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SCHOOL OF BIO & CHEMICAL ENGINEERING
DEPARTMENT OF BIO MEDICAL ENGINEERING

UNIT - I

Basic Electrical Engineering - SEEA1203

D.C.CIRCUITS

Electrical /Quantities – Definitions, Symbols and / Units

- **Charge:**

A body is said to be changed positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge pm 6.28×10^{18} electrons.

- Electric Potential:

When a body is charged, either electrons are supplied on it (or) removed on from it. In both cases the work is done. The ability of the charged body to do work is called electric Potential. The charged body has the capacity to do (or) by moving the other charges by either attraction Repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential And, the work done, to charge a body to 1 coulomb is the measure of electric Potential.

Electric Potential, $V = \dfrac{\text{Work done}}{\text{Charge}} = \dfrac{W}{Q}$

W = Work done per unit charge.

Q = Charge measured in coulombs.

Unit of electric potential is **joules / Coulomb (or) volt**. If $W_{1=}$ 1 joule; Q= 1 coulomb, then V=1/1 = 1 Volt.

A body is said to have and electric potential of 1 volt, if one joule of work is done to / charge a body to one coulomb. Hence greater the joules / coulomb on charged body, greater is electric Potential.

- **Potential Difference**:

The difference in the potentials of two charged bodies is called potential difference.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is ampere (or) Coulomb / sec.

$$\text{Current, } (i) = \frac{\text{Charge(q)}}{\text{Time(t)}} = \frac{q}{t} \text{ Coulombs / Sec}$$

In differential Form, $i = \frac{dq}{dt}$ Coulombs / Sec

- **Resistance:**

Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical Property of the substance due to which it opposes (or) Restricts the flow of electricity (ie electrons) through it. Its unit is ohms.

A wire is said to have a resistance of 1 ohm if a p.d. /of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 joule, when a current of 1A flows through it.

- **conductors**

Certain substances offer very little opposition to the flow of electric current, they are called as conductors for examples, metals, acids etc. Amongst Pure metals, Sliver, Copper and aluminium are very good conductors.

- **Insulators**

Certain substances offer very high opposition to the flow of electric current, they are called as insulators. For eg, Bakelite, mica, glass, rubber, P.V.C, dry wood etc. The substance, whose properties lies between those of Conductors and insulators are called semi-conductors, for eg, Silicon, Germanium etc.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is I inversely proportional to the Cross Sectional area of the Conductor, ie,

$$R \alpha \frac{l}{A}$$

(iii) It depends upon the nature of the matter of the Conductor.

(iv) It depends upon the temperature of the conductor.

Therefore by assuming the temperature to remain constant, we get,

$$R \alpha \frac{l}{A}$$
$$R = \rho \frac{1}{A}$$

ρ ('Rho') is a constant of proportionality called **resistivity** (or) Specific resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

Specific Resistance (or) Resistivity:

Resistance of a wire is given by $R = \rho \frac{1}{A}$

Resistivity is the property (or) nature of the material due to which it opposes the Flow of Current through it. The unit of resistivity is ohm-metre. $[\rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m \,(\text{ohm-metre})$

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the conductivity (or) specific conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows Flow of Current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G \frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{\text{m}^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric Circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done moving the electrons in a unit time is called electric power. The unit of electric Power is Joules/sec or Watt. $\left[P = VI = I^2R = V^2/R\right]$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = electric power * time.

Electrical Energy is measured in Kilowatt hour (kwh)

Problems:

1. The resistance of a conductor 1mm^2 in Cross section and 20m long is 0.346Ω. Determine the Specific resistance of the conducting material.

Given Data

Area of Cross-Section A = 1mm^2

Length, l = 20m

Resistance, R = 0.346Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A} \Rightarrow \rho = \frac{Rl}{A}$

Solution: Area of Cross-section, $A = 1mm^2$

$= 1 * 10^{-6} m^2$

$$\rho = \frac{1*10^{-6}*0.346}{20} = 1.73*10^{-8}\,\Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\Omega m.$

2. A Coil consists of 2000turns of Copper wire having a Cross-sectional area of 1mm^2. The mean length per turn is 80cm and resistivity of copper is 0.02μΩm at normal working temperature. Calculate the resistance of the Coil.

Given data:

No of turns = 2000

Length /turn = 80cm = 0.8m

Resistivity = ρ = 0.02μΩm = $0.02*10^{-6}$

P = $2*10^{-8}$Ωm.

Cross-Sectional area of the wire, A = 1mm^2 = $1*10^{-6}$m

Solution:

Mean length of the wire, l = 2000*0.8

l = 1600m.

We know that $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2*10^{-8}*1600}{1*10^{-6}} = 32\Omega$

Resistance of the Coil = 32Ω

3. A silver wire of length 12m has a resistance of 0.2Ω. Find the specific Resistivity of the wire, if the cross-sectional area of the wire is 0.01cm^2.

$R = \frac{\rho l}{A}$ $\Rightarrow$ length,l=12m

Resistance, R=0.2Ω

A=0.01cm^2

$$\rho = \frac{RA}{l} = \frac{0.2 * 0.01 * 10^{-4}}{12}$$

$$\rho = 1.688 * 10^{-8}\,\Omega m$$

Ohm's law and its limitations:

The relationship between the potential difference (V), the current (I) and Resistance (R) in a d.c. Circuit was first discovered by the scientist George Simon ohm, is called ohm's law.

Statement:

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition (eg. Temperature, etc.) do not change.

ie, $\frac{V}{I}$ = Constant

(or)

$$\frac{V}{I} = R \Rightarrow V = I * R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

Power, $P = V * I = I^2 R = \frac{V^2}{R}$

Limitations:

(i) Ohm's law does not applied to all non-metallic conductors. For eg. For Silicon carbide.

(ii) It also does not apply to non-linear devices such as zener diode, voltage Regulators.

(iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

Problems based on ohm's law:

1. An electric heater draws 8A from 250V Supply. What is the power rating? Also find the resistance of the heater Element.

Given data:

Current, $I = 8A$

Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8*250 = 2000 Watt$

Resistance $(\mathrm{R}) = \frac{V}{I} = \frac{250}{8} = 31.25\Omega$

2. What will be the current drawn by a lamp rated at 250V, 40Watt, connected to a 230V Supply.

Given Data:

Rated Power = 40W
Rated Voltage = 250V
Supply Voltage = 230V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\Omega$$

$$Current, I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472A$$

3. A Battery has an emf of 12.8 volts and supplies a current of 3.24A. What is the resistance of the circuit? How many coulombs leave the battery in 5 minutes?

Solution:

$$\text{Current Resistance, } \mathrm{R} = \frac{\mathrm{V}}{\mathrm{I}} = \frac{12.8}{3.24} = 4\Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.2×5×60 = 960 coulomb

Combination of Resistors:

Resistances in series (or) series combination:

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends A and B.

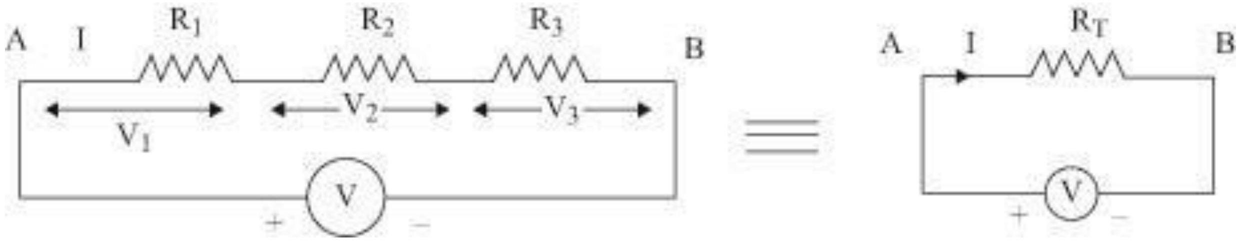


Figure 1.1

In the above circuit, there is only one closed path, so only one current flow through all the elements. In other words if the Current is same through all the resistors the combination is called series combination

To find equivalent Resistance:

Let, V = Applied voltage

I = Source current = Current through each Element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By ohms low, $V_1 = IR_1$

$V_2 = IR_2 and\ V_3 = IR_3$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\ (R_1 + R_2 + R_3)$

$V = I\ (R_1 + R_2 + R_3)$

$V = IR_T$

$\frac{V}{T} = R_T$

The ratio of $\left(V/_I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\dfrac{1}{G_T} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$

∴ Equivalent resistance (R_T) is the sum of all individual resistances.

Concepts of series circuit:

i. The current is same through all elements.

ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.

iii. The equivalent Resistance (R_T) is greater than the greatest individual resistance of that combination.

iv. Voltage drops are additive.

v. Power are additive.

vi. The applied voltage equals the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 in terms of V and R_1, R_2, R_3:

Equivalent Resistance, $RT = R_1 + R_2 + R_3$

By ohm's low, $I = \dfrac{V}{RT} = \dfrac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

∴ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms is series the total Resistance is given by,

$$R_T = n * R$$

Applications:

* When variable voltage is given to the load, a variable Resistance (Rheostat) is connected in series with the load. Example: Fan Regulator is connected in series with the fan.
* The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
* When a load of low voltage is to be Operated on a high voltage supply, a fixed value of resistance is, connected in series with the load.

Disadvantage of Series Circuit:

* If a break occurs at any point in the circuit, current coil flow and the entire circuit become useless.
* If 5 numbers of lamps, each rated 230 volts are be connected in series circuit, then the supply voltage should be 5 x 230 = 1150volts. But voltage available for lighting circuit in each and every house is only 230v. Hence, series circuit is not practicable for lighting circuits.
* Since electrical devices have different current ratings, they cannot be connected in series for efficient Operation.

Problems based on series combination:

1. Three resistors 30Ω, 25 Ω, 40 Ω are connected in series across 200v as shown in figure Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.

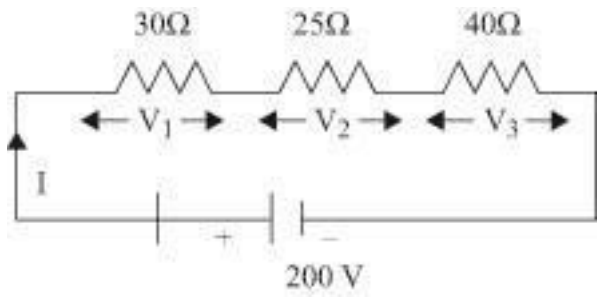


Figure 1.2

(i) Total Resistance (R_T)

(ii) Current,

(iii) Potential difference across each element,

2. An incandescent lamp is rated for 110v, 100w. Using suitable resistor how can you operate this lamp on 220v mains.

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110}$

I = 0.909A

When the voltage across lamp is 110v, then the remaining voltage must be across R

$$\begin{aligned} Supply\ voltage &= V = 220 Volts \\ Voltage\ across\ R &= V - 110 Volts \\ ie,\ V_R &= 220 - 110 = 110v \\ By\ ohm's\ law,\ V_R &= IR \\ 110 &= 0.909R \\ R &= 121\Omega \end{aligned}$$

Resistance in Parallel (or) Parallel Combination:

If one end of all the resistors is joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B,

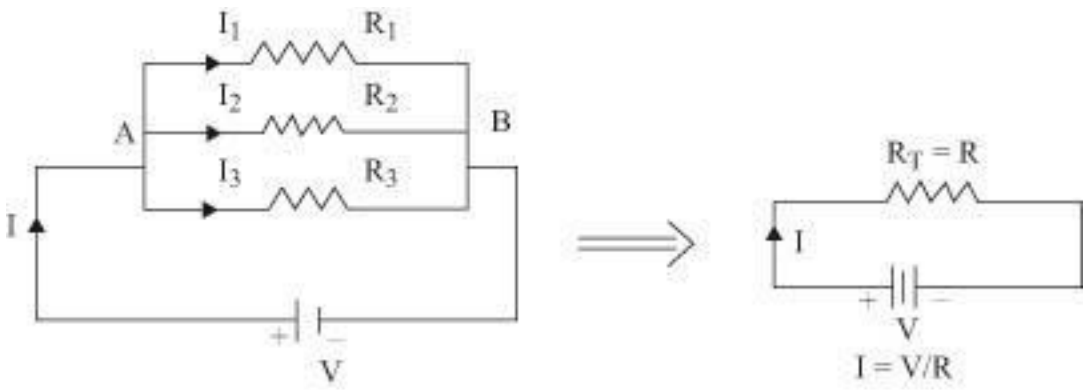


Figure 1.3

as shown in the Figure 1.3

$$I = \frac{V}{R} \quad \text{................} \quad (1)$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \text{.....................} \quad (2)$$

Total current is the sum of three individual currents, $I_T = I = T_1 + I_2 + I_3$ .... (3)

Substituting the above expression for the current in equation (3), $\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If $R_T = R$,

Then $\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ .... (4)

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie,

Power dissipated by R_1 = Power dissipated by R_1 + Power dissipated by R_2 + Power dissipated by R_3

We know that reciprocal of Resistance is called as conductance.

$$\text{Conductance} = \frac{1}{\text{Resistance}} \left[G = \frac{1}{R} \right]$$

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

Concepts of Parallel Circuit:

- Voltage is same across all the elements.
- All elements we have individual currents, depends upon the resistance of element.
- The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
- If n resistance each of R are connected in parallel, then

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + n terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

$$(or)\ R_T = \frac{R}{n}$$

Powers are additive.

Conductance are additive.

Branch currents are additive.

Current Division Technique:

Case (i) When two resistances are in parallel:

Two resistance $R_1\, and\, R_2$ ohms are connected in parallel across a battery of V (volts)

Current through $R_2\, is\, I_2$ and through $R_2\, is\, I_2$ as shown in figure 4. The total current is I.

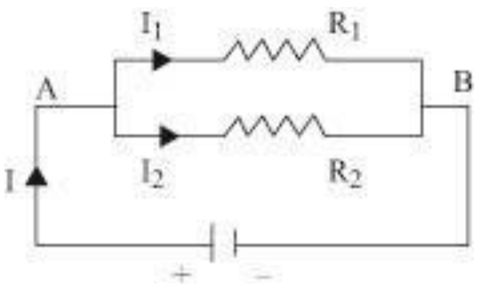


Figure 1.4

- To express I_1 and I_2 in terms of I, R_1 and R_2(or)

To find Branch Currents I_1, I_2

$$I_2R_2 = I_1R_1$$

$$I_2 = \frac{I_1R_1}{R_2} \text{..............................} (5)$$

Also, The total Current, $I = I_1 + I_2$.................(6)

Substituting (5) in (6), $I_1 + \frac{I_1R_1}{R_2} = I$

$$\frac{I_1R_2 + I_1R_1}{R_2} = I$$

$$I_1(R + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \frac{IR_1}{(R_1 + R_2)}$

To find Total equivalent Resistance, (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

$$R_T = R_1R_2$$

Hence the total value of two resistances connected parallel is equal to their product divided by their ie, Equivalent Resistance $= \frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel as shown in figure 1.5. Let I be the supply current (or) total curve I_1, I_2, I_3 are the current through R_1, R_2 and R_3.

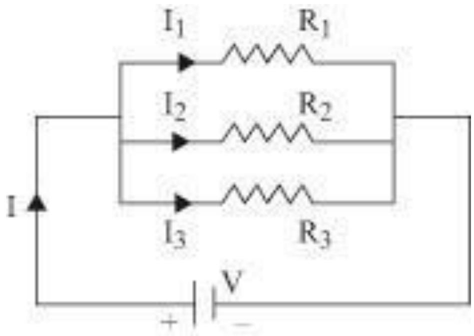


Figure 1.5

Let R_T be equivalent resistance

To find the equivalent Resistance (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3}$$

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

To find the Branch currents I_1, I_2 and I_3:

We know that, $I_1 + I_2 + I_3 = I$..................(7)

Also, $I_3R_3 = I_1R_1 = I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (7)

$$I_2 = \frac{I_1R_1}{R_2}; I_3 = \frac{I_1R_1}{R_3}$$

$$I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3} = I$$

$$I_1\left(1+\frac{R}{R_2}+\frac{R_1}{R_3}\right)=I$$

$$\frac{I_1\,(R_2R_3+R_3R_1+R_1R_2)}{R_2R_3}=I$$

$$I_1=\frac{I(R_2R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

$$I_3=\frac{I(R_1R_3)}{(R_1R_2+R_2R_3+R_3R_1)}$$

Advantages of parallel circuits:

* The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other is performance.
* If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

Applications of parallel circuits:

* All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switch.
* Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided,
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal	Reciprocal of resistance is equal to sum of

to sum of Individual Resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest of the resistance.
There is only one path for the flow of current.	There are more than one path for the flow of current.

Problems based on parallel Combinations:

1. what is the value of the unknown resistor R in Figure 6, If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

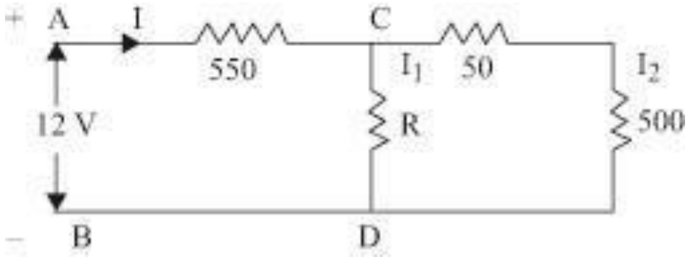


Figure1.6

Given Data:

$V_{500} = 2.5V$

$I_2 = \frac{V_{550}}{R} = \frac{2.5}{500} = 0.005A$

$V_{50} =$ Voltage across 50Ω

$V_{50} = IR = 0.005*50 = 0.25V$

$VCD = V_{50} + V500 = 0.25+2.5 = 2.75V$

$V_{550} =$ Drop across 550Ω = 12-2.75 = 9.25V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I} = \frac{2.75}{0.018} = 232.69\Omega$$

$$R = 232.69\Omega$$

2. Three resistors 2Ω, 3Ω, 4Ω are in parallel. How will a total current of 8A is divided in the circuit shown in figure1.7.

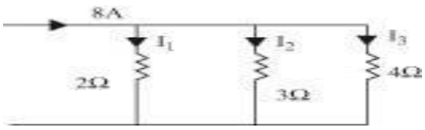


Figure1.7

3Ω and 4Ω are connected in parallel. Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\Omega$

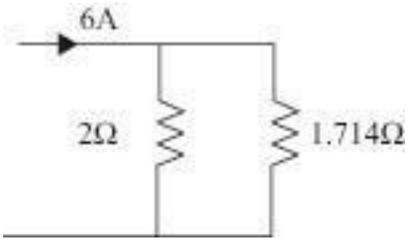


Figure1.8

1.714Ω and 2Ω are connected in parallel, its equivalent resistance is 0.923Ω

$$\frac{1.714*2}{2+1.714} = 0.923\Omega$$

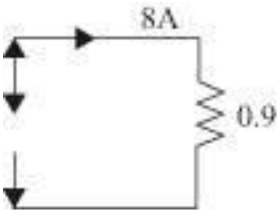


Figure 1.9

$$V = IR = 8*0.923$$
$$V = 7.385V$$

Branch Currents, $I_1 = \frac{V}{R_1} = \frac{7.385}{2} = 3.69$

$$I_2 = \frac{V}{R_1} = \frac{7.385}{3} = 2.46A$$

$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Total current, I = 8A is divided as 3.69A, 2.46A, 1.84A.

3. what resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω R is connected in parallel with 10Ω Resistor to given an equivalent Resistance of 6Ω.

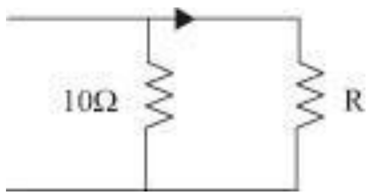


Figure 1.10

$$\frac{10*R}{10+R} = 6$$

$$10R = (10+R)6$$
$$10R = 60 + 6R$$

$$10R-6R \rightarrow 4R = 60$$

$$R = \frac{60}{4} = 15\Omega$$

$$R = 15\Omega$$

4. Calculate the current supplied by the battery the given circuit as shown in the figure 1.11.

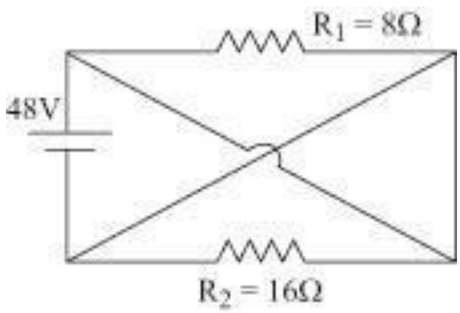


Figure 1.11

Solution: The above given Circuit can be redrawn as as shown in figure 1.12.

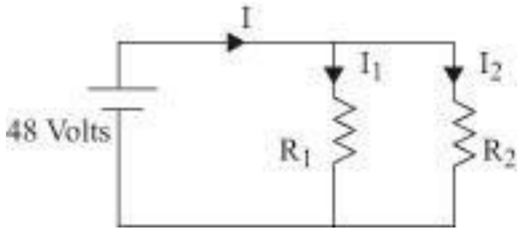


Figure 1.12

R_1 and R_2 are in Parallel across the voltage of 48volts.

Equivalent Resistance, $R_T = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{8*16}{8+16} = \dfrac{16}{3}\,\Omega$

$$R_T = 5.33A$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9A$$

5. Calculate the total resistance and battery current in the given circuit shown in figure 1.13.

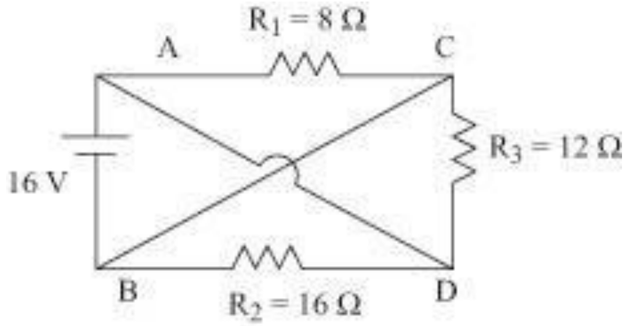


Figure1.13

The given above circuit can be re-drawn as shown in figure1.14.

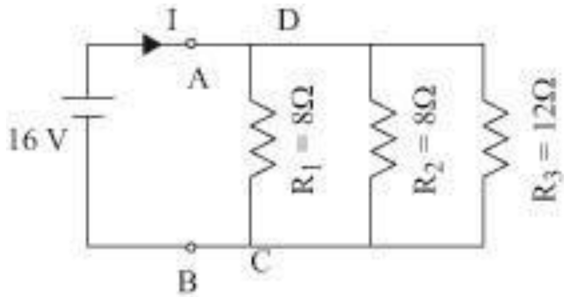


Figure1.14

8Ω, 16Ω, 12Ω are connected in parallel. Its equivalent Resistance, $R_T = \dfrac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\Omega$$

$$R_T = 3.692\Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Series — Parallel Combination

As the name suggests, this Circuit is a Combination of series and parallel Circuits. A simple example of such a Circuit is illustrated in Figure1.15. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with R_1.

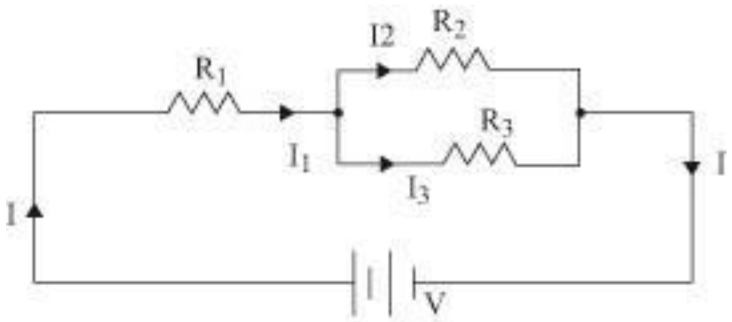


Figure 1.15

Equivalent Resistance: R_T for parallel combination.

$$R_P = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the Circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel Combination = $I * \frac{R_2 R_3}{R_2 + R_3}$

Problems based on Series – Parallel Combination:

1. In the Circuit shown in figure 1.16, find the Current in all the resist Also calculate the supply voltage.

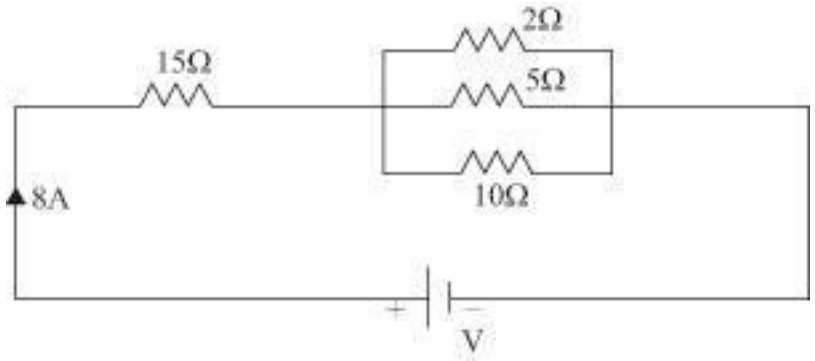


Figure 1.16

Solution: V=130 Volts.

Current through 2Ω resistor, is 5A.

Current through 5Ω Resistor, 2A

Current through 10Ω Resistor, 1A.

1. Find the equivalent resistance between the terminals A and B in the figure 1.17.

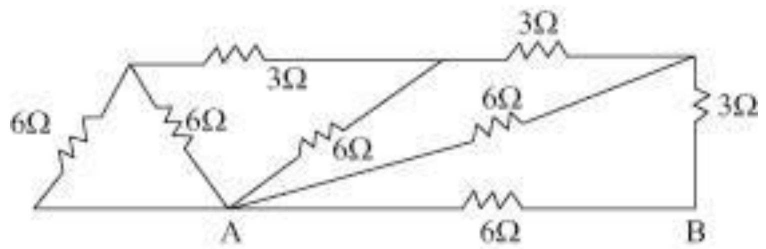


Figure 1.17

Solution: R_{AB} =3Ω.

3. Determine the value of R if the power dissipated in 10Ω Resistor is 90W in figure 1.18.

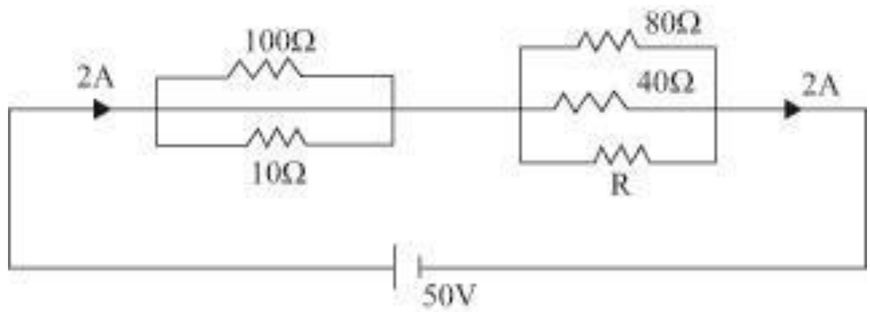


Figure1.18

Solution: R = 39.4Ω.

Kirchhoff's Law

Circuit Basics

$\boxed{V=IR}$ - Ohm's Law; applies to both the entire circuit and any resistive element within it.

The voltage across any two elements in parallel is the same;

The current through any two elements in series is the same.

The voltage across elements in series adds up to the total voltage;

The current through any elements in parallel add up to the total current.

$\boxed{R = \frac{\rho L}{A}}$; Resistance of a conductor of length L, area, A and resistivity, .

$$\boxed{\begin{aligned} R_s &= R_1 + R_2 + \ldots \\ \frac{1}{R_p} &= \frac{1}{R_1} + \frac{1}{R_2} + \ldots \end{aligned}}$$ Resistors in series and parallel;

$\boxed{Q=CV}$ - definition of capacitance; applies to the entire circuit and any capacitor within the circuit;

The charge on any capacitors in series is the same and the voltage across these elements adds up to the total voltage; the charge on any capacitors in a parallel adds up to the total charge and the voltage across these elements is the same.

$\boxed{C = \frac{\kappa \varepsilon_0 A}{d}}$ - Capacitance of a parallel plate capacitor of area A, separation d and dielectric.

$$\boxed{\begin{aligned} C_p &= C_1 + C_2 + \ldots \\ \frac{1}{C_s} &= \frac{1}{C_1} + \frac{1}{C_2} + \ldots \end{aligned}}$$ Capacitors in series and parallel;

$\boxed{P=IV}$ - the power dissipated or generated by any element is equal to the current through it multiplied by the voltage across it.

Kirchhoff's Rules

- Used when a circuit is not a simple series and parallel combination of resistors;
- Often used when there is more than one voltage source in a circuit.

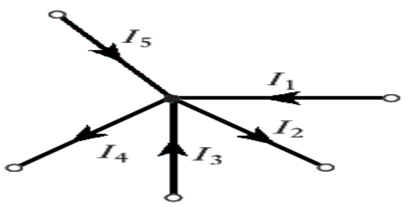
Kirchhoff's First Law:

The algebraic sum of the currents flowing through a junction is zero.

Sum of the Currents approaching the junction are equal to the sum of the currents going away from the junction are Equal.

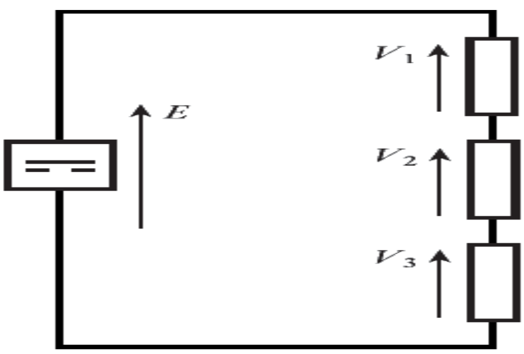
Kirchhoff's Second Law:

The algebraic sum of the potential differences in a circuit loop must be zero. Potential rises are equal to potential drops in a closed circuit.



$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

Kirchhoff's First Law



$$E - V_1 - V_2 - V_3 = 0$$

Kirchhoff's Second Law

Figure 1.19

Figure 1.20

Point Rule: the sum of all the currents into any point in the circuit is equal to the sum of all of the currents out of the same point. $\boxed{\sum I_{in} = \sum I_{out}}$

Loop Rule: the sum of the voltage drops around any closed loop is zero. $\boxed{\sum \Delta V = 0}$

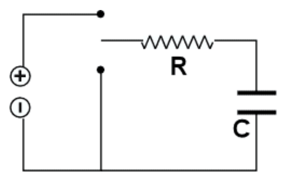
Problem Solving Strategy:

1) Label all junctions and corners in order to make solution easier.
2) Guess current directions in each branch of the circuit.
3) Apply point rule to n-1 junctions where n is the number of junctions.
4) Apply loop rule to as many loops as necessary such the total number of equations is equal to the number of unknown currents.
    a. When going with the current, $\Delta V = -IR$ through a resistor; when going against the current, $\Delta V = +IR$
    b. When going from the positive terminal to the negative terminal of a battery, $\Delta V=-V$ and when going from negative to positive terminal, $\Delta V=+V$.
5) Solve the system of equations for the unknown currents.

Rarely are the students asked to solve an entire system on an AP exam, but they are often asked to apply these principles to parts of circuits on multiple choice problems.

RC Circuits

The simplest RC circuit contains a single resistor R in series with a capacitor C and a battery of voltage V.



Apply Kirchhoff's loop rule to the loop.

Figure 1.21

$V = V_R+V_C$

$V = iR + \frac{q}{C}$ - substituting Ohm's Law and the definition of capacitance;

$V = R\frac{dq}{dt} + \frac{q}{C}$ - substituting the relationship between current and charge;

Solve the differential equation by method of separation of variables:

$$\frac{dq}{VC - q} = \frac{dt}{RC}$$

$$\int \frac{dq}{VC - q} = \int \frac{dt}{RC}$$

$$-\ln(VC - q) + const = t/RC$$

$q(t) = VC(1 - e^{-t/RC})$ The constant comes from the initial conditions: q=0 at t=0.

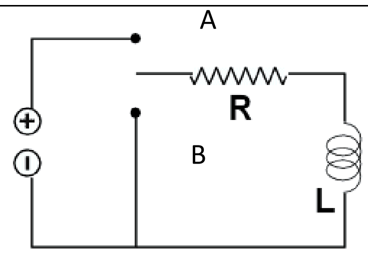
For a discharging circuit, the method of solution is similar: the only difference is that V=0. Plug that in, make the same substitutions as above, solve by separation of variables and the result is

$q(t) = VCe^{-t/RC}$.

In circuits with more than one branch, or a resistor in parallel with the capacitor, it helps to teach the students about the initial and steady state behavior of the capacitor. When the capacitor is empty, it offers no "resistance" to the flow of current; when it is full, no current will flow in the branch with the capacitor in it.

RL Circuits

The picture at right represents a resistor in series with an inductor and a battery.



The switch is initially moved to position A and the inductor is "charged."

Figure 1.22

The voltage across the inductor varies (according to Faraday's Law) with the rate of change of current according to the equation $V_L = L\frac{di}{dt}$ (magnitude only, the sign due to Lenz's law has been omitted).

Again we start with Kirchhoff's Loop Rule.

$V = V_R + V_L$

$$V = iR + L\frac{di}{dt}$$

Solve by separation of variables:

$$V - iR = L\frac{di}{dt}$$

$$\frac{di}{/R - i} = \frac{Rdt V}{L}$$

$$\int \frac{di}{/R - i} = \int \frac{Rdt V}{L}$$

$-\ln(\frac{V}{R} - i) = \frac{Rt}{L} + const$ - the constant comes from the initial conditions: i=0 at t=0.

$$i(t) = \frac{V}{R}(1 - e^{-Rt/L})$$

When the switch is flipped to position B, the inductor begins to "discharge." Again, for the discharging equation, simply start with V=0 in the loop equation as above. Then solve by separation of variables as above to get the equation $i(t) = \frac{V}{R}e^{-Rt/L}$ where the initial current in the inductor is the same as it was just before the switch was opened causing the inductor to discharge.

Again, for more complex circuits involving more than one resistor in a circuit with an inductor, either in series or parallel with the inductor, it is important for the students to understand the initial behavior of the circuit after a switch is moved, and the long-term behavior after the switch is moved (either opened or closed). Initially, it is useful for the students to think of the inductor as an open switch: it prevents any sudden changes in current so if there was no current just before the switch is moved, there will be no current immediately after. However, the ideal inductor behaves like a wire when the current has reached a steady state. These two situations will help us determine the current in any part of the circuit just after a switch is moved and in steady state conditions.

Problems on Kirchhoff's Law:

- Solve this circuit, find the value of 5 branch currents using Kirchhoff's laws and elimination method (or maybe called elimination by substitution BUT not using any matrix method)

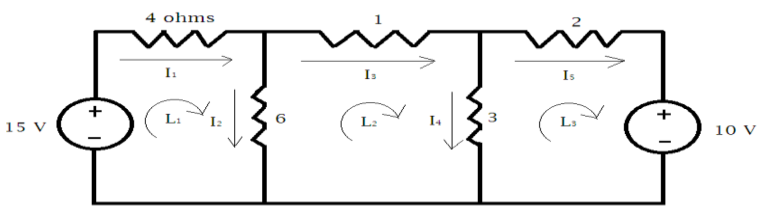


Figure 1.23

Solution:

Loop 1:15−4I1−6I2=0
Loop 2:6I2−I3−3I4=0
Loop 3:3I4−2I5−10=0

I1=1.89
I2=1.24
I3=0.65
I4=2.26
I5=−1.61

- Determine the value current in 40 Ohms resistance

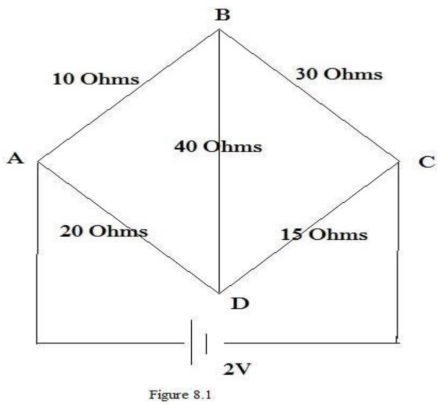


Figure 1.24

Solution:

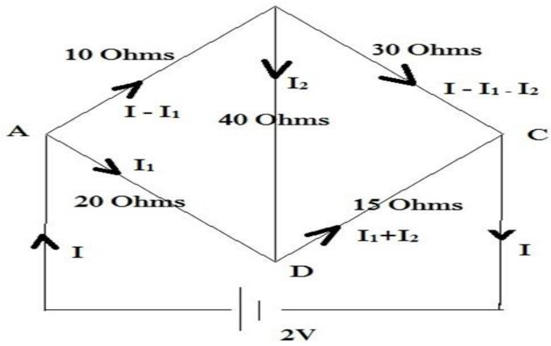


Figure 1.25

Note that there are three unknown variables (I , I1 , I2) and therefore we need to solve three equations simultaneously to find this unknown variables.

Apply KCL to Circuit ABDA

(I - I1)*10 + I2*40 - 20I = 0

10*I - 30*I1 +40*I2 = 0-------(1)

Apply KCL to Circuit BCDB

(I - I1 - I2)*30 - (I1+I2)*15 - I2*40 = 0

30*I - 45*I1 - 85*I2 = 0--------------(2)

Apply KCL to Circuit ADCEA

I1*20 + (I1+I2)*15 = 2

35*I1 + 15*I2 = 2---------- (3)

Now solve these three equations

I = 87/785 A

I1 = 41/785 A

I2 = 9/785 A

So the current in 40 Ohms resistance is 9/785 A From B to D

- Determine the current flowing in each of the batteries and the voltage difference between 10 Ohms resistance. Refer the following figure.

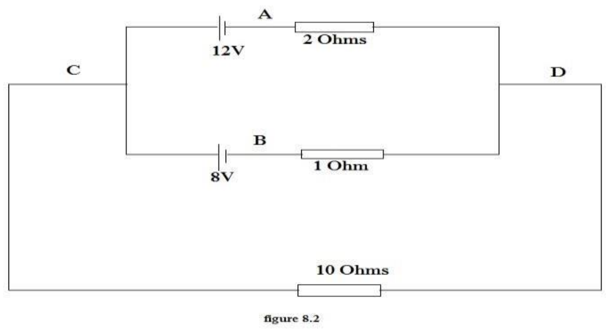


Figure 1.26

First we have to apply KCL 1 to the network.

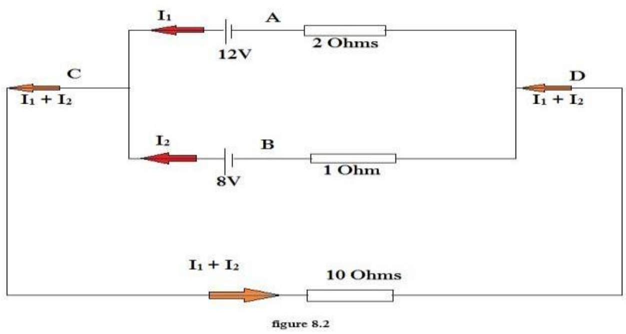


Figure 1.27

Applying KCL to loop CEDAC

(I1+I2)*10 + I1*2 = 12

12*I1 + 10*I2 = 12 --------- (1)

Applying KCL to loop CEDBC

(I1+I2)*10 + I2*1 = 8

10*I1 + 11*I2 = 8 ------------ (2)

By solving (1) & (2)

I1 = 1.625A

I2 = -0.750A

Current through 10 Ohms resistance is = I1 + I2 = 0.875A

Voltage across 10 Ohms resistance is = 0.875A x 10 Ohms = 8.750V

- Determine the current in 4 Ohms resistance.

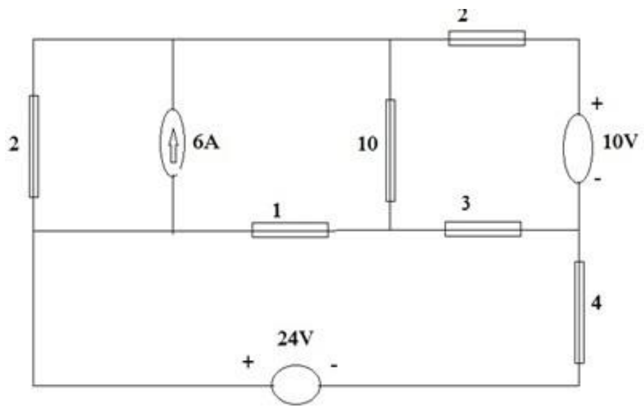


Figure 1.28

Note: This Network has a current source. So this question will teach you something new. To study about Current sources, see my previous post about "voltage and current sources".

Solution:

First we have to apply KCL 1 to the network

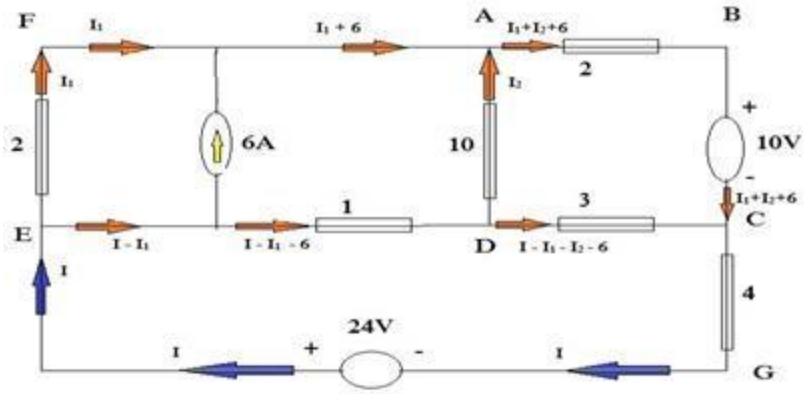


Figure 1.29

Applying KCL to loop EFADE

I1*2 - I2*10 - (I - I1 - 6)*1 = 0

I - 3*I1 + 10*I2 = 6 ------------(1)

Applying KCL to loop ABCDA

(I1+I2+6)*2 - 3*(I - I1 - I2 - 6) + I2*10 = -10

3*I - 5*I1 +15*I2 = 6 ---------(2)

Applying KCL to loop EDCGE

(I-I1-6)*1 + (I-I1-I2-6)*3 + I*4 = 24

8*I - 4*I1 3*I2 = 48 ----------(3)

By solving these three equations

I = 4.1A

So the current in 4Ohms resistance = 4.1A . Once we find the mesh currents we can use them to calculate any other currents or voltages of interest.

MESH-CURRENT METHOD

- Mesh-Current method is developed by applying KVL around meshes in the circuit.
- A mesh is a loop which doesn’t contain any other loops within it.

- Loop (mesh) analysis results in a system of linear equations which must be solved for unknown currents.
- Reduces the number of required equations to the number of meshes.
- Can be done systematically with little thinking.
- As usual, be careful writing mesh equations – follow sign convention.
- Powerful analysis method which applies KVL to find unknown currents.
- It is applicable to a circuit with no branches crossing each other.

The various steps involved in Mesh method

1. Clearly label all circuit parameters and distinguish the unknown parameters from the known.
2. Identify all meshes of the circuit.
3. Assign mesh currents and label polarities.
4. Apply KVL at each mesh and express the voltages in terms of the mesh currents.
5. Solve the resulting simultaneous equations for the mesh currents.
6. Now that the mesh currents are known, the voltages may be obtained from Ohm's law.

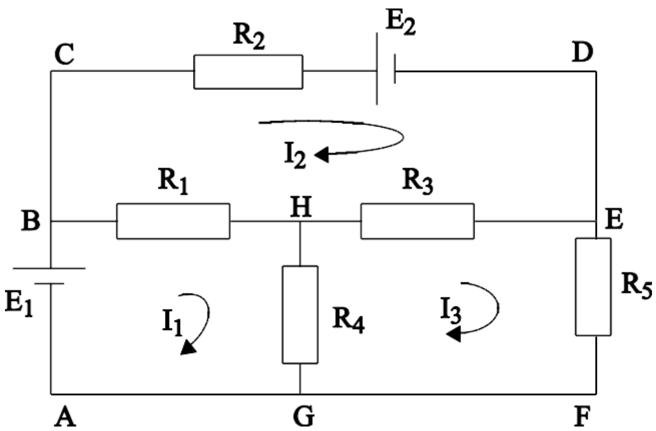


Figure 1.30

In the above figure we shall name the three loop currents I_1 , I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is $(I_1 - I_3)$ in a downward direction and R_1 is $(I_1 - I_2)$ from left to Right.

Apply KVL for the first loop ABHGA,

$$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$$

$$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$$

$$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1 \text{.............................} (1)$$

Apply KVL for the loop BEDC,

$$- R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$$

$$\therefore - R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2 \quad \ldots\ldots\ldots\ldots (2)$$

Apply KVL for the loop HEFGH,

$$R_3 (I_3 - I_2) + R_5 I_3 + R_4 (I_3 - I_1) = 0$$

$$- R_4 I_1 - R_3 I_2 + (R_3 + R_4 + R_5) I_3 = 0 \quad \ldots\ldots\ldots\ldots (3)$$

Equation (1) to (3) can be arranged in a matrix form as,

$$\begin{bmatrix} R_1 + R_4 & -R_1 & -R_4 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \\ -R_4 & -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ -E_2 \\ 0 \end{bmatrix}$$

In general equation (4) will be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \quad \ldots\ldots\ldots\ldots (5)$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop **i**.

$$R_{ij} = \begin{cases} -\sum (\text{Resistance common to the loop i and loop j}, \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances}) \\ +\sum (\text{Resistance common to the loop i and loop j}, \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances}) \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{vmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{vmatrix}; \qquad \Delta_2 = \begin{vmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{vmatrix};$$

$$\Delta_3 = \begin{vmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{vmatrix}; \qquad \Delta = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig using Mesh current method

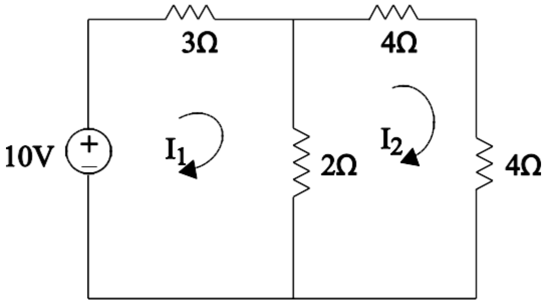


Figure 1.31

Solution

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E_1 = 10V

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the current in 3Ω resistor using Mesh Analysis.

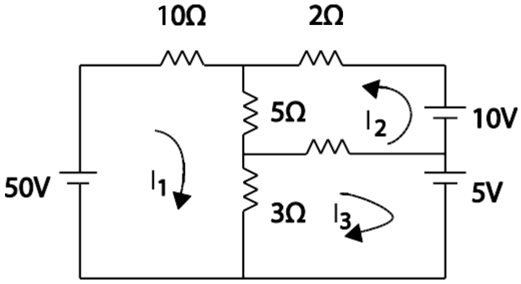


Figure 1.32

Solution

For loop 1,

$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$

$18I_1 + 5I_2 - 3I_3 = 50$ ....(1)

For loop 2,

$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$

$5I_1 + 8I_2 + I_3 = 10$...........................(2)

For loop 3,

$3(I_3 - I_1) + 1(I_3 + I_2) = -5$

$-3I_1 + I_2 + 4I_3 = -5$ ....(3)

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$= 18\ (32-1)\ -5\ (20+3)\ -3(5+24)$

$= 356$

$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$= 18\ (-40-10)\ -5\ (-25+30)\ +\ 50(5+24)$

$= -900 - 25 + 1450$

$= 525$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

3) Determine the currents in various elements of the bridge circuit as shown below.

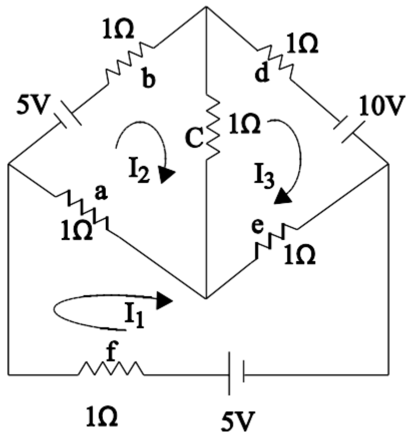


Figure 1.33

Solution

For loop 1,

$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$

$3I_1 - I_2 - I_3 = 5$ ....(1)

For loop 2,

$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$

$-I_1 + 3I_2 - I_3 = 5$… .... (2)

For loop 3,

$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$

$-I_1 - I_2 + 3I_3 = 10$............................ (3)

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$= 3(9-1) + 1(-3-1) - 1(1+3)$

$= 16$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

= 3 (15 + 10) -5 (-3 - 1) -1 (-10 + 5)

= 100

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

= 3 (30 + 5) + 1 (-10 + 5) + 5 (1 + 3)

= 120.

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2 = 6.25 - 6.25 = 0$ A

$I_b = I_2 = 6.25$ A.

$I_c = I_2 - I_3 = 6.25 - 7.5 = -1.25$ A

$I_d = I_3 = 7.5$ A

$I_e = I_1 - I_3 = 6.25 - 7.5 = -1.25$ A.

$I_f = I_1 = 6.25$ A.

NODE ANALYSIS

This method is mainly based on Kirchhoff's Current Law (KCL). This method uses the analysis of the different nodes of the network. Every junction point in a network, where two or more branches meet is called a node. One of the nodes is assumed as reference node whose potential is assumed to be zero. It is also called zero potential node or datum node. At other nodes the different voltages are to be measured with respect to this reference node. The reference node should be given a number zero and then the equations are to be written for all other nodes by applying KCL. The advantage of this method lies in the fact that we get (n – 1) equations to solve if there are 'n' nodes. This reduces calculation work.

STEPS FOR THE NODE ANALYSIS

1. Choose the nodes and node voltages to be obtained.
2. Choose the currents preferably leaving the node at each branch connected to each node.
3. Apply KCL at each node with proper sign convention.
4. If there are super nodes, obtain the equations directly in terms of node voltages which are directly connected through voltage source.
5. Obtain the equation for the each branch current in terms of node voltages and substitute in the equations obtained in step 3.
6. Solve all the equations obtained in step 4 and step 5 simultaneously to obtain the required node voltages.

CASE I.

Consider the below fig. Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node a we get

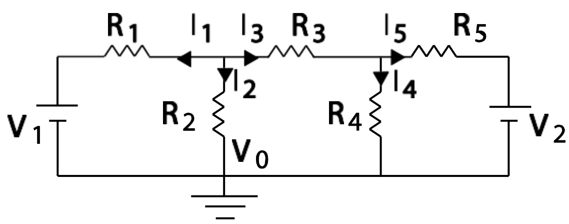


Figure 1.34

$$I_1 + I_2 + I_3 = 0 \quad \text{.................} \quad (1)$$

Where

$$I_1 = \frac{V_a - V_1}{R_1}; I_2 = \frac{V_a - V_0}{R_2}; I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ .(1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [$V_o = 0$]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$I_4 = \frac{V_b - V_o}{R_4}; I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

$V_o = 0$ [reference node]

$$V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] - V_a\left[\frac{1}{R_3}\right] = \frac{V_2}{R_5} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\Delta_1 = \begin{vmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{vmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2,

$$\begin{vmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{vmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$$V_a = \frac{\Delta_1}{\Delta}; \qquad V_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

CASE II:

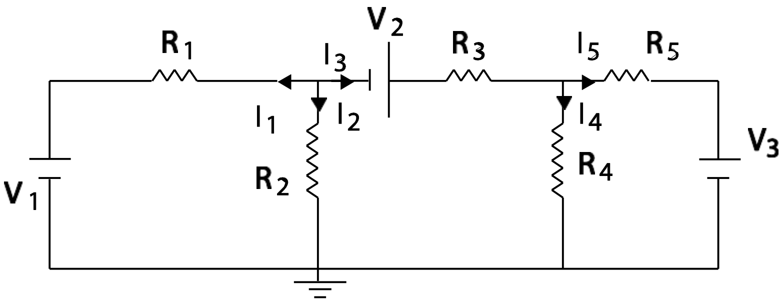


Figure 1.35

Consider the above fig

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = 0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R_1} - \frac{V_2}{R_3} \quad \text{.................................... (5)}$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a + V_2 - V_b}{R_3} = \frac{V_b}{R_4} + \frac{V_b - V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = \frac{V_b}{R_4} + \frac{V_b}{R_5} - \frac{V_3}{R_5}$$

$$\frac{V_a}{R_3} - V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] = -\frac{V_3}{R_5} - \frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3} + V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] = \frac{V_3}{R_5} + \frac{V_2}{R_3} \quad \text{................. (6)}$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} - \frac{V_2}{R_3} \\ \frac{V_2}{R_3} + \frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1 = \begin{pmatrix} \frac{V_1}{R_1} - \frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3} + \frac{V_3}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1} - \frac{V_2}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3} + \frac{V3}{R_5}\right)$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{vmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

Case iii

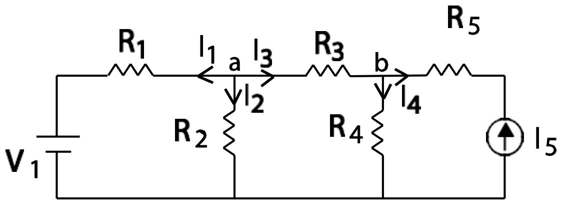


Figure 1.36

Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R_1} \text{..................................} (7)$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3}+\frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \text{....................} \mathbf{(8)}$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

1) Write the node voltage equation and calculate the currents in each branch for the network.

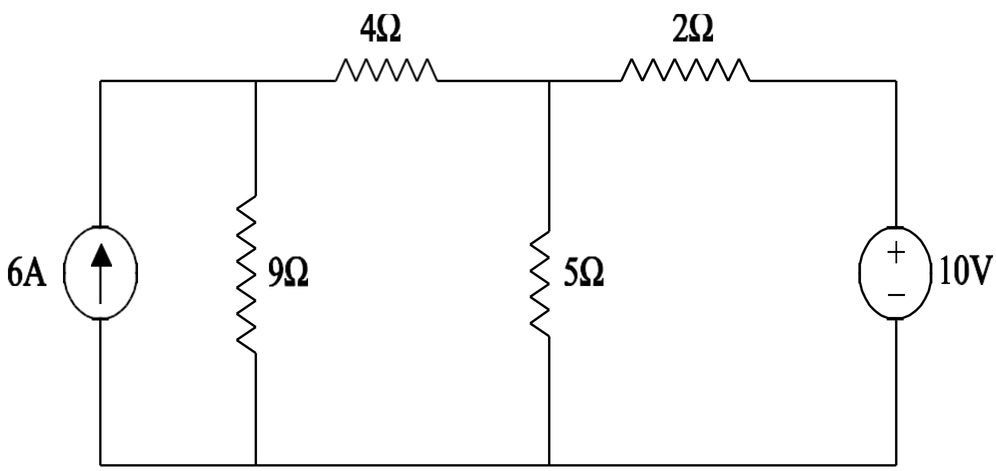


Figure 1.37

Step 1: To assign voltages at each node

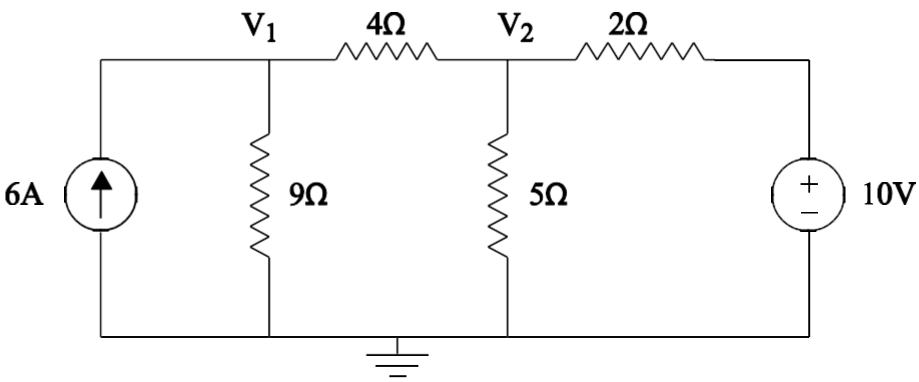


Figure 1.38

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

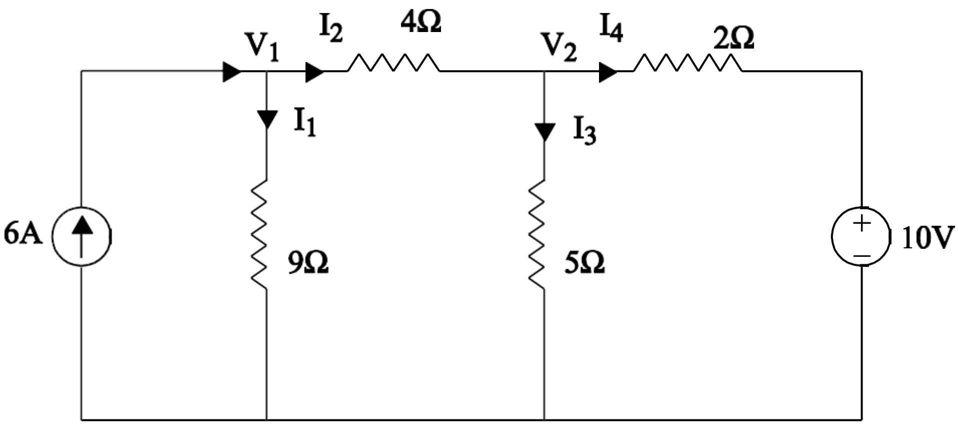


Figure 1.39

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule,

form the matrix

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86\upsilon$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ - & \end{bmatrix} = 3.3$$

$$V_{22}\frac{3.3}{.2795} = 11.8V$$

$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$

$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$

$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$

$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$

Hence currents in all the branches are found.

2) Use the Nodal Method to find V_{ba} and current through 30Ω resistor in the circuit shown.

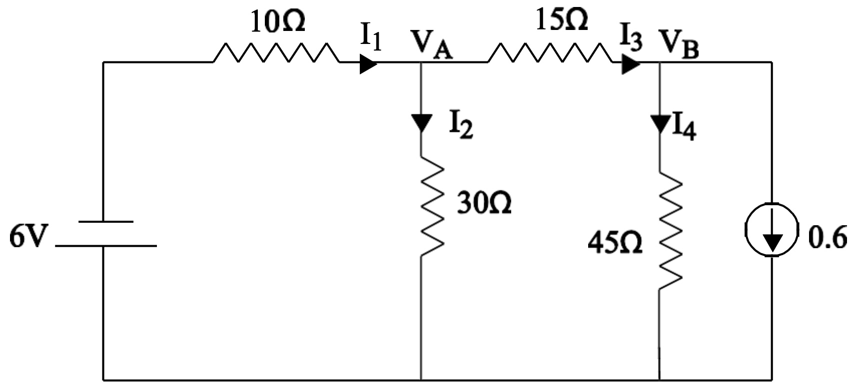


Figure 1.40

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A\left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15}\right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} + 0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35 \times 10^{-3}\right]$$

$\Delta = [0.01324]$

$\Delta = 0.01324$

$$\Delta_1 = \begin{pmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15}+\frac{1}{45} \end{pmatrix} = -0.093$$

$\Delta_1 = [-0.053 - 0.04] = -0.093$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$\Delta_2 = [-0.12 - 0.0396]$

$\Delta_2 = -0.1596$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$V_{ba} = V_A - V_B = -7 + 12 = 5V$

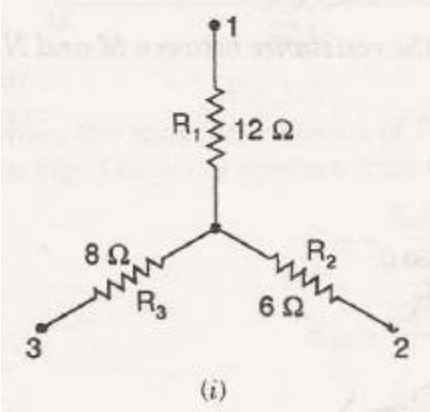
$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$I_2 = -0.233A$

Star- Delta Transformation

1. In the figure shown below a number of resistances connected in delta and star. Using star/delta conversion method complete the network resistance measured between (i) Land M (ii) M and N and (iii) N and L.

Solution. Three resistances 12 Ω, 6 Ω and 8 Ω are star connected. Transform them into delta with ends at the same points as before.



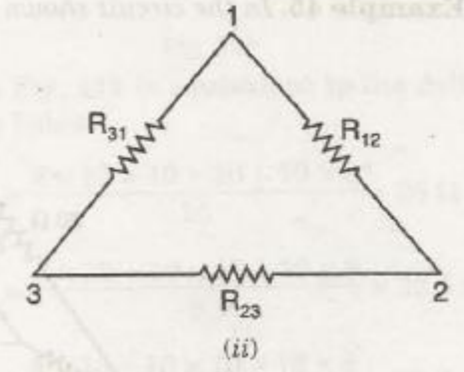
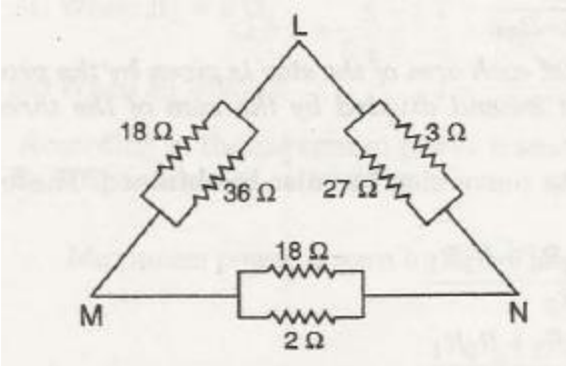


Figure 1.41 Figure 1.42

$R_{12} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_3 = 12 \times 6 + 6 \times 8 + 8 \times 12 / 8 = 27\ \Omega$

$R_{23} = R_1 R_2 + R_2R_3 + R_3R_1 / R_1 = 12 \times 6 + 6 \times 8 \times 12 / 12 = 18\ \Omega$



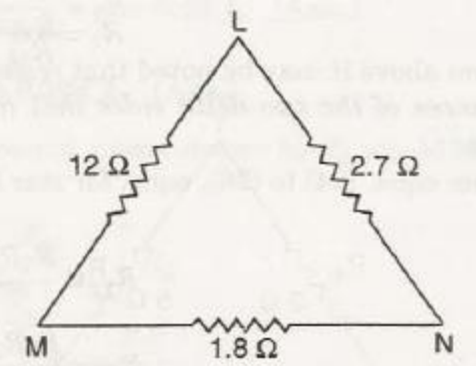


Figure 1.43 Figure 1.44

the transformed circuit connected to original delta connected resistances in the circuit 18 Ω, 3 Ω and 2 Ω.

here 18 Ω and 36 Ω are in parallel;

3 Ω and 27 Ω are in parallel, and

2 Ω and 18 Ω are in parallel.

These resistances are equivalent to:

$18 \times 36 / 18 + 36 = 12\ \Omega$; $3 \times 27 / 3 + 27 = 2.7\ \Omega$ and $2 \times 18 / 2 + 18 = 1.8\ \Omega$

(i) Resistance between L and M,

$R_{LM} = 12 \times (2.7 + 1.8) / 12 + (2.7 + 1.8) = 3.27\ \Omega$

(ii) Resistance between M and N,

$R_{MN} = 1.8 \times (12 + 1.8) / 1.8 + 12 + 2.7 = 1.6\ \Omega$

(iii) Resistance between N and L,

$R_{NL} = 2.7 \times (12 + 1.8) / 2.7 + (12 + 1.8) = 2.25\ \Omega$

2. In the circuit shown in Figure, find the resistance between M and N.

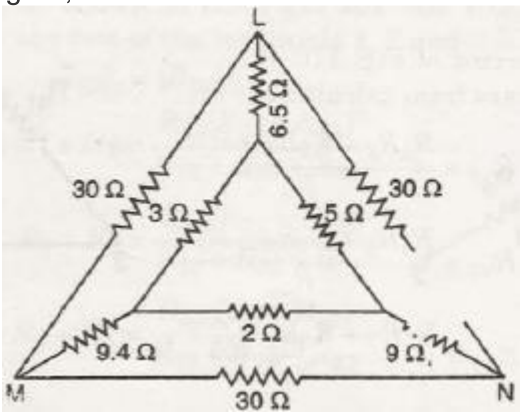


Figure 1.45

Solution.

$R_1 = R_{12}\, R_{31} / R_{12} + R_{23} + R_{31} = 5 \times 3 / 5 + 2 + 3 = 1.5$

$R_2 = R_{23}\, R_{12} / R_{12} + R_{23} + R_{31} = 2 \times 5 / 5 + 2 + 3 = 1$

$R_3 = R_{31}\, R_{23} / R_{12} + R_{23} + R_{31} = 3 \times 2 / 5 + 2 + 3 = 0.6$

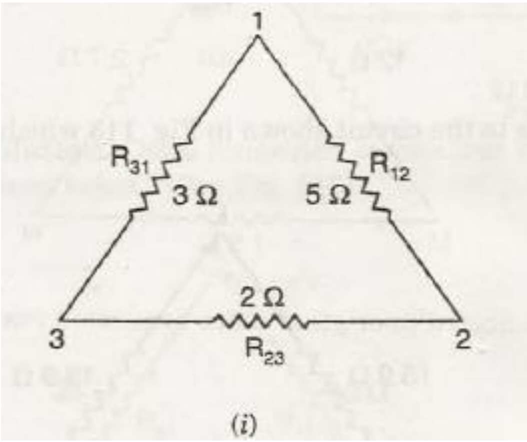


Figure 1.46

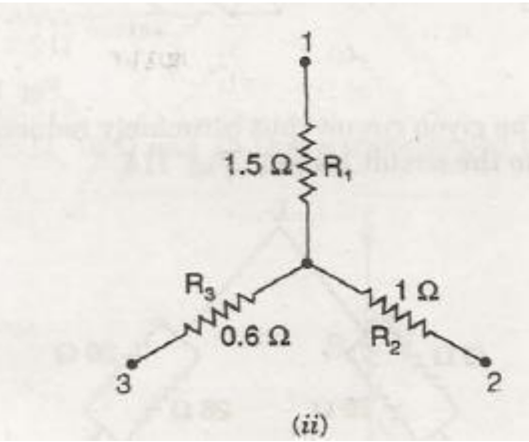


Figure 1.47

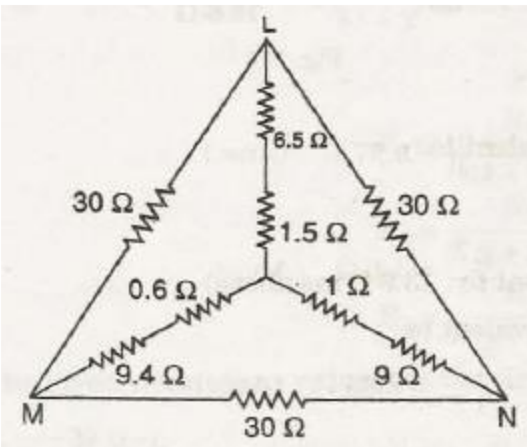


Figure 1.48

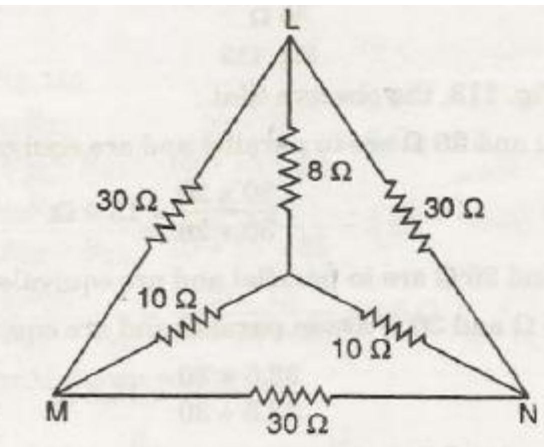


Figure 1.49

$R_{12} = R_1 R_2 + R_2 R_3 + R_3 R_1 / R_3 = 8 \times 10 + 10 \times 10 + 10 \times 8 / 10 = 26\ \Omega$

$R_{23} = R_1 R_2 + R_2R_3 + R_3R_1 / R_1 = 8 \times 10 + 10 \times 10 \times 10 / 8 = 32.5\ \Omega$

$R_{31} = 8 \times 10 + 10 \times 10 \times 10 \times 8 / 10 = 26\ \Omega$

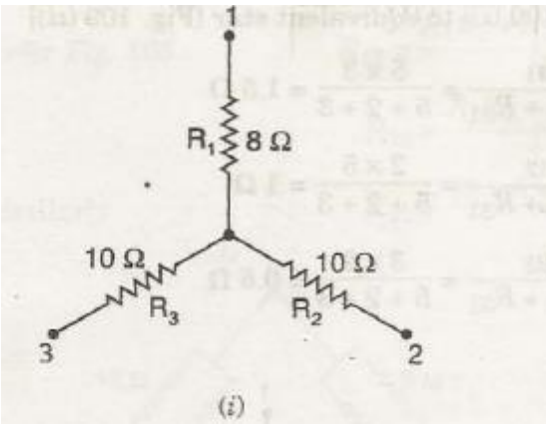


Figure 1.50

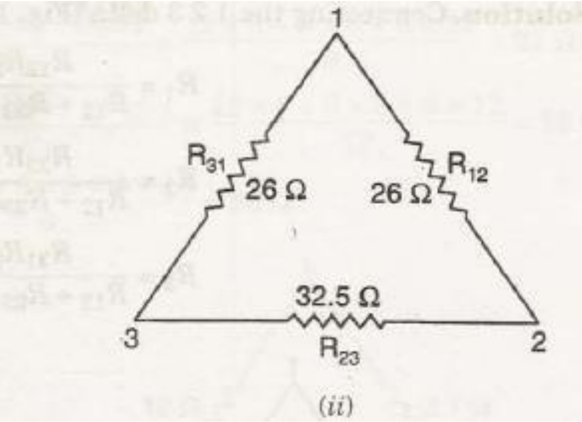


Figure 1.51

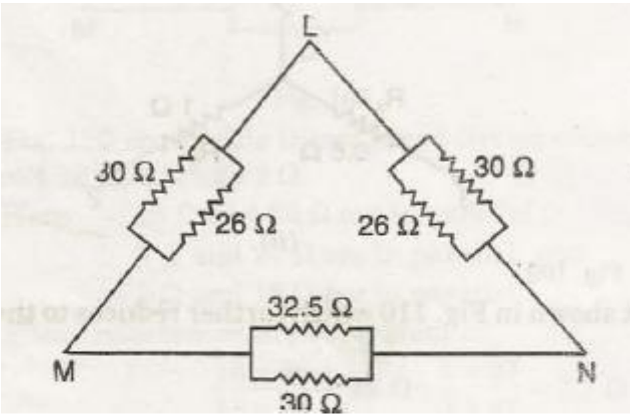


Figure 1.52

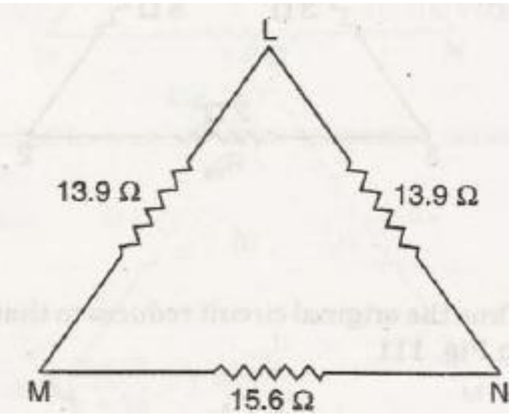


Figure 1.53

30 Ω and 26 Ω are in parallel and are equivalent to :

$30 \times 26 / 30 + 26 = 13.9$

30 and 26 Ω are in parallel and are equivalent to : 13.9 Ω (as above)

32.5 Ω and 30 Ω are in parallel and are equivalent to :

$32.5 \times 30 / 32.5 + 30 = 15.6$

Hence total resistance between M and N,

$R_{MN} = 15.6 \times (13.9 + 13.9) / 15.6 + (13.9 + 13.9)$

$= 433.69 / 43.4 = 9.99$

Example 3. Find the current I supplied by the battery, using delta/star transformation.

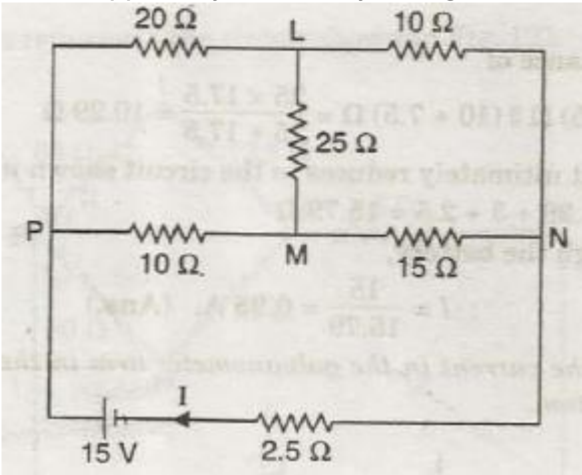


Figure 1.54

Solution. Delta connected resistances 25 Ω, 10 Ω and 15 Ω are transformed to equivalent star as given below :

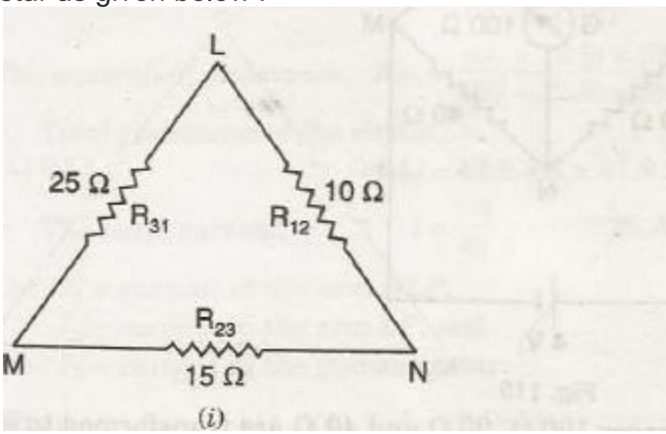


Figure 1.55

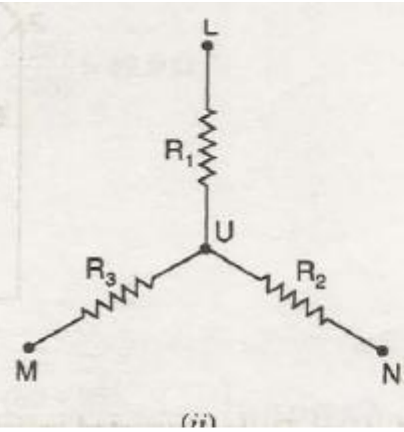


Figure 1.56

$R_1 = R_{12}\, R_{31} / R_{12} + R_{23} + R_{31} = 10 \times 25 / 10 + 15 + 25 = 5$

$R_2 = R_{23}\, R_{12} / R_{12} + R_{23} + R_{31} = 15 \times 10 / 10 + 15 + 25 = 3$

$R_3 = R_{31}\, R_{23} / R_{12} + R_{23} + R_{31} = 25 \times 15 / 10 + 15 + 25 = 7.$

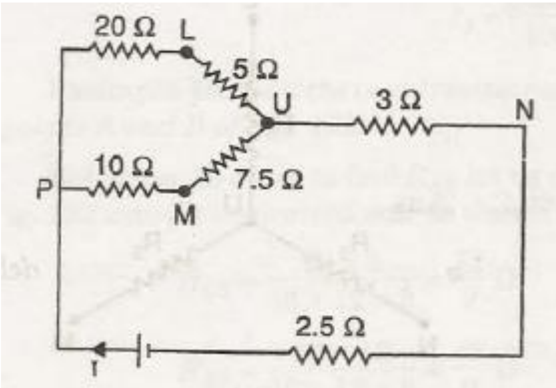


Figure 1.57

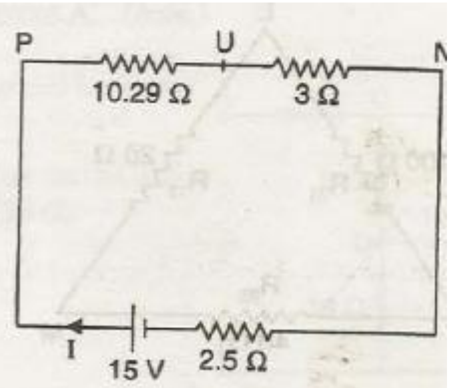


Figure 1.58

The equivalent resistance of

(20 + 5) Ω || (10 + 7.5) Ω = 25 x 17.5 / 25 + 17.5 = 10.29 Ω

Total resistance = 10.29 + 3 + 2.5 = 15.79 Ω

Hence current through the battery,

I = 15 / 15.79 = 0.95 A

Example 4. Using Delta-star conversion find resistance between terminals AB

Solution.

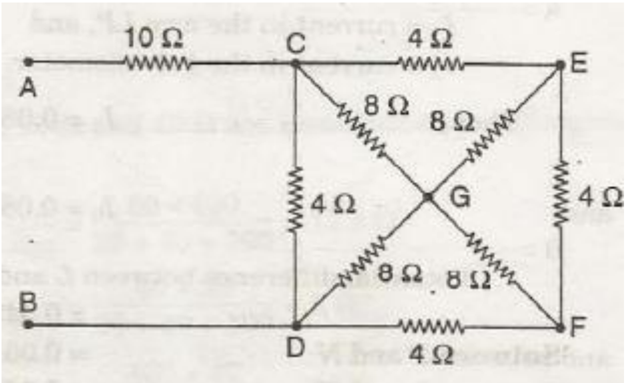
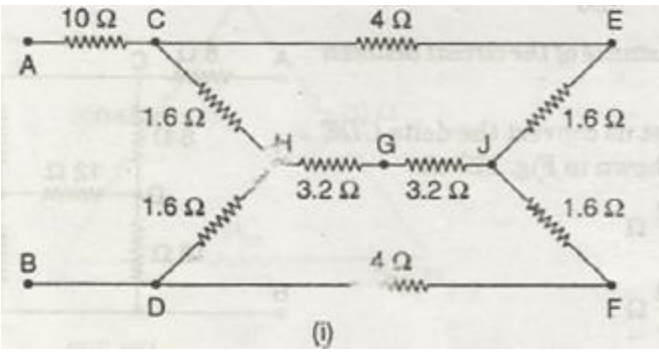


Figure 1.59

Using series-parallel combinations, we have

R_{AB} = (10 + 0.66) + [(0.752 + 1.6) || (2.64 + 5.6)]



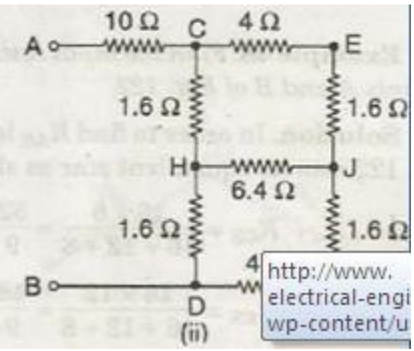


Figure 1.60

Figure 1.61

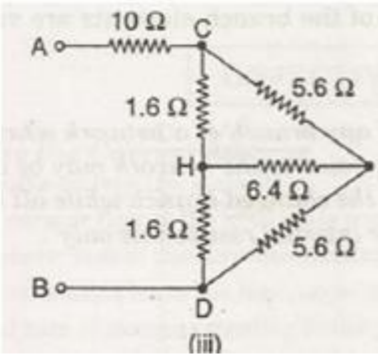


Figure 1.62

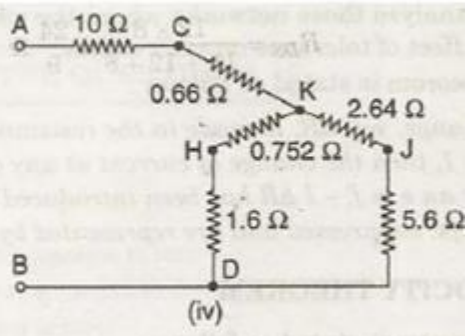


Figure 1.63

$= 10.66 + 2.352 \times 8.24 / (2.352 + 8.24 = 12.49\ \Omega$

Example 5. Find the resistance at the A-B terminals in the electric circuit, using Δ-Y transformations.

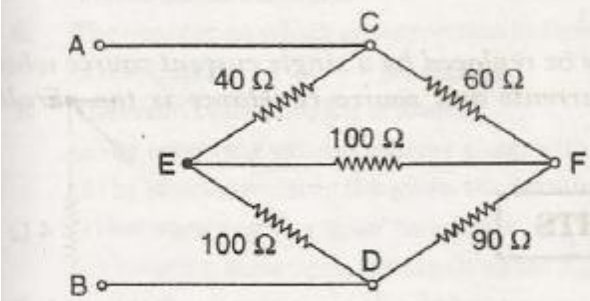


Figure 1.64

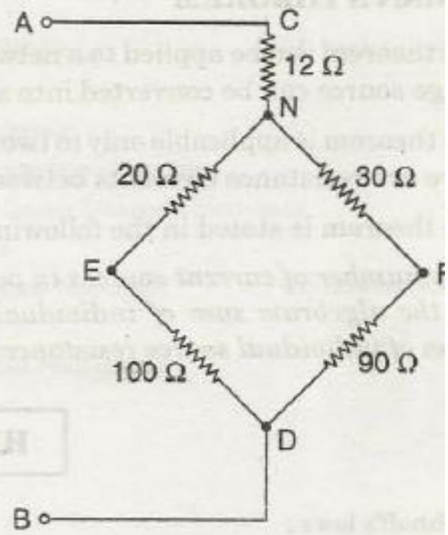


Figure 1.65

Solution. Converting CEF into star connections,

$CN = 60 \times 40 / 60 + 40 + 100 = 24\ \Omega$

$NE = 40 \times 100 / 60 + 40 + 100 = 20\ \Omega$

$NF = 60 \times 100 / 60 + 40 + 100 = 30\ \Omega$

$R_{ND} = (100 + 20) \times (90 \times 30) / 120 + 120 = 60\ \Omega$

$R_{AB} = 12 + 60 = 72\ \Omega$

Questions

Part – A

1. What is voltage ?
2. Define resistance?
3. State ohm’s law?
4. Explain the kirchoff’s law?
5. Write the conditions of series circuit?
6. Write the condition of parallel circuit?
7. State and explain Kirchhoff’s First law.
8. Give the formulas for current dividing between two parallel connected resistors.
9. Give the formulas for voltage dividing among two series connected resistors.
10. State and explain Kirchhoff’s Second law.
11. State few application of Kirchhoff’s law
12. What is a node?
13. What is a supernode?
14. Define mesh.
15. Define supermesh.
16. Write the objective of star delta transformation.
17. Given a delta circuit having resistors, write the required expressions to transform the circuit to a star circuit.
18. Given a star circuit having resistors, write the required expressions to transform the circuit to a delta circuit.
19. Three equal value resistors of value 5Ω are connected in star configuration. What is the resistance in one of the arms of equivalent delta network?
20. Three equal values resistors of value 9 Ω are connected in delta configuration. What is the resistance in one of the arms of equivalent star network?

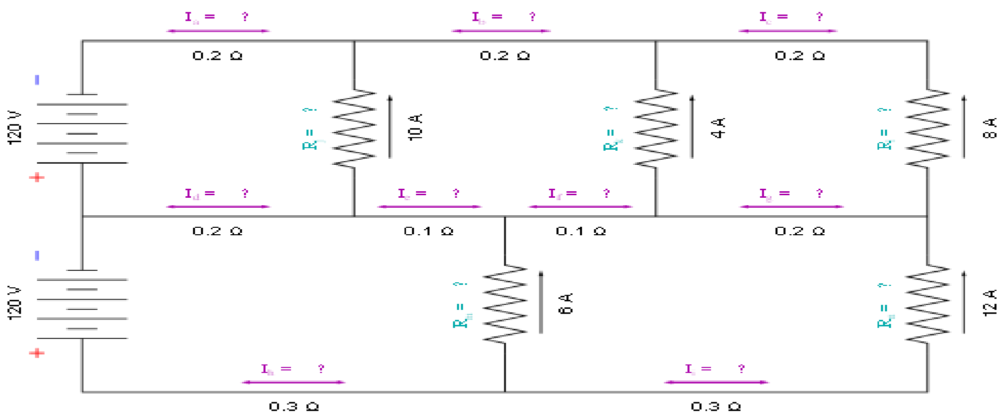
Part-B

1. A fairly complicated three-wire circuit is shown below. The source voltage is 120 V between the center (neutral) and the outside (hot) wires. Load currents on the upper half of the circuit are given as 10 A, 4 A, and 8 A for the load resistors j, k, and l, respectively. Load currents on the lower half of the circuit are given as 6 A and 12 A for the load resistors m and n, respectively. The resistances of the connecting wires a, b, c, d, e, f, g, h, and i are also given. Determine…

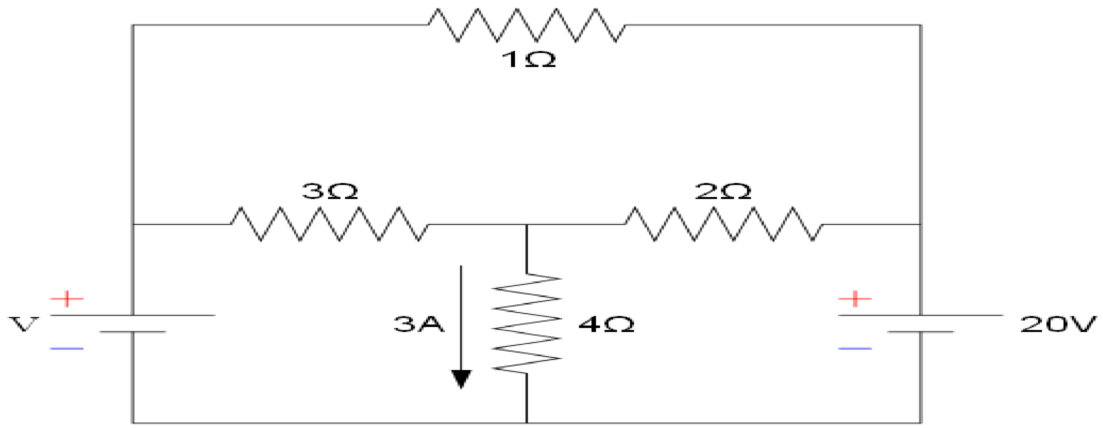
the current through each of the connecting wires (a, b, c, d, e, f, g, h, i) with the direction (left, right);

the voltage drop across each load element (j, k, l, m, n); and

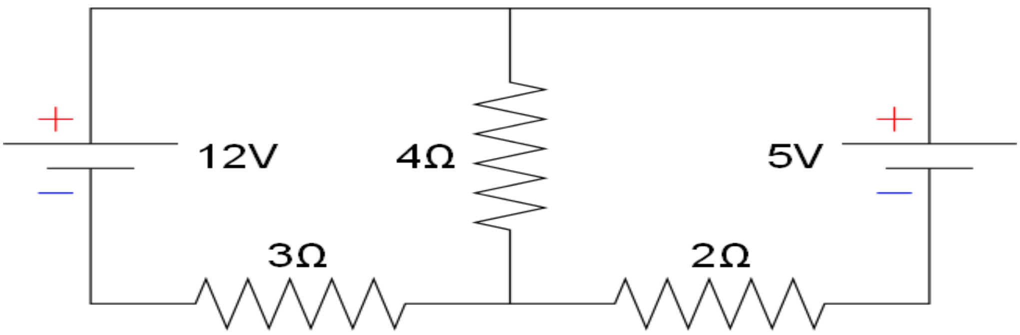
the resistance of each load element (j, k, l, m, n).



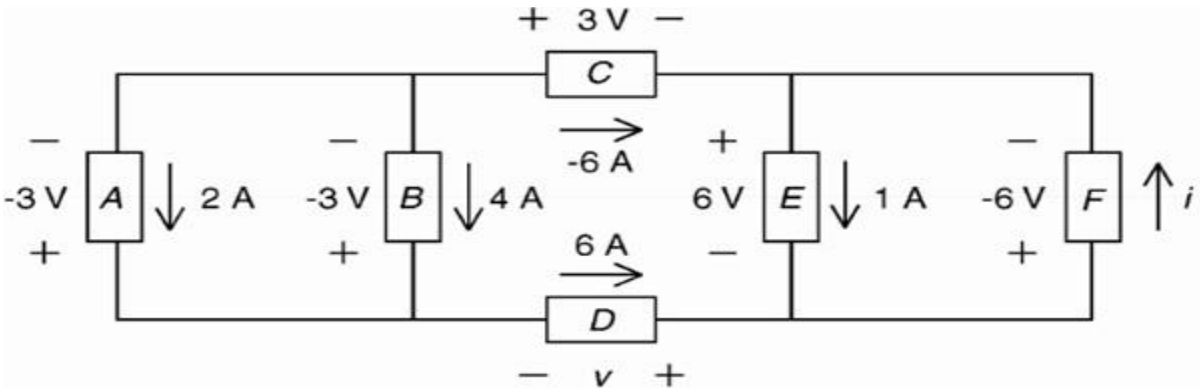
2. Given the circuit below with 3 A of current running through the 4 Ω resistor as indicated in the diagram to the right. Determine. The current through each of the other resistors, the voltage of the battery on the left, and the power delivered to the circuit by the battery on the right.



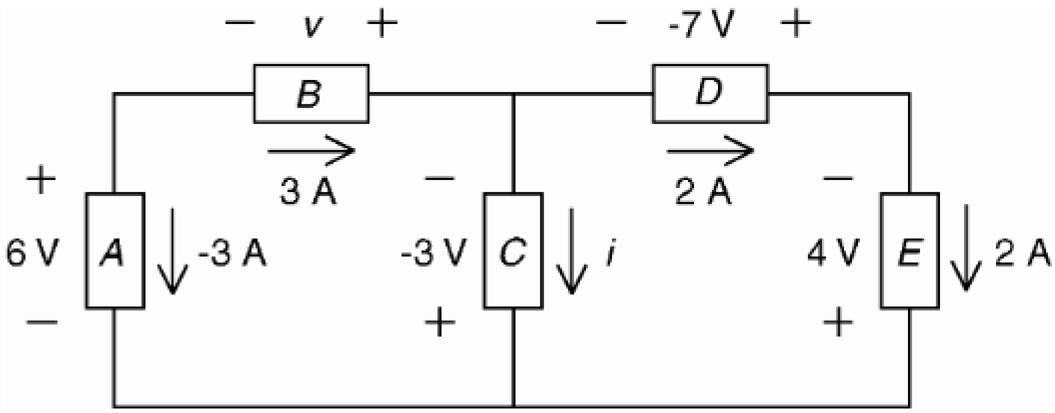
3. Determine the current through each resistor in the circuit shown below.



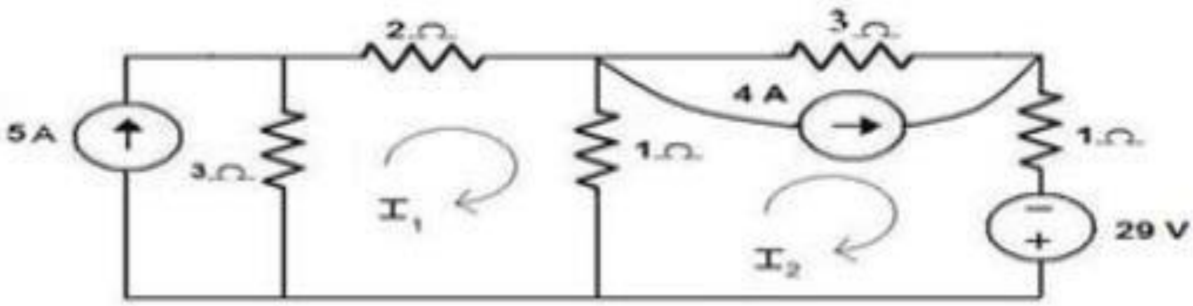
4. Consider the circuit shown below. Determine the power supplied by element D and the power received by element F.



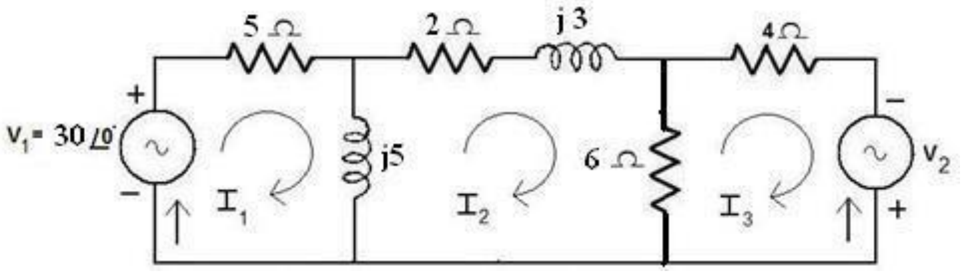
5. Consider the circuit shown below. Determine the power supplied by element B and the power supplied by element C.



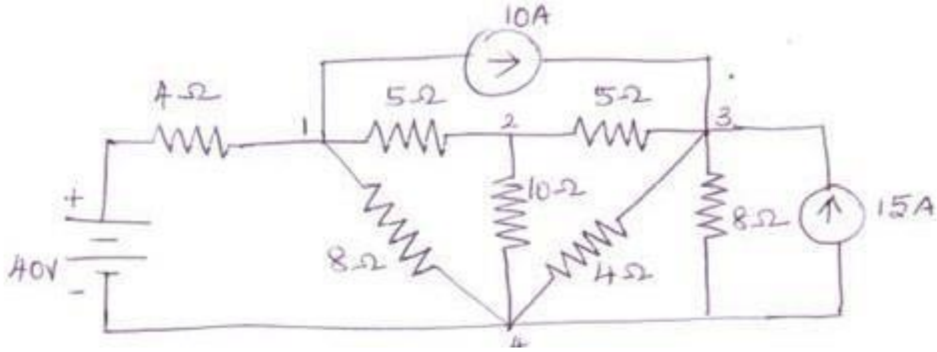
6.Determine the mesh currents I1 and I2 for the given circuit shown below



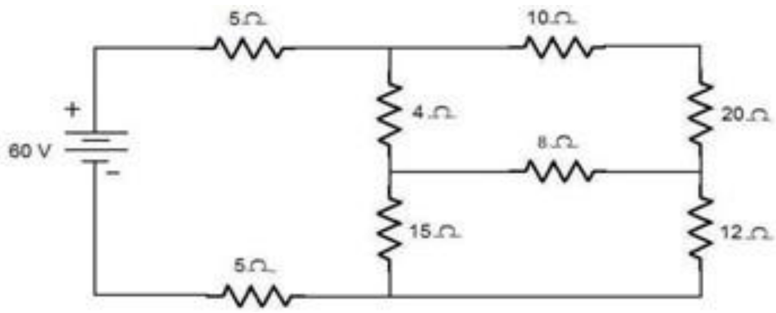
7.Determine the value of V2 such that the current through the impedance (2+j3) ohm is zero.



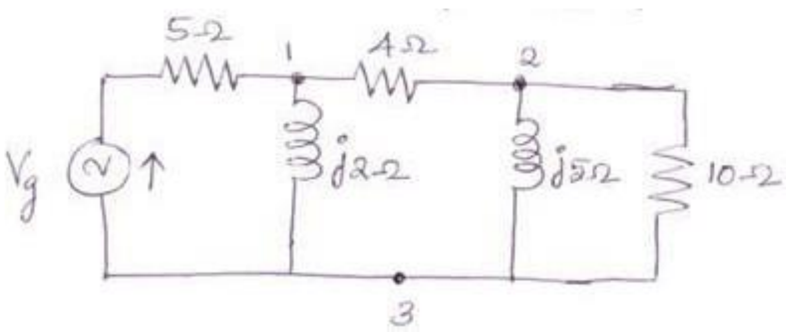
8.Use Nodal Voltage method and find thepower dissipated in the 10 Ω resistance on the circuit shown below



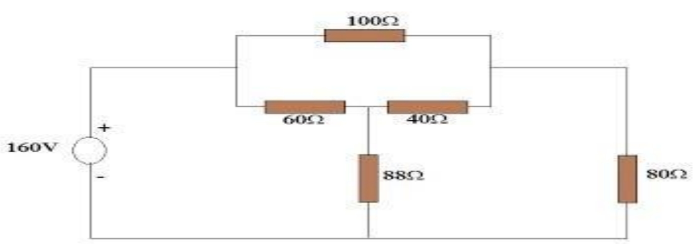
9.In the network shown below, find the current delivered by the battery.



10.Given the nodes 1 and 2 in network of figure, Find the ratio of voltage V1 / V2

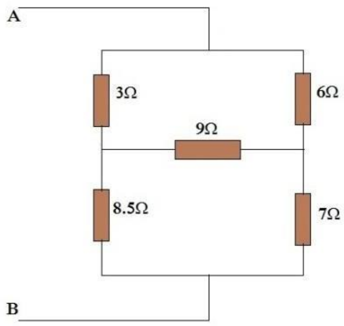


11. Determine the resistance between the terminals A&B and hence find the current through the voltage source. Refer figure1



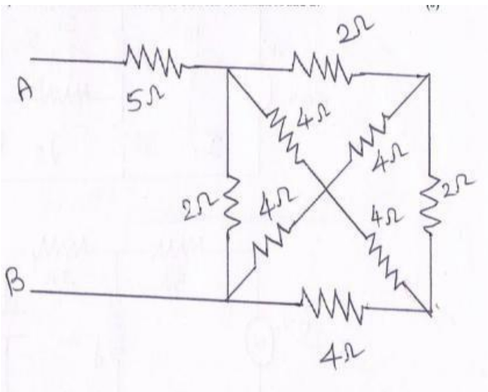
12.Derive the expression for conversion of delta into star with neat Diagram.

13. Find the total resistance between A&B terminals for the network shown in figure 2



14. Derive the expression for conversion of star into delta with the neat diagram

15. Use star to delta transformation to find resistance between terminals A and B

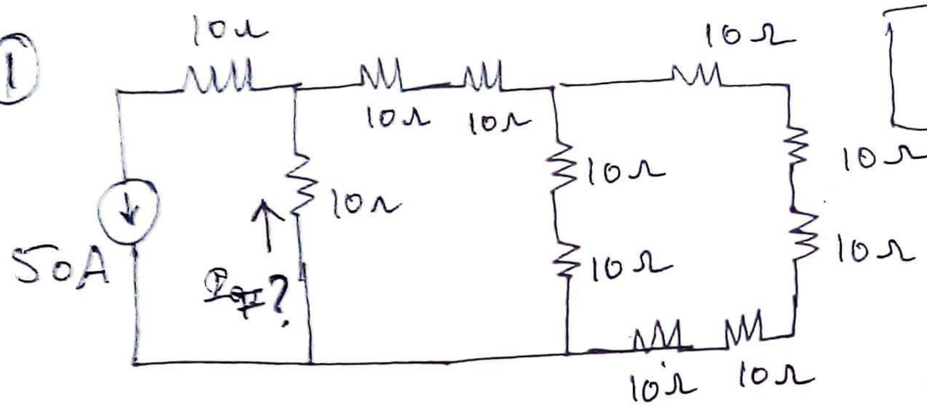


Problems on Series & Parallel Circuits

Voltage division & Current division

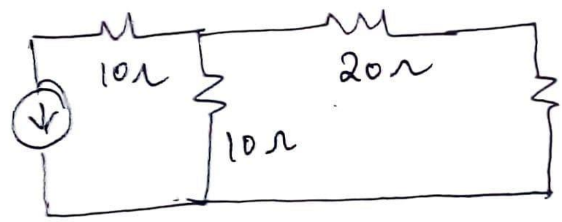
(1)

$R_T = ?$
$I = ?$



— (A)

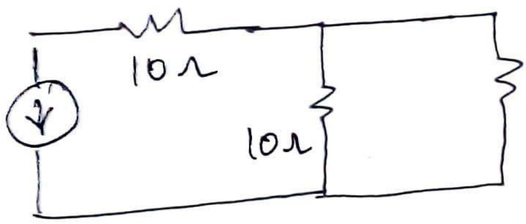
— (B)



— (C)

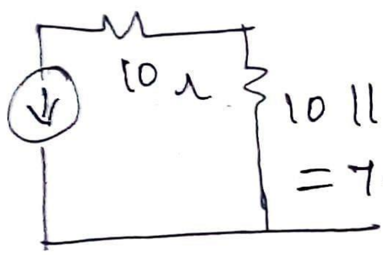
$20 \parallel 50 = 14.28\,\Omega$

$\frac{20 \times 50}{20+50} = \frac{1000}{70} = 14.28\,\Omega$



— (D)

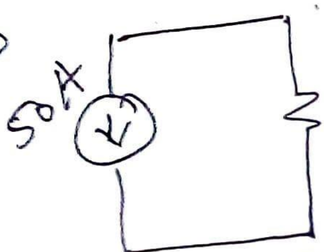
$20 + 14.28 = 34.28\,\Omega$



— (E)

$10 \parallel 34.24 = 7.74\,\Omega$

$\frac{10 \times 34.28}{10+34.28} = \frac{342.8}{44.28} = 7.74\,\Omega$

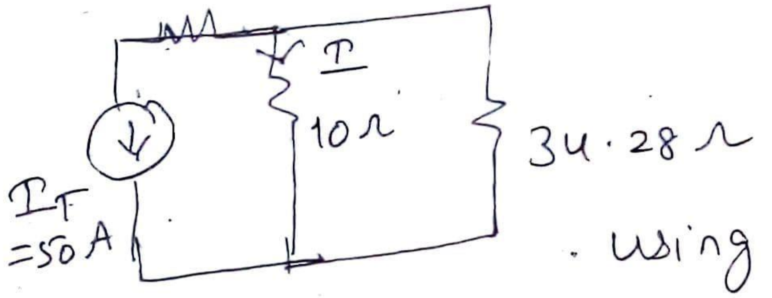


— (F)

$10 + 7.74\,\Omega = 17.74\,\Omega$

$$\boxed{R_T = 17.74\,\Omega}$$

To find current I, consider Fig ① ②



using current division,

$$I = \frac{50 \times 34.28}{34.28 + 10} = \frac{1714}{44.28}$$

$$\boxed{I = 38.71\ A}$$

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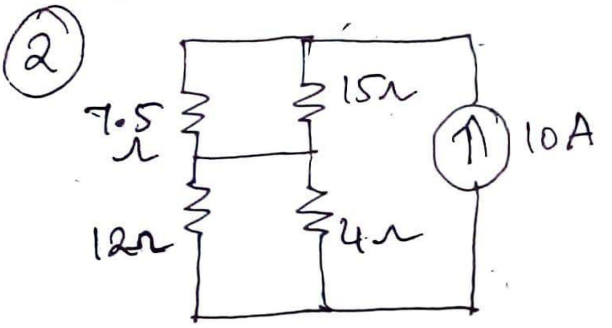
② Question

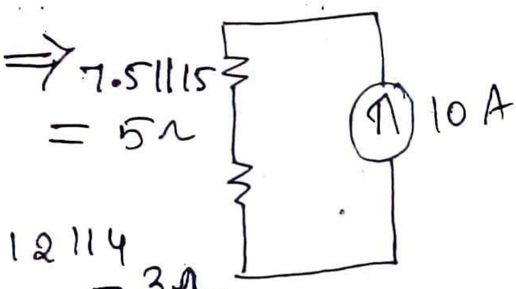
Find R_T

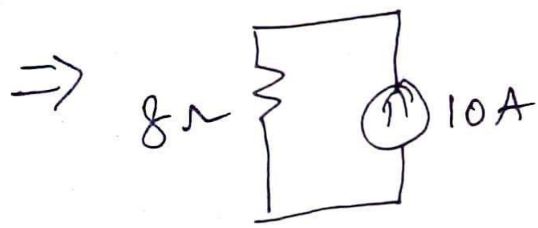
Power Supplied by Source = ?



$$\frac{7.5 \times 15}{7.5 + 15} = 5$$

$$\frac{12 \times 4}{12 + 4} = \frac{48}{16} = 3$$





$$\boxed{R_T = 8\Omega}$$

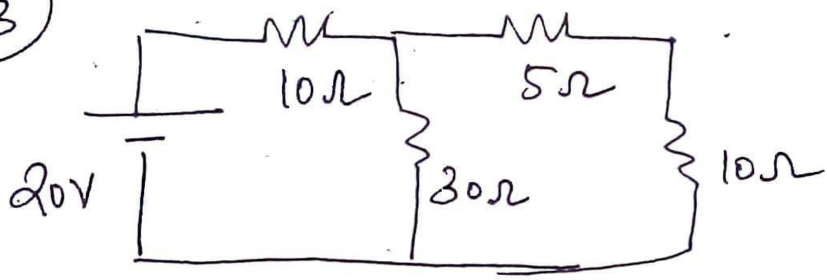
Power supplied by Source $= VI = I^2R$

$= VI = (IR_T) \times I = (10 \times 8) \times 10$

$= 800$ watts.

(or) $I^2R = (10)^2 \times 8\Omega = 800$ watts

3



Question 3

$R_T = ?$

$I_T = ?$

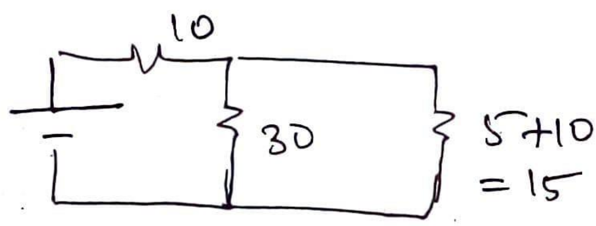
$I_{30\Omega} = ?$

$I_{5\Omega} = ?$

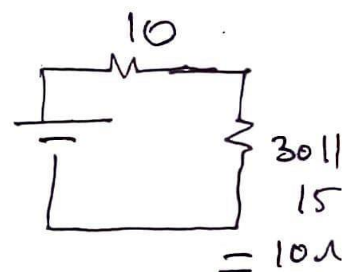
$$R_T = [(5+10) \parallel 30] + 10$$
$$= [15 \parallel 30] + 10$$
$$= \frac{15 \times 30}{15+30} + 10 = 10 + 10 = 20\Omega$$

$$\boxed{R_T = 20\Omega}$$

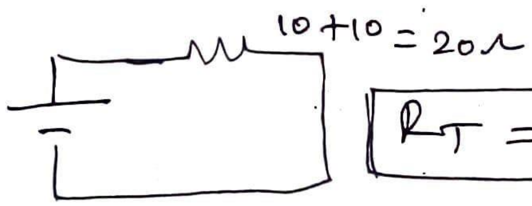
(ie) ⇒



⇒



⇒



$$\boxed{R_T = 20\Omega}$$

$$\boxed{I_T = \cancel{20} \quad \frac{V}{R_T} = \frac{20}{20} = 1A}$$

$$I_{30\Omega} = 1 \times \frac{(10+5)}{30+(10+5)} = \frac{15}{45}$$

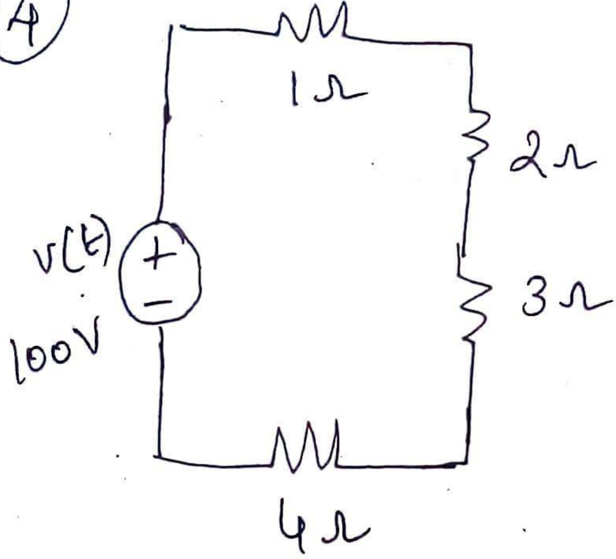
$$\boxed{I_{30\Omega} = \frac{1}{3} = 0.33\ A}$$

$$I_{5\Omega} = 1 \times \frac{30}{30+5+10} = \frac{30}{45} = \frac{2}{3}$$

$$\boxed{I_{5\Omega} = 0.67\ A}$$

→ Using Current Division

(4)



(4)

Question.

Find Voltage across all resistors.

$V_{1\Omega} = ?$

$V_{2\Omega} = ?$

$V_{3\Omega} = ?$

$V_{4\Omega} = ?$

using Voltage Division,

$$\Rightarrow V_{1\Omega} = \frac{100 \times 1}{1+2+3+4} = \frac{100}{10} = 10V$$

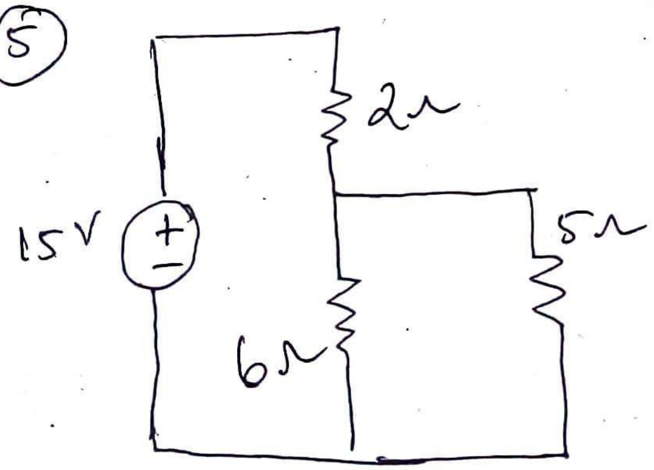
$$V_{2\Omega} = \frac{100 \times 2}{1+2+3+4} = \frac{200}{10} = 20V$$

$$V_{3\Omega} = \frac{100 \times 3}{1+2+3+4} = \frac{300}{10} = 30V$$

$$V_{4\Omega} = \frac{100 \times 4}{1+2+3+4} = \frac{400}{10} = 40V$$

---

(5)



$R_T = ?$ $I_T = ?$

$I_{5\Omega} = ?$

$$R_T = (6 \| 5) + 2$$

$$= \frac{6 \times 5}{6+5} + 2$$

$$= \frac{30}{11} + 2$$

$$\boxed{R_T = 4.73\ \Omega}$$

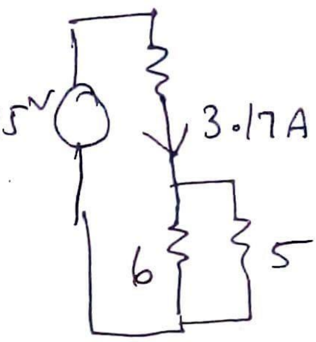
5

$$I_T = \frac{V}{R_T} = \frac{15}{4.73}$$

$$\boxed{I_T = 3.17\ A}$$

using current Division,

$$I_{6\Omega} = 3.17 \times \frac{5}{5+6}$$



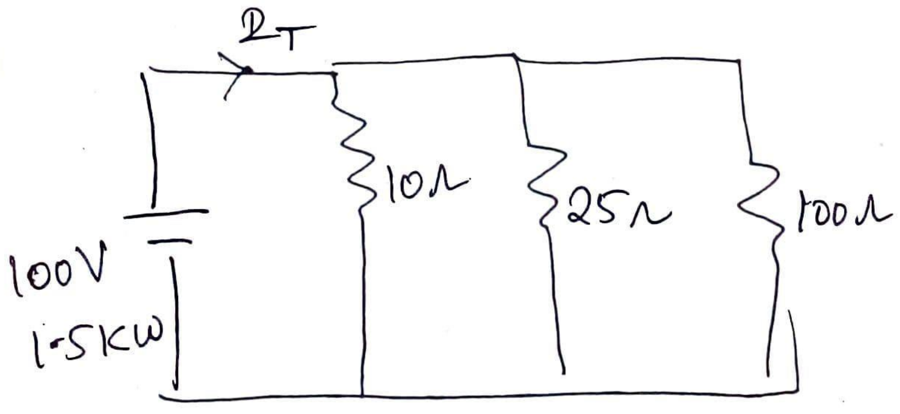
$$= 3.17 \times \frac{5}{11}$$

$$= \boxed{1.44\ A}$$

$$I_{5\Omega} = 3.17 \times \frac{6}{5+6}$$

$$= \boxed{1.73\ A}$$

6



$I_T = ?$

$I_{10\Omega} = ?$

$I_{25\Omega} = ?$

$I_{100\Omega} = ?$

$$\text{Power} = 1.5\ \text{KW} = VI$$

$$\therefore I = \frac{\text{Power}}{V} = \frac{1500 \text{ watts}}{100}$$

$$\boxed{I_T = 15\ A}$$

Using Current Division

$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{25} + \frac{1}{100}$$

$$\frac{1}{R_T} = 0.1 + 0.04 + 0.01$$

$$= 0.15$$

$$\therefore \boxed{R_T = \frac{1}{0.15} = 6.667\ \Omega}$$

Using Current Division technique.

$$I_{10\Omega} = I_T\left(\frac{R_T}{10}\right) = 15\left(\frac{6.667}{10}\right) = 10\text{ A}$$

$$I_{25\Omega} = I_T\left(\frac{R_T}{25}\right) = 15\left(\frac{6.667}{25}\right) = 4\text{ A}$$

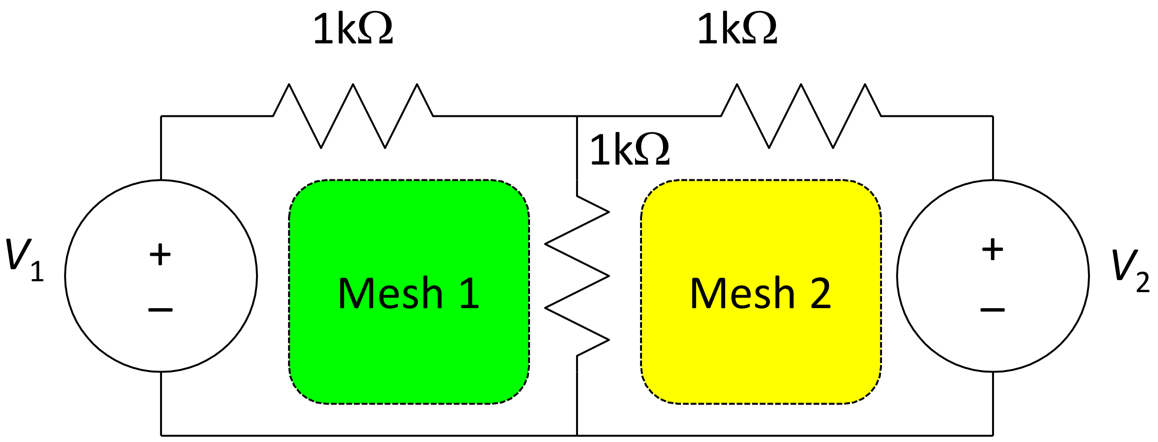
$$I_{100\Omega} = I_T\left(\frac{R_T}{100}\right) = 15\left(\frac{6.667}{100}\right) = 1\text{ A}$$

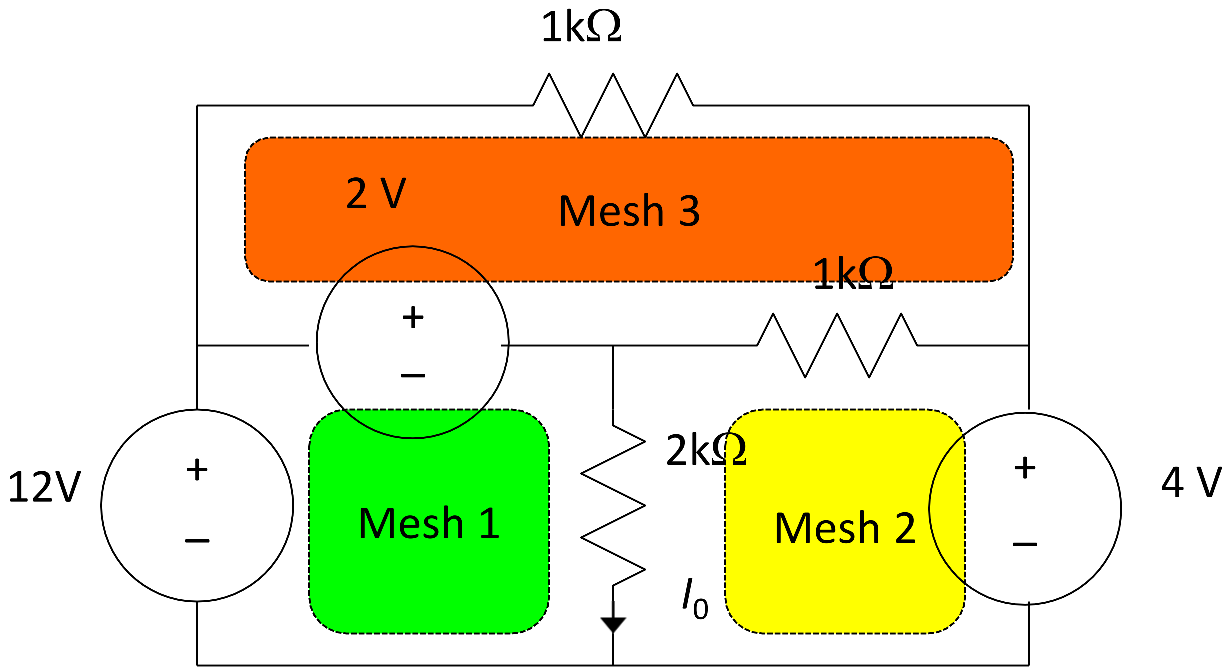



SATHYABAMA


SATHYABAMA

Identifying the Meshes





By ohms law V=IR

If the circuit contains 2 loops then matrix will be 2X2 matrix

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

If the circuit contains 3 loops then matrix will be 3X3 matrix

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{23} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

Wh

R_{11} →

R_{22} → Sum of th

R_{33} → Sum of the resistance

$R_{31} = R_{13}$ → Sum of the resistances in l

$R_{12}=R_{21}$ → Sum of the resistances in loop 1 and 2

$R_{32}=R_{23}$ → Sum of the resistances in loop 2 and 3

V_1 → Sum of the voltages in loop 1

V_2 → Sum of the voltages in loop 2

V_3 → Sum of the voltages in loop 3

I_1 → Unknown loop current in loop 1

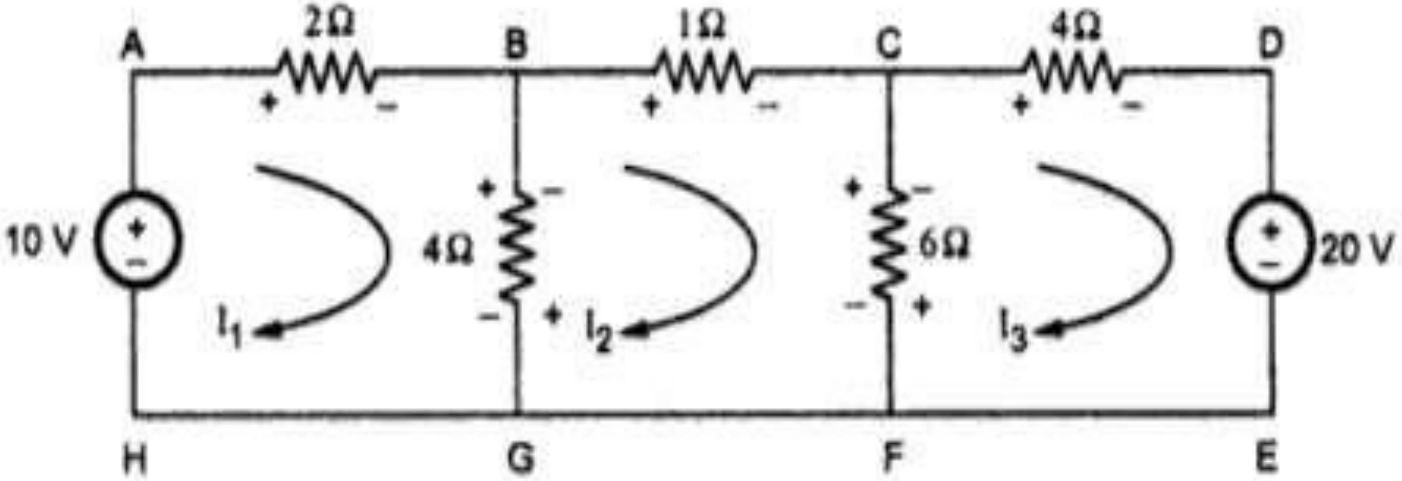
I_2 → Unknown loop current in loop 2

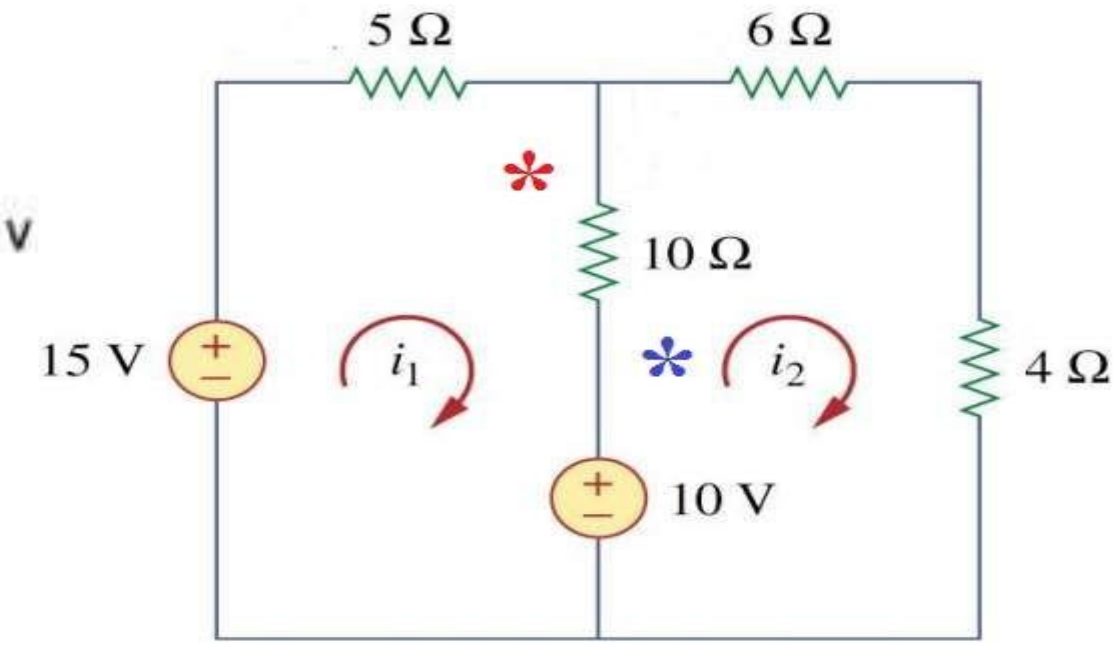
I_3 → Unknown loop current in loop 3

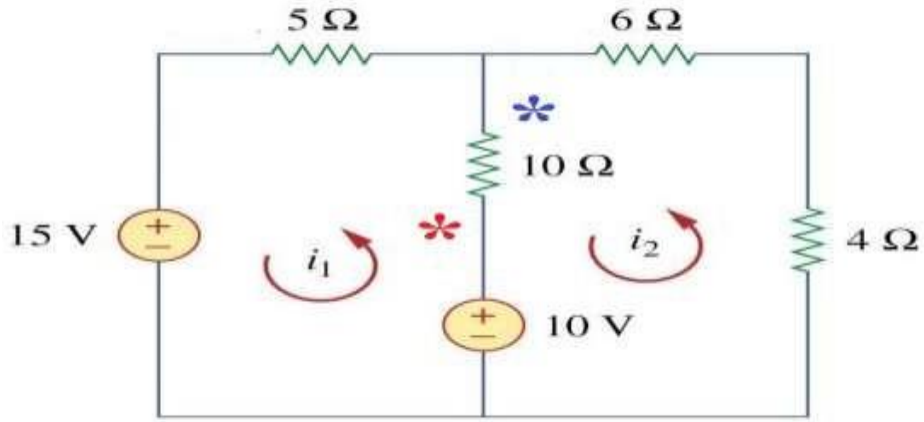
Rules:

All the diagonal elements are positive (R_{11}, R_{22}, R_{33})

All the Non diagonal elements($R_{12}, R_{23}, R_{13}, R_{21}, R_{32}, R_{31}$) are Negative if all the loop currents are in the same direction

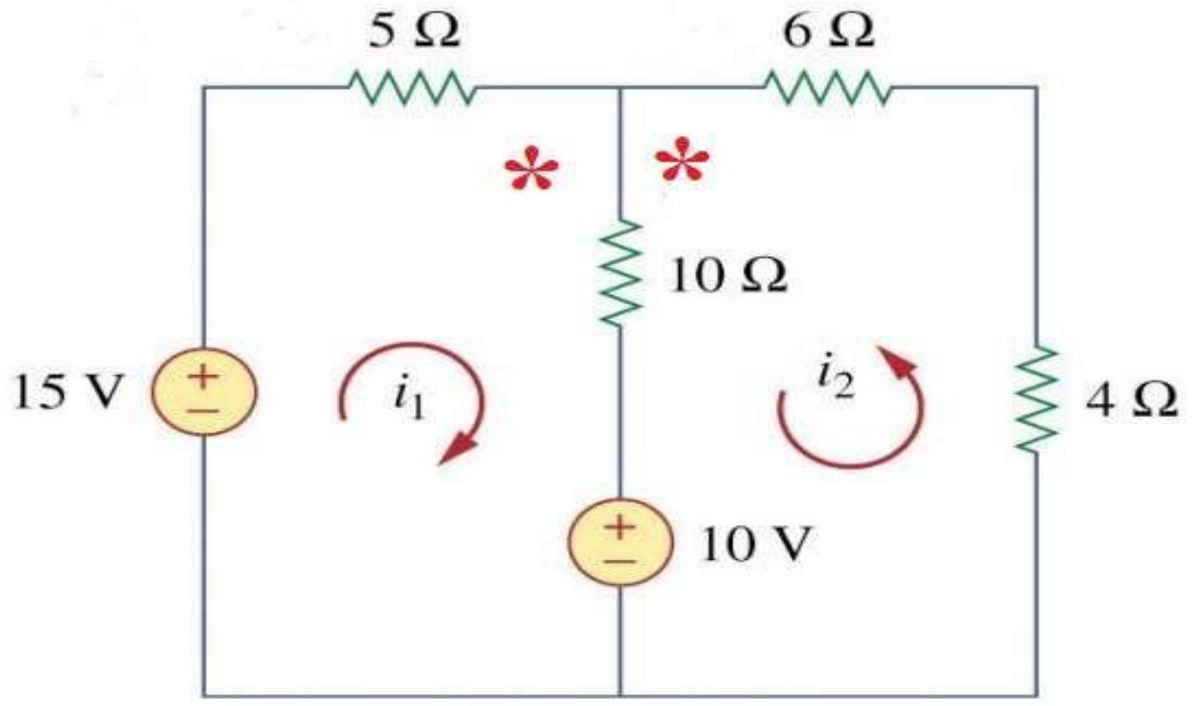






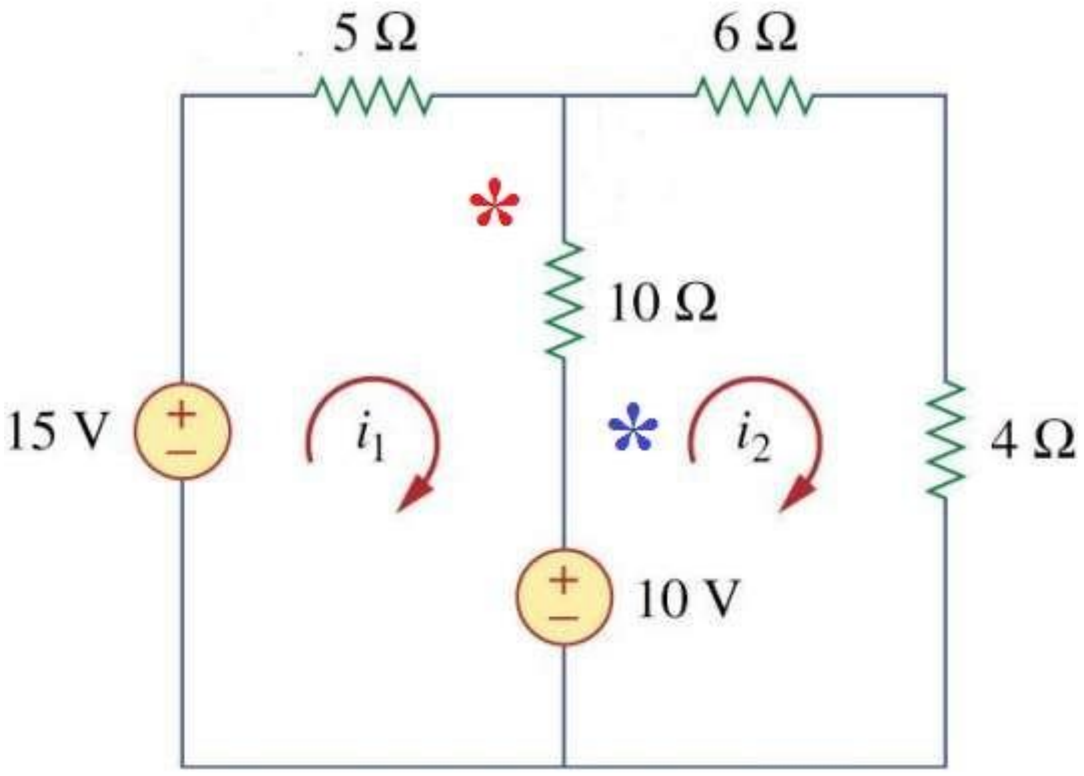
Rules continued

The Non diagonal elements(R_{12},R_{23},R_{13},R_{21},R_{32},R_{31}) are positive if the loop currents in the common element aid each other.



Problem No.1

Solve the given circuit and find the mesh currents and branch currents and branch voltages.



Solution for Problem No.1

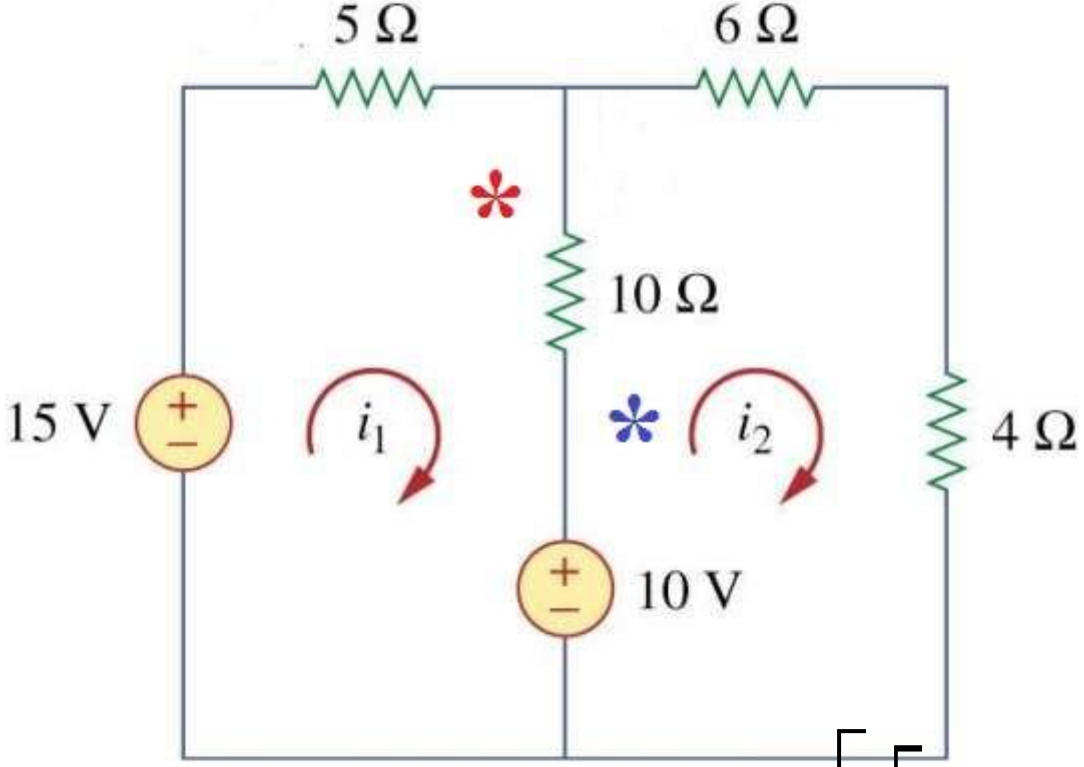
$R_{11} = 5+10 = 15$

$R_{22} = 10+6+4 = 20$

$R_{12} = R_{21} = -10$

$V_1 = 15-10 = 5$

$V_2 = 10$



$$\begin{bmatrix}15 & -10\\ -10 & 20\end{bmatrix}\begin{bmatrix}I_1\\ I_2\end{bmatrix} = \begin{bmatrix}5\\ 10\end{bmatrix}$$

$$\Delta = \begin{bmatrix}15 & -10\\ -10 & 20\end{bmatrix} = 200$$

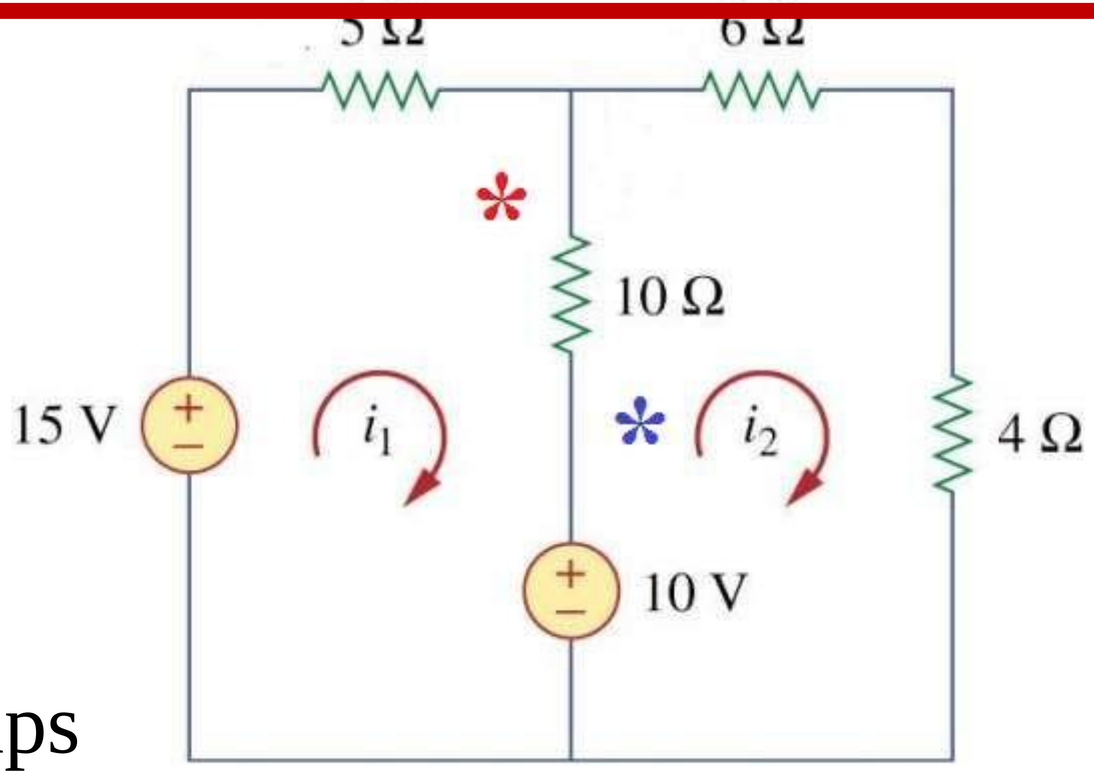
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{200}{200} = 1Amps$$

$$\Delta_1 = \begin{bmatrix}5 & -10\\ 10 & 20\end{bmatrix} = 200$$

$$\Delta_2 = \begin{bmatrix}15 & 5\\ -10 & 10\end{bmatrix} = 200$$

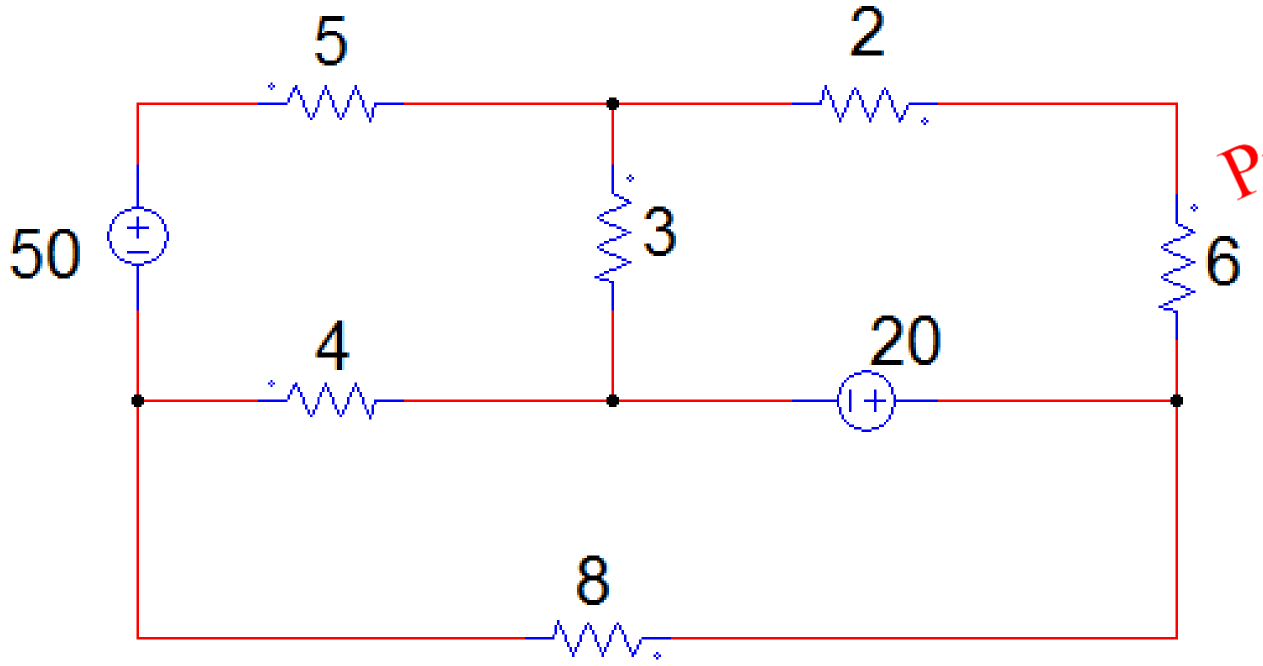
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{200}{200} = 1Amps$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{200}{200} = 1Amps$$



Current in 5Ω Resistor= I_1=1 Amps
Current in 6Ω Resistor= I_2=1 Amps
Current in 4Ω Resistor= I_2=1 Amps
Current in 10Ω Resistor= I_1-I_2=0 Amps

Voltage in 5Ω Resistor =1*5 =5 Volts
Voltage in 6Ω Resistor= 6*1= 6 Volts
Voltage in 4Ω Resistor= 4*1= 4 Volts
Voltage in 10Ω Resistor= 0 Volts



Problem No.2

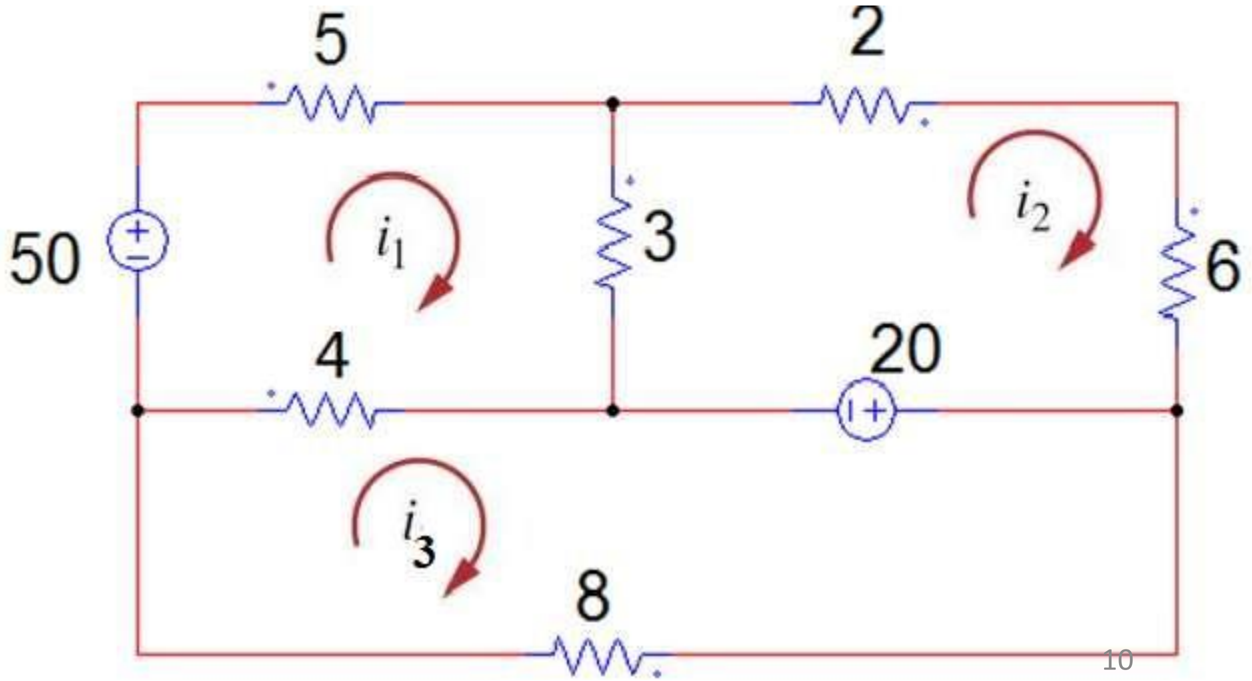
Solve the given circuit by Mesh Analysis and find the loop currents

Solution for Problem No.2

R_{11}=5+3+4=12

R_{22}=2+6+3=11

R_{33}=4+8=12

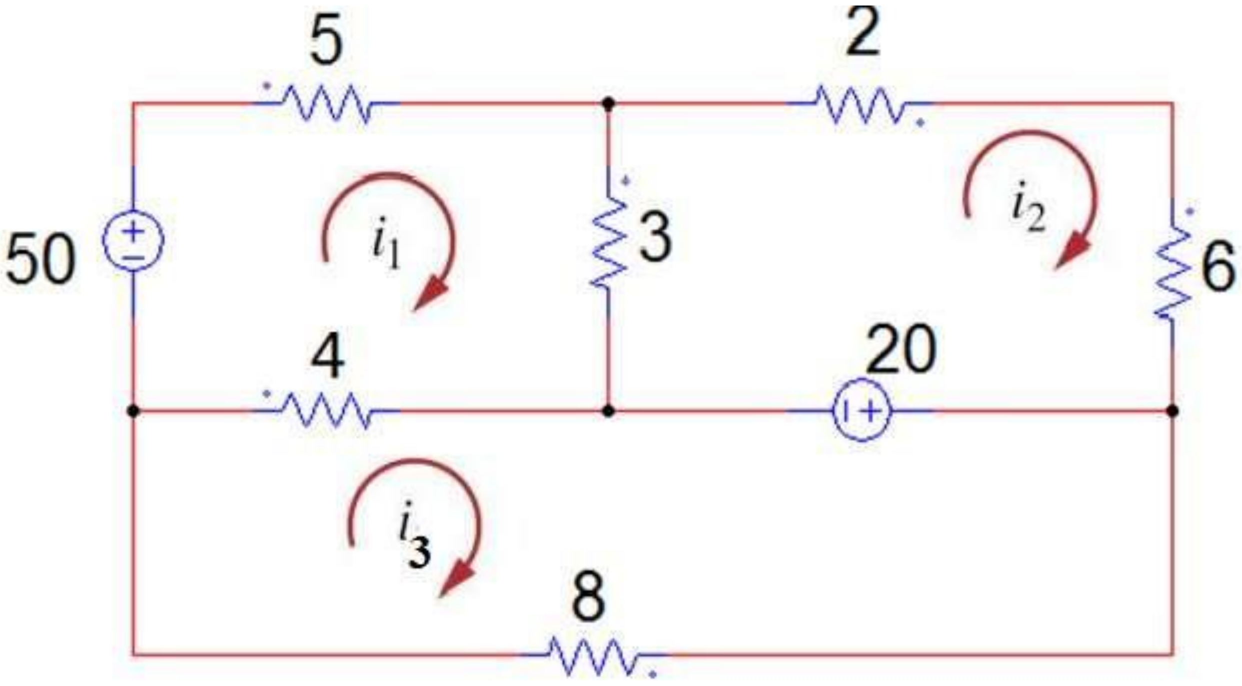


Solution for Problem No.2

$R_{12}=R_{21}= -3$

$R_{13}=R_{31}= -4$

$R_{23}=R_{32}= 0$



$V_1= 50$

$V_2= -20$

$V_3=20$

$$\begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 20 \end{bmatrix}$$

Solution for Problem No.2

$$\begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -20 \\ 20 \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{6760}{1300} = 5.2\,Amps$$

$$\Delta = \begin{bmatrix} 12 & -3 & -4 \\ -3 & 11 & 0 \\ -4 & 0 & 12 \end{bmatrix} = 1300$$

$$\Delta_2 = \begin{bmatrix} 12 & 50 & -4 \\ -3 & -20 & 0 \\ -4 & 20 & 12 \end{bmatrix} = -520$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-520}{1300} = -0.4\,Amps$$

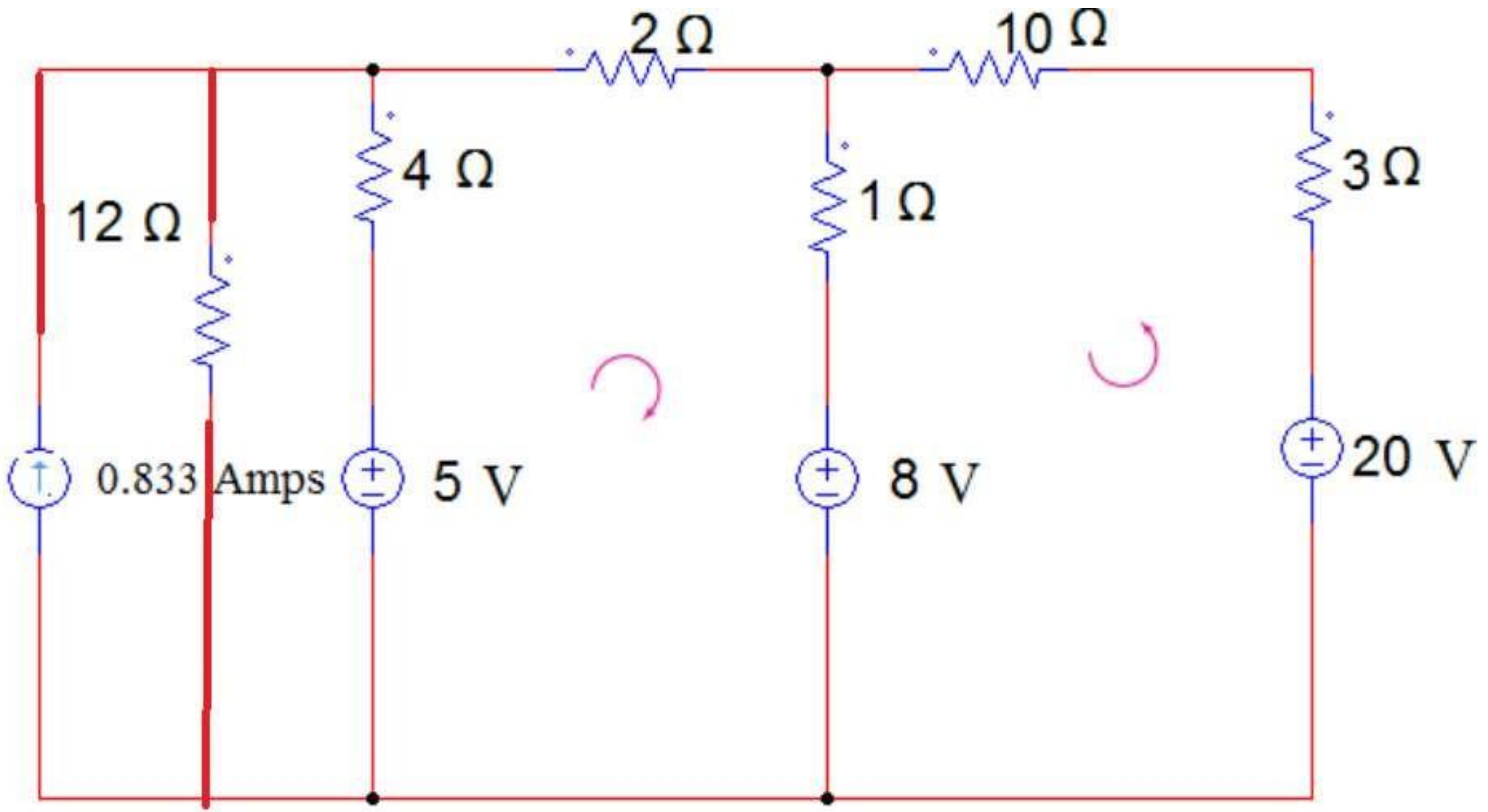
$$\Delta_1 = \begin{bmatrix} 50 & -3 & -4 \\ -20 & 11 & 0 \\ 20 & 0 & 12 \end{bmatrix} = 6760$$

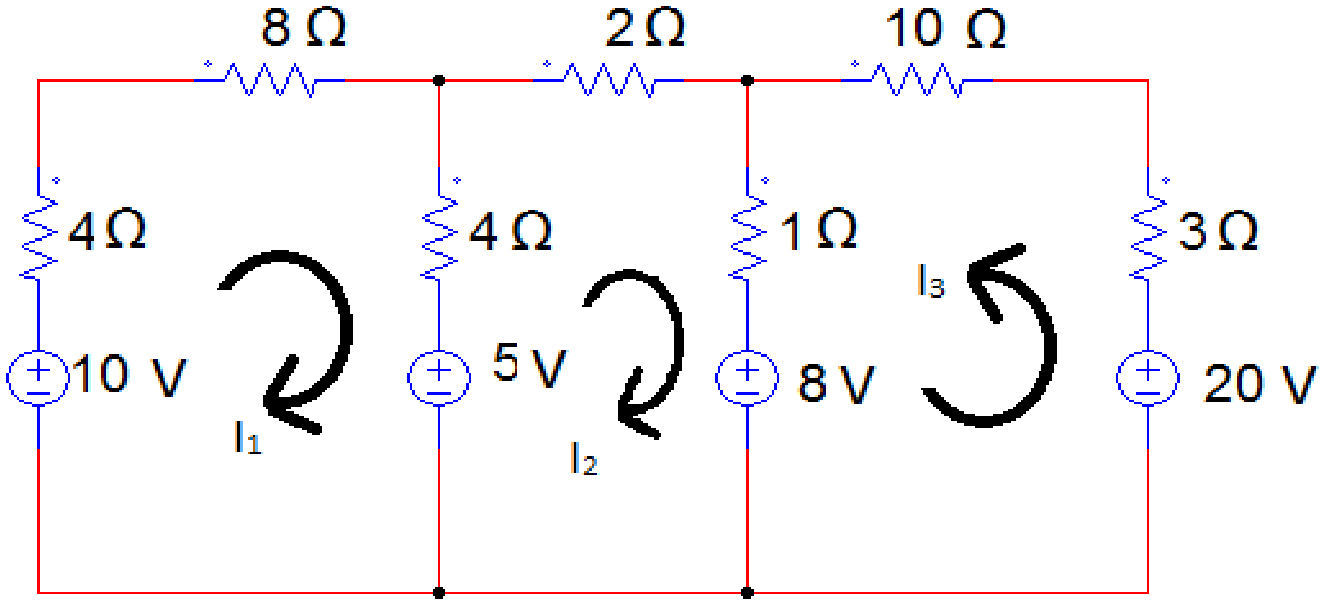
$$\Delta_3 = \begin{bmatrix} 12 & -3 & 50 \\ -3 & 11 & -20 \\ -4 & 0 & 20 \end{bmatrix} = 4420$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{4420}{1300} = 3.4\,Amps$$

Problem No.3

Find the mesh currents by using mesh analysis





Find the mesh currents by using mesh analysis

$R_{11}=4+8+4=16$

$R_{22}=4+2+1=7$

$R_{33}=1+10+3=14$

$R_{12}=R_{21}= -4$

$R_{23}=R_{32}= 1$

$R_{13}=R_{31}=0$

$V_1= 10-5 =5$

$V_2 = 5-8 = -3$

$V_3 = -8 + 20 =12$

$$\begin{bmatrix} 16 & -4 & 0 \\ -4 & 7 & 1 \\ 0 & 1 & 14 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \\ 12 \end{bmatrix}$$

$$\Delta_1 = \begin{vmatrix} 5 & -4 & 0 \\ -3 & 7 & 1 \\ 12 & 1 & 14 \end{vmatrix} = 269$$

$$\Delta = \begin{vmatrix} 16 & -4 & 0 \\ -4 & 7 & 1 \\ 0 & 1 & 14 \end{vmatrix} = 1328$$

$$\Delta_2 = \begin{vmatrix} 16 & 5 & 0 \\ -4 & -3 & 1 \\ 0 & 12 & 14 \end{vmatrix} = -584$$

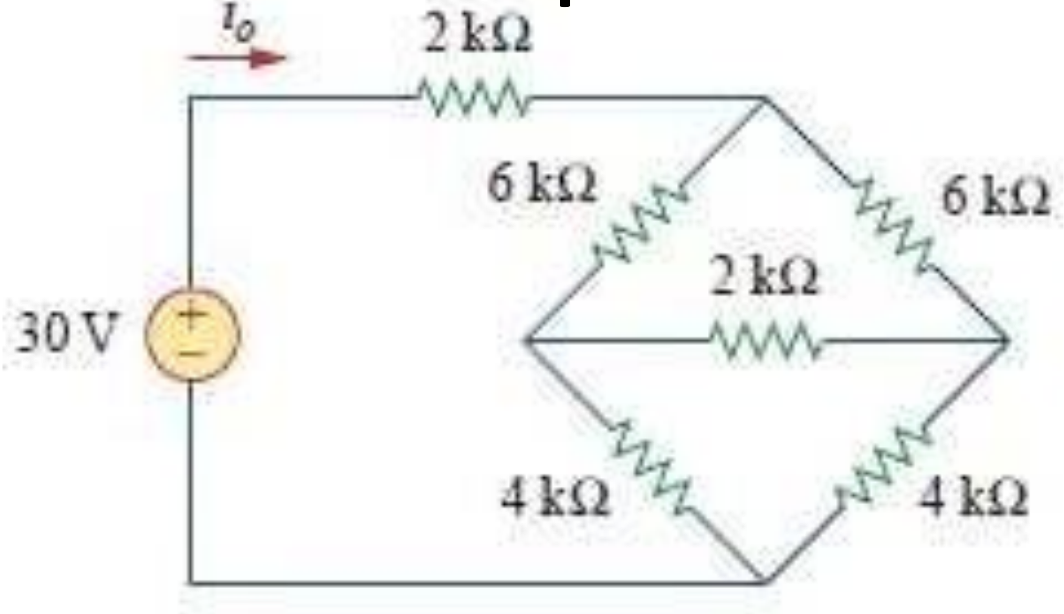
$$\Delta_3 = \begin{vmatrix} 16 & -4 & 5 \\ -4 & 7 & -3 \\ 0 & 1 & 12 \end{vmatrix} = 1180$$

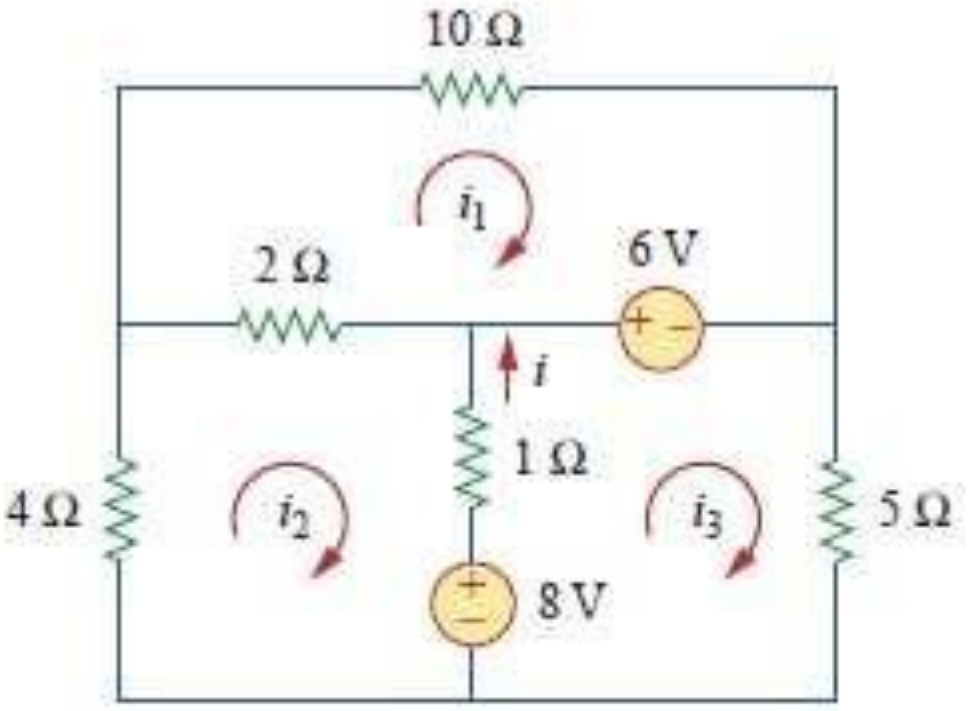
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{269}{1328} = 0.2026\,Amps$$

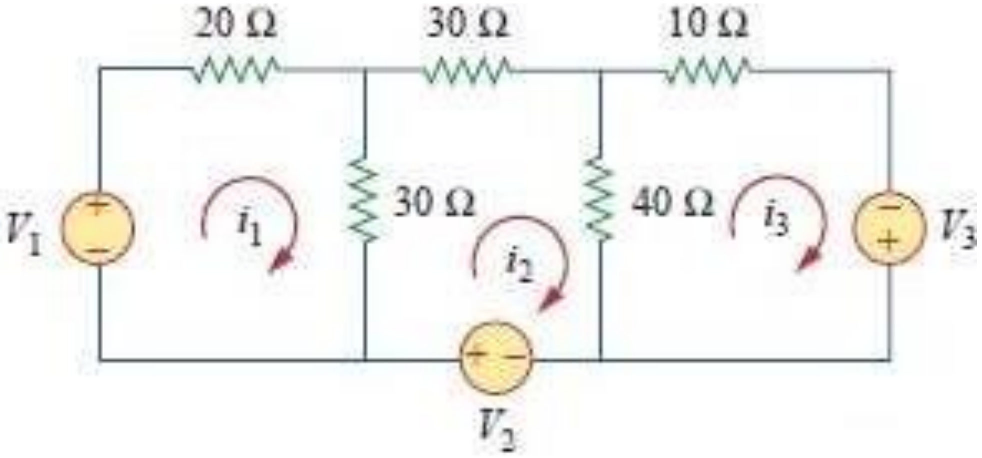
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-584}{1328} = -0.4398\,Amps$$

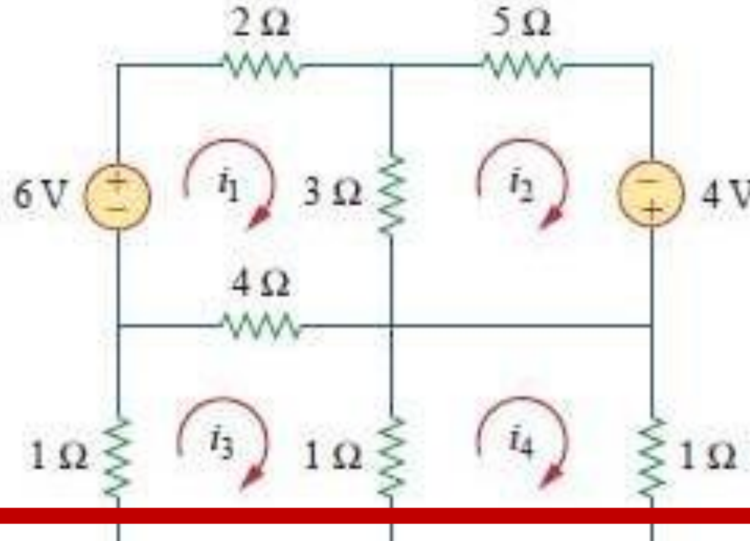
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{1180}{1328} = 0.886\,Amps$$

Additional problems

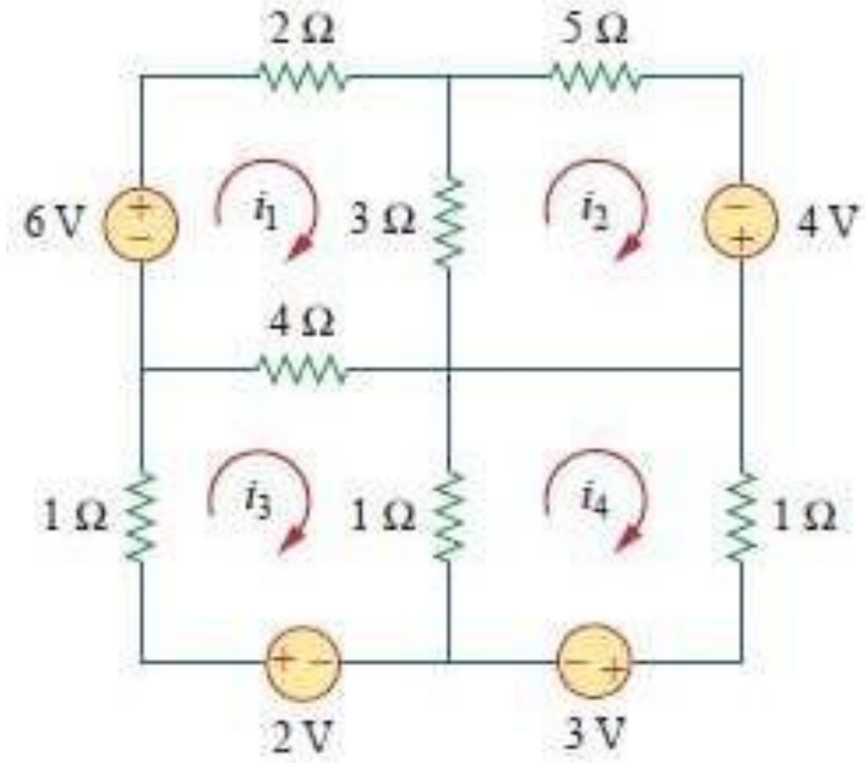


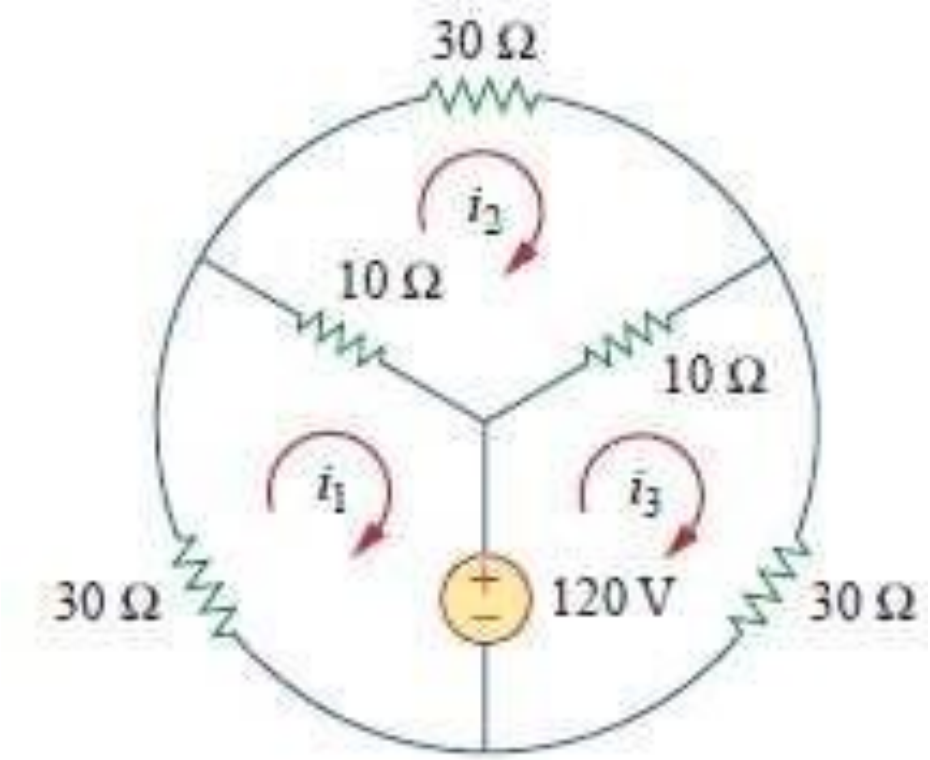






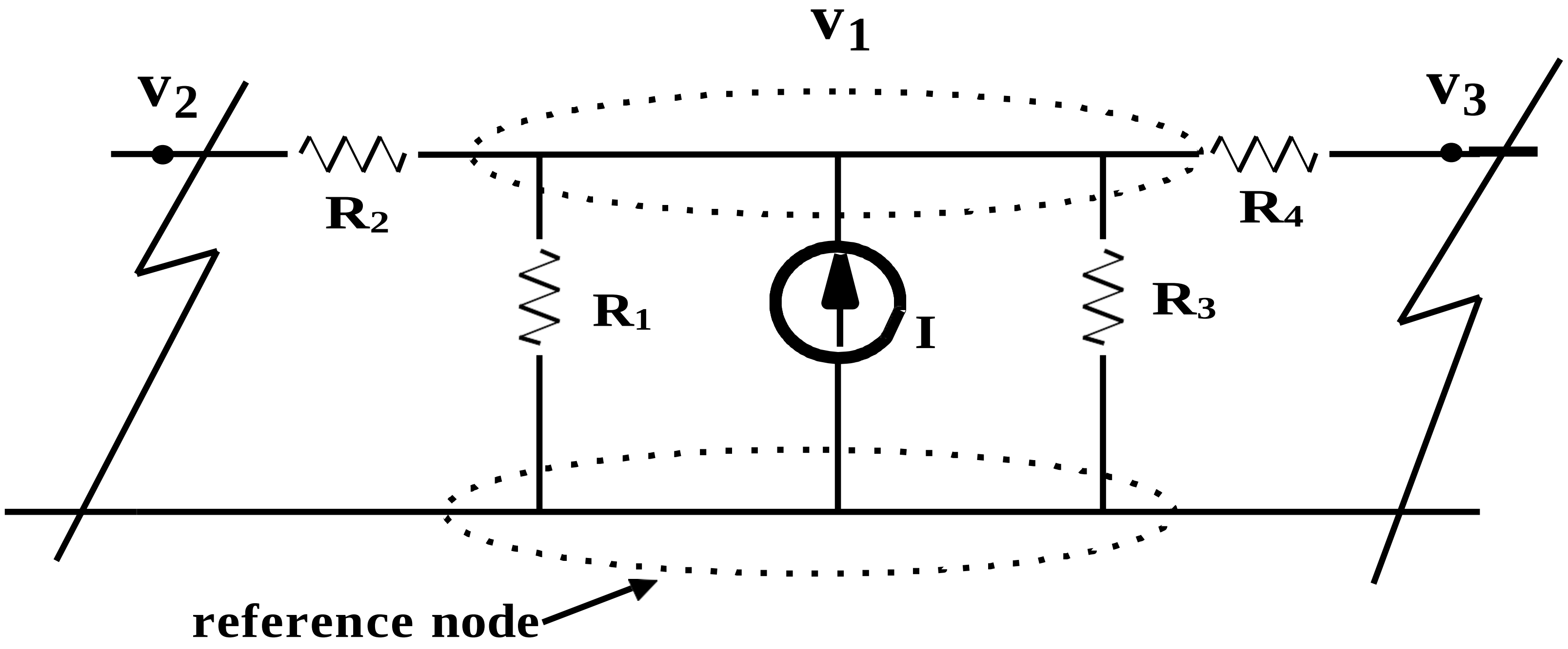
Additional problems





Nodal Analysis Concept

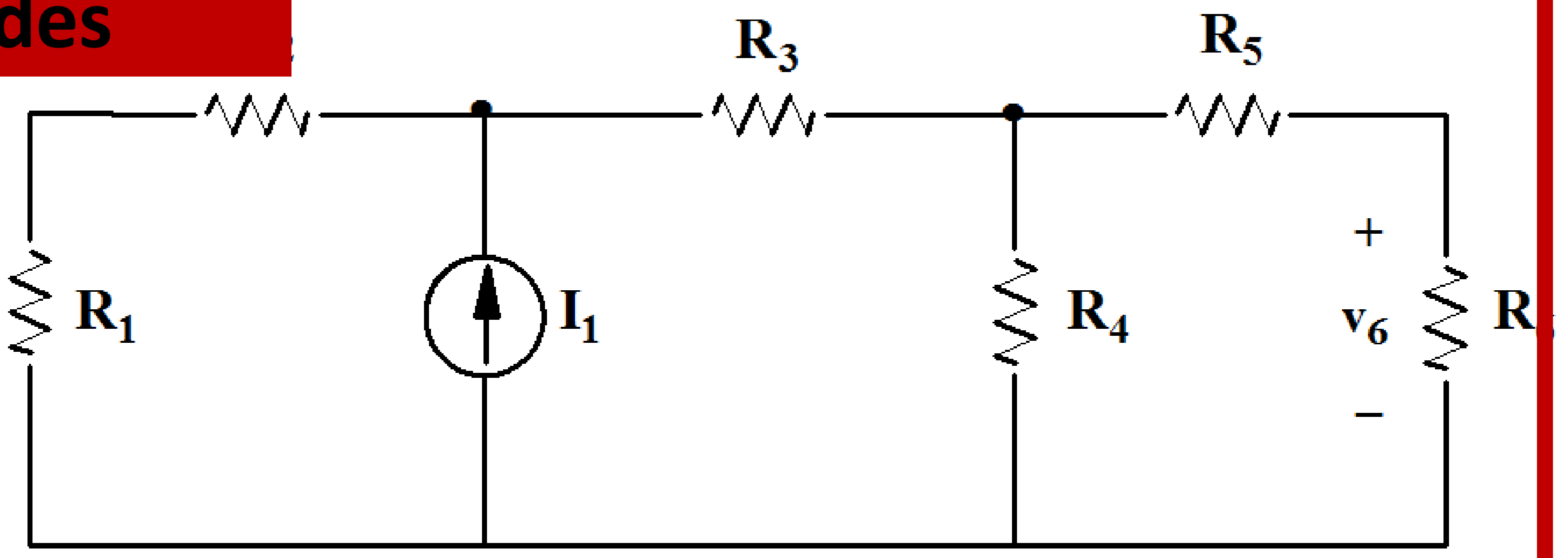
Three are more elements joined together is called as Node.

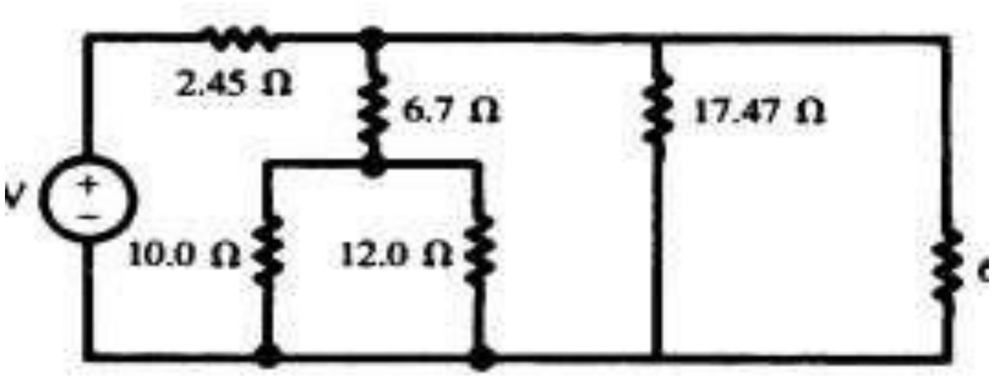


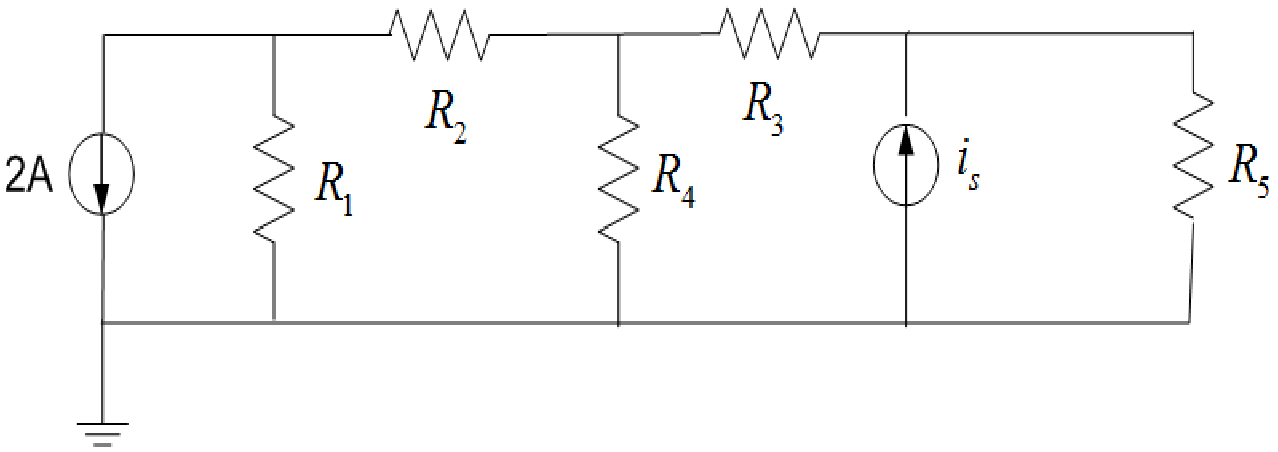
$$\frac{V_1 - V_2}{R_2} + \frac{V_1}{R_1} + \frac{V_1}{R_3} + \frac{V_1 - V_3}{R_4} = I$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) V_1 - \left(\frac{1}{R_2}\right) V_2 - \left(\frac{1}{R_4}\right) V_3 = I$$

Identify the number of nodes







Nodal Analysis by Inspection method

By ohms law V=IR

I=1/R*V

I=GV

If the circuit contains 2 major nodes then matrix will be 2X2 matrix

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

If the circuit contains 3 major nodes then matrix will be 3X3 matrix

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where

G_{11} → Sum of the conductance's in node No. 1

G_{22} → Sum of the conductance's in node No. 2

G_{33} → Sum of the conductance's in node No. 3

$G_{31} = G_{13}$ → Sum of the conductance's connected between node No. 1and 3

$G_{12} = G_{21}$ → Sum of the conductance's connected between node No. 1 and 2

$G_{32} = G_{23}$ → Sum of the conductance's connected between node No. 2 and 3

I_1 → Sum of the currents flowing towards node no. 1

I_2 → Sum of the currents flowing towards node no. 2

I_3 → Sum of the currents flowing towards node no.3

V_1 → Unknown node voltage of node no. 1

V_2 → Unknown node voltage of node no. 2

V_3 → Unknown node voltage of node no. 3

Rules:

All the diagonal(G_{11},G_{22},G_{33})elements are positive.

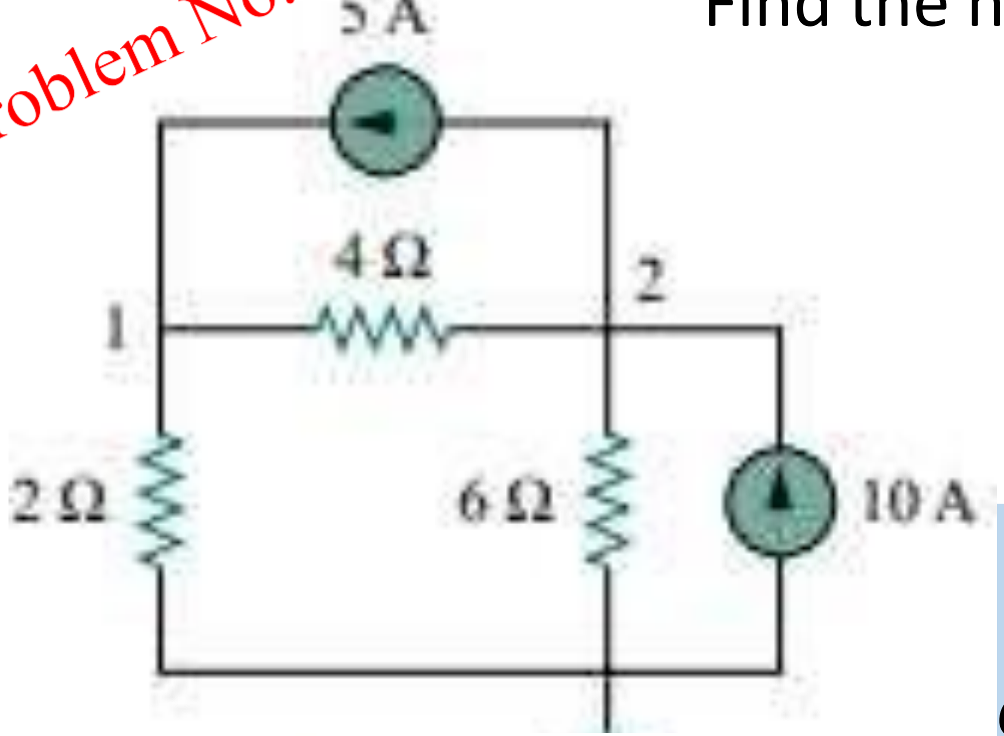
All the Non diagonal(G_{12},G_{13},G_{23},G_{32},G_{21},G_{31}) elements are negative.

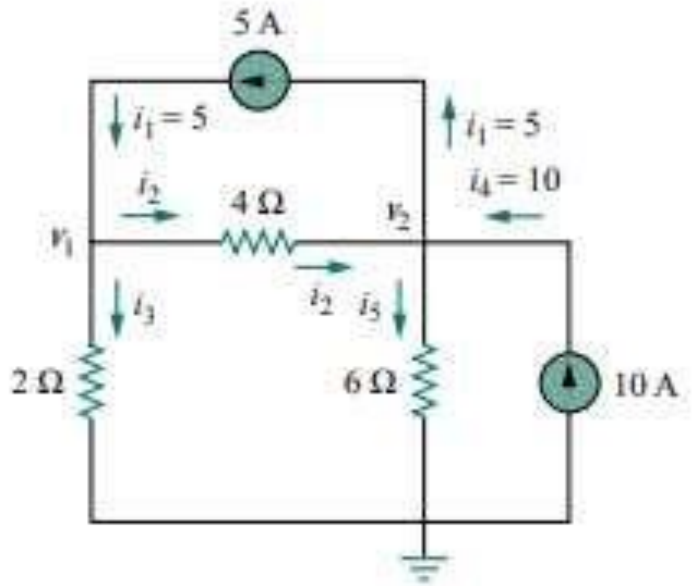
Note:

For inspection method the circuit should contain only current source. If voltage source is present convert in to current source and then apply inspection method.

Problem No.1

Find the node voltages and branch currents.





Since there are 2 nodes we have 2x2 matrix

$$\begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$G_{11} = \frac{1}{2} + \frac{1}{4} = 0.25 + 0.5 = 0.75$$

$$G_{22} = \frac{1}{4} + \frac{1}{6} = 0.25 + 0.1666 = 0.4166$$

$$G_{12} = G_{21} = \frac{1}{-4} = 0.25 = -0.25$$

$I_1=5$

$I_2 = -5 + 10=5$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.75 & -0.25 \\ -0.25 & 0.4166 \end{bmatrix} = 0.24995$$

$$\Delta_1 = \begin{bmatrix} 5 & -0.25 \\ 5 & 0.4166 \end{bmatrix} = 3.333$$

$$\Delta_2 = \begin{bmatrix} 0.75 & 5 \\ -0.25 & 5 \end{bmatrix} = 5$$

$$V_1 = \frac{\Delta_1}{\Delta} = 13.33 Volts$$

$$V_2 = \frac{\Delta_2}{\Delta} = 20 Volts$$

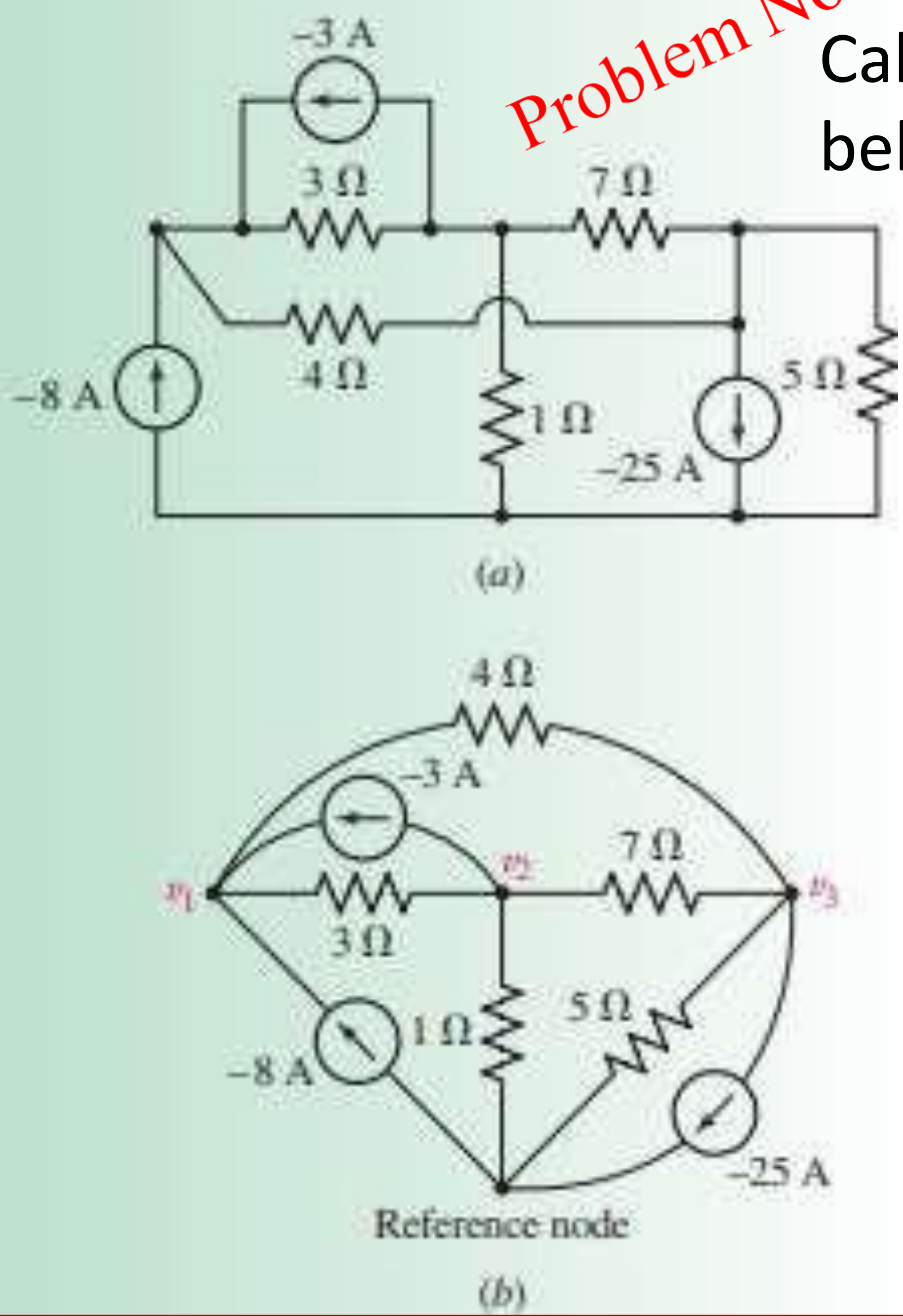
$$I_{2\Omega} = \frac{V_1}{2} = 6.665 Amps$$

$$I_{4\Omega} = \frac{V_2 - V_1}{4} = 1.6675 Amps$$

$$I_{6\Omega} = \frac{V_2}{6} = 3.33 Amps$$

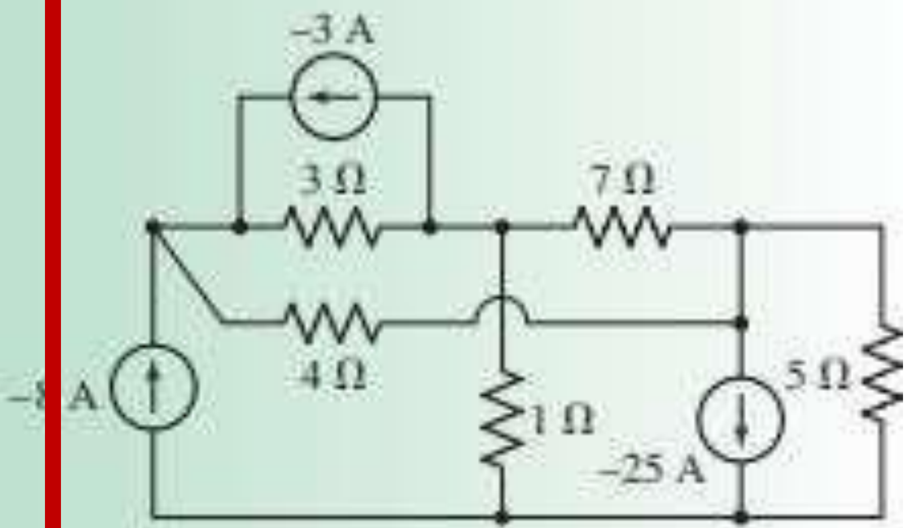
Problem No.2

Calculate the node voltages for the circuit given below

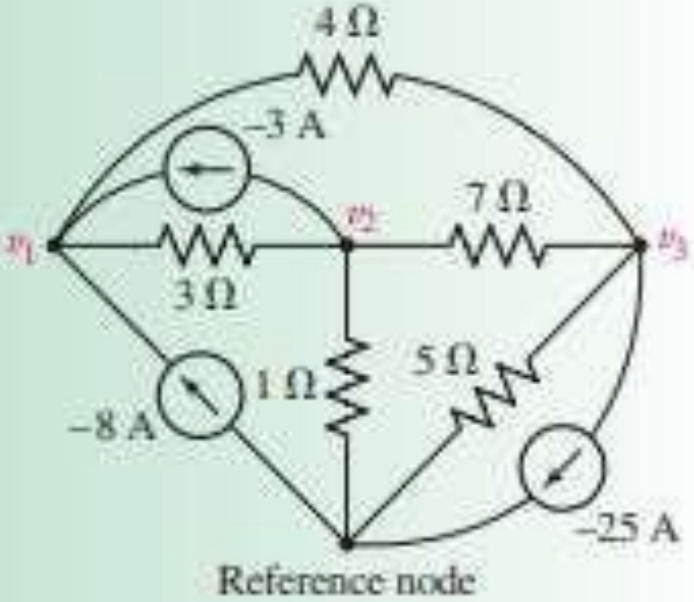


SOLUTION: Since there are three nodes we have 3x3 matrix

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$



(a)



(b)

$$G_{11} = \frac{1}{3} + \frac{1}{4} = 0.5833$$

$$G_{22} = \frac{1}{3} + \frac{1}{1} + \frac{1}{7} = 1.4762$$

$$G_{33} = \frac{1}{5} + \frac{1}{7} + \frac{1}{4} = 0.5929$$

$$G_{13} = G_{31} = \frac{1}{4} = 0.25$$

$$I_1 = -3 - 8 = -11$$

$$I_2 = -(-3) = 3$$

$$I_3 = -(-25) = 25$$

$$G_{12} = G_{21} = \frac{1}{3} = 0.33$$

$$G_{32} = G_{23} = \frac{1}{7} = 0.1429$$

$$\begin{bmatrix} 0.5833 & -0.33 & -0.25 \\ -0.33 & 1.4762 & -0.1429 \\ -0.25 & -0.1429 & 0.5929 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

$$v_1 = \frac{\begin{vmatrix} -11 & -0.3333 & -0.2500 \\ 3 & 1.4762 & -0.1429 \\ 25 & -0.1429 & 0.5929 \end{vmatrix}}{\begin{vmatrix} 0.5833 & -0.3333 & -0.2500 \\ -0.3333 & 1.4762 & -0.1429 \\ -0.2500 & -0.1429 & 0.5929 \end{vmatrix}} = \frac{1.714}{0.3167} = 5.412 \text{ V}$$

Similarly,

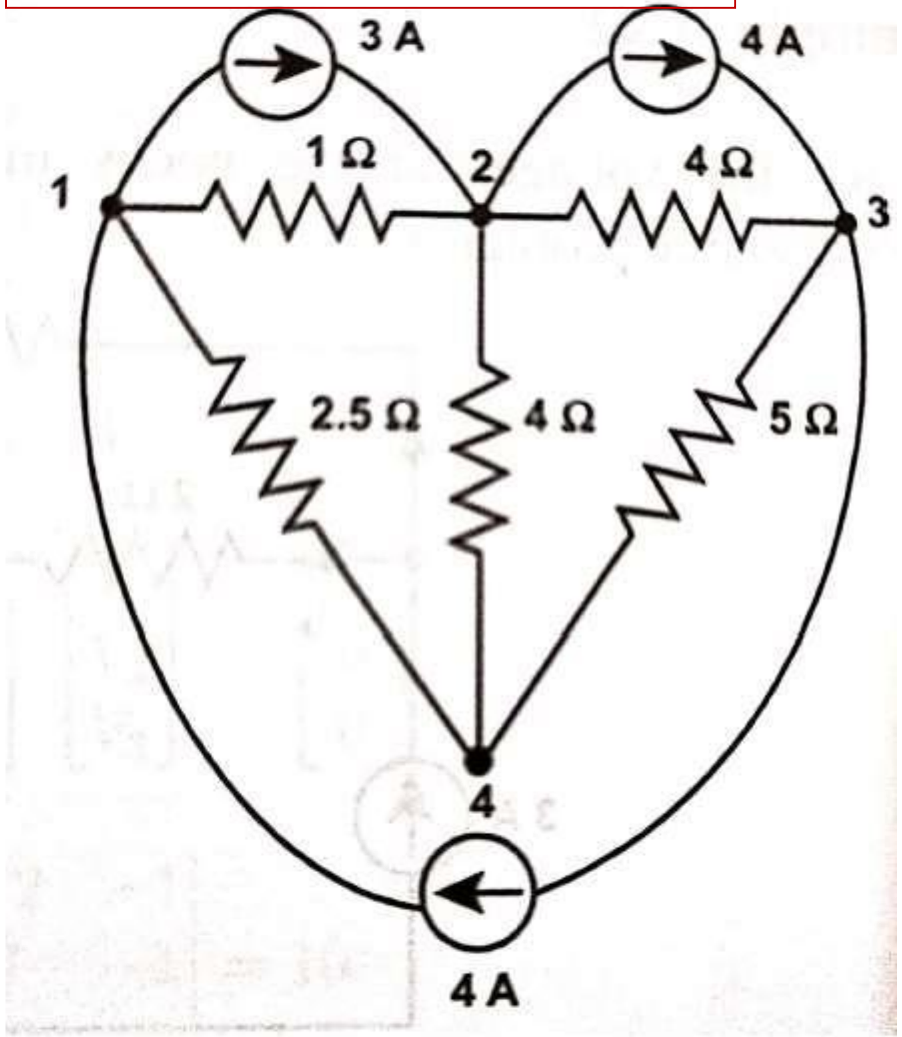
$$v_2 = \frac{\begin{vmatrix} 0.5833 & -11 & -0.2500 \\ -0.3333 & 3 & -0.1429 \\ -0.2500 & 25 & 0.5929 \end{vmatrix}}{0.3167} = \frac{2.450}{0.3167} = 7.736 \text{ V}$$

and

$$v_3 = \frac{\begin{vmatrix} 0.5833 & -0.3333 & -11 \\ -0.3333 & 1.4762 & 3 \\ -0.2500 & -0.1429 & 25 \end{vmatrix}}{0.3167} = \frac{14.67}{0.3167} = 46.32 \text{ V}$$

$$\begin{bmatrix} 0.5833 & -0.33 & -0.25 \\ -0.33 & 1.4762 & -0.1429 \\ -0.25 & -0.1429 & 0.5929 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} -11 \\ 3 \\ 25 \end{bmatrix}$$

Problem No.3



Find the voltage across the node 2 and 4 by nodal analysis

$$G_{11} = \frac{1}{1} + \frac{1}{2.5} = 1.4$$

$$G_{22} = \frac{1}{4} + \frac{1}{1} + \frac{1}{4} = 1.5$$

$$G_{33} = \frac{1}{4} + \frac{1}{5} = 0.45$$

$$G_{13} = G_{31} = 0$$

$$G_{12} = G_{21} = \frac{1}{1} = 1 = -1$$

$$G_{32} = G_{23} = \frac{1}{4} = 0.25 = -0.25$$

$$I_1 = 4 - 3 = 1$$

$$I_2 = 3 - 4 = -1$$

$$I_3 = 4 - 4 = 0$$

$$\begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 1.4 & 1 & 0 \\ -1 & -1 & -0.25 \\ 0 & 0 & 0.45 \end{bmatrix} = -0.28$$

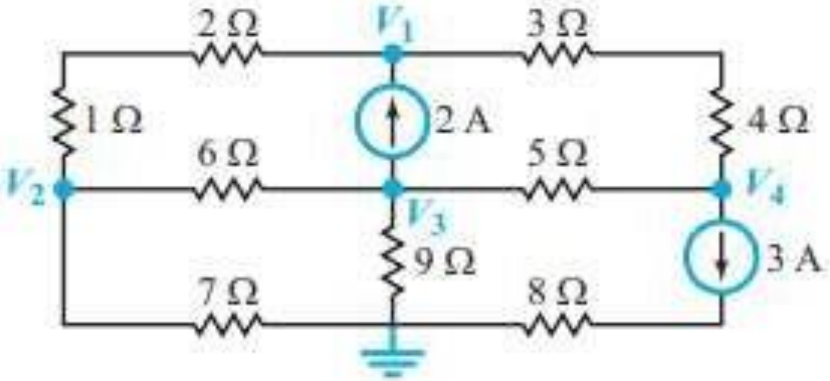
$$\Delta = \begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} = 0.665$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.28}{0.665} - 0.421 Volts$$

$$V_{4\Omega} = V_2 - V_4 = -0.421 - 0 = -0.421 Volts$$

Problem No.3

Use the by-inspection method to establish a node-voltage matri equation for the circuit to find V_1 to V_4.



$$G_{11} = \frac{1}{2+1} + \frac{1}{3+4} = 0.476$$

$$G_{12} = G_{21} = -\frac{1}{2+1} = -0.333$$

$$G_{13} = G_{31} = 0$$

$$G_{14} = G_{41} = -\frac{1}{3+4} = -0.143$$

$$G_{22} = \frac{1}{1+2} + \frac{1}{7} + \frac{1}{6} = 0.643$$

$$G_{23} = G_{32} = -\frac{1}{6} = -0.167$$

$$G_{24} = G_{42} = 0$$

$$G_{33} = \frac{1}{5} + \frac{1}{6} + \frac{1}{9} = 0.478$$

$$G_{34} = G_{43} = -\frac{1}{5} = -0.2$$

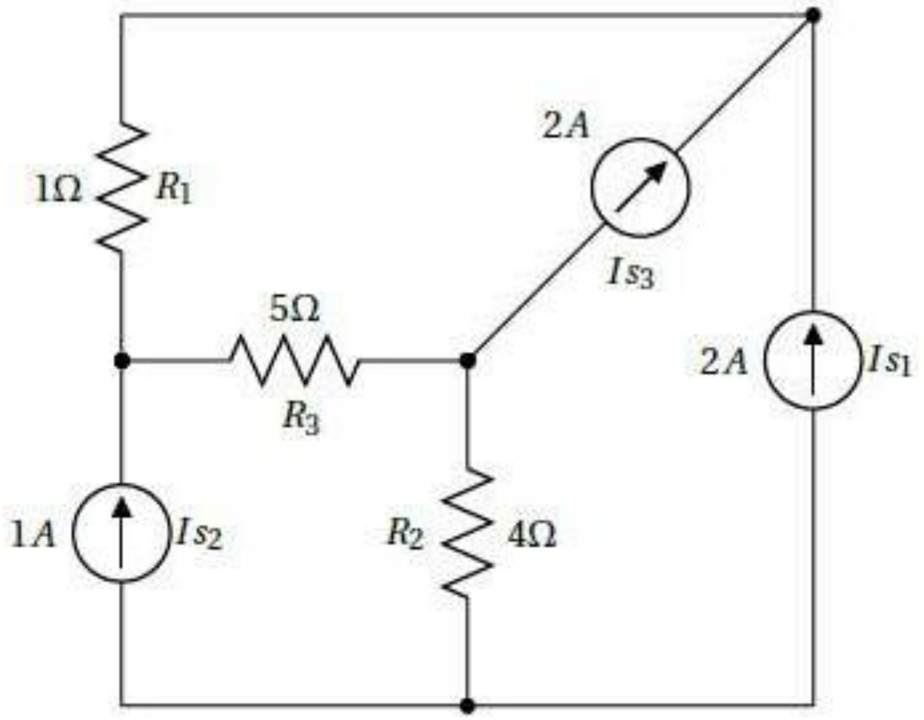
$$G_{44} = \frac{1}{3+4} + \frac{1}{5} = 0.343$$

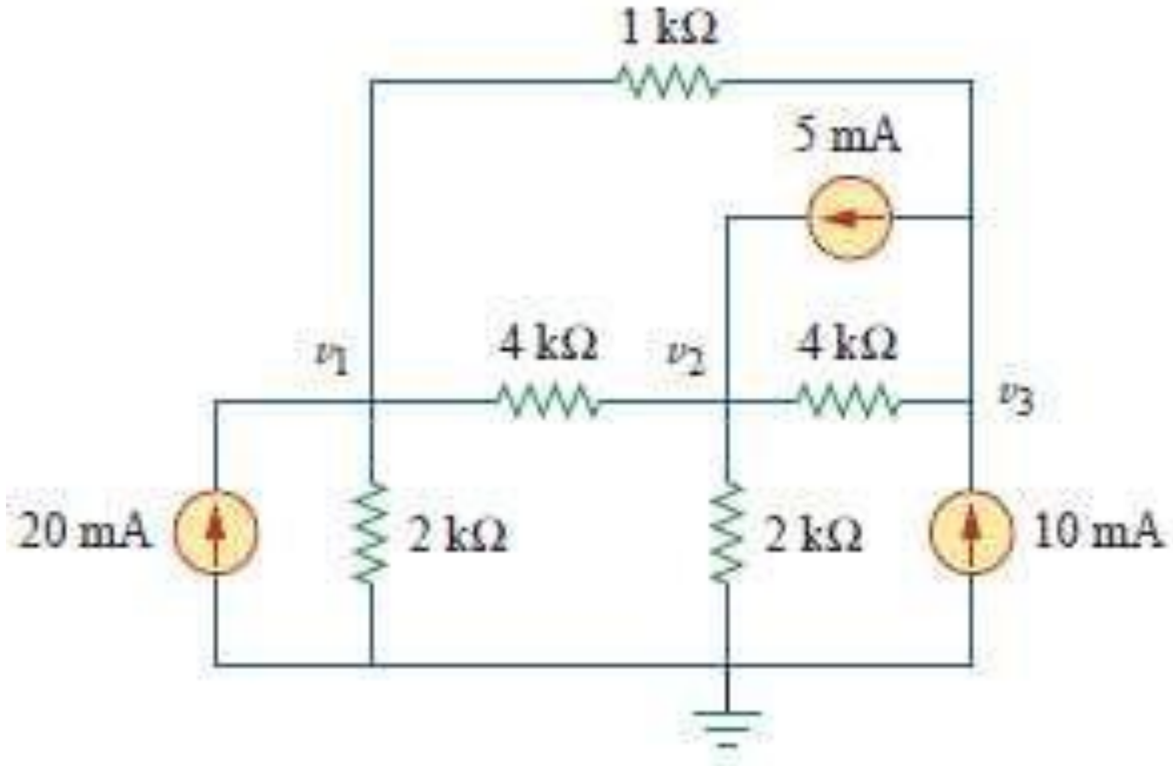
$$\begin{bmatrix} 0.476 & -0.333 & 0 & -0.143 \\ -0.333 & 0.643 & -0.167 & 0 \\ 0 & -0.167 & 0.478 & -0.2 \\ -0.143 & 0 & -0.2 & 0.343 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -2 \\ -3 \end{bmatrix}$$

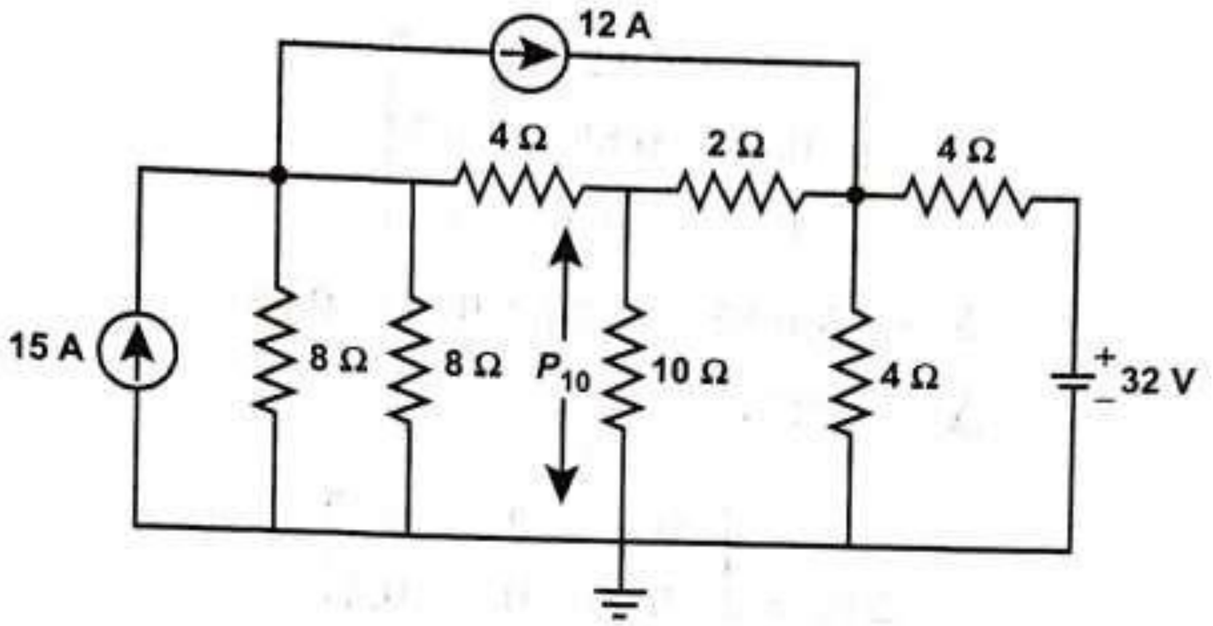
Matrix inversion gives:

$V_1 = -8.1689$ V, $\quad V_2 = -8.4235$ V, $\quad V_3 = -16.155$ V, $\quad V_4 = -21.5748$ V.

Additional problems






12 A
4 Ω
2 Ω
4 Ω
15 A
8 Ω
8 Ω
P_{10}
10 Ω
4 Ω
+
_
32 V

THANK YOU

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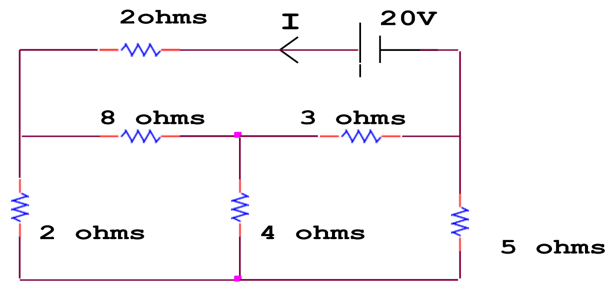
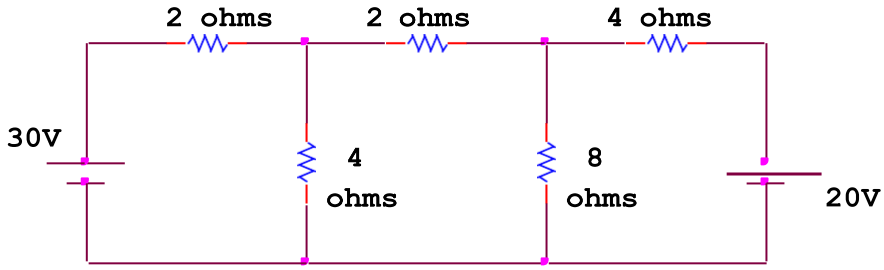
Practicing Problems

Program : B.E
Course : CSE
Course code : SEEA1103 **Sem : 1**
Batch : 2020 - 2024 **Unit : 1**

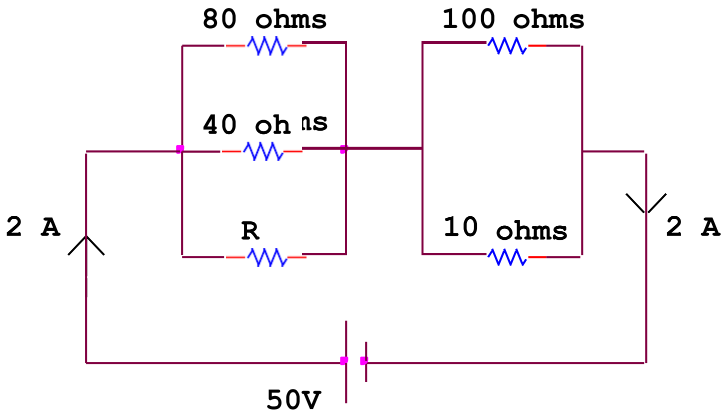
1. Evaluate the current through 2 ohms resistances.



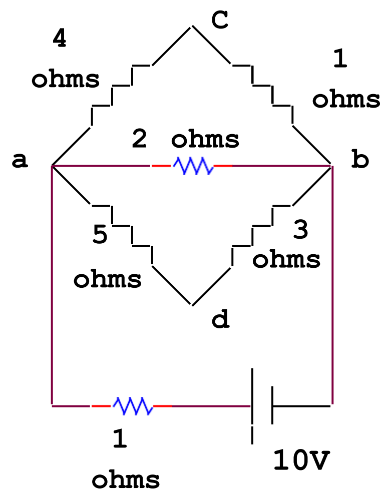
2. Measure the power dissipated across 8 ohms resistor on the circuit shown in the figure



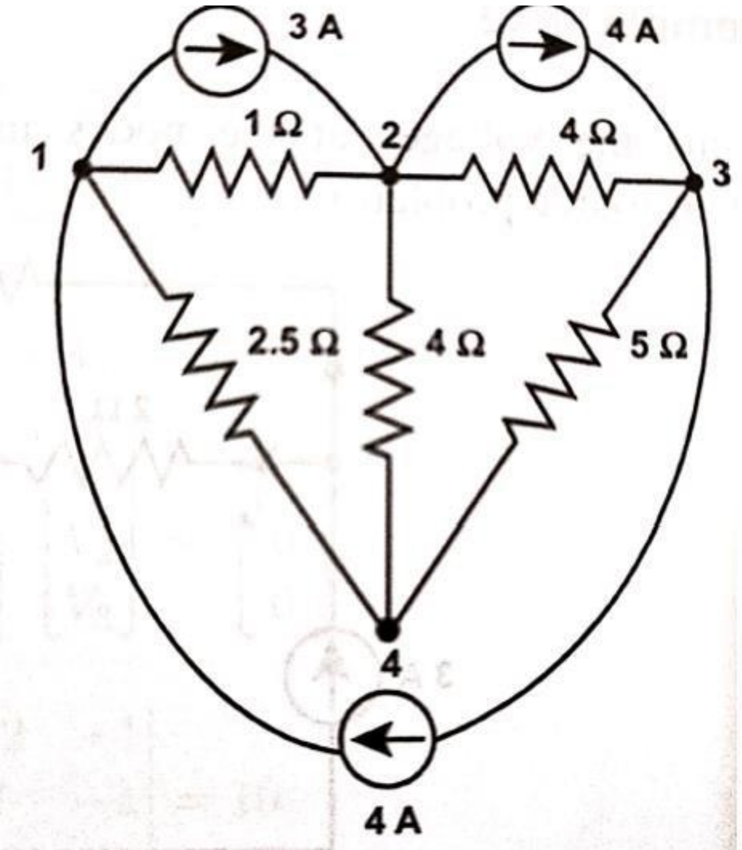
3. Design the value of R for below given circuit.



4. In the circuit shown, evaluate the voltage across 2 Ω resistor and the total current delivered by the battery. Use Mesh analysis.



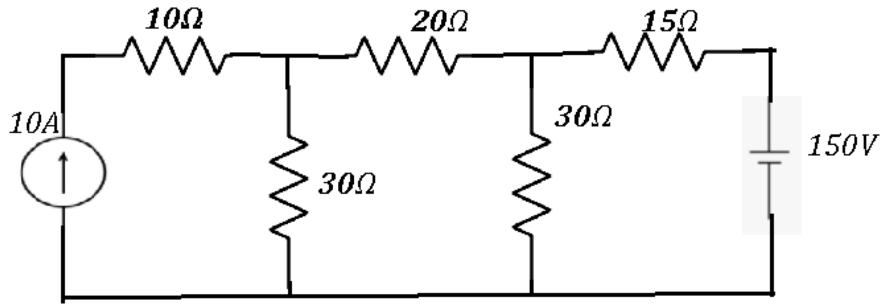
5. For the given electrical circuit, estimate the voltage across 4 Ω resistances and 2.5 Ω resistance.



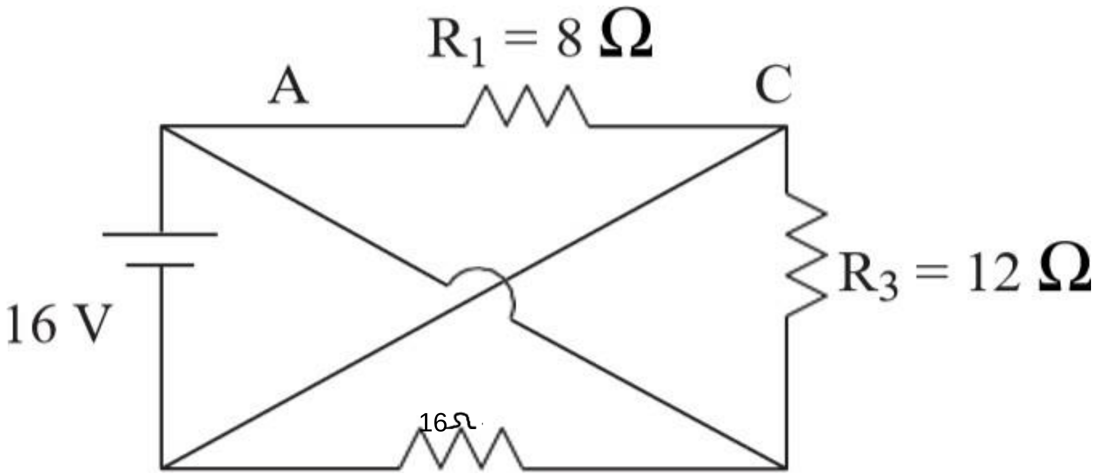
6. Four resistors are in parallel. The current in the first three resistors are 4A, 5A and 6A respectively. The voltage drop across the fourth resistor is 200V. The

total power dissipated is 5KW. Evaluate the values of resistances of the branches and the total resistance.

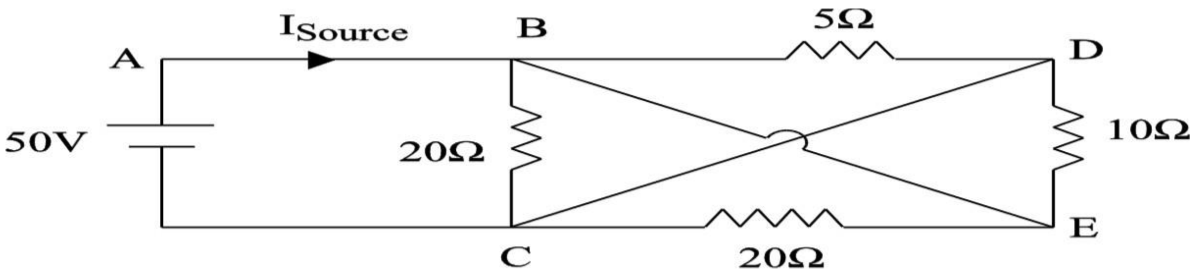
7. Evaluate the Node voltages in the given circuit using nodal analysis. Also find the Current flowing through 20Ω



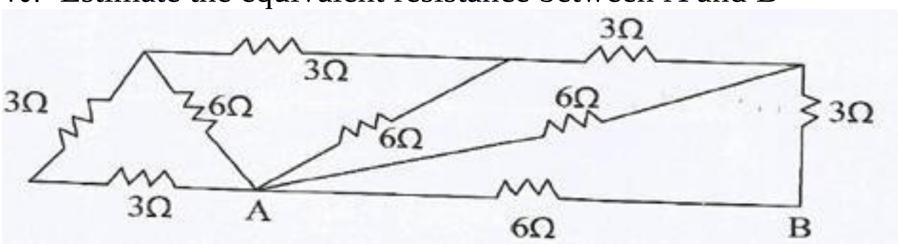
8. Calculate the total resistance and battery current in the given circuit



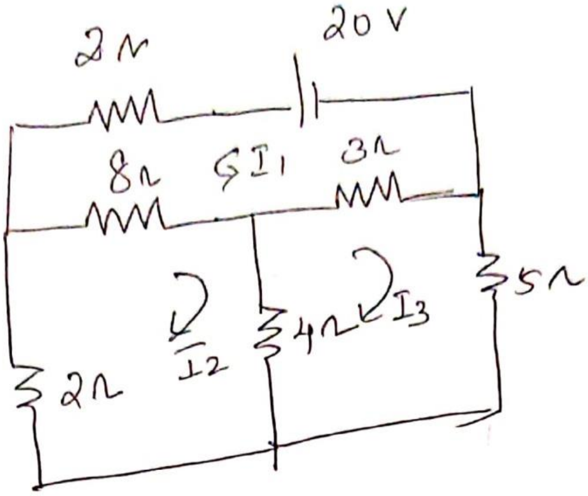
9. Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current



10. Estimate the equivalent resistance between A and B



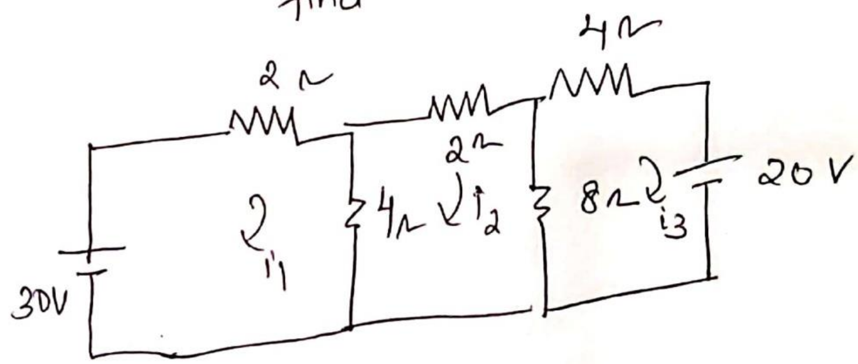
(1)



$$\begin{bmatrix} 13 & 8 & 3 \\ 8 & 14 & -4 \\ 3 & -4 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 0 \end{bmatrix}$$

Find $I_{2\Omega} = I_1 = \dfrac{\Delta_1}{\Delta}$

(2)



$$P(8\Omega) = (i_8)^2 \times 8$$

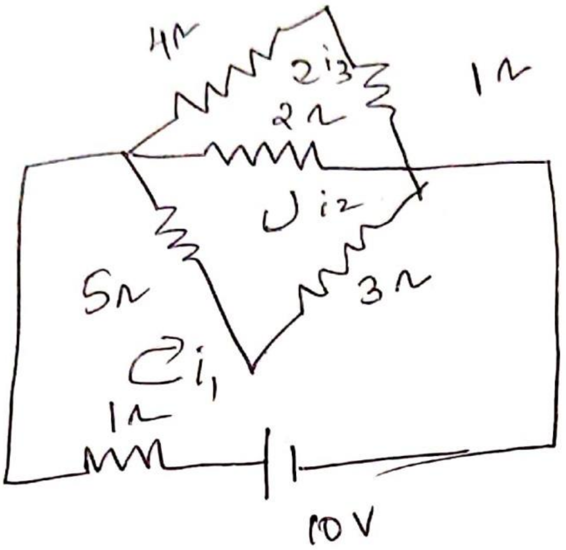
$$i_8 = i_2 - i_3$$

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 14 & -8 \\ 0 & -8 & 12 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \\ -20 \end{bmatrix}$$

$$i_2 = \frac{\Delta_2}{\Delta}$$

$$i_3 = \frac{\Delta_3}{\Delta}$$

4



$$V_{2\Omega} = ?$$

$$V_{2\Omega} = i_{2\Omega} \times 2$$

$$\begin{pmatrix} 10 & -8 & 0 \\ -8 & 10 & -2 \\ 0 & -2 & 7 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$i_{2\Omega} = i_2 \sim i_3$$

$$i_2 = \frac{\Delta_2}{\Delta} \quad i_3 = \frac{\Delta_3}{\Delta}$$

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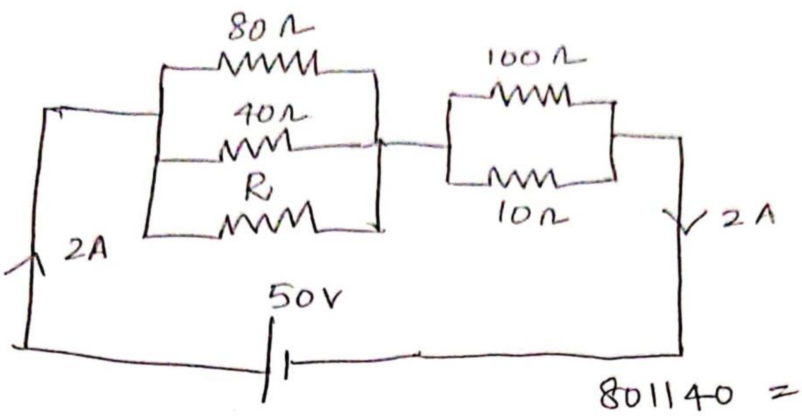
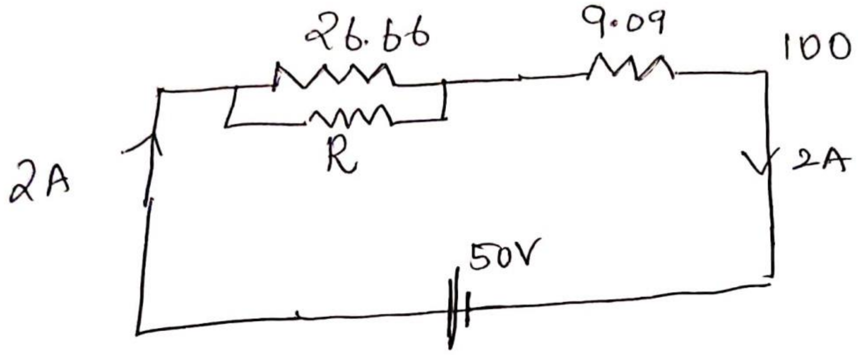
3. Design the value of R

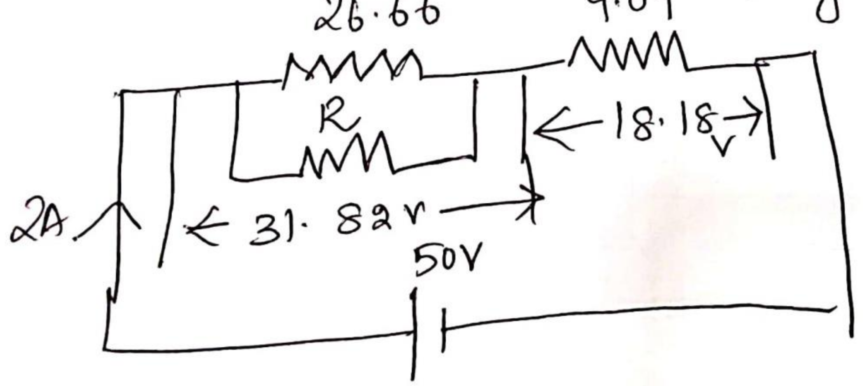


$80 \parallel 40 = \frac{80 \times 40}{80+40} = 26.66$

$100 \parallel 10 = \frac{100 \times 10}{100+10} = 9.09$



Voltage across 9.09



$V = IR$

$= 2 \times 9.09$

18.18

$50 - 18.18$

$= 31.82$

$$I_{26.66} = \frac{V}{R} = \frac{31.82}{26.66} = 1.19$$

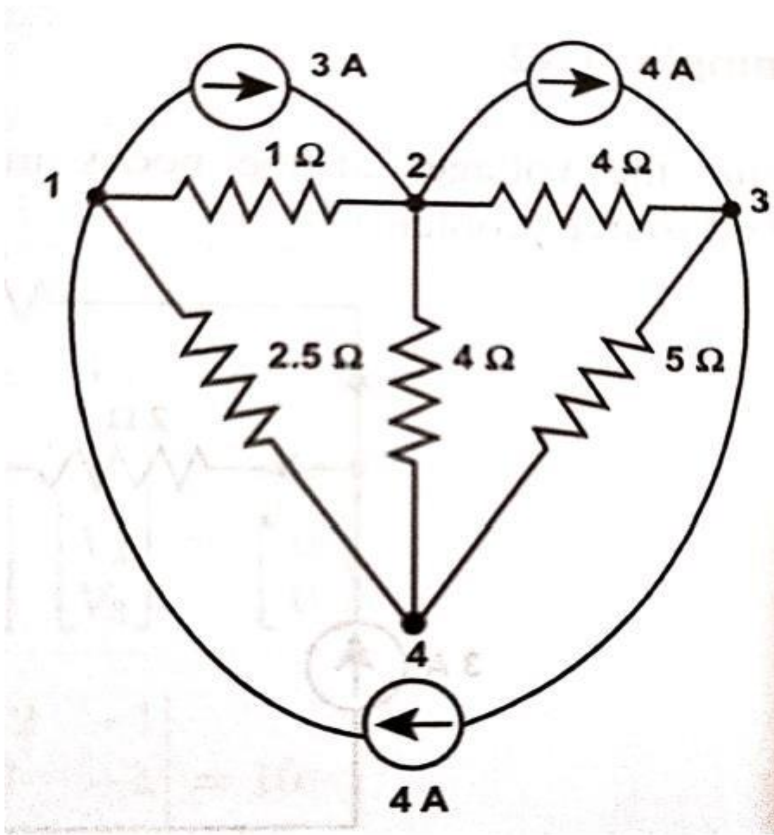
$$I_R = 2 - 1.19$$

$$= 0.81$$

$$R = \frac{V}{I} = \frac{31.82}{0.81} = 39.28$$

$$\boxed{R = 39.28}$$

5. For the given electrical circuit, estimate the voltage across 4 Ω resistances and 2.5 Ω resistance.



If the circuit contains 3 major nodes then matrix will be 3X3 matrix

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

I_1 → Sum of the currents flowing towards node no. 1

I_2 → Sum of the currents flowing towards node no. 2

I_3 → Sum of the currents flowing towards node no.3

$$G_{11} = \frac{1}{1} + \frac{1}{2.5} = 1.4$$

$$G_{22} = \frac{1}{4} + \frac{1}{1} + \frac{1}{4} = 1.5$$

$$G_{33} = \frac{1}{4} + \frac{1}{5} = 0.45$$

$$G_{13} = G_{31} = 0$$

$$G_{12} = G_{21} = \frac{1}{1} = 1 = -1$$

$$G_{32} = G_{23} = \frac{1}{4} = 0.25 = -0.25$$

$$I_1 = 4 - 3 = 1$$

$$I_2 = 3 - 4 = -1$$

$$I_3 = 4 - 4 = 0$$

$$\begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$\Delta_2 = \begin{bmatrix} 1.4 & 1 & 0 \\ -1 & -1 & -0.25 \\ 0 & 0 & 0.45 \end{bmatrix} = -0.28$$

$$\Delta = \begin{bmatrix} 1.4 & -1 & 0 \\ -1 & 1.5 & -0.25 \\ 0 & -0.25 & 0.45 \end{bmatrix} = 0.665$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.28}{0.665} - 0.421 Volts$$

$$V_{4\Omega} = V_2 - V_4 = -0.421 - 0 = -0.421 Volts$$

$V_{2.5ohm}$=V_1-V_4

V_1=0.1625/0.665=0.244Volts

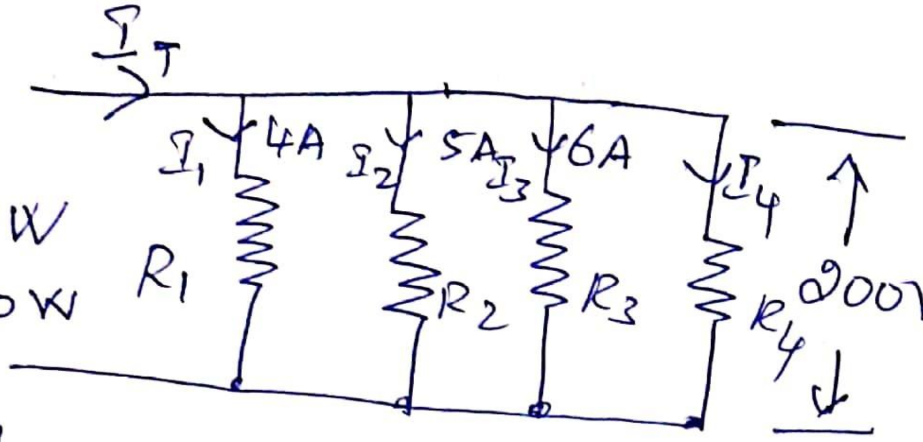
$V_{2.5ohm}$=V_1-V_4=0.244-0=0.244Volts

Ans; $V_{2.5ohm}$=0.244 Volts

V_{4ohm}=-0.412 Volts

6. Solution:

$P = 5\,kW$
$= 5000\,W$



Voltage across (since $V_{R_4} = 200V$)

$V_{R_1} = 200V$ ∵ ||el resistor has same voltage

$V_{R_2} = 200V$

$V_{R_3} = 200V$ $\boxed{V_T = 200V}$

$P_T = V_T \times I_T = 200 \times I_T$

$I_T = P/V = \frac{5000}{200} = 25A$

$R_1 = \frac{V_{R_1}}{4A} = \frac{200}{4}$; $R_2 = \frac{V_{R2}}{5A} = \frac{200}{5}$; $R_3 = \frac{V_{R3}}{6A} = \frac{200}{6}$

$\boxed{R_1 = 50\,\Omega}$ $\boxed{R2 = 40\,\Omega}$ $\boxed{R3 = 33.33\,\Omega}$

$I_4 = I_T - (I_1 + I_2 + I_3)$
$= 25 - (4 + 5 + 6)$
$\boxed{I_4 = 10A}$

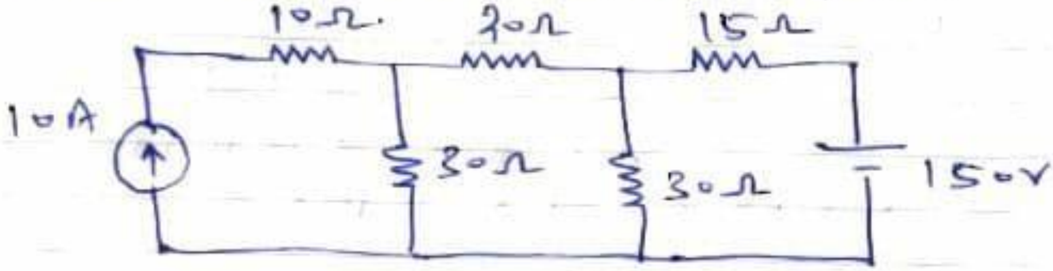
$R_T = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$
$= 1/50 + 1/40 + \frac{1}{33.33} + \frac{1}{20}$

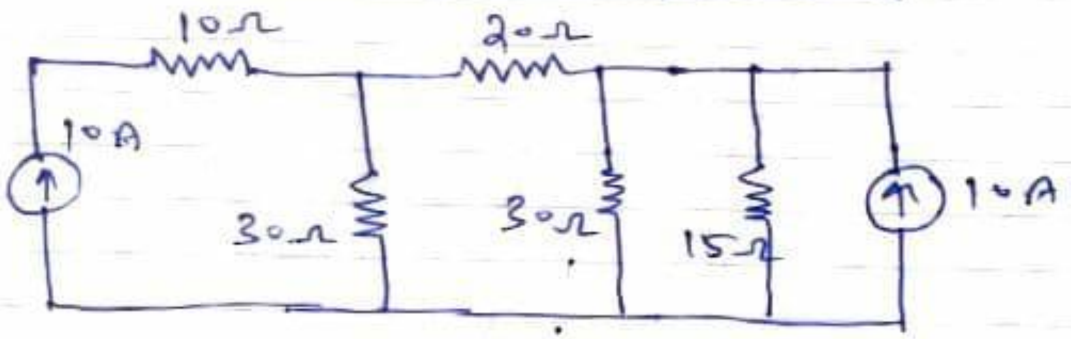
$R_4 = \frac{200}{I_4} = \frac{200}{10} = 20\,\Omega$ $\boxed{R_T = 0.125\,\Omega}$

Expt. No. ______ Page No. ______

Expt. Name ______ Date : ______

7. Evaluate the node voltages in the given circuit using Nodal analysis. Also find the current flowing through $20\,\Omega$.





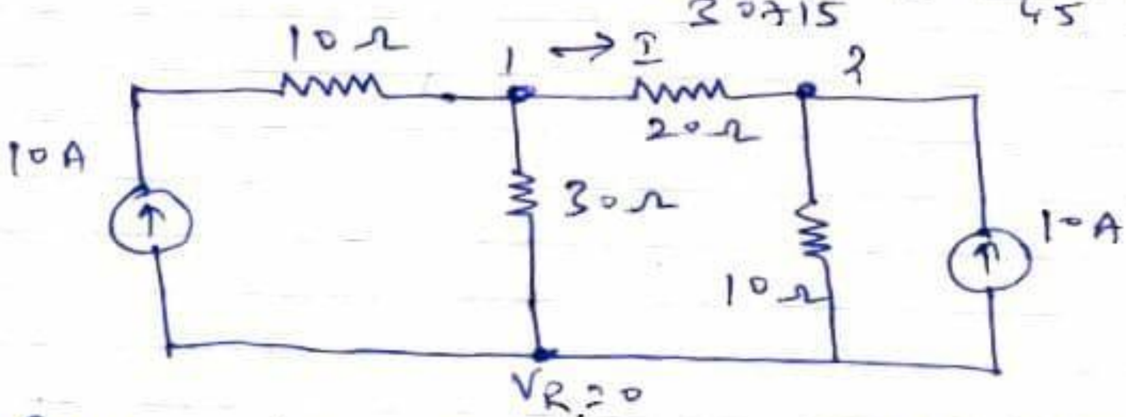
$$I = \frac{V}{R}$$

$$= \frac{150}{15}$$

$$= 10A$$

$30\,\Omega \parallel 15\,\Omega$

$$\frac{30 \times 15}{30 + 15} = \frac{450}{45} = 10\,\Omega$$



$$\begin{bmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{20} & -\frac{1}{20} \\ -\frac{1}{20} & \frac{1}{20} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 0.183 & -0.05 \\ -0.05 & 0.15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$$

$$\Delta = (0.183 \times 0.15) - (0.05 \times 0.05) = 0.027 - 0.0025$$
$$= 0.0245$$

$$\Delta_1 = \begin{bmatrix} 10 & -0.05 \\ 10 & 0.15 \end{bmatrix} = 10(0.15) + 10(0.05)$$
$$= 1.5 + 0.5$$
$$= 2$$

$$\Delta_2 = \begin{bmatrix} 0.183 & 10 \\ -0.05 & 10 \end{bmatrix} = 0.183(10) + 10(0.05)$$
$$= 1.83 + 0.5$$
$$= 2.33$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{2}{0.0245} = 81.63\,V$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{2.33}{0.0245} = 95.10\,V$$

$$I_{20\Omega} = \frac{V_1 - V_2}{20} = \frac{81.63 - 95.10}{20}$$

$$= \frac{-13.47}{20} = -0.6735\,A. \text{ (Anticlockwise Direction)}$$

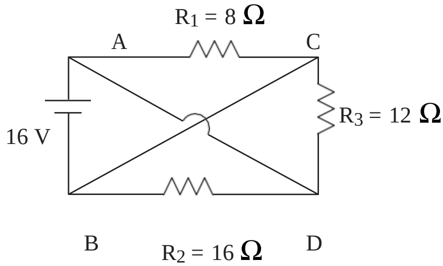
<u>Node voltages</u>

$V_1 = 81.63V$

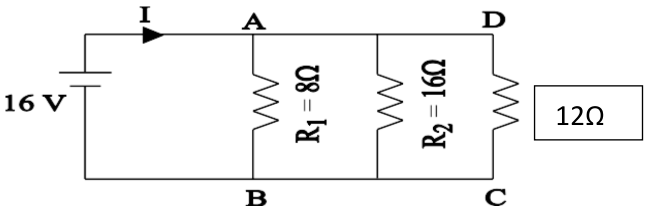
$V_2 = 95.10V$

<u>Current</u> <u>flow</u> <u>through the</u> <u>20 Ω</u> <u>Resistor is</u> 0.6735A.

8. Calculate the total resistance and battery current in the given circuit



The given above circuit can be re-drawn as,



8 Ω, 16 Ω, 12 Ω are connected in parallel. Its equivalent resistance,

$$R_T = \frac{R_1 R_2 R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

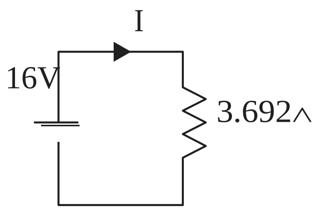


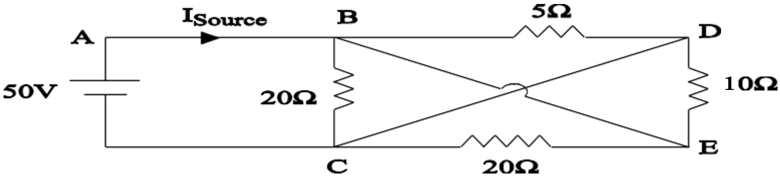
Figure 1.28

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\ \Omega$$

$$R_T = 3.692\ \Omega$$

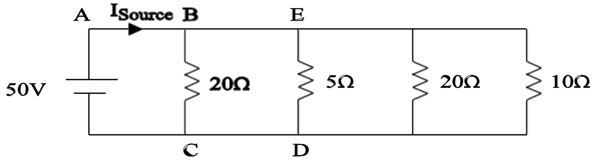
$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

9. Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current



A

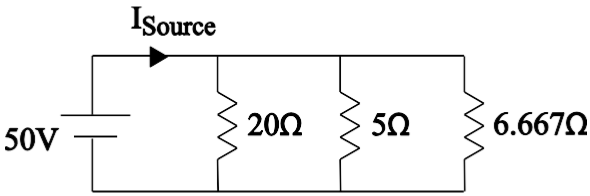
Solution: The given above circuit can be re-drawn as



20 Ω and 10 Ω resistors are connected in parallel, its equivalent resistance is

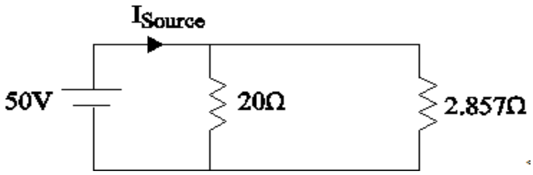
given by, $\frac{20 * 10}{20 + 10} = 6.667\ \Omega$

The given circuit is reduced as,



6.667 Ω and 5 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{6.667 * 5}{6.667 + 5} = 2.857\ \Omega$

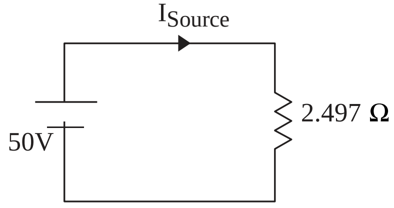
The circuit is reduced as,



20 Ω and 2.857 Ω are connected in parallel. It equivalent resistance is,

$$\frac{20 * 2.857}{20 + 2.857} = 2.497\ \Omega$$

The Circuit is re-drawn as,

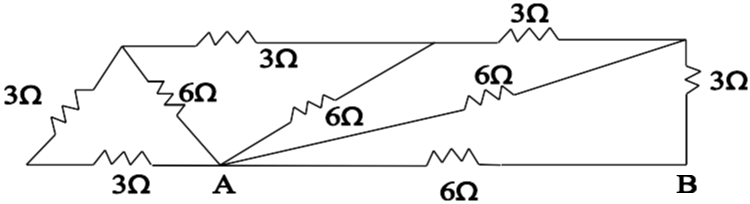


Hence the equivalent resistance of the Circuit is R_T= 2.497 Ω =2.5 Ω

Source Current of the Circuit is given by,

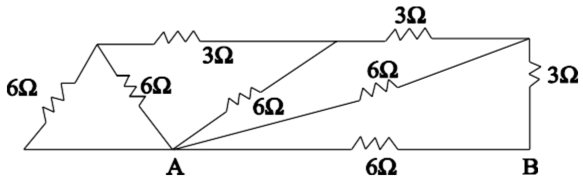
$$I_{source} = \frac{V}{R} = \frac{50}{2.5} = 20A$$

10. Find the equivalent resistance between the terminals A and B.



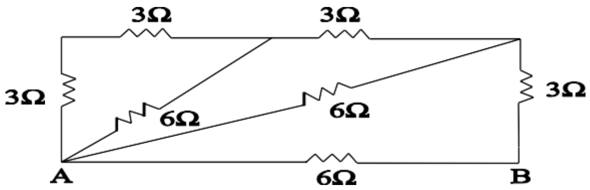
Solution:

3 Ω and 3 Ω are connected in Series, it equivalent resistance is, (3 + 3) = 6 Ω. The Circuit gets reduced as



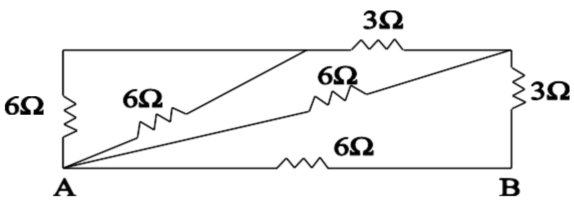
6 Ω and 6 Ω are connected in parallel. The circuit gets reduced as,

$$\frac{6*6}{6+6} = 3\text{ohms.}$$



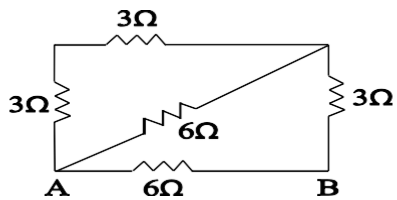
3 Ω and 3 Ω are connected in series (3 + 3 = 6 Ω).

The reduced Circuit is,

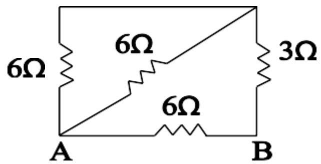


6 Ω and 6 Ω are connected in parallel. Its equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

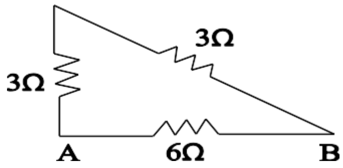
The circuit can be reduced as,

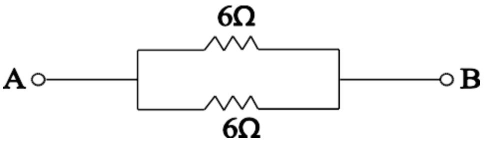


3 Ω and 3 Ω are connected in series. (3 + 3 = 6 Ω).



6 Ω and 6 Ω are connected in parallel. It equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$



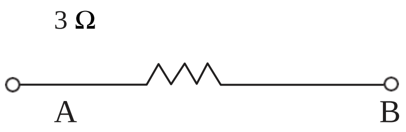


3 Ω and 3 Ω are connected in series, the reduced Circuit is 3 + 3 = 6 Ω

6 Ω and 6 Ω are connected in parallel.

The equivalent resistance between the terminals

A and Bgiven by R_{AB} = 3 Ω.



$$\therefore R_{AB} = 3\ \Omega$$

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SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF BIO MEDICAL ENGINEERING

UNIT – II

Basic Electrical Engineering – SEEA1203

UNIT II

AC CIRCUITS

2.1 INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

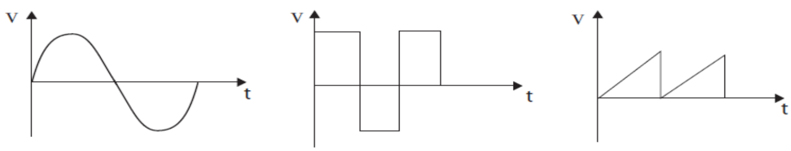

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurring interval.
3. Cycle

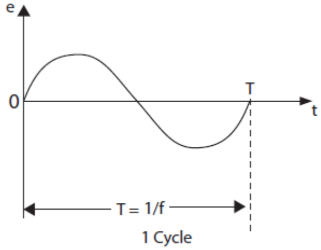

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period
   The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.
5. Frequency
   The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz

$$f = 1/T$$

6. Amplitude
   The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.
7. R.M.S value [Root Mean Square]
   The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$RMS\ Value\ = \sqrt{\begin{array}{l}Area\ Under\ the\ square\ curve\ for\\ one\ complete\ cycle / Period\end{array}}$$

8. Average Value of AC
   The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

   Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)
   It is the ratio of RMS value to average value

   Form Factor = RMS value/Average Value

10. Peak Factor (Ka)
   It is the ratio of Peak (or) maximum value to RMS value.

   Peak Factor Ka=Peak Value/RMS value

2.2 Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

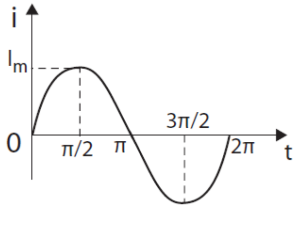

Figure 2.3

$$i = I_m \sin \omega t \ \ ; \omega t=\theta$$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \text{ [since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{id\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, \mathrm{d}\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, \mathrm{d}\theta$$

$$= -\frac{\mathrm{I_m}}{\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}\left[\cos\pi - \cos 0\right]$$

$$= \frac{I_m}{\pi}(-1-1)$$

$$= -\frac{2I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{Avg} = \frac{\mathrm{I_m}/\sqrt{2}}{2\mathrm{I_m}/\pi} = 1.11$$

$$Peak\,Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

2.2.1 Expression for RMS, Average, Form Factor, Peak factor for Half wave rectifier

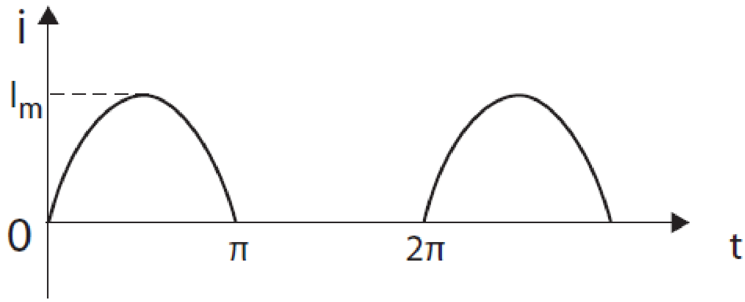

Figure 2.4

1) RMS value

$i = I_m \sin\theta \quad ; \quad 0 < \theta < \pi$

$i = 0 \quad ; \quad \pi < \theta \leq 2\pi$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi}$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i d\theta + 0\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} I_m \sin\theta d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{2\pi}(-1-1)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

$$Form\,Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{\frac{I_m}{2}} = 2$$

Examples:

2.1) The equation of an alternating current is given by

$$i = 40\sin 314\ t$$

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

$$i = 40\sin 314\ t$$

We know that $i = I_m \sin \omega t$

S_o $\quad I_m = 40$

$\omega = 314\ rad/\sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50\,Hz$

(v) $Form\ Factor \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(vi) $Peak\ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

2.2) what is the equation of a 50Hz voltage sin wave having an rms value of 50 volt

Solution:

f = 50Hz
V_{rms} = 50V
v = V_m sinωt
ω = 2πf = 2π×50=314 rad/sec
$V_m = V_{rms}\sqrt{2} = 50\times\sqrt{2} = 70.7$ volt
$\therefore v = 70.7\sin 314t$

2.3 PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

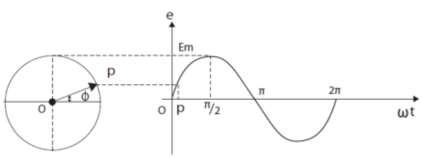

Figure 2.5

$\overline{OP}$ =E_m

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = Im_1 \sin\theta$
$i_2 = Im_2 \sin\theta$

can be represented graphically as shown in Fig 2.6(a).

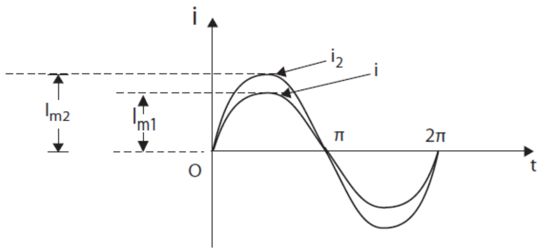

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

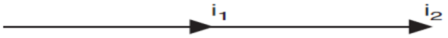

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of ‘**lag**’ and ‘**lead**’ and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = Im_1 \sin(\omega t - \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

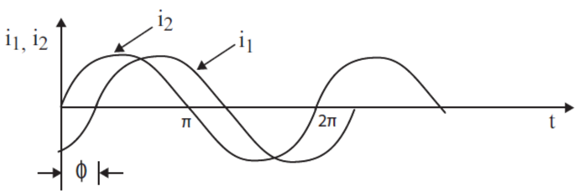

Figure 2.7a

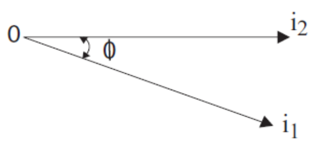

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of ‘Φ’ electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = Im_1 \sin(\omega t + \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

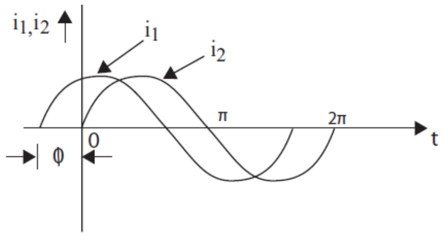

Figure 2.8(a)

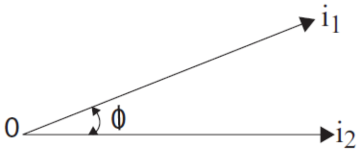

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference between them is 90°.
2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

2.4 REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

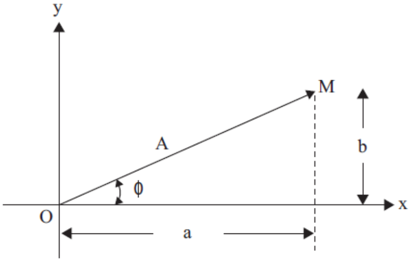

Figure 2.9

In Fig 2.9, OM represents vector A
Φ represents the phase angle of vector A

A = a + jb
a – Horizontal component or active component or in phase component
b – Vertical component or reactive component or quadrature component

The magnitude of vector $'A' = \sqrt{a^2 + b^2}$
Phase angle of Vector 'A' = $\alpha = \tan^{-1}$ (b/a)
Features of j – Operator

1. $j = \sqrt{-1}$
   It indicates anticlockwise rotation of Vector through 90°.
2. $j^2 = j \cdot j = -1$
   It indicates anticlockwise rotation of vector through 180°.
3. $j^3 = j \cdot j \cdot j = -j$
   It indicates anticlockwise rotation of vector through 270°.
4. $j^4 = j \cdot j \cdot j \cdot j = 1$
   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.
A = 2 + j3

This representation is known as rectangular form.
Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606
Phase angle of A = $\alpha = \tan^{-1}(3/2) = 56°.31$
$A = |A| \angle \alpha°$
$A = 3.606 \angle 56°.31$

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

$\therefore$ Magnitude = | A + B | = $\sqrt{6^2 + 8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.

Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

$\therefore$ Magnitude = | A – B | = $\sqrt{-2^2 + 7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.

Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 $\angle$ 56°.310

B = 4 – j5

|B| = $\sqrt{4^2 + -5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340

B= 6.403$\angle$ -51°.340

A X B = 3.606 $\angle$ 56°.310 X 6.403$\angle$ -51°.340

= 3.606 X 6.403$\angle$ (56°.310 + (-51°.340))

= 23.089 $\angle$4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

|A| = $\sqrt{4^2 + -2^2}$ = 4.472

α = $\tan^{-1}$ (-2/4) = -26°.565

A= 4.472 $\angle$ -26°.565

B = 2 + j3

$|B| = \sqrt{2^2 + 3^2} = 3.606$

$\alpha = \tan^{-1}(3/2) = 56°.310$

$B = 3.606\angle 56°.310$

$$\frac{A}{B} = \frac{4.472\angle\text{-}26^o.565}{3.606\angle 56^o.310} = \frac{4.472}{3.606}\angle\ \text{-}26^o.565 - 56^o.310 = 1.240\ \angle\text{-}82.875$$

2.5 ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

2.5.1 Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

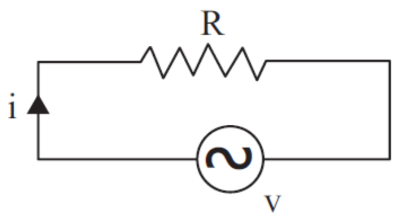

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

From Ohms law,

$$v = i\,R$$

$$I = \frac{v}{R} = \frac{V_m\ \sin\omega t}{R}$$

$$\because I_m = \frac{V_m}{R}$$

$$= I_m \sin\omega t$$

where,

$V_m \rightarrow$ Maximum value of voltage (V)
$I_m \rightarrow$ Maximum value of current (A)
$\omega \rightarrow$ Angular frequency (rad/sec)
$t \rightarrow$ Time period (sec)

Phasor Representation:

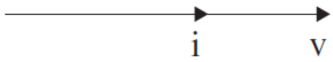

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

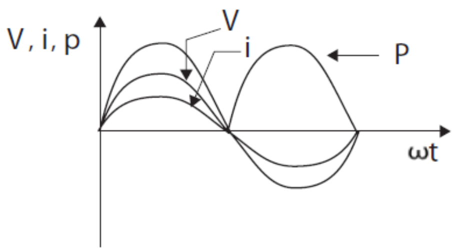

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:
In an AC circuit, impedance is the ratio of the maximum value of voltage to the maximum value of current.

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m/R} = R$$

$$\therefore Z = R$$

Power:

(i) Instantaneous power:
It is defined as the product of instantaneous voltage and instantaneous current.

p= v i

$= V_m \sin\omega t \, I_m \sin\omega t = V_m I_m \sin^2\omega t$

$[\because \omega t = \theta]$

$p = V_m I_m \sin^2\theta$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2\pi}[\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS} = V.I$$

$$P = V I$$

Power Factor:

It is defined as the cosine of the phase angle between voltage and current.

$$\cos\phi = \cos 0 = 1(unity)$$

Problems:

2.7) A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: $v = 240 \sin 377t$

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$=\frac{V_m \sin\omega t}{R}$$

$$=\frac{240}{6}\sin 377t$$

$$=40\sin 377tA$$

I. Phase angle:

$\phi = 0$

II. Impedance:

$Z = R = 6\Omega$

III. Instantaneous power:

IV. $p = V_m I_m \sin^2\omega t$

$$= 240.40.\sin^2 377t$$

$$= 9600\sin^2 377t$$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800\text{watts}$$

VI. Power factor:

$\cos\Phi = \cos 0 = 1$

2.8) A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: e = 200sinωt

$V_m = 200$

P = 100w

Average power, $P = \frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1$ A

Also, $V_m = I_m.R$

$R = 200\Omega$

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t$ A

2.9) A voltage e = 250sinωt when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:

Given: e = 250sinωt

$V_m = 250$
$P = 100W$

I. $P = \frac{V_m I_m}{2}$

$100 = \frac{250 I_m}{2}$

$I_m = 0.8$ A

II. $I_m = \frac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8\sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

2.5.2 Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

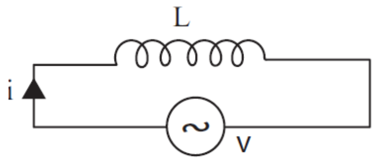

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v dt = \frac{1}{L}\int V_m \sin\omega t dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

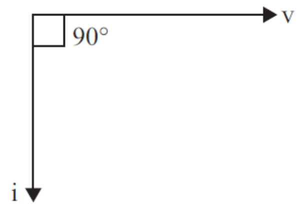

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

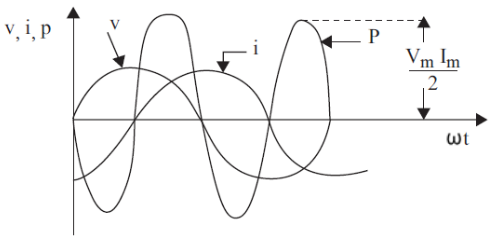

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{V_m / \omega L} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$
$$= V_m \sin\omega t\ I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= V_m I_m \sin\omega t\ (-\cos\omega t)$$
$$= -V_m I_m \sin\omega t \cos\omega t = -V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta\, d\theta$$
$$= -\frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$
$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$
$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[\cos 2\pi - \cos 0]$$
$$= \frac{V_m I_m}{4\pi}[1-1] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

2.10) A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor (iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: L = 0.225 H

V_{RMS} = V = 120 V

f = 50Hz

I. Inductive reactance, XL = 2πfL =2π x 50 x 0.225 = 70.68Ω

II. Instantaneous current, i =-I_m cosωt

$\because I_m = \frac{V_m}{\omega L} and V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$V_m = V_{RMS}\sqrt{2}$ =169.71V

$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$

Maximum power, $P_m = \frac{V_m I_m}{2}$ = 203.74 W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 344t volts

Instantaneous current, i = -2.4 cosωt A

2.11) A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: L = 0.01 H

I = 10cos1500t

I_m = 10A

ω = 1500 rad/sec

I. Inductive reactance, X_L = ωL = 1500 x 0.01 = 15Ω

II. The voltage across the inductor, e= $L\frac{di}{dt}$

$$= 0.01\frac{d(10\cos 1500t)}{dt} = 0.01 \times 10[-\sin 1500t.1500]$$

$$= -150 \sin 1500t \text{ V}$$

III. X_L = 40Ω; 2πfL = 40

$$f = \frac{40}{2\pi \times 0.01} = 637\text{Hz}$$

2.12) In the circuit, source voltage is v=$200\sin\left(314t+\frac{\pi}{6}\right)$ and the current is i = $20\sin\left(314t-\frac{\pi}{3}\right)$ Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$
$I_m = 20A$
$\omega = 314$ rad/sec

I. $\omega = 2\pi f$
$f = 50Hz$

II. $V_m = 200V$ and $I_m = 20A$

III. $$V_{RMS} = \frac{V_m}{\sqrt{2}} = 141.42V$$

$$I_{RMS} = \frac{I_m}{\sqrt{2}} = 14.142A$$

IV. For a sinusoidal wave, Average value of current, $I_{av} = \frac{2I_m}{\pi} = 12.732A$

Average value of voltage, $V_{av} = \frac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

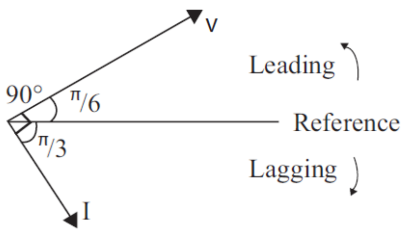

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle (90^0). So the circuit is purely inductive.

$$I_m = \frac{V_m}{\omega L}$$

$$L = \frac{200}{314 \times 20} = 31.85mH$$

2.5.3 Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

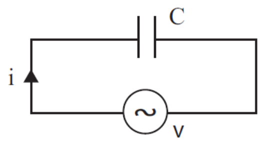

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, Q = C.v

$$= C.\ V_m \sin\omega t$$

$$\text{Current,i} = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}\left(CV_m \sin\omega t\right) = \omega CV_m \cos\omega t$$

$$= \omega CV_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\left[\because I_m = \omega CV_m\right]$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

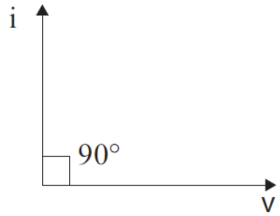

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

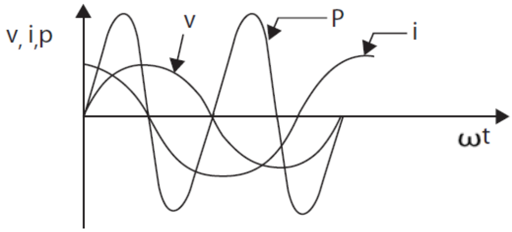

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\, i$$

$$= V_m \sin\omega t\, I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t (\cos \omega t)$$

$$= V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

2.13) A 135μF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:

Given: $V_{RMS} = V = 150V$

$C = 135\mu F$

$f = 50Hz$

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)\quad \because I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150X\sqrt{2} = 212.13V$

$I_m = 314X135X10^{-6}X212.13 = 8.99A$

$i = 8.99\sin\left(314t + \frac{\pi}{2}\right)A$

III. $p = V_m I_m \sin\omega t(\cos\omega t) = 212.13X8.99\ \sin 314t.\cos 314t$

$= 66642.6\ \sin 314t.\cos 314t = 66642.6\frac{\sin 628t}{2}$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$

$= 33321.3\ \sin 628t\ W$

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}} = 6.36A$

VI. $P_m = \frac{V_m I_m}{2} = 953.52\ W$

2.14) A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: V_{RMS}= V= 100V

C = 12μF

I = 0.5A

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

V_m=100X $\sqrt{2}$ =141.42V

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

I_m= 0.5X $\sqrt{2}$ = 0.707A

II. $I_m = \omega C V_m = 2\pi f C V_m$

f = 66.3Hz

2.5.4 RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

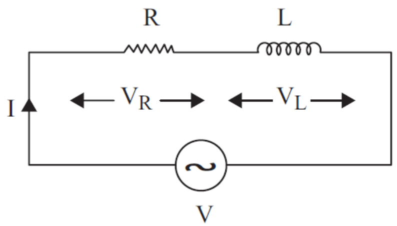

Figure 2.20

Let V = V_m Sin ωt be the applied voltage.

i = Circuit current at any constant.

I = Effective Value of Circuit Current.

V_R= Potential difference across inductor.

V_L= Potential difference across inductor.

F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

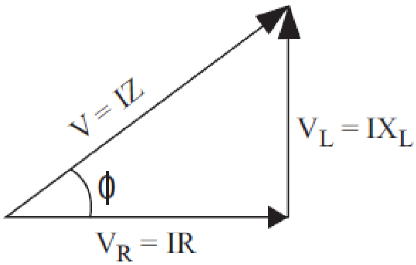

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$

$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

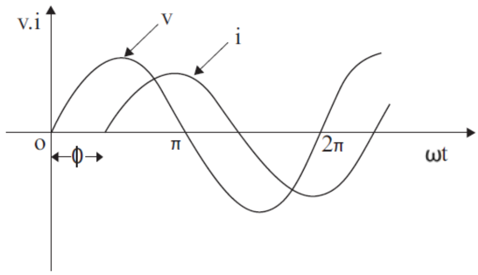

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\ \phi = \frac{R}{Z}$

Actual Power P = VI cos ϕ – Current component is phase with voltage

Reactive or Quadrature Power

Q = VI sinϕ – Current component is quadrature with voltage

Complex or Apparent Power

S = VI – Product of voltage and current

S = P + jQ

Problem

2.15) A series RL Circuit has

$$i(t) = 5\ sin\left(314t + \frac{2\pi}{3}\right) \text{ and } V(t) = 20\ sin\left(314t + \frac{5\pi}{3}\right)$$

Determine (a) the impedence of the circuit

(b) the values of R_1L and power factor

(c) average power of the circuit

Solution:

$$i\ (t) = 5 \sin (314t + \frac{2\pi}{3})$$

$$V(t) = 20 \sin (314t + \frac{5\pi}{3})$$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2\times180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5\times180}{3} = 150°$

Phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$$= 150 - 120$$

$$\theta = 30^0$$

Power factor = cos θ

$$= \cos 30$$

$$= 0.866 \text{ (lagging)}$$

Impedence of the circuit $Z = \frac{V_m}{I_m}$

$$= \frac{20}{5}$$

$$Z = 4\Omega$$

(i) But cos $\phi = \frac{R}{Z}$

$$0.866 = \frac{R}{4}$$

$$\therefore R = 4 \times 0.866$$

$$R = 3.46\Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$X_L = \sqrt{Z^2 + R^2}$$

$$= \sqrt{(4)^2 - (3.46)^2}$$

$$X_L = 2\Omega$$

$$\omega L = 2\Omega$$

$$L = \frac{2}{\omega}$$

$$= \frac{2}{3/4}$$

$$L = 6.37 \times 10^{-3} \text{ H}$$

(ii) Average power = VI cos ϕ

$$= \frac{20}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} (0.866)$$

$$= 43.3 \text{ watts}$$

2.16) A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current
(ii) The phase angle between the current and the voltage
(iii) Power factor
(iv) Power

Solution:

$R = 6\Omega$
$L = 0.03$ H
$X_L = 2\pi fL$
$X_L = 2\pi\times50\times0.03$
$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$
$$= \sqrt{(6)^2 + (9.42)^2}$$
$$|Z| = 11.17\Omega$$

(i) $I = \frac{V}{Z} = \frac{100}{11.17} = 8.95$ amps

(ii) $\phi = tan^{-1}\left(\frac{X_L}{R}\right)$

$= tan^{-1}\left(\frac{9.42}{6}\right)$

$\Phi = 57.5$ (lagging)

(iii) Power factor = cos ϕ
= cos 57.5
= 0.537 (lagging)

(iv) Power = Average power
= VI cos Φ
= 100 ×8.95×0.537
Power = 480.6 Watts

2.17) A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

$R = 10\Omega$
$L = 20$ mH $= 20\times10^{-3}$H
$X_L = 2\pi fL$
$= 2\pi\times60\times20\times10^{-3}$
$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \frac{V}{Z} = \frac{250}{12.5} = 20\, amps$

$V_R = IR = 20\times10 = 200$ volts

(iii) $V_L = I X_L = 20\times7.54 = 150.8$ volts

(iv) Apparent power S = VI
= 250×20
S= 5000VA

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \text{ (lagging)}$$

Active power = VI cos ϕ

=250×20×0.8

P = 4000 Watts

$$\sin\phi = \sqrt{1-\cos^2\Phi} = \sqrt{1-(0.8)^2} = 0.6$$

Reactive Power Q = VI sin ϕ

= 250×20×0.6

Q= 3000 KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$

$Z_2 = 10 - j7$

V = 200 volts

$Z_{Total} = Z_1 + Z_2$

$= 5 + j7 + 10 - j7$

$Z_{Total} = 15 < 0^{\cdot}$

$\therefore \phi = 0^{\cdot}$

Taking V as referenve,

V = 200 < 0°· Volts

(i) $I = \frac{V}{Z} = \frac{200\angle 0°}{15\angle 0°} = 13.33\angle 0° \, amps$

(ii) ϕ = 0

PF = cos ϕ = cos 0 = 1

(iii) Power = VI cos ϕ

= 200 ×13.33×1

Power = 2666 watts

2.5.5 RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.
I = Circuit current of any instant
I = Effective value of circuit current
V_R = Potential Difference across Resistor
V_c = Potential Difference across Capacitor
f = Frequency of applied voltage
The same Current I flows through R and C
Voltage across R = V_R = IR in phase with I
Voltage across C = $V_c = IX_c$ lagging I by 90^0
Applied voltage $V = IR - jIX_c$

$$= I (R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$

Figure 3.27

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF
R = 120Ω
V = 100V
F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\,\Omega$$

$$|Z| = \sqrt{R^2 + X_c^{\,2}}$$

$$= 340\,\Omega$$

(a) $|I| = \dfrac{|V|}{|Z|}$

$$= \frac{100}{340}$$

$$= 0.294\, amps$$

(b) $PhaseDifference\, \phi = \tan^{-1}\left(\dfrac{X_c}{R}\right)$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3^\circ \quad (Leading)$$

$$\cos\phi = \cos(69.3)^\circ$$

$$= 0.353 \quad (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38\, Watts$$

3.21 The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$
$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$
$$= \sqrt{(240)^2 - (100)^2}$$
$$|V_C| = 218.2\, volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$
$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3amps$$
$$|X_C| = \frac{V_C}{|I|} \quad (Apply\, ohm's\, law\ for\, C)$$
$$= \frac{218.2}{3} = 72.73\Omega$$
$$\frac{1}{2\pi fc} = 72.73$$
$$C = \frac{1}{2\pi \times 50 \times 72.73} = 43.77 \times 10^{-6} F$$
$$C = 43.77 \mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum $V_c = \sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

3.22 A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$$|V| = 60 volts$$

$$F = 50\,Hz$$

$$|I| = 2\,amps$$

Power Dissipated = P = 80 watts

(a) $|Z| = \frac{|V|}{|I|} = \frac{60}{2} = 30\Omega$

(b) $As\, P = I^2 R$

$$R = \frac{P}{I^2} = \frac{80}{4}$$

$$= 20\Omega$$

(c) $Since, |Z|^2 = R^2 + X_c{}^2$

$$X_c = \sqrt{(z)^2 - R^2}$$

$$= \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$\frac{1}{2\pi fc} = 22.36$$

$$c = \frac{1}{2\pi f (22.36)}$$

$$= \frac{1}{2\pi \times 50 \times 22.36}$$

$= 142 \times 10^{-6}$ F

C = 142 µF

(or) Power factor = $\cos\phi = \frac{R}{|Z|}$

$$= = \frac{20}{30}$$

= 0.67(Leading)

It is capacitive circuit.

3.23 A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

V_R = 100 volts

W = I^2R = V_RI

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\,amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$

$$V_C = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(230)^2 - (100)^2}$$

$$= 207volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$

$$= 27.6\Omega$$

$$\frac{1}{2\pi fc} = 27.6$$

$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$

$$= 96.19\,\mu F$$

$$Power\ \ factor = \cos\phi = \frac{R}{|Z|}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$

$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$

$$= 13.33\Omega$$

$$Powerfactor = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{13.33}{30.7}$$

$$= 0.434(\text{Leading})$$

3.5.6 RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$
$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$
$$= |I|^2 + [R^2 + (X_L - X_C)^2]$$
$$|V| = |I|\sqrt{R^2 + (X_L - X_C)^2}$$
$$|Z| = \frac{|V|}{|I|}$$
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2} \qquad \because X = (X_L - X_C)$$

Three cases of Z

Case 1 If $X_L > X_C$
The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

Case 2 If $X_L < X_C$
The circuit behaves like RC circuit current leads applied voltage power factor is leading.

Case 3 When $X_L = X_C$, the circuit behaves like pure resistance. Current is in phase with the applied voltage power factor is unity.
Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage
   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by
   1) $i = I_m \sin (\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin (\omega t + \theta)$, for $X_L > X_C$
   3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j\,(X_L - X_C)$$

Problems

3.24 In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V \qquad V_R = 3V \qquad V_L = 1V$

$V^2 = V_R^{\,2} + (V_L - V_C)^2$

$(V_L - V_C)^2 = V^2 - V_R^{\,2}$

$= 25 - 9 = 16$

$V_L - V_C = \pm 4$

$V_C = V_L \pm 4 = 1 + 4$

$V_C = 5V$

3.25 A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1\ H$

$C = 150\ \mu F \qquad = 150 \times 10^{-6}\ F$

$V = 200V \qquad f = 50\ Hz$

a) $X_L = 2\pi f L = 2\pi\,(50)\,0.1$

$= 31.4\ \Omega$

b) $$X_C = \frac{1}{2\pi fc} = \frac{1}{2\pi(50)(150\times10^{-6})}$$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $$|Z| = \sqrt{R^2 + X^2}$$

$$= \sqrt{10^2 + (10.2)^2}$$

$$= 14.28\Omega\,(Inductive)$$

$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

= 0.7 (lagging) (I lags behind V)

3.5.7 Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi fc_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi fL_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^2 + X_{C1}^2}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^2 + X^2_{L2}}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g

Susceptance = b

Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \ (positive)$$

$$|Y_1| = \sqrt{g_1^2 + b_1^2}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2} \text{ (Negative)}$$

$$|Y_2| = \sqrt{g_2^2 + b_2^2}$$

Total conductance $G = g_1 + g_2$
Total Suceptance $B = b_1 - b_2$

Total admittance $|Y| = \sqrt{G^2 + B^2}$

Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle $= tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

3.26 Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$
$Z_2 = 10 - j60\ \Omega$
$F = 50$ Hz
$Z_1 = R_1 + jX_L$
$Z_2 = R_2 - jX_C$

J term positive for in inductive
J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\,(50)\,(L)$

$X_L = 15$

$2\pi\,(50)\,L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$
$R_2 = 10$
$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$$C = \frac{1}{2\pi(50)60}$$

C = 53 μF.

2.3.27 Two circuits, the impedances of which are $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$
$$Z2 = (6 - j8)\Omega = 10\angle -53.13$$
$$I = 15\ A$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \qquad \text{(Current divider rule)}$$
$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$

$(Z_1 + Z_2 = 10 + j15 + 6 – j8)$

$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$
$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I – I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$
$$= 15 – (1.97 – j8.37)$$
$$= 15.5 – 32.7A$$

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$

=739.6 watts

Power taken by branch 2

$$= |I_2|^2 R_2$$
$$= (15.5)^2 \times 6$$

=1442 watts

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$Z_1 = R = 100\Omega$

$Z_2 = j\,X_L = j2\pi fL$

$= j\,(2\pi\times 50\times 0.6)$

$= j\,188.5\Omega$

$= 188.5\angle 90$

$$Z_T = Z_1 * Z_2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

$Total\ impedance\ |Z_T| = 88.33\Omega$

R = 78Ω, X_L = 41.46Ω

X_L = 2πfLeq

41.46 = 2π × 50 × Leq

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

= 30 – j40 + 24 +j32

= 54 – j8

= 54.6 ∠ – 8.43 A

Comparing 'V' and 'I_T' current I_T lag voltage 'V'

∴ ϕ = 8.43° lag

Power factor = cos ϕ= cos 8.43

= 0.99 lag

$\text{True Power} = W = |V||I|\cos\phi$

=200 × 54.6 × cos 8.43

= 10802 watts

= 10.802 KW

$\text{Apparent Power} = |V| I$

= 200× 54.6

= 10920 VA = 10.920 KVA

$\text{Reactive Power} = |V| I \sin\phi$

= 200 × 54.6 × sin 8.43

= 1601 VAR

= 1.601 KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

= 3.663 ∠ 8.43

= 3.623 + j0.54

= R +j X_L

R = 3.623Ω X_L = 0.54 Ω

(or)

$$Z_{Total} = Z1 * Z2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2.4 + j3.2)(3 - j4)}{2.4 + j3.2 + 3 - j4}$$

$$= \frac{4\angle 53.13 \times 5\angle -53.13}{5.46\angle -8.43}$$

$$= \frac{20\angle 0^0}{5.46\angle -8.43}$$

$$= 3.663 \angle 8.43$$

$$= 3.623 + j\,0.54\ \Omega$$

3.6 THREE PHASE A.C. CIRCUITS

Three Phase Connection

We have seen above only about single phase systems. Generally generation transmission and distribution of electrical energy are of three phase in nature. Three phase system is a very common poly phase system. It could be viewed combination of three single phase system with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system where as pulsating torque is produced in the case of single phase system. Because of these advantages the overall generation, transmission and distribution of electrical power is usually of three phase.

There are two possible connections in 3-phase system. One is star connection and the other one is delta or mesh connection. Each type of connection is governed by characteristics equations relating the currents and the voltages.

3.6.1 Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called star point or the neutral point. The free ends of the three phase coils will be operating at specific potential with respect to the zero potential of star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system and three phase three wire system.

Analysis

Let us analyze the relationship between currents and voltages. In a three phase circuit, the voltage across the individual coil is known as phase voltage and the voltage between two lines is called line voltage. Similarly the current flowing through the coil is called phase current and the current flowing through the line is called line current.

Notations Defined

E_R , E_Y , E_B	: Phase voltages of R, Y and B phases.
I_R , I_Y , I_B	: Phase currents
V_{RY}, V_{YB}, V_{BR}	: Line voltages
I_{L1}, I_{L2}, I_{L3}	: Line currents

Figure 3.32

A balanced system is one in which the currents in all phases are equal in magnitude and are displaced from one another by equal angles. Also the voltages in all the phases are equal in magnitude and are displaced from one another by equal angles. Thus,

$$E_R=E_Y=E_B=E_P \qquad V_{RY}=V_{YB}=V_{BR}=V_L$$
$$I_R=I_Y=I_B=I_P \qquad I_{L1}=I_{L2}=I_{L3}=I_L$$

Figure 3.33

Current Relationship:

Apply Kirchhoff's current law at nodes R_1, Y_1, B_1 We get

$$I_R = I_{LI} ;\ I_Y = I_{L1} ;\ I_B=I_{L3}$$

This means that in a balanced star connected system, phase current equals the line current

$I_P = I_L$
Phase current = Line current

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_{R}; V_{Ry} and E_y.

$$\vec{E}_R - \vec{E}_Y = \vec{V}_{RY}$$

Using law of parallelogram

$$\left|\vec{V}_{RY}\right| = V_{RY} = \sqrt{E_R^2 + E_R^2 + 2E_R E_Y \cos 60}$$

$$= \sqrt{E_P^2 + E_P^2 + 2E_P E_P \cos 60} == E_P\sqrt{3}$$

Similarly,

$$\vec{E}_Y - \vec{E}_B = \vec{V}_{YB} \; and \; \vec{E}_B - \vec{E}_R = \vec{V}_{BR}$$
$$V_{RY} = E_P\sqrt{3} \; and \; V_{BR} = E_P\sqrt{3}$$

Hence $V_L = \sqrt{3}\,E_P$

Line Voltage $= \sqrt{3}$ phase voltage

Power relationship:
Let cosϕ be the power factor of the system.

Power consumed in one phase=$E_P I_P \cos\phi$

Power consumed in three phase $= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L cos\phi$

$$= \sqrt{3} V_L I_L cos\phi \; watts$$

Reactive power in one phase $= E_P I_P sin\phi$

Total Reactive power $= 3E_P I_P \sin\phi$

$$= \sqrt{3} V_L I_L \sin\phi \quad VAR$$

Apparent power per phase=$E_P I_P$

Total Apparent Power=$3E_P I_P = \sqrt{3} V_L I_L$ Volt

3.6.2 Delta Connection:

The dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Figure 3.33

Let us analyze the relationship between currents and voltages. The system is balanced one. Notation used in the star connection are used here.

E_R, E_Y, E_B :	Phase voltages of R, Y and B phases.
I_R, l_Y, IB :	Phase currents
V_{RY}, V_{YB}, V_{BR}:	Line voltages
I_{L1}, I_{L2}, I_{L3} :	Line currents

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R,V_{RY}

We Have	E_R=V_{RY}
Similarly	$E_Y = V_{YB}$ *and* $E_B = V_{BR}$
Thus	E_P=V_L

Phase voltage= line voltage

Current Relationship:

Apply Kirchhoff's current law at node A (i.e.) R_1 , B_2 We get

$$\vec{I}_R - \vec{I}_B = \vec{I}_{1,1}$$

Referring to the phasor diagram and applying the law of parallelogram,
We get

$$I_{L1} = \sqrt{I_R^2 + I_Y^2 + 2I_R I_Y \cos 60}$$
$$= \sqrt{I_P^2 + I_P^2 + 2I_P I_P \cos 60}$$

Similarly,

$\vec{I}_Y - \vec{I}_R = \vec{I}_{1,2}$ *and* $\vec{I}_B - \vec{I}_Y = \vec{I}_{1,3}$

Hence $I_{L2} = I_P\sqrt{3}$ *and* $I_{L3} = I_P\sqrt{3}$

Thus Line current $= \sqrt{3}$ Phase current

$$I_L = I_P\sqrt{3}$$

Power relationship:
Let cosф be the power factor of the system.

Power consumed in one phase $= E_P\, l_P\, cos\phi$

Power consumed in three phase $= 3\,\mathrm{V}_L\left(\dfrac{I_L}{\sqrt{3}}\right)\cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi\, watts$$

Reactive power in one phase = $E_p\, I_p\, sin\phi$

Total Reactive power = $3E_p\, I_p\, cos\phi$

$$= \sqrt{3} V_L I_L \sin\phi VAR$$

Apparent power per phase = E_pI_p

Total Apparent Power = $3E_pI_p = \sqrt{3}\ V_LI_L$ volt

3.7 MEASUREMENT OF POWER IN THREE PHASE CIRCUITS:

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phase are not of same value. As a result, the current flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The three phase power can be measured by using three watt maters in each phases. The algebraic sum of the reading gives the total three phase power consumed. However three phase power can also measured using two watt meter.

Case I Star Connected load

In this section we analyse the measurement of three phase power using two wattmeter, when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Diagram 4

Figure 3.35

For Wattmeter 1

$$Current\ measured = \vec{I}_{L1} = \vec{I}_R$$

$$Voltage\ measured = \vec{V}_{RY}$$

$$Phase\ angle\ between\ them = 30 + \phi$$

$$Power\ measured\ = P1 = V_{RY} I_R \cos(30 + \phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{L3} = \vec{I}_B$$

$$Voltage\ measured = \vec{V}_{BY}$$

$$Phase\ angle\ between\ them = 30 - \phi$$

$$Power\ measured\ = P1 = V_{BY} I_B \cos(30 - \phi)$$

$$= \mathrm{V_L I_L} \cos(30 - \phi)$$

$$Now, \quad P1 + P2 = V_L I_L \cos(30 + \phi) + \mathrm{V_L I_L} \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\, \mathrm{V_L I_L} \cos\phi = Total\ power\ in\ a\ three\ phase\ circuit$$

$$P2 - P1 = V_L I_L[\cos(30-\phi) - \cos(30+\phi)]$$
$$= V_L I_L \times 2 \times \sin 30 s\text{in}\,\phi$$
$$= \mathrm{V_L I_L}\,\sin\phi$$

$$\frac{P2-P1}{P2+P1} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$
$$\tan\phi = \sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]$$
$$\tan\phi = \sqrt{3}\,(\mathrm{P}\,2 - P1/\mathrm{P}\,2 + P1)$$
$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]\right\}$$

Thus, two wattmeters connected appropriately in a three phase circuit can measure the total power consumed in the circuit.

Case II Delta Connected load

In this section we analyse the measurement of three phase power using two wattmeter, power when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Figure 3.36

For Wattmeter 1

$$\textit{Current measured} = \vec{I}_{1,1} = \vec{I}_R - \vec{I}_B$$
$$\textit{Voltage measured} = \vec{V}_{RY} = \vec{E}_R$$
$$\textit{Phase angle between them} = 30 + \phi$$
$$\textit{Power measured} = P1 = V_{RY} I_{L1} \cos(30+\phi)$$
$$= V_L I_L \cos(30+\phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{1,3} = \vec{I}_B - \vec{I}_Y$$

$$Voltage\ measured = \vec{V}_{BY} = -\vec{E}_Y$$

$$Phase\ angle\ between\ them = 30 - \phi$$

$$Power\ measured = P1 = V_{BY} I_{1,3} \cos(30-\phi)$$

$$= \mathrm{V_L I_L} \cos(30-\phi)$$

$$Now,\quad P1 + P2 = V_L I_L \cos(30+\phi) + \mathrm{V_L I_L}\cos(30-\phi)$$

$$= V_L I_L [\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\,\mathrm{V_L I_L} \cos\phi = Total\ power\ in\ a\ three\ phase\ circuit$$

$$P2 - P1 = V_L I_L [\cos(30-\phi) - \cos(30+\phi)]$$

$$= V_L I_L \times 2 \times \sin 30 s\ \mathrm{in}\,\phi$$

$$= \mathrm{V_L I_L} \sin\phi$$

$$\mathrm{Tan}\,\phi = \sqrt{3}\,(\mathrm{P}\,2 - P1/\mathrm{P}\,2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2-P1}{P2+P1}\right]\right\}$$

Problems 3.30

Three similar coils of Resistance of 10Ω and inductance 0.15 Henry are connected in star across a 3Φ, 440V, 50Hz supply. Find the line and phase values of current. Also find the above values when they are connected in Delta.

Solution:

Given Data

$$V_L = 440V,\ R_{ph} = 10\Omega,\ L_{ph} = 0.15H,\ f = 50Hz$$

$$X_{L\,ph} = 2\pi f\ L_{ph} = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$|Z_{ph}| = \sqrt{R_{ph}^{\ 2} + X_{L\ ph}^{\ 2}} = \sqrt{10^2 + (47.12)^2}$$

$$= 48.17\Omega$$

In star Connection

$$I_L = I_{ph} \qquad V_L = \sqrt{3} V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 230.95 \text{ Volt}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.95}{48.17} = 4.794 A$$

$$I_L = I_{ph} = 4.794\, A$$

Active power = $3V_{ph}\, I_{ph} \cos\Phi$

$$\cos\Phi = \frac{R_{ph}}{Z_{ph}} = 0.2075$$

$$\text{Active power} = 3 * 230.95 * 4.794 * 0.2075$$
$$= 689.54 W$$

$$\text{Reactive power} = 3V_{ph}\, I_{ph} \sin\Phi$$

$$\sin\Phi = \sqrt{1 - \cos^2\Phi} = 0.9782$$

$$\text{Reactive power} = 3 * 230.95 * 4.794 * 0.9782$$
$$= 3249.23 VAR$$

$$\text{Apparent power} = 3V_{ph}\, I_{ph} = 3 * 230.95 * 4.794$$
$$= 3321.52 V$$

If it is Delta connected coils, then

$$VL = V_{ph} \quad \& \quad IL = \sqrt{3}\, I_{ph}$$

$$VL = V_{ph} \quad = 440 V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} \frac{440}{48.17} = 9.134\, A$$

$$IL = \sqrt{3}\, I_{ph} = \sqrt{3} * 9.134 = 15.82\, A$$

$$\text{Active power} = 3\, V_{ph}\, I_{ph} \cos\Phi$$
$$= 3 * 440 * 9.134 * 0.2075$$
$$= 2501.80\, watt$$

$$\text{Reacive power} = 3\, V_{ph}\, I_{ph} \sin\Phi$$
$$= 3 * 440 * 9.134 * 0.9782$$

$$\text{Apparent power} = 3V_{ph}\, I_{ph} = 3 * 440 * 9.134$$
$$= 12056.88\, VA$$

Problem 3.31
Two wattmeters connected to measure the 3Φ power indicate 1000 watts and 500 watts respectively. Calculate the power factor of the ckt.

Solution:
Given data

p_1 = 500 watts, p_2 = 1000 watts,

$$p_1 + p_2 = 1000 + 500 = 1500\, watts$$
$$p_2 - p_1 = 1000 - 500 = 500\, watts$$
$$p_1 = VL\, IL \cos(30 + \Phi)$$
$$p_2 = VL\, IL \cos(30 - \Phi)$$
$$\mathrm{p_1 + p_2} = \sqrt{3}\, VL\, IL \cos\Phi$$
$$p_2 - p_1 = \sqrt{3} * \frac{(p_2 - p_1)}{(p_1 + p_2)} = \frac{\sqrt{3} * 500}{1500}$$
$$= 0.5773$$
$$\Phi = 29.99°$$

Power factor cos Φ = 0.866

Problem 3.32
A balanced star connected load of (3+j4) Ω impedance is connected to 400V, three phase supply. What is the real power consumed by the load?

Solution:
Given data

$$V_L = 400\, volt$$
$$\text{Impedence / phase} = Z_{ph} = 3 + j4 = 5\angle 53°$$
$$\text{In star connection}$$
$$\mathrm{I_L} = I_{ph} \;\&\; V_{\mathrm{L}} = \sqrt{3}\, V_{ph}$$
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\, volt$$
$$\text{Current in each } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5\angle 53°}$$
$$= 46.02\angle -53°\, A$$
$$\text{Line current } I_L = 46.02\, A$$
$$\text{Total power consumed in the load} = \sqrt{3}\, V_L I_L \cos\Phi$$
$$= \sqrt{3} * 400 * 46.02 * \cos(-53°)$$
$$= 19188\, watt$$

PART A – QUESTIONS

1. Define Form factor and Peak factor
2. What is meant by average value?
3. Give the relation between line voltage and phase voltage, line current and phase current for star and delta connection.
4. What are the advantages of polyphase system ?
5. Define power factor?
6. What is phase sequence?
7. Define inductance and write its unit.
8. What is meant by balanced system?
9. Write down the expression for power factor in two wattmeter method.

PART B – QUESTIONS

1. Explain with neat figures the power measurement in three phase circuits using two-wattmeter method.
2. A given load consisting of a resistor R & a capacitor C, takes a power of 4800W from 200V, 60HZ supply mains, Given that the voltage drop across the resistor is 120V, Calculate the (a) impedance (b) current (c) power factor (d) resistance (e) capacitance. Write down the equations for the current and voltage.
3. A coil of 10 ohms and inductance of 0.1H in series with a 150µF capacitor across 200V,250HZ supply. Calculate (i) inductive reactance, capacitive reactance and impedance of the circuit (ii) current (iii)power factor(iv)voltage across the coil and capacitor respectively.
4. An impedance z_1= (2.4+j3.2) ohms is in parallel with another impedance z_2= (3-j4) ohms. The combination is given a supply of 200 V. Calculate (i) total impedance (ii) individual & total currents (iii) power factor (iv) power in the circuit.
5. A balanced three phase load consists of 6 ohms resistor & 8 ohms reactor (inductive) in each phase. The supply is 230V, 3 phases, 50HZ. Find (a) phase current (b) line current (c) total power. Assume the load to be connected in star & delta.
6. A 3phase, 4 wire 208 V, ABC system supplies a star connected load in which $Z_A=10\angle 0$, $Z_B=15\angle 30$, $Z_C=10\angle -30$. Find the line currents, the neutral current and the load power.
7. A coil having R = 10Ω and L = 0.2H is connected to a 100V, 50 Hz supply. Calculate (i) the impedance of the coil (ii) the current (iii) the phase difference between the current and voltage and (iv) the power.
8. Three similar coils of resistance of 10Ω and inductance 0.15H are connected in star across a 3 phase 440V, 50 Hz supply. Find the line

and phase values of current. Also find the above values when they are connected in delta.

9. Each phase of a delta connected load comprises a resistor of Ohm and a capacitor of μF in series. Calculate the line current for a 3 - ϕ voltages of 400V at 50 Hz. Also evaluate the power factor and the total 3 - ϕ power absorbed by the load.

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SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF BIO MEDICAL ENGINEERING

UNIT – III

Basic Electrical Engineering – SEEA1203

NETWORK THEOREMS

Superposition Theorem - Reciprocity Theorem - Thevenin's Theorem - Norton's Theorem - Maximum power transfer Theorem..

2.1 SUPERPOSITION THEOREM

The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals. This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the circuit which contains two sources as shown in Fig. 1.

Now let us find the current passing through the 3 V resistor in the circuit. According to the superposition theorem, the current I_2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A

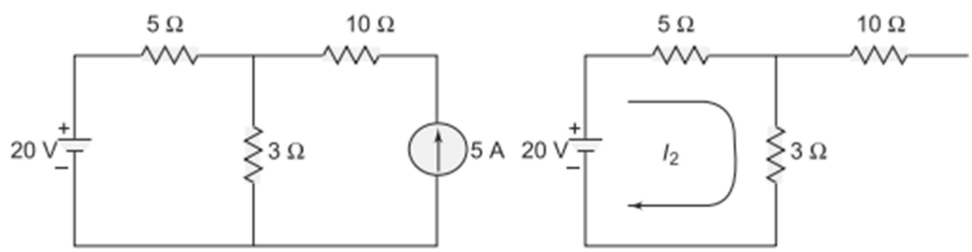


Figure 1 : Superposition theorem

The current I_5 due to the 5 A source with the 20 V source short circuited is

$$I_5 = 5 \times \frac{5}{(3+5)} = 3.125\,\text{A}$$

The total current passing through the 3 V resistor is

(2.5 + 3.125) = 5.625 A

Let us verify the above result by applying nodal analysis.

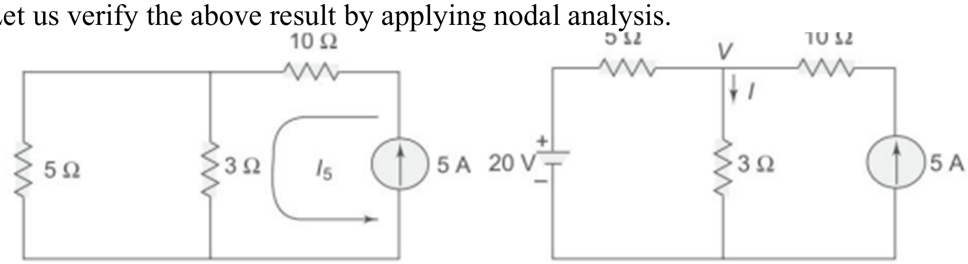


Figure 2: Superposition theorem

The current passing in the 3 V resistor due to both sources should be 5.625 A.
Applying nodal analysis to Fig. 2, we have

$$\frac{V-20}{5}+\frac{V}{3}=5$$

$$V\left[\frac{1}{5}+\frac{1}{3}\right]=5+4$$

$$V=9\times\frac{15}{8}=16.875 \text{ V}$$

The current passing through the 3 V resistor is equal to V/3,

i.e. I = 16.875/3=5.625 A

So the superposition theorem is verified.
Let us now examine the power responses.
Power dissipated in the 3 V resistor due to the voltage source acting alone

$$P_{20} = (I_2)^2R = (2.5)^2\ 3 = 18.75 \text{ W}$$

Power dissipated in the 3 V resistor due to the current source acting alone

$$P_5 = (I_5)^2R = (3.125)^2\ 3 = 29.29 \text{ W}$$

Power dissipated in the 3V resistor when both the sources are acting simultaneously is given by

$$P = (5.625)^2 \times 3 = 94.92 \text{ W}$$

From the above results, the superposition of P20 and P5 gives

$$P_{20} + P_5 = 48.04 \text{ W}$$

which is not equal to P = 94.92 W
We can, therefore, state that the superposition theorem is not valid for power responses. It is applicable only for computing voltage and current responses.

Example 1: Find the voltage across the 2 V resistor in Fig. 3 by using the superposition theorem.

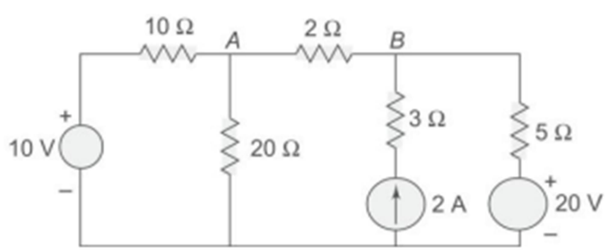


Figure 3

Solution
Let us find the voltage across the 2 V resistor due to individual sources. The algebraic sum of these voltages gives the total voltage across the 2 V resistor.

Our first step is to find the voltage across the 2 V resistor due to the 10 V source, while other sources are set equal to zero.

The circuit is redrawn as shown in Fig. 4

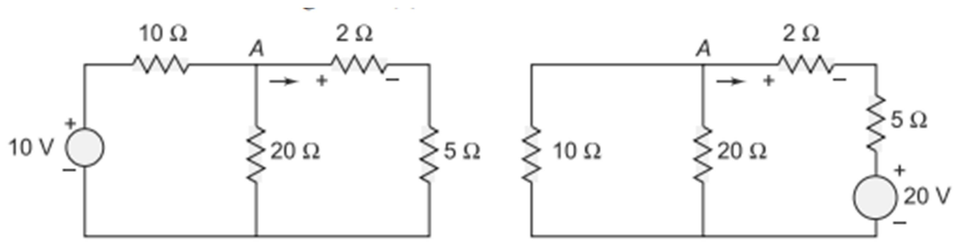


Figure 4

Assuming a voltage V at the node 'A' as shown in Fig. 4, the current equation is

$$\frac{V-10}{10}+\frac{V}{20}+\frac{V}{7}=0$$

$$V\,[0.1 + 0.05 + 0.143] = 1$$

$$\text{or} \quad V = 3.41 \text{ V}$$

The voltage across the 2 V resistor due to the 10 V source is

$$V_2=\frac{V}{7}\times 2 = 0.97 \text{ V}$$

Our second step is to find out the voltage across the 2 V resistor due to the 20 V source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 4.

Assuming voltage V at the node A as shown in Fig. 4, the current equation is

$$\frac{V-20}{7}+\frac{V}{20}+\frac{V}{10}=0$$

$$V\,[0.143 + 0.05 + 0.1] = 2.86$$

$$\text{or} \quad V=\frac{2.86}{0.293}=9.76 \text{ V}$$

The voltage across the 2 V resistor due to the 20 V source is

$$V_2=\left(\frac{V-20}{7}\right)\times 2 = -2.92 \text{ V}$$

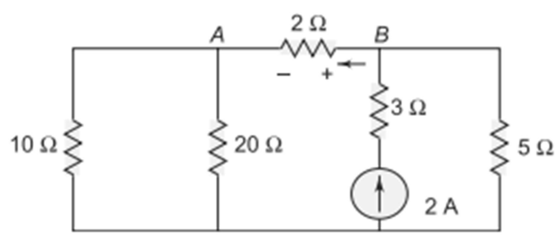


Figure 5

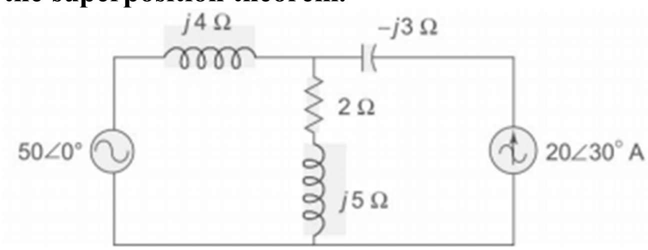
The last step is to find the voltage across the 2 V resistor due to the 2 A current source, while the other sources are set equal to zero. The circuit is redrawn as shown in Fig. 5

$$\text{The current in the 2 } \Omega \text{ resistor} = 2 \times \frac{5}{5+8.67}$$

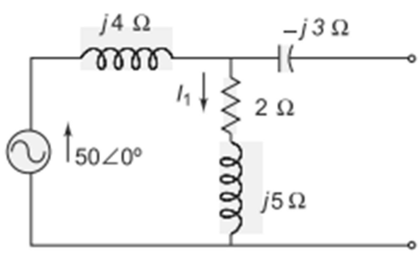
$$= \frac{10}{13.67} = 0.73\,\text{A}$$

The voltage across the 2 V resistor = 0.73 x 2 = 1.46 V
The algebraic sum of these voltages gives the total voltage across the 2 V resistor in the network
V = 0.97 – 2.92 – 1.46 5 = - 3.41 V
The negative sign of the voltage indicates that the voltage at 'A' is negative

Example2:
Determine the voltage across the (2 + j5) V impedance as shown in Fig below by using the superposition theorem.



Solution According to the superposition theorem, the current due to the 50 ∠0° V voltage source is I1 as shown in Fig below with current source 20 ∠30° A open-circuited.



$$\text{Current } I_1 = \frac{50\angle 0^\circ}{2+j4+j5} = \frac{50\angle 0^\circ}{(2+j9)}$$

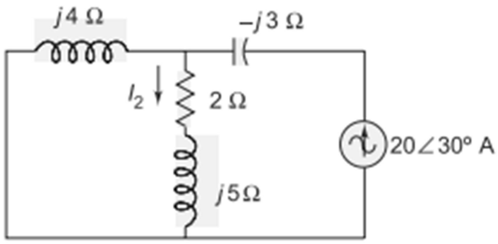
$$= \frac{50\angle 0^\circ}{9.22\angle 77.47} = 5.42\angle -77.47^\circ \text{ A}$$

Voltage across (2 + $j5$) Ω due to the current I_1 is

$$V_1 = 5.42 \angle -77.47^\circ\,(2+j5)$$

$$= (5.38)\,(5.42) \angle -77.47^\circ + 68.19^\circ$$

$$= 29.16 \angle -9.28^\circ$$



The current due to the 20 ∠30° A current source is I_2 as shown in Fig. 7.18, with the voltage source 50 ∠0° V short-circuited.

$$\text{Current } I_2 = 20\angle 30^\circ \times \frac{(j4)\,\Omega}{(2+j9)\,\Omega}$$

$$= \frac{20\angle 30^\circ \times 4\angle 90^\circ}{9.22\angle 77.47^\circ}$$

$$\therefore \quad I_2 = 8.68\angle 120^\circ - 77.47^\circ = 8.68\angle 42.53^\circ$$

Voltage across $(2 + j5)\ \Omega$ due to the current I_2 is

$$V_2 = 8.68\angle 42.53^\circ\,(2 + j5)$$
$$= (8.68)\,(5.38)\,\angle 42.53^\circ + 68.19^\circ$$
$$= 46.69\angle 110.72^\circ$$

Voltage across $(2 + j5)\ \Omega$ due to both sources is

$$V = V_1 + V_2$$
$$= 29.16\angle -9.28^\circ + 46.69\angle 110.72^\circ$$
$$= 28.78 - j4.7 - 16.52 + j43.67$$
$$= (12.26 + j38.97)\ \text{V}$$

Voltage across $(2 + j5)\ \Omega$ is $V = 40.85\angle 72.53^\circ$.

2.2 THEVENIN'S THEOREM

In many practical applications, it is always not necessary to analyse the complete circuit; it requires that the voltage, current, or power in only one resistance of a circuit be found. The use of this theorem provides a simple, equivalent circuit which can be substituted for the original network. Thevenin's theorem states that any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance, where the value of the voltage source is equal to the open-circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances. According to Thevenin's theorem, an equivalent circuit can be found to replace the circuit in Fig. 6

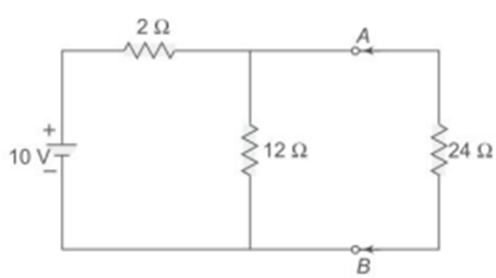


Figure 6

In the circuit, if the 24 V load resistance is connected to Thevenin's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experienced in the original circuit. To verify this, let us find the current passing through the 24 V resistance due to the original circuit.

$$I_{24} = I_T \times \frac{12}{12+24}$$

$$\text{where} \quad I_T = \frac{10}{2+(12\|24)} = \frac{10}{10} = 1 \text{ A}$$

$$\therefore \quad I_{24} = 1 \times \frac{12}{12+24} = 0.33 \text{ A}$$

The voltage across the 24 V resistor = 0.33 x 24 = 7.92 V.
Now let us find Thevenin's equivalent circuit.
The Thevenin voltage is equal to the open-circuit voltage across the terminals 'AB', i.e. the voltage across the 12 V resistor. When the load resistance is disconnected from the circuit, the Thevenin voltage

$$V_{Th} = 10 \times \frac{12}{14} = 8.57 \text{ V}$$

The resistance into the open-circuit terminals is equal to the Thevenin resistance

$$R_{Th} = \frac{12 \times 2}{14} = 1.71 \ \Omega$$

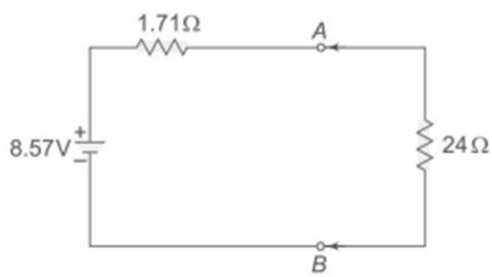


Figure 7

Thevenin's equivalent circuit is shown in Fig. 7. Now let us find the current passing through the 24 V resistance and voltage across it due to Thevenin's equivalent circuit. Fig. 7

$$I_{24} = \frac{8.57}{24+1.71} = 0.33 \text{A}$$

The voltage across the 24 V resistance is equal to 7.92 V. Thus, it is proved that RL (5 24 V) has the same values of current and voltage in both the original circuit and Thevenin's equivalent circuit.

Example : Determine the Thevenin's equivalent circuit across 'AB' for the given circuit shown in Fig. 8

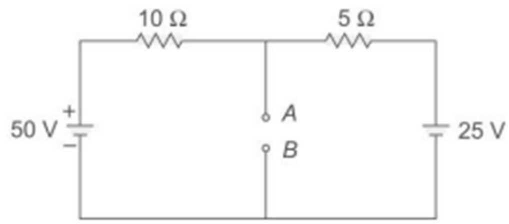


Figure 8

Solution The complete circuit can be replaced by a voltage source in series with a resistance as shown in Fig. 9

where VTh is the voltage across terminals AB, and
RTh is the resistance seen into the terminals AB.

To solve for VTh, we have to find the voltage drops around the closed path as shown in Fig. 9

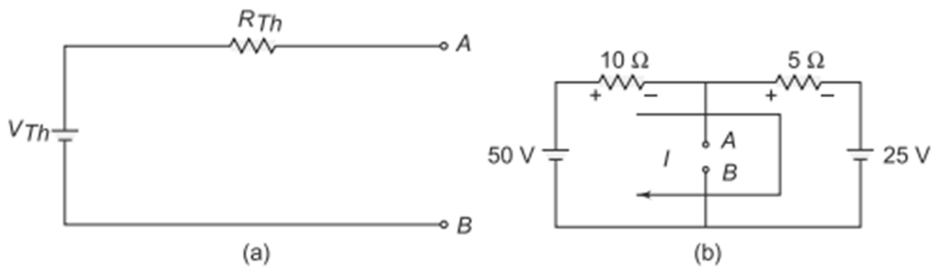


Figure 9

We have $50 - 25 = 10I + 5I$

or $15I = 25$

$$\therefore \quad I = \frac{25}{15} = 1.67\,\text{A}$$

Voltage across 10 Ω = 16.7 V

Voltage drop across 5 Ω = 8.35 V

or $V_{Th} = V_{AB} = 50 - V_{10}$

$= 50 - 16.7 = 33.3$ V

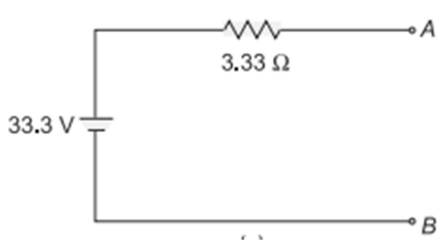


Figure 10

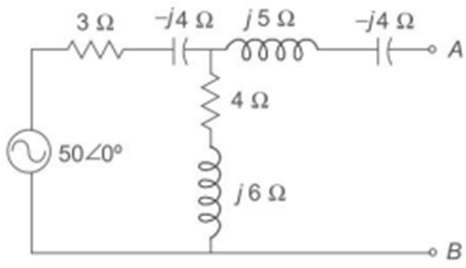
To find RTh, the two voltage sources are removed and replaced with short circuit. The resistance at terminals AB then is the parallel combination of the 10 V resistor and 5 V resistor; or

$$R_{Th} = \frac{10 \times 5}{15} = 3.33\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 9

Example 2: For the circuit shown in Fig. 7.22, determine Thèvenin's equivalent between the output terminals.

Solution The Thèvenin voltage, VTh, is equal to the voltage across the (4 + j6) V impedance. The voltage across (4 + j6) V is



$$V = 50\angle 0° \times \frac{(4+j6)}{(4+j6)+(3-j4)}$$

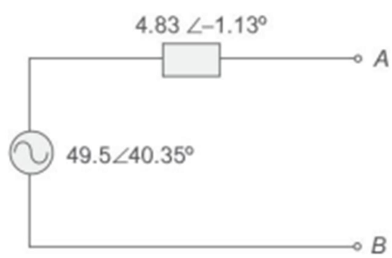
$$= 50\angle 0° \times \frac{4+j6}{7+j2}$$

$$= 50\angle 0° \times \frac{7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= 50\ \angle 0° \times 0.99\ \angle 40.35°$$

$$= 49.5\ \angle 40.35°\ \text{V}$$

The impedance seen from terminals A and B is



$$Z_{\text{Th}} = (j5 - j4) + \frac{(3-j4)(4+j6)}{3-j4+4+j6}$$

$$= j1 + \frac{5\angle 53.13° \times 7.21\angle 56.3°}{7.28\angle 15.95°}$$

$$= j1 + 4.95\ \angle -12.78° = j1 + 4.83 - j1.095$$

$$= 4.83 - j0.095$$

$$\therefore \quad Z_{\text{Th}} = 4.83\ \angle -1.13°\ \Omega$$

The Thèvenin equivalent circuit is shown in Fig above.

2.3 NORTON'S THEOREM

Another method of analysing the circuit is given by Norton's theorem, which states that any two terminal linear network with current sources, voltage sources and resistances can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance. The value of the current source is the short-circuit current between the two terminals of the network and the resistance is the equivalent resistance measured between the terminals of the network with all the energy sources are replaced by their internal resistance.

According to Norton's theorem, an equivalent circuit can be found to replace the circuit in Fig. 11
In the circuit, if the load resistance of 6 V is connected to Norton's equivalent circuit, it will have the same current through it and the same voltage across its terminals as it experiences in the original circuit. To verify this, let us find the current passing through the 6 V resistor due to the original circuit

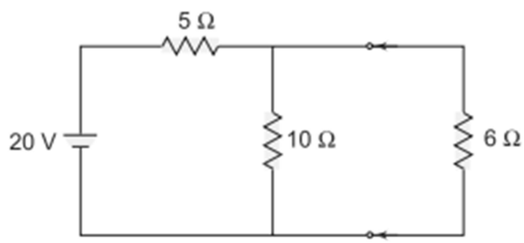


Figure 11

$$I_6 = I_T \times \frac{10}{10+6}$$

$$\text{where} \quad I_T = \frac{20}{5+(10\|6)} = 2.285\,\text{A}$$

$$\therefore \quad I_6 = 2.285 \times \frac{10}{16} = 1.43\,\text{A}$$

i.e. the voltage across the 6 V resistor is 8.58 V. Now let us find Norton's equivalent circuit. The magnitude of the current in the Norton's equivalent circuit is equal to the current passing through short-circuited terminals as shown in Fig. 12

$$\text{Here,} \quad I_N = \frac{20}{5} = 4\,\text{A}$$

Norton's resistance is equal to the parallel combination of both the 5 V and 10 V resistors

$$R_N = \frac{5 \times 10}{15} = 3.33\,\Omega$$

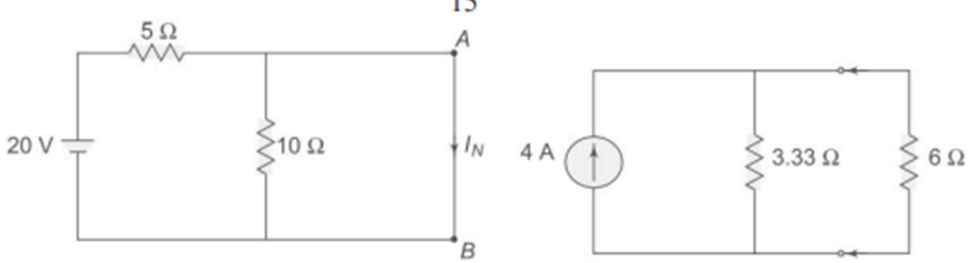


Figure 12

The Norton’s equivalent source is shown in Fig. 12.
Now let us find the current passing through the 6 V resistor and the voltage across it due to Norton’s equivalent circuit

$$I_6 = 4 \times \frac{3.33}{6+3.33} = 1.43 \text{ A}$$

The voltage across the 6 Ω resistor = 1.43 x 6 = 8.58 V
Thus, it is proved that RL (= 6 Ω) has the same values of current and voltage in both the original circuit and Norton’s equivalent circuit.

Example Determine Norton’s equivalent circuit at terminals AB for the circuit shown in Fig. 13

Solution The complete circuit can be replaced by a current source in parallel with a single resistor as shown in Fig. 14, where IN is the current passing through the short circuited output terminals AB and RN is the resistance as seen into the output terminals.

To solve for IN, we have to find the current passing through the terminals AB as shown in Fig. 14

From Fig. 14, the current passing through the terminals AB is 4 A. The resistance at terminals AB is the parallel combination of the 10 V resistor and the 5 V resistor
Norton’s equivalent circuit is shown in Fig. 14

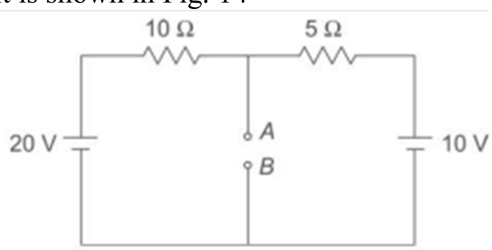


Figure 13

$$\text{or} \quad R_N = \frac{10 \times 5}{10+5} = 3.33 \ \Omega$$

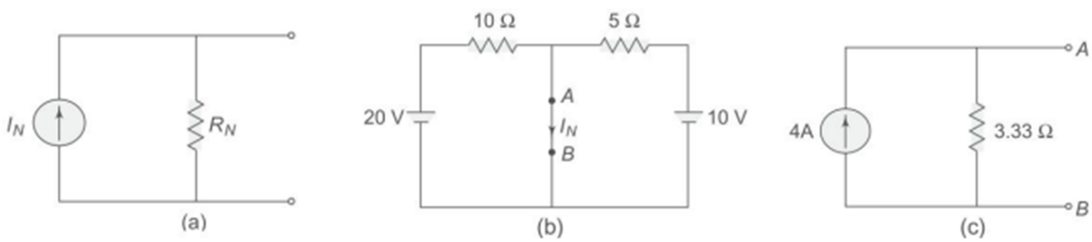
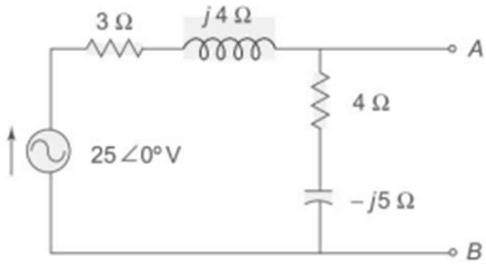
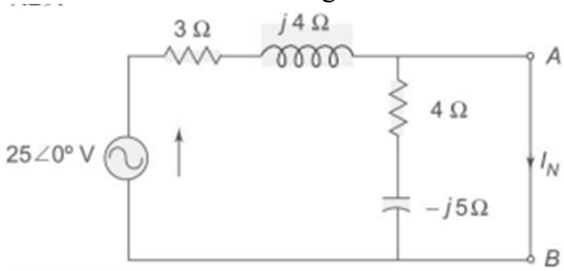


Figure 14

Example 2: For the circuit shown in Fig below, determine Norton's equivalent circuit between the output terminals, AB



Solution Norton's current IN is equal to the current passing through the short-circuited terminals AB as shown in Fig. below



The current through terminals AB is

$$I_N = \frac{25\angle 0^\circ}{3+j4} = \frac{25\angle 0^\circ}{5\angle 53.13^\circ}$$

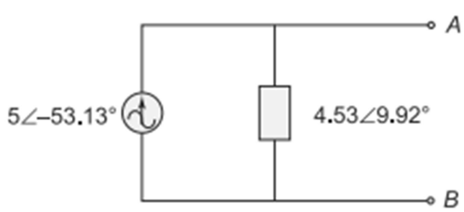
$$= 5\angle -53.13^\circ$$

The impedance seen from terminals AB is

$$Z_N = \frac{(3+j4)(4-j5)}{(3+j4)+(4-j5)}$$

$$= \frac{5\angle 53.13^\circ \times 6.4\angle -51.34^\circ}{7.07\angle -8.13^\circ}$$

$$= 4.53\angle 9.92^\circ$$



Norton's equivalent circuit is shown in Fig above.

2.4 RECIPROCITY THEOREM

In any linear bilateral network, if a single voltage source Va in branch 'a' produces a current Ib in branch 'b', then if the voltage source Va is removed and inserted in branch 'b' will produce a current Ib in branch 'a'. The ratio of response to excitation is same for the two conditions mentioned above. This is called the reciprocity theorem.

Consider the network shown in Fig. 15. AA’ denotes input terminals and BB9 denotes output terminals.

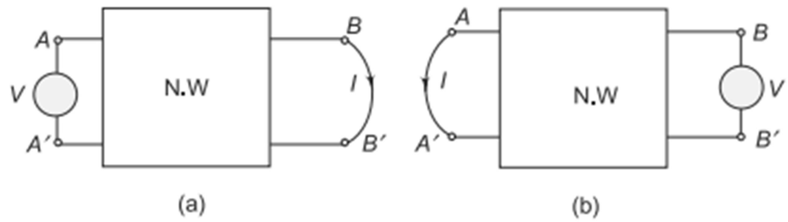


Figure 15

The application of voltage V across AA’ produces current I at BB’. Now if the positions of the source and responses are interchanged, by connecting the voltage source across BB9, the resultant current I will be at terminals AA’. According to the reciprocity theorem, the ratio of response to excitation is the same in both cases.

Example Verify the reciprocity theorem for the network shown in Fig. 16

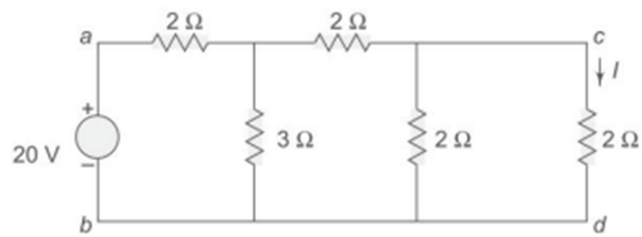


Figure 16

Solution Total resistance in the circuit = 2 + [3 || (2 + 2 || 2)] = 3.5 Ω
The current drawn by the circuit (See Fig. 17 (a))

$$I_T = \frac{20}{3.5} = 5.71\ \Omega$$

The current in the 2 V branch cd is I = 1.43 A.

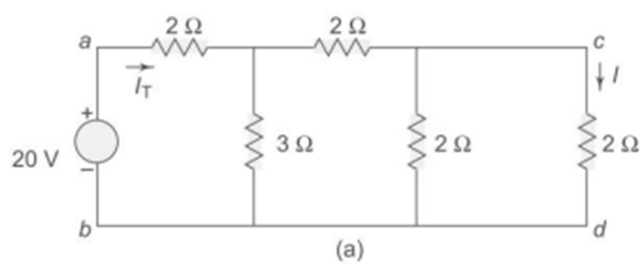


Figure 17 (a)

Applying the reciprocity theorem, by interchanging the source and response, we get Fig. 17 (b).

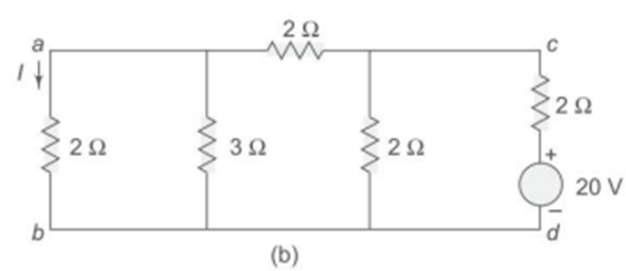


Figure 17 (b)

Total resistance in the circuit = 3.23 V.
Total current drawn by the circuit= = 20/3.23 = 6.19 A
The current in the branch ab is I = 1.43 A
If we compare the results in both cases, the ratio of input to response is the same, i.e. (20/1.43) = 13.99

2.5 MAXIMUM POWER TRANSFER THEOREM

Many circuits basically consist of sources, supplying voltage, current, or power to the load; for example, a radio speaker system, or a microphone supplying the input signals to voltage pre-amplifiers. Sometimes it is necessary to transfer maximum voltage, current or power from the source to the load. In the simple resistive circuit shown in Fig. 18, Rs is the source resistance. Our aim is to find the necessary conditions so that the power delivered by the source to the load is maximum

It is a fact that more voltage is delivered to the load when the load resistance is high as compared to the resistance of the source. On the other hand, maximum current is transferred to the load when the load resistance is small compared to the source resistance.

For many applications, an important consideration is the maximum power transfer to the load; for example, maximum power transfer is desirable from the output amplifier to the speaker of an audio sound system. The maximum power transfer theorem states that maximum power is delivered from a source to a load when the load resistance is equal to the source resistance. In Fig. 18, assume that the load resistance is variable.

Current in the circuit is I = VS /(RS + RL)

Power delivered to the load RL is $P = I^2R_L = V^2\ S\ R_L / (RS + RL)^2$

To determine the value of RL for maximum power to be transferred to the load, we have to set the first derivative of the above equation with respect to RL, i.e. when dP/dR_L equals zero.

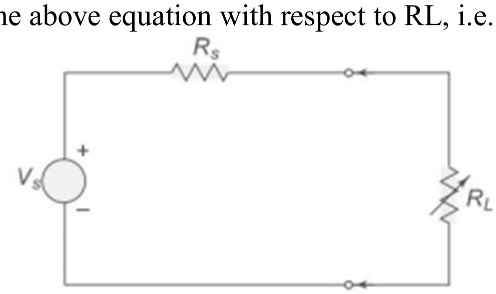


Figure 18

$$\frac{dP}{dR_L} = \frac{d}{dR_L}\left[\frac{V_S^2}{(R_S+R_L)^2}R_L\right]$$

$$= \frac{V_S^2\left\{(R_S+R_L)^2-(2R_L)(R_S+R_L)\right\}}{(R_S+R_L)^4}$$

$$\therefore \quad (R_S + R_L)^2 - 2R_L(R_S + R_L) = 0$$

$$R^2_S + R^2_L + 2R_S R_L - 2R^2_L - 2R_S R_L = 0$$

$$\therefore \quad R_S = R_L$$

So, maximum power will be transferred to the load when load resistance is equal to the source resistance

Example In the circuit shown in Fig. 19, determine the value of load resistance when the load resistance draws maximum power. Also find the value of the maximum power

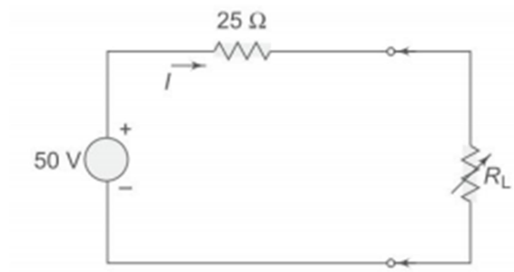
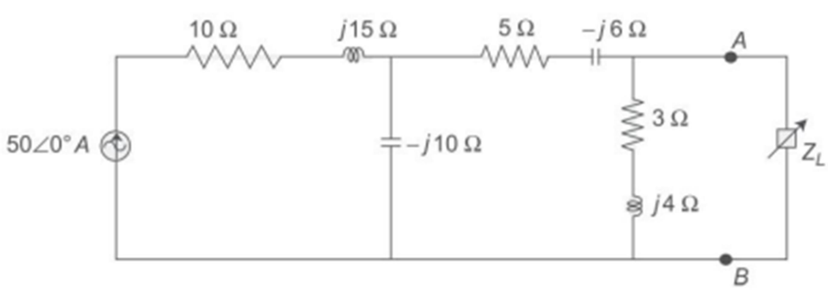


Figure 19

Solution In Fig. 19, the source delivers the maximum power when load resistance is equal to the source resistance.
RL = 25 V
The current I = 50/(25 + RL) = 50/50 = 1 A
The maximum power delivered to the load P = I^2R_L= 1 x 25= 25 W

Example 2: Determine the maximum power delivered to the load in the circuit shown in Fig below



Solution The circuit is replaced by Thèvenin's equivalent circuit in series with ZL as shown in Fig. below

where $V_{AB} = I_{(3+j4)\Omega} \times (3 + j4)$ volts

$$I_{(3+j4)\Omega} = \frac{50\angle 0^\circ \times (-j10)}{5 - j6 + 3 + j4 - j10} = 34.67\angle -33.7^\circ \text{ A}$$

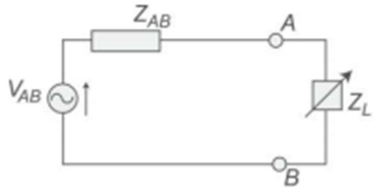
Voltage across AB is $V_{AB} = 34.67\angle{-33.7^\circ} \times 5\angle{53.13^\circ}$

$$= 173.35\angle{19.43^\circ}\ \text{V}$$

Impedance across terminals *AB* is

$$Z_{AB} = \{[(10 + j15) \parallel (-j10)] + (5 - j6)\} \parallel (3 + j4$$

$$Z_{AB} = 5.27\angle{41.16^\circ} = 4 + j3.5$$



To get the maximum power delivered to the load impedance, the load impedance must be equal to complex conjugate of source impedance. Therefore, the total impedance in the circuit shown in Fig. 7.44 is 8 Ω. The current in the circuit is

$$I_2 = \frac{V_{AB}}{8} = \frac{173.35}{8} = 21.66\ \text{A}$$

The maximum power transferred to the load is

$$P = I_L^2 R_L = (21.66)^2 \times 4 = 1874.9\ \text{watts}$$

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DEPARTMENT OF BIO MEDICAL ENGINEERING

UNIT – IV

Basic Electrical Engineering – SEEA1203

UNIT - I

MAGNETIC CIRCUITS

Introduction

In an electric circuit, electromotive force(emf) drives the current through the circuit.Similarly in a magnetic circuit,a magetomotive force(mmf) drives the flux through the circuit. The flow of current depends on the resistance while the flow of flux depends on the characteristics of the medium through which it is flowing. The flux can travel across an airgap also. Thus magnetic circuits may consist of an air gap along with the magnetic materials with which they are made up of.

- **Magnet**

A **magnet** is a material or object that produces a magnetic field. This magnetic field is invisible but is responsible for the most notable property of a magnet: a force that pulls on other ferromagnetic materials, such as iron, and attracts or repels other magnets.

- **Permanent Magnet**

A piece of magnetic material that retains its magnetism after it is removed from a magnetic field. Example- steel

- **Electromagnet**

An electromagnet is made from a coil of wire that acts as a magnet when an electric current passes through it but stops being a magnet when the current stops. Often, the coil is wrapped around a core of "soft" ferromagnetic material such as steel, which greatly enhances the magnetic field produced by the coil.

- **Magnet and its properties**

  - Like poles repel each other and unlike poles attract each other.
  - When a magnet is rolled into iron piece, maximum iron pieces accumulate at the two ends of the magnet while very few accumulate at the centre of magnet.

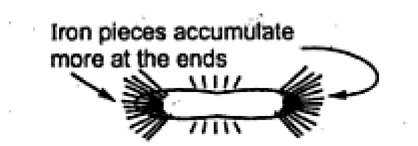


Fig. 3.1 Natural magnet

The points at which the iron pieces accumulate maximum are called poles of the magnet while imaginary line joining these poles is called axis of the magnet.

- When a magnet is placed near an iron piece, its property of attraction gets transferred to iron piece. Such property is called magnetic induction.

- **Magnetic Induction**

The phenomenon due to which a magnet can induce magnetism in a piece of magnetic material placed near it without actual physical contact is called magnetic induction.

- **Laws of Magnetism**
  - Like magnetic poles repel and unlike poles attract each other.
  - The force F exerted by one pole on other pole is

$$F \alpha M_1 M_2 / d^2$$
$$F = k\, M_1 M_2 / d^2$$

Where M_1 and M_2 are magnetic pole strengths , d is the distance between the poles and k is the nature of the surroundings.

- **Magnetic Field**

A field of force surrounding a permanent magnet or a moving charged particle, in which another permanent magnet or moving charge experiences a force

- **Magnetic Lines of Force**

The magnetic field of magnet is represented by imaginary lines around it which are called magnetic lines of force.

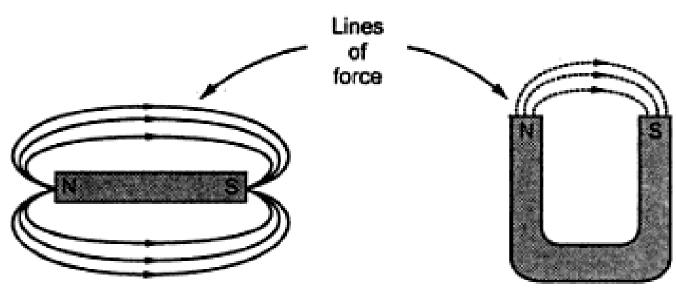


Fig. 3.2 Magnetic lines of force

- **Magnetic Flux(ϕ)**

The total number of lines existing in a particular magnetic field is called magnetic flux. Unit-Weber(Wb). Symbol for flux is ϕ.

1 weber=1×10^8 lines of force.

- **Pole Strength**

Every pole has a capacity to radiate or accept certain number of magnetic lines of force which is called pole strength.

- **Magnetic Flux Density(B)**

Magnetic flux Density is defined as the total number of **magnetic** lines of force passing through a specified area in a **magnetic** field. Symbol- B

B= Flux/ Area = ϕ/A (unit – Wb/m^2 or Tesla)

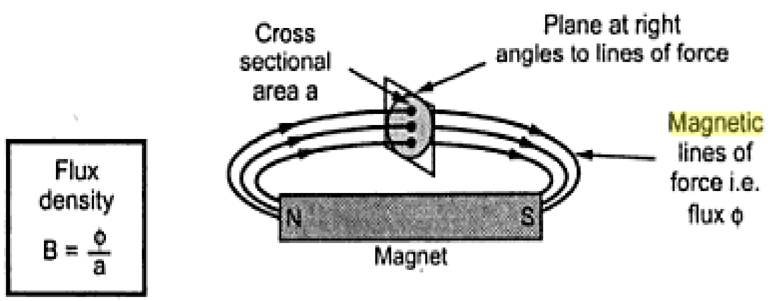


Fig. 3.3 Concept of Magnetic Flux Density

- **Magneto Motive Force(mmf)**

MMF is the cause for producing flux in a magnetic circuit. The amount of flux set up in the core depends upon current(I) and number of turns(N). The product of NI is called magneto-motive force and it determines the amount of flux set up in the magnetic circuit.

MMF= NI Unit- Ampere Turns(AT)

- **Reluctance**

The opposition that magnetic circuit offers to flux is called reluctance. It is defined as the ratio of magneto motive force to the flux.

$$S = MMF / Flux$$

S= NI / ϕ. Unit : AT/Wb

- **Permeance**

It is the reciprocal of reluctance.

$$Permeance = \frac{1}{Reluctance} = \frac{1}{S} \qquad \text{Unit : Wb/AT}$$

- **Magnetic Field Strength(H)**

This gives quantitative measure of strongness or weakness of the magnetic field. Field strength is defined as "the force experienced by a unit N-pole when placed at any point in a magnetic field ".It is denoted by H and its unit is newtons per weber(N/Wb) or ampere turns per meter(AT/m).It is defined as ampere turns per unit length.

$$H = \frac{Ampere\ turns}{lengt\ h} = \frac{N}{l} \quad \text{(unit – AT/m)}$$

Where N- no. of turns of the coil, I- current through the coil, l-length of the coil.

- **Permeability**

Permeability is the ability with which the magnetic material forces the magnetic flux through a given medium.Permeability of a material means its conductivity for magnetic flux. Greater the permeability of a material, greater the conductivity for magnetic flux.

Flux density (B) is proportional to the magnetizing force (H).

$B \alpha H$

$B = \mu H$

$\mu = B/H$

- **Relative Permeability**

Relative Permeability of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by same magnetizing force.

$$\mu_r = \frac{B}{B0} = \frac{\mu H}{\mu 0 H} = \frac{\mu}{\mu 0}$$

Therefore $\mu = \mu_0 \mu_r$

μ- absolute permeability

μ_0- absolute permeability of air or vaccum=$4\Pi x10^{-7}$ H/m

μ_r -relative permeability of the material

Problem 3.1 A bar of iron 1cm^2 in cross section has 10^{-4} Wb of magnetic flux in it. Find the flux density in the bar. If the relative permeability of iron is 2000, what is the magnetic field intensity in the bar?

Solution:

Area of iron bar(A)= 1cm2=$1x10^{-4}m^2$

Flux $\phi=10^{-4}$ Wb

$\mu_r=2000$

B= $Flux/\ Area\ =\ \phi/A$ $=10^{-4}$ / $1x10^{-4}$ = 1 Wb/m^2.

H=B/μ = B/$\mu_0\ \mu_r$ = 1/ ($4\Pi x10^{-7}$ x 2000) = 397.88 AT/m

Problem 3.2 A solenoid is wound with a coil of 200 turns. The coil is carrying a current of 1.5A. Find the value of magnetic field intensity when the length of the coil is 80cm.

Solution:

N=200

I=1.5A

l=80cm=$80x10^{-2}$m=0.8m

$$H = \frac{NI}{l} = 200 * \frac{1.5}{0.8}$$

H=375AT/m.

Problem 3.3 A current of 2A passes through a coil of 350 turns wound on an iron ring of mean diameter 12cm. The flux density established in the ring is 1.4Wb/m^2. Find the value of relative permeability of iron.

Solution:

I=2A

N=350

D=12cm=$12x10^{-2}$m=0.12m

B=1.4Wb/m^2

Length of the ring, l=2Πr = ΠD=Π*0.12

$$B=\mu H$$

$$B = \frac{\mu r \mu o N I}{l}$$

$$\mu r = \frac{Bl}{\mu oNI} = \frac{1.4 * \Pi * 0.12}{\mu o * \Pi * 350 * 2} = 600$$

Problem 3.4 A mild steel ring of mean circumference 50 cm and cross- sectional area of 5 cm^2 has a coil of 250 turns wound uniformly around it. Calculate.

(i) Reluctance
(ii) Current required to produce a flux of 700 µwb in the ring. Take μ_r of mild steel as380.

Area of steel ring A = $5cm^2 = 5\times10^{-4}m^2$ No of turns of the coil, N = 250

Length of the magnetic path=Circumference of the ring=*l* = 50 cm

$= 50\times10^{-2}m$

Flux, Φ = 700µwb

$= 700\times10^{-6}$ wb

Relative permeability of mild steel, $\mu_r = 380$, $\mu_o = 4\pi\times10^{-7}$H/m

$$S = \frac{l}{\mu o \mu r A}$$

$$= \frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 380 \times 5 \times 10^{-4}}$$

=2094144AT/Wb

ELECTROMAGNETISM

The EMF may be produced either by batteries through chemical reaction or by thermocouples by heating the junction of two dissimilar metals. Michael faraday 1831 discovered that the EMF can also be produced by electromagnetic induction, used in commercial generation of power.

- **Electromagnetic Induction**

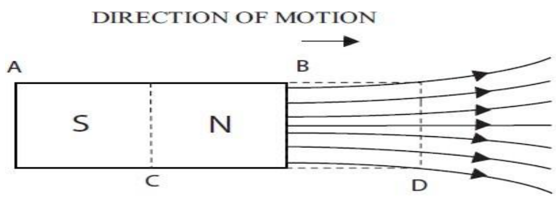
Whenever the magnetic flux linking with the coil changes, an EMF is induced in the coil. This phenomenon is called as electromagnetic induction.

Applications: microphones, telephones, transformers, generators motors etc.,

- **Production of induced EMF and current**

The change in flux linkage can be obtained by three methods

Method 1:



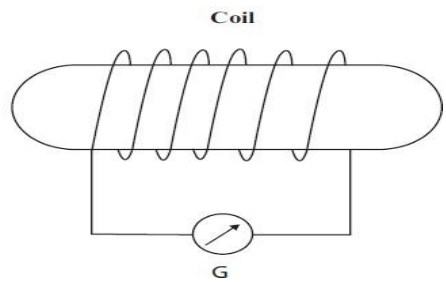


Fig.3.4

When a magnet is moved towards the coil there is deflection in the galvanometer connected across the coil thus indicating the flow of current. This current is due to the induced EMF in the coil.

- Change in flux results in production of EMF.
- Presence of EMF gives rise to flow of current.

If the magnet movement is stopped the pointer will show zero deflection. If the magnet moves with higher speed or if the number of turns of coil is increased or If we use the stronger magnet we can observe greater deflection of the pointer. As the magnet taken away from the coil, the flux linked with the coil is decreased the deflection of galvanometer is in the opposite direction.

Method2:

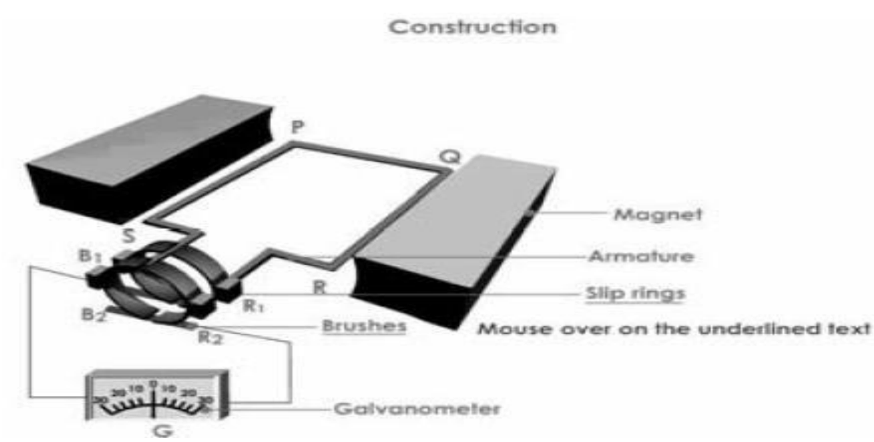


Fig.3.5

The EMF can also be induced in the coil by moving the coil and keeping the magnet stationary. These two methods of producing the emf are called the dynamic methods and the induced emf is called dynamically induced emf which is employed in generators (in ac and dc generators there is motion of conductors which results in the change in flux linkage).

Method 3:

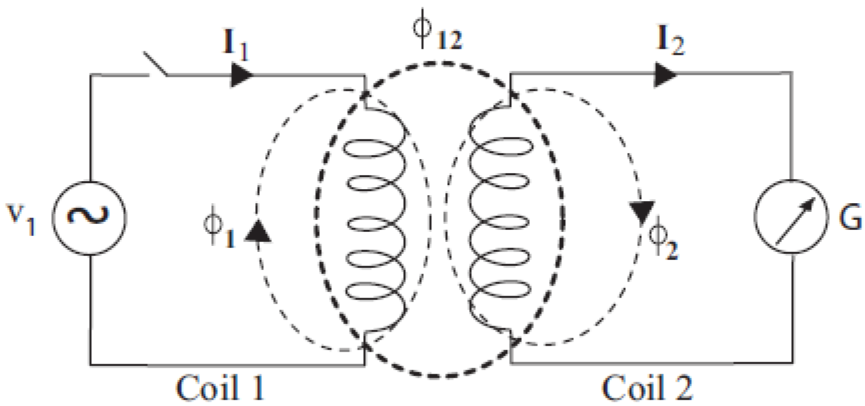


Fig.3.6

Both conductor and magnet are kept stationary and change in flux is obtained by changing the current this method of emf production is called statically induced emf which is employed in transformers.

- **Faradays law of electromagnetic induction**

First law: Whenever the conductor cut across the magnetic field,an emf is induced in the conductor or whenever the magnetic flux linking with any coil (circuit) changes an emf is induced in the coil

Second law: The magnitude of the induced emf is equal to the rate of change of flux linkage.

Suppose a coil has N turns, Let the flux through it change from Φ_1Weber to Φ_2 Weber in't' seconds product of N and Φ is called flux linkage.

The initial flux linkage = $N\,\Phi_1$

The final flux linkage = $N\,\Phi_2$

Change of flux linkage = $N\Phi_2 - N\,\Phi_1$

= $N(\Phi_2 - \Phi_1)$

Rate of change of flux linkage = $N(\Phi_2 - \Phi_1)/t$

Let e be the induced emf

According to faradays second law e=N$\frac{d\Phi}{dt}$ volts

Actually the direction of emf induced is so as to oppose the very cause producing it

$$e = - N\frac{d\Phi}{dt} \text{ volts}$$

Note:

i) To produce induced EMF in a conductor there must be a rotating magnetic field cutting the stationary conductor .EMF is also induced when a moving conductor is cut by a stationary magnetic flux.

ii) When there is no relative motion between the magnetic flux and the conductor no emf is induced in it.

iii) The direction of the induced emf in the coil depends upon the direction of the magnetic field and that of motion of the coil.

iv) If the conductor is moved parallel to the direction of flux,it does not cut the conductor. Hence no emf is induced in it.

- **Direction of induced emf:**

To determine the direction of induced EMF and current in a conductor the following rules are used

- **Fleming's right hand rule**

Statement: Stretch out the forefinger, middle finger and the thumb of the right hand such that they are mutually perpendicular to one another. If the forefinger points in the direction of magnetic field and the middle finger points in the direction of current then the thumb points in the direction of the motion of the conductor

Note: The direction of dynamically induced emf can be determined by Fleming's right hand rule This rule is in D.C. generators.

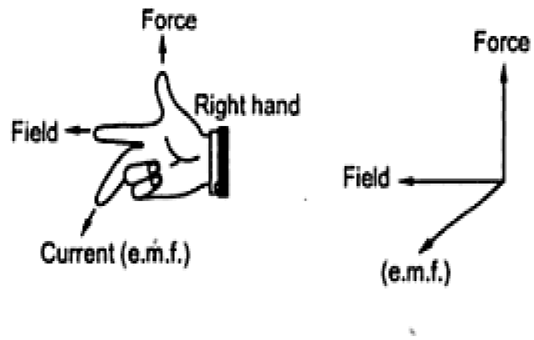


Fig. 3.7 Fleming's Right Hand Rule

- **Fleming's left hand rule**

Statement: Stretch out the forefinger middle finger and the thumb of the left hand mutually perpendicular to one another. If the forefinger points to the direction of the field and the middle finger points in the direction of the current, then, the thumb indicates the direction of the mechanical force exerted by the conductor.

Note: This rule is used in D.C. Motors.

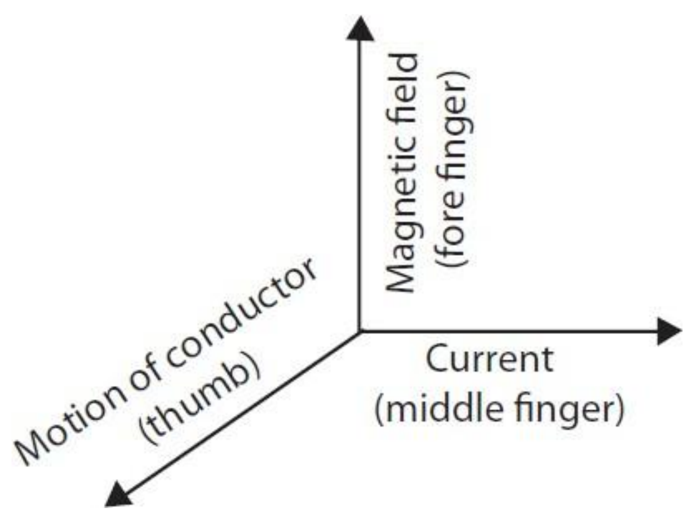


Fig. 3.8

Statement: In effect, electromagnetically induced emf and hence the current flows in a coil in such a direction that the magnetic field set up by it opposes the very cause producing it.

Note: The direction of statically induced emf lenz's law is used.

TYPES OF INDUCED EMF

When the flux linked with the coil or conductor changes an emf is induced in the coil. There are two ways to obtain the change in flux linkage. They are

Dynamically induced emf

When a conductor is moved in a stationary magnetic field or when the magnetic field is moved by keeping the conductor stationary an emf is induced provided the movement is done in such a way that the conductor is moved across the magnetic field. The emf thus induced is called as dynamically induced emf. An example of dynamically induced emf is the emf generated in D.C. and A.C. generators.

Consider the stationary magnetic field of flux density B wb/m^2 the direction of magnetic field is shown in the figure below and the conductor with circular cross section is placed let the length of the conductor in field 'l 'in meters .conductor is allowed to move at right angles to magnetic field, in a time of 'dt ' seconds the conductor is moved to a distance of 'dx' meters.

The area swept by the conductor = m^2

Magnetic flux cut by the conductor = flux density*area swept

$$= B\, l\, dx \text{ Weber}$$

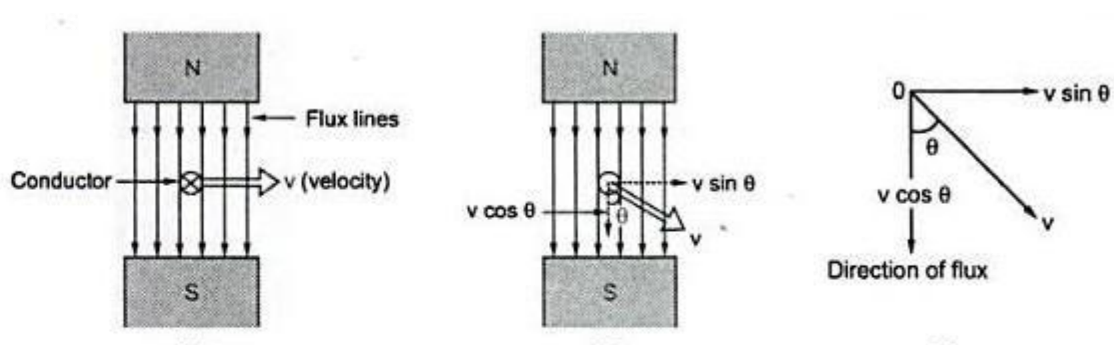


Fig.3.9. Conductor with Magnetic field

By faradays law of electromagnetic induction, the emf induced in the conductor is

$$e = N\frac{d\Phi}{dt}$$

If the number of turn in a conductor is one (N= 1)

$$e = \frac{d\Phi}{dt}$$

w.k.t

$$d\,\Phi = B\, l\, dx$$

$$e = B\, l\, dx/dt \qquad \text{since } dx/dt = v \text{(linear velocity)}$$

$$e = B\, l\, v \quad \text{volts}$$

If the conductor moves at an angle θ to the magnetic field then the velocity at which the conductor moves across the field is vsin θ ,therefore

$$e = B\,l\,v\sin\theta \text{ volts}$$

The direction of the induced emf is determined by the Flemings right hand rule.

Statically induced emf

The flux is linked with the coil(conductor) without moving either the coil or field system but by changing the current in the field system. The emf induced in this way without motion of either conductor or flux is called statically induced emf.An example of statically induced emf is the emf induced in transformer winding.

It is further classified as i)self induced emf ii)mutually induced emf

- ***Self induced emf***

The emf induced in a coil due to change in the value of its own flux linking it is called self induced emf. Consider a coil shown in figure

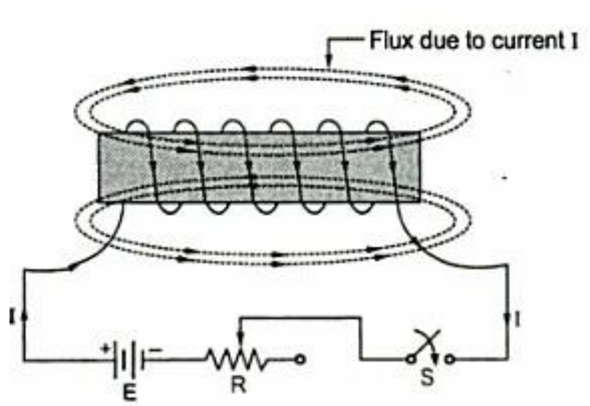


Fig. 3.10 Self Induced emf

If the current in the coil changes the flux linked with the coil also changes, which results in the production of emf, this is called self induced emf .The magnitude of this self induced emf

$$e = N\frac{d\Phi}{dt} \text{ volts}$$

The direction of the induced emf would be such as to oppose the very cause of production. Hence it is known as counter emf of self induction

- ***Self inductance (L)***

The property of a coil that opposes any change in the amount of current flowing through it is called its self inductance. It depends on the

i) Shape of the coil and number of turns

ii) Relative permeability of the magnetic material

iii) Speed in which the magnetic field changes.

Equation for self inductance

let

N= Number of turns in the coil

I=current in the coil.

If the current flowing through the coil changes the flux also changes which results in self induced emf

$$e=N\frac{d\Phi}{dt}$$
$$=\frac{d(N\Phi)}{dt}$$

Since the flux depends on the current so

$$N\Phi \quad \alpha \quad I$$

$$e \quad \alpha \quad \frac{dI}{dt}$$

$$e = L\frac{dI}{dt}$$

Where L=self inductance of the coil in henry

- ***Other expression for self inductance***

Method 1

From the above equation $e= L\frac{dI}{dt}$

$$=\frac{d(LI)}{dt} \text{ ....(1)}$$

$$e = N\frac{d\Phi}{dt}$$
$$=\frac{d(N\Phi)}{dt} \text{ ....(2)}$$

Since equation (1) and (2)

$$\frac{d(LI)}{dt} = \frac{d(N\Phi)}{dt}$$

$$LI = N\Phi$$

$$L = \frac{N\Phi}{I} \text{ ....(3)}$$

NΦ is also called as flux linkage.when N is in turns, Φ in Weber I in amperes, then L is in henrys.

Method 2

We know that magnetic field intensity $H = \frac{NI}{l}$.................. (1)

Flux density $B = \mu_0\mu_r H$......... (2)

$= \mu_0\mu_r \frac{NI}{l}$

We also know that $\Phi = B*a$

$= \mu_0\mu_r \frac{NI}{l} *a$

We know that flux linkage = N Φ

$= N^2 I\, \mu_0\mu_r a/l$

We know that $L = \frac{N\Phi}{I}$ henry

$= N^2 I\, \mu_0\mu_r a/l$

$= N^2/l/\mu o \mu r a$

$L = N^2/S$

- ***Mutually induced emf***

The emf induced in a circuit due to the charging current in the neighbouring circuit is called mutually induced emf.

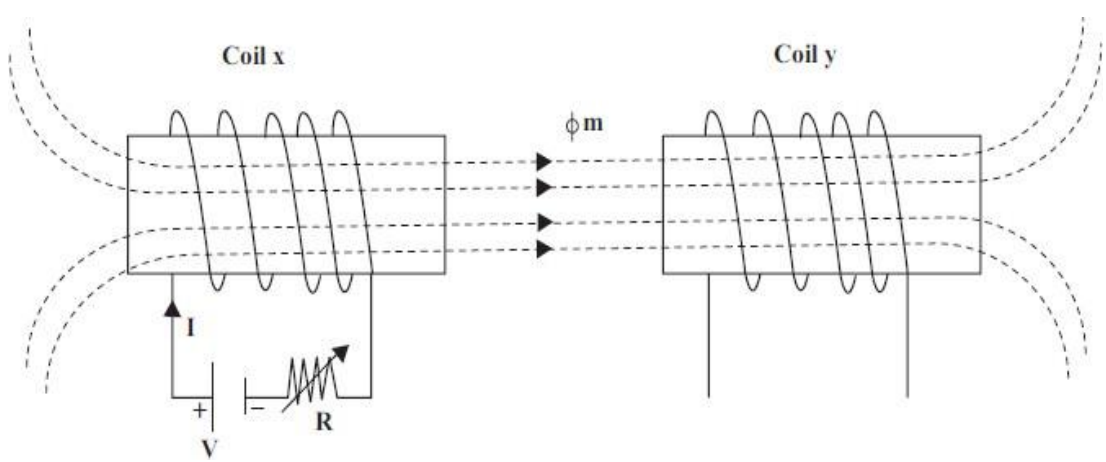


Fig. 3.11*Mutually induced emf.*

Here, coil x and coil y are close to each other, current flows through coil x due to this flux is produced in coil x part of the flux links the coil y which is called as mutual flux Φ_m. The flux common to both coil x and coil y is called mutual flux.

Note: If the current in coil x varies, emf in both the coil varies.

(i) The emf in coil x is called as self induced emf.
(ii) The emf in coil Y is called as mutually induced emf.

- ***Mutual inductance(M)***

Consider two coils X and Y placed close to each other, I_1 flows through coil X, a flux is set up and a part Φ12 of this flux links coil Y. This flux which is common to both the coils is called mutual flux (Φm). If current in coil X changes, the mutual flux alsochanges and hence emf is induced in coil Y. The emf induced in coil Y is called mutually induced emf.

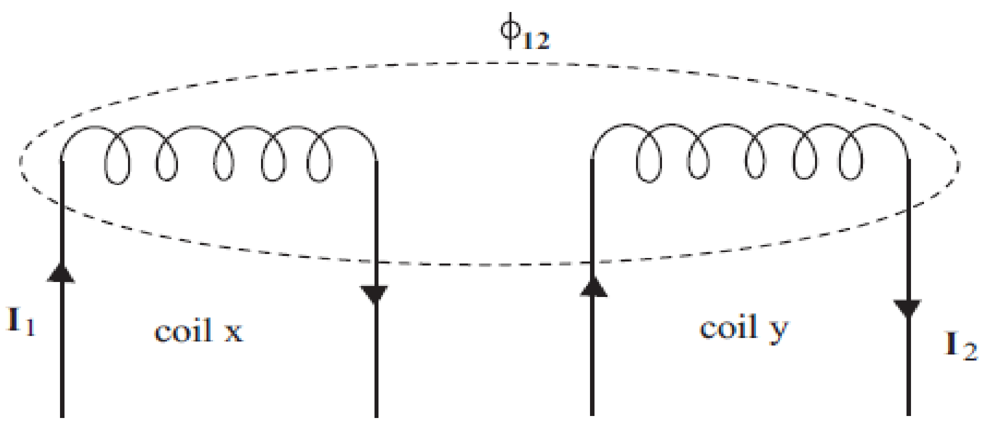


Fig.3.12

Expression for M:

Mutually induced emf in coil Y is directly propositional to the rate of changeof current in coil X.

$$e_m \, a \, \frac{dI_1}{dt}$$

$$e_m = M \frac{dI_1}{dt}$$

M = Mutual inductance between the coils

Method 1:

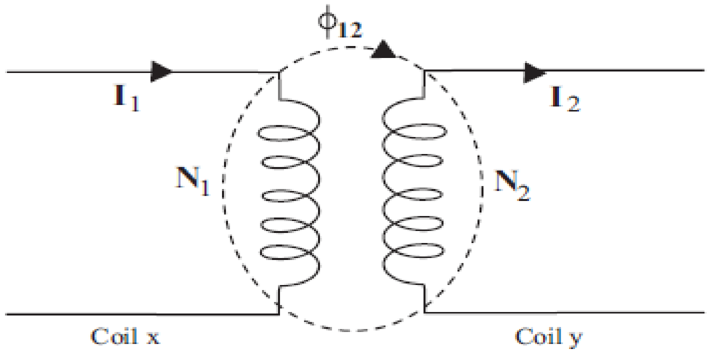


Fig.3.13

$$e_m = M\,\frac{dI_1}{dt}$$

$$= \frac{d}{dt}(MI_1)$$

Coefficicent Coupling

Co-efficient of coupling is defined as the fraction of magnetic flux produced by the current in one coil that links the other coil. If L1 and L2 are self inductances of two coils & M be the mutual inductance and K is the Co-efficient of coupling.

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Note:

When there in no mutual flux between two coils then K = 0, M = 0.

Proof:

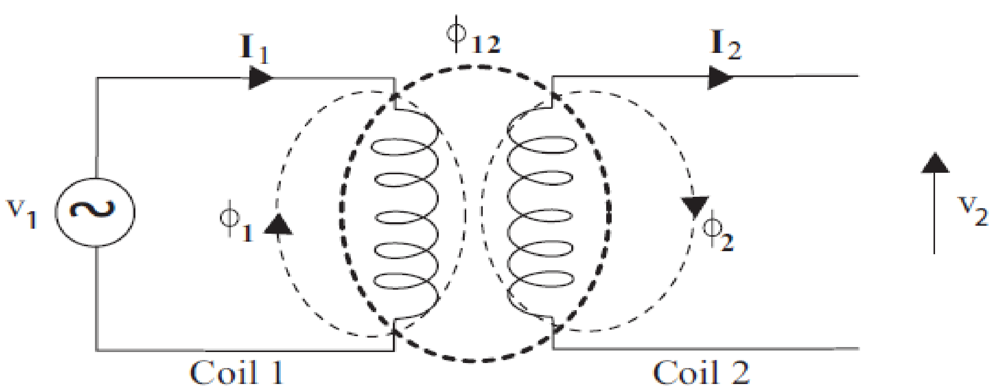


Fig.3.14

The ratio of the flux linked in the second coil to the total flux in the first coil due to current in the first coil is called co-efficient of coupling.
Let
Coil 1 and coil 2 be coupled magnetice and coil 1 is energized by a voltage of V1 Volts

- I1 produces a flux of Φ1 in coil 1
- Φ11 is the part of the flux linked only with coil1
- Φ12 is the part of the flux linking both the coil 1 and coil 2
- According to above definition

$$K = \frac{\phi_{12}}{\phi_1}$$

$$\phi_{12} = K\phi_1 \,.............(1)$$

Due to reciprocal action i.e., when source connected to coil 2.

$$K = \frac{\phi_{21}}{\phi 2}$$

$$\phi_{21} = K\phi_2 \,...........(2)$$

We know that

$$M = \frac{N_2\phi_{12}}{I_1}[coil\, 1 \text{ with source voltage}] \,.........(3)$$

$$M = \frac{N_1\phi_{21}}{I_2}[coil\, 2 \text{ with source voltage}] \,.........(3)$$

$$M^2 = M \times M$$

$$= \frac{N_2\phi_{12}}{I_1}\frac{N_1\phi_{21}}{I_2} \,..............(5)$$

$$= \frac{N_2N_1\phi_{12}\phi_{21}}{I_1I_2} \qquad \because \phi_{12} = K\phi_1$$

$$\phi_{21} = K\phi_2$$

$$= \frac{N_2\,K\,\phi_1}{I_1} \times \frac{N_1\,K\,\phi_2}{I_2}$$

$$=K^2\left[\frac{N_2\ \phi_2}{I_2}\right]\left[\frac{N_1\ \phi_1}{I_1}\right]\text{...........}(6)$$

$$M^2=K^2\ \frac{N_2\ \phi_2}{I_2}\frac{N_1\ \phi_1}{I_1}$$

$$L_1=\frac{N_1\ \phi_1}{I_1},\ \ L_2=\frac{N_2\ \phi_2}{I_2}$$

$$M^2=K^2\left[L_1\ L_2\right]$$

$$K^2=\frac{M^2}{L_1L_2}\text{............}(7)$$

$$K^2=\frac{M}{L_1}\frac{M}{L_2}\text{............}(8)\ (\text{or})$$

$$\mathrm{K}=\frac{M}{\sqrt{L_1L_2}}\text{............}(9)$$

Analogy of magnetic and electric circuits

S.No	Magnetic Circuits	Electric Circuits
1	The closed path for magnetic flux is called magnetic circuit	The closed path for electric current is called electric circuit
2	Magnetic flux Φ in webers	Electric current 'I' in amperes
3	Magneto motive force 'NI' in ampere turns	Electromotive force in volts
4	Magnetic flux $\Phi = \frac{\text{mmf}}{\text{reluctance}}$	Electric current $\mathrm{I} = \frac{\text{emf}}{\text{resistance}}$
5	Reluctance S $= \frac{l}{\mu_o \mu_r A}$ in AT/wb	Resistance R $= \frac{\rho l}{A}$ in ohms
6	Permeance $= \frac{1}{\text{reluctance}}$	Conductance $= \frac{1}{\text{resistance}}$
7	Reluctivity	Resistivity

8	Permeability	Conductivity
9	Flux density B = $\frac{\Phi}{A}$ in wb/m^2	Current density J $= \frac{I}{A}$ in A/m^2
10	Magnetic intensity H $= \frac{NI}{l}$ in AT/m	Electric intensity E = $\frac{V}{d}$ in volts/metre
11	Magnetic flux does not actually flow in a magnetic circuit.	The electric current actually flows in an electric circuit.
12	The reluctance of a magnetic circuit is not constant and it depends up on flux density in the material.	The resistance of an electric circuit is practically constant, even though it varies slightly with temperature.
13	In a magnetic circuit, energy is required to create the flux and not to maintain it.	In an electric circuit, energy is required so long as the current has to flow through it.
14	For magnetic flux, there is no perfect insulator.	There are many electrical insulators like glass, air, rubber etc.

Problem 3.5 Calculate the emf induced in a coil of 200 turns, when the flux linking with it changes from 1 milliweber to 3 milliweber in 0.1 sec

Given data:

i) No of turns (N) = 200
ii) Initial value of flux (Φ1) = 1 mwb
= 1 × 10-3 wb
iii) Final value of flux (Φ2) = 3 mwb
= 3 × 10-3 wb

Solution:

$$e = -N\frac{d\Phi}{dt}$$

change in flux $d\Phi = \Phi_2 - \Phi_1 = 3 \times 10^{-3} - 1\times10^{-3}$
= 2 × 10-3 wb

$$e = -200 \times \frac{2\times10^{-3}}{0.1} = 4 \text{ volts (in magnitude)}$$

Problem 3.6 A coil of 50 turns is linked by a flux of 20 mWb. If this flux is reversed in a time of 2 ms, Calculate the average emf induced in the coil.

Given:

i) No of turns (N) = 50
ii) Flux linked in coil (Φ) = 20 mwb
= 20X10-3 wb.
iii) Time required for flux reversal (t) = 2 ms

Solution
Emf induced in the coil (e)??

$$e = N\frac{d\Phi}{dt}$$

change in flux dΦ= 20-(-20) = 40 mWb

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{50 \times 40 \times 10^{-3}}{2 \times 10^{-3}}$$

$$= 1000V$$

emf induced e= 1000V

Problem 3.7 A coil of 100 turns of wire is wound on a magnetic circuit of reluctance 2000 AT/Wb. If a current of 1 A flowing in the coil is reversed in 10 ms, find the average emf induced in the coil.

Given:

i) No. of turns (N) = 100
ii) Reluctance (S) = 2000 ATwb
iii) Current in coil (I) = 1A
iv) Time required to reversal current t= 10 ms

Solution:

$$\text{Emf induced in coil } e = N\frac{d\Phi}{dt}$$

$$Flux = MMF\ /\ S$$
$$= NI\ /\ S$$
$$= \frac{100 \times 1}{2000}$$
$$= 0.05 \text{ Wb}$$

When a current of 1A is reversed in coil change in flux,

$$(d\varphi) = 0.05 - (0.05) = 0.1 mWb$$

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{100 \times 0.1 \times 10^{-3}}{10 \times 10^{-3}} = 1V$$

emf = 1V

Problem 3.8 A coil of 2000 turns surrounds a flux of 5mWb produced by a permanent magnet. The magnet is suddenly drawn away causing the flux inside the coil to drop to 2mWb in 0.1 sec. What is the average emf induced?

Given:

i) No of turns is coil (N) = 2000 turns

ii) Initial value of flux (Φ1) = 5 mwb
$= 5\times10^{-3}$WB
iii) Final value of flux $\Phi = 2mWb = 2 \times10^{-3}$ WB
iv) Time required for change in flux (dt) = 0.1 sec
v) change is flux $d\Phi = 5 - 2 = 3$ mwb.

Solution:

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{2000\times3\times10^{-3}}{0.1}$$

$$=60V$$

emf induced = 60 V.

Problem 3.9 A current of 5 Amperes when flowing through a coil of 1000 turns establishes a flux of 0.3 mwb. Determine the inductance of the coil.

Given:

I=5A
N=1000 turns
Φ=0.3mWb
$=0.3\text{x}10^{-3}$Wb

Solution:

$$L = \frac{N\emptyset}{I}$$

$$= \frac{1000 \times 0.3 \times 10^{-3}}{5}$$

$$=0.06H$$

Problem 3.10 A coil of 1500 turns carries a current of 10 A establishes a flux of 0.5 mWb. Find the inductance of the coil.

Given:

I =50 A
N =1500 turns
$\Phi = 5\times10^{-3}$ WB

Solution:

$$L = \frac{N\emptyset}{I}$$

$$= \frac{1500 \times5\times10^{-3}}{50}$$

$$=0.075H$$

Problem 3.11 A coil has self inductance of 10H. If a current of 200 mA is reduced to zero in a time of 1 ms. Find the average value of induced emf across the terminals of the coil.

Given:

L = 10 H

dI = 200 mA = 200×10^{-3} A

dt = 1 ms = 1×10^{-3} sec

Solution:

$$e = L\frac{dI}{dt}$$

$$= \frac{10 \times 200 \times 10^{-3}}{1 \times 10^{-3}}$$

$$=2000V$$

Problem 3.12 A air cored solenoid has 400 turns, its length is 30 cm and cross sectional area of 5 cm^2 . Calculate self inductance.

Given:

N = 400

l = 30 cm = 30×10^{-2} m

a = 5 cm2 = 5×10^{-4} m

Solution:

$$L = \frac{N^2}{S} = \frac{400^2}{S}$$

$$S = \frac{l}{\mu o \mu r A}$$

$$= \frac{30\times10^{-2}}{4\pi\times10^{-7}\ \times1\times5\times10^{-4}}$$

$$= 4.77 \times 10^8 AT/m$$

$$L = \frac{400^2}{4.77\times10^8}$$

$$= 3.354 \times 10^{-4} H$$

Problem 3.13 The number of turns in a coil is 250. When a current of 2 A flows in this coil, the flux in the coil is 0.3 mwb. When this current is reduce to zero in 2 ms. The voltage induced in a coil lying in the vicinity of coil in 63.75 volts. If the co-efficient of coupling between the coil is 0.75, find self inductances of the two coils, mutual inductance and number of turns in the second coil.

Given data:

N_1 = 250 turns

I_1 = 2 A

Φ_1 = 0.3 mwb =0.3×10^{-3} WB

di_1 = 2 A

dt = 2 ms = 2×10^{-3} s

e_m = 63.75v k = 0.75

Solution:

a) $L_1 = \frac{N_1 \emptyset_1}{I_1} = \frac{250 \times 0.3 \times 10^{-3}}{2}$

=0.0375H

b) $M = K\sqrt{L_1 L_2}$

$$L_2 = \frac{M^2}{K^2 L_1}$$

$$e_m = M \frac{di_1}{dt} = 63.75\text{V}$$

$$M = \frac{63.75}{2/2 \times 10^{-3}}$$

$= 63.75mH$

Substituting these values of M, L_1 and K

$$L_2 = \frac{(63.75 \times 10^{-3})^2}{0.75^2 \times 37.5 \times 10^{-3}}$$

=0.193H

=193mH

c) $em_2 = N_2 \frac{d\emptyset_2}{dt}$

$$\emptyset_2 = K\emptyset_1$$

$$= N_2 \frac{d(k\emptyset_1)}{dt}$$

$$= N_2 \frac{k(d\emptyset_1)}{dt}$$

$$em_2 = N_2 \times 0.75 \times \frac{0.3 \times 10^{-3}}{2 \times 10^{-3}}$$

$$63.75 = 0.1125N_2$$

$$N_2 = \frac{63.75}{0.1125}$$

$$N_2 = 567\ turns$$

Series and Parallel Magnetic circuits

2.2.1 Simple Magnetic Circuit

Consider simple magnetic circuit shown in Fig (2.4). This circuit consists of an iron core with cross sectional area of 'a' m^2 and mean length of 'l' m.

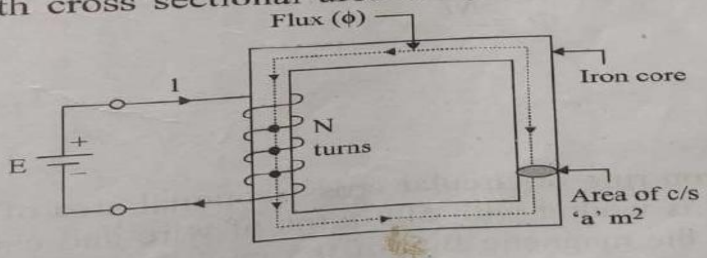


Figure 2.4 Magnetic Circuit

This is mean length of the magnetic path which flux is going to trace). A coil of N turns is wound on one of the sides of the square core which is excited by

a supply. This supply drives a current I through the coil. This Current produces the flux (□) which completes its path through the core.

Let us derive relationship between m.m.f., flux and reluctance.

I = current flowing through the coil.
N = Number of turns.
Φ = flux in webers
B = flux density in the core.

μ- absolute permeability of the material.
μ_0- absolute permeability of air or vaccum = $4\pi \times 10^{-7}$ H/m
μ_r -relative permeability of the material

Magnetic field strength inside the solenoid is given by

$$H = \frac{NI}{l}$$

$$B = \mu H$$

$$B = \mu_0 \mu_r \frac{NI}{l}$$

$$\text{Flux } \Phi = BA$$

$$\Phi = \mu_0 \mu_r \frac{NIA}{l}$$

$$\Phi = \frac{NI}{\left(\frac{l}{\mu_0 \mu_r A}\right)}$$

$$\Phi = \frac{mmf}{\text{reluctance}}$$

$$\Phi = \frac{NI}{s}$$

where $S = \dfrac{l}{\mu_0 \mu_r A}$

Problem 2.5 An iron ring of circular cross sectional area of 3 cm^2 and mean diameter of 20 cm is wound with 500 turns of wire and carries a current of 2.09 A to produce the magnetic flux of 0.5 mwb in the ring. Determine the permeability of the material.

Solution:

$A = 3\ cm^2 = 3*10^{-4}m^2 = 0.0003\ m$

Mean diameter, $d = 20\ cm = 20*10^{-2}m = 0.2\ m$

$N = 500$

$I = 2.09\ A$

$\Phi = 0.5\ mwb = 0.5*10^{-3}\ wb = 0.0005\ wb$

Mean length $l = \pi d = \pi*20*10^{-2} = 0.6283m$

$B = \mu H$

$B = \mu_o \mu_r H$

$$B = \frac{\Phi}{A} = \frac{0.0005}{0.0003} = 1.6667\, wb/m^2.$$

$$H = \frac{NI}{l}$$

$$H = \frac{500*2.09}{0.6283} = 1663.218 AT/m$$

$$B = \mu_o \mu_r H$$

$$\mu_r = \frac{B}{\mu_o H} = \frac{1.6667}{\mu_o * 1663.218} = 797.845$$

2.2.2 Composite series magnetic circuit

It consists of different parts of materials (iron or other magnetic materials) with different dimensions and relative permeability. Each part has its own reluctance. For this circuit, flux is same in all the parts. Hence the circuit can be termed as 'series magnetic circuit'. Although the flux is the same in all sections, the flux density in each section may vary, depending on its effective cross-sectional area. The total reluctance is given by the sum of reluctances of individual parts.

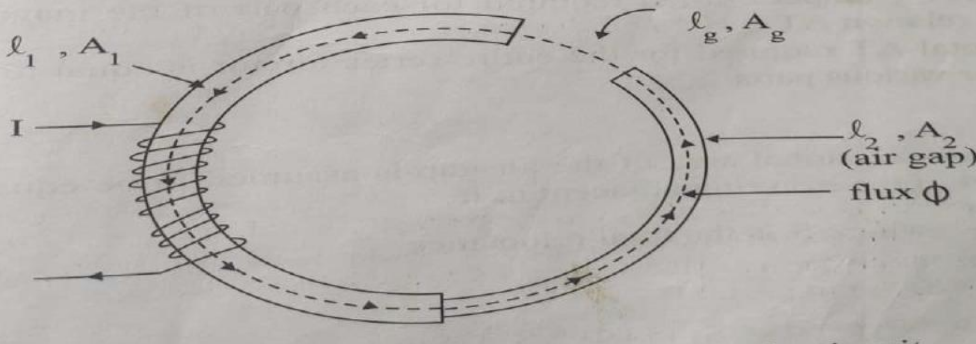


Figure 2.5 Composite series magnetic circuit

Consider a circular ring made up of different materials of lengths l_1, l_2 with air gap of length l_g and with cross sectional areas A_1, A_2 and A_g with absolute permeabilities of μ_1, μ_2 and μ_0 (since $\mu = \mu_0 \mu_r$ and for air, $\mu_r = 1$)

The total reluctance, $S_t = S_1 + S_2 + S_g$.

$$S_t = \frac{l_1}{(\mu_0 \mu_{r1} A_1)} + \frac{l_2}{(\mu_0 \mu_{r2} A_2)} + \frac{l_g}{(\mu_0 A_g)}$$

Total mmf $= \Phi * S$

$$= \Phi * \left\{ \frac{l_1}{(\mu_0 \mu_{r1} A_1)} + \frac{l_2}{(\mu_0 \mu_{r2} A_2)} + \frac{l_g}{(\mu_0 A_g)} \right\}$$

$$= \frac{\Phi}{(\mu_0 \mu_{r1} A_1)} l_1 + \frac{\Phi}{(\mu_0 \mu_{r2} A_2)} l_2 + \frac{\Phi}{(\mu_0 A_g)} l_g$$

$$= \frac{B_1}{(\mu_0 \mu_{r1})} l_1 + \frac{B_2}{(\mu_0 \mu_{r2})} l_2 + \frac{B_g}{(\mu_0)} l_g \quad (\text{Since } \frac{\Phi}{A} = B)$$

$$= H_1 l_1 + H_2 l_2 + H_g l_3 \ (\text{Since } \frac{B}{\mu_0 \mu_r} = H)$$

Steps for finding the total mmf :

- Find H for each part of the series magnetic circuit. For air, $\mu_r = 1$ $H = \frac{B}{\mu_0}$ whereas for a magnetic circuit, $H = \frac{B}{\mu_0 \mu_r}$
- Find the mean length l of magnetic path for each part of the circuit
- Find AT (ampere turns) required for each part of the magnetic circuit using relation AT = H * l
- The total AT required for the entire series circuit is equal to the sum of AT for various parts

Note:

- The cross-sectional area of the air gap is assumed to be equal to area of the part of the iron ring adjacent to it.
- $\Phi = \frac{NI}{S_t}$ where S_t is the total reluctance

  $NI = \Phi * S_t$

  $NI = (\Phi * S_1) + (\Phi * S_2) + (\Phi * S_g)$ (Since Φ is same in all the parts of the series circuit)
- Total mmf $= MMF_{iron} + MMF_{airgap}$

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Total Reluctance $= S_{iron} + S_{airgap}$

2.2.3 Parallel Magnetic circuit

If a magnetic circuit has two or more paths for the magnetic flux, it is called as parallel magnetic circuit.

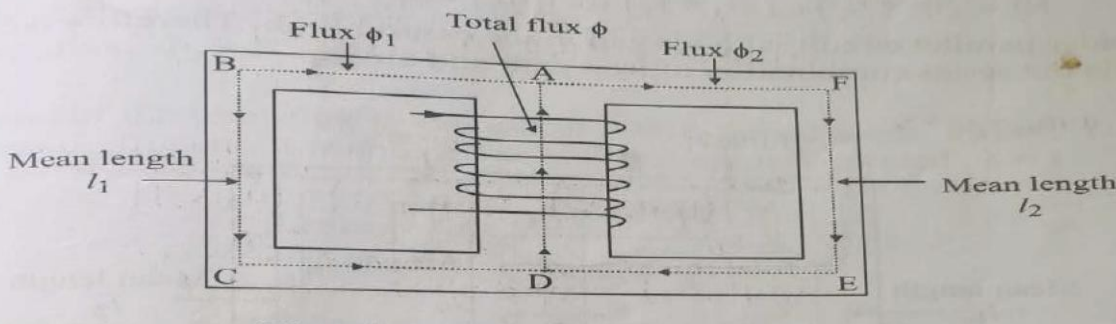


Figure 2.6 Parallel magnetic circuit

On the central limb AD, a current carrying coil is wound. The MMF in the coil sets up a magnetic flux Φ in the central limb. At point A, the total flux divides into two parts

i) Path ABCD with mean length l_1 which carries flux Φ_1

ii) Path AFED with mean length l_2 which carries flux Φ_2

$$\Phi = \Phi_1 + \Phi_2$$

Mean length of path AD = l_c

Reluctance of path ABCD = $S_1 = \frac{l_1}{(\mu_0 \mu_{r1} A_1)}$

Reluctance of path AFED = $S_2 = \frac{l_2}{(\mu_0 \mu_{r2} A_2)}$

Reluctance of path AD = $S_c = \frac{l_c}{(\mu_0 \mu_{rc} A_c)}$

For path ABCDA, MMF = $(\Phi_1 * S_1) + (\Phi * S_c)$

For path AFEDA, MMF = $(\Phi_2 * S_2) + (\Phi * S_c)$

For parallel circuit,

Total MMF = MMF required by central limb + MMF required by any one of

$$= (MMF)_{AD} + (MMF)_{ABCD} \text{ (or) } (MMF)_{AFED}$$

$$NI = (NI)_{AD} + (NI)_{ABCD} \text{ (or) } (NI)_{AFED}$$

$$NI = (\Phi * S_c) + (\Phi_1 * S_1) \text{ (or)} (\Phi_2 * S_2)$$

Consider parallel circuit with air gap in the central limb. Therefore the cent limb is the series combination of iron path and air gap

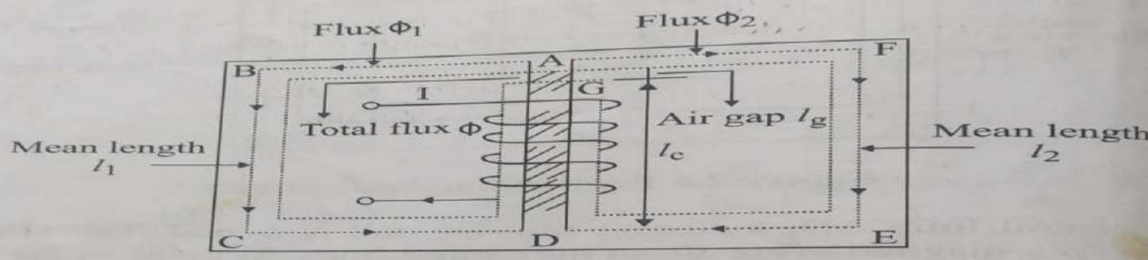


Figure 2.7 Parallel circuit with air gap

Path G-D = Iron path of length l_c

Path G-A = Air gap of length l_g

Reluctance of central limb is Sc = Si + Sg

$$= \frac{l_c}{(\mu_0 \mu_{rc} A_c)} + \frac{l_g}{(\mu_0 A_c)} \quad (\text{since } A_c = A_g \text{ and } \mu_r = 1)$$

Hence MMF of central limb, $(MMF)_{AD} = (MMF)_{GD} + (MMF)_{GA}$

Total MMF, $NI_{Total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD}$ (or) $(NI)_{AFED}$

- **Leakage Flux**

It is defined as the flux that does not follow the desired path in a magnetic circuit i.e. the amount of flux lines that do not follow the core and are lost to the surrounding air. In many practical magnetic circuits, air gaps are provided. Hence a large part of flux path is through magnetic material and the remaining part of flux path is through air. The flux through air gap is known as useful flux. The total flux produced by the coil does not pass through the air gap as some of it leaks through the air surrounding the iron. Thus, the total magnetic flux produced is equal to the sum of the useful magnetic flux and the leakage flux.

Φ_t = Total flux produced in the iron ring
Φ_g = useful flux across the air gap
Φ_l = leakage flux
$\Phi_t = \Phi_l + \Phi_g$
Leakage flux, $\Phi_l = \Phi_t - \Phi_g$

The ratio of the total flux to the useful flux is called as the leakage factor or leakage co-efficient. It is denoted as λ. If leakage is neglected , $\lambda = 1$

$$\text{Leakage flux } \lambda = \frac{\text{total flux produced}}{\text{useful flux}}$$

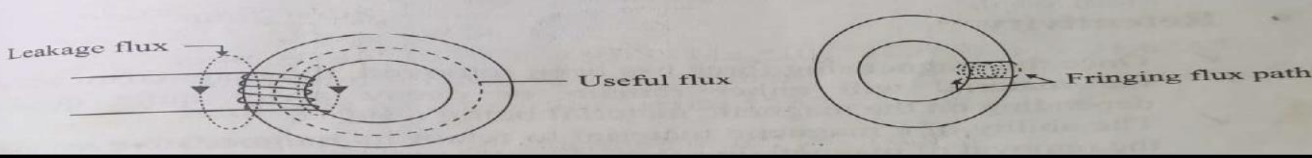


Figure 2.8 Leakage Flux

The value of λ is always greater than unity. Typical values of Leakage factor are from about 1.12 to 1.25. Leakage is a characteristic of all magnetic circuits and can never be completely eliminated.

- **Fringing**

When the flux lines cross the airgap, they tend to bulge out across the edges of the air gap. This effect is called fringing. The effect of fringing makes the effective gap area larger than that of the ring. Since the area is made larger, the flux density in the air gap is reduced $\left(B = \frac{\Phi}{A}\right)$.

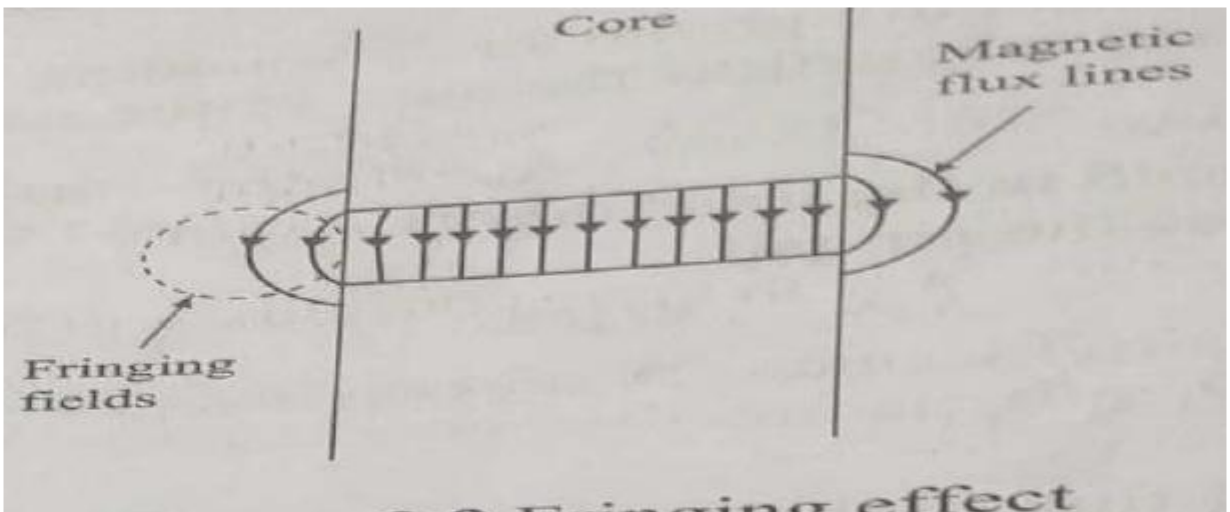


Figure 2.9 Fringing effect

The air gap length is kept as small as possible so that the reduction in the fl density is minimum. If the air gap length is small compared with the g width, the effect of fringing can be neglected.

- **Retentivity**
  - ✓ Once the magnetising force has been removed, the magnetism within the material will either remain or decay away quiet quickly depending on the magnetic material being used.
  - ✓ The ability of a magnetic material to retain its magnetism even after the removal of magnetizing force is called as retentivity.
  - ✓ The magnetism remaining in the magnetic material, even when the magneticfield is reduced to zero is called residual magnetism.
  - ✓ The materials which are required to retain their magnetism will have a high retentivity and are used to make permanent magnets, while those materials required to lose their magnetism quickly such as soft iron cores are used for relays and solenoids will have a very low retentivity.

Problem 2.6 A magnetic circuit comprises three parts in series as follows:

a. A length of 60 mm with cross section area of 50 mm^2
b. A length of 30 mm with a cross section area of 80 mm^2
c. An air gap of length 0.3 mm and cross section area of 150 mm^2.

A coil of 2500 turns is wound on part (b) and the flux density in air gap is 0.3T. Assuming that there is no leakage and the relative permeability $\mu_r = 1500$, estimate the current required in the circuit to produce the flux density.

l_a=60 mm =60*10-3 m
l_b=30 mm=30*10-3 m
l_c=0.3 mm=$0.3*10^{-3}$ m

$A_a=50mm^2 = 50*10^{-6}\ m^2$
$A_b=80mm^2=80*10^{-6}\ m^2$
$A_c=150mm^2=150*10^{-6}\ m^2$

Since this magnetic circuit is in series the flux will be the same.

$$\Phi = B_cA_c = 0.3*150*10^{-6} = 45*10^{-6}\ wb$$

$$mmf_a = \Phi S_a = \frac{\Phi l_a}{\mu_0\mu_r A_a}$$

$$mmf_a = (45*10^{-6} *60*10^{-3}) / (4\pi*10^{-7}*1500*50*10^{-6}) = 28.6AT.$$

$$mmf_b = \Phi S_b = \frac{\Phi l_b}{\mu_0\mu_r A_b}$$

$$mmf_b = (45*10^{-6} *30*10^{-3}) / (4\pi *10^{-7}*1500*80*10^{-6}) = 8.95AT.$$

$$mmf_c = \Phi S_c = \frac{\Phi l_c}{\mu_0\mu_r A_c}$$

$$mmf_c = (45*10^{-6} *0.3*10^{-3}) / (4\pi *10^{-7}*150*10^{-6}) = 71.62AT.$$

Total mmf $= mmf_a + mmf_b + mmf_c$

mmf $= 28.6+8.95+71.62 = 109.17$AT

But mmf = NI

$$I = \frac{mmf}{N} = \frac{109.17}{2500} = 0.04367A$$

Problem 2.7 A flux density of 1.2 wb/m^2 is required in the 1mm of airgap of an electromagnet having an iron path of 1.5 m long. Calculate the MMF required. Given relative permeability of iron = 1600. Neglect leakage.

Solution:

Flux density, B = 1.2 wb/m^2

Length of the magnetic path, l = 1.5 m

Relative permeability of iron, μ_r = 1600

Absolute permeability of air, $\mu_o = 4\pi\times10^{-7}$ H/m

$$MMF = NI = H \times l$$

$$= \frac{Bl}{\mu_o\mu_r} \qquad [\because B=\mu H=\mu_o\mu_r H]$$

$$MMF\ in\ iron\ path = \frac{1.2\times1.5}{4\pi\times10^{-7}\times1600}$$

$$=895.24\ AT$$

Length of air gap l_g = 1 mm = 1×10^{-3}m

For air, $\mu_r = 1$

$\mu_o = 4\pi\times10^{-7}$H/m

flux density, B = 1.2 wb/m^2

$$mmf = \Phi S$$

$$= \Phi\frac{l_g}{\mu_o\mu_r A}$$

$$= BA\frac{l_g}{\mu_o\mu_r A}$$

$$mmf\ in\ air\ gap = \frac{Bl_g}{\mu_o\mu_r}$$

$$= \frac{1.2\times1\times10^{-3}}{4\pi\times10^{-7}\times1}$$

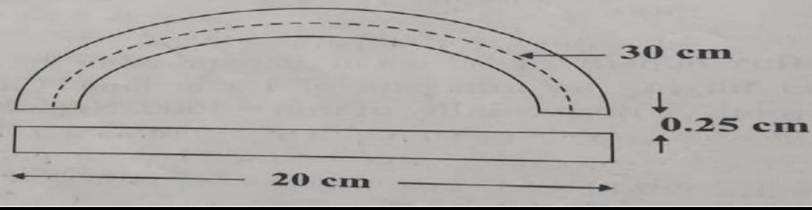
$$=954.93\ AT$$

Total mmf = mmf in iron path + mmf in airgap

= 895.24 + 954.93

= 1850.17 AT

Problem 2.8 An electromagnet is made of iron of square cross section with cm side. A flux of 1.2 mwb is required in the air gap. Find the number of ampere turns required. Take μ_r = 2000. Neglect leakage and fringing.



Solution:

Area of cross section of iron, A = 5 cm × 5 cm

= 25 cm^2

= 25 × 10^{-4} m^2

Flux, Φ = 1.2 mwb = 1.2×10^{-3} wb

Length of air gap l_g = 0.25 cm = 0 25 x 10^{-2} m

For air gap,
$\mu_r = 1$

$$mmf = \Phi S = \Phi \frac{l_g}{\mu_o \mu_r A}$$

$$= \frac{1.2\times10^{-3} \times 0.25\times10^{-2}}{4\pi\times10^{-7} \times 1\times 25\times10^{-4}}$$

$$= 954.92 AT$$

For two air gaps = 2 x 954.92
= 1909.85AT

For iron path,
$\mu_r = 2000$

Length of iron l_i = 30 cm + 20 cm
= 50 cm
= 50×10^{-2} m

$$\text{mmf for iron path} = \frac{\Phi l_i}{\mu_o \mu_r A}$$

$$= \frac{1.2\times10^{-3} \times 50\times10^{-2}}{4\pi\times10^{-7} \times 2000\times 25\times10^{-4}}$$

$$= 95.49 AT$$

Total mmf = mmf of air + mmf of iron path
= 1909.85+95.49
= 2005.34 AT

Problem 2.9 Find the ampere-turns required in produce a flux of 0.4 mwb in the airgap of a circular magnetic circuit which has air gap of 0.5 mm. The iron ring has 4 square cm cross section and 63 cm mean length. The relative permeability of iron is 1800 and the leakage co-efficient is 1.15.

Given data:

Flux, Φ = 0.4 m wb
= 0.4×10^{-3} wb

Air gap length, l_g = 0.5 mm
= 0.5×10^{-3} m

Area of an iron ring, a = 4 cm^2
= 4×10^{-4} m^2

Length of an iron ring, l = 63 cm
= 63×10^{-2} m

Relative permeability, μ_r = 1800

Leakage co-efficient, λ = 1.15

To find:

MMF required to produce a flux of 0.4 mWb
Total MMF = MMF_{iron} + $MMF_{air\ gap}$.
MMF = flux × reluctance

Solution:

(a) $mmf_{(iron)}$ = flux × reluctance

(b) $mmf_{air\ gap}$ = flux × reluctance

$= \Phi_g \times S_g$

$$=0.4\times10^{-3}\times\frac{l_g}{\mu_0\mu_r a} \quad [\because \mu_r=1$$

$$\mu_0=4\pi\times10^{-7}\ H/m]$$

$$=0.4\times10^{-3}\times\frac{0.5\times10^{-3}}{4\pi\times10^{-7}\times4\times10^{-4}}$$

$=397.88\ AT$

$\phi_i=\lambda\times\phi_{useful}$

$=1.15\times0.4\times10^{-3}$

$=0.46\times10^{-3}\ Wb$

$$Si=\frac{l_i}{\mu_0\mu_r a}=\frac{l-l_g}{\mu_0\mu_r a} \quad [\because \mu_0=4\pi\times10^{-7}\ H/m]$$

$$=\frac{0.63-0.5\times10^{-3}}{4\pi\times10^{-7}\times1800\times4\times10^{-4}}$$

$=695,750.25\ AT/Wb$

$mmf_{iron}=\phi_i\times S_i$

$=0.46\times10^{-3}\times695,750.25$

$=320.04\ AT$

Total mmf = mmf_{iron} + $mmf_{air\ gap}$

= 320.04 + 397.88

= 717.925 AT

Result:

The total mmf required is 717.925 AT

Problem 2.10 An iron ring of mean length 50 cm has an airgap of 1mm and a winding of 200 turns. If the permeability of the iron is 400. When a current of 1.25 ampere flows through the coil, find the flux density.

Given data:

Mean length, l = 50 cm = 50 × 10^{-2} m

= 0.5 m

Length of air gap, lg = 1 mm = 1× 10^{-3} m

No. of turns, N = 200

Relative permeability, μ_r = 400

Current through the coil, I = 1.25 A.

Note: It is assumed that there is no leakage of flux.

Let Φ = Flux in webers

To find: Flux density of an iron ring.

Solution:

(i) Total mmf = NI

= 200 × 1.25A

= 250 AT

(ii) Total reluctance = $S_g + S_i$

(a) $S_g = \dfrac{l_g}{\mu_0 A}$ $[\because \mu_r = 1,\ \mu_0 = 4\pi\times10^{-7}\ H/m]$

$$= \frac{0.5}{4\pi\times10^{-7}\times400\times A}$$

$$= \frac{994.7}{A}\ AT/Wb$$

$\dfrac{0.5\times10^{3}}{16\pi\times10^{-9}}$ $\dfrac{1}{6\pi}\times10$

(b) $S_i = \dfrac{l_i}{\mu_0\mu_r A}$

$$= \frac{0.5}{4\pi\times10^{-7}\times400\times A}$$

$$= \frac{994.7}{A}\ AT/Wb$$

(c) $Total\ reluctance = \dfrac{796}{A} + \dfrac{994.7}{A}$

$$= \frac{1790.7}{A}$$

(d) Total mmf = Flux × Total reluctance

$$250 = \phi\times\frac{1790.7}{A}$$

$$\frac{\phi}{A} = \frac{250}{1790.7}$$

$$\frac{\phi}{A} = 0.1396\ Wb/m^2$$

Result:

$$B = \frac{\phi}{A} = 0.1396\, Wb/m^2\ (or)\ Telsa$$

Problem 2.11 A magnetic ring has a mean circumference of 150 cm and is of 0.01 m² in cross section and it is wound with 200 turns. A saw cut of 2 mm is made in the ring. Calculate the magnetising current required to produce a flux of 800 μWb in the air gap. Assume relative permeability of iron as 600 and a leakage factor of 1.2.

Given data:

Cross sectional area, a = $0.01 m^2$

No. of turns, N = 200

Length of iron ring, l_i = 150 cm

= 150×10^{-2} m

Length of air gap, l_g = 2mm = 2×10^{-3}m

Flux in air gap, Φ_g = 800 μwb

= 800×10^{-6} wb

Relative permeability, μ_r = 600

Leakage factor, λ = 1.2

To find:

I = ?

Solution:

(a) For airgap:

AT for airgap = $H_g \times l_g$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$B = \frac{\phi}{A}$$

$$\therefore B = \frac{800 \times 10^{-6}}{0.01} = 0.08\, Wb/m^2$$

$$\therefore H = \frac{0.08}{4\pi \times 10^{-7} \times 1} = 63661.9\, AT/m$$

Now,

AT = 63661.9 × 2 × 10^{-3}
= 127.32 AT

(b) AT For iron path

We know that $\lambda = \frac{\phi_T}{\phi_g}$ $\phi_r = \phi_i$

$$\phi = \lambda \phi_g$$

$$= 1.2 \times 800 \times 10^{-6}$$

$$= 9.6 \times 10^{-4}\, Wb$$

AT For iron path = H l

$$= \frac{B}{\mu_0 \mu_r} l$$

$$= \frac{\phi_i l_i}{\mu_0 \mu_r A}$$

$$= \frac{9.6 \times 10^{-4} \times 150 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 0.01}$$

$$= 191\, AT$$

Total AT = $AT_{airgap} + AT_{iron}$
= 127.32 + 191
= 318.32 AT

(c) Now

$$\text{Magnetizing current} = \frac{TotalAT}{N}$$

$$= \frac{318.38}{200}$$

$$= 1.59\, A$$

Problem 2.12 A steel ring of circular cross-section 1 cm in radius has an airgap of 1mm length. It is wound uniformly with 500 turns of wire carrying a current of 3A. Neglect magnetic leakage. The air gap takes 60% of the total MMF. Find the total reluctance.

Solution:

Radius of the cross-section, r = 1 cm = 1×10^{-2} m

Length of air gap, l_g = 1mm = 1×10^{-3} m

No. of turns, N = 500

Current through the coil, I = 3 A

MMF taken by airgap = 60% of total MMF

Total Reluctance, $S_T = \dfrac{Total\ MMF}{Flux}$

Total MMF = NI

$= 500 \times 3 = 1500$ AT

MMF taken by air gap = 60% of 1500 AT

$$= \frac{60}{100} \times 1500$$

$$= 900 \text{ AT}$$

MMF in Airgap = $H_g \, l_g$

$$= \frac{B}{\mu_0} l_g \qquad [\because \mu_r = 1 (for\ air)]$$

$$= 900 \text{ AT}$$

$$B = \frac{900 \times \mu_0}{l_g}$$

$$= \frac{900 \times 4\pi \times 10^{-7}}{1 \times 10^{-3}}$$

$$B = 1.131\, Wb/m^2$$

Flux, Φ = BA

$= B(\pi r^2)$ [$A = \pi r^2$ for a circular ring]

$= 1.131 [\pi \times (1 \times 10^{-2})^2]$

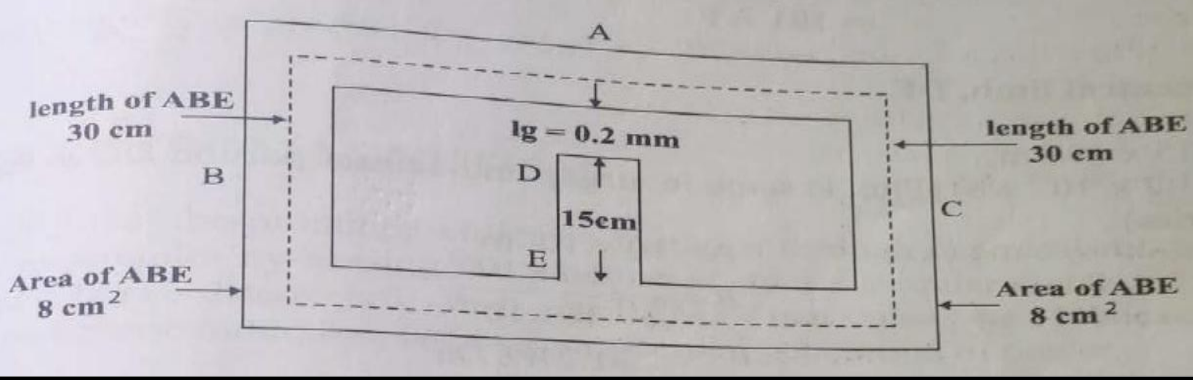
$\Phi = 3.553 \times 10^{-4}$ wb

Total Reluctance, $S_T = \dfrac{TotalMMF}{Flux(\phi)}$

$$= \frac{1500}{3.553\times10^{-4}}$$

$$S_T = 4.222 \times 10^6 \text{ AT/wb}$$

Problem 2.13 A magnetic core has the following dimensions



The length of portion DE = 15 cm
Area of cross-section = 10 cm^2
The length of portion ABE = 30 cm
Area of cross-section = 8 cm^2
The length of portion ACE = 30 cm
Area of cross-section = 8 cm^2
Length of air gap, l_g = 0.2 mm
Air gap flux = 1.2 mwb

The central limb is wound with 500 turns. Calculate the total ampere turns and the current required in the coil to produce the above flux.
The data for B-H curve for the core is

B (wb/m^2)	0.45	0.75	1.0	1.2
H (AT/m)	90	140	230	450

Solution: For air gap, $\mu_r = 1$, $\mu_o = 4\pi \times 10^{-7}$ H/m.
$l_g = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m

Cross sectional area of the air gap is assumed to be equal to area of the cross-section in DE = 10cm^2

$$\therefore A_g = 10\text{cm}^2 = 10 \times 10^{-4}\text{m}^2$$

$$\Phi_g = 1.2 \times 10^{-3} \text{ wb}$$

$$B = \frac{\phi_g}{A_g} = \frac{1.2\times10^{-3}}{10\times10^{-4}} = 1.2\,Wb/m^2$$

$$H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{1.2}{4\pi \times 10^{-7} \times 1} = 954929.66 \quad [\because \mu_0 = 1 (for\ air)]$$

AT required for air gap $= H_g\, l_g$
$= 954929.66 \times 0.2 \times 10^{-3}$
$= 191$ AT

For central limb, DE

$l = 15 \times 10^{-2}$ m
$\Phi = 1.2 \times 10^{-3}$ wb (Flux is same in airgap and central portion DE as they are in series)

$$A = 10 \times 10^{-4} m^2$$

$$B = \frac{\phi}{A} = \frac{1.2 \times 10^{-3}}{10 \times 10^{-4}}$$

$$= 1.2\, Wb/m^2$$

'H' corresponding to 1.2 wb/m^2 from the table = 450 AT/m

AT required $= H \times l$
$= 450 \times 15 \times 10^{-2}$
$= 67.5$ AT

For portions ABE & ACE,
At point 'A', flux is, divided into two equal parts

$\therefore$ Flux throughs each portion $= \frac{1.2 \times 10^{-3}}{2}$ [$\because$ Parallel circuit]
$= 0.6 \times 10^{-3}$ wb

Area of cross section, $A = 8 \times 10^{-4} m^2$

$$B = \frac{\phi}{A} = \frac{0.6 \times 10^{-3}}{8 \times 10^{-4}}$$

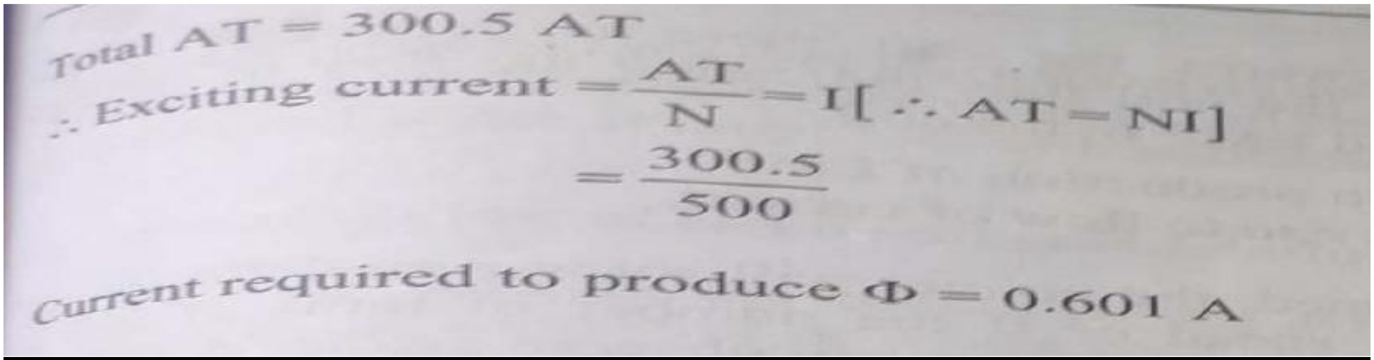
$$= 0.75\, Wb/m^2$$

'H' corresponding to 0.75 wb/m^2 from the table = 140 AT/m

$l = 30$ cm $= 30 \times 10^{-2}$ m

AT required $= H \times l = 30 \times 10^{-2} \times 140 = 42$ AT

Total AT = AT airgap + AT central limb DE + AT portion ABE (or) ACE
$= 191 + 67.5 + 42$



Total AT = 300.5 AT

$\therefore$ Exciting current $= \frac{AT}{N} = I\ [\therefore AT = NI]$

$= \frac{300.5}{500}$

Current required to produce $\Phi = 0.601$ A

PART A- Questions

1. What is electromagnet?
2. Define flux. Give its unit.
3. Define magnetic flux density. Give its unit.
4. Define magnetic field intensity. Give its unit.
5. Define mmf. Give its unit.
6. What is reluctance? Give its unit.
7. Define retentivity.
8. What is permeance. Give its unit.
9. State Ohm's law of magnetism.
10. A bar of iron $1cm^2$ in cross section has 10^{-4} wb of flux in it. Find the flux density in the bar. If the relative permeability of iron is 2000. what is the magnetic field intensity in the bar?
11. A solenoid is wound with a coil of 200 turns. The coil is carrying a current of 1.5A. Find the magnetic field intensity when the length of the coil is 80cm.
12. A current of 2A passes through a coil of 350 turns wound on an iron ring of mean diameter 12cm.The flux density established in the ring is $1.4wb/m^2$. find the value of relative permeability of iron.
13. State Faraday's law of electromagnetic induction.
14. Define Lenz's law.
15. Define self inductance and give its unit.
16. Define mutual inductance and give its unit
17. Define leakage flux and leakage factor.
18. What is fringing?
19. State Fleming's right hand rule.
20. State Fleming's left hand rule.
21. Define coefficient of coupling.

PART- B- Questions

1. State Faraday's law of electromagnetic induction. Derive the expression for dynamically induced emf .
2. Explain about statically induced emf with a neat sketch and Give the expression for self and mutual inductance.
3. An iron ring of mean length 50cm has an air gap of 1mm and a winding of 200 turns. If the permeability of the iron ring is 400 when a current of 1.25A flows through the coil, find the flux density.

4. An iron ring 8cm mean diameter is made up of round iron of diameter 1cm and permeability of 900, has an airgap of 2mm wide. It consists of winding with 400 turns carrying a current of 3.5A. Determine, i) mmf ii)total reluctance iii) flux iv)flux density.
5. Find the AT required to produce a flux of 0.4mwb in the airgap of a circular magnetic circuit which has an airgap of 0.5mm. The iron ring has $4cm^2$ cross section and 63cm mean length. The relative permeability of iron is 1800 and leakage coefficient is 1.15.
6. Compare electric and magnetic circuit.
7. A coil of 50 turns is linked by a flux of 20mwb. If this flux is reversed in a time of 2ms. Calculate the average emf induced in the coil.
8. A coil of 100 turns of wire is wound on a magnetic circuit of reluctance 2000AT/wb. If a current of 1A flowing in the coil is reversed in 10ms, find the average emf induced in the coil.
9. Two 100 turns air cored solenoids have length 20cm and cross sectional area of $3cm^2$ each. If the mutual inductance between them is 5μH. Find the coefficient of coupling.
10. Two magnetically coupled coils have self inductances L_1=100mH & L_2=400mH. If the coefficient of coupling is 0.8. Find the value of mutual inductance between the coils. What would be the max possible mutual inductance.
11. A coil of 800 turns is wound on a wooden former and a current of A produces flux of $200x10^{-6}$ wb. Calculate the inductance of a coil and induced emf when current is reversed in 0.2 sec.

i) Reluctance,

$$S = \frac{l}{\mu_o \mu_r A} \text{ AT/wb}$$

$$= \frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 380 \times 5 \times 10^{-4}}$$

$$S = 2094144 \text{ AT/wb}$$

ii) Current required to produce flux of 700 μwb in the ring = I

$$\text{Reluctance, } S = \frac{\text{MMF}}{Flux} = \frac{NI}{\Phi}$$

$$I = \frac{\Phi S}{N}$$

$$= \frac{700 \times 10^{-6} \times 2094144}{250}$$

$$I = 5.86\ A$$

2.2 MAGNETIC CIRCUITS

Magnetic circuit is defined as the closed path traced by the magnetic lines of force i.e flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f., flux, reluctance, permeability, etc.

2.2.1 Simple Magnetic Circuit

Consider simple magnetic circuit shown in Fig (2.4). This circuit consists of an iron core with cross sectional area of 'a' m^2 and mean length of 'l' m.

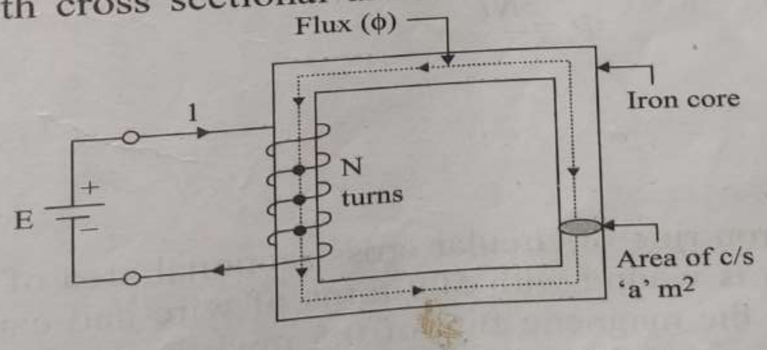


Figure 2.4 Magnetic Circuit

This is mean length of the magnetic path which flux is going to trace). A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil. This Current produces the flux (□) which completes its path through the core.

Let us derive relationship between m.m.f., flux and reluctance.

I = current flowing through the coil.
N = Number of turns.
Φ = flux in webers
B = flux density in the core.

μ- absolute permeability of the material.
μ_0- absolute permeability of air or vaccum = $4\pi \times 10^{-7}$ H/m
μ_r -relative permeability of the material

Magnetic field strength inside the solenoid is given by

$$H = \frac{NI}{l}$$

$$B = \mu H$$

$$B = \mu_0 \mu_r \frac{NI}{l}$$

$$\text{Flux } \Phi = BA$$

$$\Phi = \mu_0 \mu_r \frac{NIA}{l}$$

$$\Phi = \frac{NI}{\left(\dfrac{l}{\mu_0 \mu_r A}\right)}$$

$$\Phi = \frac{mmf}{\text{reluctance}}$$

$$\Phi = \frac{NI}{s}$$

where $S = \dfrac{l}{\mu_0 \mu_r A}$

Problem 2.5 An iron ring of circular cross sectional area of 3 cm^2 and mean diameter of 20 cm is wound with 500 turns of wire and carries a current of 2.09 A to produce the magnetic flux of 0.5 mwb in the ring. Determine the permeability of the material.

Solution:
A = 3 cm^2 = 3*10^{-4}m^2 = 0.0003 m
Mean diameter, d = 20 cm = 20*10^{-2}m = 0.2 m

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$N = 500$
$I = 2.09$ A
$\Phi = 0.5\,mwb = 0.5 * 10^{-3}\,wb = 0.0005\,wb$

Mean length $l = \pi d = \pi * 20 * 10^{-2} = 0.6283$m

$B = \mu H$
$B = \mu_o \mu_r H$

$$B = \frac{\Phi}{A} = \frac{0.0005}{0.0003} = 1.6667\,wb/m^2.$$

$$H = \frac{NI}{l}$$

$$H = \frac{500 * 2.09}{0.6283} = 1663.218\,AT/m$$

$$B = \mu_o \mu_r H$$

$$\mu_r = \frac{B}{\mu_o H} = \frac{1.6667}{\mu_o * 1663.218} = 797.845$$

2.2.2 Composite series magnetic circuit

It consists of different parts of materials (iron or other magnetic materials) with different dimensions and relative permeability. Each part has its own reluctance. For this circuit, flux is same in all the parts. Hence the circuit can be termed as 'series magnetic circuit'. Although the flux is the same in all sections, the flux density in each section may vary, depending on its effective cross-sectional area. The total reluctance is given by the sum of reluctances of individual parts.

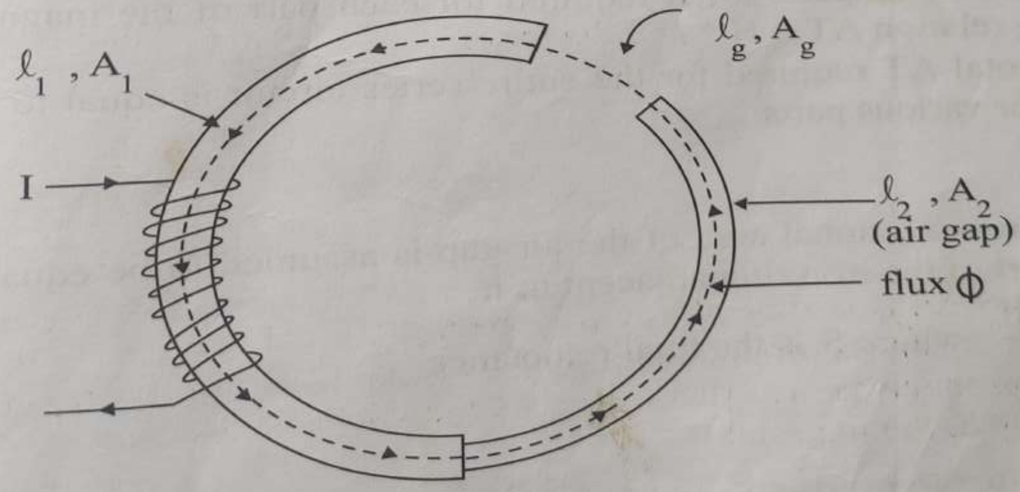

Figure 2.5 Composite series magnetic circuit

Consider a circular ring made up of different materials of lengths l_1, l_2 with air gap of length l_g and with cross sectional areas A_1, A_2 and A_g with absolute permeabilities of μ_1, μ_2 and μ_0 (since $\mu = \mu_0\,\mu_r$ and for air, $\mu_r = 1$)

The total reluctance, $S_t = S_1 + S_2 + S_g$.

$$S_t = \frac{l_1}{(\mu_0\mu_{r1}A_1)} + \frac{l_2}{(\mu_0\mu_{r2}A_2)} + \frac{l_g}{(\mu_0 A_g)}$$

Total mmf $= \Phi * S$

$$= \Phi * \left\{ \frac{l_1}{(\mu_0\mu_{r1}A_1)} + \frac{l_2}{(\mu_0\mu_{r2}A_2)} + \frac{l_g}{(\mu_0 A_g)} \right\}$$

$$= \frac{\Phi}{(\mu_0\mu_{r1}A_1)} l_1 + \frac{\Phi}{(\mu_0\mu_{r2}A_2)} l_2 + \frac{\Phi}{(\mu_0 A_g)} l_g$$

$$= \frac{B_1}{(\mu_0\mu_{r1})} l_1 + \frac{B_2}{(\mu_0\mu_{r2})} l_2 + \frac{B_g}{(\mu_0)} l_g \quad \left(\text{Since } \frac{\Phi}{A} = B\right)$$

$$= H_1 l_1 + H_2 l_2 + H_g l_3 \left(\text{Since } \frac{B}{\mu_0\mu_r} = H\right)$$

Steps for finding the total mmf :

- Find H for each part of the series magnetic circuit. For air, $\mu_r = 1$ $H = \dfrac{B}{\mu_0}$ whereas for a magnetic circuit, $H = \dfrac{B}{\mu_0\mu_r}$
- Find the mean length l of magnetic path for each part of the circuit
- Find AT (ampere turns) required for each part of the magnetic circuit using relation AT = H * l
- The total AT required for the entire series circuit is equal to the sum of AT for various parts

Note:

- The cross-sectional area of the air gap is assumed to be equal to area of the part of the iron ring adjacent to it.
- $\Phi = \dfrac{NI}{S_t}$ where S_t is the total reluctance

  $NI = \Phi * S_t$

  $NI = (\Phi * S_1) + (\Phi * S_2) + (\Phi * S_g)$ (Since Φ is same in all the parts of the series circuit)
- Total mmf = $MMF_{iron} + MMF_{airgap}$

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Total Reluctance = $S_{iron} + S_{airgap}$

2.2.3 Parallel Magnetic circuit

If a magnetic circuit has two or more paths for the magnetic flux, it is called as parallel magnetic circuit.

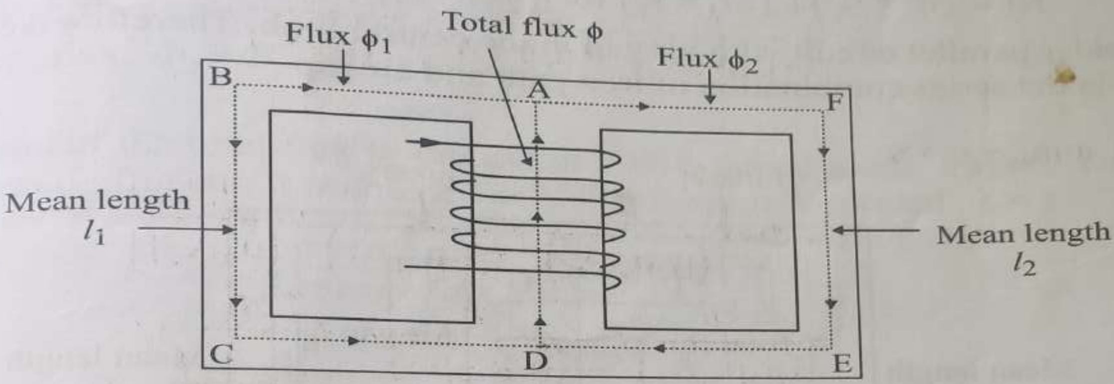

Figure 2.6 Parallel magnetic circuit

On the central limb AD, a current carrying coil is wound. The MMF in the coil sets up a magnetic flux Φ in the central limb. At point A, the total flux divides into two parts

i) Path ABCD with mean length l_1 which carries flux □$_1$

ii) Path AFED with mean length l_2 which carries flux □$_2$

$$\Phi = \Phi_1 + \Phi_2$$

Mean length of path AD = l_c

Reluctance of path ABCD = $S_1 = \frac{l_1}{(\mu_0 \mu_{r1} A_1)}$

Reluctance of path AFED = $S_2 = \frac{l_2}{(\mu_0 \mu_{r2} A_2)}$

Reluctance of path AD = $S_c = \frac{l_c}{(\mu_0 \mu_{rc} A_c)}$

For path ABCDA, MMF = $(\Phi_1 * S_1) + (\Phi * S_c)$

For path AFEDA, MMF = $(\Phi_2 * S_2) + (\Phi * S_c)$

For parallel circuit,

Total MMF = MMF required by central limb + MMF required by any one of

$= (MMF)_{AD} + (MMF)_{ABCD}$ (or) $(MMF)_{AFED}$

$NI = (NI)_{AD} + (NI)_{ABCD}$ (or) $(NI)_{AFED}$

$NI = (\Phi * S_c) + (\Phi_1 * S_1)$ (or) $(\Phi_2 * S_2)$

Consider parallel circuit with air gap in the central limb. Therefore the central limb is the series combination of iron path and air gap

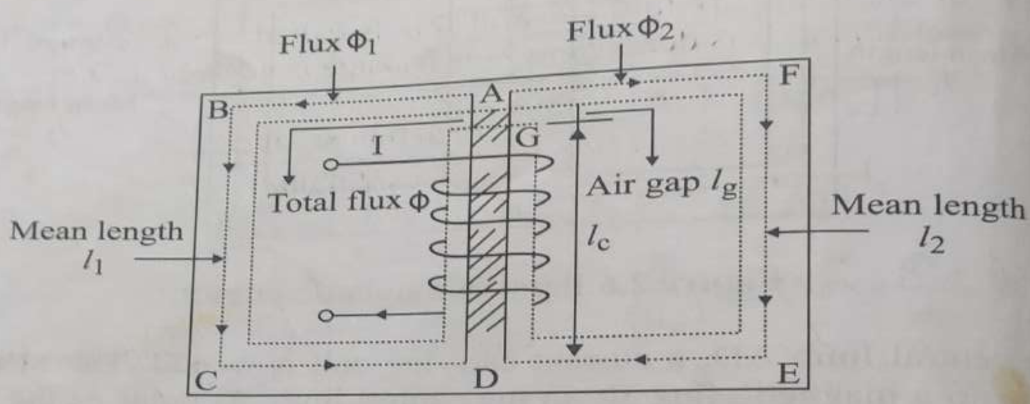

Figure 2.7 Parallel circuit with air gap

Path G-D = Iron path of length l_c

Path G-A = Air gap of length l_g

Reluctance of central limb is Sc = Si + Sg

$$= \frac{l_c}{(\mu_0 \mu_{rc} A_c)} + \frac{l_g}{(\mu_0 A_c)} \quad (\text{since } A_c = A_g \text{ and } \mu_r = 1)$$

Hence MMF of central limb, $(MMF)_{AD} = (MMF)_{GD} + (MMF)_{GA}$

Total MMF, $NI_{Total} = (NI)_{GD} + (NI)_{GA} + (NI)_{ABCD}$ (or) $(NI)_{AFED}$

- **Leakage Flux**

It is defined as the flux that does not follow the desired path in a magnetic circuit i.e. the amount of flux lines that do not follow the core and are lost to the surrounding air. In many practical magnetic circuits, air gaps are provided. Hence a large part of flux path is through magnetic material and the remaining part of flux path is through air. The flux through air gap is known as useful flux. The total flux produced by the coil does not pass through the air gap as some of it leaks through the air surrounding the iron. Thus, the total magnetic flux produced is equal to the sum of the useful magnetic flux and the leakage flux.

Φ_i = Total flux produced in the iron ring
Φ_g = useful flux across the air gap
Φ_l = leakage flux
$\Phi_i = \Phi_l + \Phi_g$
Leakage flux, $\Phi_l = \Phi_i - \Phi_g$

The ratio of the total flux to the useful flux is called as the leakage factor or leakage co-efficient. It is denoted as λ. If leakage is neglected , $\lambda = 1$

$$\text{Leakage flux } \lambda = \frac{\text{total flux produced}}{\text{useful flux}}$$

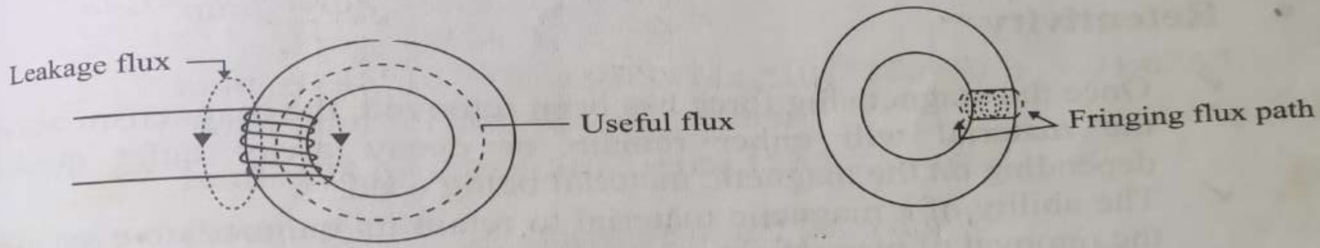

Figure 2.8 Leakage Flux

The value of λ is always greater than unity. Typical values of Leakage factor are from about 1.12 to 1.25. Leakage is a characteristic of all magnetic circuits and can never be completely eliminated.

- ## Fringing

When the flux lines cross the airgap, they tend to bulge out across the edges of the air gap. This effect is called fringing. The effect of fringing makes the effective gap area larger than that of the ring. Since the area is made larger, the flux density in the air gap is reduced $\left(B = \frac{\Phi}{A}\right)$.

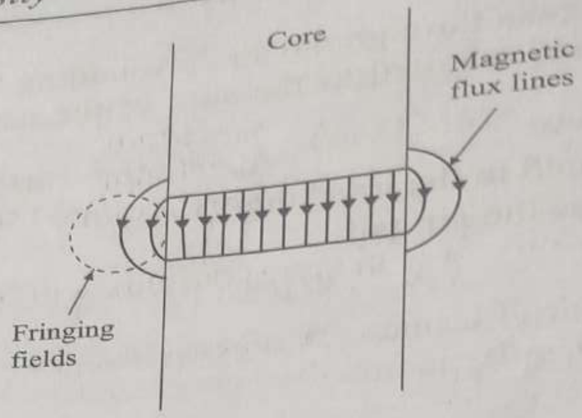

Figure 2.9 Fringing effect

The air gap length is kept as small as possible so that the reduction in the flux density is minimum. If the air gap length is small compared with the gap width, the effect of fringing can be neglected.

- **Retentivity**
  - ✓ Once the magnetising force has been removed, the magnetism within the material will either remain or decay away quiet quickly depending on the magnetic material being used.
  - ✓ The ability of a magnetic material to retain its magnetism even after the removal of magnetizing force is called as retentivity.
  - ✓ The magnetism remaining in the magnetic material, even when the magneticfield is reduced to zero is called residual magnetism.
  - ✓ The materials which are required to retain their magnetism will have a high retentivity and are used to make permanent magnets, while those materials required to lose their magnetism quickly such as soft iron cores are used for relays and solenoids will have a very low retentivity.

Problem 2.6 A magnetic circuit comprises three parts in series as follows:

a. A length of 60 mm with cross section area of 50 mm^2
b. A length of 30 mm with a cross section area of 80 mm^2
c. An air gap of length 0.3 mm and cross section area of 150 mm^2.

A coil of 2500 turns is wound on part (b) and the flux density in air gap is 0.3T. Assuming that there is no leakage and the relative permeability $\mu_r = 1500$, estimate the current required in the circuit to produce the flux density.

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Solution:

l_a=60 mm =60*10^{-3} m
l_b=30 mm=30*10^{-3} m
l_c=0.3 mm=0.3*10^{-3} m

A_a=50mm^2 =50*10^{-6} m^2
A_b=80mm^2=80*10^{-6} m^2
A_c=150mm^2=150*10^{-6} m^2

Since this magnetic circuit is in series the flux will be the same.

$$\Phi = B_c A_c = 0.3*150*10^{-6} = 45*10^{-6} \text{ wb}$$

$$\text{mmf}_a = \Phi S_a = \frac{\Phi l_a}{\mu_0 \mu_r A_a}$$

$$\text{mmf}_a = (45*10^{-6} *60*10^{-3}) / (4\pi*10^{-7}*1500*50*10^{-6}) = 28.6\text{AT}.$$

$$\text{mmf}_b = \Phi S_b = \frac{\Phi l_b}{\mu_0 \mu_r A_b}$$

$$\text{mmf}_b = (45*10^{-6} *30*10^{-3}) / (4\pi *10^{-7}*1500*80*10^{-6}) = 8.95\text{AT}.$$

$$\text{mmf}_c = \Phi S_c = \frac{\Phi l_c}{\mu_0 \mu_r A_c}$$

$$\text{mmf}_c = (45*10^{-6} *0.3*10^{-3}) / (4\pi *10^{-7}*150*10^{-6}) = 71.62\text{AT}.$$

Total mmf $= \text{mmf}_a + \text{mmf}_b + \text{mmf}_c$

mmf $= 28.6+8.95+71.62=109.17$AT

But mmf = NI

$$I = \frac{\text{mmf}}{N} = \frac{109.17}{2500} = 0.04367 A$$

Problem 2.7 A flux density of 1.2 wb/m^2 is required in the 1mm of airgap of an electromagnet having an iron path of 1.5 m long. Calculate the MMF required. Given relative permeability of iron = 1600. Neglect leakage.

Solution:

Flux density, B = 1.2 wb/m^2
Length of the magnetic path, l = 1.5 m
Relative permeability of iron, μ_r = 1600
Absolute permeability of air, $\mu_o = 4\pi\times10^{-7}$ H/m

$$MMF = NI = H \times l$$

$$= \frac{Bl}{\mu_o \mu_r} \qquad [\because B = \mu H = \mu_o \mu_r H]$$

$$MMF \text{ in iron path} = \frac{1.2\times1.5}{4\pi\times10^{-7}\times1600}$$

$$= 895.24\, AT$$

Length of air gap l_g = 1 mm = 1×10^{-3}m

For air, $\mu_r = 1$
$\mu_o = 4\pi\times10^{-7}$H/m
flux density, B = 1.2 wb/m^2

$$\text{mmf} = \Phi S$$
$$= \Phi \frac{l_g}{\mu_o \mu_r A}$$
$$= BA \frac{l_g}{\mu_o \mu_r A}$$

$$mmf\ in\ air\ gap = \frac{Bl_g}{\mu_o \mu_r}$$
$$= \frac{1.2\times1\times10^{-3}}{4\pi\times10^{-7}\times1}$$
$$= 954.93\ AT$$

Total mmf = mmf in iron path + mmf in airgap
= 895.24 + 954.93
= 1850.17 AT

Problem 2.8 An electromagnet is made of iron of square cross section with 5 cm side. A flux of 1.2 mwb is required in the air gap. Find the number of ampere turns required. Take $\mu_r = 2000$. Neglect leakage and fringing.

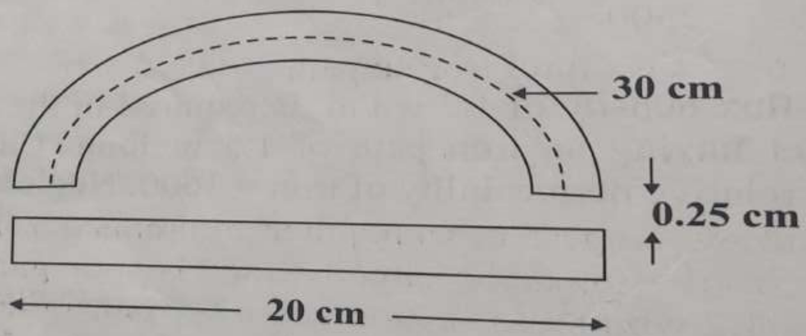

Figure 2.10

Solution:

Area of cross section of iron, A = 5 cm × 5 cm
= 25 cm^2
= 25 × 10^{-4} m^2

Flux, Φ = 1.2 mwb = 1.2×10^{-3} wb

Length of air gap l_g = 0.25 cm = 0 25 x 10^{-2} m

For air gap,
$\mu_r = 1$

$$mmf = \Phi S = \Phi \frac{l_g}{\mu_o \mu_r A}$$
$$= \frac{1.2\times10^{-3}\times0.25\times10^{-2}}{4\pi\times10^{-7}\times1\times25\times10^{-4}}$$
$$= 954.92 AT$$

For two air gaps = 2 x 954.92
= 1909.85AT

For iron path,
$\mu_r = 2000$

Length of iron l_i = 30 cm + 20 cm
= 50 cm
= 50×10^{-2} m

$$\text{mmf for iron path} = \frac{\Phi l_i}{\mu_o \mu_r A}$$
$$= \frac{1.2\times10^{-3}\times50\times10^{-2}}{4\pi\times10^{-7}\times2000\times25\times10^{-4}}$$
$$= 95.49 AT$$

Total mmf = mmf of air + mmf of iron path
= 1909.85+95.49
= 2005.34 AT

Problem 2.9 Find the ampere-turns required in produce a flux of 0.4 mwb in the airgap of a circular magnetic circuit which has air gap of 0.5 mm. The iron ring has 4 square cm cross section and 63 cm mean length. The relative permeability of iron is 1800 and the leakage co-efficient is 1.15.

Given data:

Flux, Φ = 0.4 m wb
= 0.4×10^{-3} wb

Air gap length, l_g = 0.5 mm
= 0.5×10^{-3} m

Area of an iron ring, a = 4 cm^2
= 4×10^{-4} m^2

Length of an iron ring, $l = 63$ cm
$= 63 \times 10^{-2}$ m

Relative permeability, $\mu_r = 1800$

Leakage co-efficient, $\lambda = 1.15$

To find:

MMF required to produce a flux of 0.4 mWb
Total MMF = $\text{MMF}_{\text{iron}} + \text{MMF}_{\text{air gap}}$.
MMF = flux × reluctance

Solution:

(a) $\text{mmf}_{\text{(iron)}}$ = flux × reluctance

(b) $\text{mmf}_{\text{air gap}}$ = flux × reluctance

$$= \Phi_g \times S_g$$

$$= 0.4 \times 10^{-3} \times \frac{l_g}{\mu_0 \mu_r a} \quad [\because \mu_r = 1$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m]$$

$$= 0.4 \times 10^{-3} \times \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = \quad 10^{-7} \times 10^{-4}!! \quad \frac{0.5}{4\pi \times 1} \quad 10^{4} \times$$

$$= 397.88\ AT$$

$$\phi_i = \lambda \times \phi_{useful}$$

$$= 1.15 \times 0.4 \times 10^{-3}$$

$$= 0.46 \times 10^{-3}\ Wb$$

$$Si = \frac{l_i}{\mu_0 \mu_r a} = \frac{l - l_g}{\mu_0 \mu_r a} \quad [\because \mu_0 = 4\pi \times 10^{-7}\ H/m]$$

$$= \frac{0.63 - 0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 1800 \times 4 \times 10^{-4}}$$

$$= 695{,}750.25\ AT/Wb$$

$$mmf_{iron} = \phi_i \times S_i$$

$$= 0.46 \times 10^{-3} \times 695{,}750.25$$

$$= 320.04\ AT$$

Total mmf = $\text{mmf}_{\text{iron}} + \text{mmf}_{\text{air gap}}$
$= 320.04 + 397.88$
$= 717.925$ AT

Result:

The total mmf required is 717.925 AT

Problem 2.10 An iron ring of mean length 50 cm has an airgap of 1mm and a winding of 200 turns. If the permeability of the iron is 400. When a current of 1.25 ampere flows through the coil, find the flux density.

Given data:

Mean length, $l = 50\text{ cm} = 50 \times 10^{-2}\text{ m}$
$= 0.5\text{ m}$

Length of air gap, $l_g = 1\text{ mm} = 1 \times 10^{-3}\text{ m}$

No. of turns, N = 200

Relative permeability, $\mu_r = 400$

Current through the coil, I = 1.25 A.

Note: It is assumed that there is no leakage of flux.

Let Φ = Flux in webers

To find: Flux density of an iron ring.

Solution:

(i) Total mmf = NI
$= 200 \times 1.25\text{A}$
$= 250\text{ AT}$

(ii) Total reluctance = $S_g + S_i$

(a) $S_g = \dfrac{l_g}{\mu_0 A}$ $\quad [\because \mu_r = 1,\ \mu_0 = 4\pi \times 10^{-7}\ H/m]$

$$= \frac{0.5}{4\pi \times 10^{-7} \times 400 \times A}$$

$$= \frac{994.7}{A}\ AT/Wb$$

$\dfrac{0.5}{16\pi \times 10^{-9}}$ $\quad \dfrac{1}{6\pi} \times 10$

(b) $S_i = \dfrac{l_i}{\mu_0 \mu_r A}$

$$= \frac{0.5}{4\pi \times 10^{-7} \times 400 \times A}$$

$$= \frac{994.7}{A}\ AT/Wb$$

(c) *Total reluctance* $=\dfrac{796}{A}+\dfrac{994.7}{A}$

$$=\frac{1790.7}{A}$$

(d) Total mmf = Flux × Total reluctance

$$250=\phi\times\frac{1790.7}{A}$$

$$\frac{\phi}{A}=\frac{250}{1790.7}$$

$$\frac{\phi}{A}=0.1396\ Wb/m^2$$

Result:

$$B=\frac{\phi}{A}=0.1396\,Wb/m^2\ (or)\ Telsa$$

Problem 2.11 A magnetic ring has a mean circumference of 150 cm and is of 0.01 m^2 in cross section and it is wound with 200 turns. A saw cut of 2 mm is made in the ring. Calculate the magnetising current required to produce a flux of 800 μWb in the air gap. Assume relative permeability of iron as 600 and a leakage factor of 1.2.

Given data:

Cross sectional area, $a = 0.01 m^2$

No. of turns, N = 200

Length of iron ring, l_i = 150 cm
$= 150 \times 10^{-2}$ m

Length of air gap, $l_g = 2mm = 2 \times 10^{-3} m$

Flux in air gap, $\Phi_g = 800\ \mu wb$
$= 800 \times 10^{-6}$ wb

Relative permeability, $\mu_r = 600$

Leakage factor, $\lambda = 1.2$

To find:

I = ?

Solution:

(a) For airgap:

AT for airgap = $H_g \times l_g$

$$H = \frac{B}{\mu_0 \mu_r}$$

$$B = \frac{\phi}{A}$$

$$\therefore B = \frac{800 \times 10^{-6}}{0.01} = 0.08\, Wb/m^2$$

$$\therefore H = \frac{0.08}{4\pi \times 10^{-7} \times 1} = 63661.9\, AT/m$$

Now,

$$AT = 63661.9 \times 2 \times 10^{-3}$$
$$= 127.32\ AT$$

(b) AT For iron path

We know that $\lambda = \frac{\phi_T}{\phi_g}$ $\qquad \phi_r = \phi_i$

$$\phi = \lambda \phi_g$$
$$= 1.2 \times 800 \times 10^{-6}$$
$$= 9.6 \times 10^{-4}\, Wb$$

AT For iron path = H l

$$= \frac{B}{\mu_0 \mu_r} l$$
$$= \frac{\phi_i l_i}{\mu_0 \mu_r A}$$
$$= \frac{9.6 \times 10^{-4} \times 150 \times 10^{-2}}{4\pi \times 10^{-7} \times 600 \times 0.01}$$
$$= 191\, AT$$

Total AT = $AT_{airgap} + AT_{iron}$
$= 127.32 + 191$
$= 318.32$ AT

(c) Now

$$\text{Magnetizing current} = \frac{Total AT}{N}$$
$$= \frac{318.38}{200}$$
$$= 1.59\, A$$

Problem 2.12 A steel ring of circular cross-section 1 cm in radius has an airgap of 1mm length. It is wound uniformly with 500 turns of wire carrying a current of 3A. Neglect magnetic leakage. The air gap takes 60% of the total MMF. Find the total reluctance.

Solution:

Radius of the cross-section, r = 1 cm = 1×10^{-2} m

Length of air gap, l_g = 1mm = 1×10^{-3} m

No. of turns, N = 500

Current through the coil, I = 3 A

MMF taken by airgap = 60% of total MMF

Total Reluctance, $S_T = \frac{Total\ MMF}{Flux}$

Total MMF = NI

$= 500 \times 3 = 1500$ AT

MMF taken by air gap = 60% of 1500 AT

$$= \frac{60}{100} \times 1500$$

$$= 900 \text{ AT}$$

MMF in Airgap = $H_g\, l_g$

$$= \frac{B}{\mu_0} l_g \quad [\because \mu_r = 1 (for\ air)]$$

$$= 900 \text{ AT}$$

$$B = \frac{900 \times \mu_0}{l_g}$$

$$= \frac{900 \times 4\pi \times 10^{-7}}{1 \times 10^{-3}}$$

$$B = 1.131\, Wb/m^2$$

Flux, $\Phi = BA$

$= B (\pi r^2)$ [$A = \pi r^2$ for a circular ring]

$= 1.131 [\pi \times (1 \times 10^{-2})^2]$

$\Phi = 3.553 \times 10^{-4}$ wb

Total Reluctance, $S_T = \frac{TotalMMF}{Flux(\phi)}$

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$$= \frac{1500}{3.553\times10^{-4}}$$

$$S_T = 4.222 \times 10^6 \text{ AT/wb}$$

Problem 2.13 A magnetic core has the following dimensions

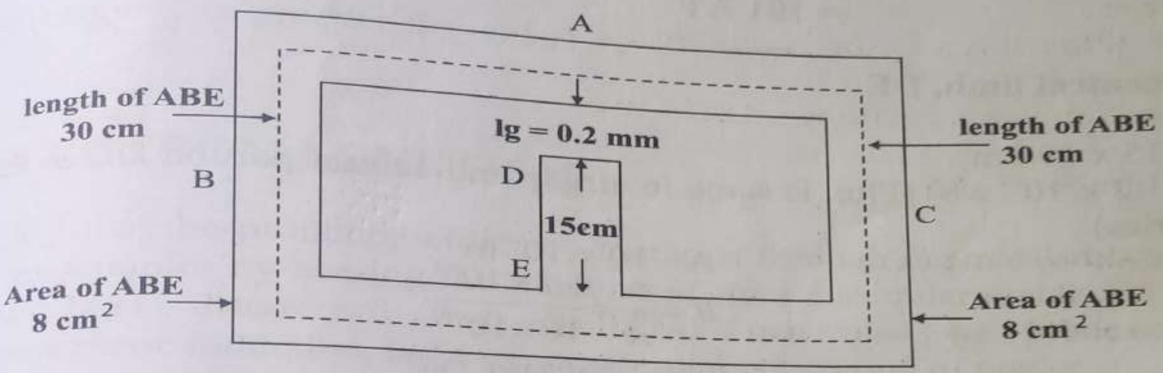

Figure 2.11

The length of portion DE = 15 cm
Area of cross-section = 10 cm^2
The length of portion ABE = 30 cm
Area of cross-section = 8 cm^2
The length of portion ACE = 30 cm
Area of cross-section = 8 cm^2
Length of air gap, l_g = 0.2 mm
Air gap flux = 1.2 mwb

The central limb is wound with 500 turns. Calculate the total ampere turns and the current required in the coil to produce the above flux.
The data for B-H curve for the core is

B (wb/m^2)	0.45	0.75	1.0	1.2
H (AT/m)	90	140	230	450

Solution: For air gap, $\mu_r = 1$, $\mu_o = 4\pi \times 10^{-7}$ H/m.
$l_g = 0.2 \text{ mm} = 0.2 \times 10^{-3}$ m

Cross sectional area of the air gap is assumed to be equal to area of the cross-section in DE = 10cm^2

$$\therefore A_g = 10\text{cm}^2 = 10 \times 10^{-4}\text{m}^2$$

$$\Phi_g = 1.2 \times 10^{-3} \text{ wb}$$

$$B = \frac{\phi_g}{A_g} = \frac{1.2\times10^{-3}}{10\times10^{-4}} = 1.2\, Wb/m^2$$

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$$H_g = \frac{B_g}{\mu_0 \mu_r} = \frac{1.2}{4\pi \times 10^{-7} \times 1} = 954929.66 \quad [\because \mu_o = 1(for\ air)]$$

AT required for air gap = $H_g\ l_g$
= $954929.66 \times 0.2 \times 10^{-3}$
= 191 AT

For central limb, DE

$l = 15 \times 10^{-2}$ m
$\Phi = 1.2 \times 10^{-3}$ wb (Flux is same in airgap and central portion DE as they are in series)

$$A = 10 \times 10^{-4} m^2$$
$$B = \frac{\phi}{A} = \frac{1.2 \times 10^{-3}}{10 \times 10^{-4}}$$
$$= 1.2 Wb/m^2$$

'H' corresponding to 1.2 wb/m^2 from the table = 450 AT/m

AT required = H × l
= $450 \times 15 \times 10^{-2}$
= 67.5 AT

For portions ABE & ACE,
At point 'A', flux is, divided into two equal parts

$$\therefore \text{Flux throughs each portion} = \frac{1.2 \times 10^{-3}}{2} \quad [\because \text{Parallel circuit}]$$
$$= 0.6 \times 10^{-3} \text{ wb}$$

Area of cross section, A = $8 \times 10^{-4} m^2$

$$B = \frac{\phi}{A} = \frac{0.6 \times 10^{-3}}{8 \times 10^{-4}}$$
$$= 0.75 Wb/m^2$$

'H' corresponding to 0.75 wb/m^2 from the table = 140 AT/m

$$l = 30 \text{ cm} = 30 \times 10^{-2} \text{ m}$$

AT required = H × l = $30 \times 10^{-2} \times 140$ = 42 AT

Total AT = AT airgap + AT central limb DE + AT portion ABE (or) ACE
= 191 + 67.5 + 42

Total AT = 300.5 AT

$$\therefore \text{Exciting current} = \frac{AT}{N} = I [\because AT = NI]$$

$$= \frac{300.5}{500}$$

Current required to produce $\Phi = 0.601$ A

2.3 ELECTROMAGNETISM

The EMF may be produced either by batteries through chemical reaction or by thermocouples by heating the junction of two dissimilar metals. Michael faraday 1831 discovered that the EMF can also be produced by electromagnetic induction, used in commercial generation of power.

- **Electromagnetic Induction**

Whenever the magnetic flux linking with the coil changes, an EMF is induced in the coil. This phenomenon is called as electromagnetic induction. **Applications**: microphones, telephones, transformers, generators, motors, etc.,

- **Production of induced EMF and current**

The change in flux linkage can be obtained by three methods

Method 1:

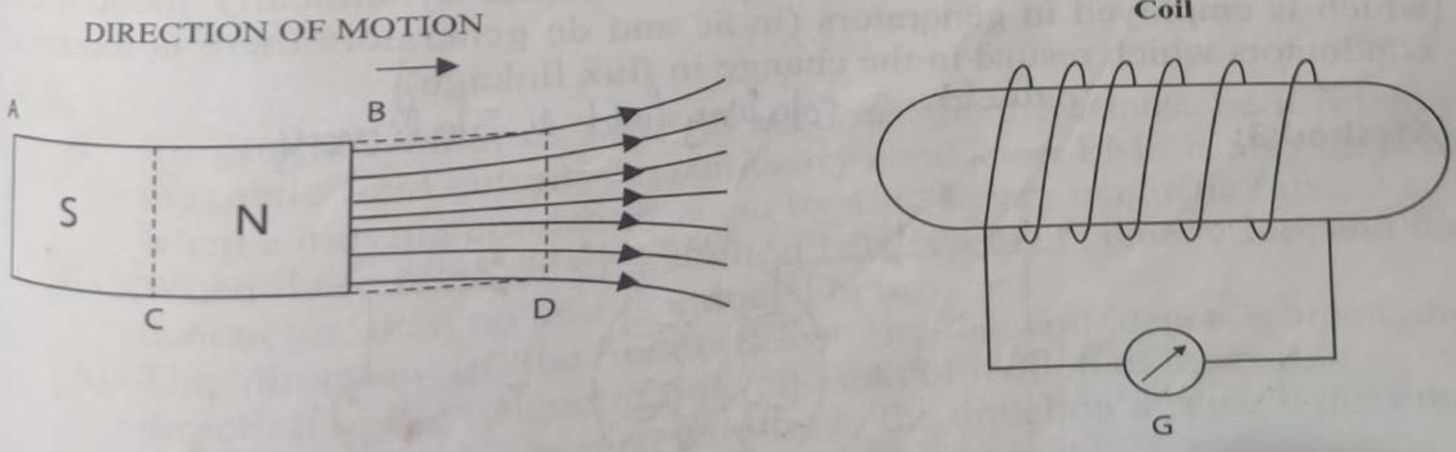

Figure 2.12

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SCHOOL OF BIO & CHEMICAL ENGINEERING

DEPARTMENT OF BIO MEDICAL ENGINEERING

UNIT – V

Basic Electrical Engineering – SEEA1203

Unit 5 – Introduction to Electric Machines

The electrical machines deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form, this process is called electromechanical energy conversion. An electrical machine which converts mechanical energy into electrical energy is called an electric generator while an electrical machine which converts electrical energy into the mechanical energy is called an electric motor. A DC generator is built utilizing the basic principle that emf is induced in a conductor when it cuts magnetic lines of force. A DC motor works on the basic principle that a current carrying conductor placed in a magnetic field experiences a force.

Working principle:

All the generators work on the principle of dynamically induced emf.

The change in flux associated with the conductor can exist only when there exists a relative motion between the conductor and the flux.

The relative motion can be achieved by rotating the conductor w.r.t flux or by rotating flux w.r.t conductor. So, a voltage gets generated in a conductor as long as there exists a relative motion between conductor and the flux. Such an induced emf which is due to physical movement of coil or conductor w.r.t flux or movement of flux w.r.t coil or conductor is called dynamically induced emf.

Whenever a conductor cuts magnetic flux, dynamically induced emf is produced in it according to Faraday's laws of Electromagnetic Induction.

This emf causes a current to flow if the conductor circuit is closed.

So, a generating action requires the following basic components to exist.

1. The conductor or a coil
2. Flux
3. Relative motion between the conductor and the flux.

In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as output, a number of conductors are connected together in a specific manner to form a winding. The winding is called armature winding of a dc machine and the part on which this winding is kept is called armature of the dc machine.

The magnetic field is produced by a current carrying winding which is called field winding.

The conductors placed on the armature are rotated with the help of some external device. Such an external device is called a prime mover.

The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc.

The purpose of the prime mover is to rotate the electrical conductor as required by Faraday's laws

The direction of induced emf can be obtained by using Flemings right hand rule. The magnitude of induced emf = e = BLV sinØ= E_m sinØ

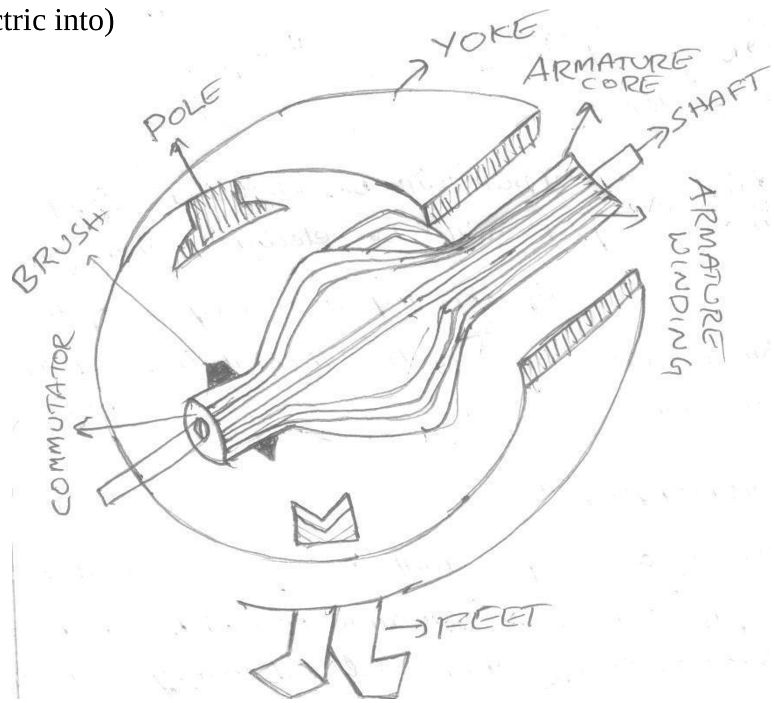
Nature of induced elf:

The nature of the induced emf for a conductor rotating in the magnetic field is alternating. As conductor rotates in a magnetic field, the voltage component at various positions is different. Hence the basic nature of induced emf in the armature winding in case of dc generator is alternating. To get dc output which is unidirectional, it is necessary to rectify the alternating induced emf. A device which is used in dc generator to convert alternating induced emf to unidirectional dc emf is called commutator.

Construction of DC machines :

A D. C. machine consists of two main parts

1. Stationary part: It is designed mainly for producing a magnetic flux.
2. Rotating part: It is called the armature, where mechanical energy is converted into electrical (electrical generate) or conversely electrical energy into mechanical (electric into)



Parts of a Dc Generator:

1) Yoke
2) Magnetic Poles
   a) Pole core
   b) Pole Shoe
3) Field Winding
4) Armature Core
5) Armature winding

6) Commutator
7) Brushes and Bearings

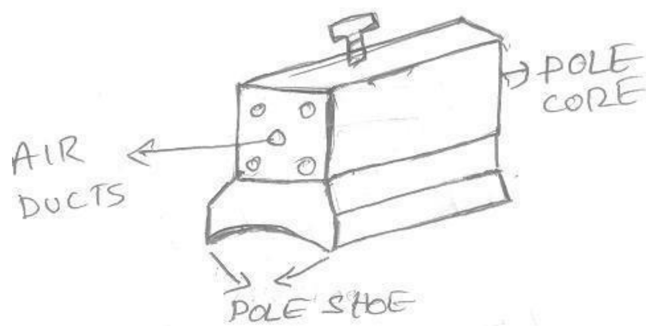
The stationary parts and rotating parts are separated from each other by an air gap. The stationary part of a D. C. machine consists of main poles, designed to create the magnetic flux, commutating poles interposed between the main poles and designed to ensure spark less operation of the brushes at the commutator and a frame / yoke. The armature is a cylindrical body rotating in the space between the poles and comprising a slotted armature core, a winding inserted in the armature core slots, a commutator and brush

Yoke:

1. It saves the purpose of outermost cover of the dc machine so that the insulating materials get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.
2. It provides mechanical support to the poles.
3. It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. Choice of material: To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is the cheapest. For large machines rolled steel or cast steel, is used which provides high permeability i.e., low reluctance and gives good mechanical strength.

Poles: Each pole is divided into two parts

a) pole core   b) pole shoe



Functions:

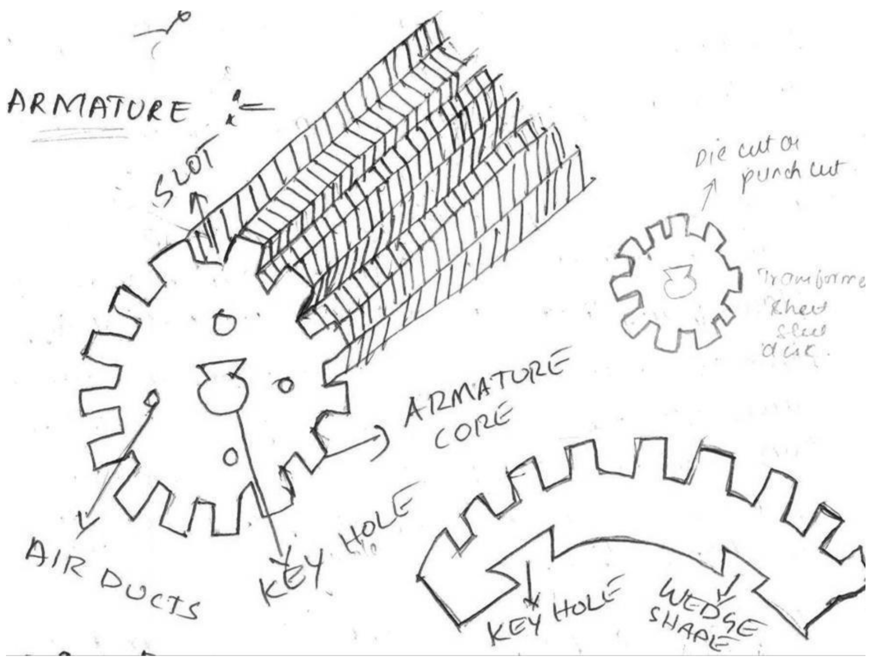
1. Pole core basically carries a field winding which is necessary to produce the flux.
2. It directs the flux produced through air gap to armature core to the next pole.
3. Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf. To achieve this, pole core has been given a particular shape.

Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to yoke.

Armature: It is further divided into two parts namely,

(1) Armature core   (2) Armature winding.

Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.



Functions:

1. Armature core provides house for armature winding i.e., armature conductors.
2. To provide a path of low reluctance path to the flux it is made up of magnetic material like cast iron or cast steel.

Choice of material: As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

It is made up of laminated construction to keep eddy current loss as low as possible.

A single circular lamination used for the construction of the armature core is shown below.

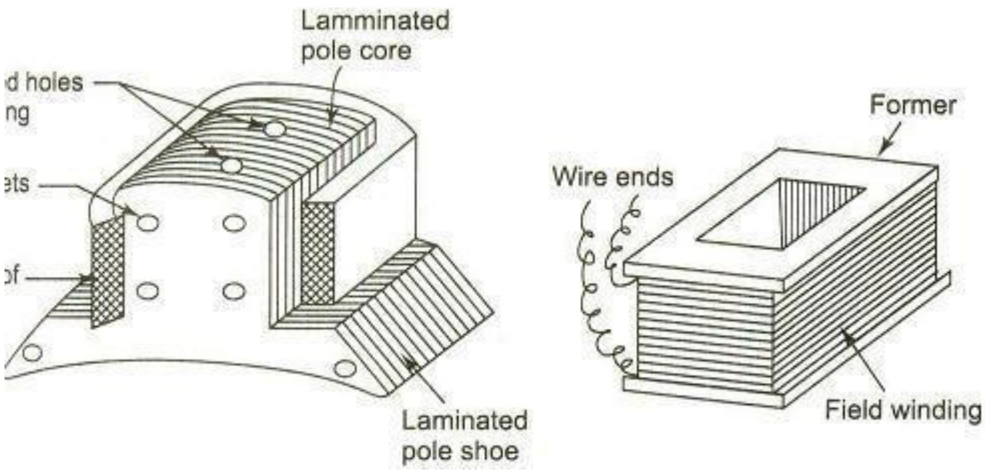
2. **Armature winding:** Armature winding is nothing but the inter connection of the armature conductors, placed in the slots provided on the armature core. When the armature is rotated, in case of generator magnetic flux gets cut by armature conductors and emf gets induced in them.

Function:

1. Generation of emf takes place in the armature winding in case of generators.
2. To carry the current supplied in case of dc motors.
3. To do the useful work it the external circuit.

Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Field winding: The field winding is wound on the pole core with a definite direction.



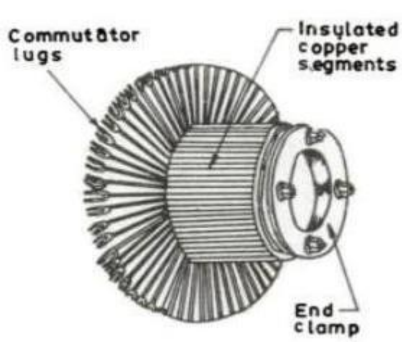
Functions: To carry current due to which pole core on which the winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called '**Field winding'** or '**Exciting winding'.**

Choice of material : As it has to carry current it should be made up of some conducting material like the aluminum or copper.

But field coils should take any type of shape should bend easily, so copper is the proper choice. Field winding is divided into various coils called as field coils. These are connected in series with each other and wound in such a direction around pole cores such that alternate N and S poles are formed.

Commutator: The rectification in case of dc generator is done by device called as commutator.



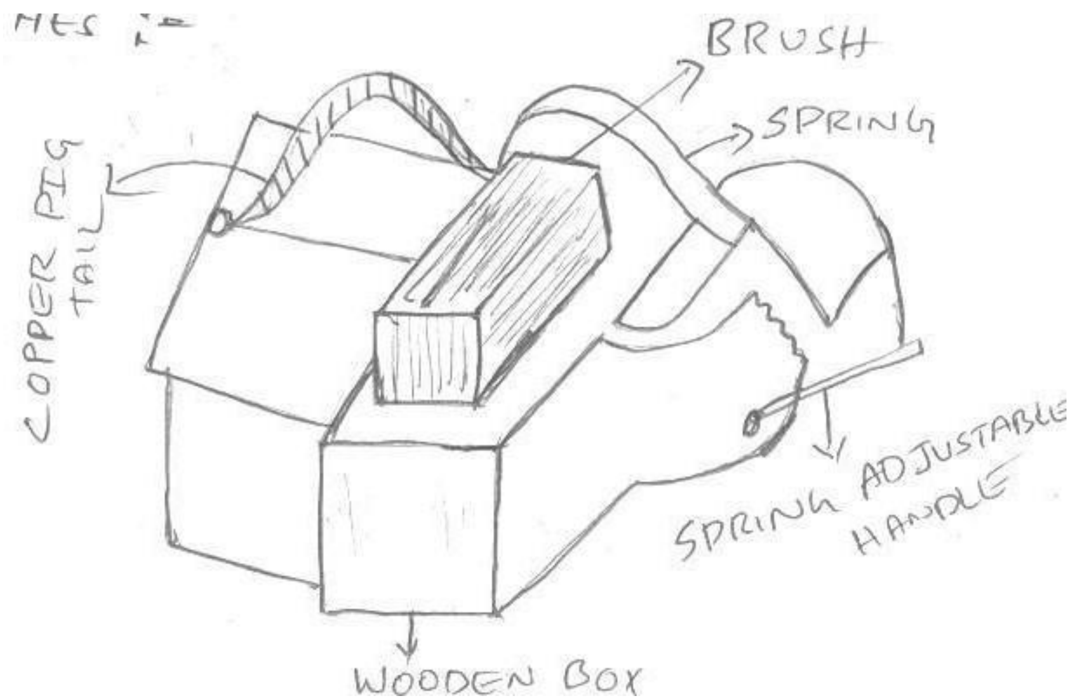
Functions: 1. To facilitate the collection of current from the armature conductors.

2. To convert internally developed alternating emf to in directional (dc) emf

3. To produce unidirectional torque in case of motor.

Choice of material: As it collects current from armature, it is also made up of copper segments. It is cylindrical in shape and is made up of wedge shaped segments which are insulated from each other by thin layer of mica.

Brushes and brush gear: Brushes are stationary and rest on the surface of the Commutator. Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pigtail is used to connect the brush to the external circuit.



Functions: To collect current from commutator and make it available to the stationary external circuit.

Choice of material: Brushes are normally made up of soft material like carbon.

Bearings: Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

Working of DC generator:

The generator is provided with a magnetic field by sending dc current through the field coils mounted on laminated iron poles and through armature winding.

A short air gap separates the surface of the rotating armature from the stationary pole surface. The magnetic flux coming out of one or more worth poles crossing the air gap , passes through the armature near the gap into one or more adjacent south poles.

The direct current leaves the generator at the positive brush, passes through the circuit and returns to the negative brush.

The terminal voltage of a dc generator may be increased by increasing the current in the field coil and may be reduced by decreasing the current.

Generators are generally run at practically constant speed by their prime mores.

Types of armature winding:

Armature conductors are connected in a specific manner called as armature winding and according to the way of connecting the conductors; armature winding is divided into two types.

Lap winding: In this case, if connection is started from conductor in slot 1 then the connections overlap each other as winding proceeds, till starting point is reached again.

There is overlapping of coils while proceeding. Due to such connection, the total number of conductors get divided into 'P' number of parallel paths, where

P = number of poles in the machine.

Large number of parallel paths indicates high current capacity of machine hence lap winding is pertained for high current rating generators.

Wave winding: In this type, winding always travels ahead avoiding over lapping. It travels like a progressive wave hence called wave winding.

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of machine.

As number of parallel paths is less, it is preferable for low current, high voltage capacity generators.

Sl. No.	**Lap winding**	**Wave winding**
1.	Number of parallel paths (A) = poles (P)	Number of parallel paths (A) = 2 (always)
2.	Number of brush sets required is equal to number of poles	Number of brush sets required is always equal to two
3.	Preferable for high current, low voltage capacity generators	Preferable for high current, low current capacity generators
4.	Normally used for generators of capacity more than 500 A	Preferred for generator of capacity less than 500 A.

EMF equation of a generator

Let P = number of poles

Ø = flux/pole in webers

Z = total number of armature conductors.

= number of slots x number of conductors/slot

N = armature rotation in revolutions (speed for armature) per minute (rpm) A = No. of parallel paths into which the ‘z’ no. of conductors are divided. E = emf induced in any parallel path

Eg = emf generated in any one parallel path in the armature.

Average emf generated/conductor = dØ/dt volt Flux current/conductor in one revolution

dt = d x p

In one revolution, the conductor will cut total flux produced by all poles = d x p No. of revolutions/second = N/60

Therefore, Time for one revolution, dt = 60/N second

According to Faraday’s laws of Electromagnetic Induction, emf generated/conductor = dØ x p x N / 60 volts /dt=

This is emf induced in one conductor. For a simplex wave-wound generator No. of parallel paths = 2

No. of conductors in (series)in one path = Z/2

EMF generated/path = ØPN/60 x Z/2 = ØZPN/120 volt

For a simple lap-wound generator Number of parallel paths = P

Number of conductors in one path = Z/P

EMF generated/path = ØPN/60 (Z/P) = ØZN/60 A = 2 for simplex – wave winding

A =P for simplex lap-winding

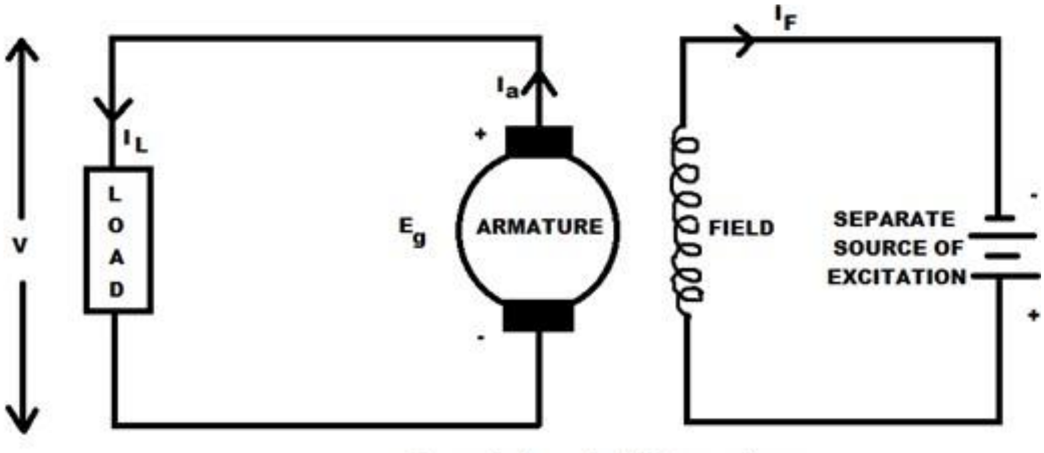
Types of DC Generators

When the flux in the magnetic circuit is established by the help of permanent magnets then it is known as Permanent magnet dc generator. It consists of an armature and one or several permanent magnets situated around the armature. This type of dc generators generates very low power. So, they are rarely found in industrial applications. They are normally used in small applications like dynamos in motor cycles.

Separately Excited DC Generator

These are the generators whose field magnets are energized by some external dc source such as battery . A circuit diagram of separately excited DC generator is shown in figure.

I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf



Separately excited DC generator

Voltage drop in the armature = $I_a \times R_a$ (R/sub>a is the armature resistance) Let, $I_a = I_L = I$ (say)

Then, voltage across the load, $V = IR_a$ Power generated, $P_g = E_g \times I$ Power delivered to the external load, $P_L = V \times I$.

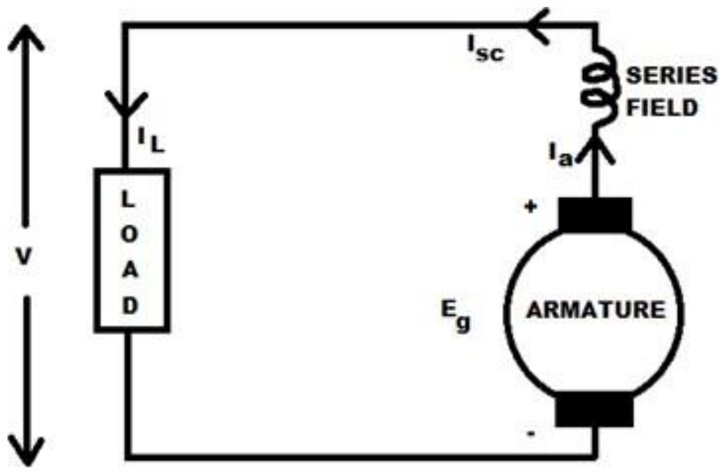
Self-excited DC Generators

These are the generators whose field magnets are energized by the current supplied by themselves. In these type of machines field coils are internally connected with the armature. Due to residual magnetism some flux is always present in the poles. When the armature is rotated some emf is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and thereby strengthening the pole flux. As the pole flux strengthened, it will produce more armature emf, which cause further increase of current through the field. This increased field current further raises armature emf and this cumulative phenomenon continues until the excitation reaches to the rated value. According to the position of the field coils the Self-excited DC generators may be classified as…

A. Series wound generators B. Shunt wound generators C. Compound wound generators

Series Wound Generator

In these type of generators, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load. As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω). Let, R_{sc} = Series winding resistance I_{sc} = Current flowing through the series field R_a = Armature resistance I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf



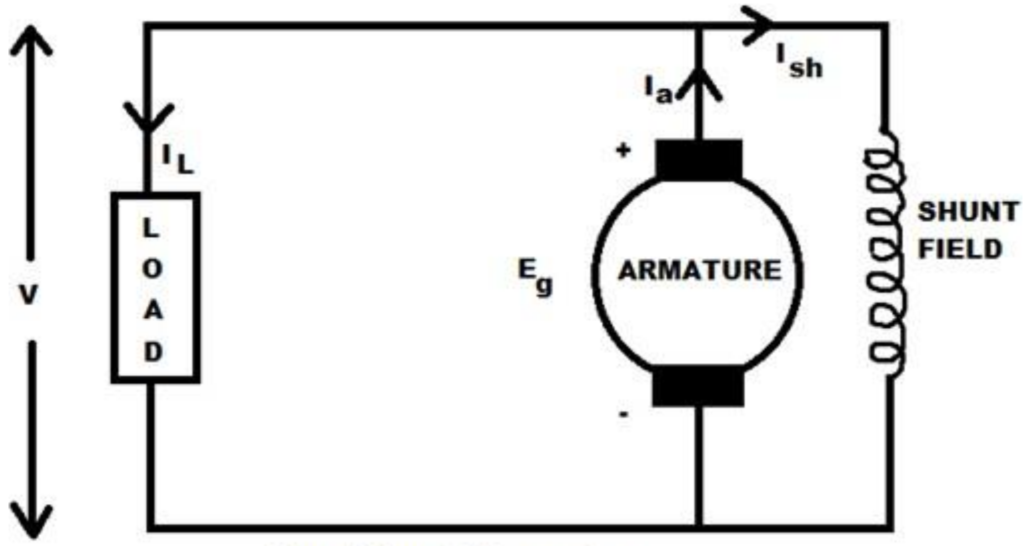
Series Wound Generator

Then, $I_a = I_{sc} = I_L = I$ (say) Voltage across the load, $V = E_g - I(I_a \times R_a)$ Power generated, P_g =

$E_g \times I$ Power delivered to the load, $P_L = V \times I$

Shunt Wound DC Generators

In these type of DC generators the field windings are connected in parallel with armature conductors as shown in figure below. In shunt wound generators the voltage in the field winding is same as the voltage across the terminal. Let, R_{sh} = Shunt winding resistance I_{sh} = Current flowing through the shunt field R_a = Armature resistance I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf



Shunt Wound Generator

Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L. So, $I_a = I_{sh} + I_L$ The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf. Shunt field current, $I_{sh} = V/R_{sh}$ Voltage across the load, $V = E_g - I_a R_a$ Power generated, $P_g = E_g \times I_a$ Power delivered to the load, $P_L = V \times I_L$

Compound Wound DC Generator

In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current. A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator. Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature. This type of DC generators may be of two types- short shunt compound wound generator and long shunt compound wound generator.

Short Shunt Compound Wound DC Generator

The generators in which only shunt field winding is in parallel with the armature winding as shown in

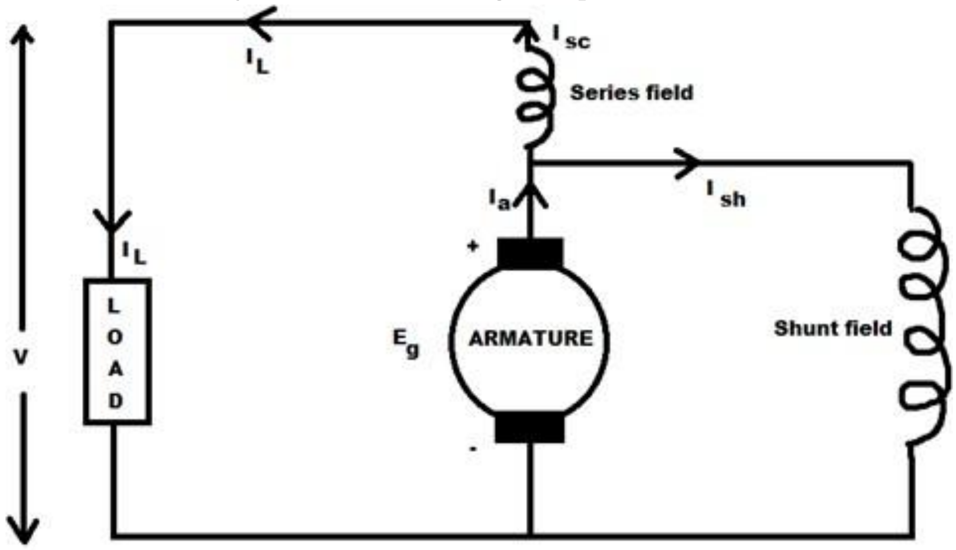


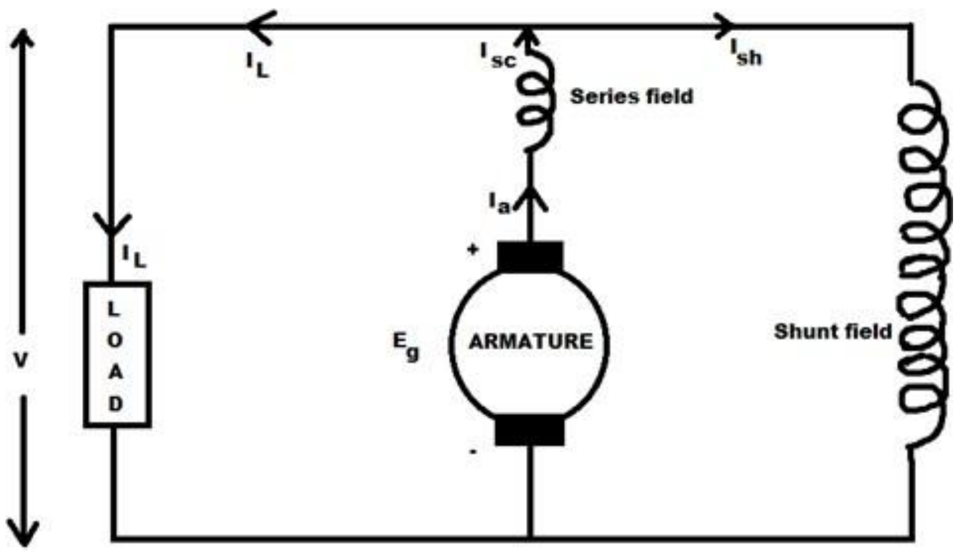
figure.

Short Shunt Compound Wound Generator

Series field current, $I_{sc} = I_L$ Shunt field current, $I_{sh} = (V+I_{sc} R_{sc})/R_{sh}$ Armature current, $I_a = I_{sh} + I_L$ Voltage across the load, $V = E_g - I_a R_a - I_{sc} R_{sc}$ Power generated, $P_g = E_g \times I_a$ Power delivered to the load, $P_L=V\times I_L$

Long Shunt Compound Wound DC Generator

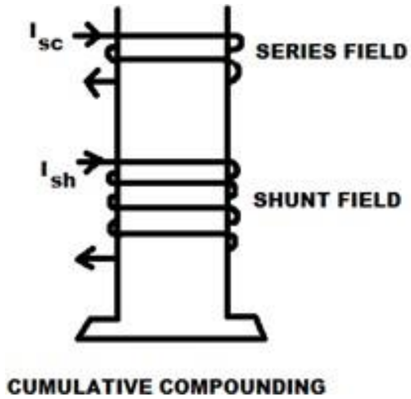
The generators in which shunt field winding is in parallel with both series field and armature winding as

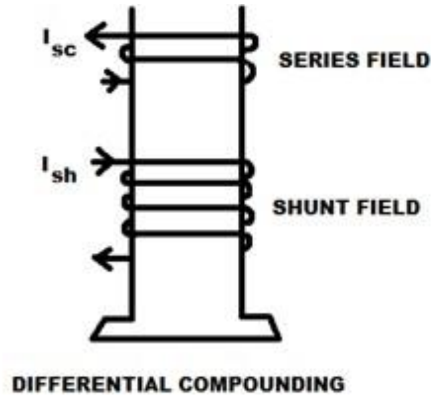


Long Shunt Compound Wound Generator

shown in figure.

Shunt field current, $I_{sh}=V/R_{sh}$ Armature current, I_a= series field current, $I_{sc}= I_L+I_{sh}$ Voltage across the load, $V=E_g-I_a R_a-I_{sc} R_{sc}=E_g-I_a (R_a+R_{sc})$ [$\therefore I_a=I_{cs}$] Power generated, $P_g= E_g\times I_a$ Power delivered to the load, $P_L=V\times I_L$ In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand if series field opposes the shunt field, the generator is said to be differentially compound wound.





Unit 3 – DC MOTOR

A dc motor is similar in construction to a dc generator. As a matter of fact a dc generator will run as a motor when its field & armature windings are connected to a source of direct current.

The basic construction is same whether it is generator or a motor.

Working principle:

The principle of operation of a dc motor can be stated as when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force. In a practical dc motor, the field winding produces the required magnetic held while armature conductor play the role of current carrying conductor and hence the armature conductors experience a force.

As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductive acts as a twisting or turning force on the armature which is called a torque.

The torque is the product of force and the radius at which this force acts, so overall armature experiences a torque and starts rotating.

Consider a single conductor placed in a magnetic field , the magnetic field is produced by a permanent magnet but in practical dc motor it is produced by the field winding when it carries a current.

Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an current. Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a conductor is clock-wise. Now, there are two fluxes present

5. Flux produced by permanent magnet called main flux
6. Flux produced by the current carrying conductor

From the figure shown below, it is clear that on one side of the conductor, both the fluxes are in the same direction in this case, on the left of the conductor there gathering of the flux lines as two fluxes help each other. A to against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened.

So on the left, there exists high flux density area while on the right of the conductor then exists low flux density area as shown.

The flux distribution around the conductor arts like a stretched ribbed bond under tension. The exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area,
i.e. from left to right from the case considered as shown above.

In the practical dc motor, the permanent magnet is replaced by the field winding which produces the required flux winding which produces the required flux called main flux and all the armature conductors, would on the periphery of the armature gram, get subjected tot he mechanical force.

Due to this, overall armature experiences a twisting force called torque and armature of the motor status rotating.

Direction of rotation of motor

The magnitude of the force experienced by the conductor in a motor is given by F = BIL newtons.

The direction of the main field can be revoked y changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding.

The direction of current through armature can be reversed by changing supply polarities of dc supplying current to the armature.

It directions of bot the currents are changed then the direction of rotation of the motor remains undamaged.

In a dc motor both the field and armature is connected to a source of direct current. The current through the armature winding establish its own magnetic flux the interaction both the main field and the armature current produces the torque, there by sensing the motor to rotate, once the motor starts rotating, already existing magnetic flux there wire be an induced emf in the armature conductors due to generator action. This emf acts in a direction apposite to supplied voltage. Therefore it is called Black emf.

Significance of Back emf

In the generating action, when a conductor cuts the lines of flux, emf gets induced in the conductor in a motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. After a motoring action, there exists a generating action there is induced emf in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced emf in the armature always acts in the opposite direction of the supply voltage. This is according tot he lenz's law which states that the direction of the induced emf is always so as to oppose the case producing it.

In a dc motor, electrical input i.e., the supply voltage is the cause and hence this induced emf opposes the supply voltage.

The emf tries to set u a current throughout he armature which is in the opposite direction to that which supply voltage is forcing through the conductor so, as this emf always opposes the supply voltage, it is called back emf and devoted as Eb.

Through it is denoted as Eb, basically it gets generated by the generating action which we have seen earlier So, $E^b \; \frac{ZNP}{60\,A}$

Voltage equation of a Motor

The voltage v applied across the motor armature has to (1) over core the back emf Eb and

3. supply the armature ohmic drop Ia Ra v = Eb + Ia Ra

This is known as voltage equation of a motor Multiplying both sides by Ia, we get

$Vi_a = E_b I_a + E_a^2 R_a$

VI_a = electrical input to the armature
E_bI_a = electrical equivalent of mechanical Power developed in the armature

$I_a^2 R_a$ = un loss in the armature

Hence, out of the armature input, some in wasted in I2 R loss and the rot is convened into mechanical power within the armature.
Motor efficiency is given by the ratio of power developed by the armature to its input i.e. $E_b I_a / vI_a = E_b/v$.

Higher the value of E_b as compared to v, higher the motor efficiency.

Conduction for maximum powers

The gross mechanical developed by a motor = $p_m = vI_a - I_a^2 R_a$

$$\frac{dP_m}{dI_a} = v - 2I_a R_a \qquad I_a R_a = v/2$$

As $v = E_b + I_a R_a$ and $I_a R_a = v/2$ $E_b = v/2$

Thus gross mechanical power developed by a motor is maximum when back emf is equal to half the applied voltage. This conduction's how ever at realized in practice, because in that case current will be much beyond the normal current of the motor.

More ova, half the input would be wasted in the form of heat and taking other losses into consideration the motor efficiency will be well below 50 %.

1. A 220v – dc machine has an armature resistance of 0.5 Ω. If the full road armature current is 20A, find the induced emf when the machine acts (1) generator (2) motor.

The dc motor is assumed to be shunt connected in cash case, short current in considered negligible because its value is not given.
(a) As generator $E_g = v + I_a R_a = 220 + 0.5 \times 20 = 230$ v
(b) As motor $E_b = v - I_a R_a = 220 - 0.5 \times 20 = 210$ v

8) A 440 v, shunt motor has armature resistance of 0.8 a and field resistance of 200 Ω. Determine the back emf when giving an output at 7.46 kw at 85% efficiency.

Motor input power = $\frac{7.46 \times 10^3}{0.85}$ w
7460

Motor input current = $\frac{7460}{0.85 \times 440} = 19.95$ A

3. A 25kw, 250 w dc such generator has armature and field resistance of 0.06 Ω and 100 Ω respectively. Determine the total armature power developed when working (1) as generator delivering 25 kw output and (2) as a motor taking 25 kw input.

Voltage equation of dc motor

For a generator, generated emf has to supply armature resistance drop and remaining part is available across the loss as a terminal voltage. But in case of dc motor, supply voltage v has to over come back emf Eb which is opposing v and also various drops are armature resistance drop Ia Ra, brush drop etc. In fact the electrical work done in overcoming the back emf gets converted into the mechanical energy, developed in the armature.

Hence, the voltage equation of a dc motor is $V = E_b + I_a R_a$ + brush drops

Or $v = E_b + I_a R_a$ neglecting brush drops

The back emf is always less than supply voltage ($E_b < v$) but R_a is very small hence under normal running conditions, the different between back emf and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back emf which decals the armature current. Hence from the voltage equation we can write $I_a = v – E_b / R_a$.

3. A 220 v dc motor has an armature resistance of 0.75 it is drawing on armature current of 30 A, during a certain load, calculate the induced emf in the motor under this condition.

$V = 200$ v, $I_a = 30$A, $R_a = 0.75$

For a motor , $v = E_b + I_a$

R_a Eb = 197.5 v

This is the induced mef called back emf in a motor.

5. A 4-pole dc motor has lap connected armature winding. The number of armature conductors is 250. When connected to 230 v dc supply it draws an armature current It 4 cm calculate the back emf and the speed with which motor is running. Assume armature is 0.6

P = 4 A = P = 4 as lap connected

= 30 m wb = 30 Ho^{-3} V = 230v, z = 250 Ia = 40 A

From voltage equation $V = E_b + I_a$

R_a 230 = Eb + 40 x 0.6

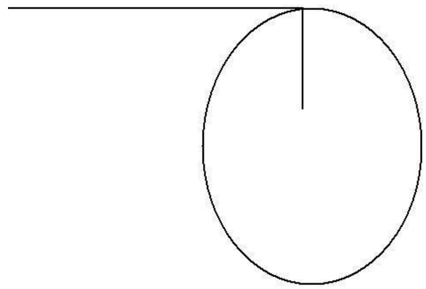
Eb = Pnz / 60A

$206 = (30 \times 10^{-3} \times 4 \times N \times 250) / (60 \times 4)$ N = 1648 rpm.

Torque: The turning or twisting movement of a body is called Torque. (Or)

It is defined as the product of force and perpendicular distance Ť=F*R

F



In case of DC motor torque is produced by the armature and shaft called as armature torque (T_a) and shaft torque(T_{sh}).

Let, N be the speed of the armature in RPM
R be the radius of the armature
Power=Work Done/Time
Work Done=Force X Distance
The distance travelled in rotating the armature for one time $=2\prod R$
If N rotations are made in 60 sec
Then time taken for one rotation is=60/N

So, Power= $(F * 2\prod R)/(60/N)$

$= (F*R)(2\prod N)/60$ $P = Ť\omega$

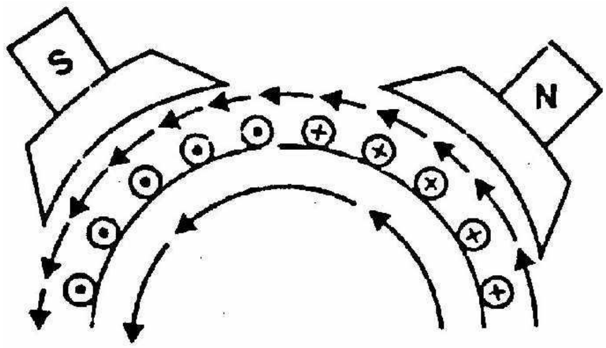
Here $P=E_bI_a$ But $E_b=ØZNP/60A$
$(ØZNP/60A)I_a= Ť\omega$
$= Ť_a(2\prod N)/60$
$Ť_a=0.159ØZ I_aP/A$
Similarly, Shaft torque T_{sh}=output/ω
T_{sh} =output/$((2\prod N)/60)$
T_{sh} =9.55(output)/N

D.C. Motor Principle

A machine that converts d.c. power into mechanical power is known as a d.c.motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by;
F BI newtons
Basically, there is no constructional difference between a d.c. motor and a d.c.generator. The same d.c. machine can be run as a generator or motor.

Working of D.C. Motor

When the terminals of the motor are connected to an external source of d.c. supply:
(i) the field magnets are excited developing alternate N and S poles;
(ii) the armature conductors carry currents.



All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction. Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig. Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it.

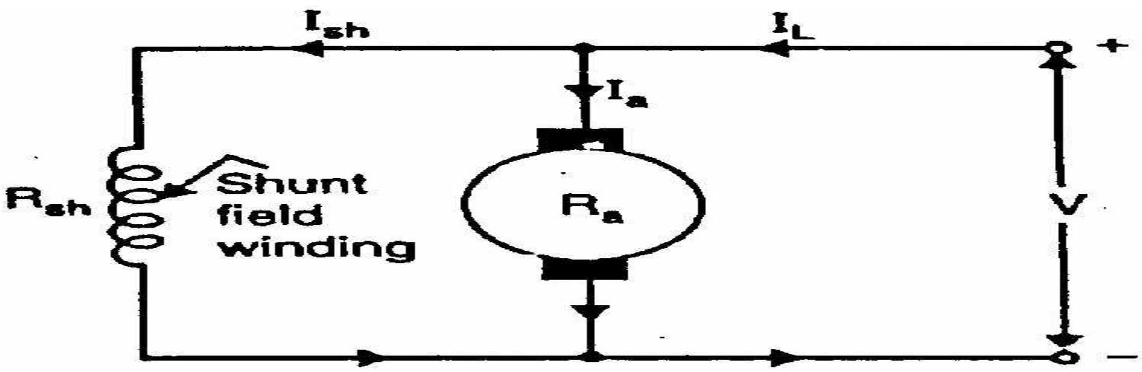
Applying Fleming’s left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating. When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

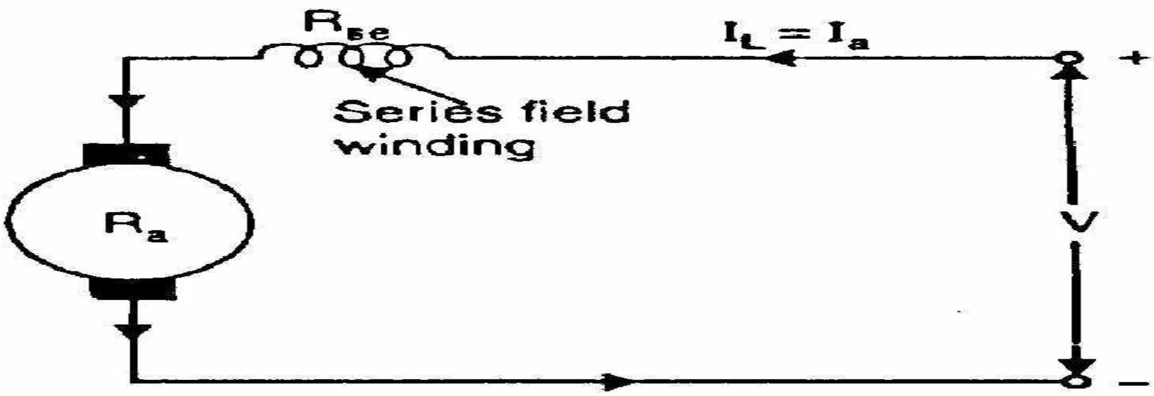
Types of D.C. Motors

Like generators, there are three types of d.c. motors characterized by the connections of field winding in relation to the armature viz.:

(i) Shunt-wound motor in which the field winding is connected in parallel with the armature. The current through the shunt field winding is not the same as the armature current. Shunt field windings are designed to produce the necessary m.m.f. by means of a relatively large

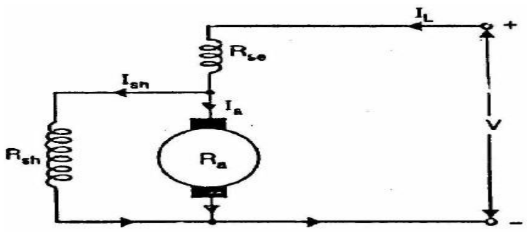
number of turns of wire having high resistance. Therefore, shunt field current is relatively small compared with the armature current.



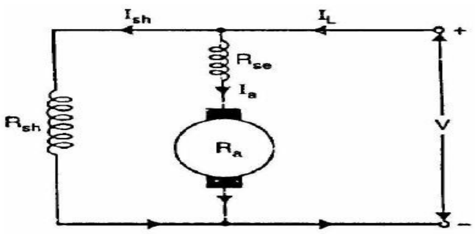


(ii) Series-wound motor in which the field winding is connected in series with the armature Therefore, series field winding carries the armature current. Since the current passing through a series field winding is the same as the armature current, series field windings must be designed with much fewer turns than shunt field windings for the same m.m.f. Therefore, a series field winding has a relatively small number of turns of thick wire and, therefore, will possess a low resistance.

(iii) Compound-wound motor which has two field windings; one connected in parallel with the armature and the other in series with it. There are two types of compound motor connections (like generators). When the shunt field winding is directly connected across the armature terminals it is called short-shunt connection. When the shunt winding is so connected that it shunts the series combination of armature and series field it is called long- shunt connection.



Short-shunt connection



Long-shunt connection

Operating Principles and Construction

What is a Transformer?

A ***transformer*** *is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.*

The use of transformers in transmission system is shown in the Figure below.

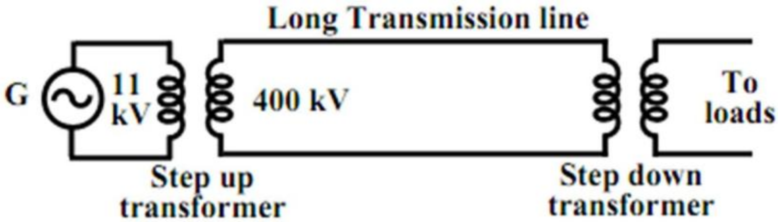


Fig.3-1 A simple single phase power system.

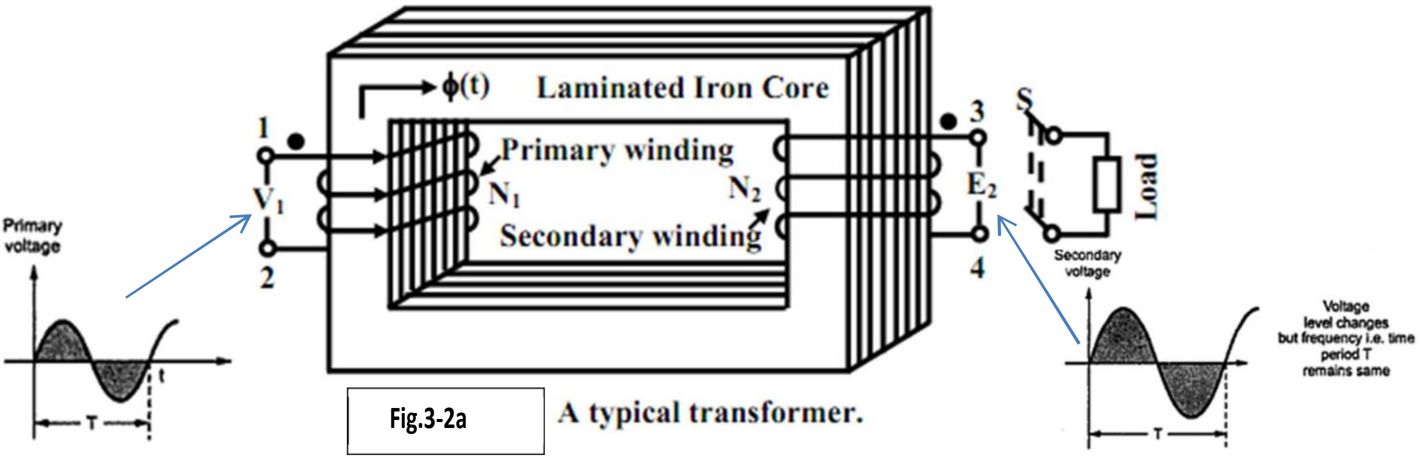


Fig.3-2a A typical transformer.

Principle of Operation

A transformer in its simplest form will consist of a rectangular laminated magnetic structure on which two coils of different number of turns are wound as shown in Figure 3.2a.

The winding to which AC voltage is impressed is called ***the primary*** of the transformer and the winding across which the load is connected is called ***the secondary*** of the transformer.

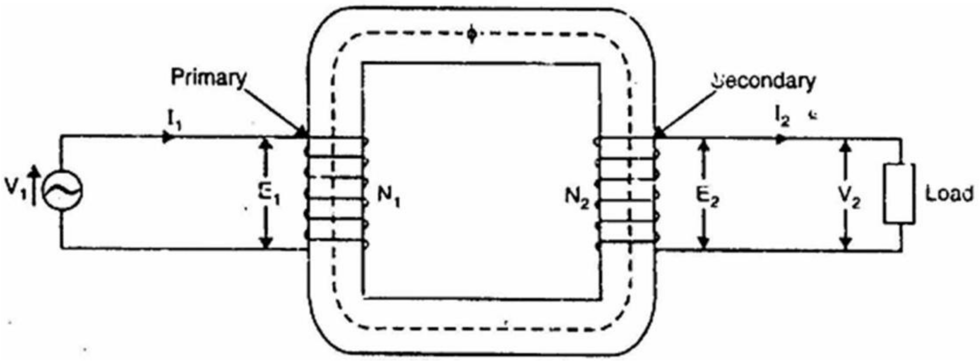


Fig.3-2b

Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating emf (E_2) is induced in the secondary. This induced emf (E2) in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a ***step up-transformer***. On the other hand, if $V_2 < V_1$, it is called a ***step-down transformer***.

When an alternating voltage V_1 is applied to the primary, an alternating flux Φ is set up in the core. This alternating flux links both the windings and induces emfs E1 and E2 in them according to ***Faraday's laws of electromagnetic induction***. The emf E1 is termed as primary emf and emf E2 is termed as Secondary emf.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. On the other hand, if $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

(i) The transformer action is based on the laws of ***electromagnetic induction***.

(ii) Thereisnoelectricalconnectionbetweentheprimaryandsecondary.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

Can DC Supply be used for Transformers?

TheDCsupplycannotbeusedforthetransformers. This is because the transformer works on the principle of ***mutual induction***, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and transformer will not work

There can be saturation of the core due to which transformer draws very large current from the supply when connected to DC.

Construction

We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:

(i) The core is made of silicon steel which has low hysteresis loss and high permeability. Further, core is laminated in order to reduce eddy current loss. These features considerably reduce the iron losses and the no-load current.

(ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to wind one-half of each winding on one limb. This ensures tight coupling between the two windings. Consequently, leakage flux is considerably reduced.

(iii) The winding resistances are minimized to reduce Copper loss and resulting rise in temperature and to ensure high efficiency.

Transformers are of two types: (i) core-type transformer (see Fig.3-3) and
(ii) shell-type transformer (see Fig.3-4).

Core-Type Transformer: In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb to reduce the leakage flux.

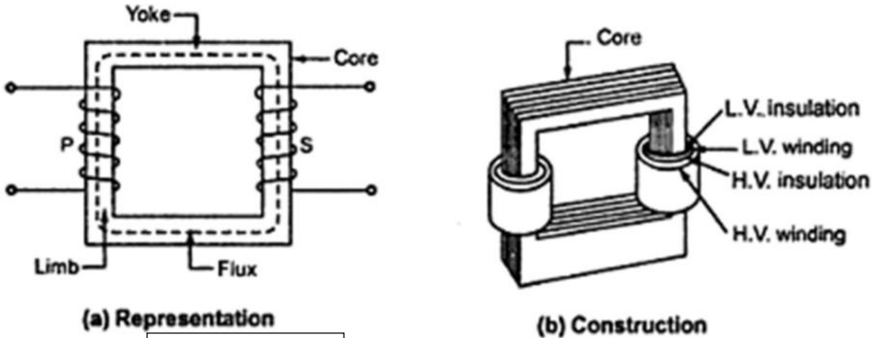


Fig.3-3 **Core type transformer**

Shell-Type Transformer: This method of construction involves the use of a double magnetic circuit. Both the windings are placed round the central limb to ensure a low-reluctance flux path.

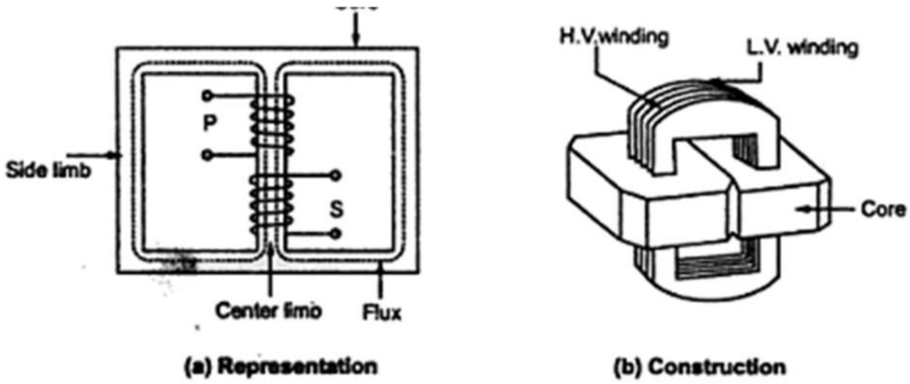


Fig.3-4 **Shell type transformer**

Comparison of Core and Shell Type Transforms

Core Type	Shell Type
The winding encircles the core.	The core encircles most part of the windin
It has single magnetic circuit	It has double magnetic circuit
The core has two limbs	The core has three limbs
The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
The winding are uniformly distributed on two limbs hence natural cooling is effective	The natural cooling does not exist as the windings are surrounded by the core.
Preferred for low voltage transformers.	Preferred for high voltage transformers.

Cooling of Transformers

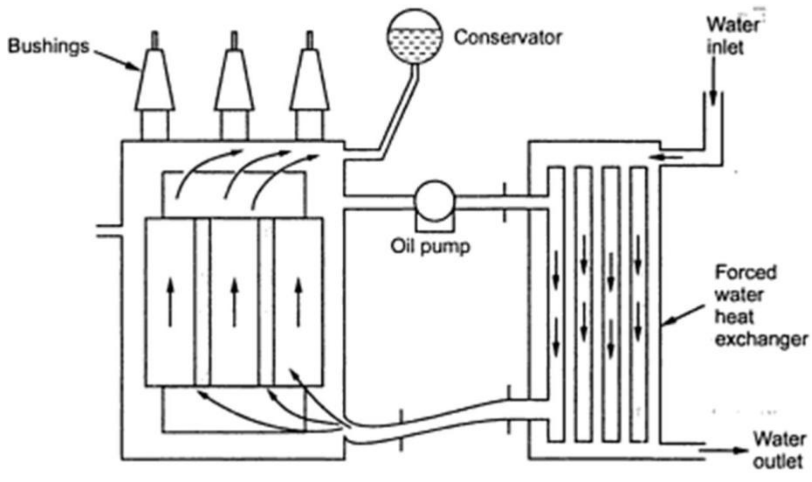
When transformer supplies a load, two types of losses occur inside the transformer. The iron losses occur in the core while copper losses occur in the windings. The power lost due to these losses appears in the form of heat. This heat increases the temperature of the transformer. ***To keep the temperature rise of the transformer within limits, a suitable coolant and cooling method is necessary.***

The various cooling methods are designated witch depended upon: A:coolingmediumusedandB:

typeofcirculationemployed.

The various coolant used such as Air, Gas, Mineral oil, and water.

One of cooling method system is shown in figure below which is called ***Oil Forced Water Forced cooling system***;



Oil forced water forced cooling method

EMF Equation of a Transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown in Fig. 3-2b. The sinusoidal flux Φ produced by the primary can be represented as:

$$\phi = \phi_m \sin\omega t$$

The instantaneous e.m.f. e_1 induced in the primary is

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin \omega t) \\ &= -\omega N_1 \phi_m \cos\omega t = -2\pi f N_1 \phi_m \cos\omega t \\ &= 2\pi f N_1 \phi_m \sin(\omega t - 90°) \qquad \text{(i)} \end{aligned}$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E^ of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi\, f\, N_1\, \phi_m}{\sqrt{2}}$$

or $$E_1 = 4.44\, f\, N_1\, \phi_m$$

Similarly $$E_2 = 4.44\, f\, N_2\, \phi_m$$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

Note. It is clear from exp. (i) above that e.m.f. E_1 induced in the primary lags behind the flux ϕ by 90°. Likewise, e.m.f. E_2 induced in the secondary lags behind flux ϕ by 90°.

Voltage Transformation Ratio (K)

From the above equations of induced emf, we have,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

TheconstantKiscalledvoltagetransformationratio.ThusifK=5(i.e. N2/N1 = 5), then E2 = 5 E1.

Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties:

i) It has no losses.

ii) Its windings have zero resistance.

iii) Leakagefluxiszeroi.e.100%fluxproducedbyprimarylinkswith the secondary.

iv) Permeabitity of core is so high that negilgiblc current is required to establish the flux init.

□ □ □ □ □

For an ideal transformer, the primary applied voltage V1 is same as the primary induced emf E1 as there are no voltage drops.

For ideal transformer:

(i) $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1I_1 = V_2I_2$$

(i) $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1I_1 = V_2I_2$$

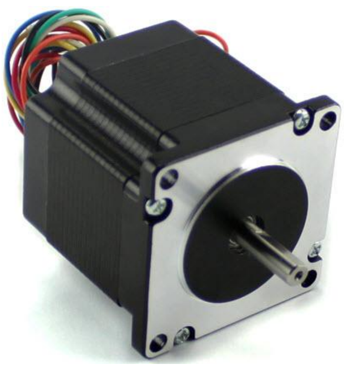
$$\text{or} \qquad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if we raise the voltage, there is a corresponding decrease of current.

What is a Stepper Motor?

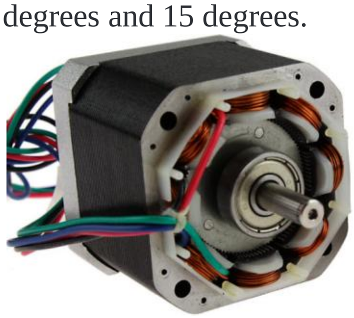
Stepper Motor is a brushless electromechanical device which converts the train of electric pulses applied at their excitation windings into precisely defined step-by-step mechanical shaft rotation. The shaft of the motor rotates through a fixed angle for each discrete pulse. This rotation can be linear or angular.It gets one step movement for a single pulse input.

When a train of pulses is applied, it gets turned through a certain angle. The angle through which the stepper motor shaft turns for each pulse is referred as the step angle, which is generally expressed in degrees.



The number of input pulses given to the motor decides the step angle and hence the position of motor shaft is controlled by controlling the number of pulses. This unique feature makes the stepper motor to be well suitable for open-loop control system wherein the precise position of the shaft is maintained with exact number of pulses without using a feedback sensor.

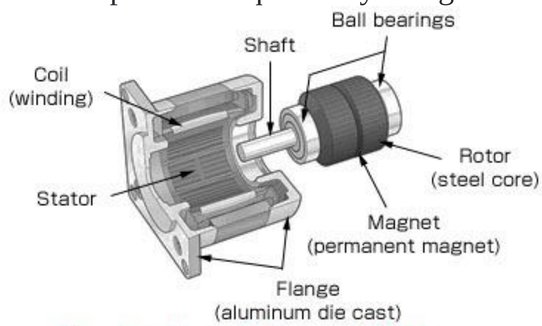
If the step angle is smaller, the greater will be the number of steps per revolutions and higher will be the accuracy of the position obtained. The step angles can be as large as 90 degrees and as small as 0.72 degrees, however, the commonly used step angles are 1.8 degrees, 2.5 degrees, 7.5 degrees and 15 degrees.



The direction of the shaft rotation depends on the sequence of pulses applied to the stator. The speed of the shaft or the average motor speed is directly proportional to the frequency (the rate of input pulses) of input pulses being applied at excitation windings. Therefore, if the frequency is low, the stepper motor rotates in steps and for high frequency, it continuously rotates like a DC motor due to inertia.

Like all electric motors, it has stator and rotor. The rotor is the movable part which has no windings, brushes and a commutator. Usually the rotors are either variable reluctance or permanent magnet kind. The stator is often constructed with multipole and multiphase windings, usually of three or four phase windings wound for a required number of poles decided by desired angular displacement per input pulse.

Unlike other motors it operates on a programmed discrete control pulses that are applied to the stator windings via an electronic drive. The rotation occurs due to the magnetic interaction between poles of sequentially energized stator winding and poles of the rotor.



Construction of Stepper Motor

There are several *types of stepper motors* are available in today's market over a wide range of sizes, step count, constructions, wiring, gearing, and other electrical characteristics. As these motors are capable to operate in discrete nature, these are well suitable to interface with digital control devices like computers.

Due to the precise control of speed, rotation, direction, and angular position, these are of particular interest in industrial process control systems, CNC machines, robotics, manufacturing automation systems, and instrumentation.

Types of Stepper Motors

There are three basic **categories of stepper motors**, namely

- **Permanent Magnet Stepper Motor**
- **Variable Reluctance Stepper Motor**
- **Hybrid Stepper Motor**

In all these motors excitation windings are employed in stator where the number of windings refer to the number of phases.

A DC voltage is applied as an excitation to the coils of windings and each winding terminal is connected to the source through a solid state switch. Depends on the type of stepper motor, its rotor design is constructed such as soft steel rotor with salient poles, cylindrical permanent magnet rotor and permanent magnet with soft steel teeth. Let us discuss these types in detail.

Variable Reluctance Stepper Motor

It is the *basic type of stepper motor* that has been in existence for a long time and it ensures easiest way to understand principle of operation from a structural point of view. As the name suggests, the angular position of the rotor depends on the reluctance of the magnetic circuit formed between the stator poles (teeth) and rotor teeth.

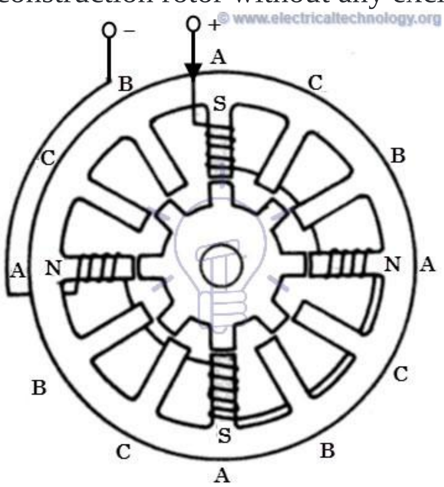
Construction of Variable Reluctance Stepper Motor

It consists of a wound stator and a soft iron multi-tooth rotor. The stator has a stack of silicon steel laminations on which stator windings are wound. Usually, it is wound for three phases which are distributed between the pole pairs.

The number of poles on stator thus formed is equal to an even multiple of the number of phases for which windings are wounded on stator. In the figure below, the stator has 12 equally spaced projecting poles where each pole is wound with an exciting coil. These three phases are energized from of a DC source with the help of solid state switches.

The rotor carries no windings and is of salient pole type made entirely of slotted steel laminations. The rotor pole's projected teeth have the same width as that of stator teeth. The number of poles on stator differs to that of rotor poles, which provides the ability to self start and bidirectional rotation of the motor.

The relation of rotor poles in terms of stator poles for a three phase stepper motor is given as, $N_r = N_s \pm (N_s / q)$. Here $N_s = 12$, and $q = 3$, and hence $N_r = 12 \pm (12 / 3) = 16$ or 8. An 8-pole construction rotor without any excitation is illustrated below.



Working of Variable Reluctance Stepper Motor

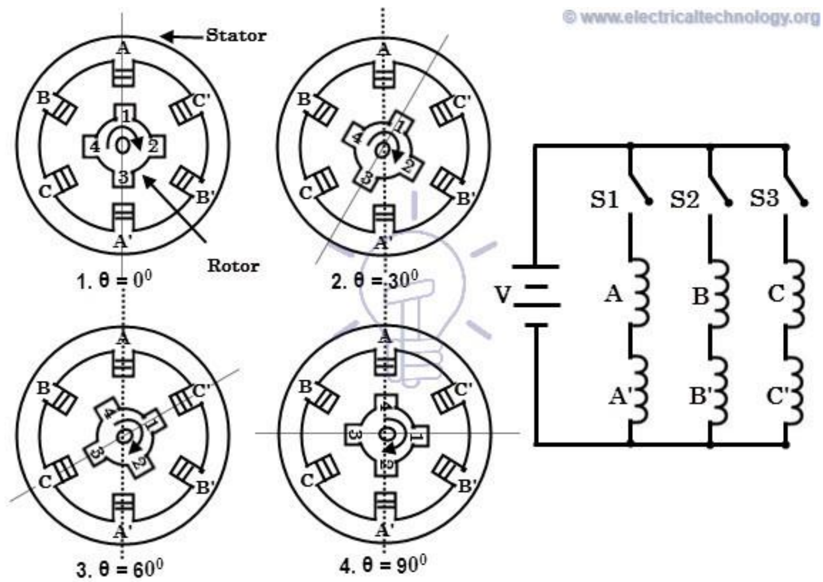
The *stepper motor works on the principle* that the rotor aligns in a particular position with the teeth of the excitation pole in a magnetic circuit wherein minimum reluctance path exist. Whenever power is applied to the motor and by exciting a particular winding, it produces its magnetic field and develops its own magnetic poles.

Due to the residual magnetism in the rotor magnet poles, it will cause the rotor to move in such a position so as to achieve minimum reluctance position and hence one set of poles of rotor aligns with the energized set of poles of the stator. At this position, the axis of the stator magnetic field matches with the axis passing through any two magnetic poles of the rotor.

When the rotor aligns with stator poles, it has enough magnetic force to hold the shaft from moving to the next position, either in clockwise or counter clockwise direction.

Consider the schematic diagram of a 3-phase, 6 stator poles and 4 rotor teeth is shown in figure below. When the phase A-A' is supplied with a DC supply by closing the switch -1, the winding

become a magnet which results one tooth become North and other South. So the stator magnetic axis lies along these poles.
Due to the force of attraction, stator coil North Pole attracts nearest rotor tooth of opposite polarity, i.e., South and South Pole attract nearest rotor tooth of opposite polarity, i.e., North. The rotor then adjusts to its minimum reluctance position where the rotor magnetic axis exactly matches with stator magnetic axis.



When the phase B-B’ is energized by closing switch -2 keeping phase A-A’ remain de-energized by opening switch-1, winding B-B’ will produce the magnetic flux and hence the stator magnetic axis shifts along the poles thus formed by it. Hence the rotor shifts to the least reluctance with magnetized stator teeth and rotates through an angle of 30 degrees in the clockwise direction.

When the phase B-B’ is energized by closing switch -2 keeping phase A-A’ remain de-energized by opening switch-1, winding B-B’ will produce the magnetic flux and hence the stator magnetic axis shifts along the poles thus formed by it. Hence the rotor shifts to the least reluctance with magnetized stator teeth and rotates through an angle of 30 degrees in the clockwise direction.

When the switch-3 is energized after opening switch-2, the phase C-C’ is energized, the rotor teeth align with new position by moving through an additional angle of 30 degrees. By this way, the rotor moves clockwise or counterclockwise direction by successively exciting stator windings in a particular sequence. The step angle of this 3-phase 4-pole rotor teeth stepper motor is expressed as, 360/ (4 × 3) = 30 degrees (as step angle = 360 / Nr × q).
The step angle can be further reduced by increasing the number of poles on the stator and rotor, in such case motors are often wound with additional phase windings. This can also be achieved

by a adopting different *construction of stepper motors* such as multistack arrangement and reduction gear mechanism.

Advantages of Stepper Motor

- At standstill position, the motor has full torque. No matter if there is no moment or changing position.
- It has a good response to starting, stopping and reversing position.
- As there is no contact brushes in the stepper motor, It is reliable and the life expectancy depends on the bearings of the motor.
- The motor rotation angle is directly proportional to the input signals.
- It is simple and less costly to control as motor provides open loop control when responding to the digital input signals.
- The motor speed is directly proportional to the input pulses frequency, this way a wide range of rotational speed can be achieved.
- When load is coupled to the shaft, it is still possible to realize the synchronous rotation with low speed.
- The exact positioning and repeatability of movement is good as it has a 3-5% accuracy of a step where the error is non cumulative from one step to another.
- Stepper motors are safer and low cost (as compared to servo motors), having high torque at low speeds, high reliability with simple construction which operates at any environment.

Disadvantages of Stepper Motors

- Stepper motors having low Efficiency.
- It has low Accuracy.
- Its torque declines very quickly with speed.
- As stepper motor operates in open loop control, there is no feedback to indicate potential missed steps.
- It has low torque to inertia ratio means it can’t accelerate the load very quickly.
- They are noisy.

Applications of Stepper Motors

- Stepper motors are used in automated production equipments and automotive gauges and industrial machines like packaging, labeling, filling and cutting etc.
- It is widely used in security devices such as security & surveillance cameras.
- In medical industry, stepper motors are widely used in samples, digital dental photography, respirators, fluid pumps, blood analysis machinery and medical scanners etc.
- They are used in consumer electronics in image scanners, photo copier and printing machines and in digital camera for automatic zoom and focus functions and positions.
- Stepper motors also used in elevators, conveyor belts and lane diverters.