the current element and is inversely proportional to the square of the distance of the given point from the current element, r.

Biot Savart Law Statement

This is Biot Savart law statement:

$$Hence,\ dB \propto \frac{Idl sin\theta}{r^2} \quad or \quad dB = k\frac{Idl sin\theta}{r^2}$$

Where, k is a constant, depending upon the magnetic properties of the medium and system of the units employed. In SI system of unit,

$$k = \frac{\mu_o \mu_r}{4\pi}$$

Therefore, final Biot Savart law derivation is,

$$dB = \frac{\mu_o \mu_r}{4\pi} \times \frac{Idl sin\theta}{r^2}$$

Let us consider a long wire carrying a current I and also consider a point p in the space. The wire is presented in the picture below, by red color. Let us also consider an infinitely small length of the wire dl at a distance r from the point P as shown. Here, r is a distance vector which makes an angle θ with the direction of current in the infinitesimal portion of the wire.

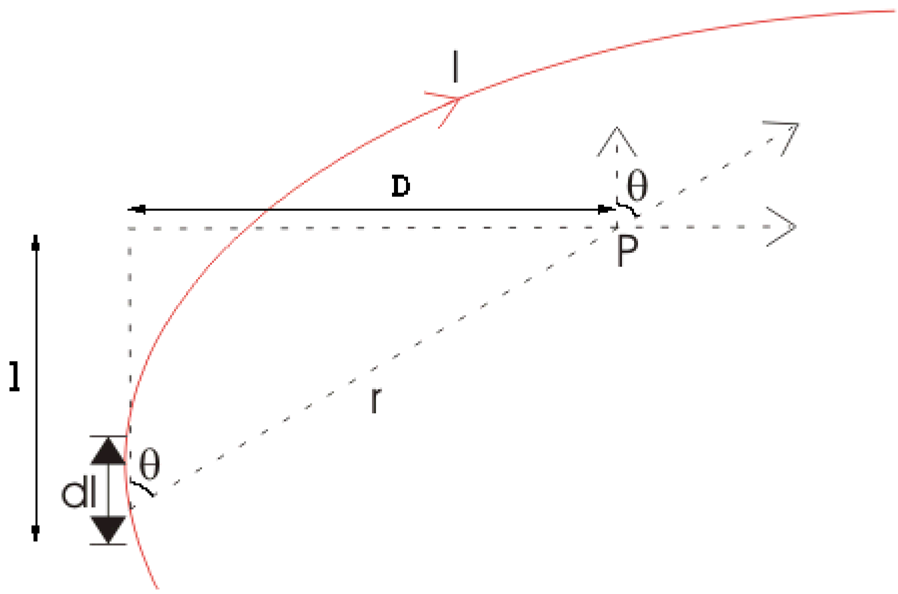
If you try to visualize the condition, you can easily understand the magnetic field density at the point P due to that infinitesimal length dl of the wire is directly proportional to current carried by this portion of the wire.

As the current through that infinitesimal length of wire is same as the current carried by the whole wire itself, we can write,

$$dB \propto I$$

It is also very natural to think that the magnetic field density at that point P due to that infinitesimal length dl of wire is inversely proportional to the square of the straight distance from point P to center of dl. Mathematically we can write this as,

$$dB \propto \frac{1}{r^2}$$



Lastly, magnetic field density at that point P due to that infinitesimal portion of wire is also directly proportional to the actual length of the infinitesimal length dl of wire. As θ be the angle between distance vector r and direction of current through this infinitesimal portion of the wire, the component of dl directly facing perpendicular to the point P is dlsinθ,

$$Hence,\ dB \propto dl sin\theta$$

Now, combining these three statements, we can write,

$$dB \propto \frac{I \cdot dl \cdot sin\theta}{r^2}$$

This is the basic form of **Biot Savart's Law**

Now, putting the value of constant k (which we have already introduced at the beginning of this article) in the above expression, we get

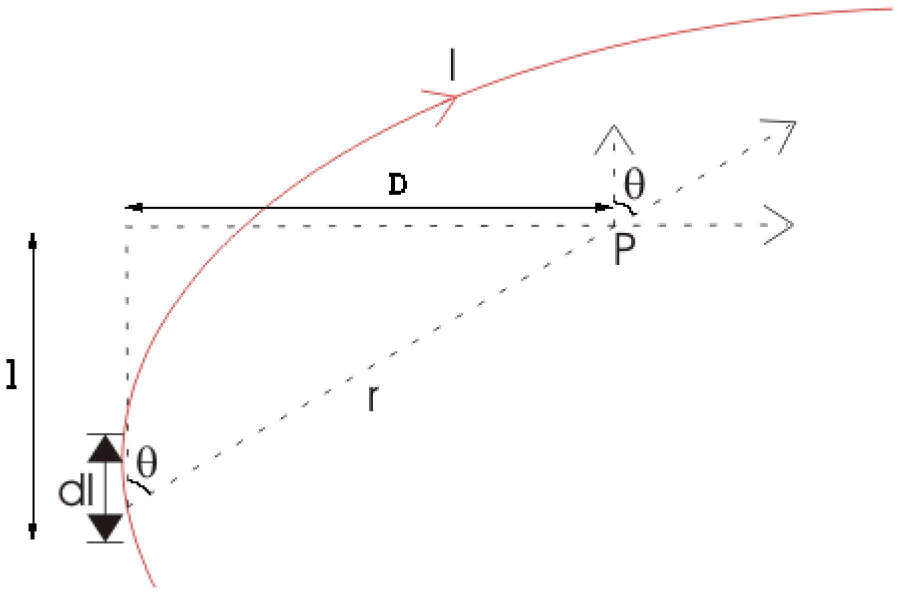
$$dB = k\frac{I \cdot dl \cdot sin\theta}{r^2}$$

$$\Rightarrow dB = \frac{\mu_o\mu_r}{4\pi} \times \frac{Idlsin\theta}{r^2}$$

Here, μ_0 used in the expression of constant k is absolute permeability of air or vacuum and it's value is $4\pi 10^{-7}$ W_b/ A-m in SI system of units. μ_r of the expression of constant k is relative permeability of the medium.

Now, flux density(B) at the point P due to total length of the current carrying conductor or wire can be represented as,

$$B = \int dB \Rightarrow dB = \int \frac{\mu_o\mu_r}{4\pi} \times \frac{Idlsin\theta}{r^2} = \frac{I\mu_o\mu_r}{4\pi}\int\frac{sin\theta}{r^2}dl$$



If D is the perpendicular distance of the point P form the wire, then

$$rsin\theta = D \; or \; r = \frac{D}{sin\theta}$$

Now, the expression of flux density B at point P can be rewritten as,

$$B = \frac{I\mu_o\mu_r}{4\pi}\int\frac{sin\theta}{r^2}dl = \frac{I\mu_o\mu_r}{4\pi}\int\frac{sin^3\theta}{D^2}dl$$

$$Again, \frac{l}{D} = cot\theta \Rightarrow l = Dcot\theta$$

As per the figure above,

$$Therefore, \; dl = -Dcsc^2\theta d\theta$$

Finally the expression of B comes as,

$$B = \frac{I\mu_o\mu_r}{4\pi}\int \frac{sin^3\theta}{D^2}\left[-Dcsc^2\theta d\theta\right]$$

$$= -\frac{I\mu_o\mu_r}{4\pi D}\int sin^3\theta csc^2\theta d\theta$$

$$= -\frac{I\mu_o\mu_r}{4\pi D}\int sin\theta d\theta$$

This angle θ depends upon the length of the wire and the position of the point P. Say for certain limited length of the wire, angle θ as indicated in the figure above varies from θ_1 to θ_2. Hence, magnetic flux density at point P due to total length of the conductor is,

$$B = -\frac{I\mu_o\mu_r}{4\pi D}\int_{\theta_1}^{\theta_2} sin\theta d\theta$$

$$= -\frac{I\mu_o\mu_r}{4\pi D}\Big[-cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$= \frac{I\mu_o\mu_r}{4\pi D}[cos\theta_1 - cos\theta_2]$$

Let's imagine the wire is infinitely long, then θ will vary from 0 to π that is $\theta_1 = 0$ to $\theta_2 = \pi$. Putting these two values in the above final expression of **Biot Savart law**, we get,

$$B = \frac{I\mu_o\mu_r}{4\pi D}[cos0 - cos\pi] = \frac{I\mu_o\mu_r}{4\pi D}[1-(-1)] = \frac{\mu_o\mu_r}{2\pi D}I$$

This is nothing but the expression of Ampere's Law.

AMPERE'S CIRCUITAL LAW

Ampere's Circuital Law states the relationship between the current and the magnetic field created by it. This law states that the integral of magnetic field density (B) along an imaginary closed path is equal to the product of current enclosed by the path and permeability of the medium. $\oint \vec{B}\cdot\vec{dl} = \mu_0 I$

James Clerk Maxwell had derived that.

It alternatively says, the integral of magnetic field intensity (H) along an imaginary closed path is equal to the current enclosed by the path.

$$\oint \overrightarrow{B} \cdot \overrightarrow{dl} = \mu_0 \, I$$

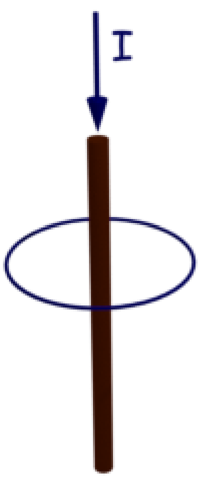
$$\Rightarrow \oint \frac{\overrightarrow{B}}{\mu_0} \cdot \overrightarrow{dl} = I$$

$$\Rightarrow \oint \overrightarrow{H} \cdot \overrightarrow{dl} = I$$

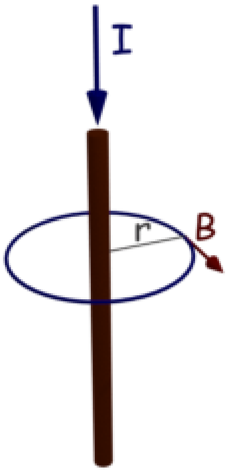
$$\left[\because \overrightarrow{H} = \frac{\overrightarrow{B}}{\mu_0} \right]$$

Let us take an electrical conductor, carrying a current of I ampere, downward as shown in the figure below.

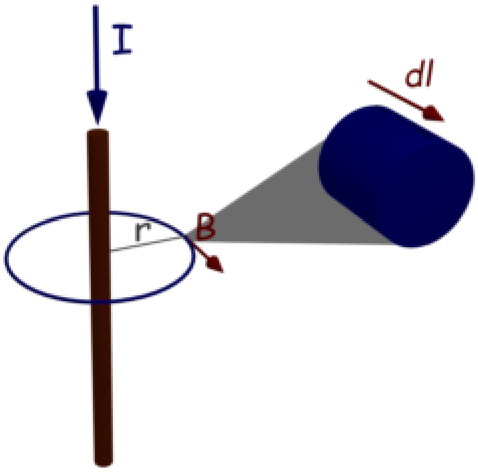
Let us take an imaginary loop around the conductor. We also call this loop as amperian loop.



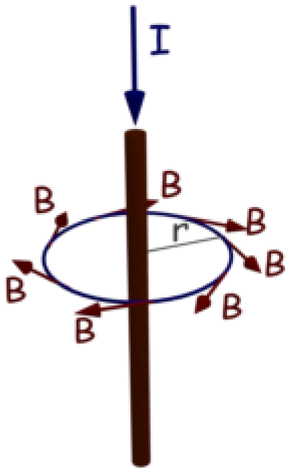
Let us also imagine the radius of the loop is r and the flux density created at any point on the loop due to current through the conductor is B.



Let us consider an infinitesimal length dl of the amperian loop at the same point.



At each point on the amperian loop, the value of B is constant since the perpendicular distance of that point from the axis of conductor is fixed, but the direction will be along the tangent on the loop at that point.



The close integral of the magnetic field density B along the amperian loop, will be,

$$\oint B \cdot dl \qquad [\, dot\ product]$$

$$[\because Direction\ of\ B\ \&\ dl\ is\ same$$
$$at\ each\ point\ on\ the\ loop.]$$

$$= B \oint dl = B \cdot (2\pi r)$$

Now, according to **Ampere's Circuital Law**

$$\oint B \cdot dl = \mu_0 \cdot I$$

Therefore,

$$2\pi r\, B = \mu_0\, I$$
$$\Rightarrow \frac{B}{\mu_0} = \frac{I}{2\pi r}$$
$$\Rightarrow H = \frac{I}{2\pi r}$$

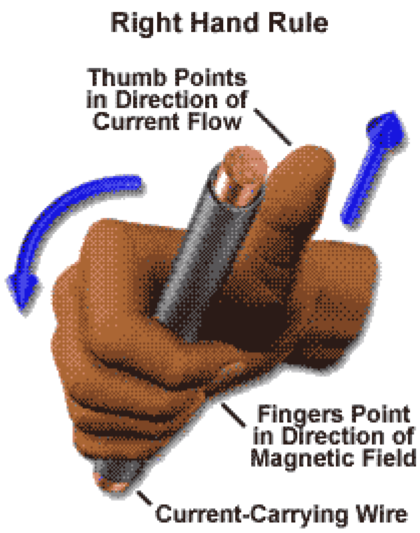
Instead of one current carrying conductor, there are N number of conductors carrying same current I, enclosed by the path, then

A CURRENT CARRYING CONDUCTOR WITHIN A MAGNETIC FIELD

Oersted had established that a compass needle gets deflected in the vicinity of a **current carrying conductor** i.e. this conductor exerts a force on the compass needle. Later on, in the year 1821, Michael Faraday discovered that a current carrying conductor also gets deflected when it is placed in a magnetic field. This can be said that magnetic field and this current carrying conductor exert a force on each other in their vicinity.

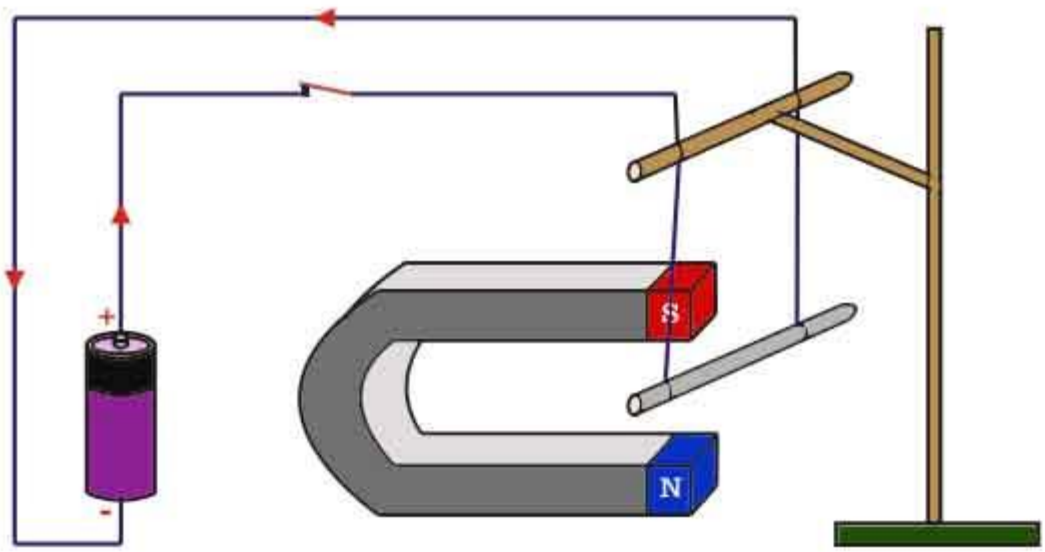
Suppose a conductor carries current I and it is with the length (l). As it is carrying current (DC), some flux lines will be generated around the conductor and they are concentric with the central axis of the conductor. So an electromagnetic field is established due to this current through this conductor.

Following right hand thumb rule the magnetic flux lines get the direction along the bent fingers when thumb denotes the direction of the current flow, i.e. shown in the figure below.



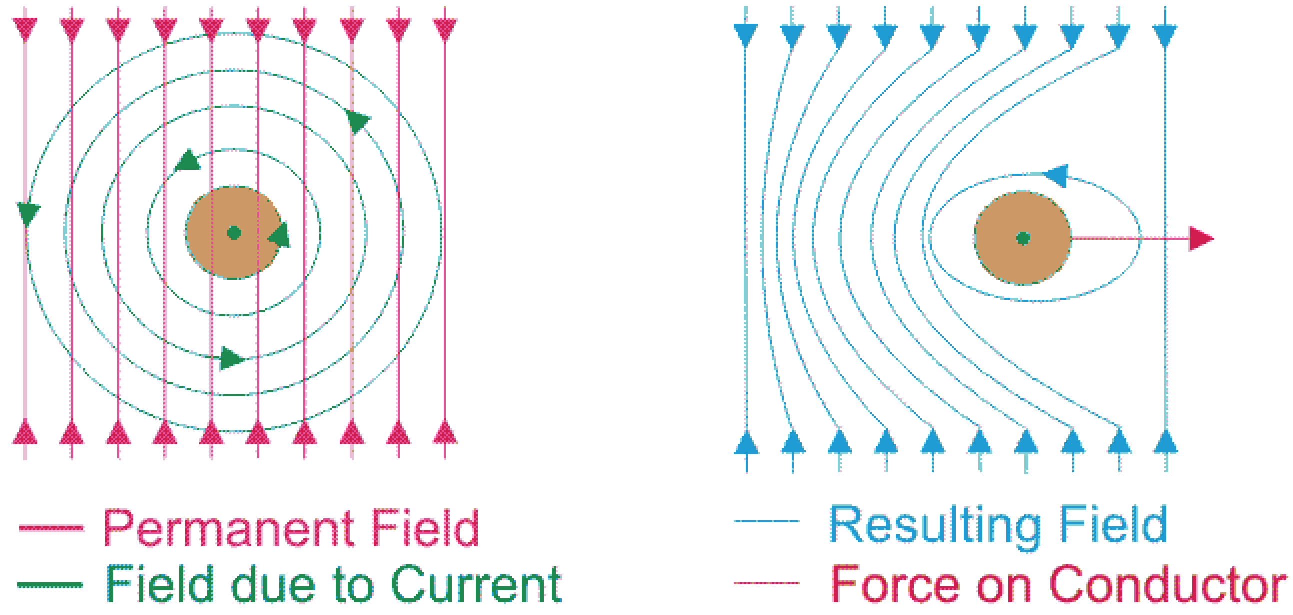
This current carrying conductor is placed between two poles of a horse shoe magnet of flux density $\overrightarrow{B}$. This magnet is tightly fixed to the ground. Conductor is not fixed, rather it is free to move. The length of the conductor is just perpendicular of the permanent magnetic field of the horse shoe.

So, it is clear that the direction of current and magnetic field is normal to each other.



Current Carrying Conductor in a Magnetic Field

Now two magnetic fields (electromagnetic field by the conductor and permanent magnetic field by the horse shoe magnet) are in their action.



The concentric circles of electromagnetic flux due to flowing current (I) through this conductor try to repel the magnetic flux of the permanent magnet at that situation.

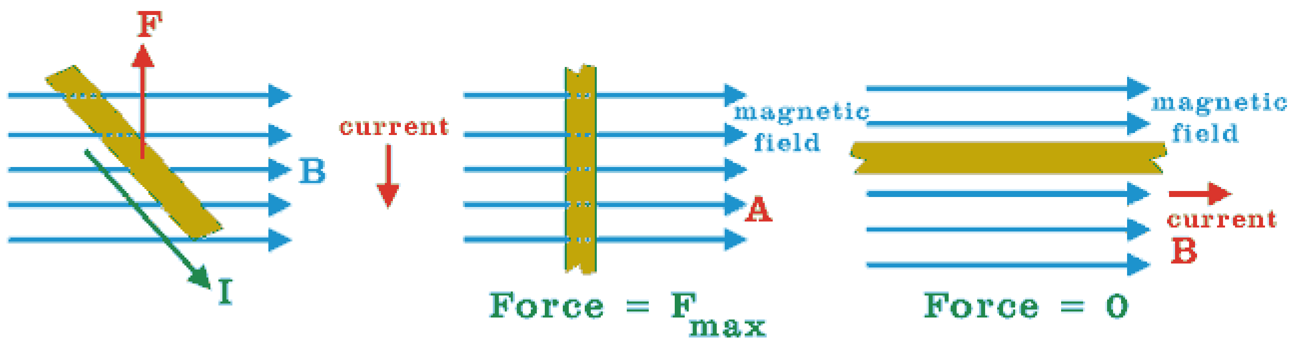
Let us consider the force is $\overrightarrow{F}$.

$$Here, \ \overrightarrow{F} = I.(\overrightarrow{l}) \times \overrightarrow{B}$$

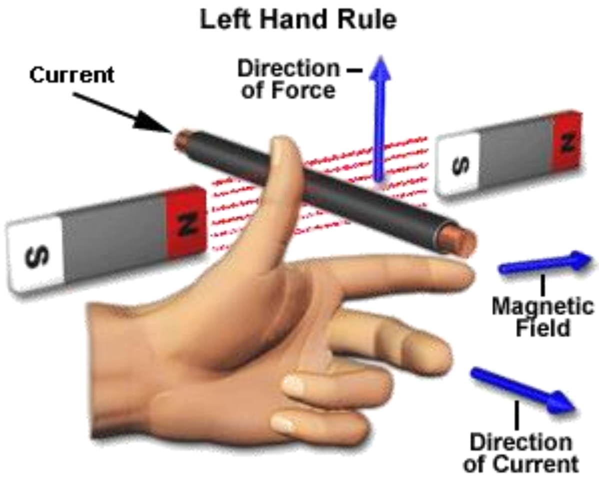
Here the direction of the current depends on the orientation of length of the current carrying conductor (l), so vector is taken for length only. The force $\overrightarrow{F}$ is the cross product of length vector ($\overrightarrow{l}$) and the flux density vector ($\overrightarrow{B}$). Now,

$$\overrightarrow{F} = I.l.B\sin\theta\,\widehat{n}$$

Here, θ is the angle between two vectors and $\widehat{n}$ is the unit vector of the force in the perpendicular direction with respect to two vectors direction.



In This direction of force the conductor will move to. This consequent can be simplified with an easy rule, i.e. Fleming's Left Hand rule. By stretching three fingers of left hand in perpendicular manner with each other, if the direction of the current is denoted by middle finger of the left hand and the second finger is for direction of the magnetic flux then the thumb of the left hand denotes the direction of the conductor’s movement.



Now the direction of the current through this conductor depends on the conductor in which orientation the conductor is placed between two poles of the magnet. So the current carrying conductor always faces a force in the vicinity of a permanent magnet or any electro-magnet. Based on this phenomenon DC motor rotates.

CURRENT AND CURRENT DENSITY

Current is the flow of electrical charge carriers like electrons. **Current** flows from negative to positive points. The SI unit for measuring electric **current** is the ampere (A). One ampere of **current** is defined as one coulomb of electrical charge moving past a unique point in a second.

The **current density** vector is defined as a vector whose magnitude is the electric **current** per cross-sectional area at a given point in space, its direction being that of the motion of the charges at this point. In SI base units, the electric **current density** is measured in amperes per square metre.

CONDUCTION AND CONVECTION CURRENT

Conduction current is the electric **current** that flows through a conductor because of an applied potential difference. **Displacement current** is the **current** that is included to explain the magnetic field inside the capacitor due to mounting up of charges on its plates.

Curl

Physical Interpretation of the **Curl**. The **curl** of a vector field measures the tendency for the vector field to swirl around. Imagine that the vector field represents the velocity vectors of water in a lake. If the vector field swirls around, then when we stick a paddle wheel into the water, it will tend to spin.

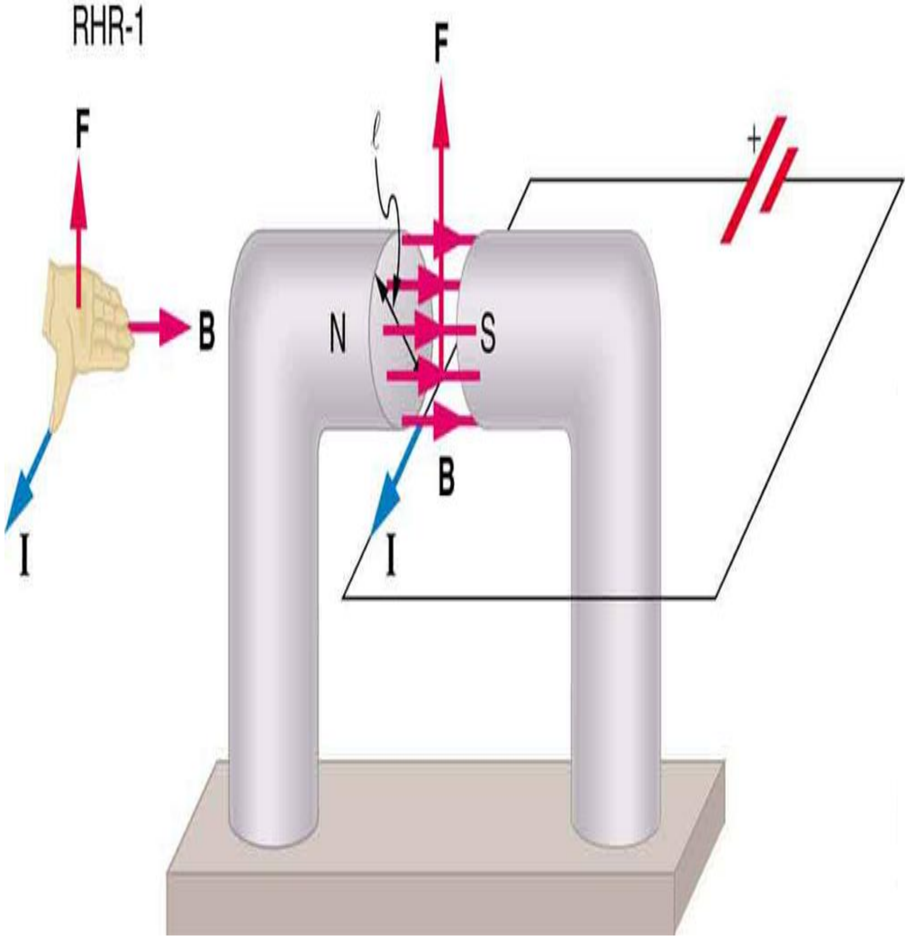
STOKES THEOREM

Stokes theorem says the surface integral of curlF over a surface S (i.e., $\iint_S \text{curl}F\cdot dS$) is the circulation of F around the boundary of the surface (i.e., $\int_C F\cdot ds$ where $C=\partial S$). force on current element in magnetic field

MAGNETIC FORCE ON A CURRENT-CARRYING CONDUCTOR

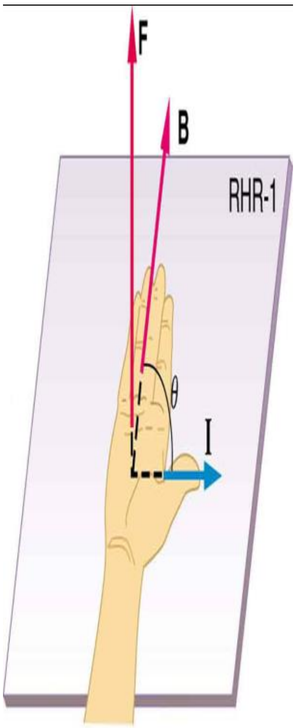
- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity vdvd is given by F = qv_dB

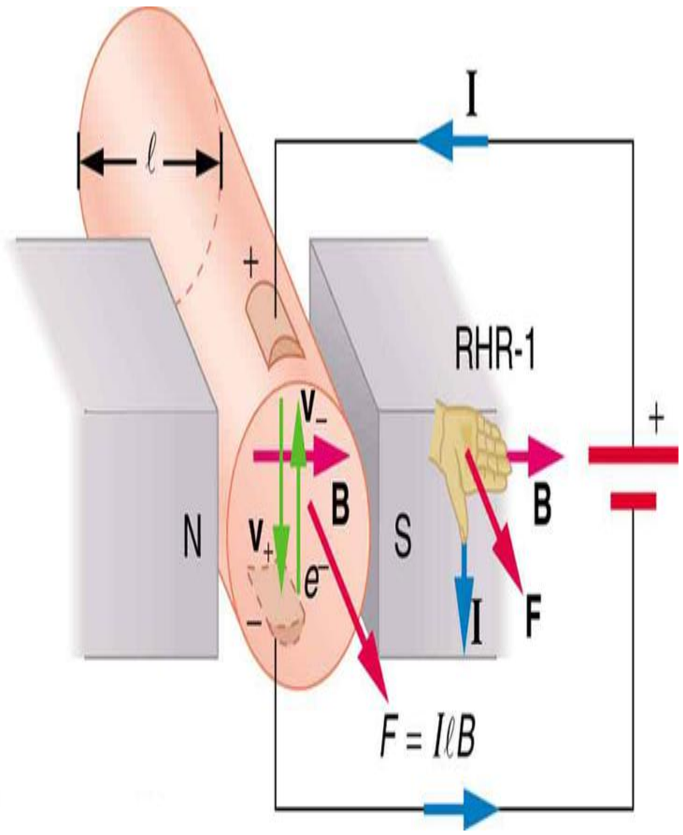


$F = I\ell B \sin\theta$

$\mathbf{F} \perp$ plane of $\mathbf{I}$ and $\mathbf{B}$

The force on a current-carrying wire in a magnetic field is ***F* = *IlB* sin *θ***. Its direction is given by RHR-1. This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See Figure 3.)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See Figure 4.) Existing MHD drives are heavy and inefficient—much development work is needed.

QUESTIONS

PART-A

1. State stokes theorem
2. State and explain ampere's law.
3. Define Curl of the vector.
4. What you mean by magnetic vector potential.
5. Define magnetic flux density.

6. Write the expressions for magnetic force, when moving charge particle in a magnetic field.
7. Define conduction current
8. Distinguish between conduction and convection current.
9. Write the boundary conditions between two magnetic media
10. Write down the expression for displacement current.

PART-B

1. State Biot–Savart's law in vector form. Also derive the Maxwells's equation from Biot–Savart's law
2. Determine the force per meter length between two long parallel wires A and B separated by 5cm in air and carrying current of 400 amps in the (a) same direction (b) opposite direction
3. (a)Show by means of Biot – Savart's law that the flux density produced by an infinitely long straight wire carrying a current I, at any point distant and normal to the wire is given by $\mu_0\mu_r \frac{I}{(2\pi a)}$.

   (b)Prove that ∇XH = 0, where **H** is the field outside of a long straight wire carrying a current I.
4. Develop an expression for the magnetic field at any point on the line through the centre at a distance 'h' from the centre and perpendicular to the plane of a plane circular loop of radius 'a' and carrying current I
5. Derive the expression for the force between two parallel wires carrying currents in the same direction.
6. (a)State and prove Stoke's theorem

   (b)Express force developed on a closed current loop due to uniform magnetic field.
7. Derive the expressions for boundary conditions in magnetic fields.
8. Express the Torque developed on a closed current loop due to steady magnetic field.
9. State Ampher's law and apply it to determine the magnetic field intensity due to an infinitely long current filament.
10. Define and apply Ampere's law to find the magnetic field intensity due to (i) Long wire. (ii)Long solenoid.

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UNIT – IV - Electromagnetic Theory - SEEA1202

I. Faraday's Law of Electromagnetic Induction

Inductor and Inductance

If a time varying current flowing through a coil there is an emf induced in it. The induced emf across the coil is directly proportional to the rate of change of current with respect to time. Due to the property inducing emf, all types of electrical coil can be referred as **inductor**. An **inductor** is an energy storage device which stores energy in form of magnetic field.

As we already told, the induced emf across a coil is directly proportional to the rate of change of current through it. The proportionality constant in that relation is known as **inductance**.

THEORY OF INDUCTOR

A current through a conductor produces a magnetic field surround it. The strength of this field depends upon the value of current passing through the conductor. The direction of the magnetic field is found using the right hand grip rule, which shown. The flux pattern for this magnetic field would be number of concentric circle perpendicular to the detection of current.

Now if we wound the conductor in the form of a coil or solenoid, it can be assumed that there will be concentric circular flux lines for each turn of the coil as shown. But it is not possible practically, as if concentric circular flux lines for each turn exist, they will intersect each other. However, since lines of flux cannot intersect, the flux lines for an individual turn will distort to form complete flux loops around the whole coil as shown. This flux pattern of a current carrying coil is similar to a flux pattern of a bar magnet

Now if the current through the coil gets changed, the magnetic flux produced by it will also get changed at the same rate. As the flux already surrounds the coil, this changing flux obviously links the coil. Now according to Faraday's law of electromagnetic induction, if changing flux links with a coil, there would be an induced emf in it. Again as per Lenz's law, this induced emf opposes every cause of producing it. Hence, the induced emf is in opposite of the applied voltage across the coil.

TYPES OF INDUCTION

There are two types of Induction self induction and mutual induction.

SELF INDUCTION

When time varying current flows in a coil the time varying flux is produced and this varying flux will link with that coil itself and as a result there will be emf induced in the coil itself. This type of Induction is called self induction.

MUTUAL INDUCTION

When time varying current flows in a coil it produces time varying flux as we have already told. This time varying flux may link with another nearby coil. Due to this flux linkage there will be an induced emf in the second coil. This type of electrical induction is called mutual induction. Hence mutual induction can be defined as the induction of emf in one coil due to time varying current flowing in any other nearby coil.

SERIES AND PARALLEL INDUCTORS WITH EFFECTS OF MUTUAL INDUCTION

When **inductors are connected in series** the equivalent inductance of the combination will be the simple sum of the inductance of all individual inductors. This is just like the equivalent resistance of series connected resistors. But in the case of inductors, we sometimes may have to consider the effect of mutual inductance between the inductors. Then the for calculating the inductance of each inductor we consider both the self inductance and mutual inductance of the inductor. The mutual inductance will be either added or subtracted from the self inductance depending on the polarity of magnetically coupled inductors. We will learn about effect of mutual inductance later in this article. Now, without considering mutual

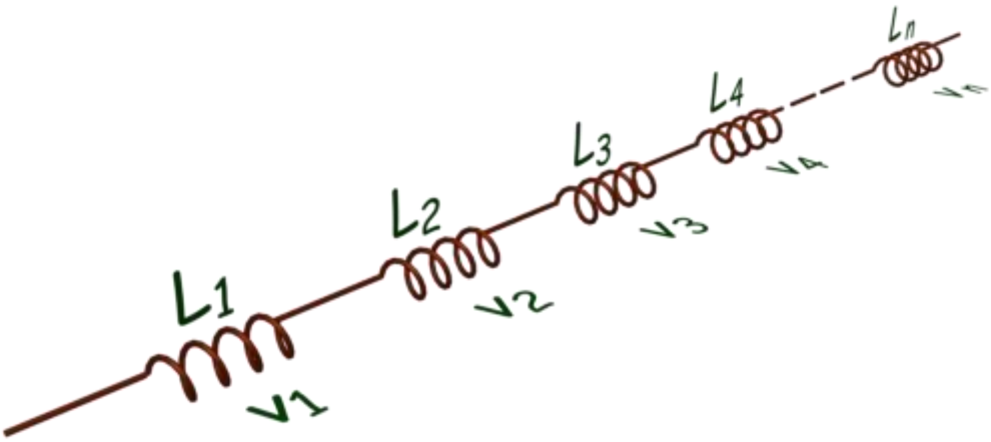
inductances we can write the equivalent inductance of series connected inductors as,

When **inductors are connected in parallel** the reciprocal of the equivalent inductance of the combination will be the sum of the reciprocal of individual inductances. This is just like the equivalent resistance of parallel connected resistors. Here also we may have to consider the effect of mutual inductance in the same way if required. We will learn the effect of mutual inductance on parallel inductors later in this article. Without considering the effect of mutual inductance we can write

An inductor is passive circuit element. Let us find out the equivalent inductance of **series connected and parallel connected inductors**.

SERIES CONNECTED INDUCTORS

Let us consider n number of **inductors connected in series** as shown below.



Let us also consider that,

the inductance of inductor 1 and voltage drop across it are L_1 and v_1 respectively,

the inductance of inductor 1 and voltage drop across it are L_2 and v_2 respectively,

the inductance of inductor 1 and voltage drop across it are L_3 and v_3 respectively,

the inductance of inductor 1 and voltage drop across it are L_4 and v_4 respectively,

the inductance of inductor 1 and voltage drop across it are L_n and v_n respectively.

Now, applying, Kirchhoff's Voltage Law, we get, total voltage drop (v) across the **series combination of the inductors**,

$$v = v_1 + v_2 + v_3 + v_4 + \cdots\cdots + v_n$$

The votage drop across an inductor of inductance L can be expressed as,

$$L\frac{di}{dt}$$

Where, i is the instanteous current through the inductor.

As all inductors of the combinations are connected in series, here, the current through each of the inductors is same, and say also it is i. So, from above KVL equation, we get,

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + L_4\frac{di}{dt} + \cdots\cdots + L_n\frac{di}{dt}$$

This equation can be rewritten as,

$$v = (L_1 + L_2 + L_3 + L_4 + \cdots\cdots + L_n)\frac{di}{dt}$$
$$= \left(\sum_{k=1}^{n} L_k\right)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

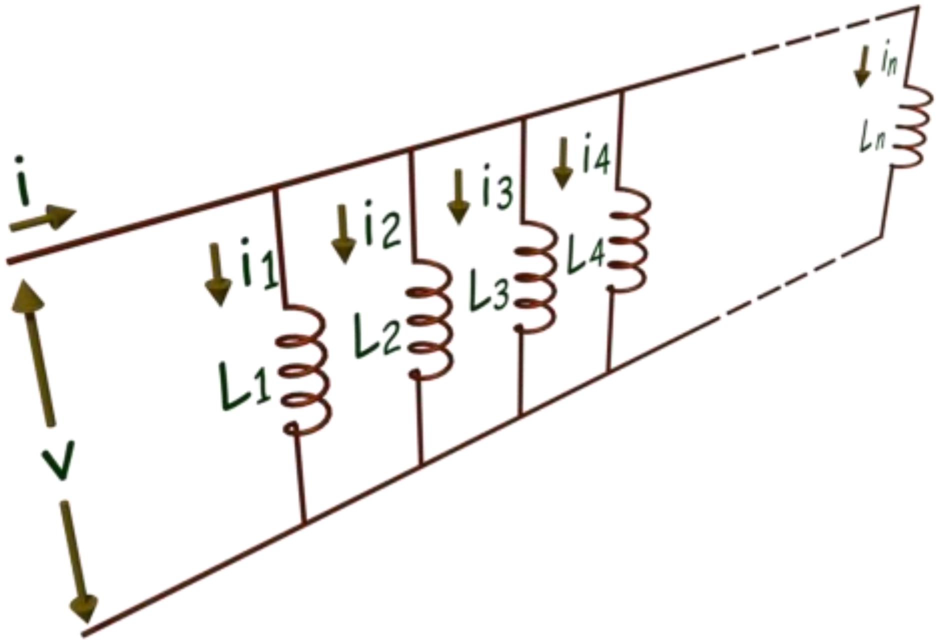
Where, L_{eq} is equivalent inductance of the **series combined inductors**. Hence,

$$\boxed{L_{eq} = L_1 + L_2 + L_3 + L_4 + \cdots\cdots + L_n}$$

Equivalent inductance of series connected inductors is simply arithmetic sum of the inductance of individual inductors.

PARALLEL CONNECTED INDUCTORS

Let us consider n number of **inductors connected in parallel** as shown below.



Let us also consider that,

the inductance of inductor 1 and current through it are L_1 and i_1 respectively,

the inductance of inductor 1 and current through it are L_2 and i_2 respectively,

the inductance of inductor 1 and current through it are L_3 and i_3 respectively,

the inductance of inductor 1 and current through it are L_4 and i_4 respectively,

the inductance of inductor 1 and current through it are L_n and i_n respectively.

Now, applying, Kirchhoff's Current Law, we get, total current (i) entering in the **parallel combination of the inductors**,

$$i = i_1 + i_2 + i_3 + i_4 + \cdots\cdots + i_n$$

The current through an inductor of inductance L can be expressed as,

$$\frac{1}{L}\int v\,dt$$

Where, v is the instantaneous voltage across the inductor.

As all inductors of the combinations are connected in parallel, here, the voltage drop across each of the inductors is same, and say also it is v. So, from above KCL equation, we get,

$$i = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \frac{1}{L_3}\int v\,dt + \frac{1}{L_4}\int v\,dt + \cdots\cdot + \frac{1}{L_n}\int v\,dt$$

This equation can be rewritten as,

$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \cdots\cdot + \frac{1}{L_n}\right)\int v\,dt$$
$$= \left(\sum_{k=1}^{n}\frac{1}{L_k}\right)\int v\,dt = \frac{1}{L_{eq}}\int v\,dt$$

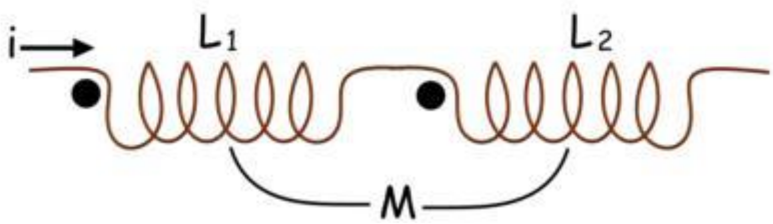
Where, L_{eq} is equivalent inductance of the **parallel combined inductors**. Hence,

$$\boxed{\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \cdots\cdot + \frac{1}{L_n}}$$

Reciprocal of equivalent inductance of parallel connected inductors is simply arithmetic sum of the reciprocal of inductance of individual inductors.

EFFECT OF MUTUAL INDUCTION IN SERIES CONNECTED INDUCTORS

Whenever more than one inductors come in closer, there may be mutual induction between them. If more than one inductors are connected in series and flux of one inductor links the other then we have to consider mutual inductance during calculations of equivalent inductance. For this purpose, we use the dot convention. Here each of the inductors is marked with a dot at one end. The current entering through the dotted terminal of one inductor will induce a voltage in the other inductor with positive polarity at the dotted terminal of the later. Let us consider the following example.

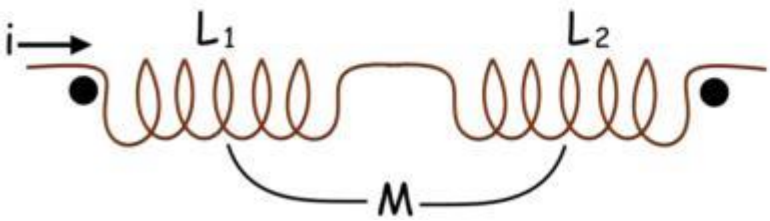


As inductors are in series same current will flow to these inductors. Hence when current enters through the dotted terminal of inductor 1, the current enters through the dotted terminal of inductor 2. Inductor 2 will induce the voltage across inductor 1 with positive polarity at dotted end of inductor 1. Current enters through the dotted terminal of inductor 1 will induce the voltage across inductor 2 with positive polarity at dotted end of the inductor 2. As the both mutually induced emfs are in the direction self induced emf the equivalent impedances the mutual inductance will simply be added to self inductance to calculate equivalent impedance.

$$v_1 = L_1\frac{di}{dt} + M\frac{di}{dt}$$
$$v_2 = L_2\frac{di}{dt} + M\frac{di}{dt}$$

$$v = v_1 + v_2 = L_e\frac{di}{dt}$$
$$= L_1\frac{di}{dt} + M\frac{di}{dt} + L_2\frac{di}{dt} + M\frac{di}{dt}$$
$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} + 2M\frac{di}{dt}$$
$$\Rightarrow L_e = L_1 + L_2 + 2M$$

Here in this second example as per dot convention provided in the figure below current is entering through the dotted terminal of one inductor and the same current is leaving the dotted terminal of the other inductor. In that case polarity of mutually induced emf differs the self induced emf. The equivalent inductance of the combination will be



$$v_1 = L_1\frac{di}{dt} - M\frac{di}{dt} \ \& \ v_2 = L_2\frac{di}{dt} - M\frac{di}{dt}$$

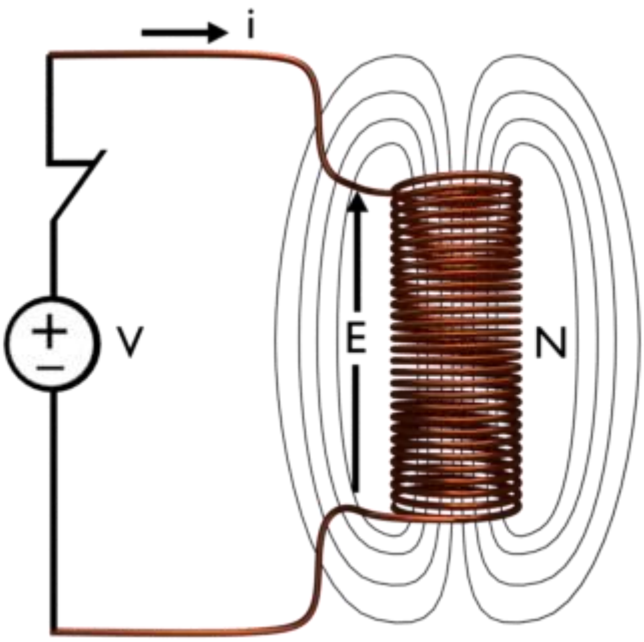
$$v = v_1 + v_2$$

$$v = L_e\frac{di}{dt} = L_1\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt} - M\frac{di}{dt}$$
$$= L_1\frac{di}{dt} + L_2\frac{di}{dt} - 2M\frac{di}{dt}$$

$$\Rightarrow L_e = L_1 + L_2 - 2M$$

DERIVATION OF INDUCTANCE

For the DC source, when the switch is ON, i.e. just at $t = 0^+$, a current starts flowing from its zero value to a certain value and with respect to time, there will be a rate of change in current momentarily. This current produces changing flux (φ) through the coil. As current changes flux (φ) also changes and the rate of change with respect to the time is $\frac{d\varphi}{dt}$



Now by apply Faraday's Law of Electromagnetic Induction, we get,

$$E = N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

Where, N is the number of turn of the coil and e is the induced EMF across this coil.

Considering Lenz's law we can write the above equation as,

$$E=-N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

Now, we can modify this equation to calculate the value of inductance.

$$E=-N\frac{\mathrm{d}\phi}{\mathrm{d}t}\ \ \&\ \ E=-L\frac{\mathrm{d}i}{\mathrm{d}t}$$
$$\Rightarrow N\ d\phi=L\ di\Rightarrow N\phi=Li$$

So, $Li=N\phi=NBA$ [B is the flux density i.e. B =φ/A, A is area of the coil],

[Nφ or Li is called magnetic flux Linkage and it is denoted by Ψ]

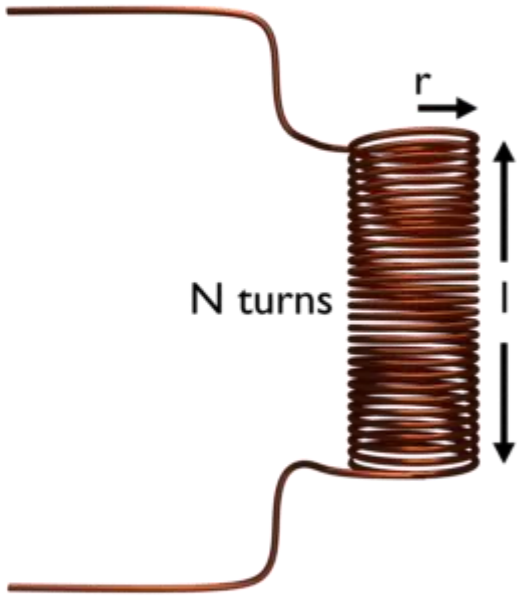
$Again,\ Hl=Ni$ Where H is the magnetizing force due to which magnetic flux lines flow from south to north pole inside the coil, l (small L) is the effective length of the coil and

$$Again,\ B=\mu H$$

$$Li=NBA\Rightarrow L=\frac{NBA}{i}=\frac{N^2BA}{Ni}$$
$$=\frac{N^2BA}{Hl}\ [\because Ni=Hl]=\frac{N^2\mu HA}{Hl}\ [\because B=\mu H]$$

$$\Rightarrow L=\frac{\mu N^2A}{l}=\frac{\mu N^2\pi r^2}{l}\quad [A=\pi r^2]$$

r is the radius of the coil cross-sectional area.



Self inductance, L is a geometric quantity; it depends only on the dimensions of the solenoid, and the number of turns in the solenoid. Furthermore, in a DC circuit when the switch is just closed, then only momentarily effect of self-inductance occurs in the coil. After some time, no effect of **self inductance** remains in the coil because after certain time the current becomes steady.

But in AC circuit, the alternating effect of current always causes the self-induction in the coil, and a certain value of this self-inductance gives the inductive reactance ($X_L = 2\pi fL$) depending on the value of supply frequency.

INDUCTANCE IN TRANSMISSION LINE

Reason of Transmission Line Inductance

Generally, electric power is transmitted through the transmission line with AC high voltage and current. High valued alternating current while flowing through the conductor sets up magnetic flux of high strength with alternating nature. This high valued alternating magnetic flux makes a linkage with other adjacent conductors parallel to the main conductor. Flux linkage in a conductor happens internally and externally. Internally flux linkage is due to self-current and externally flux linkage due to external flux. Now the term inductance is closely related to the flux linkage, denoted by λ. Suppose a coil with N number of turn is linked by flux Φ due to current I, then,

$$Inductance\ L = \frac{Flux\ linkage}{current} = \frac{N\phi}{I}$$

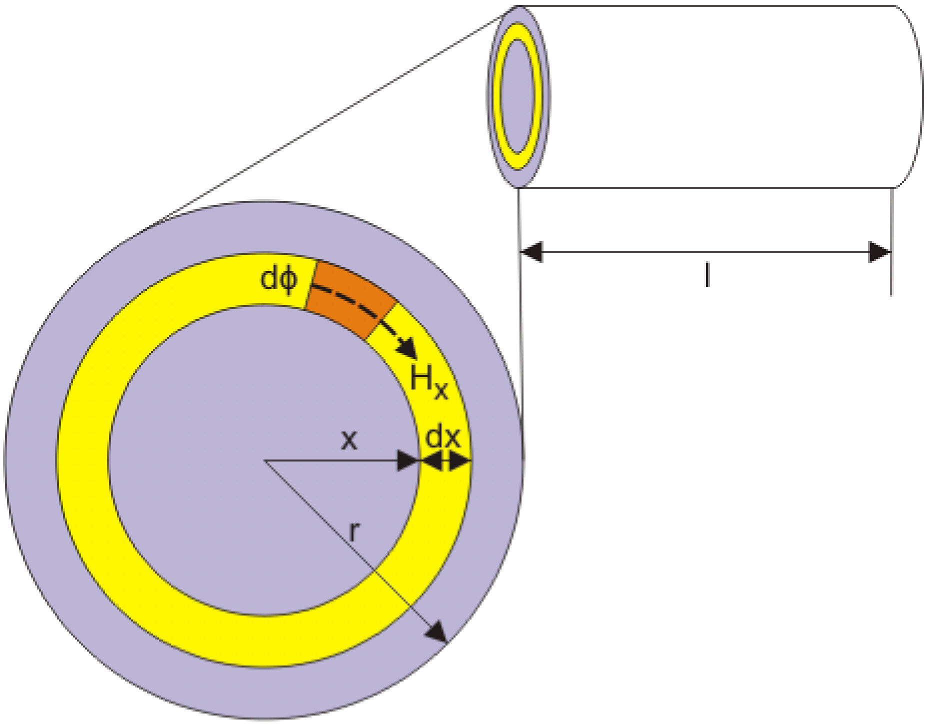
But for transmission line N = 1. We have to calculate only the value of flux Φ, and hence, we can get the transmission line inductance.

CALCULATION OF INDUCTANCE OF SINGLE CONDUCTOR

Calculation of Internal Inductance due to Internal Magnetic Flux of a Conductor

Suppose a conductor is carrying current I through its length l, x is the internal variable radius of the conductor and r is the original radius of the conductor. Now the cross-sectional area with respect to radius x is πx^2 square – unit and current I_x is flowing through this cross-sectional area. So the value of I_x can be expressed in term of original conductor current I and cross-sectional area πr^2 square – unit

$$\frac{I_x}{I} = \frac{\pi x^2}{\pi r^2} \Rightarrow I_x = I\frac{\pi x^2}{\pi r^2} = I\frac{x^2}{r^2}$$



Now consider small thickness dx with the 1m length of the conductor, where H_x is the magnetizing force due to current I_x around the area πx^2.

$$So,\ H_x = \frac{I_x}{2\pi x} = \frac{I}{2\pi r^2}\cdot x\ (A/m)\ldots\ldots\ldots\ldots \left[as\ I_x = I\frac{x^2}{r^2}\right]$$

And magnetic flux density $B_x = \mu H_x$, where μ is the permeability of this conductor. Again, $\mu = \mu_0\mu_r$. If it is considered that the relative permeability of this conductor $\mu_r = 1$, then $\mu = \mu_0$. Hence, here $B_x = \mu_0 H_x$.

$$So,\ B_x = \mu_0 H_x = \mu_0\frac{I}{2\pi r^2}\cdot x = \frac{\mu_0}{2\pi}\left(\frac{xI}{r^2}\right)\quad Tesla$$

dφ for small strip dx is expressed by

$$d\phi = B_x dx = \frac{\mu_0}{2\pi}\left(\frac{xI}{r^2}\right)dx\quad Wb/m$$

Here entire cross-sectional area of the conductor does not enclose the above expressed flux. The ratio of the cross sectional area inside the circle of radius x to the total cross section of the conductor can be thought about as fractional turn that links the flux. Therefore the flux linkage is

$$d\lambda = \frac{\pi x^2}{\pi r^2} d\phi = \frac{\mu_0}{2\pi}\left(\frac{x^3 I}{r^4}\right) \quad Wb/m$$

Now, the total flux linkage for the conductor of 1m length with radius r is given by

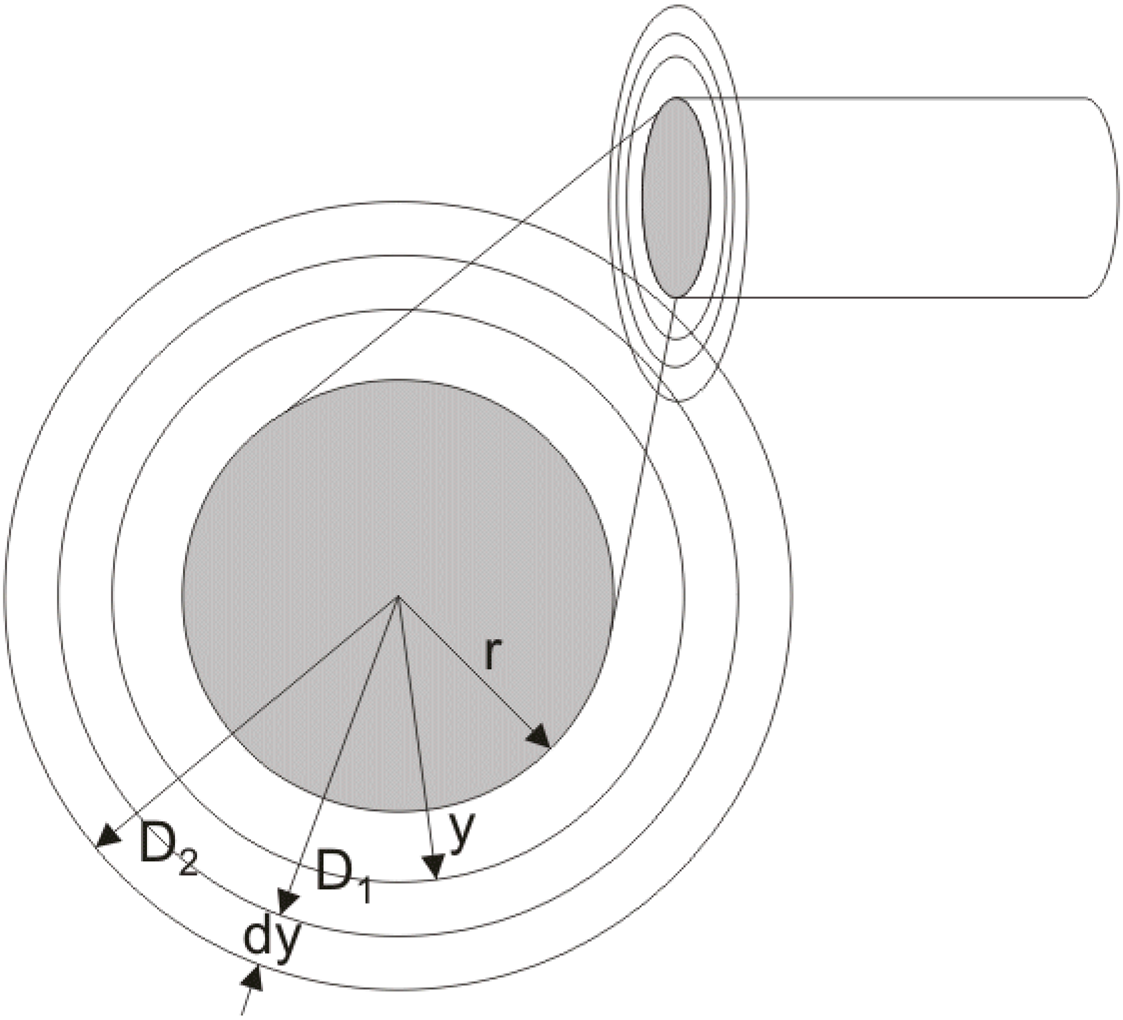
$$\lambda = \int_0^r d\lambda = \frac{\mu_0}{8\pi} I \quad Wb/m$$

Hence, the internal inductance is

$$L_{internal} = \frac{\lambda}{I} = \frac{\mu_0}{8\pi} \quad Henry/m$$

External Inductance due to External Magnetic Flux of a Conductor

Let us assume, due to skin effect conductor current I is concentrated near the surface of the conductor. Consider, the distance y is taken from the center of the conductor making the external radius of the conductor.



H_y is the magnetizing force and B_y is the magnetic field density at y distance per unit length of the conductor.

$$Now,\ H_y = \frac{I}{2\pi y}\quad A/m\ \ \&\ \ B_y = \mu_0 H_y = \frac{\mu_0}{2\pi} \times \frac{I}{y}\quad Tesla$$

Let us assume magnetic flux dφ is present within the thickness dy from D_1 to D_2 for 1 m length of the conductor as per the figure.

$$So,\ d\phi = B_y dy = \frac{\mu_0}{2\pi} \times \frac{I}{y}\, dy\quad Wb/m$$

As the total current I is assumed to flow in the surface of the conductor, so the flux linkage dλ is equal to dφ.

$$\therefore\ \ d\lambda = d\phi = \frac{\mu_0}{2\pi} \times \frac{I}{y}\ \Rightarrow\ \lambda_{1\text{-}2} = \int_{D_1}^{D_2} d\lambda = \frac{\mu_0}{2\pi} I \int_{D_1}^{D_2} \frac{1}{y} dy = \frac{\mu_0}{2\pi} I \times ln\frac{D_1}{D_2}\quad Wb/m$$

But we have to consider the flux linkage from conductor surface to any external distance, i.e. r to D

$$So,\ \lambda = \frac{\mu_0}{2\pi} I \times ln\frac{D}{r}\quad Wb/m$$

$$\therefore\ \ External\ Inductance\ L_{external} = \frac{\mu_0}{2\pi} \times ln\frac{D}{r}\quad H/m$$

$$\therefore Total\ Inductance\ L_{total} = L_{internal} + L_{external} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi} \times ln\frac{D}{r} = \frac{\mu_0}{2\pi}\left(\frac{1}{4} + ln\frac{D}{r}\right) H/m$$

$$\Rightarrow L_{total} = \frac{\mu_0}{2\pi}\left[ln(e^{\frac{1}{4}}) + ln\frac{D}{r}\right] = \frac{\mu_0}{2\pi}\left[ln\frac{D}{r} - ln(e^{-\frac{1}{4}})\right]$$

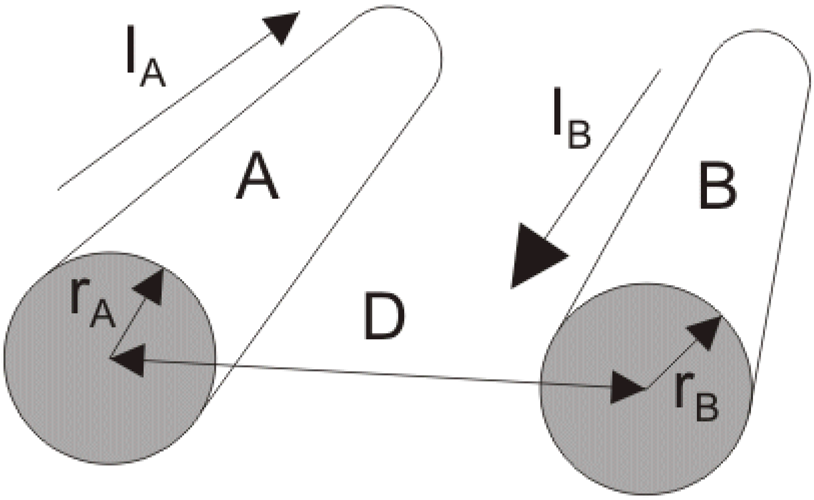
$$= \frac{\mu_0}{2\pi} ln\left(\frac{D}{e^{-\frac{1}{4}} r}\right) = \frac{\mu_0}{2\pi} ln\left(\frac{D}{0.7788r}\right)\ [\because e^{-1/4} = 0.7788]$$

$$= \frac{\mu_0}{2\pi} ln\left(\frac{D}{GMR}\right)$$

$$[\because 0.7788r = Geometric\ Mean\ Radius(GMR)]$$

INDUCTANCE OF TWO WIRE SINGLE PHASE TRANSMISSION LINE

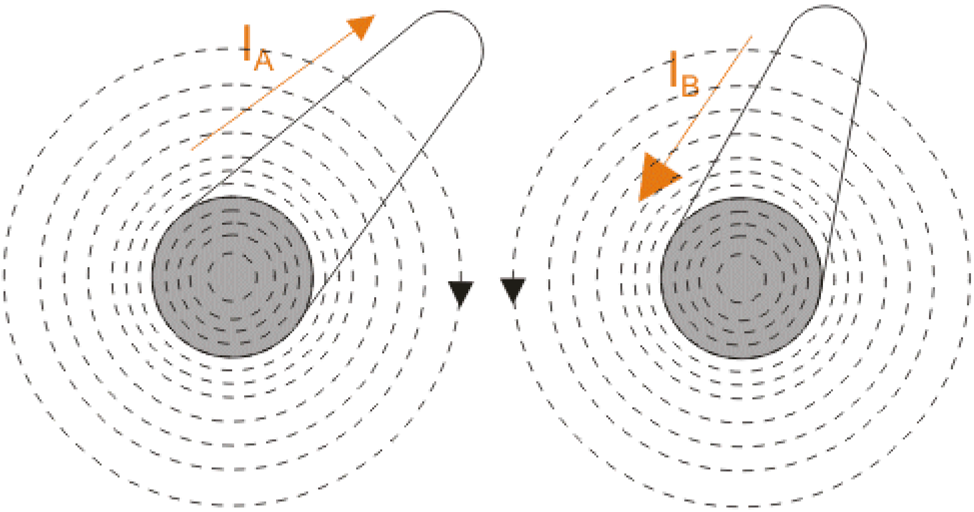
Suppose conductor A of radius r_A carries a current of I_A in opposite direction of current I_B through the conductor B of radius r_B. Conductor A is at a distance D from conductor B and both are of length l. They are in close vicinity with each other so that flux linkage takes place in both of the conductors due to their electromagnetic effects.



Let us consider the magnitude of current in both conductors are same and hence $I_A = - I_B$,

Now, total flux linkage in conductor A = flux linkage by self-current of conductor A + flux linkage on conductor A due to current in the conductor B.

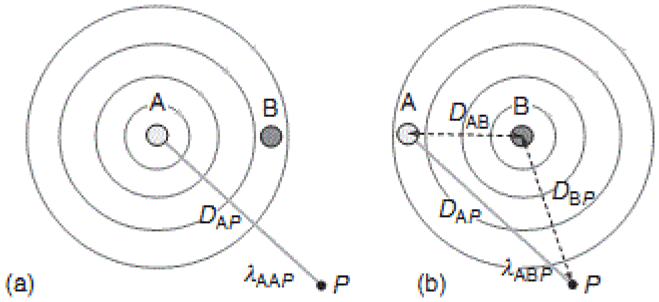
Similarly, flux linkage in conductor B = flux linkage by self-current of conductor B + flux linkage on conductor B due to current through conductor A.



Now if we consider a point P in close vicinity both conductor A and B, the flux linkage at point P would be, flux linkage at point P for current carrying conductor A + flux linkage at point P for current carrying conductor B i.e.

$$\lambda_P = \lambda_{AP} + \lambda_{BP}$$

Now, $\lambda_P = \lambda_{AP} + \lambda_{BP} = (\lambda_{AAP} + \lambda_{ABP}) + (\lambda_{BAP} + \lambda_{BBP})$........ shown in the figure below in.



- λ_{AAP} is the flux linkage at point P for conductor A due to current through conductor A itself.
- λ_{ABP} is the flux linkage at point P for conductor A due to current through conductor B.
- λ_{BAP} is the flux linkage at point P for conductor B due to current through conductor A.
- λ_{BBP} is the flux linkage at point P for conductor B due to current through conductor B itself.

$$\lambda_{AAP} = \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{AP}}{GMR_A}\right) \ \ \& \ \ \lambda_{BBP} = \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{BP}}{GMR_B}\right) \quad Wb/m$$

$$\lambda_{ABP} = \int_D^{D_{BP}} B_{BP} dP = -\frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{BP}}{D}\right) \quad Wb/m$$

$$\lambda_{BAP} = \int_D^{D_{AP}} B_{AP} dP = -\frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{AP}}{D}\right) \quad Wb/m$$

λ_{ABP} and λ_{BAP} are negative in value because the directions current are opposite with respect to each other.

$$\begin{aligned} So,\ \lambda_P &= \lambda_{AP} + \lambda_{BP} = (\lambda_{AAP} + \lambda_{ABP}) + (\lambda_{BAP} + \lambda_{BBP}) \\ &= \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{AP}}{GMR_A}\right) - \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{AP}}{D}\right) \\ &+ \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{BP}}{GMR_B}\right) - \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{BP}}{D}\right) \\ &= \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D_{AP}}{GMR_A} \times \frac{D}{D_{AP}} \times \frac{D_{BP}}{GMR_B} \times \frac{D}{D_{BP}}\right) \\ &= \frac{\mu_0}{2\pi} \times I \times ln\left(\frac{D^2}{GMR_A GMR_B}\right) \quad Wb/m \end{aligned}$$

If we consider that both conductor are with same radius, i.e. $r_A = r_B = r$ and point P is shifted to infinite distance then we can write that

$$\lambda = \frac{\mu_0}{2\pi} I \times ln\left(\frac{D^2}{GMR^2}\right) = \frac{\mu_0}{2\pi} I \times 2ln\left(\frac{D}{GMR}\right) = \frac{\mu_0}{\pi} I \times ln\left(\frac{D}{GMR}\right) \quad Wb/m$$

$$Therefore,\ inductance\ per\ phase = \frac{\lambda}{I} = L = \frac{\mu_0}{\pi} \times ln\left(\frac{D}{GMR}\right) \quad H/m$$

If conductor A becomes bundled conductor, then its geometrical mean radius (GMR) will be calculated for n number of conductors per bundle.

$$So,\ GMR_{n\text{-}buldled\ conductors} = \sqrt[n]{d^{(n-1)}GMR_{stranded}}$$

Where, d is the distance between the central axis of conductors within the bundle.

INDUCTANCE IN THREE PHASE TRANSMISSION LINE

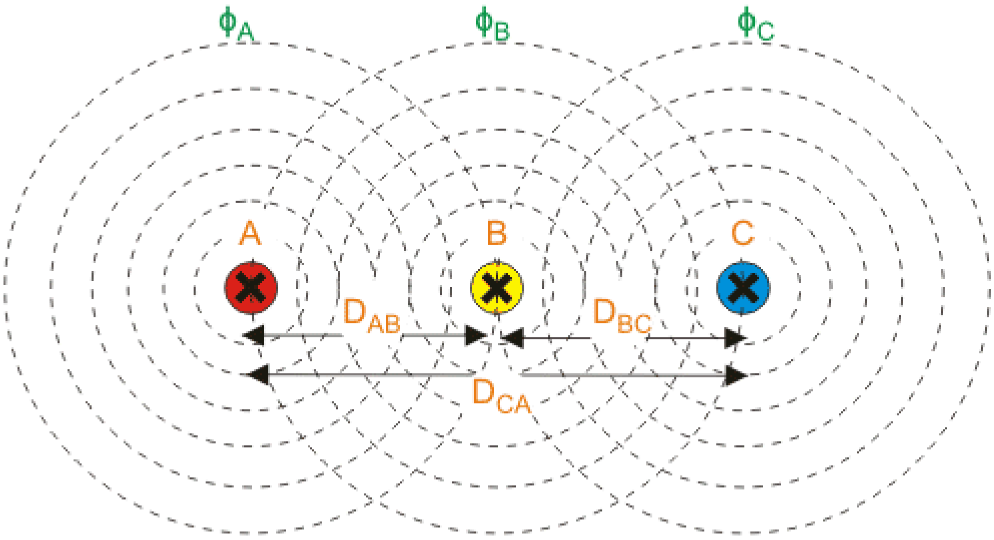
In the **three phase transmission line**, three conductors are parallel to each other. The direction of the current is same through each of the conductors.

Let us consider conductor A produces magnetic flux φ_A,

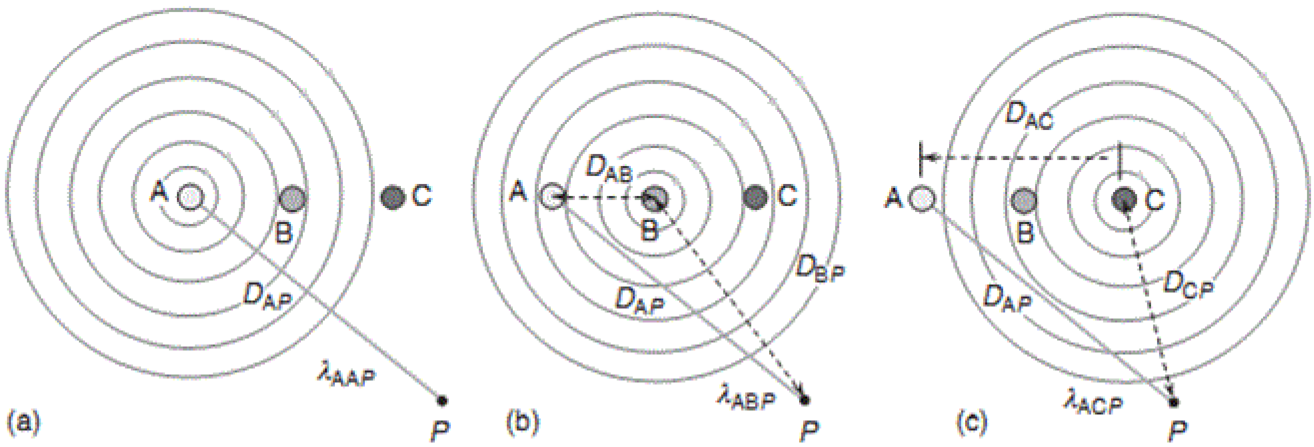
Conductor B produces magnetic flux φ_B,

And conductor C produces magnetic flux φ_C.

When they carry the current of the same magnitude "I", they are in flux linkage with each other.



Now, let us consider a point P near three conductors. So, flux linkage at point P due to current through conductor A is, $\lambda_{AP} = \lambda_{AAP} + \lambda_{ABP} + \lambda_{ACP}$



Flux linkage at point P for conductor A due to current through conductor A =

$$\lambda_{AAP} = \frac{\mu_0}{2\pi} I_A \times ln\left(\frac{D_{AP}}{GMR_A}\right) \quad Wb/m$$

Flux linkage at point P for conductor A due to current through conductor B =

$$\lambda_{ABP} = \frac{\mu_0}{2\pi} I_B \times ln\left(\frac{D_{BP}}{D_{AB}}\right) \quad Wb/m$$

Flux linkage at point P for conductor A due to current through conductor C =

$$\lambda_{ACP} = \frac{\mu_0}{2\pi} I_C \times ln\left(\frac{D_{CP}}{D_{AC}}\right) \quad Wb/m$$

Therefore, flux linkage at point P for conductor A,

$$\lambda_{AP} = \frac{\mu_0}{2\pi} I_A \times ln\left(\frac{D_{AP}}{GMR_A}\right) + \frac{\mu_0}{2\pi} I_B \times ln\left(\frac{D_{BP}}{D_{AB}}\right) + \frac{\mu_0}{2\pi} I_C \times ln\left(\frac{D_{CP}}{D_{AC}}\right) \quad Wb/m$$

$$\Rightarrow \lambda_{AP} = \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + I_B \times ln\left(\frac{1}{D_{AB}}\right) + I_C \times ln\left(\frac{1}{D_{AC}}\right)\right]$$
$$+ \frac{\mu_0}{2\pi}\left[I_A \times ln(D_{AP}) + I_B \times ln(D_{BP}) + I_C \times ln(D_{CP})\right] \quad Wb/m$$

As, $D_{AP} = D_{BP} = D_{CP}$ and $I_A + I_B + I_C = 0$ in balanced system, then we can write that $I_A = -I_B - I_C$

$$\therefore \frac{\mu_0}{2\pi}\left[I_A \times ln(D_{AP}) + I_B \times ln(D_{BP}) + I_C \times ln(D_{CP})\right]$$

$$= \frac{\mu_0}{2\pi}\left[-I_B \times ln(D_{AP}) - I_C \times ln(D_{AP}) + I_B \times ln(D_{BP}) + I_C \times ln(D_{CP})\right] = 0$$

$$\Rightarrow \lambda_{AP} = \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + I_B \times ln\left(\frac{1}{D_{AB}}\right) + I_C \times ln\left(\frac{1}{D_{AC}}\right)\right] + 0 = \lambda_A(say)$$

$$So, \ \lambda_A = \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + I_B \times ln\left(\frac{1}{D_{AB}}\right) + I_C \times ln\left(\frac{1}{D_{AC}}\right)\right]$$

$$Similarly, \ \lambda_B = \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{D_{BA}}\right) + I_B \times ln\left(\frac{1}{GMR_B}\right) + I_C \times ln\left(\frac{1}{D_{BC}}\right)\right]$$

$$and, \ \lambda_C = \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{D_{CA}}\right) + I_B \times ln\left(\frac{1}{D_{CB}}\right) + I_C \times ln\left(\frac{1}{GMR_C}\right)\right]$$

If we arrange them in matrix form, then we get

$$\begin{bmatrix} \lambda_A \\ \lambda_B \\ \lambda_C \end{bmatrix} \begin{bmatrix} L_{AA} & L_{AB} & L_{AC} \\ L_{BA} & L_{BB} & L_{BC} \\ L_{CA} & L_{CB} & L_{CC} \end{bmatrix} \begin{bmatrix} I_A \\ I_B \\ I_C \end{bmatrix}$$

Where, λ_A, λ_B, λ_C are the total flux linkages of conductor A, B and C.

L_{AA}, L_{BB} and L_{CC} are the self inductances of conductor A, B and C.

L_{AB}, L_{AC}, L_{BC}, L_{BA}, L_{CA}, L_{CB} are the mutual inductances of between conductors A, B and C.

Again balanced system

$$D_{AB} = D_{BC} = D_{CA} = D$$

And

$$I_A + I_B + I_C = 0$$

In balanced system, then we can write that

$$I_A = -I_B - I_C$$

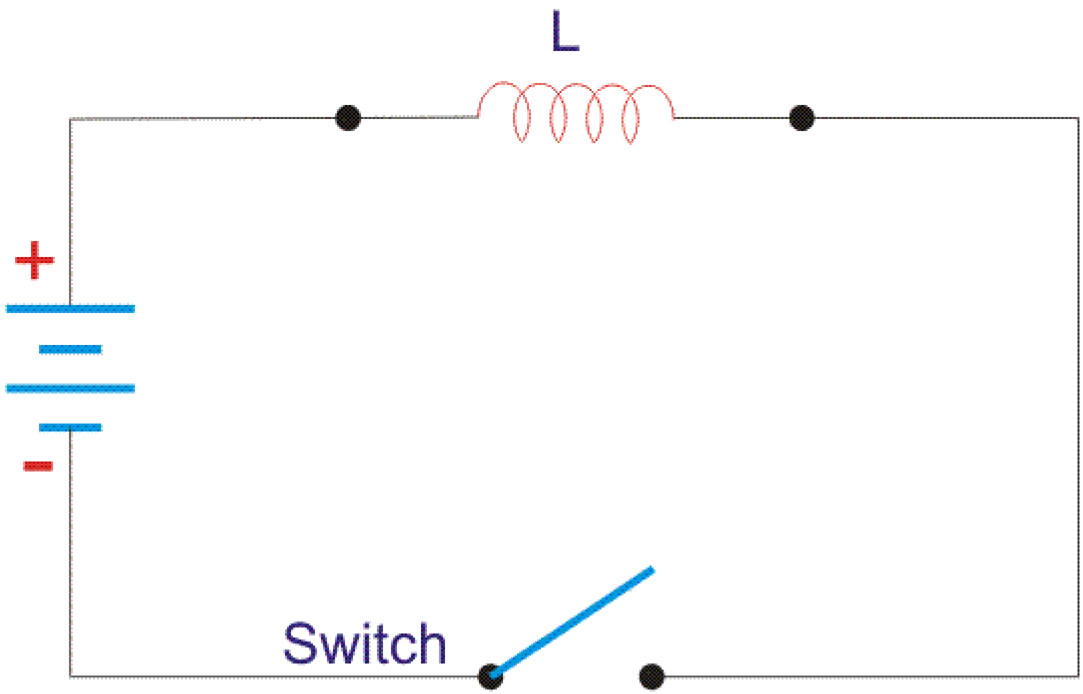
$$\begin{aligned}
\lambda_A &= \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + I_B \times ln\left(\frac{1}{D}\right) + I_C \times ln\left(\frac{1}{D}\right)\right] \\
&= \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + (I_B + I_C) \times ln\left(\frac{1}{D}\right)\right] \\
&= \frac{\mu_0}{2\pi}\left[I_A \times ln\left(\frac{1}{GMR_A}\right) + (-I_A) \times ln\left(\frac{1}{D}\right)\right] \\
&= \frac{\mu_0}{2\pi} I_A \times ln\left(\frac{D}{GMR_A}\right) \quad Wb/m
\end{aligned}$$

Similarly,

ENERGY STORED IN A MAGNETIC FIELD

Magnetic field can be of permanent magnet or electro-magnet. Both magnetic fields store some energy. Permanent magnet always creates the magnetic flux and it does not vary upon the other external factors. But electromagnet creates its variable magnetic fields based on how much current it carries. The dimension of this electro-magnet is responsible to create the strength the magnetic field and hence the energy stored in this electromagnet.

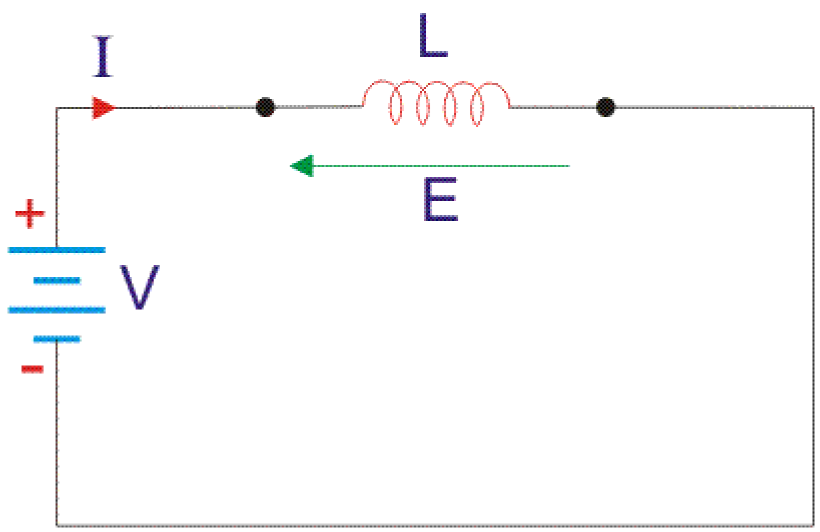
First we consider the magnetic field is due to electromagnet i.e. a coil of several no. turns. This coil or inductor is carrying current I when it is connected across a battery or voltage source through a switch.



Suppose battery voltage is V volts, value of inductor is L Henry, and current I will flow at steady state. When the switch is ON, a current will flow from zero to its steady value. But due to self induction a induced voltage appears which is

$$E = -L\frac{dI}{dt}$$

this E always in the opposite direction of the rate of change of current.



Now here the energy or work done due to this current passing through this inductor is U.
As the current starts from its zero value and flowing against the induced emf E, the energy will grow up gradually from zero value to U.

dU = W.dt, where W is the small power and W = – E.I

So, the energy stored in the inductor is given by

$$dU = W.dt = -E.Idt = L\frac{dI}{dt}.Idt = LIdI$$

Now integrate the energy from 0 to its final value.

$$U = \int_0^U dU = \int_0^I LIdI = \frac{1}{2}LI^2$$

Again, $L = \frac{\mu_0 N^2 A}{l}$

as per dimension of the coil, where N is the number of turns of the coil, A is the effective cross-sectional area of the coil and l is the effective length of the coil.

Again, $I = \frac{H.l}{N}$

Where, H is the magnetizing force, N is the number of turns of the coil and l is the effective length of the coil.

$$I = \frac{B.l}{\mu_0.N}$$

Now putting expression of L and I in equation of U, we get new expression i.e.

$$U = \frac{\frac{\mu_0 N^2 A}{l}\cdot\frac{B.l}{\mu_0.N}}{2} = \frac{B^2 Al}{2\mu_0}$$

So, the stored energy in a electromagnetic field i.e. a conductor can be calculated from its dimension and flux density.

Now let us start discussion about energy stored in the magnetic field due to permanent magnet.

Total flux flowing through the magnet cross-sectional area A is φ.

Then we can write that φ = B.A, where B is the flux density.

Now this flux φ is of two types, (a) φ_r this is remanent flux of the magnet and (b) φ_d this is demagnetizing flux.

So, $\varphi = \varphi_r + \varphi_d$

as per conservation of the magnetic flux Law.

$$\Phi = A.B_r + A.B_d$$

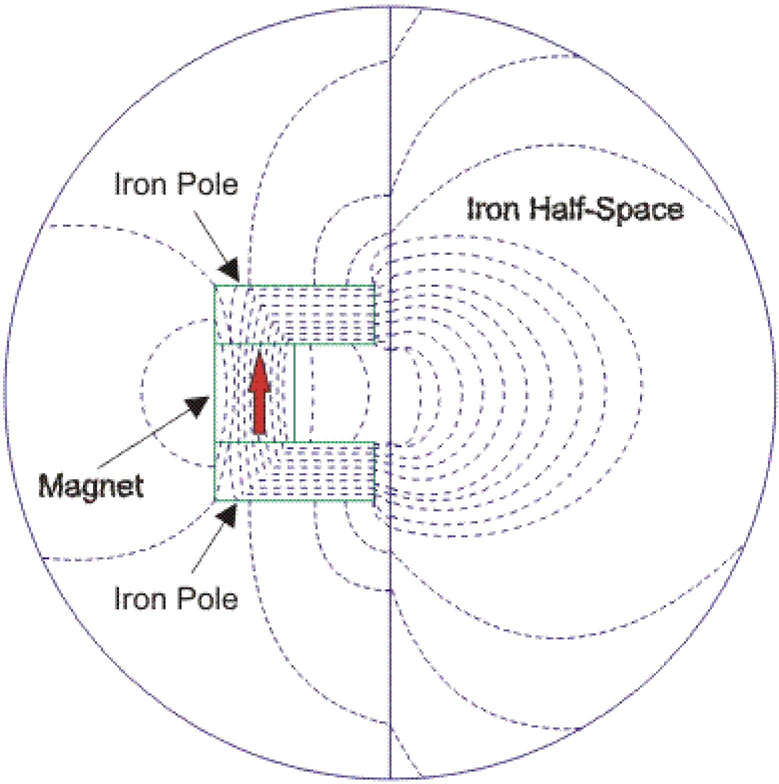
Again, $B_d = \mu. H$, here H is the magnetic flux intensity.

Now MMF or Magneto Motive Force can be calculated from H and dimension of the magnet.

$$\Delta MMF = H.l$$

where l is the effective distance between two poles.

$$\Delta MMF = \frac{Bd.l}{\mu} = \frac{l}{\mu}.\varphi d$$



Now to calculate energy we have to first go for the reluctance of the magnetic flux path.

Magnet's internal reluctance path that is for demagnetizing is denoted as R_m,

$$R_m = \frac{l}{\mu A}$$

And

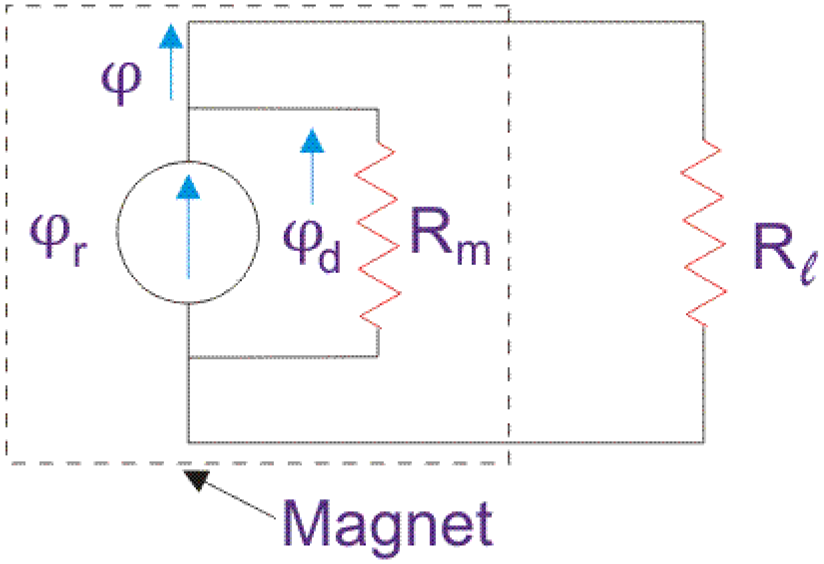
Now W_m, is the energy stored in the magnet's internal reluctance.

$$W_m = \frac{1}{2}.R_m.\varphi\frac{2}{d} = \frac{1}{2}.\frac{l}{\mu.A}.(\mu.H.A) = A.l.(\frac{1}{2}\mu H^2)$$

Now energy density

$$\frac{W_m}{A.l} = \frac{1}{2}\mu H^2 = W'_m$$

Look at the model below. Dotted lined box is the magnet and one reluctance path R_l for the mechanical load is connected across the magnet.



Now apply node equation and loop equation, we get

$$R_m.\varphi_d + \varphi.R_l = 0,$$
$$\varphi = \varphi_r + \varphi_d,$$
$$Again, \ \ \varphi = (\frac{R_m}{R_m + R_l})\varphi_r \ \ and \ \ \varphi_d = -\varphi_r(\frac{R_l}{R_m + R_l})$$

Now, If we do any mechanical work inside a magnetic field, then the energy required W.

$$W = \frac{1}{2}.(R_m.\varphi_d^2 + R_l.\varphi^2) = \frac{1}{2}(\frac{R_l R_m}{R_l + R_m})\varphi_r^2$$

Again, if we place a electromagnetic coil in the vicinity of a permanent magnet, then this coil will experience a force. To move this coil some work is done. This energy density is the co-energy with respect to the permanent magnet and the coil magnet. Magnetizing flux intensity for the permanent magnet is H and for the coil is H_C.

This co-energy is denoted as

$$W' = \frac{1}{2}\mu(H + H_C)^2 = \frac{1}{2}\mu B^2$$

Where, B is the flux density at the coil position near the permanent magnet.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION: FIRST & SECOND LAW

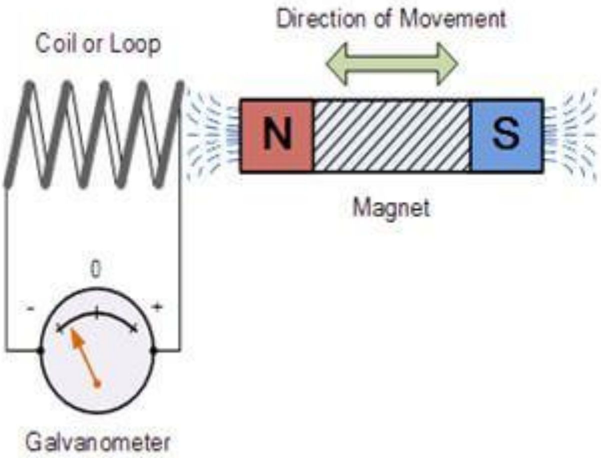
Faraday's law of electromagnetic induction (referred to as **Faraday's law**) is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (EMF). This phenomenon is known as electromagnetic induction.

Faraday’s law states that a current will be induced in a conductor which is exposed to a changing magnetic field. Lenz’s law of electromagnetic induction states that the direction of this induced current will be such that the magnetic field created by the induced current ***opposes*** the initial changing magnetic field which produced it. The direction of this current flow can be determined using Fleming’s right-hand rule.

Faraday’s law of induction explains the working principle of transformers, motors, generators, and inductors. The law is named after Michael Faraday, who performed an experiment with a magnet and a coil. During Faraday’s experiment, he discovered how EMF is induced in a coil when the flux passing through the coil changes.

Faraday’s Experiment

In this experiment, Faraday takes a magnet and a coil and connects a galvanometer across the coil. At starting, the magnet is at rest, so there is no deflection in the galvanometer i.e the needle of the galvanometer is at the center or zero position. When the magnet is moved towards the coil, the needle of the galvanometer deflects in one direction.



When the magnet is held stationary at that position, the needle of galvanometer returns to zero position. Now when the magnet moves away from the coil, there is some deflection in the needle but opposite direction, and again when the magnet becomes stationary, at that point respect to the coil, the needle of the galvanometer returns to the zero position. Similarly, if the magnet is held stationary and the coil moves away, and towards the magnet, the galvanometer similarly shows deflection. It is also seen that the faster the change in the magnetic field, the greater will be the induced EMF or voltage in the coil.

Position of magnet	Deflection in galvanometer
Magnet at rest	No deflection in the galvanometer
Magnet moves towards the coil	Deflection in galvanometer in one direction

Magnet is held stationary at same position (near the coil)	No deflection in the galvanometer
Magnet moves away from the coil	Deflection in galvanometer but in the opposite direction
Magnet is held stationary at the same position (away from the coil)	No deflection in the galvanometer

Conclusion: From this experiment, Faraday concluded that whenever there is relative motion between a conductor and a magnetic field, the flux linkage with a coil changes and this change in flux induces a voltage across a coil.

Michael Faraday formulated two laws on the basis of the above experiments. These laws are called **Faraday's laws of electromagnetic induction**.

Faraday's First Law

Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.

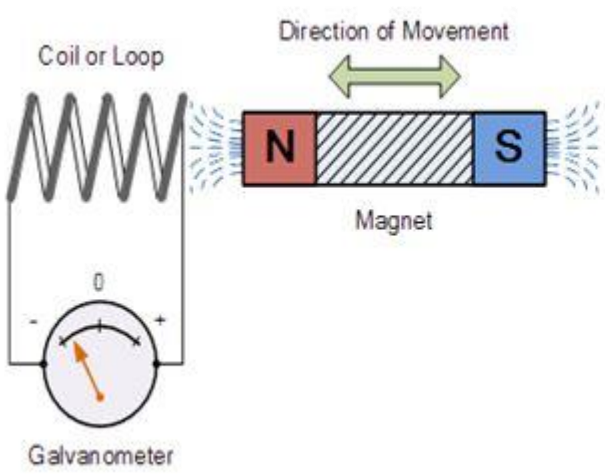
Method to change the magnetic field:

1. By moving a magnet towards or away from the coil
2. By moving the coil into or out of the magnetic field
3. By changing the area of a coil placed in the magnetic field
4. By rotating the coil relative to the magnet

Faraday's Second Law

It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of the number of turns in the coil and flux associated with the coil.

Faraday Law Formula



Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2.

Flux linkage with the coil at time,

$$T_1 = N\phi_1 \ wb$$

Flux linkage with the coil at time,

$$T_2 = N\phi_2 \ wb$$

Change in flux linkage,

$$N(\phi_2 - \phi_1)$$

Let this change in flux linkage be,

$$\phi = (\phi_2 - \phi_1)$$

So, the Change in flux linkage

$$N\phi$$

Now the rate of change of flux linkage

$$\frac{N\phi}{t}$$

Take derivative on right-hand side we will get

$$N\frac{\mathrm{d}\phi}{\mathrm{d}t}$$

The rate of change of flux linkage

$$E = N\frac{d\phi}{dt}$$

But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.

$$E = -N\frac{d\phi}{dt}$$

Considering Lenz's Law.

Where:

- Flux Φ in Wb = B.A
- B = magnetic field strength
- A = area of the coil

How To Increase EMF Induced in a Coil

- By increasing the number of turns in the coil i.e N, from the formulae derived above it is easily seen that if the number of turns in a coil is increased, the induced emf also gets increased.
- By increasing magnetic field strength i.e B surrounding the coil- Mathematically, if magnetic field increases, flux increases and if flux increases emf induced will also get increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for the coil to cut and hence there will be more emf induced.
- By increasing the speed of the relative motion between the coil and the magnet – If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

Applications of Faraday's Law

Faraday law is one of the most basic and important laws of electromagnetism. This law finds its application in most of the electrical machines, industries, and the medical field, etc.

- Power transformers function based on Faraday's law
- The basic working principle of the electrical generator is Faraday's law of mutual induction.
- The Induction cooker is the fastest way of cooking. It also works on the principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that the flow of current always produces heat in it.
- Electromagnetic Flow Meter is used to measure the velocity of certain fluids. When a magnetic field is applied to an electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to the velocity of fluid flowing.
- Form bases of Electromagnetic theory, Faraday's idea of lines of force is used in well known Maxwell's equations. According to Faraday's law, change in magnetic field gives rise to change in electric field and the converse of this is used in Maxwell's equations.
- It is also used in musical instruments like an electric guitar, electric violin, etc.

QUESTIONS

PART – A

1. What is energy density in magnetic field and Energy stored in Magnetic field?
2. Write the expression for the inductance of the toroids.
3. Calculate the inductance inside a solenoid.
4. A single turn rectangular loop with enclosed area of 1m2 is situated in air with its plane normal to a magnetic field which varies at the rate of 1 Tesla. Calculate the emf induced in the loop.
5. State Farday's law of electromagnetic induction.
6. How the motional EMF differs from the Transformer EMF?
7. Evaluate the inductance of a solenoid of 1500 turns wound uniformly over a length of 0.4m on a cylindrical paper tube, 4 cm in diameter and air is the medium.
8. Define self inductance and Mutual inductance.
9. Write the mathematical expression for the energy stored in the inductor.
10. What will be Effective Inductance, if two inductors are connected in (a) Series and (b) Parallel.

PART – B

1. State and explain Faraday's law of electromagnetic induction. Hence derive the Expressions for statically and dynamically induced emfs.
2. A solenoid has an inductance of 20 mH. If the length of the solenoid is increased by two times and the radius is decreased to half of its original value, find the new inductance.
3. Calculate the loop inductance of a ring shaped coil having a mean diameter of 20 cm wound on a wooden core of 2 cm diameter. The winding is uniformly distributed and contains 200 turns.
4. A coil has a self-inductance of 1H and a resistance of 4Ω. If it is connected to a 40V d.c. supply, estimate the energy stored in the magnetic field when the current has attained its final steady value.
5. An iron ring of relative permeability 100 is wound uniformly with 2 coils of 100 and 400 turns of wire. The cross section of ring is 4 cm2 and mean circumference is 50 cm. Calculate

   (a) Self-inductance

   (b) Mutual-inductance

   (c) total inductance,

   when the coils are connected in series in the same sense and in opposition.
6. From the basics, develop a inductance of a coaxial transmission line.

7. (a)Derive the expression for the inductance of Toroid.

   (b)Calculate the inductance per kilometer of two parallel round conductors spaced 80 cm apart and having conductor diameter of 1 cm.

8. (i)State Faraday's law of electromagnetic induction.

   (ii) Two coils with 10,000 and 12,000 turns respectively lying in parallel planes, and is found that 50% of the flux produced by coil 1 links with coil 2. The current in coil 1 of 5A produces 50μ Wb. Find mutual inductance between the coils and co-efficient of coupling.

9. Obtain the expression for the energy stored in magnetic field and energy density.
10. Step by step, derive the inductance for Solenoid, Toroids and Co-axial cables.

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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT – V - Electromagnetic Theory - SEEA1202

I. Maxwell's Equation

Definitions

PROPERTIES OF UNIFORM PLANE WAVE

1. At every point in space, the electric field E and magnetic field H are perpendicular to each other and to the direction of the travel.
2. The field vary harmonically with time and at the same frequency, everywhere in space.
3. Each field has the same direction, magnitude and phase at every point in any plane perpendicular to the direction of wave travel.

INTRINSIC IMPEDANCE

It is the ratio electric field to magnetic field.

It is the ratio of square root of permeability to permittivity of the medium.

POYNTING VECTOR

By Poynting theorem, the vector product of electric field intensity and magnetic field intensity is another product called the Poynting vector.

WAVE

If a physical phenomenon that occurs at one place at a given time is reproduced at other places at later times, the time delay being proportional to the space separation from the first location, then the group of phenomena constitutes a wave.

PROPAGATION CONSTANT

The propagation constant (γ) is a complex number and is given by $\Gamma = \alpha + j\,\beta$ α – Attenuation constant β – Phase constant

DISPLACEMENT

It is the current flowing through a capacitor when an AC voltage is applied across the capacitor.

CONDUCTION CURRENT

It is the current flowing through a conductor whose resistance is R.

MODIFIED AMPERE'S CIRCUITAL LAW

The line integral of tangential component of magnetic field intensity around a closed path is exactly equal to the current enclosed by that path.

INTRODUCTION TO MAXWELL'S EQUATIONS

Maxwell's Equations are a set of 4 complicated equations that describe the world of electromagnetics. These equations describe how electric and magnetic fields propagate, interact, and how they are influenced by objects.

James Clerk Maxwell [1831-1879] was an Einstein/Newton-level genius who took a set of known experimental laws (Faraday's Law, Ampere's Law) and unified them into a symmetric coherent set of

Equations known as Maxwell's Equations. Maxwell was one of the first to determine the speed of propagation of electromagnetic (EM) waves was the same as the speed of light - and hence to conclude that EM waves and visible light were really the same thing.

Maxwell's Equations are critical in understanding Antennas and Electromagnetics. They are formidable to look at - so complicated that most electrical engineers and physicists don't even really know what they mean. Shrouded in complex math (which is likely so "intellectual" people can feel superior in discussing them), true understanding of these equations is hard to come by.

This leads to the reason for this website - an intuitive tutorial of Maxwell's Equations. I will avoid if at all possible the mathematical difficulties that arise, and instead describe what the equations mean. And don't be afraid - the math is so complicated that those who do understand complex vector calculus still cannot apply Maxwell's Equations in anything but the simplest scenarios. For this reason, intuitive knowledge of Maxwell's Equations is far superior than mathematical manipulation-based knowledge. To understand the world, you must understand what equations mean, and not just know mathematical constructs. I believe the accepted methods of teaching electromagnetics and Maxwell's Equations do not produce understanding. And with that, let's say something about these equations.

Maxwell's Equations are laws - just like the law of gravity. These equations are rules the universe uses to govern the behavior of electric and magnetic fields. A flow of electric current will produce a magnetic field. If the current flow varies with time (as in any wave or periodic signal), the magnetic field will also give rise to an electric field. Maxwell's Equations shows that separated charge (positive and negative) gives rise to an electric field - and if this is varying in time as well will give rise to a propagating electric field, further giving rise to a propgating magnetic field.

To understand Maxwell's Equations at a more intuitive level than most Ph.Ds in Engineering or Physics, click through the links and definitions above. You'll find that the complicated math masks an inner elegance to these equations - and you'll learn how the universe operates the Electromagnetic Machine.

1. $$\nabla \cdot \mathbf{D} = \rho_V$$

2. $$\nabla \cdot \mathbf{B} = 0$$

3. $$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

4. $$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

GAUSS' LAW

Gauss' Law is the first of Maxwell's Equations which dictates how the Electric Field behaves around electric charges. Gauss' Law can be written in terms of the Electric Flux Density and the Electric Charge Density as:

$$\nabla \cdot \mathbf{D} = \rho_V \qquad \textbf{[Equation 1]}$$

In Equation [1], the symbol $\nabla \cdot$ is the divergence operator.

Equation [1] is known as Gauss' Law in point form. That is, Equation [1] is true at any point in space. That is, if there exists electric charge somewhere, then the divergence of **D** at that point is nonzero, otherwise it is equal to zero.

To get some more intuition on Gauss' Law, let's look at Gauss' Law in integral form. To do this, we assume some arbitrary volume (we'll call it *V*) which has a boundary (which is written *S*). Then integrating Equation [1] over the volume *V* gives Gauss' Law in integral form:

$$\int_V (\nabla \cdot \mathbf{D})\, dV = \int_V \rho_V\, dV$$

[Equation 2]

$$\Rightarrow \quad \int_S \mathbf{D} \cdot \mathbf{dS} = Q_{enc}$$

I probably made things less clear, but let's go through it real quick. As an example, look at Figure 1. We have a volume *V*, which is the cube. The surface *S* is the boundary of the cube (i.e. the 6 flat faces that form the boundary of the volume).

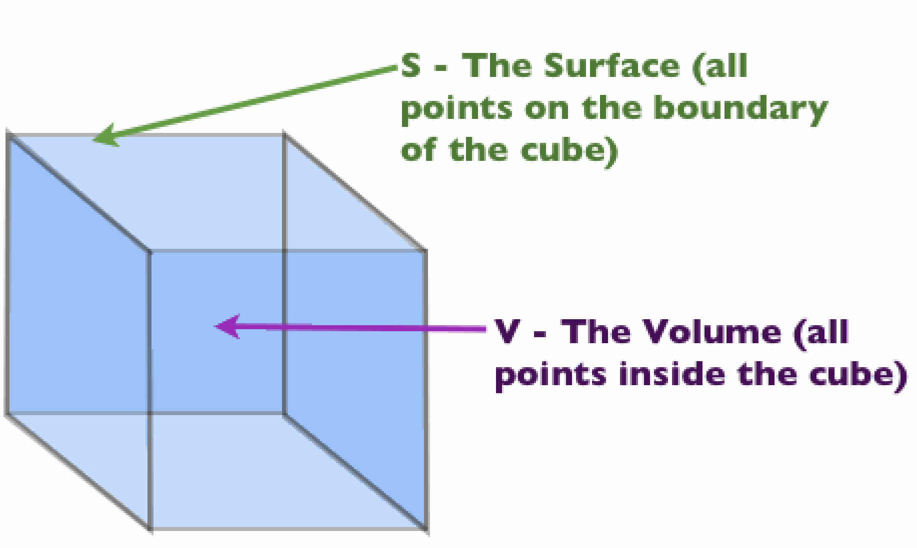

Figure 1. Illustration of a volume *V* with boundary surface *S*.

Equation [2] states that the amount of charge inside a volume *V* (= Q_{enc}) is equal to the total amount of Electric Flux (**D**) exiting the surface *S*. That is, to determine the Electric Flux leaving the region *V*, we only need to know how much electric charge is within the volume. We rewrite Equation [2] with more of the terms defined in Equation [3]:

$\mathbf{D} \cdot \mathbf{dS}$ = This means we are only interested in the component of **D** that is entering or exiting the volume V. We don't care about "the tangential" components of **D** - these circle around the volume but don't contribute to the net Electric Flux in or out of the volume.

$\int_S \mathbf{D} \cdot \mathbf{dS}$ = This means we want to sum up the $\mathbf{D} \cdot \mathbf{dS}$ values at each point along the surface *S*.

GAUSS' LAW FOR MAGNETIC FIELDS

Before you read this page, you should have read the page on Gauss' Law for Electric Fields. If that makes sense, then the second of Maxwell's Equations will be pretty easy. First, observe both of Gauss' Laws, written in Equation [1]:

$$\nabla \cdot \mathbf{D} = \rho_V$$
$$\nabla \cdot \mathbf{B} = 0$$

[Equation 1]

You see that both of these equations specify the divergence of the field in question. For the top equation, we know that Gauss' Law for Electric Fields states that the divergence of the Electric Flux Density **D** is equal to the volume electric charge density. But the second equation, Gauss' Magnetism law states that the divergence of the Magnetic Flux Density (**B**) is zero.

Why? Why isn't the divergence of **B** equal to the *magnetic charge density*?

Well - it is. But it just so happens that no one has ever found magnetic charge - not in a laboratory or on the street or on the subway. And therefore, until this hypothetical magnetic charge is found, we set the right side of Gauss' Law for Magnetic Fields to zero:

Gauss' Law For Magnetism

$$\nabla \cdot \mathbf{B} = 0 \quad \text{(Magnetic Charge Does Not Exist)}$$

$$\nabla \cdot \mathbf{H} = 0 \quad \text{(also true since } \mathbf{B} = \mu \mathbf{H}\text{)}$$

THE 3RD MAXWELL'S EQUATION

On this page, we'll explain the meaning of the 3rd of Maxwell's Equations, **Faraday's Law**, which is given in Equation [1]:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

[Equation 1]

Faraday was a scientist experimenting with circuits and magnetic coils way back in the 1830s. His experiment setup, which led to Farday's Law, is shown in Figure 1:

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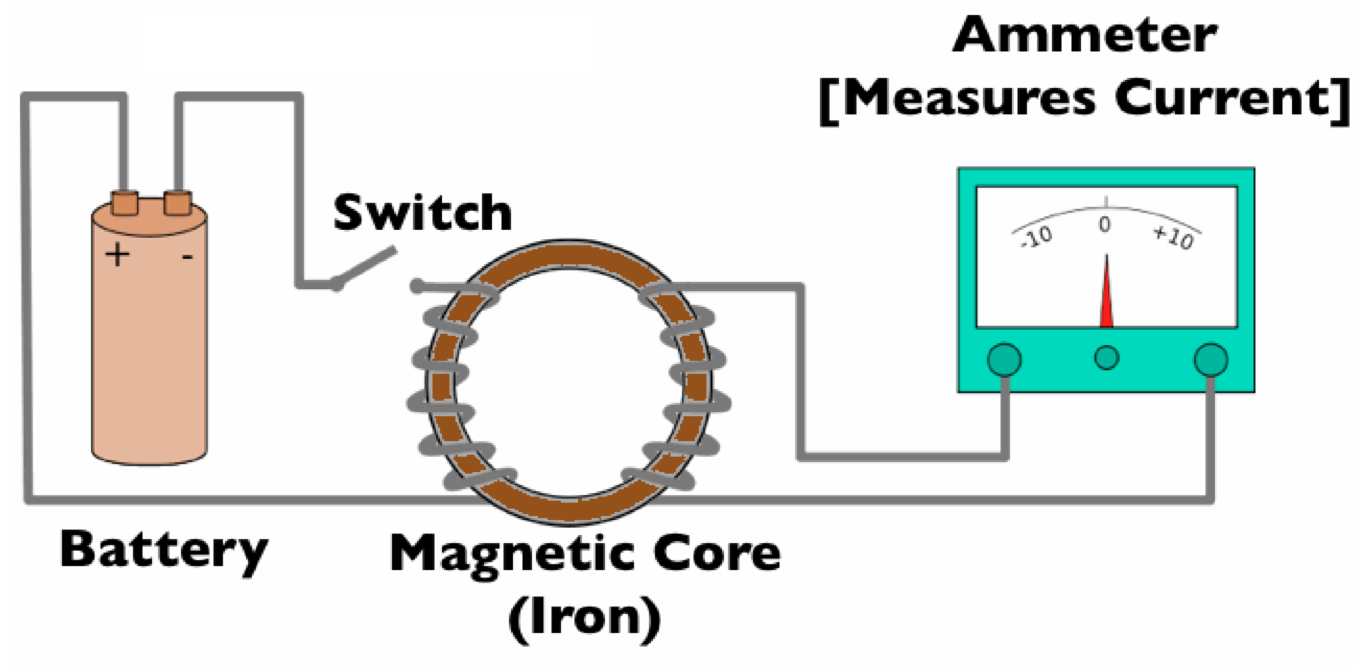


Figure 1. Experimental Setup For Faraday.

The experiment itself is somewhat simple. When the battery is disconnected, we have no electric current flowing through the wire. Hence there is no magnetic flux induced within the Iron (Magnetic Core). The Iron is like a highway for Magnetic Fields - they flow very easily through magnetic material. So the purpose of the core is to create a path for the Magnetic Flux to flow.

When the switch is closed, the electric current will flow within the wire attached to the battery. When this current flows, it has an associated magnetic field (or magnetic flux) with it. When the wire wraps around the left side of the magnetic core (as shown in Figure 1), a magnetic field (magnetic flux) is induced within the core. This flux travels around the core. So the Magnetic Flux produced by the wired coil on the left exists within the wired coil on the right, which is connected to the ammeter.

Now, a funny thing happens, which Faraday observed. When he closed the switch, then current would begin flowing and the ammeter would spike one way (say measuring +10 Amps on the other side). But this was very brief, and the current on the right coil would go to zero. When the switch was opened, the measured current would spike to the other side (say -10 Amps would be measured), and then the measured current on the right side would again be zero.

Faraday figured out what was happening. When the switch was initially changed from open to closed, the magnetic flux within the magnetic core increased from zero to some maximum number (which was a constant value, versus time). When the flux was increasing, there existed an induced current on the opposite side.

Similarly, when the switch was opened, the magnetic flux in the core would decrease from it's constant value back to zero. Hence, a decreasing flux within the core induced an opposite current on the right side.

Faraday figured out that a changing Magnetic Flux within a circuit (or closed loop of wire) produced an induced *EMF*, or voltage within the circuit. He wrote this as:

$$EMF = -\frac{d\Phi}{dt} \quad \textbf{[Equation 2]}$$

In Equation [2], Φ is the Magnetic Flux within a circuit, and *EMF* is the electro-motive force, which is basically a voltage source. Equation [2] then says that the induced voltage in a circuit is the opposite of the time-rate-of-change of the magnetic flux. For more information on derivatives, see the partial derivatives page.

Equation [2] is known as *Lenz's Law*. Lenz was the guy who figured out the minus sign. *We know that an electric current gives rise to a magnetic field - but thanks to Farady we also know that a magnetic field within a loop gives rise to an electric current.* The universe loves symmetry and Maxwell's Equations has a lot of it.

THE 4TH MAXWELL'S EQUATION

On this page, we'll explain the meaning of the last of Maxwell's Equations, **Ampere's Law**, which is given in Equation [1]:

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J} \quad \textbf{[Equation 1]}$$

Ampere was a scientist experimenting with forces on wires carrying electric current. He was doing these experiments back in the 1820s, about the same time that Farday was working on Faraday's Law. Ampere and Farday didn't know that there work would be unified by Maxwell himself, about 4 decades later.

Forces on wires aren't particularly interesting to me, as I've never had occassion to use the very complicated equations in the course of my work (which includes a Ph.D., some stints at a national lab, along with employment in the both defense and the consumer electronics industries). So, I'm going to start by presenting Ampere's Law, which relates a electric current flowing and a magnetic field wrapping around it:

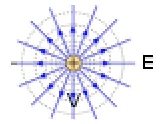
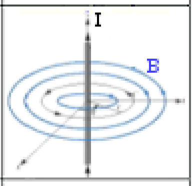
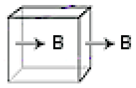
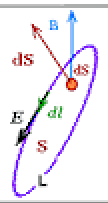
$$\oint \mathbf{H} \cdot \mathbf{dL} = I_{enc} \quad \textbf{[Equation 2]}$$

Equation [2] can be explained: Suppose you have a conductor (wire) carrying a current, *I*. Then this current produces a Magnetic Field which circles the wire.

The left side of Equation [2] means: If you take any imaginary path that encircles the wire, and you add up the Magnetic Field at each point along that path, then it will numerically equal the amount of current that is encircled by this path (which is why we write I_{enc} for encircled or enclosed current).

ELECTROMAGNETIC WAVE EQUATION

The field equations for electromagnetic radiation are a product of the Victorian Era in the 19th century. It was originated from a set of equations in electromagnetism. Maxwell's contribution is to add a "displacement current" term, i.e., the (1/c) $\frac{\partial}{\partial t}\mathbf{E}$ term in Eq.(40), and thus put the equations into a consistent set which implied new physical phenomena, at that time unknown but subsequently verified in all details by experiments. In terms of the electric field **E** and magnetic field **B** the Maxwell's equations are four first order differential equations :

Law	Differential Form	Integral Form	Pictorial Form
Coulomb's (1785)	$\nabla \cdot \mathbf{E} = \rho$	$\oiint \mathbf{E} \cdot d\mathbf{S} = \iiint \rho \, dV = Q$	
Ampere's (1820, 1861)	$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{\mathbf{J}}{c}$	$\oint \mathbf{B} \cdot d\ell - \frac{1}{c}\frac{\partial}{\partial t}\iint \mathbf{E} \cdot d\mathbf{S} = \frac{1}{c}\iint \mathbf{J} \cdot d\mathbf{S} = \frac{I}{c}$	
Gauss's (1813)	$\nabla \cdot \mathbf{B} = 0$	$\oiint \mathbf{B} \cdot d\mathbf{S} = 0$	
Faraday's (1831)	$\nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = 0$	$\oint \mathbf{E} \cdot d\ell + \frac{1}{c}\frac{\partial}{\partial t}\iint \mathbf{B} \cdot d\mathbf{S} = 0$	

The Wave Equation

One of the most fundamental equations to all of Electromagnetics is the wave equation, which shows that all waves travel at a single speed - the speed of light. On this page we'll derive it from Ampere's and Faraday's Law.

We assume we are in a source free region - so no charges or currents are flowing. We want to determine how Electric and Magnetic Fields propagate through the region.

To start, let me throw out a vector identity, which is basically a mathematical manipulation that is true for all vector fields:

$$\nabla \times \nabla \times \mathbf{H} = \nabla\left(\nabla \cdot \mathbf{H}\right) - \nabla^2 \mathbf{H} \qquad \textbf{[Equation 1]}$$

The left side of Equation 1 is simply the curl of the curl of a vector field. On the right side, I can define the terms for you in the next couple of equations. The first term on the right side of Equation [1] is known as the "gradient of the divergence". However, since we know the divergence of the fields in question will be zero because we are in a source free region. Hence, this term is zero:

$$\nabla(\underbrace{\cancel{\nabla}\cdot\mathbf{H}}_{0}) \qquad \text{(this doesn't n} \quad \textbf{[Equation 2]}$$

The second term on the right side of Equation [1] is known as the Laplacian. This is basically the sum of second-order partial derivatives, as seen in Equation [3]:

$$\nabla^2\mathbf{H} = \nabla^2\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} \\ \frac{\partial^2 H_y}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} + \frac{\partial^2 H_y}{\partial z^2} \\ \frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} \end{bmatrix} \quad \textbf{[Equation 3]}$$

OK, so now we can rewrite Equation [1] as:

$$\nabla \times \nabla \times \mathbf{H} = -\nabla^2\mathbf{H}$$

[Equation 4]

$$\nabla \times \nabla \times \mathbf{E} = -\nabla^2\mathbf{E}$$

I've written Equation [4] out as two equations to show that this is true for both the Electric and Magnetic Fields, in source free regions.

If we start now with Farday's Law, and take the curl of both sides, we get:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t}$$

[Equation 5]

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H})$$

We can rewrite the left side of equation [5] (the curl of the curl of E) with the help of Equation [4]. And we can rewrite the right side of Equation [5] by substituting in Ampere's law:

$$\begin{aligned}
\nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} = & \\
& = -\mu \frac{\partial}{\partial t}(\nabla \times \mathbf{H}) \quad \text{(substitute in Ampere's Law)} \\
& = -\mu \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}\right) \quad \text{(}\mathbf{J}\text{ is zero because source free region)} \\
& = -\mu\varepsilon \frac{\partial}{\partial t}\left(\frac{\partial \mathbf{E}}{\partial t}\right)
\end{aligned}$$

[Equation 6]

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$$\Rightarrow \boxed{\nabla^2 \mathbf{E} = \mu\varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2}} \quad \text{[The Vector Wave Equation]}$$

Equation [6] is known as the **Wave Equation** It is actually 3 equations, since we have an x-, y- and z- component for the **E** field.

To break down and understand Equation [6], let's imagine we have an E-field that exists in source-free region. Suppose we only have an E-field that is polarized in the x-direction, which means that Ey=Ez=0 (the y- and z- components of the E-field are zero). Further, let's assume that the field is travelling in the z-direction, and there is no variation in the x- and y-directions (this means the partial derivatives with respect to x- and y- are zero). Then Equation [6] simplifies to:

$$\nabla^2 E_x = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2} \quad \text{[Scalar Wave Equation]}$$

[Equation 7]

$$\frac{\partial^2 E_x}{\partial z^2} = \mu\varepsilon \frac{\partial^2 E_x}{\partial t^2}$$

Introduction to Polarization

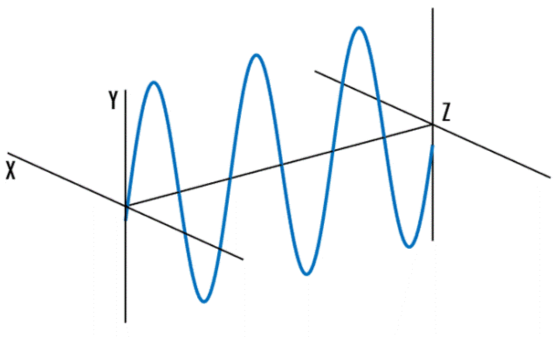
Understanding and manipulating the polarization of light is crucial for many optical applications. Optical design frequently focuses on the wavelength and intensity of light, while neglecting its polarization. Polarization, however, is an important property of light that affects even those optical systems that do not explicitly measure it. The polarization of light affects the focus of laser beams, influences the cut-off wavelengths of filters, and can be important to prevent unwanted back reflections. It is essential for many metrology applications such as stress analysis in glass or plastic, pharmaceutical ingredient analysis, and biological microscopy. Different polarizations of light can also be absorbed to different degrees by materials, an essential property for LCD screens, 3D movies, and your glare-reducing sunglasses.

Understanding Polarization

Light is an electromagnetic wave, and the electric field of this wave oscillates perpendicularly to the direction of propagation. Light is called unpolarized if the direction of this electric field fluctuates randomly in time. Many common light sources such as sunlight, halogen lighting, LED spotlights, and incandescent bulbs produce unpolarized light. If the direction of the electric field of light is well defined, it is called polarized light. The most common source of polarized light is a laser.

Depending on how the electric field is oriented, we classify polarized light into three types of polarizations:

- Linear polarization: the electric field of light is confined to a single plane along the direction of propagation *(Figure 1)*.
- Circular polarization: the electric field of light consists of two linear components that are perpendicular to each other, equal in amplitude, but have a phase difference of π/2. The resulting electric field rotates in a circle around the direction of propagation and, depending on the rotation direction, is called left- or right-hand circularly polarized light *(Figure 2)*.
- Elliptical polarization: the electric field of light describes an ellipse. This results from the combination of two linear components with differing amplitudes and/or a phase difference that is not π/2. This is the most general description of polarized light, and circular and linear polarized light can be viewed as special cases of elliptically polarized light *(Figure 3)*.



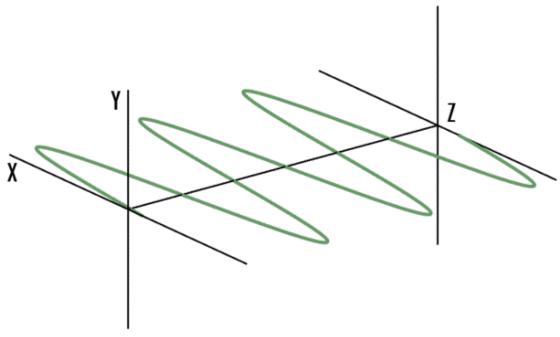


Figure 1: The electric field of linearly polarized light is confined to the y-z plane (left) and the x-z plane (right), along the direction of propagation.

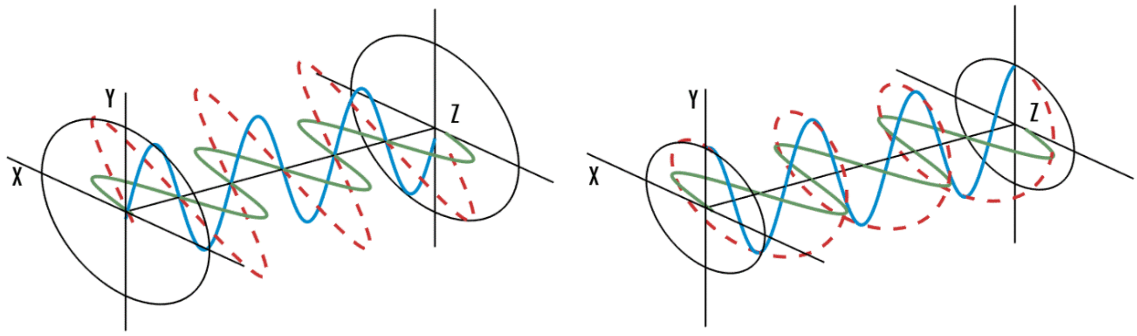


Figure 2: The electric field of linearly polarized light (left) consists of two perpendicular, equal in amplitude, linear components that have no phase difference. The resultant electric field wave propagates along the y = x plane. The electric field of circularly polarized light (right) consists of two perpendicular, equal in amplitude, linear components that have a phase difference of π/2 or 90°. The resultant electric field wave propagates circularly.

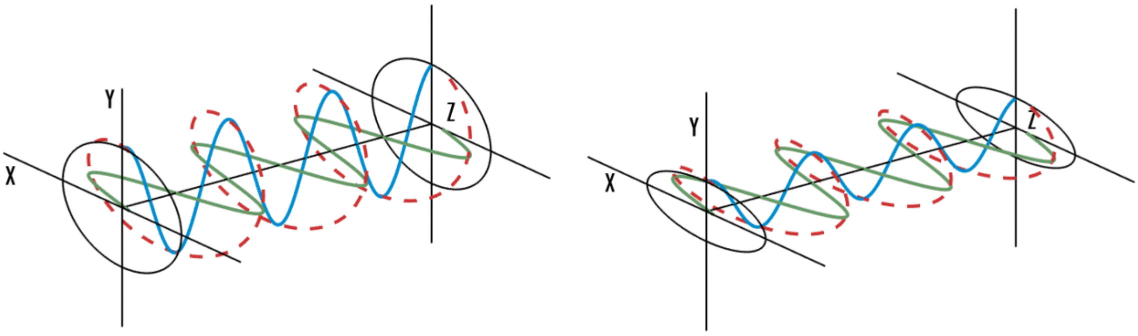


Figure 3: The circular electric field (left) has two components which are of equal amplitude and have a π/2 or 90° phase difference. If the two components however, have differing amplitudes, or if there is a phase difference other than π/2, then then they will create elliptically polarized light (right).

The two orthogonal linear polarization states that are most important for reflection and transmission are referred to as p- and s-polarization. P-polarized (from the German parallel) light has an electric field polarized parallel to the plane of incidence, while s-polarized (from the German senkrecht) light is perpendicular to this plane.

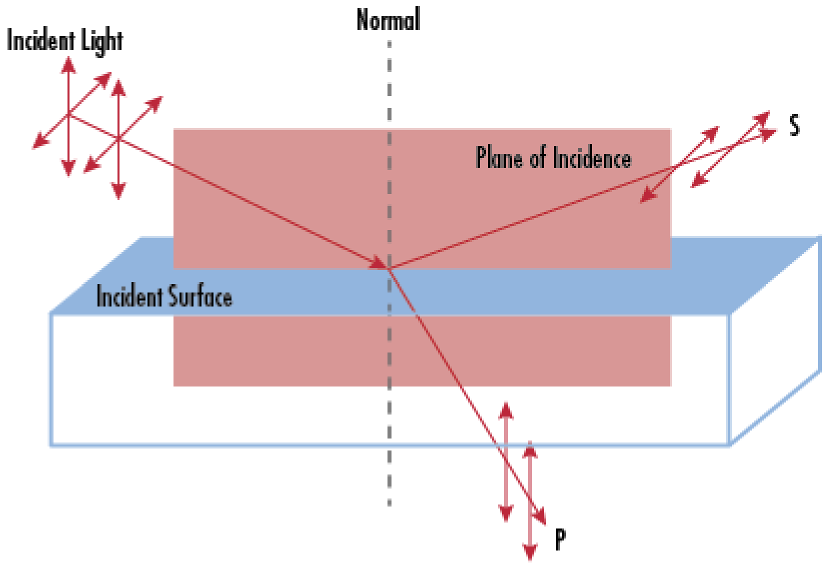


Figure 4: P and S are linear polarizations defined by their relative orientation to the plane of incidence.

Poynting vector and poynting theorem

When electromagnetic wave travels in space, it carries energy and energy density is always associated with electric fields and magnetic fields.

The rate of energy travelled through per unit area i.e. the amount of energy flowing through per unit area in the perpendicular direction to the incident energy per unit time is called poynting vector.

Mathematically poynting vector is represented as

$$\vec{P} = \vec{E} \times \vec{H} \left(= \frac{\vec{E} \times \vec{B}}{\mu} \right) \qquad \ldots(i)$$

the direction of poynting vector is perpendicular to the plane containing $\vec{E}$ and $\vec{H}$. Poynting vector is also called as instantaneous energy flux density. Here rate of energy transfer $\vec{P}$ is perpendicular to both $\vec{E}$ and $\vec{H}$. Since it represents the rate of energy transfer per unit area, its unit is W/m^2.

Poynting theorem states that the net power flowing out of a given volume V is equal to the time rate of decrease of stored electromagnetic energy in that volume decreased by the conduction losses.

i.e. total power leaving the volume = rate of decrease of stored electromagnetic energy − ohmic power dissipated due to motion of charge

Proof : The energy density carried by the electromagnetic wave can be calculated using Maxwell's equations

as $\text{div}\ \vec{D} = 0$ ...(i) $\text{div}\ \vec{B} = 0$ ...(ii) $\text{Curl}\ \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ ...(iii)

and $$\text{Curl}\ \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \qquad \text{...(iv)}$$

taking scalar product of (iii) with H and (iv) with $\vec{E}$

i.e. $$\vec{H}\ \text{curl}\ \vec{E} = -\vec{H}\cdot\frac{\partial \vec{B}}{\partial t} \qquad \text{...(v)}$$

and $$\vec{E}.\ \text{curl}\ \vec{H} = \vec{E}.\vec{J} + \vec{E}.\frac{\partial \vec{D}}{\partial t} \qquad \text{...(vi)}$$

doing (vi) − (v) i.e. $$\vec{H}.\text{curl}\ \vec{E} - \vec{E}.\ \text{curl}\ \vec{H} = -\vec{H}.\frac{\partial \vec{B}}{\partial t} - \vec{E}.\vec{J} - \vec{E}.\frac{\partial \vec{D}}{\partial t}$$

$$= -\left[\vec{H}.\frac{\partial \vec{B}}{\partial t} + \vec{E}.\frac{\partial \vec{D}}{\partial t}\right] - \vec{E}.\vec{J}$$

as $$\text{div}\left(\vec{A}\times\vec{B}\right) = \vec{B}.\ \text{curl}\ \vec{A} - \vec{A}\ \text{curl B}$$

so $$\text{div}\left(\vec{E}\times\vec{H}\right) = -\left[\vec{H}.\frac{\partial \vec{B}}{\partial t} + \vec{E}.\frac{\partial \vec{D}}{\partial t}\right] - \vec{E}.\vec{J} \qquad \text{...(vii)}$$

But $\vec{B} = \mu\vec{H}$ and $\vec{D} = \varepsilon\vec{E}$

so $$\vec{H}.\frac{\partial \vec{B}}{\partial t} = \vec{H}.\frac{\partial}{\partial t}\left(\mu\vec{H}\right) = \frac{1}{2}\mu\frac{\partial}{\partial t}\left(H^2\right)$$

$$= \frac{\partial}{\partial t}\left[\frac{1}{2}\vec{H}.\vec{B}\right]$$

and $$\vec{E}.\frac{\partial \vec{D}}{\partial t} = \vec{E}.\frac{\partial}{\partial t}\left(\varepsilon \vec{E}\right) = \frac{1}{2}\varepsilon\frac{\partial}{\partial t}\left(E\right)^2 = \frac{\partial}{\partial t}\left[\frac{1}{2}\vec{E}.\vec{D}\right]$$

so from equation (vii) $$\text{div}\left(\vec{E}\times\vec{H}\right) = -\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\vec{H}.\vec{B}+\vec{E}.\vec{D}\right)\right] - \vec{E}.\vec{J}$$

or $$\vec{E}.\vec{J} = -\frac{\partial}{\partial t}\left[\frac{1}{2}\left(\vec{H}.\vec{B}+\vec{E}.\vec{D}\right)\right] - \text{div}\left(\vec{E}\times\vec{H}\right) \qquad \text{...(viii)}$$

Integrating equation (viii) over a volume V enclosed by a surface S

$$\int_V \vec{E}.\vec{J}\,dV = -\int_V \left|\frac{\partial}{\partial t}\left\{\frac{1}{2}\left(\vec{H}.\vec{B}+\vec{E}.\vec{D}\right)\right\}\right| dV - \int_V \text{div}\left(\vec{E}\times\vec{H}\right)dV$$

or $$\int_V \vec{E}.\vec{J}\,dV = -\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right]dV - \int_S \left(\vec{E}\times\vec{H}\right).ds$$

as $\vec{B} = \mu\vec{H}$, $\vec{D} = \varepsilon\vec{E}$ and $\int_V \text{div}\left(\vec{E}\times\vec{H}\right)dV = \int_S \left(\vec{E}\times\vec{H}\right).ds$

or $$\int_V \left(\vec{E}\,.\,\vec{J}\right)dV = -\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right]dV - \int_S \left(\vec{E}\times\vec{H}\right).ds$$

or $$\int_s \left(\vec{E}\times\vec{H}\right).ds = -\int_V \frac{\partial\, U_{em}}{\partial t}\,dV - \int_V \left(\vec{E}\,.\,\vec{J}\right)dV$$

or $$\boxed{\int_s \vec{P}.ds = -\int_V \frac{\partial\, U_{em}}{\partial t}\,dV - \int_V \left(\vec{E}\,.\,\vec{J}\right)dV} \qquad \left(\text{as } \vec{P} = \vec{E}\times\vec{H}\right) \quad \text{...(ix)}$$

i.e. Total power leaving the volume = rate of decrease of stored e.m. energy - ohmic power dissipated due to charge motion

This equation (ix) represents the poynting theorem according to which the net power flowing out of a given volume is equal to the rate of decrease of stored electromagnetic energy in that volume minus the conduction losses.

In equation (ix) $\int_s \vec{P}.ds$ represents the amount of electromagnetic energy crossing the closed surface per second or the rate of flow of outward energy through the surface S enclosing volume V i.e. it is poynting vector.

The term $\int_V \frac{\partial U_{em}}{\partial t} dV$ or $\frac{\partial}{\partial t}\int_V \left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right] dV$, the terms $\frac{1}{2}\mu H^2$ and $\frac{1}{2}\varepsilon E^2$ represent the energy stored in electric and magnetic fields respectively and their sum denotes the total energy stored in electromagnetic field. So total terms gives the rate of decrease of energy stored in volume V due to electric and magnetic fields.

QUESTIONS

PART A

1. Define a wave.
2. Write down the wave equations for E and H in a conducting medium.
3. What are the properties of uniform plane wave?
4. Define intrinsic impedance or characteristic impedance.
5. Define polarization.
6. Write down the Maxwell's equation in integral form.
7. State Slepian vector.
8. State Poynting theorem.
9. Write down the Maxwell's equation in point form.
10. Define displacement current?

PART B

1. State Poynting theorem and derive it.
2. Derive Maxwell's equation in point form and integral form.
3. Write a note on conduction current, polarization and displacement current.
4. Derive the wave equations.

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