SATHYABAMA

INSTITUTE OF SCIENCE AND TECHNOLOGY

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SCHOOL OF COMPUTING

DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

DEPARTMENT OF INFORMATION TECHNOLOGTY

UNIT - I
DC CIRCUITS

DC CIRCUITS

Electrical Quantities, Ohm's law, Kirchoff's laws, Resistors in series and parallel combinations, Current and Voltage division rules, Node and Mesh Analysis.

ELECTRICAL QUANTITIES – DEFINITIONS, SYMBOLS AND UNITS

- **Charge:**

A body is said to be charged positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge on 6.28×10^{18} electrons.

- **Atom:**

To understand the basic concepts of electric current, we should know the Modern Electron Theory. Consider the matter which is in the form of solid, liquid (or) gas. Smallest particle of matter is molecule. Minute Particles are called molecules, which are themselves made up of still minute particles known as Atoms.

Atom: Minute tiny Particles with the central Part Nucleus.

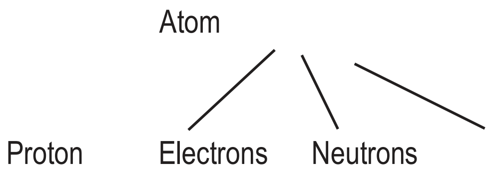


Figure 1.1

These are the types of tiny Particles in an Atom.

Protons: It is charged with positive charge.

Neutron: It is uncharged and hence it is neural.

Electron: It is revolving around nucleus. It is charged with small and constant amount of negative charge.

In an Atom, No of electrons = No of Protons

- **Electric Potential:**

When a body is charged, either electrons are supplied on it (or) removed from it. In both cases the work is done. The ability of the charged body to do work is called electric potential. The charged body has the capacity to do, by moving the other charges by either attraction (or) repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential. And the work done, to charge a body to 1 Colomb is the measure of electric potential.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

W = Work done per unit charge.
Q = Charge measured in Coulombs.

Unit of electric potential is **Joules / Coulomb (or) Volt**. If W = 1 joule; Q = 1 Coulomb, then V = 1/1 = 1 Volt.

A body is said to have an electric potential of 1 Volt, if one Joule of work is done to charge a body to one Coulomb. Hence greater the Joules / Coulomb on a charged body, greater is electric potential.

- **Potential Difference:**

The difference in the potentials of two charged bodies is called potential difference.

Consider two charged bodies A and B having Potentials of 5 Volts and 3 Volts respectively.

Potential Difference is +2v.
Unit of potential difference is Volts.
Potential difference is sometimes called Voltage.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is Ampere (or) Coulomb / sec.

$$\text{Current, } (I) = \frac{\text{Charge(q)}}{\text{Sec Time(t)}} = \frac{q}{t} \text{ Coulombs /}$$

In differential form, $i = \frac{dq}{dt}$ Coulombs / Sec

Consider a conducting material like metal, say Copper. A large number of free electrons are available. They move from one Atom to the other at random, before an electric force is applied. When an electric potential difference is applied across the metallic conductors, free electrons start moving towards the positive terminal of the cell. This continuous flow of electrons forms electric current. According to modern electronic theory, the direction of conventional current is form positive terminal to negative terminal through the external circuit.

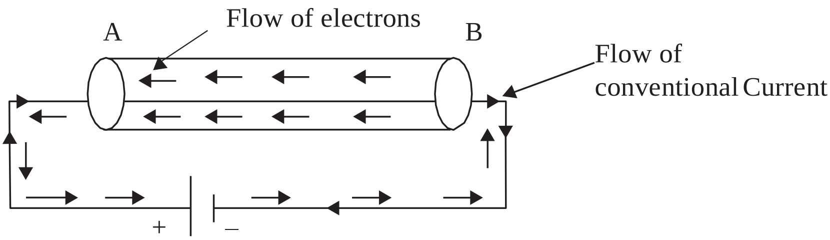


Figure 1.2

Thus, a wire is said to carry a current of 1 Ampere when charge flows through it at the rate of one Coulomb per second.

- **Resistance:**

Consider a conductor which is provided some potential difference. The free electrons start moving in a particular direction. While moving, the free electrons may collide with some Atoms (or) Molecules. They oppose the flow of electrons. Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical property of the substance due to which it opposes (or) restricts the flow of electricity (i.e. electrons) through it. Its unit is Ohms.

A wire is said to have a resistance of 1 ohm if a potential difference of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 Joule, when a current of 1A flows through it.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is inversely proportional to the cross sectional area of the Conductor, ie, $R \alpha \frac{l}{A}$

(iii) It depends upon the nature of the material of the conductor.

(iv) It depends upon the temperature of the conductor.

From the First three points and assuming the temperature to remain constant, we get,

$$R \,\alpha\, \frac{l}{A}$$

$$R = \rho \frac{1}{A}$$

ρ (‘Rho’) is a constant of proportionality called **Resistivity** (or) Specific Resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

- **Specific Resistance (or) Resistivity:**

Resistance of a wire is given by $R = \rho \frac{1}{A}$

If l = 1metre, A =1m^2 then, R = ρ . The resistance offered by a wire of length 1 metre and across sectional area of Cross-section of 1m^2 is called the Resistivity of the material of the wire.

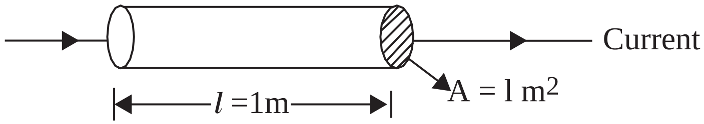


Figure 1.3

If a cube of one meter side is taken instead of wire, ρ is defined as below., Let l = 1 metre, A = 1 m^2, then R = ρ. “Hence, the resistance between the opposite faces of 1 metre cube of the given material is called the resistivity of that material”. The unit of resistivity is ohm-metre

$$\left[\, \rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m (\text{ohm-metre}) \,\right]$$

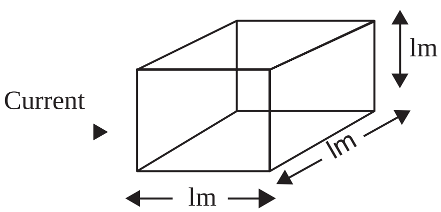


Figure 1.4

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho Symbol for its unit is (℧)

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the Conductivity (or) Specific Conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows flow of current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G\frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{\text{m}^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in an electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done inmoving the electrons in a unit time is called Electric Power. The unit of Electric Power is Joules/sec (or) Watt. $\{P = VI = I^2 R = V^2 / R\}$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = (Electrical Power)*(Time)

$$\text{Electrical Energy} = I^2Rt = \frac{V^2}{R}t$$

Electrical Energy is measured in Kilowatt hour (kwh)

Problem 1.1 The resistance of a conductor 1 mm^2 in cross section and 20 m long is 0.346 Ω. Determine the specific resistance of the conducting material.

Given Data

Area of cross-section A = 1 mm^2
Length, l = 20 m
Resistance, R = 0.346 Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A}$

$$\Rightarrow \rho = \frac{RA}{l}$$

Solution: Area of Cross-section, $A = 1mm^2$

$$= 1 * 10^{-6} m^2$$

$$\rho = \frac{1*10^{-6} * 0.346}{20} = 1.738*10^{-8}\ \Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\ \Omega m$.

Problem 1.2 A Coil consists of 2000 turns of copper wire having a cross-sectional area of 1 mm^2. The mean length per turn is 80 cm and resistivity of copper is 0.02 μΩm at normal working temperature. Calculate the resistance of the coil.

Given data:

No of turns = 2000
Length / turn = 80 cm =0.8 m

Resistivity, = 0.02 μΩm = $0.02*10^{-6}$ Ωm = $2*10^{-8}$ Ωm
Cross sectional area of the wire, A= 1mm^2 = $1*10^{-6}m^2$

Solution:
Mean length of the wire, l = 2000*0.8 =1600 m.

We know that, $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2 *10^{-8} *1600}{1*10^{-6}} = 32\Omega$

Resistance of the coil $= 32\Omega$

Problem 1.3 A wire of length 1m has a resistance of 2Ω. What is the resistance of the second wire, whose specific resistance is double that of first, if the length of wire is 3m and the diameter is double that of first?

Given Data:

For the first wire: $l_1 = 1m$, $R_1 = 2\,\Omega$, $\rho_1 = \rho$ (say)

$$d_1 = d \ (say)$$

For the Second wire: $l_2 = 3m$, $d_2 = 2d$, $\rho_2 = 2\rho$

Solution:

$$R_1 = \rho_1 \frac{l_1}{A_1} = \frac{\rho * 1}{\frac{\pi d^2}{4}} \quad [\text{Radius of the wire} = \pi r^2, \text{ where } r = \frac{d}{2}]$$

$$\text{ie, } R_1 = \frac{4\rho}{\pi d^2} = \frac{\rho_1 * 1}{\pi d^2 / 4} \ \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} = \frac{2\rho * 3}{\frac{\pi (2d)^2}{4}} = \frac{6\rho}{\pi d^2} \qquad (2)$$

Dividing equation (1) by (2),

$$\frac{4\rho}{\pi d^2} * \frac{\pi d^2}{6\rho} \Rightarrow \frac{4}{6} = \frac{R_1}{R_2}$$

$$R_2 = \frac{6R_1}{4} = \frac{6 * 2}{4} = 3\,\Omega$$

$$R_2 = 3\,\Omega$$

The Resistance of the second wire, $R_2 = 3\,\Omega$

Problem 1.4 A Rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (i) opposite ends and (ii) opposite sides. The resistivity of copper is $1.7*10^{-6}$ Ωcm.

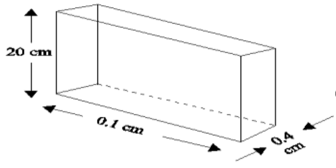


Figure 1.5

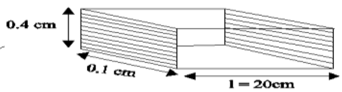


Figure 1.6

(i) Opposite Ends

Wide, w = 0.1cm

Thickness, t = 0.4cm

Length, l = 20cm

(ii) Opposite Sides:

Wide, w = 0.1cm

Thickness, t = 20 cm

Length, l = 0.4 cm

(a) Area = $w*t = 0.1*0.4 = 0.04cm^2$

$$R_1 = \frac{\rho l}{A} = \frac{1.7*10^{-6}*20}{0.04} = 0.85*10^{-3}\,\Omega$$

$R_1 = 0.85\,m\wedge$ [Opposite ends, referring to Figure 1.5]

Area, $A = w*t = 0.1*20 = 20cm^2$

$$R_2 = \frac{1.7*10^{-6}*0.4}{2} = 0.34*10^{-6}\,\Omega$$ [Opposite Sidesi referring to Figure 1.6]

$R_2 = 0.34\mu\,\Omega$

Problem 1.5 A silver wire of length 12m has a resistance of 0.2Ω. Find the specific resistivity of the material. The cross-sectional area of the wire is $0.01cm^2$.

$$R = \frac{\rho l}{A}$$

$\Rightarrow$ length, l = 12m

Resistance, R = 0.2Ω

A = $0.01cm^2$

$$\rho = \frac{RA}{l} = \frac{0.2*0.01*10^{-4}}{12}$$

$$\rho = 1.688*10^{-8}\,\Omega\, m$$

OHM'S LAW AND ITS LIMITATIONS

The relationship between DC potential difference (V) current (I) and Resistance (R) in a DC circuit was first discovered by the scientist George Simon Ohm, is called Ohm's law.

- **Statement:**

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition (eg. Temperature, etc.) do not change.

$$\text{ie, } \frac{V}{I} = \text{Constant}$$

(or)

$$\frac{V}{I} = R$$

$$\Rightarrow V = I * R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

$$\text{Power, } P = V * I = I^2 R = \frac{V^2}{R}$$

- **Illustration:**

Let the potential difference between points A and B be V volts and current flowing be I Amp. Then, $\frac{V}{I} = \text{Constant}$,

$$\frac{V}{I} = R \text{ (say)}$$

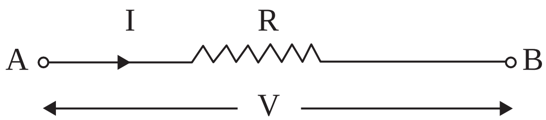


Figure 1.7

We know that, if the voltage is doubled (2V), the current flowing will also be doubled (2I). So, the ratio $\frac{V}{I}$ remains the same (ie, R). Also when voltage is measured in volts, current in ampere, then resistance will be in ohms.

- **Graphical representation of Ohm's law**

[Slope line of the graph represents the resistance]

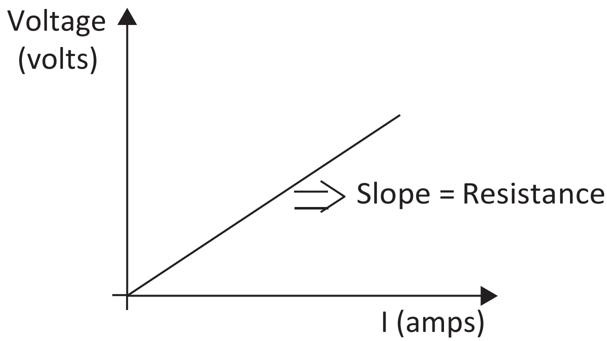


Figure 1.8

- **Limitations in ohm's law**:

(i) Ohm's law does not apply to all non-metallic conductors. For eg. Silico Carbide.
(ii) It also does not apply to non-linear devices such as Zener diode, etc.
(iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

- **Problems based on ohm's law**:

Problem 1.6. An electric heater draws 8A from 250V supply. What is the power rating? Also find the resistance of the heater element.

Given data:

Current, $I = 8A$
Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8 * 250 = 2000\,Watt$

Resistance (R) = $\frac{V}{I} = \frac{250}{8} = 31.25\ \Omega$

Problem 1.7 What will be the current drawn by a lamp rated at 250V, 40W, connected to a 230 V supply.
Given Data:

Rated Power = 40 W

Rated Voltage = 250 V

Supply Voltage = 230 V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\ \Omega$$

$$\text{Current, } I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472\ A$$

Problem 1.8 A Battery has an emf of 12.8 volts and supplies a current of 3.24 A. What is the resistance of the circuit? How many Coulombs leave the battery in 5 minutes?

Solution:

$$\text{Circuit Resistance, R} = \frac{\text{V}}{\text{I}} = \frac{12.8}{3.24} = 4\ \Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.24×5×60 = 960 Coulomb

Problem 1.9 If a resistor is to dissipate energy at the rate of 250W, find the resistance for a terminal voltage of 100V.

Given data:

Power = 250W

Voltage = 100V

Solution:

$$\text{Resistance, } R = \frac{V^2}{\rho} = \frac{100^2}{250} = 40\ \Omega$$

$$R = 40\ \Omega\ .$$

Problem 1.10 A voltmeter has a resistance of, 20,200 Ω. When connected in series with an external resistance across a 230 V supply, the instrument reads 160 V. What is the value of external resistance?

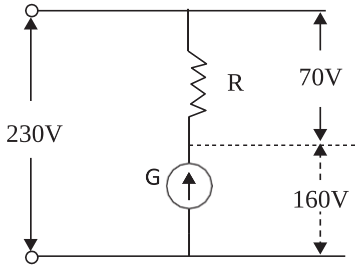


Figure 1.9

The voltage drop across external resistance, R

$$V_R = 230 - 160 = 70V$$

$$\text{Circuit current, } I = \frac{160}{20,000} = \frac{1}{125} A$$

We know that, $V = IR$

$$70 = IR$$

$$70 = \frac{1}{125} \times R$$

$$R = 8750\ \Omega$$

COMBINATION OF RESISTORS

- **Introduction:**

The closed path followed by direct Current (DC) is called a DC Circuit A d.c circuit essentially consist of a source of DC power (eg. Battery, DC generator, etc.) the conductors used to carry current and the load. The load for a DC circuit is usually a resistance. In a DC circuit, loads (i.e, resistances) may be connected in series, parallel, series – parallel. Hence the resistor has to be connected in the desired way for getting the desired resistance.

Resistances in series (or) series combination

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends. [A and B]

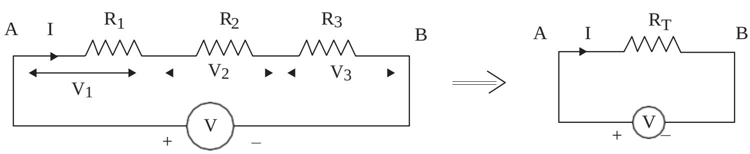


Figure 1.10

In the above circuit, there is only one closed path, so only one current flows through all the elements. In other words, if the Current is same through all the resistors, the combination is called series combination.

- **To find equivalent Resistance**:

Let, V = Applied voltage
I = Source current = Current through each element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By Ohms law, $V_1 = IR_1$

$V_2 = IR_2$ and $V_3 = IR_3$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I\left(R_1 + R_2 + R_3\right)$

$$V = I\left(R_1 + R_2 + R_3\right)$$

$$V = IR_T$$

$$\frac{V}{I} = R_T$$

The ratio of $\left(V/I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\dfrac{1}{G_T} = \dfrac{1}{G_1} + \dfrac{1}{G_2} + \dfrac{1}{G_3}$ (In terms of conductance)

$\therefore$ Equivalent resistance (R_T) is the sum of all individual resistances.

- **Concepts of series circuit**:

i. The current is same through all elements.
ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.
iii. The equivalent resistance (R_T) is greater than the greatest individual resistance of that combination.
iv. Voltage drops are additive.
v. Powers are additive.
vi. The applied voltage is equals to the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 interms of V and R_1, R_2, R_3:

Equivalent Resistance, $R_T = R_1 + R_2 + R_3$

By ohm's low, $I = \dfrac{V}{R_T} = \dfrac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

$\therefore$ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms in series, then the total Resistance is given by,

$$R_T = n * R$$

- **Applications:**
  - When variable voltage is given to the load, a variable resistance (Rheostat) is connected in series with the load. Example: Fan regulator is connected in series with the fan.
  - The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
  - When a load of low voltage is to be operated on a high voltage supply, a fixed value of resistance is connected in series with the load.
- **Disadvantage of Series Circuit:**
  - If a break occurs at any point in the circuit, no current will flow and the entire circuit becomes useless.
  - If 5 numbers of lamps, each rated 230 volts are to be connected in series circuit, then the supply voltage should be 5 x 230 = 1150 volts. But voltage available for lighting circuit in each and every house is only 230 V. Hence, series circuit is not practicable for lighting circuits.
  - Since electrical devices have different current ratings, they cannot be connected in series for efficient operation.
- **Problems based on series combination:**

Problem 1.11 Three resistors 30 Ω, 25 Ω, 45 Ω are connected in series across 200V. Calculate (i) Total resistance (ii) Current (iii) Potential difference across each element.

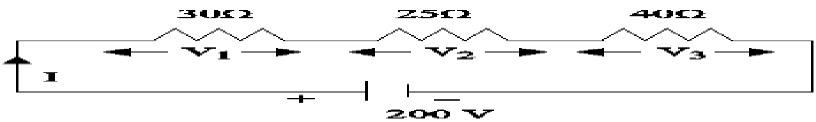


Figure 1.11

(i) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3$$
$$R_T = 30 + 25 + 45 = 100\ \Omega$$

(ii) Current, $I = \dfrac{V}{R_T} = \dfrac{200}{100} = 2\ A$

(iii) Potential difference across each element,

$$V_1 = IR_1 = 2 * 30 = 60\ V$$
$$V_2 = IR_2 = 2 * 25 = 50\ V$$
$$V_3 = IR_3 = 2 * 45 = 90\ V$$

Problem 1.12 Find the value of 'R' in the circuit diagram, given below.

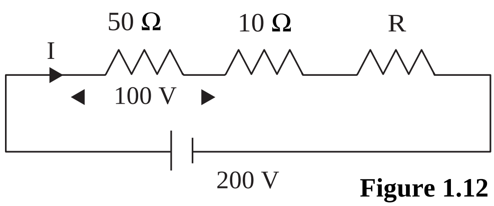


Figure 1.12

We know that, $V_1 = IR_1$

$$I = V_1 / R_1 = 100/50 = 2\ A$$

Similarly, $V_2 = IR_2 = 2 * 10 = 20\ V$

Total voltage drop, $V = V_1 + V_2 + V_3$

$$V_3 = V - (V_1 + V_2) = 200 - (100 + 20)$$
$$V_3 = 80\ V$$
$$V_3 = IR_3,\ R_3 = V_3 / I = 80/2 = 40\ \Omega$$
$$\therefore R_3 = 40\ \Omega$$

Problem 1.13 A 100W, 200V bulb is put in series with a 60W bulb across a supply. What will be the current drawn? What will be the voltage across the 60W bulb? What will be the supply voltage?

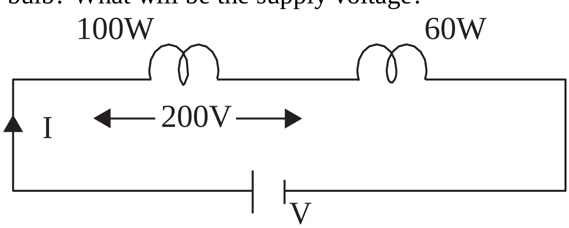


Figure 1.13

Power dissipated in the first bulb, $P_1 = V_1 I$
Current, I = P_1 / V_1 = 100/200 = 0.5 A
Power dissipated in the second bulb, $P_2 = V_2 I$
Voltage across the 60 W bulb,

$$V_2 = \frac{P_2}{I} = \frac{60}{0.5} = 120V$$

The supply voltage, $V = V_1 + V_2 = 200 + 120$

$$V = 320V$$

The supply voltage, V = 320 V.

Problem 1.14 An incandescent lamp is rated for 110V, 100W. Using suitable resistor how can you operate this lamp on 220V mains.

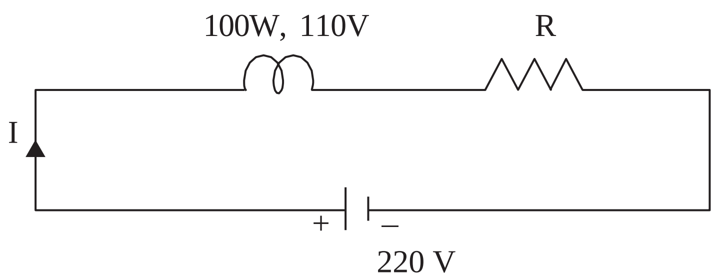


Figure 1.14

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110}$ = 0.909A, I = 0.909A

For satisfactory operation of the lamp, Current of 0.909A should flow. When the voltage across the lamp is 110V, then the remaining voltage must be across R

$$\text{Supply voltage} = V = 220 \text{ Volts}$$
$$\text{Voltage across } R = V - 110 \text{ Volts}$$
$$\text{ie, } V_R = 220 - 110 = 110V$$
$$\text{By ohm's law, } V_R = IR$$
$$110 = 0.909 \text{ R}$$
$$R = 121\ \Omega$$

Problem 1.15 The lamps in a set of decoration lights are connected in series. If there are 20 lamps and each lamp has resistance of 25Ω, calculate the total resistance of the set of lamp and hence calculate the current taken from a supply of 230 volts.

Given Data: Supply voltage, $V = 230\ volts$

Resistance of each lamp, $R = 25\ \Omega$

No of lamps in series, $n = 20$

Solution: Total Resistance, $R_T = n * R = 20 * 25$

$$R_T = 500\ \Omega$$

Current from supply. $I = \dfrac{V}{R_T} = \dfrac{230}{500} = 0.46\ A$

Problem 1.16 The field coil of a d.c generator has a resistance of 250Ω and is supplied from a 220 V source. If the current in the field coil is to be limited to 0.44 A. Calculate the resistance to be connected in series with the coil.

Given Data: Source voltage, $V = 220$ volts, $I = 0.44\ A$

Field coil resistance, $R_f = 250\ \Omega$

Solution: Let the resistance in series with R_f be R in Ohms.

Total resistance, $R_T = R_f + R = 250 + R$

Current, $I = 0.44\ A$

By ohm's law, $R_T = \dfrac{V}{I} = \dfrac{220}{0.44} = 500\ \Omega$

$$250 + R = 500\ \Omega$$

$$R = 500 - 250 = 250\ \Omega$$

$$R = 250\ \Omega$$

Resistance in Parallel (or) Parallel Combination

If one end of all the resistors are joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and is given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B, as shown in the Figure 1.15(a)..

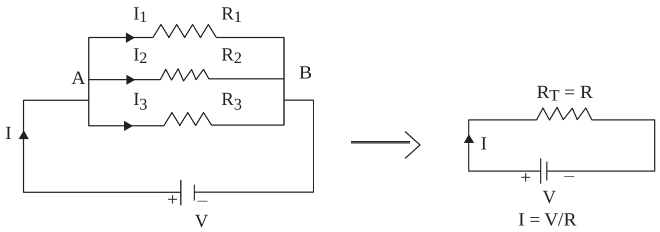


Figure 1.15

$$I = \frac{V}{R} \qquad (1)$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \qquad (2)$$

Total current is the sum of three individual currents,

$$I_T = I = I_1 + I_2 + I_3 \qquad (3)$$

Substituting the above expression for the current in equation (3),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Referring to Figure (1.15(b)), $R_T = R$

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \qquad (4)$$

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie, Power dissipated by R = Power dissipated by R_1 + Power dissipated by R_2 + Power dissipated by R_3

We know that reciprocal of Resistance is called as conductance.

Conductance = 1 / Resistance

[G = 1/R]

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

- **Concepts of Parallel Circuit:**
  - Voltage is same across all the elements.
  - All elements will have individual currents, depends upon the resistance of element.
  - The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
  - If n resistance each of R are connected in parallel then,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots n\,terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

(or)

$$R_T = \frac{R}{n}$$

  - Powers are additive.
  - Conductance are additive.
  - Branch currents are additive.

- **Current Division Technique:**

Case (i) When two resistances are in parallel:

Two resistance R_1 and R_2 ohms are connected in parallel across a battery of V (volts) Current through R_2 is I_2 and through R_2 is I_2 The total current is I.

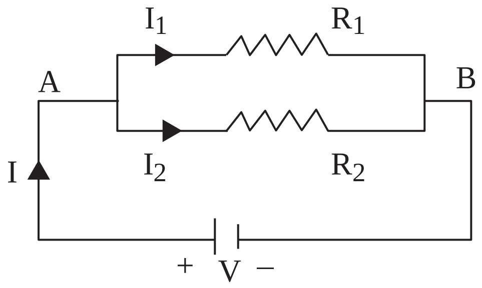


Figure 1.16

To express I_1 and I_2 interms of I, R_1 and R_2 (or) to find branch currents I_1, I_2:

$$I_2 R_2 = I_1 R_1$$

$$I_2 = \frac{I_1 R_1}{R_2} \qquad (1)$$

Also, the total current, $I = I_1 + I_2$ (2)

Substituting (1) in (2), $I_1 + \frac{I_1 R_1}{R_2} = I$

$$\frac{I_1 R_2 + I_1 R_1}{R_2} = I$$

$$I_1 (R_1 + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \frac{IR_1}{(R_1 + R_2)}$

To find the equivalent Resistance, (R_T) :

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, the total value of two resistances connected parallel is equal to their product divided by their sum i.e.,

$$\text{Equivalent Resistance} = \frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel. Let I be the supply current (or) total current. I_1, I_2, and I_3 are the currents through the resistors R_1, R_2 and R_3.

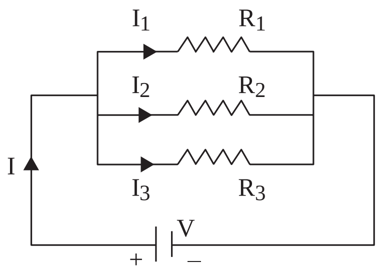


Figure 1.17

To find the equivalent Resistance (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3}$$

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

To find the branch currents I_1, I_2 and I_3:

We know that, $I_1 + I_2 + I_3 = I$ (1)

Also, $I_3R_3 = I_1R_1 = I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (1)

$$I_2 = \frac{I_1R_1}{R_2}\,;\; I_3 = \frac{I_1R_1}{R_3}$$

$$I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3} = I$$

$$I_1\left(1 + \frac{R}{R_2} + \frac{R_1}{R_3}\right) = I$$

$$\frac{I_1(R_2R_3 + R_3R_1 + R_1R_2)}{R_2R_3} = I$$

$$I_1 = \frac{I(R_2R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

$$I_3 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

- **Advantages of parallel circuits:**
  - The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other's performance.
  - If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

- **Applications of parallel circuits:**
  - ∗ All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switches.
  - ∗ Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided, inversely proportional to resistance.
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal to sum of individual resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	Reciprocal of the equivalent resistance is equal to sum of reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest resistances in the circuit.
There is only one path for the flow of current.	There are more than one path for the flow of current.

- **Problems based on parallel combinations:**

Problem 1.17 What is the value of the unknown resistor R shown in Figure 1.18. If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

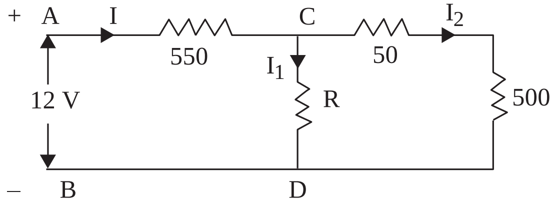


Figure 1.18

Given Data:

$$V_{500} = 2.5V$$

$$I_2 = \frac{V_{500}}{R} = \frac{2.5}{500} = 0.005A$$

V_{50} = Voltage across 50 Ω

$V_{50} = I_2 R = 0.005*50 = 0.25$ V

$V_{CD} = V_{50} + V_{500} = 0.25 + 2.5 = 2.75$ V

V_{550} = Drop across 550Ω = 12 – 2.75 = 9.25 V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 232.69\ \Omega$$

$$R = 232.69\ \Omega$$

Problem 1.18 Three resistors 2 Ω, 3 Ω and 4 Ω are in parallel. How will be a total current of 8A is divided.

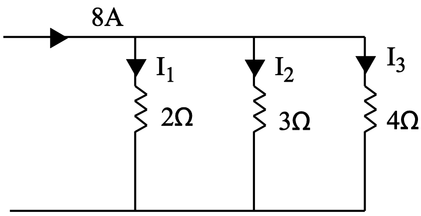


Figure 1.19

This given circuit can be reduced as, 3 Ω and 4 Ω are connected in parallel.

Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\Omega$

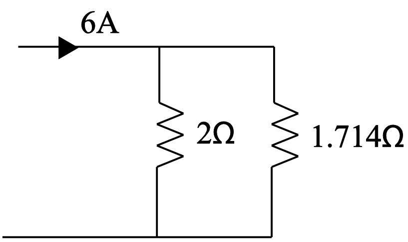


Figure 1.20

1.714 Ω and 2 Ω are connected in parallel, its equivalent resistance is 0.923 Ω

$$\frac{1.714 * 2}{2 + 1.714} = 0.923$$

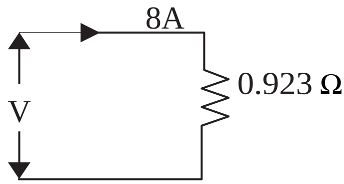


Figure 1.21

$$V = IR = 8 * 0.923$$
$$V = 7.385V$$

Branch currents, $I_1 = \dfrac{V}{R_1} = \dfrac{7.385}{2} = 3.69$ A

$$I_2 = \frac{V}{R_2} = \frac{7.385}{3} = 2.46\,A$$
$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Problem 1.19 What resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω

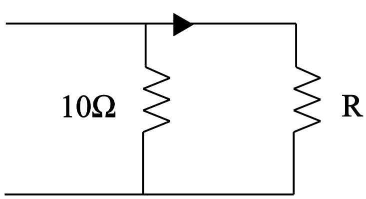


Figure 1.22

R is connected in parallel with 10 Ω Resistor to given an equivalent resistance of 6 Ω.

$$\frac{10 * R}{10 + R} = 6$$
$$10R = (10 + R)6$$
$$10R = 60 + 6R$$
$$10R - 6R = 60$$
$$R = \frac{60}{4} = 15\ \Omega$$
$$R = 15\ \Omega$$

Problem 1.20 Two resistors R_1 and R_2 are connected in Parallel and a Voltage of 200V DC is applied to the terminals. The total current drawn is 20A, R_1=30 Ω. Find R_2 and power dissipated in each resistor, for the figure 1.23.

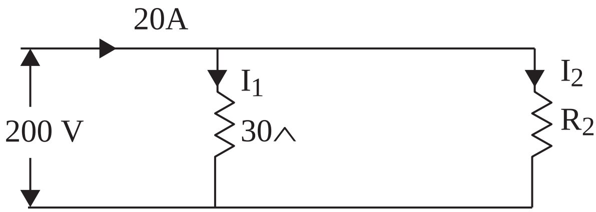


Figure 1.23

Given Data:

V = 200V, I = 20A, R_1 = 30 Ω

Solution: $I_1 = \dfrac{V}{R_1} = \dfrac{200}{30} = 6.667\ A$

$$I_1 + I_2 = I$$

$$I_2 = I - I_1$$

$$= 20 - 6.667 = 13.33 \text{ A}$$

$$I_2 = \frac{IR_1}{R_1 + R_2}$$

$$13.33 = \frac{20 * 30}{30 + R_2}$$

$$(30 + R_2)13.33 = 600$$

$$13.33R_2 = 600 - 400$$

$$13.33R_2 = 200$$

$$R_2 = \frac{200}{13.33} = 15\ \Omega$$

$$R_2 = 15\ \Omega$$

Power dissipated in 30 Ω, $P_1 = VI_1$ = 200*6.667

P_1 = 1333 W

Power dissipated in 15 Ω, $P_2 = VI_2$

P_2 = 200*13.33 = 2667

P_2 = 2667 W

Problem 1.21 Calculate the current supplied by the battery in the given circuit as shown in the figure 1.24.

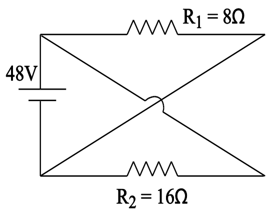


Figure 1.24

Solution: The above given circuit can be redrawn as,

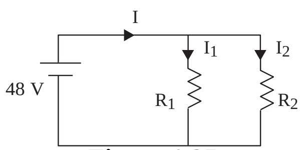


Figure 1.25

R_1 and R_2 are in parallel across the voltage of 48 volts.

Equivalent Resistance, $R_T = \dfrac{R_1 R_2}{R_1 + R_2} = \dfrac{8*16}{8+16} = \dfrac{16\ \Omega}{3}$

$$R_T = 5.33\wedge$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9\text{A}$$

Problem 1.22 Calculate the total resistance and battery current in the given circuit

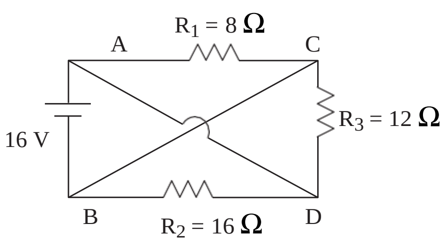


Figure 1.26

The given above circuit can be re-drawn as,

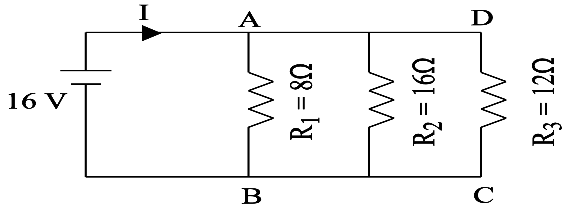


Figure 1.27

8 Ω, 16 Ω, 12 Ω are connected in parallel. Its equivalent resistance,

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

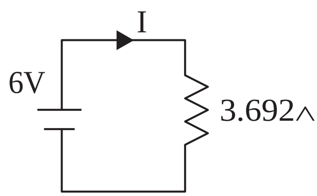


Figure 1.28

$$R_T = \frac{8*6*12}{128+192+96} = 3.692\ \Omega$$

$$R_T = 3.692\ \Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Problem 1.23 In the Circuit shown in the figure 1.29, calculate

(i) The current in all resistors.
(ii) The value of unknown resistance ‘x’
(iii) The equivalent resistance between A and B.

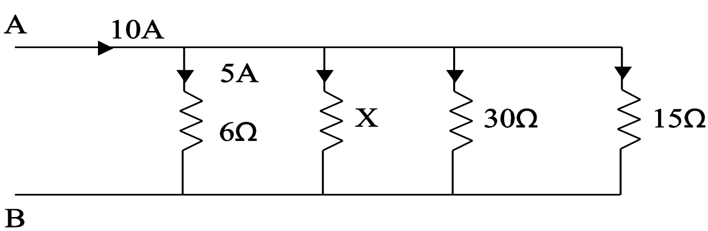


Figure 1.29

Solution: As all the resistors are in parallel, the voltage across each one is same. Give that current through 6 Ω , ie, $I_{6\,\Omega} = 5A$

Voltage across 6 Ω = 5×6 = 30volts.

Hence, current through 30 Ω, $I_{30} = \frac{V_{30}}{R} = \frac{30}{30} = 1A$

Similarly, current through 15 Ω, $I_{15} = \frac{V_{15}}{R} = \frac{30}{15} = 2A$

Total Current, $I = I_6 + I_x + I_{30} + I_{15}$

$$10 = 5 + I_x + 1 + 2$$

$$I_X = 2\ A$$

Hence, the current flowing through the 'X' Resistor is, $I_X = 2\ A$

Value of the Resistor 'X' is given by,

$$X = \frac{30}{I_x} = \frac{30}{2} = 15\ \Omega$$

Let, the equivalent resistance across AB = R_T

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{x} + \frac{1}{30} + \frac{1}{15}$$

$$\frac{1}{R_T} = \frac{5+2+1+2}{30} = \frac{1}{3}$$

$$R_T = 3\ \Omega$$

Series — Parallel Combination

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Figure 1.30. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with R_1.

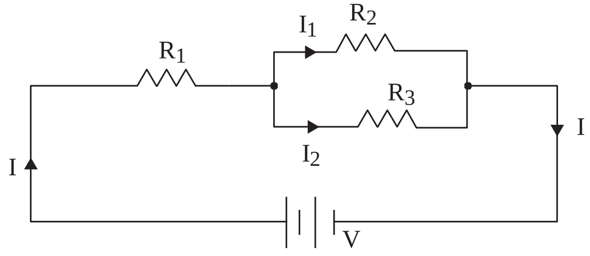


Figure 1.30

Equivalent Resistance: R_T for parallel combination.

$$R_P = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel combination = $I * \frac{R_2 R_3}{R_2 + R_3}$.

- **Problems based on Series – Parallel Combination:**

Problem 1.24 In the circuit, find the current in all the resistors. Also calculate the supply voltage.

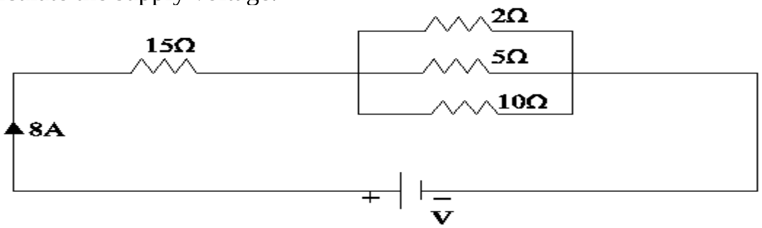


Figure 1.31

Voltage across 15∧, $V_{15} = I_{15} \times R = 8\times15 = 120V$

Resistors 2 Ω, 5 Ω, 10 Ω are connected in parallel, it equivalent resistance is given by,

$$R_P = \frac{2 * 5 * 10}{2\times5 + 5\times10 + 10\times2} = 1.25\ \Omega$$

Voltage across the parallel Combination is given by

$$V_p = V_2 = V_5 = V_{10} = I \times R_P = 8\times1.25 = 10V$$

Total supply Voltage, V = V_{15}+V_p

$$V = 120 + 10 = 130\text{V}$$

$$V = 130\text{V}$$

Hence, the Current through the parallel combination of the resistors are given by,

Current through 2 Ω resistor, $I_2 = \frac{V_2}{R} = \frac{10}{2} = 5A$

Current through 5 Ω Resistor, $I_5 = \frac{V_5}{R} = \frac{10}{5} = 2\,A$

Current through 10 Ω Resistor, $I_{10} = \frac{V_{10}}{R} = \frac{10}{10} = 1A$

The Current of 8A across the parallel combination is divided as 5A, 2A, and 1A.

Problem 1.25 Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current

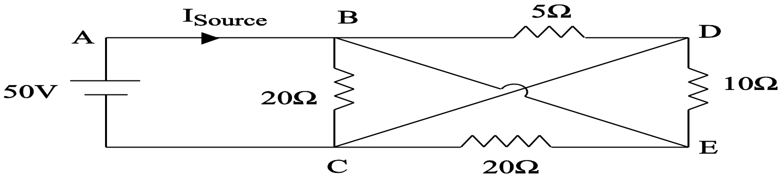


Figure 1.32

Solution: The given above circuit can be re-drawn as

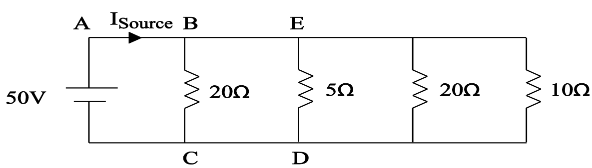


Figure 1.33

20 Ω and 10 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{20 * 10}{20 + 10} = 6.667\ \Omega$

The given circuit is reduced as,

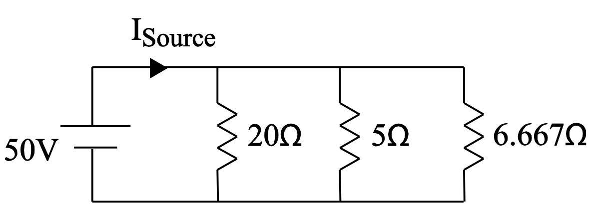


Figure 1.34

6.667 Ω and 5 Ω resistors are connected in parallel, its equivalent resistance is given by, $\dfrac{6.667 * 5}{6.667 + 5} = 2.857\ \Omega$

The circuit is reduced as,

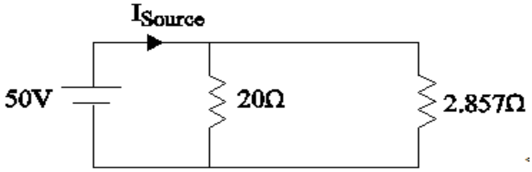


Figure 1.35

20 Ω and 2.857 Ω are connected in parallel. It equivalent resistance is,

$$\frac{20 * 2.857}{20 + 2.857} = 2.497\ \Omega$$

The Circuit is re-drawn as,

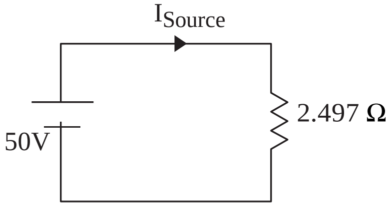


Figure 1.36

Hence the equivalent resistance of the Circuit is $R_T = 2.497\ \Omega = 2.5\ \Omega$

Source Current of the Circuit is given by,

$$I_{source} = \frac{V}{R} = \frac{50}{2.5} = 20A$$

Problem 1.26 Find the equivalent resistance between the terminals A and B.

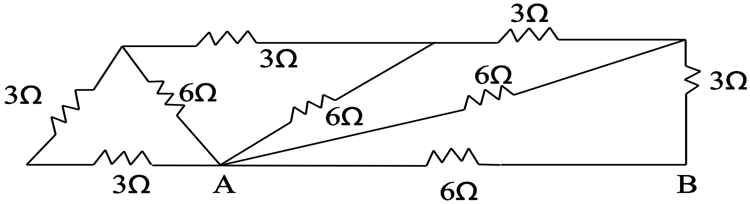


Figure 1.37

Solution:

3 Ω and 3 Ω are connected in Series, it equivalent resistance is, (3 + 3) = 6 Ω. The Circuit gets reduced as

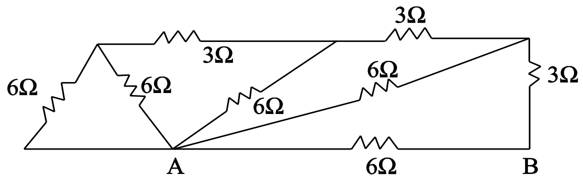


Figure 1.38

6 Ω and 6 Ω are connected in parallel. The circuit gets reduced as,

$$\frac{6*6}{6+6} = 3\text{ohms.}$$

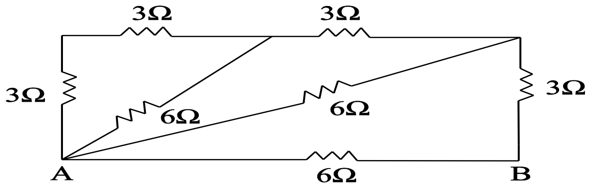


Figure 1.39

3 Ω and 3 Ω are connected in series (3 + 3 = 6 Ω).

The reduced Circuit is,

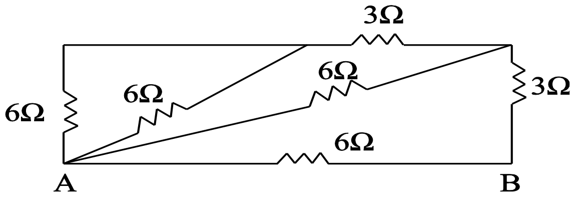


Figure 1.40

6 Ω and 6 Ω are connected in parallel. Its equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

The circuit can be reduced as,

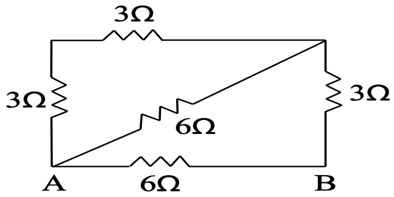


Figure 1.41

3 Ω and 3 Ω are connected in series. (3 + 3 = 6 Ω).

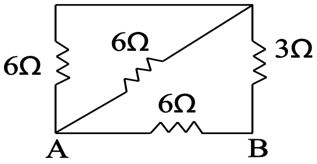


Figure 1.42

6 Ω and 6 Ω are connected in parallel. It equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

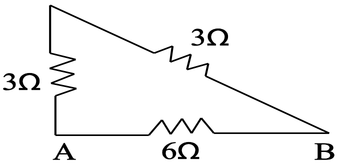


Figure 1.43

3 Ω and 3 Ω are connected in series, the reduced Circuit is 3 + 3 = 6 Ω

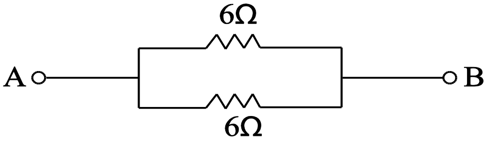


Figure 1.44

6 Ω and 6 Ω are connected in parallel.

$\frac{6*6}{6+6}$ = 3 Ω . The equivalent resistance between the terminals A and B given by R_{AB} = 3 Ω.

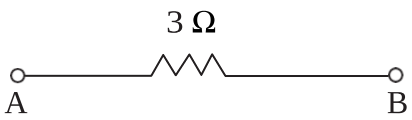


Figure 1.45

$$\therefore R_{AB} = 3\ \Omega$$

Problem 1.27 Determine the value of R if the power dissipated in 10 Ω resistor is 90 W.

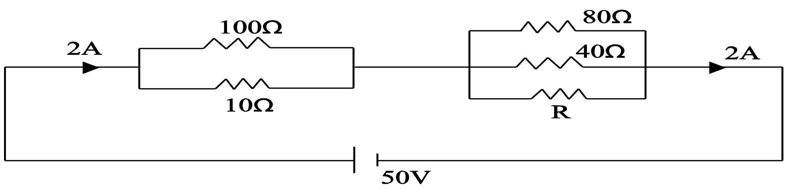


Figure 1.46

Solution:

100 Ω and 10 Ω are connected in parallel.

Its equivalent resistance is, $\frac{100*10}{100+10}$ = 9.09 Ω

The circuit is reduced as,

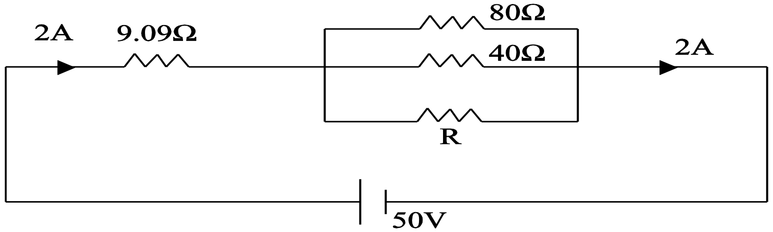


Figure 1.47

Current of 2A flows through the 9.09 Ω resistor. Voltage across 9.09 Ω is given by,

$$V_{9.09} = I_{9.09} \times R$$
$$V_{9.09} = 2 \times 9.09 = 18.18V$$

Similarly voltage across the unknown resistor V_R,

$$V_R = V - V_{9.09} = 50 - 18.18 = 31.818V$$

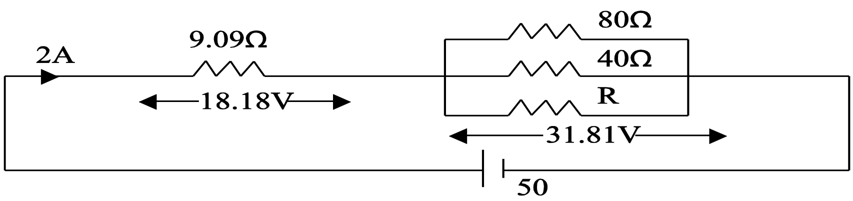


Figure 1.48

Hence the Current through 40 Ω, 80 Ω resistors can be found out with the voltage drop of 31.818V across it.

$$I_{80} = \frac{V_R}{80} = \frac{31.818}{80} = 0.397\,A$$
$$I_{40} = \frac{V_R}{40} = \frac{31.818}{40} = 0.7954\,A$$

Hence current through the unknown resistor R is I_R,

$$I_R = I - [I_{80} + I_{40}]$$
$$I_R = 2 - (0.397 + 0.7954) = 0.8075A$$

Hence, the value of the unknown Resistor R is given by

$$R = \frac{V_R}{I_R} = \frac{31.818}{0.8075} = 39.4\ \Omega$$

The value of the unknown resistor R is given by, R = 39.4 Ω.

Problem 1.28 Calculate the following for the circuits given,

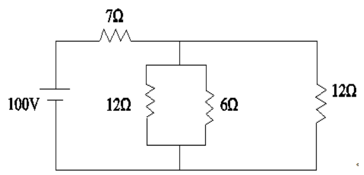


Figure 1.49

(i) Total resistance offered to the Source.
(ii) Total Current from the Source.
(iii) Power Supplied by the Source.

Solution: 12 Ω and 6 Ω are connected in Parallel.

Its equivalent resistance, $\frac{12*6}{12+6} = 4\ \Omega$. The reduced circuit is given as,

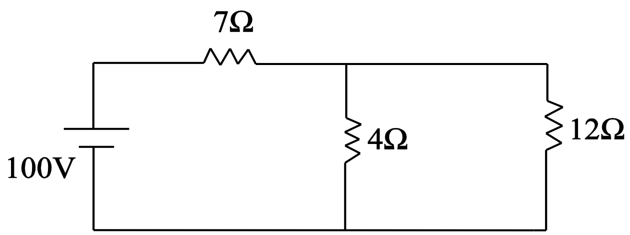


Figure 1.50

4 Ω and 12 Ω are connected in parallel. $\frac{4*12}{12+4} = 3\ \Omega$

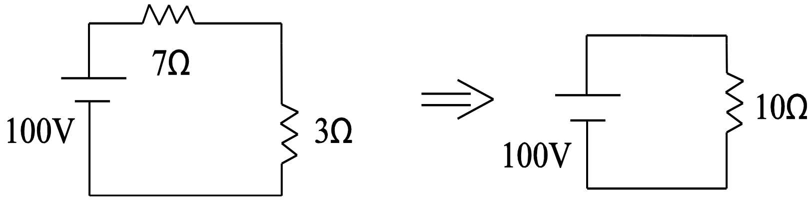


Figure 1.51

7 Ω and 3 Ω are connected in series, $7 + 3 = 10\Omega$

Total resistance offered to the Source, $R = 10\ \Omega$

Total Current from the Source, $I = \frac{100}{10} = 10$ A

$$I = 10A$$

Power supplied by the Source, $P = I^2R = 10^2 \times 10 = 1000\text{W}$

P = 1000W.

Problem 1.29 A letter A is Constructed of an uniform wire of 1 Ω resistance per cm. The signs of the letter are 60cm long and the cross piece is 30cm long, Apex angle 60°. Find the resistance of the letter between two ends of the legs.

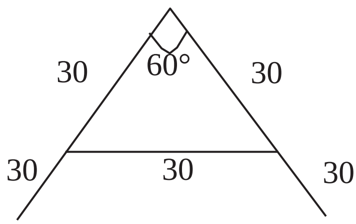


Figure 1.52

Solution:

The given circuit can be redrawn as,

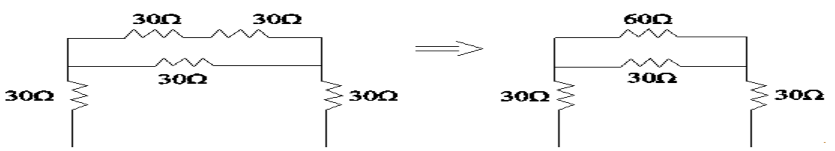


Figure 1.53

60 Ω and 30 Ω are connected in parallel

$$\frac{60 * 30}{60 + 30} = \text{2O ohms}$$

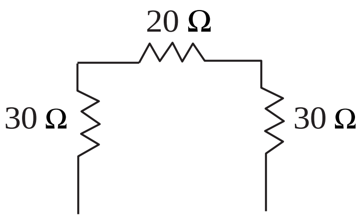


Figure 1.54

Equivalent Resistance = 80 Ω.

Problem 1.30 Find the current supplied by the battery.

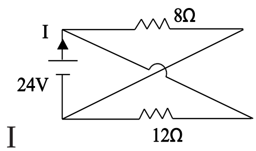


Figure 1.55

Solution:

The given circuit can be re-drawn as,

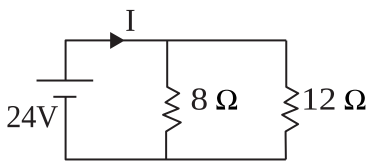


Figure 1.56

8 Ω and 12 Ω connected in parallel.

$$\frac{8*12}{8+12} = 4.8\ \Omega$$

Reduced circuit is,

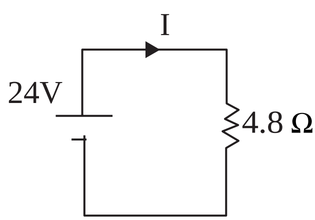


Figure 1.57

Current, $I = \frac{V}{R} = \frac{24}{4.8} = 5A$

$I = 5A$

Problem 1.31 Find the current supplied by the battery for the figure shown below.

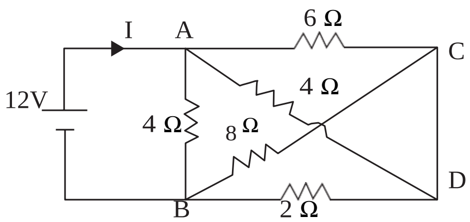


Figure 1.58

Solution:

The given above circuit can be redrawn as,

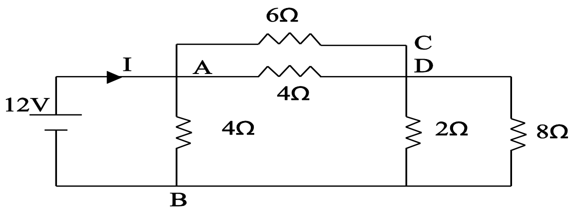


Figure 1.59

4 Ω and 6 Ω are connected in parallel. $\dfrac{6 * 4}{6+4} = 2.4\ \Omega$

Similarly, 2 Ω and 8 Ω are connected in parallel.

$$\frac{2*8}{8+2} = 1.6\ \Omega$$

The reduced circuit can be re drawn as,

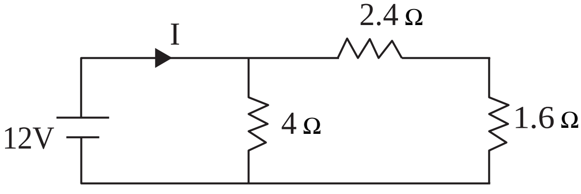


Figure 1.60

2.4 Ω and 1.6 Ω are connected in series. 2.4 +1.6 = 4 Ω

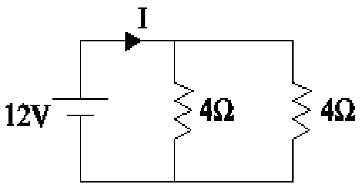


Figure 1.61

4 Ω and 4 Ω are connected in parallel $\frac{4*4}{4+4} = 2\ \Omega$

The reduced circuit is,

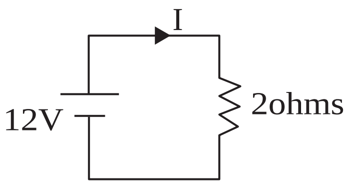


Figure 1.62

$$I = \frac{V}{R} = \frac{12}{2} = 6A$$

Current I, supplied by the battery = 6A.

Problem 1.32 Two Resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100v supply. The voltage drop across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50,000 Ω. Find the sum of the Reading.

Solution:

Case (i) A voltmeter is connected across 2500 Ω.

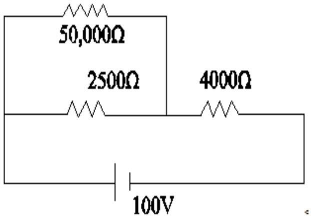


Figure 1.63

2500 Ω and 50,000 Ω are connected is parallel.

$$\frac{2500 * 50000}{2500 + 50000} = 2381 \text{ohms}$$

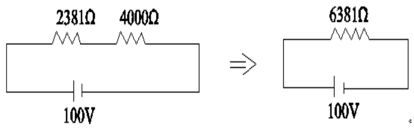


Figure 1.64

2381 Ω and 4000 Ω are connected in series.

$$2381 + 4000 = 6381\ \Omega$$

Current $I = \frac{V}{R} = \frac{100}{6381} = 0.01567\text{A}$

Voltage drop across, the Resister R_1 is measured by connecting a voltmeter having resistance of 50,000 across R_1. Hence V_A be voltage drop across R_1

$$V_A = IR = 0.01567 * 2381$$
$$V_A = 37.31\text{V}$$

Case (ii) Voltmeter is connected across 4000 Ω.

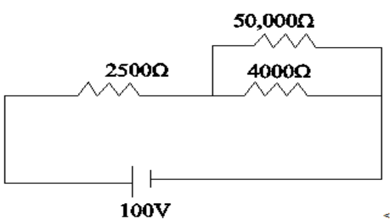


Figure 1.65

4000 Ω and 50,000 Ω are connected in parallel.

$$\frac{4000 * 50000}{4000 + 50000} = 3703.7 \text{ohms}$$

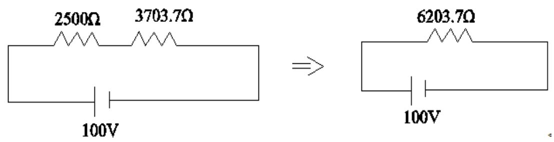


Figure 1.66

Current, $I = \dfrac{V}{R} = \dfrac{100}{6203.7} = 0.0161$ A

Voltage drop across the resistor R_2 is measured by connecting a voltmeter having resistance of 50000 across R_2.Hence, V_B be the voltage drop across R_2.

$$V_B = IR = 0.6161 * 3703.7$$
$$V_B = 59.7\text{V}$$

The total voltage drop = $V_A + V_B$

$$V = 37.31 + 59.7$$
$$V = 97 \text{ V}$$

Problem 1.33 Find the value of 'R' and the total current when the total power dissipated in the network is 16W as shown in the figure.

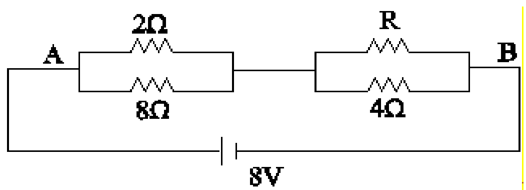


Figure 1.67

Solution:

Total Power (P) = 16w

Total Current, $I = \dfrac{P}{V} = \dfrac{16}{8} = 2$A

Total Resistance, $(R_{AB}) = \dfrac{P}{I^2} = \dfrac{16}{4} = 4\ \Omega$

Total Resistance between A and B is given by,

$$R_{AB} = \frac{2*8}{2+8} + \frac{4*R}{4+R}$$

$$4 = 1.6 + \frac{4R}{4+R}$$

$$4(4+R) = 1.6(4+R) + 4R \quad R = 6\ \Omega.$$

KIRCHOFF'S LAWS

Kirchhoff's current law

The kirchoff's current law states that the algebraic sum of currents in a node is zero.

It can also be stated that "sum of incoming currents is equal to sum of outgoing currents."

Kirchhoff's current law is applied at nodes of the circuit. A node is defined as two or more electrical elements joined together. The electrical elements may be resistors, inductors capacitors, voltage sources, current sources etc.

Consider a electrical network as shown below.

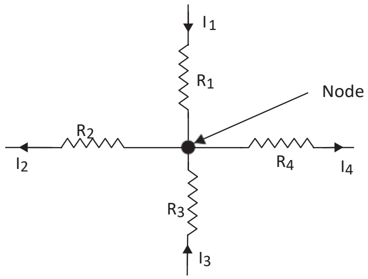


Figure 1.68

Four resistors are joined together to form a node. Each resistor carries different currents and they are indicated in the diagram.

$I_1 \rightarrow$ Flows towards the node and it is considered as positive current. $(+\ I_1)$

$I_2 \rightarrow$ Flows away from the node and it is considered as negative current. $(-\ I_2)$

$I_3 \rightarrow$ Flows towards the node and it is considered as positive current. $(+I_3)$

$I_4 \rightarrow$ Flows away from the node and hence it is considered as negative current $(-I_4)$

Applying KCL at the node, by diffinition-1 algebraic sum of currents in a node is zero.

$$+I_1 - I_2 + I_3 - I_4 = 0 \tag{1}$$

taking the I_2 & I_4 to other side

$$I_1 + I_3 = I_2 + I_4 \tag{2}$$

From equation (2) we get the definition – 2. Where I_1 & I_3 are positive currents (Flowing towards the node) I_2 & I_4 are negative currents. (Flowing away from the node).

Kirchoff's voltage Law: (KVL)

Kirchhoff's voltage law states that "sum of the voltages in a closed path (loop) is zero".

In electric circuit there will be closed path called as loops will be present.

The KVL is applied to the closed path only the loop will consists of voltage sources, resistors, inductors etc.

In the loop there will be voltage rise and voltage drop. This voltage rise and voltage drop depends on the direction traced in the loop. So it is important to understand the sign convention and the direction in which KVL is applied (Clock wise Anti clock wise).

- **Sign Conventions**

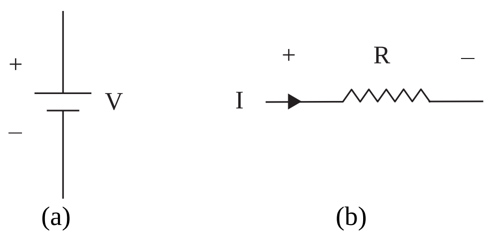
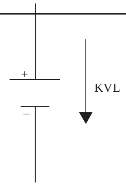


Figure 1.69

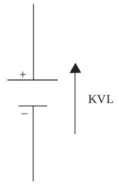
Consider a battery source V as shown in the figure 1.69(a). Here positive of the battery is marked with + sign and negative of the battery is marked with – sign.

When we move from + sign to – sign, it is called voltage drop.

When we move from – sign to + sign, it is called as voltage rise.

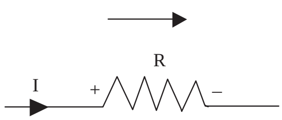


When KVL is applied in Anti clockwise direction as shown above it is called as voltage drop. A voltage drop is indicated in a loop with "—" sign (–V)

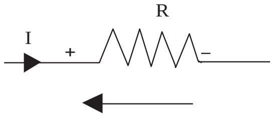


For the same battery source if the KVL is applied in clock wise direction we move from – sign to + sign. Hence it is called as Voltage Rise. A Voltage rise indicated in the loop with + sign. (+V).

Similarly in the resistor the current entry point is marked as positive (+ sign) and current leaving point is marked as negative sign. (– sign).



For the resistor shown in the diagram above, if KVL is applied in clock wise direction then it is called as voltage drop. Voltage drop in KVL equation must be indicated with negative sign (–). $\therefore$ –IR.



For the resistor shown in the diagram above, if KVL is applied in anti clockwise direction then it is called as voltage rise. A voltage rise is indicated in the KVL equation as positive. i.e. + IR.

In short the above explanation is summarized below in a Table.

S.No.	Element	KVL in clockwise	KVL in anticlockwise
1.	+ –	+ – Drop –V	+ – Rise +V
2.	I + R –	I + R – Drop –IR	I + R – Drop +IR

- **Procedure for KVL:**
  - * Identify the loops and Name them.
  - * Mark the branch currents and name them.
  - * Apply the sign convention.
  - * Select a loop & apply KVL either in clockwise or Anticlockwise and frame the equation.
  - * Solve all the equations of the loop.

- **Problems based on Kirchhoff's laws**

Problem 1.34 For the given circuit find the branch currents and voltages by applying KVL.

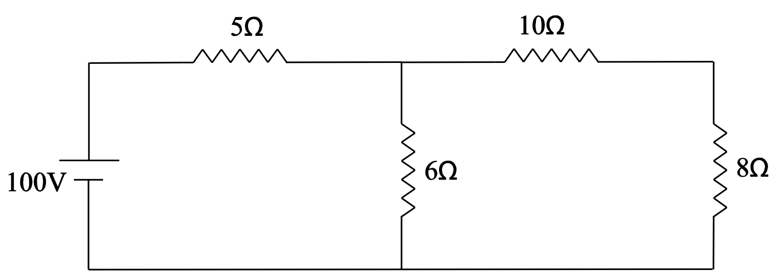


Figure 1.70

Solution:

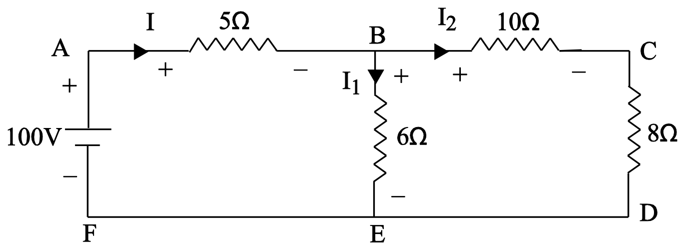


Figure 1.71

Consider loop ABEF & Apply KVL in CLK wise direction

$$100 - 5I - 6I_1 = 0$$

But $I = I_1 + I_2$

$$100 - 5\left(I_1 + I_2\right) - 6I_1 = 0$$
$$100 - 5I_1 - 5I_2 - 6I_1 = 0$$
$$-11I_1 - 5I_2 + 100 = 0$$
$$11I_1 + 5I_2 = 100 \qquad (1)$$

Consider loop BCDEB & Apply KVL in CLK wise direction

$$-10I_2 - 8I_2 + 6I_1 = 0$$
$$-18I_2 + 6I_1 = 0$$
$$6I_1 = 18I_2$$
$$I_1 = 3I_2 \qquad (2)$$

Sub I_1 in equ (1)

$$11\left(3\,I_2\right) + 5\,I_2 = 100$$
$$33\,I_2 + 5\,I_2 = 100$$
$$38\,I_2 = 100$$
$$I_2 = \frac{100}{38} = 2.63\ \text{Amps.}$$
$$I_2 = 2.63\ \text{Amps}$$

Sub I_2 in equ (2)

$$I_1 = 3(2.63) = 7.89\text{Amps}$$
$$I_1 = 7.89\text{Amps}$$
$$I = I_1 + I_2 = 10.52$$
$$I = 10.52\text{Amps.}$$

Voltage Across 5∧ = $5 \times I = 5 \times 10.52$

$= 52.6$ *volts*

Voltage Across 6∧ = $6 \times I_1 = 6 \times 7.89$

$= 47.34$ *volts*

Voltage Across 10∧ = $10 \times I_2 = 10 \times 2.63$

$= 26.3$ *volts*

Voltage Across 8∧ = $8 \times I_2 = 8 \times 2.63$

$= 21.04$ *volts*

(Or)

The above problem can be solved by applying KVL in Anti clock wise directions.

Consider loop ABEF & Apply KVL in anti clock wise direction

$$6I_1 + 5I - 100 = 0$$

But $I = I_1 + I_2$

$$6I_1 + 5(I_1 + I_2) - 100 = 0$$
$$6I_1 + 5I_1 + 5I_2 = 100$$
$$11I_1 + 5I_2 = 100 \qquad (3)$$

Consider loop BCDEB & Apply KVL in anti clockwise direction

$$8I_2 + 10I_2 - 6I_1 = 0$$
$$18I_2 = 6I_1$$
$$I_1 = 3I_2 \qquad (4)$$

equations (3) & (1) are identical

equations (2) & (4) are identical

Hence we get the same answer irrespective of directions of applying KVL.

Problem 1.35 Calculate the branch current in 15 Ω resistor by Applying kirchhoff's law

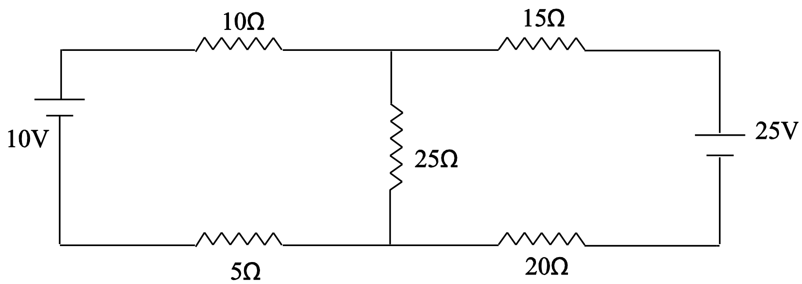


Figure 1.72

Figure 72 battery voltage value 25 volt missing

Solution:

Name the loop and Mark the current directions

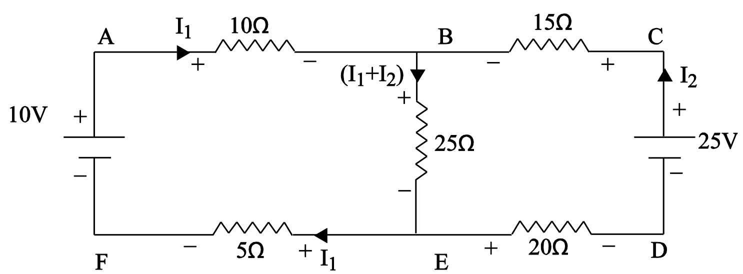


Figure 1.73

Consider the loop ABEFA & apply KVL in CLK wise

$$10 - 10I_1 - 25(I_1 + I_2) - 5I_1 = 0$$

$$10 - 10I_1 - 25I_1 - 25I_2 - 5I_1 = 0$$

$$-40I_1 - 25I_2 + 10 = 0$$

$$40I_1 + 25I_2 = 10 \qquad (1)$$

Consider the loop BCDEB and Apply KVL in CLK wise direction

Consider the loop BCDEB and Apply KVL in CLK wise direction

$$15I_2 - 25 + 20I_2 + 25\left(I_1 + I_2\right) = 0$$
$$15I_2 - 25 + 20I_2 + 25\left(I_1 + I_2\right) = 0$$
$$15I_2 - 25 + 20I_2 + 25I_1 + 25I_2 = 0$$

$$25I_1 + 60I_2 - 25 = 0$$

$$25I_1 + 60I_2 = 25 \text{ ……………..(2)}$$

Solve (1) & (2) & find I_2 alone

$(1) \times 25 \Rightarrow 1000\, I_1 + \cancel{625 I_2} = 25$

$(2) \times 40 \Rightarrow \cancel{1000}\, I_1 + 2400 I_2$

$(A) - (B) \Rightarrow -1775\, I_2 = -750$

$I_2 = 0.42$ Amps.

Current in 15 Ω resistor is 0.42Amps.

Problem 1.36 For the given network find the branch current in 8 Ω and voltage across the 3 Ω by applying KVL

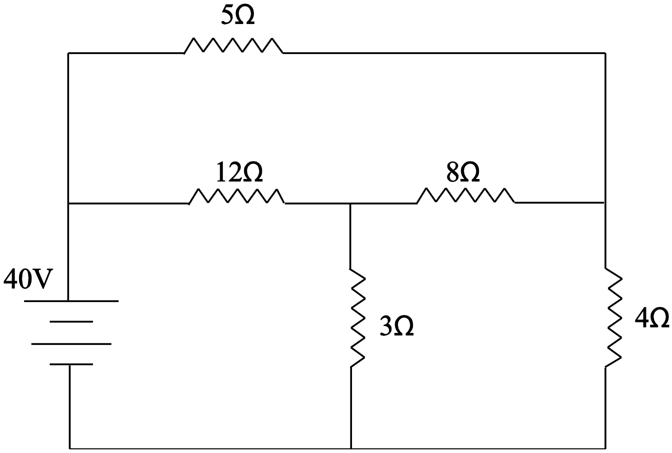


Figure 1.74

Solution:

Name the loop and mark the current directions and apply sign convention.

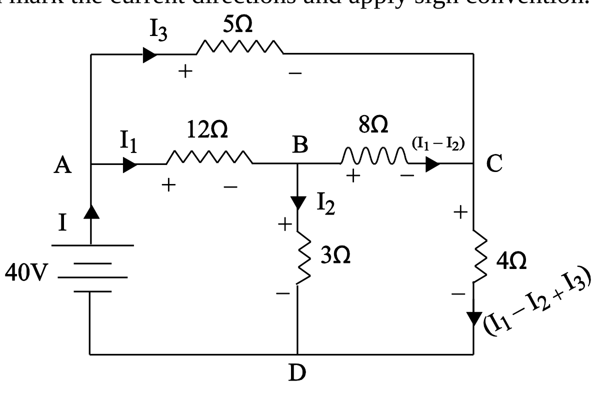


Figure 1.75

Consider loop ABDA and apply KVL $-8I_1 + 8$

$$-12I_1 - 3I_2 + 40 = 0$$

$$12I_1 + 3I_2 = 40$$

Consider loop BCDB and apply KVL

$$-8(I_1 - I_2) - 4(I_1 - I_2 + I_3) + 3I_2 = 0$$

$$I_2 - 4I_1 + 4\,I_2 - 4\,I_3 + 3\,I_2 = 0 \qquad (1)$$
$$-12I_1 + 15I_2 - 4I_3 = 0 \qquad (2).$$

Consider loop ABCA and apply KVL

$$-12I_1 - 8(I_1 - I_2) + 5I_3 = 0$$
$$-12I_1 - 8I_1 + 8I_2 + 5I_3 = 0$$
$$-20I_1 + 8I_2 + 5I_3 = 0 \qquad (3)$$

Solve equ (2) & (3) and cancel out I_3

$$(2) \times 5 \Rightarrow -60\,I_1 + 75\,I_2 - 20\,I_3 = 0$$
$$(3) \times 4 \Rightarrow -80\,I_1 + 32\,I_2 + 20\,I_3 = 0$$

Add the above two equations $\Rightarrow -140\,I_1 + 107I_2 = 0$ (4)

Solve equ (4) & (1) and find I_1 & I_2

$$12I_1 + 3I_2 = 40 \qquad (1)$$
$$-140I_1 + 107I_2 = 0 \qquad (4)$$

$$(1) \times 107 \Rightarrow 1284I_1 + 321I_2 = 4280$$
$$(4) \times 3 \Rightarrow -420I_1 + 321I_2 = 0$$

Subtract the above two $1704\,I_1 = 4280$

$$I_1 = 2.51\ Amps$$

Sub I_1 in (4)

$$-140 \times 2.51 + 107I_2 = 0$$
$$-351.4 + 107I_2 = 0$$
$$107I_2 = 351.4$$
$$I_2 = 3.28\ Amps$$

Current in 8 Ω resistor $= I_1 - I_2$

$= 2.51 - 3.28$

$= -0.77$ Amps.

Negative sign indicates that current flows in the opposite direction of our assumption.

Voltage in 3 Ω resistor = $3I_2$

$$= 3 \times 3.28 = 9.84 \text{ volts}$$

Note: Since there are 3 loops three unknown currents I_1, I_2 and I_3 should be named in the loop.

Problem 1.37 For the given network shown below find the branch currents by applying KVL and also find the voltage across 5 Ω resistor.

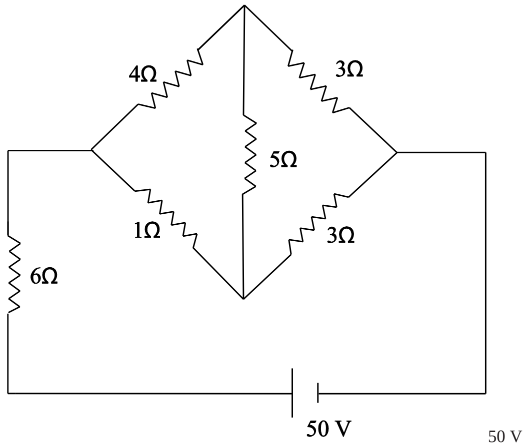


Figure 1.76

Solution:

Name the loop and assume the branch currents.

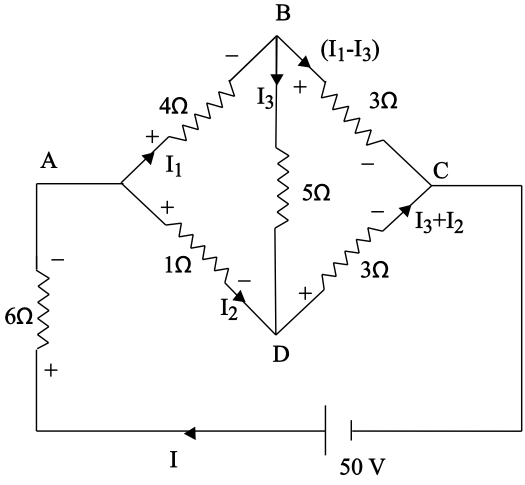


Figure 1.77

Consider the loop ABDA and apply KVL.

$$-4I_1 - 5\,I_3 + I_2 = 0$$

$$-4\,I_1 + I_2 - 5\,I_3 = 0 \qquad (1)$$

Consider the loop BCDB and apply KVL.

$$-3(I_1 - I_3) + 3(I_3 + I_2) + 5I_3 = 0$$

$$-3I_1 + 3I_3 + 3I_3 + 3I_2 + 5I_3 = 0$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (2)$$

Consider the loop ADCA and apply KVL.

$$-6(I_1 + I_2) - I_2 - 3(I_3 + I_2) + 50 = 0$$

$$-6I_1 - 6I_2 - I_2 - 3I_3 - 3I_2 = -50$$

$$-6I_1 - 10I_2 - 3I_3 = -50$$

$$6I_1 + 10I_2 + 3I_3 = 50 \qquad (3)$$

From eqn is (1) & (2) Cancel I_3

$$-4I_1 + I_2 - 5I_3 = 0 \qquad (4)$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (5)$$

(4) x 3 $\Rightarrow -12I_1 + 3I_2 - 15I_3 = 0$

(5) x 4 $\Rightarrow -12I_1 + 12I_2 - 44I_3 = 0$

By subtracting the above two equations $-9I_2 - 59I_3 = 0$

$$9I_2 = -\,59I_3$$

$$I_2 = -6.56I_3 \quad (6)$$
$$-3I_1 + 3I_2 + 11I_3 = 0 \quad (7)$$
$$6I_1 + 10I_2 + 3I_3 = 50 \quad (8)$$

(7) x 2 $\Rightarrow -6I_1 + 6I_2 + 22I_3 = 0$
(8) $\Rightarrow 6I_1 + 10I_2 + 3I_3 = 50$

By adding the above two equations $16I_2 + 28I_3 = 50$ (9)

Sub eqn (6) in (9)

$$16(-6.56\ I_3) + 25\ I_3 = 50$$
$$-104.96 + 25\ I_3 = 50$$
$$-79.96\ I_3 = 50$$
$$I_3 = -0.625 \text{ Amps} \quad (10)$$

Sub eqn (10) in (6)

$$I_2 = -6.56 \times (-0.625)$$
$$I_2 = 4.1 \text{ Amps} \quad (7)$$

Sub (10) & (11) in eqn (8)

$$6I_1 + 10I_2 + 3I_3 = 50$$
$$6I_1 + 10(4.1) + 3(-0.625) = 50$$
$$6I_1 + 41 - 1.875 = 50$$
$$6I_1 = 10.875$$
$$I_1 = 1.81 \text{ Amps}$$

Current in 6 Ω resistor = $(I_1 + I_2)$ = (1.81 + 4.1) = 5.91Amps
Current in 4 Ω resistor = I_1 = 1.81 Amps

Current in 5 Ω resistor = I_3 = –0.625 Amps
Current in 3 Ω resistor = $(I_1 - I_3)$ = 1.81 + 0.625 = 2.44 Amps
Current in 3 Ω resistor = $(I_3 + I_2)$ = 3.475 Amps
Current in 1 Ω resistor = I_2 = 4.1 Amps.

Voltage Across 5 Ω resistor= 5 × 0.625 = 3.13 volts.

Problem 1.38 For the Circuit shown below determine voltages (i) V_{df} and (ii)V_{ag}

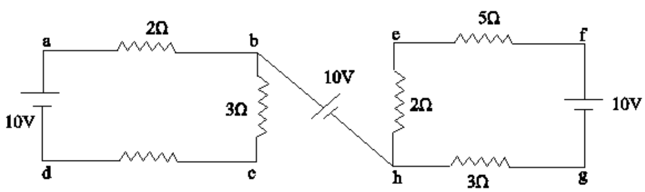


Figure 1.78

Solution:
Mark the current directions and mark the polarity

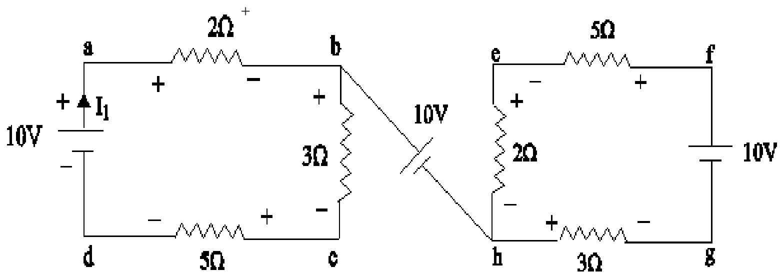


Figure 1.79

Apply KVL to loop abcda
$10 - 2I_1 - 3I_1 - 5I_1 = 0$
$-10I_1 = -10$
$I_1 = 1$ Amps

Apply KVL to loop efghe
$5I_2 - 10 + 3I_2 + 2I_2 = 0$
$10I_2 = 10$
$I_2 = 1$ Amps

To find V_{df}:
Trace the path V_{df}

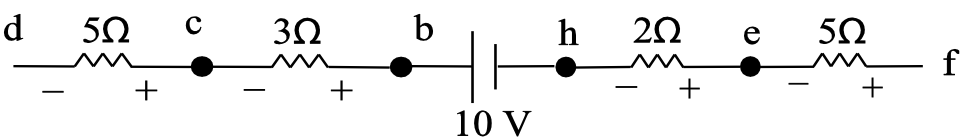


Figure 1.80

$V_{df} = -5(I_1 - 3I_1 + 10 + 2I_2 + 5I_2)$
$V_{df} = -5 - 3 + 10 + 2 + 5$
$V_{df} = 9$ Volts.
$V_{df} = -9$ Volts [$\because$ because - sign on d side + on f side]

To find V_{ag}:

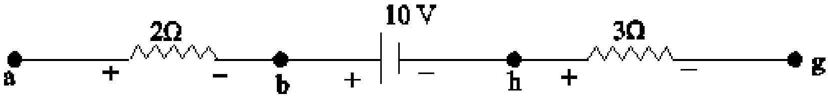


Figure 1.81

Apply KVL to the above Trace
$-2I_1 - 10 - 3I_2 = V_{ag}$
$V_{ag} = -2 - 10 - 3$
$V_{ag} = -15$

$V_{ag} = 15$ Volts. (With a side + w.r.t g)

Problem 1.39 Find the currents through R_2, R_3, R_4, R_5 and R_6 of the network.

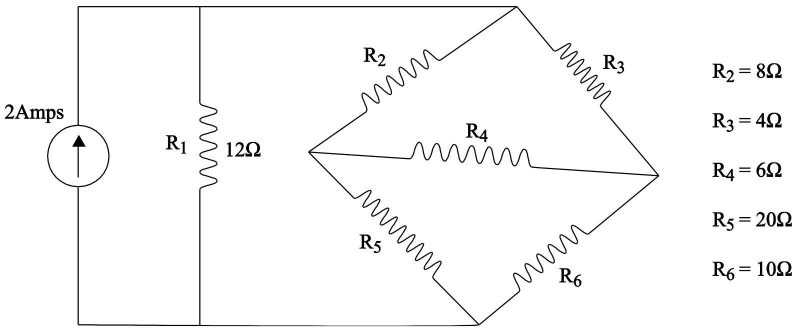


Figure 1.82

$R_2 = 8\,\Omega$
$R_3 = 4\,\Omega$
$R_4 = 6\,\Omega$
$R_5 = 20\,\Omega$
$R_6 = 10\,\Omega$

Solution:
Name the circuit and mark the current directions and polarity as shown below

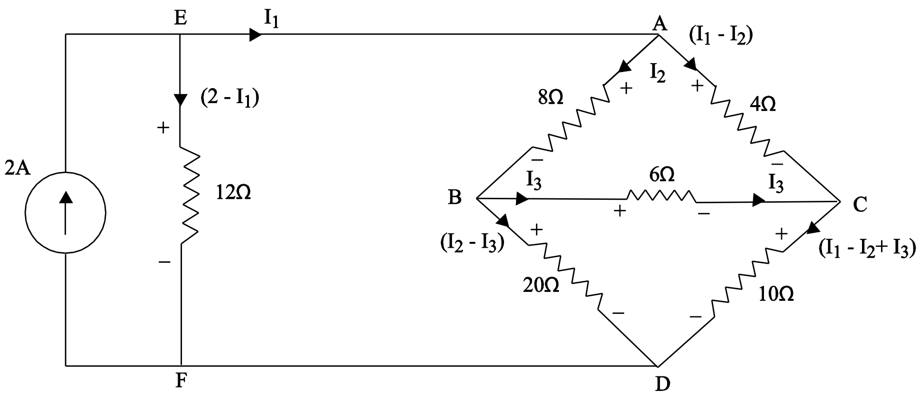


Figure 1.83

Apply KVL to the loop ACBA.

$$-4(I_1 - I_2) + 6I_3 + 8I_2 = 0.$$
$$-4I_1 + 4I_2 + 6I_3 + 8I_2 = 0$$
$$-4I_1 + 12I_2 + 6I_3 = 0 \qquad (1)$$

Apply KVL to the loop BCDB

$$-6I_3 - 10(I_1 - I_2 + I_3) + 20(I_2 - I_3) = 0$$
$$-6I_3 - 10I_1 + 10I_2 - 10I_3 + 20I_2 - 20I_3 = 0$$
$$-10I_1 + 30I_2 - 36I_3 = 0 \qquad (2)$$

Apply KVL to loop EABDFE

$$-8I_2 - 20(I_2 - I_3) + 12(2 - I_1) = 0$$
$$-8I_2 - 20I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-28I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-12I_1 - 28I_2 + 20I_3 = -24$$
$$12I_1 + 28I_2 - 20I_3 = 24 \qquad (3)$$

Solving equ. (1) (2) & (3). We get

$I_1 = 1.125$ Amps
$I_2 = 0.375$ Amps
$I_3 = 0$ Amps

$\therefore$ Current in $R_2 = 0.375$ Amps
$R_3 = 0.75$ Amps
$R_4 = 0$ Amps
$R_5 = 0.375$ Amps
$R_6 = 0.75$ Amps

NODAL ANALYSIS

- In nodal analysis, node equations relating node voltages are obtained for a multi node network.
- These node voltages are derived from kirchoff's current law (KCL)

- In this method the number of equations required to be solved is N-1, where N is the number of nodes.
- A node is a junction in a network where three or more branches meet. One of the nodes in a network is regarded as reference (datum) node and the potential of the other nodes are defined with reference to the datum node.

Case I.

Consider figure 1 Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node 'a' we get

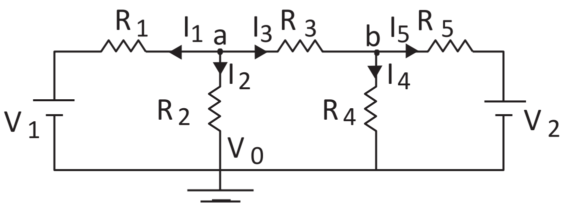


Figure 1.84

$$I_1 + I_2 + I_3 = 0 \qquad (1)$$

Where

$$I_1 = \frac{V_a - V_1}{R_1}; I_2 = \frac{V_a - V_0}{R_2}; I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ. (1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [$V_0 = 0$]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \quad \text{................ (2)}$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \quad \text{................ (3)}$$

$$I_4 = \frac{V_b - V_0}{R_4}; I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

$V_o = 0$ [reference node]

$$V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] - V_a\left[\frac{1}{R_3}\right] = \frac{V_2}{R_5} \text{ .... (4)}$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\begin{pmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{pmatrix}$$

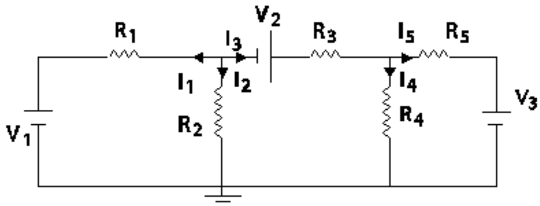
$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$$V_a = \frac{\Delta_1}{\Delta}; \qquad V_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

CASE II:



Consider fig 2

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = 0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1}-\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a+V_2-V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b-V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b}{R_5}-\frac{V_3}{R_5}$$

$$\frac{V_a}{R_3}-V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=-\frac{V_3}{R_5}-\frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3}+V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_3}{R_5}+\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1=\begin{pmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)$$

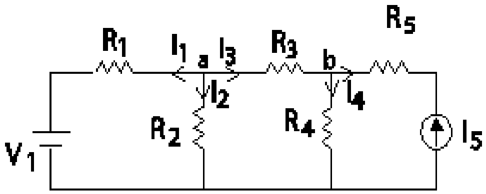
$$\Delta_2 = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

(case iii)



Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R_1} \quad \text{.................. (7)}$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3} + \frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \qquad \text{..................(8)}$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

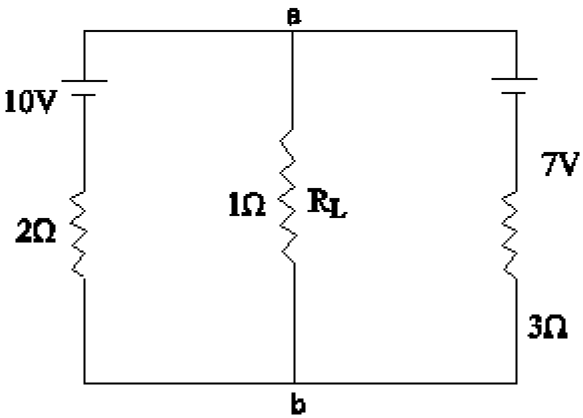
1) Two batteries having emf of 10V and 7V and internal resistances of 2Ω and 3Ω respectively, are connected in parallel across a load of resistance 1Ω . Calculate

(i) The individual battery currents

(ii) The current through the load

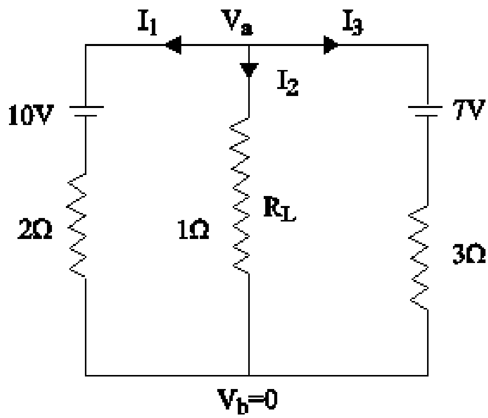
(iii) The Voltage across the load

Solution:



Step 1) Select the nodes and mark the nodes

Step 2) Select the datum or reference node.



<fig 84>

b is the ground node $V_b = 0$

Step 3: Mark the currents I_1, I_2 & I_3

Step 4: Write the node equations for node a and solve for V_a.

$$I_1 + I_2 + I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_1 = \frac{V_a - 10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$I_2 = \frac{V_a}{1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting (2), (3) & (4) in (1)

$$\frac{V_a - 10}{2} + V_a + \frac{V_a - 7}{3} = 0$$

$$V_a\left[\frac{1}{2} + 1 + \frac{1}{3}\right] = \frac{10}{2} + \frac{7}{3}$$

$$1.83\ V_a = 7.33$$

$$V_a = 4\ V$$

(i) Individual battery currents

$$I_1 = \frac{V_a - 10}{2} = \frac{4-10}{2}$$

$$= -3A$$

Ans: I_1=3 A

$$I_3 = \frac{V_a - 7}{3} = \frac{4-7}{3} = -1$$

Ans: I_3 = 1A

(ii) Current through the load

$$I_L = I_{1-2} = \frac{V_a}{1} = 4A$$

(iii)Voltage across the load

$$V_L = V_a - V_b$$
$$= 4 - 0$$
$$V_L = 4\ V$$

2) Write the node voltage equation and calculate the currents in each branch for the network.

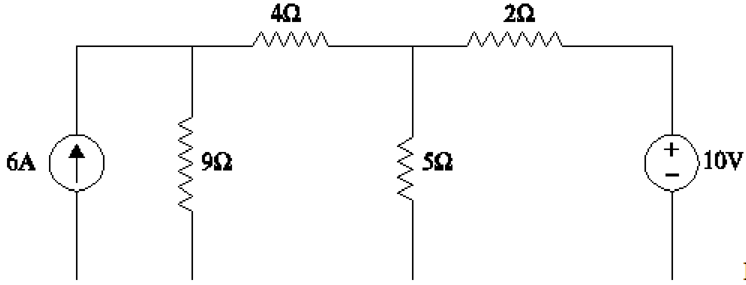


FIG85

Step 1: To assign voltages at each node

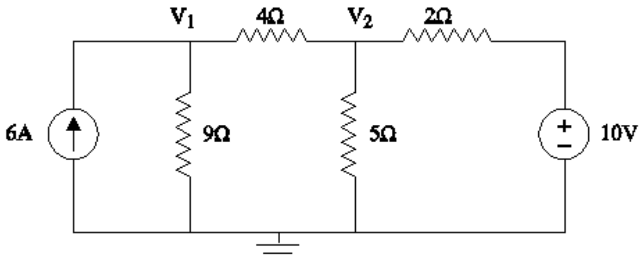


FIG86

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

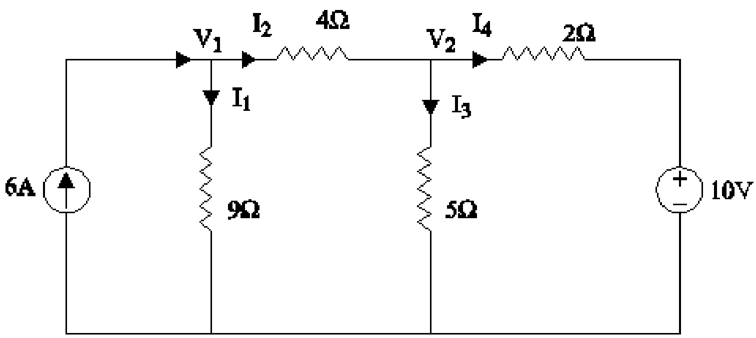


FIG87

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \dots\dots\dots\dots\dots\dots (1)$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule.

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{V_1}{V_2} \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \end{bmatrix}$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix} \begin{bmatrix} \frac{V_1}{V_2} \end{bmatrix} = \begin{bmatrix} \frac{6}{5} \end{bmatrix}$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86\upsilon$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ -.25 & 5 \end{bmatrix} = 3.3$$

$$V_{22}\frac{3.3}{.2795} = 11.8V$$

$$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$$

$$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$$

$$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$$

$$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$$

Hence currents in all the branches are found.

Problem 1.42 Use the Nodal Method to find V_{ba} and current through 30 Ω resistor in the circuit shown

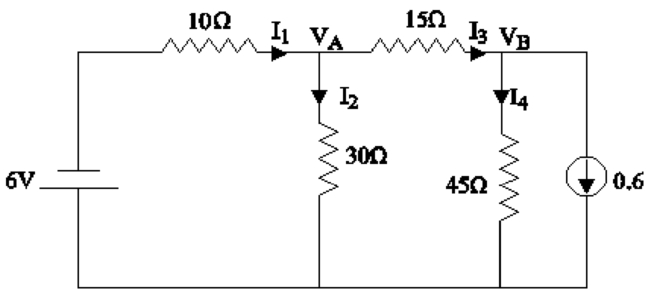


FIG88

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A\left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15}\right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} \quad +0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35\times10^{-3}\right]$$

$$\Delta = [0.01324]$$

$$\Delta = 0.01324$$

$$\Delta_1 = \begin{pmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15} + \frac{1}{45} \end{pmatrix} = -0.093$$

$$\Delta_1 = [-0.053 - 0.04] = \ -0.093$$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$$\Delta_2 = [-0.12 - 0.0396]$$

$$\Delta_2 = -0.1596$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$$V_{ba} = V_A - V_B = \ -7 + 12 \ = \ 5V$$

$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$$I_2 = -0.233A$$

Maxwell's Mesh method (Loop method).

This method was first proposed by Maxwell simplifies the solution of several networks. In this method, KVL is used. In any network, the number of independent loop equations will be

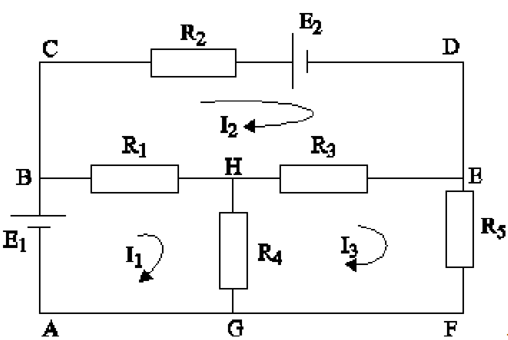
$$m = l - (j - 1)$$

Where l is the number of branches and j is the number of junctions.

Let us consider the circuit shown in fig(). for writing the mesh equations. It has

Number of junctions = 4 (B, H, E, G).

Number of branches = 6 (AB, BC, CD, DE, EF, HG).



<fig89>

In the above figure we shall name the three loop currents I_1, I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is $(I_1 - I_3)$ in a downward direction and R_1 is $(I_1 - I_2)$ from left to Right

Apply KVL for the first loop ABHGA,

$$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$$

$$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$$

$$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1 \dots\dots\dots\dots\dots\dots (1)$$

Apply KVL for the loop BEDC,

$$- R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$$

$$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$$

$$\therefore - R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2 \dots\dots\dots\dots\dots\dots (2)$$

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \dots\dots\dots\dots\dots\dots (5)$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop i.

$$R_{ij} = \begin{cases} -\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances}) \\ +\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances}) \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{bmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{bmatrix};$$

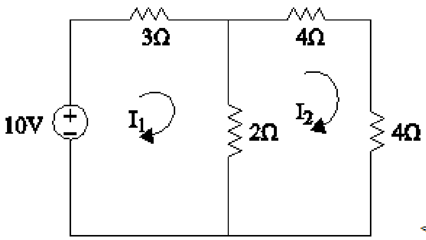
$$\Delta_3 = \begin{bmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{bmatrix}; \quad \Delta = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig () using Mesh current method

Solution:



<fig90>

Method 1:

Apply KVL for the first loop,

$$10 - 3I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

Apply KVL for the second loop,

$$-4I_2 - 4I_2 - 2(I_2 - I_1) = 0$$

$$-2I_1 + 10I_2 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Solve eqn (1) & (2), we get

$$\text{(1) X 5} \Rightarrow 25I_1 - 10I_2 = 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

$$\text{(2)} \quad \Rightarrow -2I_1 + 10I_2 = 0$$

$$\text{(3) + (2)} \Rightarrow \quad 23\, I_1 = 50$$

$$I_1 = \frac{50}{23} = 2.174 \text{ A}$$

Sub I_1 in (2)

$$I_2 = \frac{2 \times 2.174}{10} = 0.435 \text{ A}$$

$$I_3\Omega = 2.174 \text{ A}$$

$$I_2\Omega = I_1 - I_2 = 1.739 \text{ A}$$

$$I_4\Omega = 0.435 \text{ A}$$

Method 2:

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

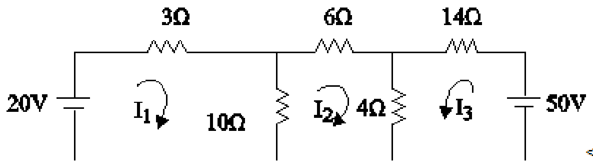
$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the loop currents for the network shown in figure below by using Loop Analysis.



<fig91>

Solution

For loop 1,

$$3I_1 + 10(I_1 - I_2) = 20$$

$13I_1 - 10I_2 = 20$ .... (1)

For loop 2,

$$10(I_2 - I_1) + 6I_2 + 4(I_2 + I_3) = 0$$

$$10I_2 - 10I_1 + 6I_2 + 4I_2 + 4I_3 = 0$$

$\div 2 \Rightarrow -5I_1 + 10I_2 + 2I_3 = 0$ .... (2)

For loop 3,

$$4(I_3 + I_2) + 14I_3 = 50$$

$$4I_2 + 18I_3 = 50$$

$\div 2 \Rightarrow 2I_2 + 9I_3 = 25$ .... (3)

$$\therefore \begin{bmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{vmatrix}$$

$$\Delta = 13(90-4) + 10(-45-0)$$

$$= 668$$

For loop 1,

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$$

$$18I_1 + 5I_2 - 3I_3 = 50 \; \ldots\ldots\ldots\ldots\ldots\ldots \; (1)$$

For loop 2,

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$$

$$5I_1 + 8I_2 + I_3 = 10 \; \ldots\ldots\ldots\ldots\ldots\ldots \; (2)$$

For loop 3,

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5$$

$$-3I_1 + I_2 + 4I_3 = -5 \; \ldots\ldots\ldots\ldots\ldots\ldots \; (3)$$

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$= 13\ (250)\ + 10\ (-125)\ + 20\ (-10)$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$= 18\ (32 - 1)\ - 5\ (20 + 3)\ - 3(5 + 24)$$
$$= 356$$

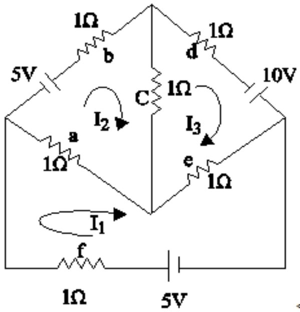
$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= 18\ (-40 - 10)\ - 5\ (-25 + 30)\ + 50(5 + 24)$$
$$= -900 - 25 + 1450$$
$$= 525$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

Solution

4) Determine the currents in various elements of the bridge circuit as shown below.



<fig92>

Solution

For loop 1,

$$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$$

$$3I_1 - I_2 - I_3 = 5 \text{ .... (1)}$$

For loop 2,

$$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$$

$$-I_1 + 3I_2 - I_3 = 5 \text{ .... (2)}$$

For loop 3,

$$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$$

$$-I_1 - I_2 + 3I_3 = 10 \text{ .... (3)}$$

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$= 3\,(9-1) + 1\,(-3-1)\ -1(1+3)$$

$$= 16$$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

$= 3\ (15 + 10) - 5\ (-3 - 1) - 1\ (-10 + 5)$

$= 100$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

$= 3\ (30 + 5) + 1\ (-10 + 5) + 5\ (1 + 3)$

$= 120.$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2 = 6.25 - 6.25 = 0$ A

$I_b = I_2 = 6.25$ A.

$I_c = I_2 - I_3 = 6.25 - 7.5 = -1.25$ A

$I_d = I_3 = 7.5$ A

$I_e = I_1 - I_3 = 6.25 - 7.5 = -1.25$ A.

$I_f = I_1 = 6.25$ A.

TEXT / REFERENCE BOOKS

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2. B.L.Theraja, Fundamentals of Electrical Engineering and Electronics, 1st edition, S.Chand & Co., 2009.

3. Smarajit Ghosh, Fundamentals of Electrical and Electronics Engineering, 2nd edition, PHI Learning Private Ltd, 2010.

UNIT II

AC CIRCUITS – SEEA1103

INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of any quantity is plotted against time.

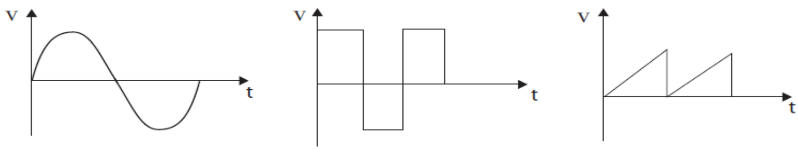

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurring interval.
3. Cycle

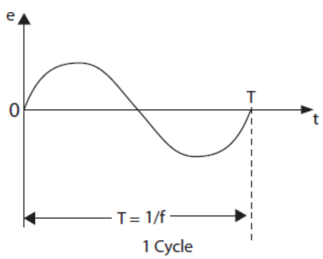

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period
   The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.
5. Frequency
   The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz

$$f = 1/\,T$$

6. Amplitude
   The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.
7. R.M.S value [Root Mean Square]
   The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$RMS\ Value = \sqrt{\frac{Area\ Under\ the\ square\ curve\ for}{one\ complete\ cycle\ /\ Period}}$$

8. Average Value of AC
   The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)
   It is the ratio of RMS value to average value

Form Factor = RMS value/Average Value

10. Peak Factor (Ka)
    It is the ratio of Peak (or) maximum value to RMS value.

Peak Factor Ka=Peak Value/RMS value

Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

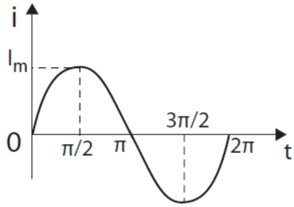

Figure 2.3

$$i = I_m \sin \omega t \quad ; \omega t=\theta$$

Mean square of AC $I^2_{RMS} = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta \quad \text{[since it is symmetrical]}$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\pi$$

$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= -\frac{I_m}{\pi}[\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$= \frac{I_m}{\pi}(-1-1)$$

$$= -\frac{2I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{\ } = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

Avg

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

Expression for RMS, Average, Form Factor, Peak factor for Half wave rectifier

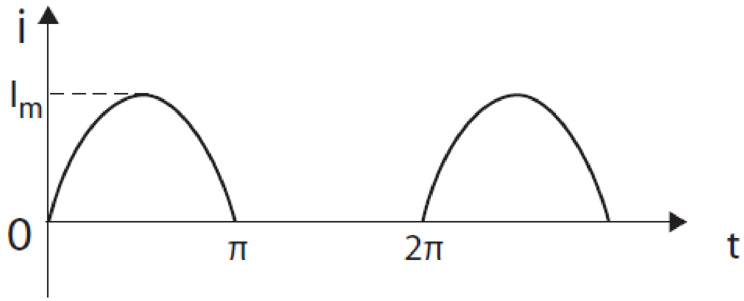

Figure 2.4

1) RMS value

$$i = I_m \sin\theta \; ; \; 0 < \theta < \pi$$

$$i = 0 \; ; \; \pi < \theta < 2\pi$$

Mean square of AC $I^2_{RMS} = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta\right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$= \frac{Im^2}{4\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\pi$$

$$I_{RMS} = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi}$$

$$= \frac{1}{\quad}\left[\int^{\pi} i d\theta + 0\right]$$

$2\pi \lfloor \; _0 \qquad \rfloor$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{2\pi}\left[\cos\theta\right]_0^{\pi}$$

$$= \frac{I_m}{2\pi}\left[\cos\pi - \cos 0\right]$$

$$= -\frac{I_m}{2\pi}(-1-1)$$

$$= \frac{2I_m}{2\pi} = \frac{I_m}{\pi}$$

$$Form\ Factor = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$Peak\ Factor = \frac{MAX}{RMS} = \frac{I_m}{RMS} / \frac{I_m}{\frac{I_m}{2}} = 2$$

Examples:

The equation of an alternating current is given by

$$i = 40\sin 314\, t$$

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

$$i = 40\sin 314\, t$$

We know that $i = I_m \sin \omega t$

S_o $I_m = 40$

$\omega = 314\ rad / \sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2\times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50Hz$

(v) $Form\ Factor\ \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(vi) $Peak\ \ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

what is the equation of a 50Hz voltage sin wave having an rms value of 50 volt

Solution:

f = 50Hz
V_{rms} = 50V
v = V_m sinωt
ω = 2πf = 2π×50=314 rad/sec
$V_m = V_{rms}\sqrt{2} = 50\times\sqrt{2} = 70.7$ volt

$\therefore v = 70.7\sin 314t$

PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

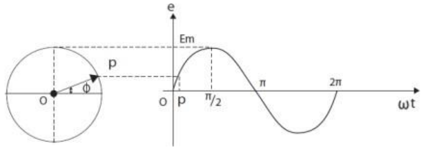

Figure 2.5

$OP = E_m$

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase

The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase

Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by the equation

$i_1 = I_{m1} \sin\theta$

$i_2 = I_{m2} \sin\theta$

can be represented graphically as shown in Fig 2.6(a).

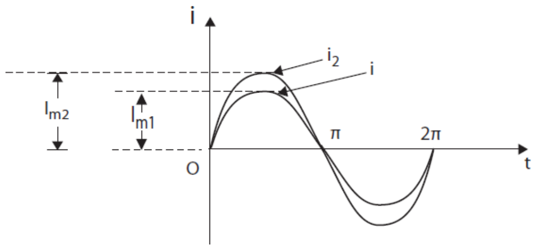

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

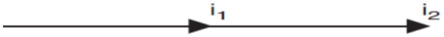

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag**' and '**lead**' and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = Im_1 \sin(\omega t - \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

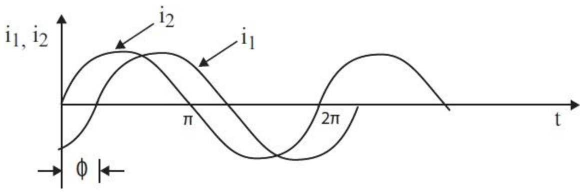

Figure 2.7a

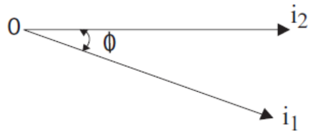

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of 'Φ' electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = Im_1 \sin(\omega t + \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

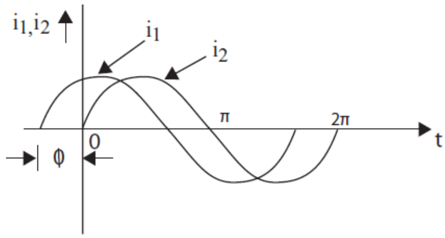

Figure 2.8(a)

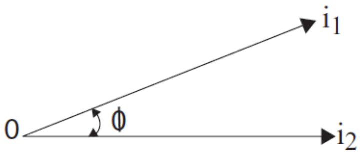

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference between them is 90°.
2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

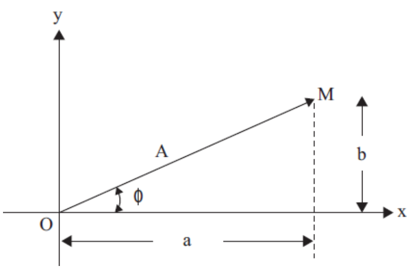

Figure 2.9

In Fig 2.9, OM represents vector A
Φ represents the phase angle of vector A

A = a + jb
a – Horizontal component or active component or in phase component
b – Vertical component or reactive component or quadrature component

The magnitude of vector 'A' = $\sqrt{a^2 + b^2}$
Phase angle of Vector 'A' = α = $\tan^{-1}$ (b/a)
Features of j – Operator

1. j = $\sqrt{-1}$
   It indicates anticlockwise rotation of Vector through 90°.
2. j^2 = j . j = -1
   It indicates anticlockwise rotation of vector through 180°.
3. j^3 = j . j . j = -j
   It indicates anticlockwise rotation of vector through 270°.
4. j^4 = j . j . j . j = 1
   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.
A = 2 + j3

This representation is known as rectangular form.
Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606
Phase angle of A = α = $\tan^{-1}$ (3/2) = 56°.31
A=|A| ∠α°
A=3.606∠56°.31

This representation is known as polar form.

Note:
1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

$\therefore$ Magnitude = | A + B | = $\sqrt{6^2+8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.
Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

$\therefore$ Magnitude = | A – B | = $\sqrt{-2^2+7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.
Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 $\angle$ 56°.310

B = 4 – j5

|B| = $\sqrt{4^2+-5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340

B= 6.403$\angle$ -51°.340

A X B = 3.606 $\angle$ 56°.310 X 6.403$\angle$ -51°.340

= 3.606 X 6.403$\angle$ (56°.310 + (-51°.340))

= 23.089 $\angle$4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

|A| = $\sqrt{4^2+-2^2}$ = 4.472

$\alpha = \tan^{-1}(-2/4) = -26°.565$

$A = 4.472 \angle -26°.565$

$B = 2 + j3$

$|B| = \sqrt{2^2 + 3^2} = 3.606$

$\alpha = \tan^{-1}(3/2) = 56°.310$

$B = 3.606\angle 56°.310$

$$\frac{A}{B} = \frac{4.472\angle -26^o.565}{3.606\angle 56^o.310} = \frac{4.472}{3.606}\angle -26^o.565 - 56^o.310 = 1.240\angle -82.875$$

ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

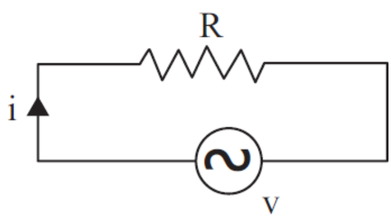

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

From Ohms law,

$$v = i R$$

$$I = \frac{v}{R} = \frac{V_m \sin\omega t}{R}$$

$$\because I_m = \frac{V_m}{R}$$

$$= I_m \sin\omega t$$

where,

$V_m \rightarrow$ Maximum value of voltage (V)

$I_m \rightarrow$ Maximum value of current (A)

$\omega \rightarrow$ Angular frequency (rad/sec)

$t \rightarrow$ Time period (sec)

Phasor Representation:

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

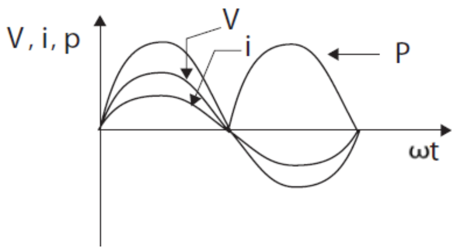

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:
In an AC circuit, impedance is the ratio of the maximum value of voltage to the maximum value of current.

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\frac{V_m}{R}} = R$$

$$\therefore Z = R$$

Power:

(i) Instantaneous power:
It is defined as the product of instantaneous voltage and instantaneous current.

$$p = v\, i$$

$= V_m \sin\omega t \, I_m \sin\omega t = V_m I_m \sin^2\omega t$

$[\because \omega t = \theta]$

$p = V_m I_m \sin^2\theta$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin^2\theta \, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta$$

$$\because \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{V_m I_m}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - 0 + \frac{\sin 0}{2}\right]$$

$$= \frac{V_m I_m}{2\pi}[\pi] = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = V_{RMS} I_{RMS} = V.I$$

$P = V\,I$

Power Factor:

It is defined as the cosine of the phase angle between voltage and current.

$$\cos\phi = \cos 0 = 1(unity)$$

Problems:

A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: v = 240 sin 377t

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin \omega t}{R}$$

$$= \frac{240}{6} \sin 377t$$

$$= 40\sin 377tA$$

I. Phase angle:

$\phi = 0$

II. Impedance:

$Z = R = 6\Omega$

III. Instantaneous power:

IV. $p = V_m I_m \sin^2 \omega t$

$= 240.40.\sin^2 377t$

$= 9600\sin^2 377t$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800\text{watts}$$

VI. Power factor:

$\cos\Phi = \cos 0 = 1$

A voltage e = 200sinωt when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: $e = 200\sin\omega t$

$V_m = 200$

$P = 100w$

Average power, $P = \frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1$ A

Also, $V_m = I_m.R$

$R = 200\Omega$

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t$ A

A voltage e = 250sinωt when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:

Given: $e = 250\sin\omega t$

$V_m = 250$

$P = 100W$

I. $P = \dfrac{V_m I_m}{2}$

$100 = \dfrac{250 I_m}{2}$

$I_m = 0.8 \text{ A}$

II. $I_m = \dfrac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8 \sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

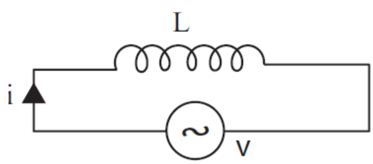

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

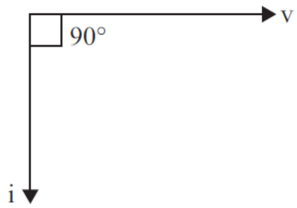

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

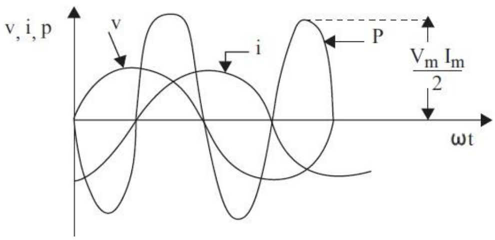

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i) Instantaneous power:

$$p = v\,i$$
$$= V_m \sin\omega t\; I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= V_m I_m \sin\omega t\,(-\cos\omega t)$$
$$= -V_m I_m \sin\omega t \cos\omega t = -V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta\, d\theta$$
$$= -\frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2}\, d\theta$$
$$\left[\because \sin 2\theta = 2\sin\theta\cos\theta\right]$$
$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}\left[\cos 2\pi - \cos 0\right]$$
$$= \frac{V_m I_m}{4\pi}\left[1-1\right] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor

(iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: L = 0.225 H

V_{RMS} = V = 120 V

f = 50Hz

I. Inductive reactance, XL = 2πfL =2π x 50 x 0.225 = 70.68Ω

II. Instantaneous current, i =-I_m cosωt

$\because I_m = \frac{V_m}{\omega L}$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$$V_m = V_{RMS}\sqrt{2} = 169.71\text{V}$$

$$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$$

Maximum power, $P_m = \frac{V_m I_m}{2}$ = 203.74 W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 344t volts

Instantaneous current, i = -2.4 cosωt A

A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: L = 0.01 H

I = 10cos1500t

I_m = 10A

ω = 1500 rad/sec

I. Inductive reactance, X_L = ωL = 1500 x 0.01 = 15Ω

II. The voltage across the inductor, e= $L\frac{di}{dt}$

$$= 0.01\frac{d(10\cos 1500t)}{dt} = 0.01 \times 10[-\sin 1500t.1500]$$

$$= -150 \sin 1500t \text{ V}$$

III. X_L = 40Ω; 2πfL = 40

$$f = \frac{40}{2\pi \times 0.01} = 637\text{Hz}$$

In the circuit, source voltage is v=200 sin $\left(314t + \frac{\pi}{6}\right)$ and the current is

$i = 20 \sin\left(314t - \frac{\pi}{3}\right)$ Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$

$I_m = 20A$

$\omega = 314$ rad/sec

I. $\omega = 2\pi f$

$f = 50Hz$

II. $V_m = 200V$ and $I_m = 20A$

III. $V_{RMS} = \frac{V_m}{\sqrt{2}} = 141.42V$

$I_{RMS} = \frac{I_m}{\sqrt{2}} = 14.142A$

IV. For a sinusoidal wave, Average value of current, $I_{av} = \frac{2I_m}{\pi} = 12.732A$

Average value of voltage, $V_{av} = \frac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

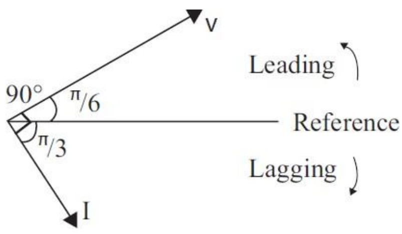

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle (90^0). So the circuit is purely inductive.

$I_m = \frac{V_m}{\omega L}$

$L = \frac{200}{314 \times 20} = 31.85mH$

Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

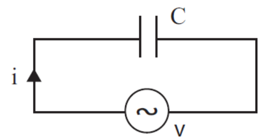

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, $Q = C.v$

$$= C.\, V_m \sin\omega t$$

$$\text{Current}, i = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}(CV_m \sin \omega t) = \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$[\because I_m = \omega C V_m]$$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

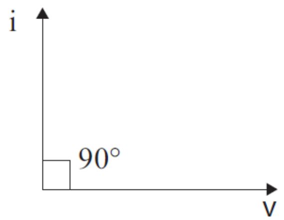

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

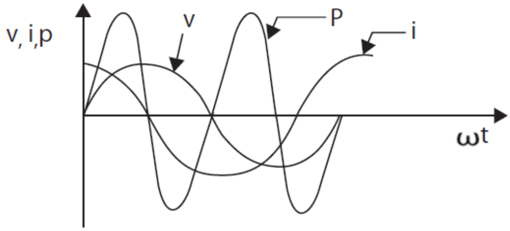

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$

$$= V_m \sin\omega t \; I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\,(\cos\omega t)$$

$$= V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta\, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta\cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:
In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

A 135µF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:
Given: V_{RMS}= V= 150V
C = 135µF
f = 50Hz

I. $X_C = \frac{1}{\omega C}$ =23.58Ω

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) \because I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150 X \sqrt{2} = 212.13V$
$I_m = 314 X 135 X 10^{-6} X 212.13 = 8.99A$
$i = 8.99 \sin\left(314t + \frac{\pi}{2}\right)$ A

III. p= $V_m I_m \sin\omega t (\cos \omega t)$=212.13X8.99 sin314t.cos314t
= 66642.6 sin 314t.cos314t = $66642.6 \frac{\sin 628t}{2}$

$[\because \sin 2\theta = 2\sin\theta\cos\theta]$
= 33321.3 sin628t W

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}}$ =6.36A

VI. $P_m = \dfrac{V_m I_m}{2} = 953.52$ W

A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: $V_{RMS} = V = 100V$

$C = 12\mu F$

$I = 0.5A$

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$V_m = 100 X \sqrt{2} = 141.42V$

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

$I_m = 0.5 X \sqrt{2} = 0.707A$

II. $I_m = \omega C V_m = 2\pi f C V_m$

$f = 66.3Hz$

RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

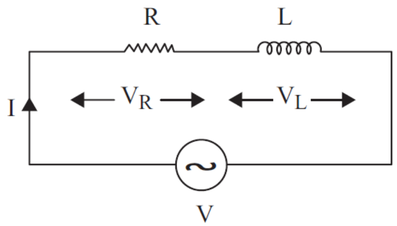

Figure 2.20

Let $V = V_m \sin \omega t$ be the applied voltage.

i = Circuit current at any constant.

I = Effective Value of Circuit Current.

V_R= Potential difference across inductor.

V_L= Potential difference across inductor.

F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

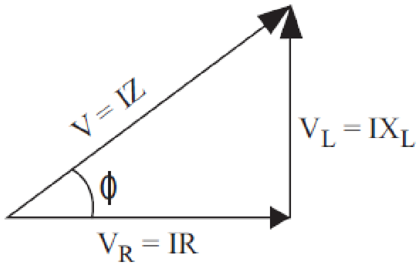

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$

$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

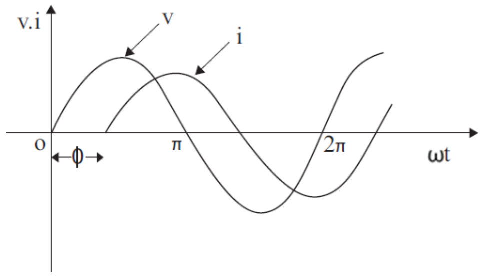

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual Power P = VI cos ϕ – Current component is phase with voltage

Reactive or Quadrature Power

Q = VI sinϕ – Current component is quadrature with voltage

Complex or Apparent Power

S = VI – Product of voltage and current

S = P + jQ

Problem

A series RL Circuit has

$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$ and $V(t) = 20\sin\left(314t + \frac{5\pi}{3}\right)$

Determine
(a) the impedence of the circuit
(b) the values of R_1L and power factor
(c) average power of the circuit

Solution:

$i(t) = 5\sin(314t + \frac{2\pi}{3})$

$V(t) = 20\sin(314t + \frac{5\pi}{3})$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2\times180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5\times180}{} = 150°$

3 3

Phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$$= 150 - 120$$

$$\theta = 30^0$$

Power factor $= \cos\theta$

$$= \cos 30$$

$$= 0.866 \text{ (lagging)}$$

Impedence of the circuit $Z = \frac{V_m}{I_m}$

$$= \frac{20}{5}$$

$$Z = 4\Omega$$

(i) But $\cos\phi = \frac{R}{Z}$

$$0.866 = \frac{R}{4}$$

$$\therefore R = 4 \times 0.866$$

$$R = 3.46\Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$X_L = \sqrt{Z^2 + R^2}$$

$$= \sqrt{(4)^2 - (3.46)^2}$$

$$X_L = 2\Omega$$

$$\omega L = 2\Omega$$

$$L = \frac{2}{\omega}$$

$$= \frac{2}{314}$$

$$L = 6.37 \times 10^{-3} \text{ H}$$

(ii) Average power = VI cos ϕ

$$= \frac{20}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} (0.866)$$

$$= 43.3 \text{ watts}$$

A coil having a resistance of 6Ω and an inductance of 0.03 H is connected across a 100V, 50Hz supply, Calculate.

(i) The current
(ii) The phase angle between the current and the voltage
(iii) Power factor

(iv) Power

Solution:

$R = 6\Omega$
$L = 0.03$ H
$X_L = 2\pi fL$
$X_L = 2\pi \times 50 \times 0.03$
$X_L = 9.42\Omega$

$$|Z| = \sqrt{(R)^2 + (X_L)^2}$$
$$= \sqrt{(6)^2 + (9.42)^2}$$
$$|Z| = 11.17\Omega$$

(i) $I = \frac{V}{Z} = \frac{100}{11.17} = 8.95$ amps

(ii) $\phi = tan^{-1}\left(\frac{X_L}{R}\right)$

$= tan^{-1}\left(\frac{9.42}{6}\right)$

$\Phi = 57.5$ (lagging)

(iii) Power factor = cos ϕ
= cos 57.5
= 0.537 (lagging)

(iv) Power = Average power
= VI cos Φ
= 100 ×8.95×0.537

Power = 480.6 Watts

A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

$R = 10\Omega$
$L = 20\ mH = 20 \times 10^{-3} H$
$X_L = 2\pi fL$
$= 2\pi \times 60 \times 20 \times 10^{-3}$
$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \frac{V}{Z} = \frac{250}{12.5} = 20\ amps$

$V_R = IR = 20 \times 10 = 200$ volts

(iii) $V_L = I X_L = 20 \times 7.54 = 150.8$ volts

(iv) Apparent power S = VI

$$= 250 \times 20$$

$$S = 5000VA$$

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \text{ (lagging)}$$

Active power = VI cos ф
=250×20×0.8
P = 4000 Watts

$$\sin\phi = \sqrt{1 - \cos^2 \Phi} = \sqrt{1 - (0.8)^2} = 0.6$$

Reactive Power Q = VI sin ф
= 250×20×0.6
Q= 3000 KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$
$Z_2 = 10 - j7$
V = 200 volts
$Z_{Total} = Z_1 + Z_2$
$= 5 + j7 + 10 - j7$
$Z_{Total} = 15 < 0^{\cdot}$
$\therefore \quad \phi = 0^{\cdot}$

Taking V as referenve,

V = 200 < 0°· Volts

(i) $I = \frac{V}{Z} = \frac{200\angle 0°}{15\angle 0°} = 13.33\angle 0°\ amps$

(ii) ф = 0
PF = cos ф = cos 0 = 1

(iii) Power = VI cos ф
= 200 ×13.33×1
Power = 2666 watts

RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.
I = Circuit current of any instant
I = Effective value of circuit current
V_R = Potential Difference across Resistor
V_c = Potential Difference across Capacitor
f = Frequency of applied voltage
The same Current I flows through R and C
Voltage across R = V_R = IR in phase with I
Voltage across C = $V_c = IX_c$ lagging I by 90^0
Applied voltage $V = IR - jIX_c$

$$= I(R - jx_c)$$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin (\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \dfrac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

$= VI$

Figure 3.27

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF
R = 120Ω
V = 100V
F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$$= 318\Omega$$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$$= 340\ \Omega$$

(a) $$|I| = \frac{|V|}{|Z|}$$

$$= \frac{100}{340}$$

$$= 0.294\ amps$$

(b) $$Phase Difference\ \phi = \tan^{-1}\left(\frac{X_c}{R}\right)$$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3^\circ\ (Leading)$$

$$\cos\phi = \cos(69.3)^\circ$$

$$= 0.353\ (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38\ Watts$$

The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$

$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$

$$= \sqrt{(240)^2 - (100)^2}$$

$$|V_C| = 218.2 \ volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$

$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3 amps$$

$$|X_C| = \frac{V_C}{|I|} \ (Apply \ ohm's \ law \ for \ C)$$

$$= \frac{218.2}{3} = 72.73\Omega$$

$$\frac{1}{2\pi fc} = 72.73$$

$$C = \frac{1}{2\pi \times 50 \times 72.73} = 43.77 \times 10^{-6} \ F$$

$$C = 43.77 \mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum V_c = $\sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$$|V| = 60 volts$$
$$F = 50\ Hz$$
$$|I| = 2\ amps$$

Power Dissipated = P = 80 watts

(a) $|Z| = \frac{|V|}{|I|} = \frac{60}{2} = 30\Omega$

(b) $As\ P = I^2 R$

$$R = \frac{P}{I^2} = \frac{80}{4}$$
$$= 20\Omega$$

(c) $Since, |Z|^2 = R^2 + X_c^2$

$$X_c = \sqrt{(z)^2 - R^2}$$
$$= \sqrt{30^2 - 20^2} = 22.36\Omega$$

$$\frac{1}{2\pi fc} = 22.36$$

$$c = \frac{1}{2\pi f(22.36)}$$

$$= \frac{1}{2\pi \times 50 \times 22.36}$$

$= 142 \times 10^{-6}$ F

C = 142 μF

(or) Power factor = $\cos\phi = \frac{R}{|Z|}$

$$= \ = \frac{20}{30}$$

= 0.67(Leading)

It is capacitive circuit.

A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

$V_R = 100$ volts

$W = I^2R = V_RI$

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\ amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$

$$V_C = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(230)^2 - (100)^2}$$

$$= 207 volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$

$$= 27.6\Omega$$

$$\frac{1}{2\pi fc} = 27.6$$

$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$

$$= 96.19\ \mu F$$

$$Power\ factor = \cos\phi = \frac{R}{|Z|}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$

$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$

$$= 13.33\Omega$$

$$Powerfactor = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{13.33}{30.7}$$

= 0.434(Leading)

Solution

RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$
$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$
$$= |I|^2 [R^2 + (X_L - X_C)^2]$$
$$|V| = |I| \sqrt{R^2 + (X_L - X_C)^2}$$
$$|Z| = \frac{|V|}{|I|} \qquad \because X = (X_L - X_C)$$
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + X^2}$$

Three cases of Z

Case 1 If $X_L > X_C$

The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

Case 2 If $X_L < X_C$

The circuit behaves like RC circuit current leads applied voltage power factor is leading.

Case 3 When $X_L = X_C$, the circuit behaves like pure resistance.

Current is in phase with the applied voltage power factor is unity.

Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage

   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by

   1) $i = I_m \sin(\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin(\omega t + \theta)$, for $X_L > X_C$

3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j\,(X_L - X_C)$$

Problems

In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V \qquad V_R = 3V \qquad V_L = 1V$

$V^2 = V_R^2 + (V_L - V_C)^2$

$(V_L - V_C)^2 = V^2 - V_R^2$

$= 25 - 9 = 16$

$V_L - V_C = \pm 4$

$V_C = V_L \pm 4 = 1 + 4$

$V_C = 5V$

A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1\ H$

$C = 150\ \mu F \qquad = 150 \times 10^{-6}\ F$

$V = 200V \qquad f = 50\ Hz$

a) $X_L = 2\pi f L = 2\pi\,(50)\,0.1$

$= 31.4\ \Omega$

b) $X_C = \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi(50)(150\times10^{-6})}$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $|Z| = \sqrt{R^2 + X^2}$

$= \sqrt{10^2 + (10.2)^2}$

$= 14.28\Omega(\mathit{Inductive})$

$$I = \frac{|V|}{|Z|} = \frac{200}{14.28} = 14 amps$$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

$= 0.7$ (lagging) (I lags behind V)

Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi f c_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi f L_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^2 + X_{C1}^2}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^2 + X_{L2}^2}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g

Susceptance = b

Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \quad (positive)$$

$$|Y_1| = \sqrt{g_1^2 + b_1^2}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2} \quad \text{(Negative)}$$

$$|Y_2| = \sqrt{g_2^{\,2} + b_2^{\,2}}$$

Total conductance $G = g_1 + g_2$
Total Suceptance $B = b_1 - b_2$
Total admittance $|Y| = \sqrt{G^2 + B^2}$
Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle = $tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$
$Z_2 = 10 - j60\ \Omega$
$F = 50$ Hz
$Z_1 = R_1 + jX_L$
$Z_2 = R_2 - jX_C$

J term positive for in inductive
J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi\,(50)\,(L)$

$X_L = 15$

$2\pi\,(50)\,L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$
$R_2 = 10$
$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

$$C = \frac{1}{2\pi(50)60}$$

$C = 53\ \mu F$.

2.3.27 Two circuits, the impedances of which are $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$
$$Z\,2 = (6 - \mathrm{j}8)\Omega = 10\angle -53.13$$
$$\mathrm{I} = 15\ \mathrm{A}$$
$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \qquad \text{(Current divider rule)}$$
$$= \frac{15\angle 0^0 \times 10\angle -53.13^0}{16 + j7}$$
$$(Z_1 + Z_2 = 10 + j15 + 6 - j8)$$
$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$
$$I_1 = 8.6\angle -76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle -76.76$$
$$= 15 - (1.97 - j8.37)$$
$$= 15.5 - 32.7\mathrm{A}$$

Power taken by branch 1

= power dissipated in resistance of branch 1

$$= |I_1|^2 R_1 = (8.6)^2 \times 10$$
$$= 739.6 \text{ watts}$$

Power taken by branch 2

$$= |I_2|^2 R_2$$
$$= (15.5)^2 \times 6$$
$$= 1442 \text{ watts}$$

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$$Z_1 = R = 100\Omega$$
$$Z_2 = j\,X_L = j2\pi fL$$
$$= j\,(2\pi \times 50 \times 0.6)$$
$$= j\,188.5\Omega$$
$$= 188.5\angle 90$$
$$Z_T = Z_1 * Z_2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

$Total\ impedance\ |Z_T| = 88.33\Omega$

$R = 78\Omega$, $X_L = 41.46\Omega$

$X_L = 2\pi fLeq$

$41.46 = 2\pi \times 50 \times Leq$

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

$= 30 - j40 + 24 + j32$

$= 54 - j8$

$= 54.6 \angle -8.43$ A

Comparing 'V' and 'I_T' current I_T lag voltage 'V'

$\therefore \phi = 8.43°$ lag

Power factor = $\cos\phi = \cos 8.43$

= 0.99 lag

True Power = $W = |V||I|\cos\phi$

$= 200 \times 54.6 \times \cos 8.43$

= 10802 watts

= 10.802 KW

Apparent Power $= |V|I$

$= 200 \times 54.6$

= 10920 VA = 10.920 KVA

Reactive Power $= |V| I \sin\phi$

$= 200 \times 54.6 \times \sin 8.43$

= 1601 VAR

= 1.601 KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

$= 3.663 \angle 8.43$

$= 3.623 + j0.54$

$= R + j\,X_L$

$R = 3.623\Omega \qquad X_L = 0.54\ \Omega$

(or)

$$Z_{Total} = Z1*Z2$$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2.4 + j3.2)(3 - j4)}{2.4 + j3.2 + 3 - j4}$$

$$= \frac{4\angle 53.13 \times 5\angle -53.13}{5.46\angle -8.43}$$

$$= \frac{20\angle 0^0}{5.46\angle -8.43}$$

$$= 3.663 \angle 8.43$$

$$= 3.623 + j\,0.54\ \Omega$$

THREE PHASE A.C. CIRCUITS

Three Phase Connection

We have seen above only about single phase systems. Generally generation transmission and distribution of electrical energy are of three phase in nature. Three phase system is a very common poly phase system. It could be viewed combination of three single phase system with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system where as pulsating torque is produced in the case of single phase system. Because of these advantages the overall generation, transmission and distribution of electrical power is usually of three phase.

There are two possible connections in 3-phase system. One is star connection and the other one is delta or mesh connection. Each type of connection is governed by characteristics equations relating the currents and the voltages.

Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called star point or the neutral point. The free ends of the three phase coils will be operating at specific potential with respect to the zero potential of star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system and three phase three wire system.

Analysis

Let us analyze the relationship between currents and voltages. In a three phase circuit, the voltage across the individual coil is known as phase voltage and the voltage between two lines is called line voltage. Similarly the current flowing through the coil is called phase current and the current flowing through the line is called line current.

Notations Defined

E_R , E_Y , E_B	: Phase voltages of R, Y and B phases.
I_R , I_Y, I_B	: Phase currents
V_{RY}, V_{YB}, V_{BR}	: Line voltages
I_{L1}, I_{L2}, I_{L3}	: Line currents

Figure 3.32

A balanced system is one in which the currents in all phases are equal in magnitude and are displaced from one another by equal angles. Also the voltages in all the phases are equal in magnitude and are displaced from one another by equal angles. Thus,

$$E_R=E_Y=E_B=E_P \qquad V_{RY}=V_{YB}=V_{BR}=V_L$$
$$I_R=I_Y=I_B=I_P \qquad I_{L1}=I_{L2}=I_{L3}=I_L$$

Figure 3.33

Current Relationship:

Apply Kirchhoff's current law at nodes R_1, Y_1, B_1 We get

$$I_R = I_{LI};\ I_Y = I_{L1}\ ;\ I_B=I_{L3}$$

This means that in a balanced star connected system, phase current equals the line current

$$I_P = I_L$$

Phase current = Line current

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R; V_{Ry} and E_y.

$$\vec{E}_R - \vec{E}_Y = \vec{V}_{RY}$$

Using law of parallelogram

$$\left|\vec{V}_{RY}\right| = V_{RY} = \sqrt{E_R^2 + E_R^2 + 2E_R E_Y \cos 60}$$

$$= \sqrt{E_P^2 + E_P^2 + 2E_P E_P \cos 60} = E_P\sqrt{3}$$

Similarly,

$$\vec{E}_Y - \vec{E}_B = \vec{V}_{YB} \text{ and } \vec{E}_B - \vec{E}_R = \vec{V}_{BR}$$

$$V_{RY} = E_P\sqrt{3} \text{ and } V_{BR} = E_P\sqrt{3}$$

Hence $V_L = \sqrt{3}\, E_P$

Line Voltage $= \sqrt{3}$ phase voltage

Power relationship:

Let cosϕ be the power factor of the system.

Power consumed in one phase $= E_P I_P \cos\phi$

Power consumed in three phase $= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi \text{ watts}$$

Reactive power in one phase $= E_P I_P \sin\phi$

Total Reactive power $= 3E_P I_P \sin\phi$

$= \sqrt{\ }$

Apparent Power $= 3E_P I_P =$

Apparent power per phase $= E_P I_P \sqrt{\ }$ Total

$3V_L I_L \sin\phi$ VAR

$3V_L I_L$ Volt

Delta Connection:

The dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Figure 3.33

Let us analyze the relationship between currents and voltages. The system is balanced one. Notation used in the star connection are used here.

E_R, E_Y, E_B : Phase voltages of R, Y and B phases.
I_R, I_Y, I_B : Phase currents
V_{RY}, V_{YB}, V_{BR}: Line voltages
I_{L1}, I_{L2}, I_{L3} : Line currents

Voltage relationship:
Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R,V_{RY}

We Have $E_R = V_{RY}$
Similarly $E_Y = V_{YB}$ *and* $E_B = V_{BR}$

Thus $E_P = V_L$
Phase voltage= line voltage

Current Relationship:
Apply Kirchhoff's current law at node A (i.e.) R_1 , B_2 We get

$$\vec{I}_R - \vec{I}_B = \vec{I}_{1,1}$$

Referring to the phasor diagram and applying the law of parallelogram,
We get

$$I_{L1} = \sqrt{I_R^2 + I_Y^2 + 2 I_R I_Y \cos 60}$$
$$= \sqrt{I_P^2 + I_P^2 + 2 I_P I_P \cos 60}$$

Similarly,

$$\vec{I}_Y - \vec{I}_R = \vec{I}_{1,2} \text{ and } \vec{I}_B - \vec{I}_Y = \vec{I}_{1,3}$$

Hence $I_{L2} = I_P\sqrt{3}$ and $I_{L3} = I_P\sqrt{3}$

Thus Line current $= \sqrt{3}$ Phase current

$$I_L = I_P\sqrt{3}$$

Power relationship:
Let cosϕ be the power factor of the system.

Power consumed in one phase $= E_P I_P cos\phi$

Power consumed in three phase $= 3V_L\left(\frac{I_L}{\sqrt{3}}\right)\cos\phi$

$$= \sqrt{3}V_L I_L \cos\phi \ watts$$

Reactive power in one phase $= E_p I_p sin\phi$

$$Total\ Reactive\ power = 3E_p I_p cos\phi$$

$$= \sqrt{3}V_L I_L \sin\phi\ VAR$$

Apparent power per phase = E_pI_p

$Total\ Apparent\ Power = 3E_p I_p = \sqrt{3}$ V_LI_L volt

MEASUREMENT OF POWER IN THREE PHASE CIRCUITS:

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phase are not of same value. As a result, the current flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The three phase power can be measured by using three watt maters in each phases. The algebraic sum of the reading gives the total three phase power consumed. However three phase power can also measured using two watt meter.

Case I Star Connected load

In this section we analyse the measurement of three phase power using two wattmeter, when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Diagram 4

Figure 3.35

For Wattmeter 1

$$\textit{Current measured} = \vec{I}_{L1} = \vec{I}_R$$

$$\textit{Voltage measured} = \vec{V}_{RY}$$

$$\textit{Phase angle between them} = 30 + \phi$$

$$\textit{Power measured} = P1 = V_{RY} I_R \cos(30 + \phi)$$

For Wattmeter 2

$$\textit{Current measured} = \vec{I}_{L3} = \vec{I}_B$$

$$\textit{Voltage measured} = \vec{V}_{BY}$$

$$\textit{Phase angle between them} = 30 - \phi$$

$$\textit{Power measured} = P1 = V_{BY} I_B \cos(30 - \phi)$$

$$= \mathrm{V_L I_L} \cos(30 - \phi)$$

$$\textit{Now,} \quad P1 + P2 = V_L I_L \cos(30 + \phi) + \mathrm{V_L I_L} \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= \mathrm{V_L I_L} \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\,\mathrm{V_L I_L} \cos\phi = \textit{Total power in a three phase circuit}$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$
$$= V_L I_L \times 2 \times \sin 30 s \text{ in} \phi$$
$$= V_L I_L \sin\phi$$

$$\frac{P2 - P1}{P2 + P1} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]$$

$$\tan\phi = \sqrt{3}\ (P\,2 - P1/P\,2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Thus, two wattmeters connected appropriately in a three phase circuit can measure the total power consumed in the circuit.

Case II Delta Connected load

In this section we analyse the measurement of three phase power using two wattmeter, power when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Figure 3.36

For Wattmeter 1

$$\textit{Current measured} = I_{1,1} = \vec{I}_R - \vec{I}_B$$

$$\textit{Voltage measured} = \vec{V}_{RY} = \vec{E}_R$$

$$\textit{Phase angle between them} = 30 + \phi$$

$$\textit{Power measured} = P1 = V_{RY} I_{L1} \cos(30 + \phi)$$

$$= V_L I_L \cos(30 + \phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{1,3} = \vec{I}_B - \vec{I}_Y$$

$$Voltage\ measured = \vec{V}_{BY} = -\vec{E}_Y$$

$$Phase angle between them = 30 - \phi$$

$$Power\ measured = P1 = V_{BY} I_{1,3} \cos(30 - \phi)$$

$$= V_L I_L \cos(30 - \phi)$$

$$Now,\quad P1 + P2 = V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi = Total\ power\ in\ a three\ phase circuit$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$= V_L I_L \times 2 \times \sin 30 s\ in\phi$$

$$= V_L I_L \sin\phi$$

$$\mathrm{Tan}\,\phi = \sqrt{3}(P2 - P1/P2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Problems 3.30

Three similar coils of Resistance of 10Ω and inductance 0.15 Henry are connected in star across a 3Φ, 440V, 50Hz supply. Find the line and phase values of current. Also find the above values when they are connected in Delta.

Solution:

Given Data

$$V_L = 440V,\ R_{ph} = 10\Omega,\ L_{ph} = 0.15H,\ f = 50Hz$$

$$X_{L\,ph} = 2\pi f L_{ph} = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$\left|Z_{ph}\right| = \sqrt{R_{ph}^2 + X_{L\,ph}^2} = \sqrt{10^2 + (47.12)^2}$$

$$= 48.17\Omega$$

In star Connection

$$I_L = I_{ph} \qquad V_L = \sqrt{3}V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 230.95 \text{ Volt}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.95}{48.17} = 4.794\,A$$

$$I_L = I_{ph} = 4.794\,A$$

Active power = $3V_{ph}I_{ph}\cos\Phi$

$$\cos\Phi = \frac{R_{ph}}{Z_{ph}} = 0.2075$$

$$\text{Activepower} = 3*230.95*4.794*0.2075$$
$$= 689.54\,W$$

$$\text{Reactive power} = 3V_{ph}I_{ph}\sin\Phi$$

$$\sin\Phi = \sqrt{1-\cos^2\Phi} = 0.9782$$

$$\text{Reactive power} = 3*230.95*4.794*0.9782$$
$$= 3249.23\,VAR$$

$$\text{Apparent power} = 3V_{ph}I_{ph} = 3*230.95*4.794$$
$$= 3321.52\,V$$

If it is Delta connected coils, then

$$VL = V_{ph} \ \ \& \ IL = \sqrt{3}\,I_{ph}$$

$$VL = V_{ph} = 440V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} \frac{440}{48.17} = 9.134\,A$$

$$IL = \sqrt{3}\,I_{ph} = \sqrt{3}*9.134 = 15.82\,A$$

$$\text{Active power} = 3\,V_{ph}I_{ph}\cos\Phi$$
$$= 3*440*9.134*0.2075$$
$$= 2501.80\,watt$$

$$\text{Reacive power} = 3\, V_{ph} I_{ph} \sin\Phi$$

$$= 3 * 440 * 9.134 * 0.9782$$

$$\text{Apparent power} = 3V_{ph} I_{ph} = 3 * 440 * 9.134$$

$$= 12056.88\ VA$$

Problem 3.31

Two wattmeters connected to measure the 3Φ power indicate 1000 watts and 500 watts respectively. Calculate the power factor of the ckt.

Solution:

Given data

$$p_1 = 500 \text{ watts},\ p_2 = 1000 \text{ watts},$$
$$p_1 + p_2 = 1000 + 500 = 1500\ watts$$
$$p_2 - p_1 = 1000 - 500 = 500\ watts$$
$$p_1 = VL\ IL \cos(30 + \Phi)$$
$$p_2 = VL\ IL \cos(30 - \Phi)$$
$$p_1 + p_2 = \sqrt{3} VL\ IL \cos\Phi$$

$$p_2 - p_1 = \sqrt{3} * \frac{(p_2 - p_1)}{(p_1 + p_2)} = \frac{\sqrt{3} * 500}{1500}$$
$$= 0.5773$$
$$\Phi = 29.99°$$

Power factor $\cos\Phi = 0.866$

Problem 3.32

A balanced star connected load of (3+j4) Ω impedance is connected to 400V, three phase supply. What is the real power consumed by the load?

Solution:

Given data

$$V_L = 400\ volt$$
$$\text{Impedence / phase} = Z_{ph} = 3 + j4 = 5\angle 53°$$
In starconnection
$$I_L = I_{ph}\ \&\ V_L = \sqrt{3} V_{ph}$$
$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 volt$$
$$\text{Current in each } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5\angle 53°}$$
$$= 46.02\angle -53°\ A$$
$$\text{Linecurrent } I_L = 46.02\ A$$

$$\text{Total power consumed in the load} = \sqrt{3}\, V_L I_L \cos\Phi$$
$$= \sqrt{3} * 400 * 46.02 * \cos(-53°)$$
$$= 19188\ watt$$

PART A – QUESTIONS

1. Define Form factor and Peak factor
2. What is meant by average value?
3. Give the relation between line voltage and phase voltage, line current and phase current for star and delta connection.
4. What are the advantages of polyphase system ?
5. Define power factor?
6. What is phase sequence?
7. Define inductance and write its unit.
8. What is meant by balanced system?
9. Write down the expression for power factor in two wattmeter method.

PART B – QUESTIONS

1. Explain with neat figures the power measurement in three phase circuits using two-wattmeter method.
2. A given load consisting of a resistor R & a capacitor C, takes a power of 4800W from 200V, 60HZ supply mains, Given that the voltage drop across the resistor is 120V, Calculate the (a) impedance (b) current (c) power factor (d) resistance (e) capacitance. Write down the equations for the current and voltage.
3. A coil of 10 ohms and inductance of 0.1H in series with a 150µF capacitor across 200V,250HZ supply. Calculate (i) inductive reactance, capacitive reactance and impedance of the circuit (ii) current (iii)power factor(iv)voltage across the coil and capacitor respectively.
4. An impedance z_1= (2.4+j3.2) ohms is in parallel with another impedance z_2= (3-j4) ohms. The combination is given a supply of 200 V. Calculate (i) total impedance (ii) individual & total currents (iii) power factor (iv) power in the circuit.
5. A balanced three phase load consists of 6 ohms resistor & 8 ohms reactor (inductive) in each phase. The supply is 230V, 3 phases, 50HZ. Find (a) phase current (b) line current (c) total power. Assume the load to be connected in star & delta.
6. A 3phase, 4 wire 208 V, ABC system supplies a star connected load in which $Z_A=10\angle 0$, $Z_B=15\angle 30$, $Z_C=10\angle -30$. Find the line currents, the neutral current and the load power.
7. A coil having R = 10Ω and L = 0.2H is connected to a 100V, 50 Hz supply. Calculate (i) the impedance of the coil (ii) the current (iii) the phase difference between the current and voltage and (iv) the power.
8. Three similar coils of resistance of 10Ω and inductance 0.15H are connected in star across a 3 phase 440V, 50 Hz supply. Find the line

and phase values of current. Also find the above values when they are connected in delta.

9. Each phase of a delta connected load comprises a resistor of Ohm and a capacitor of μF in series. Calculate the line current for a 3 - ϕ voltages of 400V at 50 Hz. Also evaluate the power factor and the total 3 - ϕ power absorbed by the load.

SCHOOL OF COMPUTING

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UNIT -III

Electrical and Electronics Engineering – SEEA1103

UNIT - III

MAGNETIC CIRCUITS

Introduction

In an electric circuit, electromotive force(emf) drives the current through the circuit.Similarly in a magnetic circuit,a magetomotive force(mmf) drives the flux through the circuit. The flow of current depends on the resistance while the flow of flux depends on the characteristics of the medium through which it is flowing. The flux can travel across an airgap also. Thus magnetic circuits may consist of an air gap along with the magnetic materials with which they are made up of.

- **Magnet**

A **magnet** is a material or object that produces a magnetic field. This magnetic field is invisible but is responsible for the most notable property of a magnet: a force that pulls on other ferromagnetic materials, such as iron, and attracts or repels other magnets.

- **Permanent Magnet**

A piece of magnetic material that retains its magnetism after it is removed from a magnetic field. Example- steel

- **Electromagnet**

An electromagnet is made from a coil of wire that acts as a magnet when an electric current passes through it but stops being a magnet when the current stops. Often, the coil is wrapped around a core of "soft" ferromagnetic material such as steel, which greatly enhances the magnetic field produced by the coil.

- **Magnet and its properties**

- Like poles repel each other and unlike poles attract each other.
- When a magnet is rolled into iron piece, maximum iron pieces accumulate at the two ends of the magnet while very few accumulate at the centre of magnet.

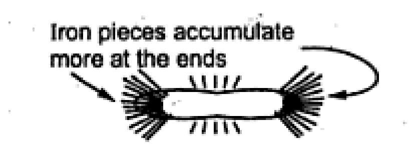

Fig. 3.1 Natural magnet

The points at which the iron pieces accumulate maximum are called poles of the magnet while imaginary line joining these poles is called axis of the magnet.

- When a magnet is placed near an iron piece, its property of attraction gets transferred to iron piece. Such property is called magnetic induction.

- **Magnetic Induction**

The phenomenon due to which a magnet can induce magnetism in a piece of magnetic material placed near it without actual physical contact is called magnetic induction.

- **Laws of Magnetism**
  - Like magnetic poles repel and unlike poles attract each other.
  - The force F exerted by one pole on other pole is

$$F \alpha M_1 M_2 / d^2$$
$$F = k\, M_1 M_2 / d^2$$

Where M_1 and M_2 are magnetic pole strengths , d is the distance between the poles and k is the nature of the surroundings.

- **Magnetic Field**

A field of force surrounding a permanent magnet or a moving charged particle, in which another permanent magnet or moving charge experiences a force

- **Magnetic Lines of Force**

The magnetic field of magnet is represented by imaginary lines around it which are called magnetic lines of force.

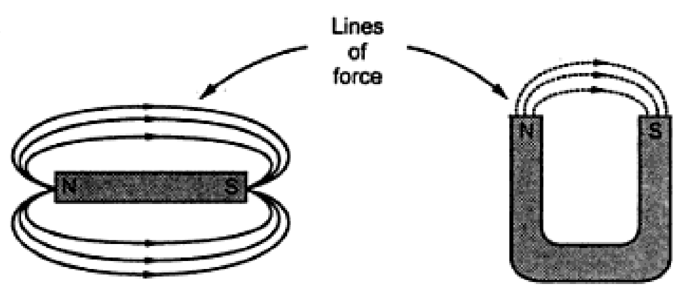

Fig. 3.2 Magnetic lines of force

- **Magnetic Flux(ϕ)**

The total number of lines existing in a particular magnetic field is called magnetic flux. Unit-Weber(Wb). Symbol for flux is ϕ.

1 weber=$1x10^8$ lines of force.

- **Pole Strength**

Every pole has a capacity to radiate or accept certain number of magnetic lines of force which is called pole strength.

- **Magnetic Flux Density(B)**

Magnetic flux Density is defined as the total number of **magnetic** lines of force passing through a specified area in a **magnetic** field. Symbol- B

B= Flux/ Area = ϕ/A (unit – Wb/m^2 or Tesla)

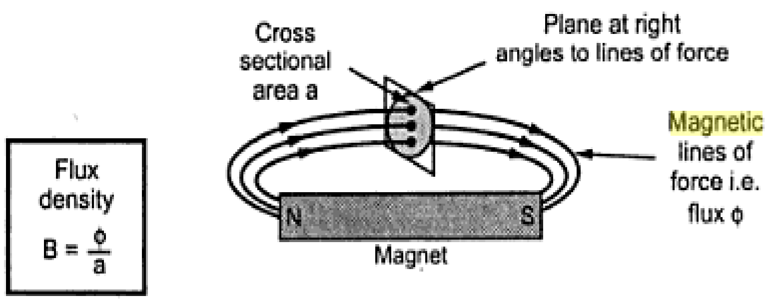

Fig. 3.3 Concept of Magnetic Flux Density

- **Magneto Motive Force(mmf)**

MMF is the cause for producing flux in a magnetic circuit. The amount of flux set up in the core depends upon current(I) and number of turns(N). The product of NI is called magneto-motive force and it determines the amount of flux set up in the magnetic circuit.

MMF= NI Unit- Ampere Turns(AT)

- **Reluctance**

The opposition that magnetic circuit offers to flux is called reluctance. It is defined as the ratio of magneto motive force to the flux.

$$S = MMF / Flux$$

S= NI / ϕ. Unit : AT/Wb

- **Permeance**

It is the reciprocal of reluctance.

$$Permeance = \frac{1}{Reluctance} = \frac{1}{S} \qquad \text{Unit : Wb/AT}$$

- **Magnetic Field Strength(H)**

This gives quantitative measure of strongness or weakness of the magnetic field. Field strength is defined as "the force experienced by a unit N-pole when placed at any point in a magnetic field ".It is denoted by H and its unit is newtons per weber(N/Wb) or ampere turns per meter(AT/m).It is defined as ampere turns per unit length.

$$H = \frac{Ampere\ turns}{length} = \frac{N}{l} \quad \text{(unit – AT/m)}$$

Where N- no. of turns of the coil, I- current through the coil, ℓ-length of the coil.

- **Permeability**

Permeability is the ability with which the magnetic material forces the magnetic flux through a given medium.Permeability of a material means its conductivity for magnetic flux. Greater the permeability of a material, greater the conductivity for magnetic flux.

Flux density (B) is proportional to the magnetizing force (H).

$$B \alpha H$$

$$B=\mu H$$

$$\mu=B/H$$

- **Relative Permeability**

Relative Permeability of a material is equal to the ratio of flux density produced in that material to the flux density produced in air by same magnetizing force.

$$\mu_r = \frac{B}{B0} = \frac{\mu H}{\mu 0 H} = \frac{\mu}{\mu 0}$$

Therefore $\mu = \mu_0 \, \mu_r$

μ- absolute permeability

μ_0- absolute permeability of air or vaccum=$4\Pi x10^{-7}$ H/m

μ_r -relative permeability of the material

Problem 3.1 A bar of iron 1cm^2 in cross section has 10^{-4} Wb of magnetic flux in it. Find the flux density in the bar. If the relative permeability of iron is 2000, what is the magnetic field intensity in the bar?

Solution:

Area of iron bar(A)= 1cm2=$1x10^{-4}m^2$

Flux $\phi=10^{-4}$ Wb

$\mu_r=2000$

$$B= Flux/\,Area\ =\ \phi/A \quad =10^{-4} / 1x10^{-4} = 1\ Wb/m^2.$$

$$H=B/\mu = B/\mu_0\ \mu_r = 1/\ (4\Pi x10^{-7}\ x\ 2000) = 397.88\ AT/m$$

Problem 3.2 A solenoid is wound with a coil of 200 turns. The coil is carrying a current of 1.5A. Find the value of magnetic field intensity when the length of the coil is 80cm.

Solution:

N=200

I=1.5A

$\ell=80cm=80x10^{-2}m=0.8m$

$$H = \frac{NI}{l} = 200 * \frac{1.5}{0.8}$$

H=375AT/m.

Problem 3.3 A current of 2A passes through a coil of 350 turns wound on an iron ring of mean diameter 12cm. The flux density established in the ring is 1.4Wb/m^2. Find the value of relative permeability of iron.

Solution:

I=2A

N=350

$D=12cm=12x10^{-2}m=0.12m$

$B=1.4Wb/m^2$

Length of the ring, $l=2\Pi r = \Pi D=\Pi*0.12$

$$B = \mu H$$

$$B = \frac{\mu r \mu o N I}{l}$$

$$\mu r = \frac{Bl}{\mu o NI} = \frac{1.4 * \Pi * 0.12}{\mu o * \Pi * 350 * 2} = 600$$

Problem 3.4 A mild steel ring of mean circumference 50 cm and cross- sectional area of 5 cm^2 has a coil of 250 turns wound uniformly around it. Calculate.

(i) Reluctance
(ii) Current required to produce a flux of 700 μwb in the ring. Take μ_r of mild steel as380.

Area of steel ring A = $5cm^2 = 5\times10^{-4}m^2$ No of turns of the coil, N = 250

Length of the magnetic path=Circumference of the ring=l = 50 cm

$= 50\times10^{-2}m$

Flux, $\Phi = 700\mu wb$

$= 700\times10^{-6}$ wb

Relative permeability of mild steel, $\mu_r = 380$, $\mu_o = 4\pi\times10^{-7}H/m$

$$S = \frac{l}{\mu o \mu r A}$$

$$= \frac{50 \times 10^{-2}}{4\pi \times 10^{-7} \times 380 \times 5 \times 10^{-4}}$$

=2094144AT/Wb

ELECTROMAGNETISM

The EMF may be produced either by batteries through chemical reaction or by thermocouples by heating the junction of two dissimilar metals. Michael faraday 1831 discovered that the EMF can also be produced by electromagnetic induction, used in commercial generation of power.

- **Electromagnetic Induction**

Whenever the magnetic flux linking with the coil changes, an EMF is induced in the coil. This phenomenon is called as electromagnetic induction.

Applications: microphones, telephones, transformers, generators motors etc.,

- **Production of induced EMF and current**

The change in flux linkage can be obtained by three methods

Method 1:

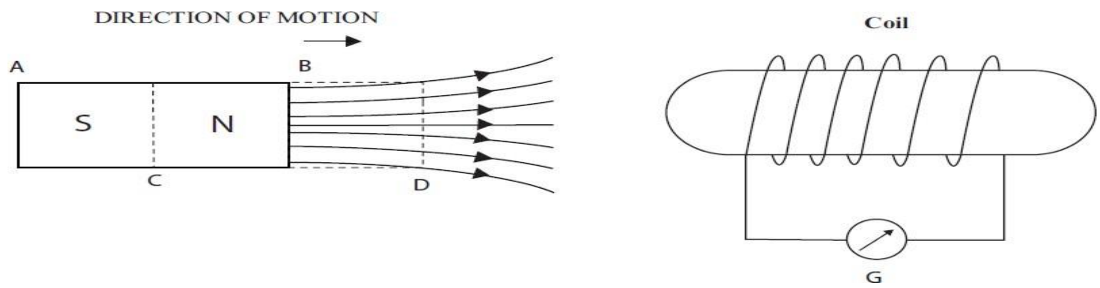

Fig.3.4

When a magnet is moved towards the coil there is deflection in the galvanometer connected across the coil thus indicating the flow of current. This current is due to the induced EMF in the coil.

- Change in flux results in production of EMF.
- Presence of EMF gives rise to flow of current.

If the magnet movement is stopped the pointer will show zero deflection. If the magnet moves with higher speed or if the number of turns of coil is increased or If we use the stronger magnet we can observe greater deflection of the pointer. As the magnet taken away from the coil, the flux linked with the coil is decreased the deflection of galvanometer is in the opposite direction.

Method2:

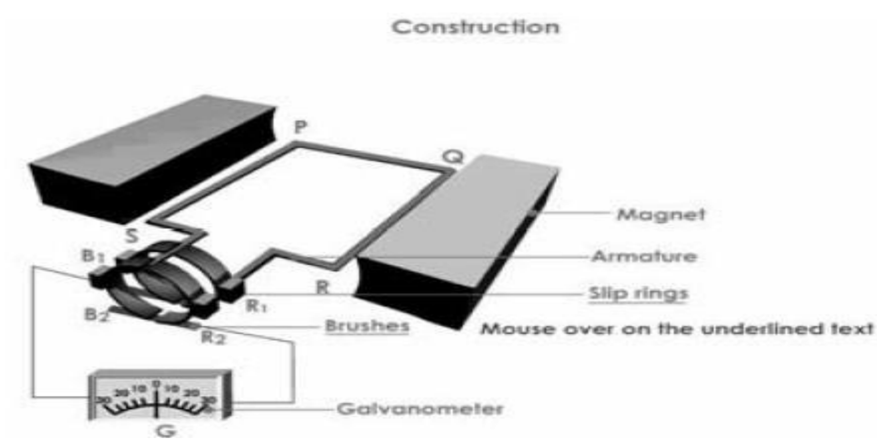

Fig.3.5

The EMF can also be induced in the coil by moving the coil and keeping the magnet stationary. These two methods of producing the emf are called the dynamic methods and the induced emf is called dynamically induced emf which is employed in generators (in ac and dc generators there is motion of conductors which results in the change in flux linkage).

Method 3:

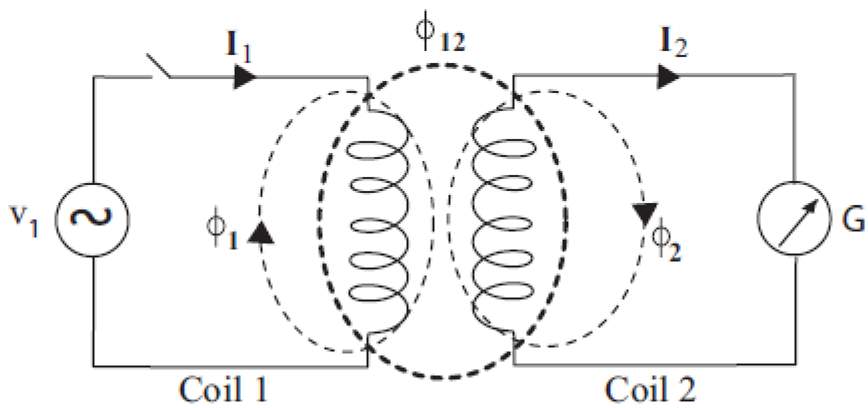

Fig.3.6

Both conductor and magnet are kept stationary and change in flux is obtained by changing the current this method of emf production is called statically induced emf which is employed in transformers.

- **Faradays law of electromagnetic induction**

First law: Whenever the conductor cut across the magnetic field,an emf is induced in the conductor or whenever the magnetic flux linking with any coil (circuit) changes an emf is induced in the coil

Second law: The magnitude of the induced emf is equal to the rate of change of flux linkage.

Suppose a coil has N turns, Let the flux through it change from Φ_1Weber to Φ_2 Weber in‘t’ seconds product of N and Φ is called flux linkage.

The initial flux linkage = $N\,\Phi_1$

The final flux linkage = $N\,\Phi_2$

Change of flux linkage = $N\Phi_2 - N\,\Phi_1$

= $N(\Phi_2 - \Phi_1)$

Rate of change of flux linkage = $N(\Phi_2 - \Phi_1)/t$

Let e be the induced emf

According to faradays second law $e=N\frac{d\Phi}{dt}$ volts

Actually the direction of emf induced is so as to oppose the very cause producing it

$$e = -N\frac{d\Phi}{dt} \text{ volts}$$

Note:

i) To produce induced EMF in a conductor there must be a rotating magnetic field cutting the stationary conductor .EMF is also induced when a moving conductor is cut by a stationary magnetic flux.

ii) When there is no relative motion between the magnetic flux and the conductor no emf is induced in it.

iii) The direction of the induced emf in the coil depends upon the direction of the magnetic field and that of motion of the coil.

iv) If the conductor is moved parallel to the direction of flux,it does not cut the conductor. Hence no emf is induced in it.

- **Direction of induced emf:**

To determine the direction of induced EMF and current in a conductor the following rules are used

- **Fleming's right hand rule**

Statement: Stretch out the forefinger, middle finger and the thumb of the right hand such that they are mutually perpendicular to one another. If the forefinger points in the direction of magnetic field and the middle finger points in the direction of current then the thumb points in the direction of the motion of the conductor

Note: The direction of dynamically induced emf can be determined by Fleming's right hand rule This rule is in D.C. generators.

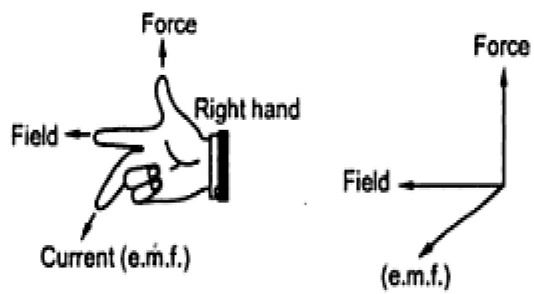

Fig. 3.7 Fleming's Right Hand Rule

- **Fleming's left hand rule**

Statement: Stretch out the forefinger middle finger and the thumb of the left hand mutually perpendicular to one another. If the forefinger points to the direction of the field and the middle finger points in the direction of the current, then, the thumb indicates the direction of the mechanical force exerted by the conductor.

Note: This rule is used in D.C. Motors.

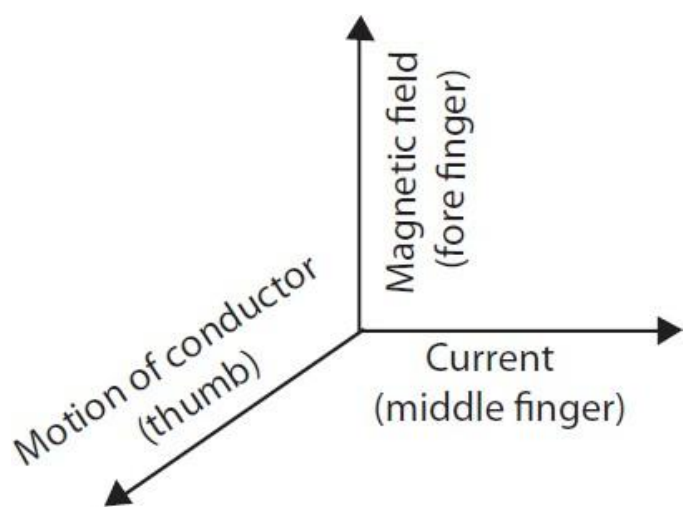

Fig. 3.8

Statement: In effect, electromagnetically induced emf and hence the current flows in a coil in such a direction that the magnetic field set up by it opposes the very cause producing it.

Note: The direction of statically induced emf lenz's law is used.

TYPES OF INDUCED EMF

When the flux linked with the coil or conductor changes an emf is induced in the coil. There are two ways to obtain the change in flux linkage. They are

Dynamically induced emf

When a conductor is moved in a stationary magnetic field or when the magnetic field is moved by keeping the conductor stationary an emf is induced provided the movement is done in such a way that the conductor is moved across the magnetic field. The emf thus induced is called as dynamically induced emf. An example of dynamically induced emf is the emf generated in D.C. and A.C. generators.

Consider the stationary magnetic field of flux density B wb/m^2 the direction of magnetic field is shown in the figure below and the conductor with circular cross section is placed let the length of the conductor in field 'ℓ 'in meters .conductor is allowed to move at right angles to magnetic field, in a time of 'dt ' seconds the conductor is moved to a distance of 'dx' meters.

The area swept by the conductor = m^2

Magnetic flux cut by the conductor = flux density*area swept

$$= B\,\ell\,dx \quad \text{Weber}$$

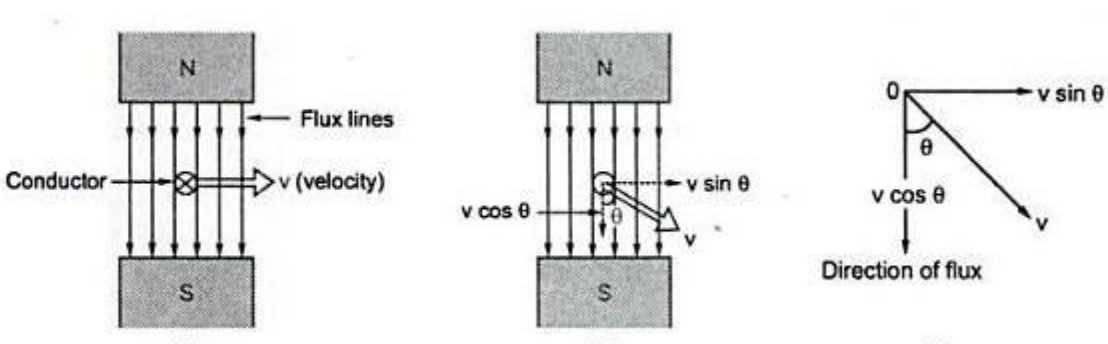

Fig.3.9. Conductor with Magnetic field

By faradays law of electromagnetic induction, the emf induced in the conductor is

$$e = N\frac{d\Phi}{dt}$$

If the number of turn in a conductor is one (N= 1)

$$e = \frac{d\Phi}{dt}$$

w.k.t

$$d\,\Phi = B\,\ell\,dx$$

$$e = B\,\ell\,dx/dt \qquad \text{since } dx/dt = v \text{(linear velocity)}$$

$$e = B\,\ell\,v \quad \text{volts}$$

If the conductor moves at an angle θ to the magnetic field then the velocity at which the conductor moves across the field is vsin θ ,therefore

$$e = B\,\ell\, v\sin\theta \text{ volts}$$

The direction of the induced emf is determined by the Flemings right hand rule.

Statically induced emf

The flux is linked with the coil(conductor) without moving either the coil or field system but by changing the current in the field system. The emf induced in this way without motion of either conductor or flux is called statically induced emf.An example of statically induced emf is the emf induced in transformer winding.

It is further classified as i)self induced emf ii)mutually induced emf

- ***Self induced emf***

The emf induced in a coil due to change in the value of its own flux linking it is called self induced emf. Consider a coil shown in figure

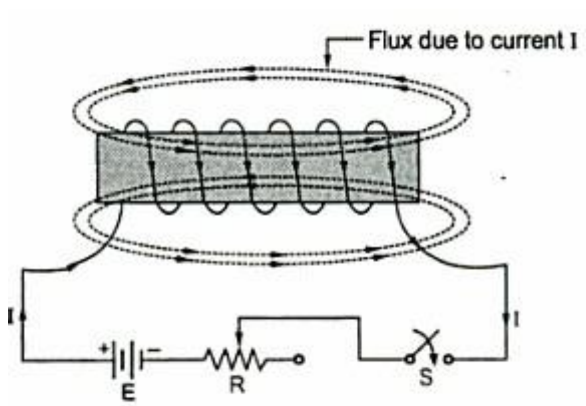

Fig. 3.10 Self Induced emf

If the current in the coil changes the flux linked with the coil also changes, which results in the production of emf, this is called self induced emf .The magnitude of this self induced emf

$$e = N\frac{d\Phi}{dt} \text{ volts}$$

The direction of the induced emf would be such as to oppose the very cause of production. Hence it is known as counter emf of self induction

- ***Self inductance (L)***

The property of a coil that opposes any change in the amount of current flowing through it is called its self inductance. It depends on the

i) Shape of the coil and number of turns

ii) Relative permeability of the magnetic material

iii) Speed in which the magnetic field changes.

Equation for self inductance

let

N= Number of turns in the coil

I=current in the coil.

If the current flowing through the coil changes the flux also changes which results in self induced emf

$$e=N\frac{d\Phi}{dt}$$

$$=\frac{d(N\Phi)}{dt}$$

Since the flux depends on the current so

$$N\Phi \quad \alpha \quad I$$

$$e \quad \alpha \quad \frac{dI}{dt}$$

$$e = L\frac{dI}{dt}$$

Where L=self inductance of the coil in henry

- ***Other expression for self inductance***

Method 1

From the above equation

$$e= L\frac{dI}{dt}$$

$$=\frac{d(\text{LI})}{dt} \text{ ....(1)}$$

$$e = N\frac{d\Phi}{dt}$$

$$=\frac{d(N\Phi)}{dt} \text{ ....(2)}$$

Since equation (1) and (2)

$$\frac{d(\text{LI})}{dt} = \frac{d(N\Phi)}{dt}$$

$$LI = N\Phi$$

$$L = \frac{N\Phi}{I} \text{ ....(3)}$$

NΦ is also called as flux linkage.when N is in turns, Φ in Weber I in amperes, then L is in henrys.

Method 2

We know that magnetic field intensity $H = \frac{NI}{l}$ .... (1)

Flux density $B = \mu_0\mu_r H$ ....(2)

$= \mu_0\mu_r\frac{NI}{l}$

We also know that $\Phi = B*a$

$= \mu_0\mu_r\frac{NI}{l} *a$

We know that flux linkage = N Φ

$= N^2 I\ \mu_0\mu_r a/\ell$

We know that $L = \frac{N\Phi}{I}$ henry

$= N^2 I\ \mu_0\mu_r a/\ell$

$= N^2/l/\mu o \mu r a$

$L = N^2/S$

- ***Mutually induced emf***

The emf induced in a circuit due to the charging current in the neighbouring circuit is called mutually induced emf.

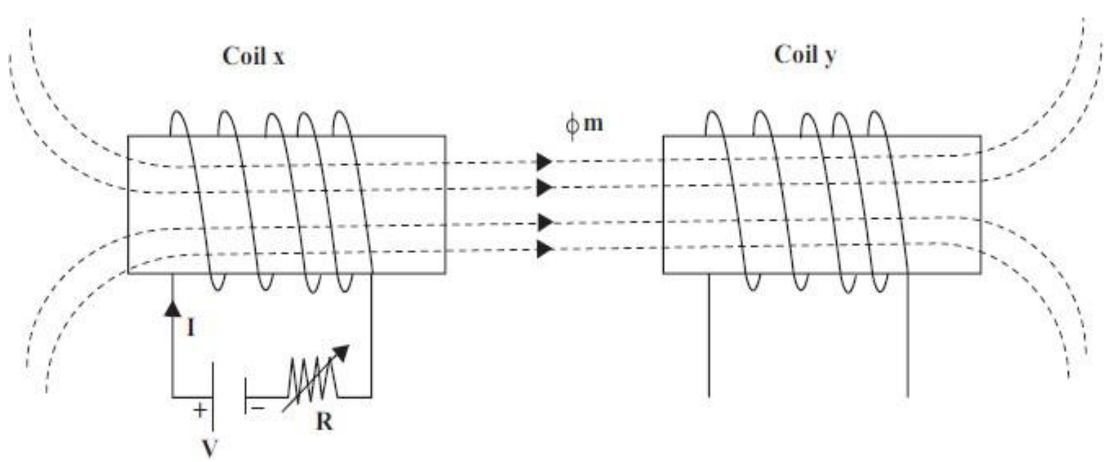

Fig. 3.11*Mutually induced emf.*

Here, coil x and coil y are close to each other, current flows through coil x due to this flux is produced in coil x part of the flux links the coil y which is called as mutual flux Φ_m. The flux common to both coil x and coil y is called mutual flux.

Note: If the current in coil x varies, emf in both the coil varies.

(i) The emf in coil x is called as self induced emf.
(ii) The emf in coil Y is called as mutually induced emf.

- ***Mutual inductance(M)***

Consider two coils X and Y placed close to each other, I_1 flows through coil X, a flux is set up and a part Φ12 of this flux links coil Y. This flux which is common to both the coils is called mutual flux (Φm). If current in coil X changes, the mutual flux alsochanges and hence emf is induced in coil Y. The emf induced in coil Y is called mutually induced emf.

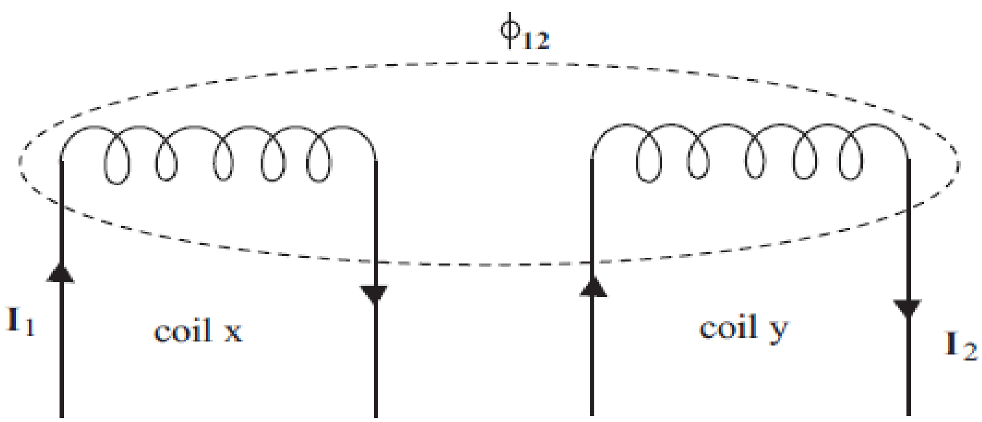

Fig.3.12

Expression for M:

Mutually induced emf in coil Y is directly propositional to the rate of changeof current in coil X.

$$e_m \, \alpha \frac{dI_1}{dt}$$

$$e_m = M \frac{dI_1}{dt}$$

M = Mutual inductance between the coils

Method 1:

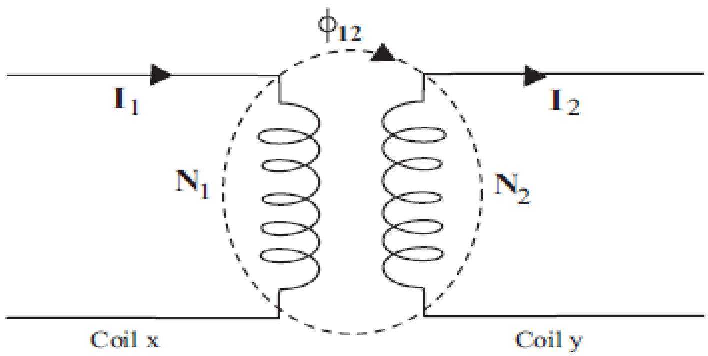

Fig.3.13

$$e_m = M\frac{dI_1}{dt}$$

$$= \frac{d}{dt}MI_1$$

Coefficicent Coupling

Co-efficient of coupling is defined as the fraction of magnetic flux produced by the current in one coil that links the other coil. If L1 and L2 are self inductances of two coils & M be the mutual inductance and K is the Co-efficient of coupling.

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

Note:

When there in no mutual flux between two coils then K = 0, M = 0.

Proof:

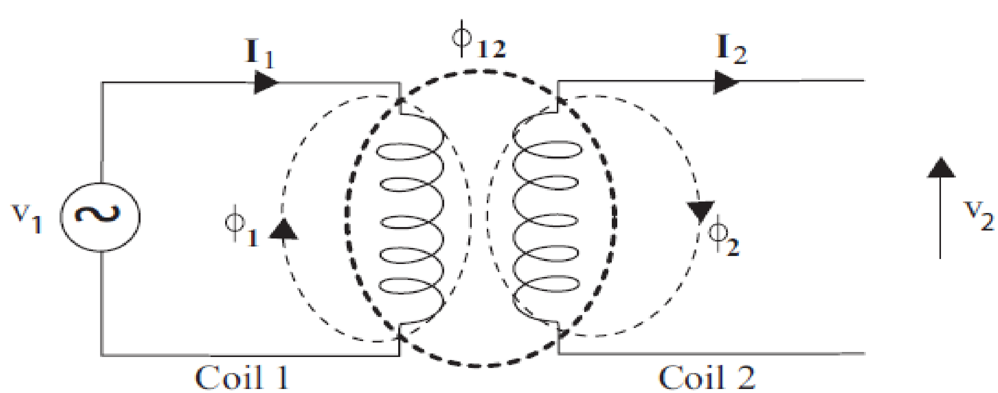

Fig.3.14

The ratio of the flux linked in the second coil to the total flux in the first coil due to current in the first coil is called co-efficient of coupling.

Let

Coil 1 and coil 2 be coupled magnetice and coil 1 is energized by a voltage of V1 Volts

- I1 produces a flux of Φ1 in coil 1
- Φ11 is the part of the flux linked only with coil1
- Φ12 is the part of the flux linking both the coil 1 and coil 2
- According to above definition

$$K = \frac{\phi_{12}}{\phi_1}$$

$$\phi_{12} = K\phi_1 \;.............(1)$$

Due to reciprocal action i.e., when source connected to coil 2.

$$K = \frac{\phi_{21}}{\phi 2}$$

$$\phi_{21} = K\phi_2 \;...........(2)$$

We know that

$$M = \frac{N_2\phi_{12}}{I_1}[coil\, 1 \text{ with source voltage}] \;.........(3)$$

$$M = \frac{N_1\phi_{21}}{I_2}[coil\, 2 \text{ with source voltage}] \;.........(3)$$

$$M^2 = M \times M$$

$$= \frac{N_2\phi_{12}}{I_1}\frac{N_1\phi_{21}}{I_2} \;..............(5)$$

$$= \frac{N_2N_1\phi_{12}\phi_{21}}{I_1I_2} \qquad \because \phi_{12} = K\phi_1$$

$$\phi_{21} = K\phi_2$$

$$= \frac{N_2\,K\,\phi_1}{I_1} \times \frac{N_1\,K\,\phi_2}{I_2}$$

$$= K^2 \left[\frac{N_2\ \phi_2}{I_2}\right]\left[\frac{N_1\ \phi_1}{I_1}\right] \text{............}(6)$$

$$M^2 = K^2 \frac{N_2\ \phi_2}{I_2} \frac{N_1\ \phi_1}{I_1}$$

$$L_1 = \frac{N_1\ \phi_1}{I_1},\quad L_2 = \frac{N_2\ \phi_2}{I_2}$$

$$M^2 = K^2 \left[L_1\ L_2\right]$$

$$K^2 = \frac{M^2}{L_1 L_2} \text{.............}(7)$$

$$K^2 = \frac{M}{L_1} \frac{M}{L_2} \text{.............}(8)\ (\text{or})$$

$$\mathrm{K} = \frac{M}{\sqrt{L_1 L_2}} \text{.............}(9)$$

Analogy of magnetic and electric circuits

S.No	Magnetic Circuits	Electric Circuits
1	The closed path for magnetic flux is called magnetic circuit	The closed path for electric current is called electric circuit
2	Magnetic flux Φ in webers	Electric current 'I' in amperes
3	Magneto motive force 'NI' in ampere turns	Electromotive force in volts
4	Magnetic flux $\Phi = \frac{\text{mmf}}{\text{reluctance}}$	Electric current $I = \frac{\text{emf}}{\text{resistance}}$
5	Reluctance S $= \frac{l}{\mu_o \mu_r A}$ in AT/wb	Resistance R $= \frac{\rho l}{A}$ in ohms
6	Permeance $= \frac{1}{\text{reluctance}}$	Conductance $= \frac{1}{\text{resistance}}$
7	Reluctivity	Resistivity

8	Permeability	Conductivity
9	Flux density B = $\frac{\Phi}{A}$ in wb/m^2	Current density J = $\frac{I}{A}$ in A/m^2
10	Magnetic intensity H = $\frac{NI}{l}$ in AT/m	Electric intensity E = $\frac{V}{d}$ in volts/metre
11	Magnetic flux does not actually flow in a magnetic circuit.	The electric current actually flows in an electric circuit.
12	The reluctance of a magnetic circuit is not constant and it depends up on flux density in the material.	The resistance of an electric circuit is practically constant, even though it varies slightly with temperature.
13	In a magnetic circuit, energy is required to create the flux and not to maintain it.	In an electric circuit, energy is required so long as the current has to flow through it.
14	For magnetic flux, there is no perfect insulator.	There are many electrical insulators like glass, air, rubber etc.

Problem 3.5 Calculate the emf induced in a coil of 200 turns, when the flux linking with it changes from 1 milliweber to 3 milliweber in 0.1 sec

Given data:

i) No of turns (N) = 200
ii) Initial value of flux (Φ1) = 1 mwb
= 1 × 10-3 wb
iii) Final value of flux (Φ2) = 3 mwb
= 3 × 10-3 wb

Solution:

$$e = -N\frac{d\Phi}{dt}$$

change in flux dΦ = $\Phi_2 - \Phi_1 = 3 \times 10^{-3} - 1\times10^{-3}$
= 2 × 10-3 wb

$$e = -200 \times \frac{2\times10^{-3}}{0.1} = 4 \text{ volts (in magnitude)}$$

Problem 3.6 A coil of 50 turns is linked by a flux of 20 mWb. If this flux is reversed in a time of 2 ms, Calculate the average emf induced in the coil.

Given:

i) No of turns (N) = 50
ii) Flux linked in coil (Φ) = 20 mwb
= 20X10-3 wb.
iii) Time required for flux reversal (t) = 2 ms

Solution
Emf induced in the coil (e)??

$$e = N\frac{d\Phi}{dt}$$

change in flux dΦ= 20-(-20) = 40 mWb

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{50 \times 40 \times 10^{-3}}{2 \times 10^{-3}}$$

$$= 1000V$$

emf induced e= 1000V

Problem 3.7 A coil of 100 turns of wire is wound on a magnetic circuit of reluctance 2000 AT/Wb. If a current of 1 A flowing in the coil is reversed in 10 ms, find the average emf induced in the coil.

Given:

i) No. of turns (N) = 100
ii) Reluctance (S) = 2000 ATwb
iii) Current in coil (I) = 1A
iv) Time required to reversal current t= 10 ms

Solution:

$$\text{Emf induced in coil } e = N\frac{d\Phi}{dt}$$

$$Flux = MMF / S$$
$$= NI / S$$
$$= \frac{100 \times 1}{2000}$$
$$= 0.05 \text{ Wb}$$

When a current of 1A is reversed in coil change in flux,

$$(d\varphi) = 0.05 - (0.05) = 0.1 mWb$$

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{100 \times 0.1 \times 10^{-3}}{10 \times 10^{-3}} = 1V$$

emf = 1V

Problem 3.8 A coil of 2000 turns surrounds a flux of 5mWb produced by a permanent magnet. The magnet is suddenly drawn away causing the flux inside the coil to drop to 2mWb in 0.1 sec. What is the average emf induced?

Given:

i) No of turns is coil (N) = 2000 turns

ii) Initial value of flux ($\Phi 1$) = 5 mwb

$= 5\times10^{-3}$WB

iii) Final value of flux Φ = 2mWb = 2×10^{-3} WB

iv) Time required for change in flux (dt) = 0.1 sec

v) change is flux $d\Phi$ = 5 - 2 = 3 mwb.

Solution:

$$e = N\frac{d\Phi}{dt}$$

$$= \frac{\mathbf{2000\times3\times10^{-3}}}{\mathbf{0.1}}$$

$$=60V$$

emf induced = 60 V.

Problem 3.9 A current of 5 Amperes when flowing through a coil of 1000 turns establishes a flux of 0.3 mwb. Determine the inductance of the coil.

Given:

I=5A

N=1000 turns

Φ=0.3mWb

=$0.3\text{x}10^{-3}$Wb

Solution:

$$L = \frac{N\emptyset}{I}$$

$$= \frac{1000 \times 0.3 \times 10^{-3}}{5}$$

$$=0.06H$$

Problem 3.10 A coil of 1500 turns carries a current of 10 A establishes a flux of 0.5 mWb. Find the inductance of the coil.

Given:

I =50 A

N =1500 turns

$\Phi = 5\times10^{-3}$ WB

Solution:

$$L = \frac{N\emptyset}{I}$$

$$= \frac{1500 \times5\times10^{-3}}{50}$$

$$=0.075H$$

Problem 3.11 A coil has self inductance of 10H. If a current of 200 mA is reduced to zero in a time of 1 ms. Find the average value of induced emf across the terminals of the coil.

Given:

L = 10 H

dI = 200 mA = 200×10^{-3} A

dt = 1 ms = 1×10^{-3} sec

Solution:

$$e = L\frac{dI}{dt}$$

$$= \frac{10 \times 200 \times 10^{-3}}{1 \times 10^{-3}}$$

=2000V

Problem 3.12 A air cored solenoid has 400 turns, its length is 30 cm and cross sectional area of 5 cm^2 . Calculate self inductance.

Given:

N = 400

l = 30 cm = 30×10^{-2} m

a = 5 cm2 = 5×10^{-4} m

Solution:

$$L = \frac{N^2}{S} = \frac{400^2}{S}$$

$$S = \frac{l}{\mu o \mu r A}$$

$$= \frac{30 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 5 \times 10^{-4}}$$

$$= 4.77 \times 10^8 AT/m$$

$$L = \frac{400^2}{4.77 \times 10^8}$$

$$= 3.354 \times 10^{-4} H$$

Problem 3.13 The number of turns in a coil is 250. When a current of 2 A flows in this coil, the flux in the coil is 0.3 mwb. When this current is reduce to zero in 2 ms. The voltage induced in a coil lying in the vicinity of coil in 63.75 volts. If the co-efficient of coupling between the coil is 0.75, find self inductances of the two coils, mutual inductance and number of turns in the second coil.

Given data:

N_1 = 250 turns
I_1 = 2 A
Φ_1 = 0.3 mwb =0.3×10^{-3} WB
di_1 = 2 A
dt = 2 ms = 2×10^{-3} s
e_m = 63.75v k = 0.75

Solution:

a) $L_1 = \frac{N_1 \emptyset_1}{I_1} = \frac{250 \times 0.3\times10^{-3}}{2}$

$=0.0375H$

b) $M = K\sqrt{L_1 L_2}$

$$L_2 = \frac{M^2}{K^2 L_1}$$

$$e_m = M\frac{di_1}{dt} = 63.75V$$

$$M = \frac{63.75}{2/2\times10^{-3}}$$

$= 63.75mH$

Substituting these values of M, L_1 and K

$$L_2 = \frac{(63.75\times10^{-3})^2}{0.75^2\times37.5\times10^{-3}}$$

$=0.193H$

$=193mH$

c) $em_2 = N_2\frac{d\emptyset_2}{dt}$

$$\emptyset_2 = K\emptyset_1$$

$$= N_2\frac{d(k\emptyset_1)}{dt}$$

$$= N_2\frac{k(d\emptyset_1)}{dt}$$

$$em_2 = N_2\times0.75\times\frac{0.3\times10^{-3}}{2\times10^{-3}}$$

$$63.75 = 0.1125N_2$$

$$N_2 = \frac{63.75}{0.1125}$$

$$N_2 = 567\ turns$$

PART A- Questions

1. What is electromagnet?
2. Define flux. Give its unit.
3. Define magnetic flux density. Give its unit.
4. Define magnetic field intensity. Give its unit.
5. Define mmf. Give its unit.
6. What is reluctance? Give its unit.
7. Define retentivity.
8. What is permeance. Give its unit.
9. State Ohm's law of magnetism.
10. A bar of iron 1cm^2 in cross section has 10^{-4} wb of flux in it. Find the flux density in the bar. If the relative permeability of iron is 2000. what is the magnetic field intensity in the bar?
11. A solenoid is wound with a coil of 200 turns. The coil is carrying a current of 1.5A. Find the magnetic field intensity when the length of the coil is 80cm.
12. A current of 2A passes through a coil of 350 turns wound on an iron ring of mean diameter 12cm.The flux density established in the ring is 1.4wb/m^2. find the value of relative permeability of iron.
13. State Faraday's law of electromagnetic induction.
14. Define Lenz's law.
15. Define self inductance and give its unit.
16. Define mutual inductance and give its unit
17. Define leakage flux and leakage factor.
18. What is fringing?
19. State Fleming's right hand rule.
20. State Fleming's left hand rule.
21. Define coefficient of coupling.

PART- B- Questions

1. State Faraday's law of electromagnetic induction. Derive the expression for dynamically induced emf .
2. Explain about statically induced emf with a neat sketch and Give the expression for self and mutual inductance.
3. An iron ring of mean length 50cm has an air gap of 1mm and a winding of 200 turns. If the permeability of the iron ring is 400 when a current of 1.25A flows through the coil, find the flux density.

4. An iron ring 8cm mean diameter is made up of round iron of diameter 1cm and permeability of 900, has an airgap of 2mm wide. It consists of winding with 400 turns carrying a current of 3.5A. Determine, i) mmf ii)total reluctance iii) flux iv)flux density.
5. Find the AT required to produce a flux of 0.4mwb in the airgap of a circular magnetic circuit which has an airgap of 0.5mm. The iron ring has 4cm^2 cross section and 63cm mean length. The relative permeability of iron is 1800 and leakage coefficient is 1.15.
6. Compare electric and magnetic circuit.
7. A coil of 50 turns is linked by a flux of 20mwb. If this flux is reversed in a time of 2ms. Calculate the average emf induced in the coil.
8. A coil of 100 turns of wire is wound on a magnetic circuit of reluctance 2000AT/wb. If a current of 1A flowing in the coil is reversed in 10ms, find the average emf induced in the coil.
9. Two 100 turns air cored solenoids have length 20cm and cross sectional area of 3cm^2 each. If the mutual inductance between them is 5μH. Find the coefficient of coupling.
10. Two magnetically coupled coils have self inductances L_1=100mH & L_2=400mH. If the coefficient of coupling is 0.8. Find the value of mutual inductance between the coils. What would be the max possible mutual inductance.
11. A coil of 800 turns is wound on a wooden former and a current of A produces flux of 200x10^{-6} wb. Calculate the inductance of a coil and induced emf when current is reversed in 0.2 sec.

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DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING

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Electrical and Electronics Engineering – SEEA1103
UNIT – IV
SEMICONDUCTOR DEVICES

SEMICONDUCTOR DEVICES

Characteristics of PN-junction diodes and Zener diodes, BJT and its configurations – input/output Characteristics, Junction Field Effect Transistor – Drain and Transfer Characteristics, MOSFET – Depletion type and Enhancement type, Uni Junction Transistors - Silicon Controlled Rectifiers.

CLASSIFICATION OF MATERIALS :

Materials can be classified based on its conductivity property as:

Conductor: A conductor is a material which allows free flow of charge when a voltage is applied across its terminals. i.e. it has very high conductivity. Eg: Copper, Aluminum, Silver, Gold.
Semiconductor: A semiconductor is a material that has its conductivity somewhere between the insulator and conductor. The resistivity level is in the range of 10 and 104 Ω-cm. Eg: Silicon and Germanium. Both have 4 valance electrons. Electronic devices like PN diode, Zener diode Bipolar Junction Transistor are made using these semiconductors.
Insulator: An insulator is a material that offers a very low level (or negligible) of conductivity when voltage is applied. Eg: Paper, Mica, glass, quartz.

Classification of semiconductors

- **Intrinsic semiconductor**
- **Extrinsic semiconductor**

Intrinsic semiconductor
They are semi-conducting materials which are pure and no impurity atoms are added to it. Eg: Germanium and Silicon.

Properties:

- **Number of electrons is equal to the number of holes. I.e., ne=nh.**
- **Electrical conductivity is low.**
- **Electrical conductivity of intrinsic semiconductors depends on their temperatures.**

Extrinsic semiconductors
Intrinsic semiconductor has very limited applications as they conduct very small amounts of current at room temperature. The current conduction capability of intrinsic semiconductor can be increased significantly by adding a small amounts impurity to the intrinsic semiconductor. By adding impurities it becomes impure or extrinsic semiconductor. This process of adding impurities is called as doping. The amount of impurity added is 1 part in 106 atoms

Properties:

- **The number of electrons is not equal to the number of holes.**
- **The electrical conductivity is high.**
- **The electrical conductivity depends on the temperature and the amount of impurity added in them. They are further subdivided as**

- **P type semiconductor**
- **N type semiconductor**

P type semiconductor:

When a intrinsic semiconductor is added with Trivalent impurity it becomes a P- Type semiconductor. Examples of trivalent impurities are Boron, Gallium, indium etc.

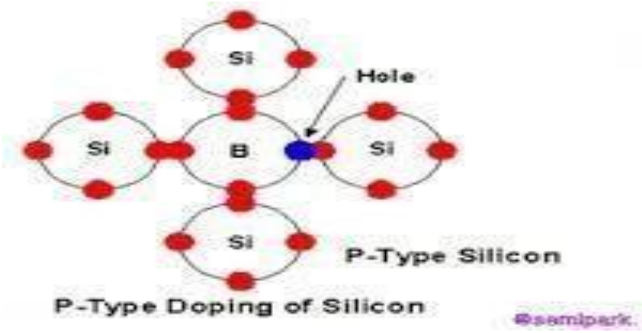


Fig. 4.1 P type Semiconductor

The crystal structure of P type semiconductor is shown in the Fig. 4.1. The three valance electrons of the impurity (boron).

forms three covalent bonds with the neighboring atoms and a vacancy exists in the fourth bond giving rise to the holes. The hole is ready to accept an electron from the neighboring atoms. Each trivalent atom contributes to one hole generation and thus introduces a large no. of holes in the valance band. At the same time the no. electrons are decreased compared to those available in intrinsic semiconductor because of increased recombination due to creation of additional holes. Thus in P type semiconductor,

- **Holes are majority carriers and electrons are minority carriers.**
- **The semiconductor is rich in holes.**

N type semiconductor:

If the added impurity is a pentavalent atom then the resultant semiconductor is called N-type semiconductor. Examples of pentavalent impurities are Phosphorus, Arsenic, Bismuth, Antimony etc.

A pentavalent impurity has five valance electrons. Fig 4.2 shows the crystal structure of N type semiconductor material where four out of five valance electrons of the impurity atom(antimony) forms covalent bond with the four intrinsic semiconductor atoms. The fifth electron is loosely bound to the impurity atom. This loosely bound electron can be easily excited from the valance band to the conduction band by the application of electric field or increasing the thermal energy. The energy required to detach the fifth electron form the impurity atom is very small of the order of 0.01ev for Ge and 0.05 eV for Si. Thus in a N type semiconductor

- **Electrons are majority carriers and holes are minority carriers.**
- **The semiconductor is rich in electrons.**

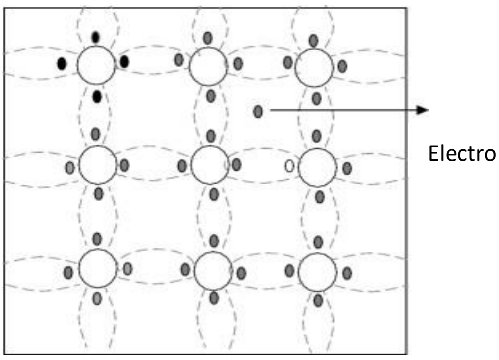


Fig. 4.2 N type Semiconductor

PN JUNCTION THEORY

When P and N type semiconductors are fused together, we obtain PN junction. When first joined together, very large density gradient exists between both sides of the PN junction. Therefore at the junction there is a tendency of free electrons from N side to diffuse over to the P side and the holes to the N side. This process is called diffusion. Hence some of the free electrons from the N side begin to migrate across this newly formed junction to fill up the holes in the P- type material.

As the free electrons move across the junction from N type to P type, they leave behind positively charge (donor ions) on the negative side and hence a positive charge is built on the N-side of the junction. Similarly, the holes from the P side migrate across the junction in the opposite direction into the N region where there are large numbers of free electrons. As a result, the charge density of the P-type along the junction is filled with negatively charged acceptor ions, and hence a negative charge is built on the P-side of the junction. The width of these layers depends on how heavily each side is doped with acceptor density and donor density respectively.

The electrostatic field across the junction caused by the positively charged N-Type region tends to drive the holes away from the junction and negatively charged P type regions tend to drive the electrons away from the junction. Thus near the junction, a region depleted of mobile charge carriers is formed. This is called depletion layer, space region, and transition region. The depletion region is of the order of 0.5µm thick. There are no mobile carriers in this narrow depletion region. Hence no current flows across the junction and the system is in equilibrium.

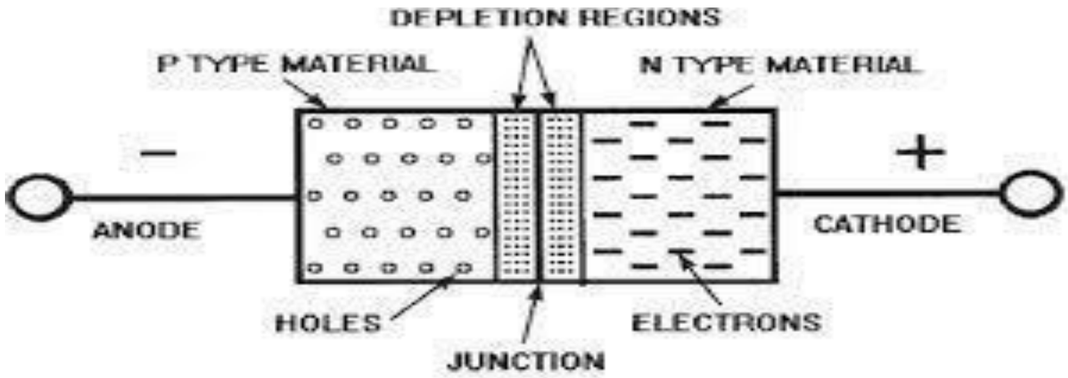


Fig. 4.3 PN junction

FORWARD BIASED OPERATION

When external voltage is applied then the potential difference is altered between the P and N regions. Positive terminal of the source is connected to the P side and the negative terminal is connected to N side then the PN junction diode is said to be connected in forward bias condition. This lowers the potential across the junction. The majority charge carriers in N and P regions are attracted towards the PN junction and the width of the depletion layer decreases with diffusion of the majority

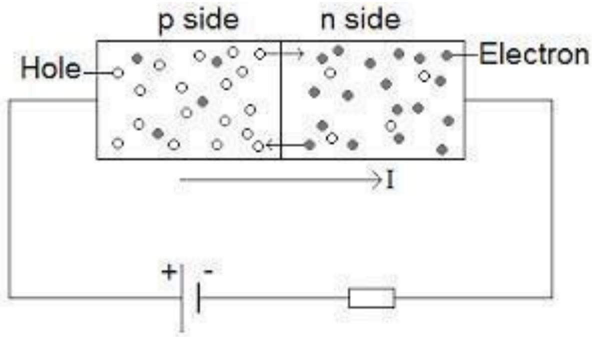


Fig. 4.4 Forward bias PN junction diode

charge carriers. The external biasing causes a departure from the state of equilibrium and also in the depletion layer. With the increase in forward bias greater than the built in potential, at a particular value the depletion region becomes very much thinner so that a large number of majority charge carriers can cross the PN junction and conducts an electric current. The current flowing up to built in potential is called as ZERO current or KNEE current.

4.2.3.Reverse Bias Operation

Positive terminal of the source is connected to the N side and the negative terminal is connected to P side. Here majority charge carriers are attracted away from the depletion layer by their respective battery terminals connected to PN junction. Positive terminal attracts the electrons away from the junction in N side and negative terminal attracts the holes away from the junction in P side. As a result of it, the width of the potential barrier increases that impedes the flow of majority carriers in N side and P side. The width of the free space charge layer increases, thereby electric field at the PN junction increases and the PN junction diode acts as a resistor. The current that flows in a PN junction diode is the small leakage current, due to minority carriers generated at the depletion layer or minority carriers which drift across the PN junction. The growth in the width of the depletion layer presents a high impedance path which acts as an insulator.

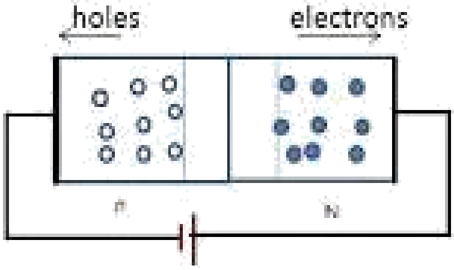


Fig. 4.5 Reverse bias PN junction diode

4.2.4 .VI characteristics of PN Diode

The VI characteristics of PN junction diode in forward bias are non linear, that is, not a straight line. This nonlinear characteristic illustrates that during the operation of the PN junction, the resistance is not constant. The slope of the PN junction diode in forward bias shows the resistance is very low. When forward bias is applied to the diode if this external voltage becomes greater than the value of the potential barrier, approx. 0.7 volts for silicon and 0.3 volts for germanium, then it causes a low impedance path and permits to conduct a large amount of current. Thus the current starts to flow above the knee point with a small amount of external potential.

In reverse bias condition, the P-type of the PN junction is connected to the negative terminal and N-type is connected to the positive terminal of the external voltage. This results in increased potential barrier at the junction. Hence, the junction resistance becomes very high and as a result practically no current flows through the circuit. However, a very small current of the order of µA, flows through the circuit in practice. This is knows as reverse saturation current and it is due to the minority carriers in the junction.

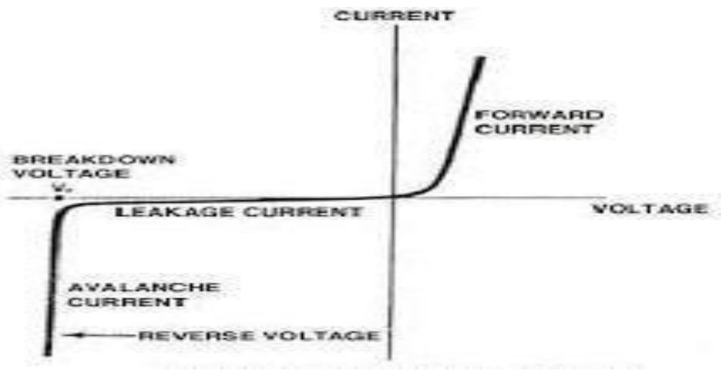


Fig. 4.6 VI characteristics of PN Diode

Applications of PN junction Diode:

The P-N junction diode has many applications.

- **P-N junction diode in reverse biased configuration is sensitive to light from a range between 400nm to 1000nm, which includes VISIBLE light. Therefore, it can be used as a photodiode.**
- **It can also be used as a solar cell.**
- **P-N junction forward bias condition is used in all LED lighting applications.**
- **The voltage across the P-N junction biased is used to create Temperature Sensors, and Reference voltages.**
- **It is used in many circuits“ rectifiers, varactor for voltage controlled oscillators.**

ZENER DIODE:

A zener diode is a special type of device designed to operate in the zener breakdown region which is heavily doped than the normal PN junction diode. Hence, it has very thin depletion region. Therefore, Zener diode allow more electric current than the normal PN junction diodes under forward bias like a normal diode but also allows electric current in the reverse direction if the applied reverse voltage is greater than the zener voltage. Thus they are always connected in reverse direction because it is specifically designed to work in reverse direction. The breakdown voltage of a Zener diode is carefully set by controlling the doping level during manufacture. The name Zener diode was named after the American physicist Clarance Melvin Zener who discovered the zener effect.

Diode symbol

Breakdown in Zener diode

There are two types of reverse breakdown regions in a Zener diode: Avalanche breakdown and Zener breakdown.

Avalanche breakdown

The avalanche breakdown occurs at high reverse voltage. When high reverse voltage is applied to the diode, the free electrons gains large amount of energy and accelerated to greater velocities. The free electrons moving at high speed will collides with the atoms and knock off more electrons. These electrons are again accelerated and collide with other atoms. Because of this continuous collision with the atoms, a large number of free electrons are generated. This cumulative process is referred to as avalanche multiplication which results in the flow of large reverse current and this breakdown of the diode is called avalanche breakdown. Avalanche breakdown occurs in zener diodes with zener voltage greater than 6V.

Zener breakdown:

The zener breakdown occurs in heavily doped diodes because of their narrow depletion region. When reverse biased voltage applied to the diode is increased, the narrow depletion region generates strong electric field. When it reaches close to zener voltage, the electric field in the depletion region is strong enough to pull electrons from their valence band. The valence electrons which gains sufficient energy from the strong electric field of depletion region will breaks bonding with the parent atom. The valance electrons which break bonding with parent atom will become free electrons. This free electrons results in large electric current, a small increase in voltage will rapidly increases the electric current. This breakdown is referred to as Zener breakdown.

Note:

- **Zener breakdown occurs at low reverse voltage whereas avalanche breakdown occurs at high reverse voltage.**
- **Zener breakdown occurs in Zener diodes because they have very thin depletion region.**
- **Breakdown region is the normal operating region for a zener diode.**
- **Zener breakdown occurs in Zener diodes with Zener voltage less than 6V.**

ZENER DIODE CHARACTERISTICS:

When a Zener diode is biased in the forward direction it behaves just like a normal PN junction diode.

Under reverse-biased condition, the reverse voltage is applied. As the reverse bias voltage is increased, breakdown of the junction occurs. The breakdown voltage depends upon the amount of doping. If the diode is heavily doped, depletion layer will be thin and consequently, breakdown occurs at lower reverse voltage and further, the breakdown voltage is sharp. A lightly doped diode has a higher breakdown voltage. Thus breakdown voltage can be selected with the amount of doping .This breakdown voltage point is called the "Zener voltage or breakdown voltage " and a large amount of current flows through the Zener diodes. This Zener breakdown voltage on the I-V curve is almost a vertical straight line.

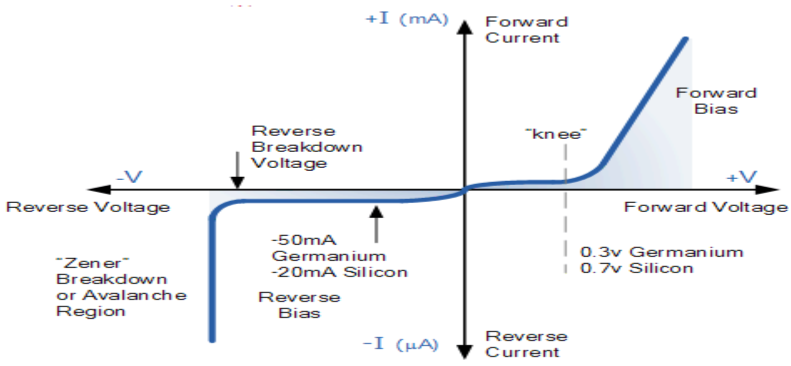


Fig. 4.7 VI characteristics of Zener diode ZENER DIODE AS A VOLTAGE

REGULATOR

From the Zener Characteristics shown, under reverse bias condition, the voltage across the diode remains constant although the current through the diode increases as shown. Thus the voltage across the zener diode serves as a reference voltage. Hence the diode can be used as a voltage regulator.

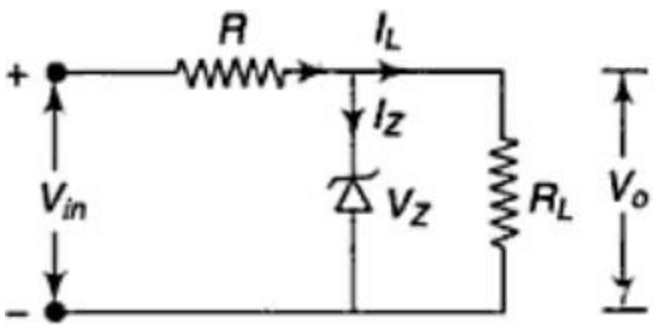


Fig. 4.8 Voltage regulator

It is required to provide constant voltage across load resistance RL , whereas the input voltage may be varying over a range. As shown, Zener diode is reverse biased and as long as the input voltage does not fall below Zener breakdown voltage, the voltage across the diode will be constant and hence the load voltage will also be constant.

BIPOLAR JUNCTION TRANSISTOR INTRODUCTION

The transistor was developed by Dr. Shockley along with Bell Laboratories team in 1951. It is a three terminal device whose output current, voltage and power are controlled by its input current. In communication systems it is the primary component in the amplifier. The important property of the transistor is that it can raise the strength of a weak signal. This property is called amplification. Transistors are used in digital computers, satellites, mobile phones and other communication systems, control systems etc., A transistor consists of two P-N junction. The junction are formed by sand witching either p-type or n-type semiconductor layers between a pair of opposite types which is shown below

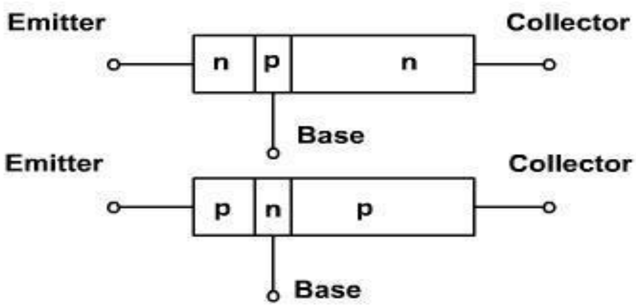


Fig. 4.9 Transistor

TRANSISTOR CONSTRUCTION:

A transistor has three regions known as emitter, base and collector.
Emitter:

- **It is a region situated in one side of a transistor, which supplies charge carriers (ie., electrons and holes) to the other two regions**
- **Emitter is heavily doped region**

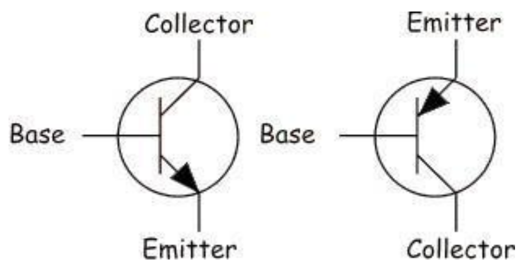
Base:

- **It is the middle region that forms two P-N junction in the transistor**
- **The base of the transistor is thin as compared to the emitter and is a lightly doped region**

Collector:

- **It is a region situated in the other side of a transistor (ie., side opposite to the emitter) which collects the charge carriers.**
- **The collector of the transistor is always larger than the emitter and base of a transistor**
- **The doping level of the collector is intermediate between the heavy doping of emitter and the light doping of the base.**

TRANSISTOR SYMBOLS



NPN Transistor **PNP transistor**

Fig. 4.10 Transistor

- **The transistor symbol carries an arrow head in the emitter pointing from the P-**

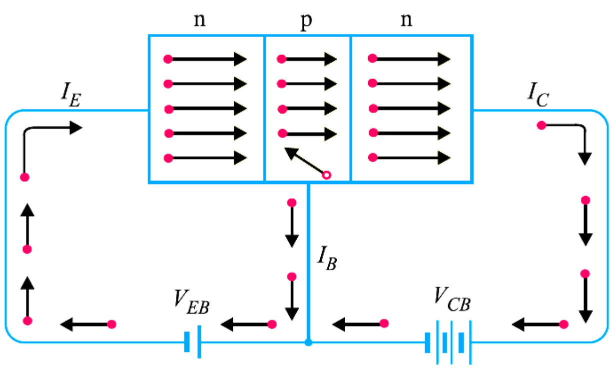
region towards the N- region

- The arrow head indicates the direction of a conventional current flow in a transistor.
- The direction of arrow heads at the emitter in NPN and PNP transistor is opposite to each other.
- The PNP transistor is a complement of the NPN transistor.
- In NPN transistor the majority carriers are free electrons, while in PNP transistor these are the holes.

UNBIASED TRANSISTORS

A transistor with three terminals (Emitter, Base, Collector) left open is called an unbiased transistor or an open – circuited transistor. The diffusion of free electrons across the junction produces two depletion layers. The barrier potential of three layers is approximately 0.7v for silicon transistor and 0.3v for germanium transistor. Since the regions have different doping levels therefore the layers do not have the same width. The emitter base depletion layer penetrates slightly into the emitter as it is a heavily doped region where as it penetrates deeply into the base as it is a lightly doped region. Similarly the collector- base depletion layer penetrates more into the base region and less into the collector region. The emitter- base depletion layer width is smaller than that of collector base depletion layer. The unbiased transistor is never used in actual practice. Because of this we went for transistor biasing.

OPERATION OF NPN TRANSISTOR



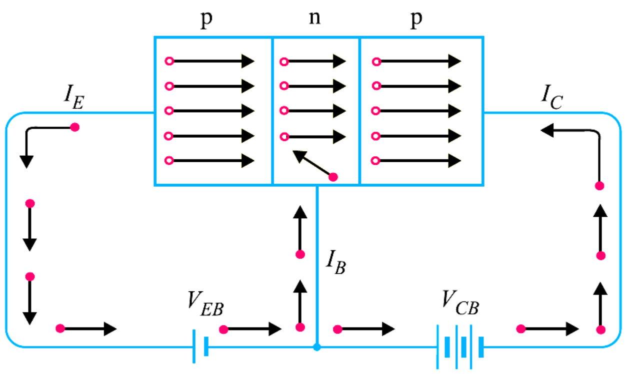
Basic connection of *npn* transistor

Fig. 4.11 NPN Transistor

- The NPN transistor is biased in forward active mode ie., emitter – base of transistor is forward biased and collector base junction is reverse biased
- The emitter – base junction is forward biased only if V is greater than barrier potential which is 0.7v for silicon and 0.3v for germanium transistor
- The forward bias on the emitter- base junction causes the free electrons in the N – type emitter to flow towards the base region. This constitutes the emitter current (IE). Direction of conventional current is opposite to the flow of electrons.
- Electrons after reaching the base region tend to combine with the holes.
- If these free electrons combine with holes in the base, they constitute base current (IB).
- Most of the free electrons do not combine with the holes in the base.
- This is because of the fact that the base and the width is made extremely small and electrons do not get sufficient holes for recombination.

- Thus most of the electrons will diffuse to the collector region and constitutes collector current (IC). This collector current is also called injected current, because of this current is produced due to electrons injected from the emitter region
- There is another component of collector current due to the thermal generated carriers.
- This is called as reverse saturation current and is quite small.

OPERATION OF PNP TRANSISTOR



Basic connection of *pnp* transistor

Fig. 4.12 PNP Transistor

- Operation of a PNP transistor is similar to npn transistor
- The current within the PNP transistor is due to the movement of holes where as, in an NPN transistor it is due to the movement of free electrons
- In PNP transistor, its emitter – base junction is forward biased and collector base junction is reverse biased.
- The forward bias on the emitter – base junction causes the holes in the emitter region to flow towards the base region
- This constitutes the emitter current (IE).
- The holes after reaching the base region combine with the electrons in the base and constitute base current (IB).
- Most of the holes do not combine with the electrons in the base region
- This is due to the fact that base width is made extremely small, and holes does not get sufficient electrons for recombination.
- Thus most of the holes diffuse to the collector region and constitutes collector current (IC).
- This current is called injected current, because it is produced due to the holes injected from the emitter region
- There is small component of collector current due to the thermally generated carriers
- This is also called as reverse saturation current.

TRANSISTOR CONFIGURATIONS

- A transistor is a three terminal device, but we require four terminals (two for input and two for output) for connecting it in a circuit.
- Hence one of the terminal is made common to the input and output circuits.
- The common terminal is grounded.

- **There are three types of configuration for the operation of a transistor.**

Common base configuration

- **This is also called grounded base configuration**
- **In this configuration emitter is the input terminal, collector is the output terminal and base is the common terminal**

Common emitter configuration(CE)

- **This is also called grounded emitter configuration**
- **In this configuration base is the input terminal, collector is the output terminal and emitter is the common terminal**

Common collector configuration(CC)

- **This is also called grounded collector configuration**
- **In this configuration, base is the input terminal, emitter is the output terminal and collector is the common terminal.**

COMMON BASE CONFIGURATION (CB)

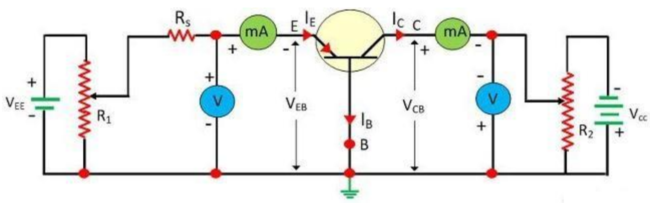


Fig. 4.13 CB Configuration

- **The input is connected between emitter and base and output is connected across collector and base**
- **The emitter – base junction is forward biased and collector – base junction is reverse biased.**
- **The emitter current, flows in the input circuit and the collector current flows in the output circuit.**
- **The ratio of the collector current to the emitter current is called current amplification factor.**

CHARACTERISTICS OF CB CONFIGURATION

- **The performance of transistors determined from their characteristic curves that relate different d.c currents and voltages of a transistor**
- **Such curves are known as static characteristics curves There are two important characteristics of a transistor**

➢ **Input characteristics**
➢ **Output characteristics**

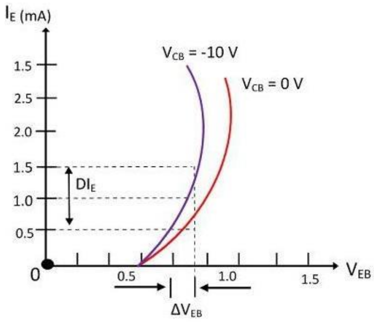


Fig. 4.14 Characteristics of CB Configuration

INPUT CHARACTERISTICS

The curve drawn between emitter current and emitter – base voltage for a given value of collector – base voltage is known as input Characteristic curves.

The following points are taken into consideration from the characteristic curve.

- **For a specific value of VCB, the curve is a diode characteristic in the forward region. The PN emitter junction is forward biased.**
- **When the value of the voltage base current increases the value of emitter current increases slightly. The junction behaves like a better diode. The emitter and collector current is independent of the collector base voltage VCB.**
- **The emitter current IE increases with the small increase in emitter-base voltage VEB. It shows that input resistance is small.**

  Input Resistance

  The ratio of change in emitter-base voltage to the resulting change in emitter current at constant collector base voltage VCB is known as $r_i = \frac{\Delta V_{EB}}{\Delta I_E}$ input resistance. The input resistance is expressed by the formula.

Base width modulation (or) Early effect

- **In a transistor, since the emitter – base junction is forward biased there is no effect on the width of the depletion region.**
- **However, since collector – base junction is reverse biased, as the reverse bias voltage across the collector – base junction increases the width of the depletion region also increases.**
- **Since the base is lightly doped the depletion region penetrates deeper into the base region.**
- **This reduces the effective width of the base region.**
- **This variation or modulation of the effective base width by the collector – base voltage is known as base width modulation or early effect.**

The decrease in base width by the collector voltage has the following three effects

- **It reduces the chances of recombination of electrons with the holes in the** base region. Hence current gain increases with increase in collector – base voltage.

- The concentration gradient of minority carriers within the base increases. This increases the emitter current.
- For extremely collector voltage, the effective base width may be reduced to zero, resulting in voltage breakdown of a transistor. This phenomenon is known as punch through.

Output characteristics

The curve drawn between collector current and collector – base voltage, for a given value of emitter current is known as output characteristics.

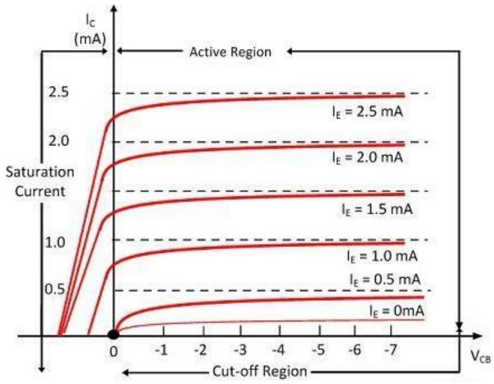


Fig. 4.15 Characteristics of CB Configuration

- **The active region of the collector-base junction is reverse biased, the collector current IC is almost equal to the emitter current IE. The transistor is always operated in this region.**
- **The curve of the active regions is almost flat. The large charges in VCB produce only a tiny change in IC. The circuit has very high output resistance ro.**
- **When VCB is positive, the collector-base junction is forward bias and the collector current decrease suddenly. This is the saturation state in which the collector current does not depend on the emitter current.**
- **When the emitter current is zero, the collector current is not zero. The current which flows through the circuit is the reverse leakage current, i.e., ICBO. The current is temperature depends and its value range from 0.1 to 1.0 µA for silicon transistor and 2 to 5 µA for germanium transistor.**

Output Resistance

$$r_0 = \frac{\Delta V_{CB}}{\Delta I_C}$$

The ratio of change in collector-base voltage to the change in collector current at constant emitter current IE is known as output resistance.

COMMON – EMITTER CONFIGURATION

- **The input is connected between base and emitter, while output is connected between collector and emitter**
- **Emitter is common to both input and output circuits.**
- **The bias voltage applied are Vce and Vbe.**
- **The emitter-base junction is forward biased and collector-emitter junction is reverse biased.**
- **The base current Ib flows in the input circuit and collector current Ic flows sin the output circuit.**

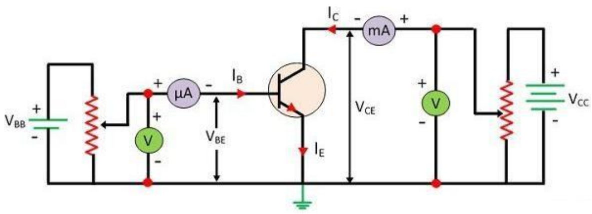


Fig. 4.16 CE Configuration

INPUT CHARACTERISTICS

- **The curve plotted between base current IB and the base-emitter voltage VEB is called Input characteristics curve.**
- **For drawing the input characteristic the reading of base currents is taken through the ammeter on emitter voltage VBE at constant collector-emitter current.**
- **The curve for different value of collector-base current is shown in the figure below.**

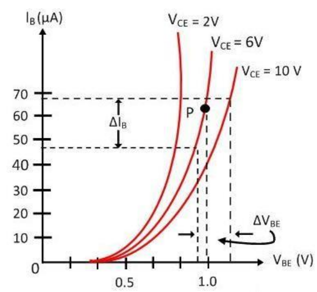


Fig. 4.17 Characteristics of CE Configuration

- **The curve for common Emitter configuration is similar to a forward diode characteristic.**
- **The base current IB increases with the increases in the emitter-base voltage VBE. Thus the input resistance of the CE configuration is comparatively higher that of CB configuration.**
- **The effect of CE does not cause large deviation on the curves, and hence the effect of a change in VCE on the input characteristic is ignored.**

Input Resistance:

The ratio of change in base-emitter voltage VBE to the change in base current ΔIB at

constant collector-emitter voltage VCE is known as input resistance, i.e.,

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} \text{ at constant } V_{CE}$$

OUTPUT CHARACTERISTIC

In CE configuration the curve draws between collector current IC and collector-emitter voltage VCE at a constant base current IB is called output characteristic. The characteristic curve for the typical NPN transistor in CE configuration is shown in the figure below.

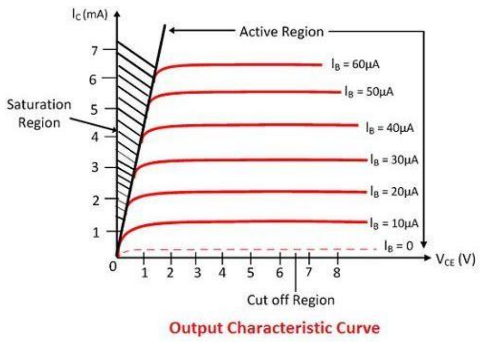


Fig. 4.18 Characteristics of CE Configuration

- In the active region, the collector current increases slightly as collector-emitter VCE current increases. The slope of the curve is quite more than the output characteristic of CB configuration. The output resistance of the common base connection is more than that of CE connection.
- The value of the collector current IC increases with the increase in VCE at constant voltage IB, the value β of also increases.
- When the VCE falls, the IC also decreases rapidly. The collector-base junction of the transistor always in forward bias and work saturate. In the saturation region, the collector current becomes independent and free from the input current IB
- In the active region IC = βIB, a small current IC is not zero, and it is equal to reverse leakage current ICEO.

Output Resistance:

The ratio of the variation in collector-emitter voltage to the collector-emitter current is known at collector currents at a constant base current IB is called output resistance ro.

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} \text{ at constant } I_B$$

The value of output resistance of CE configuration is more than that of CB.

COMMON - COLLECTOR CONFIGURATION

The configuration in which the collector is common between emitter and base is known as

CC configuration. In CC configuration, the input circuit is connected between emitter and base and the output is taken from the collector and emitter. The collector is common to both the input and output circuit and hence the name common collector connection or common collector configuration.

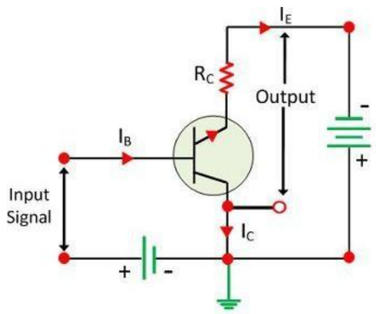


Fig. 4.19 CB Configuration

INPUT CHARACTERISTICS

The input characteristic of the common collector configuration is drawn between collector base voltage VCE and base current IB at constant emitter current voltage VCE. The value of the output voltage VCE changes with respect to the input voltage VBC and IB With the help of these values, input characteristic curve is drawn. The input characteristic curve is shown below.

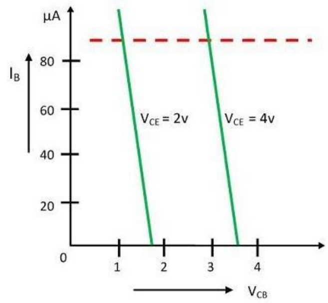


Fig. 4.20 Input Characteristics of CC Configuration

OUTPUT CHARACTERISTICS

The output characteristic of the common emitter circuit is drawn between the emitter-collector voltage VEC and output current IE at constant input current IB. If the input current IB is zero, then the collector current also becomes zero, and no current flows through the transistor.

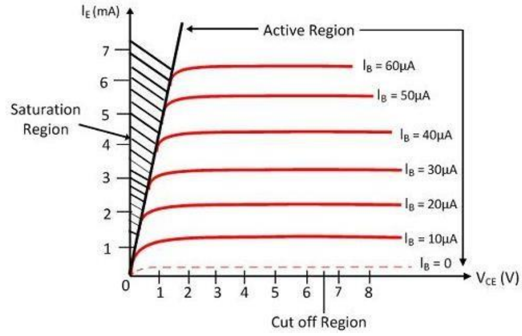


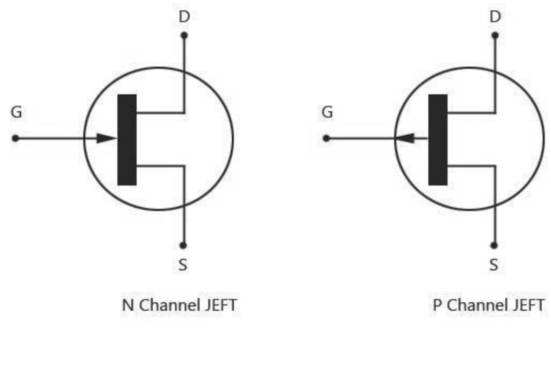
Fig. 4.21 Output Characteristics of CC Configuration

The transistor operates in active region when the base current increases and reaches to saturation region. The graph is plotted by keeping the base current IB constant and varying the emitter-collector voltage VCE, the values of output current IE are noticed with respect to VCE. By using the VCE and IE at constant IB the output characteristic curve is drawn.

JUNCTION FIELD EFFECT TRANSISTOR (JFET)

JFET is a unipolar-transistor, which acts as a voltage controlled current device and is a device in which current at two electrodes is controlled by the action of an electric field at a PN junction.

A JFET, or junction field-effect transistor, or JUGFET, is a FET in which the gate is created by reverse-biased junction (as opposed to the MOSFET which creates a junction via a field generated by conductive gate, separated from the gate region by a thin insulator).



JFET-N-Channel and P-channel Schematic Symbol

Fig. 4.22 JFET

CONSTRUCTION

N-Channel JFET

The figure shows construction and symbol of N-channel JFET. A small bar of extrinsic semiconductor material, N type is taken and its two ends, two ohmic contacts are made which is the drain and source terminals of FET.

Heavily doped electrodes of P type material form PN junctions on each side of the bar. The thin region between the two P gates is called the channel. Since this channel is in the N type bar, the FET is known as N-channel JFET.

The electrons enter the channel through the terminal called source and leave through the terminal called drain. The terminals taken out from heavily doped electronics of P type material are called gates. These electrodes are connected together and only one terminal is taken out, which is called gate, as shown below.

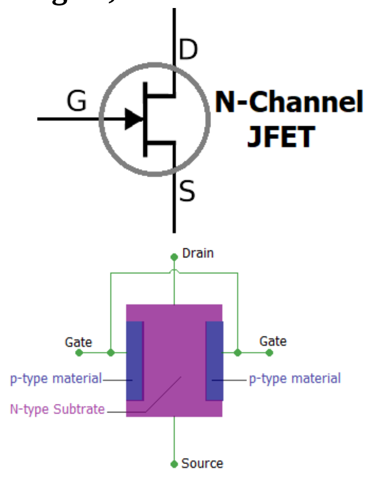


Fig. 4.23 N-Channel JFET

P-Channel JFET

The device could be made of P type bar with two N type gates as shown in the figure below. This will be P- channel JFET. The principle of working of N-channel JFET

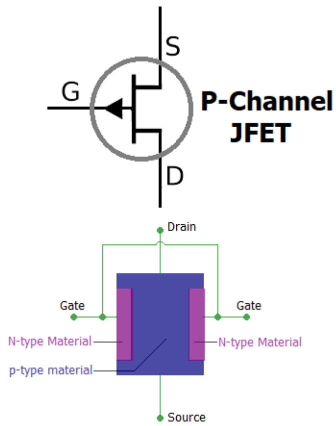


Fig. 4.24 P-Channel JFET

and P-channel JFET are similar. The only difference being that in N-channel JFET the current is carried by electrons while in P-channel JFET, it is carried by holes.

WORKING OF JFET

One best example to understand the working of a JFET is to imagine the garden hose pipe. Suppose a garden hose is providing a water flow through it. If we squeeze the hose the water flow will be less and at a certain point if we squeeze it completely there will be zero water flow. JFET works exactly in that way. If we interchange the hose with a JFET and the water flow with a current and then construct the current-carrying channel, we could control the current flow.

When there is no voltage across gate and source, the channel becomes a smooth path which is wide open for electrons to flow. But the reverse thing happens when a voltage is applied between gate and source in reverse polarity, which makes the P-N junction, reversed biased and makes the channel narrower by increasing the depletion layer and could put the JFET in cut-off or pinch off region.

In the below image we can see the saturation mode and pinch off mode and we will be

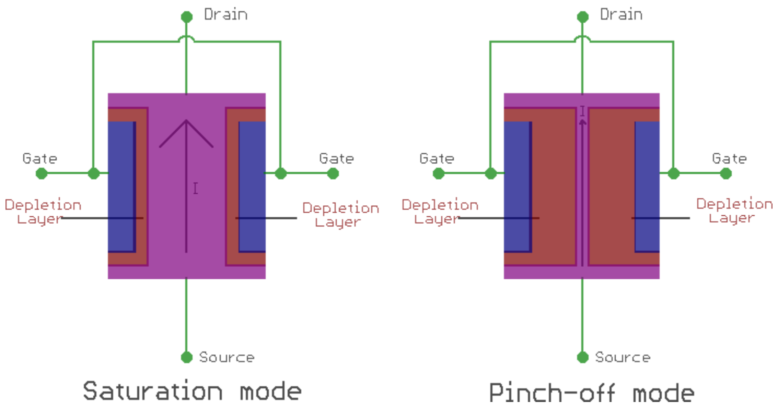


Fig. 4.25 Operation of JFET

able to understand the depletion layer became wider and the current flow becomes less.If we want to switch off a JFET we need to provide a negative gate to source voltage denoted as VGS for an N-type JFET. For a P-type JFET, we need to provide positive VGS.

JFET only works in the depletion mode.

CHARACTERISTICS OF JFETS

There are two types of static characteristics viz

1. Output or drain characteristics and
2. Transfer characteristic.

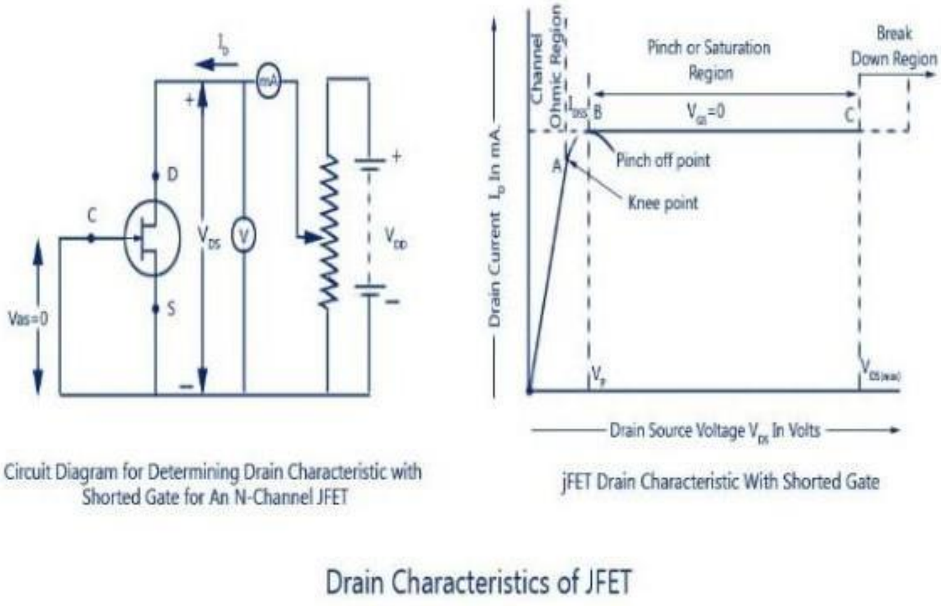


Fig. 4.26 Drain Characteristics of JFET

OUTPUT OR DRAIN CHARACTERISTICS.

- **The curve drawn between drain current Ip and drain-source voltage VDS with gate-to source voltage VGS as the parameter is called the drain or output characteristic. This characteristic is analogous to collector characteristic of a BJT:**

TRANSFER CHARACTERISTICS OF JFET

- **The transfer characteristic for a JFET can be determined experimentally, keeping drain-source voltage, VDS constant and determining drain current, ID for various values of gate-source voltage, VGS.**
- **The circuit diagram is shown in fig. The curve is plotted between gate-source voltage, VGS and drain current, ID , as illustrated in fig. It is similar to the transconductance characteristics of a vaccum tube or a transistor.**
- **It is observed that**
  1. **Drain current decreases with the increase in negative gate-source bias**
  2. **Drain current, ID = IDSS when VGS = 0**
  3. **Drain current, ID = 0 when VGS = VD**

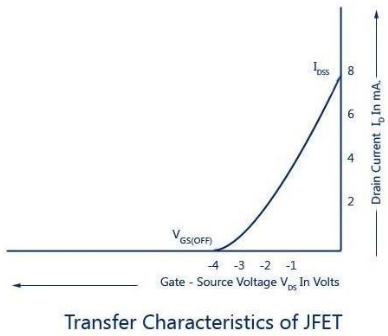


Fig. 4.27 Transfer Characteristics of JFET

The pinch off voltage is the value of VDS at which the drain current reaches its constant saturation value. Any further increase in VDS does not have any effect on the value of ID.

METAL OXIDE SEMICONDUCTOR FIELD EFFECT TRANSISTOR (MOSFET)

MOSFET stands for metal oxide semiconductor field effect transistor which is widely used for switching and amplifying electronic signals in the electronic devices. It is capable of voltage gain and signal power gain. The MOSFET is a core of integrated circuit and it can be designed and fabricated in a single chip because of these very small sizes.

The MOSFET is a four terminal device with source(S), gate (G), drain (D) and body (B) or substrate terminals. The body of the MOSFET is frequently connected to the source terminal so making it a three terminal device like field effect transistor. The MOSFET is very far the most common transistor and can be used in both analog and digital circuits.

The drain and source terminals are connected to the heavily doped regions. The gate terminal is connected top on the oxide layer.The metal of the gate terminal and the semiconductor acts the parallel and the oxide layer acts as insulator of the state MOS capacitor. Between the drain and source terminal inversion layer is formed and due to the flow of carriers in it, the current flows in MOSFET the inversion layer is properties are controlled by gate voltage. Thus it is a voltage controlled device.

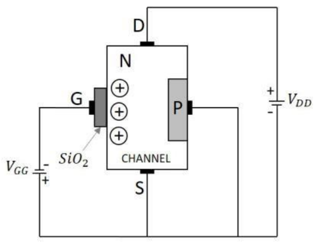


Fig. 4.28 MOSFET

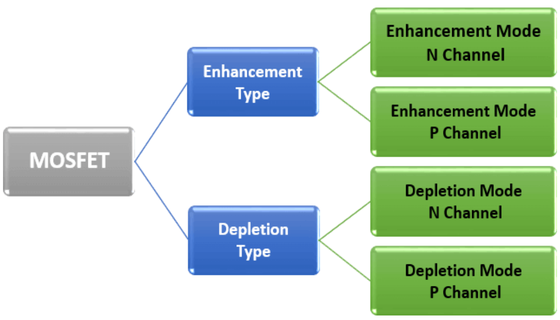


Fig. 4.29 Types of MOSFET

TYPES OF MOSFET

The MOSFET is classified into two types such as;

- Depletion type MOSFET
  - Depletion mode – Negative Gate - Source Voltage (VGS) is applied
  - Enhancement mode – Positive Gate - Source Voltage (VGS) is applied.
- Enhancement type MOSFET

Depletion Type MOSFET:

When there is zero voltage on the gate terminal, the channel shows its maximum conductance. As the voltage on the gate is negative or positive, then decreases the channel conductivity.

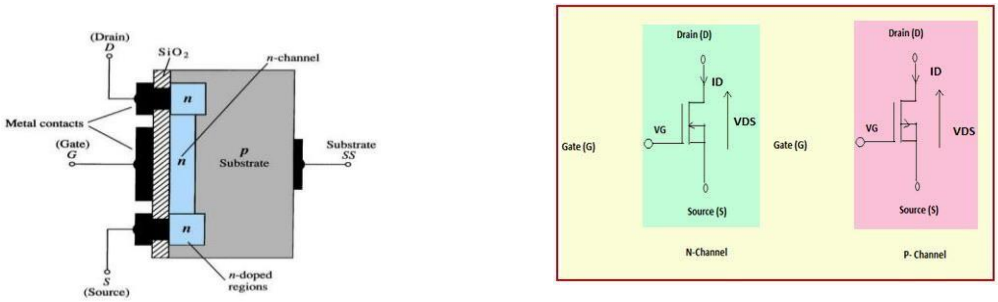
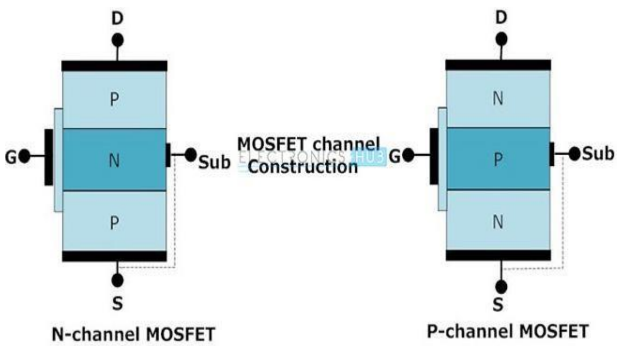


Fig. 4.30 Construction of Depletion Type MOSFET & Symbols

- **The Drain (D) and source (S) leads connect to the n – doped regions**
- **The n doped regions are connected by an N – Channel**
- **This N-Channel is connected to the Gate (G) through a thin insulating layer of SiO2.**
- **The n-doped material lies on a p-doped substrate that may have an additional terminal connection called SS.**

Enhancement type MOSFET:

When there is no voltage on the gate terminal the device does not conduct. More voltage applied on the gate terminal, the device has good conductivity.



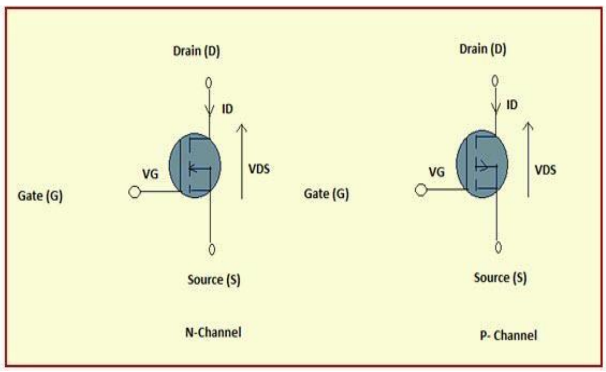


Fig. 4.31Construction of Enhancement Type of MOSFET & Symbols

- **The Enhancement type MOSFET is equivalent to "Normally Open" switch and these types of transistors require gate-source voltage to switch ON the device.**
- **The broken line is connected between the source and drain which represents the enhancement type. In enhancement type MOSFETs the conductivity increases by increasing the oxide layer which adds the carriers to the channel.**
- **Generally, this oxide layer is called as „Inversion layer". The channel is formed between the drain and source in the opposite type to the substrate, such as N-channel is made with a P-type substrate and P-channel is made with an N-type substrate. The conductivity of the channel due to electrons or holes depends on N-type or P-type channel respectively.**

WORKING OF DEPLETION TYPE MOSFET:

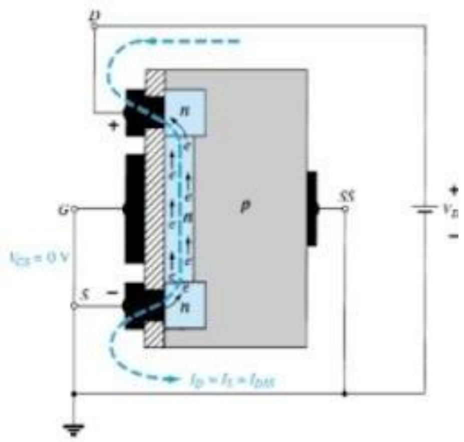


Fig. 4.32 Construction of Enhancement Type of MOSFET & Symbols

Working of Depletion Type MOSFET

The gate-to-source voltage is set to zero volts by the direct connection from one terminal to the other and a voltage VDS is applied across the drain to source terminals. The result is an attraction for the positive potential at the drain by the free electrons of the n-channel and a current similar to that established through the channel of the JFET. In fact, the resulting current with VGS = 0 V continues to be labeled IDSS, as shown in the characteristics of depletion type MOSFET in the below figure.

VGS has been set at a negative voltage such as 1 V. The negative potential at the gate will tend to pressure electrons toward the p-type substrate (like charges repel) and attract holes from the p-type substrate (opposite charges attract) as shown in the above figure

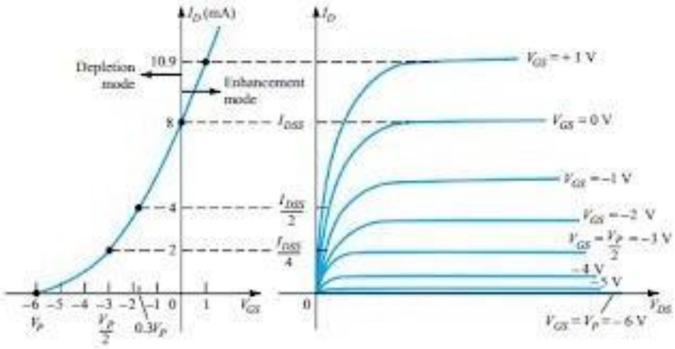


Fig. 4.33   Drain and Transfer Characteristics of Depletion type MOSFET

Depending on the magnitude of the negative bias established by VGS, a level of recombination between electrons and holes will occur that will reduce the number of free electrons in the n-channel available for conduction. The more negative the bias, the higher the rate of recombination. The resulting level of drain current is therefore reduced with increasing negative bias for VGS as shown in the figure below for VGS = - 1 V, - 2 V, and so on, to the pinch-off level of 6 V. The resulting levels of drain current and the plotting of the transfer curve proceeds exactly as described for the JFET.

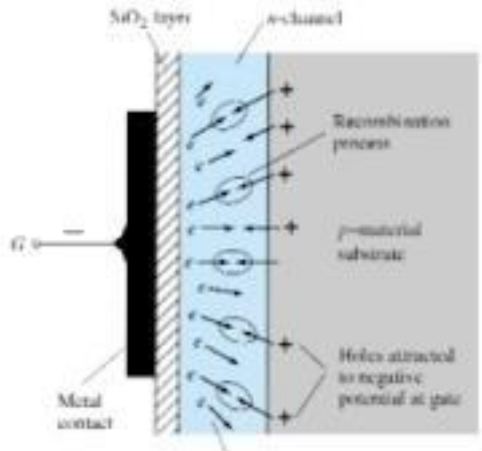


Fig. 4.34   Depletion type MOSFET

For positive values of VGS, the positive gate will draw additional electrons (free carriers) from the p-type substrate due to the reverse leakage current and establish new carriers through the collisions resulting between accelerating particles. As the gate to source voltage continues to increase in the positive direction, characteristics of depletion type MOSFET reveals that the drain current will increase at a rapid rate for the reasons listed above.

The vertical spacing between the VGS = 0 V and VGS = 1 V curves in the characteristic curve is a clear indication of how much the current has increased for the 1-V change in VGS. Due to the rapid rise, the user must be aware of the maximum drain

current rating since it could be exceeded with a positive gate voltage. That is, for the device of figure showing characteristics of depletion type MOSFET, the application of a voltage VGS = 4 V would result in a drain current of

mA, which could possibly exceed the maximum rating (current or power) for the device.

As revealed above, the application of a positive gate-to-source voltage has "enhanced" the level of free carriers in the channel compared to that encountered with VGS = 0 V. For this reason the region of positive gate voltages on the drain or transfer characteristics is often referred to as the enhancement region, with the region between cutoff and the saturation level of IDSS referred to as the depletion region.

WORKING OF ENHANCEMENT TYPE MOSFET(EMOSFET):

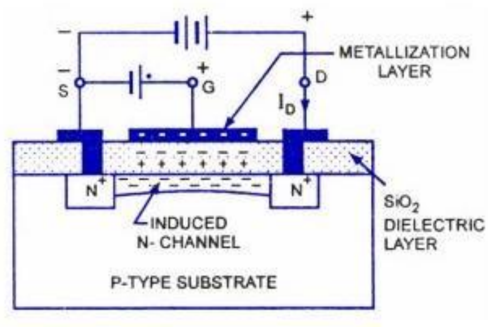


Fig. 4.35 Operation of Enhancement type MOSFET

Working of an EMOSFET

As its name indicates, this MOSFET operates only in the *enhancement mode* and has no depletion mode. It operates with large positive gate voltage only. It does not conduct when the gate-source voltage VGS = 0. This is the reason that it is called normally-off MOSFET. In these MOSFETâ€™s drain current ID flows only when VGS exceeds VGST [gate-to-source threshold voltage].

When drain is applied with positive voltage with respect to source and no potential is applied to the gate two N-regions and one P-substrate from two P-N junctions connected back to back with a resistance of the P-substrate. So a very small drain current that is, reverse leakage current flows. If the P-type substrate is now connected to the source terminal, there is zero voltage across the source substrate junction, and the⁻drain-substrate junction remains reverse biased.

When the gate is made positive with respect to the source and the substrate, negative (i.e. minority) charge carriers within the substrate are attracted to the positive gate and accumulate close to the-surface of the substrate. As the gate voltage is increased, more and more electrons accumulate under the gate. Since these electrons can not flow across the insulated layer of silicon dioxide to the gate, so they accumulate at the surface of the substrate just below the gate. These accumulated minority charge carriers N -type channel

stretching from drain to source. When this occurs, a channel is induced by forming what is termed an *inversion layer* (N-type). Now a drain current start flowing. The strength of the drain current depends upon the channel resistance which, in turn, depends upon the number of charge carriers attracted to the positive gate. Thus drain current is controlled by the gate potential.

Since the conductivity of the channel is enhanced by the positive bias on the gate so this device is also called the *enhancement MOSFET* or E- MOSFET.

The minimum value of gate-to-source voltage VGS that is required to form the inversion layer (N-type) is termed the *gate-to- source threshold voltage* VGST. For VGS below VGST, the drain current ID = 0. But for VGS exceeding VGST an N-type inversion layer connects the source to drain and the drain current ID is large. Depending upon the device being used, VGSTmay vary from less than 1 V to more than 5 V.
JFETs and DE-MOSFETs are classified as the depletion-mode devices because their conductivity depends on the action of depletion layers. E-MOSFET is classified as an enhancement-mode device because its conductivity depends on the action of the inversion layer. Depletion-mode devices are normally ON when the gate-source voltage VGS = 0, whereas the enhancement-mode devices are normally OFF when VGS = 0.

CHARACTERISTICS OF AN EMOSFET.

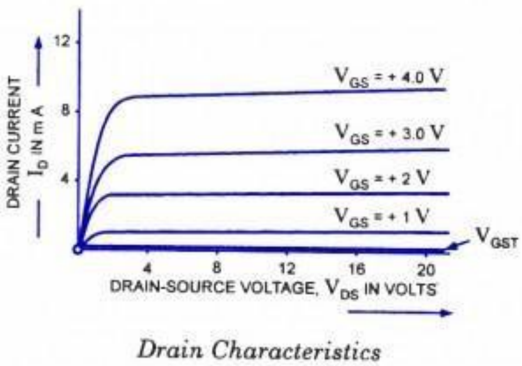


Fig. 4.36 Characteristics of Enhancement type MOSFET

Drain Characteristics-EMOSFET

Drain characteristics of an N-channel E-MOSFET are shown in the above figure. The lowest curve is the VGST curve. When VGS is lesser than VGST, ID is approximately zero. When VGS is greater than VGST, the device turns- on and the drain current ID is controlled by the gate voltage. The characteristic curves have almost vertical and almost horizontal parts. The almost vertical components of the curves correspond to the ohmic region, and the horizontal components correspond to the constant current region. Thus E-MOSFET can be operated in either of these regions i.e. it can be used as a variable-voltage resistor (WR) or as a constant current source.

EMOSFET-Transfer Characteristics

The above figure shows a typical transconductance curve. The current I_{DSS} at V_{GS} <=0 is very small, being of the order of a few nano-amperes. When the V_{GS} is made positive, the drain current I_D increases slowly at first, and then much more rapidly with an increase in V_{GS}. The equation for the transfer characteristic does not obey equation. However it does follow a similar "square law type" of relationship. The equation for the transfer characteristic of E-MOSFETs is given as: $I_D=K(V_{GS}-V_{GST})^2$

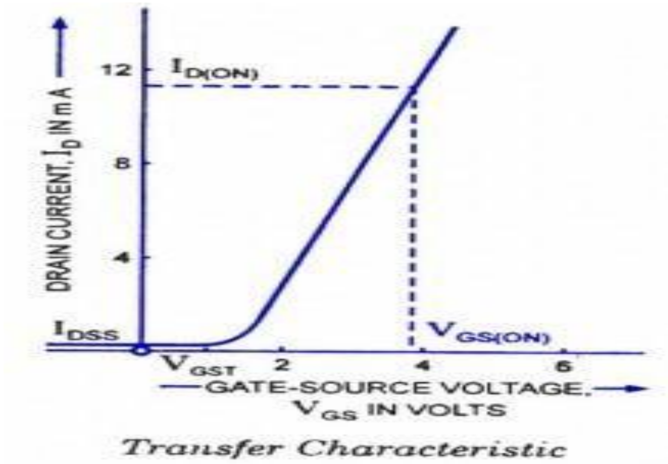


Fig. 4.37 Transfer Characteristics of Enhancement type MOSFET

Handling Precautions for MOSFET

The MOSFET has the drawback of being very susceptible to overload voltage and may require special handling during installation. The MOSFET gets damaged easily if it is not properly handled. A very thin layer of SiO_2, between the gate and channel is damaged due to high voltage and even by static electricity. The static electricity may result from the sliding of a device in a plastic bag. If a person picks up the transistor by its case and brushes the gate against some grounded objects, a large electrostatic discharge may result. In a relatively dry atmosphere, a static potential of 300V is not uncommon on a person who has high resistance soles on his footwear.

MOSFETs are protected by a shorting ring that is wrapped around all four terminals during shipping and must remain in place until after the devices soldered in position. prior to soldering ,the technician should use a shorting strap to discharge his static electricity and make sure that the tip of the soldering iron is grounded. Once in circuit, there are usually low resistances present to prevent any excessive accumulation of electro static charge .However, the MOSFET should never be inserted into or removed from a circuit with the power ON.JFET is not subject to these restrictions, and even some MOSFETs have a built in gate protection known as "integral gate protection", a system built into the device to get around the problem of high voltage on the gate causing a puncturing of the oxide layer. The manner in which this is done is shown in the cross sectional view of Fig.7.11.The symbol clearly shows that between each and the source is placed a back-to-back (or front-to-front) pair of diodes, which are built right into P type substrate.

FET as Voltage-Variable Resistor

FET is operated in the constant-current portion of its output characteristics for the linear applications. In the region before pinch-off, where VDS is small, the drain to source resistance rd can be controlled by the bias voltage VGS. The FET is useful as a voltage variable resistor (VVR) or voltage dependent resistor (VDR).

In JFET, the drain to source conductance gd =ID/VDS for small values of VDS, which may also be expressed as gd=g do [1-(VGS/VP)$^{1/2}$]

Where, gdo is the value of drain conductance when the bias voltage VGS is zero. The variation of the rd with VGS can be closely approximated by the empirical expression,

rd =ro/(1-KVGS)

Where ro=drain resistance at zero gate bias, and K=a constant, dependent upon FET type.

Comparison of MOSFET and JFET

1. **In enhancement and depletion types of MOSFET, the transverse electric field induced across an insulating layer deposited on the semiconductor material controls the conductivity of the channel. In the JFET the transverse electric field across the reverse biased PN junction controls the conductivity of the channel.**
2. **The gate leakage current in a MOSFET is of the order of 10^{-12}A.Hence the input resistance of a MOSFET is very high in the order of 10^{10} to 10^{15} ohm. The gate leakage current of a JFET is of the order of 10^{-9}A and its input resistance is of the order of 10^{8} ohm.**
3. **The output characteristics of the JFET are flatter than those of the MOSFET and hence, the drain resistance of a JFET(0.1 to 1Mohm) is much higher than that of a MOSFET(1 to 50 K ohm)**
4. **JFETs are operated only in the depletion mode. The depletion type MOSFET may be operated in both depletion and enhancement mode.**
5. **Comparing to JFET, MOSFETs are easier to fabricate.**
6. **MOSFET is very susceptible to overload voltage and needs special handling during installation. It gets damaged easily if it is not properly handled.**
7. **MOSFET has zero offset voltage. As it is a symmetrical device, the source and drain can be interchanged. These two properties are very useful in analog signal switching.**
8. **Special digital CMOS circuits are available which involves near –zero power dissipation and very low voltage and current requirements. This makes them most suitable for portable systems.**

Comparison of JFET and BJT

1. **FET operations depend only on the flow of majority carrier-holes for P-channel FETs and electrons for N-channel FETs. Therefore, they are called Unipolar devices. Bipolar transistor (BJT) operation depends on both minority and majority current carrier.**
2. **As FET has no junctions and the conduction is through an N-type or P-type semiconductor material, FET is less noisy than BJT.**

3. As the input circuit of FET is reverse biased, FET exhibits as much higher input impedance (in the order of 100MOHM) and lower output impedance and there will be a high degree of isolation between input and output. So, FET can act as excellent buffer amplifier but the BJT has low input impedance because its input circuit is forward biased.
4. FET is a voltage control device, i.e. voltage at the input terminal controls the output current, whereas BJT is a current control device, i.e. the input current controls the output current.
5. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
6. The performance of BJT is degraded by neutron radiations because of reduction in minority carrier life time, whereas FET can tolerate a much higher level of radiation since they do not rely on minority carrier for their operation.
7. The performance of FET is relatively unaffected by ambient temperature changes. As it has a negative temperature coefficient at high current levels, it prevents the FET from thermal break down. The BJT has a positive temperature coefficient at high current levels which leads to thermal break down.
8. Since FET does not suffer from minority carrier storage effects, it has a higher switching speeds and cut off frequencies.BJT suffers a minority carrier storage effects and therefore has lower switching speed and cut off frequencies.
9. FET amplifiers have low gain bandwidth product due to the junction capacitive effects and produce more signal distortion except for small signal operation.
10. BJT are cheaper to produce than FETs.

Silicon Controlled Rectifier (SCR)

A silicon controlled rectifier is a semiconductor device that acts as a true electronic switch. It can change alternating current into direct current and at the same time can control the amount of power fed to the load. Thus *SCR* combines the features of a rectifier and a transistor.

Constructional details.

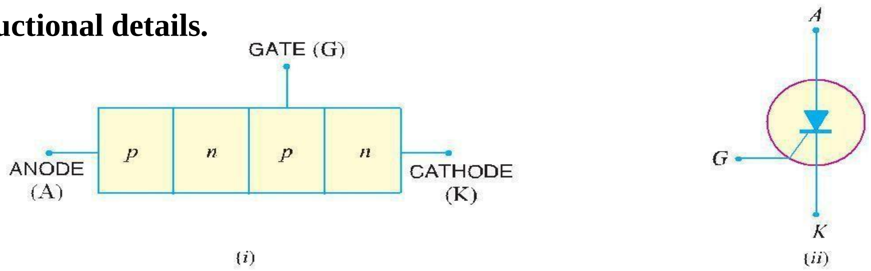


Fig. 4.38 SCR

When a PN junction is added to a junction transistor, the resulting three pn junction device is called a silicon controlled rectifier. Fig. I shows its construction. It is clear that it is essentially an ordinary rectifier (pn) and a junction transistor (npn) combined in one unit to form pnpn device. Three terminals are taken; one from the outer p-type material called anode A, second from the outer n-type material called cathode K and the third from the base of transistor section and is called gate
G. In the normal operating conditions of SCR, anode is held at high positive potential with respect to cathode and gate at small positive potential with respect to cathode. Fig. (ii)

shows the symbol of SCR. The silicon controlled rectifier is a solid state equivalent of thyratron. The gate, anode and cathode of SCR correspond to the grid, plate and cathode of thyratron. For this reason, SCR is sometimes called thyristor.

Working of SCR

In a silicon controlled rectifier, load is connected in series with anode. The anode is always kept at positive potential *w.r.t.*
cathode. The working of *SCR* can be studied under the following two heads:
When gate is open. The below diagram shows that the SCR circuit with gate open i.e. no voltage applied to the gate. Under this condition, junction J2 is reverse biased while junctions J1 and J3 are forward biased. Hence, the situation in the junctions J1 and J3 is just as in a npn transistor with base open. Consequently, no current flows through the load RL and the SCR. However, if the applied voltage is gradually increased, a stage is reached when reverse biased junction J2 breaks down. The SCR now conducts heavily and is said to be in the ON state. The applied voltage at which SCR conducts heavily without gate voltage is called Break over voltage.

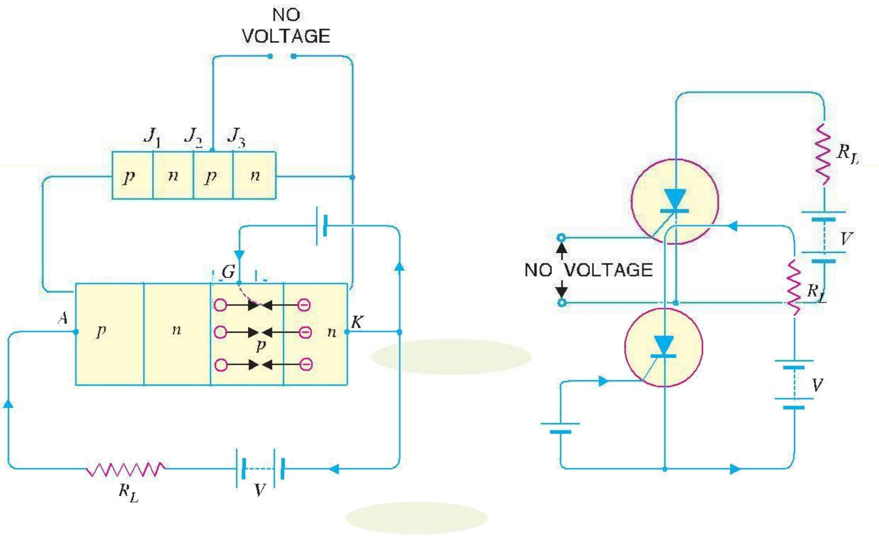


Fig. 4.39 Operation of SCR

(*ii*) When gate is positive w.r.t. cathode.

The SCR can be made to conduct heavily at smaller applied voltage by applying a small positive potential to the gate as shown in Now junction J3 is forward biased and junction J2 is reverse biased. The electrons from n-type material start moving across junction J3 towards left whereas holes from p-type towards the right. Consequently, the electrons from junction J3 are attracted across junction J2 and gate current starts flowing. As soon as the gate current flows, anode current increases. The increased anode current in turn makes more electrons available at junction J2. This process continues and in an extremely small time, junction J2 breaks down and the SCR starts conducting heavily.

Once SCR starts conducting, the gate (the reason for this name is obvious) loses all control. Even if gate voltage is removed, the anode current does not decrease at all. The only way to stop conduction (i.e. bring SCR in off condition) is to reduce the applied voltage to zero.

The whole applied voltage V appears as reverse bias across junction J2 as junctions J1 and J3 are forward biased. Because J1 and J3 are forward biased and J2 has broken down.

Conclusion. The following conclusions are drawn from the working of SCR : An SCR has two states i.e. either it does not conduct or it conducts heavily. There is no state in between. Therefore, SCR behaves like a switch.There are two ways to turn on the SCR. The first method is to keep the gate open and make the supply voltage equal to the breakover voltage. The second method is to operate SCR with supply voltage less than breakover voltage and then turn it on by means of a small voltage (typically 1.5 V, 30 mA) applied to the gate.Applying small positive voltage to the gate is the normal way to close an SCR because the breakover voltage is usually much greater than supply voltage.To open the SCR (i.e. to make it non- conducting), reduce the supply voltage to zero.

VI CHARACTERISTICS OF SCR:

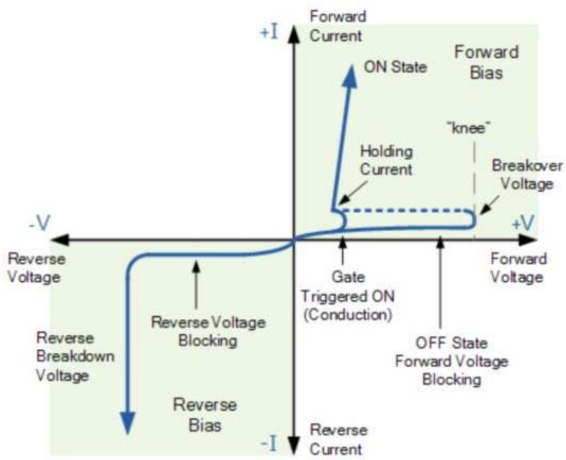


Fig. 4.40 V-I characteristics of SCR

The V-I characteristics of the SCR reveal that the SCR can be operated in three modes. Forward blocking mode (off state) Forward conduction mode (on state) Reverse blocking mode (off state) Forward blocking mode

In this mode of operation, the anode is given a positive potential while the cathode is given a negative voltage, keeping the gate at zero potential i.e. disconnected. In this case junction J1 and J3 are forward biased while J2 is reversed biased due to which only a small leakage current exists from the anode to the cathode until the applied voltage reaches its breakover value, at which J2 undergoes avalanche breakdown and at this breakover voltage it starts conducting, but below breakover voltage it offers very high resistance to the current and is said to be in the off state.

Forward conduction mode

SCR can be brought from blocking mode to conduction mode in two ways: either by increasing the voltage across anode to cathode beyond break over voltage or by applying of positive pulse at gate. Once it starts conducting, no more gate voltage is required to maintain it in the on state. There are two ways to turn it off: 1. Reduce the current through it below a minimum value called the holding current and 2. With the Gate turned off, short out the Anode and Cathode momentarily with a push- button switch or transistor across the junction.

Reverse blocking mode

In this mode SCR is reversed biased , ie when anode is negative compared to cathode. The characteristic of this region are similar to those of an ordinary PN junction diode. in this region ,junctionJ1and J3 are reversed biased whereas j2 is farward biased

.the device behaves as if two diodes are connected in series with a reverse voltage applied to them. A small leakage current of the order of mill amperes or micro amperes flow in the device. This reverse blocking mode is called the OFF state of the thyristor .when the reverse voltage of the SCR increases to a large extent breakdown occurs and the current in the device increases rapidly. Thus when the SCR is biased in this region the power dissipated is very high, if the power dissipated is more than the rated value of the SCR , the SCR is permanently damaged .thus in the reverse bias condition the voltage should never cross the breakdown voltage.

Characteristics of SCR
It is the curve between anode-cathode voltage (V) and anode current (I)of an SCR at constant gate current

Forward characteristics.

When anode is positive w.r.t. cathode, the curve between V and I is called the forward characteristic. In the above fig, OABC is the forward characteristic of SCR at IG = 0.If the supply voltage is increased from zero, a point is reached (point A) when the SCR starts conducting. Under this condition, the voltage across SCR suddenly drops as shown by dotted curve AB and most of supply voltage appears across the load resistance RL. If proper gate current is made to flow,SCR can close at much smaller supply voltage.

Reverse characteristics.
When anode is negative w.r.t. cathode, the curve between V and I is known as reverse characteristic. The reverse voltage does come across SCR when it is operated witha.c. supply. If the reverse voltage is gradually increased, at first the anode current remains small (i.e.leakage current) and at some reverse voltage, avalanche breakdown occurs and the SCR starts conducting heavily in the reverse direction as shown by the curve DE. This maximum reverse voltage at which SCR starts conducting heavily is known as reverse breakdown voltage.

EQUIVALENT CIRCUIT OF SCR

The *SCR* shown in Fig. 20.4 (*i*) can be visualised as separated into two transistors as shown in

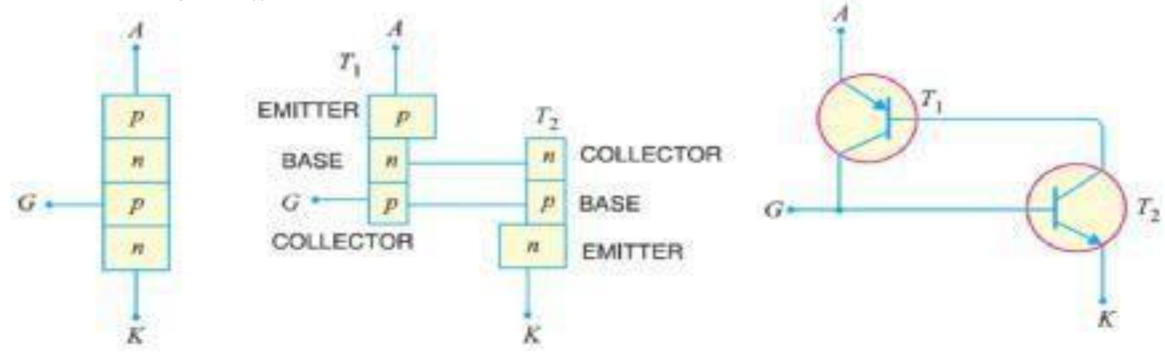


Fig. 4.41 Equivalent circuit of SCR

Thus, the equivalent circuit of *SCR* is composed of *pnp* transistor and *npn* transistor connected as shown in the above fig. It is clear that collector of each transistor is coupled to the base of the other, thereby making a positive feedback loop. The working of *SCR* can be easily explained from its equivalent circuit. The above fig shows the equivalent circuit of *SCR* with supply voltage *V* and load resistance *RL*. Assume the supply voltage *V* is less than break over voltage as is usually the case.
With gate open (*i.e.* switch *S* open), there is no base current in transistor *T*2. Therefore, no current flows in the collector of *T*2 and hence that of *T*1. Under such conditions, the

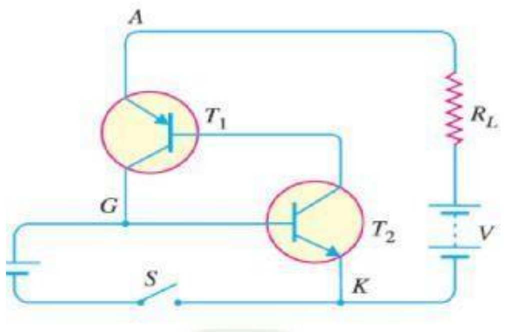


Fig. 4.42 Equivalent circuit of SCR **– Two Transistor model**

***SCR* is open. However, if switch *S* is closed, a small gate current will flow through the base of *T*2 which means its collector current will increase.**

The collector current of *T*2 is the base current of *T*1.Therefore, collector current of *T*1 increases. But collector current of *T*1 is the base current of *T*2. This action is accumulative since an increase of current in one transistor causes an increase of current in the other transistor. As a result of this action, both transistors are driven to saturation, and heavy current flows through the load *RL*. Under such conditions, the *SCR* closes.

APPLICATIONS

SCRs are used in many areas of electronics where they find uses in a variety of different applications. Some of the more common applications for them are outlined below:

- **AC power control (including lights, motors,etc).**
- **Overvoltage protection crowbar for power supplies.**
- **AC power switching.**
- **Control elements in phase angle triggered controllers.**
- **Within photographic flash lights where they act as the switch to discharge a stored voltage through the flash lamp, and then cut it off at the required time.**

Thyristors are able to switch high voltages and withstand reverse voltages making them ideal for switching applications, especially within AC scenarios.

UNI JUNCTION TRANSISTOR (UJT)

A unijunction transistor (abbreviated as UJT) is a three-terminal semiconductor switching device. This device has a unique characteristic that when it is triggered, the emitter current increases regeneratively until it is limited by emitter power supply. Due to this characteristic, the unijunction transistor can be employed in a variety of applications e.g., switching, pulse generator, saw-tooth generator etc

CONSTRUCTION.

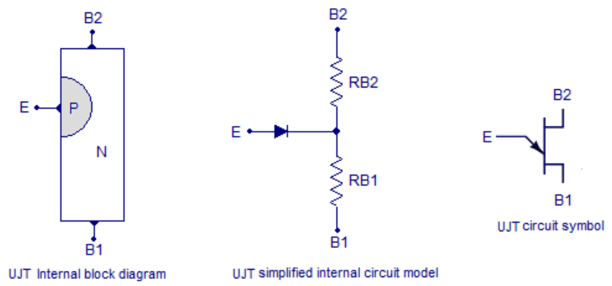


Fig. 4.43 UJT

The above Fig shows the basic structure of a unijunction transistor. It consists of an n-type silicon bar with an electrical connection on each end. The lead to these connections arecalled base leads base-one B1 and base two B2. Part way along the bar between the two bases, nearer to B2 than B1, a pn junction is formed between a p-type emitter and the bar. The lead to this junction is called the emitter lead E. Fig shows the symbol of unijunction transistor. Note that emitter is shown closer to B2 than B1.
i)Since the device has one pn junction and three leads, it is commonly called a unijunction transistor (uni means single).

With only one pn-junction, the device is really a form of diode. Because the two base terminals are taken from one section of the diode, this device is also called double-based diode.

The emitter is heavily doped having many holes. The n region, however, is lightly doped. For this reason, the resistance between the base terminals is very high (5 to 10 kΩ) when emitter lead is open.

WORKING PRINCIPLE OF UJT

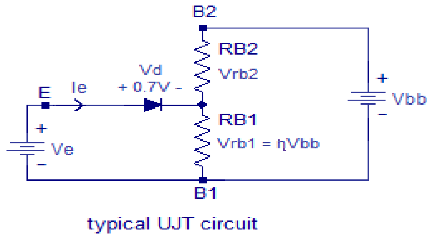


Fig. 4.44 Operation of UJT

The above fig shows the basic circuit operation of a unijunction transistor. The device has normally B2 positive w.r.t. B1.If voltage VBB is applied between B2 and B1 with emitter open, a voltage gradient is established along the n-type bar. Since the emitter is located nearer to B2, more than half of VBB appears between the emitter and B1. The voltage V1 between emitter and B1 establishes a reverse bias on the pn junction and the emitter current is cut off. Of course, a small leakage current flows from B2 to emitter due to minority carriers.

If a positive voltage is applied at the emitter, the pn junction will remain reverse biased so long as the input voltage is less than V1. If the input voltage to the emitter exceeds V1, the pn junction becomes forward biased. Under these conditions, holes are injected from p-type material into the n-type bar. These holes are repelled by positive B2 terminal and they are attracted towards B1 terminal of the bar. This accumulation of holes in the emitter to B1 region results in the decrease of resistance in this section of the bar. The result is that internal voltage drop from emitter to B1 is decreased and hence the emitter current IE increases. As more holes are injected, a condition of saturation will eventually be reached. At this point, the emitter current is limited by emitter power supply only. The device is now in the ON state.

If a negative pulse is applied to the emitter, the pn junction is reverse biased and the emitter current is cut off. The device is then said to be in the OFF state.

Intrinsic standoff ratio:

For ease of understanding, the internal model of the UJT is used in the circuit. B2 terminal of the UJT is made positive with respect to B1 terminal using the voltage source Vbb. Emitter terminal E of the UJT is forward biased using the voltage source Ve. Current starts flowing into the emitter only when the bias voltage Ve has exceeded the forward drop of the internal diode (Vd) plus the voltage drop across RB1 (Vrb1). This condition can be expressed using the following equation.

Ve = Vd + Vrb1

Vrb1 = Vbb* (RB1/(RB1+RB2))

Considering the intrinsic stand off ratio η = RB1/(RB1+RB2), the equation becomes

Ve = Vd+ η ·Vbb

A typical silicon diode has a forward voltage drop of 0.7V. When this factor is considered, the equation can be re written as

Ve = 0.7V + η *·Vbb*

CHARACTERISTICS OF UJT

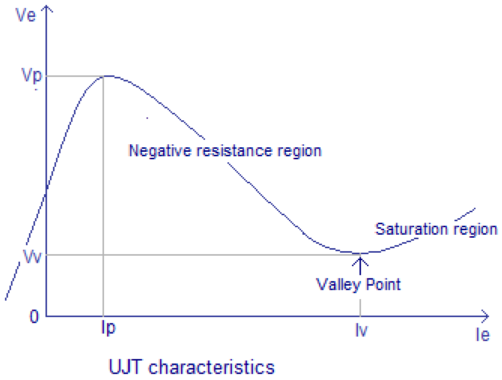


Fig. 4.45 Characteristics of UJT

The above Fig. shows the curve between emitter voltage (VE) and emitter current (IE) of a UJT at a given voltage VBB between the bases. This is known as the emitter characteristic of UJT. The following points may be noted from the characteristics :
(I)Initially, in the cut-off region, as VE increases from zero, slight leakage current flows from terminal B2 to the emitter. This current is due to the minority carriers in the reverse biased diode.
Above a certain value of VE, forward IE begins to flow, increasing until the peak voltage VP and current IP are reached at point P.
After the peak point P, an attempt to increase VE is followed by a sudden increase in emitter current IE with a corresponding decrease in VE. This is a negative resistance portion of the curve because with increase in IE, VE decreases. The device, therefore, has a negative resistance region which is stable enough to be used with a great deal of reliability in many areas e.g., trigger circuits, sawtooth generators, timing circuits.

ADVANTAGES OF UJT

The UJT was introduced in 1948 but did not become commercially available until 1952. Since then, the device has achieved great popularity due to the following reasons :

- **It is a low cost device.**
- **It has excellent characteristics.**
- **It is a low-power absorbing device under normal operating conditions.**

APPLICATIONS OF UJT

Due to above reasons, this device is being used in a variety of applications. A few include oscillators, trigger circuits, saw- tooth generators, bistable network etc.
The UJT is very popular today mainly due to its high switching speed.

A few selected applications of the UJT are as follows:
It is used to trigger SCRs and TRIACs It is used in non-sinusoidal oscillators

It is used in phase control and timing circuits It is used in saw tooth generators It is used in oscillator circuit design.

TEXT / REFERENCE BOOKS

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2. **G.K.Mithal, Basic Electronic Devices and circuits, 2nd Edition, G.K.Publishers Pvt, 2008**

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UNIT – V
RECTIFIERS, AMPLIFIERS AND OSCILLATORS

UNIT-5

RECTIFIERS, AMPLIFIERS AND OSCILLATORS

RECIFIERS

A rectifier is nothing but a simple diode or group of diodes which converts the Alternating Current (AC) into Direct Current (DC).

We know that a diode allows electric current in one direction and blocks electric current in another direction. We are using this principle to construct various types of rectifiers. Rectifiers are classified into different types based on the number of diodes used in the circuit or arrangement of diodes in the circuit.

The basic types of rectifiers are:

- Half wave rectifier
- Full wave rectifier.

Half wave rectifier definition

A half wave rectifier is a type of rectifier which converts the positive half cycle (positive current) of the input signal into pulsating DC (Direct Current) output signal.

or

A half wave rectifier is a type of rectifier which allows only half cycle (either positive half cycle or negative half cycle) of the input AC signal while the half cycle is blocked.

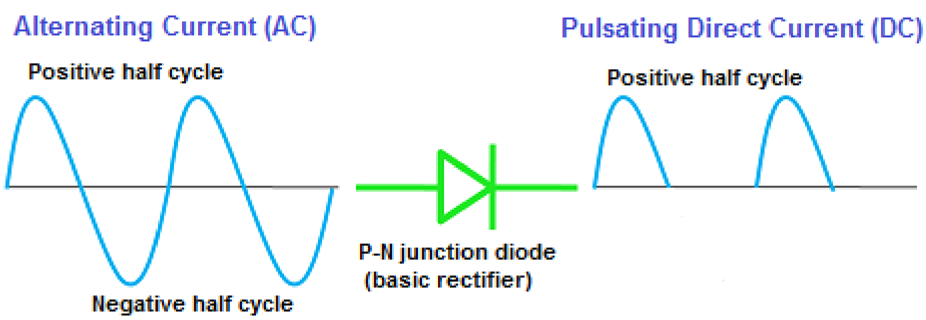


Fig 5.1: Waveforms of AC and DC

For example, if the positive half cycle is allowed then the negative half cycle is blocked. Similarly, if the negative half cycle is allowed then the positive half cycle is blocked. However, a half wave rectifier will not allow both positive and negative half cycles at the same time. Therefore, the half cycle (either positive or negative) of the input signal is wasted.

HALF WAVE RECTIFIER OPERATION

The half wave rectifier is the simplest form of the rectifier. We use only a single diode to construct the half wave rectifier. The half wave rectifier is made up of an AC source, transformer (step-down), diode, and resistor (load). The diode is placed between the transformer and resistor (load).

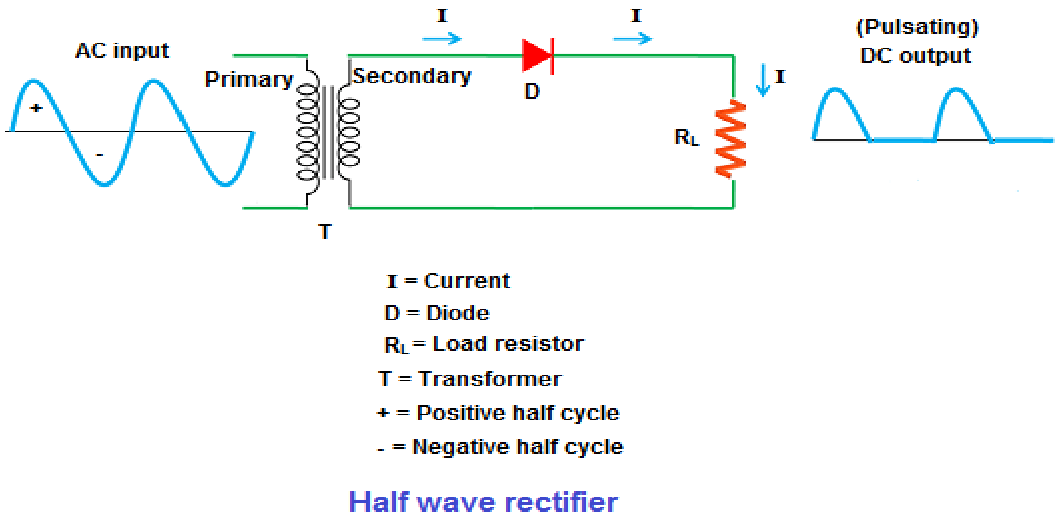


Fig 5.2: Half wave Rectifier

AC source

The AC source supplies Alternating Current to the circuit. The alternating current is often represented by a sinusoidal waveform.

Transformer

Transformer is a device which reduces or increases the AC voltage. The step-down transformer reduces the AC voltage from high to low whereas the step-up transformer increases the AC voltage from low to high. In half wave rectifier, we generally use a step-down transformer because the voltage needed for the diode is very small. Applying a large AC voltage without using transformer will permanently destroy the diode. So we use step-down transformer in half wave rectifier. However, in some cases, we use a step-up transformer.In the step-down transformer, the primary winding has more turns than the secondary winding. So the step-down transformer reduces the voltage from primary winding to secondary winding.

Diode

A diode is a two terminal device that allows electric current in one direction and blocks electric current in another direction.

Resistor

A resistor is an electronic component that restricts the current flow to a certain level.

Half wave rectifier operation

Positive half wave rectifier

When high AC voltage (60 Hz) is applied, the step-down transformer reduces this high voltage into low voltage. Thus, a low voltage is produced at the secondary winding of the transformer. The low voltage produced at the secondary winding of the transformer is called secondary voltage (V_S). The AC voltage or AC signal applied to the transformer is nothing but an input AC signal or input AC voltage.

The low AC voltage produced by the step-down transformer is directly applied to the diode.

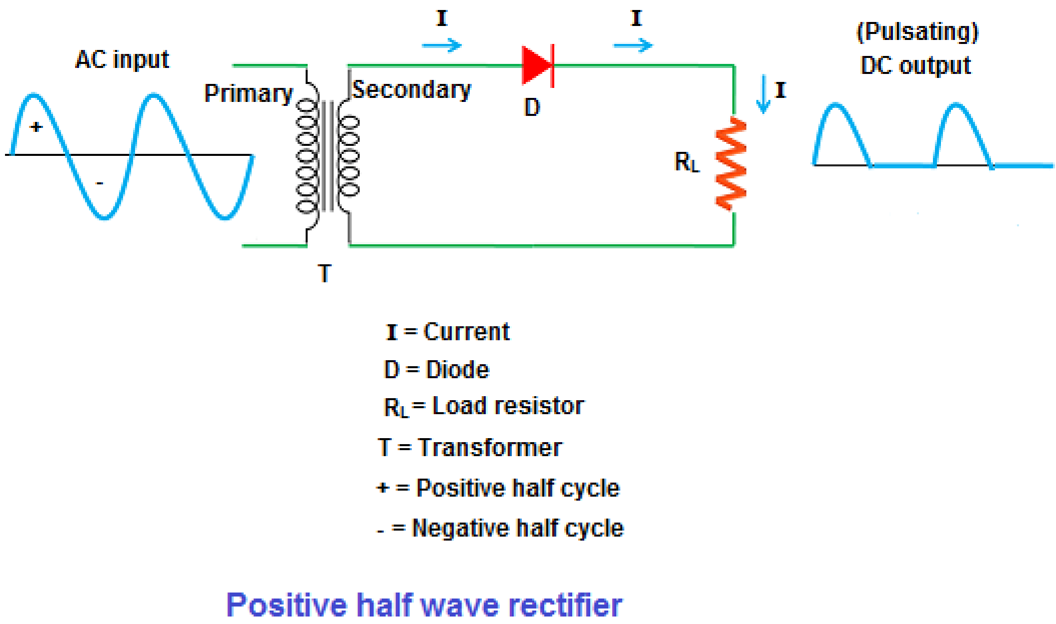


Fig 5.3: Positive Half wave Rectifier

When low AC voltage is applied to the diode (D), during the positive half cycle of the signal, the diode is forward biased and allows electric current whereas, during the negative half cycle, the diode is reverse biased and blocks electric current. In simple words, the diode allows the positive half-cycle of the input AC signal and blocks the negative half-cycle of the input AC signal.

The positive half-cycle of the input AC signal or AC voltage applied to the diode is analogous to the forward DC voltage applied to the p-n junction diode similarly the negative half-cycle of the input AC signal applied to the diode is analogous to the reverse DC voltage applied to the p-n junction diode.

We know that diode allows electric current when it is forward biased and blocks electric current when it is reverse biased. Similarly, in an AC circuit, the diode allows electric current during the positive half cycle (forward biased) and blocks electric current during the negative half cycle (reverse biased).

The positive half wave rectifier does not completely block the negative half cycles. It allows a small portion of negative half cycles or small negative current. This current is produced by the minority carriers in the diode.The current produced by the minority carriers is very small. So it is neglected. We can’t visually see the small portion of negative half cycles at the output.

In an ideal diode, the negative half cycles or negative current is zero.The resistor placed at the output consumes the DC current generated by the diode. Hence, the resistor is also known as an electrical load. The output DC voltage or DC current is measured across the load resistor RL.

The electrical load is nothing but an electrical component of a circuit that consumes electric current. In half waverectifier, the resistor consumes the DC current generated by the diode. So the resistor in half wave rectifier is known as a load. Sometimes, the load is also refers to the power consumed by the circuit.

The load resistors are used in half wave rectifiers to restrict or block the unusual excess DC current produced by the diode.

Thus, the half wave rectifier allows positive half cycles and blocks negative half cycles. The half wave rectifier which allows positive half cycles and blocks negative half cycles is called a positive half wave rectifier. The output DC current or DC signal produced by a positive half wave rectifier is a series of positive half cycles or positive sinusoidal pulses.

Negative half wave rectifier

The construction and working of negative half wave rectifier is almost similar to the positive half wave rectifier. The only thing we change here is the direction of a diode.When AC voltage is applied, the step-down transformer reduces the high voltage to low voltage. This low voltage is applied to the diode.

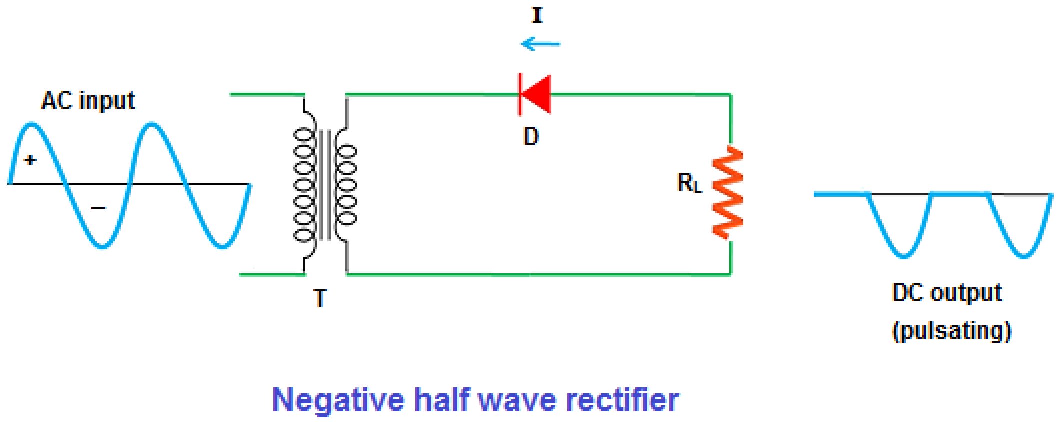


Fig 5.4: Negative Half wave Rectifier

Unlike the positive half wave rectifier, the negative half wave rectifier allows electric current during the negative half-cycle of input AC signal and blocks electric current during the positive half-cycle of the input AC signal.

During the negative half cycle, the diode is forward biased and during the positive half cycle the diode is reverse biased, so the negative half wave rectifier allows electric current only during the negative half cycle.

Thus, the negative half wave rectifier allows negative half cycles and blocks positive half cycles.

The negative half wave rectifier does not completely block the positive half cycles. It allows a small portion of positive half cycles or small positive current. This current is produced by the minority carriers in the diode.

The current produced by the minority carriers is very small. So it is neglected. We can't visually see this small positive half cycles at the output. In an ideal diode, the positive half cycle or positive current is zero.

The DC current or DC voltage produced by the negative half wave rectifier is measured across the load resistor RL. The output DC current or DC signal produced by a negative half wave rectifier is a series of negative half cycles or negative sinusoidal pulses.

Thus, a negative half wave rectifier produces a series of negative sinusoidal pulses.

In a perfect or ideal diode, the positive half cycle or negative half cycle at the output is exactly same as the input positive half cycle or negative half cycle. However, in practice, the positive half cycle or negative half cycle at the output is slightly different from the input positive half cycle or negative half cycle. But this difference is negligible. So we can't see the difference with our eyes.

Thus, the half wave rectifier produces a series of positive sinusoidal pulses or negative sinusoidal pulses. This series of positive pulses or negative pulses is not a pure direct current. It is a pulsating direct current.

The pulsating direct current changes its value over a short period of time. But our aim is to produce a direct current which does not change its value over a short period of time. Therefore, the pulsating direct current is not much useful. Half wave rectifier with capacitor filter

A filter converts the pulsating direct current into pure direct current. In half wave rectifiers, a capacitor or inductor is used as a filter to convert the pulsating DC to pure DC. The output voltage produced by a half wave rectifier is not constant; it varies with respect to time. In practical applications, a constant DC supply voltage is needed. In order to produce a constant DC voltage, we need to suppress the ripples of a DC voltage. This can be achieved by using either a capacitor filter or inductor filter at the output side. In the below circuit, we are using the capacitor filter. The capacitor placed at the output side smoothen the pulsating DC to pure DC.

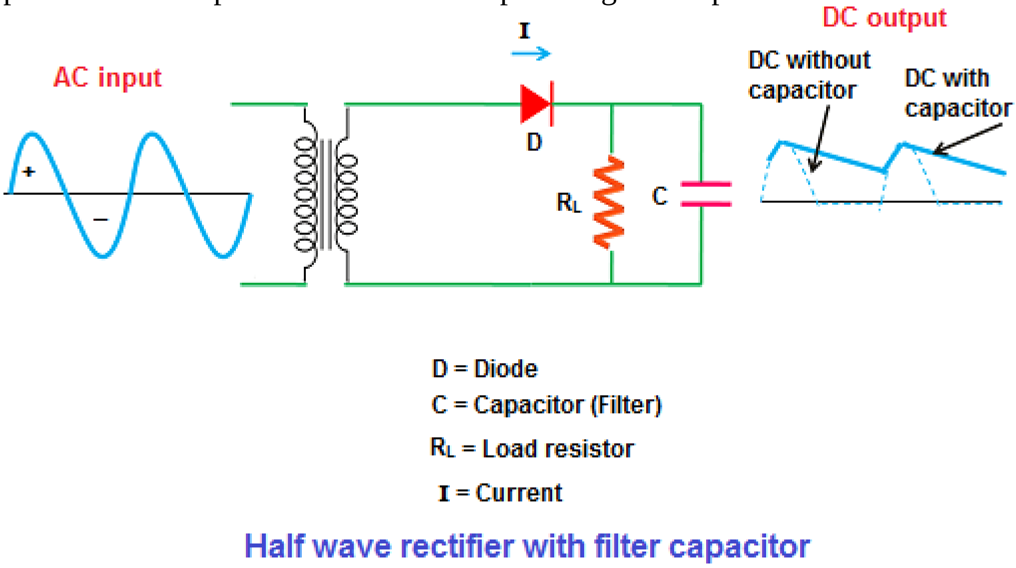


Fig 5.5: Half wave Rectifier with filter Capacitor

Characteristics of half wave Rectifier

Ripple factor

The direct current (DC) produced by a half wave rectifier is not a pure DC but a pulsating DC. In the output pulsating DC signal, we find ripples. These ripples in the output DC signal can be reduced by using filters such capacitors and inductors.

In order to measure how much ripples are there in the output DC signal we use a factor known as ripple factor. The ripple factor is denoted by γ.

The ripple factor tells us the amount of ripples present in the output DC signal.

A large ripple factor indicates a high pulsating DC signal while a low ripple factor indicates a low pulsating DC signal. If the ripple factor is very low then it indicates that the output DC current is closer to the pure DC current. In simple words, the lower the ripple factor the smoother the output DC signal.

Ripple factor can be mathematically defined as the ratio of rms value of AC component of the output voltage to the DC component of the output voltage.

Ripples factor = rms value of AC component of the output voltage / DC component of the output voltage Where, rms = root mean square

or

The ripple factor is also simply defined as the ratio of ripple voltage to the DC voltage Ripple factor = Ratio of ripple voltage / DC voltage

The ripple factor should be kept as minimum as possible to construct a good rectifier. The ripple factor is given as

$$\gamma = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

Finally, we get

$\gamma = 1.21$

The unwanted ripple present in the output along with the DC voltage is 121% of the DC magnitude. This indicates that the half wave rectifier is not an efficient AC to DC converter. The high ripples in the half wave rectifier can be reduced by using filters.

DC current The DC current is given by,

$$I_{DC} = \frac{I_{max}}{\pi}$$

Where,

Imax = maximum DC load current

Output DC voltage (VDC)

The output DC voltage (VDC) is the voltage appeared at the load resistor (RL). This voltage is obtained by multiplying the output DC current with load resistance RL.

VDC = IDC RL

$$V_{DC} = \frac{V_{Smax}}{\pi}$$

Where, VSmax = Maximum secondary voltage Peak inverse voltage (PIV)

Peak inverse voltage is the maximum reverse bias voltage up to which a diode can withstand. If the applied voltage is greater than the peak inverse voltage, the diode will be destroyed.

During the positive half cycle, the diode is forward biased and allows electric current. This current is dropped at the resistor load (RL). However, during the negative half cycle, the diode is reverse biased and does not allows electric current, so the input AC current or AC voltage is dropped at the diode.

The maximum voltage dropped at the diode is nothing but an input voltage. Therefore, peak inverse voltage (PIV) of diode = VSmax

Rectifier efficiency

Rectifier efficiency is defined as the ratio of output DC power to the input AC power. The rectifier efficiency of a half wave rectifier is 40.6%

Root mean square (RMS) value of load current IRMS

The root mean square (RMS) value of load current in a half wave rectifier is

$$I_{RMS} = \frac{I_m}{2}$$

Root mean square (RMS) value of output load voltage VRMS

The root mean square (RMS) value of output load voltage in a half wave rectifier is

$$V_{RMS} = I_{RMS}\ R_L = \frac{I_m}{2}\ R_L$$

Form factor

Form factor is defined as the ratio of RMS value to the DC value It can be mathematically written as

F.F = RMS value / DC value

The form factor of a half wave rectifier is

F.F = 1.57

Full wave rectifier

The process of converting the AC current into DC current is called rectification. Rectification can be achieved by using a single diode or group of diodes. These diodes which convert the AC current into DC current are called rectifiers.

Rectifiers are generally classified into two types: half wave rectifier and full wave rectifier.

A half wave rectifier uses only a single diode to convert AC to DC. So it is very easy to construct the half wave rectifier. However, a single diode in half wave rectifier only allows either a positive half cycle or a negative half cycle of the input AC signal and the remaining half cycle of the input AC signal is blocked. As a result, a large amount of power is wasted. Furthermore, the half wave rectifiers are not suitable in the applications which need a steady and smooth DC voltage. So the half wave rectifiers are not efficient AC to DC converters.

We can easily overcome this drawback by using another type of rectifier known as a full wave rectifier. The full wave rectifier has some basic advantages over the half wave rectifier. The average DC output voltage produced by the full wave rectifier is higher than the half wave rectifier. Furthermore, the DC output signal of the full wave rectifier has fewer ripples than the half wave rectifier. As a result, we get a smoother output DC voltage.

Full wave rectifier definition

A full wave rectifier is a type of rectifier which converts both half cycles of the AC signal into pulsating DC signal.

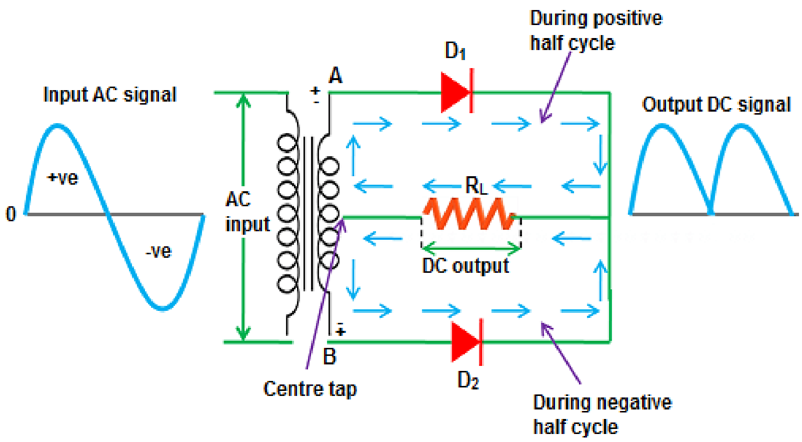


Fig 5.6: Full wave Rectifier

As shown in the above figure, the full wave rectifier converts both positive and negative half cycles of the input AC signal into output pulsating DC signal.

The full wave rectifier is further classified into two types:

- Centre tapped full wave rectifier
- Full wave bridge rectifier.

In this tutorial, centre tapped full wave rectifier is explained. Before going to the working of a centre tapped full wave rectifier, let's first take a look at the centre tapped transformer. Because the centre tapped transformer plays a key role in the centre tapped full wave rectifier.

Centre tapped transformer

When an additional wire is connected across the exact middle of the secondary winding of a

The wire is adjusted in such a way that it falls in the exact middle point of the secondary winding. So the wire is exactly at zero volts of the AC signal.

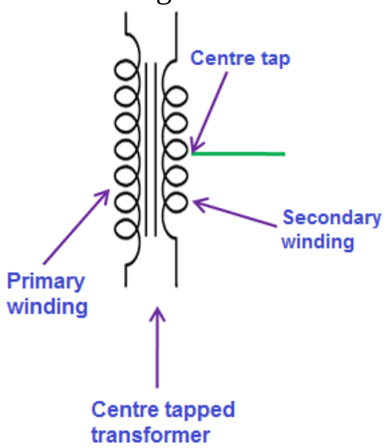


Fig 5.7**: Centre tapped transformer**

This wire is known as the centre tap. The centre tapped transformer works almost similar to a normal transformer. Like a normal transformer, the centre tapped transformer also increases or reduces the AC voltage. However, a centre tapped transformer has another important feature. That is the secondary winding of the centre tapped transformer divides the input AC current or AC signal (V_P) into two parts.

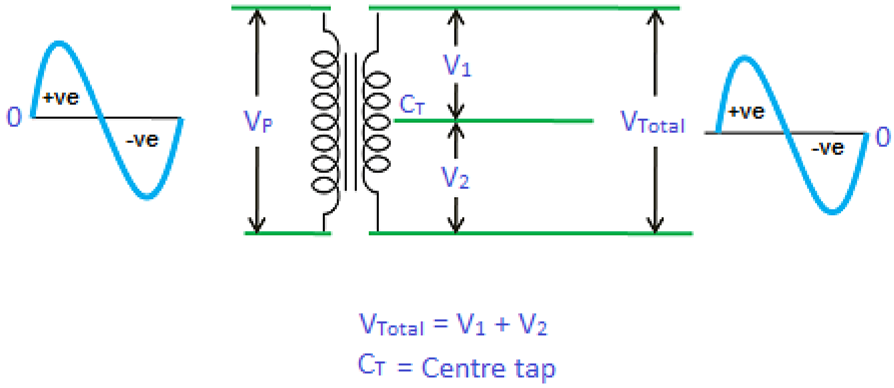


Fig 5.**8: Centre tapped transformer with Input supply**

The upper part of the secondary winding produces a positive voltage V1 and the lower part of the secondary winding produces a negative voltage V2. When we combine these two voltages at output load, we get a complete AC signal.

I.e. VTotal = V1 + V2

The voltages V1 and V2 are equal in magnitude but opposite in direction. That is the voltages (V1 and V2) produced by the upper part and lower part of the secondary winding are 180 degrees out of phase with each other. However, by using a full wave rectifier with centre tapped transformer, we can produce the voltages that are in phase with each other. In simple words, by using a full wave rectifier with centre tapped transformer, we can produce a current that flows only in single direction.

What is centre tapped full wave rectifier

A center tapped full wave rectifier is a type of rectifier which uses a centre tapped transformer and two diodes to convert the complete AC signal into DC signal.

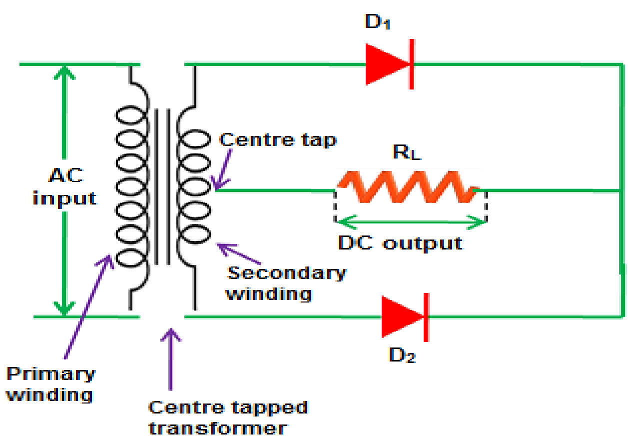


Fig 5.9**: Centre tapped full wave rectifier**

The centre tapped full wave rectifier is made up of an AC source, a center tapped transformer, two diodes, and a load resistor.

The AC source is connected to the primary winding of the centre tapped transformer. A centre tap (additional wire) connected at the exact middle of the the secondary winding divides the input voltage into two parts. The upper part of the secondary winding is connected to the diode D_1 and the lower part of the secondary winding is connected to the diode D_2. Both diode D_1 and diode D_2 are connected to a common load R_L with the help of a centre tap transformer. The centre tap is generally considered as the ground point or the zero voltage reference point.

How centre tapped full wave rectifier works

The centre tapped full wave rectifier uses a centre tapped transformer to convert the input AC voltage into output DC voltage. When input AC voltage is applied, the secondary winding of the centre tapped transformer divides this input AC voltage into two parts: positive and negative.

During the positive half cycle of the input AC signal, terminal A become positive, terminal B become negative and centre tap is grounded (zero volts). The positive terminal A is connected to the p-side of the diode D_1 and the negative terminal B is connected to the n-side of the diode D_1. So the diode D_1 is forward biased during the positive half cycle and allows electric current through it.

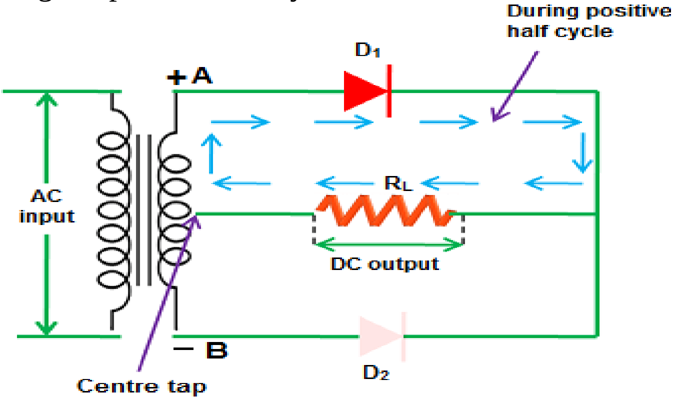


Fig 5.10: Centre tapped full wave rectifier during Positive half cycle

On the other hand, the negative terminal B is connected to the p-side of the diode D_2 and the positive terminal A is connected to the n-side of the diode D_2. So the diode D_2 is reversed biased during the positive half cycle and does not allow electric current through it.

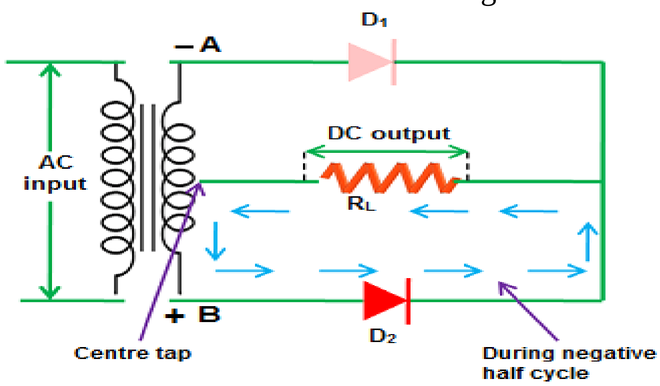


Fig 5.11: Centre tapped full wave rectifier during Negative half cycle

The diode D_1 supplies DC current to the load R_L. The DC current produced at the load R_L will return to the secondary winding through a centre tap. During the positive half cycle, current flows only in the upper part of the circuit while the lower part of the circuit carry no current to the load because the diode D_2 is reverse biased. Thus, during the positive half cycle of the input AC signal, only diode D_1 allows electric current while diode D_2 does not allow electric current. During the negative half cycle of the input AC signal, terminal A become negative, terminal B become positive and centre tap is grounded (zero volts). The negative terminal A is connected to the p-side of the diode D_1 and the positive terminal B is connected to the n-side of the diode D_1. So the diode D_1 is reverse biased during the negative half cycle and does not allow electric current through it.On the other hand, the positive terminal B is connected to the p-side of the diode D_2 and the negative terminal A is connected to the n-side of the diode D_2. So the diode D_2 is forward biased during the negative half cycle and allows electric current through it. The diode D_2 supplies DC current to the load R_L.

The DC current produced at the load R_L will return to the secondary winding through a centre tap.

During the negative half cycle, current flows only in the lower part of the circuit while the upper part of the circuit carry no current to the load because the diode D_1 is reverse biased. Thus, during the negative half cycle of the input AC signal, only diode D_2 allows electric current while diode D_1 does not allow electric current. Thus, the diode D_1 allows electric current during the positive half cycle and diode D_2 allows electric current during the negative half cycle of the input AC signal. As a result, both half cycles (positive and negative) of the input AC signal are allowed. So the output DC voltage is almost equal to the input AC voltage.

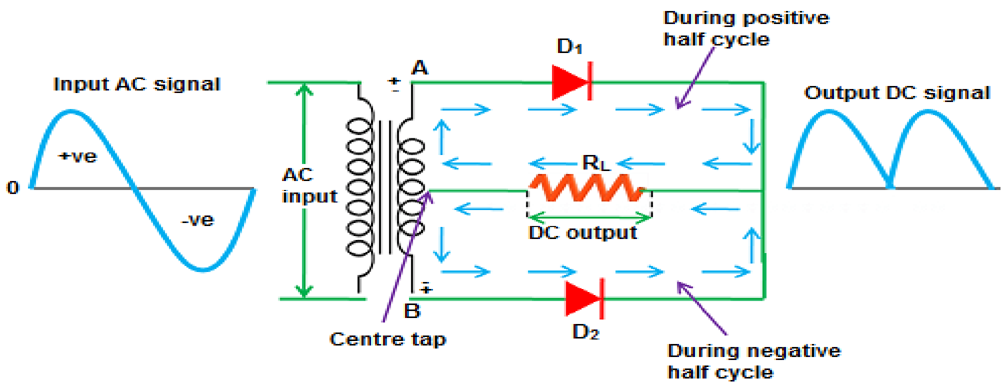


Fig 5.12**: Centre tapped full wave rectifier during Positive and Negative half cycle**

A small voltage is wasted at the diode D_1 and diode D_2 to make them conduct. However, this voltage is very small as compared to the voltage appeared at the output. So this voltage is neglected. The diodes D_1 and D_2 are commonly connected to the load R_L. So the load current is the sums of individual diode currents. We know that a diode allows electric current in only one direction. From the above diagram, we can see that both the diodes D_1 and D_2 are allowing current in the same direction.
We know that a current that flows in only single direction is called a direct current. So the resultant current at the output (load) is a direct current (DC). However, the direct current appeared at the output is not a pure direct current but a pulsating direct current. The value of the pulsating direct current changes with respect to time. This is due to the ripples in the output signal. These ripples can be reduced by using filters such as capacitor and inductor. The average output DC voltage across the load resistor is double that of the single half wave rectifier circuit.

Output waveforms of full wave rectifier

The output waveforms of the full wave rectifier is shown in the below figure.

The first waveform represents an input AC signal. The second waveform and third waveform represents the DC signals or DC current produced by diode D_1 and diode D_2. The last waveform represents the total output DC current produced by diodes $D_1$and D_2. From the above waveforms, we can conclude that the output current produced at the load resistor is not a pure DC but a pulsating DC.

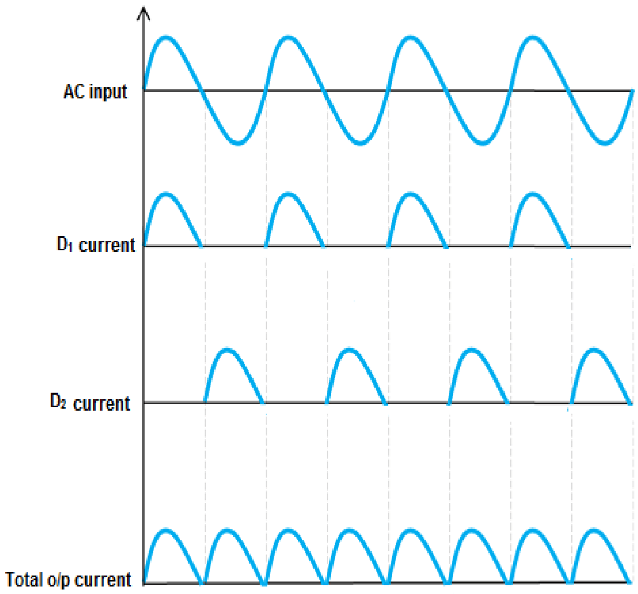


Fig 5.13**: Output waveforms of full wave rectifier**

Characteristics of full wave rectifier
Ripple factor

The ripple factor is used to measure the amount of ripples present in the output DC signal. A high ripple factor indicates a high pulsating DC signal while a low ripple factor indicates a low pulsating DC signal.
Ripple factor is defined as the ratio of ripple voltage to the pure DC voltage The ripple factor is given by

$$\gamma = \sqrt{\left(\frac{V_{rms}}{V_{DC}}\right)^2 - 1}$$

Finally, we get

$$\gamma = 0.48$$

Rectifier efficiency

Rectifier efficiency indicates how efficiently the rectifier converts AC into DC. A high percentage of rectifier efficiency indicates a good rectifier while a low percentage of rectifier efficiency indicates an inefficient rectifier.
Rectifier efficiency is defined as the ratio of DC output power to the AC input power.
It can be mathematically written as

$$\eta = \text{output } P_{DC} / \text{input } P_{AC}$$

The rectifier efficiency of a full wave rectifier is 81.2%.

The rectifier efficiency of a full wave rectifier is twice that of the half wave rectifier. So the full wave rectifier is more efficient than a half wave rectifier

Peak inverse voltage (PIV)

Peak inverse voltage or peak reverse voltage is the maximum voltage a diode can withstand in the reverse bias condition. If the applied voltage is greater than the peak inverse voltage, the diode will be permanently destroyed.
The peak inverse voltage (PIV) = $2V_{smax}$

DC output current

At the output load resistor R_L, both the diode D_1 and diode D_2 currents flow in the same direction. So the output current is the sum of D_1 and D_2 currents. The current produced by D_1 is I_{max} / π and the current produced by D_2 is I_{max} / π. So the output current

$$I_{DC} = 2I_{max} / \pi$$

Where,

I_{max} = maximum DC load current

DC output voltage

The DC output voltage appeared at the load resistor R_L is given as

$$V_{DC} = 2V_{max} / \pi$$

Where,

V_{max} = maximum secondary voltage

Root mean square (RMS) value of load current I_{RMS}

The root mean square (RMS) value of load current in a full wave rectifier is

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

Root mean square (RMS) value of the output load voltage V_{RMS}

The root mean square (RMS) value of output load voltage in a full wave rectifier is

$$V_{RMS} = I_{RMS} R_L = \frac{I_m}{\sqrt{2}} R_L$$

Form factor

Form factor is the ratio of RMS value of current to the DC output current It can be mathematically written as

F.F = RMS value of current / DC output current

The form factor of a full wave rectifier is

F.F = 1.11

Advantages of full wave rectifier with centre tapped transformer

- High rectifier efficiency
- Full wave rectifier has high rectifier efficiency than the half wave rectifier. That means the full wave rectifier converts AC to DC more efficiently than the half wave rectifier.
- Low power loss
- In a half wave rectifier, only half cycle (positive or negative half cycle) is allowed and the remaining half cycle is blocked. As a result, more than half of the voltage is wasted. But in full wave rectifier, both half cycles (positive and negative half cycles) are allowed at the

same time. So no signal is wasted in a full wave rectifier.

- Low ripples
- The output DC signal in full wave rectifier has fewer ripples than the half wave rectifier.

Disadvantages of full wave rectifier with center tapped transformer

- High cost
- The centre tapped transformers are expensive and occupy a large space.

RC Coupled Amplifier

A Resistance Capacitance (RC) Coupled Amplifier is basically a multi-stage amplifier circuit extensively used in electronic circuits. Here the individual stages of the amplifier are connected together using a resistor–capacitor combinationdue to which it bears its name as RC Coupled. Figure 1 shows such a two-stage amplifier whose individual stages are nothing but the common emitter amplifiers. Hence the design of individual stages of the RC coupled amplifiers is similar to that in the case of common emitter amplifiers in which the resistors R1 and R2 form the biasing network while the emitter resistor RE form the stabilization network. Here the CE is also called bypass capacitor which passes only AC while restricting DC, which causes only DC voltage to drop across RE while the entire AC voltage will be coupled to the next stage.Further, the coupling capacitor CC also increases the stability of the network as it blocks the DC while offers a low resistance path to the AC signals, thereby preventing the DC bias conditions of one stage affecting the other. In addition, in this circuit, the drop across the collector-emitter terminal is chosen to be 50% of the supply voltage VCC in order to ensure appropriate biasing point.

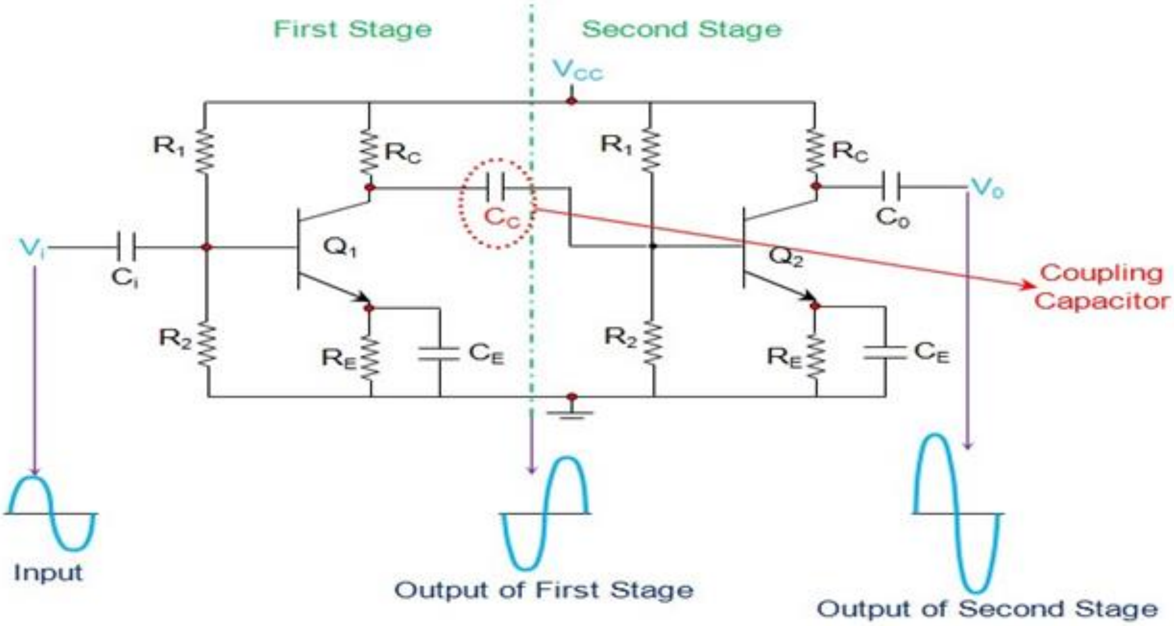


Fig 5.14**: Two stage RC Coupled amplifier**

In this kind of amplifier, the input signal applied at the base of the transistor in stage 1 (Q1) is amplified and appears at its collector terminal with a phase-shift of 180o.

The AC component of this signal is coupled to the second stage of the RC coupled amplifier through the coupling capacitor CC and thus appears as an input at the base of the second transistor Q2. This is further amplified and is passed-on as an output of the second stage and is available at the collector terminal of Q2 after being shift by 180o in its phase. This means that the output of the second stage will be 360o out-of-phase with respect to the input, which in turn indicates that the phase of the input signal and the phase of the output signal obtained at stage II will be identical.

Further it is to be noted that the cascading of individual amplifier stages increases the gain of the overall circuit as the net gain will be the product of the gain offered by the individual stages. However in real scenario, the net gain will be slightly less than this, due to the loading effect. In addition, it is important to note that by following the pattern exhibited by Figure 1, one can cascade any number of common emitter amplifiers but by keeping in mind that when the number of stages are even, the output will be in-phase with the input while if the number of stages are odd, then the output and the input will be out-of-phase.

The frequency response of a RC coupled amplifier (a curve of amplifier's gain v/s frequency), shown by Figure 2, indicates that the gain of the amplifier is constant over a wide range of mid-frequencies while it decreases considerably both at low and high frequencies. This is because, at low frequencies, the reactance of coupling capacitor CC is high which causes a small part of the signal to couple from one stage to the other. Moreover for the same case, even the reactance of the emitter capacitor CE will be high due to which it fails to shunt the emitter resistor RE effectively which in turn reduces the voltage gain.

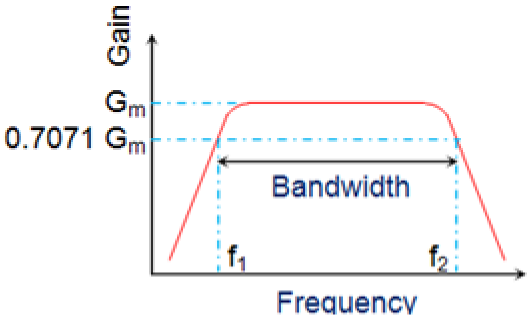


Fig 5.15**: Frequency Response of RC Coupled amplifier**

On the other hand, at high frequencies, the reactance of CC will be low which causes it to behave like a short circuit. This results in an increase in the loading effect of the next stage and thus reduces the voltage gain. In addition to this, for this case, the capacitive reactance of the base-emitter junction will be low. This results in a reduced voltage gain as it causes the base current to increase which inturn decreases the current amplification factor β. However, in mid-frequency range, as the frequency increases, the reactance of CC goes on decreasing which would lead to the increase in gain if not compensated by the fact that the reduction in reactance leads to an increase in the loading effect. Due to this reason, the gain of the amplifier remains uniform/constant throughout the mid-frequency band.

Advantages of RC Coupled Amplifier

- Cheap, economical and compact as it uses only resistors and capacitors.
- Offers a constant gain over a wide frequency band.

Disadvantages of RC Coupled Amplifier

- Unsuitable for low-frequency amplification.
- Low voltage and power gain as the effective load resistance (and hence the gain) is reduced due to the fact that the input of each stage presents a low resistance to its next stage.
- Moisture-sensitive, making them noisy as time elapses.
- Poor impedance matching as it has the output impedance several times larger than the device at its end- terminal (for example, a speaker in the case of a public address system).
- Narrow bandwidth when compared to JFET amplifier.

Applications of RC Coupled Amplifier

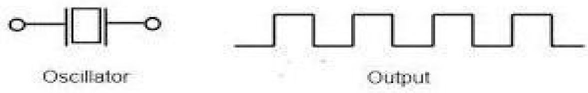
1. RF Communications.
2. Optical Fiber Communications.
3. Public address systems as pre-amplifiers.
4. Controllers.

5. Radio or TV Receivers as small signal amplifiers.

OSCILLATORS

Introduction

An oscillator is a circuit that produces a repetitive signal from a dc voltage. The feedback type oscillator which rely on a positive feedback of the output to maintain the oscillations. The relaxation oscillator makes use of an RC timing circuit to generate a non-sinusoidal signal such as square wave.



The requirements for oscillation are described by the Baukhausen criterion:
The magnitude of the loop gain Aβ must be 1
The phase shift of the loop gain A β must be 0° or 360° or integer multiple of 2pi

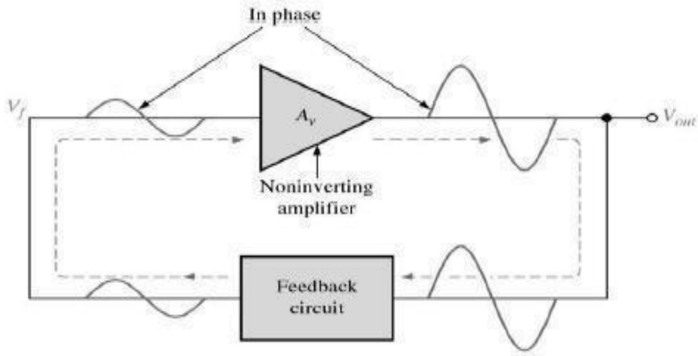


Fig 5.16: Oscillator Output
Amplitude stabilization

Amplitude stabilization:

- In both the oscillators above, the loop gain is set by component values
- In practice the gain of the active components is very variable
- If the gain of the circuit is too high it will saturate
- If the gain of the circuit is too low the oscillation will die

Real circuits need some means of stabilizing the magnitude of the oscillation to cope with variability in the gain of the circuit

Barkhausan criterion

The conditions for oscillator to produce oscillation are given by Barkhausan criterion. They are: The total phase shift produced by the circuit should be 360° or 0°

The Magnitude of loop gain must be greater than or equal to 1 (ie)$|A\beta| \geq 1$

In practice loop gain is kept slightly greater than unity to ensure that oscillator work even if there is a slight change in the circuit parameters

Mechanism of start of oscillation

The starting voltage is provided by noise, which is produced due to random motion of electrons in resistors used in the circuit. The noise voltage contains almost all the sinusoidal frequencies. This low amplitude noise voltage gets amplified and appears at the output terminals. The amplified noise drives the feedback network which is the phase shift network. Because of this the feedback voltage is maximum at a particular frequency, which in turn represents the frequency of oscillation.

LC Oscillator:

Oscillators are used in many electronic circuits and systems providing the central "clock" signal that controls that controls the sequential operation of the entire system. Oscillators convert a DC input (the supply voltage) into an AC output (the waveform), which can have a wide range of different wave shapes and frequencies that can be either complicated in nature or simple sine waves depending upon the application.

Oscillators are also used in many pieces of test equipment producing sinusoidal sine wave, square, saw tooth or triangular shaped waveforms or just a train of pulse of a variable or constant width. LC Oscillators are commonly used in radio-frequency circuits because of their good phase noise characteristics and their ease of implementation.

An Oscillator is basically an Amplifier with "Positive Feedback", or regenerative feedback (in-phase) and one of the many problems in electronic circuit design is stooping amplifiers from oscillating while trying to get oscillators to oscillate. Oscillators work because they overcome the losses of their feedback resonator circuit either in the form of a capacitor or both in the same circuit by applying DC energy at the required frequency into this resonator circuit.

In other words, an oscillator is an amplifier which uses positive feedback that generates an output frequency without the use of an input signal.

It is self sustaining. Then an oscillator has a small signal feedback amplifier with an open-loop gain equal to or slightly greater than one for oscillations to start but to continue oscillations the average loop gain must return to unity. In addition to these reactive components, an amplifying device such as an Operational Amplifier or Bipolar Transistors required. Unlike an amplifier there is no external AC input required to cause the Oscillator to work as the DC supply energy is converted by the oscillator into AC energy at the required frequency.

Basic Oscillator Feedback Circuit

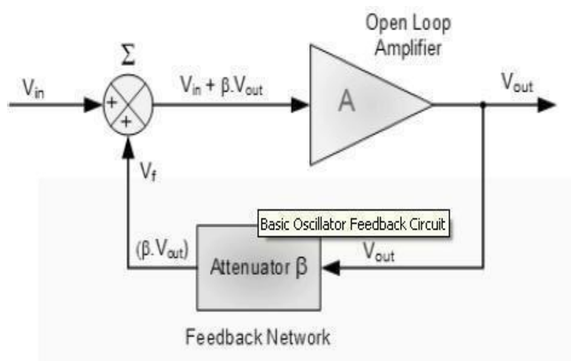


Fig 5.17: **Basic Oscillator Feedback Circuit**

Where: β is a feedback fraction.

$$\text{Gain, } Av = \frac{Vout}{Vin} \qquad A = \text{open loop voltage gain}$$

$$Av \times Vin = Vout$$

With Feedback

$$Av(Vin - \beta Vout) = Vout \qquad \beta \text{ is the feedback fraction}$$

$$Av.Vin - Av.\beta.Vout = Vout \qquad A\beta = \text{the loop gain}$$

$$Av.Vin = Vout(1 + A\beta) \qquad 1 + A\beta = \text{the feedback factor}$$

$$\therefore \frac{Vout}{Vin} = Gv = \frac{A}{1 + A\beta} \qquad Gv = \text{the closed loop gain}$$

Oscillators are circuits that generate a continuous voltage output waveform at a required frequency with the values of the inductors, capacitors or resistors forming a frequency selective LC resonant tank circuit and feedback network. This feedback network is an attenuation network which has a gain of less than one ($\beta<1$) and starts oscillations when A $\beta>1$ which returns to unity (A $\beta=1$) once oscillations commence. The LC oscillator's frequency is controlled using a tuned or resonant inductive/capacitive (LC) circuit with the resulting output frequency being known as the Oscillation Frequency.
By making the oscillators feedback a reactive network the phase angle of the feedback will vary as a function of frequency and this is called Phase-shift.

There are basically types of Oscillators:

1. **Sinusoidal Oscillators** - these are known as Harmonic Oscillators and are generally a: LC Tuned-feedback" or "RC tuned-feedback" type Oscillator that generates a purely sinusoidal waveform which is of constant amplitude and frequency.

2. **Non-Sinusoidal Oscillators** – these are known as Relaxation Oscillators and generate complex non- sinusoidal waveforms that changes very quickly from one condition of stability to another such as "Square-wave", "Triangular- wave" or "Saw-toothed-wave" type waveforms.

- **Resonance**

When a constant voltage but of varying frequency is applied to a circuit consisting of an inductor, capacitor and resistor the reactance of both the Capacitor/Resistor and Inductor/Resistor circuits is to change both the amplitude and the phase of the output signal due to the reactance of the components used.

At high frequencies the reactance of a capacitor is very low acting as a short circuit while the reactance of the inductor is high acting as an open circuit. At low frequencies the reverse is true, the reactance of the capacitor acts as an open circuit and the reactance of the inductor acts as a short circuit.

Between these two extremes the combination of the inductor and capacitor produces a “Tuned” or “Resonant” circuit that has a Resonant Frequency, (fr) in which the capacitive and inductive reactance’s
are equal and cancel out each other, leaving only the resistance of the circuit to oppose the flow of current. This means that there is no phase shift as the current is in phase with the voltage. Consider the circuit below.

- **Basic LC Oscillator Tank Circuit**

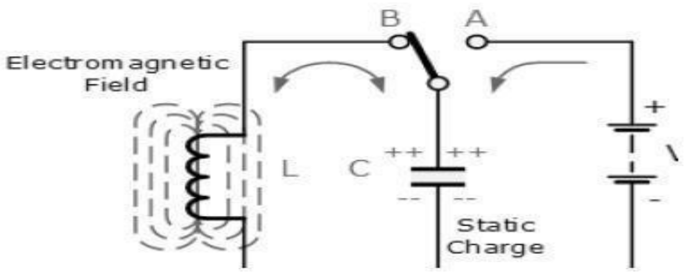


Fig 5.18: **Basic LC Oscillator Tank Circuit**

The circuit consists of an inductive coil, L and a capacitor, C. The capacitor stores energy in the form of an electrostatic field and which produces a potential (static voltage) across its plates, while the inductive coil stores its energy in the form of an electromagnetic field.

The capacitor is charged up to the DC supply voltage, V by putting the switch in position A. When the capacitor is fully charged the switch changes to position B. The charged capacitor is now connected in parallel across the inductive coil so the capacitor begins to discharge itself through the coil.

The voltage across C starts falling as the current through the coil begins to rise. This rising current sets up an electromagnetic field around the coil which resists this flow of current. When the capacitor, C is completely discharged the energy that was originally stored in the capacitor, C as an electrostatic filed is now stored in the inductive coil, L as an electromagnetic field around the coils windings.

As there is now no external voltage in the circuit to maintain the current within the coil, it starts to fall as the electromagnetic field begins to collapse. A back emf is induced in the coil (e= -Ldi/dt) keeping the current flowing in the original direction. This current now charges up the capacitor, c with the opposite polarity to its original charge.

C continues to chare up until the current reduces to zero and the electromagnetic field of the coil has collapsed completely. The energy originally introduced into the circuit through the switch, has been returned to the capacitor which again has an electrostatic voltage potential across it, although it is now of the opposite polarity. The capacitor now starts to discharge again back through the coil and the whole process os repeated. The polarity of the voltage changes as the energy is passed back and forth between the capacitor and inductor producing an AC type sinusoidal voltage and current waveform.

This then forms the basis of an LC oscillator’s tank circuit and theoretically this cycling back and forth will continue indefinitely. However, every time energy is transferred from C to L or from L to C losses occur which decay the oscillations.

This oscillatory action of passing energy back and forth between the capacitor, C to the inductor, L would continue indefinitely if it was not for energy losses within the circuit. Electrical energy is lost in the DC or real resistance of the inductors coil, in the dielectric of the capacitor, and in radiation from the circuit so the oscillation steadily decreases until they die away completely and the process stops.

- **Resonant Frequency of a LC Oscillator**

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Where:

L is the Inductance in Henries C is the Capacitance
in Farads fr is the Output Frequency in Hertz

This equation shows that if either L or C are decreased, the frequency increases. This output frequency is commonly given the abbreviation of (fr) to identify it as the "resonant frequency". To keep the oscillations going in an LC tank circuit, we have to replace all the energy lost in each oscillation and also maintain the amplitude of these oscillations at a constant level.

The amount of energy replaced must therefore be equal to the energy lost during each cycle. If the energy replaced is too large the amplitude would increase until clipping of the supply rails occurs. Alternatively, if the amount of energy replaced is too small the amplitude would eventually decrease to zero over time and the oscillations would stop.

The simplest way of replacing this lost energy is to take part of the output from the LC tank circuit, amplify it and then feed it back into the LC circuit again. This process can be achieved using a voltage amplifier using an op- amp, FET or bipolar transistor as its active device.

However, if the loop gain of the feedback amplifier is too small, the desired oscillation decays to zero and if it is too large, the waveform becomes distorted. To produce a constant oscillation, the level of the energy fed back to the LC network must be accurately controlled.

Then there must be some form of automatic amplitude or gain control when the amplitude tries to vary from a reference voltage either up or down. To maintain a stable oscillation the overall gain of the circuit must be equal to one or unity. Any less and the oscillations will not start or die away to zero, any more the oscillations will occur but the amplitude will become clipped by the supply rails causing distortion. Consider the circuit below.

Basic Transistor LC Oscillator Circuit

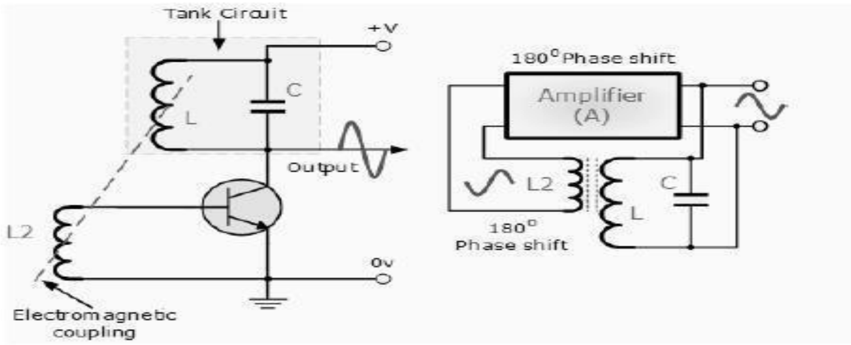


Fig 5.19: **Basic Transistor LC Oscillator Circuit**

A Bipolar Transistor is used as the LC oscillator's amplifier with the tuned LC tank circuit acts as the collector load. Another coil L2 is connected between the base and the emitter of the transistor whose electromagnetic field is "mutually" coupled with that of coil L. Mutual inductance exists between the two circuits.

The changing current flowing in one coil circuit induces, by electromagnetic induction, a potential voltage in the other (transformer effect) so as the oscillations occur in the tuned circuit, electromagnetic energy is transferred from coil L to coil L2 and a voltage of the same frequency as that in the tuned circuit is applied between the base and emitter of the transistor.

In this way the necessary automatic feedback voltage is applied to the amplifying transistor. The amount of feedback can bre increased or decreased by altering the coupling between the two Coils L and L2. When the circuit is oscillating its impedance is resistive and the collector and base voltages are 180 out of phase. In order to maintain oscillations (called frequency stability) the voltage applied to the tuned circuit must be "in-phase" with the oscillations occurring in the tuned circuit.

Therefore, we must introduce an additional 180° phase shift into the feedback path between the collector and the base. This is achieved by winding the coil of L2 in the correct direction relative to coil L giving us the correct amplitude and phase relationships for the Oscillators circuit or by connecting a phase shift network between the output and input of the amplifier.

The LC Oscillator is therefore a "Sinusoidal Oscillator" or a "Harmonic Oscillator" as it is more commonly called. LC oscillators can generate high frequency sine waves for use in radio frequency (RF) type applications with the transistor amplifier being of a Bipolar Transistor or FET.Harmonic Oscillators come in many different forms because there are many different ways to construct an LC filter network and amplifier with the most common being the Hartley LC Oscillator**,** Colpitts LC Oscillator**,** Armstrong Oscillator and Clapp Oscillator to name a few.

The Hartley Oscillator

The main disadvantages of the basic LC Oscillator circuit we looked at in the previous tutorial is that they have no means of controlling the amplitude of the oscillations and also, it is difficult to tune the oscillator to the required frequency. However, it is possible to feed back exactly the right amount of voltage for constant amplitude oscillations. If we feed back more than is necessary the amplitude of the oscillations can be controlled by biasing the amplifier in such a way that if the oscillations increase in amplitude, the bias is increased and the gain of the amplifier is reduced.

If the amplitude of the oscillations decreases the bias decreases and the gain of the amplifier increases, thus increasing the feedback. In this way the amplitude of the oscillations are kept constant using a process known as Automatic Base Bias.

One big advantage of automatic base bias in a voltage controlled oscillator, is that the oscillator can be made more efficient by providing a Class-B bias or even a Class-C bias condition of the transistor. This has the advantage that the collector current only flows during part of the oscillation cycle so the quiescent collector current is very small.

Then this "self-tuning" base oscillator circuit forms one of the most common types of LC parallel resonant feedback oscillator configurations called the Hartley Oscillator circuit.

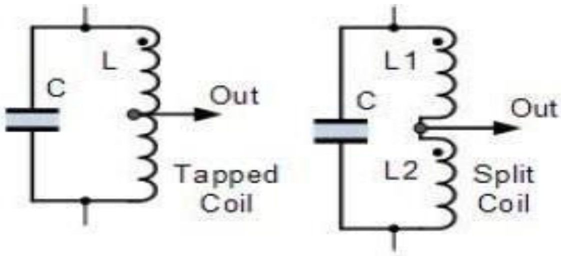


Fig 5.20: Hartley Oscillator Tuned Circuit

In the Hartley Oscillator the tuned LC circuit is connected between the collector and the base of the transistor amplifier. As far as the oscillatory voltage is concerned, the emitter is connected to a tapping point on the tuned circuit coil. The feedback of the tuned tank circuit is taken from the centre tap of the inductor coil or even two separate coils in series which are in parallel with a variable capacitor, C as shown.The Hartley circuit is often referred to as a split- inductance oscillator because coil L is centre-tapped. In effect, inductance L acts like two separate coils in very close proximity with the current flowing through coil section XY induces a signal into coil section YZ below.A Hartley Oscillator circuit can be made from any configuration that uses either a single tapped coil (similar to an autotransformer) or a pair of series connected coils in parallel with a single capacitor as shown below.

Basic Hartley Oscillator Circuit

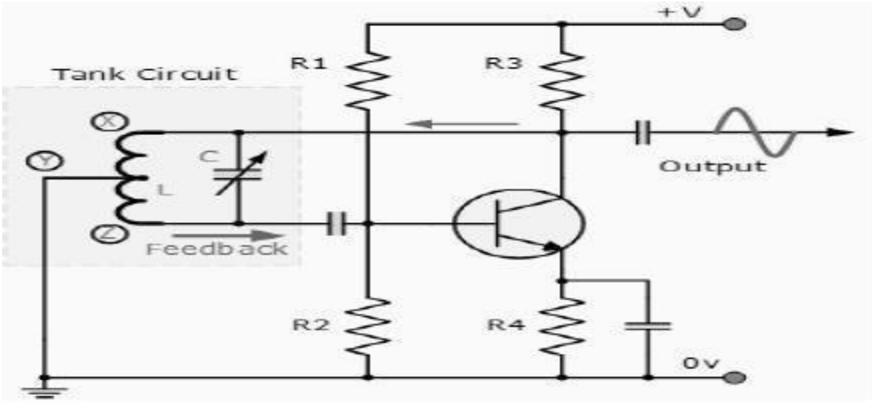


Fig 5.21: Basic Hartley Oscillator Circuit

When the circuit is oscillating, the voltage at point X (collector), relative to point Y (emitter), is 180 degree out- of-phase with the voltage at point Z(base) relative to point Y. At the frequency of oscillation, the impedance of the collector load is resistive and an increase in base voltage causes a decrease in the collector voltage. Then there is a 180 phase change in the voltage between the base and collector and this along with the original 180 phase shift in the feedback loop provides the correct and along with the original 180 phase shift in the feedback loop provides the correct phase relationship of positive feedback for oscillations to be maintained. The amount of feedback depends upon the position of the "tapping point" of the inductor. If this is moved nearer to the collector the amount of feedback is increased, but the output taken between the Collector and earth is reduced and vice versa. Resistors, R1 and R2 provide the usual stabilizing DC bias for the transistor in the normal manner while the capacitors act as DC-blocking capacitors.

In this Hartley Oscillator circuit, the DC Collector current flows through part of the coil and for this reason the circuit is said to be "Series-fed" with the frequency of oscillation of the Hartley Oscillator being given as.

$$f = \frac{1}{2\pi\sqrt{L_T C}}$$

where: $L_T = L_1 + L_2 + 2M$

The frequency of oscillations can be adjusted by varying the "tuning" capacitor, C or by varying the position of the iron-dust core inside the coil (inductive tuning) giving an output over a wide range of frequencies making it very easy to tune. Also the Hartley Oscillator produces output amplitude which is constant over the entire frequency range.

As well as the Series-fed Hartley Oscillator above, it is also possible to connect the tuned tank circuit across the amplifier as a shunt-fed oscillator as shown below.

The Colpitts Oscillator

The Colpitts Oscillator, named after its inventor Edwin Colpitts is another type of LC oscillator design. In many ways, the Colpitts oscillator is the exact opposite of the Hartley Oscillator we looked at in the previous tutorial. Just like the Hartley oscillator, the tuned tank circuit consists of an LC resonance sub-circuit connected between the collector and the base of a single stage transistor

amplifier producing a sinusoidal output waveform.
The basic configuration of the Colpitts Oscillator resembles that of the Hartley Oscillator but the difference this time is that the centre tapping of the tank sub-circuit is now made at the junction of a "capacitive voltage divider" network instead of a tapped autotransformer type inductor as in the Hartley oscillator.

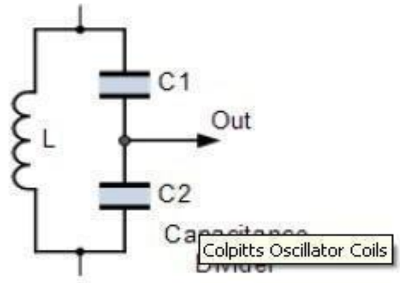


Fig 5.22: The Colpitts Oscillator

Colpitts Oscillator Circuit

The Colpitts oscillator uses a capacitor voltage divider as its feedback source.

The two capacitors, C1 and C2 are placed across a common inductor, L as shown so that C1, C2 and L forms the tuned tank circuit the same as for the Hartley oscillator circuit. The advantage of this type of tank circuit configuration is that with less self and mutual inductance in the tank circuit, frequency stability is improved along with a more simple design. As with the Hartley oscillator, the colpitts oscillator uses a single stage bipolar transistor amplifier as the gain element which produces a sinusoidal output. Consider the circuit below.

Basic Colpitts Oscillator Circuit

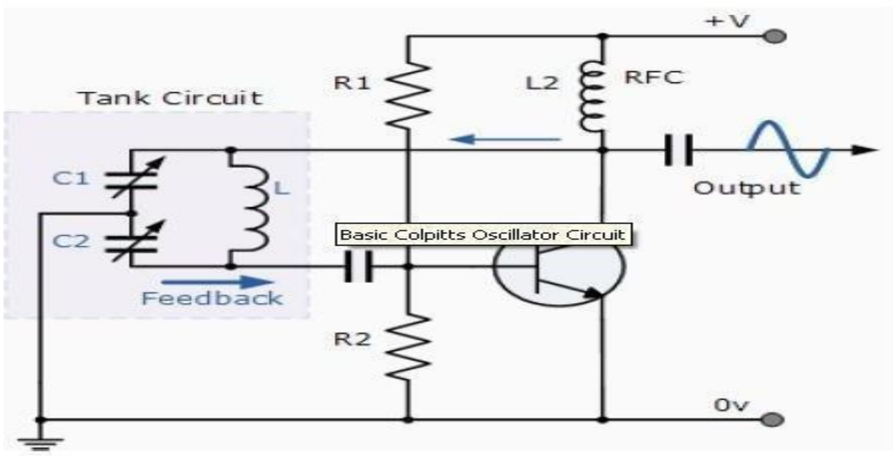


Fig 5.23: Basic Colpitts Oscillator Circuit

The transistor amplifiers emitter is connected to the junction of capacitors, C1 and C2 which are connected in series and act as a simple voltage divider. When the power supply is firstly applied, capacitors C1 and C2 charge up and then discharge through the coil L. The oscillations across the capacitors are applied to the base-emitter junction and appear in the amplified at the collector output. The amount of feedback depends on the values of C1 and C2 with the smaller the values of C the greater will be the feedback.

The required external phase shift is obtained in a similar manner to that in the Hartley oscillator circuit with the required positive feedback obtained for sustained un-damped oscillations. The amount of feedback is determined by the ratio of C1 and C2 which are generally "ganged" together to provide a constant amount of feedback so as one is adjusted the other automatically follows.

The frequency of oscillations for a Colpitts oscillator is determined by the resonant frequency of the LC tank circuit and is given as:

$$f_r = \frac{1}{2\pi\sqrt{L\,C_T}}$$

where C_T is the capacitance of C1 and C2 connected in series and is given as:.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C_T = \frac{C_1 \times C_2}{C_1 + C_2}$$

The configuration of the transistor amplifier is of a Common Emitter Amplifier with the output signal 180° out of phase with regards to the input signal. The additional 180° phase shift require for oscillation is achieved by the fact that the two capacitors are connected together in series but in parallel with the inductive coil resulting in overall phase shift of the circuit being zero or 360 °. Resistors, R1 and R2 provide the usual stabilizing DC bias for the transistor in the normal manner while the capacitor acts as a DC- blocking capacitors. The radio-frequency choke (RFC) is used to provide a high reactance (ideally open circuit) at the frequency of oscillation, (fr) and a low resistance at DC.

RC Phase-Shift Oscillator

In a RC Oscillator the input is shifted 180° through the amplifier stage and180° again through a second inverting stage giving us "180° + 180° = 360°" of phase shift which is the same as 0° thereby giving us the required positive feedback. In other words, the phase shift of the feedback loop should be "0".

In a Resistance-Capacitance Oscillator or simply an RC Oscillator, we make use of the fact that a phase shift occurs between the input to a RC network and the output from the same network by using RC elements in the feedback branch, for example.

RC Phase-Shift Network

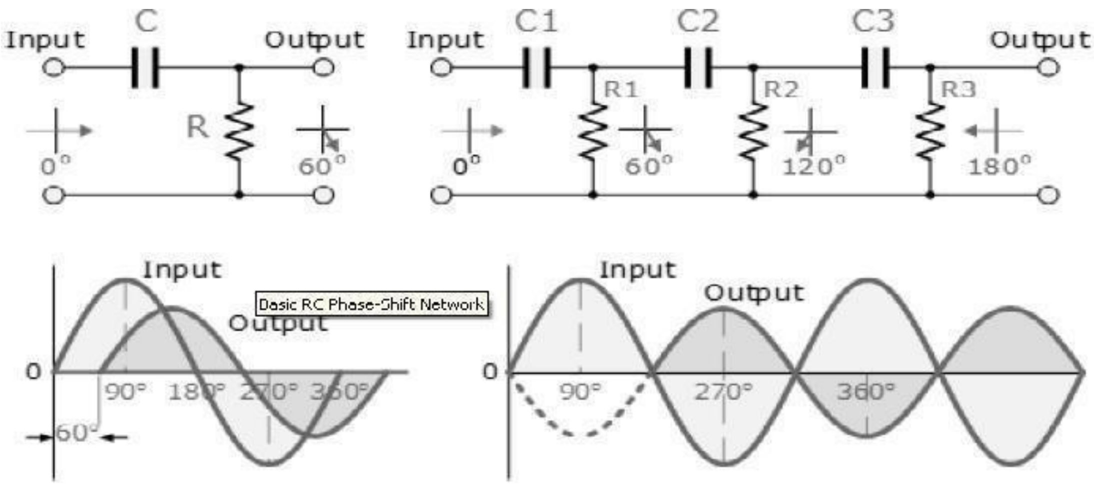


Fig 5.24: RC Phase-Shift Network

The circuit on the left shows a single resistor -capacitor network and whose output voltage "leads" the input voltage by some angle less than 90 °. An ideal RC circuit would produce a phase shift of exactly 90 degree. The amount of actual phase shift in the circuit depends upon the values of the resistor and the capacitor,the chosen frequency of oscillations with the phase angle (Φ) being given as:

$$\phi = \tan^{-1} \frac{X_C}{R}$$

RC Oscillator Circuit

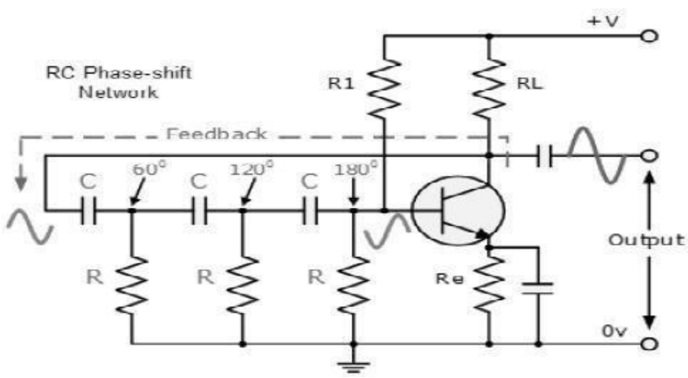


Fig 5.25: RC Oscillator Circuit

The RC Oscillator which is also called a Phase Shift Oscillator**,** produces a sine wave output signal using regenerative feedback from the resistor- capacitor combination. This regenerative feedback from the RC network is due to the ability of the capacitor to store an electric charge, (similar to the LC tank circuit).

This resistor-capacitor feedback network can be connected as shown above to produce a leading phase shift (phase advance network) or interchanged to produce a lagging phase shift (phase retard network) the outcome is still the same as the sine wave oscillations only occur at the frequency at which the overall phase-shift is 360^o.
By varying one or more of the resistors or capacitors in the phase-shift network, the frequency can be varied and generally this is done using a 3-ganged variable capacitor

If all the resistors, R and the capacitors, C in the phase shift network are equal in value, then the frequency of oscillations produced by the RC oscillator is given as:

$$f = \frac{1}{2\pi CR\sqrt{6}}$$

WIEN BRIDGE OSCILLATOR

One of the simplest sine wave oscillators which uses a RC network in place of the conventional LC tuned tank circuit to produce a sinusoidal output waveform, is the Wien Bridge Oscillator.The Wien Bridge Oscillator is so called because the circuit is based on a frequency-selective form of the Whetstone bridge circuit. The Wien Bridge oscillator is a two-stage RC coupled amplifier circuit that has good stability at its resonant frequency, low distortion and is very easy to tune making it a popular circuit as an audio frequency oscillator

Wien Bridge Oscillator

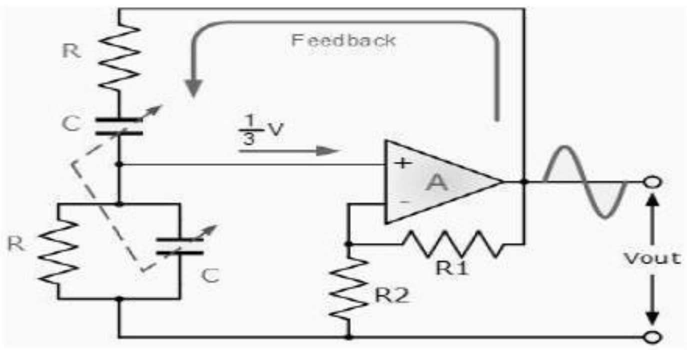


Fig 5.26: **Wien Bridge Oscillator**

The output of the operational amplifier is fed back to both the inputs of the amplifier. One part of the feedback signal is connected to the inverting input terminal (negative feedback) via the resistor divider network of R1 and R2 which allows the amplifiers voltage gain to be adjusted within narrow limits. The other part is fed back to the non- inverting input terminal (positive feedback) via the RC Wien Bridge network. The RC network is connected in the positive feedback path of the amplifier and has zero phase shift a just one frequency. Then at the selected resonant frequency, (fr) the voltages applied to the inverting and non-inverting inputs will be equal and "in-phase" so the positive feedback will cancel out the negative feedback signal causing the circuit to oscillate.

Also the voltage gain of the amplifier circuit MUST be equal to three "Gain =3" for oscillations to start. This value is set by the feedback resistor network, R1 and R2 for an inverting amplifier and is given as the ratio -R1/R2.also due to the open loop gain limitations of operational amplifiers; frequencies above 1MHZ are unachievable without the use of special high frequency op-amps. Then for oscillations to occur in a weinbridge oscillator circuit the following conditions must apply.

1. With no input signal the Wien Bridge Oscillator produces output oscillations.

2. The Wien Bridge Oscillator can produce a large range of frequencies.

3. The Voltage gain of the amplifier must be at least 3.

4. The network can be used with a Non-inverting amplifier.

5. The input resistance of the amplifier must be high compared to R so that the RC network is not overloaded and alter the required conditions.

6. The output resistance of the amplifier must be low so that the effect of external loading is minimized.

7. Some method of stabilizing the amplitude of the oscillations must be provided because if the voltage gain of the amplifier is too small the desired oscillation will decay and stop and if it is too large the output amplitude rises to the value of the supply rails, which saturates the op-amp and causes the output waveform to become distorted.

8. With amplitude stabilization in the form of feedback diodes, oscillations from the oscillator can go on indefinitely.

9. Frequency of oscillation $F=1/2*\pi*RC$

TEXT / REFERENCE BOOKS

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