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SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING
DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

UNIT - I

DC Machines and Transformers – SEEA1202

Unit 1 – MAGNETIC CIRCUITS

Definition of MMF, Flux and Reluctance - Leakage Factor - Reluctances in Series and Parallel (Series and Parallel Magnetic Circuits) - Electromagnetic Induction - Fleming's Rule - Lenz's Law - Faraday's laws - statically and dynamically induced EMF - Self and mutual inductance - Analogy of Electric and Magnetic Circuits.

Definition of MMF, Flux and Reluctance - Leakage Factor

Magneto motive force (MMF)

Magneto motive force, also known as magnetic potential, is the property of certain substances or phenomena that gives rise to magnetic fields. Magneto motive force is analogous to electromotive force or voltage in electricity .The standard unit of magneto motive force is the ampere-turn (AT), represented by a steady, direct electrical current of one ampere (1A) flowing in a single-turn loop of electrically conducting material in a vacuum .

$$\text{Magneto motive force (mmf)}\, F = NI(\text{ampere} - \text{turns})$$

Sometimes a unit called the gilberts (G) is used to quantify magneto motive force. The gilberts is defined differently, and is a slightly smaller unit than the ampere- turn. To convert from ampere-turns to gilberts, multiply by 1.25664. Conversely, multiply by 0.795773.

Flux

Magnetic flux (most often denoted as Φ_m), is the amount of magnetic field (also called "magnetic flux density") passing through a surface (such as a conducting coil). The SI unit of magnetic flux is the Weber (Wb) (in derived units: volt-seconds). The CGS unit is the Maxwell.

Reluctance

Magnetic reluctance, or magnetic resistance, is a concept used in the analysis of magnetic circuits. It is analogous to resistance in an electrical circuit, but rather than dissipating electric energy it stores magnetic energy. In likeness to the way an electric field causes an electric current to follow the path of least resistance, a magnetic field causes magnetic flux to follow the path of least magnetic reluctance. It is a scalar, extensive quantity, akin to electrical resistance. The unit for magnetic reluctance is inverse henry, H^{-1}.Reluctance depends on the dimensions of the core as well as its materials.

Reluctance = l/μA.(A- t/Wb)

Permeability

The total magnetic flux in an electric rotating machine or transformer divided by the useful flux that passes through the armature or secondary winding. Also known as leakage coefficient. There are three categories of magnetic materials: diamagnetic, in which the material tends to exclude magnetic fields; paramagnetic, in which the material is slightly magnetized by a magnetic field; and ferromagnetic, which are materials that very easily become magnetized. The vast majority of materials do not respond to magnetic fields, and their permeability is very close to that of free space. The materials that readily accept magnetic flux—that is, ferromagnetic materials—are principally iron, cobalt, and nickel and various alloys that include these elements. The units of permeability are webers per amp-turn-meter (Wb/A-t-m). The permeability of free space is given by

Permeability of free space $\mu_0 = 4\pi \times 10{-}7$ Wb/A-t-m

Oftentimes, materials are characterized by their relative permeability, μ_r, which for ferromagnetic materials may be in the range of hundreds to hundreds of thousands. As will be noted later, however, the relative permeability is not a constant for a given material: It varies with the magnetic field intensity. In this regard, the magnetic analogy deviates from its electrical counterpart and so must be used with some caution.

Relative permeability = $\mu_r = \mu\mu_0$

Magnetic flux density

Another important quantity of interest in magnetic circuits is the magnetic flux *density*, *B*. As the name suggests, it is simply the -density‖ of flux .Unit is Tesla.

Magnetic flux density $B = \varphi/A$ webers/m^2 or tesla *(*T*)*

Magnetic field intensity

The magnetic field intensity is defined as the magnetomotive force (mmf) per unit of length around the magnetic loop. With *N* turns of wire carrying current i, the mmf created in the circuit is *Ni* ampere-turns. With *l* representing the mean path length for the magnetic flux, the magnetic field intensity is therefore

Magnetic field intensity H =NI/L ampere-turns/meter

We arrive at the following relationship between magnetic flux density *B* and magnetic field intensity as B = μH

Problems:

1. **Given a copper core with: Susceptibility as -9.7*10^6 ,Length of core L = 1 m, Gap length g = .01 m, Cross sectional area A = .1 m, Current I = 10A,N = 5 turns. Find: Bg**

$\mu = \mu_0(1 + \chi_m)$ **, Now using the length, cross sectional area, and permeability of the core we can solve for reluctance** R_c **by:**

$$R_C = \frac{L}{\mu_A} = \frac{1}{1.2566 * 10^{-6} * .1} = \mathbf{7.96 * 10^{6}}$$

Similarly, to get the reluctance of the gap

$$R_g = \frac{g}{\mu_o(\sqrt{A}+g)^2} = \frac{.01}{4\pi\times 10^{-7}(\sqrt{.1}+.01)^2} = 74.8\times 10^3$$

Now recall the equation for the magnetic field of a gap as seen in

$$B_g = \frac{NI}{(R_g+R_c)(\sqrt{A}+G)^2}$$

$$B_g = \frac{5\times 10}{74.8\times 10^3 \times 7.96\times 10^6 \times (\sqrt{.1}+.01)^2}$$

2. **A coils of 200 turns is wound uniformly over a wooden ring having a mean circumference of**

Solution:

First we need to find the permeabilit of copper given by the equation

y

600 mm and a uniform cross sectional area of 500 mm2 . If the current through the coil is 4 A, calculate: (a) the magnetic field strength, (b) the flux density, and (c) the total flux

Answer: 1333 A/m, 1675×10-6 T, 0.8375 mWb

3. A mild steel ring having a cross sectional area of 500 m2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. Calculate: (a) the reluctance of the ring and (b) the current required to produce a flux of 800 mWb in the ring. (Given that mr is about 380).

Answer: 1.677×106 A/Wb, 6.7 A.

Reluctance in Series (Composite Magnetic Circuit)

A magnetic circuit having a number of parts of different magnetic materials and different dimensions carrying the same magnetic field is called a Series Magnetic Circuit. It is also known as Composite Magnetic Circuit. One such circuit is shown in figure.

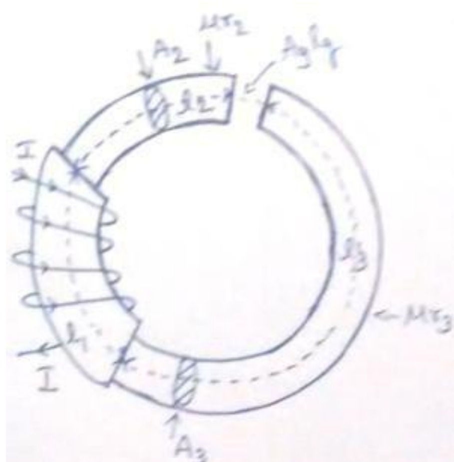


Figure. Reluctance in series

It consist of 3 different magnetic material and one air gap . Since materials are different the permeability are different. Assume that the length and the areas of cross-section are also different . Then the reluctance of each path will be different.

As the reluctance are in series, the total reluctance is the sum of the reluctance of different paths.

Total reluctance = $S = S_1 + S_2 + S_3 + S_g$

Total mmf = flux x reluctance

$$= \phi \times S = \phi\,(l_1/\mu_0\mu_{r1}A_1 + l_2/\mu_0\mu_{r2}A_2 + l_3/\mu_0\mu_{r3}A_3 + l_g/\mu_0A_g)$$

$$= l_1\,\phi / A_1\mu_0\mu_{r1} + l_2\,\phi / A_2\mu_0\mu_{r2} + l_3\,\phi / A_3\mu_0\mu_{r3} + l_g\,\phi / A_g\mu_0$$

$$= l_1B_1/\mu_0\mu_{r1} + l_2\,B_2/\mu_0\mu_{r2} + l_3\,B_3/\mu_0\mu_{r3} + l_g\,B_g/\mu_0$$

Total mmf = $H_1l_1 + H_2l_2 + H_3l_3 + H_gl_g$

Note: The following formulae are used in the above expression

1. **$\phi / A = B$**
2. **$B/\mu_0\mu_r = H$**

Reluctance in Parallel (Parallel Magnetic Circuits)

If a magnetic circuit has 2 or more paths for the magnetic flux, it is called a parallel magnetic flux.

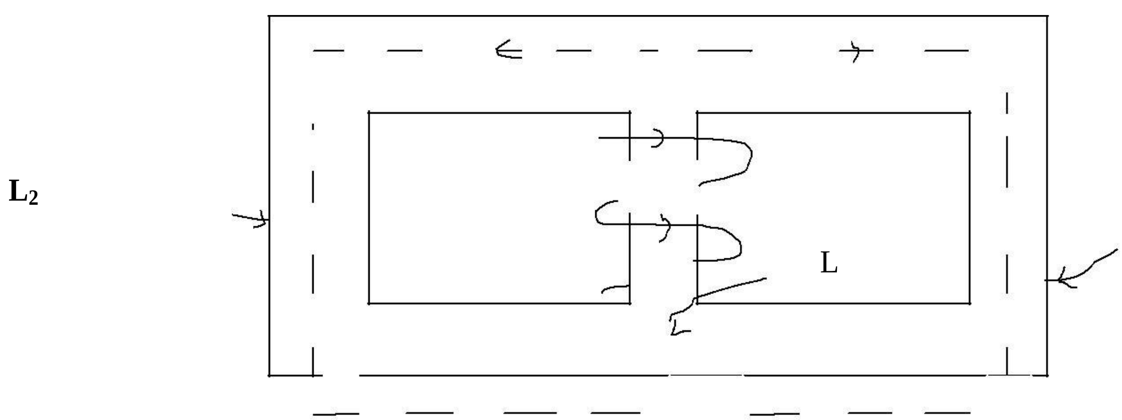


Figure 2.2 Reluctance in Parallel

On the central limb AB, a current carrying coil ia wound. The mmf in the coil sets up a magnetic flux ϕ_1 in the central limb. It is further divided in to 2 paths. They are

1. **The path ADCB which carries flux Φ_2 and**
2. **The path AFEB which carries flux Φ_3**

These 2 path are in parallel. The ampere turns (mmf) for this circuit is equal to the ampere turns required for any one of these paths.

$$Ø1 = Ø2 + Ø3$$

Reluctance of path BA = $S_1 = l_1/\mu_0\mu_{r1}A_1$

Reluctance of path ADCB = $S_2 = l_2/\mu_0\mu_{r2}A_2$

Reluctance of path AFEB = $S_3 = l_3/\mu_0\mu_{r3}A_3$

Mmf required for path ADCB = $Ø_2 \times S_2$

Mmf required for path AFEB = $Ø_3 \times S_3$

Mmf for parallel path = $Ø_2 \times S_2 = Ø_3 \times S_3$
Mmf required for path BA = $Ø_1 \times S_1$

Total mmf required = mmf for path BA + mmf required for path ADCB or path AFEB Total mmf (or) AT = $Ø_1 \times S_1 + Ø_2 \times S_2 = Ø_1 \times S_1 + Ø_3 \times S_3$

Worked Example

1. **Find the ampere turns required to produce a flux of 0.4 milliweber in the airgap of a circular magnetic circuit which has an airgap of 0.5mm. The iron ring has 4sq.cm cross section and 63cm mean length. The relative permeability of iron is 1800 and the leakage co-efficient is 1.15**

Sol. Given Data:

Flux in the airgap	**= Øg= Øuseful= o.4 weber**
Length of airgap lg	**= .5mm**
Cross-section of the iron ring A	**= $4\times10^{-4}m^2$**

Mean length of iron ring = l = 63 cm

Relative permeability of iron = 1800

Leakage co-efficient λ = 1.15

This magnetic circuit has two materials airgap and iron

Total mmf = mmf in airgap + mmf in iron

Flux = mmf/reluctance

Mmf = flux x reluctance

a) For airgap: mmf = Øuseful x S_g

$= 0.4\times10^{-4} \times (lg/\mu_0 A)$

$= 0.4\times10^{-4} ((0.5\times10^{-3})/(4\pi\times10^{-7}\times4\times10^{-4}))$

= 397.88 AT

b) For iron path flux = Øi = λ x Øuseful

$= 1.15\times 0.4\times10^{-3}$

$= 0.46\times10^{-3}$ wb

Reluctance, Si = $(l/(\mu_0 \mu_r A))$

$= 0.63/(4\pi\times10^{-7}\times1800\times4\times10^{-4})$

= 696302.876 AT/Wb

Mmf = $Ø_i$ x S_i

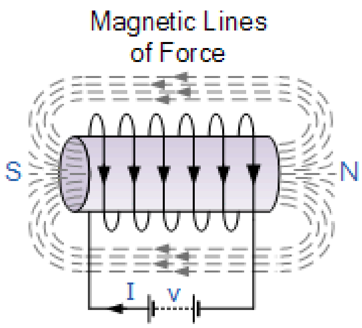
$= 0.46\times10^{-3}\times696302.876$

= 320.29AT

Total ampere turns required : 397.88+320.29 = 718 AT

Electromagnetic Induction

We have seen previously that when a DC current pass through a long straight conductor a magnetising force, H and a static magnetic field, B is developed around the wire. If the wire is then wound into a coil, the magnetic field is greatly intensified producing a static magnetic field around itself forming the shape of a bar magnet giving a distinct North and South pole.



Air-core Hollow Coil

The magnetic flux developed around the coil being proportional to the amount of current flowing in the coils windings as shown. If additional layers of wire are wound upon the same coil with the same current flowing through them, the static magnetic field strength would be increased.

Therefore, the [Magnetic Field Strength]() of a coil is determined by the *ampere turns* of the coil. With more turns of wire within the coil, the greater the strength of the static magnetic field around it.

But what if we reversed this idea by disconnecting the electrical current from the coil and instead of a hollow core we placed a bar magnet inside the core of the coil of wire. By moving this bar magnet -in‖ and -out‖ of the coil a current would be induced into the coil by the physical movement of the magnetic flux inside it.

Likewise, if we kept the bar magnet stationary and moved the coil back and forth within the magnetic field an electric current would be induced in the coil. Then by either moving the wire or changing the magnetic field we can induce a voltage and current within the coil and this process is known as Electromagnetic Induction and is the basic principal of operation of transformers, motors and generators.

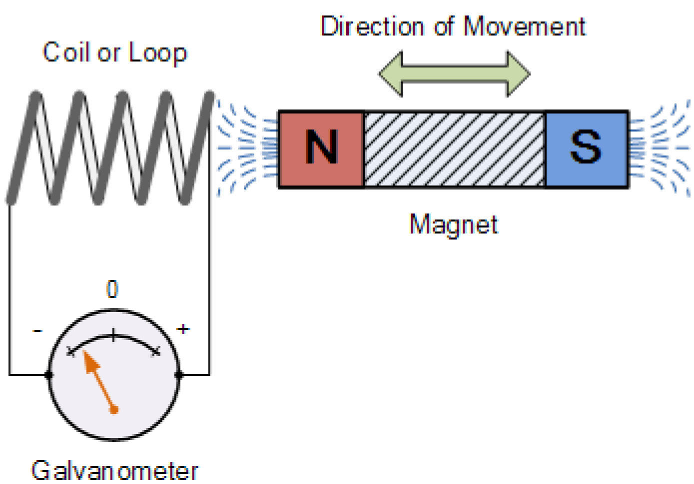
Electromagnetic Induction was first discovered way back in the 1830‘s by Michael Faraday. Faraday noticed that when he moved a permanent magnet in and out of a coil or a single loop of wire it induced an ElectroMotive Force or emf, in other words a Voltage, and therefore a current was produced.

So what Michael Faraday discovered was a way of producing an electrical current in a circuit by using only the force of a magnetic field and not batteries. This then lead to a very important law linking electricity with magnetism, Faraday's Law of Electromagnetic Induction. So how does this work?.

When the magnet shown below is moved -towards‖ the coil, the pointer or needle of the Galvanometer, which is basically a very sensitive centre zero‘ed moving-coil ammeter, will deflect away from its centre position in one direction only. When the magnet stops moving and is held stationary with regards to the coil the needle of the galvanometer returns back to zero as there is no physical movement of the magnetic field.

Likewise, when the magnet is moved -away‖ from the coil in the other direction, the needle of the galvanometer deflects in the opposite direction with regards to the first indicating a change in polarity. Then by moving the magnet back and forth towards the coil the needle of the galvanometer will deflect left or right, positive or negative, relative to the directional motion of the magnet.

Electromagnetic Induction by a Moving Magnet



Likewise, if the magnet is now held stationary and ONLY the coil is moved towards or away from the magnet the needle of the galvanometer will also deflect in either direction. Then the action of moving a coil or loop of wire through a magnetic field induces a voltage in the coil with the magnitude of this induced voltage being proportional to the speed or velocity of the movement.

Then we can see that the faster the movement of the magnetic field the greater will be the induced emf or voltage in the coil, so for Faraday's law to hold true there must be -relative motion‖ or movement between the coil and the magnetic field and either the magnetic field, the coil or both can move.

Faraday's Law of Induction

From the above description we can say that a relationship exists between an electrical voltage and a changing magnetic field to which Michael Faraday's famous law of electromagnetic induction states: -that a voltage is induced in a circuit whenever relative motion exists between a conductor and a magnetic field and that the magnitude of this voltage is proportional to the rate of change of the flux‖.

In other words, Electromagnetic Induction is the process of using magnetic fields to produce voltage, and in a closed circuit, a current.

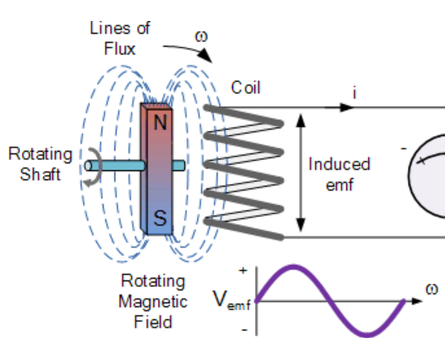
So how much voltage (emf) can be induced into the coil using just magnetism. Well this is determined by the following 3 different factors.

- **1). Increasing the number of turns of wire in the coil. – By increasing the amount of individual conductors cutting through the magnetic field, the amount of induced emf produced will be the sum of all the individual loops of the coil, so if there are 20 turns in the coil there will be 20 times more induced emf than in one piece of wire.**
- **2). Increasing the speed of the relative motion between the coil and the magnet. – If the same coil of wire passed through the same magnetic field but its speed or velocity is increased, the wire will cut the lines of flux at a faster rate so more induced emf would be produced.**
- **3). Increasing the strength of the magnetic field. – If the same coil of wire is moved at the same speed through a stronger magnetic field, there will be more emf produced because there are more lines of force to cut.**

If we were able to move the magnet in the diagram above in and out of the coil at a constant speed and distance without stopping we would generate a continuously induced voltage that would alternate between one positive polarity and a negative polarity producing an alternating or AC output voltage and this is the basic principal of how a Generator works similar to those used in dynamos and car alternators.

In small generators such as a bicycle dynamo, a small permanent magnet is rotated by the action of the bicycle wheel inside a fixed coil. Alternatively, an electromagnet powered by a fixed DC voltage can be made to rotate inside a fixed coil, such as in large power generators producing in both cases an alternating current.

Simple Generator using Magnetic Induction



The simple dynamo type generator above consists of a permanent magnet which rotates around a central shaft with a coil of wire placed next to this rotating magnetic field. As the magnet spins, the magnetic field around the top and bottom of the coil constantly changes between a north and a south pole. This rotational movement of the magnetic field results in an alternating emf being induced into the coil as defined by Faraday's law of electromagnetic induction.

The magnitude of the electromagnetic induction is directly proportional to the flux density, β the number of loops giving a total length of the conductor, l in meters and the rate or velocity, ν at which the magnetic field changes within the conductor in meters/second or m/s, giving by the motional emf expression:

Faraday's Motional emf Expression

$$\varepsilon = -\beta.\ell.\upsilon \quad \text{volts}$$

If the conductor does not move at right angles (90°) to the magnetic field then the angle θ° will be added to the above expression giving a reduced output as the angle increases:

$$\varepsilon = -\beta.\ell.\upsilon \sin\theta \quad \text{volts}$$

Lenz's Law of Electromagnetic Induction

Faraday's Law tells us that inducing a voltage into a conductor can be done by either passing it through a magnetic field, or by moving the magnetic field past the conductor and that if this conductor is part of a closed circuit, an electric current will flow. This voltage is called an induced emf as it has been induced into the conductor by a changing magnetic field due to electromagnetic induction with the negative sign in Faraday's law telling us the direction of the induced current (or polarity of the induced emf).

But a changing magnetic flux produces a varying current through the coil which itself will produce its own magnetic field as we saw in the Electromagnets tutorial. This self- induced emf opposes the change that is causing it and the faster the rate of change of current the greater is the opposing emf. This self-induced emf will, by Lenz's law oppose the change in current in the coil and because of its direction this self-induced emf is generally called a back-emf.

Lenz's Law states that: ‖ the direction of an induced emf is such that it will always opposes the change that is causing it‖. In other words, an induced current will always OPPOSE the motion or change which started the induced current in the first place and this idea is found in the analysis of Inductance.

Likewise, if the magnetic flux is decreased then the induced emf will oppose this decrease by generating and induced magnetic flux that adds to the original flux.

Lenz's law is one of the basic laws in electromagnetic induction for determining the

direction of flow of induced currents and is related to the law of conservation of energy.

According to the law of conservation of energy which states that the total amount of energy in the universe will always remain constant as energy can not be created nor destroyed. Lenz‘s law is derived from Michael Faraday‘s law of induction.

One final comment about Lenz‘s Law regarding electromagnetic induction. We now know that when a relative motion exists between a conductor and a magnetic field, an emf is induced within the conductor.

But the conductor may not actually be part of the coils electrical circuit, but may be the coils iron core or some other metallic part of the system, for example, a transformer. The induced emf within this metallic part of the system causes a circulating current to flow around it and this type of core current is known as an Eddy Current.

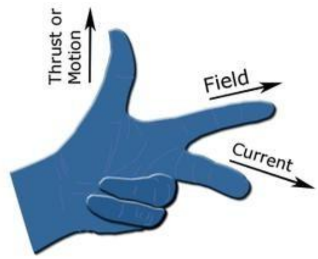
Eddy currents generated by electromagnetic induction circulate around the coils core or any connecting metallic components inside the magnetic field because for the magnetic flux they are acting like a single loop of wire. Eddy currents do not contribute anything towards the usefulness of the system but instead they oppose the flow of the induced current by acting like a negative force generating resistive heating and power loss within the core. However, there are electromagnetic induction furnace applications in which only eddy currents are used to heat and melt ferromagnetic metals.

Fleming’s Rule

Whenever a current carrying conductor comes under a magnetic field, there will be force acting on the conductor and on the other hand, if a conductor is forcefully brought under a magnetic field, there will be an induced current in that conductor. In both of the phenomenon, there is a relation between magnetic field, current and force. This relation is directionally determined by Fleming Left Hand rule and Fleming Right Hand rule respectively. 'Directionally' means these rules do not show the magnitude but show the direction of any of the three parameters (magnetic field, current, force) if the direction of other two are known. Fleming Left Hand rule is mainly applicable for electric motor and Fleming Right Hand rule is mainly applicable for electric generator. In late 19th century, John Ambrose Fleming introduced both these rules and as per his name, the rules are well known as Fleming left and right hand rule.

Fleming’s Left Hand Rule

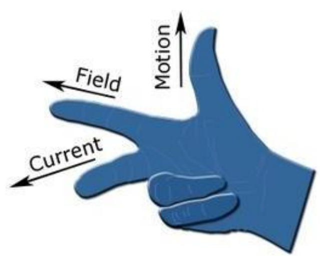
According to Fleming's left hand rule, if the thumb, fore-finger and middle finger of left hand are stretched perpendicular to each other as shown the figure above, and if fore finger represent the direction of magnetic field, the middle finger represents the direction of current, then the thumb represents the direction of force.



Fleming's left hand rule is applicable for electric motors. Whenever a current carrying conductor is placed in a magnetic field, the conductor experiences a force.

Fleming's Right Hand Rule

According to Fleming's right hand rule, the thumb, fore finger and middle finger of right hand are stretched perpendicular to each other as shown in the figure at right, and if thumb represents the direction of the movement of conductor, fore-finger represents direction of the magnetic field, then the middle finger represents direction of the induced current.



Fleming's right hand rule is applicable for electrical generators. As per Faraday's law of electromagnetic induction, whenever a conductor is moved in an electromagnetic field, and closed path is provided to the conductor, current gets induced in it.

Types of induced emf

There are two type of induced emf based on the nature, they are:-

- Dynamically induced emf
- Statically induced emf:

Further classified into:

- Self induced emf
- Mutually induced emf

Dynamically induced emf

An emf induced due to a physical movement of either conductor or flux. Here field is stationary and conductors cut across it. Either the coil or magnet moves. Magnitude of dynamically induced emf is as shown the conductor of length l is placed in the magnetic

field produced by a permanent magnet. The conductor moves in a plane which is parallel to the plane of the magnetic flux. Therefore induced emf is zero. Now if the plane of direction of motion of the conductor is perpendicular to the plane of magnetic flux then the induced emf is maximum. So the expression for magnitude of emf is given by: E =blv sin θ

2 Statically induced emf

Due to ac there is a change in the coil current with respect to time. This will result in alternating flux. Hence there will be change in flux w.r.t. time. This change in flux w.r.t. time will induce emf in this coil which is known as statically induced emf.

Self Induce E.M.F.

Consider a coil having N turns and carrying current I when Switch S is in closed position.The flux produced by the coil links with coil itself. The total flux linkages of coil will be NΦ Wb-turns. Now if the current I is changed with the help of variable resistance, then flux produced will also change, due to which flux linkages will also change.

Hence according to Faraday's law, due to rate of chang of flux linkages there will be induced emf in the coil. So without physically moving coil or flux there is induced emf in the coil. The phenomenon is called self induction

The emf induced in a coil due to the change of its own flux linked with it is called self induced emf.

Self Inductance

According to Lenz's law the direction of this induced emf will be so as to oppose the cause producing it. The cause is the current I hence the self induced emf will try to set up a current which is in opposite direction to that of current I. When current is increased, self induced emf reduces the current tries to keep it to its original value. If current is decreased, self induced emf increases the current and tries to maintain it back to its original value. So any change in current through coil is opposed by the coil.

This property of the coil which opposes any change in the current passing through it is called Self Inductance or only inductance.

Magnitude of Self induce EMF

From the Faraday's law of electromagnetic induction, self induced emf can be expressed as,

$$e = -N\frac{d\phi}{dt}$$

Negative sign indicates that direction of this emf is opposing change in current due to which it exists.

The flux can be expressed as,

$$\phi = (flux/ampere) \times ampere = \frac{\phi}{I} \times I$$

Now for a circuit, as long as permeability μ is constant, ratio of flux to current remains constant.

$$\therefore Rate\ of\ change\ of\ flux = \frac{\phi}{I} \times rate\ of\ change\ of\ current$$

$$\therefore \frac{d\phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$= -\left(\frac{N\phi}{I}\right)\frac{dI}{dt}$$

e

The constant $\frac{N\phi}{I}$ in this expression is nothing but the quantitative measure of the property due to which coil opposes any change in current. This constant is called coefficient of self inductance and denoted by ‗L‘

$$\therefore L = \frac{N\phi}{I}$$

Expression for self inductance ‘L’

$$L = \frac{N\phi}{I}$$

$$\phi = \frac{mmf}{\text{Reluctance}} = \frac{NI}{S}$$

$$L = \frac{N.NI}{I.S}$$

$$L = \frac{N^2}{s}\ henries$$

$$Now,\ S = \frac{l}{\mu a}$$

$$L = \frac{N^2}{\left(\frac{l}{\mu a}\right)}$$

$$L = \frac{N^2 \mu a}{l} = \frac{N^2 \mu_0 \mu_r a}{l}\ henries$$

Mutually induced EMF

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced emf in the second coil, and then such an emf is called mutually induced emf.

Magnitude of Mutually Induced EMF

Let

N_1 = Number of turns of coil A
N_2= Number of turns of coil B
I_1 = current flowing through coil A
Φ_1 = flux produce due to current I_1 in webers Φ_2
= flux linking with coil B

According to Faradays law , the induced emf in coil B is ,

$$e_2 = -N_2 \frac{d\phi_2}{dt}$$

Negative sign indicates that this emf will set up a current which will oppose the change of flux linking with it.

Now

$$\phi_2 = \frac{\phi_2}{I_1} \times I_1$$

If permeability of the surrounding is assumed constant then φ_2/I_1 is constant.

$$\therefore \text{Rate of Change of } \phi_2 = \frac{\phi_2}{I_1} \times \text{Rate of change of current } I_1$$

$$\therefore \frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$e_2 = -N_2 \cdot \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$e_2 = -\left(\frac{N_2\phi_2}{I_1}\right) \cdot \frac{dI_1}{dt}$$

This $\left(\frac{N_2\phi_2}{I_1}\right)$ is called coefficient of mutual inductance denoted by M.

$$\therefore e_2 = -M \frac{dI_1}{dt} \text{ volts}$$

Coefficient of mutual inductance is defined as the property by which emf gets induced in the second coil because of change in current through first coil.

Analogy of Electric and Magnetic Circuits

Electric Circuit	Magnetic Circuit
Path traced by the current is known as	Path traced by the magnetic flux is called
electric current.	as magnetic circuit.
EMF is the driving force in the electric circuit. The unit is Volts.	MMF is the driving force in the magnetic circuit. The unit is ampere turns.
There is a current I in the electric circuit which is measured in amperes.	There is flux φ in the magnetic circuit which is measured in the weber.
The flow of electrons decides the current in conductor.	The number of magnetic lines of force decides the flux.
Resistance (R) oppose the flow of the current. The unit is Ohm	Reluctance (S) is opposed by magnetic path to the flux. The Unit is ampere turn/weber.
$R= \rho.l/a$ Directly proportional to l. Inversely proportional to a. Depends on nature of material.	$S=l/(\mu_0\mu_r a)$. Directly proportional to l. Inversely proportional to $\mu = \mu_0\mu_r$. Inversely proportional to a
The current I = EMF/ Resistance	The Flux = MMF/ Reluctance
The current density	The flux density
Kirchhoff current law and voltage law is applicable to the electric circuit.	Kirchhoff mmf law and flux law is applicable to the magnetic flux.

Questions

PART A

1. What is a Series Magnetic circuit?
2. What is a parallel magnetic circuit?
3. Give the expression for the Total MMF of a composite magnetic circuit.
4. Give the expression for the Total MMF of a parallel magnetic circuit.
5. What is reluctance?
6. Define magneto motive force.
7. Define magnetic flux
8. What is leakage factor?
9. Give the correlation between flux, mmf and reluctance.
10. Write the correlation between magnetic flux density and magnetic field intensity.
11. Define magnetic flux density.
12. State Faraday's law of Electromagnetic induction.
13. State Fleming right hand rule.

14. **State Lenz's law.**
15. **What is Self inductance and Mutual inductance?**
16. **Reluctance and flux density are the two terms in magnetic circuits. What are the corresponding analogous terms in an electrical circuit?**
17. **A conductor of length 1m moves at right angles to a uniform magnetic field of flux density 1.5 wb/m^2, with a velocity of 50 m/sec. calculate the emf induced in it.**
18. **Write the expression for energy stored in magnetic field.**

PART B

1. **Explain in detail the analysis of simple and composite magnetic circuits.**

2. **a) Compare Magnetic circuits & Electric circuits.**
   b) An iron ring has a cross-sectional area of 400mm^2 and a mean diameter of 20 cm. It is wound with 500 turns. If the value of relative permeability is 250, find the total flux set up in the ring. The coil resistance is 480 ohms and the supply voltage is 240 V.

3. **A coil is wound uniformly with 300 turns over a steel ring of relative permeability 900 having a mean circumference of 40 cm and a cross- sectional area of 5cm^2. If the coil has a resistance of 100 ohms and is connected to a 250 V dc supply, calculate (i) the coil mmf (ii) the magnetic field intensity (iii) total flux (iv) permeance of the ring.**

4. **A no of turns in a coil is 250. When a current of 20A flows in this coil, the flux in the coil is milliwebers. When this current is reduced to zero in 2 milliseconds, the voltage induced in the coil lying in the vicinity of coil is 63.75V. If the co-efficient of coupling between the coil is 0.75 Find (i) Self inductance of the two coils**
   (ii) Mutual inductance
   (iii) Number of turns in the second coil

5. **a) A mild steel ring having a cross-sectional area of 500mm^2 and a mean circumference of 400mm has a coil of 200 turns wound uniformly around it. Calculate (a) reluctance of the ring and (b) the current required to produce a flux of 800μwb in the ring. Assume relative permeability of mild steel to be 380.**

6. **(a) Derive the expression for self and mutual inductance**
   (b) Explain the types of induced emfs.

7. **An iron ring 30 cm mean diameter is made of square iron of 2 cm x 2 cm across section**

and is uniformly wound with 400 turns of wire of 2mm2 cross section. Calculate the value of the
self inductance of the coil. Assume μ_r = 800.

8. **A ring shaped electromagnet has an air gap of 6mm long and 20cm2 in area, the mean length of the core being 50cm and its cross-section is 10cm2. Calculate the ampere turns required to produce a flux density of 0.5Wb/m2 in the air gap. Assume the permeability of iron as 1800.**

9. **A circular iron ring, having a cross – sectional area Of 10cm2 and a length of 4 cm in iron, has an air gap of 0.4 mm made by a saw-cut. The relative permeability of iron is 1000 and permeability of free space is 4 x 10-7 H/m. The ring is wound with a coil of 2000 turns and carries 2mA current. Determine the air gap flux neglecting leakage and fringing.**

10. **State and explain the Faraday's laws of electromagnetic induction. Also give the Fleming's Right hand and Left hand rules.**

SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING
DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

UNIT - I I

DC Machines and Transformers – SEEA1202

Unit 2 – DC GENERATOR

The electrical machines deals with the energy transfer either from mechanical to electrical form or from electrical to mechanical form, this process is called electromechanical energy conversion. An electrical machine which converts mechanical energy into electrical energy is called an electric generator while an electrical machine which converts electrical energy into the mechanical energy is called an electric motor. A DC generator is built utilizing the basic principle that emf is induced in a conductor when it cuts magnetic lines of force. A DC motor works on the basic principle that a current carrying conductor placed in a magnetic field experiences a force.

Working principle:

All the generators work on the principle of dynamically induced emf. The change in flux associated with the conductor can exist only when there exists a relative motion between the conductor and the flux. The relative motion can be achieved by rotating the conductor w.r.t flux or by rotating flux w.r.t conductor. So, a voltage gets generated in a conductor as long as there exists a relative motion between conductor and the flux. Such an induced emf which is due to physical movement of coil or conductor w.r.t flux or movement of flux w.r.t coil or conductor is called dynamically induced emf. Whenever a conductor cuts magnetic flux, dynamically induced emf is produced in it according to Faraday's laws of Electromagnetic Induction. This emf causes a current to flow if the conductor circuit is closed.

So, a generating action requires the following basic components to exist.

1. **The conductor or a coil**
2. **Flux**
3. **Relative motion between the conductor and the flux.**

In a practical generator, the conductors are rotated to cut the magnetic flux, keeping flux stationary. To have a large voltage as output, a number of conductors are connected together in a specific manner to form a winding. The winding is called armature winding of a dc machine and the part on which this winding is kept is called armature of the dc machine.

The magnetic field is produced by a current carrying winding which is called field winding.

The conductors placed on the armature are rotated with the help of some external device. Such an external device is called a prime mover.

The commonly used prime movers are diesel engines, steam engines, steam turbines, water turbines etc.

The purpose of the prime mover is to rotate the electrical conductor as required by Faraday's laws

The direction of induced emf can be obtained by using Flemings right hand rule. The magnitude of induced emf = $e = BLV \sin Ø = E_m \sin Ø$

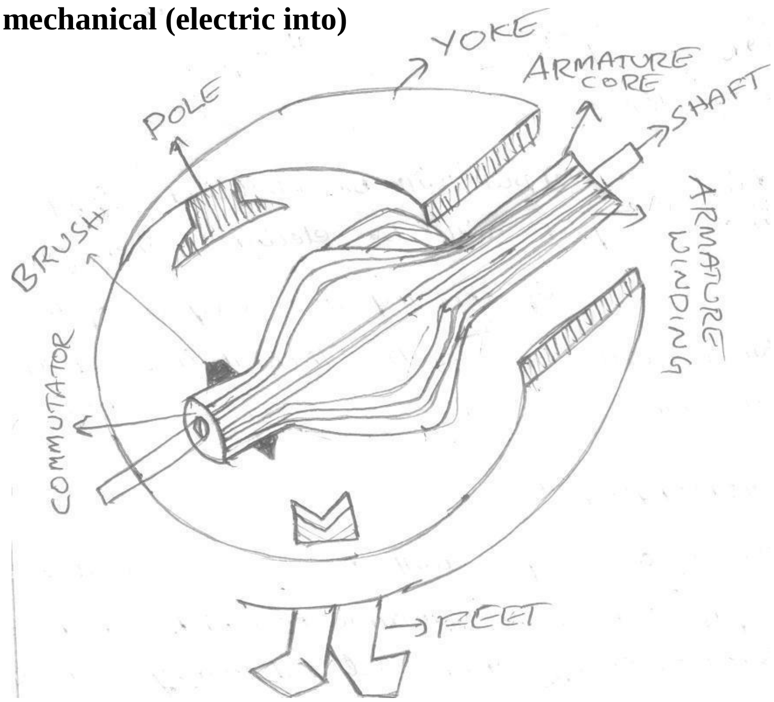
Nature of induced elf:

The nature of the induced emf for a conductor rotating in the magnetic field is alternating. As conductor rotates in a magnetic field, the voltage component at various positions is different. Hence the basic nature of induced emf in the armature winding in case of dc generator is alternating. To get dc output which is unidirectional, it is necessary to rectify the alternating induced emf. A device which is used in dc generator to convert alternating induced emf to unidirectional dc emf is called commutator.

Construction of DC machines :

A D. C. machine consists of two main parts

1. Stationary part: It is designed mainly for producing a magnetic flux.
2. Rotating part: It is called the armature, where mechanical energy is converted into electrical (electrical generate) or conversely electrical energy into mechanical (electric into)



Parts of a Dc Generator:

1) **Yoke**
2) **Magnetic Poles**

a) **Pole core**
b) **Pole Shoe**

3) **Field winding**
4) **Armature Core**
5) **Armature winding**
6) **Commutator**
7) **Brushes and Bearings**

The stationary parts and rotating parts are separated from each other by an air gap. The stationary part of a D. C. machine consists of main poles, designed to create the magnetic flux, commutating poles interposed between the main poles and designed to ensure spark less operation of the brushes at the commutator and a frame / yoke. The armature is a cylindrical body rotating in the space between the poles and comprising a slotted armature core, a winding inserted in the armature core slots, a commutator and brush

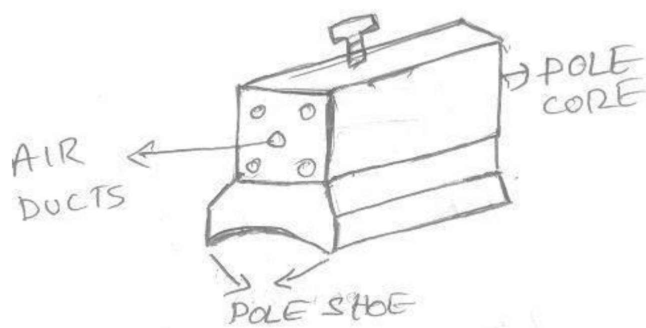
Yoke:

1. **It saves the purpose of outermost cover of the dc machine so that the insulating materials**

   get protected from harmful atmospheric elements like moisture, dust and various gases like SO_2, acidic fumes etc.
2. **It provides mechanical support to the poles.**
3. **It forms a part of the magnetic circuit. It provides a path of low reluctance for magnetic flux. Choice of material: To provide low reluctance path, it must be made up of some magnetic material. It is prepared by using cast iron because it is the cheapest. For large machines rolled steel or cast steel, is used which provides high permeability i.e., low reluctance and gives good mechanical strength.**

Poles: Each pole is divided into two parts

a) pole core　　　　b) pole shoe



Functions:

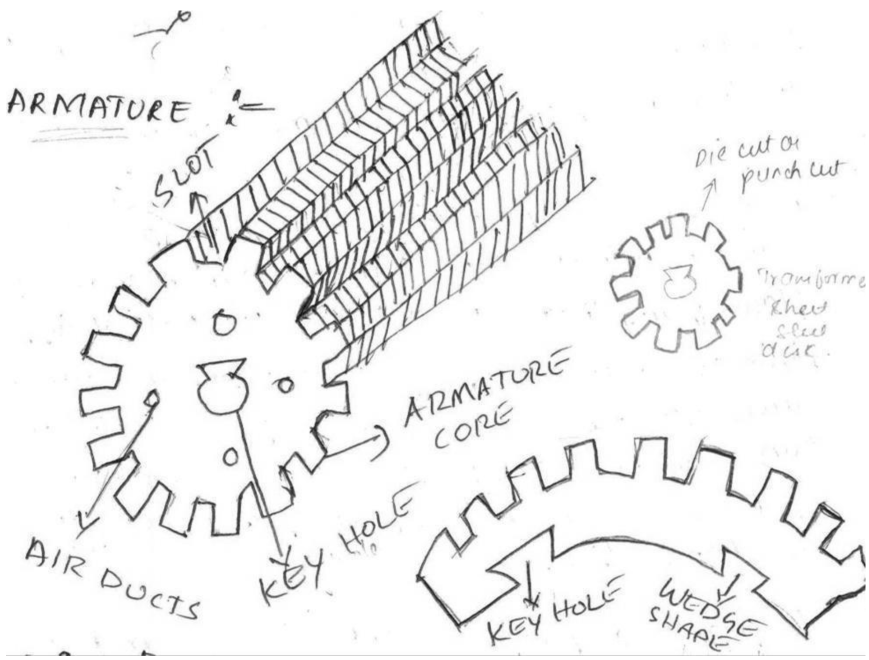
1. **Pole core basically carries a field winding which is necessary to produce the flux.**
2. **It directs the flux produced through air gap to armature core to the next pole.**
3. **Pole shoe enlarges the area of armature core to come across the flux, which is necessary to produce larger induced emf. To achieve this, pole core has been given a particular shape.**

Choice of material: It is made up of magnetic material like cast iron or cast steel. As it requires a definite shape and size, laminated construction is used. The laminations of required size and shape are stamped together to get a pole which is then bolted to yoke.

Armature: It is further divided into two parts namely,

(1) Armature core (2) Armature winding.

Armature core is cylindrical in shape mounted on the shaft. It consists of slots on its periphery and the air ducts to permit the air flow through armature which serves cooling purpose.



Functions:

1. **Armature core provides house for armature winding i.e., armature conductors.**
2. **To provide a path of low reluctance path to the flux it is made up of magnetic material like cast iron or cast steel.**

Choice of material: As it has to provide a low reluctance path to the flux, it is made up of magnetic material like cast iron or cast steel.

It is made up of laminated construction to keep eddy current loss as low as possible.

A single circular lamination used for the construction of the armature core is shown below.

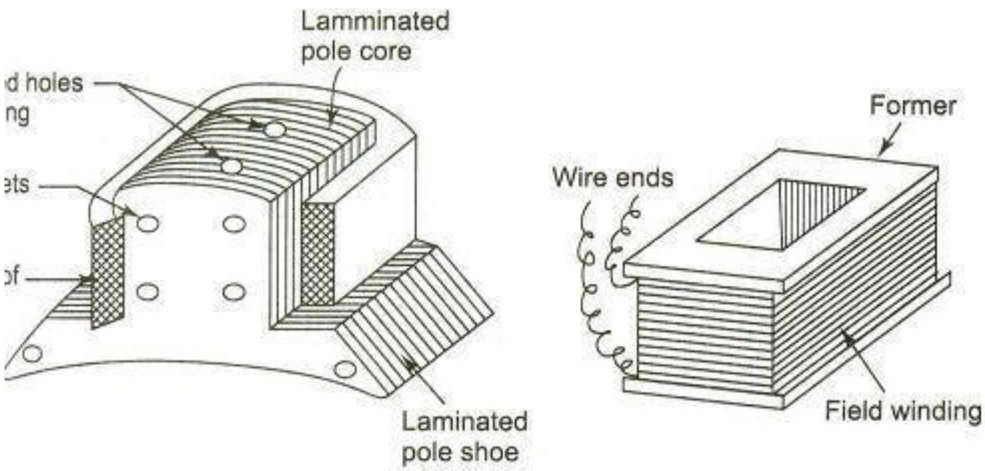
2. Armature winding: Armature winding is nothing but the inter connection of the armature conductors, placed in the slots provided on the armature core. When the armature is rotated, in case of generator magnetic flux gets cut by armature conductors and emf gets induced in them.

Function:

1. **Generation of emf takes place in the armature winding in case of generators.**
2. **To carry the current supplied in case of dc motors.**
3. **To do the useful work it the external circuit.**

Choice of material : As armature winding carries entire current which depends on external load, it has to be made up of conducting material, which is copper.

Field winding: The field winding is wound on the pole core with a definite direction.



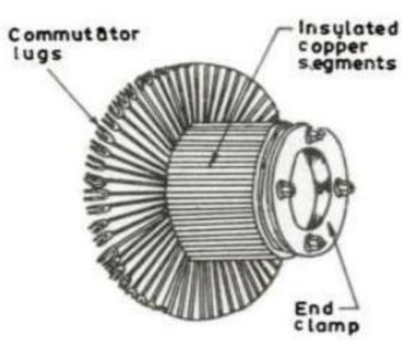
Functions: To carry current due to which pole core on which the winding is placed behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field i.e. exciting the pole as electromagnet it is called 'Field winding' or 'Exciting winding'.

Choice of material : As it has to carry current it should be made up of some conducting material like the aluminum or copper.

But field coils should take any type of shape should bend easily, so copper is the proper choice. Field winding is divided into various coils called as field coils. These are connected in series with each other and wound in such a direction around pole cores such that alternate N and S poles are formed.

Commutator: The rectification in case of dc generator is done by device called as commutator.

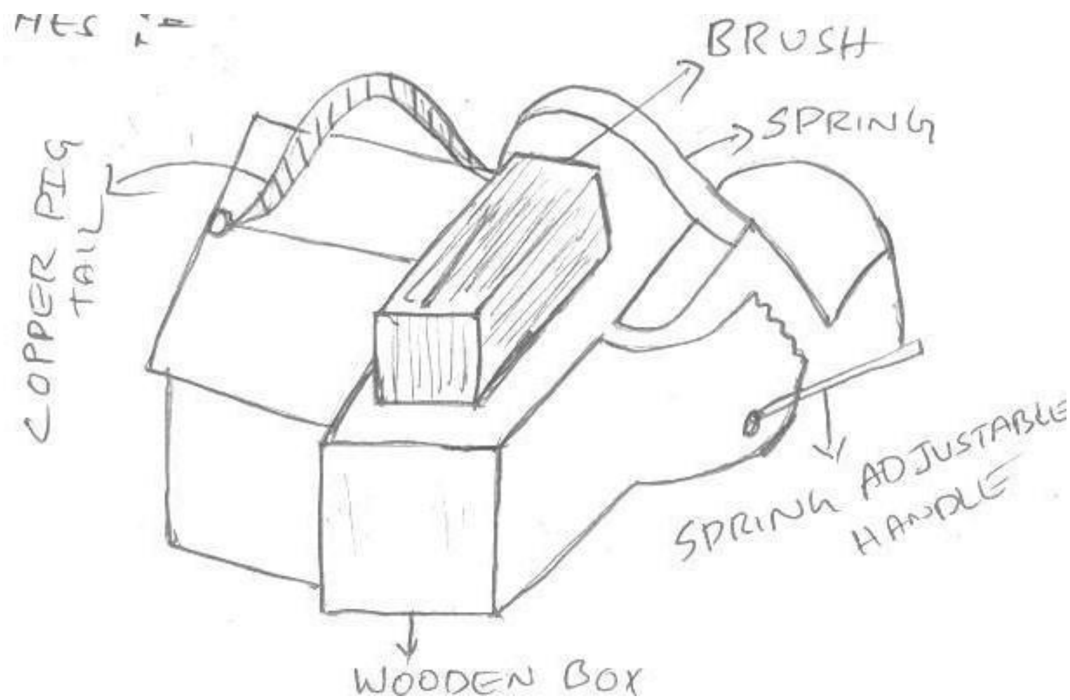


Functions: 1. To facilitate the collection of current from the armature conductors.

2. **To convert internally developed alternating emf to in directional (dc) emf**
3. **To produce unidirectional torque in case of motor.**

Choice of material: As it collects current from armature, it is also made up of copper segments. It is cylindrical in shape and is made up of wedge shaped segments which are insulated from each other by thin layer of mica.

Brushes and brush gear: Brushes are stationary and rest on the surface of the Commutator. Brushes are rectangular in shape. They are housed in brush holders, which are usually of box type. The brushes are made to press on the commutator surface by means of a spring, whose tension can be adjusted with the help of lever. A flexible copper conductor called pigtail is used to connect the brush to the external circuit.



Functions: To collect current from commutator and make it available to the stationary external circuit.

Choice of material: Brushes are normally made up of soft material like carbon.

Bearings: Ball-bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

Working of DC generator:

The generator is provided with a magnetic field by sending dc current through the field coils mounted on laminated iron poles and through armature winding.

A short air gap separates the surface of the rotating armature from the stationary pole surface. The magnetic flux coming out of one or more worth poles crossing the air gap , passes through the armature near the gap into one or more adjacent south poles.

The direct current leaves the generator at the positive brush, passes through the circuit and returns to the negative brush.

The terminal voltage of a dc generator may be increased by increasing the current in the field coil and may be reduced by decreasing the current.

Generators are generally run at practically constant speed by their prime mores.

Types of armature winding:

Armature conductors are connected in a specific manner called as armature winding and according to the way of connecting the conductors; armature winding is divided into two types.

Lap winding: In this case, if connection is started from conductor in slot 1 then the connections overlap each other as winding proceeds, till starting point is reached again.

There is overlapping of coils while proceeding. Due to such connection, the total number of conductors get divided into 'P' number of parallel paths, where

P = number of poles in the machine.

Large number of parallel paths indicates high current capacity of machine hence lap winding is pertained for high current rating generators.

Wave winding: In this type, winding always travels ahead avoiding over lapping. It travels like a progressive wave hence called wave winding.

Both coils starting from slot 1 and slot 2 are progressing in wave fashion.

Due to this type of connection, the total number of conductors get divided into two number of parallel paths always, irrespective of number of poles of machine.

As number of parallel paths is less, it is preferable for low current, high voltage capacity generators.

Sl. No.	Lap winding	Wave winding
1.	Number of parallel paths (A) = poles (P)	Number of parallel paths (A) = 2 (always)
2.	Number of brush sets required is equal to number of poles	Number of brush sets required is always equal to two
3.	Preferable for high current, low voltage capacity generators	Preferable for high current, low current capacity generators
4.	Normally used for generators of capacity more than 500 A	Preferred for generator of capacity less than 500 A.

EMF equation of a generator

Let P = number of poles

Ø = flux/pole in webers

Z = total number of armature conductors.

= number of slots x number of conductors/slot

N = armature rotation in revolutions (speed for armature) per minute (rpm) A = No. of parallel paths into which the ‘z’ no. of conductors are divided. E = emf induced in any parallel path

Eg = emf generated in any one parallel path in the armature.

Average emf generated/conductor = dØ/dt volt Flux current/conductor in one revolution

dt = d x p

In one revolution, the conductor will cut total flux produced by all poles = d x p No. of revolutions/second = N/60

Therefore, Time for one revolution, dt = 60/N second

According to Faraday’s laws of Electromagnetic Induction, emf/dt=□ generated/conductor = dØ x p x N / 60 volts

This is emf induced in one conductor. For a simplex wave-wound generator No. of parallel paths = 2

No. of conductors in (series)in one path = Z/2

EMF generated/path = ØPN/60 x Z/2 = ØZPN/120 volt

For a simple lap-wound generator Number of parallel paths = P

Number of conductors in one path = Z/P

EMF generated/path = ØPN/60 (Z/P)
= ØZN/60 A = 2 for simplex – wave winding

A =P for simplex lap-winding

Armature Reaction and Commutation

Introduction

In a d.c. generator, the purpose of field winding is to produce magnetic field (called main flux) whereas the purpose of armature winding is to carry armature current. Although the armature winding is not provided for the purpose of producing a magnetic field, nevertheless the current in the armature winding will also produce magnetic flux (called armature flux). The armature flux distorts and weakens the main flux posing problems for the proper operation of the d.c. generator. The action of armature flux on the main flux is called armature
reaction.

2.1 Armature Reaction

So far we have assumed that the only flux acting in a d.c. machine is that due to the main poles called main flux. However, current flowing through armature conductors also creates a magnetic flux (called armature flux) that distorts and weakens the flux coming from the poles. This distortion and field weakening takes place in both generators and motors. The action of armature flux on the main flux is known as armature reaction.
The phenomenon of armature reaction in a d.c. generator is shown in Fig.(2.1)

Only one pole is shown for clarity. When the generator is on no-load, a small current flowing in the armature does not appreciably affect the main flux f1 coming from the pole [See Fig 2.1 (i)]. When the generator is loaded, the current flowing through armature conductors sets up flux f1. Fig. (2.1) (ii) shows flux due to armature current alone. By superimposing f1 and f2, we obtain the resulting flux f3 as shown in Fig. (2.1) (iii). Referring to Fig (2.1) (iii), it is clear that flux density at; the trailing pole tip (point B) is increased while at the leading pole tip (point

4. it is decreased. This unequal field distribution produces the following
two effects: The main flux is distorted.

Due to higher flux density at pole tip B, saturation sets in. Consequently, the increase in flux at pole tip B is less than the decrease in flux under pole tip A. Flux f3 at full load is, therefore, less than flux f1 at no load. As we shall see, the weakening of flux due to armature reaction depends upon the position of brushes.

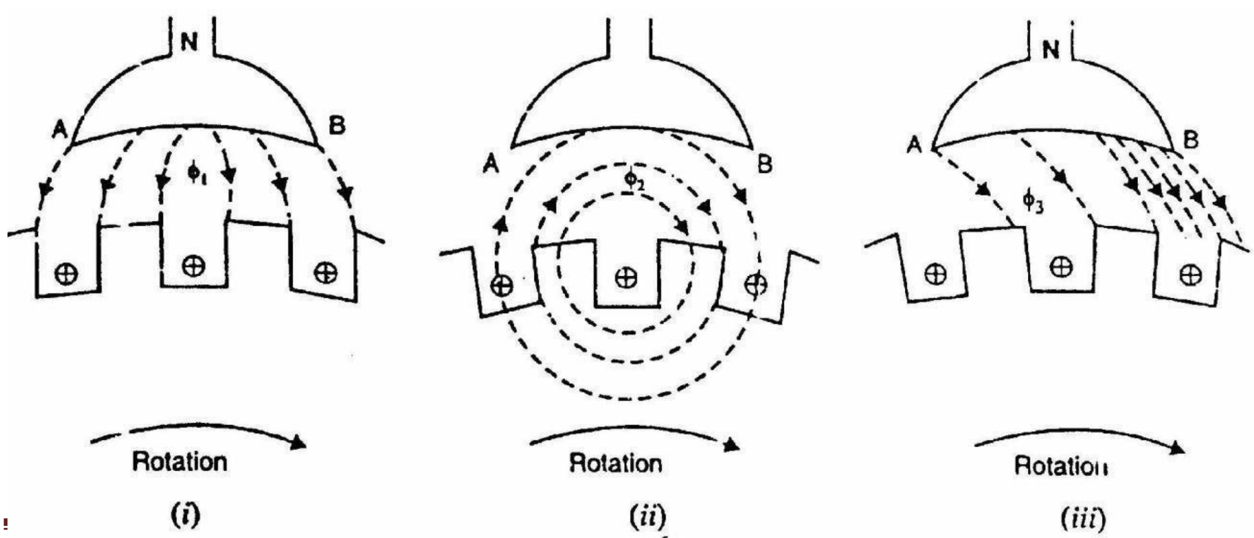


Fig. (2.1)
Geometrical and Magnetic Neutral Axes

4. The geometrical neutral axis (G.N.A.) is the axis that bisects the angle between the centre line of

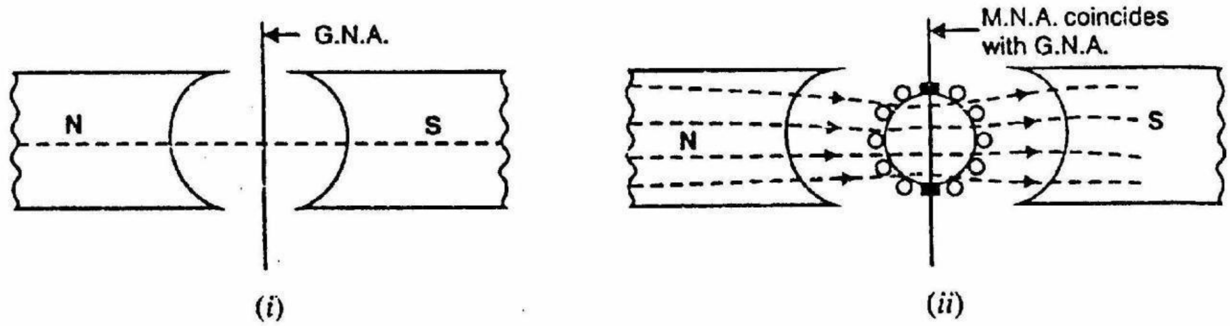


Fig. (2.2)

4. **The magnetic neutral axis (M. N. A.) is the axis drawn perpendicular to the mean direction of the flux passing through the centre of the armature. Clearly, no e.m.f. is produced in the armature conductors along this axis because then they cut no flux. With no current in the armature conductors, the M.N.A. coincides with G, N. A. as shown in Fig. (2.2).**

5. **In order to achieve sparkless commutation, the brushes must lie along M.N.A.**

Explanation of Armature Reaction

With no current in armature conductors, the M.N.A. coincides with G.N.A. However, when current flows in armature conductors, the combined action of main flux and armature flux shifts the
M.N.A. from G.N.A. In case of a generator, the M.N.A. is shifted in the direction of rotation of the machine. Inorder to achieve sparkless commutation, the brushes have to be moved along the new M.N.A. Under such a condition, the armature reaction produces the following two effects:

1. **It demagnetizes or weakens the main flux.**
2. **It cross-magnetizes or distorts the main flux.**

Let us discuss these effects of armature reaction by considering a 2-pole generator (though the following remarks also hold good for a multipolar generator).

- **(i)** **Fig. (2.3) (i) shows the flux due to main poles (main flux) when the armature conductors carry no current. The flux across the air gap is uniform. The m.m.f. producing the main flux is represented in magnitude and direction by the vector OFm in Fig. (2.3) (i). Note that OFm is perpendicular to G.N.A.**
- **(ii)** **Fig. (2.3) (ii) shows the flux due to current flowing in armature conductors alone (main poles unexcited). The armature conductors to the left of G.N.A. carry current "in" (′) and those to the right carry current "out" (•). The direction of magnetic lines of force can be found by cork screw rule. It is clear that armature flux is directed downward parallel to the brush axis. The m.m.f. producing the armature flux is represented in magnitude and direction by the vector OFA in Fig. (2.3) (ii).**
- **(iii)** **Fig. (2.3) (iii) shows the flux due to the main poles and that due to current in armature conductors acting together. The resultant m.m.f. OF is the vector sum of OFm and OFA as shown in Fig. (2.3) (iii). Since M.N.A. is always perpendicular to the resultant m.m.f., the M.N.A. is shifted through an angle q. Note that M.N.A. is shifted in the direction of rotation of the generator.**
- **(iv)** **In order to achieve sparkless commutation, the brushes must lie along the M.N.A. Consequently, the brushes are shifted through an angle q so as to lie along the new**

M.N.A. as shown in Fig. (2.3) (iv). Due to brush shift, the m.m.f. FA of the armature is also rotated through the same angle q. It is because some of the conductors which were earlier under N-pole now come under S-pole and vice-versa. The result is that armature
m.m.f. FA will no longer be vertically downward but will be rotated in the direction of rotation through an angle q as shown in Fig. (2.3) (iv). Now FA can be resolved into

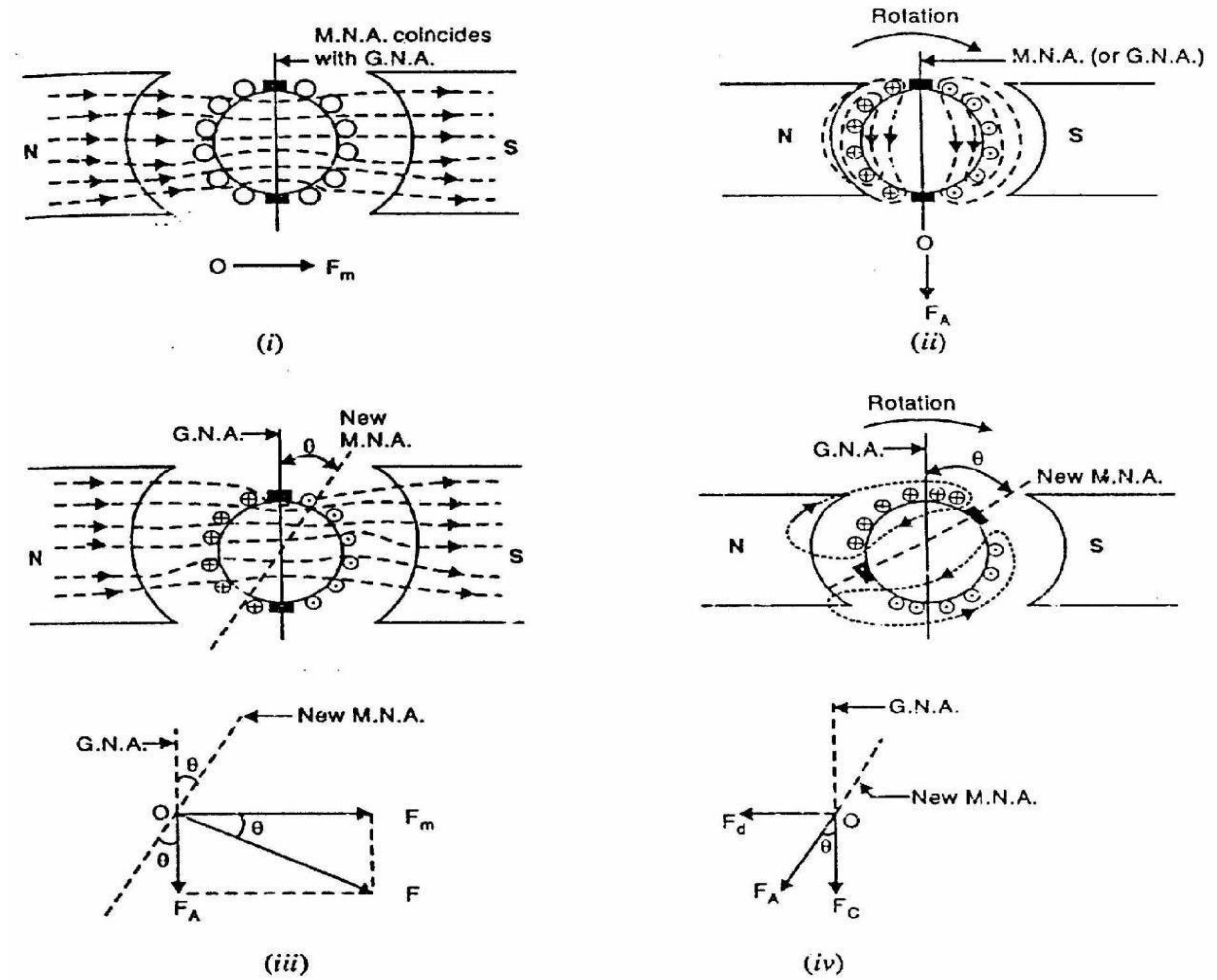


Fig. (2.3)

(a) **The component Fd is in direct opposition to the m.m.f. OFm due to main poles. It has a demagnetizing effect on the flux due to main poles. For this reason, it is called the demagnetizing or weakening component of armature reaction.**

(b) **The component Fc is at right angles to the m.m.f. OFm due to main poles. It distorts the main field. For this reason, it is called the cross magnetizing or distorting component of armature reaction. It**

may be noted that with the increase of armature current, both demagnetizing and distorting effects will increase.

Conclusions

(i) **With brushes located along G.N.A. (i.e., q = 0°), there is no demagnetizing component of armature reaction (Fd = 0). There is only distorting or cross magnetizing effect of armature reaction.**

(ii) **With the brushes shifted from G.N.A., armature reaction will have both demagnetizing and distorting effects. Their relative magnitudes depend on the amount of shift. This shift is directly proportional to the Armature current.**

(iii) **The demagnetizing component of armature reaction weakens the main flux. On the other hand, the distorting component of armature reaction distorts the main flux.**

(iv) **The demagnetizing effect leads to reduced generated voltage while cross magnetizing effect leads to sparking at the brushes.**

Demagnetizing and Cross-Magnetizing Conductors

With the brushes in the G.N.A. position, there is only cross-magnetizing effect of armature reaction. However, when the brushes are shifted from the G.N.A. position, the armature reaction will have both demagnetizing and cross magnetizing effects. Consider a 2-pole generator with brushes shifted (lead) θm mechanical degrees from G.N.A. We shall identify the armature conductors that produce demagnetizing effect and those that produce cross-magnetizing effect.

(i) The armature conductors θm on either side of G.N.A. produce flux in direct opposition to main flux as shown in Fig. (2.4) (i). Thus the conductors lying within angles AOC = BOD = 2 θm at the top and bottom of the armature produce demagnetizing effect. These are called demagnetizing armature conductors and constitute the demagnetizing ampere-turns of armature reaction (Remember two conductors constitute a turn).

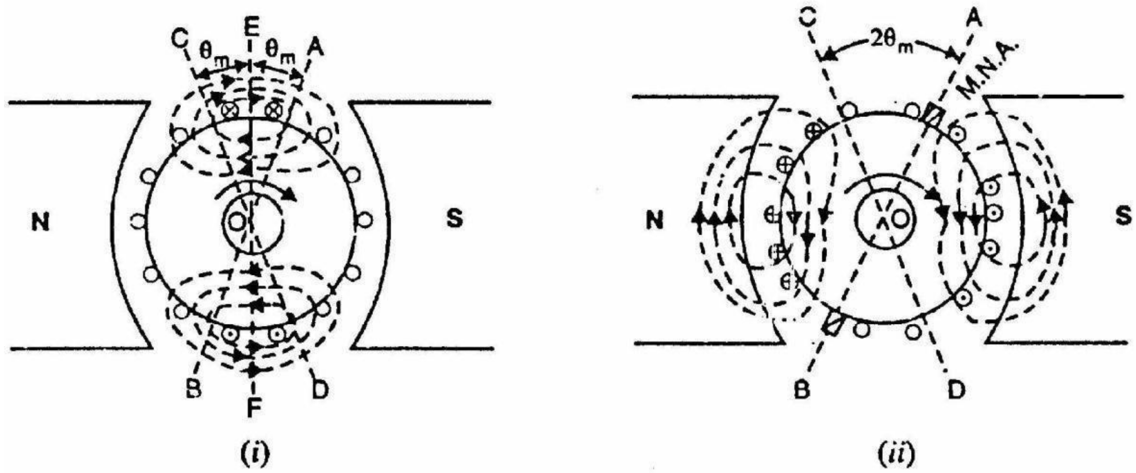


Fig.(2.4)

(ii) The axis of magnetization of the remaining armature conductors lying between angles AOD and COB is at right angles to the main flux as shown in Fig. (2.4) (ii). These conductors produce the cross-magnetizing (or distorting) effect i.e., they produce uneven flux distribution on each pole. Therefore, they are called cross-magnetizing conductors and constitute the cross-magnetizing ampere- turns of armature reaction.

Calculation of Demagnetizing Ampere-Turns Per Pole (ATd/Pole)

It is sometimes desirable to neutralize the demagnetizing ampere-turns of armature reaction. This is achieved by adding extra ampere-turns to the main field winding. We shall now calculate the demagnetizing ampere-turns per pole (ATd/pole).

Let Z = total number of armature conductors

I = current in each armature conductor

= Ia/2 ... for simplex wave winding

= Ia/P ... for simplex lap winding

Θm = forward lead in mechanical degrees

Referring to Fig. (2.4) (i) above, we have,
Total demagnetizing armature conductors

$$= \text{Conductors in angles AOC and BOD} = \frac{4\theta_m}{360} \times Z$$

Since two conductors constitute one turn,

$$\therefore \quad \text{Total demagnetizing ampere-turns} = \frac{1}{2}\left[\frac{4\theta_m}{360} \times Z\right] \times I = \frac{2\theta_m}{360} \times ZI$$

These demagnetizing ampere-turns are due to a pair of poles.

$$\therefore \quad \text{Demagnetizing ampere-turns/pole} = \frac{\theta_m}{360} \times ZI$$

i.e., $$AT_d / \text{pole} = \frac{\theta_m}{360} \times ZI$$

As mentioned above, the demagnetizing ampere-turns of armature reaction can be neutralized by putting extra turns on each pole of the generator.

$$\therefore \quad \text{No. of extra turns/pole} = \frac{AT_d}{I_{sh}} \qquad \text{for a shunt generator}$$

$$= \frac{AT_d}{I_a} \qquad \text{for a series generator}$$

Note. When a conductor passes a pair of poles, one cycle of voltage is generated. We say one cycle contains 360 electrical degrees. Suppose there are P poles in a generator. In one revolution, there are 360 mechanical degrees and 360 *P/2 electrical degrees.

$$\therefore \quad 360^\circ \text{ mechanical} = 360 \times \frac{P}{2} \text{ electrical degrees}$$

or $$1^\circ \text{ Mechanical} = \frac{P}{2} \text{ electrical degrees}$$

$$\therefore \quad \theta \text{ (mechanical)} = \frac{\theta\text{(electrical)}}{\text{Pair of pols}}$$

or $$\theta_m = \frac{\theta_e}{P/2} \qquad \therefore \quad \theta_m = \frac{2\theta_e}{P}$$

Cross-Magnetizing Ampere-Turns Per Pole (ATc/Pole)

We now calculate the cross-magnetizing ampere-turns per pole (ATc/pole).

Total armature reaction ampere-turns per pole

$$= \frac{Z/2}{P} \times I = \frac{Z}{2P} \times I \qquad (\because \quad \text{two conductors make one turn})$$

Demagnetizing ampere-turns per pole is given by;

$$AT_d / \text{pole} = \frac{\theta_m}{360} \times ZI$$

(found as above)

Cross-magnetizing ampere-turns/pole are

$$AT_d / \text{pole} = \frac{Z}{2P} \times I - \frac{\theta_m}{360} \times ZI = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

$$\therefore \quad AT_d / \text{pole} = ZI\left(\frac{1}{2P} - \frac{\theta_m}{360}\right)$$

Compensating Windings

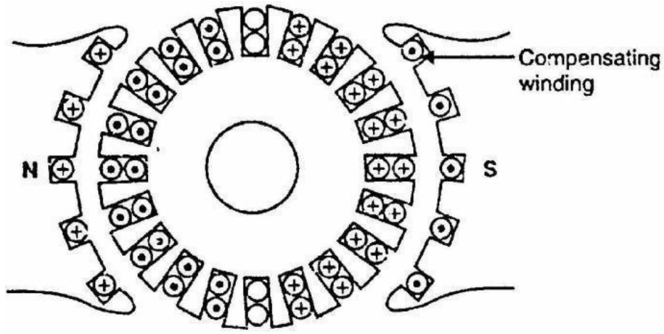


Fig. (2.5)

The cross-magnetizing effect of armature reaction may cause trouble in d.c. machines subjected to large fluctuations in load. In order to neutralize the cross magnetizing effect of armature reaction, a compensating winding is used. A compensating winding is an auxiliary winding embedded in slots in the pole faces as shown in Fig. (2.5). It is connected in series with armature in a manner so that the direction of current through the compensating conductors in any one pole face will be opposite to the direction of the current through the adjacent armature conductors [See Fig. 2.5].

Let us now calculate the number of compensating conductors/ pole face. In calculating the conductors per pole face required for the compensating winding, it should be remembered that the current in the compensating conductors is the armature current Ia whereas the current in armature conductors is Ia/A where A is the number of parallel paths.

Let Z_c = No. of compensating conductors/pole face
Z_a = No. of active armature conductors
I_a = Total armature current
I_a/A = Current in each armature conductor

$$\therefore \quad Z_c I_a = Z_a \times \frac{I_a}{A}$$

or $$Z_c = \frac{Z_a}{A}$$

The use of a compensating winding considerably increases the cost of a machine and is justified only for machines intended for severe service e.g., for high speed and high voltage machines.

AT/Pole for Compensating Winding

Only the cross-magnetizing ampere-turns produced by conductors under the pole face are effective in producing the distortion in the pole cores. If Z is the total number of armature conductors and P is the number of poles, then,

$$\text{No. of armature conductors/pole} = \frac{Z}{P}$$

$$\text{No. of armature turns/pole} = \frac{Z}{2P}$$

$$\text{No. of armature turns under pole face} = \frac{Z}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

If I is the current through each armature conductor, then,

$$\text{AT/pole required for compensating winding} = \frac{ZI}{2P} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

$$= \text{Armature AT/pole} \times \frac{\text{Pole arc}}{\text{Pole pitch}}$$

Commutation

Fig. (2.6) shows the schematic diagram of 2-pole lap-wound generator. There are two parallel paths between the brushes. Therefore, each coil of the winding carries one half (Ia/2 in this case) of the total current (Ia) entering or leaving the armature.

Note that the currents in the coils connected to a brush are either all towards the brush (positive brush) or all directed away from the brush (negative brush). Therefore, current in a coil will reverse as the coil passes a brush. This reversal of current as the coil passes & brush is called commutation.

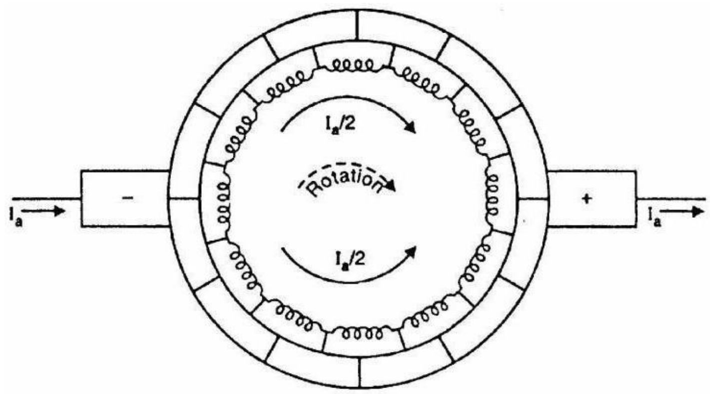


Fig. (2.6)

The reversal of current in a coil as the coil passes the brush axis is called commutation. When commutation takes place, the coil undergoing commutation is short circuited by the brush. The brief period during which the coil remains short circuited is known as commutation period Tc. If the current reversal is completed by the end of commutation period, it is called ideal commutation. If the current reversal is not completed by that time, then sparking occurs between the brush and the commutator which results in progressive damage to both.

Ideal commutation

Let us discuss the phenomenon of ideal commutation (i.e., coil has no inductance) in one coil in the armature winding shown in Fig. (2.6) above. For this purpose, we consider the coil A. The brush width is equal to the width of one commutator segment and one mica insulation. Suppose the total armature current is 40 A. Since there are two parallel paths, each coil carries a current of 20 A.

(i) In Fig. (2.7) (i), the brush is in contact with segment 1 of the commutator. The commutator segment 1 conducts a current of 40 A to the brush; 20 A from coil A and 20 A from the adjacent coil as shown. The coil A has yet to undergo commutation.

(ii) As the armature rotates, the brush will make contact with segment 2 and thus short-circuits the coil A as shown in Fig. (2.7) (ii). There are now two parallel paths into the brush as long as the short-circuit of coil A exists. Fig. (2.7) (ii) shows the instant when the brush is one- fourth on segment 2 and three-fourth on segment 1. For this condition, the resistance of the path through segment 2 is three times the resistance of the path through segment 1 (Q contact resistance varies inversely as the area of contact of brush with the segment). The brush again conducts a current of 40 A; 30 A through segment 1 and 10 A through segment 2. Note that current in coil A (the coil undergoing commutation) is reduced from 20 A to 10 A.

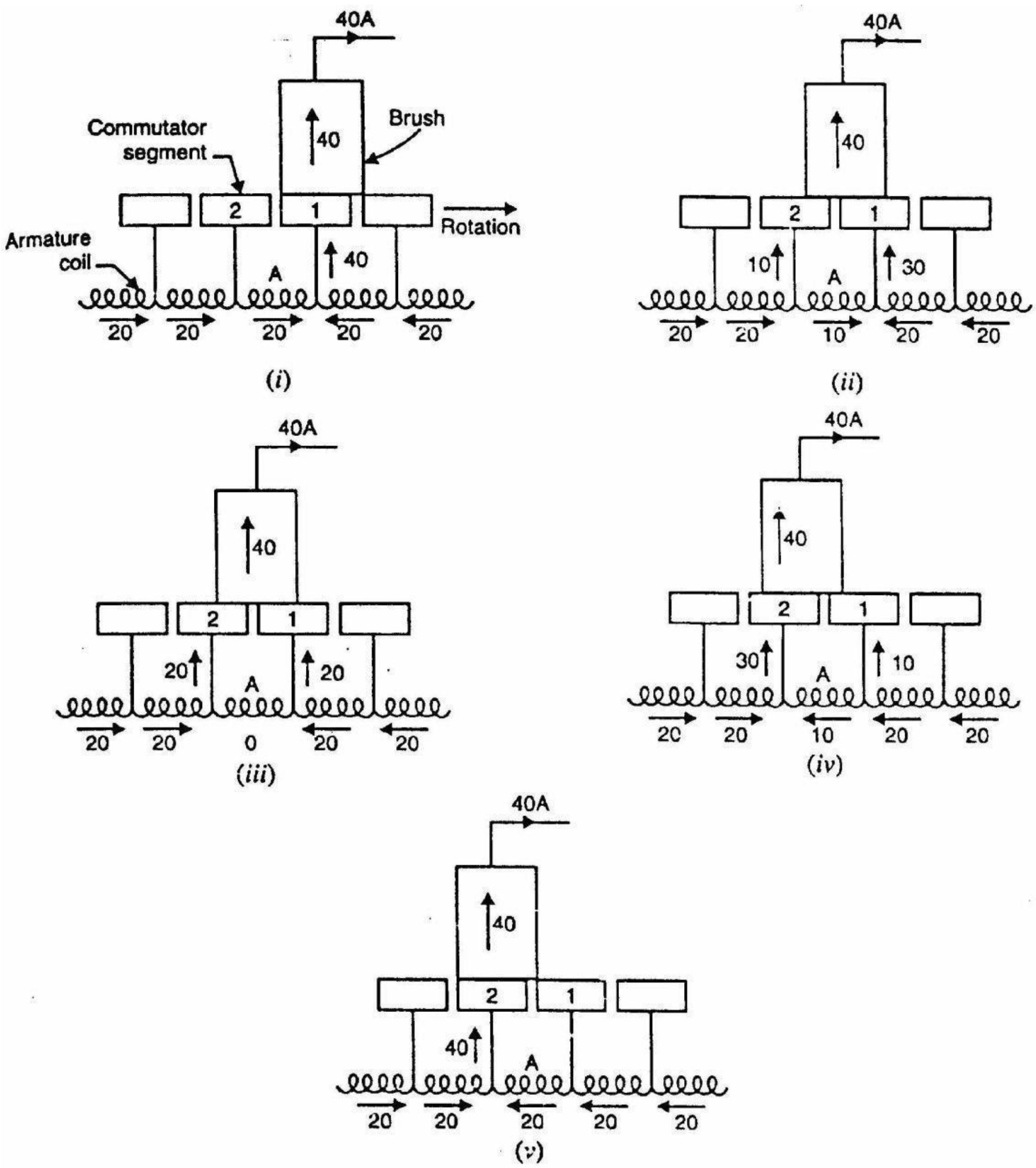
(iii) Fig. (2.7) (iii) shows the instant when the brush is one-half on segment 2 and one-half on segment 1. The brush again conducts 40 A; 20 A through segment 1 and

20 A through segment 2 (Q now the resistances of the two parallel paths are equal). Note that now. current in coil A is zero.

(iv) **Fig. (2.7) (iv) shows the instant when the brush is three-fourth on segment 2 and one-fourth on segment 1. The brush conducts a current of 40 A; 30 A through segment 2 and 10 A through segment 1. Note that current in coil A is 10 A but in the reverse direction to that before the start of commutation. The reader may see the action of the commutator in**

(v) **Fig. (2.7) (v) shows the instant when the brush is in contact only with segment 2. The brush again conducts 40 A; 20 A from coil A and 20 A from the adjacent coil to coil A. Note that now current in coil A is 20 A but in the reverse direction. Thus the coil A has undergone commutation. Each coil undergoes commutation in this way as it passes the brush axis. Note that during commutation, the coil under consideration remains short circuited by the brush.**

Fig. (2.8) shows the current-time graph for the coil A undergoing commutation. The horizontal line AB represents a constant current of 20 A upto the beginning of commutation. From the finish of commutation, it is represented by another horizontal line CD on the opposite side of the zero line and the same distance from it as AB i.e., the current has exactly reversed (- 20 A). The way in which current changes from B to C depends upon the conditions under which the coil undergoes commutation. If the current changes at a uniform rate (i.e., BC is a straight line), then it is called ideal commutation as shown in Fig. (2.8). Under such conditions, no sparking will take place between the brush and the commutator.



40A
40
Brush
Commutator segment
2
1
Rotation
Armature coil
A
20
10
30
0
(i)
(ii)
(iii)
(iv)
(v)

Fig. (2.7)

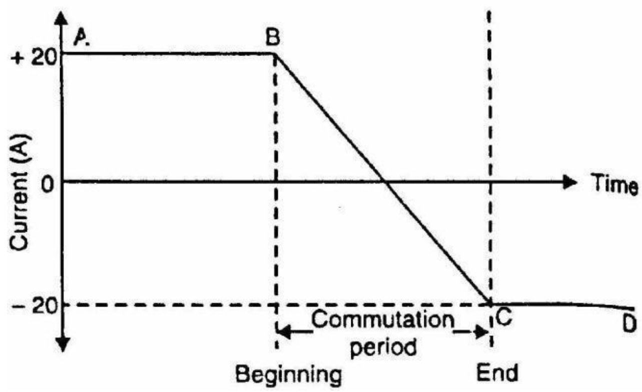


Fig. (2.8)

Practical difficulties

The ideal commutation (i.e., straight line change of current) cannot be attained in practice. This is mainly due to the fact that the armature coils have appreciable inductance. When the current in the coil undergoing commutation changes, self-induced e.m.f. is produced in the coil. This is generally called reactance voltage. This reactance voltage opposes the change of current in the coil undergoing commutation. The result is that the change of current in the coil undergoing commutation occurs more slowly than it would be under ideal commutation.

This is illustrated in Fig. (2.9). The straight line RC represents the ideal commutation whereas the curve BE represents the change in current when self-inductance of the coil is taken into account. Note that current CE (= 8A in Fig. 2.9) is flowing from the commutator segment 1 to the brush at the instant when they part company. This results in sparking just as when any other current carrying circuit is broken. The sparking results in overheating of commutators brush contact and causing damage to both.

Fig. (2.10) illustrates how sparking takes place between the commutators segment and the brush. At the end of commutation or short-circuit period, the current in coil A is reversed to a value of 12 A (instead of 20 A) due to inductance of the coil. When the brush breaks contact with segment 1, the remaining 8 A current jumps from segment 1 to the brush through air causing sparking between segment 1 and the brush.

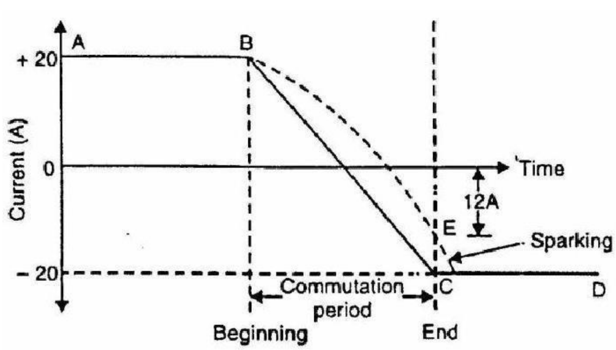


Fig. (2.9)

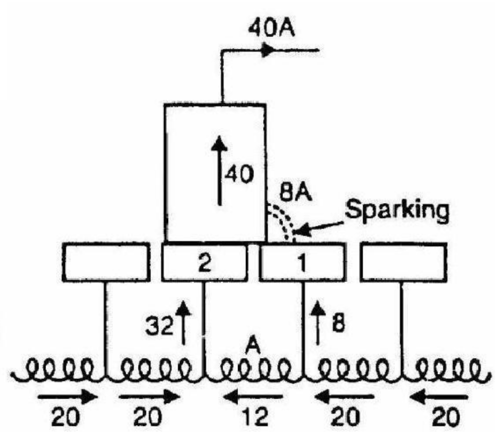


Fig. (2.10)

Calculation of Reactance Voltage

Reactance voltage = Coefficient of self-inductance *Rate of change of current

When a coil undergoes commutation, two commutator segments remain short circuited

by the brush. Therefore, the time of short circuit (or commutation period Tc) is equal to the time required by the commutator to move a distance equal to the circumferential thickness of the brush minus the thickness of one insulating strip of mica

Let W_b = brush width in cm;

W_m = mica thickness in cm

v = peripheral speed of commutator in cm/s

$\therefore$ Commutation period, $T_c = \dfrac{W_b - W_m}{v}$ seconds

The commutation period is very small, say of the order of 1/500 second.

Let the current in the coil undergoing commutation change from + I to – I (amperes) during the commutation. If L is the inductance of the coil, then reactance voltage is given by;

Reactance voltage, $E_R = L*2I/T_c$

Methods of Improving Commutation

Improving commutation means to make current reversal in the short-circuited coil as sparkless as possible. The following are the two principal methods of improving commutation:

(i) Resistance commutation

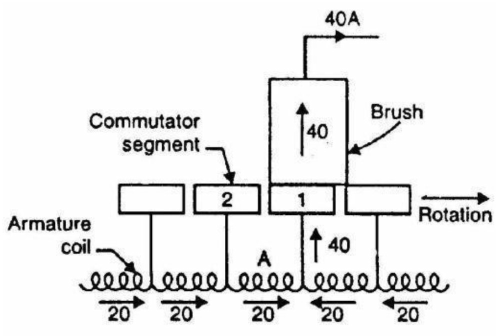
(ii) E.M.F. commutation

Resistance Commutation

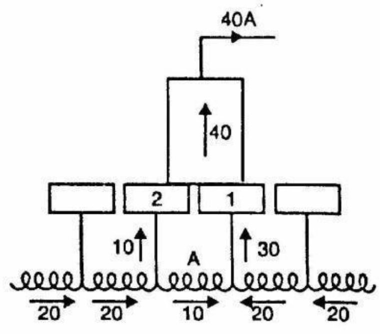
The reversal of current in a coil (i.e., commutation) takes place while the coil is short-circuited by the brush. Therefore, there are two parallel paths for the current as long as the short circuit exists. If the contact resistance between the brush and the commutator is made large, then current would divide in the inverse ratio of contact resistances (as for any two resistances in parallel). This is the key point in improving commutation. This is achieved by using carbon brushes (instead of Cu brushes) which have high contact resistance. This method of improving commutation is called resistance commutation. Figs. (2.11) and (2.12) illustrates how high contact resistance of carbon brush improves commutation (i.e., reversal of current) in coil A.

In Fig. (2.11) (i), the brush is entirely on segment 1 and, therefore, the current in coil A is 20 A. The coil A is yet to undergo commutation. As the armature rotates, the brush short circuits the coil A and there are two parallel paths for the current into the brush.

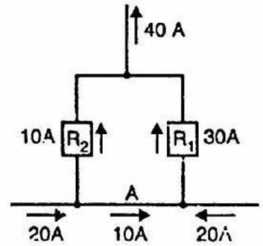
Fig. (2.11) (ii) shows the instant when the brush is one-fourth on segment 2 and three-fourth on segment 1. The equivalent electric circuit is shown in Fig. (2.11) (iii) where R1 and R2 represent the brush contact resistances on segments 1 and 2. A resistor is not shown for coil A since it is assumed that the coil resistance is negligible as compared to the brush contact resistance


40A
Brush
Commutator segment
40
2
1
Rotation
Armature coil
A
40
20
20
20
20
20

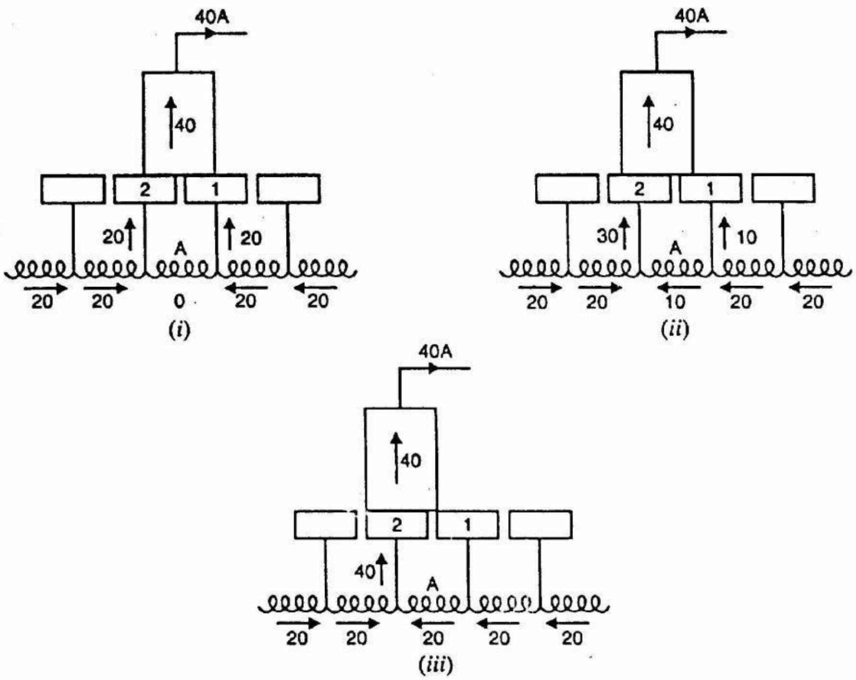
(i)


40A
40
2
1
10
A
30
20
20
10
20
20

(ii)


40 A
10A
R_2
R_1
30A
A
20A
10A
20A

(iii)


40A
40
2
1
20
A
20
20 20 0 20 20
(i)
40A
40
2
1
30
A
10
20 20 10 20 20
(ii)
40A
40
2
1
40
A
20 20 20 20 20
(iii)

The values of current in the parallel paths of the equivalent circuit are determined by the respective resistances of the paths. For the condition shown in Fig. (2.11) (ii), resistor R2 has three times the resistance of resistor R1. Therefore, the current distribution in the paths will be as shown. Note that current in coil A is reduced from 20 A to10 A due to division of current in (he inverse ratio of contact resistances. If the Cu brush is used (which has low contact resistance), R1 R2 and the current in coil A would not have reduced to 10 A.

As the carbon brush passes over the commutator, the contact area with segment 2 increases and that with segment 1 decreases i.e., R2 decreases and R1 increases. Therefore, more and more current passes to the brush through segment 2. This is illustrated in Figs. (2.12) (i) and (2.12) (ii), When the break between the brush and the segment 1 finally occurs [See Fig. 2.12 (iii)], the current in the coil is reversed and commutation is achieved. It may be noted that the main cause of sparking during commutation is the production of reactance voltage and carbon brushes cannot prevent it.

Nevertheless, the carbon brushes do help in improving commutation. The other minor advantages of carbon brushes are:

(i) The carbon lubricates and polishes the commutator.

(ii) If sparking occurs, it damages the commutator less than with copper brushes and the damage to the brush itself is of little importance.

E.M.F. Commutation

In this method, an arrangement is made to neutralize the reactance voltage by producing a reversing voltage in the coil undergoing commutation. The reversing voltage acts in opposition to the reactance voltage and neutralizes it to some extent. If the reversing voltage is equal to the reactance voltage, the effect of the latter is completely wiped out and we get sparkless commutation. The reversing voltage may be produced in the following two ways:

(i) By brush shifting

(ii) By using interpoles or compoles

(i) By brush shifting

In this method, the brushes are given sufficient forward lead (for a generator) to bring the short-circuited coil (i.e., coil undergoing commutation) under the influence of the next pole of opposite polarity. Since the short-circuited coil is now in the reversing field, the reversing voltage produced cancels the reactance voltage. This method suffers from the following drawbacks:

(a) The reactance voltage depends upon armature current. Therefore, the brush shift will depend on the magnitude of armature current which keeps on changing. This necessitates frequent shifting of brushes.

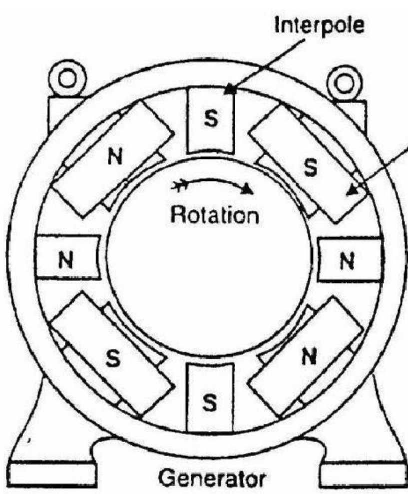
(b) The greater the armature current, the greater must be the forward lead for a generator. This increases the demagnetizing effect of armature reaction and further weakens the main field.

(ii) By using interpoles or compotes

The best method of neutralizing reactance voltage is by, using interpoles or compoles.

Interpoles or Compoles

The best way to produce reversing voltage to neutralize the reactance voltage is by using interpoles or compoles. These are small poles fixed to the yoke and spaced mid-way between the main poles (See Fig. 2.13). They are wound with comparatively few turns and connected in series with the armature so that they carry armature current. Their polarity is the same as the next main pole ahead in the direction of rotation for a generator (See Fig. 2.13). Connections for a d.c. generator with interpoles is shown in Fig. (2.14).



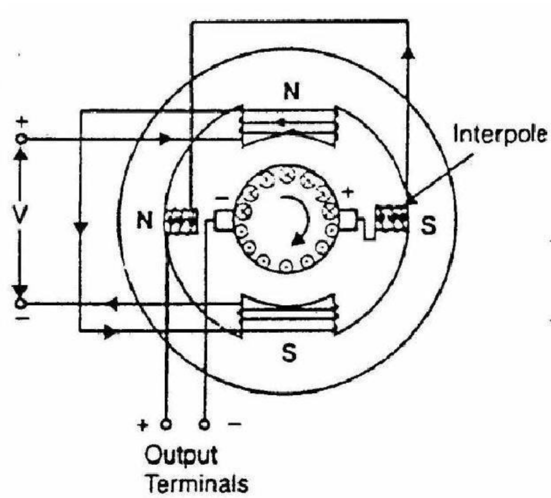


Fig. (2.13) Fig. (2.14)

Functions of Interpoles

The machines fitted with interpoles have their brushes set on geometrical neutral axis (no lead). The interpoles perform the following two functions:

(i) As their polarity is the same as the main pole ahead (for a generator), they induce an e.m.f. in the coil (undergoing commutation) which opposes reactance voltage. This leads to sparkless commutation. The
e.m.f. induced by compoles is known as commutating or reversing e.m.f. Since the interpoles carry the armature current and the reactance voltage is also proportional to armature current, the neutralization of reactance voltage is automatic.

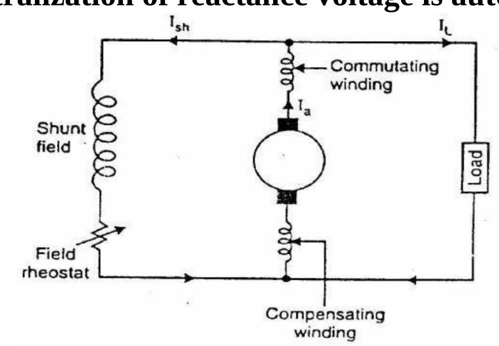


Fig. (2.15)

(ii) **The m.m.f. of the compoles neutralizes the cross-magnetizing effect of armature reaction in small region in the space between the main poles. It is because the two m.m.f.s oppose each other in this region. Fig. (2.15) shows the circuit diagram of a shunt generator with commutating winding and compensating winding. Both these windings are connected in series with the armature and so they carry the armature current. However, the functions they perform must be understood clearly. The main function of commutating winding is to produce reversing (or commutating) e.m.f. in order to cancel the reactance voltage. In addition to this, the m.m.f. of the commutating winding neutralizes the cross magnetizing ampere-turns in the space between the main poles. The compensating winding neutralizes the cross- magnetizing effect of armature reaction under the pole faces.**

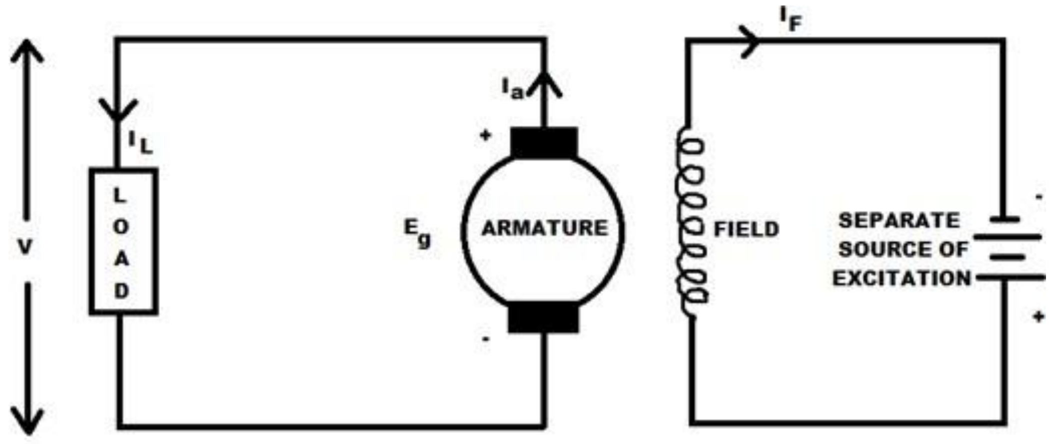
Types of DC Generators

When the flux in the magnetic circuit is established by the help of permanent magnets then it is known as Permanent magnet dc generator. It consists of an armature and one or several permanent magnets situated around the armature. This type of dc generators generates very low power. So, they are rarely found in industrial applications. They are normally used in small applications like dynamos in motor cycles.

Separately Excited DC Generator

These are the generators whose field magnets are energized by some external dc source such as battery . A circuit diagram of separately excited DC generator is shown in figure.

I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf



Separately excited DC generator

Voltage drop in the armature = $I_a \times R_a$ (R/sub>a is the armature resistance) Let, $I_a = I_L$ = I (say) Then, voltage across the load, V = IR_a Power generated, $P_g = E_g \times I$ Power delivered to the external load, $P_L = V \times I$.

Self-excited DC Generators

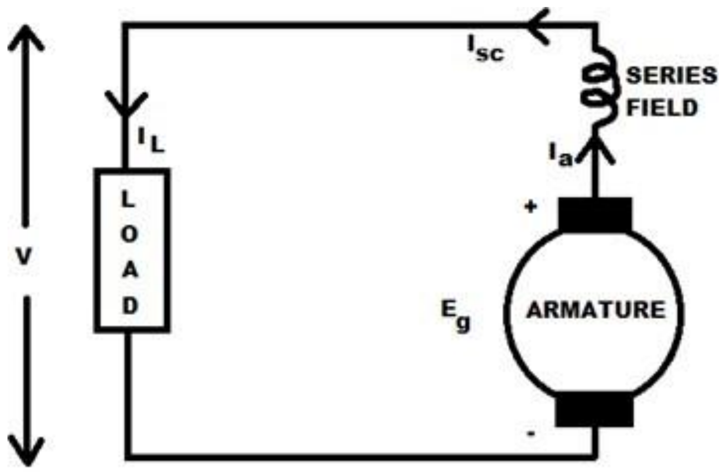
These are the generators whose field magnets are energized by the current supplied by themselves. In these type of machines field coils are internally connected with the armature. Due to residual magnetism some flux is always present in the poles. When the armature is rotated some emf is induced. Hence some induced current is produced. This small current flows through the field coil as well as the load and

thereby strengthening the pole flux. As the pole flux strengthened, it will produce more armature emf, which cause further increase of current through the field. This increased field current further raises armature emf and this cumulative phenomenon continues until the excitation reaches to the rated value. According to the position of the field coils the Self-excited DC generators may be classified as…

A. Series wound generators B. Shunt wound generators C. Compound wound generators

Series Wound Generator

In these type of generators, the field windings are connected in series with armature conductors as shown in figure below. So, whole current flows through the field coils as well as the load. As series field winding carries full load current it is designed with relatively few turns of thick wire. The electrical resistance of series field winding is therefore very low (nearly 0.5Ω). Let, R_{sc} = Series winding resistance I_{sc} = Current flowing through the series field R_a = Armature resistance I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf

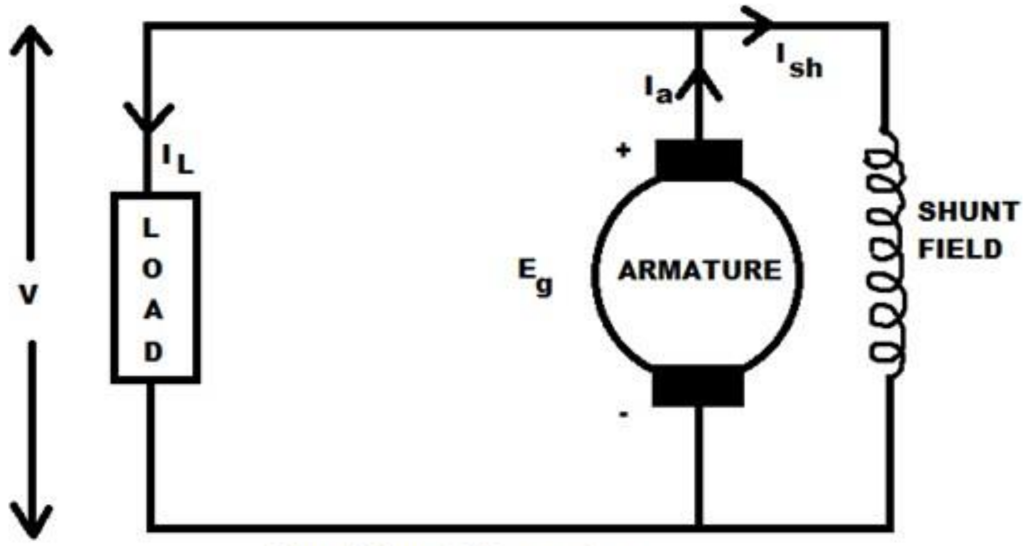


Series Wound Generator

Then, $I_a = I_{sc} = I_L = I$ (say) Voltage across the load, $V = E_g - I(I_a \times R_a)$ Power generated, $P_g = E_g \times I$ Power delivered to the load, $P_L = V \times I$

Shunt Wound DC Generators

In these type of DC generators the field windings are connected in parallel with armature conductors as shown in figure below. In shunt wound generators the voltage in the field winding is same as the voltage across the terminal. Let, R_{sh} = Shunt winding resistance I_{sh} = Current flowing through the shunt field R_a = Armature resistance I_a = Armature current I_L = Load current V = Terminal voltage E_g = Generated emf



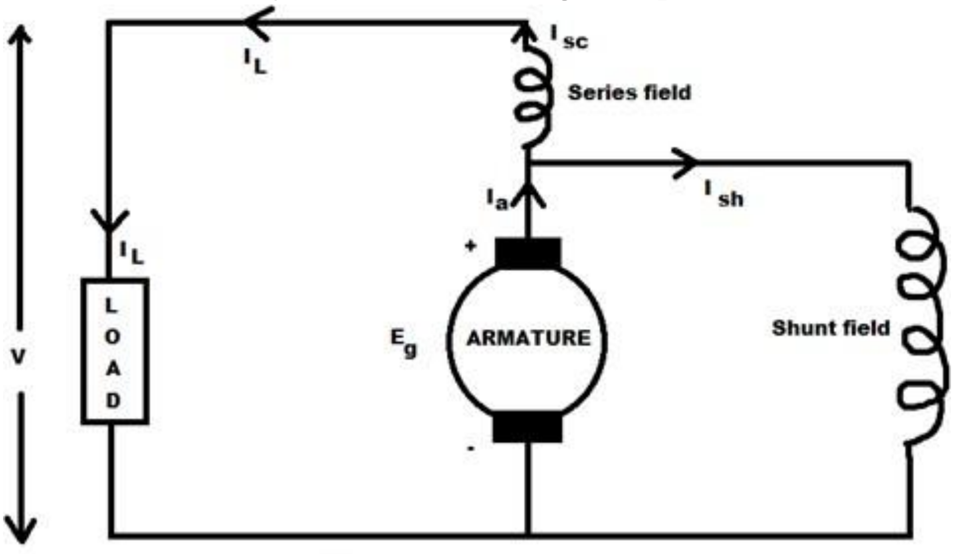
Shunt Wound Generator

Here armature current I_a is dividing in two parts, one is shunt field current I_{sh} and another is load current I_L. So, $I_a = I_{sh} + I_L$ The effective power across the load will be maximum when I_L will be maximum. So, it is required to keep shunt field current as small as possible. For this purpose the resistance of the shunt field winding generally kept high (100 Ω) and large no of turns are used for the desired emf. Shunt field current, $I_{sh} = V/R_{sh}$ Voltage across the load, $V = E_g - I_a R_a$ Power generated, $P_g = E_g \times I_a$ Power delivered to the load, $P_L = V \times I_L$

Compound Wound DC Generator

In series wound generators, the output voltage is directly proportional with load current. In shunt wound generators, output voltage is inversely proportional with load current. A combination of these two types of generators can overcome the disadvantages of both. This combination of windings is called compound wound DC generator. Compound wound generators have both series field winding and shunt field winding. One winding is placed in series with the armature and the other is placed in parallel with the armature. This type of DC generators may be of two types- short shunt compound wound generator and long shunt compound wound generator.

Short Shunt Compound Wound DC Generator
The generators in which only shunt field winding is in parallel with the armature winding as shown in



Short Shunt Compound Wound Generator

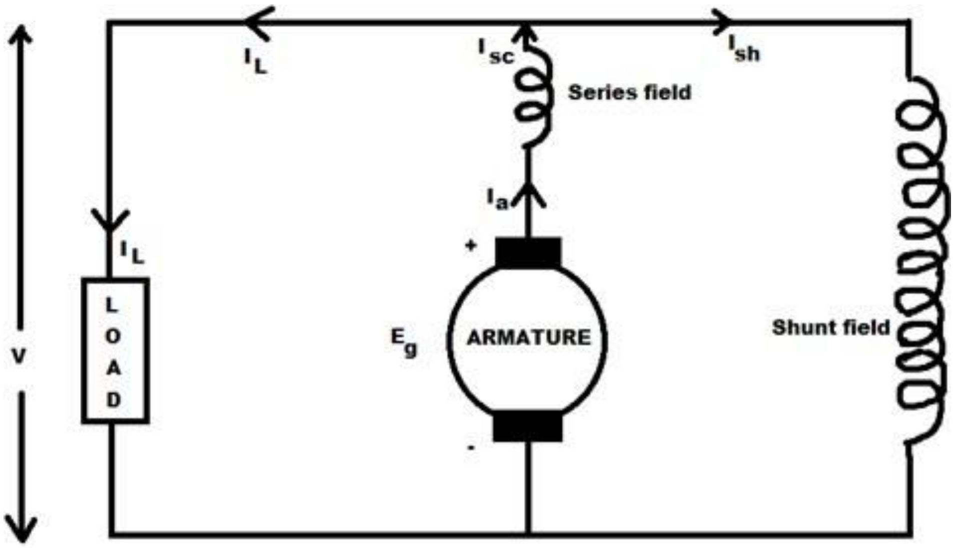
figure.

Series field current, $I_{sc} = I_L$ Shunt field current, $I_{sh} = (V+I_{sc} R_{sc})/R_{sh}$ Armature current,

$I_a = I_{sh} + I_L$ Voltage across the load, $V = E_g - I_a R_a - I_{sc} R_{sc}$ Power generated, $P_g = E_g \times I_a$

Power delivered to the load, $P_L = V \times I_L$

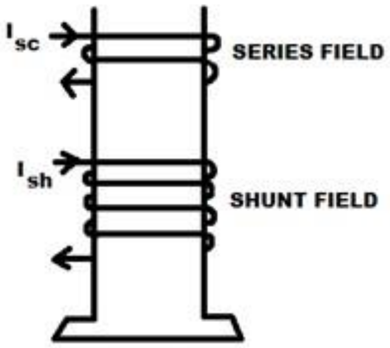
Long Shunt Compound Wound DC Generator
The generators in which shunt field winding is in parallel with both series field and armature winding as



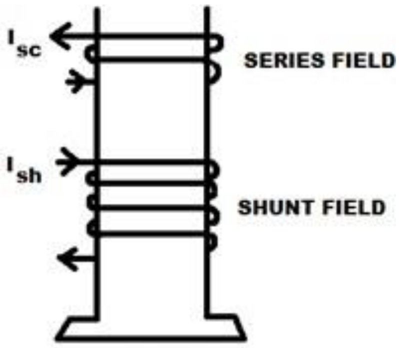
Long Shunt Compound Wound Generator

shown in figure.
~~**Shunt field current, $I_{sh}=V/R_{sh}$ Armature current, I_a= series field current, I_{sc}= I_L+I_{sh}**~~
Voltage across the load, $V=E_g-I_a R_a-I_{sc} R_{sc}=E_g-I_a (R_a+R_{sc})$ [$\therefore I_a=I_{cs}$] Power generated, $P_g= E_g \times I_a$ Power delivered to the load, $P_L=V \times I_L$ In a compound wound generator, the shunt field is stronger than the series field. When the series field assists the shunt field, generator is said to be commutatively compound wound. On the other hand if series field

opposes the shunt field, the generator is said to be differentially compound wound.



CUMULATIVE COMPOUNDING



DIFFERENTIAL COMPOUNDING

D.C. GENERATOR CHARACTERISTICS

Introduction

The speed of a d.c. machine operated as a generator is fixed by the prime mover. For general- purpose operation, the prime mover is equipped with a speed governor so that the speed of the generator is practically constant. Under such condition, the generator performance deals primarily with the relation between excitation, terminal voltage and load. These relations can be best exhibited graphically by means of curves known as generator characteristics. These characteristics show at a glance the behaviour of the generator under different load conditions.

D.C. Generator Characteristics

The following are the three most important characteristics of a d.c. generator:

Open Circuit Characteristic (O.C.C.)

This curve shows the relation between the generated e.m.f. at no-load (E_0) and the field current (I_f) at constant speed. It is also known as magnetic characteristic or no-load saturation curve. Its shape is practically the same for all generators whether separately or self- excited. The data for O.C.C. curve are obtained experimentally by operating the generator at no load and constant speed and recording the change in terminal voltage as the field current is varied.

External characteristic (V/I_L)

This curve shows the relation between the terminal voltage (V) and load current (I_L). The terminal voltage V will be less than E due to voltage drop in the armature circuit. Therefore, this curve will lie below the internal characteristic. This characteristic is very important in determining the suitability of a generator for a given purpose. It can be obtained by making simultaneous measurements of terminal voltage and load current (with voltmeter and ammeter) of a loaded generator.

Internal or Total characteristic (E/I_a)

This curve shows the relation between the generated e.m.f. on load (E) and the armature current (I_a). The e.m.f. E is less than E_0 due to the demagnetizing effect of armature reaction. Therefore, this curve will lie below the open circuit characteristic (O.C.C.). The internal characteristic is of interest chiefly to the designer. It cannot be obtained directly by experiment. It is because a voltmeter cannot read the e.m.f. generated on load due to the voltage drop in armature resistance. The internal characteristic can be obtained from external characteristic if winding resistances are known because armature reaction effect is included in both characteristics.

Open Circuit Characteristic of a D.C. Generator

The O.C.C. for a d.c. generator is determined as follows. The field winding of the d.c. generator (series or shunt) is disconnected from the machine and is separately excited from an external d.c. source as shown in Fig. (3.1) (ii). The generator is run at fixed speed (i.e., normal speed). The field current (I_f) is increased from zero in steps and the corresponding values of generated e.m.f. (E_0) read off on a voltmeter connected across the armature terminals. On plotting the relation between E_0 and I_f, we get the open circuit characteristic as shown in Fig. (3.1) (i).

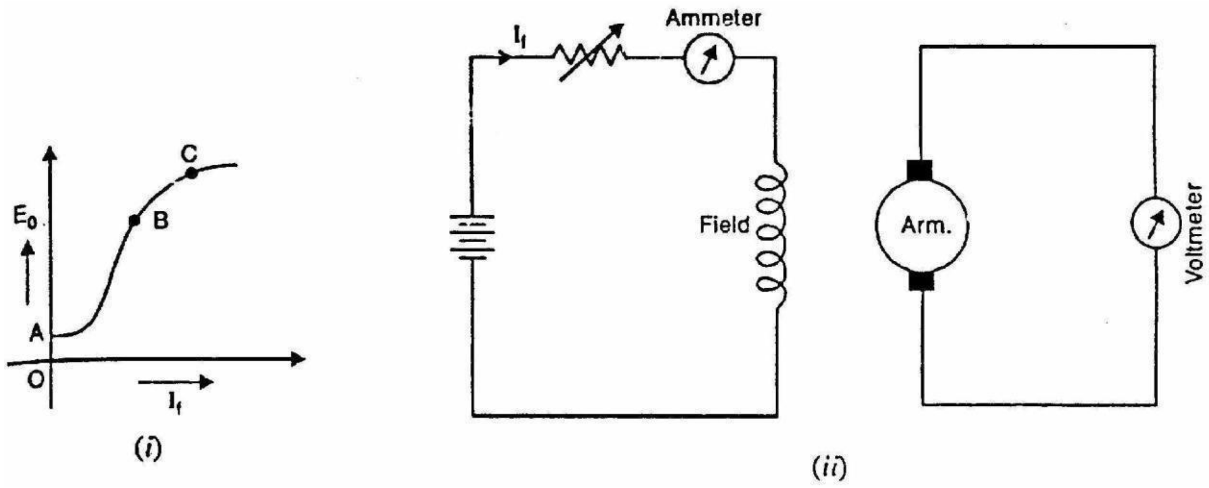


Fig. (3.1)

The following points may be noted from O.C.C.:

4. When the field current is zero, there is some generated e.m.f. OA. This is due to the residual magnetism in the field poles.

5. Over a fairly wide range of field current (upto point B in the curve), the curve is linear. It is because in this range, reluctance of iron is negligible as compared with that of air gap. The air gap reluctance is constant and hence linear relationship.

6. After point B on the curve, the reluctance of iron also comes into picture. It is because at higher flux densities r for iron decreases and reluctance

of iron is no longer negligible. Consequently, the curve deviates from linear relationship.
(iv) After point C on the curve, the magnetic saturation of poles begins and E_0 tends to level off.

The reader may note that the O.C.C. of even self-excited generator is obtained by running it as a separately excited generator.

Characteristics of a Separately Excited D.C. Generator

The obvious disadvantage of a separately excited d.c. generator is that we require an external d.c. source for excitation. But since the output voltage may be controlled more easily and over a wide range (from zero to a maximum), this type of excitation finds many applications.

(i) Open circuit characteristic.

The O.C.C. of a separately excited generator is determined in a manner described in Sec. (3.2). Fig. (3.2) shows the variation of generated e.m f. on no load with field current for various fixed speeds. Note that if the value of constant speed is increased, the steepness of the curve also increases. When the field current is zero, the residual magnetism in the poles will give rise to the small initial e.m.f. as shown.

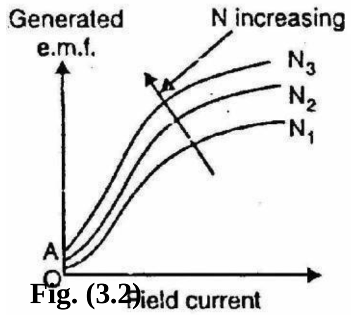


Fig. (3.2)

(ii) Internal and External Characteristics

The external characteristic of a separately excited generator is the curve between the terminal voltage (V) and the load current I_L (which is the same as armature current in this case). In order to determine the external characteristic, the circuit set up is as shown in Fig. (3.3) (i). As the load current increases, the terminal voltage falls due to two reasons:

(a) The armature reaction weakens the main flux so that actual e.m.f. generated E on load is less than that generated (E_0) on no load.
(b) There is voltage drop across armature resistance ($= I_L R_a = I_a R_a$).

Due to these reasons, the external characteristic is a drooping curve [curve 3 in Fig. 3.3 (ii)]. Note that in the absence of armature reaction and armature drop, the generated e.m.f. would have been E_0 (curve 1).

The internal characteristic can be determined from external characteristic by adding $I_L R_a$ drop to the external characteristic. It is because armature reaction drop is included in the external characteristic. Curve 2 is the internal

characteristic of the generator and should obviously lie above the external characteristic.

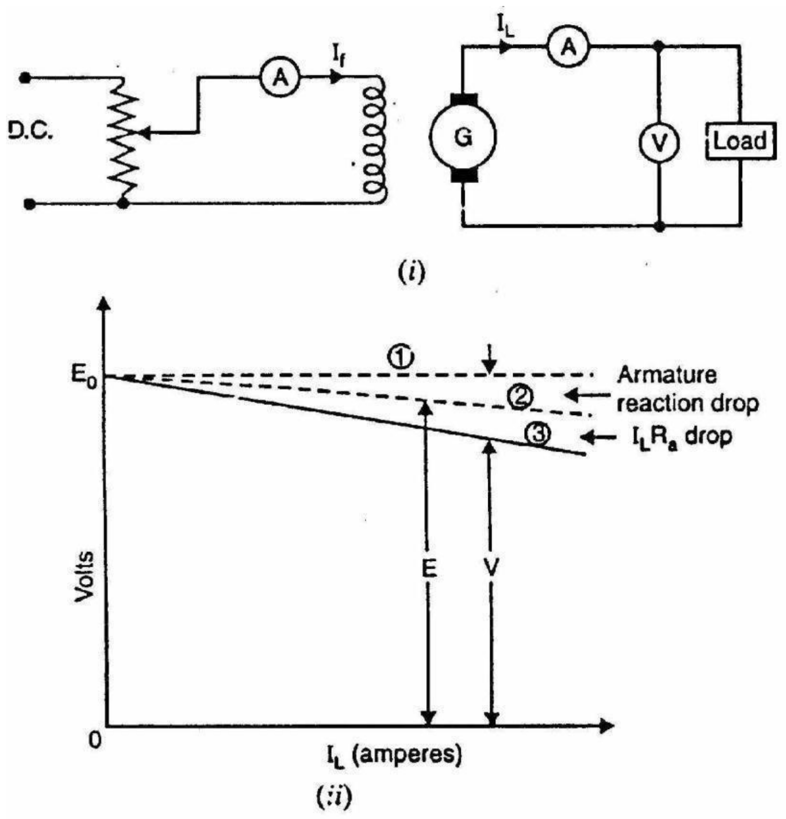


Fig. (3.3)

Voltage Build-Up in a Self-Excited Generator

Let us see how voltage builds up in a self-excited generator.

(i) Shunt generator

Consider a shunt generator. If the generator is run at a constant speed, some e.m.f. will be generated due to residual magnetism in the main poles. This small e.m.f. circulates a field current which in turn produces additional flux to reinforce the original residual flux (provided field winding connections are correct). This process continues and the generator builds up the normal generated voltage following the O.C.C. shown in Fig. (3.4) (i).

The field resistance Rf can be represented by a straight line passing through the origin as shown in Fig. (3.4) (ii). The two curves can be shown on the same diagram as they have the same ordinate [See Fig. 3.4 (iii)].

Since the field circuit is inductive, there is a delay in the increase in current upon closing the field circuit switch The rate at which the current increases depends

upon the voltage available for increasing it. Suppose at any instant, the field current is i (= OA) and is increasing at the rate di/dt. Then,

$$E_0 - iR_f = L\frac{di}{dt}$$

where R_f = total field circuit resistance L = inductance of field circuit

At the considered instant, the total e.m.f. available is AC [See Fig. 3.4 (iii)]. An amount AB of the c.m.f. AC is absorbed by the voltage drop iR_f and the remainder part BC is available to overcome L di/dt. Since this surplus voltage is available, it is possible for the field current to increase above the value OA.

However, at point D, the available voltage is OM and is all absorbed by i R_f drop. Consequently, the field current cannot increase further and the generator build up stops.

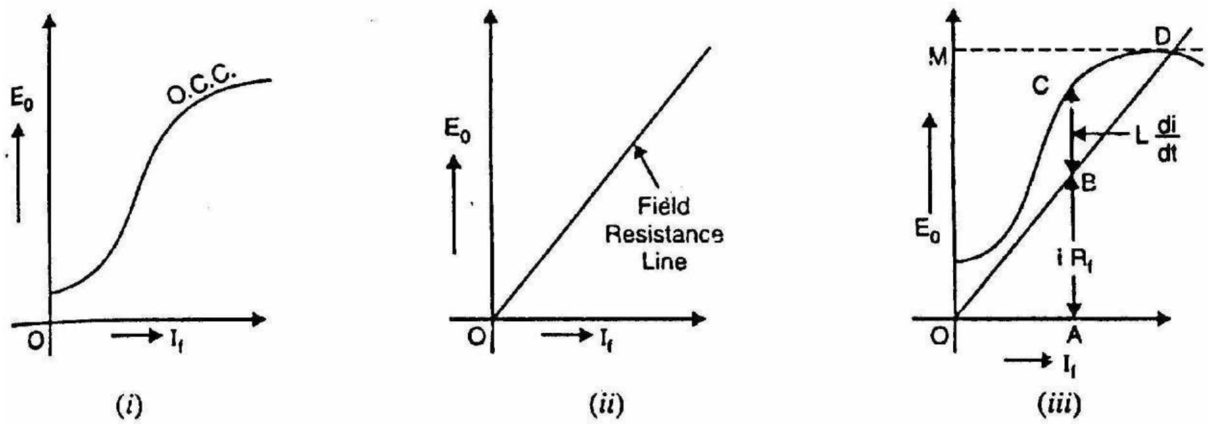


Fig. (3.4)

We arrive at a very important conclusion that the voltage build up of the generator is given by the point of intersection of O.C.C. and field resistance line. Thus in Fig. (3.4) (iii), D is point of intersection of the two curves. Hence the generator will build up a voltage OM.

(ii) Series generator

During initial operation, with no current yet flowing, a residual voltage will be generated exactly as in the case of a shunt generator. The residual voltage will cause a current to flow through the whole series circuit when the circuit is closed. There will then be voltage build up to an equilibrium point exactly analogous to the build up of a shunt generator. The voltage build up graph will be similar to that of shunt generator except that now load current (instead of field current for shunt generator) will be taken along x-axis.

(iii) Compound generator

When a compound generator has its series field flux aiding its shunt field flux, the machine is said to be cumulative compound. When the series field is connected in reverse so that its field flux opposes the shunt field flux, the generator is then differential compound.

The easiest way to build up voltage in a compound generator is to start under no load conditions. At no load, only the shunt field is effective. When no-load voltage build up is achieved, the generator is loaded. If under load, the voltage rises, the series field connection is cumulative. If the voltage drops significantly, the connection is differential compound.

Critical Field Resistance for a Shunt Generator

We have seen above that voltage build up in a shunt generator depends upon field circuit resistance. If the field circuit resistance is R_1 (line OA), then generator will build up a voltage OM as shown in Fig. (3.5). If the field circuit resistance is increased to R_2 (tine OB), the generator will build up a voltage OL, slightly less than OM. As the field circuit resistance is increased, the slope of resistance line also increases. When the field resistance line becomes tangent (line OC) to O.C.C., the generator would just excite. If the field circuit resistance is increased beyond this point (say line OD), the generator will fail to excite. The field circuit resistance represented by line OC (tangent to O.C.C.) is called critical field resistance R_C for the shunt generator. It may be defined as under:

***The maximum field circuit resistance (for a given speed) with which the shunt generator would just excite is known as its* critical field resistance.**

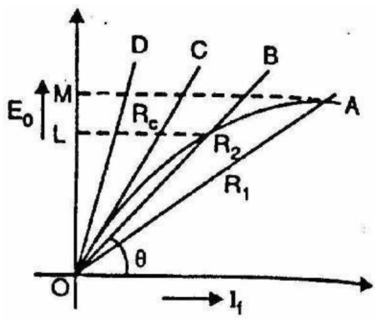


Fig. (3.5)

It should be noted that shunt generator will build up voltage only if field circuit resistance is less than critical field resistance.

Critical Resistance for a Series Generator

Fig. (3.6) shows the voltage build up in a series generator. Here R_1, R_2 etc. represent the total circuit resistance (load resistance and field winding resistance). If the total circuit resistance is R_1, then series generator will build up a voltage OL. The

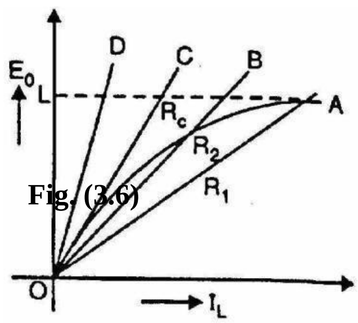


Fig. (3.6)

line OC is tangent to O.C.C. and represents the critical resistance R_C for a series generator. If the total resistance of the circuit is more than R_C (say line OD), the generator will fail to build up voltage. Note that Fig. (3.6) is similar to Fig. (3.5) with the following differences:

(i) In Fig. (3.5), R_1, R_2 etc. represent the total field circuit resistance. However, R_1, R_2 etc. in Fig. (3.6) represent the total circuit resistance (load resistance and series field winding resistance etc.).

(ii) In Fig (3.5), field current alone is represented along X-axis. However, in Fig. (3.6) load current I_L is represented along Y-axis. Note that in a series generator, field current = load current I_L.

Characteristics of Series Generator

Fig. (3.7) (i) shows the connections of a series wound generator. Since there is only one current (that which flows through the whole machine), the load current is the same as the exciting current.

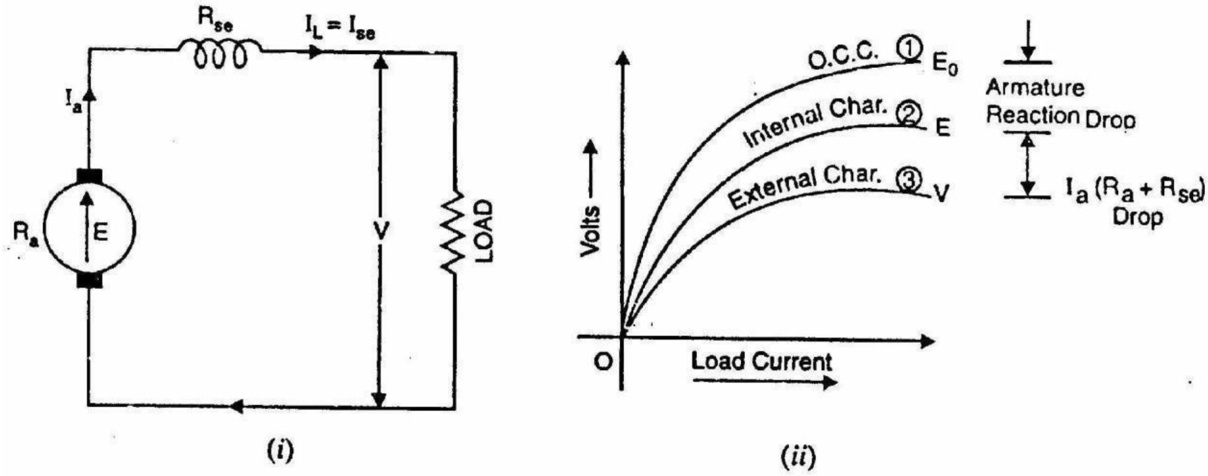


Fig. (3.7)

(i) O.C.C.

Curve 1 shows the open circuit characteristic (O.C.C.) of a series generator. It can be obtained experimentally by disconnecting the field winding from the machine and exciting it from a separate d.c. source as discussed in Sec. (3.2).

(ii) Internal characteristic

Curve 2 shows the total or internal characteristic of a series generator. It gives the relation between the generated e.m.f. E. on load and armature current. Due to armature reaction, the flux in the machine will be less than the flux at no load.

Hence, e.m.f. E generated under load conditions will be less than the e.m.f. E_0 generated under no load conditions. Consequently, internal characteristic curve

lies below the O.C.C. curve; the difference between them representing the effect of armature reaction [See Fig. 3.7 (ii)].

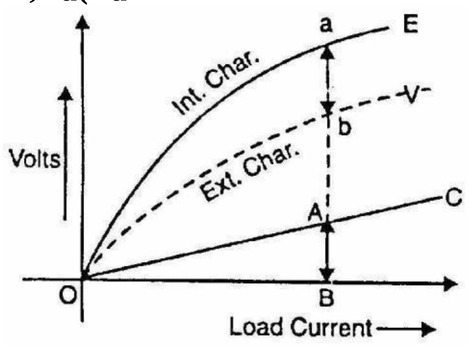
(iii) External characteristic

Curve 3 shows the external characteristic of a series generator. It gives the relation between terminal voltage and load current I_L:.

$$V = E - I_a(R_a + R_{se})$$

Therefore, external characteristic curve will lie below internal characteristic curve by an amount equal to ohmic drop [i.e., $I_a(R_a + R_{se})$] in the machine as shown in Fig. (3.7) (ii).

The internal and external characteristics of a d.c. series generator can be plotted from one another as shown in Fig. (3.8). Suppose we are given the internal characteristic of the generator. Let the line OC represent the resistance of the whole machine i.e. $R_a + R_{se}$. If the load current is OB, drop in the machine is AB i.e.



AB = Ohmic drop in the machine = OB($R_a + R_{se}$)

Now raise a perpendicular from point B and mark a point b on this line such that ab = AB. Then point b will lie on the external characteristic of the generator.

Following similar procedure, other points of external characteristic can be located. It is easy to see that we can also plot internal characteristic from the external characteristic.

Fig. (3.8)

Characteristics of a Shunt Generator

Fig (3.9) (i) shows the connections of a shunt wound generator. The armature current I_a splits up into two parts; a small fraction I_{sh} flowing through shunt field winding while the major part I_L goes to the external load.

(i) O.C.C.

The O.C.C. of a shunt generator is similar in shape to that of a series generator as shown in Fig. (3.9) (ii). The line OA represents the shunt field circuit resistance. When the generator is run at normal speed, it will build up a voltage OM. At no-load, the terminal voltage of the generator will be constant (= OM) represented by the horizontal dotted line MC.

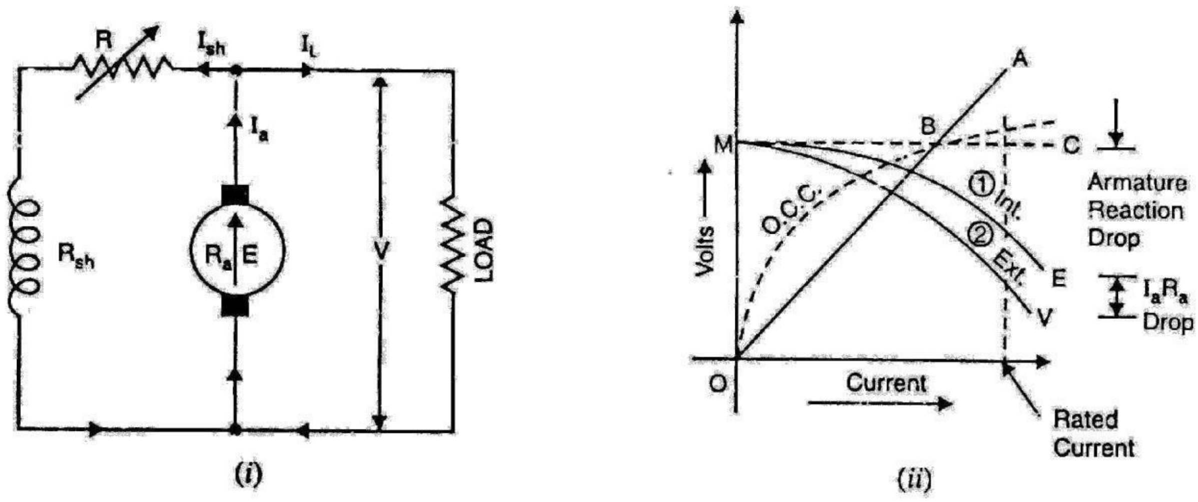


Fig. (3.9)

(ii) Internal characteristic

When the generator is loaded, flux per pole is reduced due to armature reaction. Therefore, e.m.f. E generated on load is less than the e.m.f. generated at no load. As a result, the internal characteristic (E/I_a) drops down slightly as shown in Fig. (3.9) (ii).

(iii) External characteristic

Curve 2 shows the external characteristic of a shunt generator. It gives the relation between terminal voltage V and load current I_L.

$$V = E - I_a R_a = E - (I_L + I_{sh}) R_a$$

Therefore, external characteristic curve will lie below the internal characteristic curve by an amount equal to drop in the armature circuit [i.e., $(I_L + I_{sh})R_a$] as shown in Fig. (3.9) (ii).

Note. It may be seen from the external characteristic that change in terminal voltage from no-load to full load is small. The terminal voltage can always be maintained constant by adjusting the field rheostat R automatically

Critical External Resistance for Shunt Generator

If the load resistance across the terminals of a shunt generator is decreased, then load current increase? However, there is a limit to the increase in load current with the decrease of load resistance. Any decreaseof load resistance beyond this point, instead of increasing the current, ultimatelyresults in

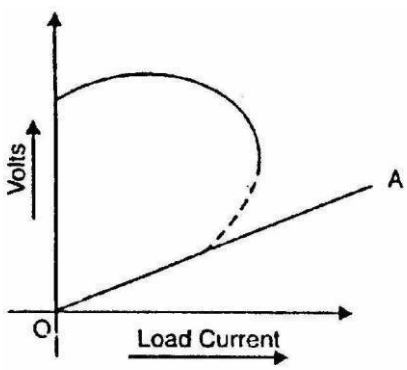


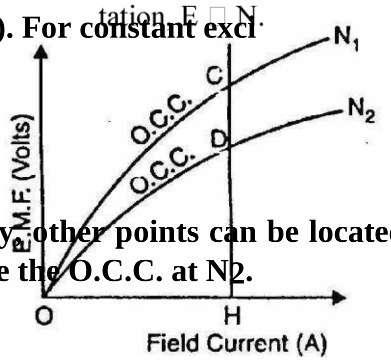
Fig. (3.10)

reduced current. Consequently, the external characteristic turns back (dotted curve) as shown in Fig. (3.10). The tangent OA to the curve represents the minimum external resistance required to excite the shunt generator on load and is called critical external resistance. If the resistance of the external circuit is less than the critical external resistance (represented by tangent OA in Fig. 3.10), the machine will refuse to excite or will de-excite if already running This means that external resistance is so low as virtually to short circuit the machine and so doing away with its excitation.

Note. There are two critical resistances for a shunt generator viz., (i) critical field resistance (ii) critical external resistance. For the shunt generator to build up voltage, the former should not be exceeded and the latter must not be gone below.

How to Draw O.C.C. at Different Speeds?

If we are given O.C.C. of a generator at a constant speed N_1, then we can easily draw the O.C.C. at any other constant speed N_2. Fig (3.11) illustrates the procedure. Here we are given O.C.C. at a constant speed N_1. It is desired to find the O.C.C. at constant speed N_2 (it is assumed that $n_1 < N_2$). For constant excitation $E \propto N$.



This locates the point D on the new O.C.C. at N_2. Similarly, other points can be located taking different values of I_f. The locus of these points will be the O.C.C. at N_2.

Critical Speed (N_C)

The critical speed of a shunt generator is the minimum speed below which it fails to excite. Clearly, it is the speed for which the given shunt field resistance represents the critical resistance. In Fig. (3.12), curve 2 corresponds to critical speed because the shunt field resistance (R_{sh}) line is tangential to it. If the generator runs at full speed N, the new O.C.C. moves upward and the R'sh line represents critical resistance for this speed.

 Speed Critical resistance

In order to find critical speed, take any convenient point C on excitation axis and erect a perpendicular so as to cut R_{sh} and R'_{sh} lines at points B and A respectively. Then,

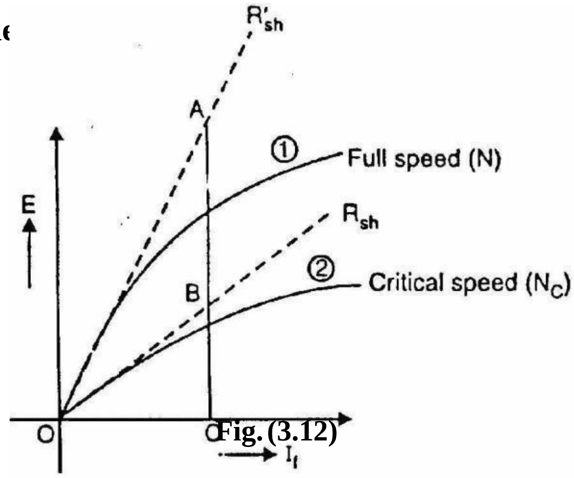


Fig. (3.12)

N

C

AC N

N
C AC

or

Conditions for Voltage Build-Up of a Shunt Generator

The necessary conditions for voltage build-up in a shunt generator are:

(v) There must be some residual magnetism in generator poles.

(vi) The connections of the field winding should be such that the field current strengthens the residual magnetism.

(vii) The resistance of the field circuit should be less than the critical resistance. In other words, the speed of the generator should be higher than the critical speed.

Compound Generator Characteristics

In a compound generator, both series and shunt excitation are combined as shown in Fig. (3.13). The shunt winding can be connected either across the armature only (short-shunt connection S) or across armature plus series field (long-shunt connection G). The compound generator can be cumulatively compounded or differentially compounded generator. The latter is rarely used in practice. Therefore, we shall discuss the characteristics of cumulatively-compounded generator. It may be noted that external characteristics of long and short shunt compound generators are almost identical.

External characteristic

Fig. (3.14) shows the external characteristics of a cumulatively compounded generator. The series excitation aids the shunt excitation. The degree of

compounding depends upon the increase in series excitation with the increase in load current.

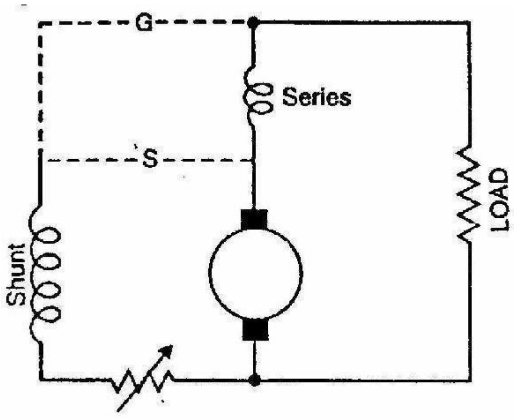


Fig. (3.13)

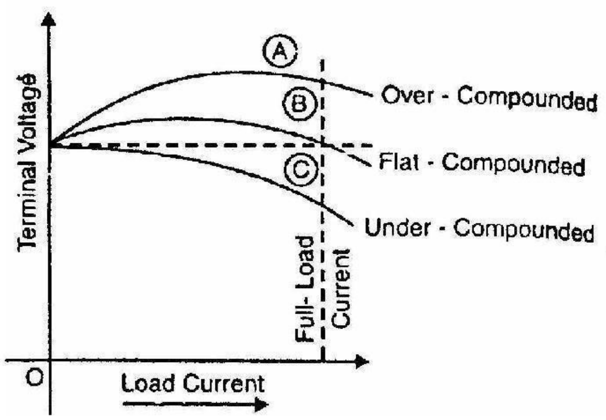


Fig. (3.14)

(i) If series winding turns are so adjusted that with the increase in load current the terminal voltage increases, it is called over-compounded generator. In such a case, as the load current increases, the series field m.m.f. increases and tends to increase the flux and hence the generated voltage. The increase in generated voltage is greater than the I_aR_a drop so that instead of decreasing, the terminal voltage increases as shown by curve A in Fig. (3.14).

(c) If series winding turns are so adjusted that with the increase in load current, the terminal voltage substantially remains constant, it is called flat-compounded generator. The series winding of such a machine has lesser number of turns than the one in over-compounded machine and, therefore, does not increase the flux as much for a given load current. Consequently, the full-load voltage is nearly equal to the no-load voltage as indicated by curve B in Fig (3.14).

(d) If series field winding has lesser number of turns than for a flat-compounded machine, the terminal voltage falls with increase in load current as indicated by curve C m Fig. (3.14). Such a machine is called under-compounded generator.

Voltage Regulation

The change in terminal voltage of a generator between full and no load (at constant speed) is called the voltage regulation, usually expressed as a percentage of the voltage at full-load.

$$\% \text{ Voltage regulation} = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100$$

where V_{NL} = Terminal voltage of generator at no load

V_{FL} = Terminal voltage of generator at full load

Note that voltage regulation of a generator is determined with field circuit and speed held constant. If the voltage regulation of a generator is 10%, it means that terminal voltage increases 10% as the load is changed from full load to no load.

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UNIT - III

DC Machines and Transformers – SEEA1202

Unit 3 – DC MOTOR

A dc motor is similar in construction to a dc generator. As a matter of fact a dc generator will run as a motor when its field & armature windings are connected to a source of direct current.

The basic construction is same whether it is generator or a motor.

Working principle:

The principle of operation of a dc motor can be stated as when a current carrying conductor is placed in a magnetic field; it experiences a mechanical force. In a practical dc motor, the field winding produces the required magnetic held while armature conductor play the role of current carrying conductor and hence the armature conductors experience a force.

As conductors are placed in the slots which are on the periphery, the individual force experienced by the conductive acts as a twisting or turning force on the armature which is called a torque.

The torque is the product of force and the radius at which this force acts, so overall armature experiences a torque and starts rotating.

Consider a single conductor placed in a magnetic field , the magnetic field is produced by a permanent magnet but in practical dc motor it is produced by the field winding when it carries a current.

Now this conductor is excited by a separate supply so that it carries a current in a particular direction. Consider that it carries a current away from an current. Any current carrying conductor produces its own magnetic field around it, hence this conductor also produces its own flux, around. The direction of this flux can be determined by right hand thumb rule. For direction of current considered the direction of flux around a conductor is clock-wise. Now, there are two fluxes present

5. **Flux produced by permanent magnet called main flux**
6. **Flux produced by the current carrying conductor**

From the figure shown below, it is clear that on one side of the conductor, both the fluxes are in the same direction in this case, on the left of the conductor there gathering of the flux lines as two fluxes help each other. A to against this, on the right of the conductor, the two fluxes are in opposite direction and hence try to cancel each other. Due to this, the density of the flux lines in this area gets weakened.

So on the left, there exists high flux density area while on the right of the conductor then exists low flux density area as shown.

The flux distribution around the conductor arts like a stretched ribbed bond under tension. The exerts a mechanical force on the conductor which acts from high flux density area towards low flux density area,

i.e. from left to right from the case considered as shown above.

In the practical dc motor, the permanent magnet is replaced by the field winding which produces the required flux winding which produces the required flux called main flux and all the armature conductors, would on the periphery of the armature gram, get subjected tot he mechanical force.

Due to this, overall armature experiences a twisting force called torque and armature of the motor status rotating.

Direction of rotation of motor

The magnitude of the force experienced by the conductor in a motor is given by F = BIL newtons.

The direction of the main field can be revoked y changing the direction of current passing through the field winding, which is possible by interchanging the polarities of supply which is given to the field winding.

The direction of current through armature can be reversed by changing supply polarities of dc supplying current to the armature.

It directions of bot the currents are changed then the direction of rotation of the motor remains undamaged.

In a dc motor both the field and armature is connected to a source of direct current. The current through the armature winding establish its own magnetic flux the interaction both the main field and the armature current produces the torque, there by sensing the motor to rotate, once the motor starts rotating, already existing magnetic flux there wire be an induced emf in the armature conductors due to generator action. This emf acts in a direction apposite to supplied voltage. Therefore it is called Black emf.

Significance of Back emf

In the generating action, when a conductor cuts the lines of flux, emf gets induced in the conductor in a motor, after a motoring action, armature starts rotating and armature conductors cut the main flux. After a motoring action, there exists a generating action there is induced emf in the rotating armature conductors according to Faraday's law of electromagnetic induction. This induced emf in the armature always acts in the opposite direction of the supply voltage. This is according tot he lenz's law which states that the direction of the induced emf is always so as to oppose the case producing it.

In a dc motor, electrical input i.e., the supply voltage is the cause and hence this induced emf opposes the supply voltage.

The emf tries to set u a current throughout he armature which is in the opposite direction to that which supply voltage is forcing through the conductor so, as this emf always opposes the supply voltage, it is called back emf and devoted as Eb.

Through it is denoted as Eb, basically it gets generated by the generating action which we have seen earlier So, $E_b = \frac{ZNP}{60\,A}$

Voltage equation of a Motor

The voltage v applied across the motor armature has to (1) over core the back emf Eb and

3. supply the armature ohmic drop

Ia Ra v = Eb + Ia Ra

This is known as voltage equation of a motor Multiplying both sides by Ia, we get

Via =Eb Ia + Ea^2 Ra

~~VIa = electrical input to the armature~~

EbIa = electrical equivalent of mechanical Power developed in the armature

$I_a^2 R_a$ = un loss in the armature

Hence, out of the armature input, some in wasted in I2 R loss and the rot is convened into mechanical power within the armature.
Motor efficiency is given by the ratio of power developed by the armature to its input i.e. $E_b I_a / vI_a = E_b/v$.

Higher the value of E_b as compared to v, higher the motor efficiency.

Conduction for maximum powers

The gross mechanical developed by a motor = $p_m = vI_a - I_a^2 R_a$

$$\frac{dP_m}{dI_a} \quad v \square 2I_a R_a \qquad I_a R_a = v/2$$

As $v = E_b + I_a R_a$ and $I_a R_a = v/2$ $\quad E_b = v/2$

Thus gross mechanical power developed by a motor is maximum when back emf is equal to half the applied voltage. This conduction's how ever at realized in practice, because in that case current will be much beyond the normal current of the motor.

More ova, half the input would be wasted in the form of heat and taking other losses into consideration the motor efficiency will be well below 50 %.

1. A 220v – dc machine has an armature resistance of 0.5 □. If the full road armature current is 20A, find **the induced emf when the machine acts (1) generator (2) motor.**

The dc motor is assumed to be shunt connected in cash case, short current in considered negligible because its value is not given.

(a) As generator $\quad E_g = v + I_a R_a = 220 + 0.5 \times 20 = 230$ v

(b) As motor $\quad E_b = v - I_a R_a = 220 - 0.5 \times 20 = 210$ v

8) A 440 v, shunt motor has armature resistance of 0.8 a and field resistance of 200 □. Determine **the back emf when giving an output at 7.46 kw at 85% efficiency.**

Motor input power = $\frac{7.46 \times 103}{}$ w 0.85

7460

Motor current = input $\overline{0.85 \times 440}$ □ 19.95 A

3. A 25kw, 250 w dc such generator has armature and field resistance of 0.06 □ and 100 □ **respectively. Determine the total armature power developed when working (1) as generator delivering 25 kw output and (2) as a motor taking 25 kw input.**

Voltage equation of dc motor

For a generator, generated emf has to supply armature resistance drop and remaining part is available across the loss as a terminal voltage. But in case of dc motor, supply voltage v has to over come back

emf Eb which is opposing v and also various drops are armature resistance drop Ia Ra, brush drop etc. In fact the electrical work done in overcoming the back emf gets converted into the mechanical energy, developed in the armature.

Hence, the voltage equation of a dc motor is $V = E_b + I_a R_a$ + brush drops
Or $v = E_b + I_a R_a$ neglecting brush drops

The back emf is always less than supply voltage ($E_b < v$) but R_a is very small hence under normal running conditions, the different between back emf and supply voltage is very small. The net voltage across the armature is the difference between the supply voltage and back emf which decals the armature current. Hence from the voltage equation we can write $I_a = v – E_b / R_a$.

3. A 220 v dc motor has an armature resistance of 0.75□ it is drawing on armature current of 30 A, **during a certain load, calculate the induced emf in the motor under this condition.**

$V = 200$ v, $I_a = 30A$, R_a = 0.75□
For a motor , $v = E_b + I_a$

R_a $E_b = 197.5$ v

This is the induced mef called back emf in a motor.

5. **A 4-pole dc motor has lap connected armature winding. The number of armature conductors is 250. When connected to 230 v dc supply it draws an armature current It 4 cm calculate the back emf and the speed with which motor is** running. Assume armature is 0.6□

P = 4 A = P = 4 as lap connected

□ = 30 m wb = 30 Ho^{-3} **V = 230v, z = 250** **Ia = 40 A**

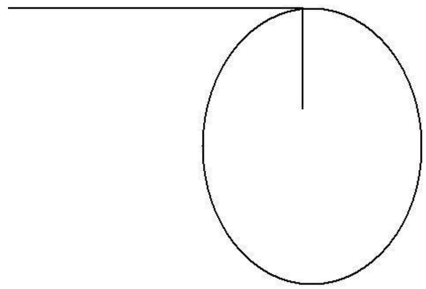
From voltage equation $V = E_b + I_a R_a$ 230 = Eb + 40 x 0.6

Eb = □ Pnz / 60A

206 = (30 x 10^{-3} x 4 x N x 250) / (60 x 4) N = 1648 rpm.

Torque: The turning or twisting movement of a body is called Torque. (Or)

It is defined as the product of force and perpendicular distance Ť=F*R F



In case of DC motor torque is produced by the armature and shaft called as armature torque (T_a) and shaft torque(T_{sh}). Let, N be the speed of the armature in RPM R be the radius of the armature Power=Work Done/Time

Work Done=Force X Distance

The distance travelled in rotating the armature for one time =2∏R

If N rotations are made in 60 sec

Then time taken for one rotation is=60/N

So, Power= (F * 2∏R)/ (60/N)

=

(F*R)(2∏N)/

60 P = Ťω

Here P=$E_b I_a$

But

E_b=ØZNP/60

A

(ØZNP/60A)I

a=

Ťω

=

Ťa(2∏N)/6

0

Ťa=0.159Ø

Z

I_aP/A

Similarly, Shaft torque T_{sh}=output/ω

T_{sh} =output/((2∏N)/60)

T_{sh} =9.55(output)/N

Testing of DC Motors

Introduction

In the previous sections we have learnt about the principle of operation of d.c. generators and motors, (starting and speed control of d.c motor). Motors convert *electrical* power (input power) into *mechanical* power (output power) while generators convert *mechanical* power (input power) into *electrical* power (output power). Whole of the input power can not be converted into the output power in a practical machine due to various losses that take place within the machine. Efficiency η being the ratio of output power to input power, is always less than 1 (or 100 %). Designer of course will try to make η as large as possible. Order of efficiency of rotating d.c machine is about 80 % to 85 %. It is therefore important to identify the losses which make efficiency poor.

In this lesson we shall first identify the losses and then try to estimate them to get an idea of efficiency of a given d.c machine.

Major losses

Take the case of a loaded d.c motor. There will be copper losses ($I_a^2 r_a$ and $I_f^2 R_f = VI_f$) in armature and field circuit. The armature copper loss is variable and depends upon degree of loading of the machine. For a shunt machine, the field copper loss will be constant if field resistance is not varied. Recall that rotor body is made of iron with slots in which armature conductors are placed. Therefore when armature rotates in presence of field produced by stator field coil, eddy current and hysteresis losses are bound to occur on the rotor body made of iron. The sum of eddy current and hysteresis losses is called the *core* loss or *iron* loss. To reduce *core* loss, circular varnished and slotted laminations or *stamping* are used to fabricate the armature. The value of the core loss will depend on the strength of the field and the armature speed. Apart from these there will be power loss due to *friction* occurring at the bearing & shaft and air friction (windage loss) due to rotation of the armature. To summarise following major losses occur in a d.c machine.

7. Field copper loss: It is power loss in the field circuit and equal to $I_f^2 R_f = VI_f$. During the course of loading if field circuit resistance is not varied, field copper loss remains constant.

8. Armature copper loss: It is power loss in the armature circuit and equal to $I_a^2 R_a$. Since the value of armature current is decided by the load, armature copper loss becomes a function of time.

9. Core loss: It is the sum of eddy current and hysteresis loss and occurs mainly in the rotor iron parts of armature. With constant field current and if speed does not vary much with loading, core loss may be assumed to be constant.

10. Mechanical loss: It is the sum of bearing friction loss and the windage loss (friction loss due to armature rotation in air). For practically constant speed operation, this loss too, may be assumed to be constant.

Apart from the major losses as enumerated above there may be a small amount loss called *stray* loss occur in a machine. Stray losses are difficult to account. Power flow diagram of a d.c motor is shown in figure 40.1. A portion of the input power is consumed by the field circuit as field copper loss. The remaining power is the power which goes to the armature; a portion of which is lost as core loss in the armature core and armature copper loss. Remaining power is the gross mechanical power developed of which a portion will be lost as friction and remaining power will be the net mechanical power developed. Obviously efficiency of the motor will be given by:

$$\eta = \frac{P_{net\ mech}}{P_{in}}$$

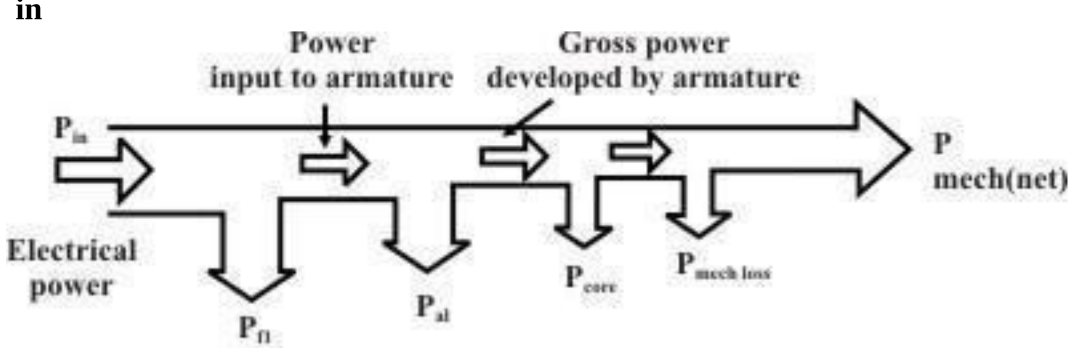
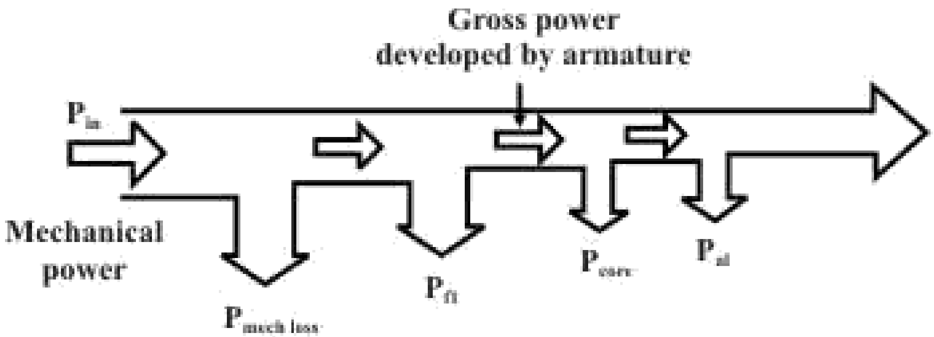


Fig. 40.1: Power flow diagram of a D.C. motor

Similar power flow diagram of a d.c generator can be drawn to show various losses and input, output power (figure 40.2).



kg. 4 2: Power now diagram of a D.C. generator

It is important to note that the name plate kW (or hp) rating of a d.c machine always corresponds to the net output at rated condition for both generator and motor.

Swinburne's Test

For a d.c shunt motor change of speed from no load to full load is quite small. Therefore, mechanical loss can be assumed to remain same from no load to full load. Also if field current is held constant during loading, the core loss too can be assumed to remain same.

In this test, the motor is run at rated speed under *no load* condition at rated voltage. The current drawn from the supply I_{L0} and the field current I_f are recorded (figure 40.3). Now we note that:

Input power to the motor, P_{in} Cu loss in the field circuit P_{fl} Power input to the armature,

4. VI_{L0}
5. VI_f
6. $VI_{L0} - VI_f$
7. $V(I_{L0} - I_f)$
8. VI_{a0}

Cu loss in the armature circuit Gross power developed by armature

$= I_{a0}^2 r_a$

$= VI_{a0} - I_{a0}^2 r_a$

$= (V - I_{a0} r_a) I_{a0}$

7. $I_{b0} I_{a0}$

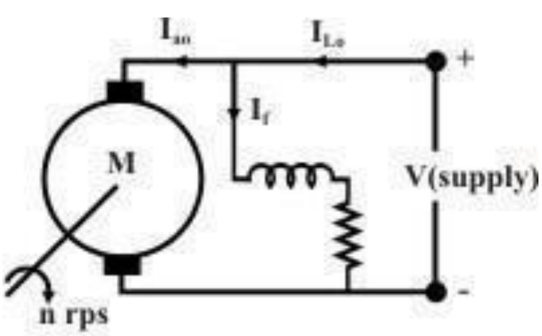


Fig. 40.3: Motor under no load

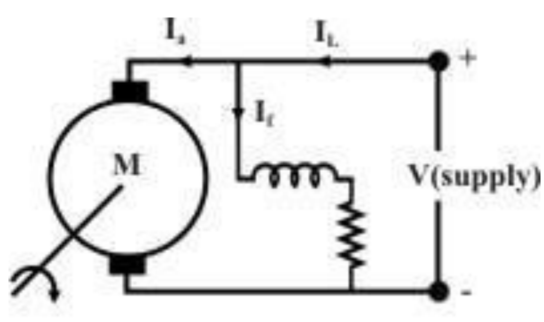


Fig. 40.4: Motor Loaded

Since the motor is operating under no load condition, net mechanical output power is zero. Hence the gross power developed by the armature must supply the core loss and friction & windage losses of the motor. Therefore,

$$P_{core} + P_{friction} = (V - I_{a0} r_a) I_{a0} = E_{b0} I_{a0}$$

Since, both P_{core} **and** $P_{friction}$ **for a shunt motor remains practically constant from no load to full load, the sum of these losses is called constant rotational loss i.e.,**

$$\text{constant rotational loss, } P_{rot} = P_{core} + P_{friction}$$

In the Swinburne's test, the constant rotational loss comprising of core and friction loss is estimated from the above equation.

After knowing the value of P_{rot} **from the Swinburne's test, we can fairly estimate the efficiency of the motor at any loading condition. Let the motor be loaded such that new current drawn from the supply is** I_L **and the new armature current is** I_a **as shown in figure 40.4. To estimate the efficiency of the loaded motor we proceed as follows:**

Input power to the motor, $P_{in} = VI_L$

Cu loss in the field circuit $P_{fl} = VI_f$

Power input to the armature, $= VI_L - VI_f$

$$= V(I_L - I_f)$$

$$= VI_a$$

Cu loss in the armature circuit $= I_a^2 r_a$

Gross power developed by armature $= VI_a - I_a^2 r_a$

$$= (V - I_a r_a) I_a$$

$$= E_b I_a$$

Net mechanical output power, $P_{net\,mech} = E_b I_a - P_{rot}$

$$\therefore \text{ efficiency of the loaded motor, } \eta = \frac{E_b I_a - P_{rot}}{VI_L} = \frac{P_{net\,mech}}{P_{in}}$$

The estimated value of P_{rot} **obtained from Swinburne's test can also be used to estimate the efficiency of the shunt machine operating as a generator. In figure 40.5 is shown to deliver a**

load current I_L to a load resistor R_L. In this case output power being known, it is easier to add the losses to estimate the input mechanical power.

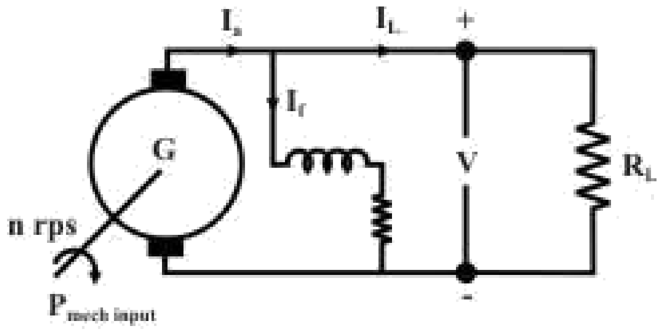


Fig. 40.5: Loaded d.c. generator

Output power of the generator, $P_{out} = VI_L$

Cu loss in the field circuit $P_{fl} = VI_f$

Output power of the armature, $= VI_L + VI_f = VI_a$

Mechanical input power, $P_{in\,mech} = VI_a + I_a^2 r_a + P_{rot}$

∴ Efficiency of the generator, η $= \dfrac{VI_L}{P_{in\,mech}}$

$$= \frac{VI_L}{VI_a + I_a^2 r_a + P_{arot}} \quad \text{(viii)}$$

The biggest advantage of Swinburne's test is that the shunt machine is to be run as motor under *no load* condition requiring little power to be drawn from the supply; based on the no load reading, efficiency can be predicted for any load current. However, this test is not sufficient if we want to know more about its performance (effect of armature reaction, temperature rise, commutation etc.) when it is actually loaded. Obviously the solution is to load the machine by connecting mechanical load directly on the shaft for motor or by connecting loading rheostat across the terminals for generator operation. This although sounds simple but difficult to implement in the laboratory for high rating machines (say above 20 kW), Thus the laboratory must have proper supply to deliver such a large power corresponding to the rating of the machine. Secondly, one should have loads to absorb this power.

Hopkinson's test

This as an elegant method of testing d.c machines. Here it will be shown that while power drawn from the supply only corresponds to no load losses of the machines, the armature physically carries any amount of current (which can be controlled with ease). Such a scenario can be created using two similar mechanically coupled shunt machines. Electrically these two machines are eventually connected in parallel and controlled in such a way that one machine acts as a generator and the other as motor. In other words two similar machines are required to carry out this testing which is not a bad proposition for manufacturer as large numbers of similar machines are manufactured.

Procedure

Connect the two similar (same rating) coupled machines as shown in figure 40.6. With switch S opened, the first machine is run as a shunt motor at rated speed. It may be noted that the second machine is operating as a separately excited generator because its field winding is excited and it is driven by the first machine. Now the question is what will be the reading of the voltmeter connected across the opened switch S? The reading may be (i) either close to twice supply voltage or (ii) small voltage. In fact the voltmeter practically reads the difference of the induced voltages in the armature of the machines. The upper armature terminal of the generator may have either + ve or negative polarity. If it happens to be +ve, then voltmeter reading will be small otherwise it will be almost double the supply voltage.

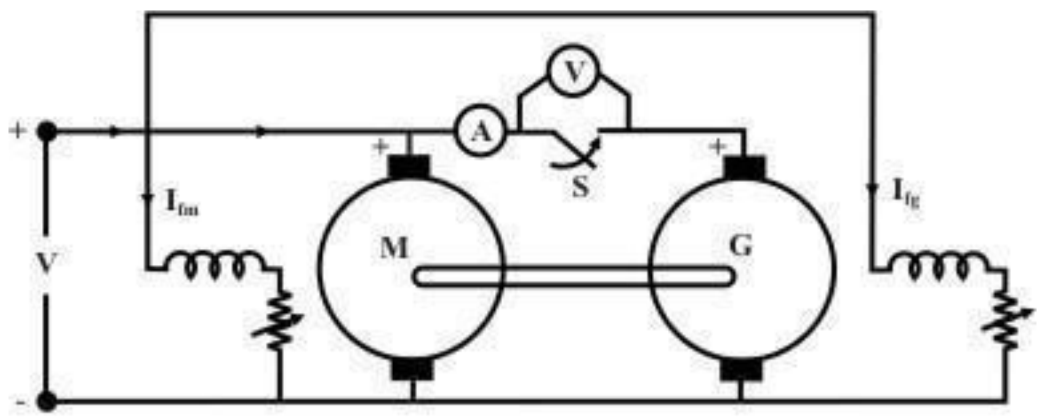


Fig. 40.6: Hopkinson's test: machines before paralleling

Since the goal is to connect the two machines in parallel, we must first ensure voltmeter reading is small. In case we find voltmeter reading is high, we should switch off the supply, reverse the armature connection of the generator and start afresh. Now voltmeter is found to read small although time is still not ripe enough to close S for paralleling the machines. Any attempt to close the switch may result into large circulating current as the armature resistances are small.
Now by adjusting the field current I_{fg} of the generator the voltmeter reading may be adjusted to zero ($E_g \approx E_b$) and S is now closed. Both the machines are now connected in parallel as shown in figure 40.7.

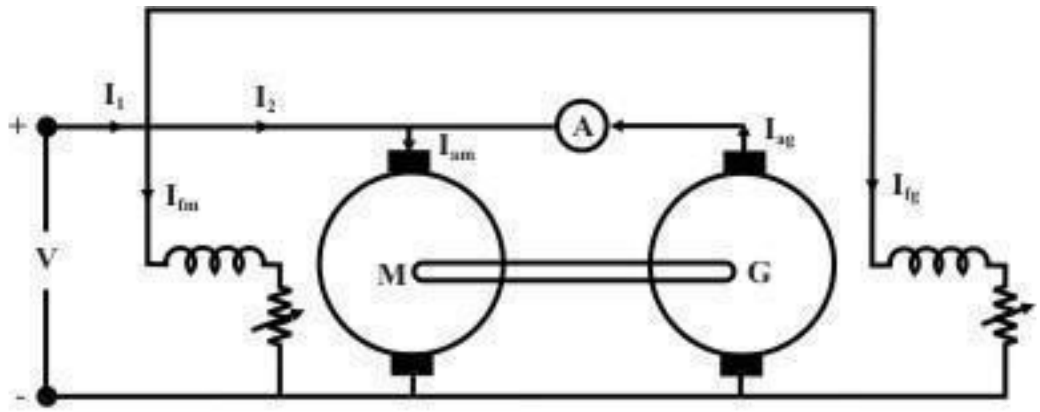


Fig. 40.7: Hopkinson's test: machines paralled

Loading the machines

After the machines are successfully connected in parallel, we go for loading the machines i.e., increasing the armature currents. Just after paralleling the ammeter reading A will be close to zero as $E_g \approx E_b$. Now if I_{fg} is increased (by decreasing R_{fg}), then E_g becomes greater than E_b and both I_{ag} and I_{am} increase, Thus by increasing field current of generator (alternatively decreasing field current of motor) one can make $E_g > E_b$ so as to make the second machine act as generator and first machine as motor. In practice, it is also required to control the field current of the motor I_{fm} to maintain speed constant at rated value. The interesting point to be noted here is that I_{ag} and I_{am} do not reflect in the supply side line. Thus current drawn from supply remains small (corresponding to losses of both the machines). The loading is sustained by the output power of the generator running the motor and vice versa. The machines can be loaded to full load current without the need of any loading arrangement.

Calculation of efficiency

Let field currents of the machines be are so adjusted that the second machine is acting as generator with armature current I_{ag} and the first machine is acting as motor with armature current I_{am} as shown in figure 40.7. Also let us assume the current drawn from the supply be I_1. Total power drawn from supply is VI_1 which goes to supply all the losses (namely Cu losses in armature & field and rotational losses) of both the machines, Now:

Power drawn from supply = VI_1

Field Cu loss for motor = VI_{fm}

Field Cu loss for generator = VI_{fg}

Armature Cu loss for motor = $I_{am}^2 r_{am}$

Armature Cu loss for generator = $I_{ag}^2 r_{ag}$

$$\therefore \text{Rotational losses of both the machines} = VI_1 - (VI_{fm} + VI_{fg} + I_{am}^2 r_{am} + I_{ag}^2 r_{ag}) \qquad (40.1)$$

Since speed of both the machines are same, it is reasonable to assume the rotational losses of both the machines are equal; which is strictly not correct as the field current of the generator will be a bit more than the field current of the motor, Thus,

$$\text{Rotational loss of each machine, } P_{rot} = \frac{VI_1 - (VI_{fm} + VI_{fg} + I_{am}^2 r_{am} + I_{ag}^2 r_{ag})}{2}$$

Once P_{rot} is estimated for each machine we can proceed to calculate the efficiency of the machines as follows,

Efficiency of the motor

As pointed out earlier, for efficiency calculation of motor, first calculate the input power and then subtract the losses to get the output mechanical power as shown below,

Total power input to the motor = power input to its field + power input to its armature

$$P_{inm} = VI_{fm} + VI_{am}$$

Losses of the motor = $VI_{fm} + I_{am}^2 r_{am} + P_{rot}$

Net mechanical output power $P_{outm} = P_{inm} - (VI_{fm} + I_{am}^2 r_{am} + P_{rot})$

$$\therefore \eta_m = \frac{P_{outm}}{P_{inm}}$$

Efficiency of the generator

For generator start with output power of the generator and then add the losses to get the input mechanical power and hence efficiency as shown below,

Output power of the generator, $P_{outg} = VI_{ag}$

Losses of the generator = $VI_{fg} + I_{ag}^2 r_{ag} + P_{rot}$

Input power to the generator, $P_{ing} = P_{outg} + (VI_{fg} + I_{ag}^2 r_{ag} + P_{rot})$

$$\therefore \eta_g = \frac{P_{outg}}{P_{ing}}$$

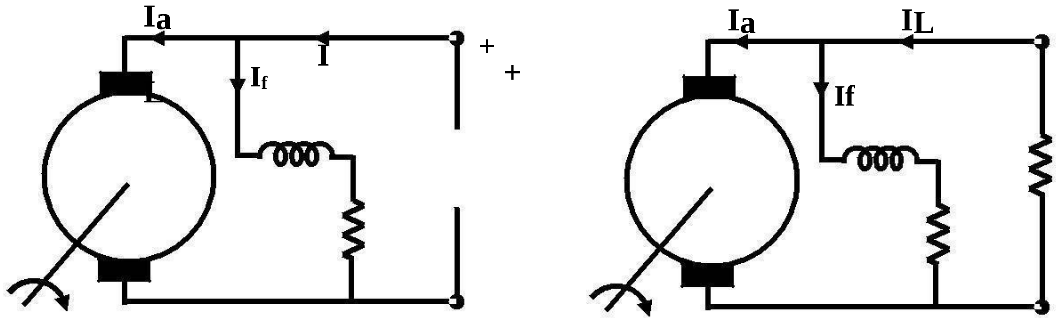
Condition for maximum efficiency

We have seen that in a transformer, maximum efficiency occurs when copper loss = core loss, where, copper loss is the variable loss and is a function of loading while the core loss is practically constant independent of degree of loading. This condition can be stated in a different way: maximum efficiency occurs when the variable loss is equal to the constant loss of the transformer.

Here we shall see that similar condition also exists for obtaining maximum efficiency in a d.c shunt machine as well.

Maximum efficiency for motor mode

Let us consider a loaded shunt motor as shown in figure 40.8. The various currents along with their directions are also shown in the figure.



We assume that field current I_f remains constant during change of loading. Let,

P_{rot} = constant rotational loss

$V I_f$ = constant field copper loss

Constant loss $P_{const} = P_{rot} + V I_f$

Now, input power drawn from supply = $V I_L$

Power loss in the armature, = $I_a^2 r_a$

Net mechanical output power $= VI_L - I_a^2 r_a - (VI_f + P_{rot})$

$$= VI_L - I_a^2 r_a - P_{const}$$

so, efficiency at this load current $\eta_m = \dfrac{VI_L - I_a^2 r_a - P_{const}}{VI_L}$

Now the armature copper loss $I_a^2 r_a$ can be approximated to $I_L^2 r_a$ as $I_a \approx I_L$. This is because the order of field current may be 3 to 5% of the rated current. Except for very lightly loaded motor, this assumption is reasonably fair. Therefore replacing I_a by I_f in the above expression for efficiency η_m, we get,

$$\eta_m = \frac{VI_L - I_L^2 r_a - P_{const}}{VI_L} = 1 - \frac{I_L r_a}{V} - \frac{P_{const}}{VI_L}$$

$$\frac{d\eta_m}{dI_L} = 0$$

or, $\dfrac{d}{dI_L}\left(\dfrac{I_L r_a}{V} - \dfrac{P_{const}}{VI_L}\right) = 0$

or, $-\dfrac{r_a}{V} + \dfrac{P_{const}}{VI_L^2}$

∴ Condition for maximum efficiency is $I_a^2 r_a \approx I_L^2 r_a =$

$$L \quad a \quad a\,a \quad P_{const} = P_{cons\,t}\, r_a$$

So, the armature current at which efficiency becomes maximum is I_a

Thus, we get a simplified expression for motor efficiency η_m in terms of the variable current (which depends on degree of loading) I_L, current drawn from the supply. So to find out the condition for maximum efficiency, we have to differentiate η_m with respect to I_L and set it to zero as shown below.

Maximum efficiency for Generation mode

Similar derivation is given below for finding the condition for maximum efficiency in generator mode by referring to figure 40.9.

We assume that field current I_f remains constant during change of loading. Let,

$$\frac{d\eta_g}{dI_L} = 0$$

$$\text{or, } \frac{d}{dI_L}\left(\frac{VI_L}{VI_L + I_L^2 r_a + P_{const}}\right) = 0$$

∴ Simplifying we get the condition as $I_L^2 r_a \approx I_a^2 r_a = P_{const}$

So, the armature current at which efficiency becomes maximum is $I_a = \sqrt{\frac{P_{const}}{r_a}}$

P_{rot} **= constant rotational loss**

$V I_f$ **= constant field copper loss**

Constant loss $P_{const} = P_{rot} + V I_f$

Net output power to load = VI_L

Power loss in the armature, = $I_a^2 r_a$

Mechanical input power $= VI_L + I_a^2 r_a + (VI_f + P_{rot})$

$= VI_L + I_a^2 r_a + P_{const}$

so, efficiency at this load current $\eta_g = \frac{VI_L}{VI_L + I_a^2 r_a + P_{const}}$

As we did in case of motor, the armature copper loss $I_a^2 r_a$ **can be approximated to** $I_L^2 r_a$ **as** $I_a \approx I_L$ **. So expression for** η_g **becomes,**

$$\eta_g = \frac{VI_L}{VI_L + I_L^2 r_a + P_{const}}$$

Thus, we get a simplified expression for motor efficiency ηg in terms of the variable current (which depends on degree of loading) I_L, current delivered to the load. So to find out the condition for maximum efficiency, we have to differentiate ηg with respect to I_L and set it to zero as shown below.

Thus maximum efficiency both for motoring and generating are same in case of shunt machines. To state we can say at that armature current maximum efficiency will occur which will make variable loss = constant loss. Eventually this leads to the expression for armature
current for maximum efficiency as $I_a = \sqrt{P_{const}/r_a}$.

D.C. Motor Principle

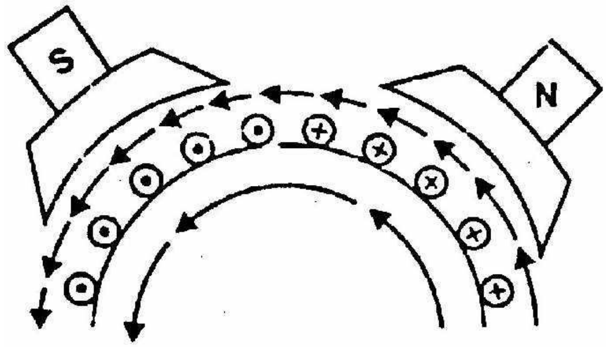
A machine that converts d.c. power into mechanical power is known as a d.c.motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by;

$F = BIl$ newtons

Basically, there is no constructional difference between a d.c. motor and a d.c.generator. The same d.c. machine can be run as a generator or motor.

Working of D.C. Motor

When the terminals of the motor are connected to an external source of d.c. supply:
(i) the field magnets are excited developing alternate N and S poles;
(ii) the armature conductors carry currents.

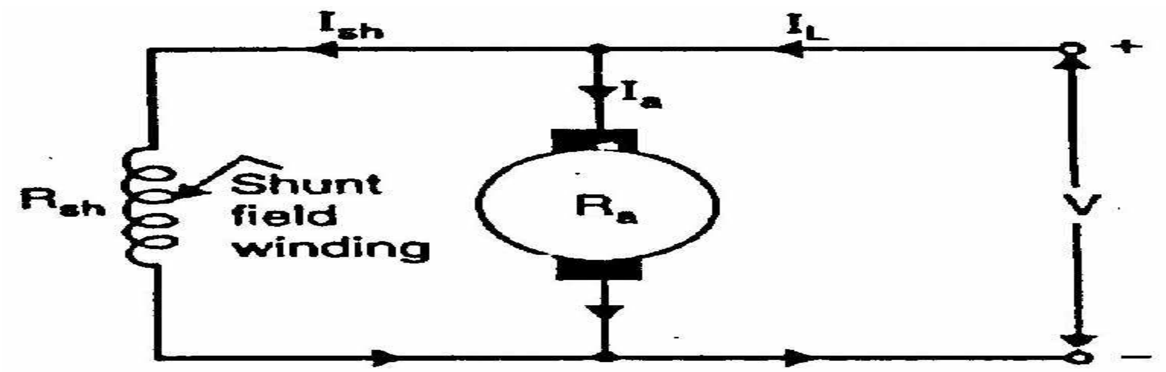


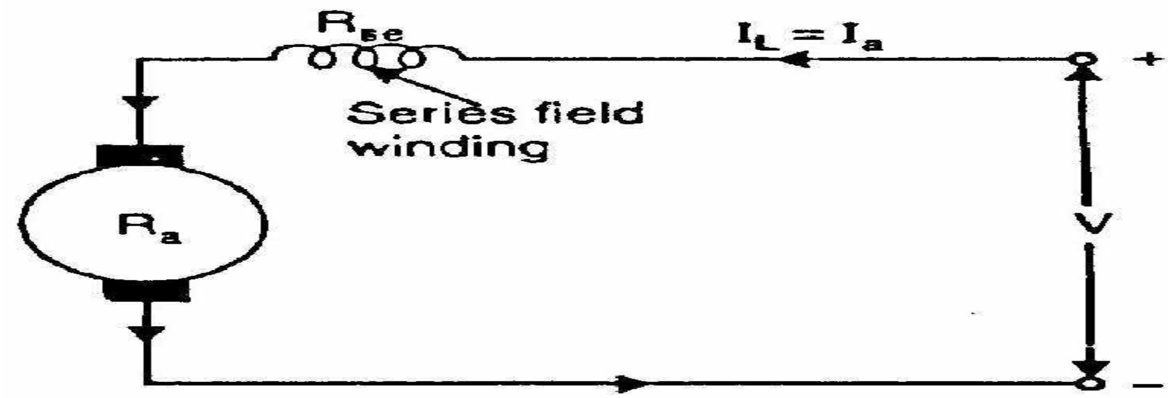
All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction. Suppose the conductors under N-pole carry currents into the plane of the paper and those under S- pole carry currents out of the plane of the paper as shown in Fig. Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it.

Applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating. When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same.

Types of D.C. Motors

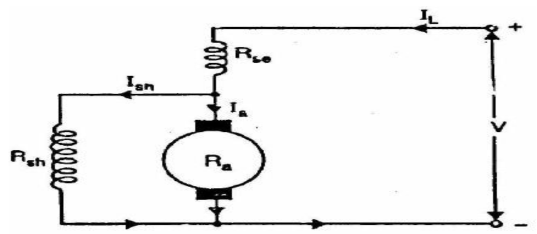
Like generators, there are three types of d.c. motors characterized by the connections of field winding in relation to the armature viz.:
(i) Shunt-wound motor in which the field winding is connected in parallel with the armature. The current through the shunt field winding is not the same as the armature current. Shunt field windings are designed to produce the necessary m.m.f. by means of a relatively large
number of turns of wire having high resistance. Therefore, shunt field current is relatively small compared with the armature current.



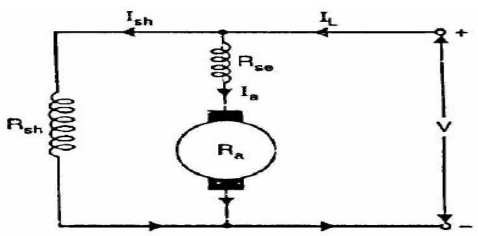


(ii) Series-wound motor in which the field winding is connected in series with the armature Therefore, series field winding carries the armature current. Since the current passing through a series field winding is the same as the armature current, series field windings must be designed with much fewer turns than shunt field windings for the same m.m.f. Therefore, a series field winding has a relatively small number of turns of thick wire and, therefore, will possess a low resistance.

(iii) Compound-wound motor which has two field windings; one connected in parallel with the armature and the other in series with it. There are two types of compound motor connections (like generators). When the shunt field winding is directly connected across the armature terminals it is called short-shunt connection. When the shunt winding is so connected that it shunts the series combination of armature and series field it is called long- shunt connection.



Short-shunt connection



Long-shunt connection

Motor Characteristics

Torque/Speed Curves

In order to effectively design with D.C. motors, it is necessary to understand their characteristic curves. For every motor, there is a specific Torque/Speed curve and Power curve.

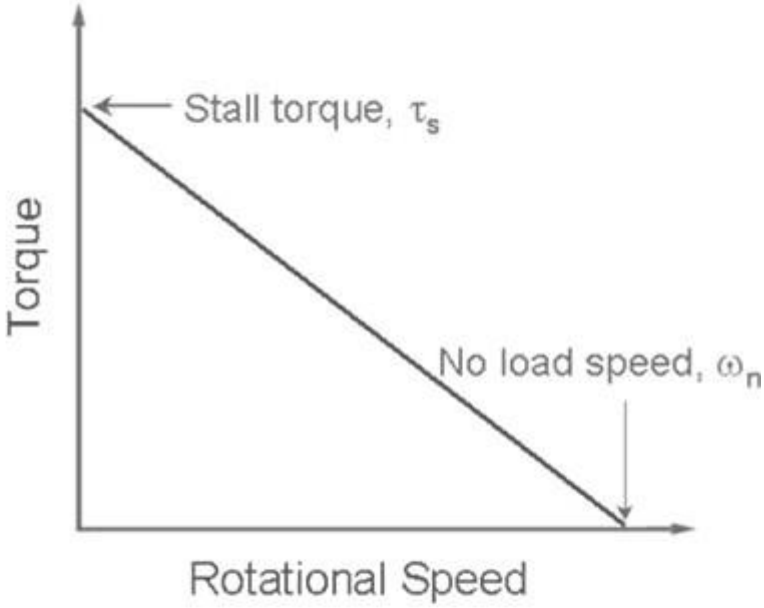


Figure 5.13 Speed Vs Torque Curve

The graph above shows a torque/speed curve of a typical D.C. motor. Note that torque is inversely proportional to the speed of the output shaft. In other words, there is a tradeoff between how much torque a motor delivers, and how fast the output shaft spins. Motor characteristics are frequently given as two points on this graph:

- **The stall torque represents the point on the graph at which the torque is a maximum, but the shaft is not rotating.**
- **The no load speed, is the maximum output speed of the motor (when no torque is applied to the output shaft).**

- **The linear model of a D.C. motor torque/speed curve is a very good approximation. The torque/speed curves shown below are actual curves for the green maxon motor (pictured at right) used by students in 2.007. One is a plot of empirical data, and the other was plotted mechanically using a device developed at MIT.**

Figure 5.14Maxon Motor

Note that the characteristic torque/speed curve for this motor is quite linear. This is generally true as long as the curve represents the direct output of the motor, or a simple gear reduced output. If the specifications are given as two points, it is safe to assume a linear curve.

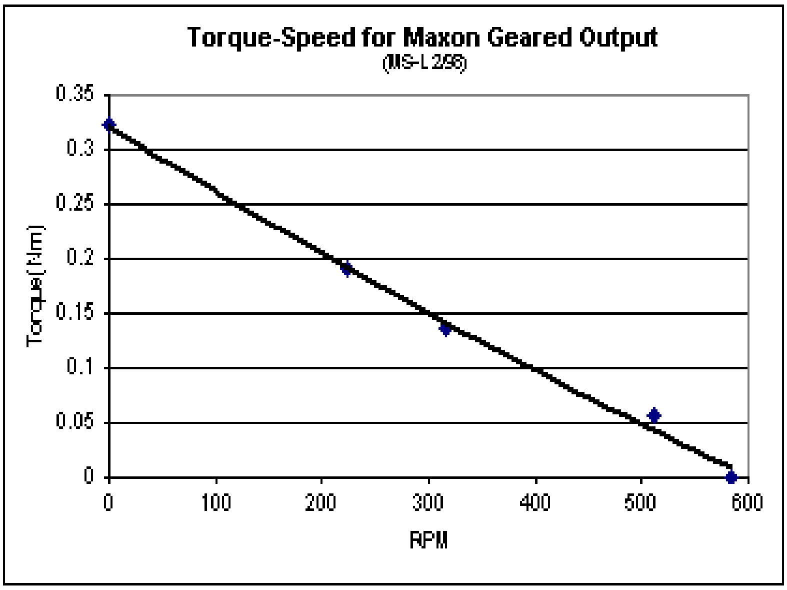


Figure 5.15Speed Vs Torque Characteristics

Recall that earlier we defined power as the product of torque and angular velocity. This corresponds to the area of a rectangle under the torque/speed curve with one corner attheorigin and another corner at a point on the curve. Due to the linear inverse relationship between torque and speed, the maximum power occurs at the point where Recall that earlier we defined power as the product of torque and angular velocity.

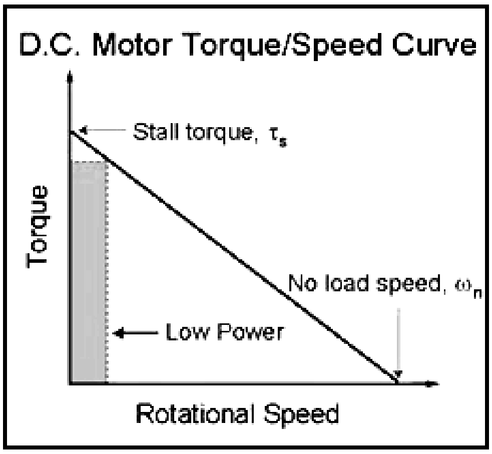


Figure 5.16Speed Vs Torque Characteristics

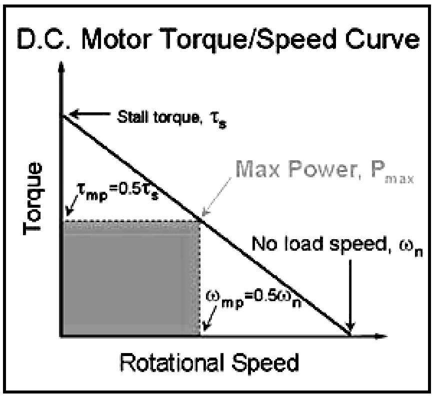


Figure 5.17Speed Vs Torque Characteristics

This corresponds to the area of a rectangle under the torque/speed curve with one corner at the origin and another corner at a point on , and = ½ .

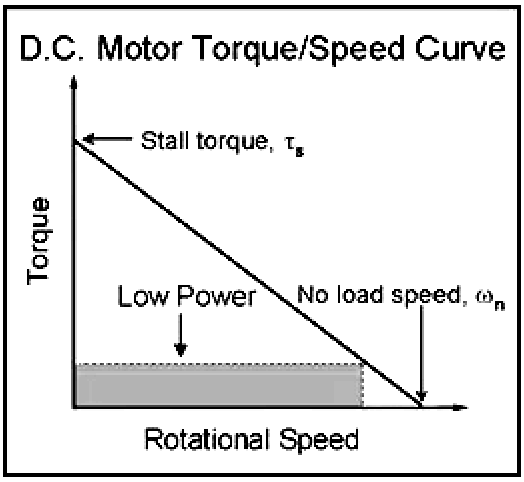


Figure 5.18Speed Vs Torque Characteristics

Power/Torque And Power/Speed Curves

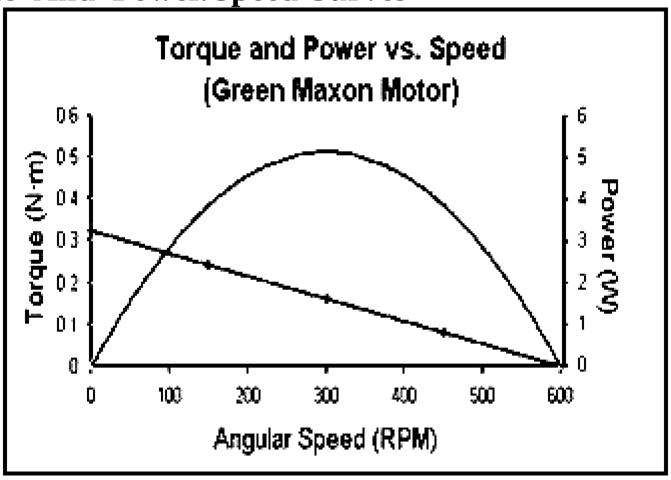


Figure5.19Power Vs Torque Curve

Speed Control Of Dc Shunt Motor

We know that the speed of shunt motor is given by:

Where,Va is the voltage applied across the armature,

N is the rotor speed and φ is the flux perpole and is proportional to the field current If. As explained earlier, armature current Iaisdecided by the mechanical load present on the shaft. Therefore, by varying Va and If we canvary n. For fixed supply voltage and the motor connected as shunt we can vary Vabycontrolling an external resistance connected in series with the armature. If of course can bevaried by controlling external field resistance Rfconnected with the field circuit. Thus for.shunt motor we have essentially two methods for controlling speed, namely by:

1. **Varying Armature Resistance**
2. **Varying Field Resistance**

Speed Control by Varying Armature Resistance

The inherent armature resistance Ra being small, speed n versus armature current (Ia) characteristic will be a straight line with a small negative slope as shown in figure.

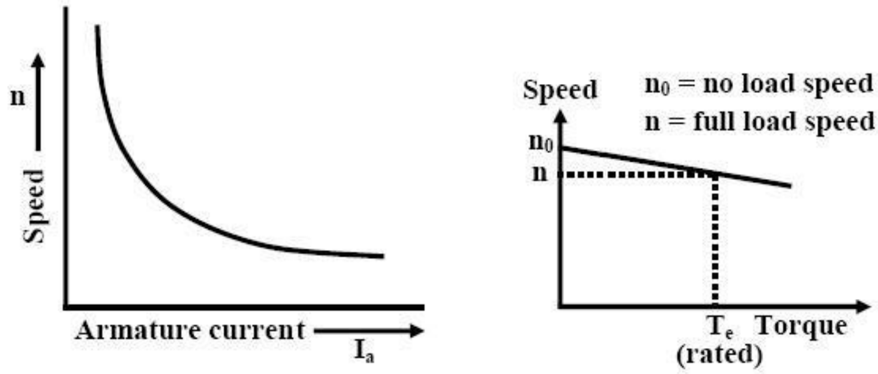


Figure 5.20(i) Speed Vs Armature Current. (ii) Speed Vs Torque Characteristics

Note that for shunt motor voltage applied to the field and armature circuit are sameand equal to the supply voltage V. However, as the motor is loaded, IaRa drop increasesmaking speed a little less than the no load speed n0. For a well designed shunt motor thisdrop in speed is small and about 3 to 5% with respect to no load speed. This drop in speedfrom no load to full load condition expressed as a percentage of no load speed is called theinherent speed regulation of the motor. It is for this reason, a d.c shunt motor is said to be practically a constant speed motorsince speed drops by a small amount fromno load to full load condition.

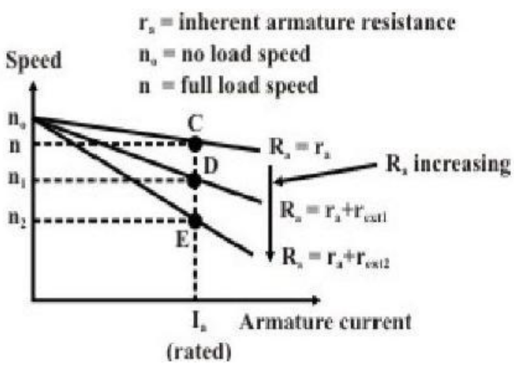


Figure 5.21Speed Vs Armature Current Characteristics

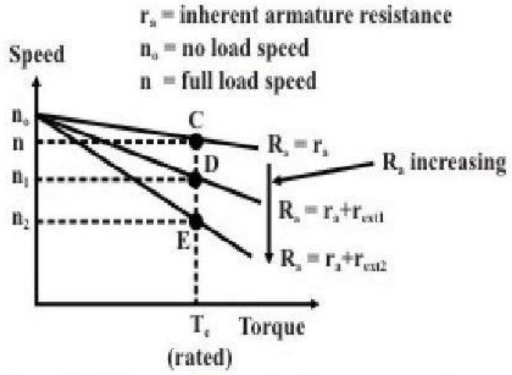


Figure 5.22Speed Vs Torque Characteristics

From these characteristic it can be explained how speed control is achieved. Let usassume that the load torque TL is constant and field current is also kept constant. Therefore,since steady state operation demands Te = TL, Te= kIaφ too will remain constant; which meansIa will not change. Suppose Rest = 0, then at rated load torque, operating point will be at C andmotor speed will be n. If additional resistance rext1 is introduced in the armature circuit, newsteady state operating speed will be n1 corresponding to the operating point D.

This same load torque is supplied at various speed. Variation of thespeed is smooth and speed will decrease smoothly if Rest is increased. Obviously, this methodis suitable for controlling speed below the base speed and for supplying constant rated loadtorque which ensures rated armature current always. Although, this method provides smoothwide range speed control (from base speed down to zero speed), has a serious draw backsince energy loss takes place in the external resistance Rest reducing the efficiency of themotor.

Speed Control by Varying Field Current

In this method field circuit resistance is varied to control the speed of a d.cshuntmotor. Let us rewrite .the basic equation to understand the method.

If flux φ will change, hence speed will vary. To change If an external resistance is connected in series with the field windings. The field coil produces rated flux when no external resistance is connected and rated voltage is applied across field coil. It should be understood that we can only decrease flux from its rated value by adding external resistance. Thus the speed of the motor will rise as we decrease the field current and speed control above the base speed will be achieved. Speed versus armature current characteristic is shown in figure for two flux values φ and φ1. Since φ1<φ, the no load speed no' for flux value φ1 is more than the no load speed no corresponding to φ.

However, this method will not be suitable for constant load torque .To make this point clear, let us assume that the load torque is constant at rated value. So from the initial steady condition, we have TL rated=Ta1= k=Ia rated .If load torque remains constant and flux is reduced to φ1, new armature current in the steady state is obtained from kI a1=TL rated . Therefore new armature current is but this fraction is less than 1. Hence new armature current will be greater than the rated armature current and the motor will be overloaded. This method therefore, will be suitable for a load whose torque demand decreases with the rise in speed keeping the output power constant as shown in figure. Obviously this method is based on flux weakening of the main field.

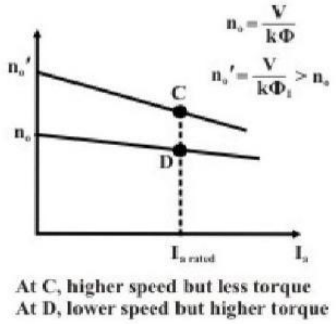


Figure 5.23 Speed Vs Armature Current Characteristics

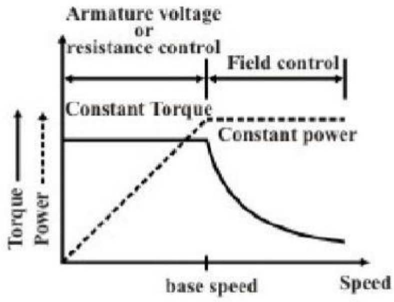


Figure 5.24 Constant Torque and Power Operation

Starting Of Dc Motors

The speed of the machine has to be increased from zero and brought to the operating speed. This is called starting of the motor. The operating speed itself should be varied as per the requirements of the load. This is called speed control. Finally, the running machine has to be brought to rest, by decelerating the same. This is called braking.

At the instant of starting, rotor speed n = 0, hence starting armature current is Ist=V/ra. Since, armature resistance is quite small, starting current may be quite high (many times larger than the rated current). A large machine, characterized by large rotor inertia (J), will pick up speed rather slowly. Thus the level of high starting current may be maintained for quite some time so as to cause serious damage to the brush/commutator and to the armature winding. Also the source should be capable of supplying this burst of large current. The other loads already connected to the same source, would experience a dip in the terminal voltage, every time a D.C motor is attempted to start with full voltage. This dip in supply voltage is caused due to sudden rise in voltage drop in the source's internal resistance. The duration for which this drop in voltage will persist once again depends on inertia of the motor. Hence, for small D.C motors extra precaution may not be necessary during starting as large starting current will very quickly die down because of fast rise in the back emf. However, for large motor, a starter is to be used during starting.

A simple starter to limit the starting current, a suitable external resistance R is connected in series, as shown in the figure, with the armature so that Ist=V/(R+ra) At the time of starting, to have sufficient starting torque, field current is maximized by keeping external field resistance Rf to zero value. As the motor picks up speed, the value of R is gradually decreased to zero so that during running no external resistance remains in the armature circuit. But each time one has to restart the motor, the external armature resistance must be set to maximum value by moving the jockey manually. Now if the supply goes off, motor will come to a stop. All on a sudden, let us imagine, supply is restored. This is then nothing but

full voltage starting. In other words, one should be constantly alert to set the resistance to maximum value

whenever the motor comes to a stop. This is one major limitation of a simple rheostatic starter.

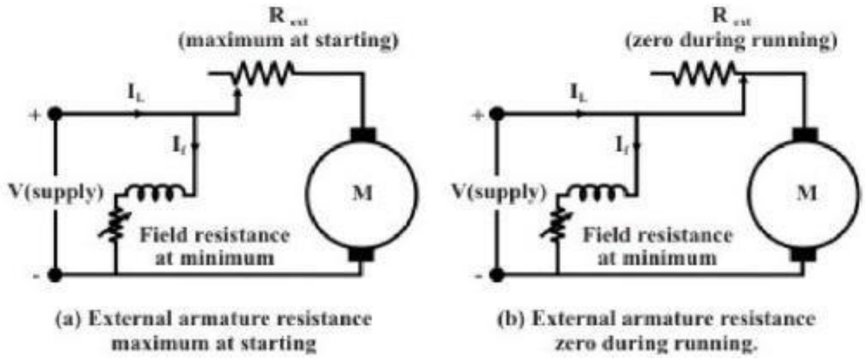
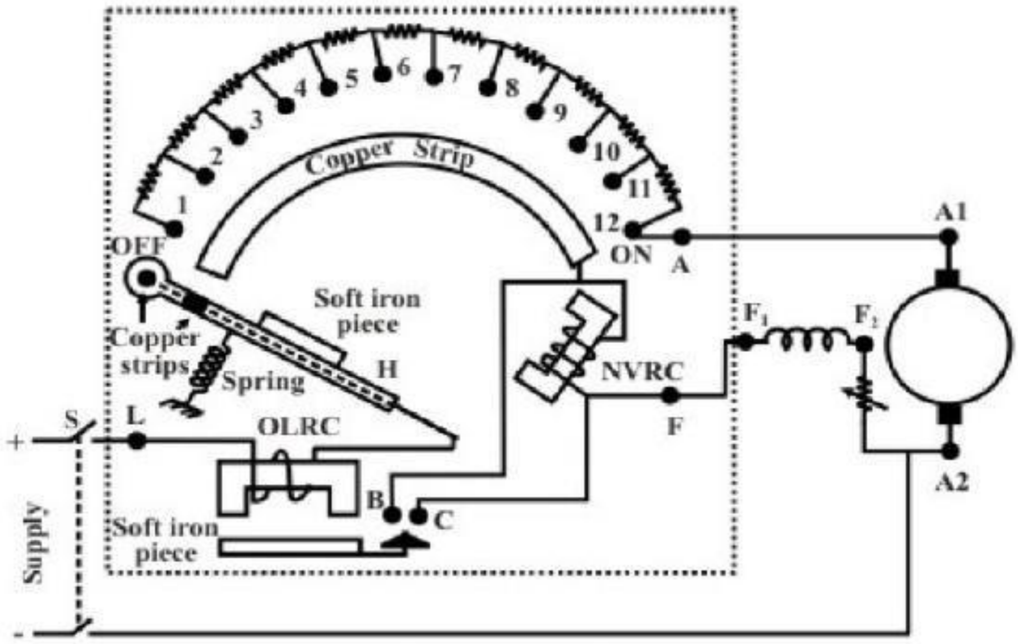


Figure 1.25 Starting Using External Resistance

Three Point Starter

A “3-point starter” is extensively used to start a D.C shunt motor. It not only overcomes the difficulty of a plain resistance starter, but also provides additional protective features such as over load protection and no volt protection. The diagram of a 3-point starter connected to a shunt motor is shown in figure. Although, the circuit looks a bit clumsy at a first glance, the basic working principle is same as that of plain resistance starter. The starter is shown enclosed within the dotted rectangular box having three terminals marked as A, L and F for external connections. Terminal A is connected to one armature terminal Al of the motor. Terminal F is connected to one field terminal F1 of the motor and terminal L is connected to one supply terminal as shown. F2 terminal of field coil is connected to A2 through an external variable field resistance and the common point connected to supply (-ve). The external armatures resistances consist of several resistances connected in series and are shown in the form of an arc. The junctions of the resistances are brought out as terminals and marked. Just beneath the resistances, a continuous copper strip also in the form of an arc is present.



Copper Strip
1
2
3
4
5
6
7
8
9
10
11
12
OFF
ON
A
A1
Soft iron piece
Copper strips
Spring
H
NVRC
F1
F2
F
S
L
OLRC
+
B
C
A2
Soft iron piece
Supply
-

Figure 5.26Three Point Starter

There is a handle which can be moved in the clockwise direction against the spring tension. The spring tension keeps the handle in the OFF position when no one attempts to move it. Now let us trace the circuit from terminal L (supply + ve). The wire from L passes through a small electro magnet called OLRC, (the function of which we shall discuss a little later) and enters through the handle shown by dashed lines. Near the end of the handle two copper strips are firmly connected with the wire.

The furthest strip is shown circular shaped and the other strip is shown to be rectangular. When the handle is moved to the right, the circular strip of the handle will make contacts with resistance terminals 1, 2 etc. Progressively. On the other hand, the rectangular strip will make contact with the continuous arc copper strip. The other end of this strip is brought as terminal F after going through an electromagnet coil (called NVRC). Terminal F is finally connected to motor field terminal Fl.

Working principle

In the operation of the starter, initially the handle is in the OFF position. Neither armature nor the field of the motor gets supply. Now the handle is moved to stud number 1. In this position armature and all the resistances in series gets connected to the supply. Field coil gets full supply as the rectangular strip makes contact with arc copper strip. As the machine picks up speed handle is moved further to stud number 2. In this position the external resistance in the armature circuit is less as the first resistance is left out. Field however, continues to get full voltage by virtue of the continuous arc strip. Continuing in this way, all resistances will be left out when stud number 12 (ON) is reached. In this position, the electromagnet (NVRC) will attract the soft iron piece attached to the handle. Even if the operator removes his hand from the handle, it will still remain in the ON position as spring restoring force will be balanced by the force of attraction between NVRC and the soft iron piece of the handle. The no volt release coil (NVRC) carries same current as that of the field coil. In case supply voltage goes off, field coil current will decrease to zero. Hence NVRC will be de-energized and will not be able to exert any force on the soft iron piece of the handle. Restoring force of the spring will bring the handle back in the OFF position.

The starter also provides over load protection for the motor. The other electromagnet, OLRC overload release coil along with a soft iron piece kept under it, is used to achieve this. The current flowing through OLRC is the line current IL drawn by the motor. As the motor is loaded, Ia hence IL increases. Therefore, IL is a measure of loading of the motor. Suppose we want that the motor should not be over loaded beyond rated current. Now gap between the electromagnet and the soft iron piece is so adjusted that for IL≤Irated the iron piece will not be pulled up. However, if IL≤Irated force of attraction will be sufficient to pull up iron piece. This upward movement of the iron piece of OLRC is utilized to de-energize NVRC. To the iron a copper strip is attached. During over loading condition, this copper strip will also move up and put a short circuit between two terminals B and C. Carefully note that B and C are nothing but the two ends of the NVRC. In other words, when over load occurs a short circuit path is created across the NVRC. Hence NVRC will not carry any current now and gets deenergized. The moment it gets deenergised, spring action will bring the

handle in the OFF position thereby disconnecting the motor from the supply. Three point starter has one disadvantage. If we want to run the machine at

higher speed (above rated speed) by field weakening (i.e., by reducing field current), the strength of NVRC magnet may become so weak that it will fail to hold the handle in the ON position and the spring action will bring it back in the OFF position. Thus we find that a false disconnection of the motor takes place even when there is neither over load nor any sudden disruption of supply.

Four-Point Starter

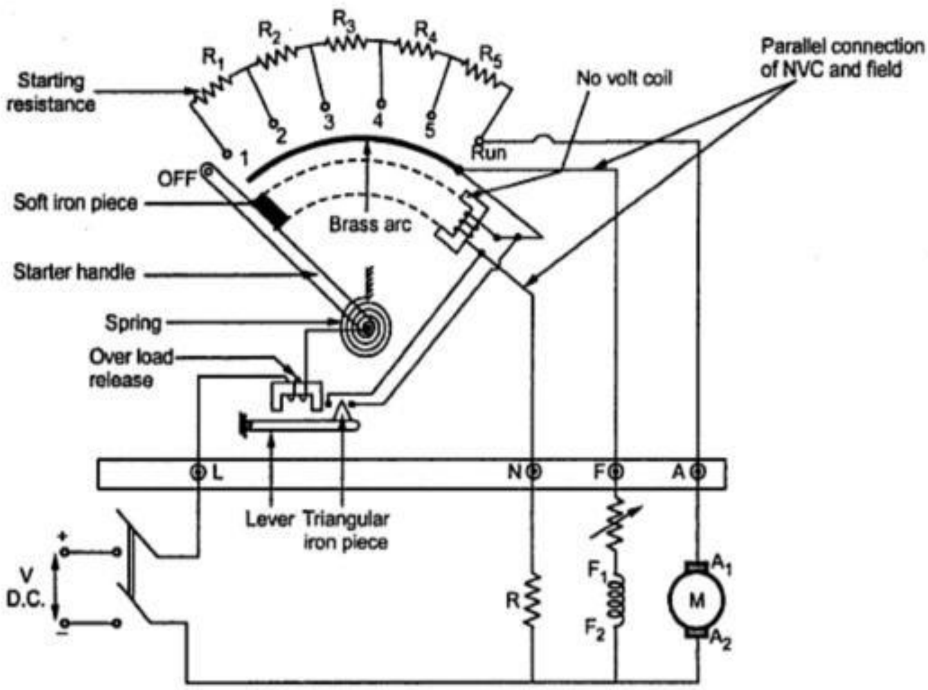


Figure 5.27 Four point starter

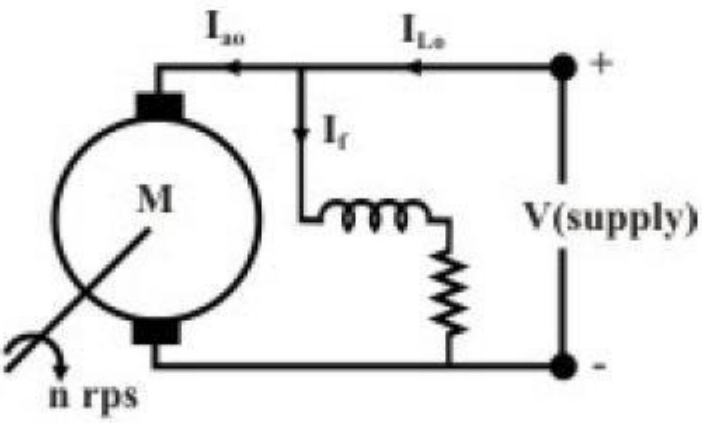
The four-point starter eliminates the drawback of the three-point starter. In addition to the same three points that were in use with the three-point starter, the other side of the line, L1, is the fourth point brought to the starter when the arm is moved from the "Off" position. The coil of the holding magnet is connected across the line. The holding magnet and starting resistors function identical as in the three- point starter.

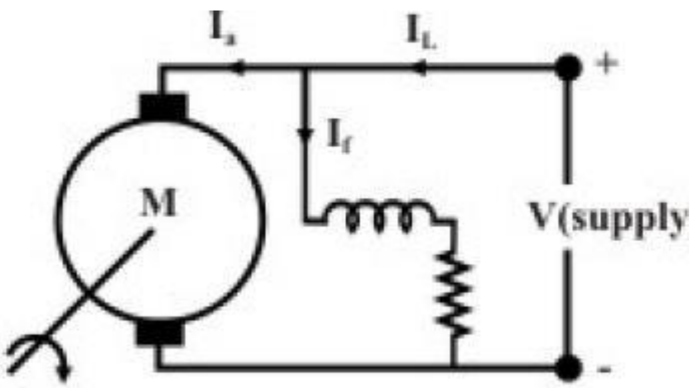
The possibility of accidentally opening the field circuit is quite remote. The four-point starter provides the no-voltage protection to the motor. If the power fails, the motor is disconnected from the line.

Swinburne's Test

- **For a d.c shunt motor change of speed from no load to full load is quite small. Therefore, mechanical loss can be assumed to remain same from no load to full load. Also if field current is held constant during loading, the core loss too can be assumed to remain same.**
- **In this test, the motor is run at rated speed under *no load* condition at rated voltage. The current drawn from the supply I_{L0} and the field current I_f are recorded (figure 40.3). Now we note that:**

$$\begin{aligned}
\text{Input power to the motor, } P_{in} &= VI_{L0} \\
\text{Cu loss in the field circuit } P_{fl} &= VI_f \\
\text{Power input to the armature, } &= VI_{L0} - VI_f \\
&= V(I_{L0} - I_f) \\
&= VI_{a0} \\
\text{Cu loss in the armature circuit } &= I_{a0}^2 r_a \\
\text{Gross power developed by armature } &= VI_{a0} - I_{a0}^2 r_a \\
&= \left(V - I_{a0} r_a\right) I_{a0} \\
&= E_{b0} I_{a0}
\end{aligned}$$



- 

-
- **Since the motor is operating under no load condition, net mechanical output power is zero. Hence the gross power developed by the armature must supply the core loss and friction & windage losses of the motor. Therefore,**

$$P_{core} + P_{friction} = \left(V - I_{a0} r_a\right) I_{a0} = E_{b0} I_{a0}$$

-
- **Since, both P_{core} and $P_{friction}$ for a shunt motor remains practically constant from no load to full load, the sum of these losses is called constant rotational loss i.e.,**

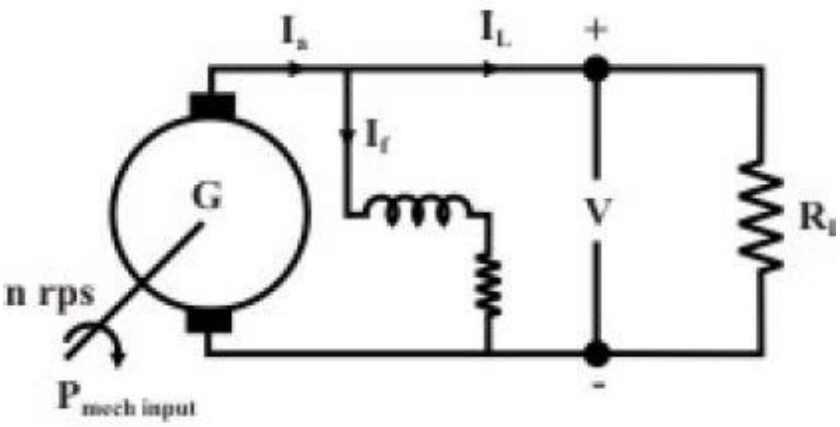
$$\text{constant rotational loss, } P_{rot} = P_{core} + P_{friction}$$

-

- **In the Swinburne's test, the constant rotational loss comprising of core and friction loss is estimated from the above equation.**
- **After knowing the value of P_{rot} from the Swinburne's test, we can fairly estimate the efficiency of the motor at any loading condition. Let the motor be loaded such that new current drawn from the supply is I_L and the new armature current is I_a as shown in figure 40.4. To estimate the efficiency of the loaded motor we proceed as follows:**

$$
\begin{aligned}
\text{Input power to the motor, } P_{in} &= VI_L \\
\text{Cu loss in the field circuit } P_{fl} &= VI_f \\
\text{Power input to the armature, } &= VI_L - VI_f \\
&= V(I_L - I_f) \\
&= VI_a \\
\text{Cu loss in the armature circuit } &= I_a^2 r_a \\
\text{Gross power developed by armature } &= VI_a - I_a^2 r_a \\
&= \left(V - I_a r_a\right) I_a \\
&= E_b I_a \\
\text{Net mechanical output power, } P_{net\,mech} &= E_b I_a - P_{rot} \\
\therefore \text{ efficiency of the loaded motor, } \eta &= \frac{E_b I_a - P_{rot}}{VI_L} \\
&= \frac{P_{net\,mech}}{P_{in}}
\end{aligned}
$$

- **The estimated value of P_{rot} obtained from Swinburne's test can also be used to estimate the efficiency of the shunt machine operating as a generator. In figure**
40.5 is shown to deliver a
- **load current I_L to a load resistor R_L. In this case output power being known, it is easier to add the losses to estimate the input mechanical**



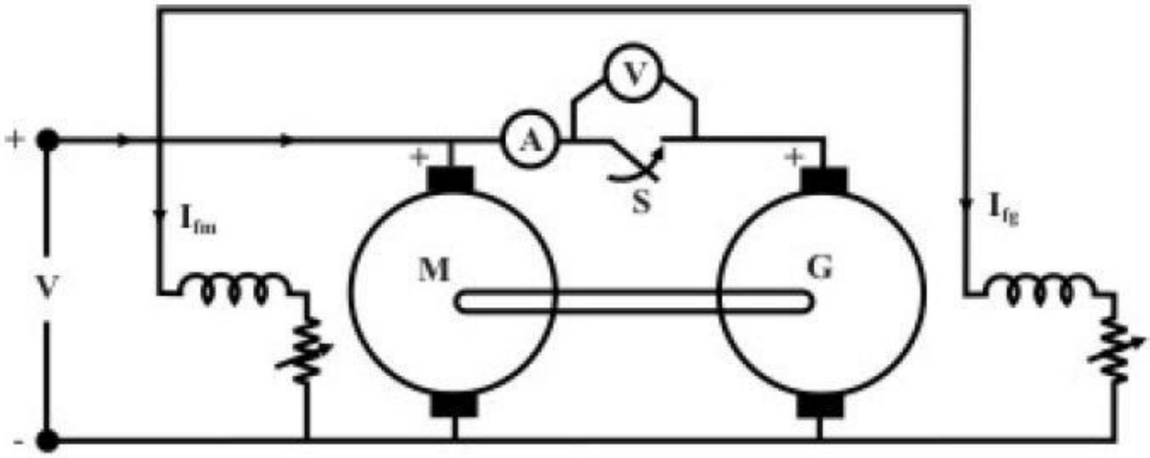
power.

$$
\begin{aligned}
\text{Output power of the generator, } P_{out} &= VI_L \\
\text{Cu loss in the field circuit } P_{fl} &= VI_f \\
\text{Output power of the armature, } &= VI_L + VI_f \\
&= VI_a \\
\text{Mechanical input power, } P_{in\,mech} &= VI_a + I_a^2 r_a + P_{rot} \\
\therefore \text{ Efficiency of the generator, } \eta &= \frac{VI_L}{P_{in\,mech}} \\
&= \frac{VI_L}{VI_a + I_a^2 r_a + P_{rot}}
\end{aligned}
$$

- **The biggest advantage of Swinburne's test is that the shunt machine is to be run as motor under *no load* condition requiring little power to be drawn from the supply; based on the no load reading, efficiency can be predicted for any load current. However, this test is not sufficient if we want to know more about its performance (effect of armature reaction, temperature rise, commutation etc.) when it is actually loaded. Obviously the solution is to load the machine by connecting mechanical load directly on the shaft for motor or by connecting loading rheostat across the terminals for generator operation. This although sounds simple but difficult to implement in the laboratory for high rating machines (say above 20 kW), Thus the laboratory must have proper supply to deliver such a large power corresponding to the rating of the machine. Secondly, one should have loads to absorb this power.**

Hopkinson's test

- **This as an elegant method of testing d.c machines. Here it will be shown that while power drawn from the supply only corresponds to no load losses of the machines, the armature physically carries any amount of current (which can be controlled with ease). Such a scenario can be created using two similar mechanically coupled shunt machines. Electrically these two machines are eventually connected in parallel and controlled in such a way that one machine acts as a generator and the other as motor. In other words two similar machines are required to carry out this testing which is not a bad proposition for manufacturer as large numbers of similar machines are manufactured.**



Procedure

- **Connect the two similar (same rating) coupled machines as shown in figure 40.6. With switch S opened, the first machine is run as a shunt motor at rated speed. It may be noted that the second machine is operating as a separately excited generator because its field winding is excited and it is driven by the first machine. Now the question is what will be the reading of the voltmeter connected across the opened switch S? The reading may be (i) either close to twice supply voltage or (ii) small voltage. In fact the voltmeter practically reads the difference of the induced voltages in the armature of the machines. The upper armature terminal of the generator may have either + ve or negative polarity. If it happens to be +ve, then voltmeter reading will be small otherwise it will be almost double the supply voltage**
- **Since the goal is to connect the two machines in parallel, we must first ensure voltmeter reading is small. In case we find voltmeter reading is high, we should switch off the supply, reverse the armature connection of the generator and start afresh. Now voltmeter is found to read small although time is still not ripe enough to close S for paralleling the machines. Any attempt to close the switch may result into large circulating current as the armature resistances are small. Now by adjusting the field current I_{fg} of the generator the voltmeter reading may be adjusted to zero ($E_g \approx E_b$) and S is now closed. Both the machines are now connected in parallel**

Loading the machines

After the machines are successfully connected in parallel, we go for loading the machines i.e., increasing the armature currents. Just after paralleling the ammeter reading A will be close to zero as $E_g \approx E_b$. Now if I_{fg} is increased (by decreasing R_{fg}), then E_g becomes greater than E_b and both I_{ag} and I_{am} increase, Thus by increasing field current of generator (alternatively decreasing field current of motor) one can make $E_g > E_b$ so as to make the second machine act as generator and first machine as motor. In practice, it is ~~also required to control the field current of the motor I_{fm} to maintain speed~~ constant at rated value. The interesting point to be noted here is that I_{ag} and I_{am} do not reflect in the supply side line. Thus current drawn from supply

remains small (corresponding to losses of both the machines). The loading is sustained by the output power of the generator running

the motor and vice versa. The machines can be loaded to full load current without the need of any loading arrangement

- **Calculation of efficiency**

Let field currents of the machines be are so adjusted that the second machine is acting as generator with armature current I_{ag} and the first machine is acting as motor with armature current I_{am} as shown in figure 40.7. Also let us assume the current drawn from the supply be I_1. Total power drawn from supply is VI_1 which goes to supply all the losses (namely Cu losses in armature & field and rotational losses) of both the machines

$$
\begin{aligned}
\text{Power drawn from supply} &= VI_1 \\
\text{Field Cu loss for motor} &= VI_{fm} \\
\text{Field Cu loss for generator} &= VI_{fg} \\
\text{Armature Cu loss for motor} &= I_{am}^2 r_{am} \\
\text{Armature Cu loss for generator} &= I_{ag}^2 r_{ag} \\
\therefore \text{Rotational losses of both the machines} &= VI_1 - \left(VI_{fm} + VI_{fg} + I_{am}^2 r_{am} + I_{ag}^2 r_{ag}\right)
\end{aligned}
$$

Since speed of both the machines are same, it is reasonable to assume the rotational losses of both the machines are equal; which is strictly not correct as the field current of the generator will be a bit more than the field current of the motor, Thus, Once P_{rot} is estimated for each machine we can proceed to calculate the efficiency of the machines as follows,

$$\text{Rotational loss of each machine, } P_{rot} = \frac{VI_1 - \left(VI_{fm} + VI_{fg} + I_{am}^2 r_{am} + I_{ag}^2 r_{ag}\right)}{2}$$

Efficiency of the motor

- **As pointed out earlier, for efficiency calculation of motor, first calculate the input power and then subtract the losses to get the output mechanical power as shown below,**

$$
\begin{aligned}
\text{Total power input to the motor} &= \text{power input to its field} + \text{power input to its armature} \\
P_{inm} &= VI_{fm} + VI_{am} \\
\text{Losses of the motor} &= VI_{fm} + I_{am}^2 r_{am} + P_{rot} \\
\text{Net mechanical output power } P_{outm} &= P_{inm} - \left(VI_{fm} + I_{am}^2 r_{am} + P_{rot}\right) \\
\therefore \eta_m &= \frac{P_{outm}}{P_{inm}}
\end{aligned}
$$

EFFICIENCY OF GENERATOR

$$\text{Losses of the generator} = VI_{fg} + I_{ag}^2 r_{ag} + P_{rot}$$

$$\text{Input power to the generator, } P_{ing} = P_{outg} + \left(VI_{fg} + I_{ag}^2 r_{ag} + P_{rot}\right)$$

$$\therefore \eta_g = \frac{P_{outg}}{P_{ing}}$$

Advantages of Hopkinson's Test

- **The merits of this test are…**
1. This test requires very small power compared to full-load power of the motor-generator coupled system. That is why it is economical.
- **2. Temperature rise and commutation can be observed and maintained in the limit because this test is done under full load condition.**
- **3. Change in iron loss due to flux distortion can be taken into account due to the advantage of its full load condition**

Disadvantages of Hopkinson's Test

- **The demerits of this test are**
- **1. It is difficult to find two identical machines needed for Hopkinson's test.**
- **2. Both machines cannot be loaded equally all the time.**
- **3. It is not possible to get separate iron losses for the two machines though they are different because of their excitations.**
- **4. It is difficult to operate the machines at rated speed because field currents vary widely.**
- **39.8 Braking of d.c shunt motor: basic idea**
- **It is often necessary in many applications to stop a running motor rather quickly. We know that any moving or rotating object acquires kinetic energy. Therefore, how fast we can bring the object to rest will depend essentially upon how quickly we can extract its kinetic energy and make arrangement to dissipate that energy somewhere else. If you stop pedaling your bicycle, it will eventually come to a stop eventually after moving quite some distance. The initial kinetic energy stored, in this case dissipates as heat in the friction of the road. However, to make the stopping faster, brake is applied with the help of rubber brake shoes on the rim of the wheels. Thus stored K.E now gets two ways of getting dissipated, one at the wheel-brake shoe interface (where most of the energy is dissipated) and the other at the road-tier interface. This is a good method no doubt, but regular maintenance of brake shoes due to wear and tear is necessary.**
- **If a motor is simply disconnected from supply it will eventually come to stop no doubt, but will take longer time particularly for large motors having high rotational inertia. Because here the stored energy has to dissipate mainly through bearing friction and wind friction. The situation can be improved, by forcing the motor to operate as a generator during braking. The idea can be understood remembering that in motor mode electromagnetic torque acts along the direction of rotation while in generator the electromagnetic**

torque acts in

the opposite direction of rotation. Thus by forcing the machine to operate as generator during the braking period, a torque opposite to the direction of rotation will be imposed on the shaft, thereby helping the machine to come to stop quickly. During braking action, the initial K.E stored in the rotor is either dissipated in an external resistance or fed back to the supply or both.

Rheostatic braking

- **Consider a d.c shunt motor operating from a d.c supply with the switch S connected to position 1 as shown in figure 39.23. S is a *single pole double throw switch* and can be connected either to position 1 or to position 2. One end of an external resistance R_b is connected to position 2 of the switch S as shown.**

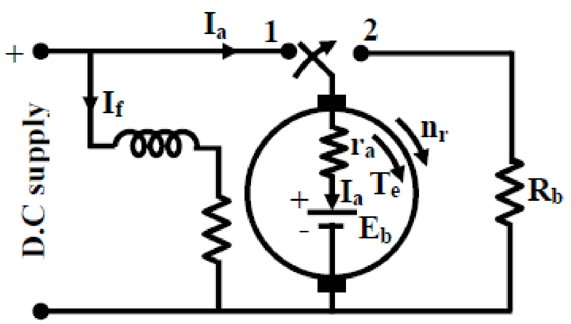


Figure 39.23: Machine operates as motor

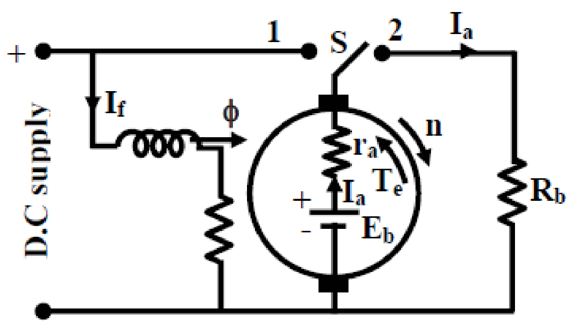


Figure 39.24: Machine operates as generator during braking

- **Let with S in position 1, motor runs at n rpm, drawing an armature current I_a and the back emf is $E_b = k\varphi n$. Note the polarity of E_b which, as usual for motor mode in opposition with the supply voltage. Also note T_e and n have same clock wise direction.**
- **Now if S is suddenly thrown to position 2 at $t = 0$, the armature gets disconnected from the supply and terminated by R_b with field coil remains energized from the supply. Since speed of the rotor can not change instantaneously, the back emf value E_b is still maintained with same polarity prevailing at $t = 0_-$. Thus at $t = 0_+$, armature current will be $I_a = E_b/(r_a + R_b)$ and with reversed direction compared to direction prevailing during motor mode at t $= 0_-$.**
- **Obviously for $t > 0$, the machine is operating as generator dissipating power to R_b and now the electromagnetic torque T_e must act in the opposite direction to that of n since I_a has changed direction but φ has not (recall $T_e \propto \varphi I_a$). As time passes after switching, n decreases reducing K.E and as a consequence both E_b and I_a decrease. In other words value of braking torque will be highest at $t = 0_+$, and it decreases progressively and becoming zero when the machine finally come to a stop.**

Plugging or dynamic braking

- **This method of braking can be understood by referring to figures 39.25 and 39.26. Here S is a double pole double throw switch. For usual motoring mode, S is connected to positions 1 and 1'. Across terminals 2 and 2', a series combination of an external resistance R_b and supply voltage with**

polarity as indicated is connected. However, during motor mode this part of the circuit remains inactive. To initiate braking, the switch is thrown to position 2 and 2' at

t = 0, thereby disconnecting the armature from the left hand supply. Here at $t = 0_+$, the armature current will be $I_a = (E_b + V)/(r_a + R_b)$ as E_b and the right hand supply voltage have additive polarities by virtue

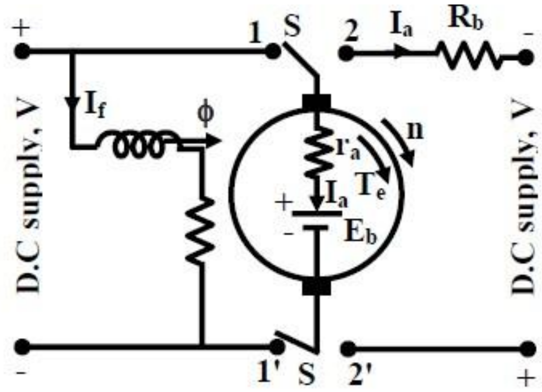


Figure 39.25: Machine operates as motor

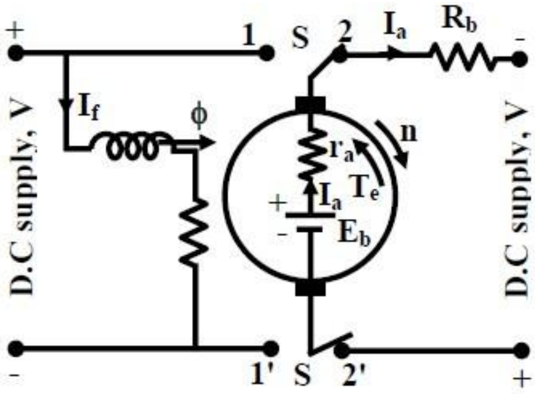


Figure 39.26: Machine operates as generator during braking (plugging).

of the connection. Here also I_a reverses direction producing T_e in opposite direction to n. I_a decreases as E_b decreases with time as speed decreases. However, I_a can not become zero at any time due to presence of supply V. So unlike rheostatic braking, substantial magnitude of braking torque prevails. Hence stopping of the motor is expected to be much faster then rheostatic breaking. But what happens, if S continuous to be in position 1' and 2' even after zero speed has been attained? The answer is rather simple, the machine will start picking up speed in the reverse direction operating as a motor. So care should be taken to disconnect the right hand supply, the moment armature speed becomes zero.

Regenerative braking

- **A machine operating as motor may go into regenerative braking mode if its speed becomes sufficiently high so as to make back emf greater than the supply voltage i.e., $E_b > V$. Obviously under this condition the direction of I_a will reverse imposing torque which is opposite to the direction of rotation. The situation is explained in figures 39.27 and 39.28. The normal motor operation is shown in figure 39.27 where armature motoring current I_a is drawn from the supply and as usual $E_b < V$. Since $E_b = k\varphi\, n_1$. The question is how speed on its own become large enough to make $E_b < V$ causing regenerative braking. Such a situation may occur in practice when the mechanical load itself becomes active. Imagine the d.c motor is coupled to the wheel of locomotive which is moving along a plain track without any gradient as shown in figure 39.27. Machine is running as a motor at a speed of n_1 rpm. However, when the track has a downward gradient, component of gravitational force along the track also**

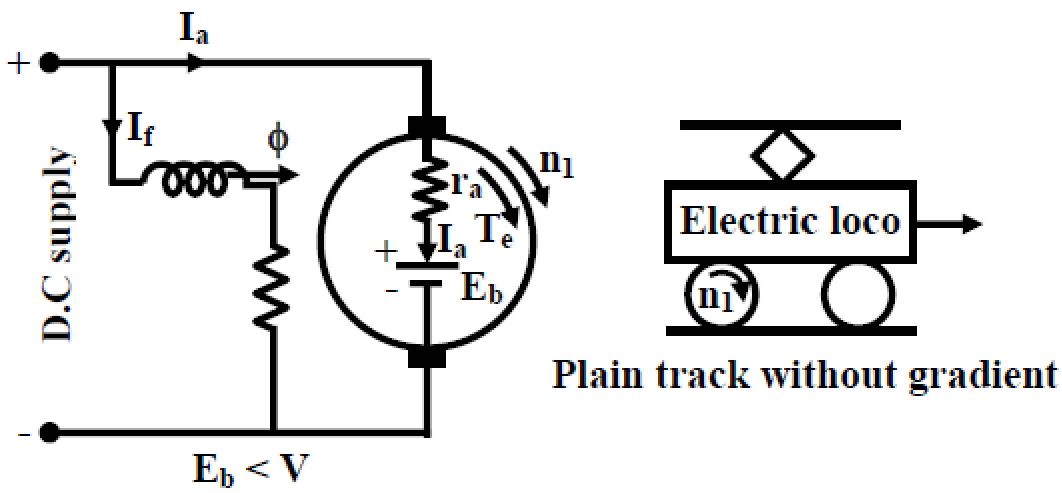
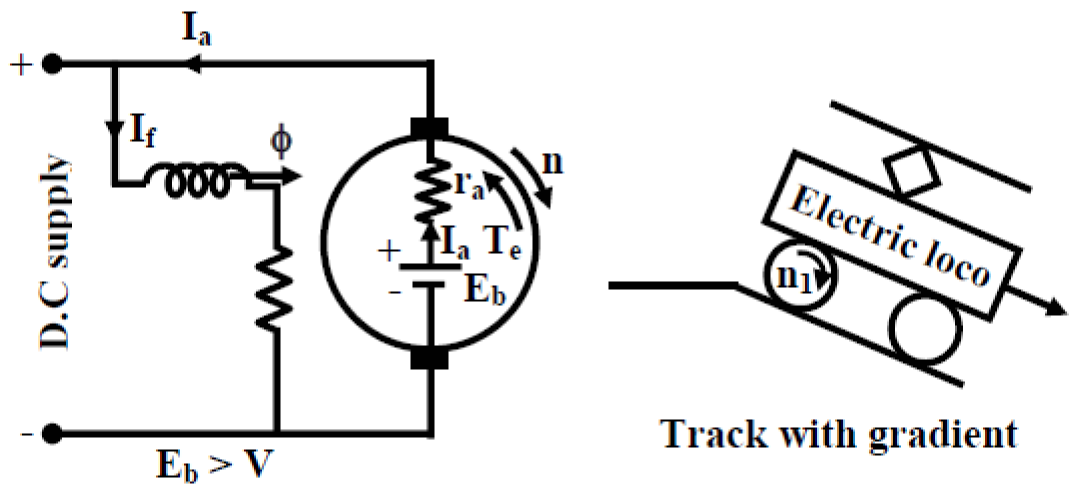


Figure 39.27: Machine operates as motor



appears which will try to accelerate the motor and may increase its speed to n_2 such that $E_b = k\varphi\ n_2 > V$. In such a scenario, direction of I_a reverses, feeding power back to supply. Regenerative braking here will not stop the motor but will help to arrest rise of dangerously high speed.

SOLVED PROBLEMS

1. **A 10KW,240V dc shunt motor draws a line current of 5.2 amps while running at no load of 1200rpm from a 240V dc supply. It has an armature resistance of 0.25 ohms and field resistance of 160 ohms . Estimate the efficiency of motor when it delivers rated load.**

GIVEN DATA:

Output power = 10KW Supply Voltage V = 240 V

No-Load current = 5.2A No- Load Speed N = 1200 rpm

Armature resistance = 0.25 Ω Field resistance = 160 Ω

TO FIND:

Efficiency of the motor at rated load.

SOLUTION:

No-load input power = V × W)

= 240× 5.2

=1248 W

This no-load input power to meet all kinds of no-load losses is armature copper loss and constant loss

Shunt field current = — = —

= 1.5 A

No-load armature current = = 5.2 1.5 = 3.7A

Now no-load armature copper loss = = × 0.25

= 3.4 W

Constant loss = 1248 -3.4 = 1244.6W

Rated current (Load) I_L = —— = 41.667 A

Full Load Armature Current I_a= I_L = 41.667 -1.5

= 40.16A

Full Load Armature Copper loss = R_a × 0.25

= 403.3W

Motor Output = -Total Loss

= 10000 - (1244.6 + 403.3)

= 8352W

% Efficiency = — = ×100

= 83.52%

Ans: η= 98.40%

2. **In a brake test the efficiency load on the branch pulley was 40Kg, the effective diameter of the pulley 73.5 cm and speed 15 rps. The motor takes 60A at 230V. Calculate the output power and efficiency at this load.**

3. **A 480 V, 20kW, shunt motor of rows 2.5A, when running at with light load**

.Taking the armature resistance to be 0.6Ω,field resistance to be 800 Ω and brush drops at 2V and find full load eff

Ans : η= 94.83%

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UNIT - IV

DC Machines and Transformers – SEEA1202

Unit 4 – TRANSFORMERS

Operating Principles and Construction

What is a Transformer?

***A transformer** is a static piece of equipment used either for raising or lowering the voltage of an AC supply with a corresponding decrease or increase in current.*

The use of transformers in transmission system is shown in the Figure below.

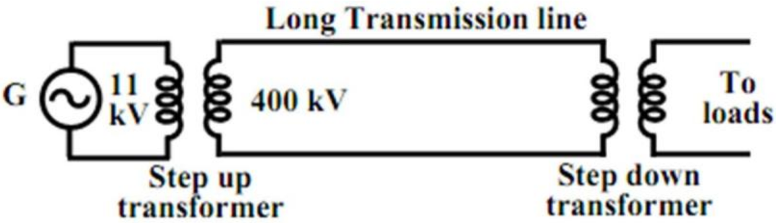


Fig.3-1 A simple single phase power system.

Principle of Operation

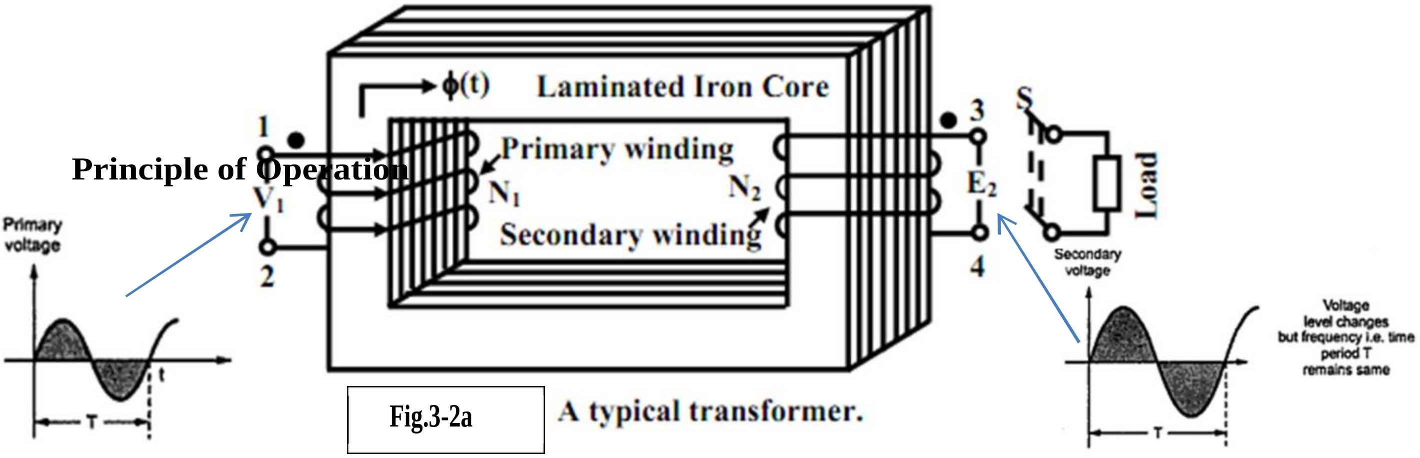


Fig.3-2a A typical transformer.

A transformer in its simplest form will consist of a rectangular laminated magnetic structure on which two coils of different number of turns are wound as shown in Figure 3.2a.

The winding to which AC voltage is impressed is called *the primary* of the transformer and the winding across which the load is connected is called *the secondary* of the transformer.

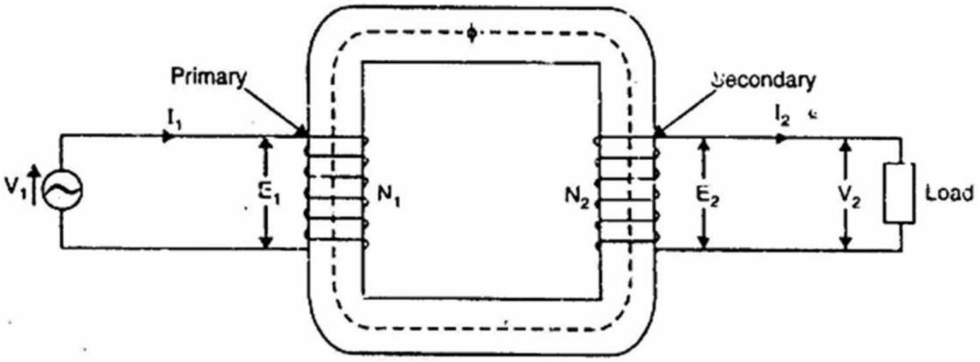


Fig.3-2b

Depending upon the number of turns of the primary (N_1) and secondary (N_2), an alternating emf (E_2) is induced in the secondary. This induced emf (E2) in the secondary causes a secondary current I_2. Consequently, terminal voltage V_2 will appear across the load. If $V_2 > V_1$, it is called a *step up-transformer*. On the other hand, if $V_2 < V_1$, it is called a *step-down transformer*.

When an alternating voltage V_1 is applied to the primary, an alternating flux Φ is set up in the core. This alternating flux links both the windings and induces emfs E1 and E2 in them according to *Faraday's laws of electromagnetic induction*. The emf E1 is termed as primary emf and emf E2 is termed as Secondary emf.

Clearly, $$E_1 = -N_1 \frac{d\phi}{dt}$$

and $$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Note that magnitudes of E_2 and E_1 depend upon the number of turns on the secondary and primary respectively. If $N_2 > N_1$, then $E_2 > E_1$ (or $V_2 > V_1$) and we get a step-up transformer. On the other hand, if $N_2 < N_1$, then $E_2 < E_1$ (or $V_2 < V_1$) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f. E_2 will cause a current I_2 to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level.

(i) The transformer action is based on the laws of *electromagnetic induction*.

(ii) There is no electrical connection between the primary and secondary.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

Can DC Supply be used for Transformers?

The DC supply cannot be used for the transformers. This is because the transformer works on the principle of *mutual induction*, for which current in one coil must change uniformly. If DC supply is given, the current will not change due to constant supply and transformer will not work

There can be saturation of the core due to which transformer draws very large current from the supply when connected to DC.

Construction

We usually design a power transformer so that it approaches the characteristics of an ideal transformer. To achieve this, following design features are incorporated:

(i) The core is made of silicon steel which has low hysteresis loss and high permeability. Further, core is laminated in order to reduce eddy current loss. These features considerably reduce the iron losses and the no-load current.

(ii) Instead of placing primary on one limb and secondary on the other, it is a usual practice to wind one-half of each winding on one limb. This ensures tight coupling between the two windings. Consequently, leakage flux is considerably reduced.

(iii) The winding resistances are minimized to reduce Copper loss and resulting rise in temperature and to ensure high efficiency.

Transformers are of two types: (i) core-type transformer (see Fig.3-3) and (ii) shell-type transformer (see Fig.3-4).

Core-Type Transformer: **In a core-type transformer, half of the primary winding and half of the secondary winding are placed round each limb to reduce the leakage flux.**

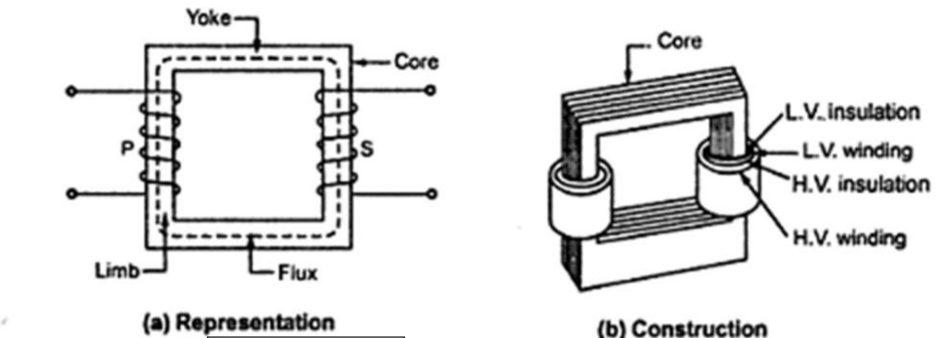


Fig.3-3 **Core type transformer**

Shell-Type Transformer: **This method of construction involves the use of a double magnetic circuit. Both the windings are placed round the central limb to ensure a low-reluctance flux path.**

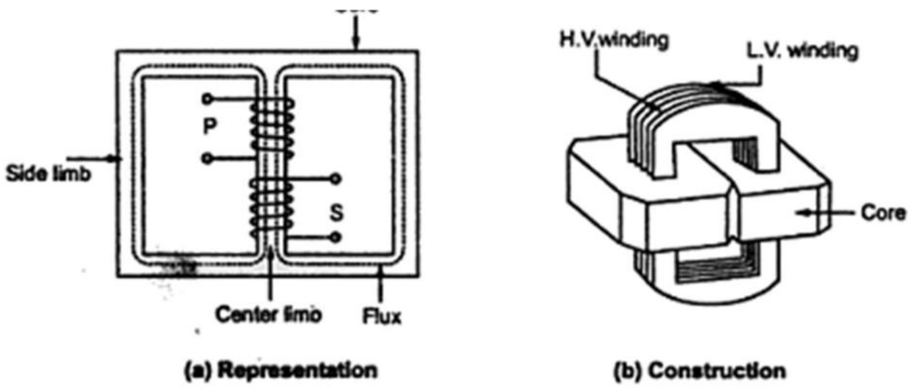


Fig.3-4 **Shell type transformer**

Comparison of Core and Shell Type Transforms

Core Type	Shell Type
The winding encircles the core.	Thecore encircles mostpartofthe windin
It has single magnetic circuit	It has double magnetic circuit
The core has two limbs	The core has three limbs
The cylindrical coils are used.	The multilayer disc or sandwich type coils are used.
The winding are uniformly distributed on two limbs hence natural cooling is effective	The natural cooling does not exist as the windings are surrounded by the core.
Preferred for low voltage transformers.	Preferred for high voltage transformers.

Cooling of Transformers

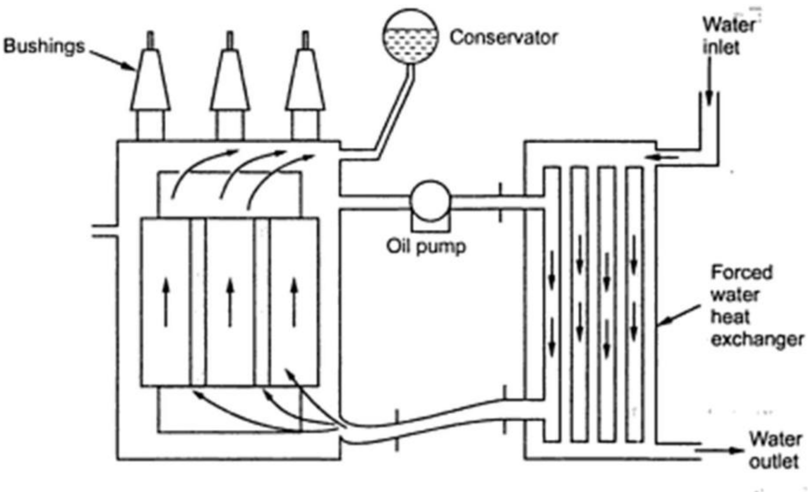
When transformer supplies a load, two types of losses occur inside the transformer. Theironlossesoccur inthecorewhilecopperlossesoccurinthe windings. The power lost due to these losses appears in the form of heat. This heat increases the temperature of the transformer. *To keep the temperature rise of the transformer within limits, a suitable coolant and cooling method is necessary.*

The various cooling methods are designated witch depended upon: A:coolingmediumused

andB:typeofcirculationemployed.

The various coolant used such as Air, Gas, Mineral oil, and water.

One of cooling method system is shown in figure below which is called *Oil Forced Water Forced cooling system*;



Oil forced water forced cooling method

EMF Equation of a Transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown in Fig. 3-2b. The sinusoidal flux Φ produced by the primary can be represented as:

$$\phi = \phi_m \sin\omega t$$

The instantaneous e.m.f. e_1 induced in the primary is

$$\begin{aligned} e_1 &= -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t) \\ &= -\omega N_1 \phi_m \cos\omega t = -2\pi f N_1 \phi_m \cos\omega t \\ &= 2\pi f N_1 \phi_m \sin(\omega t - 90^\circ) \qquad \text{(i)} \end{aligned}$$

It is clear from the above equation that maximum value of induced e.m.f. in the primary is

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E^ of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi\ f\ N_1\ \phi_m}{\sqrt{2}}$$

or $\quad E_1 = 4.44\ f\ N_1\ \phi_m$

Similarly $\quad E_2 = 4.44\ f\ N_2\ \phi_m$

In an ideal transformer, $E_1 = V_1$ and $E_2 = V_2$.

Note. It is clear from exp. (i) above that e.m.f. E_1 induced in the primary lags behind the flux ϕ by 90°. Likewise, e.m.f. E_2 induced in the secondary lags behind flux ϕ by 90°.

Voltage Transformation Ratio (K)

From the above equations of induced emf, we have,

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K$$

The constant K is called voltage transformation ratio. Thus if K = 5 (i.e. N2/N1 = 5), then E2 = 5

E1.

Concept of Ideal Transformer

A transformer is said to be ideal if it satisfies following properties:

i) **It has no losses.**
ii) **Its windings have zero resistance.**
iii) **Leakage flux is zero i.e. 100 % flux produced by primary links with the secondary.**
iv) **Permeabitity of core is so high that negilgiblc current is required to establish the flux init.**

□ □ □ □ □

For an ideal transformer, the primary applied voltage V1 is same as the primary induced emf E1 as there are no voltage drops.

For ideal transformer:

(i) $E_1 = V_1$ and $E_2 = V_2$ as there is no voltage drop in the windings.

$$\therefore \quad \frac{E_2}{E_1} = \frac{V_2}{V_1} = \frac{N_2}{N_1} = K$$

(ii) there are no losses. Therefore, volt-amperes input to the primary are equal to the output volt-amperes i.e.

$$V_1 I_1 = V_2 I_2$$

$$\text{or} \qquad \frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$$

Hence, currents are in the inverse ratio of voltage transformation ratio. This simply means that if we raise the voltage, there is a corresponding decrease of current.

Volt-Ampere Rating

Transformer rating is specified as the product of voltage and current and called
□ □ □ □ □ □ □ □ □

$$\text{kVA rating of a transformer} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

The full load primaty and secondary currents which indicate the safe maximum values of currents which transformer windings can carry can be given as:

$$I_1 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_1} \qquad \text{... (1000 to convert kVA to VA)}$$

$$I_2 \text{ full load} = \frac{\text{kVA rating} \times 1000}{V_2}$$

Ideal Transformer on No Load

Consider an ideal transformer in Fig. 3-5. For no load $I_2 = 0$. I_1 is just necessary to produce flux in the core, which is called *magnetising* current denoted as I_m. I_m is very small and lags V_1 by 90^0 as the winding is purely inductive.

According to Lenz's law, the induced e.m.f opposes the cause producing it which is supply voltage V_1. Hence E_1 and E_2 are in antiphase with V_1 but equal in magnitude and E_1 and E_2 are in phase.

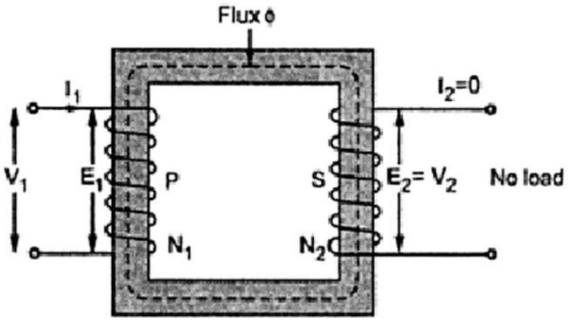
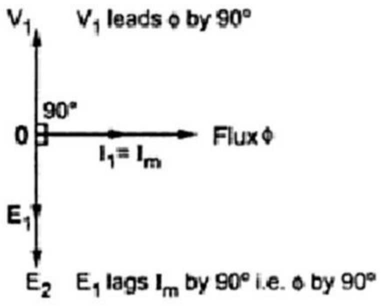


Fig.3-5

This can be illustrated in the phase diagram as shown below:

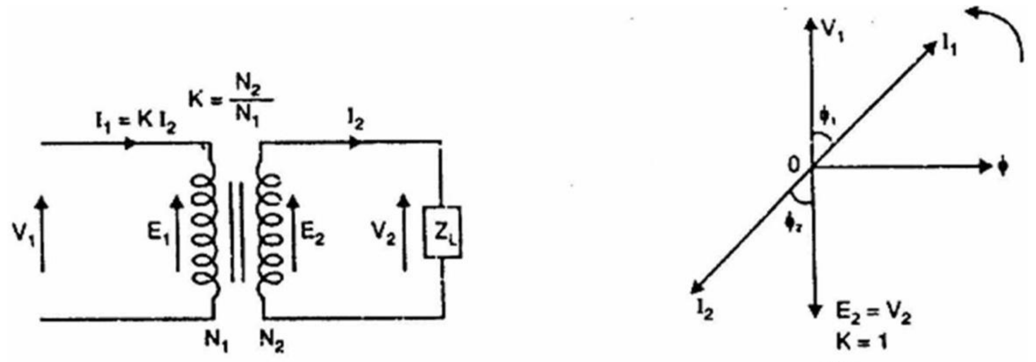


Phasor diagram for ideal transformer on no load

Ideal Transformer on Load

Let us connect a load Z_L across the secondary of an ideal transformer as shown in Figure below:

The secondary emf E2 will cause a current I_2 to flow through the load:



(i) (ii)

$$I_2 = \frac{E_2}{Z_L} = \frac{V_2}{Z_L}$$

The angle at which I_2 leads or lags V_2 (or E_2) depends upon the resistance and reactance of the load. In the present case, we have considered inductive load so that current I_2 lags behind V_2 (or E_2) by ϕ_2.

The secondary current I_2 sets up an m.m.f. N_2I_2 which produces a flux in the opposite direction to the flux ϕ originally set up in the primary by the magnetizing current. This will change the flux in the core from the original value. However, the flux in the core should not change from the original value.

Thus when a transformer is loaded and carries a secondary current I_2, then a current I_1, (= K I_2) must flow in the primary to maintain the m.m.f. balance. In other words, the primary must draw enough current to neutralize the demagnetizing effect of secondary current so that mutual flux ϕ remains constant. Thus as the secondary current increases, the primary current I_1 (= KI_2) increases in unison and keeps the mutual flux ϕ constant. The power input, therefore, automatically increases with the output. For example if $K = 2$ and $I_2 =$ 2A, then primary will draw a current $I_1 = KI_2 = 2 \times 2 = 4A$. If secondary current is increased to 4A, then primary current will become $I_1 = KI_2 = 2 \times 4 = 8A$.

The Phasor diagram for the ideal transformer on load is shown in Figure (ii) above.

The secondary current I_2 lags behind V_2 (or E_2) by Φ_2. It causes a primary current $\boxed{I_1=KI_2=I2}$ **(for K=1) which is in antiphase with it.**

(i) $\phi_1 = \phi_2$

or $\cos \phi_1 = \cos \phi_2$

Thus, power factor on the primary side is equal to the power factor on the secondary side.

(ii) Since there are no losses in an ideal transformer, input primary power is equal to the secondary output power i.e.,

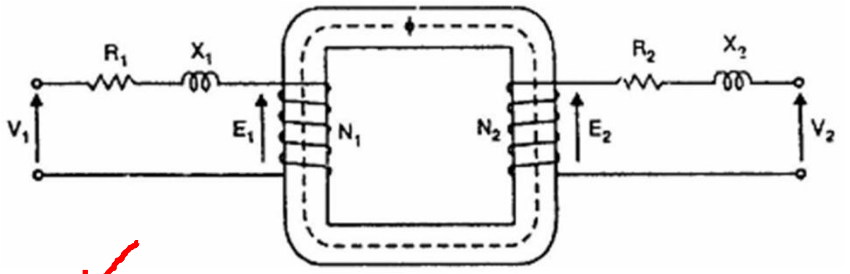
$$V_1 I_1 \cos\phi_1 = V_2 I_2 \cos\phi_2$$

Practical Transformer

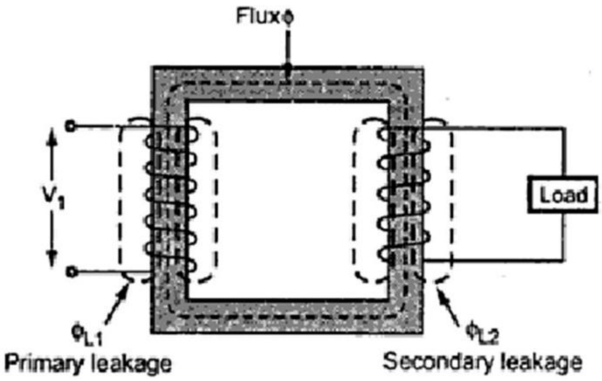
A practical transformer differs from the ideal transformer in many respects. The practical transformer has (i) *iron losses* (ii) *winding resistances* and (iii) *magnetic leakage*, giving rise to leakage reactance.

(i) ***Iron losses*. Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it.**

(ii) ***Winding resistances*. Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R1 and secondary resistance R2 act in series with the respective windings as shown below:**

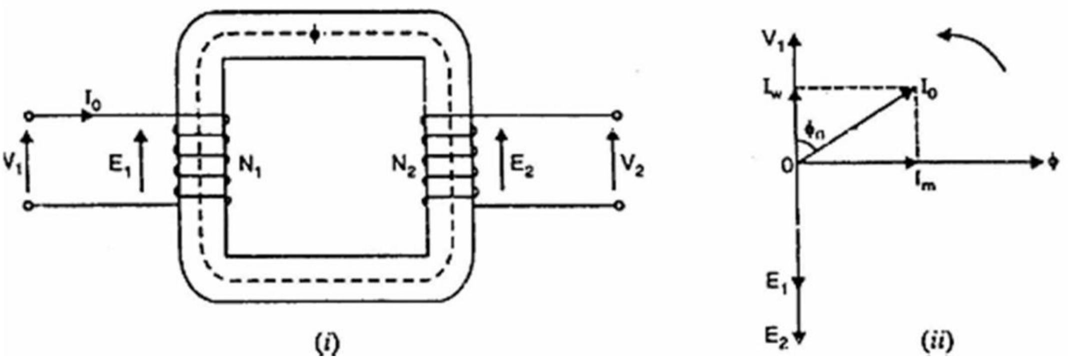


(iii) ***Leakage reactance*. Both primary and secondary currents produce flux. The flux Φ which links both the windings is the useful flux However, primary current would produce some flux Φ which would not link the secondary winding and is called *mutual flux* (*for more information review Lecture Note 2*) (See Fig. below).**



Practical Transformer on No Load

Consider the figure below:



The primary will draw a small current I_0 to supply (i) the iron losses and (ii) a very small amount of copper loss in the primary. Hence the primary no load current I_0 is not 90° behind the applied voltage V_1 but lags it by an angle $\Phi_0 < 90°$ as shown in the phasor diagram.

The no-load primary current I_0 can be resolved into two rectangular components:

(i) The component I_W in phase with the applied voltage V_1. This is known as active or working or iron loss component and supplies the iron loss and a very small primary copper loss.

$$\boxed{I_W = I_0 \cos \phi_0}$$

ii)

The component I_m lagging behind V_1 by 90° and is known as magnetizing component. It is this component which produces the mutual flux ϕ in the core.

$$\boxed{I_m = I_0 \sin \phi_0}$$

Clearly, I_0 is phasor sum of I_m and I_W,

$$\therefore \quad I_0 = \sqrt{I_m^2 + I_W^2}$$

No load p.f., $\cos\phi_0 = \dfrac{I_W}{I_0}$

It is emphasized here that no load primary copper loss (i.e. $I_0^2 R_1$) is very small and may be neglected. Therefore, the no load primary input power is practically equal to the iron loss in the transformer i.e.,

No load input power, W_0 = Iron loss

Note. At no load, there is no current in the secondary so that $V_2 = E_2$. On the primary side, the drops in R_1 and X_1, due to I_0 are also very small because of the smallness of I_0. Hence, we can say that at no load, $V_1 = E_1$.

Practical Transformer on Load

We shall consider two cases (i) when such a transformer is assumed to have no winding resistance and leakage flux (ii) when the transformer has winding resistance and leakage flux.

No winding resistance and leakage flux

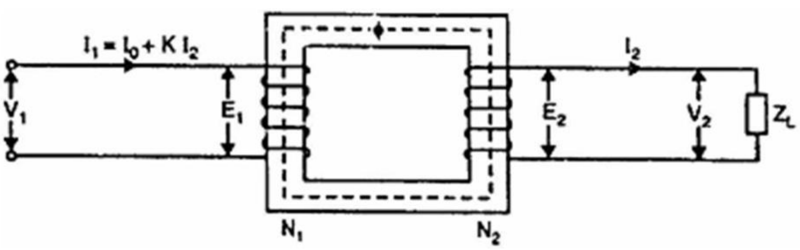


Fig. above shows a practical transformer with the assumption that resistances and leakage reactances of the windings are negligible. With this assumption, V_2 = E_2 and V_1 = E_1.

Let us take the usual case of inductive load which causes the I_2 to lag V_2 by Φ_2.

The total primary current I_1 must meet two requirements:

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I'_0 to counteract the demagnetizing effect of secondary currently I_2. The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

or
$$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$

The total primary current I_1 is the phasor sum of I'_2 and I_0 i.e.,

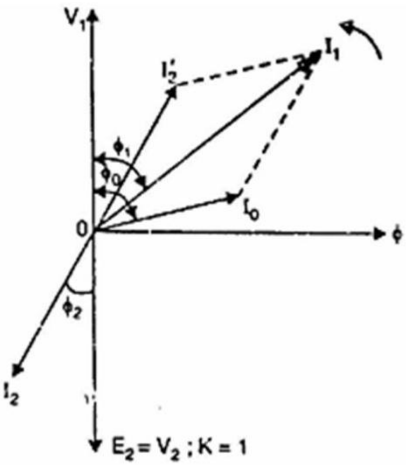
$$I_1 = I'_2 + I_0$$

where
$$I'_2 = -K I_2$$

Note that I'_2 is 180° out of phase with I_2.

ϕ

Phasor Diagram: **Both E_1 and E_2 lag behind the mutual flux f by 90°. The current I'_2 represents the primary current to neutralize the demagnetizing effect of secondary current I_2. Now $I'_2 = K I_2$ and is antiphase with I_2. I_0 is the no-load current of the transformer. The phasor sum of I'_2 and I_0 gives the total primary current I_1. Note that in drawing the phasor diagram, the value of K is assumed to be unity so that primary phasors are equal to secondaryphasors.**



Transformer with resistance and leakage reactance

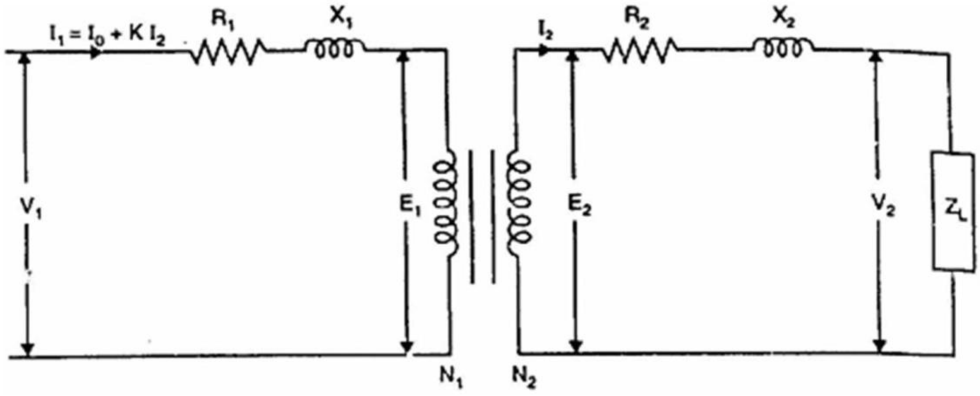
The total primary current I_1 must meet two requirements:

(a) It must supply the no-load current I_0 to meet the iron losses in the transformer and to provide flux in the core.

(b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary current I_2. The magnitude of I'_2 will be such that:

$$N_1 I'_2 = N_2 I_2$$

or $$I'_2 = \frac{N_2}{N_1} I_2 = K I_2$$



The total primary current I_1 will be the phasor sum of I'_2 and I_0 i.e.,

$$I_1 = I'_2 + I_0 \qquad \text{where} \qquad I'_2 = -KI_2$$

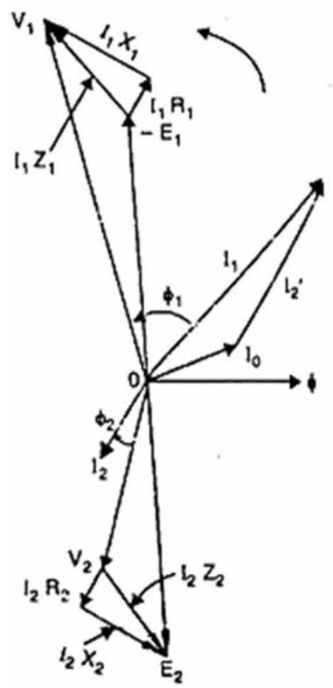
$$V_1 = -E_1 + I_1(R_1 + jX_1) \qquad \text{where} \qquad I_1 = I_0 + (-KI_2)$$

$$= -E_1 + I_1 Z_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2)$$

$$= E_2 - I_2 Z_2$$

Note that counter emf that opposes the applied voltage V_1 is $-E_1$. Therefore, if we add I_1R_1 (in phase with I_1) and $I_1 X_1$ (90° ahead of I_1) to $-E_1$, we get the applied primary voltage V_1. The phasor E_2 represents the induced emf in the secondary by the mutual flux. The secondary terminal voltage V_2 will be what is left over after subtracting I_2R_2 and I_2X_2 from E_2.



Load power factor = $\cos \phi_2$

Primary power factor = $\cos \phi_1$

Input power to transformer, $P_1 = V_1 I_1 \cos \phi_1$

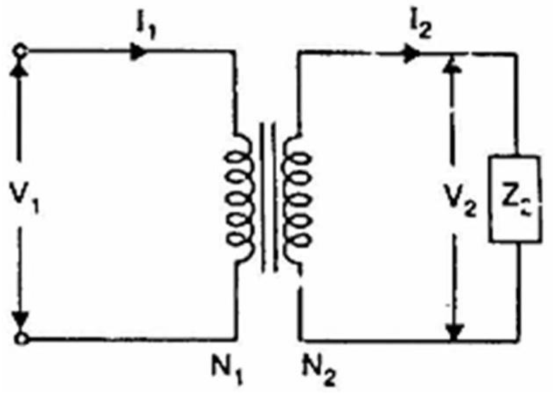
Output power of transformer, $P_2 = V_2 I_2 \cos \phi_2$

Modelling and Equivalent Circuits of Single Phase Transformers

The term equivalent circuit of a transformer means the combination of fixed and variable resistances and reactances, which exactly simulates performance and working of the transformer.

Impedance Ratio

Consider a transformer having impedance Z_2 in the secondary as shown in the figure below:



$$Z_2 = \frac{V_2}{I_2}$$

$$Z_1 = \frac{V_1}{I_1}$$

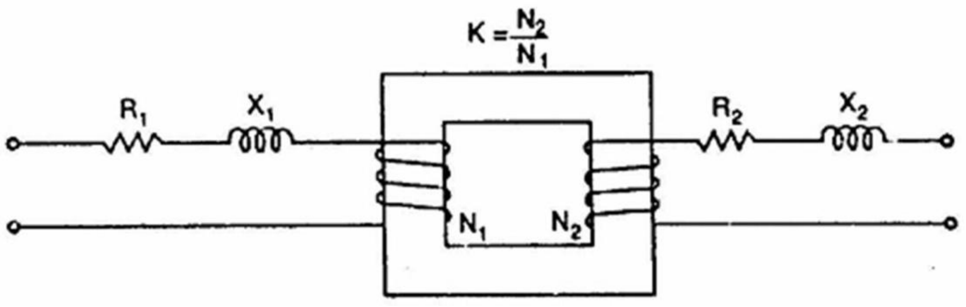
$$\therefore \quad \frac{Z_2}{Z_1} = \left(\frac{V_2}{V_1}\right) \times \left(\frac{I_1}{I_2}\right)$$

or $$\boxed{\frac{Z_2}{Z_1} = K^2}$$

Shifting Impedances in a Transformer

NOTE:

Consider the following figure:



We can transfer the parameters from one winding to the other. Thus:

- **A resistance R_1 in the primary becomes K^2R_1 *when transferred to the secondary*.**

- **A resistance R_2 in the secondary becomes R_2/K^2 when transferred to the primary.**

- **A reactance X_1 in the primary becomes K^2X_1 when transferred to the secondary.**

- **A reactance X_2 in the secondary becomes X_2/K^2 when transferred to the primary.**

NOTE:

- **When transferring resistance or reactance from primary to secondary, multiply it by K^2**

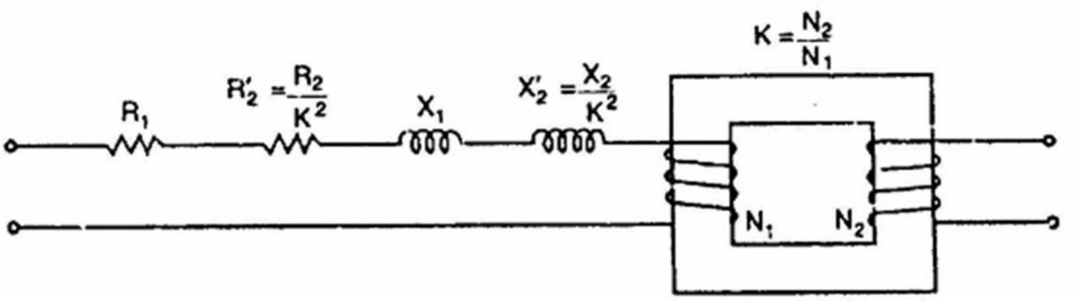
.

- **When transferring resistance or reactance from secondary to primary, divide it by K^2**

.

- **When transferring voltage or current from one winding to the other, only K is used.**

A- Referred to primary



► *Equivalent resistance of transformer referred to primary*

$$\boxed{R_{01} = R_1 + R'_2 = R_1 + \frac{R_2}{K^2}}$$

► *Equivalent reactance of transformer referred to primary*

$$\boxed{X_{01} = X_1 + X'_2 = X_1 + \frac{X_2}{K^2}}$$

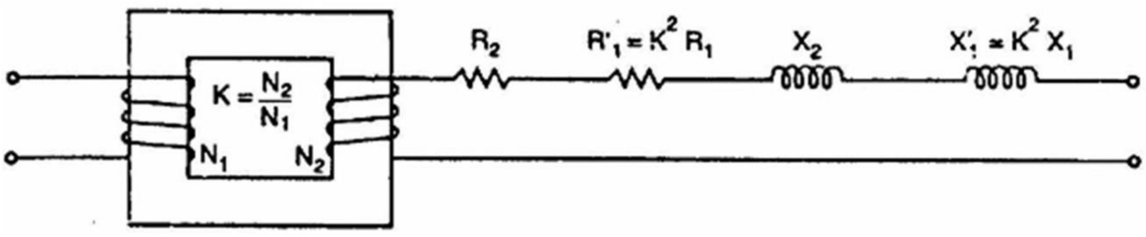
► *Equivalent impedance of transformer referred to primary*

$$\boxed{Z_{01} = \sqrt{R_{01}^2 + X_{01}^2}}$$

► *The value of primary current I_1*

$$\boxed{I_1 = KI_2}$$

B- Referred to secondary



► ***Equivalent resistance of transformer referred to secondary***

$$\boxed{R_{02} = R_2 + R'_1 = R_2 + K^2 R_1}$$

► ***Equivalent reactance of transformer referred to secondary***

$$\boxed{X_{02} = X_2 + X'_1 = X_2 + K^2 X_1}$$

► ***Equivalent impedance of transformer referred to secondary***

$$\boxed{Z_{02} = \sqrt{R_{02}^2 + X_{02}^2}}$$

► ***The value of secondary voltage referred to primary***

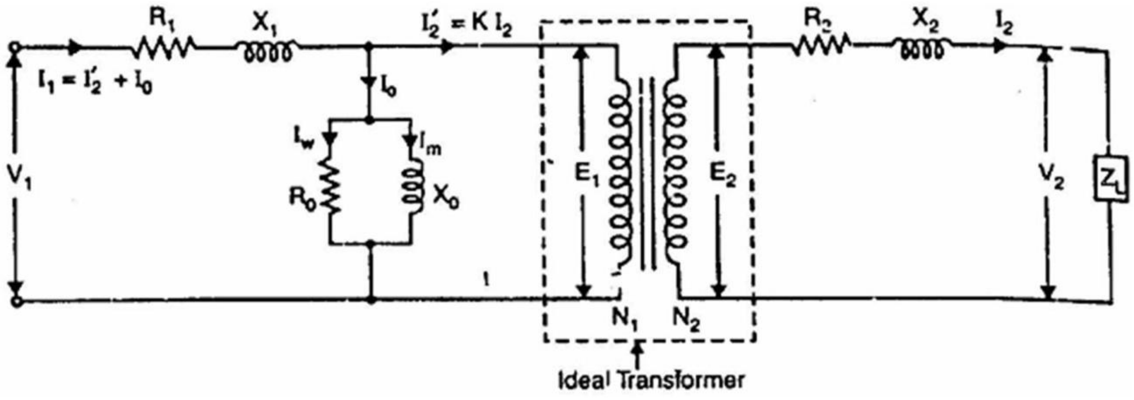
$$\boxed{V_2 = KV_1}$$

What the Importance of Shifting Impedances?

If we shift all the impedances from one winding to the other, the transformer is eliminated and we get an equivalent electrical circuit. ***Various voltages and currentscanbereadilyobtainedbysolvingthiselectricalcircuit.***

Exact Equivalent Circuit of a Loaded Transformer

Theequivalentcircuitforthetransformercanberepresentedasshowninthe figure.



Where:

R_1: primary winding resistance R_2:

secondary winding resistance

X_1: leakage reactance of primary winding

X_2: leakage reactance of the secondary winding

R_o: represents the core losses (hysteresis and eddy current losses) X_o: represents

magnetisingreactanceofthecore

I_m: magnetizing current (to create magnetic flux in the core)

I_w: active current (required to supply the core losses) I_o = no load primary

current

- **NOTE[1]: Parallel circuit R_0 - X_0 is the no-load equivalent circuit of the transformer or called exciting circuit.**

- **NOTE[2]:** ***The equivalent circuit has created two normal electrical circuits separated only by an ideal transformer whose function is to change values according to the equation:***

$$\boxed{\frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I'_2}{I_2}}$$

- **NOTE[3]: If Z_L is the external load across the secondary circuit, voltage E_2 induced in the secondary by mutual flux will produce a secondary current I_2, hence:**

$$\boxed{V_2 = E_2 - I_2(R_2 + j\,X_2) = E_2 - I_2 Z_2}$$

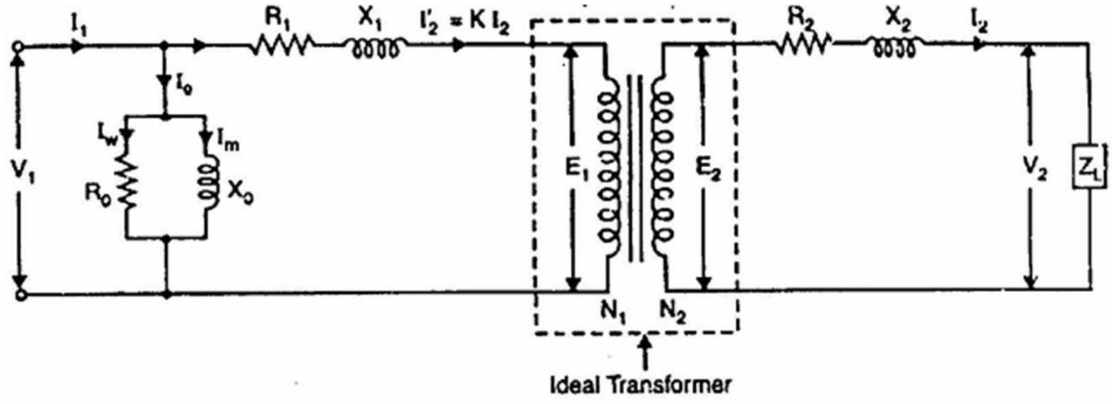
Similarly supply voltage can be given as

$$\boxed{V_1 = -E_1 + I_1(R_1 + j\,X_1) = -E_1 + I_1 Z_1}$$

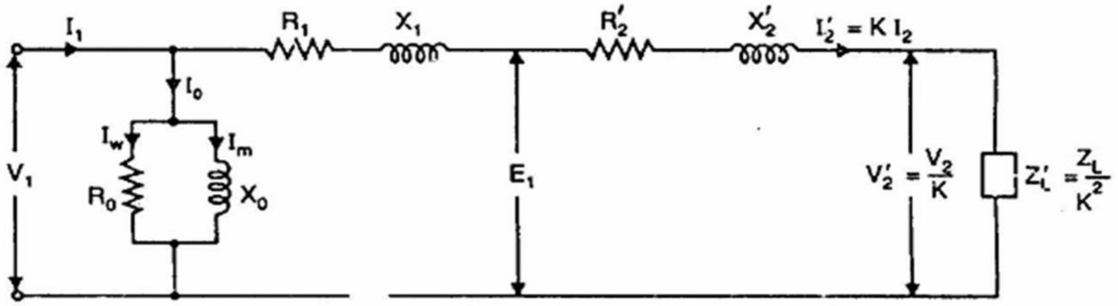
- **NOTE[5]: When the transformer is loaded to carry the secondary current I_2, the primary current consists of two components*:***
  - I_0 *to provide magnetizing current and the current required to supply the core losses.*
  - **primary current I'_2 (= K I_2)** ***required to supply the load connected to thesecondary***

Simplified Equivalent Circuit of a Loaded Transformer

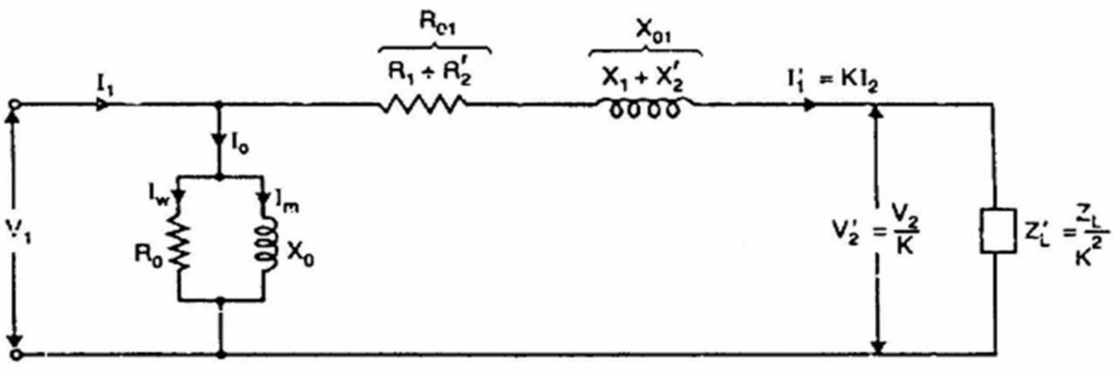
If I_0 of a transformer is small as compared to the rated primary current I_1, voltage drops in R_1 and X_1 due to I_0 are negligible. Hence, the exact equivalent circuit can be simplified by transferring the shunt circuit R_0 - X_0 to the input terminals as shown below:



If all the secondary quantities are referred to the primary, we can get the ***simplified equivalent circuit of the transformer referred to the primary*** **as shown below**



(i)



(ii)

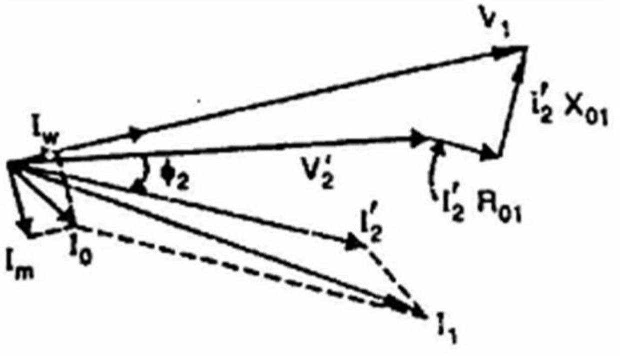
From the above circuits:

$$\boxed{R'_2 = \frac{R_2}{K^2}; \quad X'_2 = \frac{X_2}{K^2}; \quad Z'_L = \frac{Z_L}{K^2}; \quad V'_2 = \frac{V_2}{K}; \quad I'_2 = K\, I_2}$$

$$\boxed{Z_{01} = R_{01} + j\, X_{01}}$$

where $$\boxed{R_{01} = R_1 + R'_2; \quad X_{01} = X_1 + X'_2}$$

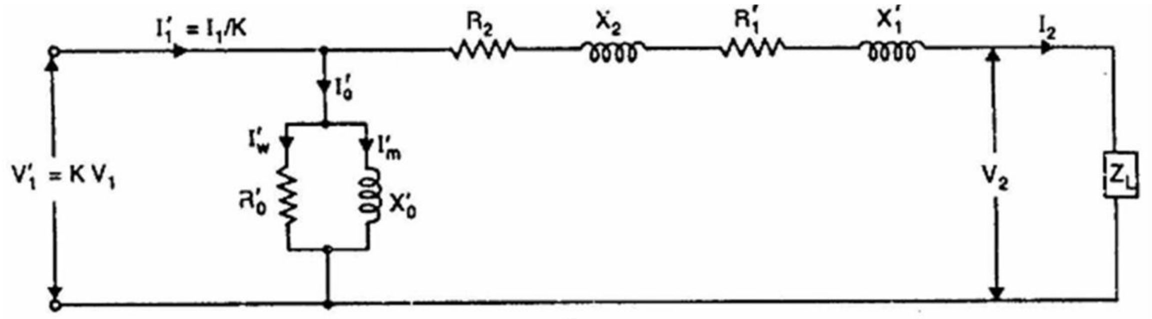
Hence the phasor diagram can be obtained as:



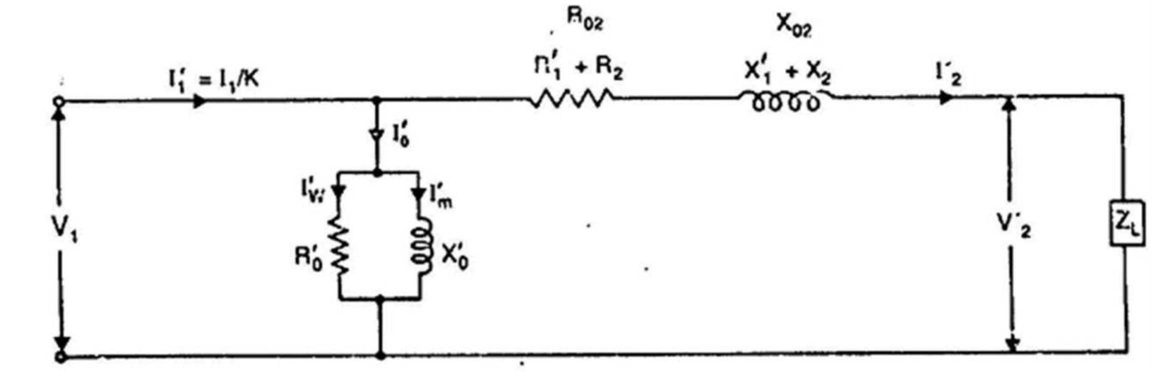
Based on the above phasor diagram we can notice the following:

- *The referred value of load voltage V'_2 is chosen as the reference phasor.*
- *I'_2 is lagging V'_2 by phase angle \$$_2$.*
- *I'_2R_{01} is in phase with I'_2 and the voltage drop I'_2X_{01}, leads I'_2 by 90°.*
- *I_W is in phase with V_1 while I_m lags behind V_1 by 90°.*
- *The phasor sum of I_W and I_m is I_0.*
- *The phasor sum of I_0 and I'_2 is the input current I_1.*

If all the primary quantities are referred to the secondary, we can get the ***simplified equivalent circuit of the transformer referred to the secondary*** **as shown below**



(i)



(ii)

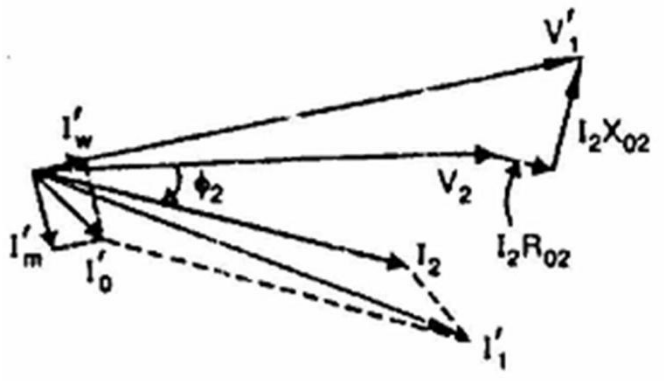
From the above circuit:

$$\boxed{R'_1 = K^2 R_1; \qquad X'_1 = K^2 X_1; \qquad V'_2 = K\ V_1;\ \ I'_1 = \frac{I_1}{K}}$$

$$\boxed{Z_{02} = R_{02} + j\,X_{02}}$$

Where $$\boxed{R_{02} = R_2 + R'_1; \quad X_{02} = X_2 + X'_1}$$

Hence the phasor diagram can be obtained as:

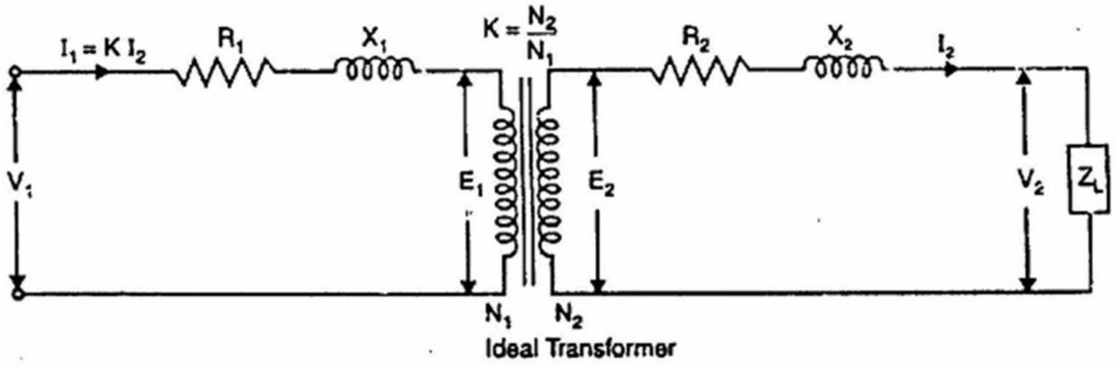


Based on the above phasor diagram we can notice the following:

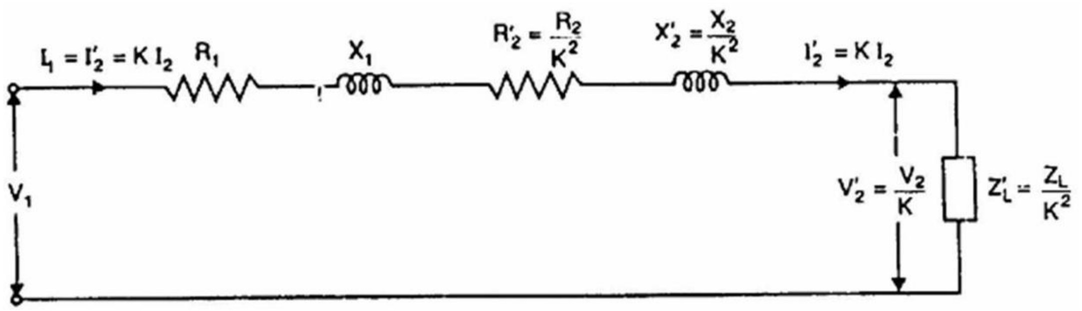
- ***The referred value of load voltage V_2 is chosen as the reference phasor.***
- ***I_2 is lagging V_2 by phase angle $\$_2$.***
- ***I_2R_{02} is in phase with I_2 and the voltage drop I_2X_{02}, leads I_2 by 90°.***
- ***I'_W is in phase with V'_1 while I'_m lags behind V'_1 by 90°.***
- ***The phasor sum of I'_W and I'_m is I'_0.***
- ***The phasor sum of I'_0 and I_2 is the input current I'_1.***

Approximate Equivalent Circuit of a Loaded Transformer

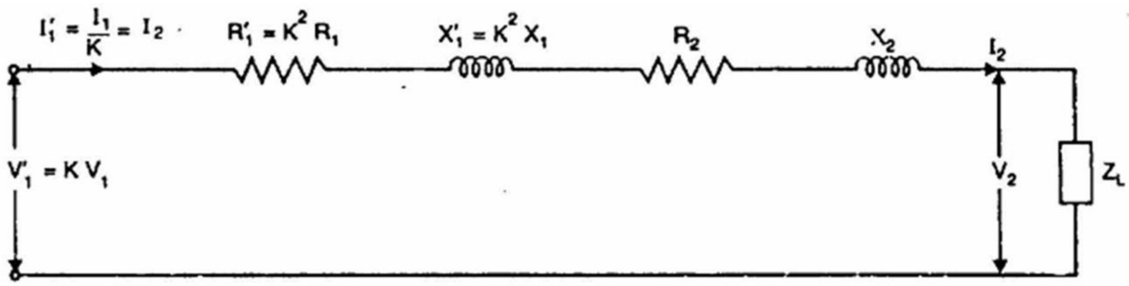
The no-load current I_0 in a transformer is only 1-3% of the rated primary current and *may be neglected without any serious error*. The transformer can then be shown as in the figure below:



If all the secondary quantities are referred to the primary, we can get the *approximate equivalent circuit of the transformer referred to the primary* as shown below

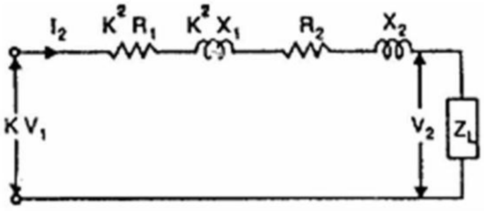


If all the primary quantities are referred to the secondary, we can get the *approximate equivalent circuit of the transformer referred to the secondary* as shown below



Approximate Voltage Drop in a Transformer

The approximate equivalent circuit of transformer referred to secondary is shown below

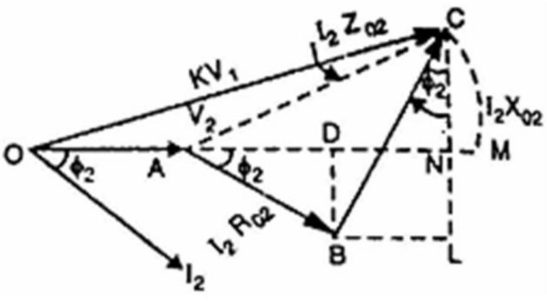


At no-load, the secondary voltage is KV_1. When a load having a lagging p.f. cos $\$_2$ is applied, the secondary carries a current I_2 and voltage drops occur in $(R_2 + K_2R_1)$ and $(X_2 + K_2 X_1)$. Consequently, the secondary voltage falls from KV_1 to V_2.

Hence, we have,

$$\boxed{\begin{aligned} V_2 &= KV_1 - I_2\left[\left(R_2 + K^2R_1\right) + j\left(X_2 + K^2X_1\right)\right] \\ &= KV_1 - I_2\left(R_{02} + j\,X_{02}\right) \\ &= KV_1 - V_2 = I_2 Z_{02} \\ \text{Drop in secondary voltage} &= KV_1 - V_2 = I_2 Z_{02} \end{aligned}}$$

It is clear from the phasor diagram below that drop in secondary voltage is AC $= I_2\ Z_{02}$.



Approximate drop in secondary voltage

$$= AN = AD + DN$$

$$= AD + BL \qquad (\because \quad BL = DN)$$

Approximate drop in secondary voltage is

$$\boxed{= I_2R_{02}\ \cos\phi_2 + I_2X_{02}\ \sin\phi_2}$$

For a load having a leading p.f. cos $_2$, we have,

$$\boxed{\text{Approximate voltage drop } = I_2R_{02}\ \cos\phi_2 - I_2X_{02}\ \sin\phi_2}$$

Note: **If the circuit is referred to primary, then it can be easily established that**

$$\boxed{\text{Approximate voltage drop } = I_1R_{01}\ \cos\phi_2 \pm I_1X_{01}\ \sin\phi_2}$$

Testing, Efficiency, and Voltage Regulation Voltage

Regulation of Transformer

The voltage regulation of a transformer *is the arithmetic difference (not phasor difference) between the no-load secondary voltage ($_0V_2$) and the secondary voltage V_2 on load expressed as percentage of no-load voltage* i.e.

$$\boxed{\text{voltage regulation} = \frac{_0V_2 - V_2}{_0V_2} \times 100}$$

where

$_0V_2$ = No-load secondary voltage = K V_1 V_2 = Secondary voltage on load

As shown in *Lecture 4*

$$\boxed{_0V_2 - V_2 = I_2R_{02}\cos\phi_2 \pm I_2X_{02}\sin\phi_2}$$

The +ve sign is for lagging p.f. and -ve sign for leading p.f.

NOTE: It may be noted that % voltage regulation of the transformer will be the same whether primary or secondary side is considered.

Losses in a Transformer

The power losses in a transformer are of two types, namely;

1. Core or Iron losses 2. Copper losses

NOTE: The above losses appear in the form of heat and produce (i) *an increase in temperature* and (ii) a *drop in efficiency*.

A- Core or Iron losses (P_i)

These consist of *hysteresis and eddy current losses* and occur in the transformer core due to the alternating flux. <u>These can be determined by open-circuit test</u> (see next sections).

$$\boxed{\begin{aligned} &\text{Hysteresis loss, } = k_h\ f\ B_m^{1.6} \quad \text{watts}/m^3 \\ &\text{Eddy current loss, } = k_e\ f^2\ B_m^2 t^2 \qquad \text{watts}/m^3 \end{aligned}}$$

Both hysteresis and eddy current losses depend upon

- **Maximum flux density B_m in the core and**
- **Supply frequency f.**

<u>NOTE</u>: Since transformers are connected to constant-frequency, constant voltage supply, both f and B_m are constant. Hence, *core or iron losses are practically the same at all loads.* Hence,

$$\boxed{\begin{aligned} \text{Iron or Core losses, } P_i &= \text{Hysteresis loss} + \text{Eddy current loss} \\ &= \text{Constant losses} \end{aligned}}$$

<u>NOTE</u>: The hysteresis loss can be minimized *by using steel of high silicon content* whereas eddy current loss can be reduced *by using core of thin laminations*.

B- Copper losses (P_C)

These losses occur in both the primary and secondary windings due to their ohmic resistance. These can be determined by short-circuit test

Total copper Cu losses:

$$\boxed{\begin{aligned} P_C &= I_1^2 R_1 + I_2^2 R_2 \\ &= I_1^2 R_{01} \text{ or } I_2^2 R_{02} \end{aligned}}$$

Hence, total losses in a transformer are:

$$\boxed{\begin{aligned} \text{Total losses in a transformer} &= P_1 + P_C \\ &= \text{Constant losses} + \text{Variable losses} \end{aligned}}$$

Efficiency of a Transformer

Like any other electrical machine, the efficiency of a transformer is defined as the ratio of output power (in watts or kW) to input power (watts or kW) i.e.

$$\text{Efficiency} = \frac{\text{Output power}}{\text{Input power}}$$

In practice, open-circuit and short-circuit tests are carried out to find the efficiency,

$$\boxed{\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output} + \text{Losses}}}$$

NOTE: **The losses can be determined by transformer tests.**

Condition for Maximum Efficiency

Output power = $V_2 I_2 \cos\phi_2$

If R_{02} is the total resistance of the transformer referred to secondary, then,

$$\text{Total Cu loss, } P_C = I_2^2 R_{02}$$

$$\text{Total losses} = P_i + P_C$$

$$\text{Transformer } \eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{02}}$$

$$= \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}}$$

For a load of given pf, efficiency depends upon load current I_2. Hence, the efficiency to be maximum *the denominator should be minimum* i.e.

$$\frac{d}{dI_2}\left(V_2 \cos\phi_2 + P_i / I_2 + I_2 R_{02}\right) = 0$$

$$\text{or} \qquad 0 - \frac{P_i}{I_2^2} + R_{02} = 0$$

$$\text{or} \qquad P_i = I_2^2 R_{02}$$

i.e., Iron losses = Copper losses

Henceefficiencyofatransformerwillbemaximumwhencopperlossesare equal to constant or iron losses.

Fromabove,theloadcurrentI2correspondingtomaximumefficiencyis:

$$\boxed{I_2 = \sqrt{\frac{P_i}{R_{02}}}}$$

NOTE: In a transformer, iron losses are constant whereas copper losses are variable. In order to obtain **maximum efficiency**, *the load current should be such that total Cu losses become equal to iron losses*.

Output kVA corresponding to Maximum Efficiency

Let P_C = Copper losses at full-load kVA P_i = Iron

losses

x = Fraction of full-load kVA at which efficiency is maximum Total Cu losses = x^2

$\times P_C$

for maximum efficiency $x^2 \times P_C = P_i$ or

$$x = \sqrt{\frac{P_i}{P_C}} = \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

⇒Output kVA corresponding to maximum efficiency:

$$\text{Full - load kVA} \times \sqrt{\frac{\text{Iron loss}}{\text{F.L. Cu loss}}}$$

NOTE: The value of kVA at which the efficiency is maximum, is independent of pf of the load.

All-Day Efficiency

All-day efficiency is of special importance for those transformers whose primaries are never open-circuited but the secondaries carry little or no load much of the time during the day.

The ratio of output in kWh to the input in kWh of a transformer over a 24-hour period is known as all-day efficiency i.e.:

$$\boxed{\eta_{\text{all-day}} = \frac{\text{kWh output in 24 hours}}{\text{kWh input in 24 hours}}}$$

NOTE: *Efficiency of a transformer means commercial efficiency unless stated otherwise*.

Transformer Tests

The circuit constants, efficiency and voltage regulation of a transformer can be determined by two simple tests:

(i) open-circuit test and (ii) short-circuit test

Open-Circuit Test

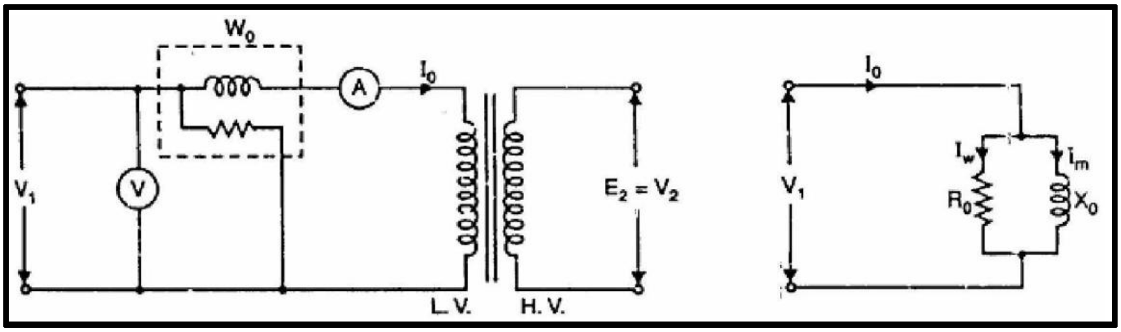
This test is conducted to determine:

- The iron losses (or core losses) and
- Parameters R_0 and X_0 of the transformer.

In this test (see Figure below), the rated voltage is applied to the primary (usually low-voltage winding) while the secondary is left open-circuited.

As the normal rated voltage is applied to the primary, therefore, normal iron losses will occur in the transformer core.

Cu losses in the primary under no-load condition are negligible as compared with iron losses.



For the figure above:

- Iron losses, P_i = Wattmeter reading = W_0
- No load current = Ammeter reading = I_0
- Applied voltage = Voltmeter reading = V_1
- Input power, $W_0 = V_1 I_0 \cos\phi_0$

⇒

$$\text{No - load p.f., } \cos\phi_0 = \frac{W_0}{V_1 I_0}$$
$$I_W = I_0 \cos\phi_0; \qquad I_m = I_0 \sin\phi_0$$
$$R_0 = \frac{V_1}{I_W} \quad \text{and} \quad X_0 = \frac{V_1}{I_m}$$

Short-Circuit or Impe

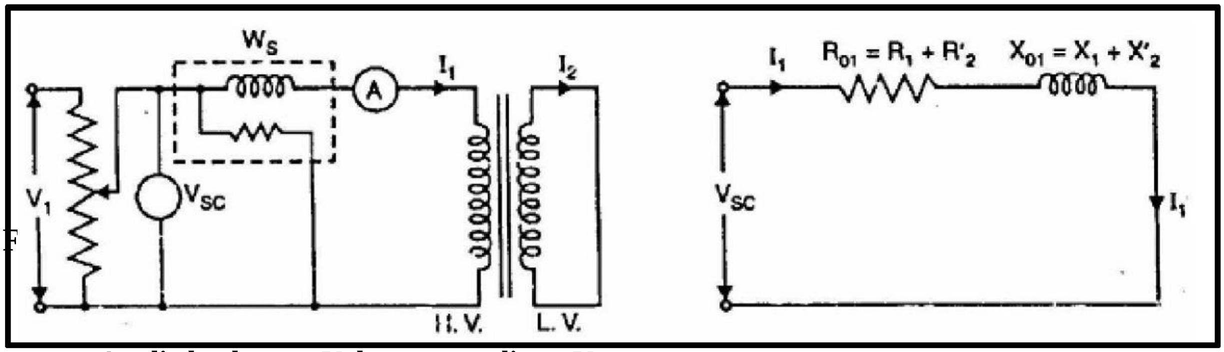
This test is conducted to determine:

- **Full-load copper losses of the transformer and**
- **R_{01} (or R_{02}), X_{01} (or X_{02}).**

In this test (see Figure below), the secondary (usually low-voltage winding) is short-circuited by a thick conductor and variable low voltage is applied to the primary.

The low input voltage is gradually raised till at voltage V_{SC}, full-load current I_1 flows in the primary. Then I_2 in the secondary also has full-load value since $I_1/I_2 = N_2/N_1$. Under such conditions, the copper loss in the windings is the same as that on full load.

F



- **Applied voltage = Voltmeter reading = V_{SC}**
- **F.L. primary current = Ammeter reading = I_1**

⇒

$$P_C = I_1^2 R_1 + I_1^2 R'_2 = I_1^2 R_{01}$$
$$R_{01} = \frac{P_C}{I_1^2}$$

where R_{01} is the total resistance of transformer referred to primary Total impedance referred toprimary,

$$Z_{01} = \frac{V_{SC}}{I_1}$$

Total leakage reactance referred to primary, Short-circuit

$$X_{01} = \sqrt{Z_{01}^2 - R_{01}^2}$$

pf

$$\cos\phi_2 = \frac{P_C}{V_{SC} I_1}$$

Efficiency from Transformer Tests

The full-load efficiency of the transformer at any pf can be obtained as:

$$\text{F.L. efficiency, } \eta_{F.L.} = \frac{\text{Full - load VA} \times \text{p.f.}}{(\text{Full - load VA} \times \text{p.f.}) + P_i + P_C}$$

where: P_i = Iron loss can be obtained from open-circuit test

P_c = Copper loss can be obtained from short-circuit test

F.L. = Full Load Also

theefficiencyforanyload,

$$\text{Corresponding total losses} = P_i + x^2 P_C$$

$$\text{Corresponding } \eta_x = \frac{(\text{xx Full - load VA}) \times \text{p.f.}}{(\text{xx Full - load VA} \times \text{p.f.}) + P_i + x^2 P_C}$$

where xx= Fraction offull-load

NOTE: Iron loss remains the same at all loads.

SCHOOL OF ELECTRICAL & ELECTRONICS ENGINEERING
DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING

UNIT - V

DC Machines and Transformers – SEEA1202

Unit 5 – SPECIAL TRANSFORMERS

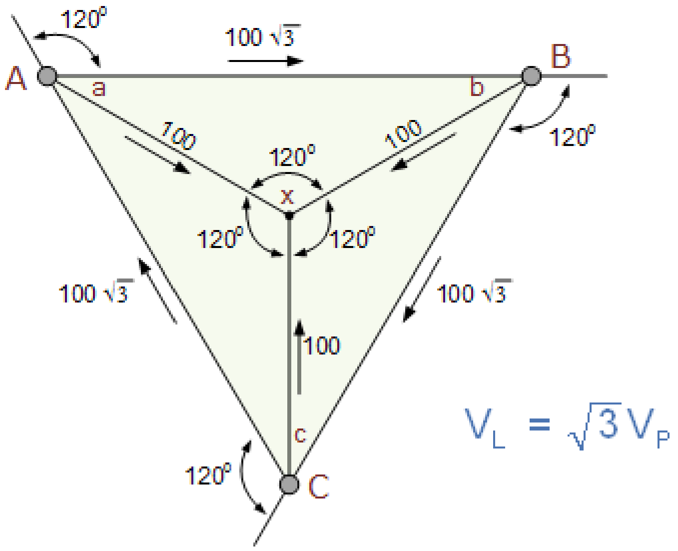
Three phase transformers :

Thus far we have looked at the construction and operation of the single-phase, two winding voltage transformer which can be used increase or decrease its secondary voltage with respect to the primary supply voltage. But voltage transformers can also be constructed for connection to not only one single phase, but for two-phases, three-phases, six-phases and even elaborate combinations up to 24-phases for some DC rectification transformers.

If we take three single-phase transformers and connect their primary windings to each other and their secondary windings to each other in a fixed configuration, we can use the transformers on a three-phase supply.

Three-phase, also written as 3-phase or 3φ supplies are used for electrical power generation, transmission, and distribution, as well as for all industrial uses. Three-phase supplies have many electrical advantages over single-phase power and when considering three-phase transformers we have to deal with three alternating voltages and currents differing in phase-time by 120 degrees as shown below.

Three Phase Voltages and Currents



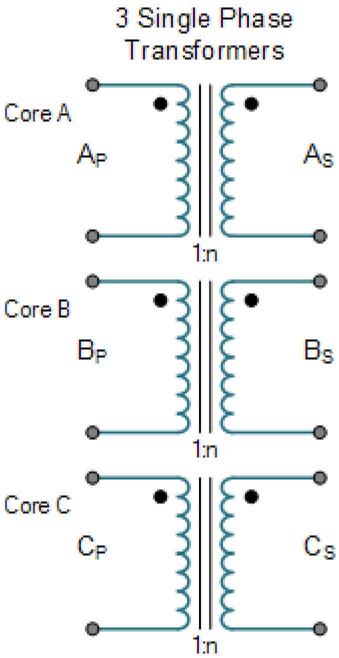
Where: V_L is the line-to-line voltage, and V_P is the phase-to-neutral voltage.

A transformer can not act as a phase changing device and change single-phase into three-phase or three-phase into single phase. To make the transformer connections compatible with three-phase supplies we need to connect them together in a particular way to form a Three Phase Transformer Configuration.

A *three phase transformer* or 3φ transformer can be constructed either by connecting together three single-phase transformers, thereby forming a so-called three phase transformer bank, or by using one pre-assembled and balanced three phase transformer which consists of three pairs of single phase windings mounted onto one single laminated core.

The advantages of building a single three phase transformer is that for the same kVA rating it will be smaller, cheaper and lighter than three individual single phase transformers connected together because the copper and iron core are used more effectively. The methods of connecting the primary and secondary windings are the same, whether using just one Three Phase Transformer or three separate *Single Phase Transformers*. Consider the circuit below:

Three Phase Transformer Connections



Primary Configuration	Secondary Configuration
Delta (Mesh)	Delta (Mesh)
Delta (Mesh)	Star (Wye)
Star (Wye)	Delta (Mesh)
Star (Wye)	Star (Wye)
Interconnected Star	Delta (Mesh)
Interconnected Star	Star (Wye)

The primary and secondary windings of a transformer can be connected in different configuration as shown to meet practically any requirement. In the case of *three phase transformer* windings, three forms of connection are possible: “star” (wye), “delta” (mesh) and “interconnected-star” (zig-zag).

The combinations of the three windings may be with the primary delta-connected and the secondary star-connected, or star-delta, star-star or delta-delta, depending on the transformers use. When transformers are used to provide three or more phases they are generally referred to as a Polyphase Transformer.

Three Phase Transformer Star and Delta Configurations

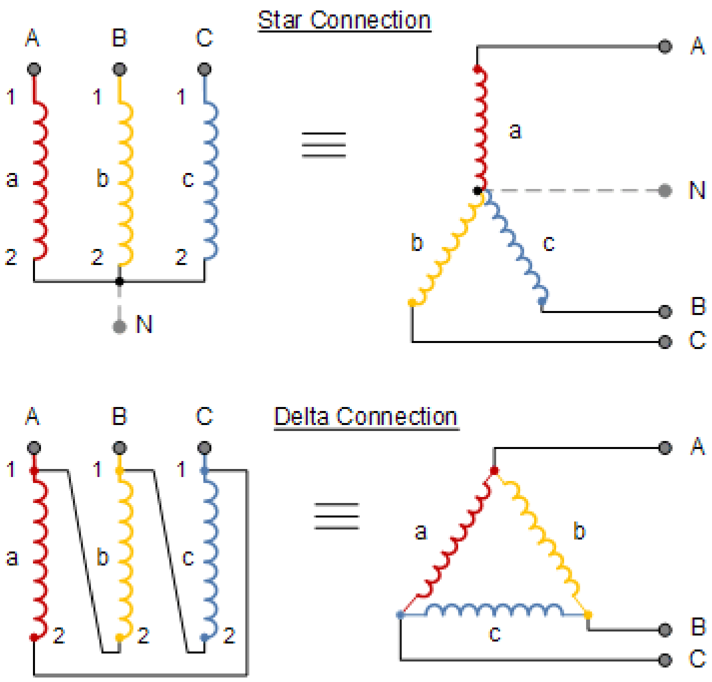
But what do we mean by “star” (also known as Wye) and “delta” (also known as Mesh) when dealing with three-phase transformer connections. A three phase transformer has three sets of primary and secondary windings. Depending upon how these sets of windings are interconnected, determines whether the connection is a star or delta configuration.

The three available voltages, which themselves are each displaced from the other by 120 electrical degrees, not only decided on the type of the electrical connections used on both the primary and secondary sides, but determine the flow of the transformers currents.

With three single-phase transformers connected together, the magnetic flux’s in the three transformers differ in phase by 120 time-degrees. With a single the three-phase transformer there are three magnetic flux’s in the core differing in time-phase by 120 degrees.

The standard method for marking three phase transformer windings is to label the three primary windings with capital (upper case) letters A, B and C, used to represent the three individual phases of RED, YELLOW and BLUE. The secondary windings are labelled with small (lower case) letters a, b and c. Each winding has two ends normally labelled 1 and 2 so that, for example, the second winding of the primary has ends which will be labelled B1 and B2, while the third winding of the secondary will be labelled c1 and c2 as shown.

Transformer Star and Delta Configurations



Symbols are generally used on a three phase transformer to indicate the type or types of connections used with upper case Y for star connected, D for delta connected and Z for interconnected star primary windings, with lower case y, d and z for their respective secondaries. Then, Star-Star would be labelled Yy, Delta-Delta would be labelled Dd and interconnected star to interconnected star would be Zz for the same types of connected transformers.

Transformer Winding Identification

Connection	Primary Winding	Secondary Winding
Delta	D	d
Star	Y	y

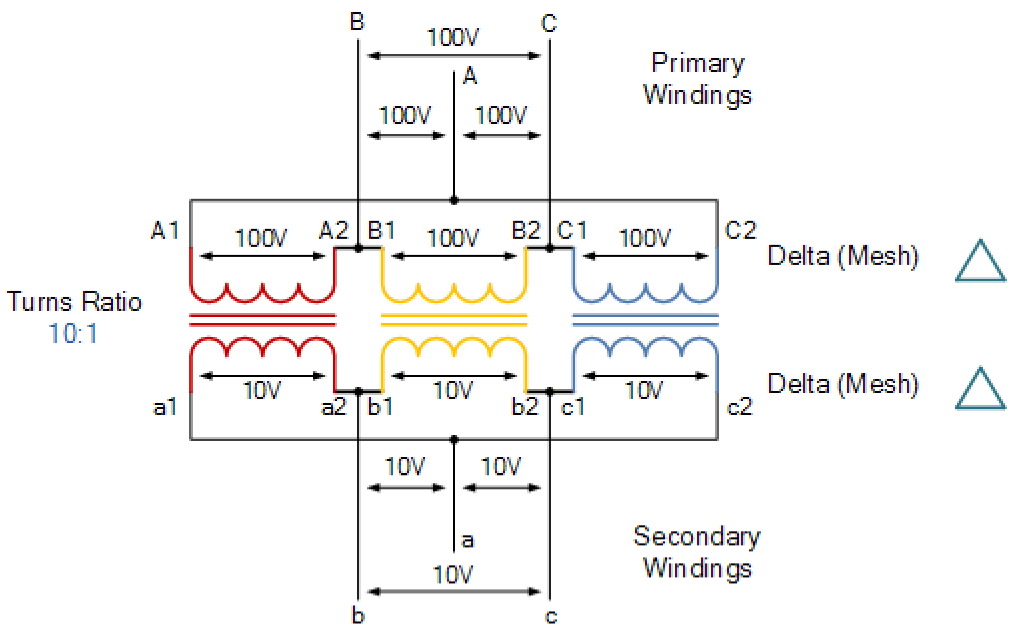
Interconnected	**Z**	**z**

We now know that there are four different ways in which three single-phase transformers may be connected together between their primary and secondary three-phase circuits. These four standard configurations are given as: Delta-Delta (Dd), Star-Star (Yy), Star-Delta (Yd), and Delta-Star (Dy).

Transformers for high voltage operation with the star connections has the advantage of reducing the voltage on an individual transformer, reducing the number of turns required and an increase in the size of the conductors, making the coil windings easier and cheaper to insulate than delta transformers.

The delta-delta connection nevertheless has one big advantage over the star-delta configuration, in that if one transformer of a group of three should become faulty or disabled, the two remaining ones will continue to deliver three-phase power with a capacity equal to approximately two thirds of the original output from the transformer unit.

Transformer Delta and Delta Connections



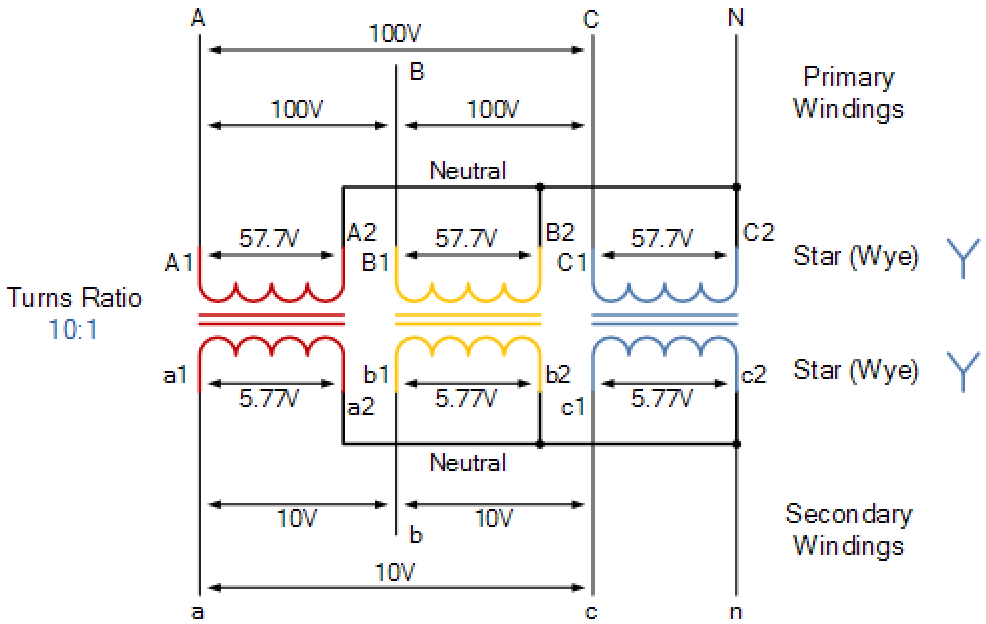
In a delta connected (Dd) group of transformers, the line voltage, V_L is equal to the supply voltage, $V_L = V_S$. But the current in each phase winding is given as: $1/\sqrt{3} \times I_L$ of the line current, where I_L is the line current.

One disadvantage of delta connected three phase transformers is that each transformer must be wound for the full-line voltage, (in our example above 100V) and for 57.7 per cent, line current. The greater number of turns in the winding, together with the insulation between turns, necessitate a larger and more expensive coil than the star connection. Another disadvantage with delta connected three phase transformers is that there is no "neutral" or common connection.

In the star-star arrangement (Yy), (wye-wye), each transformer has one terminal connected to a common junction, or neutral point with the three remaining ends of the primary windings connected to the three-phase mains supply. The number of turns in a transformer winding for star connection is 57.7 per cent, of that required for delta connection.

The star connection requires the use of three transformers, and if any one transformer becomes fault or disabled, the whole group might become disabled. Nevertheless, the star connected three phase transformer is especially convenient and economical in electrical power distributing systems, in that a fourth wire may be connected as a neutral point, (n) of the three star connected secondaries as shown.

Transformer Star and Star Connections



The voltage between any line of the three-phase transformer is called the "line voltage", V_L, while the voltage between any line and the neutral point of a star connected transformer is called the "phase voltage", V_P. This phase voltage between the neutral point and any one of the line connections is $1/\sqrt{3} \times V_L$ of the line voltage. Then above, the primary side phase voltage, V_P is given as.

$$V_P = \frac{1}{\sqrt{3}} \times V_L = \frac{1}{\sqrt{3}} \times 100 = 57.7 \text{ Volts}$$

The secondary current in each phase of a star-connected group of transformers is the same as that for the line current of the supply, then $I_L = I_S$.

Then the relationship between line and phase voltages and currents in a three-phase system can be summarised as:

Three-phase Voltage and Current

Connection	Phase Voltage	Line Voltage	Phase Current	Line Current
Star	$V_P = V_L \div \sqrt{3}$	$V_L = \sqrt{3} \times V_P$	$I_P = I_L$	$I_L = I_P$

Delta	$V_P = V_L$	$V_L = V_P$	$I_P = I_L \div \sqrt{3}$	$I_L = \sqrt{3} \times I_P$

Where again, V_L is the line-to-line voltage, and V_P is the phase-to-neutral voltage on either the primary or the secondary side.

Other possible connections for three phase transformers are star-delta Yd, where the primary winding is star-connected and the secondary is delta-connected or delta-star Dy with a delta-connected primary and a star-connected secondary.

Delta-star connected transformers are widely used in low power distribution with the primary windings providing a three-wire balanced load to the utility company while the secondary windings provide the required 4th-wire neutral or earth connection.

When the primary and secondary have different types of winding connections, star or delta, the overall turns ratio of the transformer becomes more complicated. If a three-phase transformer is connected as delta-delta (Dd) or star-star (Yy) then the transformer could potentially have a 1:1 turns ratio. That is the input and output voltages for the windings are the same.

However, if the 3-phase transformer is connected in star–delta, (Yd) each star-connected primary winding will receive the phase voltage, V_P of the supply, which is equal to $1/\sqrt{3} \times V_L$.

Then each corresponding secondary winding will then have this same voltage induced in it, and since these windings are delta-connected, the voltage $1/\sqrt{3} \times V_L$ will become the secondary line voltage. Then with a 1:1 turns ratio, a star–delta connected transformer will provide a $\sqrt{3}$:1 step-down line-voltage ratio.

Then for a star–delta (Yd) connected transformer the turns ratio becomes:

Star-Delta Turns Ratio

$$TR = \frac{N_P}{N_S} = \frac{V_P}{\sqrt{3}V_S}$$

Likewise, for a delta–star (Dy) connected transformer, with a 1:1 turns ratio, the transformer will provide a 1:$\sqrt{3}$ step-up line-voltage ratio. Then for a delta-star connected transformer the turns ratio becomes:

Delta-Star Turns Ratio

$$TR = \frac{N_P}{N_S} = \frac{\sqrt{3}V_P}{V_S}$$

Then for the four basic configurations of a three-phase transformer, we can list the transformers secondary voltages and currents with respect to the primary line voltage, V_L and its primary line current I_L as shown in the following table.

Three-phase Transformer Line Voltage and Current

Primary-Secondary	Line Voltage Primary or	Line Current Primary or

Configuration	Secondary	Secondary
Delta – Delta	$V_L \Rightarrow nV_L$	$I_L \Rightarrow \frac{I_L}{n}$
Delta – Star	$V_L \Rightarrow \sqrt{3}.nV_L$	$I_L \Rightarrow \frac{I_L}{\sqrt{3}.n}$
Star – Delta	$V_L \Rightarrow \frac{nV_L}{\sqrt{3}}$	$I_L \Rightarrow \sqrt{3}.\frac{I_L}{n}$
Star – Star	$V_L \Rightarrow nV_L$	$I_L \Rightarrow \frac{I_L}{n}$

Where: n equals the transformers "turns ratio" (T.R.) of the number of secondary windings N_S, divided by the number of primary windings N_P. (N_S/N_P) and V_L is the line-to-line voltage with V_P being the phase-to-neutral voltage.

Three Phase Transformer Example

The primary winding of a delta-star (Dy) connected 50VA transformer is supplied with a 100 volt, 50Hz three-phase supply. If the transformer has 500 turns on the primary and 100 turns on the secondary winding, calculate the secondary side voltages and currents.

Given Data: transformer rating, 50VA, supply voltage, 100v, primary turns 500, secondary turns, 100.

$$n = \frac{N_S}{N_P} = \frac{100}{500} = 0.2$$

$$\begin{aligned} V_{L(sec)} &= \sqrt{3} \times n \times V_{L(pri)} \\ &= \sqrt{3} \times 0.2 \times 100 \\ &= 34.64 \text{Volts} \end{aligned}$$

$$V_{P(sec)} = \frac{V_{L(sec)}}{\sqrt{3}} = \frac{34.64}{\sqrt{3}} = 20 \text{Volts}$$

$$I_{L(pri)} = \frac{VA}{\sqrt{3} V_{L(pri)}} = \frac{50}{\sqrt{3} \times 100} = 0.289 \text{Amps}$$

$$I_{sec} = \frac{I_{L(pri)}}{\sqrt{3} \times n} = \frac{0.289}{\sqrt{3} \times 0.2} = 0.834 \text{Amps}$$

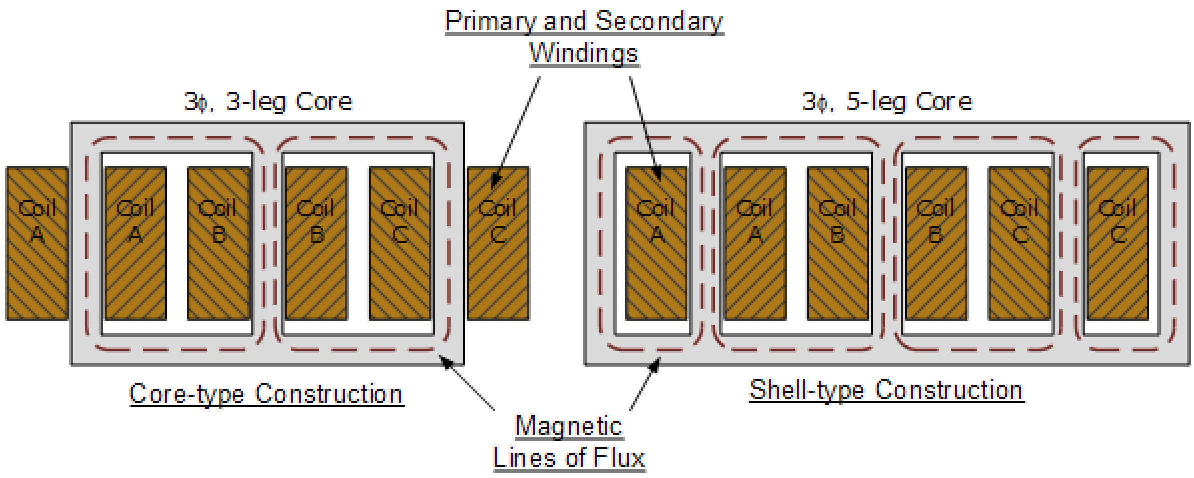
Then the secondary side of the transformer supplies a line voltage, V_L of about 35v giving a phase voltage, V_P of 20v at 0.834 amperes.

Three Phase Transformer Construction

We have said previously that the three-phase transformer is effectively three interconnected single phase transformers on a single laminated core and considerable savings in cost, size and weight can be achieved by combining the three windings onto a single magnetic circuit as shown.

A three-phase transformer generally has the three magnetic circuits that are interlaced to give a uniform distribution of the dielectric flux between the high and low voltage windings. The exception to this rule is a three-phase shell type transformer. In the shell type of construction, even though the three cores are together, they are non-interlaced.

Three Phase Transformer Construction



The three-limb core-type three-phase transformer is the most common method of three-phase transformer construction allowing the phases to be magnetically linked. Flux of each limb uses the other two limbs for its return path with the three magnetic flux's in the core generated by the line voltages differing in time-phase by 120 degrees. Thus the flux in the core remains nearly sinusoidal, producing a sinusoidal secondary supply voltage.

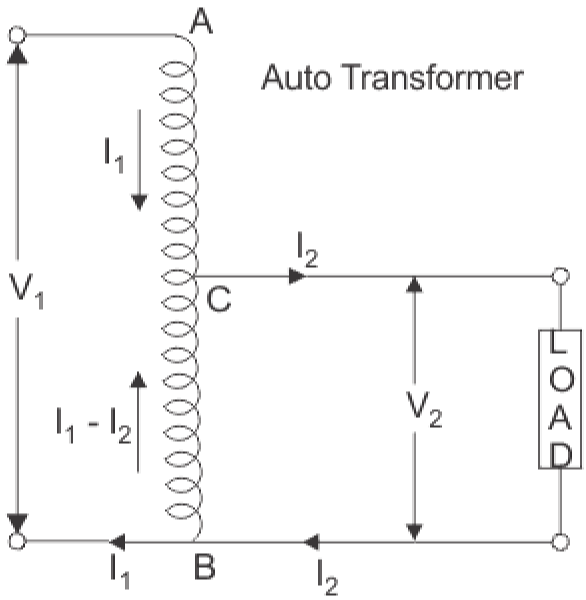
The shell-type five-limb type three-phase transformer construction is heavier and more expensive to build than the core-type. Five-limb cores are generally used for very large power transformers as they can be made with reduced height. A shell-type transformers core materials, electrical windings, steel enclosure and cooling are much the same as for the larger single-phase types.

What is an Autotransformer?
February 15, 2019 by Electrical4U

An autotransformer is a kind of electrical transformer where primary and secondary shares same common single winding. So basically it's a one winding transformer.

Autotransformer Theory

In an auto transformer, one single winding is used as primary winding as well as secondary winding. But in two windings transformer two different windings are used for primary and secondary purpose. A circuit diagram of auto transformer is shown below.



The winding AB of total turns N_1 is considered as primary winding. This winding is tapped from point 'C' and the portion BC is considered as secondary. Let's assume the number of turns in between points 'B' and 'C' is N_2.
If V_1 voltage is applied across the winding i.e. in between 'A' and 'C'. $So\ voltage\ per\ turn\ in\ this\ winding\ is\ \frac{V_1}{N_1}$
Hence, the voltage across the portion BC of the winding, will be, $\frac{V_1}{N_1} X N_2\ and\ from\ the\ figure\ above,\ this\ voltage\ is\ V_2$

$$Hence,\ \frac{V_1}{N_1} X N_2 = V_2$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{N_2}{N_1} = Constant = K$$

As BC portion of the winding is considered as secondary, it can easily be understood that value of constant 'k' is nothing but turns ratio or voltage ratio of that auto transformer. When load is connected between secondary terminals i.e.between 'B' and 'C', load current I_2 starts flowing. The current in the secondary winding or common winding is the difference of I_2 and I_1.
Copper Savings in Auto Transformer

Now we will discuss the savings of copper in auto transformer compared to conventional two winding transformer.
We know that weight of copper of any winding depends upon its length and cross-sectional area. Again length of conductor in winding is proportional to its number of turns and cross-sectional area varies with rated current.
So weight of copper in winding is directly proportional to product of number of turns and rated current of the winding.

Therefore, weight of copper in the section AC proportional to, $(N_1 - N_2)I_1$
and similarly, weight of copper in the section BC proportional to, $N_2(I_2 - I_1)$

Hence, total weight of copper in the winding of auto transformer proportional to,
$(N_1 - N_2)I_1 + N_2(I_2 - I_1)$

$\Rightarrow N_1 I_1 - N_2 I_1 + N_2 I_2 - N_2 I_1$

$\Rightarrow N_1 I_1 + N_2 I_2 - 2N_2 I_1$

$\Rightarrow 2N_1 I_1 - 2N_2 I_1 (Since,\ N_1 I_1 = N_2 I_2)$

$\Rightarrow 2(N_1 I_1 - N_2 I_1)$

In similar way it can be proved, the weight of copper in two winding transformer is proportional to,
$N_1 I_1 - N_2 I_2$

$\Rightarrow 2N_1 I_1 \qquad (Since,\ in\ a\ transformer\ N_1 I_1 = N_2 I_2)$

$N_1I_1 + N_2I_2$
$\Rightarrow 2N_1I_1$ (Since, in a transformer $N_1I_1 = N_2I_2$)

Let's assume, W_a and W_{tw} are weight of copper in auto transformer and two winding transformer respectively,

$$Hence,\ \frac{W_a}{W_{tw}} = \frac{2(N_1 I_1 - N_2 I_1)}{2(N_1 I_1)}$$

$$= \frac{N_1 I_1 - N_2 I_1}{N_1 I_1} = 1 - \frac{N_2 I_1}{N_1 I_1}$$

$$= 1 - \frac{N_2}{N_1} = 1 - k$$

$$\therefore W_a = W_{tw}(1 - k)$$

$$\Rightarrow W_a = W_{tw} - kW_{tw}$$

∴ Saving of copper in auto transformer compared to two winding transformer,
$\Rightarrow W_{tw} - W_a = kW_{tw}$

Advantages of using Auto Transformers

1. **For transformation ratio = 2, the size of the auto transformer would be approximately 50% of the corresponding size of two winding transformer. For transformation ratio say 20 however the size would be 95 %. The saving in cost of the material is of course not in the same proportion. The saving of cost is appreciable when the ratio of transformer is low, that is lower than 2. Thus auto transformer is smaller in size and cheaper.**
2. **An auto transformer has higher efficiency than two winding transformer. This is because of less ohmic loss and core loss due to reduction of transformer material.**
3. **Auto transformer has better voltage regulation as voltage drop in resistance and reactance of the single winding is less.**

Disadvantages of Using Auto Transformer

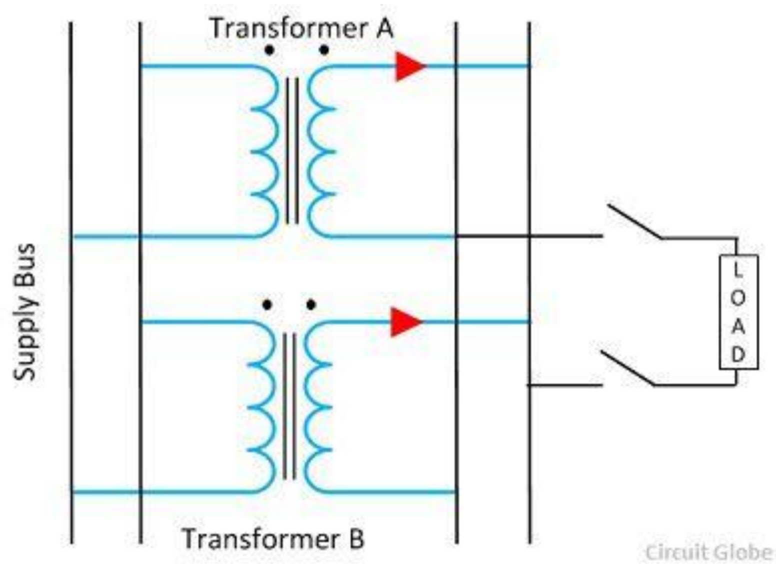
1. **Because of electrical conductivity of the primary and secondary windings the lower voltage circuit is liable to be impressed upon by higher voltage. To avoid breakdown in the lower voltage circuit, it becomes necessary to design the low voltage circuit to withstand higher voltage.**
2. **The leakage flux between the primary and secondary windings is small and hence the impedance is low. This results into severer short circuit currents under fault conditions.**
3. **The connections on primary and secondary sides have necessarily needs to be same, except when using interconnected starring connections. This introduces complications due to changing primary and secondary phase angle particularly in the case of delta/delta connection.**
4. **Because of common neutral in a star/star connected auto transformer it is not possible to earth neutral of one side only. Both their sides should have their neutrality either earth or isolated.**
5. **It is more difficult to maintain the electromagnetic balance of the winding when voltage adjustment tappings are provided. It should be known that the provision of tapping on an auto transformer increases considerably the frame size of the transformer. If the range of tapping is very large, the advantages gained in initial cost is lost to a great event.**

Applications of Auto Transformers

1. **Compensating voltage drops by boosting supply voltage in distribution systems.**
2. **Auto transformers with a number of tapping are used for starting induction and synchronous motors.**
3. **Auto transformer is used as variac in laboratory or where continuous variable over broad ranges are required.**

Parallel Operation of a Transformer

The Transformer is said to be in Parallel Operation when their primary windings are connected to a common voltage supply, and the secondary windings are connected to a common load. The connection diagram of the parallel operation of a transformer is shown in the figure below.



The parallel operation of a transformer has some advantages likes it increases the efficiency of the system, makes the system more flexible and reliable. But it

increases the short-circuit current of the transformers.

Parallel operation of a transformer is necessary because of the following reasons are given below

- **It is impractical and uneconomical to have a single large transformer for heavy and large loads. Hence, it will be a wise decision to connect a number of transformers in parallel.**
- **In substations, the total load required may be supplied by an appropriate number of the transformer of standard size. As a result, this reduces the spare capacity of the substation.**
- **If the transformers are connected in parallel, so there will be scope in future, for expansion of a substation to supply a load beyond the capacity of the transformer already installed.**
- **If there will be any breakdown of a transformer in a system of transformers connected in parallel, there will be no interruption of power supply, for essential services.**
- **If any of the transformer from the system is taken out of service for its maintenance and inspection, the continuity of the supply will not get disturbed.**

Necessary Conditions For Parallel Operation

For the satisfactory parallel operation of the transformer, the two main conditions are necessary. One is that the Polarities of the transformers must be same. Another condition is that the Turn Ratio of the transformer should be equal.

The other two desirable conditions are as follows:-

- **The voltage at full load across the transformer internal impedance should be equal.**
- **The ratio of their winding resistances to reactances should be equal for both the transformers. This condition ensures that both transformers operate at the same power factor, thus sharing their active power and reactive volt-amperes according to their ratings.**