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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

NETWORK ANALYSIS AND SYNTHESIS-SEEA1301

UNIT I-NETWORK THEOREMS BOTH DC&AC

NETWORK THEOREMS

Superposition Theorem - Reciprocity Theorem - Thevenin's Theorem - Norton's Theorem - Maximum power transfer Theorem.

1.1 SUPERPOSITION THEOREM

The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals. This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the circuit which contains two sources as shown in Fig. 1.

Now let us find the current passing through the 3 V resistors in the circuit. According to the superposition theorem, the current I_2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A

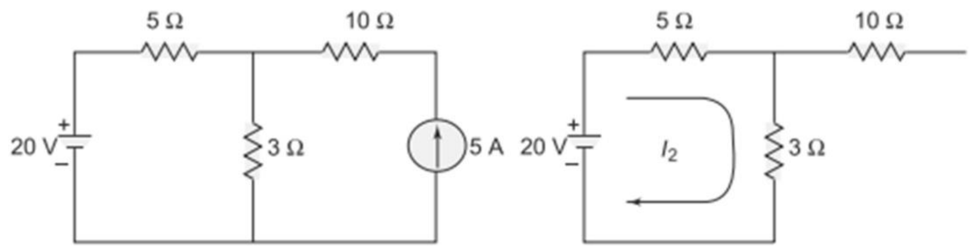


Figure 1 : Superposition theorem

The current I_5 due to the 5 A source with the 20 V source short circuited is

$$I_5 = 5 \times \frac{5}{(3+5)} = 3.125\,\text{A}$$

The total current passing through the 3 V resistor is

(2.5 + 3.125) = 5.625 A

Let us verify the above result by applying nodal analysis.

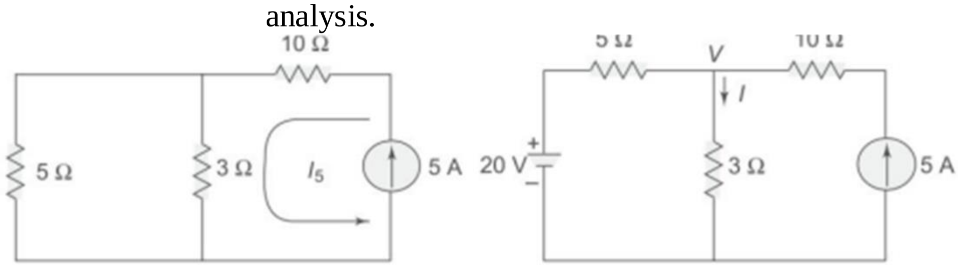


Figure 2: Superposition theorem

The current passing in the 3 V resistor due to both sources should be 5.625 A.
Applying nodal analysis to Fig. 2, we have

$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$V\left[\frac{1}{5}+\frac{1}{3}\right]=5+4$$

$$V=9\times\frac{15}{8}=16.875\ \text{V}$$

The current passing through the 3 V resistor is equal to V/3,

i.e. I = 16.875/3=5.625 A

So the superposition theorem is verified.
Let us now examine the power responses.
Power dissipated in the 3 V resistor due to the voltage source acting alone

$$P_{20}=(I_2)^2R=(2.5)^2\,3=18.75\ \text{W}$$

Power dissipated in the 3 V resistor due to the current source acting alone

$$P_5=(I_5)^2R=(3.125)^2\,3=29.29\ \text{W}$$

Power dissipated in the 3V resistor when both the sources are acting simultaneously is given by

$$P=(5.625)^2\times 3=94.92\ \text{W}$$

From the above results, the superposition of P20 and P5 gives

$$P_{20}+P_5=48.04\ \text{W}$$

which is not equal to P = 94.92 W
We can, therefore, state that the superposition theorem is not valid for power responses. It is applicable only for computing voltage and current responses.

Example 1: Find the voltage across the 2 V resistor in Fig. 3 by using the superposition theorem.

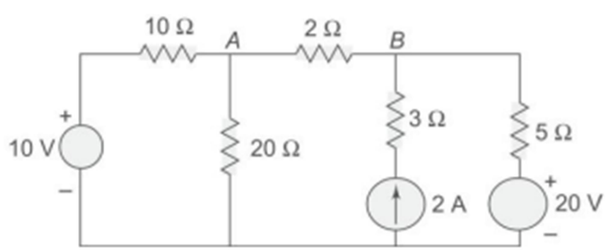


Figure 3

Solution
Let us find the voltage across the 2 V resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2 V resistor.

Our first step is to find the voltage across the 2 V resistor due to the 10 V source, while other sources are set equal to zero.

The circuit is redrawn as shown in Fig. 4

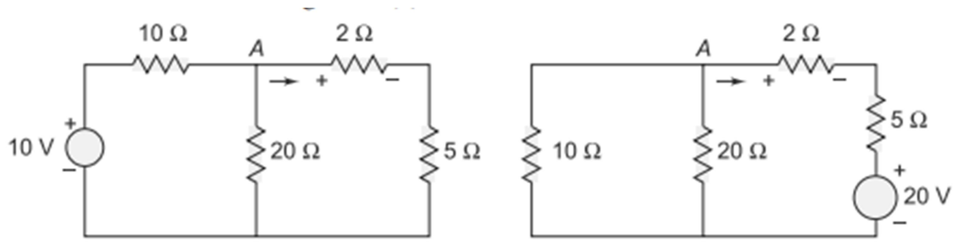


Figure 4

Assuming a voltage V at the node 'A' as shown in Fig. 4, the current equation is

$$\frac{V-10}{10}+\frac{V}{20}+\frac{V}{7}=0$$

$$V\,[0.1 + 0.05 + 0.143] = 1$$

$$\text{or} \quad V = 3.41\text{ V}$$

The voltage across the 2 V resistor due to the 10 V source is

$$V_2=\frac{V}{7}\times 2=0.97\text{ V}$$

Our second step is to find out the voltage across the 2 V resistor due to the 20 V source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 4.

Assuming voltage V at the node A as shown in Fig. 4, the current equation is

$$\frac{V-20}{7}+\frac{V}{20}+\frac{V}{10}=0$$

$$V\,[0.143 + 0.05 + 0.1] = 2.86$$

$$\text{or} \quad V=\frac{2.86}{0.293}=9.76\text{ V}$$

The voltage across the 2 V resistor due to the 20 V source is

$$V_2=\left(\frac{V-20}{7}\right)\times 2=-2.92\text{ V}$$

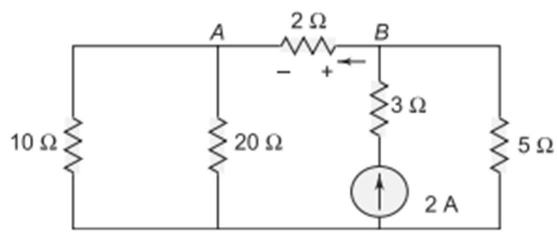


Figure 5

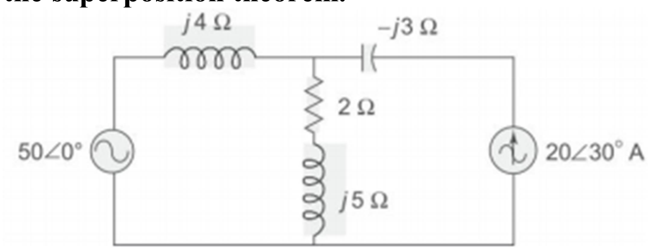
The last step is to find the voltage across the 2 V resistor due to the 2 A current source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 5

$$\text{The current in the } 2\ \Omega \text{ resistor} = 2\times\frac{5}{5+8.67}$$

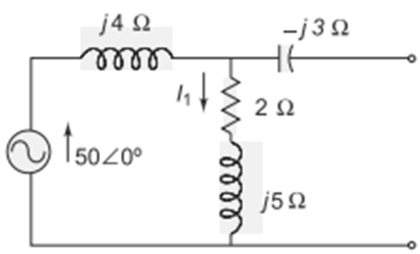
$$= \frac{10}{13.67} = 0.73\,\text{A}$$

The voltage across the 2 V resistor = 0.73 x 2 = 1.46 V
The algebraic sum of these voltages gives the total voltage across the 2 V resistor in the network
V = 0.97 – 2.92 – 1.46 5 = - 3.41 V
The negative sign of the voltage indicates that the voltage at ‘A’ is negative

Example2:
Determine the voltage across the (2 + j5) V impedance as shown in Fig below by using the superposition theorem.



Solution According to the superposition theorem, the current due to the 50 ∠0° V voltage source is I1 as shown in Fig below with current source 20 ∠30° A open-circuited.



$$\text{Current } I_1 = \frac{50\angle 0^\circ}{2+j4+j5} = \frac{50\angle 0^\circ}{(2+j9)}$$

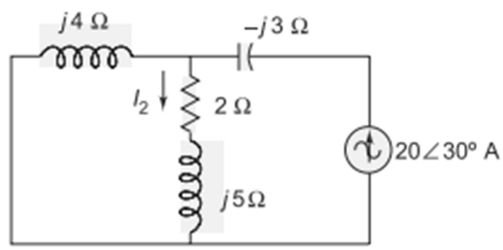
$$= \frac{50\angle 0^\circ}{9.22\angle 77.47} = 5.42\angle -77.47^\circ\ \text{A}$$

Voltage across $(2 + j5)\ \Omega$ due to the current I_1 is

$$V_1 = 5.42 \angle -77.47^\circ (2 + j5)$$

$$= (5.38)(5.42) \angle -77.47^\circ + 68.19^\circ$$

$$= 29.16 \angle -9.28^\circ$$



The current due to the $20\angle 30°$ A current source is I_2 as shown in Fig. 7.18, with the voltage source $50\angle 0°$ V short-circuited.

$$\text{Current } I_2 = 20\angle 30° \times \frac{(j4)\,\Omega}{(2+j9)\,\Omega}$$

$$= \frac{20\angle 30° \times 4\angle 90°}{9.22\angle 77.47°}$$

$$\therefore \quad I_2 = 8.68\angle 120° - 77.47° = 8.68\angle 42.53°$$

Voltage across $(2 + j5)\,\Omega$ due to the current I_2 is

$$V_2 = 8.68\angle 42.53°\,(2 + j5)$$
$$= (8.68)\,(5.38)\angle 42.53° + 68.19°$$
$$= 46.69\angle 110.72°$$

Voltage across $(2 + j5)\,\Omega$ due to both sources is

$$V = V_1 + V_2$$
$$= 29.16\angle -9.28° + 46.69\angle 110.72°$$
$$= 28.78 - j4.7 - 16.52 + j43.67$$
$$= (12.26 + j38.97)\text{ V}$$

Voltage across $(2 + j5)\,\Omega$ is $V = 40.85\angle 72.53°$.

2.2 THEVENIN'S THEOREM

In many practical applications, it is always not necessary to analyse the complete circuit; it requires that the voltage, current, or power in only one resistance of a circuit be found. The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network. Thevenin's theorem states that any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the open-circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances. According to Thevenin's theorem, an equivalent circuit can be found to replace the circuit in Fig. 6

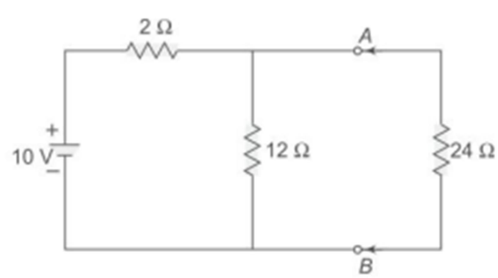


Figure 6

In the circuit, if the 24 V load resistance is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 V resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

$$\text{where} \quad I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1 \text{ A}$$

$$\therefore \quad I_{24} = 1 \times \frac{12}{12+24} = 0.33 \text{ A}$$

The voltage across the 24 V resistor = 0.33 x 24 = 7.92 V.

Now let us find Thevenin's equivalent circuit.

The Thevenin voltage is equal to the open-circuit voltage across the terminals 'AB', i.e. the voltage across the 12 V resistor. When the load resistance is disconnected from the circuit, the Thevenin voltage

$$V_{Th} = 10 \times \frac{12}{14} = 8.57 \text{ V}$$

The resistance into the open-circuit terminals is equal to the Thevenin resistance

$$R_{Th} = \frac{12 \times 2}{14} = 1.71 \ \Omega$$

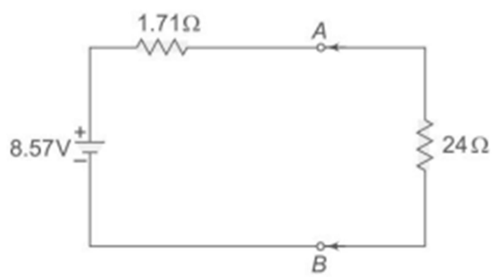


Figure 7

Thevenin's equivalent circuit is shown in Fig. 7. Now let us find the current passing through the 24 V resistance and voltage across it due to Thevenin's equivalent circuit. Fig. 7

$$I_{24} = \frac{8.57}{24+1.71} = 0.33 \text{A}$$

The voltage across the 24 V resistance is equal to 7.92 V. Thus, it is proved that RL (5 24 V) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Example : Determine the Thevenin's equivalent circuit across 'AB' for the given circuit shown in Fig. 8

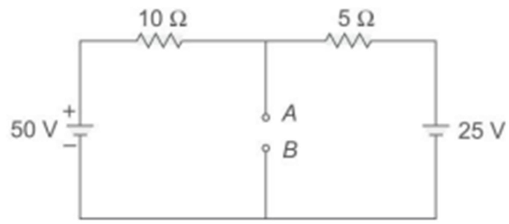


Figure 8

Solution The complete circuit can be replaced by a voltage source in series with a resistance as shown in Fig. 9

where VTh is the voltage across terminals AB, and

RTh is the resistance seen into the terminals AB.

To solve for VTh, we have to find the voltage drops around the closed path as shown in Fig. 9

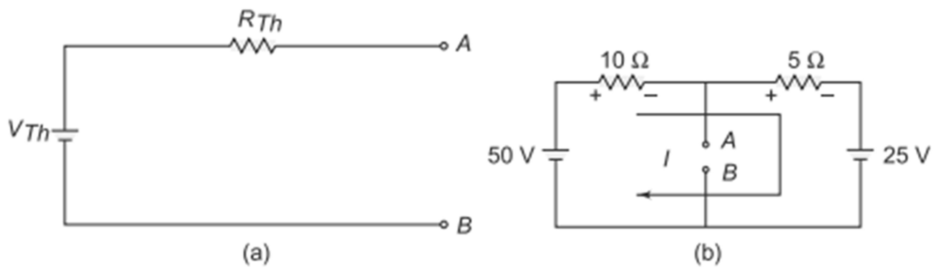


Figure 9

We have $50 - 25 = 10I + 5I$

or $15I = 25$

$$\therefore \quad I = \frac{25}{15} = 1.67\,\text{A}$$

Voltage across 10 Ω = 16.7 V

Voltage drop across 5 Ω = 8.35 V

or $V_{Th} = V_{AB} = 50 - V_{10}$

$= 50 - 16.7 = 33.3$ V

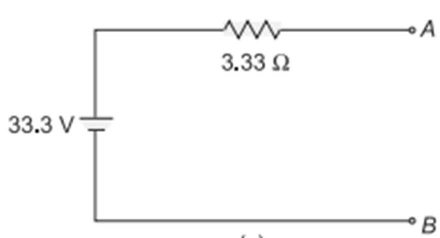


Figure 10

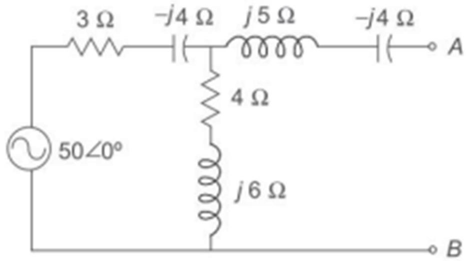
To find RTh, the two voltage sources are removed and replaced with short circuit. The resistance at terminals AB then is the parallel combination of the 10 V resistor and 5 V resistor; or

$$R_{Th} = \frac{10 \times 5}{15} = 3.33\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 9

Example 2: For the circuit shown in Fig. 7.22, determine Thèvenin's equivalent between the output terminals.

Solution The Thèvenin voltage, VTh, is equal to the voltage across the (4 + j6) V impedance. The voltage across (4 + j6) V is



$$V = 50\angle 0° \times \frac{(4+j6)}{(4+j6)+(3-j4)}$$

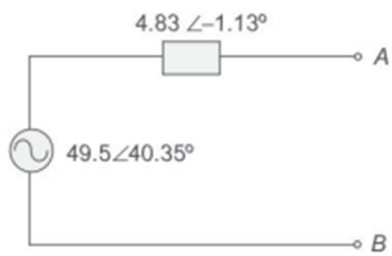
$$= 50\angle 0° \times \frac{4+j6}{7+j2}$$

$$= 50\angle 0° \times \frac{7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= 50\ \angle 0° \times 0.99\ \angle 40.35°$$

$$= 49.5\ \angle 40.35°\ \text{V}$$

The impedance seen from terminals A and B is



$$Z_{\text{Th}} = (j5 - j4) + \frac{(3-j4)(4+j6)}{3-j4+4+j6}$$

$$= j1 + \frac{5\angle 53.13° \times 7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= j1 + 4.95\ \angle -12.78° = j1 + 4.83 - j1.095$$

$$= 4.83 - j0.095$$

$$\therefore\ \ Z_{\text{Th}} = 4.83\ \angle -1.13°\ \Omega$$

The Thèvenin equivalent circuit is shown in Fig above.

2.3 NORTON'S THEOREM

Another method of analysing the circuit is given by Norton's theorem, which states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short-circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.

According to Norton's theorem, an equivalent circuit can be found to replace the circuit in Fig. 11

In the circuit, if the load resistance of 6 V is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6 V resistor due to the original circuit

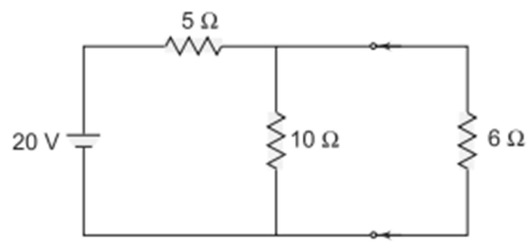


Figure 11

$$I_6 = I_T \times \frac{10}{10+6}$$

$$\text{where} \quad I_T = \frac{20}{5+(10\|6)} = 2.285\,\text{A}$$

$$\therefore \quad I_6 = 2.285 \times \frac{10}{16} = 1.43\,\text{A}$$

i.e. the voltage across the 6 V resistor is 8.58 V. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short-circuited terminals as shown in Fig. 12

$$\text{Here,} \quad I_N = \frac{20}{5} = 4\,\text{A}$$

Norton's resistance is equal to the parallel combination of both the 5 V and 10 V resistors

$$R_N = \frac{5 \times 10}{15} = 3.33\,\Omega$$

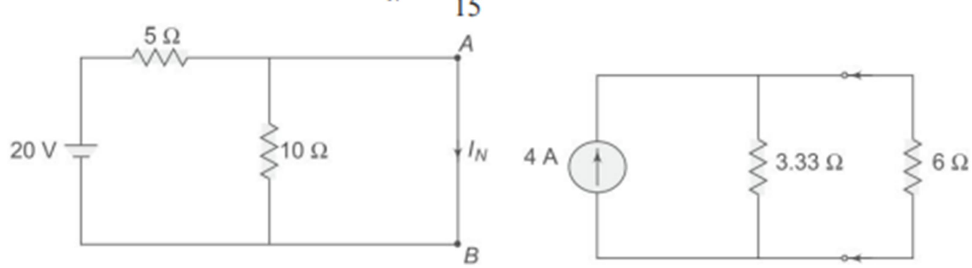


Figure 12

The Norton's equivalent source is shown in Fig. 12.
Now let us find the current passing through the 6 V resistor and the voltage across it due to Norton's equivalent circuit

$$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43 \text{ A}$$

The voltage across the 6 Ω resistor = 1.43 x 6 = 8.58 V
Thus, it is proved that RL (= 6 Ω) has the same values of current and voltage in both the original circuit and Norton's equivalent circuit.

Example Determine Norton's equivalent circuit at terminals AB for the circuit shown in Fig. 13

Solution The complete circuit can be replaced by a current source in parallel with a single resistor as shown in Fig. 14, where IN is the current passing through the short circuited output terminals AB and RN is the resistance as seen into the output terminals.

To solve for IN, we have to find the current passing through the terminals AB as shown in Fig. 14

From Fig. 14, the current passing through the terminals AB is 4 A. The resistance at terminals AB is the parallel combination of the 10 V resistor and the 5 V resistor
Norton's equivalent circuit is shown in Fig. 14

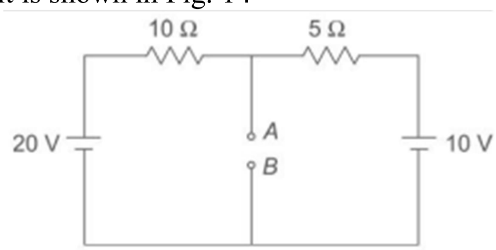


Figure 13

$$\text{or} \quad R_N = \frac{10 \times 5}{10+5} = 3.33 \ \Omega$$

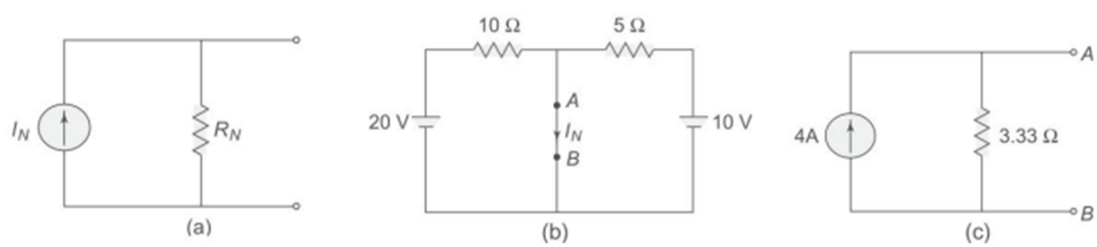
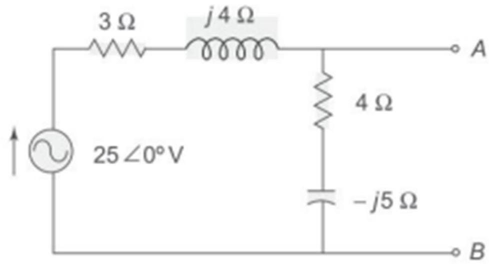
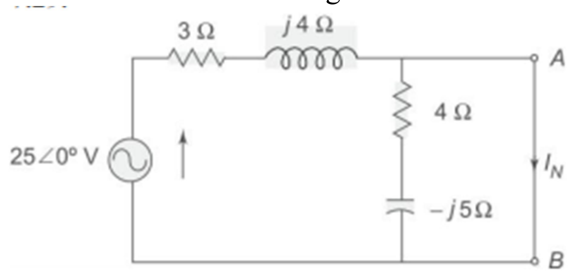


Figure 14

Example 2: For the circuit shown in Fig below, determine Norton's equivalent circuit between the output terminals, AB



Solution Norton's current IN is equal to the current passing through the short-circuited terminals AB as shown in Fig. below

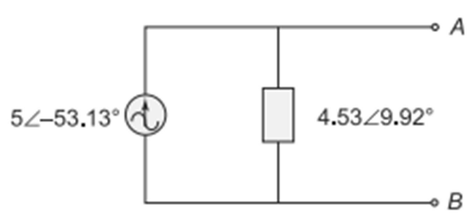


The current through terminals AB is

$$I_N = \frac{25\angle 0^\circ}{3+j4} = \frac{25\angle 0^\circ}{5\angle 53.13^\circ}$$
$$= 5\angle -53.13^\circ$$

The impedance seen from terminals AB is

$$Z_N = \frac{(3+j4)(4-j5)}{(3+j4)+(4-j5)}$$
$$= \frac{5\angle 53.13^\circ \times 6.4\angle -51.34^\circ}{7.07\angle -8.13^\circ}$$
$$= 4.53\angle 9.92^\circ$$



Norton's equivalent circuit is shown in Fig above.

2.4 RECIPROCITY THEOREM

In any linear bilateral network, if a single voltage source Va in branch 'a' produces a current Ib in branch 'b', then if the voltage source Va is removed and inserted in branch 'b' will produce a current Ib in branch 'a'. The ratio of response to excitation is same for the two conditions mentioned above. This is called the reciprocity theorem.

Consider the network shown in Fig. 15. AA' denotes input terminals and BB9 denotes output terminals.

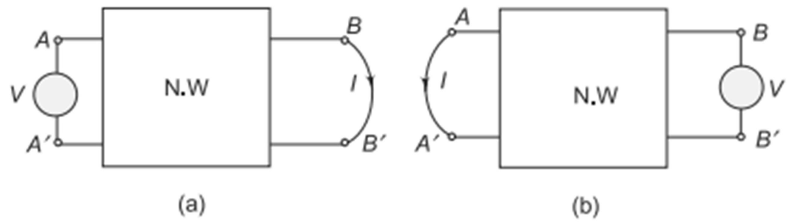


Figure 15

The application of voltage V across AA' produces current I at BB'. Now if the positions of the source and responses are interchanged, by connecting the voltage source across BB9, the resultant current I will be at terminals AA'. According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

Example Verify the reciprocity theorem for the network shown in Fig. 16

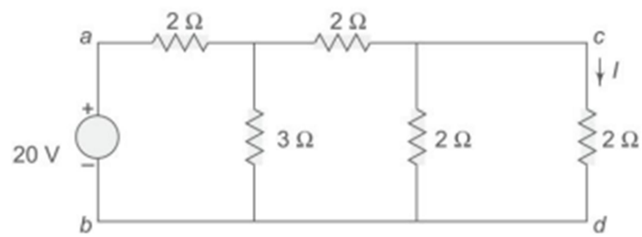


Figure 16

Solution Total resistance in the circuit = 2 + [3 || (2 + 2 || 2)] = 3.5 Ω
The current drawn by the circuit (See Fig. 17 (a))

$$I_T = \frac{20}{3.5} = 5.71\ \Omega$$

The current in the 2 V branch cd is I = 1.43 A.

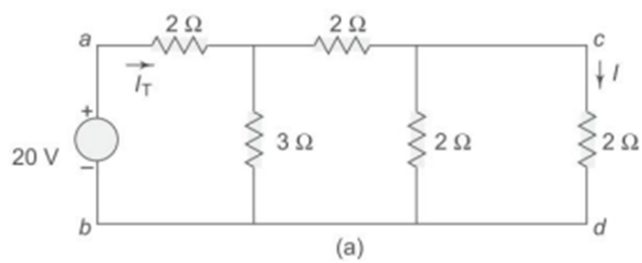


Figure 17 (a)

Applying the reciprocity theorem, by interchanging the source and response, we get Fig. 17 (b).

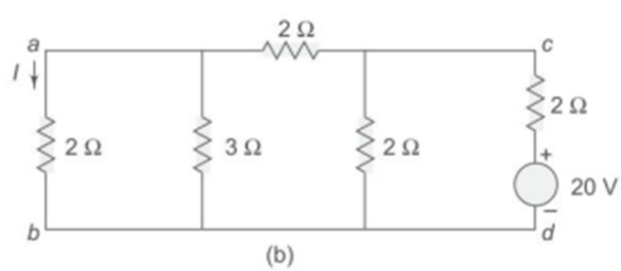


Figure 17 (b)

Total resistance in the circuit = 3.23 V.
Total current drawn by the circuit= = 20/3.23 = 6.19 A
The current in the branch ab is I = 1.43 A
If we compare the results in both cases, the ratio of input to response is the same, i.e. (20/1.43) = 13.99

2.5 MAXIMUM POWER TRANSFER THEOREM

Many circuits basically consist of sources, supplying voltage, current, or power to the load; for example, a radio speaker system, or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from the source to the load. In the simple resistive circuit shown in Fig. 18, Rs is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to the load is maximum

It is a fact that more voltage is delivered to the load when the load resistance is high as compared to the resistance of the source. On the other hand, maximum current is transferred to the load when the load resistance is small compared to the source resistance.

For many applications, an important consideration is the maximum power transfer to the load; for example, maximum power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum power transfer theorem states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance. In Fig. 18, assume that the load resistance is variable.

Current in the circuit is I = VS /(RS + RL)

Power delivered to the load RL is $P = I^2R_L = V^2\ S\ R_L / (RS + RL)^2$

To determine the value of RL for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to RL, i.e. when dP/dR_L equals zero.

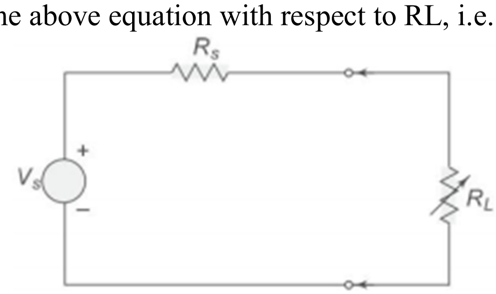


Figure 18

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2}{(R_S+R_L)^2}R_L\right]$$

$$= \frac{V_S^2\left\{(R_S+R_L)^2-(2R_L)(R_S+R_L)\right\}}{(R_S+R_L)^4}$$

$$\therefore \quad (R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$$

$$R^2{}_S + R^2{}_L + 2R_SR_L - 2R^2{}_L - 2R_SR_L = 0$$

$$\therefore \quad R_S = R_L$$

So, maximum power will be transferred to the load when load resistance is equal to the source resistance

Example In the circuit shown in Fig. 19, determine the value of load resistance when the load resistance draws maximum power. Also find the value of the maximum power

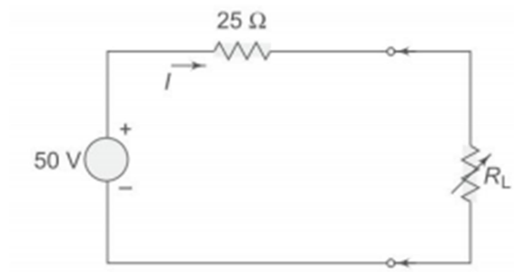
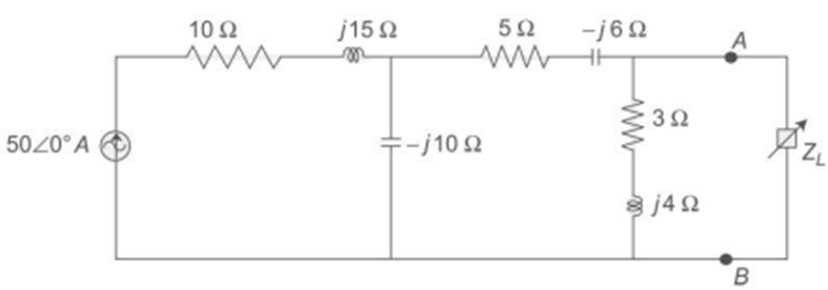


Figure 19

Solution In Fig. 19, the source delivers the maximum power when load resistance is equal to the source resistance.
RL = 25 V
The current I = 50/(25 + RL) = 50/50 = 1 A
The maximum power delivered to the load $P = I^2R_L = 1 \times 25 = 25$ W

Example 2: Determine the maximum power delivered to the load in the circuit shown in Fig below



Solution The circuit is replaced by Thèvenin's equivalent circuit in series with ZL as shown in Fig. below

where $V_{AB} = I_{(3+j4)\Omega} \times (3+j4)$ volts

$$I_{(3+j4)\Omega} = \frac{50\angle 0^\circ \times (-j10)}{5-j6+3+j4-j10} = 34.67\angle -33.7^\circ \text{ A}$$

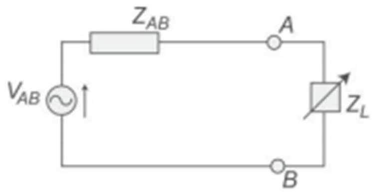
Voltage across AB is $V_{AB} = 34.67\angle{-33.7^\circ} \times 5\angle{53.13^\circ}$

$$= 173.35\angle{19.43^\circ}\ \text{V}$$

Impedance across terminals *AB* is

$$Z_{AB} = \{[(10 + j15) \parallel (-j10)] + (5 - j6)\} \parallel (3 + j4$$

$$Z_{AB} = 5.27\angle{41.16^\circ} = 4 + j3.5$$



To get the maximum power delivered to the load impedance, the load impedance must be equal to complex conjugate of source impedance. Therefore, the total impedance in the circuit shown in Fig. 7.44 is 8 Ω. The current in the circuit is

$$I_2 = \frac{V_{AB}}{8} = \frac{173.35}{8} = 21.66\ \text{A}$$

The maximum power transferred to the load is

$$P = I_L^2 R_L = (21.66)^2 \times 4 = 1874.9\ \text{watts}$$

SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – II – NETWORK FUNCTIONS AND ITS PARAMETERS

Two-Port Networks

2.1 Terminals and Ports

In a two-terminal network, the terminal voltage is related to the terminal current by the impedance $Z = V/I$. In a four-terminal network, if each terminal pair (or port) is connected separately to another circuit as in Fig. 4-1, the four variables i , i , u , and u are related by two equations called the terminal characteristics. These two equations, plus the terminal characteristics of the connected circuits, provide the necessary and sufficient number of equations to solve for the four variables.

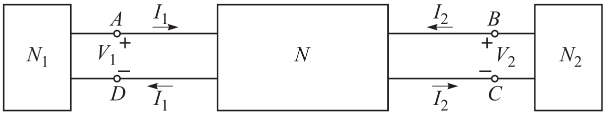


Fig. 2-1

2.2 Z-Parameters

The terminal characteristics of a two-port network, having linear elements and dependent sources, may be written in the **s**-domain as

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2 \end{aligned} \qquad (1)$$

The coefficients $\mathbf{Z}_{ij}$ have the dimension of impedance and are called the **Z**-*parameters* of the network. The **Z**-parameters are also called *open-circuit impedance parameters* since they may be measured at one terminal while the other terminal is open. They are

$$\begin{aligned} \mathbf{Z}_{11} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{Z}_{12} &= \left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \\ \mathbf{Z}_{21} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_1}\right|_{\mathbf{I}_2=0} \\ \mathbf{Z}_{22} &= \left.\frac{\mathbf{V}_2}{\mathbf{I}_2}\right|_{\mathbf{I}_1=0} \end{aligned} \qquad (2)$$

EXAMPLE 2.1 Find the Z-parameters of the two-port circuit in Fig. 2-2.

Apply KVL around the two loops in Fig. 4-2 with loop currents I and I to obtain

$$\mathbf{V}_1 = 2\mathbf{I}_1 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = (2 + \mathbf{s})\mathbf{I}_1 + \mathbf{s}\mathbf{I}_2$$

$$\mathbf{V}_2 = 3\mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{s}\mathbf{I}_1 + (3 + \mathbf{s})\mathbf{I}_2$$

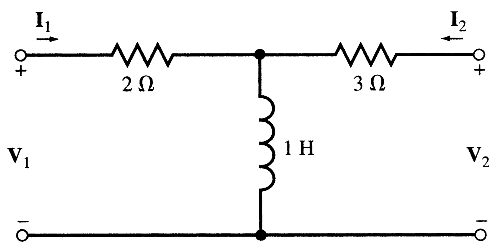


Fig. 2-2

By comparing (1) and (3), the **Z**-parameters of the circuit are found to be

$$\mathbf{Z}_{11} = \mathbf{s} + 2$$

$$\mathbf{Z}_{12} = \mathbf{Z}_{21} = \mathbf{s} \qquad (4)$$

$$\mathbf{Z}_{22} = \mathbf{s} + 3$$

Note that in this example $\mathbf{Z}_{12} = \mathbf{Z}_{21}$.

Reciprocal and Nonreciprocal Networks

A two-port network is called *reciprocal* if the open-circuit transfer impedances are equal: $\mathbf{Z}_{12} = \mathbf{Z}_{21}$. Consequently, in a reciprocal two-port network with current **I** feeding one port, the open-circuit voltage measured at the other port is the same, irrespective of the ports. The voltage is equal to $\mathbf{V} = \mathbf{Z}_{12}\mathbf{I} = \mathbf{Z}_{21}\mathbf{I}$. Networks containing resistors, inductors, and capacitors are generally reciprocal. Networks that additionally have 4dependent sources are generally nonreciprocal (see Example 4.2).

EXAMPLE 4.2 The two-port circuit shown in Fig. 4-3 contains a current-dependent voltage source. Find its **Z**-parameters.

As in Example 4.1, we apply Kirchhoff's Voltage Law (KVL) around the two loops:

$$\mathbf{V}_1 = 2\mathbf{I}_1 - \mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = (2 + \mathbf{s})\mathbf{I}_1 + (\mathbf{s} - 1)\mathbf{I}_2$$

$$\mathbf{V}_2 = 3\mathbf{I}_2 + \mathbf{s}(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{s}\mathbf{I}_1 + (3 + \mathbf{s})\mathbf{I}_2$$

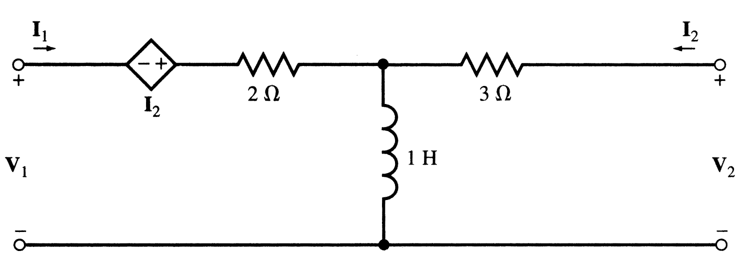


Fig. 4-3

The **Z**-parameters are

$$\begin{aligned} \mathbf{Z}_{11} &= \mathbf{s} + 2 \\ \mathbf{Z}_{12} &= \mathbf{s} - 1 \\ \mathbf{Z}_{21} &= \mathbf{s} \\ \mathbf{Z}_{22} &= \mathbf{s} + 3 \end{aligned} \tag{5}$$

With the dependent source in the circuit, $\mathbf{Z}_{12} \neq \mathbf{Z}_{21}$ and so the two-port circuit is nonreciprocal.

4.3 T-Equivalent of Reciprocal Networks

4A reciprocal network may be modeled by its T-equivalent as shown in the circuit of Fig. 4-4. Z , Z , and Z are obtained from the **Z**-parameters as follows.

$$\begin{aligned} \mathbf{Z}_a &= \mathbf{Z}_{11} - \mathbf{Z}_{12} \\ \mathbf{Z}_b &= \mathbf{Z}_{22} - \mathbf{Z}_{21} \\ \mathbf{Z}_c &= \mathbf{Z}_{12} = \mathbf{Z}_{21} \end{aligned} \tag{6}$$

The T-equivalent network is not necessarily realizable.

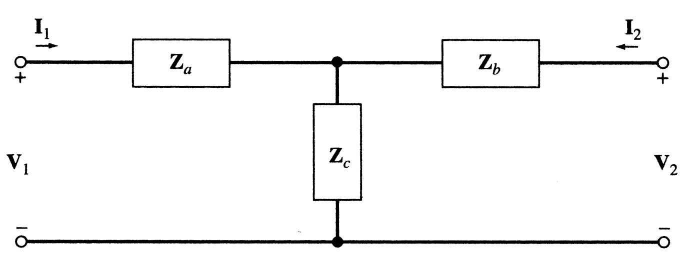


Fig. 4-4

EXAMPLE 4.3 Find the Z-parameters of Fig. 4-4.

Again we apply KVL to obtain

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{Z}_a\mathbf{I}_1 + \mathbf{Z}_c(\mathbf{I}_1 + \mathbf{I}_2) = (\mathbf{Z}_a + \mathbf{Z}_c)\mathbf{I}_1 + \mathbf{Z}_c\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{Z}_b\mathbf{I}_2 + \mathbf{Z}_c(\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{Z}_c\mathbf{I}_1 + (\mathbf{Z}_b + \mathbf{Z}_c)\mathbf{I}_2 \end{aligned} \tag{7}$$

By comparing (1) and (7) the **Z**-parameters are found to be

$$\begin{aligned} \mathbf{Z}_{11} &= \mathbf{Z}_a + \mathbf{Z}_c \\ \mathbf{Z}_{12} &= \mathbf{Z}_{21} = \mathbf{Z}_c \\ \mathbf{Z}_{22} &= \mathbf{Z}_b + \mathbf{Z}_c \end{aligned} \tag{8}$$

4.4 Y-Parameters

The terminal characteristics may also be written as in (9), where $\mathbf{I}_1$ and $\mathbf{I}_2$ are expressed in terms of $\mathbf{V}_1$ and $\mathbf{V}_2$.

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2 \\ \mathbf{I}_2 &= \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2 \end{aligned} \tag{9}$$

The coefficients $\mathbf{Y}_{ij}$ have the dimension of admittance and are called the **Y**-*parameters* or *short-circuit admittance parameters* because they may be measured at one port while the other port is short-circuited. The **Y**-parameters are

$$
\begin{aligned}
\mathbf{Y}_{11} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}\\
\mathbf{Y}_{12} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}\\
\mathbf{Y}_{21} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_1}\right|_{\mathbf{V}_2=0}\\
\mathbf{Y}_{22} &= \left.\frac{\mathbf{I}_2}{\mathbf{V}_2}\right|_{\mathbf{V}_1=0}
\end{aligned}
\tag{10}
$$

EXAMPLE 4.4 Find the Y-parameters of the circuit in Fig. 4-5.

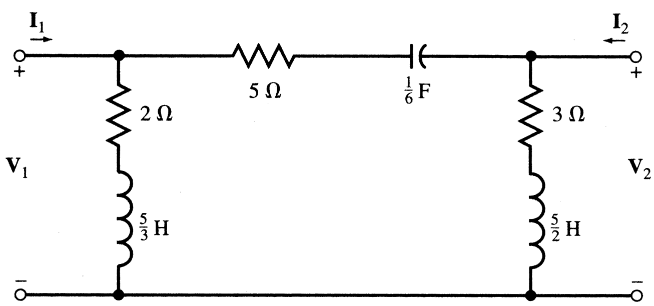


Fig. 4-5

We apply Kirchhoff's Current Law (KCL) to the input and output nodes (for convenience, we designate the admittances of the three branches of the circuit by Y , Y , and Y as shown in Fig. 4-6). Thus,

$$
\begin{aligned}
\mathbf{Y}_a &= \frac{1}{2 + 5\mathbf{s}/3} = \frac{3}{5\mathbf{s} + 6}\\
\mathbf{Y}_b &= \frac{1}{3 + 5\mathbf{s}/2} = \frac{2}{5\mathbf{s} + 6}\\
\mathbf{Y}_c &= \frac{1}{5 + 6/\mathbf{s}} = \frac{\mathbf{s}}{5\mathbf{s} + 6}
\end{aligned}
\tag{11}
$$

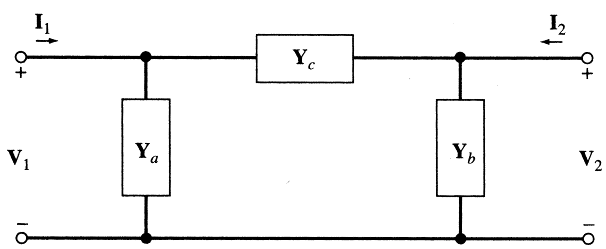


Fig. 2-6

The node equations are

$$
\begin{aligned}
\mathbf{I}_1 &= \mathbf{V}_1\mathbf{Y}_a + (\mathbf{V}_1 - \mathbf{V}_2)\,\mathbf{Y}_c = (\mathbf{Y}_a + \mathbf{Y}_c)\mathbf{V}_1 - \mathbf{Y}_c\mathbf{V}_2\\
\mathbf{I}_2 &= \mathbf{V}_2\mathbf{Y}_b + (\mathbf{V}_2 - \mathbf{V}_1)\,\mathbf{Y}_c = -\mathbf{Y}_c\mathbf{V}_1 + (\mathbf{Y}_b + \mathbf{Y}_c)\mathbf{V}_2
\end{aligned}
\tag{12}
$$

By comparing (9) with (12), we get

$$\begin{aligned} \mathbf{Y}_{11} &= \mathbf{Y}_a + \mathbf{Y}_c \\ \mathbf{Y}_{12} &= \mathbf{Y}_{21} = -\mathbf{Y}_c \\ \mathbf{Y}_{22} &= \mathbf{Y}_b + \mathbf{Y}_c \end{aligned} \tag{13}$$

Substituting $\mathbf{Y}_a$, $\mathbf{Y}_b$, and $\mathbf{Y}_c$ from (11) into (13), we find

$$\begin{aligned} \mathbf{Y}_{11} &= \frac{\mathbf{s}+3}{5\mathbf{s}+6} \\ \mathbf{Y}_{12} &= \mathbf{Y}_{21} = \frac{-\mathbf{s}}{5\mathbf{s}+6} \\ \mathbf{Y}_{22} &= \frac{\mathbf{s}+2}{5\mathbf{s}+6} \end{aligned} \tag{14}$$

Since $\mathbf{Y}_{12} = \mathbf{Y}_{21}$, the two-port circuit is reciprocal.

2.5 Pi-Equivalent of Reciprocal Networks

A reciprocal network may be modeled by its Pi-equivalent as shown in Fig. 4-6. The three elements of the Pi-equivalent network can be found by reverse solution. We first find the Y-parameters of Fig. 4-6. From (10) we have

$$\begin{aligned} \mathbf{Y}_{11} &= \mathbf{Y}_a + \mathbf{Y}_c && \text{[Fig. 4-7-(a)]} \\ \mathbf{Y}_{12} &= -\mathbf{Y}_c && \text{[Fig. 4- 7-(b)]} \\ \mathbf{Y}_{21} &= -\mathbf{Y}_c && \text{[Fig. 4- 7-(a)]} \\ \mathbf{Y}_{22} &= \mathbf{Y}_b + \mathbf{Y}_c && \text{[Fig. 4-7-(b)]} \end{aligned} \tag{15}$$

from which

$$\mathbf{Y}_a = \mathbf{Y}_{11} + \mathbf{Y}_{12} \qquad \mathbf{Y}_b = \mathbf{Y}_{22} + \mathbf{Y}_{12} \qquad \mathbf{Y}_c = -\mathbf{Y}_{12} = -\mathbf{Y}_{21} \tag{16}$$

The Pi-equivalent network is not necessarily realizable.

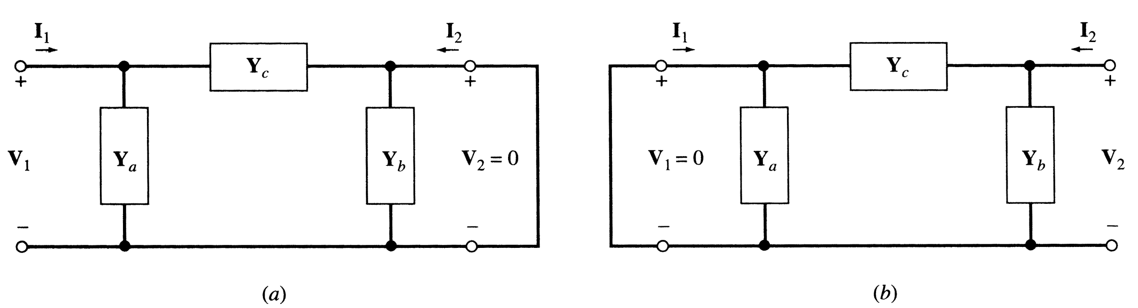


Fig. 2-7

2.6 Application of Terminal Characteristics

The four terminal variables $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ in a two-port network are related by the two equations (1) or (9). By connecting the two-port circuit to the outside as shown in Fig. 4-1, two additional equations are obtained. The four equations then can determine $\mathbf{I}_1$, $\mathbf{I}_2$, $\mathbf{V}_1$, and $\mathbf{V}_2$ without any knowledge of the inside structure of the circuit.

EXAMPLE 4.5 The Z-parameters of a two-port network are given by

$$\mathbf{Z}_{11} = 2\mathbf{s} + 1/\mathbf{s} \qquad \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s} \qquad \mathbf{Z}_{22} = 2\mathbf{s} + 4$$

4The network is connected to a source and a load as shown in Fig. 4-8. Find I , I , V , and V .

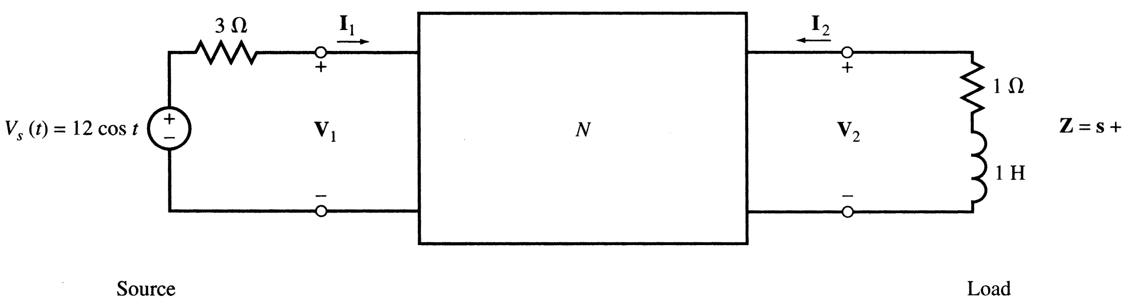


Fig. 2-8

The terminal characteristics are given by

$$\begin{aligned} \mathbf{V}_1 &= (2\mathbf{s} + 1/\mathbf{s})\mathbf{I}_1 + 2\mathbf{s}\mathbf{I}_2 \\ \mathbf{V}_2 &= 2\mathbf{s}\mathbf{I}_1 + (2\mathbf{s} + 4)\mathbf{I}_2 \end{aligned} \tag{17}$$

The phasor representation of voltage $\upsilon_s(t)$ is $\mathbf{V}_s = 12$ V with $\mathbf{s} = j$. From KVL around the input and output loops we obtain the two additional equations

$$\begin{aligned} \mathbf{V}_s &= 3\mathbf{I}_1 + \mathbf{V}_1 \\ 0 &= (1 + \mathbf{s})\mathbf{I}_2 + \mathbf{V}_2 \end{aligned} \tag{18}$$

Substituting $\mathbf{s} = j$ and $\mathbf{V}_s = 12$ in (17) and in (18) we get

$$\begin{aligned} \mathbf{V}_1 &= j\mathbf{I}_1 + 2j\mathbf{I}_2 \\ \mathbf{V}_2 &= 2j\mathbf{I}_1 + (4 + 2j)\mathbf{I}_2 \\ 12 &= 3\mathbf{I}_1 + \mathbf{V}_1 \\ 0 &= (1 + j)\mathbf{I}_2 + \mathbf{V}_2 \end{aligned}$$

from which

$$\mathbf{I}_1 = 3.29 \underline{/-10.2^\circ} \qquad \mathbf{I}_2 = 1.13 \underline{/-131.2^\circ}$$

$$\mathbf{V}_1 = 2.88 \underline{/37.5^\circ} \qquad \mathbf{V}_2 = 1.6 \underline{/93.8^\circ}$$

2.7 Conversion between Z- and Y-Parameters

The **Y**-parameters may be obtained from the **Z**-parameters by solving (1) for $\mathbf{I}_1$ and $\mathbf{I}_2$. Applying Cramer's rule to (1), we get

$$\begin{aligned} \mathbf{I}_1 &= \frac{\mathbf{Z}_{22}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_1 - \frac{\mathbf{Z}_{12}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_2 \\ \mathbf{I}_2 &= \frac{-\mathbf{Z}_{21}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_1 + \frac{\mathbf{Z}_{11}}{\mathbf{D}_{\mathbf{ZZ}}}\mathbf{V}_2 \end{aligned} \tag{19}$$

where $\mathbf{D_{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21}$ is the determinant of the coefficient matrix in (1). By comparing (19) with (9) we have

$$
\begin{aligned}
\mathbf{Y}_{11} &= \frac{\mathbf{Z}_{22}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{12} &= \frac{-\mathbf{Z}_{12}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{21} &= \frac{-\mathbf{Z}_{21}}{\mathbf{D_{ZZ}}} \\
\mathbf{Y}_{22} &= \frac{\mathbf{Z}_{11}}{\mathbf{D_{ZZ}}}
\end{aligned}
\tag{20}
$$

Given the **Z**-parameters, for the **Y**-parameters to exist, the determinant $\mathbf{D_{ZZ}}$ must be nonzero. Conversely, given the **Y**-parameters, the **Z**-parameters are

$$
\begin{aligned}
\mathbf{Z}_{11} &= \frac{\mathbf{Y}_{22}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{12} &= \frac{-\mathbf{Y}_{12}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{21} &= \frac{-\mathbf{Y}_{21}}{\mathbf{D_{YY}}} \\
\mathbf{Z}_{22} &= \frac{\mathbf{Y}_{11}}{\mathbf{D_{YY}}}
\end{aligned}
\tag{21}
$$

where $\mathbf{D_{YY}} = \mathbf{Y}_{11}\mathbf{Y}_{22} - \mathbf{Y}_{12}\mathbf{Y}_{21}$ is the determinant of the coefficient matrix in (9). For the **Z**-parameters of a two-port circuit to be derived from its **Y**-parameters, $\mathbf{D_{YY}}$ should be nonzero.

EXAMPLE 4.6 Referring to Example 4.4, find the Z-parameters of the circuit of Fig. 4-5 from its Y-parameters.
The **Y**-parameters of the circuit were found to be [see (14)]

$$\mathbf{Y}_{11} = \frac{\mathbf{s}+3}{5\mathbf{s}+6} \qquad \mathbf{Y}_{12} = \mathbf{Y}_{21} = \frac{-\mathbf{s}}{5\mathbf{s}+6} \qquad \mathbf{Y}_{22} = \frac{\mathbf{s}+2}{5\mathbf{s}+6}$$

Substituting into (21), where $\mathbf{D_{YY}} = 1/(5\mathbf{s}+6)$, we obtain

$$
\begin{aligned}
\mathbf{Z}_{11} &= \mathbf{s} + 2 \\
\mathbf{Z}_{12} = \mathbf{Z}_{21} &= \mathbf{s} \\
\mathbf{Z}_{22} &= \mathbf{s} + 3
\end{aligned}
\tag{22}
$$

The Z-parameters in (22) are identical to the Z-parameters of the circuit of Fig. 4-2. The two circuits are equivalent as far as the terminals are concerned. This was by design. Figure 4-2 is the T-equivalent of Fig. 4-5. The equivalence between Fig. 4-2 and Fig. 4-5 may be verified directly by applying (6) to the **Z**-parameters given in (22) to obtain the T-equivalent network.

2.8 h-Parameters

Some two-port circuits or electronic devices are best characterized by the following terminal equations:

$$
\begin{aligned}
\mathbf{V}_1 &= \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2 \\
\mathbf{I}_2 &= \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2
\end{aligned}
\tag{23}
$$

where the $\mathbf{h}_{ij}$ coefficients are called the *hybrid* or **h**-*parameters*.

EXAMPLE 4.7 Find the h-parameters of Fig. 4-9.

This is the simple model of a bipolar junction transistor in its linear region of operation. By inspection, the terminal characteristics of Fig. 4-9 are

$$\mathbf{V}_1 = 50\mathbf{I}_1 \qquad \text{and} \qquad \mathbf{I}_2 = 300\mathbf{I}_1 \tag{24}$$

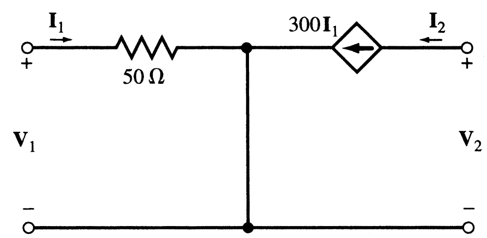


Fig. 4-9

By comparing (24) and (23) we get

$$\mathbf{h}_{11} = 50 \qquad \mathbf{h}_{12} = 0 \qquad \mathbf{h}_{21} = 300 \qquad \mathbf{h}_{22} = 0 \tag{25}$$

4.9 g-Parameters

The terminal characteristics of a two-port circuit may also be described by still another set of hybrid parameters as given in (26).

$$\begin{aligned} \mathbf{I}_1 &= \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2 \\ \mathbf{V}_2 &= \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2 \end{aligned} \tag{26}$$

where the coefficients $\mathbf{g}_{ij}$ are called *inverse hybrid* or **g**-*parameters.*

EXAMPLE 4.8 Find the g-parameters in the circuit shown in Fig. 4-10.

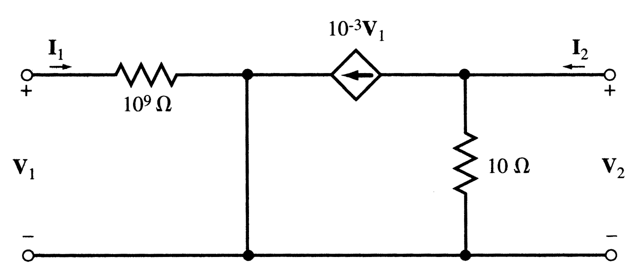


Fig. 4-10

This is the simple model of a field effect transistor in its linear region of operation. To find the **g**-parameters, we first derive the terminal equations by applying Kirchhoff's laws at the terminals:

At the input terminal: $\qquad \mathbf{V}_1 = 10^9\,\mathbf{I}_1$

At the output terminal: $\qquad \mathbf{V}_2 = 10(\mathbf{I}_2 - 10^{-3}\,\mathbf{V}_1)$

or

$$\mathbf{I}_1 = 10^{-9}\,\mathbf{V}_1 \qquad \text{and} \qquad \mathbf{V}_2 = 10\mathbf{I}_2 - 10^{-2}\,\mathbf{V}_1 \tag{27}$$

By comparing (27) and (26) we get

$$\mathbf{g}_{11} = 10^{-9} \qquad \mathbf{g}_{12} = 0 \qquad \mathbf{g}_{21} = -10^{-2} \qquad \mathbf{g}_{22} = 10 \tag{28}$$

2.10 Transmission Parameters

The transmission parameters **A**, **B**, **C**, and **D** express the required source variables $\mathbf{V}_1$ and $\mathbf{I}_1$ in terms of the existing destination variables $\mathbf{V}_2$ and $\mathbf{I}_2$. They are called **ABCD** or T-parameters and are defined by

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2 \\ \mathbf{I}_1 &= \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2 \end{aligned} \tag{29}$$

EXAMPLE 4.9 Find the T-parameters of Fig. 4-11 where Z and Z are nonzero.

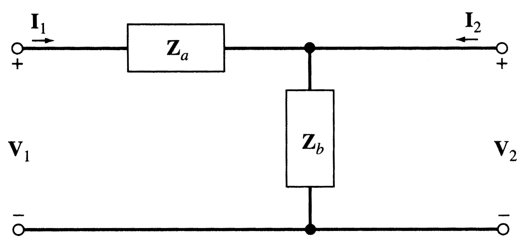


Fig. 4-11

This is the simple lumped model of an incremental segment of a transmission line. From (29) we have

$$\begin{aligned} \mathbf{A} &= \left.\frac{\mathbf{V}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \frac{\mathbf{Z}_a + \mathbf{Z}_b}{\mathbf{Z}_b} = 1 + \mathbf{Z}_a\mathbf{Y}_b \\ \mathbf{B} &= -\left.\frac{\mathbf{V}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = \mathbf{Z}_a \\ \mathbf{C} &= \left.\frac{\mathbf{I}_1}{\mathbf{V}_2}\right|_{\mathbf{I}_2=0} = \mathbf{Y}_b \\ \mathbf{D} &= -\left.\frac{\mathbf{I}_1}{\mathbf{I}_2}\right|_{\mathbf{V}_2=0} = 1 \end{aligned} \tag{30}$$

4.11 Interconnecting Two-Port Networks

Two-port networks may be interconnected in various configurations, such as series, parallel, or cascade connections, resulting in new two-port networks. For each configuration, a certain set of parameters may be more useful than others to describe the network.

Series Connection

Figure 4-12 shows a series connection of two two-port networks a and b with open-circuit impedance parameters $\mathbf{Z}_a$ and $\mathbf{Z}_b$, respectively. In this configuration, we use the **Z**-parameters since they are combined as a series connection of two impedances. The Z-parameters of the series connection are (see Problem 4.10):

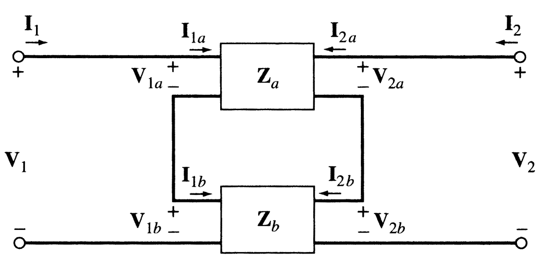


Fig. 4-12

$$\mathbf{Z}_{11} = \mathbf{Z}_{11,a} + \mathbf{Z}_{11,b}$$
$$\mathbf{Z}_{12} = \mathbf{Z}_{12,a} + \mathbf{Z}_{12,b}$$
$$\mathbf{Z}_{21} = \mathbf{Z}_{21,a} + \mathbf{Z}_{21,b} \qquad (31a)$$
$$\mathbf{Z}_{22} = \mathbf{Z}_{22,a} + \mathbf{Z}_{22,b}$$

or, in the matrix form,

$$[\mathbf{Z}] = [\mathbf{Z}_a] + [\mathbf{Z}_b] \qquad (31b)$$

Parallel Connection

Figure 4-13 shows a parallel connection of the two-port networks a and b with short-circuit admittance parameters $\mathbf{Y}_a$ and $\mathbf{Y}_b$, respectively. In this case, the **Y**-parameters are convenient to work with. The **Y**-parameters of the parallel connection are (see Problem 4.11):

$$\mathbf{Y}_{11} = \mathbf{Y}_{11,a} + \mathbf{Y}_{11,b}$$
$$\mathbf{Y}_{12} = \mathbf{Y}_{12,a} + \mathbf{Y}_{12,b}$$
$$\mathbf{Y}_{21} = \mathbf{Y}_{21,a} + \mathbf{Y}_{21,b} \qquad (32a)$$
$$\mathbf{Y}_{22} = \mathbf{Y}_{22,a} + \mathbf{Y}_{22,b}$$

or, in matrix form,

$$[\mathbf{Y}] = [\mathbf{Y}_a] + [\mathbf{Y}_b] \qquad (32b)$$

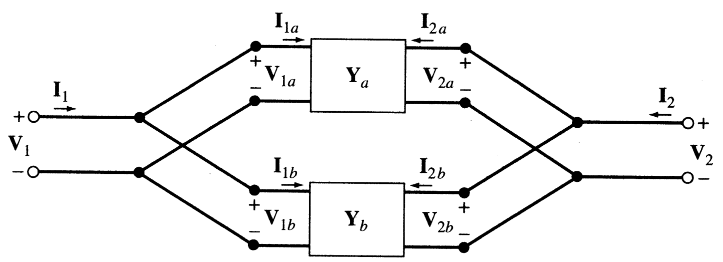


Fig. 4-13

Cascade Connection

The cascade connection of the two-port networks a and b is shown in Fig. 4-14. In this case the T-parameters are particularly convenient. The T-parameters of the cascade combination are

$$\mathbf{A} = \mathbf{A}_a\mathbf{A}_b + \mathbf{B}_a\mathbf{C}_b$$
$$\mathbf{B} = \mathbf{A}_a\mathbf{B}_b + \mathbf{B}_a\mathbf{D}_b$$
$$\mathbf{C} = \mathbf{C}_a\mathbf{A}_b + \mathbf{D}_a\mathbf{C}_b \qquad (33a)$$
$$\mathbf{D} = \mathbf{C}_a\mathbf{B}_b + \mathbf{D}_a\mathbf{D}_b$$

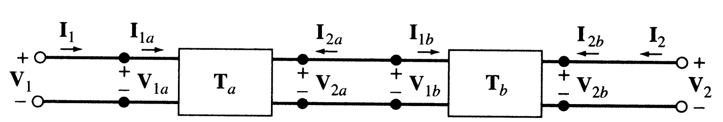


Fig. 4-14

or, in matrix form,

$$[\mathbf{T}] = [\mathbf{T}_a][\mathbf{T}_b] \tag{33b}$$

4.12 Choice of Parameter Type

What types of parameters are appropriate for and can best describe a given two-port network or device? Several factors influence the choice of parameters. (1) It is possible that some types of parameters do not exist as they may not be defined at all (see Example 4.10). (2) Some parameters are more convenient to work with when the network is connected to other networks, as shown in Section 4.11. In this regard, by converting the two-port network to its T- and Pi-equivalents and then applying the familiar analysis techniques, such as element reduction and current division, we can greatly reduce and simplify the overall circuit. (3) For some networks or devices, a certain type of parameter produces better computational accuracy and better sensitivity when used within the interconnected circuit.

EXAMPLE 4.10 Find the Z- and Y-parameters of Fig. 4-15.

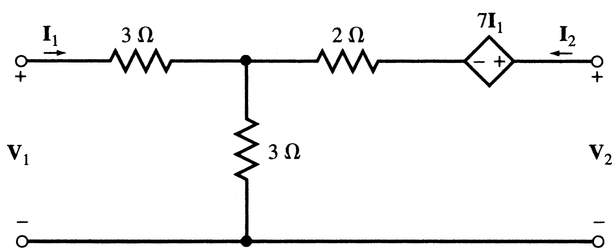


Fig. 4-15

We apply KVL to the input and output loops. Thus,

Input loop: $\mathbf{V}_1 = 3\mathbf{I}_1 + 3(\mathbf{I}_1 + \mathbf{I}_2)$

Output loop: $\mathbf{V}_2 = 7\mathbf{I}_1 + 2\mathbf{I}_2 + 3(\mathbf{I}_1 + \mathbf{I}_2)$

or

$$\mathbf{V}_1 = 6\mathbf{I}_1 + 3\mathbf{I}_2 \qquad \text{and} \qquad \mathbf{V}_2 = 10\mathbf{I}_1 + 5\mathbf{I}_2 \tag{34}$$

By comparing (34) and (2) we get

$$\mathbf{Z}_{11} = 6 \qquad \mathbf{Z}_{12} = 3 \qquad \mathbf{Z}_{21} = 10 \qquad \mathbf{Z}_{22} = 5$$

The **Y**-parameters are, however, not defined, since the application of the direct method of (10) or the conversion from **Z**-parameters (19) produces $\mathbf{D}_{zz} = 6(5) - 3(10) = 0$.

4.13 Summary of Terminal Parameters and Conversion

The various terminal parameters are defined by the following equations:

Z-parameters

$$\mathbf{V}_1 = \mathbf{Z}_{11}\mathbf{I}_1 + \mathbf{Z}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{Z}_{21}\mathbf{I}_1 + \mathbf{Z}_{22}\mathbf{I}_2$$
$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{I}]$$

h-parameters

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

T-parameters

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$
$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

Y-parameters

$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1 + \mathbf{Y}_{12}\mathbf{V}_2$$
$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1 + \mathbf{Y}_{22}\mathbf{V}_2$$
$$[\mathbf{I}] = [\mathbf{Y}][\mathbf{V}]$$

g-parameters

$$\mathbf{I}_1 = \mathbf{g}_{11}\mathbf{V}_1 + \mathbf{g}_{12}\mathbf{I}_2$$
$$\mathbf{V}_2 = \mathbf{g}_{21}\mathbf{V}_1 + \mathbf{g}_{22}\mathbf{I}_2$$

Table 4-1 summarizes the conversion between the Z-, Y-, h-, g-, and T-parameters. For the conversion to be possible, the determinant of the source parameters must be nonzero.

Table 4-1

	Z		**Y**		**h**		**g**		**T**	
Z	$\mathbf{Z}_{11}$	$\mathbf{Z}_{12}$	$\frac{\mathbf{Y}_{22}}{\mathbf{D_{YY}}}$	$\frac{-\mathbf{Y}_{12}}{\mathbf{D_{YY}}}$	$\frac{\mathbf{D_{hh}}}{\mathbf{h}_{22}}$	$\frac{\mathbf{h}_{12}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{g}_{11}}$	$\frac{-\mathbf{g}_{12}}{\mathbf{g}_{11}}$	$\frac{\mathbf{A}}{\mathbf{C}}$	$\frac{\mathbf{D}_{TT}}{\mathbf{C}}$
	$\mathbf{Z}_{21}$	$\mathbf{Z}_{22}$	$\frac{-\mathbf{Y}_{21}}{\mathbf{D_{YY}}}$	$\frac{\mathbf{Y}_{11}}{\mathbf{D_{YY}}}$	$\frac{-\mathbf{h}_{21}}{\mathbf{h}_{22}}$	$\frac{1}{\mathbf{h}_{22}}$	$\frac{\mathbf{g}_{21}}{\mathbf{g}_{11}}$	$\frac{\mathbf{D_{gg}}}{\mathbf{g}_{11}}$	$\frac{1}{\mathbf{C}}$	$\frac{\mathbf{D}}{\mathbf{C}}$
Y	$\frac{\mathbf{Z}_{22}}{\mathbf{D_{zz}}}$	$\frac{-\mathbf{Z}_{12}}{\mathbf{D_{zz}}}$	$\mathbf{Y}_{11}$	$\mathbf{Y}_{12}$	$\frac{1}{\mathbf{h}_{11}}$	$\frac{-\mathbf{h}_{12}}{\mathbf{h}_{11}}$	$\frac{\mathbf{D_{gg}}}{\mathbf{g}_{22}}$	$\frac{\mathbf{g}_{12}}{\mathbf{g}_{22}}$	$\frac{\mathbf{D}}{\mathbf{B}}$	$\frac{-\mathbf{D}_{TT}}{\mathbf{B}}$
	$\frac{-\mathbf{Z}_{21}}{\mathbf{D_{zz}}}$	$\frac{\mathbf{Z}_{11}}{\mathbf{D_{zz}}}$	$\mathbf{Y}_{21}$	$\mathbf{Y}_{22}$	$\frac{\mathbf{h}_{21}}{\mathbf{h}_{11}}$	$\frac{-\mathbf{D_{nn}}}{\mathbf{h}_{11}}$	$\frac{-\mathbf{g}_{21}}{\mathbf{g}_{22}}$	$\frac{1}{\mathbf{g}_{22}}$	$\frac{-1}{\mathbf{B}}$	$\frac{\mathbf{A}}{\mathbf{B}}$
h	$\frac{\mathbf{D_{ZZ}}}{\mathbf{Z}_{22}}$	$\frac{\mathbf{Z}_{12}}{\mathbf{Z}_{22}}$	$\frac{1}{\mathbf{Y}_{11}}$	$\frac{-\mathbf{Y}_{12}}{\mathbf{Y}_{11}}$	$\mathbf{h}_{11}$	$\mathbf{h}_{12}$	$\frac{\mathbf{g}_{22}}{\mathbf{D}_{gg}}$	$\frac{\mathbf{g}_{12}}{\mathbf{D}_{gg}}$	$\frac{\mathbf{B}}{\mathbf{D}}$	$\frac{\mathbf{D}_{TT}}{\mathbf{D}}$
	$\frac{-\mathbf{Z}_{21}}{\mathbf{Z}_{22}}$	$\frac{1}{\mathbf{Z}_{22}}$	$\frac{\mathbf{Y}_{21}}{\mathbf{Y}_{11}}$	$\frac{\mathbf{D}_{yy}}{\mathbf{Y}_{11}}$	$\mathbf{h}_{21}$	$\mathbf{h}_{22}$	$\frac{\mathbf{g}_{21}}{\mathbf{D}_{gg}}$	$\frac{\mathbf{g}_{11}}{\mathbf{D}_{gg}}$	$\frac{-1}{\mathbf{D}}$	$\frac{\mathbf{C}}{\mathbf{D}}$
g	$\frac{1}{\mathbf{Z}_{11}}$	$\frac{-\mathbf{Z}_{12}}{\mathbf{Z}_{11}}$	$\frac{\mathbf{D_{YY}}}{\mathbf{Y}_{22}}$	$\frac{\mathbf{Y}_{12}}{\mathbf{Y}_{22}}$	$\frac{\mathbf{h}_{22}}{\mathbf{D_{hh}}}$	$\frac{-\mathbf{h}_{12}}{\mathbf{D_{hh}}}$	$\mathbf{g}_{11}$	$\mathbf{g}_{12}$	$\frac{\mathbf{C}}{\mathbf{A}}$	$\frac{-\mathbf{D}_{TT}}{\mathbf{A}}$
	$\frac{\mathbf{Z}_{21}}{\mathbf{Z}_{11}}$	$\frac{\mathbf{D_{ZZ}}}{\mathbf{Z}_{11}}$	$\frac{-\mathbf{Y}_{21}}{\mathbf{Y}_{22}}$	$\frac{1}{\mathbf{Y}_{22}}$	$\frac{-\mathbf{h}_{21}}{\mathbf{D_{hh}}}$	$\frac{\mathbf{h}_{11}}{\mathbf{D_{hh}}}$	$\mathbf{g}_{21}$	$\mathbf{g}_{22}$	$\frac{1}{\mathbf{A}}$	$\frac{\mathbf{B}}{\mathbf{A}}$
T	$\frac{\mathbf{Z}_{11}}{\mathbf{Z}_{21}}$	$\frac{\mathbf{D_{ZZ}}}{\mathbf{Z}_{21}}$	$\frac{-\mathbf{Y}_{22}}{\mathbf{Y}_{21}}$	$\frac{-1}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{D_{hh}}}{\mathbf{h}_{21}}$	$\frac{-\mathbf{h}_{11}}{\mathbf{h}_{21}}$	$\frac{1}{\mathbf{g}_{21}}$	$\frac{\mathbf{g}_{22}}{\mathbf{g}_{21}}$	$\mathbf{A}$	$\mathbf{B}$
	$\frac{1}{\mathbf{Z}_{21}}$	$\frac{\mathbf{Z}_{22}}{\mathbf{Z}_{21}}$	$\frac{-\mathbf{D_{YY}}}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{Y}_{11}}{\mathbf{Y}_{21}}$	$\frac{-\mathbf{h}_{22}}{\mathbf{h}_{21}}$	$\frac{-1}{\mathbf{h}_{21}}$	$\frac{\mathbf{g}_{11}}{\mathbf{g}_{21}}$	$\frac{\mathbf{D_{gg}}}{\mathbf{g}_{21}}$	$\mathbf{C}$	$\mathbf{D}$

$\mathbf{D_{PP}} = \mathbf{P}_{11}\mathbf{P}_{22} - \mathbf{P}_{12}\mathbf{P}_{21}$ is the determinant of the coefficient matrix for the **Z**–, **Y**–, **h**–, **g**–, or T-parameters.

SOLVED PROBLEMS

4.1. Find the Z-parameters of the circuit in Fig. 4-16(a).

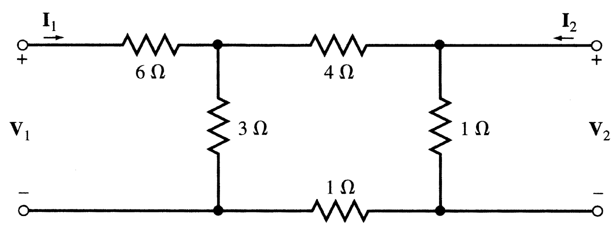
Z and Z are obtained by connecting a source to port #1 and leaving port #2 open [Fig. 4-16(b)]. The parallel and series combination of resistors produces

$$\mathbf{Z}_{11} = \frac{\mathbf{V}_1}{\mathbf{I}_1}\bigg|_{\mathbf{I}_{22}=0} = 8 \qquad \text{and} \qquad \mathbf{Z}_{21} = \frac{\mathbf{V}_2}{\mathbf{I}_1}\bigg|_{\mathbf{I}\ =0} = \frac{1}{3}$$

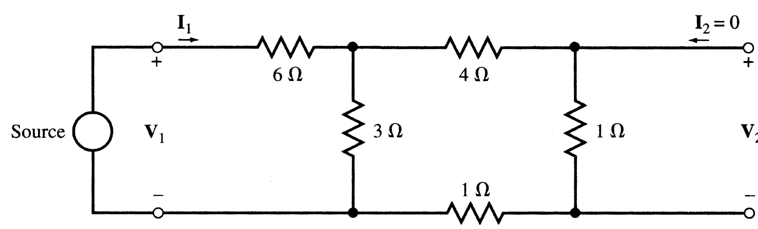
Similarly, $\mathbf{Z}_{22}$ and $\mathbf{Z}_{12}$ are obtained by connecting a source to port #2 and leaving port #1 open [Fig. 4-16(c)].

$$\mathbf{Z}_{22} = \frac{\mathbf{V}_2}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0} = \frac{8}{9} \qquad \mathbf{Z}_{12} = \frac{\mathbf{V}_1}{\mathbf{I}_2}\bigg|_{\mathbf{I}_1=0} = \frac{1}{3}$$

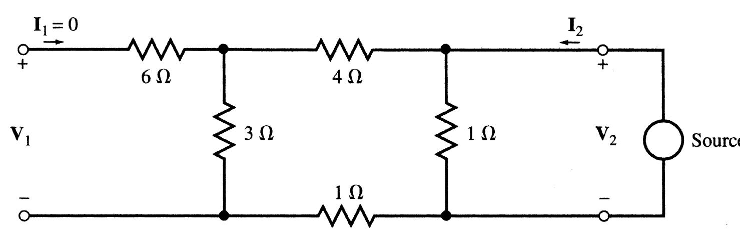
The circuit is reciprocal, since $\mathbf{Z}_{12} = \mathbf{Z}_{21}$.



(*a*)



(*b*)



(*c*)

Fig. 4-16

4.2. The Z-parameters of a two-port network N are given by

$$\mathbf{Z}_{11} = 2\mathbf{s} + 1/\mathbf{s} \qquad \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s} \qquad \mathbf{Z}_{22} = 2\mathbf{s} + 4$$

(*a*) Find the T-equivalent of *N*. (*b*) The network *N* is connected to a source and a load as shown in the circuit of Fig. 4-8. Replace N by its T-equivalent and then solve for i , i , u , and u .

(a) The three branches of the T-equivalent network (Fig. 4-4) are

$$\mathbf{Z}_a = \mathbf{Z}_{11} - \mathbf{Z}_{12} = 2\mathbf{s} + \frac{1}{\mathbf{s}} - 2\mathbf{s} = \frac{1}{\mathbf{s}}$$

$$\mathbf{Z}_b = \mathbf{Z}_{22} - \mathbf{Z}_{12} = 2\mathbf{s} + 4 - 2\mathbf{s} = 4$$

$$\mathbf{Z}_c = \mathbf{Z}_{12} = \mathbf{Z}_{21} = 2\mathbf{s}$$

(*b*) The T-equivalent of *N*, along with its input and output connections, is shown in the phasor domain in Fig. 4-17.

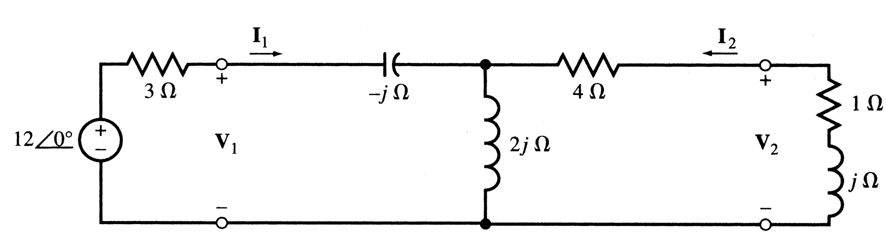


Fig. 4-17

By applying the familiar analysis techniques, including element reduction and current division, to Fig. 4-17 we find i , i , u , and u .

In the phasor domain	In the time domain:
$\mathbf{I}_1 = 3.29 \underline{/-10.2^\circ}$	$i_1 = 3.29 \cos(t - 10.2^\circ)$
$\mathbf{I}_2 = 1.13 \underline{/-131.2^\circ}$	$i_2 = 1.13 \cos(t - 131.2^\circ)$
$\mathbf{V}_1 = 2.88 \underline{/37.5^\circ}$	$v_1 = 2.88 \cos(t + 37.5^\circ)$
$\mathbf{V}_2 = 1.6 \underline{/93.8^\circ}$	$v_2 = 1.6 \cos(t + 93.8^\circ)$

4.3. Find the Z-parameters of the two-port network in Fig. 4-18.

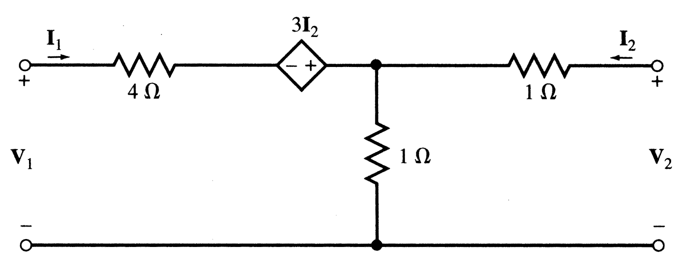


Fig. 4-18

KVL applied to the input and output ports results in the following:

Input port: $\mathbf{V}_1 = 4\mathbf{I}_1 - 3\mathbf{I}_2 + (\mathbf{I}_1 + \mathbf{I}_2) = 5\mathbf{I}_1 - 2\mathbf{I}_2$

Output port: $\mathbf{V}_2 = \mathbf{I}_2 + (\mathbf{I}_1 + \mathbf{I}_2) = \mathbf{I}_1 + 2\mathbf{I}_2$

By applying (1) to the above, $\mathbf{Z}_{11} = 5$, $\mathbf{Z}_{12} = -2$, $\mathbf{Z}_{21} = 1$, and $\mathbf{Z}_{22} = 2$.

4.4. Find the Z-parameters of the two-port network in Fig. 4-19 and compare the results with those of Problem 4.3.

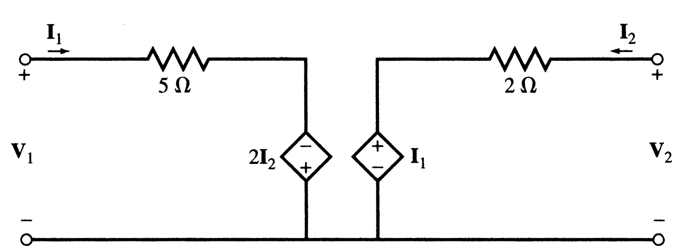


Fig. 4-19

KVL gives

$$\mathbf{V}_1 = 5\mathbf{I}_1 - 2\mathbf{I}_2 \qquad \text{and} \qquad \mathbf{V}_2 = \mathbf{I}_1 + 2\mathbf{I}_2$$

The above equations are identical with the terminal characteristics obtained for the network of Fig. 13-18. Thus, the two networks are equivalent.

4.5. Find the Y-parameters of Fig. 4-19 using its Z-parameters.

From Problem 4.4,

$$\mathbf{Z}_{11} = 5, \mathbf{Z}_{12} = -2, \mathbf{Z}_{21} = 1, \mathbf{Z}_{22} = 2$$

Since $\mathbf{D_{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21} = (5)(2) - (-2)(1) = 12$,

$$\mathbf{Y}_{11} = \frac{\mathbf{Z}_{22}}{\mathbf{D}_{zz}} = \frac{2}{12} = \frac{1}{6} \qquad \mathbf{Y}_{12} = \frac{-\mathbf{Z}_{12}}{\mathbf{D}_{zz}} = \frac{2}{12} = \frac{1}{6} \qquad \mathbf{Y}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{D}_{zz}} = \frac{-1}{12} \qquad \mathbf{Y}_{22} = \frac{\mathbf{Z}_{11}}{\mathbf{D}_{zz}} = \frac{5}{12}$$

4.6. Find the Y-parameters of the two-port network in Fig. 4-20 and thus show that the networks of Figs. 4-19 and 4-20 are equivalent.

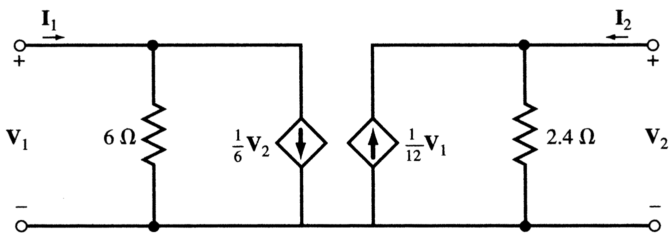


Fig. 4-20

Apply KCL at the ports to obtain the terminal characteristics and **Y**-parameters. Thus,

Input port: $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{6} + \frac{\mathbf{V}_2}{6}$$

Output port: $$\mathbf{I}_2 = \frac{\mathbf{V}_2}{2.4} - \frac{\mathbf{V}_1}{12}$$

and $$\mathbf{Y}_{11} = \frac{1}{6} \qquad \mathbf{Y}_{12} = \frac{1}{6} \qquad \mathbf{Y}_{21} = \frac{-1}{12} \qquad \mathbf{Y}_{22} = \frac{1}{2.4} = \frac{5}{12}$$

which are identical with the **Y**-parameters obtained in Problem 13.5 for Fig. 13-19. Thus, the two networks are equivalent.

4.7. Apply the short-circuit equations (10) to find the Y-parameters of the two-port network in Fig. 4-21.

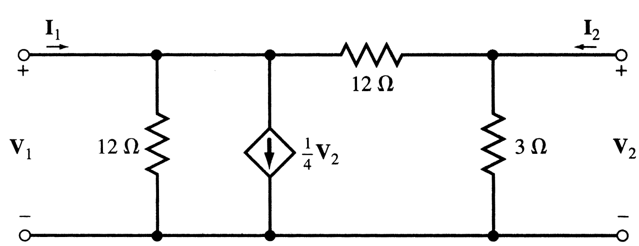


Fig. 4-21

$$\mathbf{I}_1 = \mathbf{Y}_{11}\mathbf{V}_1\Big|_{\mathbf{V}_2=0} = \left(\frac{1}{12} + \frac{1}{12}\right)\mathbf{V}_1 \qquad \text{or} \qquad \mathbf{Y}_{11} = \frac{1}{6}$$

$$\mathbf{I}_1 = \mathbf{Y}_{12}\mathbf{V}_2\Big|_{\mathbf{V}_1=0} = \frac{\mathbf{V}_2}{4} - \frac{\mathbf{V}_2}{12} = \left(\frac{1}{4} - \frac{1}{12}\right)\mathbf{V}_2 \qquad \text{or} \qquad \mathbf{Y}_{12} = \frac{1}{6}$$

$$\mathbf{I}_2 = \mathbf{Y}_{21}\mathbf{V}_1\Big|_{\mathbf{V}_2=0} = -\frac{\mathbf{V}_1}{12} \qquad \text{or} \qquad \mathbf{Y}_{21} = -\frac{1}{12}$$

$$\mathbf{I}_2 = \mathbf{Y}_{22}\mathbf{V}_2\Big|_{\mathbf{V}_1=0} = \frac{\mathbf{V}_2}{3} + \frac{\mathbf{V}_2}{12} = \left(\frac{1}{3} + \frac{1}{12}\right)\mathbf{V}_2 \qquad \text{or} \qquad \mathbf{Y}_{22} = \frac{5}{12}$$

4.8. Apply KCL at the nodes of the network in Fig. 4.21 to obtain its terminal characteristics and Y-parameters. Show that the two-port networks of Figs. 4.18 to 4.21 are all equivalent.

Input node: $$\mathbf{I}_1 = \frac{\mathbf{V}_1}{12} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{12} + \frac{\mathbf{V}_2}{4}$$

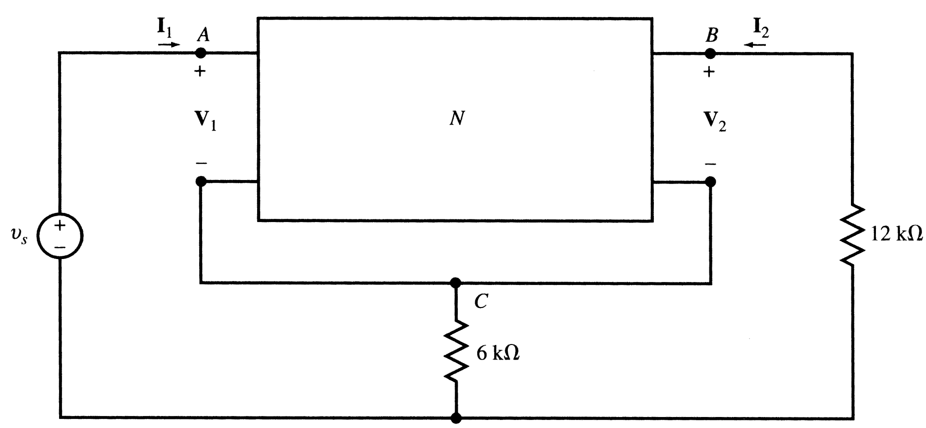
Output node: $$\mathbf{I}_2 = \frac{\mathbf{V}_2}{3} + \frac{\mathbf{V}_2 - \mathbf{V}_1}{12}$$

$$\mathbf{I}_1 = \frac{1}{6}\mathbf{V}_1 + \frac{1}{6}\mathbf{V}_2 \qquad \mathbf{I}_2 = -\frac{1}{12}\mathbf{V}_1 + \frac{5}{12}\mathbf{V}_2$$

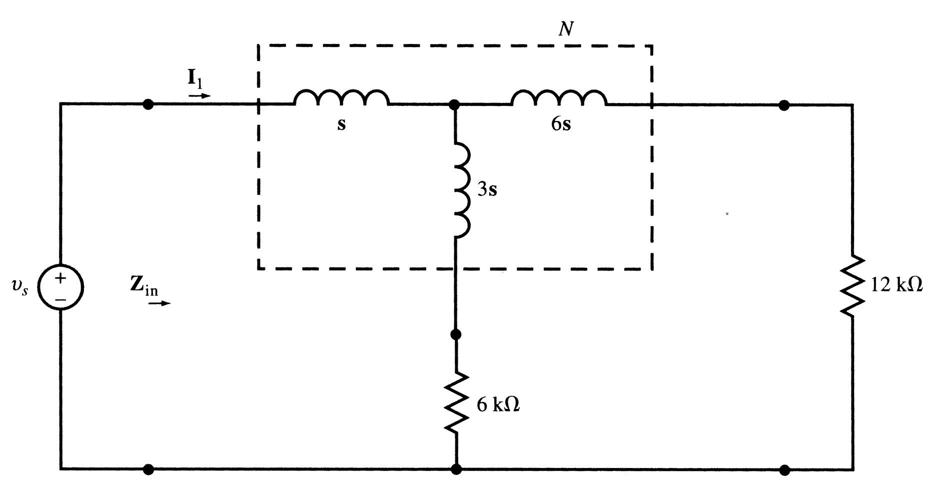
The **Y**-parameters observed from the above characteristic equations are identical with the **Y**-parameters of the circuits in Figs. 4.18, 4.19, and 4.20. Therefore, the four circuits are equivalent.

4.9. Z-parameters of the two-port network N in Fig. 4.22(a) are Z = 4s, Z =Z = 3s, and Z = 9s. (*a*) Replace *N* by its T-equivalent. (*b*) Use part (*a*) to find input current i_1 for $\upsilon_s = \cos 1000t$ (V).

(*a*) The network is reciprocal. Therefore, its T-equivalent exists. Its elements are found from (6) and shown in the circuit of Fig. 4.22(b).



(*a*)



(*b*)

Fig. 4.22

$$\mathbf{Z}_a = \mathbf{Z}_{11} - \mathbf{Z}_{12} = 4\mathbf{s} - 3\mathbf{s} = \mathbf{s}$$

$$\mathbf{Z}_b = \mathbf{Z}_{22} - \mathbf{Z}_{21} = 9\mathbf{s} - 3\mathbf{s} = 6\mathbf{s}$$

$$\mathbf{Z}_c = \mathbf{Z}_{12} = \mathbf{Z}_{21} = 3\mathbf{s}$$

(b) We repeatedly combine the series and parallel elements of Fig. 4.22(b), with resistors being in kΩ and **s** in krad/s, to find $\mathbf{Z}_{\text{in}}$ in kΩ:

$$\mathbf{Z}_{\text{in}}(\mathbf{s}) = \mathbf{V}_s/\mathbf{I}_1 = \mathbf{s} + \frac{(3\mathbf{s}+6)(6\mathbf{s}+12)}{9\mathbf{s}+18} = 3\mathbf{s} + 4 \qquad \text{or} \qquad \mathbf{Z}_{\text{in}}(j) = 3j + 4 = 5\underline{/36.9^\circ}\ \text{k}\Omega$$

and $i_1 = 0.2\cos(1000t - 36.9^\circ)$ (mA).

4.10. Two two-port networks a and b, with open-circuit impedances Z and Z , respectively, are connected in series (see Fig. 4.12). Derive the Z-parameter equations (31a).

From network **a** we have

$$\mathbf{V}_{1a} = \mathbf{Z}_{11,a}\mathbf{I}_{1a} + \mathbf{Z}_{12,a}\mathbf{I}_{2a}$$

$$\mathbf{V}_{2a} = \mathbf{Z}_{21,a}\mathbf{I}_{1a} + \mathbf{Z}_{22,a}\mathbf{I}_{2a}$$

From network **b** we have

$$\mathbf{V}_{1b} = \mathbf{Z}_{11,b}\mathbf{I}_{1b} + \mathbf{Z}_{12,b}\mathbf{I}_{2b}$$

$$\mathbf{V}_{2b} = \mathbf{Z}_{21,b}\mathbf{I}_{1b} + \mathbf{Z}_{22,b}\mathbf{I}_{2b}$$

From the connection between **a** and **b** we have

$$\mathbf{I}_1 = \mathbf{I}_{1a} = \mathbf{I}_{1b} \qquad \mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1b}$$

$$\mathbf{I}_2 = \mathbf{I}_{2a} = \mathbf{I}_{2b} \qquad \mathbf{V}_2 = \mathbf{V}_{2a} + \mathbf{V}_{2b}$$

Therefore,

$$\mathbf{V}_1 = (\mathbf{Z}_{11,a} + \mathbf{Z}_{11,b})\mathbf{I}_1 + (\mathbf{Z}_{12,a} + \mathbf{Z}_{12,b})\mathbf{I}_2$$

$$\mathbf{V}_2 = (\mathbf{Z}_{21,a} + \mathbf{Z}_{21,b})\mathbf{I}_1 + (\mathbf{Z}_{22,a} + \mathbf{Z}_{22,b})\mathbf{I}_2$$

from which the **Z**-parameters (31*a*) are derived.

13.11. Two two-port networks **a** and **b**, with short-circuit admittances $\mathbf{Y}_a$ and $\mathbf{Y}_b$, respectively, are connected in parallel (see Fig. 4.13). Derive the Y-parameter equations (32a).

From network **a** we have

$$\mathbf{I}_{1a} = \mathbf{Y}_{11,a}\mathbf{V}_{1a} + \mathbf{Y}_{12,a}\mathbf{V}_{2a}$$

$$\mathbf{I}_{2a} = \mathbf{Y}_{21,a}\mathbf{V}_{1a} + \mathbf{Y}_{22,a}\mathbf{V}_{2a}$$

and from network **b** we have

$$\mathbf{I}_{1b} = \mathbf{Y}_{11,b}\mathbf{V}_{1b} + \mathbf{Y}_{12,b}\mathbf{V}_{2b}$$

$$\mathbf{I}_{2b} = \mathbf{Y}_{21,b}\mathbf{V}_{1b} + \mathbf{Y}_{22,b}\mathbf{V}_{2b}$$

From the connection between **a** and **b** we have

$$\mathbf{V}_1 = \mathbf{V}_{1a} = \mathbf{V}_{1b} \qquad \mathbf{I}_1 = \mathbf{I}_{1a} + \mathbf{I}_{1b}$$

$$\mathbf{V}_2 = \mathbf{V}_{2a} = \mathbf{V}_{2b} \qquad \mathbf{I}_2 = \mathbf{I}_{2a} + \mathbf{I}_{2b}$$

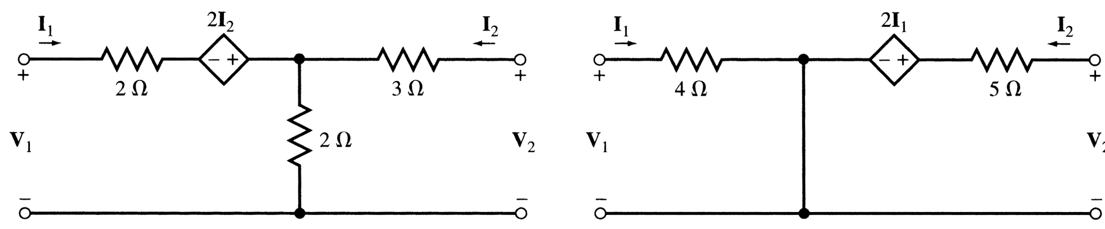
Therefore,

$$\mathbf{I}_1 = (\mathbf{Y}_{11,a} + \mathbf{Y}_{11,b})\mathbf{V}_1 + (\mathbf{Y}_{12,a} + \mathbf{Y}_{12,b})\mathbf{V}_2$$

$$\mathbf{I}_2 = (\mathbf{Y}_{21,a} + \mathbf{Y}_{21,b})\mathbf{V}_1 + (\mathbf{Y}_{22,a} + \mathbf{Y}_{22,b})\mathbf{V}_2$$

from which the **Y**-parameters of (32*a*) result.

13.12. Find (a) the Z-parameters of the circuit of Fig. 4.23(a) and (b) an equivalent model which uses three positive-valued resistors and one dependent voltage source.



(*a*) (*b*)

Fig. 4.23

(*a*) From application of KVL around the input and output loops we find, respectively,

$$\mathbf{V}_1 = 2\mathbf{I}_1 - 2\mathbf{I}_2 + 2(\mathbf{I}_1 + \mathbf{I}_2) = 4\mathbf{I}_1$$

$$\mathbf{V}_2 = 3\mathbf{I}_2 + 2(\mathbf{I}_1 + \mathbf{I}_2) = 2\mathbf{I}_1 + 5\mathbf{I}_2$$

The **Z**-parameters are $\mathbf{Z}_{11} = 4$, $\mathbf{Z}_{12} = 0$, $\mathbf{Z}_{21} = 2$, and $\mathbf{Z}_{22} = 5$.

(b) The circuit of Fig. 4.23(b), with two resistors and a voltage source, has the same Z-parameters as the circuit of Fig. 4.23(a). This can be verified by applying KVL around its input and output loops.

4.13. (a) Obtain the Y-parameters of the circuit in Fig. 13-23(a) from its Z-parameters. (b) Find an equivalent model which uses two positive-valued resistors and one dependent current source.

(*a*) From Problem 13.12, $\mathbf{Z}_{11} = 4$, $\mathbf{Z}_{12} = 0$, $\mathbf{Z}_{21} = 2$, $\mathbf{Z}_{22} = 5$, and so $\mathbf{D}_{\mathbf{ZZ}} = \mathbf{Z}_{11}\mathbf{Z}_{22} - \mathbf{Z}_{12}\mathbf{Z}_{21} = 20$. Hence,

$$\mathbf{Y}_{11} = \frac{\mathbf{Z}_{22}}{\mathbf{D}_{\mathbf{ZZ}}} = \frac{5}{20} = \frac{1}{4} \qquad \mathbf{Y}_{12} = \frac{-\mathbf{Z}_{12}}{\mathbf{D}_{\mathbf{ZZ}}} = 0 \qquad \mathbf{Y}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{D}_{\mathbf{ZZ}}} = \frac{-2}{20} = -\frac{1}{10} \qquad \mathbf{Y}_{22} = \frac{\mathbf{Z}_{11}}{\mathbf{D}_{\mathbf{ZZ}}} = \frac{4}{20} = \frac{1}{5}$$

(*b*) Figure 13-24, with two resistors and a current source, has the same **Y**-parameters as the circuit in Fig.13-23(*a*). This can be verified by applying KCL to the input and output nodes.

4.14. Referring to the network of Fig. 4.23(b), convert the voltage source and its series resistor to its Norton equivalent and show that the resulting network is identical to that in Fig. 4.24.

The Norton equivalent current source is $\mathbf{I}_N = 2I_1/5 = 0.4I_1$. But $I_1 = V_1/4$. Therefore, $\mathbf{I}_N = 0.4I_1 = 0.1V_1$. The 5-Ω resistor is then placed in parallel with $\mathbf{I}_N$. The circuit is shown in Fig. 13-25 which is the same as the circuit in Fig. 13-24.

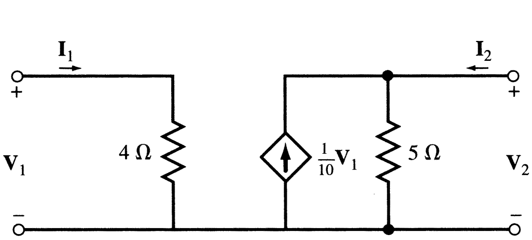


Fig. 4.24

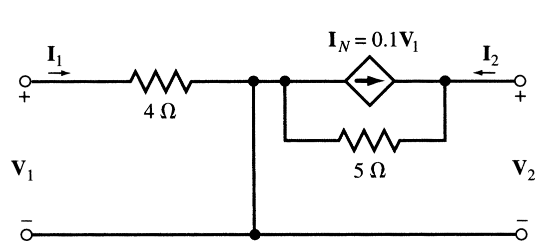


Fig. 4.25

4.15. The h-parameters of a two-port network are given. Show that the network may be modeled by the network in Fig. 13-26 where $\mathbf{h}_{11}$ is an impedance, $\mathbf{h}_{12}$ is a voltage gain, $\mathbf{h}_{21}$ is a current gain, and $\mathbf{h}_{22}$ is an admittance.

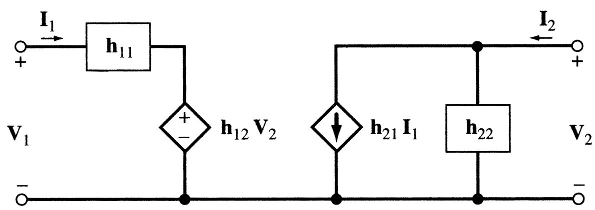


Fig. 4.26

Apply KVL around the input loop to get

$$\mathbf{V}_1 = \mathbf{h}_{11}\mathbf{I}_1 + \mathbf{h}_{12}\mathbf{V}_2$$

Apply KCL at the output node to get

$$\mathbf{I}_2 = \mathbf{h}_{21}\mathbf{I}_1 + \mathbf{h}_{22}\mathbf{V}_2$$

These results agree with the definition of **h**-parameters given in (23).

4.16. Find the h-parameters of the circuit in Fig. 4.25.

By comparing the circuit in Fig. 4.25 with that in Fig. 4.26, we find

$$\mathbf{h}_{11} = 4\,\Omega, \qquad \mathbf{h}_{12} = 0, \qquad \mathbf{h}_{21} = -0.4, \qquad \mathbf{h}_{22} = 1/5 = 0.2\ \Omega^{-1}$$

4.17. Find the h-parameters of the circuit in Fig. 4.25 from its Z-parameters and compare with the results of Problem 4.16.

Refer to Problem 4.13 for the values of the Z-parameters and D . Use Table 4.1 for the conversion of the **Z**-parameters to the **h**-parameters of the circuit. Thus,

$$\mathbf{h}_{11} = \frac{\mathbf{D}_{ZZ}}{\mathbf{Z}_{22}} = \frac{20}{5} = 4 \qquad \mathbf{h}_{12} = \frac{\mathbf{Z}_{12}}{\mathbf{Z}_{22}} = 0 \qquad \mathbf{h}_{21} = \frac{-\mathbf{Z}_{21}}{\mathbf{Z}_{22}} = \frac{-2}{5} = -0.4 \qquad \mathbf{h}_{22} = \frac{1}{\mathbf{Z}_{22}} = \frac{1}{5} = 0.2$$

The above results agree with the results of Problem 4.16.

4.18. The simplified model of a bipolar junction transistor for small signals is shown in Fig. 4.27. Find its **h**-parameters.

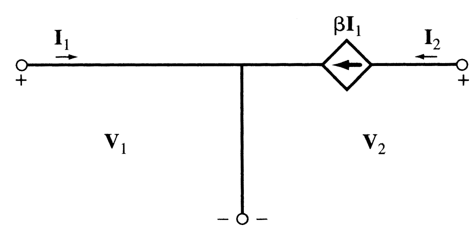


Fig. 4.27

The terminal equations are $\mathbf{V}_1 = 0$ and $\mathbf{I}_2 = \beta\mathbf{I}_1$. By comparing these equations with (23), we conclude that $\mathbf{h}_{11} = \mathbf{h}_{12} = \mathbf{h}_{22} = 0$ and $\mathbf{h}_{21} = \beta$.

4.19. The h-parameters of a two-port device H are given by

$$\mathbf{h}_{11} = 500\ \Omega \qquad \mathbf{h}_{12} = 10^{-4} \qquad \mathbf{h}_{21} = 100 \qquad \mathbf{h}_{22} = 2(10^{-6})\ \Omega^{-1}$$

Draw a circuit model of the device made of two resistors and two dependent sources. Include the values of each element.

From a comparison with Fig. 4.26, we draw the model of Fig. 4.28.

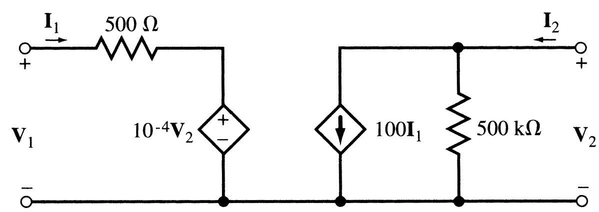
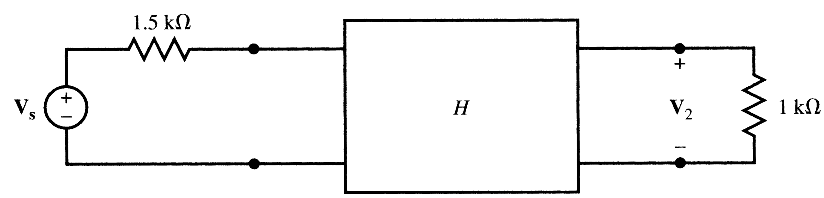
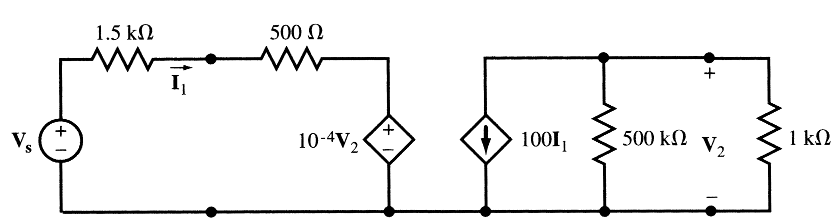


Fig. 4.28

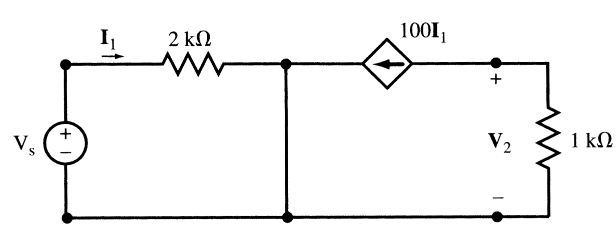
4.20. The device H of Problem 4.19 is placed in the circuit of Fig. 4.29(a). Replace H by its model of Fig. 4.28 and find V /V.



(*a*)



(*b*)



(*c*)

Fig. 4.29

The circuit of Fig. 4.29(b) contains the model. With good approximation, we can reduce it to Fig. 4.29(c) from which

$$\mathbf{I}_1 = \mathbf{V}_s/2000 \qquad \mathbf{V}_2 = -1000(100\mathbf{I}_1) = -1000(100\mathbf{V}_s/2000) = -50\ \mathbf{V}_s$$

Thus, $\mathbf{V}_2/\mathbf{V}_s = -50$.

4.21. A load Z is connected to the output of a two-port device N (Fig. 13-30) whose terminal characteristics are given by $\mathbf{V}_1 = (1/N)\mathbf{V}_2$ and $\mathbf{I}_1 = -N\mathbf{I}_2$. Find (*a*) the T-parameters of *N* and (*b*) the input impedance $\mathbf{Z}_{in} = \mathbf{V}_1/\mathbf{I}_1$.

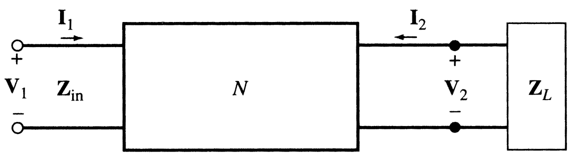


Fig. 4.30

(*a*) The T-parameters are defined by [see (29)]

$$\mathbf{V}_1 = \mathbf{A}\mathbf{V}_2 - \mathbf{B}\mathbf{I}_2$$

$$\mathbf{I}_1 = \mathbf{C}\mathbf{V}_2 - \mathbf{D}\mathbf{I}_2$$

The terminal characteristics of the device are

$$\mathbf{V}_1 = (1/N)\mathbf{V}_2$$

$$\mathbf{I}_1 = -N\mathbf{I}_2$$

By comparing the two pairs of equations we get $\mathbf{A} = 1/N$, $\mathbf{B} = 0$, $\mathbf{C} = 0$, and $\mathbf{D} = N$.

(*b*) Three equations relating $\mathbf{V}_1$, $\mathbf{I}_1$, $\mathbf{V}_2$, and $\mathbf{I}_2$ are available: two equations are given by the terminal characteristics of the device and the third equation comes from the connection to the load,

$$\mathbf{V}_2 = -\mathbf{Z}_L\mathbf{I}_2$$

By eliminating $\mathbf{V}_2$ and $\mathbf{I}_2$ in these three equations, we get

$$\mathbf{V}_1 = \mathbf{Z}_L\mathbf{I}_1/N^2 \qquad \text{from which} \qquad \mathbf{Z}_{in} = \mathbf{V}_1/\mathbf{I}_1 = \mathbf{Z}_L/N^2$$

SUPPLEMENTARY PROBLEMS

4.22. The Z-parameters of the two-port network N in Fig. 4.22(a) are Z = 4s, Z =Z = 3s, and Z = 9s. Find the input current i_1 for $\upsilon_s = \cos 1000t$ (V) by using the open circuit impedance terminal characteristic equations of *N*, together with KCL equations at nodes *A*, *B*, and *C*.

Ans. $i_1 = 0.2 \cos (1000t - 36.9°)$ (A)

4.23. Express the reciprocity criteria in terms of h-, g-, and T-parameters.

Ans. $\mathbf{h}_{12} + \mathbf{h}_{21} = 0$, $\mathbf{g}_{12} + \mathbf{g}_{21} = 0$, and $\mathbf{AD} - \mathbf{BC} = 1$

4.24. Find the T-parameters of a two-port device whose Z-parameters are Z =s, Z =Z = 10s, and Z = 100s.

Ans. $\mathbf{A} = 0.1$, $\mathbf{B} = 0$, $\mathbf{C} = 10^{-1}/\mathbf{s}$, and $\mathbf{D} = 10$

4.25. Find the T-parameters of a two-port device whose Z-parameters are Z = 10 s, Z =Z = 10 s, and Z = 10 s. Compare with the results of Problem 4.21.

Ans. $\mathbf{A} = 0.1$, $\mathbf{B} = 0$, $\mathbf{C} = 10^{-7}/\mathbf{s}$ and $\mathbf{D} = 10$. For high frequencies, the device is similar to the device of Problem 13.21, with $N = 10$.

4.26. The Z-parameters of a two-port device N are Z = $k\mathbf{s}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = 10k\mathbf{s}$, and $\mathbf{Z}_{22} = 100k\mathbf{s}$. A 1-Ω resistor is connected across the output port (see Fig. 4.30). (a) Find the input impedance Z =V /I and construct its equivalent circuit. (*b*) Give the values of the elements for $k = 1$ and 10 .

Ans. (*a*) $\mathbf{Z}_{\text{in}} = \dfrac{k\mathbf{s}}{1 + 100k\mathbf{s}} = \dfrac{1}{100 + 1/k\mathbf{s}}$

The equivalent circuit is a parallel RL circuit with $R = 10^{-2}\,\Omega$ and $L = 1$ kH.

(*b*) For $k = 1$, $R = \dfrac{1}{100}\,\Omega$ and $L = 1$ H. For $k = 10^6$, $R = \dfrac{1}{100}\,\Omega$ and $L = 10^6$ H.

4.27. The device N in Fig. 4.30 is specified by the following Z-parameters: Z =N Z and $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \sqrt{\mathbf{Z}_{11}\mathbf{Z}_{22}} = N\mathbf{Z}_{11}$. Find $\mathbf{Z}_{\text{in}} = \mathbf{V}_1/\mathbf{I}_1$ when a load $\mathbf{Z}_L$ is connected to the output terminal. Show that if $\mathbf{Z}_{11} \gg \mathbf{Z}_L/N^2$ we have impedance scaling such that $\mathbf{Z}_{\text{in}} = \mathbf{Z}_L/N^2$.

Ans. $\mathbf{Z}_{\text{in}} = \dfrac{\mathbf{Z}_L}{N^2 + \mathbf{Z}_L/\mathbf{Z}_{11}}$. For $\mathbf{Z}_L \ll N^2\mathbf{Z}_{11}$, $\mathbf{Z}_{\text{in}} = \mathbf{Z}_L/N^2$.

4.28. Find the Z-parameters in the circuit of Fig. 4.31. Hint: Use the series connection rule.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \mathbf{s} + 3 + 1/\mathbf{s}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \mathbf{s} + 1$

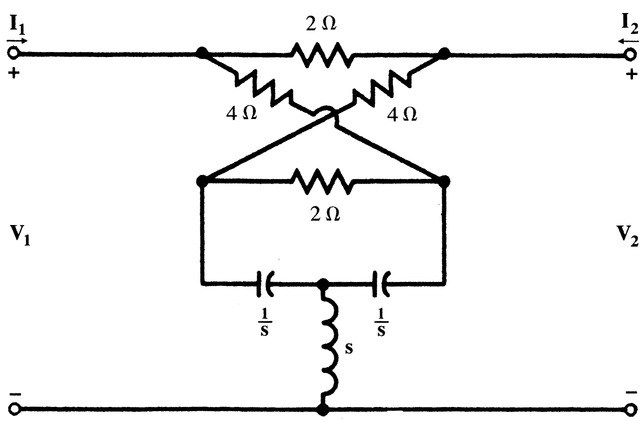


Fig. 4.31

4.29. Find the Y-parameters in the circuit of Fig. 4.32. Hint: Use the parallel connection rule.

Ans. $\mathbf{Y}_{11} = \mathbf{Y}_{22} = 9(\mathbf{s} + 2)/16$, $\mathbf{Y}_{12} = \mathbf{Y}_{21} = -3(\mathbf{s} + 2)/16$

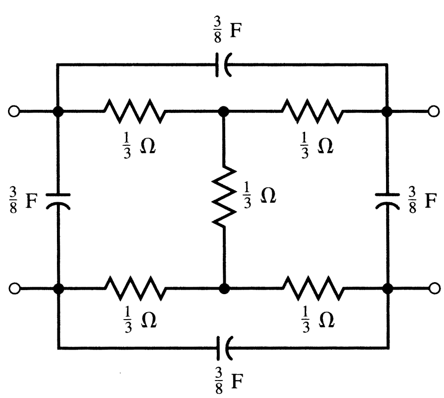


Fig. 4.32

4.30. Two two-port networks a and b with transmission parameters T and T are connected in cascade (see Fig. 13-14). Given $I_{2a} = -I_{1b}$ and $V_{2a} = V_{1b}$, find the T-parameters of the resulting two-port network.

Ans. $\mathbf{A} = \mathbf{A}_a\mathbf{A}_b + \mathbf{B}_a\mathbf{C}_b$, $\mathbf{B} = \mathbf{A}_a\mathbf{B}_b + \mathbf{B}_a\mathbf{D}_b$, $\mathbf{C} = \mathbf{C}_a\mathbf{A}_b + \mathbf{D}_a\mathbf{C}_b$, $\mathbf{D} = \mathbf{C}_a\mathbf{B}_b + \mathbf{D}_a\mathbf{D}_b$

4.31. Find the T- and Z-parameters of the network in Fig. 4.33. The impedances of the capacitors are given. Hint: Use the cascade connection rule.

Ans. $\mathbf{A} = 5j - 4$, $\mathbf{B} = 4j + 2$, $\mathbf{C} = 2j - 4$, and $\mathbf{D} = j3$, $\mathbf{Z}_{11} = 1.3 - 0.6j$, $\mathbf{Z}_{22} = 0.3 - 0.6j$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = -0.2 - 0.1j$

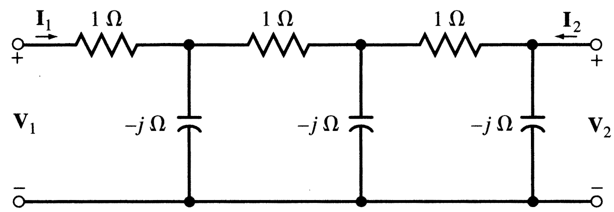


Fig. 4.33

4.32. Find the Z-parameters of the two-port circuit of Fig. 4.34.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \frac{1}{2}(\mathbf{Z}_b + \mathbf{Z}_a)$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \frac{1}{2}(\mathbf{Z}_b - \mathbf{Z}_a)$

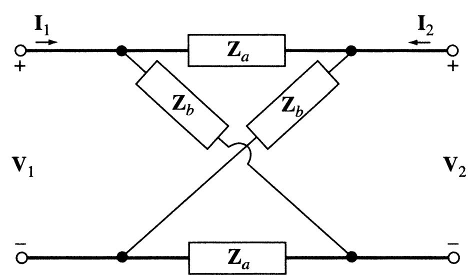


Fig. 4.34

4.33. Find the Z-parameters of the two-port circuit of Fig. 4.35.

Ans. $\mathbf{Z}_{11} = \mathbf{Z}_{22} = \dfrac{1}{2}\dfrac{\mathbf{Z}_b(2\mathbf{Z}_a + \mathbf{Z}_b)}{\mathbf{Z}_a + \mathbf{Z}_b}$, $\mathbf{Z}_{12} = \mathbf{Z}_{21} = \dfrac{1}{2}\dfrac{\mathbf{Z}_b^2}{\mathbf{Z}_a + \mathbf{Z}_b}$

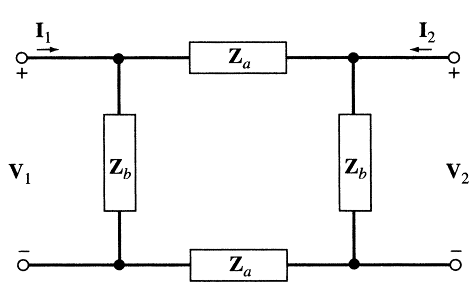


Fig. 4.35

4.34. Referring to the two-port circuit of Fig. 4.36, find the T-parameters as a function of w and specify their values at $\omega = 1$, 10^3, and 10^6 rad/s.

Ans. $\mathbf{A} = 1 - 10^{-9}\,\omega^2 + j10^{-9}\,\omega$, $\mathbf{B} = 10^{-3}\,(1 + j\omega)$, $\mathbf{C} = 10^{-6}\,j\omega$, and $\mathbf{D} = 1$. At $\omega = 1$ rad/s, $\mathbf{A} = 1$, $\mathbf{B} = 10^{-3}\,(1 + j)$, $\mathbf{C} = 10^{-6}\,j$, and $\mathbf{D} = 1$. At $\omega = 10^3$ rad/s, $\mathbf{A} \approx 1$, $\mathbf{B} \approx j$, $\mathbf{C} = 10^{-3}j$, and $D = 1$. At $\omega = 10^6$ rad/s, $\mathbf{A} \approx -10^3$, $\mathbf{B} \approx 10^3\,j$, $\mathbf{C} = j$, and $\mathbf{D} = 1$.

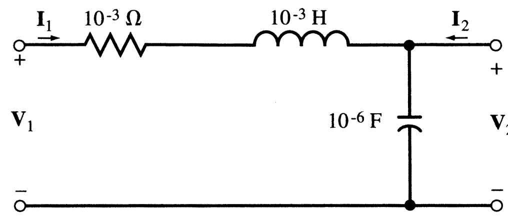


Fig. 4.36

4.35. A two-port network contains resistors, capacitors, and inductors only. With port #2 open [Fig. 4.37(a)], a unit step voltage $v_1 = u(t)$ produces $i_1 = e^{-t}u(t)$ (μA) and $v_2 = (1 - e^{-t})\,u(t)$ (V). With port #2 short-circuited [Fig. 13-37(*b*)], a unit step voltage $v_1 = u(t)$ delivers a current $i_1 = 0.5(1 + e^{-2t})u(t)$ (μA). Find i_2 and the T-equivalent network. *Ans.* $i_2 = 0.5(-1 + e)u(t)$ *[see Fig. 4.37(c)]*

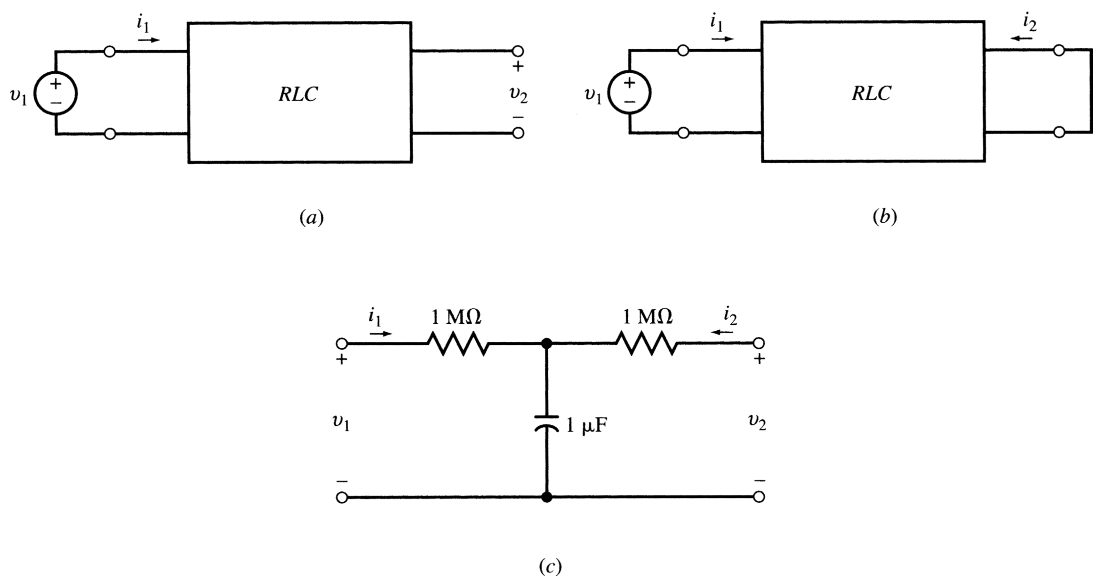


Fig. 4.37

4.36. The two-port network N in Fig. 4.38 is specified by $Z_{11} = 2$, $Z_{12} = Z_{21} = 1$, and $Z_{22} = 4$. Find I_1, I_2, and I_3.

Ans. $I_1 = 24$ A, $I_2 = 1.5$ A, and $I_3 = 6.5$ A

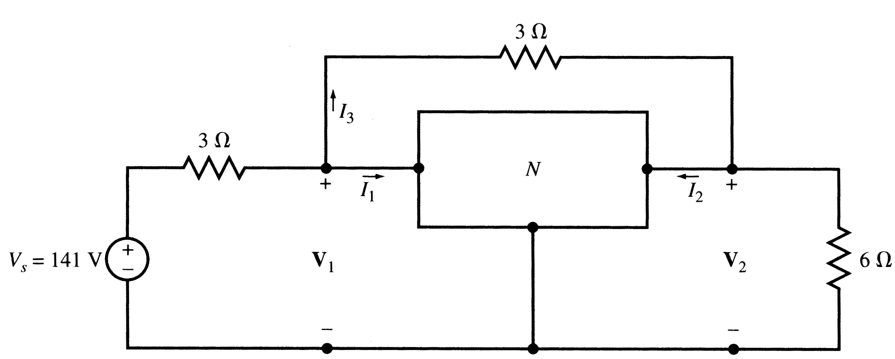


Fig. 4.38

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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – III – TWO PORT NETWORK

UNIT –III

TWO PORT NETWORKS

1. Asymmetrical Networks

If a network is asymmetrical the input and output terminals cannot be interchanged without affecting the electrical properties of the network. In this situation the characteristic impedance has a different value when looking at either input or output. In this situation we have to introduce the concept of the image impedance.

2.Image Impedance

It is the impedance which when connected to the input and the output of the transducer makes both the impedances equal at the input and the output terminal. It is basically the concept which is used in the field of the network analysis and design and also in filter design methods. It applies to the seen impedance which is determined by looking through the ports of the network.

The Two-port network shown in Fig.3.1 describes the concept of the image impedance in the better way.

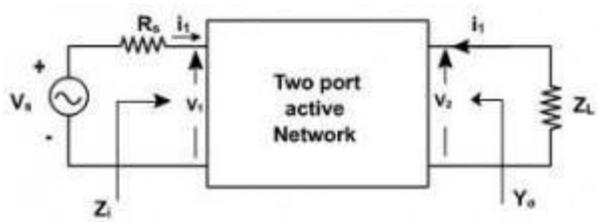


Fig.3.1 Two port network

The impedance z_{i1} – when considered from the port 1

Z_{i2} –image impedance when considered from the port 2

The image impedance will not be equal until the network is the symmetrical network or anti-symmetrical with respect to the ports.

Characteristic impedance

The characteristics impedance also known as the surge impedance is usually considered in the case of the transmission line and is represented as Z_0. The characteristics impedance is defined as the ratio of the amplitude of the voltage and the current taking the consideration of the single wave through the line. The surge impedance is usually allocated through the transmission line with its geometry and the material. It is to be noted that this impedance is independent of the line length.SI unit – ohm

Iterative impedance

It is defined as the particular value of the load impedance which has the ability to produce an input impedance with the value as same as the value of the load impedance. In the two ports system when it is connected at the one end then it produces equal impedance when looking at each other.

Image transfer coefficient

It is usually considered for the linear passive type of the two-port network, such network must be terminated with the image impedance of the network. Let

V_1 – voltage at the input terminal
I_1 – current at the input terminal
I_2 – current at the output terminal
V_2 – voltage at the input terminal
Hence, the image transfer coefficient can be calculated as half the logarithm of the product of V_1 andI_1 divided by the product of the V_2 and I_2.

Propagation constant

This constant is usually considered for the wave and is defined as change in the phase angle with respect to the per unit change in the distance travelled by the wave. In other words we can say the rate of the change in the phase of wave with distance. This constant is represented by the term K.

Image Impedance for asymmetrical 'T' Network

Let Z_{i1} & Z_{i2} be the image impedance of asymmetrical T network.

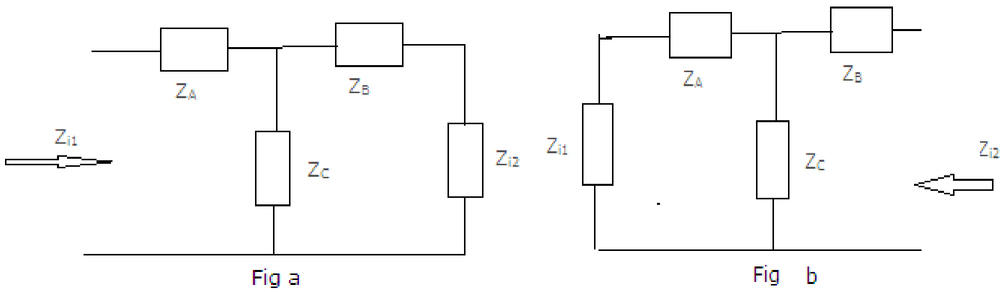


Fig 2.2. Image Impedance of Asymmetrical T network

From Fig 2.2. a

$Z_{i1} = (Z_A+Z_C)//(Z_B+Z_{i2})$

By Simplifying

$Z_{i1}(Z_B+Z_C)+Z_{i1}Z_{i2}-\sum Z_AZ_B-Z_{i2}(Z_A+Z_C)=0$----------------1

From Fig 2.2.b

$Z_{i2}= (Z_A+Z_{i1})//Z_C+Z_B$

By Simplifying

$Z_{i2}(Z_A+Z_C)+Z_{i1}Z_{i2}-\sum Z_AZ_B-Z_{i1}(Z_B+Z_C)=0$----------------2

Add 1and 2

$Z_{i1}Z_{i2}=\sum Z_AZ_B$----------------3

Sub 1and 2

$Z_{i1}/Z_{i2}=Z_A + Z_C / Z_B + Z_C$----------------4

MULTIPLY 3 and 4

$$Z_{i1} = \sqrt{\sum Z_A Z_B \left(\frac{Z_A + Z_C}{Z_B + Z_C} \right)}$$

Divide 3and 4

$$Z_{i2}= \sqrt{\sum Z_A Z_B \left(\frac{Z_B + Z_C}{Z_A + Z_C} \right)}$$

Image Impedance for asymmetrical 'π' Network

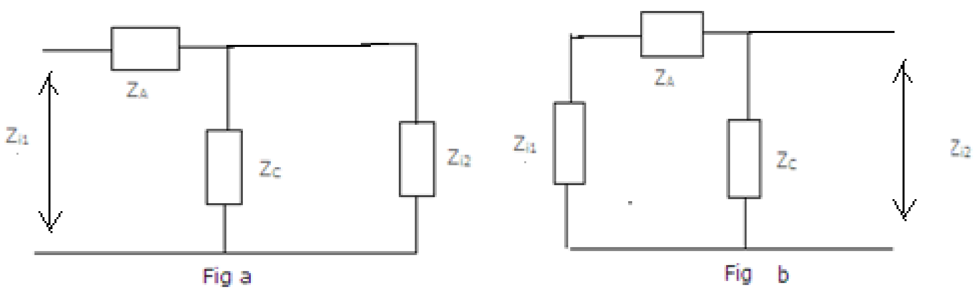


Fig.2.3 Image Impedance for asymmetrical 'π' Network

From Fig 2.3.a

$Z_{i1} = Z_C // Z_{i2} + Z_A$

By Simplifying

$Z_{i1}Z_C + Z_{i1}Z_{i2} - Z_{i2}(Z_A + Z_C) - Z_A Z_C = 0$----------------1

From Fig 2.3.b

$Z_{i2} = (Z_A + Z_{i1}) // Z_C$.

By Simplifying

$Z_{i2}(Z_A + Z_C) + Z_{i1}Z_{i2} - Z_{i1}Z_C - Z_A Z_C = 0$----------------2

Add 1and 2

$Z_{i1}Z_{i2} = Z_A Z_C$----------------3

Sub 1and 2

$Z_{i1}/Z_{i2} = Z_A + Z_C / Z_C$----------------4

MULTIPLY 3 and 4

$Z_{i1} = \sqrt{Z_A(Z_A + Z_C)}$

Divide 3and 4

$Z_{i2} = Z_C \sqrt{\frac{Z_A}{Z_A + Z_C}}$

Iterative Impedance for asymmetrical 'T' Network

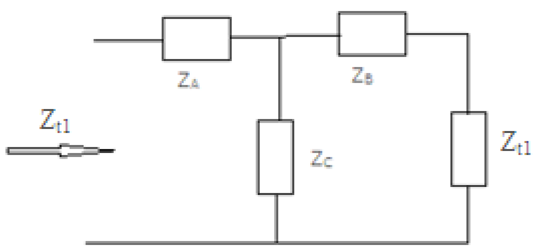


Fig.2.4 Iterative Impedance Z_{t1} for asymmetrical 'T' Network

$Z_{t1} = Z_A + (Z_B + Z_{t1}) // Z_C$

$$Z_{t1} = \frac{(Z_A - Z_B) + \sqrt{(Z_B - Z_A)^2 + 4(\sum Z_A Z_B)}}{2}$$

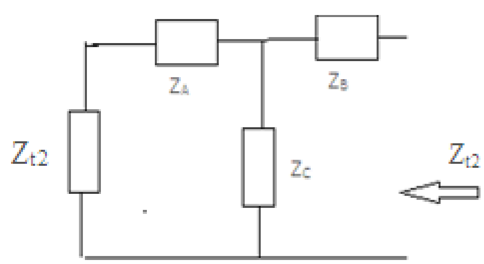


Fig.2.5 Iterative Impedance Z_{t2} for asymmetrical 'T' Network

$Z_{t2} = Z_A + (Z_B + Z_{t2}) // Z_C$

$$Z_{t2} = \frac{-(Z_A - Z_B) + \sqrt{(Z_A - Z_B)^2 + 4(\sum Z_A Z_B)}}{2}$$

Iterative Impedance for asymmetrical ‘π’ Network

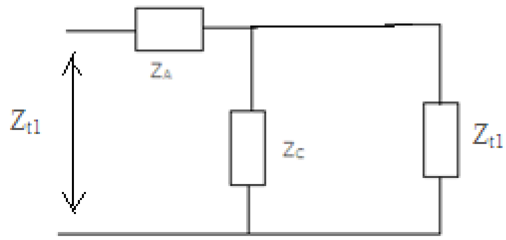


Fig.2.6 Iterative Impedance Z_{t1} for asymmetrical ‘π’ Network

$Z_{t1} = Z_A + Z_{t1} // Z_C$

$$Z_{t1} = \frac{Z_A + \sqrt{Z_A^{\;2} + 4Z_A Z_c}}{2}$$

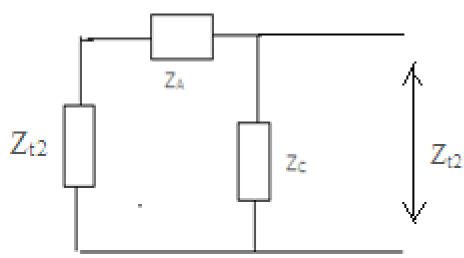


Fig.2.7 Iterative Impedance Z_{t2} for asymmetrical ‘π’ Network

$Z_{t2} = (Z_A + Z_{t2}) // Z_C$

$$Z_{t2} = \frac{-Z_A + \sqrt{Z_A^{\;2} + 4Z_A Z_c}}{2}$$

Iterative Impedance in terms of ABCD parameter

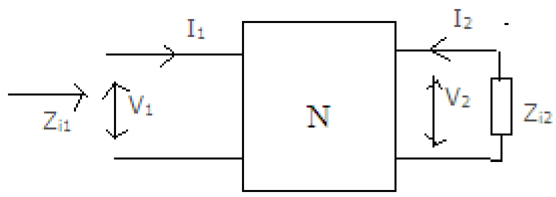


Fig.2.8 Iterative Impedance for Z_{i1} open

$V_2 = -I_2 Z_{i2}$

$Z_{i1} = V_1 / I_1$

CONSIDER LINEAR EQUATION FOR ABCD

$V_1 = AV_2 - BI_2$------------------------- 1

$I_1 = CV_2 - DI_2$------------------------- 2

Divide Equation 1 by 2

$$Z_{i1} = \left[\frac{Z_{i2}A + B}{CZ_{i2} + D}\right] \text{------------------ } 3$$

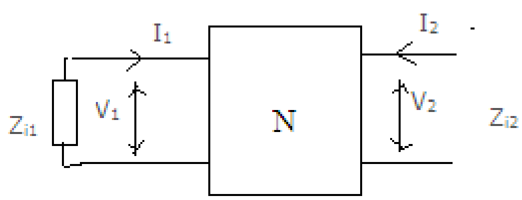


Fig.2.9 Iterative Impedance for Z_{i2} open

$Z_{i2} = V_1 / I_2$

$V_1 = -I_1 Z_{i1}$

$$-Z_{i1} = \left[\frac{Z_{i2}A - B}{CZ_{i2} - D} \right]$$ **------------------ 4**

Rearranging Equation 3 & 4

$CZ_{i1}Z_{i2}+DZ_{i2}=AZ_{i2}+B$------------------ 5

$-CZ_{i1}Z_{i2}+DZ_{i1}-AZ_{i2}+B=0$ ---------------- 6

Adding equation 5 & 6

$$\frac{Z_{i1}}{Z_{i2}} = \frac{A}{D}$$ **--------------- 7**

Subtracting equation 5 from 6

$$\frac{Z_{i1}}{Z_{i2}} = \frac{B}{C}$$ **---------------- 8**

Multiply equation 7 and 8

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

Divide Equation 7 by8

$$Z_{i2} = \sqrt{\frac{DB}{CA}}$$

Lattice and bridged network

A network that is made up of four branches connected in series to form a mesh; two nonadjacent junction points serve as input terminals, and the remaining two junction points serve as output terminals. The lattice networks are being widely used in the areas like grid computing, sensor networks and in many more areas. The main points which highlights the lattice networks are

1. **Its optimal routing policies**

2. **limits on the capacities of its elements**
3. **Its performance with the finite amount of the buffers**

A bridge-T network has a fourth branch connected between an input and an output terminal and across two branches of the network.

Insertion Loss

The insertion loss can be explained as the loss in load power because of the insertion of a particular component or device in a transmission system. It is represented in the ratio of the decibels of the power received at the side of the load before the insertion of the component to the power received at the load side after the insertion of the component or the device.

LATTICE NETWORKS

One of the common four-terminal two-port networks is the lattice, or bridge network, shown in Figure 2.10 (a). Lattice networks are used in filter sections and are also used as attenuators filter and attenuators. Lattice structures are sometimes used in preference to ladder structures in some special applications. Z_a and Z_d are called the series arms; Z_b and Z_c are called the diagonal arms. The lattice network is redrawn as a bridge network as shown in Figure 2.10 (b). It can be observed that if Z_d is zero, the lattice structure as shown in Figure 2.10 (c).

Z-parameters,

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}.$$

When

$$I_2 = 0,\ V_1 = I_1 \frac{(Z_a+Z_b)(Z_d+Z_c)}{Z_a+Z_b+Z_c+Z_d)}$$

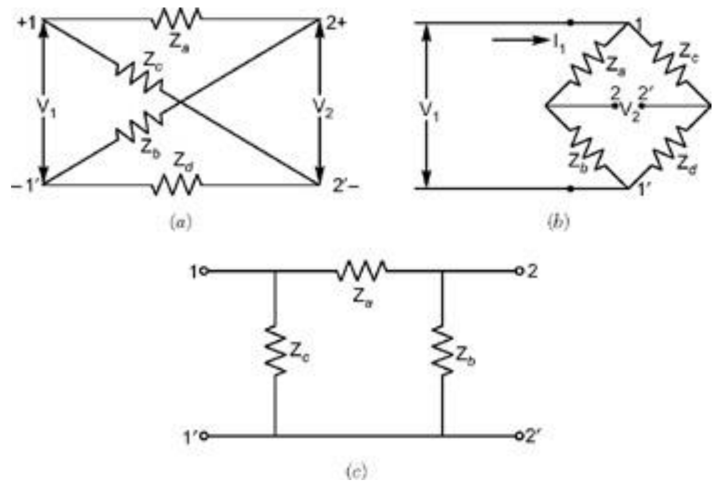


Fig.2.10 Lattice networks

Therefore

$$I_1 1 = \frac{(Z_a+Z_b)(Z_d+Z_c)}{Z_a+Z_b+Z_c+Z_d)}.$$

If the network is symmetric, then $Z_a = Z_d$, $Z_b = Z_c$

Therefore

$$\begin{aligned} Z_{11} &= \frac{Z_a+Z_b}{2} \\ Z_{21} &= \left.\frac{V2}{I_1}\right|_{I_2=0}. \end{aligned}$$

When I2 = 0, V2 is the voltage across 2-2

$$V_2 = V_1\left[\frac{Z_b}{Z_a+Z_b} - \frac{Z_d}{Z_c+Z_d}\right].$$

Substituting the value of V_1 from Equation, we have

$$V_2 = \left[\frac{I_1(Z_a+Z_b)(Z_d+Z_c)}{Z_a+Z_b+Z_d+Z_c}\right]\left[\frac{Z_b(Z_c+Z_d)-Z_d(Z_a+Z_b)}{(Z_a+Z_b)(Z_c+Z_d)}\right]$$

$$\frac{V_2}{I_1} = \left[\frac{Z_b(Z_c+Z_d)-Z_d(Z_a+Z_b)}{Z_a+Z_b+Z_d+Z_d}\right] = \frac{Z_bZ_c-Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}$$

Therefore

$$Z_{21} = \frac{Z_bZ_c-Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}$$

If the network is symmetric, $Z_a = Z_d$, $Z_b = Z_c$

$$Z_{21} = \frac{Z_b-Z_a}{2}$$

When the input port is open, $I_1 = 0$,

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

The network can be redrawn as shown in Figure 2.11

$$V_1 = V_2\left[\frac{Z_c}{Z_a+Z_c} - \frac{Z_d}{Z_b+Z_d}\right]$$

$$V_1 = V_2\left[\frac{(Z_a+Z_c)(Z_d+Z_b)}{Z_a+Z_b+Z_c+Z_d}\right]$$

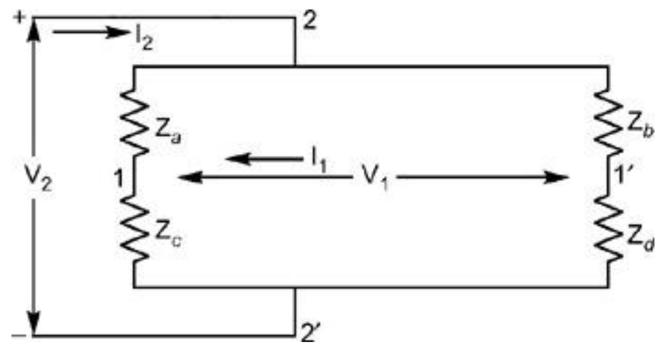


Fig.2.11 Modified Lattice networks

Substituting the value of V2 into V1, we get

$$V_1 = I_2\left[\frac{Z_c(Z_b+Z_d)-Z_d(Z_a+Z_c)}{Z_a+Z_b+Z_c+Z_d}\right]$$

$$\frac{V_1}{I_2} = \frac{Z_cZ_b-Z_aZ_d}{Z_a+Z_b+Z_c+Z_d}.$$

If the network is symmetric, $Z_a = Z_d$, $Z_b = Z_c$,

$$\frac{V_1}{I_2} = \frac{Z_b^2-Z_a^2}{2(Z_a+Z_b)}$$

Therefore

$$Z_{12} = \frac{Z_b-Z_a}{2}$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_2=0}.$$

We have

$$\frac{V_2}{I_2} = \frac{(Z_a+Z_c)(Z_d+Z_b)}{Z_a+Z_d+Z_c+Z_d}$$

If the network is symmetric, $Z_a = Z$

π and T networks

Transforming from π to T and vice versa

Any pi network can be transformed to an equivalent T network. This is also known as the Wye-Delta transformation, which is the terminology used in power distribution and electrical engineering. The π is equivalent to the Delta and the T is equivalent to the Wye (or Star) form.

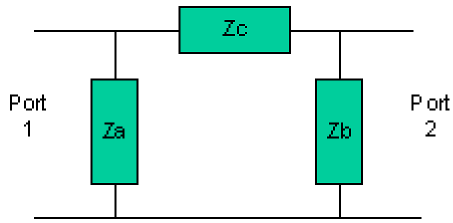


Fig.2.12 π Network

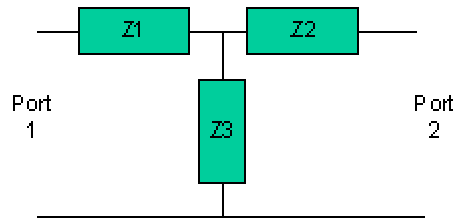


Fig.2.13 T Network

The impedances of the π network (Za, Zb, and Zc) can be found from the impedances of the T network with the following equations:

Za = ((Z1*Z2)+(Z1*Z3)+(Z2*Z3)) / Z2
Zb = ((Z1*Z2)+(Z1*Z3)+(Z2*Z3)) / Z1
Zc = ((Z1*Z2)+(Z1*Z3)+(Z2*Z3)) / Z3

Note the common numerator in all these expressions which can prove useful in reducing the amount of computation necessary.

The impedances of the T network (Z1, Z2, Z3) can be found from the impedances of the equivalent pi network with the following equations:

Z1 = (Za * Zc) / (Za + Zb + Zc)
Z2 = (Zb * Zc) / (Za + Zb + Zc)
Z3 = (Za * Zb) / (Za + Zb + Zc)

The Twin-T Bridge

The twin-T bridge shown in Fig. 2.14 is frequently used as a feedback element in selective amplifiers, oscillators and for many other purposes. It consists of two T-circuits connected in parallel. The analysis of this circuit is best carried out by transforming both T into equivalent Π-connection and connecting them parallel as shown in Fig. 2.15, where

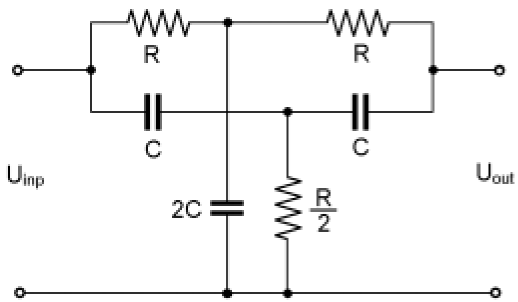
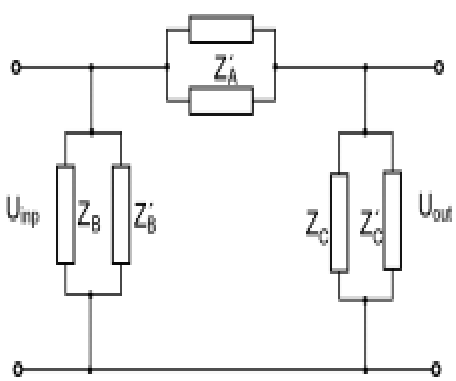


Fig.2.14 Twin –T bridge network

$$Z_A = 2(R + j\omega CR),$$

$$Z'_A = 2\left(\frac{1}{j\omega C} - \frac{1}{\omega^2 C^2 R}\right),$$

$$Z_B = Z'_B = Z_C = Z'_C = R + \frac{1}{j\omega C}.$$



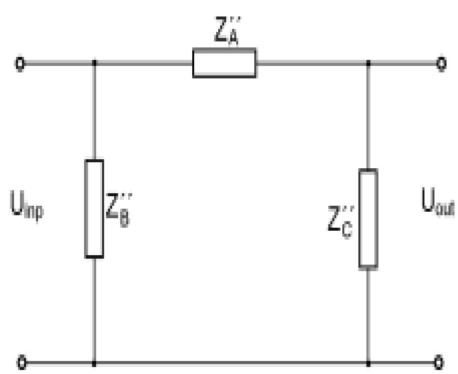


Fig 2.15 T to π bridge network **Fig 2.16 Simplified T to π bridge network**

Adding the impedances in Fig.2.15 in parallel we get a new circuit as shown in Fig.2.16

$$\mathbf{Z}_A'' = 2R\frac{1 + j\omega CR}{1 - \omega^2 C^2 R^2},$$

$$\mathbf{Z}_B'' = \mathbf{Z}_C'' = \frac{1}{2}\left(R + \frac{1}{j\omega C}\right).$$

The complex transmission coefficient is

$$K(\omega) = \frac{U_{out}(\omega)}{U_{inp}(\omega)} = \frac{\mathbf{Z}_C''}{\mathbf{Z}_A'' + \mathbf{Z}_C''} = \frac{1 - \omega^2 C^2 R^2}{1 - \omega^2 C^2 R^2 + j4\omega CR}.$$

The absolute value of transmission coefficient is given by

$$K(\omega) = \frac{U_{out}(\omega)}{U_{inp}(\omega)} = \left|\frac{\mathbf{Z}_C''}{\mathbf{Z}_A'' + \mathbf{Z}_C''}\right| = \frac{1 - \omega^2 C^2 R^2}{\sqrt{\left(1 - \omega^2 C^2 R^2\right)^2 + \left(4\omega CR\right)^2}} =$$

$$= \frac{1 - \left(\dfrac{\omega}{\omega_0}\right)^2}{\sqrt{\left[1 - \left(\dfrac{\omega}{\omega_0}\right)^2\right]^2 + \left(4\dfrac{\omega}{\omega_0}\right)^2}}.$$

where $\omega_0 = 1/(RC)$. If the resistors and capacitors in Fig. 2.14 are fixed, the output voltage is dependent on the frequency of the input voltage. The dependence of $U_{out}(\omega/\omega_0)$ is shown in Fig. 2.17.

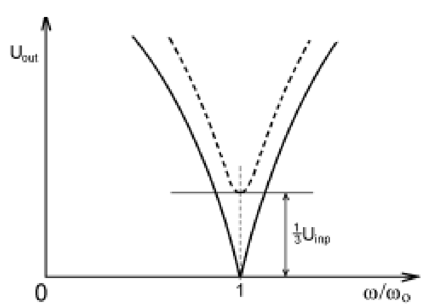


Fig.2.17 Dependance of Uout

We see, that there is a single frequency

$$\omega_0 = \frac{1}{RC},$$
$$f_0 = \frac{1}{2\pi RC},$$

at which the output voltage is zero. In the vicinity of this frequency the circuit behaves itself as a resonant circuit with relatively high Q- factor. The circuit is particularly useful at low frequencies where the equivalent RLC-circuit request large values of L and C.

Bartlett's Bisection Theorem

It is an electrical theorem in network analysis due to Albert Charles Bartlett. The theorem shows that any symmetrical two-port network can be transformed into a lattice network. The theorem often appears in filter theory where the lattice network is sometimes known as a filter X-section following the common filter theory practice of naming sections after alphabetic letters to which they bear a resemblance.

The theorem as originally stated by Bartlett required the two halves of the network to be topologically symmetrical. The theorem was later extended by Wilhelm Cauer to apply to all networks which were electrically symmetrical. That is, the physical implementation of the network is not of any relevance. It is only required that its response in both halves are symmetrical.

Applications of Bartlett's Bisection Theorem

Lattice topology filters are not very common. The reason for this is that they require more components (especially inductors) than other designs. Ladder topology is much more popular. However, they do have the property of being intrinsically balanced and a balanced version of another topology, such as T-sections, may actually end up using more

inductors. One application is for all-pass phase correction filters on balanced telecommunication lines. The theorem also makes an appearance in the design of crystal filters at RF frequencies. Here ladder topologies have some undesirable properties, but a common design strategy is to start from a ladder implementation because of its simplicity. Bartlett's theorem is then used to transform the design to an intermediate stage as a step towards the final implementation (using a transformer to produce an unbalanced version of the lattice topology).

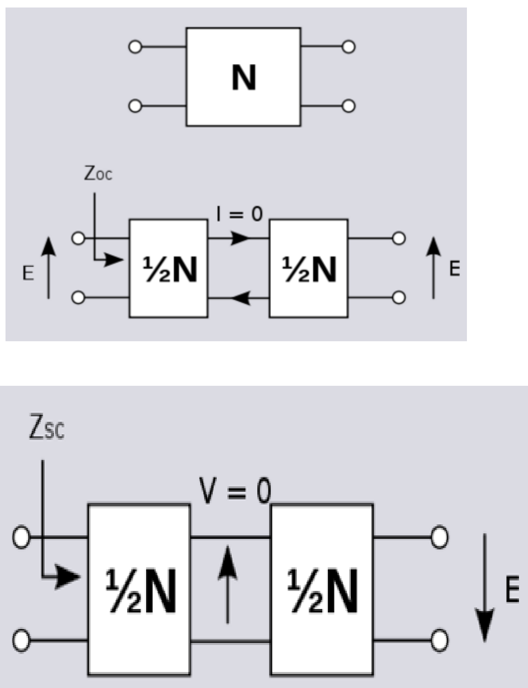


Fig.2.18 Bartlett's bisection theorem

Definition

Start with a two-port network, N, with a plane of symmetry between the two ports. Next cut N through its plane of symmetry to form two new identical two-ports, ½N. Connect two identical voltage generators to the two ports of N. It is clear from the symmetry that no current is going to flow through any branch passing through the plane of symmetry. The impedance measured into a port of N under these circumstances will be the same as the impedance measured if all the branches passing through the plane of symmetry

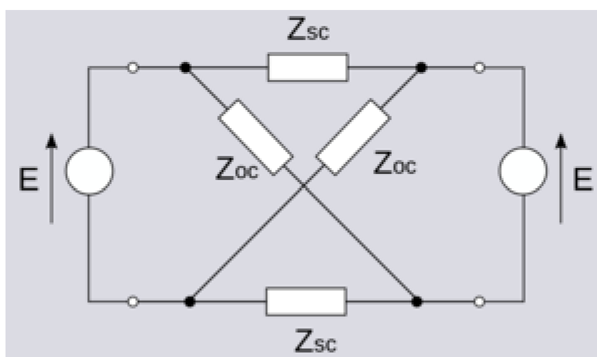
were open circuit. It is therefore the same impedance as the open circuit impedance of ½N. Let us call that impedanceZ_{oc}.

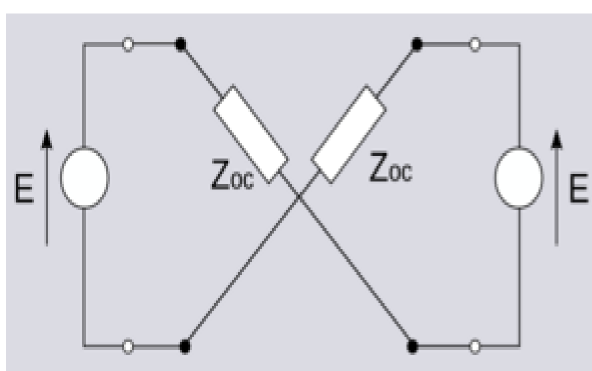
Now consider the network N with two identical voltage generators connected to the ports but with opposite polarity. Just as superposition of currents through the branches at the plane of symmetry must be zero in the previous case, by analogy and applying the principle of duality, superposition of voltages between nodes at the plane of symmetry must likewise be zero in this case. The input impedance is thus the same as the short circuit impedance of ½N. Let us call that impedanceZ_{sc}.

Bartlett's bisection theorem states that the network N is equivalent to a lattice network with series branches of Z_{sc} and cross branches of Z_{oc}.

Proof

Consider the lattice network shown with identical generators, E, connected to each port. It is clear from symmetry and superposition that no current is flowing in the series branchesZ_{sc}.





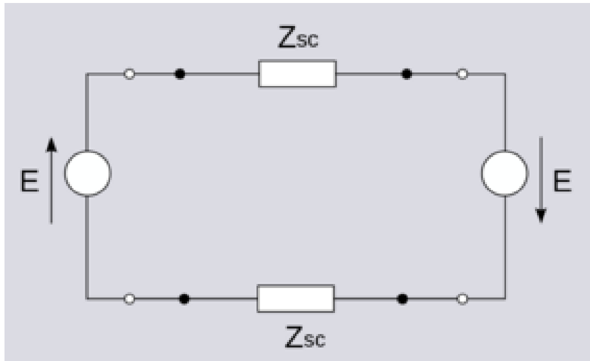


Fig.2.19 Lattice network

Those branches can thus be removed and left open circuit without any effect on the rest of the circuit. This leaves a circuit loop with a voltage of 2E and an impedance of $2Z_{oc}$ giving a current in the loop of;

$$I = \frac{2E}{2Z_{oc}}$$

and an input impedance of;

$$\frac{E}{I} = Z_{oc}$$

as it is required to be for equivalence to the original two-port.

Similarly, reversing one of the generators results, by an identical argument, in a loop with an impedance of $2Z_{sc}$ and an input impedance of;

$$\frac{E}{I} = Z_{sc}$$

Recalling that these generator configurations are the precise way in which Z_{oc} and Z_{sc} were defined in the original two-port it is proved that the lattice is equivalent for those two cases. It is proved that this is so for all cases by considering that all other input and output conditions can be expressed as a linear superposition of the two cases already proved.

Interconnections of two-port networks

Two-port networks may be interconnected in various configurations, such as series, parallel, cascade, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

Series Connection

Figure shows a series connection of two-port networks N $_a$ and N $_b$.

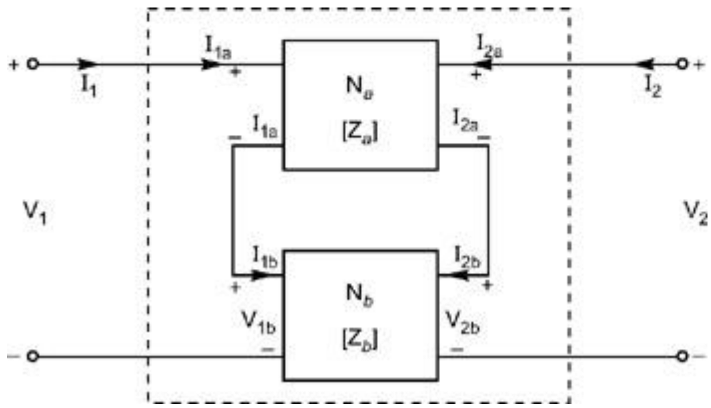


Figure.2.20 Series connection of two two-port networks

For network N $_a$,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$V_{1a} = Z_{11a}I_{1a} + Z_{12a}I_{2a}$$

$$V_{2a} = Z_{21a}I_{1a} + Z_{22a}I_{2a}.$$

For network N $_b$,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22}b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$V_{1b} = Z_{11b}I_{1b} + Z_{12b}I_{2b}$$

$$V_{2b} = Z_{21b}I_{1b} + Z_{22b}I_{2b}$$

The condition for series connection is

$$I_{1a} = I_{1b} = I_1,\ \ and\ I_2 = I_{2b} = I_2\ (current\ same)$$

$$V_1 = V_{1a} + V_{1b}$$

$$V_2 = V_{2a} + V_{2b}$$

Putting the values of V $_{1a}$ and V $_{1b}$ into V1,we get

$$\begin{aligned} V_1 &= Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b} \\ &= Z_{11a}I_1 + Z_{12a}I_2 + Z_{11b}I_1 + Z_{12b}I_2\ [I_{1a} = I_{1b} = I_1,\ I_{2a} = I_{2b} = I_2] \\ V_1 &= (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2. \end{aligned}$$

Putting the values of V $_{2a}$ and V $_{2b}$ into V2, we get

$$V_2 = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

The Z-parameters of the series-connected combined network can be written as

$$\begin{aligned} V_1 &= Z_{11}I_1 + Z_{12}I_2 \\ V_2 &= Z_{21}I_1 + Z_{22}I_2, \end{aligned}$$

where

$$\begin{aligned} Z_1 1 &= Z_{11a} + Z_{11b} \\ Z_1 2 &= Z_{12a} + Z_{12b} \\ Z_2 1 &= Z_{21a} + Z_{21b} \\ Z_2 2 &= Z_{22a} + Z_{22b} \end{aligned}$$

or in the matrix form,

$$[Z] = [Z_a] + [Z_b].$$

The overall Z-parameter matrix for series connected two-port networks is simply the sum of Z-parameter matrices of each individual two-port network connected in series.

Parallel Connection

A parallel connection of two two-port networks N a and N b is shown in Fig.2.21. The resultant of two admittances connected in parallel is Y 1 + Y 2. So in parallel connection, the parameters are Y-parameters.

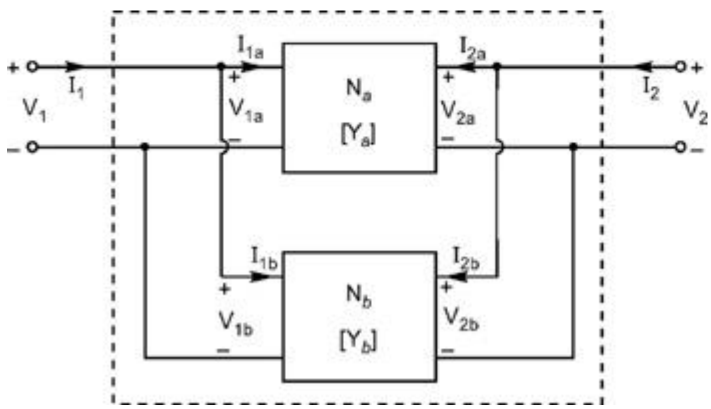


Figure.2.21 Parallel connections for two two-port networks

For network N_a,

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

Or

$$I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$

$$I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}.$$

For network N_b,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{22b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$I_{1b} = Y_{11b}V_{1b} + Y_{12b}V_{2b}$$

$$I_{2b} = Y_{21b}V_{1b} + Y_{22b}V_{2b}$$

Now the condition for parallel,

$$V_{1a} = V_{1b} = V_1,\ V_{2a} = V_{2b} = V_2\ [\textit{Same voltage}]$$

and

$$I_1 = I_{1a} + I_{1b},$$

$$I_2 = I_{2a} + I_{2b}$$

$$\begin{aligned} I_1 &= Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b} \\ &= Y_{11a}V_1 + Y_{12a}V_2 + Y_{11b}V_1 + Y_{12b}V_2 \end{aligned}$$

$$I_1 = (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$

Similarly,

$$I_2 = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

The Y-parameters of the parallel connected combined network can be written as

$$\begin{aligned} I_1 &= Y_{11}V_1 + Y_{12}V_2 \\ I_2 &= Y_{21}V_1 + Y_{22}V_2 \end{aligned}$$

Where

$\mathbf{Y_{11} = Y_{11a} + Y_{11b}}$

$\mathbf{Y_{12} = Y_{12a} + Y_{12b}}$

$\mathbf{Y_{21} = Y_{21a} + Y_{21b}}$

$\mathbf{Y_{22} = Y_{22a} + Y_{22b}}$

$Y = [Y_a] + [Y_b]$

Cascade connection of two port networks

Consider two networks N’ and N’’ are connected in cascade as shown in figure 2.22. When two port are connected in cascade, we can multiply their individual transmission parameter to get overall transmission parameters of the cascade connection.

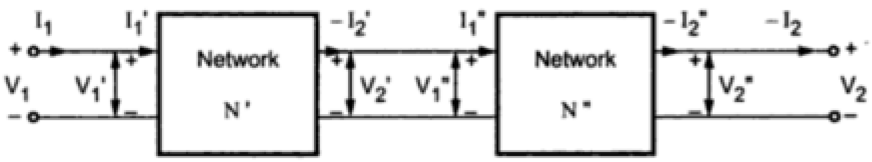


Figure.2.22 Cascade connections for two two-port networks

Let the transmission parameters of network N’ be A’, B’, C’, D’. Let the transmission parameters of the network N’’ be A’’, B’’, C’’, D’’. Let the overall transmission parameters of the cascade connection be A, B, C, D. For cascade connection we have,

$V_1 = V_1^1$, $V_2^1 = V_1^{11}$, $V_2 = V_2^{11}$,

$I_1 = I_1^1$, $I_2^1 = -I_1^{11}$, $I_2 = I_2^{11}$,

For the network N_1^1, transmission parameter equations are

$$V_1' = A' V_2' + B'(-I_2')$$

$$I_1' = C' V_2' + D'(-I_2')$$

For the network N_1^{11}, transmission parameter equations are,

$$V_1'' = A'' V_2'' + B''(-I_2'')$$
$$I_1'' = C'' V_2'' + D''(-I_2'')$$

The overall transmission parameters of the cascade connection can be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} V_1' \\ I_1' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} V_1'' \\ I_1'' \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}\begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix}\begin{bmatrix} V_2'' \\ -I_2'' \end{bmatrix}$$

$$= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Hence, the transmission parameters for the cascaded two port network is simply the matris product of the transmission parameter matrix of each individual two port network,

Problems

1. A network has the following open-circuit and short circuit impedances:

Z_{1oc}= (600+j300)Ω, Z_{2oc}500Ω, Z_{1sc}=(500+j400)Ω, Z_{2sc}=(450+j150)Ω

Find its image parameters.

Solution:

Z_{I1}=√(600+j300)(500+j400)

=552.05+j353

Z_{I2}=√ Z_{2oc} Z_{2sc}

= 480.7+j78.02

Tanhθ==√ Z_{1sc}/ Z_{1oc}

2. The Z parameters of the two port are Z_{11} =10 Ω, Z_{22} =15Ω, Z_{12}=Z_{21}=5 Ω .Compute the equivalent T network ABCD parameters.

Solution:

ABCD parameters

A= Z_{11}/ Z_{21} =3 Ω

B= ΔZ/ Z_{21}=70 Ω

C=1/ Z_{21}=.2

$D = Z_{22}/ Z_{21}=5$

Equivalent T network

$Z_{11}=Z_A+Z_C=15$

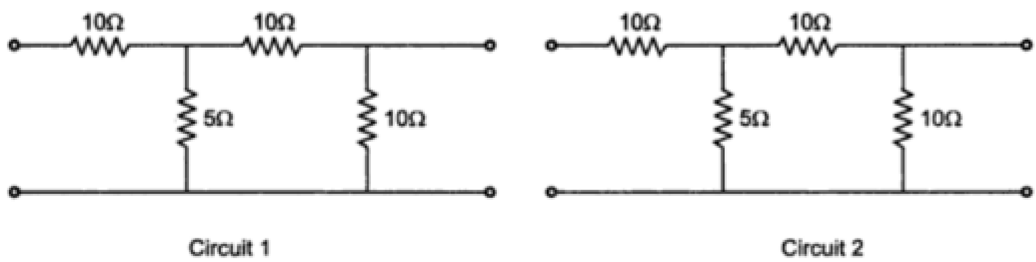
$Z_{12}=Z_{21}=Z_C=5$

$Z_A=10$

$Z_{22}=Z_B+Z_C$

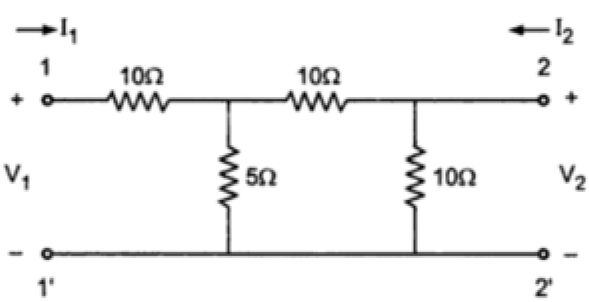
$Z_B=20$

3. Two networks have been shown in figure. Obtain the transmission parameters of the resulting circuit when both the circuits are in cascade.



Solution :

Consider the network as shown in circuit 1



By definition, transmission parameters are given by,

$$V_1 = AV_2 + B(-I_2)$$

$$I_1 = CV_2 + D(-I_2)$$

[A] Let $-I_2 = 0$,

Open circuiting the port 2 as shown in circuit 2

By current divider rule,

$$-I_2 = I_1\left[\frac{5}{5+10}\right] = \frac{1}{3}I_1$$

$$\therefore D = \left.\frac{I_1}{-I_2}\right|_{V_2=0} = 3$$

Applying KVL to outer loop.

$$-10I_1 - 10(-I_2) + V_1 = 0$$

$$\therefore -10I_1 - 10(-I_2) = -V_1$$

$$\therefore 10I_1 + 10(-I_2) = V_1$$

But $I_1 = 3(-I_2)$

$$\therefore 10(3)(-I_2) + 10(-I_2) = V_1$$

$$\therefore 40(-I_2) = V_1$$

$$\therefore B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = 40\Omega$$

Hence transmission parameters for the given network are given by,

$$[T] = \begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix}$$

Two identical networks are connected in cascade. Hence for cascade connection, the overall transmission parameters can be obtained by finding matrix product of two networks.

$$[T_{overall}] = \begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix}\begin{bmatrix} 7 & 40 \\ \frac{1}{2} & 3 \end{bmatrix} = \begin{bmatrix} 69 & 400 \\ 5 & 29 \end{bmatrix}$$

Related Questions

Part-A

1. What are the different types of connection in two port network?

2. Draw the symmetrical lattice network.

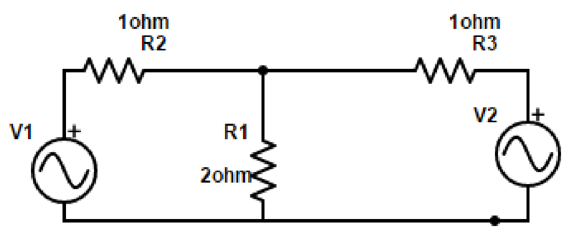
3. Define Iterative impedance?

4. What is Lattice network?

5. Draw the symmetrical Twin–T network.

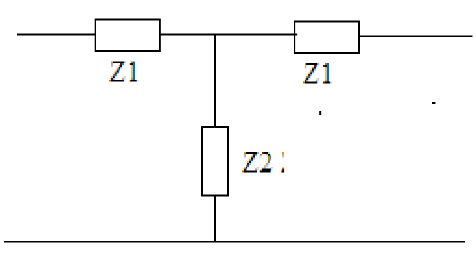
6. Define propagation constant.

7. Find the image parameters of the network shown in figure



8. The currents of a two port network are given by $I_1=6V_1-V_2$, $I_2=-V_1+2V_2$.Find the equivalent π network.

9. Find the lattice network by using Barlett's theorem.



10.The Z-parameters of two port network are $Z_{11}=15\Omega$, $Z_{22}=25\Omega$, $Z_{12}=Z_{21}=5\Omega$.Determine the ABCD parameters.

Part-B

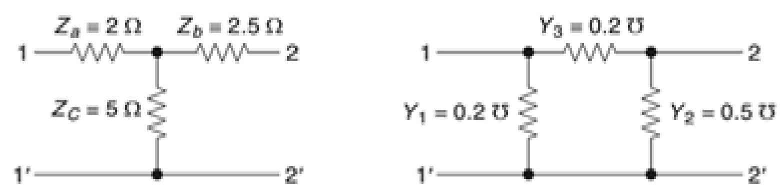
1. Derive the expression for image impedance for asymmetrical T and π network.

2. Derive the expression for iterative impedance for asymmetrical T and π network.

3. Derive the expression for image impedance in terms of ABCD parameter.

4. a) Find the equivalent π network for the T-network shown in Figure (a)

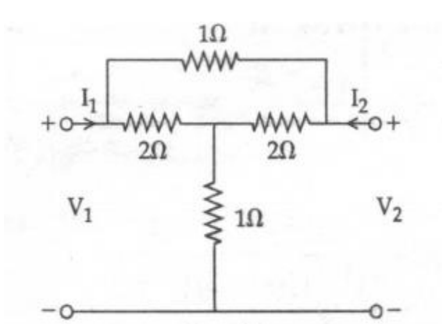
   b) Find the equivalent T network for the π -network shown in Figure (b)



(a) (b)

5. A network has two input terminals a, b and two output terminals c, d. The input impedance with c-d open circuited is (250+j100) Ω and with c-d short circuited is (400+j300) Ω. The impedance across c-d with a-b open circuited is 200 Ω. Determine the equivalent T-network parameters.

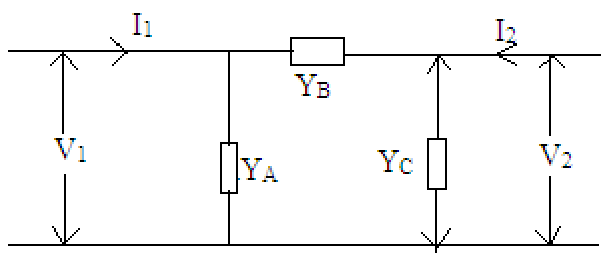
6. Find the open circuit and short circuit impedance of the network shown in figure. Also obtain its π equivalent.



7.Explain in detail about interconnection of two port networks.

8.Derive the expression for types of interconnection of two port networks.

9. Find the short circuit admittance parameter for the circuit shown in figure.



10. Derive the expression for Lattice networks

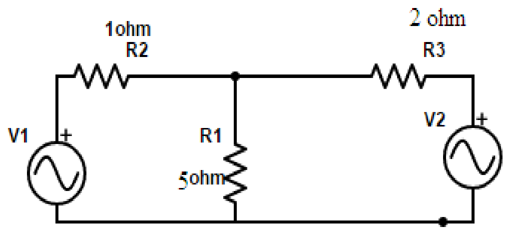
11. Derive the expression for Twin T networks.

12. State and prove Bartlett's Bisection Theorem.

13. The Z parameters of the two port are Z_{11} =10 Ω, Z_{22} =15Ω, Z_{12}=Z_{21}=5 Ω .Compute the equivalent T network and Y and ABCD parameters.

14. Explain how the overall parameters are calculated, if two different two- port networks are connected in cascade and Series-Parallel.

15. Determine the Image parameters of the T-network shown in figure.



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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – IV – SYNTHESIS OF LC, RL AND RC NETWORK

Unit IV

Synthesis of LC, RL & RC Network

Hurwitz polynomial:-

A polynomial is said to be Hurwitz polynomial, if its zeros lie on the left hand side of S-plane or on jω axis provided on the jω axis, zeros are simple.

Conditions:

The conditions for the polynomial

$P(S) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$ to be Hurwitz are

i) P(S) is real, when 's' is real
ii) The roots of polynomial P(S) must have zero or negative real parts.

Properties:-

Let $P(S) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0$ be a Hurwitz polynomial where $a_0, a_1, \ldots a_n$ are the coefficients

i) All the coefficients of the polynomial a_i where i = 0, 1 … n are positive.
ii) All the terms starting from highest power of S to lowest power of S must be present.

$$P(S) = S^4 + 2S^2 + 8 \Longrightarrow \text{evenpolynomial}$$
$$P(S) = S^5 + 2S^3 + 5 \Longrightarrow \text{evenpolynomial}$$

iii) If the polynomial is even or odd then all the roots must be on imaginary jω axis.
iv) Given Hurwitz polynomial can be separated into even and odd parts. Odd part is denoted by O(S) & even part denoted by E(S). $P(S) = E(S) + O(S)$
v) If the ratio of odd to even parts of P(S) or even to odd parts of P(S) is expressed in the continued fraction expansion then all the quotient terms must be positive.

$$P(S) = \frac{O(S)}{E(S)} \quad \text{or} \quad \frac{E(S)}{O(S)}$$
$$= q_1(S) + \cfrac{1}{q_2(S) + \cfrac{1}{q_3(S) + \cfrac{1}{\ldots + \cfrac{1}{q_n(S)}}}}$$

$$
\begin{array}{l}
E(S)\overline{)\ O(S)\ }\left(q_1(S)\right. \\
\quad\quad --- \\
\quad\quad \overline{R_1(S)}\overline{)\,E(S)}\left(q_2(S)\right. \\
\quad\quad\quad\quad --- \\
\quad\quad\quad\quad \overline{R_2(S)}\overline{)\,R_1(S)}\left(q_3(S)\right. \\
\quad\quad\quad\quad\quad\quad --- \\
\quad\quad\quad\quad\quad\quad \overline{R_3}(S)
\end{array}
$$

vi) If P(S) is either odd or even function then the continued fraction expansion is obtained from the ratio of polynomial P(S) to its derivative$P'(S)$.

$$\text{Eg: } P(S) = S^4 + 3S^2 + 2. \quad \text{Then} P'(S) = 4S^4 + 6S$$

vii) If the continued fraction expansion terminates prematurely, then that indicates the functions E(S) & O(S) contain a common factor X(S).In that case, P(S) = X(S)Y(S).If X(S) & Y(S) are Hurwitz, then P(S) is Hurwitz.

1) Examine whether the given polynomials are Hurwitz or not

a) $S^5 + 4S^4 + 7S^2 + 6S + 2 \quad \Rightarrow S^3$ term is missing. So not Hurwitz.

b) $S^6 + 7S^5 + 5S^4 - 3S^3 + 2S^2 + S + 4 \quad \Rightarrow S^3$term is negative. So not hurwitz.

2) Test whether the following polynomials are Hurwitz.

a) $P(S) = S^3 + 2S^2 + 4S + 2$

$O(S) = S^3 + 4S$; $\quad E(S) = 2S^2 + 2$.

$$
\begin{array}{l}
2S^2+2\overline{)\,S^3+4S\,}\left(S/2\right. \\
\quad\quad \underline{S^3+S} \\
\quad\quad\quad\quad 3S\overline{)\,2S^2+2\,}\left(2/3\,S\right. \\
\quad\quad\quad\quad\quad \underline{2S^2} \\
\quad\quad\quad\quad\quad\quad\quad 2\overline{)\,3S\,}\left(3/2\,S\right. \\
\quad\quad\quad\quad\quad\quad\quad\quad \underline{3S} \\
\quad\quad\quad\quad\quad\quad\quad\quad \underline{0}
\end{array}
$$

$C(s) = \frac{s}{2} + \frac{1}{\frac{2s}{3} + \frac{1}{\frac{3s}{2}}}$.All quotient terms are positive. So P(s) is a Hurwitz polynomial.

(b) $P(S) = S^3 + 2S^2 + 3S + 6$

$$O(S) = S^3 + 3S$$

$$E(S) = 2S^2 + 6$$

$$2S^2 + 6\overline{)S^3 + 3S}\left(S/2\right.$$

$$\underline{S^3 + 3S}$$

$$\underline{0}$$

Continued fraction expansion is terminated abruptly. There exists a common factor.

P(S)=(S^3+3S)(1+$\frac{2}{S}$). Here(1+$\frac{2}{s}$) is Hurwitz. All quotients are positive.

S^3+3s is odd polynomial and be m(S) and $m^{/}$ (s) = $3S^2$+3.

$$3S^2 + 3\overline{)S^3 + 3S}\left(S/3\right.$$

$$\underline{S^3 + S}$$

$$2S\overline{)3S^2 + 3}\left(3S/2\right.$$

$$\underline{3S^2}$$

$$3\overline{)2S}\left(2S/3\right.$$

$$\underline{2S}$$

$$\underline{0}$$

Continued fraction expansion, $C(S) = \frac{5}{3} + \cfrac{1}{\frac{3S}{2} + \cfrac{1}{\frac{2S}{3}}}$

All quotient terms are positive. Hence P(S) is Hurwitz.

3) Test whether the polynomial is Hurwitz or not.

$$P(S) = S^4 + S^3 + 5S^2 + 3S + 4$$

$$E(S) = S^4 + 5S^2 + 4$$

$$O(S) = S^3 + 3S$$

$$C(S) = \frac{E(S)}{O(S)}$$

$$S^3 + 3S \,\big)\, S^4 + 5S^2 + 4 \,\big(\, S$$

$$\underline{S^4 + 3S^2}$$

$$2S^2 + 4 \,\big)\, S^3 + 3S \,\big(\, S/2$$

$$\underline{S^3 + 2S}$$

$$S \,\big)\, 2S^2 + 4 \,\big(\, 2S$$

$$\underline{2S^2}$$

$$4 \,\big)\, S \,\big(\, S/4$$

$$\underline{S}$$

$$\underline{0}$$

$$C(S) = S + \cfrac{1}{\cfrac{S}{2} + \cfrac{1}{2S + \cfrac{1}{\cfrac{S}{4}}}}$$

All the quotient terms are positive. So P(S) is Hurwitz.

.

4) Prove that the polynomial $P(S) = S^4 + S^3 + 2S^2 + 3S + 2$ is not Hurwitz.

$$C(S) = S^4 + 2S^2 + 2$$

$$O(S) = S^3 + 3S$$

$$C(S) = \frac{E(S)}{O(S)}$$

$$S^3 + 3S \,\big)\, S^4 + 2S^2 + 2 \,\big(\, S$$

$$\underline{S^4 + 3S^2}$$

$$-S^2 + 2 \,\big)\, S^3 + 3S \,\big(\, -S$$

$$\underline{S^3 - 2S}$$

$$5S \,\big)\, -S^2 + 2 \,\big(\, -S/5$$

$$\underline{-S^2}$$

$$2 \,\big)\, 5S \,\big(\, 5/2$$

$$\underline{5S}$$

$$\underline{0}$$

$$C(S) = S + \cfrac{1}{-S + \cfrac{1}{-S/5 + \cfrac{1}{5/2\,S}}}$$

Two quotient terms are negative. So, P(S) is not Hurwitz.

5) Test whether the polynomial P(S) = $S^5 + S^3 + S$ is Hurwitz.

Given polynomial contains odd function only,

$$C(S) = \frac{P(S)}{P'(S)}$$

$$P^1(S) = 5S^4 + 3S^2 + 1$$

$$5S^4 + 3S^2 + 1\big)\, S^5 + S^3 + S \left(\frac{S}{5} \right.$$

$$\underline{S^5 + \frac{3}{5}S^3 + \frac{S}{5}}$$

$$\frac{2}{5}S^3 + \frac{4S}{5}\Big)\, 5S^4 + 3S^2 + 1 \left(\frac{25}{2}S \right.$$

$$\underline{5S^4 + 10S^2}$$

$$-7S^2 + 1\big)\, \frac{2}{5}S^3 + \frac{4S}{5} \left(\frac{-2}{35}S^3 \right.$$

$$\underline{\frac{2}{5}S^3 - \frac{2S}{35}}$$

$$\frac{26}{35} - S\Big) - 7S^2 + 1 \left(\frac{-269S}{35} \right.$$

$$\underline{-7S^2}$$

$$1\big)\, \frac{26S}{35} \left(\frac{26}{35}S \right.$$

$$\underline{\frac{26S}{35}}$$

$$\underline{0}$$

$$C(S) = \frac{S}{5} + \cfrac{1}{\frac{25}{2}S + \cfrac{1}{\frac{-2}{35}S^3 + \cfrac{1}{\frac{-269S}{35} + \cfrac{1}{\frac{26}{35}S}}}}$$

All the quotient terms are not positive. P(S) is not Hurwitz.

6) Test whether the polynomials are Hurwitz or not

$$Y(S) = S^7 + 2S^6 + 2S^5 + S^4 + 4S^3 + 8S^2 + 8S + 4$$

$$O(S) = S^7 + 2S^5 + 4S^3 + 8S$$

$$E(S) = 2S^6 + S^4 + 8S^2 + 4$$

$$P_1(S) = \frac{O(S)}{E(S)}$$

$$2S^6 + S^4 + 8S^2 + 4 \,\big)\, S^7 + 2S^5 + 4S^3 + 8S \,\big(\, S/2$$

$$\underline{S^7 + \frac{S^5}{2} + 4S^3 + 2S}$$

$$\frac{3}{2}S^5 \quad +6S \,\big)\, 2S^6 + S^4 + 8S^2 + 4 \,\big(\, 4/3\, S$$

$$\underline{2S^6 + O + 8S^2}$$

$$S^4 + 2 \,\big)\, \frac{3S^5}{2} + 6S \,\big(\, 3S/2$$

$$\underline{\frac{3S^5}{2} + 6S}$$

$$\underline{0}$$

Continued fraction expansion is terminated abruptly. So there is a common factor in the function Y(S).

$$Y(S) = S^7 + 2S^6 + 2S^5 + S^4 + 4S^3 + 8S^2 + 8S + 4$$

Taking S^4 as common in the 1st 4 terms & 4 as common in the last 4 terms.

$$Y(S) = S^4\left(S^3 + 2S^2 + 2S + 1\right) + 4\left(S^3 + 2S^2 + 2S + 1\right)$$

$$Y(S) = \left(S^4 + 4\right)\left(S^3 + 2S^2 + 2S + 1\right)$$

$$Y(S) = F_1(S) \cdot F_2(S)$$

$$F_1(S) = S^4 + 4$$

Missing terms are there. So $F_1(S)$ is not Hurwitz.

$$F_2(S) = S^3 + 2S^2 + 2S + 1$$

$$O(S) = S^3 + 2S$$

$$E(S) = 2S^2 + 1$$

Continued fraction expansion is,

$$2S^6 + 1\big)\, S^3 + 2S \,\big(\, S/2$$

$$\underline{S^3 + \frac{S}{2}}$$

$$\frac{3}{2}S\,\big)\, 2S^2 + 1 \,\big(\, 4/3\, S$$

$$\underline{2S^2}$$

$$1\big)\, \frac{3}{2} + S \,\big(\, 3/2\, S$$

$$\underline{\frac{3}{2}S}$$

$$\underline{0}$$

$$P_1(S) = \frac{5}{2} + \cfrac{1}{\cfrac{4S}{3} + \cfrac{1}{\cfrac{3S}{2}}}$$

Since all the quotient terms are positive. $F_2(S)$ is Hurwitz.

$\because$ $F_1(S)$ is not Hurwitz & $F_2(S)$ is Hurwitz, the polynomial Y(S) is not Hurwitz.

Routh Hurwitz array method:

- In this method, an array is constructed using the coefficients of given polynomial in a specific way. By the inspection of such an array formed, the polynomial can be decided to be Hurwitz or not.

Let

$$\boxed{P(s) = a_n\, s^n + a_{n-1}\, s^{n-1} + a_{n-2}\, s^{n-2} + a_{n-3}\, s^{n-3} + \ldots\ldots + a_1\, s + a_0}$$

Routh Hurwitz array is

$$
\begin{array}{c|ccc}
S^{n} & a_{n} & a_{n-2} & a_{n-4} \\
S^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} \\
S^{n-2} & b_{n} & b_{n-1} & b_{n-2} \\
\vdots & c_{n} & c_{n-1} & c_{n-2} \\
 & \vdots & & \\
S^{0} & a_{0} & &
\end{array}
$$

- 1^{st} row consists of all the coefficients of alternate power of S starting from n.
- Next row consists of all the coefficients of alternate power of S starting from n–1.
- A row corresponding to s^{n-2} is generated from first two rows as

$$b_n = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}}, \quad b_{n-1} = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}}$$

- Row corresponding to S^{h-3} is generated from the two previous rows

$$c_n = \frac{b_n a_{n-3} - a_{n-1} b_{n-1}}{a_n}; \quad c_{n-1} = \frac{b_n a_{n-5} - a_{n-1} a_{n-2}}{a_n}$$

- Procedure is continued until the row corresponding to S^0 is obtained.
- Last row a_0 = constant term of the polynomial.
- For the given polynomial P(S) to be Hurwitz
  1) All the elements in the 1^{st} column should be non zero
  2) There should not be any sign change in the 1^{st} column.

Special case:

$$
\begin{array}{c|ccc}
S^{n} & a_1 & a_2 & a_3 \\
S^{n-1} & b_1 & b_2 & b_3 \\
S^{n-2} & c_1 & c_2 & c_3 \\
\hline
S^{n-3} & 0 & 0 & 0 \\
\hline
\end{array}
\quad \leftarrow \text{Rows of zeros}
$$

If there occur a complete row as row of zeros while generating an array, an equation is formed using the coefficients of a row which is just above the row of zeros. Such an equation is called the auxiliary equation A(S). A(S) is always odd or even polynomial in S.

$$A(S) = c_1 S^{n-2} + c_2 S^{n-4} + c_2 S^{n-6} + \cdots$$

Find $\frac{dA(S)}{dS}$ and replace the row of zeros by the coefficients of equation $\frac{dA(S)}{dS}$

The special case is of repeated roots of P(S) on imaginary axis. Such case can be identified by solving the equation A(S) = 0. This is because the roots of A(S) = 0 are some of the roots of P(S) = 0, which decide whether polynomial is Hurwitz or not. If there is any sign change in the 1st column of the completed array, then the given polynomial is not Hurwitz or else it is Hurwitz.

(1) Test whether $P(S) = S^5 + 8S^4 + 24S^3 + 28S^2 + 23S + 6$ is Hurwitz or not using Routh array method.

Routh array can be obtained as

S^5	1	24	23
S^4	8	28	6
S^3	20.5 $\frac{8\times 24-1\times 28}{8}$	22.25 $\frac{8\times 23-1\times 6}{8}$	0
S^2	19.32	6	
S^1	15.88	0	
S^0	6		

No sign change in the 1st column,
all coefficients are positive.
So P(S) is Hurwitz.

(2) Test whether $F(S) = 2S^6 + S^5 + 13S^4 + 6S^3 + 56S^2 + 26S + 25$ is Hurwitz or not using Routh array method.

S^6	2	13	56	25	
S^5	1	6	25	0	
S^4	1	6	25		
S^3	0	0	0	$\leftarrow$ Rows of zeros	

$$A(S) = S^4 + 6S^2 + 25$$

$$\frac{dA}{dS} = 4S^3 + 12S$$

S^3	4	12	0
S^2	3	25	
S^1	–21.33	0	
S^0	25		

There is sign change in the 1st column. So, P(S) is not Hurwitz.

Positive Real Function:-

Significance of positive real function is that if the driving point imitance (i.e.) [admittance or impedance] is a positive real function, then only it is physically realizable using passive R, L & C components. Hence imitance function must be checked for positive realness before synthesizing.

For a function to be positive real function it has to satisfy the following basic properties.

i) Given function F(S) is real for real S.

ii) Real part of F(S) is greater than or equal to zero, when the real part of S is greater than or equal to zero

$$\text{Re}\left[F(S)\right] \geq 0 \quad \text{for } \text{Re}\,|S| \geq 0.$$

Let
$$F(S) = \frac{P(S)}{Q(S)} = \frac{a_0 S^n + a_1 S^{n-1} + \cdots + a_n}{b_0 S^m + b_1 S^{m-1} + \cdots + a_m}$$
$$= \frac{a_0 (S - Z_1)(S - Z_2)\cdots(S - Z_n)}{b_0 (S - P_1)(S - P_2)\cdots(S - P_m)}$$

Where Z_1, Z_2, … Z_n are zeros & P_1, P_2, … P_m are poles.

iii) F(S) must not have any poles on the right half of S plane.

iv) F(S) may have simple poles on the jω axis with real & positive residues

v) Real part of H(jω) is greater than or equal to zero for all ω values

(ie) $\text{Re}\left[H(j\omega)\right] \geq 0 \quad \forall \omega$

Properties:-

1) Coefficients of Numerator & Denominator polynomials P(S) & Q(S) are real and positive.
2) Poles & zeros of F(S) have zero or negative real parts.

3) Poles of F(S) or $\frac{1}{F(S)}$ lying on the imaginary axis must be simple & their residues must be real & positive.

4) Poles & zeros are real or they occur in complex conjugate pairs.

5) Highest degree of F(S) & Q(S) differ almost by 1.

6) Lowest degree of F(S) & Q(S) differ almost by 1.

7) IF F(S) is positive real, then $\frac{1}{F(S)}$ is also positive real function

8) Sum of positive real function is also positive real.

9) Difference of positive real function is not necessarily positive real.

1. Derive the condition for positive realness?

$$\text{Let } F(S) = \frac{P(S)}{Q(S)}$$

$$F(S) = \frac{E_1(S) + O_1(S)}{E_2(S) + O_2(S)}$$

Where $E_1(S)$ & $O_1(S)$ are even & odd functions of P(S) & $E_2(S)$ & $O_2(S)$ are even & odd functions of Q(S). For simplicity let us drop S.

$$F(S) = \frac{E_1 + O_1}{E_2 + O_2}$$

By dividing& multiplying by $E_2 - O_2$.

$$F(S) = \frac{(E_1 + O_1)(E_2 - O_2)}{(E_2 + O_2)(E_2 - O_2)}$$

$$F(S) = \frac{E_1E_2 - O_1O_2}{E_2^2 - O_2^2} + \frac{E_2O_1 - E_1O_2}{E_2^2 - O_2^2}$$

$\because$ E_1E_2 & O_1O_2 are even functions, while E_2O_1 & E_1O_2 are odd functions

Where $\text{Even } F(S) = \frac{E_1E_2 - O_2}{E_2^2 - O_2^2}$, $\text{odd } F(S) = \frac{E_2O_1 - E_1O_2}{E_2^2 - O_2^2}$

$$\therefore \quad F(S) = \text{Even}\left[F(S)\right] + \text{odd } F(S)$$

Let S = jω,

$$F(j\omega) = \text{Even}\left[F(j\omega)\right] + \text{odd}\left[F(j\omega)\right]$$

Even function of F(jω) gives real value, as even power of jω removes j. odd function of F(jω) gives imaginary value. $F(j\omega) = \text{Re}\left[F(j\omega)\right] + j\,\text{Im}\left[F(j\omega)\right]$

Where $\mathrm{Re}\left[F(j\omega)\right] = \mathrm{Even}\left[F(j\omega)\right] \& \mathrm{Im}\left[F(j\omega)\right] = \mathrm{odd}\left[F(j\omega)\right]$

Conditions for positive realness, is $\mathrm{Re}\left[F(j\omega)\right] \geq 0 \quad \forall\omega$

$\mathrm{Even}\left[F(j\omega)\right] \geq 0$ for all values of ω

$$\frac{E_1E_2 - O_1O_2}{{E_2}^2 - {O_2}^2} \geq 0$$

The condition to be satisfied is $E_1E_2 - O_1O_2 \geq 0$

2. Find the condition for the given function to be a positive real function.

a) $H(S) = \dfrac{S+a}{S^2 + bS + c}$

$E_1(S) = a, \quad O_1(S) = S$

$E_2(S) = S^2 + c, \quad O_2(S) = bS$

For positive realness, the condition to be satisfied is $E_1E_2 - O_1O_2 \geq 0$

$a\left(S^2 + c\right) - S(bS) \geq 0$

$a\left(S^2 + c\right) - bS^2 \geq 0$

Let S = jω,

$a\left((j\omega)^2 + c\right) - b(j\omega)^2 \geq 0$

$a\left(-\omega^2 + c\right) + b\omega^2 \geq 0$

$\omega^2\left(b - a\right) + ca \geq 0$

ω lies between 0 to infinity,

a, b, c ≥ 0 & b – a ≥ 0 (i.e) b ≥ a.

b) $H(S) = \dfrac{S^2 + a_1S + a_0}{S^2 + b_1S + b_0}$

$E_1(S) = S^2 + a_0, \quad O_1(S) = a_1S$

$$E_2(S) = S^2 + b_0, \quad O_2(S) = b_1 S$$

$$E_1 E_2 - O_1 O_2 \geq 0$$

$$\left(S^2 + a_0\right)\left(S^2 + b_0\right) - (a_1 S)(b_1 S) \geq 0$$

$$S^4 + a_0 S^2 + b_0 S^2 + a_0 b_0 - a_1 b_1 S^2 \geq 0$$

$$S^4 + \left(a_0 + b_0 - a_1 b_1\right) S^2 + a_0 b_0 \geq 0$$

Let S = jω, Therefore $\left(j\omega\right)^4 + \left(a_0 + b_0 - a_1 b_1\right)\left(j\omega\right)^2 + a_0 b_0 \geq 0$

$$\omega^4 + \left[\left(-\left(a_0 + b_0 - a_1 b_1\right)\omega\right)\right]^2 + a_0 b_0 \geq 0$$

$$\omega^2 = \frac{\left(a_0 + b_0 - a_1 b_1\right) \pm \sqrt{\left(-\left(a_0 + b_0\right) + a_1 b_1\right)^2 - 4 a_0 b_0}}{2}$$ → should be either zero or negative

(ie) $\left[a_1 b_1 - \left(a_0 + b_0\right)\right]^2 - 4 a_0 b_0 \geq 0$

$$\left[a_1 b_1 - \left(a_0 + b_0\right)\right]^2 \geq 4 a_0 b_0$$

Taking square root on both sides,

$$a_1 b_1 - \left(a_0 + b_0\right) \geq \pm 2\sqrt{a_0 b_0}$$

As –ve value is lesser than +ve value,

$$a_1 b_1 - \left(a_0 + b_0\right) \geq -2\sqrt{a_0 b_0}$$

$$a_1 b_1 \geq a_0 + b_0 - 2\sqrt{a_0 b_0}$$

$$a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$$

So condition for positive realness is,

$$a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$$

(3). Check the positive realness of the function, $N(S) = \dfrac{S + 4}{S^2 + 2S + 1}$

This is of the form, $\dfrac{S + a}{S^2 + bS + c}$

a = 4, b = 2, c = 1.

For +ve realness, $a, b, c \geq 0$ & $b \geq a$

But a > b, the function is not a p.r.f. (or)

$$N(S) = \frac{S+4}{S^2+2S+1} = \frac{S+4}{(S+1)^2}$$

$$(S+1)^2 = 0$$

$$S = -1.$$

There are multiple poles at S = –1. Hence the function is not a p.r.f.

(4). Prove that the given function is a p.r.f.

$$N(S) = \frac{S+2}{S^2+3S+2}$$

a = 2, b = 3, c = 2.

a, b, c ≥ 0, b > a, the given function is a p.r.f.

(5). Check the positive realness of $N(S) = \dfrac{S^2+S+6}{S^2+S+1}$

The given function is of the form, $\dfrac{S^2+a_1S+a_0}{S^2+b_1S+b_0}$

Condition for p.r.f. is

$$a_1 b_1 \geq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2$$

$$a_0 = 6, b_0 = 6, a_1 = 1, b_1 = 1$$

$$a_1 b_1 = 1$$

$$\left(\sqrt{a_0} - \sqrt{b_0}\right)^2 = \left(\sqrt{6} - \sqrt{1}\right)^2 = 2.1$$

$$a_1 b_1 \ngeq \left(\sqrt{a_0} - \sqrt{b_0}\right)^2.$$

So it is not a p.r.f.

(6) Prove that the function $Z(S)=\dfrac{(S+2)(S+4)}{(S+1)(S+3)}$ is +ve real.

$$Z(S)=\frac{(S+2)(S+4)}{(S+1)(S+3)}=\frac{S^2+6S+8}{S^2+4S+3}$$

This function is of the form, $\dfrac{S^2+a_1S+a_0}{S^2+b_1S+b_0}$

$a_0=8,\ a_1=6;\quad b_0=3,\ b_1=4.$

$$\left(\sqrt{a_0}-\sqrt{b_0}\right)^2=\left(\sqrt{8}-\sqrt{3}\right)^2=1.2$$

$$a_1b_1=(6)(4)=24$$

$a_1b_1\geq\left(\sqrt{a_0}-\sqrt{b_0}\right)^2$, the given function is a p.r.f.

(7) Find whether the given function is +ve real or not $Z(S)=\dfrac{3S+5}{S(S^2+1)}$

$$Z(S)=\frac{3S^2+5}{S(S^2+1)}=\frac{A}{S}+\frac{BS+C}{S^2+1}$$

$$3S^2+5=A(S^2+1)+BS^2+CS$$

$$3S^2+5=(A+B)S^2+CS+A$$

Equating constant terms, $A=5$

Equating S terms, $C=0$

Equating S^2 terms, $A+B=3$

$B=3-A=3-5=-2$

Residue value B is –ve, given function is not a p.r.f.

(8) Check the positive realness of the following function

$$H(S)=\frac{S^3+5S}{S^4+2S^2+1}$$

$$H(S)=\frac{S^3+5S}{S^4+2S^2+1}=\frac{S\left(S^2+5\right)}{\left(S^2+1\right)^2}$$

$$\left(S^2+1\right)^2=0$$

$$\left(S^2+1\right)\left(S^2+1\right)=0$$

$$S^2=-1$$

$S=\pm j\ \ \&\ \ \pm j$ Multiple poles, not a p.r.f.

Elementary Synthesis Procedure:

Properties of LC driving point functions:

1) LC imitance function is the ratio of odd to even or even to odd polynomials.
2) Poles and zeros are simple & on the imaginary axis.
3) Poles & zeros are alternating.
4) At origin (i.e.) at S = 0, there is a pole or zero
5) At infinity (i.e.) at $S = \infty$, there is a pole or zero
6) $\mathrm{Re}\left[F(j\omega)\right] = 0 \quad \forall\omega$
7) The residues of imaginary axis poles are positive & real
8) Highest power of Numerator & Denominator differ by unity. The lowest powers also differ by unity.

The main methods for realizing a reactance function as a network are

1) Foster form I
2) Foster form II
3) Cauer form I
4) Cauer form II

Foster form I: $Z(S) = \dfrac{H\left(S^2+\omega_1{}^2\right)\left(S^2+\omega_3{}^2\right)\left(S^2+\omega_5{}^2\right)\cdots}{S\left(S^2+\omega_2{}^2\right)\left(S^2+\omega_4{}^2\right)\left(S^2+\omega_0{}^2\right)\cdots}$

- Used to realize impedance function. If admittance function is given, the reciprocal of the function is realized.

  By partial fraction

  $$Z(S) = \frac{K_0}{S} + \frac{2K_2S}{S^2+\omega_2{}^2} + \frac{2K_4S}{S^2+\omega_4{}^2} + \cdots + K_\infty S$$

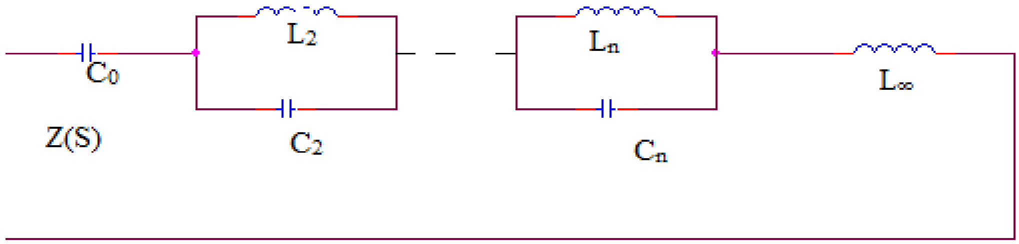
  $$Z(S) = \frac{K_0}{S} + \sum_{i=2,4}\frac{2K_iS}{S^2+\omega_i{}^2} + K_\infty S$$

  K_0, K_i, K_∞ are the residues of Z(S) at origin, ω and ∞ respectively.

  $\dfrac{K_0}{S}$ represent a capacitor of $\dfrac{1}{K_0}$ farads

  $K_\infty S$ represent an inductor of K_∞ Henrys

$\frac{2K_iS}{S^2+\omega_i^{\ 2}}$ represent a parallel combination of a capacitor of $\frac{1}{2K_i}$ F & inductance of $\frac{2K_i}{\omega_i^{\ 2}}$ H



Foster form II:-

- Used to realize admittance function.
- If impedance function is given, reciprocal of the function which gives admittance is realized.

$$Y(S)=\frac{H\left(S^2+\omega_1^{\ 2}\right)\left(S^2+\omega_3^{\ 2}\right)}{S\left(S^2+\omega_3^{\ 2}\right)\left(S^2+\omega_4^{\ 2}\right)}$$

$$Y(S)=\frac{K_0}{S}+\frac{2K_2S}{S^2+\omega_2^{\ 2}}+\frac{2K_4S}{S^2+\omega_4^{\ 2}}+\ldots K_\infty S.$$

$$Y(S)=\frac{K_0}{S}+\sum_{i=2,4}\frac{2K_iS}{S^2+\omega_i^{\ 2}}+K_\infty S.$$

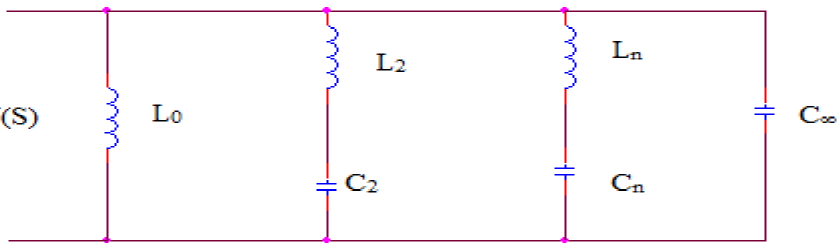
Where K_0, K_i & K_∞ are the residues of Y(S) at origin, ω_i and ∞ respectively.

$\frac{K_0}{S}$ represent a inductor of $\frac{1}{K_0}$ Henrys

$K_\infty S$ represent an inductor of K_∞ farads

$\frac{2K_iS}{S^2+\omega_i^{\ 2}}$ represent a series combination of a inductor of $\frac{1}{2K_i}$ H &capacitor of $\frac{2K_i}{\omega_i^{\ 2}}$ F.

Y(S) is the parallel combination of elemental admittance.



1) The driving point impedance of a one-port reactive network is given by

$$Z(S)=\frac{5\left(S^2+4\right)\left(S^2+25\right)}{S\left(S^2+16\right)}$$ Obtain the 1st & 2nd Foster networks.

Foster 1:

$$Z(S)=\frac{5\left(S^2+4\right)\left(S^2+25\right)}{S\left(S^2+1b\right)}=\frac{5\left(S^4+4S^2+25S^2+100\right)}{S^3+16S}$$

$$=\frac{5S^4+145S^2+500}{S^3+16S}$$

$$S^3+16S\overline{\smash{\big)}\,5S^4+145S^2+500}\big(5S$$

$$\frac{5S^4+80S^2}{\qquad 65S^2+500}$$

$$Z(S)=5S+\frac{65S^2+500}{S^3+16S}=5S+\frac{65S^2+500}{S\left(S^2+16\right)}$$

$$\frac{65S^2+500}{S\left(S^2+16\right)}=\frac{A}{S}+\frac{BS+C}{S^2+16}$$

$$Z(S)=5S+\frac{125}{4S}+\frac{\frac{135S}{4}}{S^2+16}$$

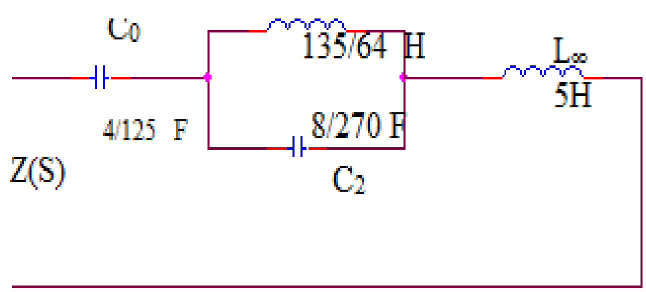
$$Z(S)=\frac{K_0}{S}+\sum\frac{2K_iS}{S^2+\omega_i^{\,2}}+\cdots+K_\infty S.$$

$$C_0=\frac{1}{K_0}==\frac{4}{125}F$$

$$L_1=\frac{2K_2}{\omega_2^{\,2}}=\frac{135\!\!\diagup\!4}{16}=\frac{135}{64}+H$$

$$C_2=\frac{1}{2K_2}=\frac{1}{135\!\!\diagup\!4}=\frac{4}{135}F$$

$$L_\infty=K_\infty=2H$$



Foster form 2:

$$Y(S)=\frac{S\left(S^2+16\right)}{5\left(S^2+4\right)\left(S^2+25\right)}$$

$$Y(S)=\frac{S\left(S^2+16\right)}{5\left(S^2+4\right)\left(S^2+25\right)}=\frac{AS+B}{S^2+4}+\frac{CS+D}{S^2+25}$$

$$Y(S)=\frac{\frac{4}{35}S}{S^2+4}+\frac{\frac{3}{35}S}{S^2+25}$$

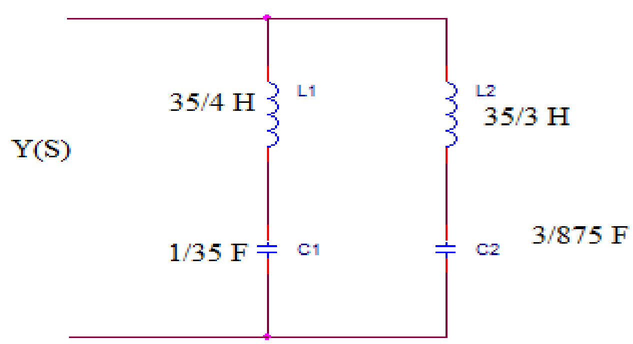
$$Y(S)=K_0S+\sum_{i=2,4}\frac{2K_iS}{S^2+\omega_i^{\,2}}+K_\infty S$$

$$L_1=\frac{1}{2K_i}=\frac{35}{4}H$$

$$C_1=\frac{2K_i}{\omega_i^{\,2}}=\frac{1}{35}F$$

$$L_2=\frac{1}{2K_i}=\frac{35}{3}H$$

$$C_2=\frac{2K_i}{\omega_i^{\,2}}=\frac{\frac{3}{35}}{25}=\frac{3}{875}F$$

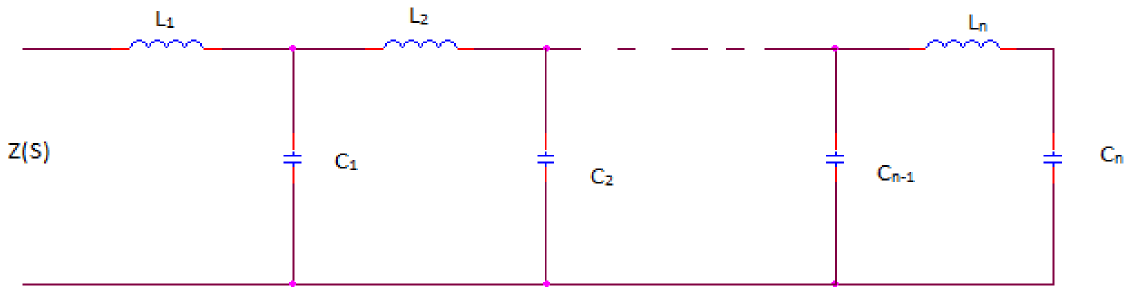


<u>Cauer form I</u>:-

- Network is realized in Cauer I form by continuous fraction expansion. Highest power of numerator & denominator differ by unity.
- Nr &Dr are arranged in the form of descending power of S.

$$Z(S) = \underset{(\text{series})}{Z_1(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_2(S)} + \cfrac{1}{\underset{(\text{series})}{Z_3(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_4(S)} + \cfrac{1}{Z_5(S)\cdots}}}}$$

$$Z(S) = 4(S) + \cfrac{1}{C_1S + \cfrac{1}{L_2S + C_2}}$$

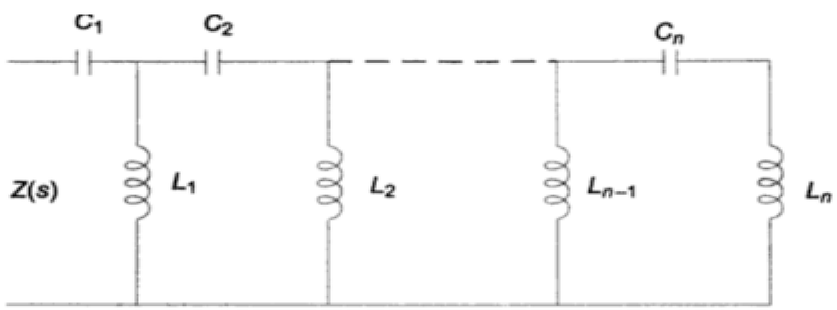


- It gives a ladder network with series arm as inductors & shunt arm as capacitors
- If Numerator power is less than its denominator power, then driving point function is inverted.
- In that case, continued fraction will give capacitive admittance as 1st shunt element and a series inductance.

<u>Cauer II form</u>:

Here Numerator &Denominator are arranged in the ascending power of S.

$$Z(S) = \underset{(\text{series})}{Z_1(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_2(S)} + \cfrac{1}{\underset{(\text{series})}{Z_3(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_4(S)}}}}$$



Here the series arms are capacitors and shunt arm are inductors.

$$Z(S)=\frac{1}{C_1(S)}+\cfrac{1}{\cfrac{1}{4S+\cfrac{1}{2S+\cfrac{1}{\cfrac{1}{L_2(S)}+\cdots}}}}$$

1) Realise the network in both Cauer forms. $Z(S)=\dfrac{S\left(S^2+4\right)}{\left(S^2+1\right)\left(S^2+9\right)}$

Cauer I:

Given Numerator degree should be higher than Denominator.

$$Y(S)=\frac{\left(S^2+1\right)\left(S^2+9\right)}{S\left(S^2+4\right)}=\frac{S^4+10S^2+9}{S^3+4S}$$

$$S^3+4S\,\big)\,S^2+10S^2+9\,\big(S \qquad \rightarrow Y$$

$$\underline{S^4+4S^2}$$

$$6S^2+9\,\big)\,S^3+4S\,\big(S/6 \qquad \rightarrow Z$$

$$\underline{S^3+\frac{3S}{2}}$$

$$\frac{5}{2}S\,\big)\,6S^2+9\,\big(\frac{12}{5} \qquad \rightarrow Y$$

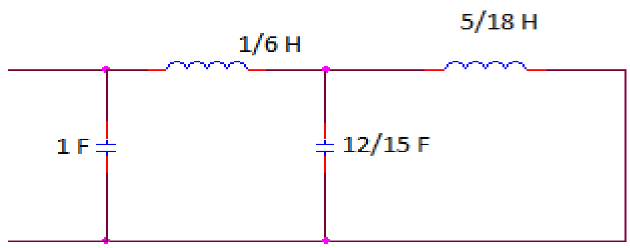
$$\underline{6S^2}$$

$$9\,\big)\,\frac{5}{2}S\,\big(\frac{5}{18}S \qquad \rightarrow Z$$

$$\underline{\frac{5}{2}S}$$

$$\underline{0}$$

$$Y(S)=S+\cfrac{1}{\cfrac{S}{6+\cfrac{1}{12S/5+\cfrac{1}{5S/18}}}}$$



Cauer II:

$$Y(S)=\frac{S^4+10S^2+9}{S^3+4S}=\frac{9+10S^2+S^4}{4S+S^3}$$

$$4S+S^3\overline{)\,9+10S^2+S^2}\left(\frac{9}{4S}\right.\qquad\rightarrow Y$$

$$\underline{9+\frac{9}{4}S^2}$$

$$\frac{31}{4}S^2+S^4\overline{)\,4S+S^3}\left(16/31S\right.\qquad\rightarrow Z$$

$$\underline{4S+\frac{16}{31}S}$$

$$\frac{15}{31}S^3\overline{)\,\frac{31}{4}S^2+S^4}\left(\frac{961}{60S}\right.\qquad\rightarrow Y$$

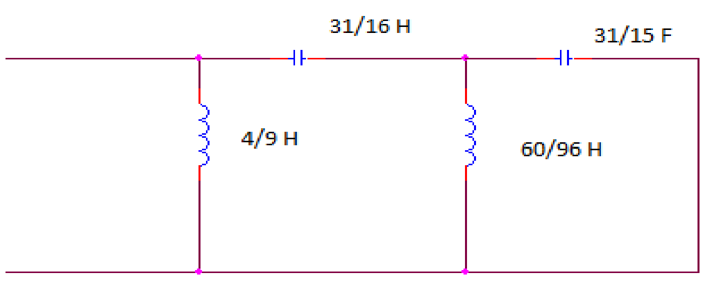
$$\underline{\frac{31}{4}S^2}$$

$$S^4\overline{)\,\frac{15}{31}S^3}\left(\frac{15}{31S}\right.\qquad\rightarrow Z$$

$$\underline{\frac{15}{31}S^3}$$

$$\underline{0}$$

$$Y(S)=\frac{9}{4S}+\cfrac{1}{\cfrac{16}{31S}+\cfrac{1}{\cfrac{961}{60S}+\cfrac{1}{\cfrac{15}{31S}}}}$$

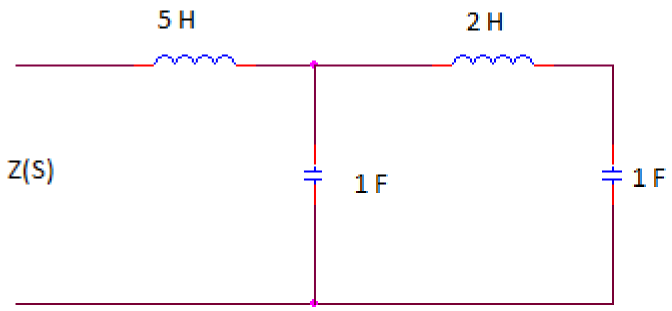


(2) Find the two Cauer realisations of driving point function given by $Z(S)=\dfrac{10S^4+12S^2+1}{2S^3+2S}$

Cauer 1:

$$
\begin{array}{l}
2S^3+2S\big)\,10S^4+12S^2+1\,\big(5S \\
\qquad\qquad \underline{10S^4+10S^2} \\
\qquad\qquad\qquad 2S^2+1\big)\,2S^3+2S\,\big(S \\
\qquad\qquad\qquad\qquad \underline{2S^3+S} \\
\qquad\qquad\qquad\qquad\qquad S\big)\,2S^2+1\,\big(2S \\
\qquad\qquad\qquad\qquad\qquad \underline{2S^2} \\
\qquad\qquad\qquad\qquad\qquad\qquad 1\big)\,S\,\big(S \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{S} \\
\qquad\qquad\qquad\qquad\qquad\qquad \underline{0}
\end{array}
$$

$$Z(S)=5S+\cfrac{1}{S+\cfrac{1}{2S+\cfrac{1}{S}}}$$



Cauer II

$$Z(S)=\frac{10S^4+12S^2+1}{2S^3+2S}=\frac{1+12S^2+10S^4}{2S+2S^3}$$

$$2S+2S^3\overline{)1+12S^2+10S^4}\left(\frac{1}{2S}\right.$$

$$\underline{1+S^2}$$

$$11S^2+10S^4\overline{)2S+2S^3}\left(\frac{2}{11S}\right.$$

$$\underline{2S+\frac{20S^3}{11}}$$

$$\frac{2S^3}{11}\overline{)11S^2+10S^4}\left(\frac{121}{2S}\right.$$

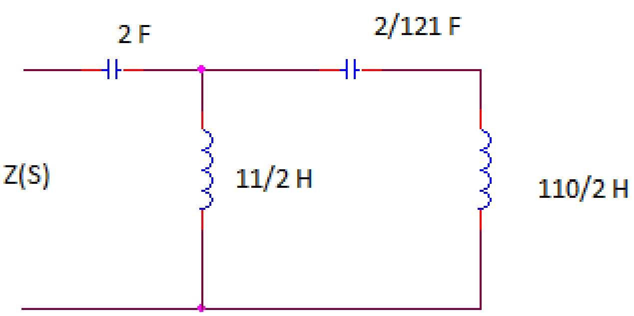
$$\underline{11S^2}$$

$$10S^4\overline{)\frac{2}{11}S^3}\left(\frac{2}{110S}\right.$$

$$\underline{\frac{2}{11}S^3}$$

$$\underline{0}$$

$$Z(S)=\frac{1}{2S}+\cfrac{1}{\cfrac{2}{11S}+\cfrac{1}{\cfrac{121}{2S}+\cfrac{1}{\cfrac{2}{110S}}}}$$



<u>Properties of RC Driving point function</u>

- RC network consists of R & C components.
- Driving point impedance of RC network is denoted as $Z_{RC}(S)$.
- Properties of driving point admittance of RL network are identical.

eg:– $z_{RC}(S)=\dfrac{(S+1)(S+4)}{S(S+2)}$ Poles are $S=0,-2$

Zeros are $S=-1,-4$

Properties of RC Driving point Impedance fns:

1) Poles and zeros are simple, no multiple poles & zeros.
2) Poles & zeros are located on negative real axis & alternating.
3) Critical frequency nearest to the origin is a pole (located at origin).
4) Critical frequency farthest from the origin is a zero (located at ∞).
5) The partial fraction expansion gives the residues values which are real & positive.
6) If $Z_{RC}(0)\geq Z_{RC}(\infty)$.
7) There is no zero at the origin & no pole located at infinity.

Synthesis of RC network:

Foster form I:

Driving point impedance RC n/w, Z(S) is given by $Z(S)=\dfrac{H(S+\sigma_1)(S+\sigma_3)\cdots}{S(S+\sigma_2)(S+\sigma_4)\cdots}$

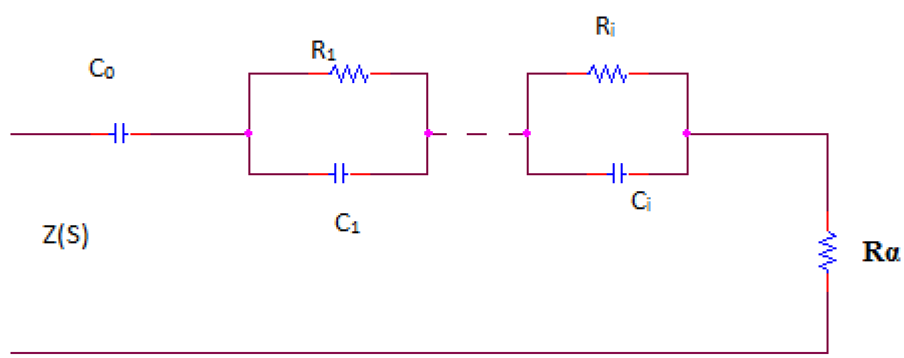
$$Z_{RC}(S)=\frac{K_0}{S}+\frac{K_1}{S+\sigma_1}+\frac{K_2}{S+\sigma_2}+\cdots+\frac{K_i}{S+\sigma_i}+\cdots+K_\infty$$

Where K_0, K_∞, K_i are the residues at origin, infinity & σ_i respectively.

$\dfrac{K_0}{S}$ represents a capacitor of $\dfrac{1}{K_0}$ F.

K_∞ represents a capacitor of K_∞ ohms.

$\dfrac{K_i}{S+\sigma_i}$ represents a parallel combination of capacitor of $\dfrac{1}{K_i}$ & resistance of $\dfrac{K_i}{\sigma_i}$



Foster Form II:

It is used to realize $Y_{RC}(S)=\dfrac{1}{Z_{RC}(S)}$ [negative residues at poles]

$$\frac{Y_{RC}(S)}{S}=\frac{K_0}{S}+\frac{K_1}{S+\sigma_1}+\frac{K_2}{S+\sigma_2}+\cdots+\frac{K_i}{S+\sigma_i}+\cdots+K_\infty$$

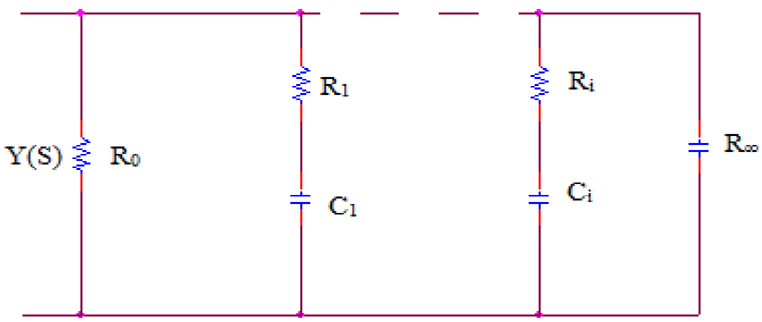
Residues of the expansion K_i will be negative, to make positive $\dfrac{Y_{RC}(S)}{S}$.

$$\therefore Y_{RC}(S)=K_0+\frac{K_1S}{S+\sigma_1}+\frac{K_2S}{S+\sigma_2}+\cdots+\frac{K_iS}{S+\sigma_i}+\cdots+K_\infty S$$

K_0 represents a resistance of $\dfrac{1}{K_0}\Omega$.

$\dfrac{K_iS}{S+\sigma_i}$ represents a series combination of resistance of $\dfrac{1}{K_i}$ ohms & a capacitance of $\dfrac{K_i}{\sigma_i}$.F

K_∞ represents a capacitance of $\dfrac{K_i}{\sigma_i}$.F



1) Find the Foster I & II form for the function $Z(S)=\dfrac{3(S+2)(S+4)}{(S+1)(S+3)}$

Numerator degree>denominator degree

$$Z(S)=\frac{3\left(S^2+2S+4S+8\right)}{S^2+S+3S+3}=\frac{3S^2+18S+24}{S^2+4S+3}$$

$$S^2+4S+3\overline{\big)3S^2+18S+24}\big(3$$
$$\underline{S^2+12S+9}$$
$$6S+15$$

$$Z(S)=3+\frac{6S+15}{S^2+4S+3}=3+\frac{6S+15}{(S+1)(S+3)}$$

$$Z(S) = 3 + \frac{A}{S+1} + \frac{B}{S+3}$$

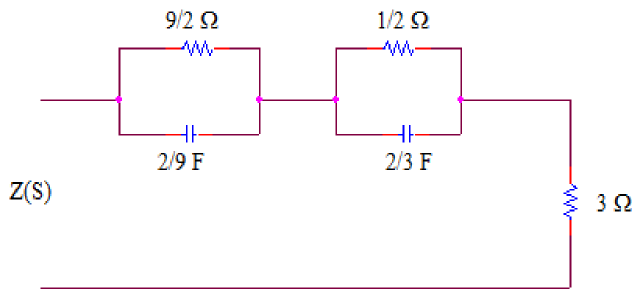
$$Z(S) = 3 + \frac{9/2}{S+1} + \frac{3/2}{S+3}$$

$$Z(S) = \frac{K_0}{S} + \frac{K_i}{S+\sigma_i} + K_\infty$$

$$R_\infty = 3$$

$$\frac{9/2}{S+1} \Rightarrow \begin{aligned} R_i &= \frac{K_i}{\sigma_i} = \frac{9}{2}\Omega \\ C_i &= \frac{1}{K_i} = \frac{2}{9}F \end{aligned}$$

$$\frac{3/2}{S+3} \Rightarrow \begin{aligned} R_i &= \frac{K_i}{\sigma_i} = \frac{1}{2}\Omega \\ C_i &= \frac{1}{K_i} = \frac{2}{3}F \end{aligned}$$



Foster II:

$$Y(S) = \frac{S^2 + 4S + 3}{3S^2 + 18S + 24}$$

$$3S^2 + 18S + 24 \overline{\big) S^2 + 4S + 3} \left(\frac{1}{3} \right.$$

$$\frac{S^2 + 6S + 8}{-2S - 5}$$

Negative terms appear.

$$\frac{Y(S)}{S} = \frac{(S+1)(S+3)}{3S(S+2)(S+4)} = \frac{A}{S} + \frac{B}{S+2} + \frac{C}{S+4}$$

$$\frac{Y(S)}{S} = \frac{1/8}{S} + \frac{1/12}{S+2} + \frac{1/8}{S+4}$$

$$Y(S) = \frac{1}{8} + \frac{1/12\,S}{S+2} + \frac{1/8\,S}{S+4}$$

$$= \frac{1}{8} + \frac{1}{12/S\,(S+2)} + \frac{1}{8/S\,(S+4)}$$

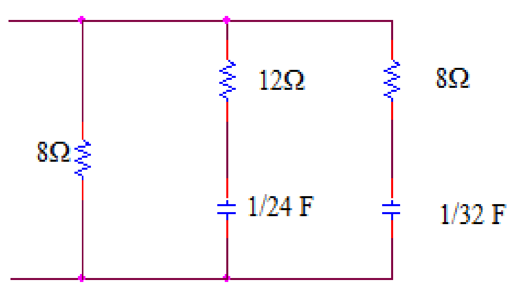
$$= \frac{1}{8} + \frac{1}{12 + 24/S} + \frac{1}{8 + 32/S}$$

$$Y(S) = K_0 + \frac{K_i S}{S+\sigma_i} + K_\infty S$$

$$K_0 = \frac{1}{8} \Rightarrow R_0 = 8$$

$$\frac{K_i S}{S+\sigma_i} \Rightarrow \begin{array}{l} K_i \Rightarrow \frac{1}{12} \Rightarrow C_i = \frac{K_i}{\sigma_i} = \frac{1}{24}F \\ \sigma_i \Rightarrow 2 \qquad R_i = \frac{1}{K_i} = 12\Omega \end{array}$$

$$\frac{1/8\,S}{S+4} \Rightarrow \frac{K_i S}{S+\sigma_i} \Rightarrow \begin{array}{l} K_i = \frac{1}{8}, \quad \therefore\ C_i = \frac{K_i}{\sigma_i} = \frac{1}{32}F \\ \sigma_i \Rightarrow 4, \qquad R_i = \frac{1}{K_i} = 8\Omega \end{array}$$

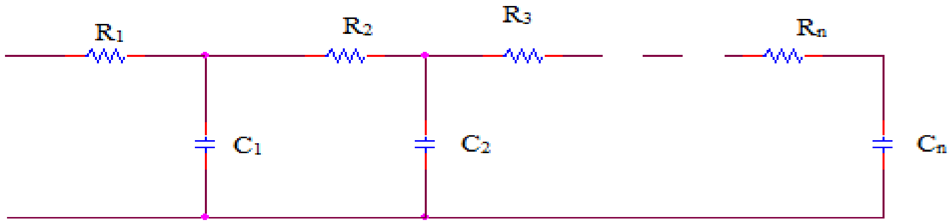


Synthesis of RC network by Cauer method:

Cauer I method:-

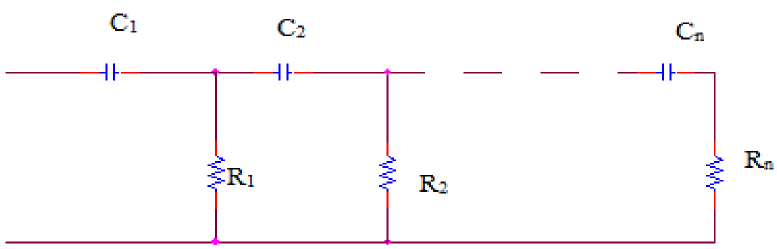
$$F(S) = q_1 + \cfrac{1}{q_2 S + \cfrac{1}{q_3 + \cfrac{1}{q_4 S} + \cdots + \cfrac{1}{q_n S}}}$$

$$= R_1 + \cfrac{1}{C_1 S + \cfrac{1}{R_2 + \cfrac{1}{C_2 S} + \cdots}}$$



Cauer II method:-

$$F(S) = \frac{1}{C_1 S} + \cfrac{1}{\cfrac{1}{R_1} + \cfrac{1}{\cfrac{1}{C_2 S} + \cfrac{1}{\cfrac{1}{R_2}} + \cdots}}$$



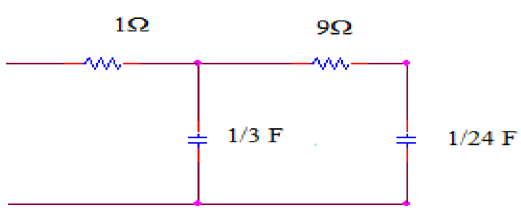
(1)Find the 1st& 2nd Cauer form of $Z(S) = \frac{(S+2)(S+4)}{S(S+3)}$

Foster 1:

$$Z(S) = \frac{S^2 + 6S + 8}{S^2 + 3S}$$

$$S^2+3S\,\big)\,S^2+6S+18\,\big(1 \qquad \rightarrow Z$$
$$\underline{S^2+3S}$$
$$3S+18\,\big)\,S^3+3S\,\big(S/3 \qquad \rightarrow Y$$
$$\underline{S^2+8S/3}$$
$$S/3\,\big)\,3S+8\,\big(9 \qquad \rightarrow Z$$
$$\underline{3S}$$
$$8\,\big)\,S/3\,\big(S/24 \qquad \rightarrow Y$$
$$\underline{S/3}$$
$$\underline{0}$$

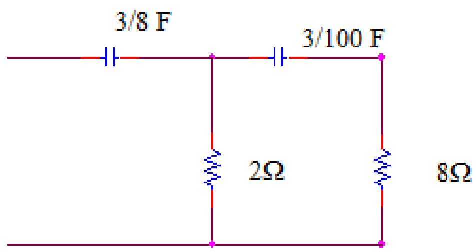
$$Z(S)=1+\cfrac{1}{\cfrac{S}{3}+\cfrac{1}{9+\cfrac{1}{\cfrac{S}{24}}+\cdots}}$$



Cauer II:

$$3S+S^2\,\big)\,8+8S+S^2\,\big(8/35 \qquad \rightarrow Z$$
$$\underline{8+8S/3}$$
$$10S/3+S^2\,\big)\,3S+S^2\,\big(9/10 \qquad \rightarrow Y$$
$$\underline{3S+9S^2/10}$$
$$S^2/10\,\big)\,10S/3+S^2\,\big(100/3S \qquad \rightarrow Z$$
$$\underline{10S/3}$$
$$S^2\,\big)\,S^2/10\,\big(1/10 \qquad \rightarrow Y$$
$$\underline{S^2/10}$$
$$\underline{0}$$

$$Z(S) = \frac{8}{3S} + \cfrac{1}{\cfrac{9}{10} + \cfrac{1}{\cfrac{100}{3S} + \cfrac{1}{\cfrac{1}{10}}}}$$



Synthesis of Driving point Impedance functions of RL Network's:-

- Driving point impedance is denoted as $Z_{RL}(S)$.
- $Z_{RL}(S)$ & $Y_{RC}(S)$ are identical

Properties:-

1) Poles & zeros are on the negative real axis of S-plane and are simple.

   $Z(S) = \frac{(S+1)(S+3)}{(S+2)(S+4)}$

   Poles at $S = -2, -4$

   Zeros at $S = -1, -3$

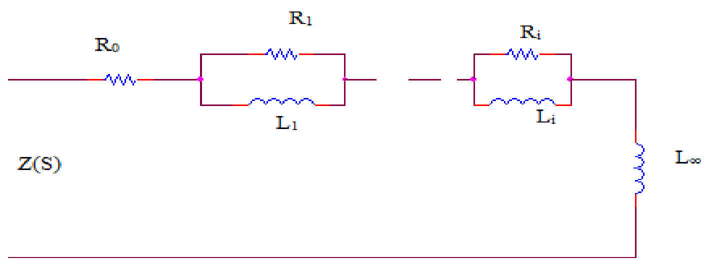
2) Poles & zeros are alternating.
3) Poles & zeros are the critical frequencies.
4) Critical frequency nearest to the origin is a zero.
5) Critical frequency farthest from the origin is a pole.
6) There cannot be a pole at the origin and cannot be a zero at infinity.
7) $Z_{RL}(\infty) \geq Z_{RL}(0)$
8) Residues of $Z_{RL}(S)$ at its poles are real & negative and those of $\frac{Z_{RC}(S)}{S}$ are real and positive.

Synthesis:-Foster form I:-

$$Z_{RL}(S) = K_0 + \frac{K_1 S}{S+\sigma_1} + \frac{K_2 S}{S+\sigma_2} + \cdots + \frac{K_i S}{S+\sigma_i} + \cdots + K_\infty S$$

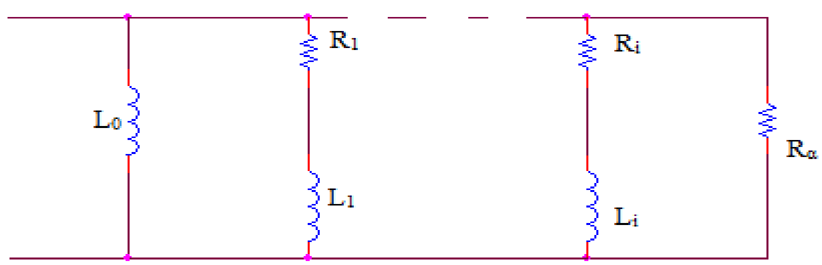
$$\frac{Z_{RL}(S)}{S}=\frac{K_0}{S}+\frac{K_1}{S+\sigma_1}+\cdots+\frac{K_i}{S+\sigma_i}+\cdots+K_\infty$$

$$Z_{RL}(S)=K_0+\frac{K_1S}{S+\sigma_1}+\frac{K_2S}{S+\sigma_2}+\cdots+\frac{K_iS}{S+\sigma_i}+\cdots+K_\infty S.$$



Foster form II:-

$$Y_{RL}(S)=\frac{K_0}{S}+\frac{K_1}{S+\sigma_1}+\frac{K_2}{S+\sigma_2}+\cdots+\frac{K_i}{S+\sigma_i}+\cdots+K_\infty$$



1) Find the 1st foster form of the driving point function:-

$$Z(S)=\frac{S(S+1)(S+4)}{(S+3)(S+5)}=\frac{S(S^2+S+4S+4)(S+4)}{S^2+3S+5S+15}=\frac{S^2+25S+20}{S^2+8S+15}$$

$$\begin{array}{r l}
S^2+8S+15\big) & 5S^2+25S+20\big(5 \\
 & \underset{(-)}{5S^2}+\underset{(-)}{40S}+75 \\
 & \overline{-15S-55}
\end{array}$$

$$\frac{Z(S)}{S}=\frac{5(S+1)(S+4)}{S(S+3)(S+5)}=\frac{A}{S}+\frac{B}{S+3}+\frac{C}{S+5}$$

$$A=\left.\frac{5(S+1)(S+4)}{S(S+3)(S+5)}\right|_{S=0}=\frac{5(1)(4)}{(3)(5)}=\frac{4}{3}$$

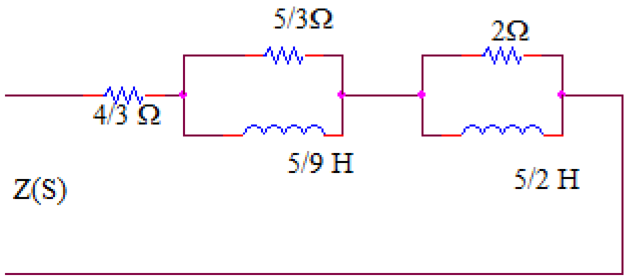
$$B = \left.\frac{5(S+1)(S+4)}{(S+3)(S+5)}\right|_{S=-3} = \frac{5(-2)(1)}{(-3)(2)} = \frac{5}{3}$$

$$C = \left.\frac{5(S+1)(S+4)}{S(S+3)}\right|_{S=-5} = \frac{5(-4)(-1)}{(-5)(-2)} = 2$$

$$\frac{Z(S)}{S} = \frac{4/3}{S} + \frac{5/3}{S+3} + \frac{2}{S+5}$$

$$Z(S) = \frac{4}{3} + \frac{5/3\,S}{S+3} + \frac{2S}{S+5}$$

$$= \frac{4}{3} + \frac{1}{3/5S\,(S+3)} + \frac{1}{1/2S\,(S+5)} = \frac{4}{3} + \frac{1}{3/5 + 9/5S} + \frac{1}{1/2 + 5/2S}$$



2) Find the 2nd foster form of the driving point function:-

$$Y(S) = \frac{2S^2+16S+30}{S^2+6S+8}$$

$$S^2+6S+8\,\big)\,2S^2+16S+30\,\big(\,2$$
$$\underline{2S^2+12S+16}$$
$$4S+14$$

$$Y(S) = \frac{2S^2+16S+30}{S^2+6S+8} = 2 + \frac{4S+14}{S^2+6S+8} = 2 + \frac{4S+14}{(S+4)(S+2)}$$

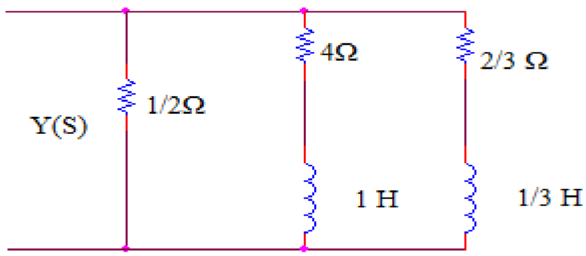
$$\frac{4S+14}{(S+4)(S+2)} = \frac{A}{S+4} + \frac{B}{S+2}$$

$$A = \left.\frac{4S+14}{S+2}\right|_{S=-4} = \frac{-16+14}{-2} = 1$$

$$B = \left.\frac{4S+14}{S+4}\right|_{S=-2} = \frac{-8+14}{2} = 3$$

$$Y(S) = 2 + \frac{1}{S+4} + \frac{3}{S+2}$$

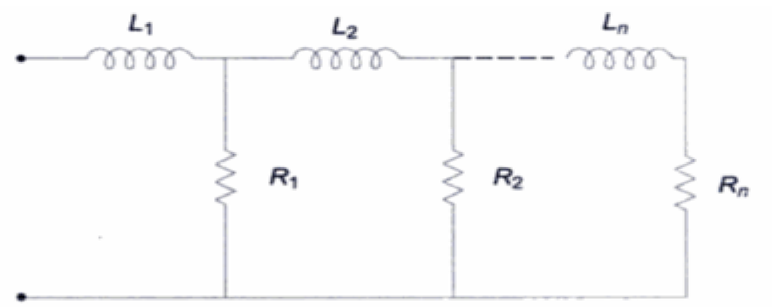
$$= 2 + \frac{1}{S+4} + \frac{1}{S/3 + 2/3}$$



<u>Cauer I</u>:-

Descending power of S.

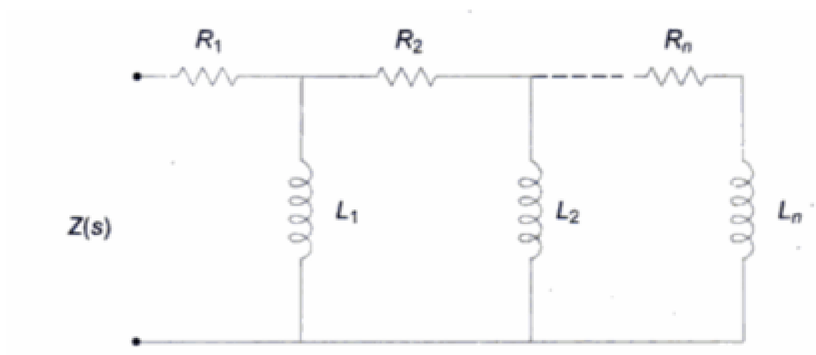
$$F(S) = \underset{(\text{series})}{q_1(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_2(S)} + \cfrac{1}{\underset{(\text{series})}{Z_2(S)} + \cfrac{1}{\underset{(\text{shunt})}{Y_2(S)}}}} \quad = \quad L_1S + \cfrac{1}{R_2 + \cfrac{1}{L_3(S) + \cfrac{1}{R_4} + \cdots + \cfrac{1}{R_\infty}}}$$



<u>Cauer II</u>:-

Ascending power of S.

$$Z(S) = R_1 + \cfrac{1}{SL_1 + \cfrac{1}{R_2 + \cfrac{1}{SL_2 + \cfrac{1}{R_3} + \cdots}}}$$

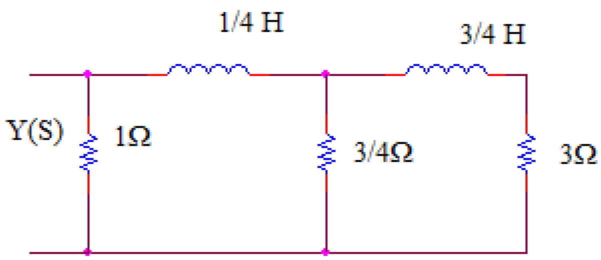


1) Find the 1[st] Cauer form of $Y(S)=\dfrac{(S+4)(S+8)}{(S+2)(S+6)}$

$$Y(S)=\frac{S^2+12S+32}{S^2+8S+12}$$

$$S^2+8S+12\overline{)S^2+12S+32}(1 \qquad \rightarrow Y$$
$$\underline{S^2+8S+12}$$
$$4S+20\overline{)S^2+8S+12}\left(S/4\right. \qquad \rightarrow Z$$
$$\underline{S^2+5S}$$
$$3S+12\overline{)4S+20}\left(4/3\right. \qquad \rightarrow Y$$
$$\underline{4S+16}$$
$$4\overline{)3S+12}\left(3/4\,S\right. \qquad \rightarrow Z$$
$$\underline{3S}$$
$$12\overline{)4}\left(1/3\right. \qquad \rightarrow Y$$
$$\underline{4}$$
$$\underline{0}$$

$$Y(S)=1+\cfrac{1}{\cfrac{S}{4}+\cfrac{1}{\cfrac{4}{3}+\cfrac{1}{\cfrac{3S}{4}+\cfrac{1}{3}}}}$$



(2)Find the 2nd Cauer form for $Z(S) = \dfrac{2S^2+8S+6}{S^2+8S+12}$

$$Z(S) = \frac{6+8S+2S^2}{12+8S+S^2}$$

$$12+8S+S^2 \,\big)\, 6+8S+2S^2 \,\Big(\, \frac{1}{2} \qquad \rightarrow Z$$

$$\underline{6+4S+\frac{1}{2}S^2}$$

$$4S+\frac{3}{2}S^2 \,\big)\, 12+8S+S^2 \,\big(\, 3/S \qquad \rightarrow Y$$

$$\underline{12+9/2\,S}$$

$$7/2\,S+S^2 \,\big)\, 4S+3/2\,S^2 \,\big(\, 8/7 \qquad \rightarrow Z$$

$$\underline{4S+8/7\,S^2}$$

$$5/14\,S^2 \,\big)\, 7/2\,S+S^2 \,\big(\, 49/5S$$

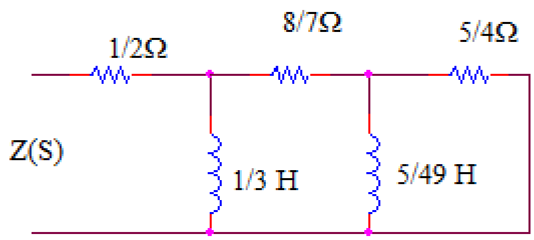
$$\underline{7/2\,S}$$

$$S^2 \,\big)\, 5/14\,S^2 \,\big(\, 5/14 \qquad \rightarrow Z$$

$$\underline{5/14\,S^2}$$

$$\underline{0}$$

$$Z(S)=\frac{1}{2}+\cfrac{1}{\frac{3}{S}+\cfrac{1}{\frac{8}{7}+\cfrac{1}{\frac{49}{5S}+\cfrac{1}{\frac{5}{14}}}}}$$



<u>Synthesis of RLC networks</u>:-

1) Synthesize the impedance function $Z(S)=\dfrac{S^2+7S+70}{S(S+10)}$.

$$Z(S)=\frac{S^2+7S+70}{S(S+10)} \text{ has a pole at origin.}$$

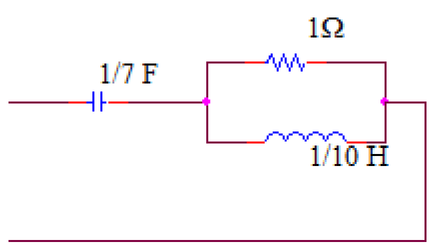
$$10S+S^2\overline{)70+7S+S^2}\left(7/S\right.$$

$$\underline{70+7S}$$

$$S^2$$

$$Z(S)=\frac{7}{S}+\frac{S^2}{S^2+10S}$$

$$Z_1(S)=\frac{7}{S},\quad Z_2(S)=\frac{S}{S+10}=\frac{1}{1+10/S}$$



2) Synthesize a network having impedance function,

$$Z(S) = \frac{6S + 3S^2 + 3S + 1}{6S^3 + 3S}$$

$$6S^3 + 3S \,\big)\, 6S^3 + 3S^2 + 3S + 1 \,\big(\, 1 \quad \rightarrow \quad Z$$

$$\underline{6S^3 + \qquad 3S}$$

$$3S^2 \qquad +1$$

$$Z(S) = 1 + \frac{3S^2 + 1}{6S^3 + 3S}$$

$$Z(S) = Z_1(S) + Z_1(S)$$

$$Z_2(S) = \frac{3S^2 + 1}{6S^3 + 3S}$$

$$Y_2(S) = \frac{6S^3 + 3S}{3S^3 + 1}$$

$$3S^2 + 1 \,\big)\, 6S^3 + 3S \,\big(\, 2S \qquad \rightarrow Y$$

$$\underline{S^3 + 2S}$$

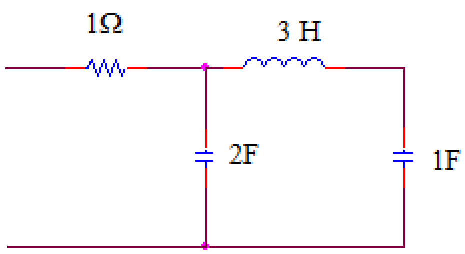
$$S \,\big)\, 3S^2 + 1 \,\big(\, 3S \qquad \rightarrow Z$$

$$\underline{3S^2}$$

$$1 \,\big)\, S \,\big(\, S \qquad \rightarrow Y$$

$$\underline{S}$$

$$\underline{0}$$



Part A

1. What are the properties of RC network?
2. What is mean by synthesis of network?
3. Test whether the polynomial $H(S) = S^5+7S^4+5S^3+S^2+S$ is Hurwitz. Give reason
4. Maintain the difference between first cauer form and second cauer form of LC network
5. What are the two foster forms?
6. For the given function determine cauer form of realization $Y(S) = (S(S+2)(S+4))/(S(S+3))$
7. Give any two conditions for a polynomial to be Hurwitz.
8. What are the properties of impedance function?
9. Write the conditions of the positive real function.
10. List out the properties of the RL impedance function

Part B

1. Check whether the given polynomial is Hurwitz or not $P(S)=S^6+3S^5+8S^4+15S^3+17S^2+12S+4$
2. Realize the given RC network impedance function using foster I and Cauer II forms $Z(S)=((S+1)(S+4))/(S(S+2))$
3. Synthesis the transfer impedance $Z_{21}=1/(S^3+3S^2+3S+2)$ with 1Ω termination.
4. Discuss the synthesis of RL network by cauer method and obtain first and second cauer of the network
5. Test the following polynomial are Hurwitz
   a) $P(S)= S^3+4S^2+5S+2$
   b) $P(S)= S^4+S^3+S^2+2S+12$
6. Synthesis the network using foster method II. Give admittance $Y(S)=((S(S^2+2)(S^2+4))/((S^2+1)(S^2+3))$
7. Find the Two foster realization of $Z(S)=(4(S^2+1)(S^2+16))/(S(S^2+4)$
8. Test whether the give equation is Hurwitz polynomial or not
   a) P(S)= S5+8S4+24S3+28S2+23S+6
   b) $F(S)= 2S^6+S^5+13S^4+6S^3+56S^2+25S+25$
9. Find two cauer realization of driving point function given by $Z(S)=(10S^4+12S^2+1)/(2S^3+2S)$
10. The driving point impedance of a one port reactive network is given by $Z(S)= (5(S^2+4)(S^2+25))/(S(S^2+16))$ obtain first and second foster networks

SCHOOL OF ELECTRICAL AND ELECTRONICS
DEPARTMENT OF ELECTRICAL AND ELECTRONICS

UNIT – V – NETWORK TOPOLOGY

NETWORK TOPOLOGY

1. Basic definitions:

Network Topology:

- Is another method of solving electric circuits
- Is generalized approach

Network:

A combination of two or more network elements is called a network.

Topology:

Topology is a branch of geometry which is concerned with the properties of a geometrical figure, which are not changed when the figure is physically distorted, provided that, no parts of the figure are cut open or joined together.

The geometrical properties of a network are independent of the types of elements and their values.

Every element of the network is represented by a line segment with dots at the ends irrespective of its nature and value.

Circuit:

If the network has at least one closed path it is a circuit.

Note that every circuit is a network but every network is not a circuit.

Branch:

Representation of each element (component) of a electric network by a line segment is a branch.

Node:

A point at which two or more elements are joined is a node. End points of the branches are called nodes.

Graph:

It is collection of branches and nodes in which each branch connects two nodes.

Graph of a Network:

The diagram that gives network geometry and uses lines with dots at the ends to represent network element is usually called a graph of a given network. For example,

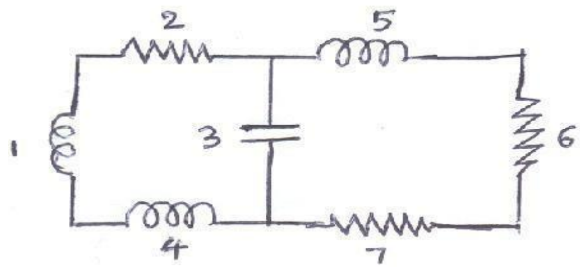


Fig.5.1 Network

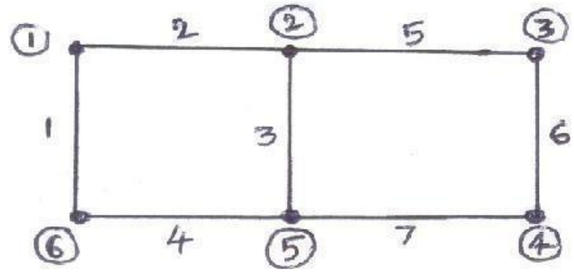


Fig.5.2 Graph

SUB GRAPH

A sub-graph is a subset of branches and nodes of a graph for example branches 1, 2, 3 & 4 forms a sub-graph. The sub-graph may be connected or unconnected. The sub- graph of graph shown in figure 2 is shown in figure 3.

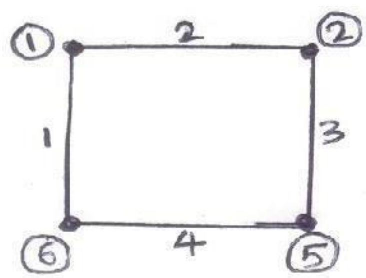


Fig.5.3 Sub-graph

Connected Graph:

If there exists at least one path from each node to every other node, then graph is said to be connected. Example,

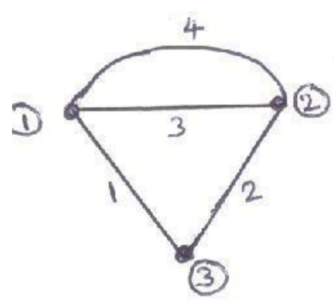


Fig.5.4 Connected Graph

Un-connected Graph:

If there exists no path from each node to every other node, the graph is said to be un-connected graph. For example, the network containing a transformer (inductively coupled parts) its graph could be un-connected.

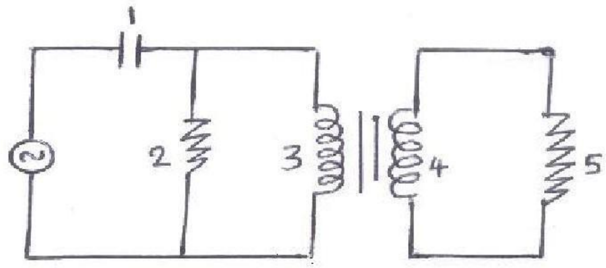


Fig.5.5

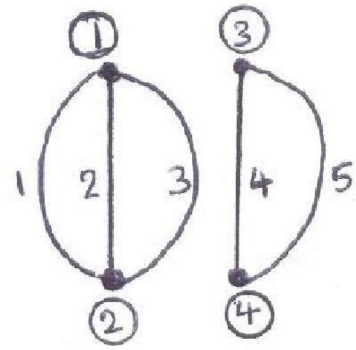


Fig.5.6 Un-connected Graph

A sequence of branches going from one node to other is called path. The node once considered should not be again considered the same node.

Loop (Closed Path):

Loop may be defined as a connected sub-graph of a graph, which has exactly two branches of the sub-graph connected to each of its node.

For example, the

branches1, 2 & 3 in figure 7 constitute a loop.

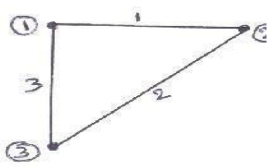


Fig 5.7 Graph

Planar and Non-planar Graphs:

A planar graph is one where the branches do not cross each other while drawn on a plain sheet of paper. If they cross, they are non-planar.

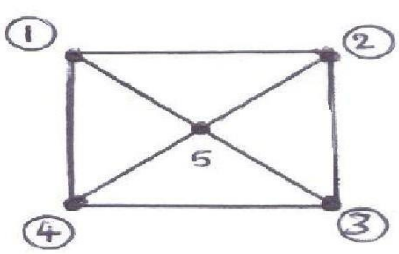


Fig.5.8 Planar Graph

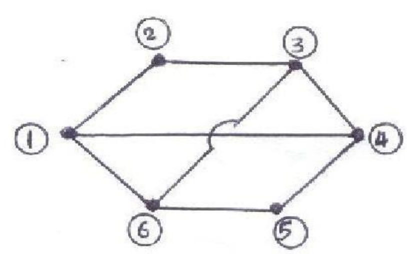


Fig.5.9 Non-planar graph

Oriented Graph:

The graph whose branches carry an orientation is called an oriented graph

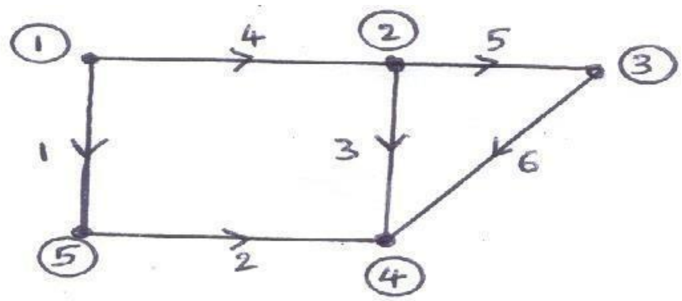


Fig.5.10 Oriented Graph

The current and voltage references for a given branches are selected with a +ve sign at tail side and –ve sign at head

Tree:

Tree of a connected graph is defined as any set of branches, which together Connect all the nodes of the graph without forming any loops. The branches of a tree are called **Twigs.**

Co-tree:

Remaining branches of a graph, which are not in the tree, form a co-tree. The branches of a co-tree are called **links** or **chords.**

The tree and co-tree for a given oriented graph shown in figure5.11 is shown in Figure 5.12 and figure5.13.

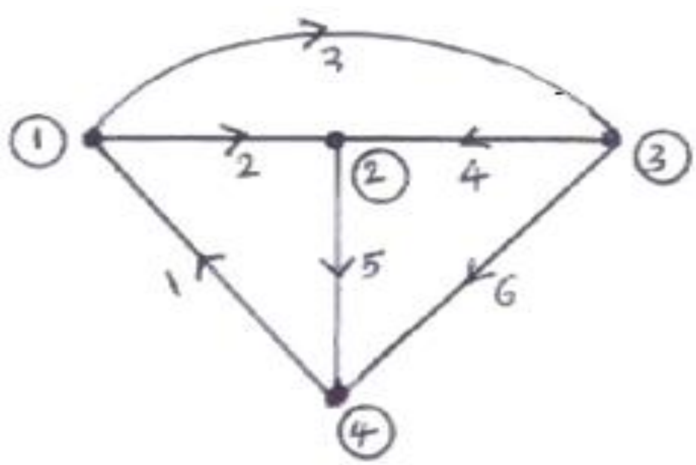


Fig. 5.11 Oriented Graph

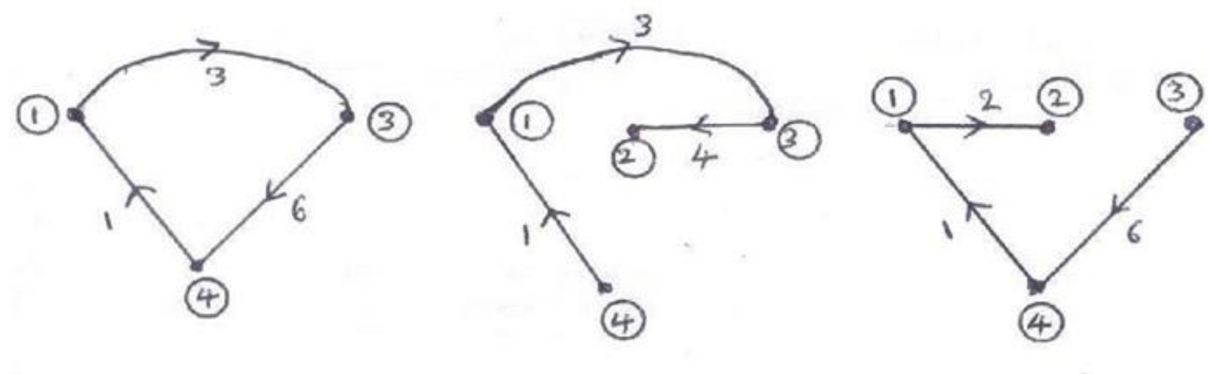


Fig. 5.12 Trees

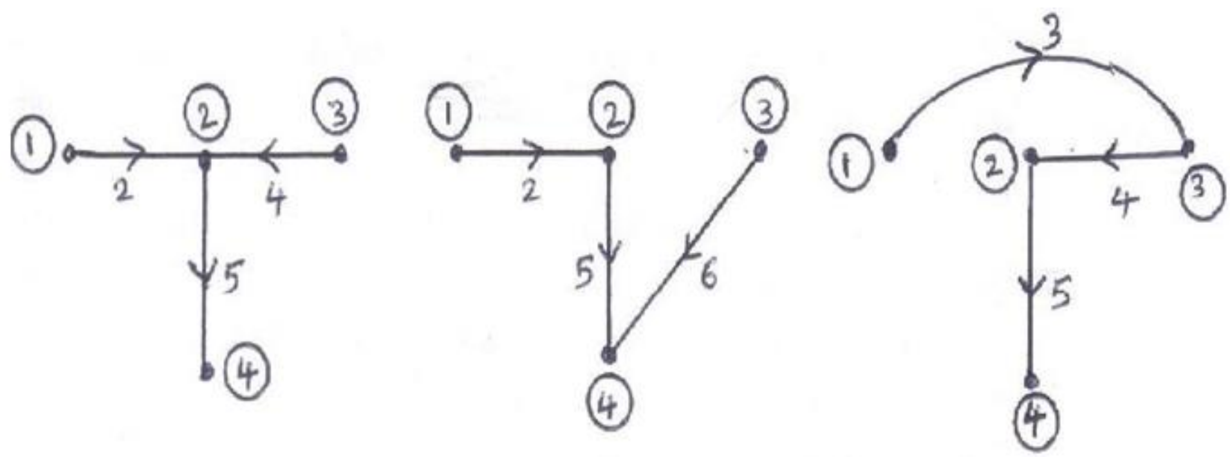


Fig.5.13 Co-trees

Tree	Twigs	Links (Chords)
1	2, 4 & 5	1, 3 & 6
2	3, 4 & 5	1, 2 & 6
3	2, 5 & 6	1, 3 & 4

Properties of Tree:

i) It contains all the nodes of the graph.

ii) It contains (n_t-1) branches. Where 'n_t' is total number of nodes in the given graph.

iii) There are no closed paths.

Total number of tree branches, n = (n_t-1)

Where n_t = Total number of nodes Total number of

links, l = (b-n)

Where b = Total number of branches in the graph.

Degree of Node:

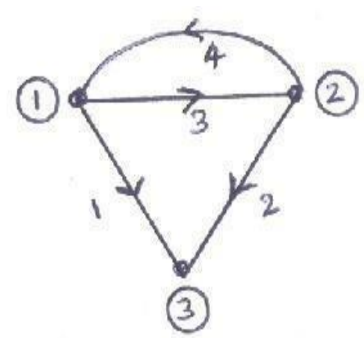
The number of branches attached to the node is degree of node.

II. Complete Incidence Matrix (A_a):

Incidence matrix gives us the information about the branches, which are joined to the nodes and the orientation of the branch, which may be towards a node or away from it.

Nodes of the graph form the rows and branches form the columns. If the branch is not connected to node, corresponding element in the matrix is given the value **'0'**. If a branch is joined, it has two possible orientations. If the orientation is away from the node, the corresponding matrix element is written as **'+1'**. If it is towards the node, the corresponding matrix element is written as **'-1'**.

Example: 1) Obtain complete incidence matrix for the graph shown



Solution: Aa =

Nodes	Branches			
	1	2	3	4
1	1	0	1	-1
2	0	1	-1	1
3	-1	-1	0	0

$$A_a = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ -1 & -1 & 0 & 0 \end{bmatrix}$$

Properties of Incidence Matrix:

i) Each column has only two non-zero elements and all other elements are zero.

ii) If all the rows of **'A_a'** are added, the sum will be a row whose elements equal zero.

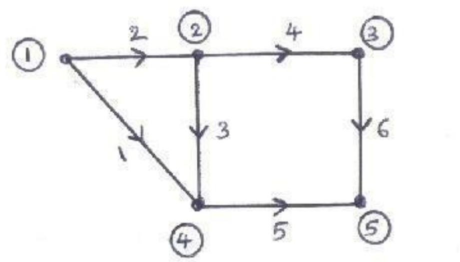
If the graph has **'b'** branches and **'n_t'** nodes, the complete incidence matrix is of the order (n_t x b).

III. Reduced Incidence Matrix (A):

When one row is eliminated from the complete incidence matrix, the remaining matrix is called **reduced incidence matrix**

If the graph has **'b'** branches and **'n_t'** nodes, the reduced incidence matrix is of the order $(n_t-1) \times b$.

Example: 2) write the complete and reduced incidence matrix for the given graph shown



Solution:

Aa =

Nodes	Branches					
	1	2	3	4	5	6
1	1	1	0	0	0	0
2	0	-1	1	1	0	0
3	0	0	0	-1	0	1
4	-1	0	-1	0	1	0
5	0	0	0	0	-1	-1

Complete Incidence Matrix, $A_a = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix}$

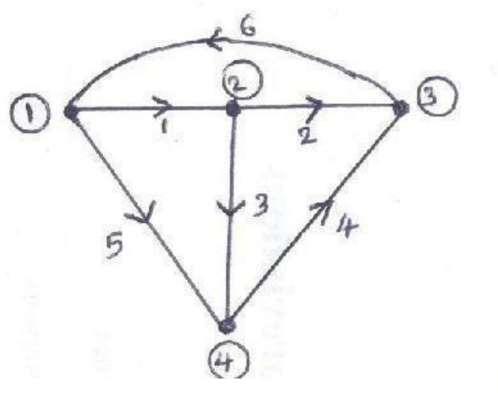
Reduced Incidence Matrix, $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & -1 & 0 & 1 & 0 \end{bmatrix}$

Example: 3) Draw the oriented graph of incidence matrix shown below

$$A_a = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Solution:

Total number of nodes = n_t = 4 Total number of branches = b = 6



Oriented Graph

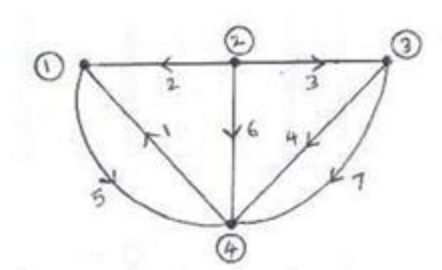
Example: 4) Draw the oriented graph of incidence matrix shown below

$$A = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Solution: The given matrix is a reduced incidence matrix. Obtain the complete incidence matrix in order to draw the oriented graph.

$$A_a = \begin{bmatrix} -1 & -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 & -1 \end{bmatrix}$$

Total number of nodes = n_t = 4
Total number of branches = b = 7



Oriented Graph

TIESET

A tie-set is a set of branches contained in a loop such that each loop contains one link or chord and remainder are tree branches.

Or

The set of branches forming the closed loop in which link or loop current circulate is called a Tie-set.

The tie-set consists of only one link and remaining are Twigs.

- The fundamental loop formed by one link has a unique path in the tree joining the two nodes of the link.

This loop is also called f-loop or a tie set.

- The orientation of the cut-set is same as orientation of link.

TIE-SET SCHEDULE

For a given network tree, a systematic way of indicating the links through the use of a schedule is called tie-set schedule

To write the tie-set for network graph,

(i) Consider an oriented network graph

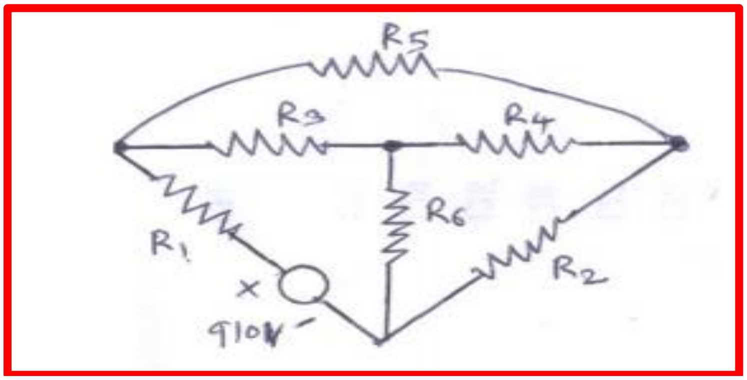
(ii) Write any one possible tree of the network graph

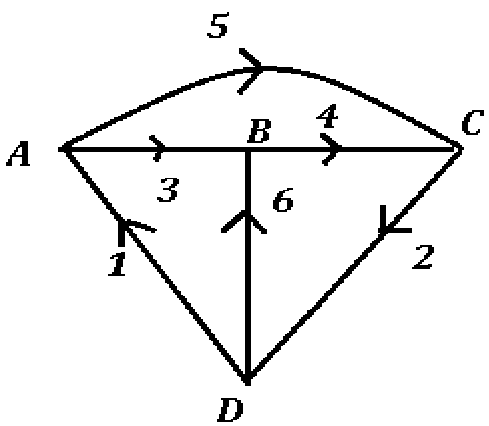
(iii)Connect a link to the tree branches to form a loop. In the same way form all Fundamental loops.

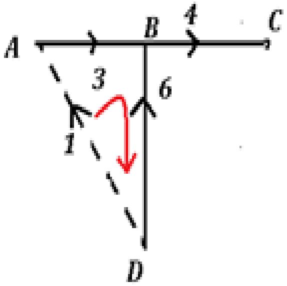
(iv)The loop current direction is same as that of the link.

(v) Form the Matrix the rows denotes the loop and columns denotes the branches

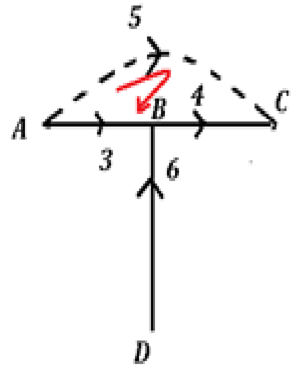
Problem 1:For the Given Network, Write a tie-set Schedule.



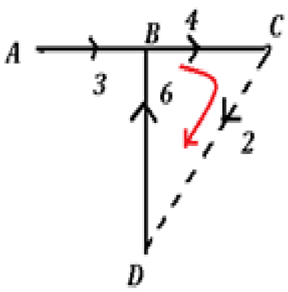




Loop	1	2	3	4	5	6
ABD	1	0	1	0	0	-1



Loop	1	2	3	4	5	6
ABC	0	0	-1	-1	1	0

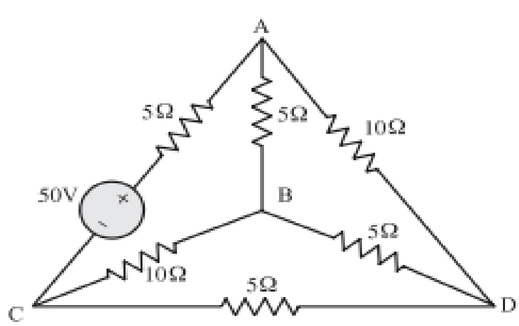


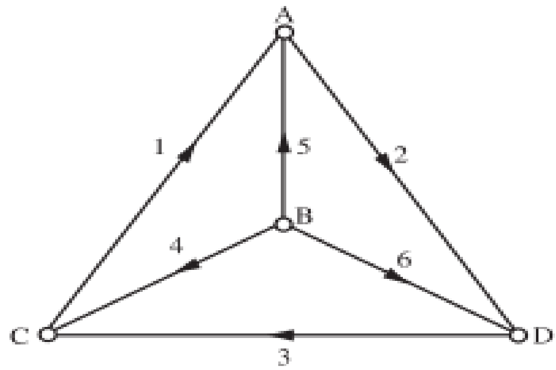
Loop	1	2	3	4	5	6
BCD	0	1	0	1	0	1

LOOPS	1	2	3	4	5	6
ABD	1	0	1	0	0	-1
ABC	0	0	-1	-1	1	0
BCD	0	1	0	1	0	1

$$M = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Problem 2: For the Given Network, Write a tie-set Schedule.

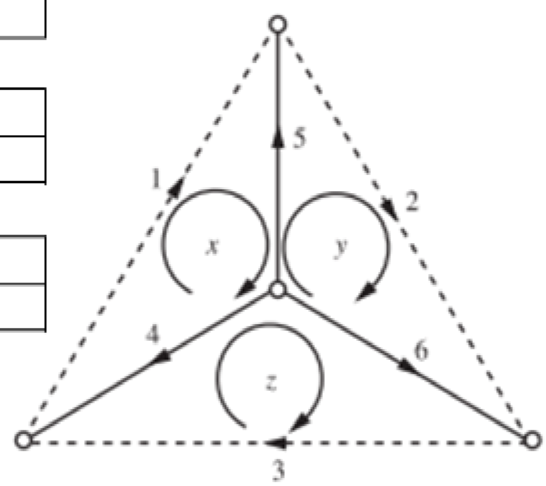




Loop	1	2	3	4	5	6
x	1	0	0	1	-1	0

Loop	1	2	3	4	5	6
y	0	1	0	0	1	-1

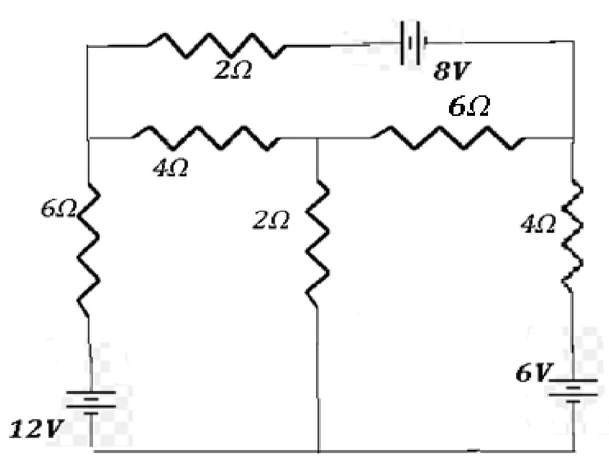
Loop	1	2	3	4	5	6
z	0	0	1	-1	0	1

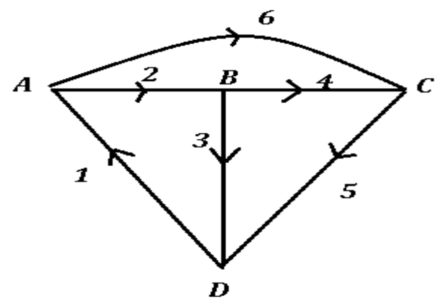


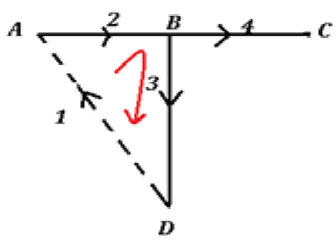
LOOPS	1	2	3	4	5	6
x	1	0	0	1	-1	0
y	0	1	0	0	1	-1
z	0	0	1	-1	0	1

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

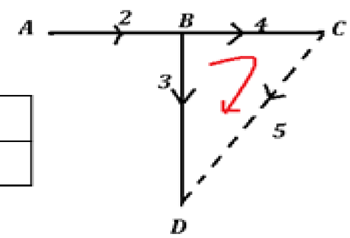
Problem 3: Find the Tieset Matrix and the Branch Voltages.



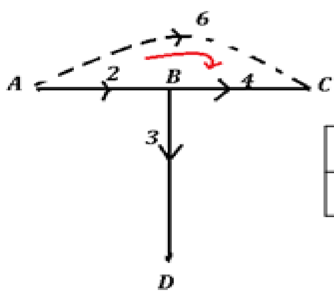




Loop	1	2	3	4	5	6
ABD	1	1	1	0	0	0



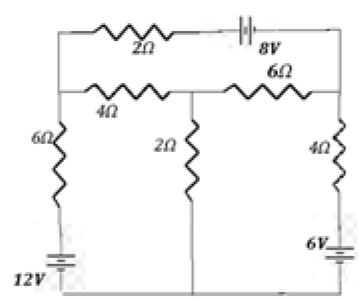
Loop	1	2	3	4	5	6
BCD	0	0	-1	1	1	0



Loop	1	2	3	4	5	6
AC	0	-1	0	-1	0	1

LOOPS	1	2	3	4	5	6
ABD	1	1	1	0	0	0
BCD	0	0	-1	1	1	0
AC	0	-1	0	-1	0	1

$$\mathrm{M} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \end{bmatrix}$$



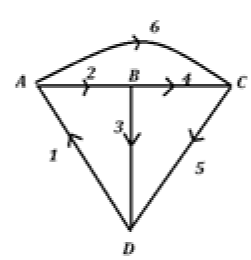
Rowwise

$$\boxed{\begin{gathered} e_1 + e_2 + e_3 = 0 \\ -e_3 + e_4 + e_5 = 0 \\ -e_2 - e_4 + e_6 = 0 \end{gathered}} \rightarrow \textbf{(I)}$$

Columnwise

$$\boxed{\begin{gathered} i_1 = I_1 \\ i_2 = I_1 - I_3 \\ i_3 = I_1 - I_2 \\ i_4 = I_2 - I_3 \\ i_5 = I_2 \\ i_6 = I_3 \end{gathered}} \rightarrow \textbf{(II)}$$

$$\boxed{\begin{gathered} e_1 + 12 = 6i_1 \\ e_2 = 4i_2 \\ e_3 = 2i_3 \\ e_4 = 6i_4 \\ e_5 - 6 = 4i_5 \\ e_6 - 8 = 2i_6 \end{gathered}} \rightarrow \textbf{(III)}$$



Comparing (II) and (III),

$$
\begin{aligned}
e_1 + 12 &= 6I_1 \\
e_2 &= 4(I_1 - I_3) \\
e_3 &= 2(I_1 - I_2 \\
e_4 &= 6(I_2 - I_3) \\
e_5 - 6 &= 4I_2 \\
e_6 - 8 &= 2I_3
\end{aligned}
$$

(IV)

Substituting (IV) in (I),

$$
\begin{gathered}
6I_1 - 12 + 4I_1 - 4I_3 + 2I_1 - 2I_2 = 0 \\
\mathbf{12I_1 - 2I_2 - 4I_3 = 12} \\
-2I_1 + 2I_2 + 6I_2 - 6I_3 + 6 + 4I_2 = 0 \\
\mathbf{-2I_1 + 12I_2 - 6I_3 = -6} \\
\mathbf{-4I_1 + 4I_3 - 6I_2 + 6I_3 + 8 + 2I_3 = 0} \\
\mathbf{-4I_1 - 6I_2 + 12I_3 = -8}
\end{gathered}
$$

(V)

$$
\begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix}
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}
=
\begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}
$$

$$
\Delta = \begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} = 960
$$

$$
\Delta_1 = \begin{bmatrix} 12 & -2 & -4 \\ -6 & 12 & -6 \\ -8 & -6 & 12 \end{bmatrix} = 528
$$

$$
\Delta_2 = \begin{bmatrix} 12 & 12 & -4 \\ -2 & -6 & -6 \\ -4 & -8 & 12 \end{bmatrix} = -832
$$

$$
\Delta_3 = \begin{bmatrix} 12 & -2 & 12 \\ -2 & 12 & -6 \\ -4 & -6 & -8 \end{bmatrix} = -880
$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{528}{960} = 0.55\text{A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-832}{960} = -0.866\text{A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-880}{960} = -0.916\text{A}$$

(IV)

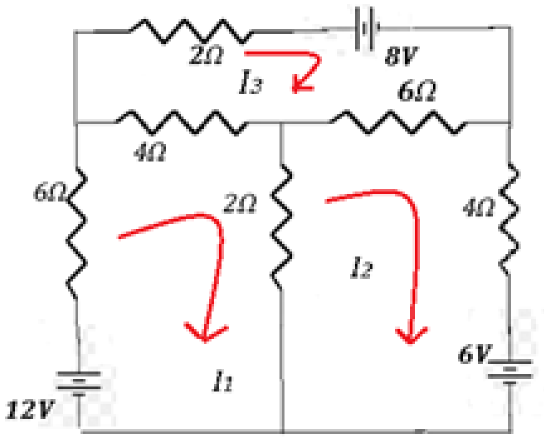
$$
\begin{aligned}
e_1 + 12 &= 6I_1 \\
e_2 &= 4(I_1 - I_3) \\
e_3 &= 2(I_1 - I_2 \\
e_4 &= 6(I_2 - I_3) \\
e_5 - 6 &= 4I_2 \\
e_6 - 8 &= 2I_3
\end{aligned}
$$

$$
\begin{gathered}
e_1 + e_2 + e_3 = 0 \\
-e_3 + e_4 + e_5 = 0 \\
-e_2 - e_4 + e_6 = 0
\end{gathered}
$$

(I)

$$
\begin{gathered}
6I_1 - 12 + 4I_1 - 4I_3 + 2I_1 - 2I_2 = 0 \\
\mathbf{12I_1 - 2I_2 - 4I_3 = 12} \\
-2I_1 + 2I_2 + 6I_2 - 6I_3 + 6 + 4I_2 = 0 \\
\mathbf{-2I_1 + 12I_2 - 6I_3 = -6} \\
\mathbf{-4I_1 + 4I_3 - 6I_2 + 6I_3 + 8 + 2I_3 = 0} \\
\mathbf{-4I_1 - 6I_2 + 12I_3 = -8}
\end{gathered}
$$

Verification by Mesh Analysis



$$\begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -6 \\ -8 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 12 & -2 & -4 \\ -2 & 12 & -6 \\ -4 & -6 & 12 \end{bmatrix} = 960$$

$$\Delta_1 = \begin{bmatrix} 12 & -2 & -4 \\ -6 & 12 & -6 \\ -8 & -6 & 12 \end{bmatrix} = 528$$

$$\Delta_2 = \begin{bmatrix} 12 & 12 & -4 \\ -2 & -6 & -6 \\ -4 & -8 & 12 \end{bmatrix} = -832$$

$$\Delta_3 = \begin{bmatrix} 12 & -2 & 12 \\ -2 & 12 & -6 \\ -4 & -6 & -8 \end{bmatrix} = -880$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{528}{960} = 0.55A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-832}{960} = -0.866A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-880}{960} = -0.916A$$

CUTSET

The cut set is a minimal set of branches of the graph, removal of which cuts the graph into two parts. It separates the nodes of the graph into two groups.

- The cut-set consists of only one tree branch and remainders are links.
- Each branch of the cut-set has one of its terminal incident at a node in one group and its other end at a node in the other group and its other end at a node in the other group.
- The orientation of the cut-set is same as orientation of tree branch.

CUT-SET SCHEDULE

For a given network tree, a systematic way of indicating the tree branch voltage through use of a schedule called cut-set schedule

To write the cut-set schedule for network graph,

(i) Consider an oriented network graph

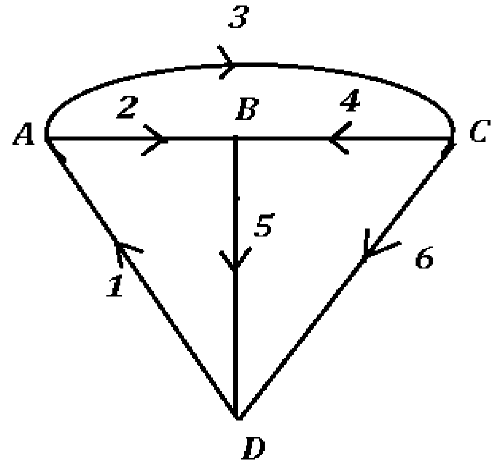
(ii) Write any one possible tree of the network graph

(iii)Assume tree branch voltages as (e1, e2…en) independent variables.

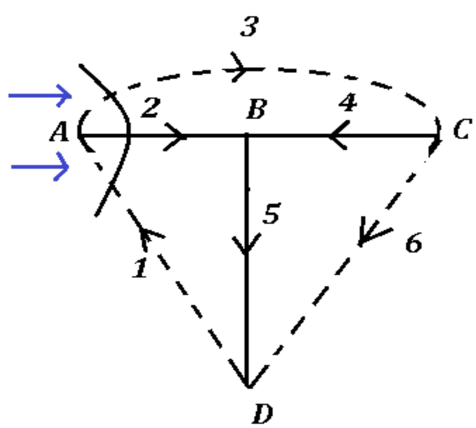
(iv)Assume the independent voltage variable is same direction as that of a tree branch voltage

(v) Mark the cut-sets (recognize) in the network graph.

PROBLEM 5: Determine the Cut set Schedule

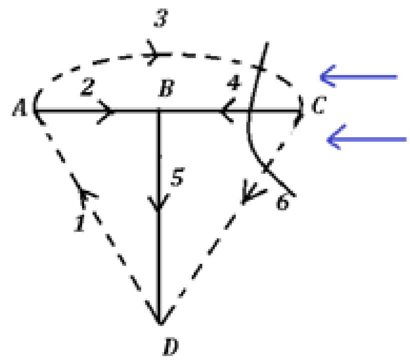


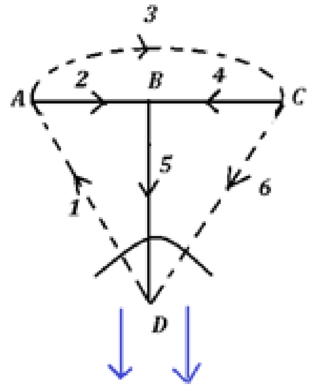
Number of Nodes= n =4
Number of Cutsets = n-1= 3



BRANCHES	1	2	3	4	5	6
NODE A	-1	1	1	0	0	0

BRANCHES	1	2	3	4	5	6
NODE C	0	0	-1	1	0	1



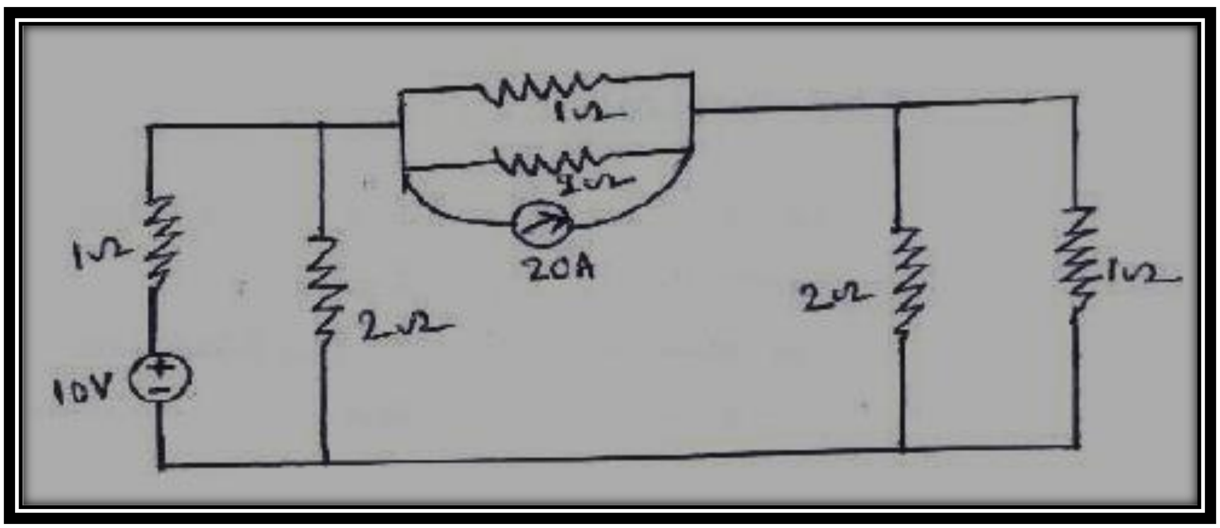


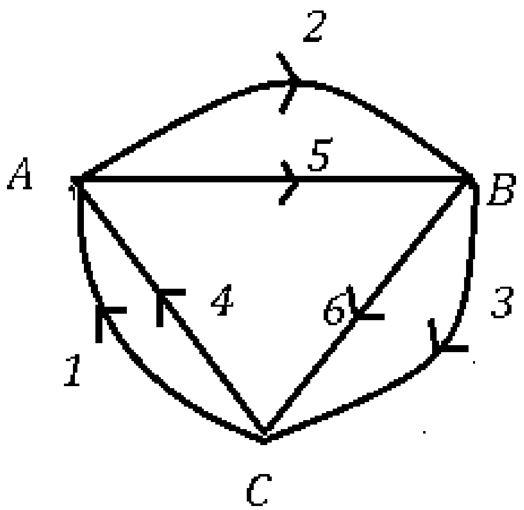
BRANCHES	1	2	3	4	5	6
NODE D	-1	0	0	0	1	1

CUTSET SCHEDULE

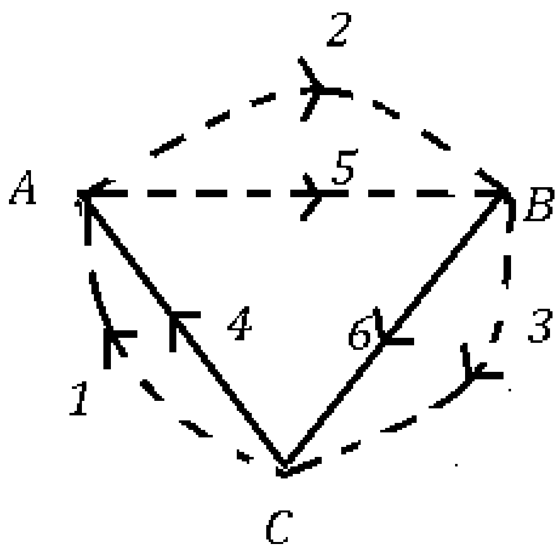
BRANCHES	1	2	3	4	5	6
NODE A	-1	1	1	0	0	0
NODE C	0	0	-1	1	0	1
NODE D	-1	0	0	0	1	1

Problem 6: Find the Cutset Matrix and the Branch Voltages

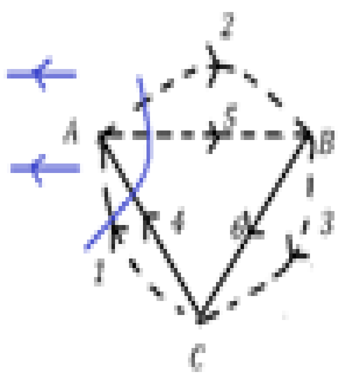




Oriented Graph

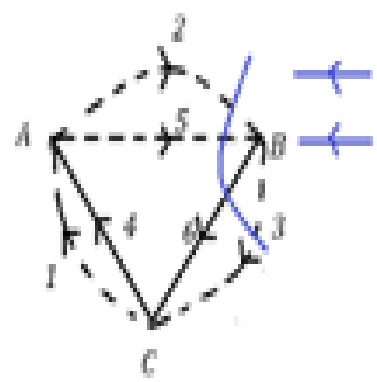


TREE & Co-TREE



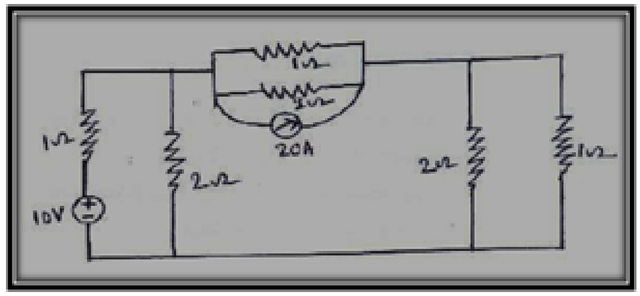
BRANCHES	1	2	3	4	5	6
NODE A	1	-1	0	1	-1	0

BRANCHES	1	2	3	4	5	6
NODE B	0	-1	1	0	-1	1



BRANCHES	1	2	3	4	5	6
NODE A	1	-1	0	1	-1	0
NODE B	0	-1	1	0	-1	1

Column----Branch Voltage(e)
Row -----Branch Current(i)



Rows of Cutset

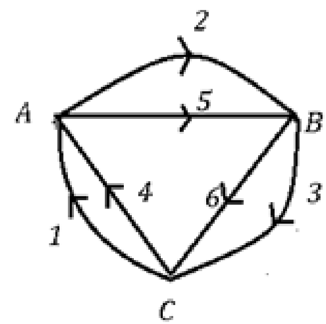
$$\boxed{\begin{array}{l} i_1 - i_2 + i_4 - i_5 = 0 \\ -i_2 + i_3 - i_5 + i_6 = 0 \end{array}} \Rightarrow \text{(I)}$$

Columns of Cutset

$$\boxed{\begin{array}{c} e_1 = E_1 \\ e_2 = -E_1 - E_2 \\ e_3 = E_2 \\ e_4 = E_1 \\ e_5 = -E_1 - E_2 \\ e_6 = E_2 \end{array}} \Rightarrow \text{(II)}$$

From Circuit,

$$\boxed{\begin{array}{c} e_1 + 10 = i_1 \\ e_2 = i_2 \\ e_3 = i_3 \\ e_4 = 2i_4 \\ e_5 + 40 = 2i_5 \\ e_6 = 2i_6 \end{array}} \Rightarrow \text{(III)}$$



Substituting (II) in (III),

$$\boxed{\begin{array}{c} E_1 + 10 = i_1 \\ -E_1 - E_2 = i_2 \\ E_2 = i_3 \\ E_1 = 2i_4 \\ -E_1 - E_2 + 40 = 2i_5 \\ E_2 = 2i_6 \end{array}} \Rightarrow \text{(IV)}$$

Substituting (IV) in (I),

$$\boxed{\begin{array}{c} E_1 + 10 + E_2 + E_1 + 0.5E_1 + 0.5E_1 - 20 + 0.5E_2 = 0 \\ 3E_1 + 1.5E_2 = 10 \\ 2E_2 + E_1 + 0.5E_1 + 0.5E_2 + 0.5E_2 - 20 = 0 \\ 1.5E_1 + 3E_2 = 20 \end{array}} \Rightarrow \text{(V)}$$

$$\begin{bmatrix} 3 & 1.5 \\ 1.5 & 3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 3 & 1.5 \\ 1.5 & 3 \end{bmatrix} = 6.75$$

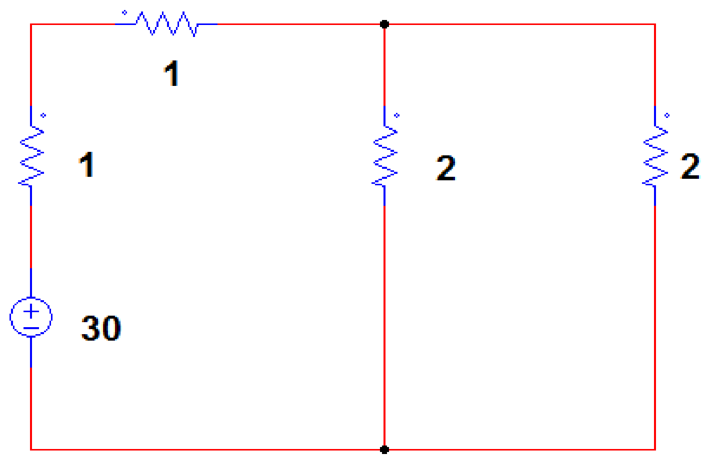
$$\Delta_1 = \begin{bmatrix} 10 & -1.5 \\ 20 & 3 \end{bmatrix} = 0$$

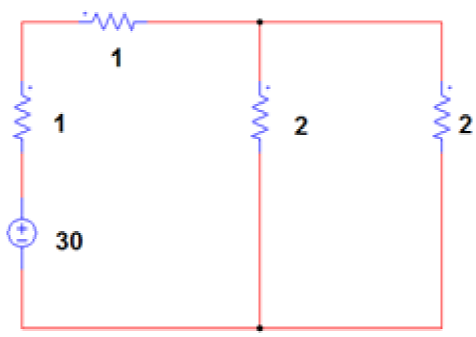
$$\Delta_2 = \begin{bmatrix} 3 & 10 \\ 1.5 & -20 \end{bmatrix} = 45$$

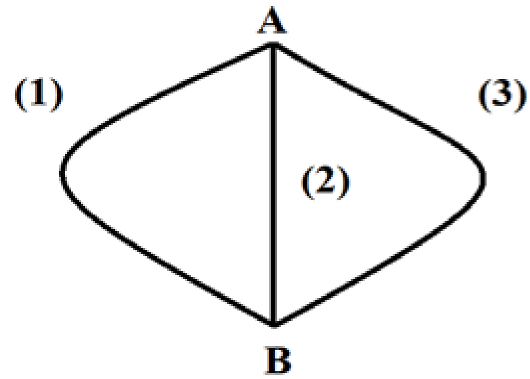
$$E_1 = \frac{\Delta_1}{\Delta} = 0$$

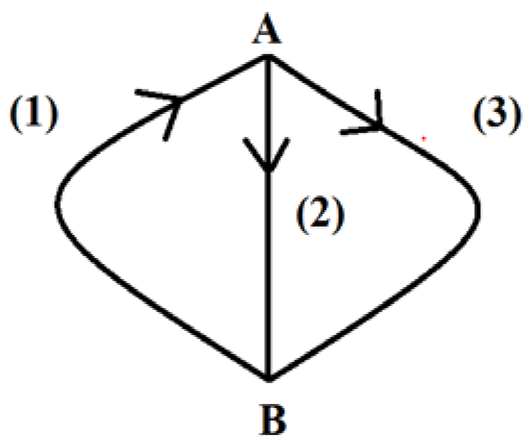
$$E_2 = \frac{\Delta_2}{\Delta} = 6.667$$

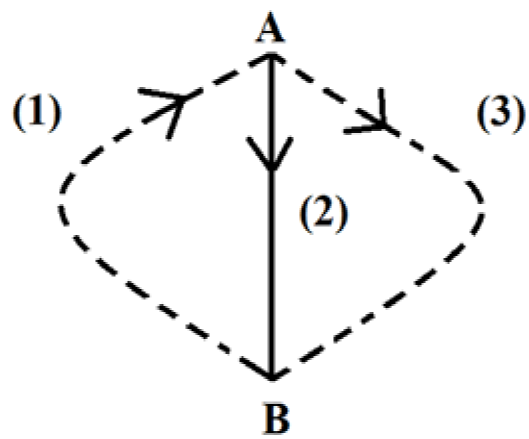
Problem 8:For the circuit shown frame the Cutset schedule and find the branch currents

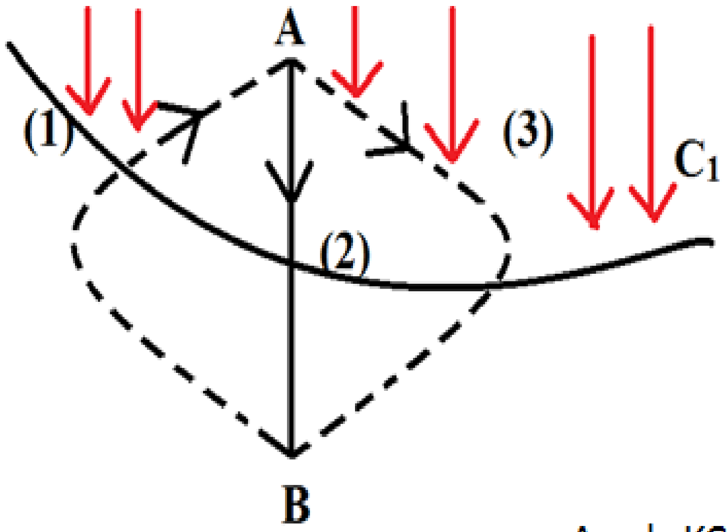












No. of cut set	Branches		
	1	2	3
C_1	-1	1	1

Apply KCL in rows

$-i_1 + i_2 + i_3 = 0$ (I)

APPLY KVL in column

Let the branch voltages be $e_1, e_2,$ & e_3

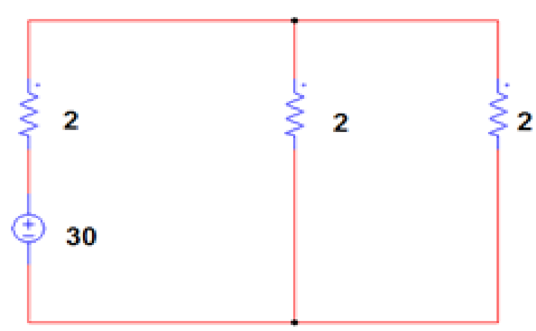
E_1 be the tree branch voltages

$$e_1 = -E_1$$
$$e_2 = E_1 \qquad \text{(II)}$$
$$e_3 = E_1$$

from the circuit

$$e_1 = -E_1$$
$$e_2 = E_1 \qquad \text{(II)}$$
$$e_3 = E_1$$

$$e_1 + 30 = 2i_1$$
$$e_2 = 2i_2 \qquad \text{(III)}$$
$$e_3 = 2i_3$$



Comparing (II) and (III)

$$-E_1 + 30 = 2i_1$$
$$E_1 = 2i_2 \qquad \text{(IV)}$$
$$E_1 = 2i_3$$

Simplifying,

$$i_1 = \frac{-E_1 + 30}{2} \qquad i_2 = \frac{E_1}{2} \qquad i_3 = \frac{E_1}{2}$$

$-i_1 + i_2 + i_3 = 0$ **(I)**

$$0.5E_1 - 15 + 0.5E_1 + 0.5E_1 = 0$$

$-15 = -1.5E_1$ $\qquad E_1 = 10V$

Substituting in Previous equation to obtain the branch current

$$i_1 = \frac{-E_1+30}{2} = 10A$$

$$i_2 = \frac{E_1}{2} = 5A$$

$$i_3 = \frac{E_1}{2} = 5A$$

DUALITY AND DUAL NETWORK:

The network is said to be dual network of each other if the mesh equations of given network are the node equations of other network. The property of duality is a mutual property. If network A is dual network B, then the network B is also dual of network A.

Some of the dual pairs are given in the following table:

	Element	Dual Element
1	Resistance	Conductance
2	Capacitance	Inductance
3	Inductance	Capacitance
4	Series Branch	Parallel Branch
5	Voltage Source	Current Source
6	Current Source	Voltage Source
7	Switch Closed (at t = 0)	Switch Opened (at t = 0)
8	Charge	Flux linkage
9	Mesh	Node
10	Link	Twig

Methods of drawing the dual of a network

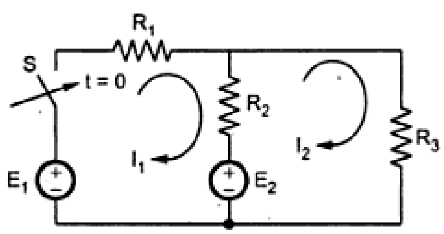
The following steps are followed to draw the dual of given electrical network:

1. A dot is placed in each independent loop of the original network. These dots placed inside the loops correspond to the independent nodes in dual network.
2. A dot is placed outside the given network. This corresponds to the reference node of the dual network.
3. All the dots are connected by dotted lines crossing all the branches. The dotted lines should cross only one branch at a time. The dual elements will form the branches connecting the corresponding nodes in the dual network.

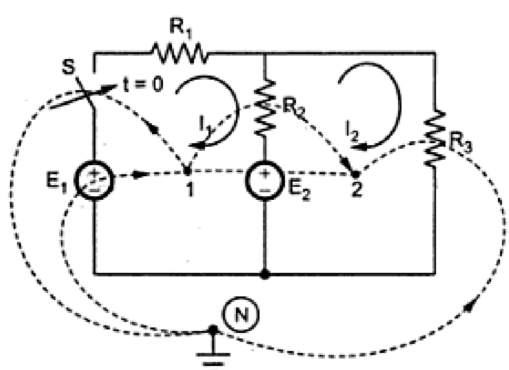
Note A: The voltage rise in the clockwise direction corresponds to a current flowing towards the independent network.

Note B: A clockwise current in a loop corresponds to positive polarity for the at the dual independent node.

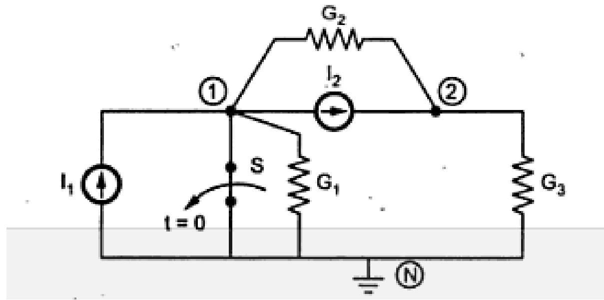
Example Draw a dual network for the given network below.



The procedure for drawing the dual network is given below:



The dual network is given below:



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