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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

BASIC ELECTRICAL AND ELECTRONICS ENGINEERING-SEEA1101

UNIT I DC CIRCUITS

UNIT – I

DC CIRCUITS

Electrical Quantities, Ohm's law, Kirchoff's laws, Resistors in series and parallel combinations, Current and Voltage division rules, Node and Mesh Analysis.

ELECTRICAL QUANTITIES – DEFINITIONS, SYMBOLS AND UNITS

- **Charge:**

A body is said to be charged positively, if it has deficit of electrons. It is said to be charged negatively if it has excess of electrons. The charge is measured in Coulombs and denoted by Q (or) q.

1 Coulomb = Charge on 6.28×10^{18} electrons.

- **Atom:**

To understand the basic concepts of electric current, we should know the Modern Electron Theory. Consider the matter which is in the form of solid, liquid (or) gas. Smallest particle of matter is molecule. Minute Particles are called molecules, which are themselves made up of still minute particles known as Atoms.

Atom: Minute tiny Particles with the central Part Nucleus.

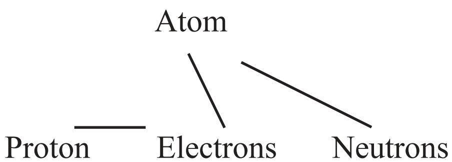


Figure 1.1

These are the types of tiny Particles in an Atom.

Protons: It is charged with positive charge.

Neutron: It is uncharged and hence it is neural.

Electron: It is revolving around nucleus. It is charged with small and constant amount of negative charge.

In an Atom, No of electrons = No of Protons

- **Electric Potential:**

When a body is charged, either electrons are supplied on it (or) removed from it. In both cases the work is done. The ability of the charged body to do work is called electric potential. The charged body has the capacity to do, by moving the other charges by either attraction (or) repulsion.

The greater the capacity of a charged body to do work, the greater is its electric potential. And the work done, to charge a body to 1 Colomb is the measure of electric potential.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{W}{Q}$$

W = Work done per unit charge.
Q = Charge measured in Coulombs.

Unit of electric potential is **Joules / Coulomb (or) Volt**. If W = 1 joule; Q = 1 Coulomb, then V = 1/1 = 1 Volt.

A body is said to have an electric potential of 1 Volt, if one Joule of work is done to charge a body to one Coulomb. Hence greater the Joules / Coulomb on a charged body, greater is electric potential.

- **Potential Difference:**

The difference in the potentials of two charged bodies is called potential difference.

Consider two charged bodies A and B having Potentials of 5 Volts and 3 Volts respectively.

Potential Difference is +2v.
Unit of potential difference is Volts.
Potential difference is sometimes called Voltage.

- **Electric Current:**

Flow of free electrons through a conductor is called electric current. Its unit is Ampere (or) Coulomb / sec.

$$\text{Current, } (I) = \frac{\text{Charge}(q)}{\text{Time}(t)} = \frac{q}{t} \text{ Coulombs / Sec}$$

In differential form, $i = \frac{dq}{dt}$ Coulombs / Sec

Consider a conducting material like metal, say Copper. A large number of free electrons are available. They move from one Atom to the other at random, before an electric force is applied. When an electric potential difference is applied across the metallic conductors, free electrons start moving towards the positive terminal of the cell. This continuous flow of electrons forms electric current. According to modern electronic theory, the direction of conventional current is form positive terminal to negative terminal through the external circuit.

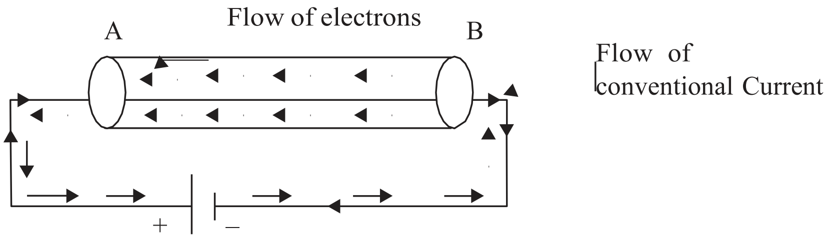


Figure 1.2

Thus, a wire is said to carry a current of 1 Ampere when charge flows through it at the rate of one Coulomb per second.

- **Resistance:**

Consider a conductor which is provided some potential difference. The free electrons start moving in a particular direction. While moving, the free electrons may collide with some Atoms (or) Molecules. They oppose the flow of electrons. Resistance is defined as the property of the substance due to which restricts the flow of electrons through the conductor. Resistance may, also be defined as the physical property of the substance due to which it opposes (or) restricts the flow of electricity (i.e. electrons) through it. Its unit is Ohms.

A wire is said to have a resistance of 1 ohm if a potential difference of 1V across the ends causes current of 1 Amp to flow through it (or) a wire is said to have a resistance of 1 ohm if it releases 1 Joule, when a current of 1A flows through it.

- **Laws of Resistance:**

The electrical resistance (R) of a metallic conductor depends upon the various Factors as given below,

(i) It is directly proportional to length l, ie, $R \alpha l$

(ii) It is inversely proportional to the cross sectional area of the Conductor, ie, $R \alpha \frac{1}{A}$

(iii) It depends upon the nature of the material of the conductor.

(iv) It depends upon the temperature of the conductor.

From the First three points and assuming the temperature to remain constant, we get,

$$R \propto \frac{l}{A}$$

$$R = \rho \frac{l}{A}$$

ρ ('Rho') is a constant of proportionality called **Resistivity** (or) Specific Resistance of the material of the conductor. The value of ρ depend upon the nature of the material of the conductor.

- **Specific Resistance (or) Resistivity:**

Resistance of a wire is given by $R = \rho \frac{l}{A}$

If l = 1metre, A =1m² then, R = ρ . The resistance offered by a wire of length 1 metre and across sectional area of Cross-section of 1m² is called the Resistivity of the material of the wire.

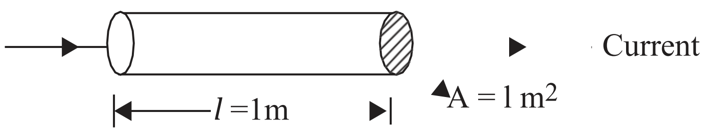


Figure 1.3

If a cube of one meter side is taken instead of wire, ρ is defined as below., Let l = 1 metre, A = 1 m², then R = ρ. "Hence, the resistance between the opposite faces of 1 metre cube of the given material is called the resistivity of that material". The unit of resistivity is ohm-metre

$$[\rho = \frac{RA}{l} = \frac{\Omega m^2}{m} = \Omega m(\text{ohm-metre})\ ]1$$

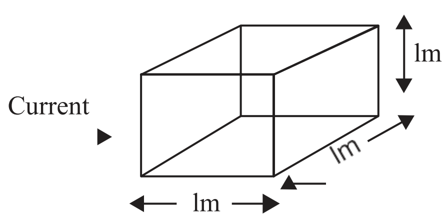


Figure 1.4

- **Conductance (or) Specific Conductance:**

Conductance is the inducement to the flow of current. Hence, Conductance is the reciprocal of resistance. It is denoted by symbol G.

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

G is measured in mho Symbol for its unit is (U)

$$\sigma = \frac{1}{\rho}$$

Here, σ is called the Conductivity (or) Specific Conductance of the material

- **Conductivity (or) Specific Conductance:**

Conductivity is the property (or) nature of the material due to which it allows flow of current through it.

$$G = \sigma \frac{A}{l} \text{ (or) } \sigma = G \frac{l}{A}$$

Substituting the units of various quantities we get

$$\sigma = \frac{\text{mho*m}}{m^2} = \text{mho/metre}$$

∴ The S.I unit of Conductivity is mho/metre.

- **Electric Power:**

The rate at which the work is done in an electric circuit is called electric power.

$$\text{Electric Power} = \frac{\text{Work done in an electric circuit}}{\text{Time}}$$

When voltage is applied to a circuit, it causes current to flow through it. The work done inmoving the electrons in a unit time is called Electric Power. The unit of Electric Power is Joules/sec (or) Watt. $\leq \Upsilon P = VI = I^2 R = V^2 / R_f$

- **Electrical Energy:**

The total work done in an electric circuit is called electrical energy.

ie, Electrical Energy = (Electrical Power)*(Time)

$$\text{Electrical Energy} = I^2Rt = \frac{V^2}{R}t$$

Electrical Energy is measured in Kilowatt hour (kwh)

Problem 1.1 The resistance of a conductor 1 mm^2 in cross section and 20 m long is 0.346 **Ω**. Determine the specific resistance of the conducting material.

Given Data

Area of cross-section A = 1 mm^2
Length, l = 20 m
Resistance, R = 0.346 Ω

Formula used: Specific resistance of the Conducting Material, $R = \frac{\rho l}{A}$

$$\Rightarrow \rho = \frac{RA}{l}$$

Solution: Area of Cross-section, $A = 1 mm^2$

$$= 1 * 10^{-6} m^2$$

$$\rho = \frac{1*10^{-6} * 0.346}{20} = 1.738*10^{-8}\ \Omega m$$

Specific Resistance of the conducting Material, $\rho = 1.738*10^{-8}\ \Omega m$.

Problem 1.2 A Coil consists of 2000 turns of copper wire having a cross-sectional area of 1 $mm^{2.}$ The mean length per turn is 80 cm and resistivity of copper is 0.02 μΩm at normal working temperature. Calculate the resistance of the coil.

Given data:

No of turns = 2000
Length / turn = 80 cm =0.8 m

Resistivity, = 0.02 μΩm = $0.02*10^{-6}$ Ωm = $2*10^{-8}$ Ωm
Cross sectional area of the wire, A= 1mm^2 = $1*10^{-6}m^2$

Solution:
Mean length of the wire, l = 2000*0.8 =1600 m.

We know that, $R = \rho \frac{l}{A}$

Substituting the Values, $R = \frac{2*10^{-8}*1600}{1*10^{-6}} = 32\Omega$

Resistance of the coil = 32Ω

Problem 1.3 A wire of length 1m has a resistance of 2Ω. What is the resistance of the second wire, whose specific resistance is double that of first, if the length of wire is 3m and the diameter is double that of first?

Given Data:

For the first wire: $l_1 = 1m$, $R_1 = 2\,\Omega$, $\rho_1 = \rho$ (say)

$$d_1 = d\ (say)$$

For the Second wire: $l_2 = 3m$, $d_2 = 2d$, $\rho_2 = 2\rho$

Solution:

$$R_1 = \rho_1 \frac{l_1}{A_1} = \frac{\rho * 1}{\frac{\pi d^2}{4}}$$ [Radius of the wire = πr^2, where $r = \frac{d}{2}$]

$$\text{ie, } R_1 = \frac{4\rho}{\pi d^2} = \frac{\rho_1 * 1}{\frac{\pi d^2}{4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} = \frac{2\rho * 3}{\frac{\pi (2d)^2}{4}} = \frac{6\rho}{\pi d^2} \qquad (2)$$

Dividing equation (1) by (2),

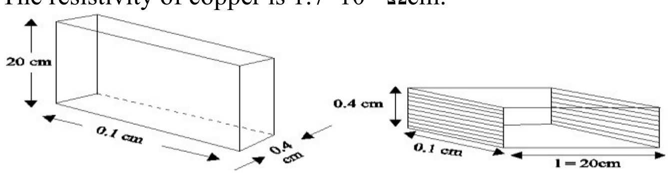
$$\frac{4\rho}{\pi d^2} * \frac{\pi d^2}{6\rho} \Rightarrow \frac{4}{6} = \frac{R_1}{R_2}$$

$$R_2 = \frac{6R_1}{4} = \frac{6 * 2}{4} = 3\,\Omega$$

$$R_2 = 3\,\Omega$$

The Resistance of the second wire, $R_2 = 3\,\Omega$

Problem 1.4 A Rectangular copper strip is 20 cm long, 0.1 cm wide and 0.4 cm thick. Determine the resistance between (i) opposite ends and (ii) opposite sides. The resistivity of copper is $1.7*10^{-6}\,\Omega$cm.



(i) Opposite Ends

Wide, w = 0.1cm

Thickness, t = 0.4cm

Length, l = 20cm

(ii) Opposite Sides:

Wide, w = 0.1cm

Thickness, t = 20 cm

Length, l = 0.4 cm

(a) Area = $w*t = 0.1*0.4 = 0.04cm^2$

$$R_1 = \frac{\rho l}{A} = \frac{1.7*10^{-6}*20}{0.04} = 0.85*10^{-3}\Omega$$

$R_1 = 0.85\, m\wedge$ [Opposite ends, referring to Figure 1.5]

Area, $A = w*t = 0.1*20 = 20cm^2$

$$R_2 = \frac{1.7*10^{-6}*0.4}{2} = 0.34*10^{-6}\,\Omega$$ [Opposite Sidesi referring to Figure 1.6]

$R_2 = 0.34\mu\,\Omega$

Problem 1.5 A silver wire of length 12m has a resistance of 0.2Ω. Find the specific resistivity of the material. The cross-sectional area of the wire is 0.01 cm^2.

$$R = \frac{\rho l}{A}$$

⇒ length, l =12m

Resistance, R= 0.2Ω

A = 0.01cm^2

$$\rho = \frac{RA}{l} = \frac{0.2*0.01*10^{-4}}{12}$$

$$\rho = 1.688*10^{-8}\,\Omega\, m$$

OHM'S LAW AND ITS LIMITATIONS

The relationship between DC potential difference (V) current (I) and Resistance (R) in a DC circuit was first discovered by the scientist George Simon Ohm, is called Ohm's law.

- **Statement:**

The ratio of potential difference between any two points of a conductor to the current following between them is constant, provided the physical condition

$$\text{ie, } \frac{V}{I} = \text{Constant}$$

(or)

$$\frac{V}{I} = R$$
$$\Rightarrow V = I * R$$

Where, R is the resistance between the two points of the conductor.

It can also be stated as, provided Resistance is kept constant, current is directly proportional to the potential difference across the ends of the conductor.

$$\text{Power, } P = V * I = I^2 R = \frac{V^2}{R}$$

- **Illustration:**

Let the potential difference between points A and B be V volts and current flowing be I Amp. Then, $\frac{V}{I}$ = Constant ,

$$\frac{V}{I} = R \text{ (say)}$$

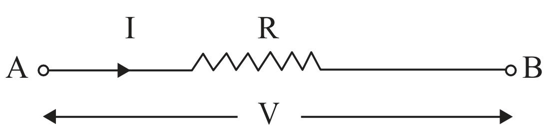


Figure 1.7

We know that, if the voltage is doubled (2V), the current flowing will also be doubled (2I). So, the ratio $\frac{V}{I}$ remains the same (ie, R). Also when voltage is measured in volts, current in ampere, then resistance will be in ohms.

- **Graphical representation of Ohm's law**

[Slope line of the graph represents the resistance]

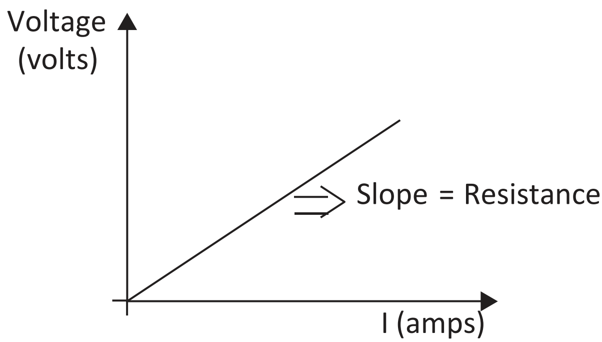


Figure 1.8

- **Limitations in ohm's law**:

(i) Ohm's law does not apply to all non-metallic conductors. For eg. Silico Carbide.
(ii) It also does not apply to non-linear devices such as Zener diode, etc.
(iii) Ohm's law is true for metal conductor at constant temperature. If the temperature changes the law is not applicable.

- **Problems based on ohm's law**:

Problem 1.6. An electric heater draws 8A from 250V supply. What is the power rating? Also find the resistance of the heater element.

Given data:

Current, $I = 8A$
Voltage, $V = 250V$

Solution:

Power rating, $P = VI = 8 * 250 = 2000\,Watt$

Resistance (R) $= \frac{V}{I} = \frac{250}{8} = 31.25\ \Omega$

Problem 1.7 What will be the current drawn by a lamp rated at 250V, 40W, connected to a 230 V supply.

Given Data:

Rated Power = 40 W

Rated Voltage = 250 V

Solution:

Resistance,

$$R = \frac{V^2}{P} = \frac{250^2}{40} = 1562.5\ \Omega$$

$$\text{Current, } I = \frac{V}{P} = \frac{230}{1562.5} = 0.1472\ A$$

Problem 1.8 A Battery has an emf of 12.8 volts and supplies a current of 3.24 A. What is the resistance of the circuit? How many Coulombs leave the battery in 5 minutes?

Solution:

$$\text{Circuit Resistance, } R = \frac{V}{I} = \frac{12.8}{3.24} = 4\ \Omega$$

Charge flowing in 5 minutes = Current × time in seconds

Charge flowing in 5 minutes = 3.24×5×60 = 960 Coulomb

Problem 1.9 If a resistor is to dissipate energy at the rate of 250W, find the resistance for a terminal voltage of 100V.

Given data:

Power = 250W

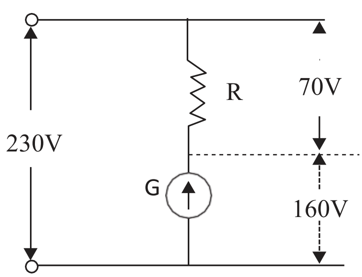
Voltage = 100V

Solution:

$$\text{Resistance, } R = \frac{V^2}{\rho} = \frac{100^2}{250} = 40\ \Omega$$

$$R = 40\ \mathbf{\Omega}.$$

Problem 1.10 A voltmeter has a resistance of, 20,200 Ω. When connected in series with an external resistance across a 230 V supply, the instrument reads 160 V. What is the value of external resistance?



The voltage drop across external resistance, R

$$V_R = 230 - 160 = 70V$$

$$\text{Circuit current, } I = \frac{160}{20,000} = \frac{1}{125}A$$

We know that, $V = IR$

$$70 = IR$$

$$70 = \frac{1}{125} \times R$$

$$R = 8750\ \Omega$$

COMBINATION OF RESISTORS

- **Introduction:**

The closed path followed by direct Current (DC) is called a DC Circuit A d.c circuit essentially consist of a source of DC power (eg. Battery, DC generator, etc.) the conductors used to carry current and the load. The load fora DC circuit is usually a resistance. In a DC circuit, loads (i.e, resistances) may be connected in series, parallel, series – parallel. Hence the resistor has to be connected in the desired way for getting the desired resistance.

Resistances in series (or) series combination

The circuit in which resistances are connected end to end so that there is one path for the current flow is called **series circuit**. The voltage source is connected across the free ends. [A and B]

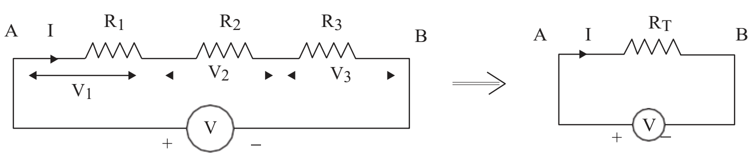


Figure 1.10

In the above circuit, there is only one closed path, so only one current flows through all the elements. In other words, if the Current is same through all the resistors, the combination is called series combination.

- **To find equivalent Resistance**:

Let, V = Applied voltage

I = Source current = Current through each element

V_1, V_2, V_3 are the voltage across R_1, R_2 and R_3 respectively.

By Ohms law, $V_1 = IR_1$

$V_2 = IR_2$ and $V_3 = IR_3$

But $V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$

$$V = I(R_1 + R_2 + R_3)$$

$$V = IR_T$$

$$\frac{V}{I} = R_T$$

The ratio of $\left(V/I\right)$ is the total resistance between points A and B and is called the total (or) equivalent resistance of the three resistances

$$R_T = R_1 + R_2 + R_3$$

Also, $\frac{1}{G_T} = \frac{1}{G_1} + \frac{1}{G_2} + \frac{1}{G_3}$ (In terms of conductance)

∴ Equivalent resistance (R_T) is the sum of all individual resistances.

- **Concepts of series circuit**:

i. The current is same through all elements.
ii. The voltage is distributed. The voltage across the resistor is directly proportional to the current and resistance.
iii. The equivalent resistance (R_T) is greater than the greatest individual resistance of that combination.
iv. Voltage drops are additive.
v. Powers are additive.
vi. The applied voltage is equals to the sum of different voltage drops.

Voltage Division Technique: (or) To find V_1, V_2, V_3 interms of V and R_1, R_2, R_3:

Equivalent Resistance, $R_T = R_1 + R_2 + R_3$

By ohm's low, $I = \frac{V}{R_T} = \frac{V}{R_1 + R_2 + R_3}$

$$V_1 = IR_1 = \frac{V}{R_T} R_1 = \frac{VR_1}{R_1 + R_2 + R_3}$$

$$V_2 = IR_2 = \frac{V}{R_T} R_2 = \frac{VR_2}{R_1 + R_2 + R_3}$$

$$V_3 = IR_3 = \frac{V}{R_T} R_3 = \frac{VR_3}{R_1 + R_2 + R_3}$$

∴ Voltage across any resistance in the series circuit,

$$\Rightarrow V_x = \frac{R_x}{R_T} V$$

Note: If there are n resistors each value of R ohms in series, then the total Resistance is given by,

$$R_T = n * R$$

- **Applications:**
  - * When variable voltage is given to the load, a variable resistance (Rheostat) is connected in series with the load. Example: Fan regulator is connected in series with the fan.
  - * The series combination is used where many lamp of low voltages are to be operated on the main supply. Example: Decoration lights.
  - * When a load of low voltage is to be operated on a high voltage supply, a fixed value of resistance is connected in series with the load.
- **Disadvantage of Series Circuit:**
  - * If a break occurs at any point in the circuit, no current will flow and the entire circuit becomes useless.
  - * If 5 numbers of lamps, each rated 230 volts are to be connected in series circuit, then the supply voltage should be 5 x 230 = 1150 volts. But voltage available for lighting circuit in each and every house is only 230 V. Hence, series circuit is not practicable for lighting circuits.
  - * Since electrical devices have different current ratings, they cannot be connected in series for efficient operation.
- **Problems based on series combination:**

Problem 1.11 Three resistors 30 Ω, 25 Ω, 45 Ω are connected in series across 200V. Calculate (i) Total resistance (ii) Current (iii) Potential differenceacross each element.

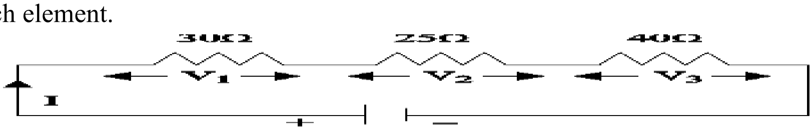


Figure 1.13

(i) Total Resistance (R_T)

$$R_T = R_1 + R_2 + R_3$$

$$R_T = 30 + 25 + 45 = 100\ \Omega$$

(ii) Current, $I = \dfrac{V}{R_T} = \dfrac{200}{100} = 2\ A$

(iii) Potential difference across each element,

$$V_1 = IR_1 = 2 * 30 = 60\ V$$

$$V_2 = IR_2 = 2 * 25 = 50\ V$$

$$V_3 = IR_3 = 2 * 45 = 90\ V$$

Problem 1.12 Find the value of 'R' in the circuit diagram, given below.

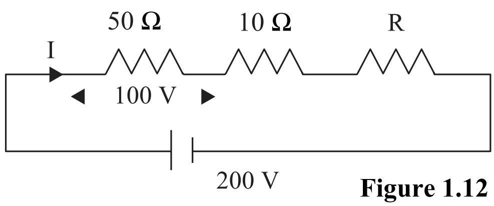


Figure 1.12

We know that, $V_1 = IR_1$

$$I = V_1 / R_1 = 100/50 = 2\ A$$

Similarly, $V_2 = IR_2 = 2 * 10 = 20\ V$

Total voltage drop, $V = V_1 + V_2 + V_3$

$$V_3 = V - (V_1 + V_2) = 200 - (100 + 20)$$

$$V_3 = 80\ V$$

$$V_3 = IR_3\ ,\ R_3 = V_3 / I = 80/2 = 40\ \Omega$$

$$\therefore R_3 = 40\ \Omega$$

Problem 1.13 A 100W, 200V bulb is put in series with a 60W bulb across a supply. What will be the current drawn? What will be the voltage across the 60W bulb? What will be the supply voltage?

100W 60W

Figure 1.13

Power dissipated in the first bulb, $P_1 = V_1 I$
Current, I = P_1 / V_1 = 100/200 = 0.5 A

Power dissipated in the second bulb, $P_2 = V_2 I$
Voltage across the 60 W bulb,

$$V_2 = \frac{P_2}{I} = \frac{60}{0.5} = 120V$$

The supply voltage, $V = V_1 + V_2 = 200 + 120$

$$V = 320V$$

The supply voltage, V = 320 V.

Problem 1.14 An incandescent lamp is rated for 110V, 100W. Using suitable resistor how can you operate this lamp on 220V mains.

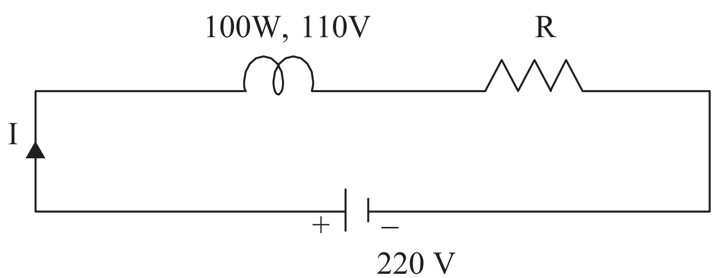


Figure 1.14

Rated current of the lamp, $I = \frac{Power}{Voltage} = \frac{100}{110} = 0.909A$, I = 0.909A

For satisfactory operation of the lamp, Current of 0.909A should flow. When the voltage across the lamp is 110V, then the remaining voltage must be across R

$$\text{Supply voltage} = V = 220 \text{ Volts}$$

$$\text{Voltage across } R = V - 110 \text{ Volts}$$

$$\text{ie, } V_R = 220 - 110 = 110V$$

$$\text{By ohm's law, } V_R = IR$$

$$110 = 0.909\,R$$

$$R = 121\ \Omega$$

Problem 1.15 The lamps in a set of decoration lights are connected in series. If there are 20 lamps and each lamp has resistance of 25Ω, calculate the total resistance of the set of lamp and hence calculate the current taken from a supply of 230 volts.

Given Data: Supply voltage, $V = 230$ *volts*

Resistance of each lamp, $R = 25\ \Omega$

No of lamps in series, $n = 20$

Solution: Total Resistance, $R_T = n * R = 20 * 25$

$R_T = 500\ \Omega$

Current from supply. $I = \frac{V}{R_T} = \frac{230}{500} = 0.46\ A$

Problem 1.16 The field coil of a d.c generator has a resistance of 250Ω and is supplied from a 220 V source. If the current in the field coil is to be limited to 0.44 A. Calculate the resistance to be connected in series with the coil.

Given Data: Source voltage, $V = 220$ volts, $I = 0.44\ A$

Field coil resistance, $R_f = 250\ \Omega$

Solution: Let the resistance in series with R_f be R in Ohms.

Total resistance, $R_T = R_f + R = 250 + R$

Current, $I = 0.44\ A$

By ohm's law, $R_T = \frac{V}{I} = \frac{220}{0.44} = 500\ \Omega$

$250 + R = 500\ \Omega$

$R = 500 - 250 = 250\ \Omega$

$R = 250\ \Omega$

Resistance in Parallel (or) Parallel Combination

If one end of all the resistors are joined to a common point and the other ends are joined to another common point, the combination is said to be parallel combination. When the voltage source is applied to the common points, the voltage across each resistor will be same. Current in the each resistor is different and is given by ohm's law.

Let R_1, R_2, R_3 be three resistors connected between the two common terminals A and B, as shown in the Figure 1.15(a)..

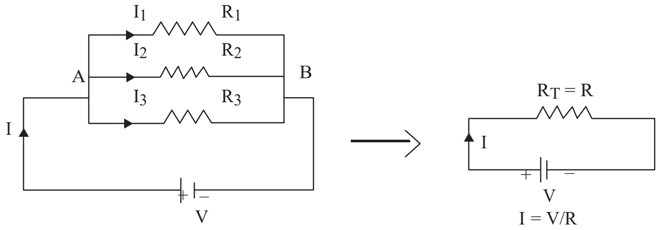


Figure 1.15

$$I = \frac{V}{R} \quad (1)$$

Let I_1, I_2, I_3 are the currents through R_1, R_2, R_3 respectively. By ohm's law,

$$\left[I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3} \right] \quad (2)$$

Total current is the sum of three individual currents,

$$I_T = I = I_1 + I_2 + I_3 \quad (3)$$

Substituting the above expression for the current in equation (3),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Referring to Figure (1.15(b)), $R_T = R$

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (4)$$

Hence, in the case of parallel combination the reciprocal of the equivalent resistance is equal to the sum of reciprocals of individual resistances. Multiplying both sides of equation (4) by V^2, we get

$$\frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2} + \frac{V^3}{R_3}$$

ie, Power dissipated by R = Power dissipated by R_1 + Power dissipated by R_2 + Power dissipated by R_3

We know that reciprocal of Resistance is called as conductance.

Conductance = 1 / Resistance

[G = 1/R]

Equation (4) can be written as,

$$G = G_1 + G_2 + G_3$$

- **Concepts of Parallel Circuit:**
  - Voltage is same across all the elements.
  - All elements will have individual currents, depends upon the resistance of element.
  - The total resistance of a parallel circuit is always lesser than the smallest of the resistance.
  - If n resistance each of R are connected in parallel then,

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots\ldots\ldots n\,terms$$

$$\frac{1}{R_T} = \frac{n}{R}$$

(or)

$$R_T = \frac{R}{n}$$

  - Powers are additive.
  - Conductance are additive.
  - Branch currents are additive.

- **Current Division Technique:**

Case (i) When two resistances are in parallel:

Two resistance $R_1$and R_2 ohms are connected in parallel across a battery of V (volts) Current through R_2 is I_2 and through R_2 is I_2 The total current is I.

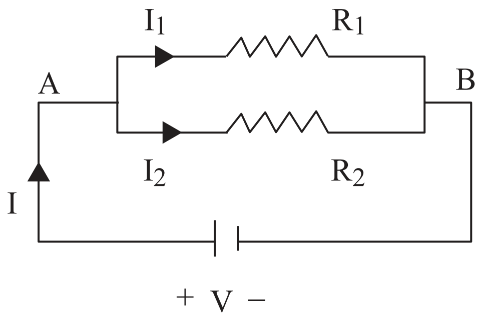


Figure 1.16

To express I_1 and I_2 interms of I, R_1 and R_2 (or) to find branch currents I_1, I_2:

$$I_2 R_2 = I_1 R_1$$

$$I_2 = \frac{I_1 R_1}{R_2} \qquad (1)$$

Also, the total current, $I = I_1 + I_2$ (2)

Substituting (1) in (2), $I_1 + \frac{I_1 R_1}{R_2} = I$

$$\frac{I_1 R_2 + I_1 R_1}{R_2} = I$$

$$I_1 (R_1 + R_2) = IR_2$$

$$I_1 = \frac{IR_2}{(R_1 + R_2)}$$

Similarly, $I_2 = \frac{IR_1}{(R_1 + R_2)}$

To find the equivalent Resistance, (R_T) :

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_2 + R_1}{R_1 R_2}$$

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Hence, the total value of two resistances connected parallel is equal to their product divided by their sum i.e.,

$$\text{Equivalent Resistance} = \frac{\text{Product of the two Resistance}}{\text{Sum of the two Resistane}}$$

Case (ii) When three resistances are connected in parallel. Let R_1, R_2 and R_3 be resistors in parallel. Let I be the supply current (or) total current. I_1, I_2, and I_3 are the currents through the resistors R_1, R_2 and R_3.

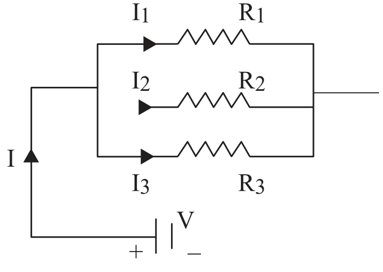


Figure 1.17

To find the equivalent Resistance (R_T):

$$\frac{1}{R} = \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_T} = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1R_2R_3}$$

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

To find the branch currents I_1, I_2 and I_3:

We know that, $I_1 + I_2 + I_3 = I$ (1)

Also, $I_3R_3 = I_1R_1 = I_2R_2$

From the above expression, we can get expressions for I_2 and I_3 interms of I_1 and substitute them in the equation (1)

$$I_2 = \frac{I_1R_1}{R_2}; I_3 = \frac{I_1R_1}{R_3}$$

$$I_1 + \frac{I_1R_1}{R_2} + \frac{I_1R_1}{R_3} = I$$

$$I_1\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = I$$

$$\frac{I_1(R_2R_3 + R_3R_1 + R_1R_2)}{R_2R_3} = I$$

$$I_1 = \frac{I(R_2R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

Similarly we can express I_2 and I_3 as,

$$I_2 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

$$I_3 = \frac{I(R_1R_3)}{(R_1R_2 + R_2R_3 + R_3R_1)}$$

- **Advantages of parallel circuits:**
  - ∗ The electrical appliances rated for the same voltage but different powers can be connected in parallel without affecting each other's performance.
  - ∗ If a break occurs in any one of the branch circuits, it will have no effect on the other branch circuits.

- **Applications of parallel circuits:**
  - * All electrical appliances are connected in parallel. Each one of them can be controlled individually will the help of separate switches.
  - * Electrical wiring in Cinema Halls, auditoriums, House wiring etc.

Comparison of series and parallel circuits:

Series Circuit	Parallel Circuit
The current is same through all the elements.	The current is divided, inversely proportional to resistance.
The voltage is distributed. It is proportional to resistance.	The voltage is the same across each element in the parallel combination.
The total (or) equivalent resistance is equal to sum of individual resistance, ie. $R_T = R_1 + R_2 + R_3$ Hence, the total resistance is greater than the greatest resistance in the circuit.	Reciprocal of the equivalent resistance is equal to sum of reciprocals of individual resistances, ie, $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ Total resistance is lesser than the smallest resistances in the circuit.
There is only one path for the flow of current.	There are more than one path for the flow of current.

- **Problems based on parallel combinations:**

Problem 1.17 What is the value of the unknown resistor R shown in Figure 1.18. If the voltage drop across the 500Ω resistor is 2.5V. All the resistor are in ohms.

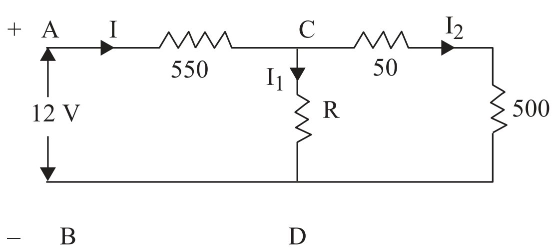


Figure 1.18

Given Data:

$$V_{500} = 2.5V$$

$$I_2 = \frac{V_{500}}{R} = \frac{2.5}{500} = 0.005A$$

V_{50} = Voltage across 50 Ω

$V_{50} = I_2 R = 0.005*50 = 0.25$ V

$V_{CD} = V_{50} + V_{500} = 0.25 + 2.5 = 2.75$ V

V_{550} = Drop across 550Ω = 12 − 2.75 = 9.25 V

$$I = \frac{V_{550}}{R} = \frac{9.25}{550} = 0.0168A$$

$$I = I_1 + I_2 \rightarrow I_1 = I - I_2 = 0.0168 - 0.005$$

$$I_1 = 0.0118A$$

$$R = \frac{V_{CD}}{I_1} = \frac{2.75}{0.0118} = 232.69\ \Omega$$

$R = 232.69\ \Omega$

Problem 1.18 Three resistors 2 Ω, 3 Ω and 4 Ω are in parallel. How will be a total current of 8A is divided.

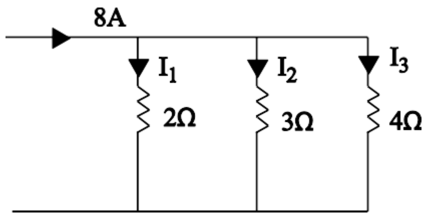


Figure 1.19

This given circuit can be reduced as, 3 Ω and 4 Ω are connected in parallel.

Its equivalent resistances are, $\frac{3*4}{3+4} = \frac{12}{7} = 1.714\,\Omega$

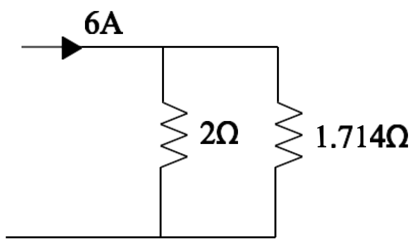


Figure 1.20

1.714 Ω and 2 Ω are connected in parallel, its equivalent resistance is 0.923 Ω

$$\frac{1.714 * 2}{2 + 1.714} = 0.923$$

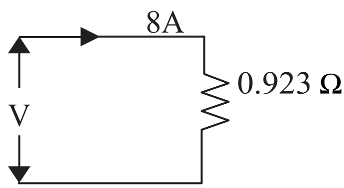


Figure 1.21

$$V = IR = 8 * 0.923$$
$$V = 7.385V$$

Branch currents, $I_1 = \frac{V}{R_1} = \frac{7.385}{2} = 3.69$ A

$$I_2 = \frac{V}{R_2} = \frac{7.385}{3} = 2.46A$$
$$I_3 = \frac{V}{R_3} = \frac{7.385}{4} = 1.84A$$

Problem 1.19 What resistance must be connected in parallel with 10Ω to give an equivalent resistance of 6Ω

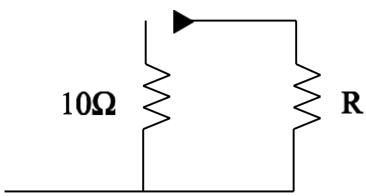


Figure 1.22

R is connected in parallel with 10 Ω Resistor to given an equivalent resistance of 6 Ω.

$$\frac{10 * R}{10 + R} = 6$$
$$10R = (10 + R)6$$
$$10R = 60 + 6R$$
$$10R - 6R = 60$$
$$R = \frac{60}{4} = 15\ \Omega$$
$$R = 15\ \Omega$$

Problem 1.20 Two resistors R_1 and R_2 are connected in Parallel and a Voltage of 200V DC is applied to the terminals. The total current drawn is 20A, R_1=30 Ω. Find R_2 and power dissipated in each resistor, for the figure 1.23.

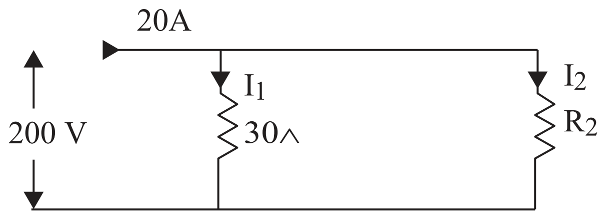


Figure 1.23

Given Data:

V = 200V, I = 20A, R_1 = 30 Ω

Solution: $I_1 = \frac{V}{R_1} = \frac{200}{30} = 6.667\ A$

$$I_1 + I_2 = I$$

$$I_2 = I - I_1$$

$$= 20 - 6.667 = 13.33 \text{ A}$$

$$I_2 = \frac{IR_1}{R_1 + R_2}$$

$$13.33 = \frac{20 * 30}{30 + R_2}$$

$$(30 + R_2)13.33 = 600$$

$$13.33R_2 = 600 - 400$$

$$13.33R_2 = 200$$

$$R_2 = \frac{200}{13.33} = 15\ \Omega$$

$$R_2 = 15\ \Omega$$

Power dissipated in 30 Ω, $P_1 = VI_1$ = 200*6.667

P_1 = 1333 W

Power dissipated in 15 Ω, $P_2 = VI_2$

P_2 = 200*13.33 = 2667

P_2 = 2667 W

Problem 1.21 Calculate the current supplied by the battery in the given circuit as shown in the figure 1.24.

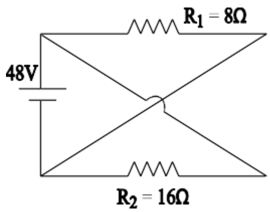


Figure 1.24

Solution: The above given circuit can be redrawn as,

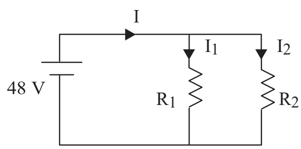


Figure 1.25

R_1 and R_2 are in parallel across the voltage of 48 volts.

Equivalent Resistance, $R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{8*16}{8+16} = \frac{16}{3}\ \Omega$

$$R_T = 5.33\wedge$$

$$I = \frac{V}{R} = \frac{48}{5.33} = 9\text{A}$$

Problem 1.22 Calculate the total resistance and battery current in the given circuit

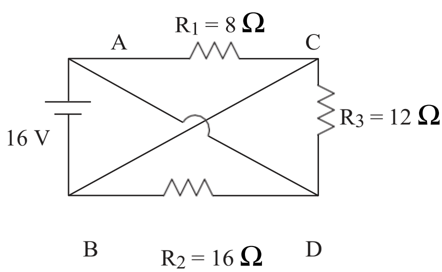


Figure 1.26

The given above circuit can be re-drawn as,

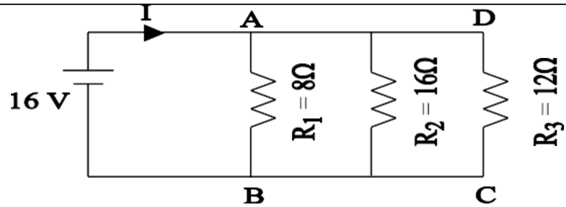


Figure 1.27

8 Ω, 16 Ω, 12 Ω are connected in parallel. Its equivalent resistance,

$$R_T = \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$$

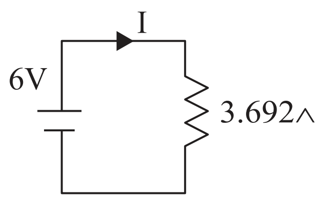


Figure 1.28

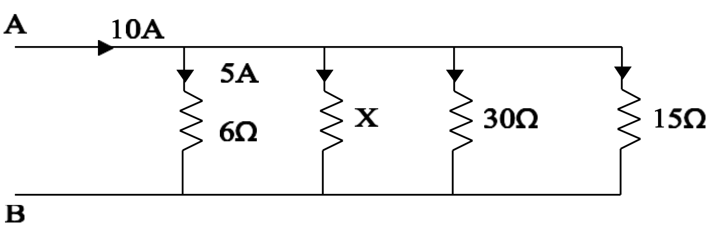
$$R_T = \frac{8*6*12}{128+192+96} = 3.692\ \Omega$$

$$R_T = 3.692\ \Omega$$

$$I = \frac{V}{R} = \frac{16}{3.692} = 4.33A$$

Problem 1.23 In the Circuit shown in the figure 1.29, calculate

(i) The current in all resistors.
(ii) The value of unknown resistance ‘x’
(iii) The equivalent resistance between A and B.



Solution: As all the resistors are in parallel, the voltage across each one is same. Give that current through 6 Ω , ie, $I_{6\,\Omega} = 5A$

Voltage across 6 Ω = 5×6 = 30volts.

Hence, current through 30 Ω, $I_{30} = \frac{V_{30}}{R} = \frac{30}{30} = 1A$

Similarly, current through 15 Ω, $I_{15} = \frac{V_{15}}{R} = \frac{30}{15} = 2A$

Total Current, $I = I_6 + I_x + I_{30} + I_{15}$

$$10 = 5 + I_x + 1 + 2$$

$$I_X = 2\ A$$

Hence, the current flowing through the 'X' Resistor is, $I_X = 2\ A$

Value of the Resistor 'X' is given by,

$$X = \frac{30}{I_x} = \frac{30}{2} = 15\ \Omega$$

Let, the equivalent resistance across AB = R_T

$$\frac{1}{R_T} = \frac{1}{6} + \frac{1}{x} + \frac{1}{30} + \frac{1}{15}$$

$$\frac{1}{R_T} = \frac{5 + 2 + 1 + 2}{30} = \frac{1}{3}$$

$$R_T = 3\ \Omega$$

Series — Parallel Combination

As the name suggests, this circuit is a combination of series and parallel circuits. A simple example of such a circuit is illustrated in Figure 1 30. R_3 and R_2 are resistors connected in parallel with each other and both together are connected in series with R_1.

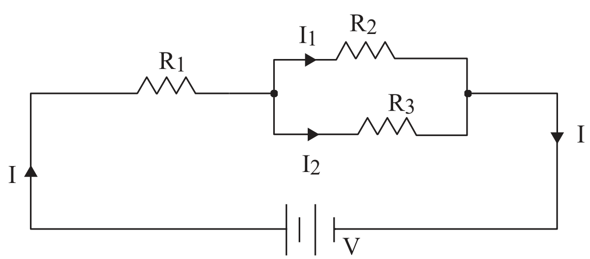


Figure 1.30

Equivalent Resistance: R_T for parallel combination.

$$R_P = \frac{R_2 R_3}{R_2 + R_3}$$

Total equivalent resistance of the circuit is given by,

$$R_T = R_1 + R_P$$

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3}$$

Voltage across parallel combination = $I * \frac{R_2 R_3}{R_2 + R_3}$.

- **Problems based on Series – Parallel Combination:**

Problem 1.24 In the circuit, find the current in all the resistors. Also calculate the supply voltage.

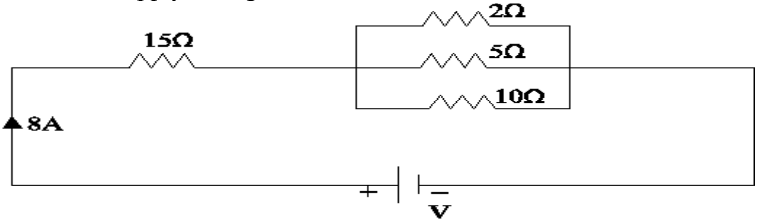


Figure 1.31

Voltage across 15∧, $V_{15} = I_{15} \times R = 8\times15 = 120V$

Resistors 2 Ω, 5 Ω, 10 Ω are connected in parallel, it equivalent resistance is given by,

$$R_P = \frac{2*5*10}{2\times5+5\times10+10\times2} = 1.25\ \Omega$$

Voltage across the parallel Combination is given by

$$V_p = V_2 = V_5 = V_{10} = I \times R_P = 8\times1.25 = 10V$$

Total supply Voltage, $V = V_{15}+V_p$

$$V = 120 + 10 = 130V$$

$$V = 130V$$

Hence, the Current through the parallel combination of the resistors are given by,

Current through 2 Ω resistor, $I_2 = \frac{V_2}{R} = \frac{10}{2} = 5A$

Current through 5 Ω Resistor, $I_5 = \frac{V_5}{R} = \frac{10}{5} = 2\,A$

Current through 10 Ω Resistor, $I_{10} = \frac{V_{10}}{R} = \frac{10}{10} = 1A$

The Current of 8A across the parallel combination is divided as 5A, 2A, and 1A.

Problem 1.25 Calculate the equivalent resistance offered by the circuit to the voltage source and also find its source current

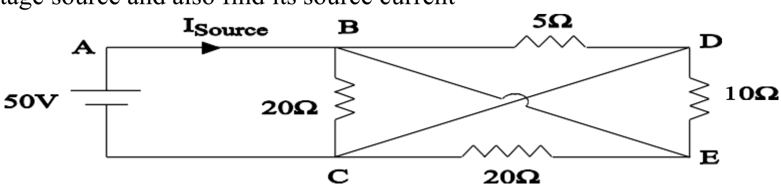


Figure 1.32

Solution: The given above circuit can be re-drawn as

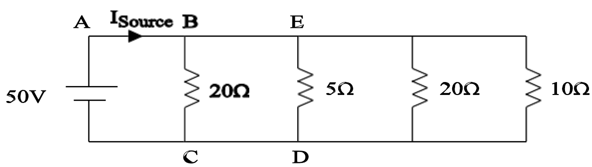


Figure 1.33

20 Ω and 10 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{20 * 10}{20 + 10} = 6.667\ \Omega$

The given circuit is reduced as,

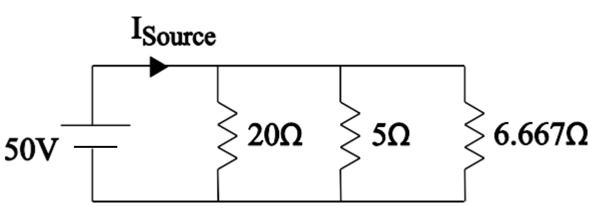


Figure 1.34

6.667 Ω and 5 Ω resistors are connected in parallel, its equivalent resistance is given by, $\frac{6.667 * 5}{6.667 + 5} = 2.857\ \Omega$

The circuit is reduced as,

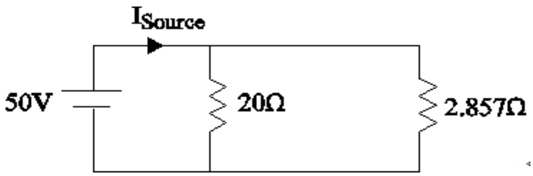


Figure 1.35

20 Ω and 2.857 Ω are connected in parallel. It equivalent resistance is,

$$\frac{20 * 2.857}{20 + 2.857} = 2.497\ \Omega$$

The Circuit is re-drawn as,

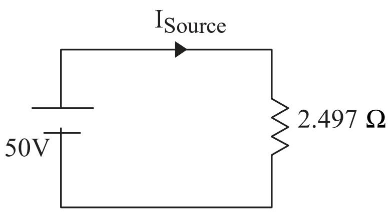


Figure 1.36

Hence the equivalent resistance of the Circuit is R_T= 2.497 Ω =2.5 Ω

Source Current of the Circuit is given by,

$$I_{source} = \frac{V}{R} = \frac{50}{2.5} = 20A$$

Problem 1.26 Find the equivalent resistance between the terminals A and B.

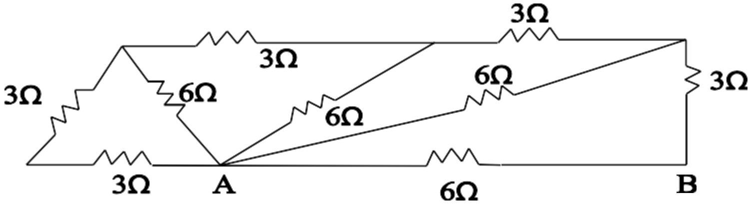


Figure 1.34

Solution:

3 Ω and 3 Ω are connected in Series, it equivalent resistance is, (3 + 3) = 6 Ω. The Circuit gets reduced as

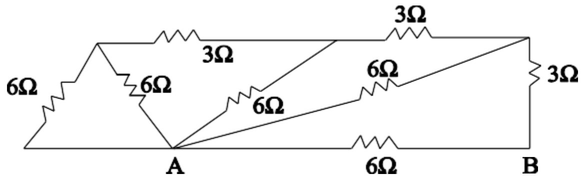


Figure 1.38

6 Ω and 6 Ω are connected in parallel. The circuit gets reduced as,

$$\frac{6*6}{6+6} = 3\text{ohms.}$$

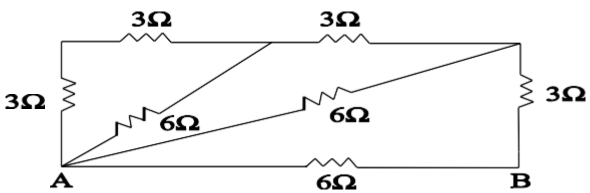


Figure 1.39

3 Ω and 3 Ω are connected in series (3 + 3 = 6 Ω).

The reduced Circuit is,

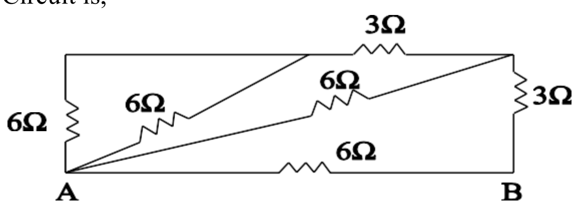


Figure 1.40

6 Ω and 6 Ω are connected in parallel. Its equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

The circuit can be reduced as,

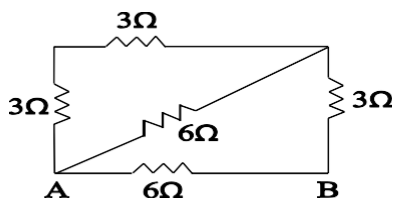


Figure 1.41

3 Ω and 3 Ω are connected in series. (3 + 3 = 6 Ω).

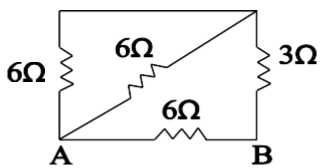


Figure 1.42

6 Ω and 6 Ω are connected in parallel. It equivalent resistance, $\frac{6*6}{6+6} = 3\ \Omega$

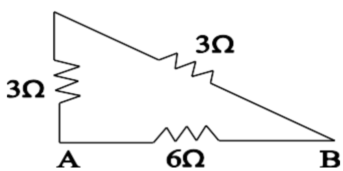


Figure 1.43

3 Ω and 3 Ω are connected in series, the reduced Circuit is 3 + 3 = 6 Ω

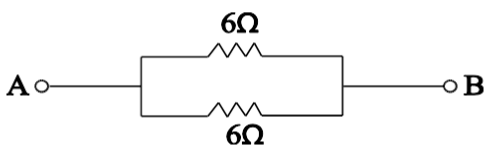


Figure 1.44

6 Ω and 6 Ω are connected in parallel.

$\frac{6*6}{6+6} = 3\ \Omega$. The equivalent resistance between the terminals A and B given by $R_{AB} = 3\ \Omega$.

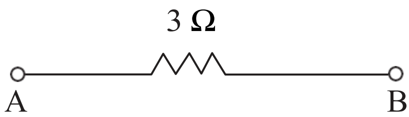


Figure 1.45

$$\therefore R_{AB} = 3\ \Omega$$

Problem 1.27 Determine the value of R if the power dissipated in 10 Ω resistor is 90 W.

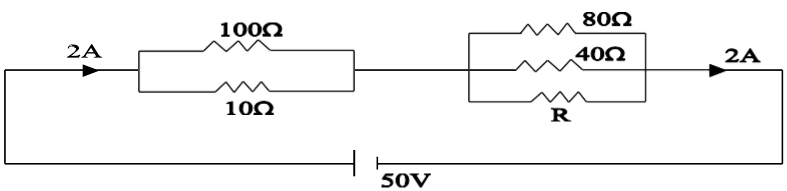


Figure 1.46

Solution:

100 Ω and 10 Ω are connected in parallel.

Its equivalent resistance is, $\frac{100*10}{100+10} = 9.09\ \Omega$

The circuit is reduced as,

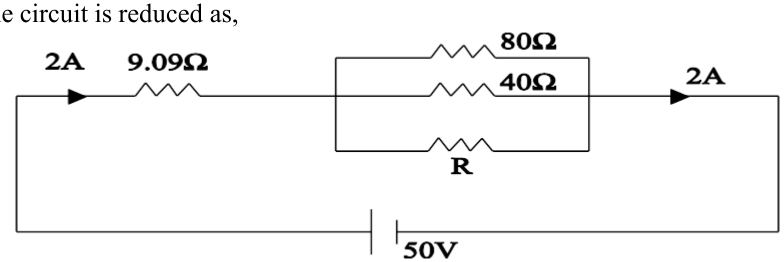


Figure 1.47

Current of 2A flows through the 9.09 Ω resistor. Voltage across 9.09 Ω is given by,

$$V_{9.09} = I_{9.09} \times R$$
$$V_{9.09} = 2 \times 9.09 = 18.18V$$

Similarly voltage across the unknown resistor V_R,

$$V_R = V - V_{9.09} = 50 - 18.18 = 31.818V$$

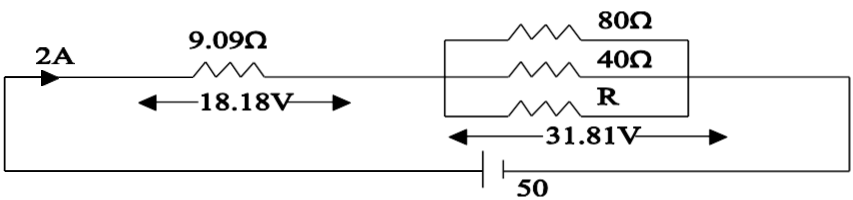


Figure 1.48

Hence the Current through 40 Ω, 80 Ω resistors can be found out with the voltage drop of 31.818V across it.

$$I_{80} = \frac{V_R}{80} = \frac{31.818}{80} = 0.397\ A$$

$$I_{40} = \frac{V_R}{40} = \frac{31.818}{40} = 0.7954\ A$$

Hence current through the unknown resistor R is I_R,

$$I_R = I - [I_{80} + I_{40}]$$
$$I_R = 2 - (0.397 + 0.7954) = 0.8075A$$

Hence, the value of the unknown Resistor R is given by

$$R = \frac{V_R}{I_R} = \frac{31.818}{0.8075} = 39.4\ \Omega$$

The value of the unknown resistor R is given by, R = 39.4 Ω.

Problem 1.28 Calculate the following for the circuits given,

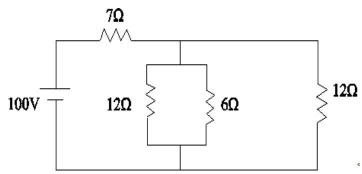


Figure 1.49

(i) Total resistance offered to the Source.
(ii) Total Current from the Source.
(iii) Power Supplied by the Source.

Solution: 12 Ω and 6 Ω are connected in Parallel.

Its equivalent resistance, $\frac{12*6}{12+6} = 4\ \Omega$. The reduced circuit is given as,

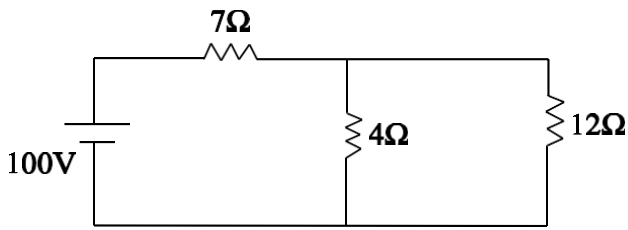


Figure 1.50

4 Ω and 12 Ω are connected in parallel. $\frac{4*12}{12+4} = 3\ \Omega$

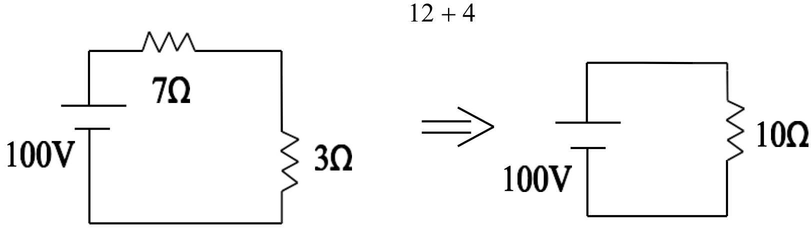


Figure 1.51

7 Ω and 3 Ω are connected in series, 7 + 3 = 10Ω

Total resistance offered to the Source, $R = 10\ \Omega$

Total Current from the Source, $I = \dfrac{100}{10} = 10$ A

$$I = 10A$$

Power supplied by the Source, $P = I^2R = 10^2 \times 10 = 1000$W

P = 1000W.

Problem 1.29 A letter A is Constructed of an uniform wire of 1 Ω resistance per cm. The signs of the letter are 60cm long and the cross piece is 30cm long, Apex angle 60°. Find the resistance of the letter between two ends of the legs.

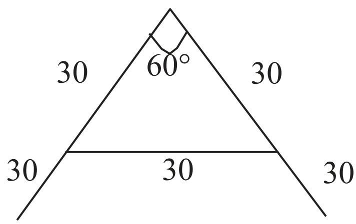


Figure 1.52

Solution:

The given circuit can be redrawn as,

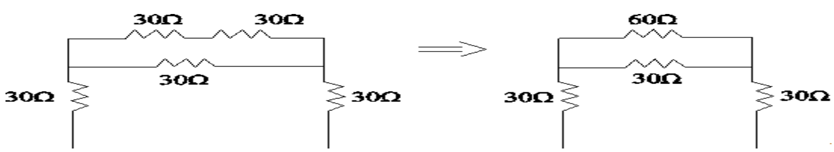


Figure 1.53

60 Ω and 30 Ω are connected in parallel

$$\frac{60 * 30}{60 + 30} = 2O \text{ ohms}$$

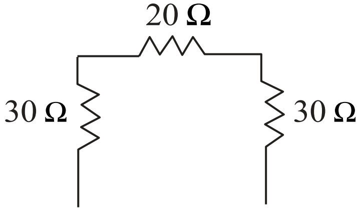


Figure 1.54

Equivalent Resistance = 80 Ω.

Problem 1.30 Find the current supplied by the battery.

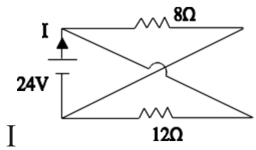


Figure 1.55

Solution:

The given circuit can be re-drawn as,

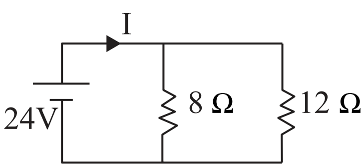


Figure 1.56

8 Ω and 12 Ω connected in parallel.

$$\frac{8*12}{8+12} = 4.8\ \Omega$$

Reduced circuit is,

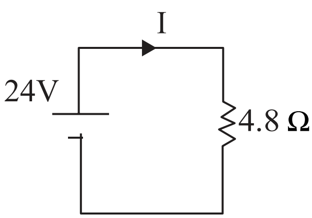


Figure 1.57

Current, $I = \frac{V}{R} = \frac{24}{4.8} = 5A$

$I = 5A$

Problem 1.31 Find the current supplied by the battery for the figure shown below.

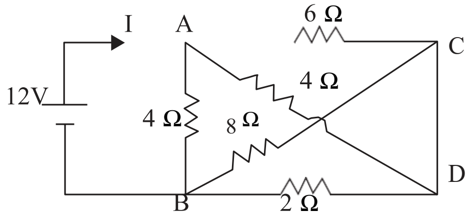


Figure 1.58

Solution:

The given above circuit can be redrawn as,

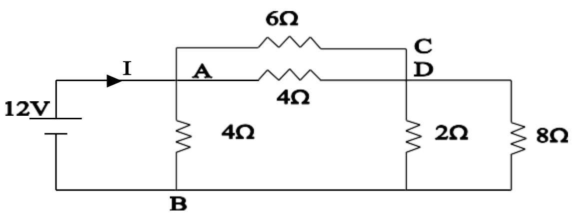


Figure 1.59

4 Ω and 6 Ω are connected in parallel. $\frac{6*4}{6+4} = 2.4\ \Omega$

Similarly, 2 Ω and 8 Ω are connected in parallel.

$$\frac{2*8}{8+2} = 1.6\ \Omega$$

The reduced circuit can be re drawn as,

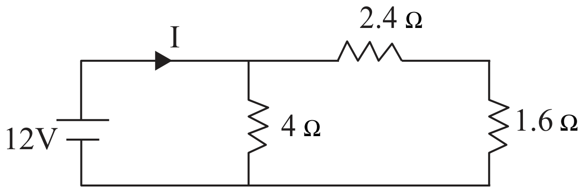


Figure 1.60

2.4 Ω and 1.6 Ω are connected in series. 2.4 +1.6 = 4 Ω

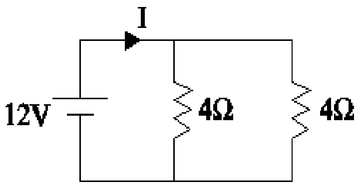


Figure 1.61

4 Ω and 4 Ω are connected in parallel $\frac{4*4}{4+4} = 2\ \Omega$

The reduced circuit is,

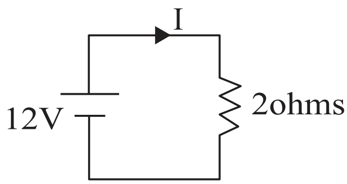


Figure 1.62

$$I = \frac{V}{R} = \frac{12}{2} = 6\text{A}$$

Current I, supplied by the battery = 6A.

Problem 1.32 Two Resistors R_1 = 2500 Ω and R_2 = 4000 Ω are joined in series and connected to a 100v supply. The voltage drop across R_1 and R_2 are measured successively by a voltmeter having a resistance of 50,000 Ω. Find the sum of the Reading.

Solution:

Case (i) A voltmeter is connected across 2500 Ω.

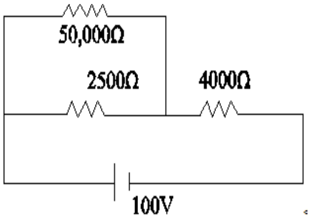


Figure 1.63

2500 Ω and 50,000 Ω are connected is parallel.

$$\frac{2500 * 50000}{2500 + 50000} = 2381 \text{ohms}$$

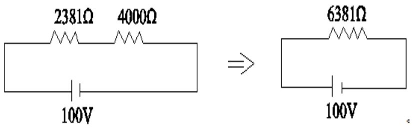


Figure 1.64

2381 Ω and 4000 Ω are connected in series.

$$2381 + 4000 = 6381\ \Omega$$

Current $I = \frac{V}{R} = \frac{100}{6381} = 0.01567\text{A}$

Voltage drop across, the Resister R_1 is measured by connecting a voltmeter having resistance of 50,000 across $R_{1.}$ Hence V_A be voltage drop across R_1

$$V_A = IR = 0.01567 * 2381$$

$$V_A = 37.31\text{V}$$

Case (ii) Voltmeter is connected across 4000 Ω.

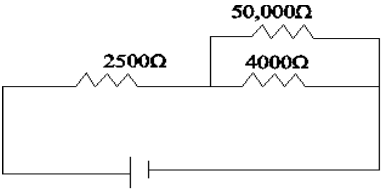


Figure 1.65

4000 Ω and 50,000 Ω are connected in parallel.

$$\frac{4000 * 50000}{4000 + 50000} = 3703.7 \text{ohms}$$

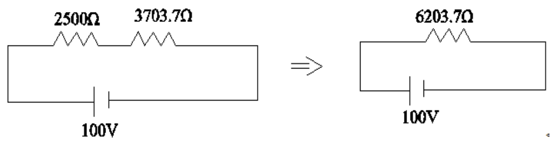


Figure 1.66

$$\text{Current, } I = \frac{V}{R} = \frac{100}{6203.7} = 0.0161 \text{ A}$$

Voltage drop across the resistor R_2 is measured by connecting a voltmeter having resistance of 50000 across R_2.Hence, V_B be the voltage drop across R_2.

$$V_B = IR = 0.6161 * 3703.7$$
$$V_B = 59.7\text{V}$$

The total voltage drop = $V_A + V_B$

$$V = 37.31 + 59.7$$
$$V = 97 \text{ V}$$

Problem 1.33 Find the value of 'R' and the total current when the total power dissipated in the network is 16W as shown in the figure.

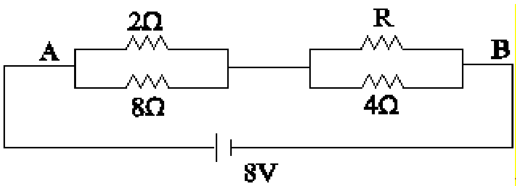


Figure 1.67

Solution:

Total Power (P) = 16w

$$\text{Total Current, } I = \frac{P}{V} = \frac{16}{8} = 2\text{A}$$

$$\text{Total Resistance, } (R_{AB}) = \frac{P}{I^2} = \frac{16}{4} = 4\ \Omega$$

Total Resistance between A and B is given by,

$$R_{AB} = \frac{2*8}{2+8} + \frac{4*R}{4+R}$$

$$4 = 1.6 + \frac{4R}{4+R}$$

$$4(4+R) = 1.6(4+R) + 4R \;\; R = 6\ \Omega.$$

KIRCHOFF'S LAWS

Kirchhoff's current law

The kirchoff's current law states that the algebraic sum of currents in a node is zero.

It can also be stated that "sum of incoming currents is equal to sum of outgoing currents."

Kirchhoff's current law is applied at nodes of the circuit. A node is defined as two or more electrical elements joined together. The electrical elements may be resistors, inductors capacitors, voltage sources, current sources etc.

Consider a electrical network as shown below.

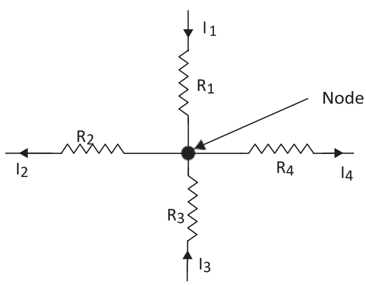


Figure 1.68

Four resistors are joined together to form a node. Each resistor carries different currents and they are indicated in the diagram.

$I_1 \rightarrow$ Flows towards the node and it is considered as positive current. $(+\ I_1)$

$I_2 \rightarrow$ Flows away from the node and it is considered as negative current. $(-\ I_2)$

$I_3 \rightarrow$ Flows towards the node and it is considered as positive current. $(+I_3)$

$I_4 \rightarrow$ Flows away from the node and hence it is considered as negative current $(-I_4)$

Applying KCL at the node, by diffinition-1 algebraic sum of currents in a node is zero.

$$+I_1 - I_2 + I_3 - I_4 = 0 \qquad (1)$$

taking the I_2 & I_4 to other side

$$I_1 + I_3 = I_2 + I_4 \qquad (2)$$

From equation (2) we get the definition – 2. Where I_1 & I_3 are positive currents (Flowing towards the node) I_2 & I_4 are negative currents. (Flowing away from the node).

Kirchoff's voltage Law: (KVL)

Kirchhoff's voltage law states that "sum of the voltages in a closed path (loop) is zero".

In electric circuit there will be closed path called as loops will be present.

The KVL is applied to the closed path only the loop will consists of voltage sources, resistors, inductors etc.

In the loop there will be voltage rise and voltage drop. This voltage rise and voltage drop depends on the direction traced in the loop. So it is important to understand the sign convention and the direction in which KVLis applied (Clock wise Anti clock wise).

- **Sign Conventions**

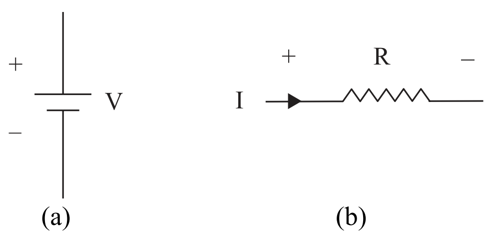


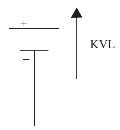
Figure 1.69

Consider a battery source V as shown in the figure 1.69(a). Here positive of the battery is marked with + sign and negative of the battery is marked with – sign.

When we move from + sign to – sign, it is called voltage drop.

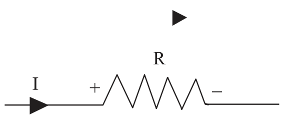
When we move from – sign to + sign, it is called as voltage rise.

When KVL is applied in Anti clockwise direction as shown above it is called as voltage drop. A voltage drop is indicated in a loop with "—" sign (–V)

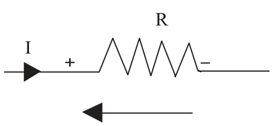


For the same battery source if the KVL is applied in clock wise direction we move from – sign to + sign. Hence it is called as Voltage Rise. A Voltage rise indicated in the loop with + sign. (+V).

Similarly in the resistor the current entry point is marked as positive (+ sign) and current leaving point is marked as negative sign. (– sign).



For the resistor shown in the diagram above, if KVL is applied in clock wise direction then it is called as voltage drop. Voltage drop in KVL equation must be indicated with negative sign (–). $\therefore$ –IR.



For the resistor shown in the diagram above, if KVL is applied in anti clockwise direction then it is called as voltage rise. A voltage rise is indicated in the KVL equation as positive. i.e. + IR.

In short the above explanation is summarized below in a Table.

S.No.	Element	KVL in clockwise	KVL in anticlockwise
1.	+ –	+ – Drop –V	+ – Rise +V
2.	I + R –	I + R – Drop IR	I + R – Drop +IR

- **Procedure for KVL:**
  - * Identify the loops and Name them.
  - * Mark the branch currents and name them.
  - * Apply the sign convention.
  - * Select a loop & apply KVL either in clockwise or Anticlockwise and frame the equation.
  - * Solve all the equations of the loop.

- **Problems based on Kirchhoff's laws**

Problem 1.34 For the given circuit find the branch currents and voltages by applying KVL.

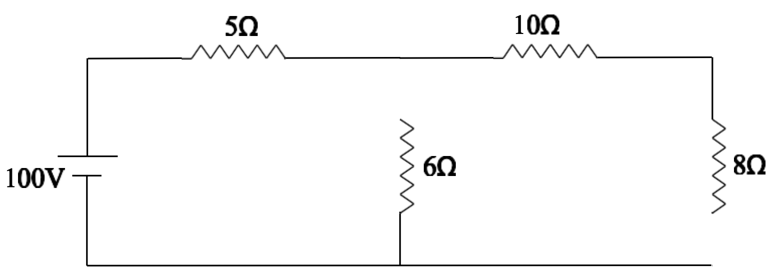


Figure 1.70

Solution:

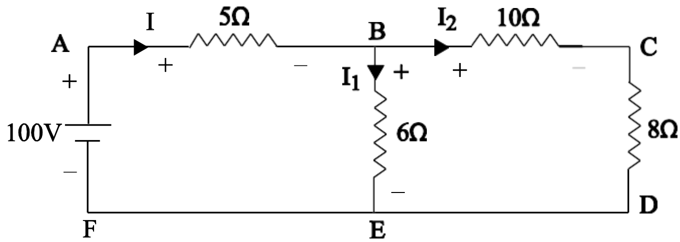


Figure 1.71

Consider loop ABEF & Apply KVL in CLK wise direction

$$100 - 5I - 6I_1 = 0$$

But $I = I_1 + I_2$

$$100 - 5\left(I_1 + I_2\right) - 6I_1 = 0$$

$$100 - 5I_1 - 5I_2 - 6I_1 = 0$$

$$-11I_1 - 5I_2 + 100 = 0$$

$$11I_1 + 5I_2 = 100 \qquad (1)$$

Consider loop BCDEB & Apply KVL in CLK wise direction

$$-10I_2 - 8I_2 + 6I_1 = 0$$

$$-18I_2 + 6I_1 = 0$$

$$6I_1 = 18I_2$$

$$I_1 = 3I_2 \qquad (2)$$

Sub I_1 in equ (1)

$$11\left(3I_2\right) + 5I_2 = 100$$

$$33I_2 + 5I_2 = 100$$

$$38I_2 = 100$$

$$I_2 = \frac{100}{38} = 2.63\ Amps.$$

$$I_2 = 2.63\ Amps$$

Sub I_2 in equ (2)

$$I_1 = 3(2.63) = 7.89\text{Amps}$$
$$I_1 = 7.89\text{Amps}$$
$$I = I_1 + I_2 = 10.52$$
$$I = 10.52\text{Amps.}$$

Voltage Across $5\wedge = \quad 5 \times I = 5 \times 10.52$

$= 52.6\ volts$

Voltage Across $6\wedge = \quad 6 \times I_1 = \quad 6 \times 7.89$

$= 47.34\ volts$

Voltage Across $10\wedge = 10 \times I_2 = 10 \times 2.63$

$= 26.3\ volts$

Voltage Across $8\wedge = \quad 8 \times I_2 = \quad 8 \times 2.63$

$= 21.04\ volts$

(Or)

The above problem can be solved by applying KVL in Anti clock wise directions.

Consider loop ABEF & Apply KVL in anti clock wise direction

$$6I_1 + 5I \quad -100 = 0$$

But $I = I_1 + I_2$

$$6I_1 + 5(I_1 + I_2) \; -100 = 0$$
$$6I_1 + 5I_1 + 5I_2 = 100$$
$$11I_1 + 5I_2 = 100 \qquad (3)$$

Consider loop BCDEB & Apply KVL in anti clockwise direction

$$8I_2 + \; 10I_2 - \; 6I_1 = 0$$
$$18I_2 = \quad 6I_1$$
$$I_1 = \; 3I_2 \qquad (4)$$

equations (3) & (1) are identical

equations (2) & (4) are identical

Hence we get the same answer irrespective of directions of applying KVL.

Problem 1.35 Calculate the branch current in 15 Ω resistor by Applying kirchhoff's law

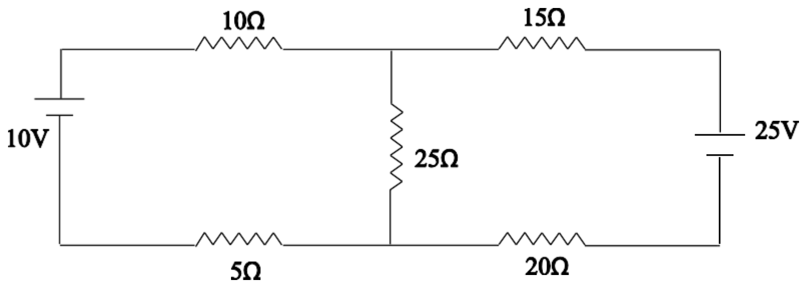


Figure 1.72

Figure 72 battery voltage value 25 volt missing

Solution:

Name the loop and Mark the current directions

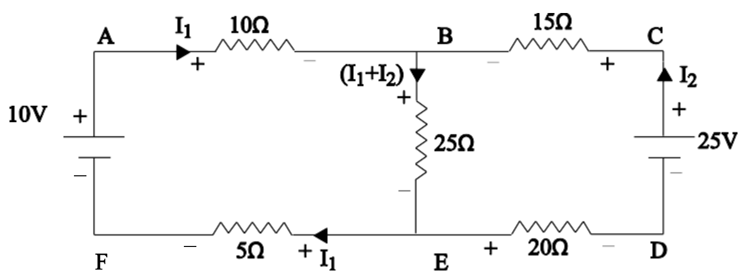


Figure 1.73

Consider the loop ABEFA & apply KVL in CLK wise

$$10 - 10I_1 - 25\left(I_1 + I_2\right) - 5I_1 = 0$$

$$10 - 10I_1 - 25I_1 - 25I_2 - 5I_1 = 0$$

$$-40I_1 - 25I_2 + 10 = 0$$

$$40I_1 + 25I_2 = 10 \quad (1)$$

Consider the loop BCDEB and Apply KVL in CLK wise direction

Consider the loop BCDEB and Apply KVL in CLK wise direction

$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$

$$15I_2 - 25 + 20I_2 + 25(I_1 + I_2) = 0$$

$$15I_2 - 25 + 20I_2 + 25I_1 + 25I_2 = 0$$

$$25I_1 + 60I_2 - 25 = 0$$

$$25I_1 + 60I_2 = 25 \ldots\ldots\ldots\ldots\ldots(2)$$

Solve (1) & (2) & find I_2 alone

$(1) \times 25 \Rightarrow 1000\, I_1 + 625I_2 = 25$

$(2) \times 40 \Rightarrow 1000\, I_1 + 2400I_2$

$(A) - (B) \Rightarrow -1775\, I_2 = -750$

$I_2 = 0.42$ Amps.

Current in 15 Ω resistor is 0.42Amps.

Problem 1.36 For the given network find the branch current in 8 Ω and voltage across the 3 Ω by applying KVL

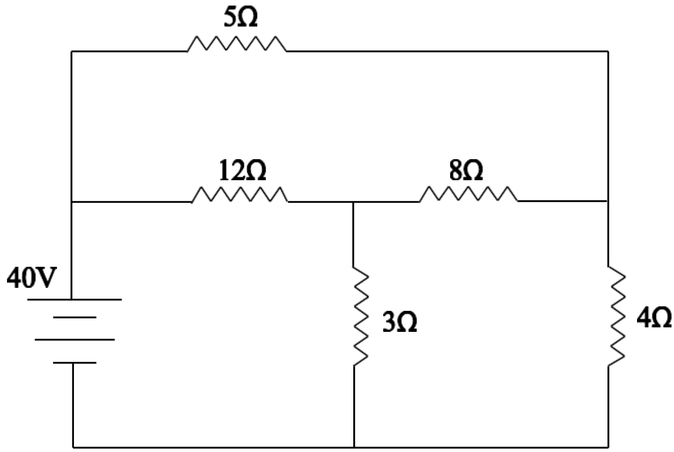


Figure 1.74

Solution:

Name the loop and mark the current directions and apply sign convention.

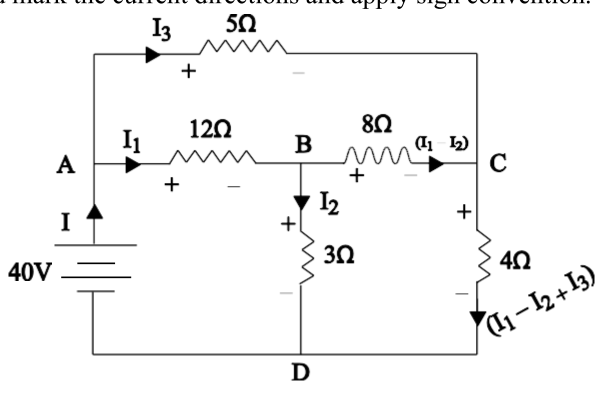


Figure 1.75

Consider loop ABDA and apply KVL -8I1 +8

$$-12I_1 - 3I_2 + 40 = 0$$

$$12I_1 + 3I_2 = 40$$

Consider loop BCDB and apply KVL

$$-8(I_1 - I_2) - 4(I_1 - I_2 + I_3) + 3\,I_2 = 0$$

$$I_2 - 4I_1 + 4 I_2 - 4 I_3 + 3 I_2 = 0 \qquad (1)$$
$$-12I_1 + 15I_2 - 4I_3 = 0 \qquad (2).$$

Consider loop ABCA and apply KVL

$$-12I_1 - 8(I_1 - I_2) + 5I_3 = 0$$
$$-12I_1 - 8I_1 + 8I_2 + 5I_3 = 0$$
$$-20I_1 + 8I_2 + 5I_3 = 0 \qquad (3)$$

Solve equ (2) & (3) and cancel out I_3

$$(2) \text{ x } 5 \Rightarrow -60 I_1 + 75 I_2 - 20 I_3 = 0$$
$$(3) \text{ x } 4 \Rightarrow -80 I_1 + 32 I_2 + 20 I_3 = 0$$

Add the above two equations $\Rightarrow -140 I_1 + 107 I_2 = 0$ (4)

Solve equ (4) & (1) and find I_1 & I_2

$$12I_1 + 3I_2 = 40 \qquad (1)$$
$$-140I_1 + 107I_2 = 0 \qquad (4)$$

$$(1) \text{ x } 107 \Rightarrow 1284I_1 + 321I_2 = 4280$$
$$(4) \text{ x } 3 \Rightarrow -420I_1 + 321I_2 = 0$$

Subtract the above two $1704 I_1 = 4280$

$$I_1 = 2.51 \; Amps$$

Sub I_1 in (4)

$$-140 \times 2.51 + 107I_2 = 0$$
$$-351.4 + 107I_2 = 0$$
$$107I_2 = 351.4$$
$$I_2 = 3.28 \; Amps$$

Current in 8 Ω resistor = $I_1 - I_2$

= 2.51- 3.28

= -0.77 Amps.

Negative sign indicates that current flows in the opposite direction of our assumption.

Voltage in 3 Ω resistor = $3I_2$

$= 3 \times 3.28 = 9.84$ volts

Note: Since there are 3 loops three unknown currents I_1, I_2 and I_3 should be named in the loop.

Problem 1.37 For the given network shown below find the branch currents by applying KVL and also find the voltage across 5 Ω resistor.

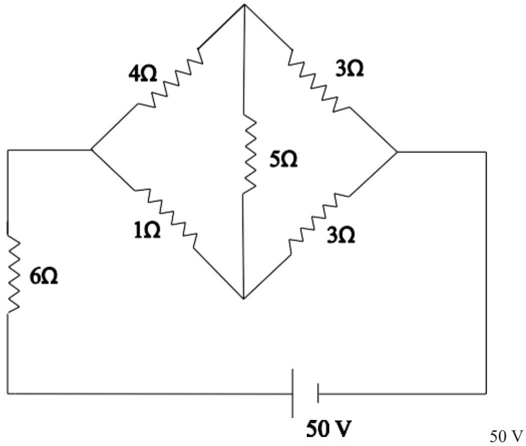


Figure 1.76

Solution:

Name the loop and assume the branch currents.

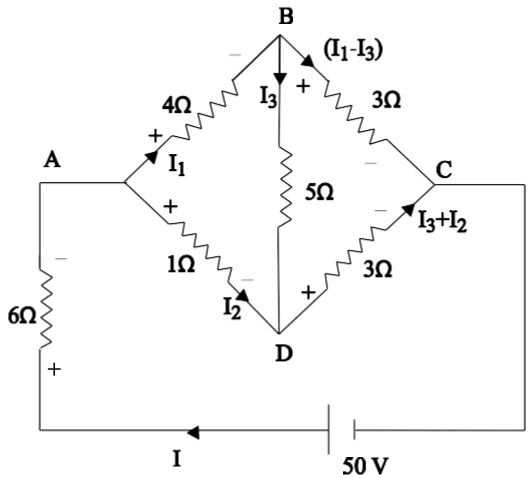


Figure 1.77

Consider the loop ABDA and apply KVL.

$$-4I_1 - 5\,I_3 + I_2 = 0$$

$$-4\,I_1 + I_2 - 5\,I_3 = 0 \qquad (1)$$

Consider the loop BCDB and apply KVL.

$$-3(I_1 - I_3) + 3(I_3 + I_2) + 5I_3 = 0$$

$$-3I_1 + 3I_3 + 3I_3 + 3I_2 + 5I_3 = 0$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (2)$$

Consider the loop ADCA and apply KVL.

$$-6(I_1 + I_2) - I_2 - 3(I_3 + I_2) + 50 = 0$$

$$-6I_1 - 6I_2 - I_2 - 3I_3 - 3I_2 = -50$$

$$-6I_1 - 10I_2 - 3I_3 = -50$$

$$6I_1 + 10I_2 + 3I_3 = 50 \qquad (3)$$

From eqn is (1) & (2) Cancel I_3

$$-4I_1 + I_2 - 5I_3 = 0 \qquad (4)$$

$$-3I_1 + 3I_2 + 11I_3 = 0 \qquad (5)$$

$$(4) \times 3 \Rightarrow -12I_1 + 3I_2 - 15I_3 = 0$$

$$(5) \times 4 \Rightarrow -12I_1 + 12I_2 - 44I_3 = 0$$

By subtracting the above two equations $-9I_2 - 59I_3 = 0$

$$9I_2 = -\,59I_3$$

$I_2 = -6.56 I_3$ (6)

$-3I_1 + 3I_2 + 11I_3 = 0$ (7)

$6I_1 + 10I_2 + 3I_3 = 50$ (8)

(7) x 2 $\Rightarrow -6I_1 + 6I_2 + 22I_3 = 0$

(8) $\Rightarrow 6I_1 + 10I_2 + 3I_3 = 50$

By adding the above two equations $16I_2 + 28I_3 = 50$ (9)

Sub eqn (6) in (9)

$16(-6.56\ I_3) + 25\ I_3 = 50$

$-104.96 + 25\ I_3 = 50$

$-79.96\ I_3 = 50$

$I_3 = -0.625$ Amps (10)

Sub eqn (10) in (6)

$I_2 = -6.56 \times (-0.625)$

$I_2 = 4.1$ Amps (7)

Sub (10) & (11) in eqn (8)

$6I_1 + 10I_2 + 3I_3 = 50$

$6I_1 + 10(4.1) + 3(-0.625) = 50$

$6I_1 + 41 - 1.875 = 50$

$6I_1 = 10.875$

$I_1 = 1.81$ Amps

Current in 6 Ω resistor = $(I_1 + I_2) = (1.81 + 4.1) = 5.91$Amps

Current in 4 Ω resistor = $I_1 = 1.81$ Amps

Current in 5 Ω resistor = $I_3 = -0.625$ Amps

Current in 3 Ω resistor = $(I_1 - I_3) = 1.81 + 0.625 = 2.44$ Amps

Current in 3 Ω resistor = $(I_3 + I_2) = 3.475$ Amps

Current in 1 Ω resistor = $I_2 = 4.1$ Amps.

Voltage Across 5 Ω resistor= $5 \times 0.625 = 3.13$ volts.

Problem 1.38 For the Circuit shown below determine voltages (i) V_{df} and (ii)V_{ag}

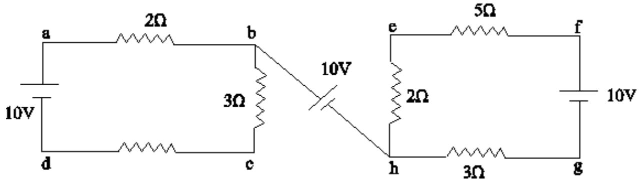


Figure 1.78

Solution:
Mark the current directions and mark the polarity

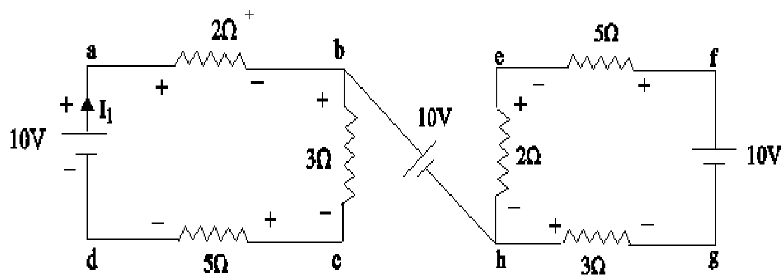


Figure 1.79

Apply KVL to loop abcda

$10 - 2I_1 - 3I_1 - 5I_1 = 0$

$-10I_1 = -10$

$I_1 = 1$ Amps

Apply KVL to loop efghe

$5I_2 - 10 + 3I_2 + 2I_2 = 0$

$10I_2 = 10$

$I_2 = 1$ Amps

To find V_{df}:

Trace the path V_{df}

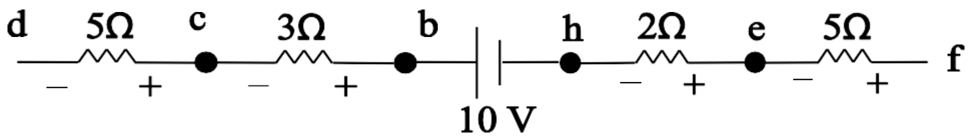


Figure 1.80

$V_{df} = –5(I_1 –3I_1 +10 + 2I_2 +5I_2)$
$V_{df} = –5 –3 +10 +2 +5$
$V_{df} = 9$ Volts.

$V_{df}= –9$ Volts [∵ because - sign on d side + on f side]

To find V_{ag}:

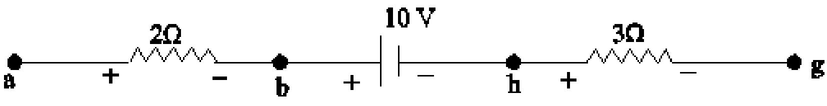


Figure 1.81

Apply KVL to the above Trace

$–2I_1 -10 –3I_2 = V_{ag}$
$V_{ag} = –2 – 10 – 3$
$V_{ag} = –15$

V_{ag} = 15 Volts. (With a side + w.r.t g)

Problem 1.39 Find the currents through R_2, R_3, R_4, R_5 and R_6 of the network.

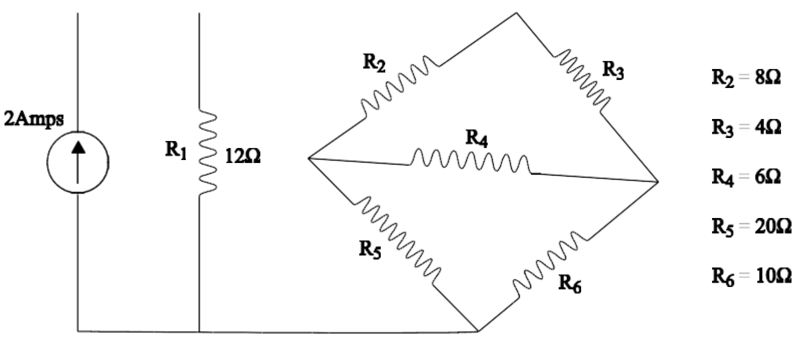


Figure 1.82

R_2 =8 Ω
R_3 =4 Ω
R_4 =6 Ω
R_5 =20 Ω
R_6 =10 Ω

Solution:
Name the circuit and mark the current directions and polarity as shown below

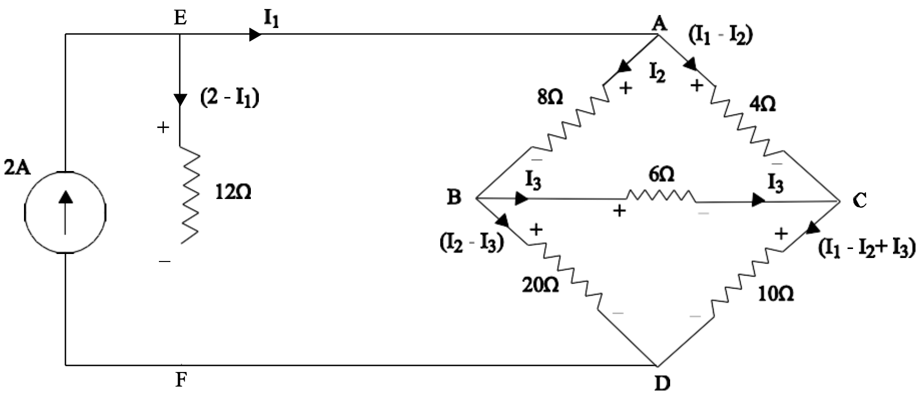


Figure 1.83

Apply KVL to the loop ACBA.

$$-4(I_1 - I_2) + 6I_3 + 8I_2 = 0.$$
$$-4I_1 + 4I_2 + 6I_3 + 8I_2 = 0$$
$$-4I_1 + 12I_2 + 6I_3 = 0 \qquad (1)$$

Apply KVL to the loop BCDB

$$-6I_3 - 10(I_1 - I_2 + I_3) + 20(I_2 - I_3) = 0$$
$$-6I_3 - 10I_1 + 10I_2 - 10I_3 + 20I_2 - 20I_3 = 0$$
$$-10I_1 + 30I_2 - 36I_3 = 0 \qquad (2)$$

Apply KVL to loop EABDFE

$$-8I_2 - 20(I_2 - I_3) + 12(2 - I_1) = 0$$
$$-8I_2 - 20I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-28I_2 + 20I_3 + 24 - 12I_1 = 0$$
$$-12I_1 - 28I_2 + 20I_3 = -24$$
$$12I_1 + 28I_2 - 20I_3 = 24 \qquad (3)$$

Solving equ. (1) (2) & (3). We get

$I_1 = 1.125$ Amps
$I_2 = 0.375$ Amps
$I_3 = 0$ Amps

∴ Current in $R_2 = 0.375$ Amps
$R_3 = 0.75$ Amps
$R_4 = 0$ Amps
$R_5 = 0.375$ Amps
$R_6 = 0.75$ Amps

NODAL ANALYSIS

- In nodal analysis, node equations relating node voltages are obtained for a multi node network.
- These node voltages are derived from kirchoff's current law (KCL)

- In this method the number of equations required to be solved is N-1, where N is the number of nodes.
- A node is a junction in a network where three or more branchesmeet. One of the nodes in a network is regarded as reference (datum) node and the potential of the other nodes are defined with reference to the datum node.

Case I.

Consider figure 1 Let the voltages at nodes a and b be V_a and V_b. Applying Kirchoff's current law (KCL) at node 'a' we get

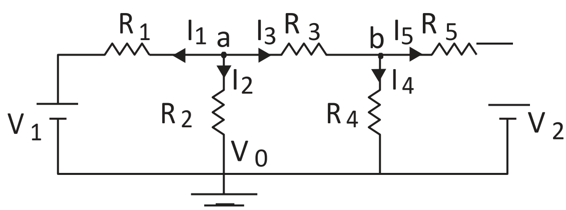


Figure 1.84

$$I_1 + I_2 + I_3 = 0 \quad (1)$$

Where

$$I_1 = \frac{V_a - V_1}{R_1}; I_2 = \frac{V_a - V_0}{R_2}; I_3 = \frac{V_a - V_b}{R_3};$$

Substituting in equ.(1)

$$\frac{V_a - V_1}{R_1} + \frac{V_a - V_0}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

On simplifying [$V_0 = 0$]

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R} \quad \text{................. (2)}$$

Similarly for node b we have

$$I_4 + I_5 = I_3 \quad \text{................. (3)}$$

$$I_4 = \frac{V_b - V_0}{R_4}; I_5 = \frac{V_b - V_2}{R_5}$$

On substituting in equ (3)

$$\frac{V_b - V_o}{R_4} + \frac{V_b - V_2}{R_5} = \frac{V_a + V_b}{R_3}$$

WKT

$V_o = 0$ [reference node]

$$V_b\left[\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right] - V_a\left[\frac{1}{R_3}\right] = \frac{V_2}{R_5} \quad \text{................. (4)}$$

Solving equations (2) and (4) we get the values as V_a and V_b.

Method for solving V_a and V_b by Cramers rule.

$$\begin{pmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_1}{R_1} \\ \frac{V_2}{R_2} \end{bmatrix}$$

$$\Delta = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) - \left(\frac{-1}{R_3}\right)\left(\frac{-1}{R_3}\right)$$

To find Δ_1

$$\begin{pmatrix} \frac{V_1}{R_1} & \frac{-1}{R_3} \\ \frac{V_2}{R_5} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \end{pmatrix}$$

$$\Delta_1 = \left(\frac{V_1}{R_1}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_2}{R_5}\right)$$

To find Δ_2,

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1} \\ \frac{-1}{R_3} & \frac{V_2}{R_5} \end{pmatrix}$$

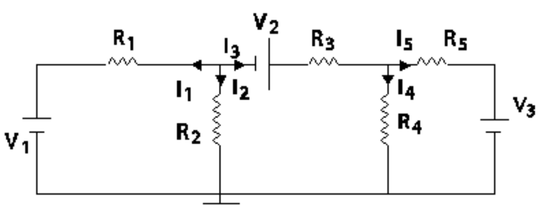
$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_5}\right)-\left(\frac{-1}{R_3}\right)\left(\frac{V_1}{R_1}\right)$$

To find v_a: To find v_b:

$$V_a = \frac{\Delta_1}{\Delta}; \qquad V_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

CASE II:



Consider fig 2

Let the voltages at nodes a and b be V_a and V_b.

The node equation at node a are

$$I_1 + I_2 + I_3 = 0$$

Where $I_1 = \frac{V_a - V_1}{R_1}$; $I_2 = \frac{V_a}{R_2}$; $I_3 = \frac{V_a + V_2 - V_b}{R_3}$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a + V_2 - V_b}{R_3} = 0$$

Simplifying

$$\frac{V_a}{R_1} - \frac{V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a}{R_3} + \frac{V_2}{R_3} - \frac{V_b}{R_3} = 0$$

Combining the common terms.

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-V_b\left[\frac{1}{R_3}\right]=\frac{V_1}{R_1}-\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The nodal equations at node b are

$$I_3 = I_4 + I_5$$

$$\frac{V_a+V_2-V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b-V_3}{R_5}$$

On simplifying

$$\frac{V_a}{R_3}+\frac{V_2}{R_3}-\frac{V_b}{R_3}=\frac{V_b}{R_4}+\frac{V_b}{R_5}-\frac{V_3}{R_5}$$

$$\frac{V_a}{R_3}-V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=-\frac{V_3}{R_5}-\frac{V_2}{R_3}$$

$$-\frac{V_a}{R_3}+V_b\left[\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right]=\frac{V_3}{R_5}+\frac{V_2}{R_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Solving equ (5) and (6) we get V_a and V_b

Method to solve V_a and V_b.

Solve by cramers rule.

$$\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{-1}{R_3} \\ \frac{-1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}\begin{bmatrix} V_a \\ V_b \end{bmatrix}=\begin{bmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{bmatrix}$$

$$\Delta=\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(-\frac{1}{R_3}\right)$$

$$\Delta_1=\begin{pmatrix} \frac{V_1}{R_1}-\frac{V_2}{R_3} & -\frac{1}{R_3} \\ \frac{V_2}{R_3}+\frac{V_3}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{pmatrix}$$

$$\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)$$

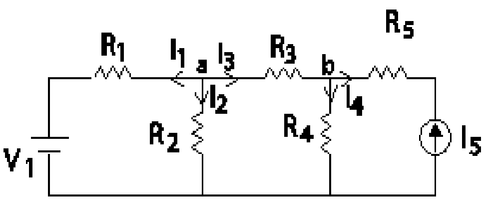
$$\Delta_2 = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & \frac{V_1}{R_1}-\frac{V_2}{R_3} \\ \frac{-1}{R_3} & \frac{V_2}{R_3}+\frac{V_3}{R_5} \end{pmatrix}$$

$$\Delta_2 = \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)\left(\frac{V_2}{R_3}+\frac{V3}{R_5}\right)-\left(-\frac{1}{R_3}\right)\left(\frac{V_1}{R_1}-\frac{V_2}{R_3}\right)$$

$$\Delta_a = \frac{\Delta_1}{\Delta}; \qquad \Delta_b = \frac{\Delta_2}{\Delta}$$

Hence V_a and V_b are found.

(case iii)



Let the voltages at nodes a and b be V_a and V_b as shown in fig

Node equations at node a are

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

$$V_a\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_2}\right] - V_b\left[\frac{1}{R_3}\right] = \frac{V_1}{R_1} \quad \text{..................} (7)$$

Similarly Node equations at node b

$$I_3 + I_5 = I_4$$

$$\frac{V_a - V_b}{R_3} + I_5 = \frac{V_b}{R_4}$$

$$I_5 = V_b\left[\frac{1}{R_3} + \frac{1}{R_4}\right] - V_a\left[\frac{1}{R_3}\right] \quad \text{..................(8)}$$

Solving eqn (7) and (8)

V_a and V_b has been found successfully.

Problems

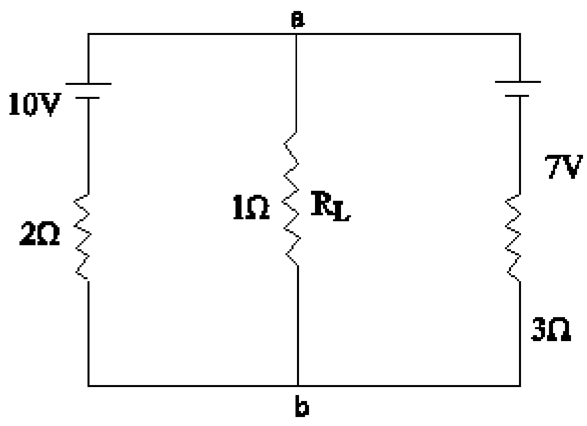
1) Two batteries having emf of 10V and 7V and internal resistances of 2Ω and 3Ω respectively, are connected in parallel across a load of resistance 1Ω. Calculate

(i) The individual battery currents

(ii) The current through the load

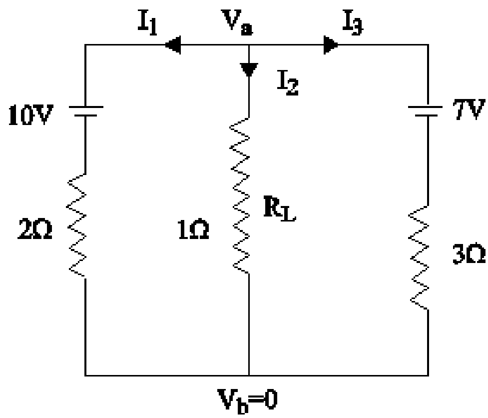
(iii) The Voltage across the load

Solution:



Step 1) Select the nodes and mark the nodes

Step 2) Select the datum or reference node.



<fig 84>

b is the ground node $V_b = 0$

Step 3: Mark the currents I_1, I_2 & I_3

Step 4: Write the node equations for node a and solve for V_a.

$$I_1 + I_2 + I_3 = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$I_1 = \frac{V_a - 10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$I_2 = \frac{V_a}{1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Substituting (2), (3) & (4) in (1)

$$\frac{V_a - 10}{2} + V_a + \frac{V_a - 7}{3} = 0$$

$$V_a\left[\frac{1}{2} + 1 + \frac{1}{3}\right] = \frac{10}{2} + \frac{7}{3}$$

$$1.83\ V_a = 7.33$$

$$V_a = 4\ V$$

(i) Individual battery currents

$$I_1 = \frac{V_a - 10}{2} = \frac{4-10}{2}$$

$= -3A$

Ans: $I_1 = 3$ A

$$I_3 = \frac{V_a - 7}{3} = \frac{4-7}{3} = -1$$

Ans: $I_3 = 1A$

(ii) Current through the load

$$I_L = I_{1-2} = \frac{V_a}{1} = 4A$$

(iii)Voltage across the load

$$V_L = V_a - V_b$$
$$= 4 - 0$$
$$V_L = 4\ V$$

2) Write the node voltage equation and calculate the currents in each branch for the network.

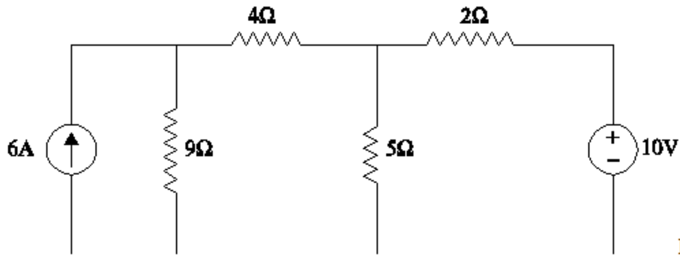


FIG85

Step 1: To assign voltages at each node

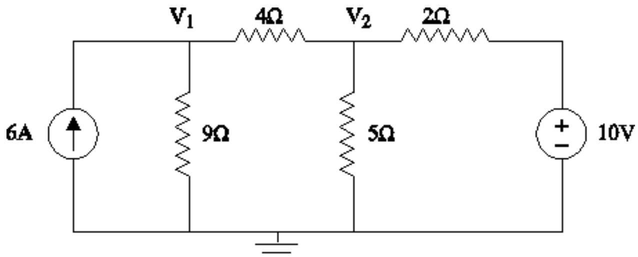


FIG86

V_1 & V_2 are active nodes

V_3 is a reference node on datum node.

Hence $V_3 = 0$.

Step 2: Mark the current directions in all the branches.

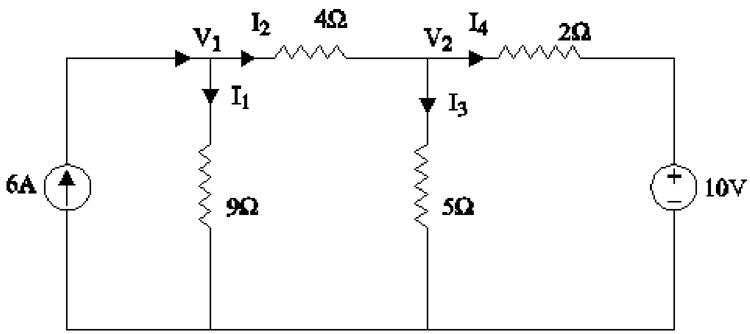


FIG87

Step 3: Write the node equations for node (1) and (2)

Node 1

$I_1 + I_2 = 6$

$$\frac{v_1}{9} + \frac{v_1 - v2}{4} = 6$$

$$V_1\left[\frac{1}{9} + \frac{1}{4}\right] - V_2\left[\frac{1}{4}\right] = 6 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Node 2:

$I_2 = I_3 + I_4$

$$\frac{V_1 - V_2}{4} = \frac{V_2}{5} + \frac{V_2 - 10}{2}$$

$$V_1\left[\frac{1}{4}\right] = V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] - \frac{10}{2}$$

$$-V_1\left[\frac{1}{4}\right] + V_2\left[\frac{1}{4} + \frac{1}{5} + \frac{1}{2}\right] = \frac{10}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Step 4: Solving equ (1) and (2) and finding V_1 and V_2 by Cramers rule,

$$\begin{bmatrix} \frac{1}{9}+\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{5}+\frac{1}{4}+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$\begin{pmatrix} .36 & -.25 \\ -.25 & .95 \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$\Delta = 0.2795$

To find Δ_1

$$\begin{pmatrix} 6 & -.25 \\ 5 & .95 \end{pmatrix} = 6.95$$

$$V_1 = \frac{6.95}{.279} = 24.86v$$

To find Δ_2

$$\begin{bmatrix} .36 & 6 \\ -.25 & 5 \end{bmatrix} = 3.3$$

$$V_{22} \frac{3.3}{.2795} = 11.8V$$

$$I_{9\Omega} = \frac{V_1}{9} = \frac{24.86}{9} = 2.76A$$

$$I_{4\Omega} = \frac{V_1 - V_2}{4} = \frac{24.86 - 11.8}{4} = 3.26A$$

$$I_{5\Omega} = \frac{V_2}{5} = \frac{11.86}{5} = 2.37A$$

$$I_{2\Omega} = \frac{V_2 - 10}{2} = \frac{11.86 - 10}{2} = 0.93A$$

Hence currents in all the branches are found.

Problem 1.42 Use the Nodal Method to find V_{ba} and current through 30 Ω resistor in the circuit shown

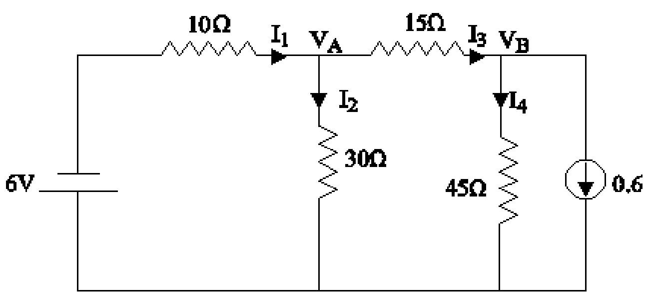


FIG88

At node A

$$\frac{V_A + 6}{10} + \frac{V_A}{30} + \frac{V_A - V_B}{15} = 0$$

$$V_A \left[\frac{1}{10} + \frac{1}{30} + \frac{1}{15} \right] - \frac{V_B}{15} = -0.6$$

At node B

$$\frac{V_B - V_A}{15} + \frac{V_B}{45} \quad +0.6 = 0$$

$$V_B\left[\frac{1}{15} + \frac{1}{45}\right] - \frac{V_A}{15} = -0.6$$

$$\begin{pmatrix} \frac{1}{10} + \frac{1}{30} + \frac{1}{15} & -\frac{1}{15} \\ -\frac{1}{15} & \frac{1}{15} + \frac{1}{45} \end{pmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -0.6 \\ -0.6 \end{bmatrix}$$

$$\Delta = \begin{bmatrix} 0.2 & -0.066 \\ -0.066 & 0.088 \end{bmatrix} = \left[0.0176 - 4.35 \times 10^{-3}\right]$$

$$\Delta = [0.01324]$$
$$\Delta = 0.01324$$

$$\Delta_1 = \begin{pmatrix} -0.6 & -\frac{1}{15} \\ -0.6 & \frac{1}{15} + \frac{1}{45} \end{pmatrix} = -0.093$$

$$\Delta_1 = [-0.053 - 0.04] = \ -0.093$$

$$V_A = \frac{\Delta_1}{\Delta} = -\frac{0.093}{0.01324} = -7.02V$$

$$\Delta_2 = \begin{bmatrix} 0.2 & -0.6 \\ -0.066 & -0.6 \end{bmatrix}$$

$$\Delta_2 = [-0.12 - 0.0396]$$

$$\Delta_2 = -0.1596$$

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{-0.1596}{0.01324} = -12.05V$$

$$V_{ba} = \ V_A - \ V_B = \ -7 + 12 \ = \ 5V$$

$$I_2 = \frac{V_A}{30} = \frac{-7}{30} = -0.233A$$

$$I_2 = -0.233A$$

Maxwell's Mesh method (Loop method).

This method was first proposed by Maxwell simplifies the solution of several networks. In this method, KVL is used. In any network, the number of independent loop equations will be

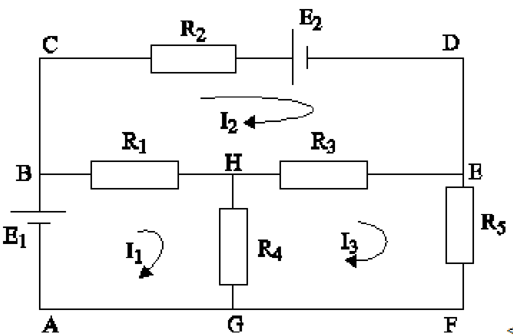
$$m = l - (j - 1)$$

Where l is the number of branches and j is the number of junctions.

Let us consider the circuit shown in fig(). for writing the mesh equations. It has

Number of junctions = 4 (B, H, E, G).

Number of branches = 6 (AB, BC, CD, DE, EF, HG).



<fig89>

In the above figure we shall name the three loop currents I_1, I_2 and I_3. The directions of the loop current are arbitrarily chosen. Note that the actual current flowing through R_4 is $(I_1 - I_3)$ in a downward direction and R_1 is $(I_1 - I_2)$ from left to Right

Apply KVL for the first loop ABHGA,

$$E_1 - R_1 (I_1 - I_2) - R_4 (I_1 - I_3) = 0$$

$R_1 (I_1 - I_2) + R_4 (I_1 - I_3) = E_1$

$\therefore (R_1 + R_4) I_1 - R_1 I_2 - R_4 I_3 = E_1$ .... (1)

Apply KVL for the loop BEDC,

$- R_2 I_2 - E_2 - R_3 (I_2 - I_3) - R_1 (I_2 - I_1) = 0$

$R_2 I_2 + R_3 (I_2 - I_3) + R_1 (I_2 - I_1) = -E_2$

$\therefore - R_1 I_1 + (R_1 + R_2 + R_3) I_2 - R_3 I_3 = -E_2$ .... (2)

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} \text{ .... (5)}$$

It can be seen that the diagonal elements of the matrix is the sum of the resistances of the mesh, where as the off diagonal elements are the negative of the sum of the resistance common to the loop.

Thus,

R_{ii} = the sum of the resistances of loop i.

$$R_{ij} = \begin{cases} -\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in opposite direction in common resistances)} \\ +\sum(\text{Resistance common to the loop i and loop j,} \\ \quad \text{if } I_i \text{ and } I_j \text{ are in same direction in common resistances)} \end{cases}$$

The above equation is only true when all the mesh currents are taken in clockwise direction. The sign of voltage vector is decided by the considered current direction. If the mesh current is entering into the positive terminal of the voltage source, the direction of voltage vector elements will be negative otherwise it will be positive.

Equation (5) can be solved by Cramer's rule as

$$\Delta_1 = \begin{bmatrix} E_1 & R_{12} & R_{13} \\ E_2 & R_{22} & R_{23} \\ E_3 & R_{32} & R_{33} \end{bmatrix}; \quad \Delta_2 = \begin{bmatrix} R_{11} & E_1 & R_{13} \\ R_{21} & E_2 & R_{23} \\ R_{31} & E_3 & R_{33} \end{bmatrix};$$

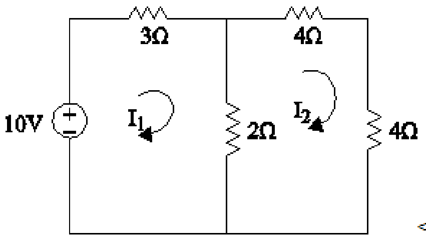
$$\Delta_3 = \begin{bmatrix} R_{11} & R_{12} & E_1 \\ R_{21} & R_{22} & E_2 \\ R_{31} & R_{32} & E_3 \end{bmatrix}; \quad \Delta = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$I_1 = \frac{\Delta_1}{\Delta}; \quad I_2 = \frac{\Delta_2}{\Delta}; \quad I_3 = \frac{\Delta_3}{\Delta}$$

Problems:

1) Find the branch currents of fig () using Mesh current method

Solution:



<fig90>

Method 1:

Apply KVL for the first loop,

$$10 - 3I_1 - 2(I_1 - I_2) = 0$$

$$5I_1 - 2I_2 = 10 \text{ .... } (1)$$

Apply KVL for the second loop,

$$-4I_2 - 4I_2 - 2(I_2 - I_1) = 0$$

$$-2I_1 + 10I_2 = 10 \text{ .... } (2)$$

Solve eqn (1) & (2), we get

$$(1) \text{ X } 5 \Rightarrow 25I_1 - 10I_2 = 50 \text{ ....} (3)$$

$$(2) \quad \Rightarrow -2I_1 + 10I_2 = 0$$

$$(3) + (2) \Rightarrow \quad 23\, I_1 = 50$$

$$I_1 = \frac{50}{23} = 2.174 \text{ A}$$

Sub I_1 in (2)

$$I_2 = \frac{2 \times 2.174}{10} = 0.435 \text{ A}$$

$$I_3\Omega = 2.174 \text{ A}$$

$$I_2\Omega = I_1 - I_2 = 1.739 \text{ A}$$

$$I_4\Omega = 0.435 \text{ A}$$

Method 2:

R_{11} = Sum of resistances of loop 1 = 3+2 =5Ω

R_{12} = - (common resistance between loop 1 and loop 2) = -2Ω

= R_{21}

R_{22} = Sum of resistance is loop 2 = 4+ 4+ 2 =10

E2 = 0

$$\begin{bmatrix} 5 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ -2 & 10 \end{vmatrix} = 50 - 4 = 46$$

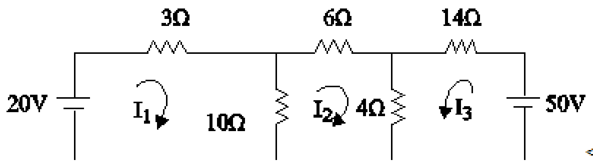
$$\Delta_1 = \begin{vmatrix} 10 & -2 \\ 0 & 10 \end{vmatrix} = 100$$

$$\Delta_2 = \begin{vmatrix} 5 & 10 \\ -2 & 0 \end{vmatrix} = 20$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{46} = 2.174A$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{20}{46} = 0.435A$$

2) Find the loop currents for the network shown in figure below by using Loop Analysis.



<fig91>

Solution

For loop 1,

$$3I_1 + 10(I_1 - I_2) = 20$$

$13I_1 - 10I_2 = 20$ .... (1)

For loop 2,

$$10(I_2 - I_1) + 6I_2 + 4(I_2 + I_3) = 0$$

$$10I_2 - 10I_1 + 6I_2 + 4I_2 + 4I_3 = 0$$

$\div 2 \Rightarrow -5I_1 + 10I_2 + 2I_3 = 0$ .... (2)

For loop 3,

$$4(I_3 + I_2) + 14I_3 = 50$$

$$4I_2 + 18I_3 = 50$$

$\div 2 \Rightarrow 2I_2 + 9I_3 = 25$ .... (3)

$$\therefore \begin{bmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \\ 25 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 13 & -10 & 0 \\ -5 & 10 & 2 \\ 0 & 2 & 9 \end{vmatrix}$$

$$\Delta = 13(90-4) + 10(-45-0)$$

$=668$

For loop 1,

$$10I_1 + 5(I_1 + I_2) + 3(I_1 - I_3) = 50$$

$$18I_1 + 5I_2 - 3I_3 = 50 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For loop 2,

$$2I_2 + 5(I_2 + I_1) + 1(I_2 + I_3) = 10$$

$$5I_1 + 8I_2 + I_3 = 10 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For loop 3,

$$3(I_3 - I_1) + 1(I_3 + I_2) = -5$$

$$-3I_1 + I_2 + 4I_3 = -5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$$\begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ 10 \\ -5 \end{bmatrix}$$

$$= 13\ (250)\ +\ 10\ (-125)\ +\ 20\ (-10)$$

$$\Delta = \begin{bmatrix} 18 & 5 & -3 \\ 5 & 8 & 1 \\ -3 & 1 & 4 \end{bmatrix}$$

$$= 18\ (32-1)\ -5\ (20+3)\ -3(5+24)$$
$$= 356$$

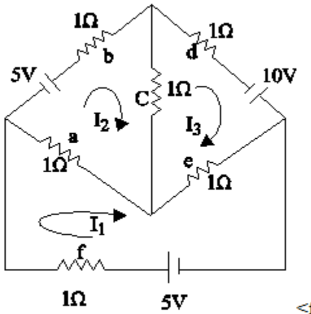
$$\Delta_3 = \begin{vmatrix} 18 & 5 & 50 \\ 5 & 8 & 10 \\ -3 & 1 & -5 \end{vmatrix}$$

$$= 18\ (-40-10)\ -5\ (-25+30)\ +\ 50(5+24)$$
$$= -900 - 25 + 1450$$
$$= 525$$

$$I_3 = \frac{\Delta I_3}{\Delta} = \frac{525}{356} = 1.47A$$

Solution

4) Determine the currents in various elements of the bridge circuit as shown below.



<fig92>

Solution

For loop 1,

$1I_1 + 1(I_1 - I_2) + 1(I_1 - I_3) = 5$

$3I_1 - I_2 - I_3 = 5$ .... (1)

For loop 2,

$1I_2 + 1(I_2 - I_3) + 1(I_2 - I_1) = 5$

$-I_1 + 3I_2 - I_3 = 5$.................... (2)

For loop 3,

$1I_3 + 1(I_3 - I_1) + 1(I_3 - I_2) = 10$

$-I_1 - I_2 + 3I_3 = 10$ .... (3)

$$\rightarrow \begin{bmatrix} -3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$= 3\,(9-1) + 1\,(-3-1)\ -1(1+3)$

$= 16$

$$\Delta_1 = \begin{vmatrix} 5 & -1 & -1 \\ 5 & 3 & -1 \\ 10 & -1 & 3 \end{vmatrix}$$

$= 40 + 25 + 35$

$= 100$

$$\Delta_2 = \begin{vmatrix} 3 & 5 & -1 \\ -1 & 5 & -1 \\ -1 & 10 & 3 \end{vmatrix}$$

$= 3\ (15 + 10)\ -5\ (-3\ -\ 1)\ -1\ (-10 + 5)$

$= 100$

$$\Delta_3 = \begin{vmatrix} 3 & -1 & 5 \\ -1 & 3 & 5 \\ -1 & -1 & 10 \end{vmatrix}$$

$= 3\ (30 + 5) + 1\ (-10 + 5) + 5\ (1 + 3)$

$= 120.$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{100}{16} = 6.25A$$
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{100}{16} = 6.25A$$
$$I_3 = \frac{\Delta_3}{\Delta} = \frac{120}{16} = 7.5A$$

$I_a = I_1 - I_2 = 6.25 - 6.25 = 0$ A

$I_b = I_2 = 6.25$ A.

$I_c = I_2 - I_3 = 6.25 - 7.5 = -1.25$ A

$I_d = I_3 = 7.5$ A

$I_e = I_1 - I_3 = 6.25 - 7.5 = -1.25$ A.

$I_f = I_1 = 6.25$ A.

Questions

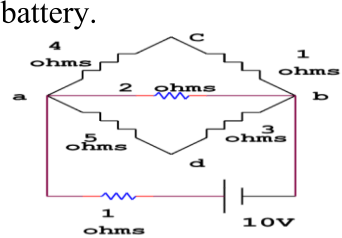
Part A

S.NO	QUESTION
1	Define Electric Current
2	What is Electric Potential?
3	List the applications of series Circuits.
4	State Ohm's Law
5	State Kirchhoff's Voltage and Current Law
6	What is the Equivalent Resistance when two resistors are connected in parallel?
7	What are the limitations of Ohm's Law?
8	What is Resistance?
9	Define Electric Charge.
10	Differentiate Series and Parallel Circuits.

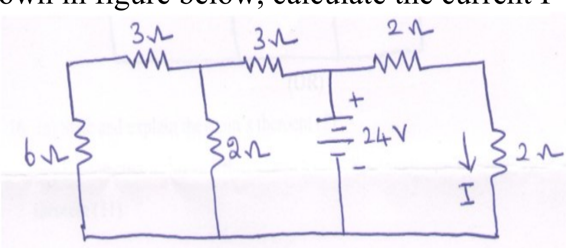
Part B

S.No. Questions

1.In the circuit shown, determine the current through the 2 Ω resistor and the total current delivered by the battery.

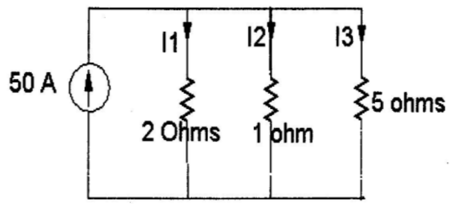


2.For the network shown in figure below, calculate the current I

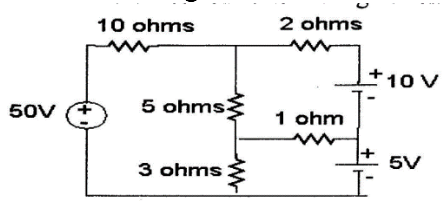


(or)

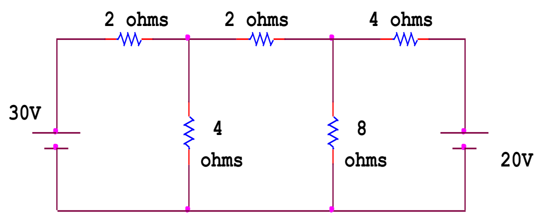
3.For the given circuit determine the currents in all resistors



4.Determine the mesh currents in the given circuit.



5.Apply nodal voltage method and find the power dissipated in the 8 ohms resistor on the circuit shown



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DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT-2 AC CIRCUITS

SEEA1101- BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

INTRODUCTION

We have seen so far about the analysis of DC circuit. A DC quantity is one which has a constant magnitude irrespective of time. But an alternating quantity is one which has a varying magnitude and angle with respect to time. Since it is time varying in nature, at any time it can be represented in three ways 1) By its effective value 2) By its average value and 3) By its peak value.

Some important terms

1. Wave form
   A wave form is a graph in which the instantaneous value of anyquantity is plotted against time.

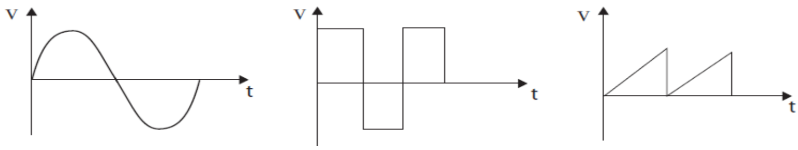

Fig 2.1(a-c)

2. Alternating Waveform
   This is wave which reverses its direction at regularly recurringinterval.
3. Cycle

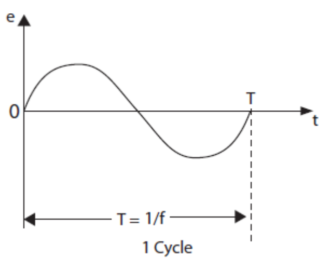

Figure 2.2

It is a set of positive and negative portion of waveforms.

4. Time Period

The time required for an alternating quantity, to complete one cycle is called the time period and is denoted by T.

5. Frequency

The number of cycles per second is called frequency and is denoted by f. It is measured in cycles/second (cps) (or) Hertz

$$f = 1/T$$

6. Amplitude

The maximum value of an alternating quantity in a cycle is called amplitude. It is also known as peak value.

7. R.M.S value [Root Mean Square]

The steady current when flowing through a given resistor for a given time produces the same amount of heat as produced by an alternating current when flowing through the same resistor for the same time is called R.M.S value of the alternating current.

$$RMS\ Value = \sqrt{\text{Area Under the square curve for one complete cycle / Period}}$$

8. Average Value of AC

The average value of an alternating current is defined as the DC current which transfers across any circuit the same change as is transferred by that alternating current during the same time.

Average Value = Area Under one complete cycle/Period.

9. Form Factor (Kf)

It is the ratio of RMS value to average value

Form Factor = RMS value/Average Value

10. Peak Factor (Ka)

It is the ratio of Peak (or) maximum value to RMS value.

Peak Factor Ka=Peak Value/RMS value

Analytical method to obtain the RMS, Average value, Form Factor and Peak factor for sinusoidal current (or) voltage

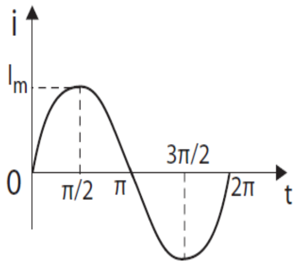

Figure 2.3

$i = I_m \sin\theta$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$= \frac{1}{\pi}\int_0^{\pi} i^2 d\theta$ [since it is symmetrical]

$= \frac{I_m^2}{\pi}\int_0^{\pi} Sin^2 d\theta$

$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-cos2\theta}{2} \, d\theta$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{Sin2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\,\pi = \frac{I_m}{\sqrt{2}}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i\,d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \mathrm{Sin}\theta d\theta$$

$$= \frac{I_m}{\pi}\int_0^{\pi} I_m \mathrm{Sin}\theta d\theta$$

$$=- \frac{I_m}{\pi}[\cos\theta]_0^{\pi}$$

$$=- \frac{I_m}{\pi}[\cos\pi - \cos 0]$$

$$=- \frac{I_m}{\pi}(-1-1)$$

$$=- \frac{2I_m}{\pi}$$

$$\text{Form Factor} = \frac{RMS}{Avg} = 1.11\theta$$

$$\text{Peak Factor} = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = 1.414$$

Expression for RMS, Average, Form Factor, Peak factor for Half waverectifier

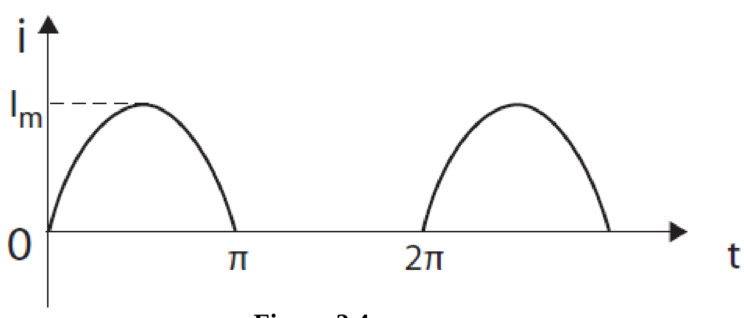

Figure 2.4

1) RMS value

$$i = I_m \text{Sin}\theta \qquad 0 < \theta < \pi$$

$$i = 0 \qquad \pi < \theta \leq 2\pi$$

Mean square of AC $I_{RMS}^2 = \frac{1}{2\pi}\int_0^{2\pi} i^2 d\theta$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + \int_{\pi}^{2\pi} i^2 d\theta\ \right]$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} i^2 d\theta + 0\right]$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} Sin^2 d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-cos2\theta}{2}\ d\theta$$

$$= \frac{I_m^2}{4\pi}\left[\theta - \frac{Sin2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\ \pi = \frac{I_m}{2}$$

Average Value:

$$I_{av} = \int_0^{\pi} \frac{i d\theta}{2\pi}$$

$$= \frac{1}{2\pi} [\int_0^{\pi} i \ \ d\theta + 0 \ ]$$

$$= \frac{1}{2\pi} \int_0^{\pi} I_m \text{Sin}\theta d\theta$$

$$= \frac{I_m}{2\pi} \int_0^{\pi} I_m \text{Sin}\theta d\theta$$

$$= - \frac{I_m}{2\pi} [\cos\theta]_0^{\pi}$$

$$= - \frac{I_m}{2\pi} [\cos\pi - \cos 0]$$

$$= - \frac{I_m}{2\pi} (-1 - 1)$$

$$= \frac{2I_m}{2\pi} \quad = \frac{I_m}{\pi}$$

$$\text{Form Factor} = \frac{RMS}{Avg} = \frac{I_m}{2} / \frac{I_m}{\pi} = 1.57$$

$$\text{Peak Factor} = \frac{MAX}{RMS} = \frac{I_m}{RMS} = \frac{I_m}{\frac{I_m}{2}} = 2$$

Examples:

The equation of an alternating current is given by

$$i = 40 \sin 314\, t$$

Determine

(i) Max value of current
(ii) Average value of current
(iii) RMS value of current
(iv) Frequency and angular frequency
(v) Form Factor
(vi) Peak Factor

Solution:

$$i = 40 \sin 314\, t$$

We know that $i = I_m \sin \omega t$

So $I_m = 40$

$\omega = 314\ rad / \sec$

(i) Maximum value of current = 40A

(ii) Average value of current

$$I_{Avg} = \frac{2I_m}{\pi} = \frac{2 \times 40}{\pi} = 25.464A$$

(iii) RMS value of current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{40}{\sqrt{2}} = 28.28\ Amp$$

(iv) Frequency $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} \approx 50 Hz$

(v) $Form\ Factor \frac{RMS}{Avg} = \frac{28.28}{25.46} = 1.11$

(vi) $Peak\ \ Factor = \frac{\max}{RMS} = \frac{40}{28.28} = 1.414$

PHASOR REPRESENTATION OF SINUSOIDAL VARYING ALTERNATING QUANTITIES

The Phasor representation is more convenient in handling sinusoidal quantities rather than by using equations and waveforms. This vector or Phasor representation of alternating quantity simplifies the complexity of the problems in the AC circuit.

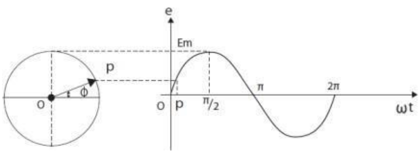

Figure 2.5

OP $=E_m$

E_m – the maximum value of alternating voltage which varies sinusoidally

Any alternating sinusoidal quantity (Voltage or Current) can be represented by a rotating Phasor, if it satisfies the following conditions.

1. The magnitude of rotating phasor should be equal to the maximum value of the quantity.
2. The rotating phasor should start initially at zero and then move in anticlockwise direction. (Positive direction)
3. The speed of the rotating phasor should be in such a way that during its one revolution the alternating quantity completes one cycle.

Phase
The phase is nothing but a fraction of time period that has elapsed from reference or zero position.

In Phase
Two alternating quantities are said to be in phase, if they reach their zero value and maximum value at the same time.

Consider two alternating quantities represented by theequation

$i_1=Im_1\sin\theta$
$i_2=Im_2\sin\theta$

can be represented graphically as shown in Fig 2.6(a).

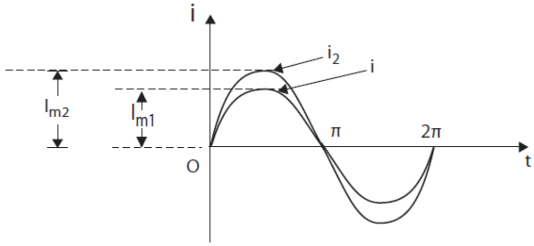

Figure 2.6(a) Graphical representation of sinusoidal current

From Fig 2.6(a), it is clear that both i_1 and i_2 reaches their zero and their maximum value at the same time even though both have different maximum values. It is referred as both currents are in phase meaning that no phase difference is between the two quantities. It can also be represented as vector as shown in Fig 2.6(b).

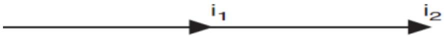

Figure 2.6(b) Vector diagram

Out of Phase

Two alternating quantities are said to be out of phase if they do not reach their zero and maximum value at the same time. The Phase differences between these two quantities are represented in terms of '**lag'** and '**lead'** and it is measured in radians or in electrical degrees.

Lag

Lagging alternating quantity is one which reaches its maximum value and zero value later than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:

$i_1 = Im_1 \sin(\omega t - \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.7(a) and Fig 2.7(b) respectively.

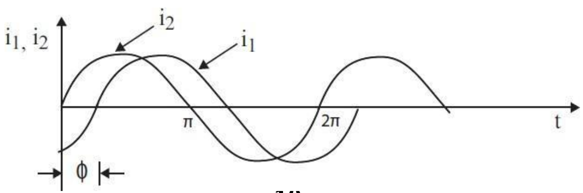

Figure 2.7a

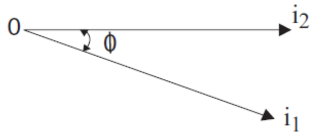

Figure 2.7b

It is clear from the Fig 2.7(a), the current i_1 reaches its maximum value and its zero value with a phase difference of 'Φ' electrical degrees or radians after current i_2. (ie) i_1 lags i_2 and it is represented by a minus sign in the equation.

Lead

Leading alternating quantity is one which reaches its maximum value and zero value earlier than that of the other alternating quantity.

Consider two alternating quantities represented by the equation:
$i_1 = Im_1 \sin(\omega t + \Phi)$
$i_2 = Im_2 \sin(\omega t)$

These equations can be represented graphically and in vector form as shown in Fig 2.8(a) and Fig 2.8(b) respectively.

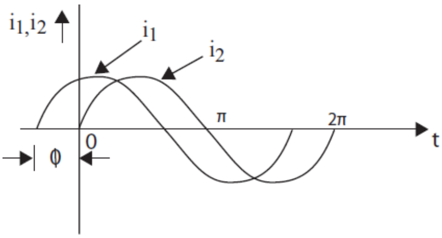

Figure 2.8(a)

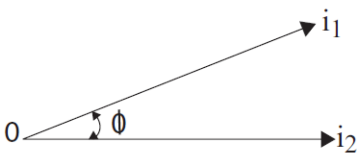

Figure 2.8(b)

The Fig 2.8(a) clearly illustrates that current i_1 has started already and reaches its maximum value before the current i_2. (ie) i_1 leads i_2 and it is represented by a positive sign in the equation.

Note:

1. Two vectors are said to be in quadrature, if the Phase difference

between them is 90°.

2. Two vectors are said to be in anti phase, if the phase difference between them is 180°.

REVIEW OF 'J' OPERATOR

A vector quantity has both magnitude and direction. A vector' A' is represented in two axis plane as shown in Fig 3.10

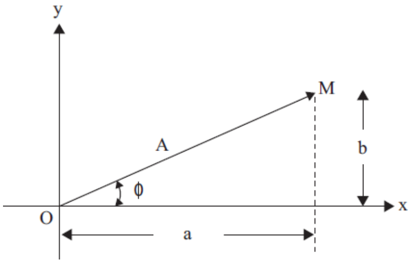

Figure 2.9

In Fig 2.9, OM represents vector A

Φ represents the phase angle of vector A

A = a + jb

a – Horizontal component or active component or in phase component

b – Vertical component or reactive component or quadrature component

The magnitude of vector ' A' = $\sqrt{a^2 + b^2}$

Phase angle of Vector 'A' = α = $\tan^{-1}$ (b/a)

Features of j – Operator

1. j = $\sqrt{-1}$

   It indicates anticlockwise rotation of Vector through 90°.
2. j^2 = j . j = -1

   It indicates anticlockwise rotation of vector through 180°.
3. j^3 = j . j . j = -j

   It indicates anticlockwise rotation of vector through 270°.
4. j^4 = j . j . j . j = 1

   It indicates anticlockwise rotation of vector through 360°.
5. –j indicates clockwise rotation of vector through 90°.
6. $\frac{1}{j} = \frac{1.j}{j.j} = \frac{j}{j^2} = \frac{j}{-1} = -j$

A vector can be written both in polar form and in rectangular form.

A = 2 + j3

This representation is known as rectangular form.

Magnitude of A = |A| = $\sqrt{2^2 + 3^2}$ =3.606

Phase angle of A = α = $\tan^{-1}$ (3/2) = 56°.31

A=|A| ∠α°

A=3.606∠56°.31

This representation is known as polar form.

Note:

1. Addition and Subtraction can be easily done in rectangular form.
2. Multiplication and division can be easily done in polar form.

Examples:

2.3) A= 2 + j3; B = 4 + j5.

Add Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A + B = 2 + j3 + 4 + j5 = 6 + j8

∴ Magnitude = | A + B | = $\sqrt{6^2 + 8^2}$ = 10.0

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (8/6) = 53°.13

2.4) A= 2 + j5; B = 4 – j2.

Subtract Vector A and Vector B and determine the magnitude and Phase angle of resultant vector.

Solution:

A – B = 2 + j5 – (4 – j2) = 2 + j5 – 4 + j2 = -2 + j7

∴ Magnitude = | A – B | = $\sqrt{-2^2 + 7^2}$ = 7.280

Phase angle = α = $\tan^{-1}$ (B /A) = $\tan^{-1}$ (7/-2) = -74°.055

2.5) A= 2 + j3; B = 4 – j5.

Perform A x B and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 2 + j3

|A| = $\sqrt{2^2+3^2}$ =3.606

α = $\tan^{-1}$ (3/2) = 56°.310

A= 3.606 ∠ 56°.310
B = 4 – j5

$|B| = \sqrt{4^2 + -5^2}$ =6.403

α = $\tan^{-1}$ (-5/4) = -51°.340
B= 6.403∠ -51°.340

A X B = 3.606 ∠ 56°.310 X 6.403∠ -51°.340
= 3.606 X 6.403∠ (56°.310 + (-51°.340))
= 23.089 ∠4°.970

2.6) A= 4 – j2; B = 2 + j3.

Perform $\frac{A}{B}$ and determine the magnitude and Phase angle of resultant vector.

Solution:

A= 4 – j2

$|A| = \sqrt{4^2 + -2^2}$ = 4.472

α = $\tan^{-1}$ (-2/4) = -26°.565
A= 4.472 ∠ -26°.565
B = 2 + j3

$|B| = \sqrt{2^2 + 3^2}$ = 3.606

α = $\tan^{-1}$ (3/2) = 56°.310
B= 3.606∠ 56°.310

$$\frac{A}{B} = \frac{4.472\angle -26^\circ.565}{3.606\angle 56^\circ.310} = \frac{4.472}{3.606}\angle -26^\circ.565 - 56^\circ.310 = 1.240\ \angle -82.875$$

ANALYSIS OF AC CIRCUIT

The response of an electric circuit for a sinusoidal excitation can be studied by passing an alternating current through the basic circuit elements like resistor (R), inductor (L) and capacitor (C).

Pure Resistive Circuit:

In the purely resistive circuit, a resistor (R) is connected across an alternating voltage source as shown in Fig.2.10

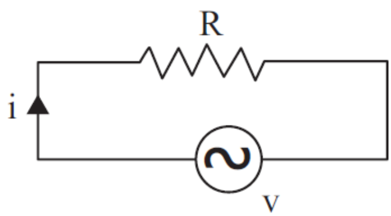

Figure 2.10

Let the instantaneous voltage applied across the resistance (R) be

$$V = V_m \sin\omega t$$

Let the applied voltage across the resistance be $V = V_m$ Sin ωt

The resulting current has instantaneous value I by ohm's law $V = iR$

$$i = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$$

Where, $I_m = \frac{V_m}{R} = Peak\ value\ of\ the\ circuit\ current$

Comparing the voltage and the current we find voltage and the current are in phase with each other.

Phasor Representation:

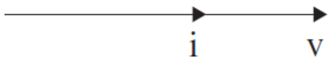

Figure 2.11

Comparing equations, we find that applied voltage and the resulting current are **inphase** with each other. Therefore in a purely resistive circuit there is no phase difference between voltage and current i.e., phase angle is zero (Φ=0).

If voltage is taken as reference, the phasor diagram for purely resistive circuit is shown in Fig.2.11

Waveform Representation:

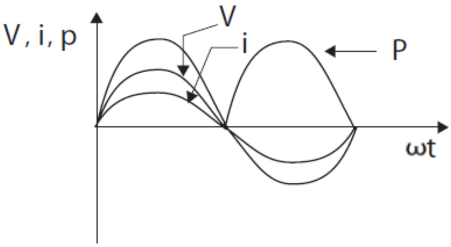

Figure 2.12

The waveform for applied voltage and the resulting current and power were shown in Fig.2.12. Since the current and voltage are inphase the waveforms reach their maximum and minimum values at the same instant.

Impedance:

$$Z = \frac{V}{I} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m}{I_m} = \frac{V_m}{V_m/_R} = R$$

Average Power:

$$P = Vi = V_m \sin \omega t \; I_m \cos \omega t = V_m \, I_m sin^2 \omega t$$

ωt=θ, $P = V_m I_m sin^2 \, \omega t$

Average power for one cycle $= \frac{V_m I_m}{\Pi} \int_0^{\Pi} sin^2\theta . d\theta = \frac{VmIm}{\pi} \int_0^{\pi} \frac{1 - \cos 2\theta}{2} \, d\theta$

$$= \frac{VmIm}{2\pi} \left[0 - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{VmIm}{2\pi} \left[\pi - \frac{\sin 2\pi}{2} - 0 + sin\frac{\theta}{2}\right]$$

$$= \frac{VmIm}{2\pi} . \pi = \frac{VmIm}{2} = \frac{Vm}{\sqrt{2}} . \frac{Im}{\sqrt{2}} = V . I$$

Average Power = VI watts

Power Factor: It is the cosine of the phase angle between voltage and current.

Cos Φ =cos 0= 1 (unity)

Problems:

A voltage of 240 sin 377t is applied to a 6Ω resistor. Find the instantaneous current, phase angle, impedance, instantaneous power, average power and power factor.

Solution:

Given: $v = 240 \sin 377t$

$V_m = 240$ V

$\omega = 377$ rad/sec

$R = 6\Omega$

Instantaneous current:

$$= \frac{V_m \sin\omega t}{R}$$

$$= \frac{240}{6} \sin 377t$$

$$= 40 \sin 377t A$$

I. Phase angle:

$\phi = 0$

II. Impedance:

$Z = R = 6\Omega$

III. Instantaneous power:

IV. $p = V_m I_m \sin^2\omega t$

$$= 240.40.\sin^2 377t$$

$$= 9600 \sin^2 377t$$

V. Average power:

$$P = \frac{V_m I_m}{2} = 4800 \text{watts}$$

VI. Power factor:

$\cos\Phi = \cos 0 = 1$

A voltage $e = 200\sin\omega t$ when applied to a resistor is found to give a power 100 watts. Find the value of resistance and the equation of current.

Solution:

Given: $e = 200\sin\omega t$

$V_m = 200$

$P = 100$w

Average power, $P = \frac{V_m I_m}{2}$

$$100 = \frac{200 I_m}{2}$$

$I_m = 1$ A

Also, $V_m = I_m.R$

$R = 200\Omega$

Instantaneous current, $I = I_m \sin\omega t = 1.\sin\omega t$ A

A voltage $e = 250\sin\omega t$ when applied to a resistor is found to give a power of 100W. Find the value of R and write the equation for current. State whether the value of R varies when the frequency is changed.

Solution:
Given: e = 250sinωt

$V_m = 250$
$P = 100W$

I. $P = \dfrac{V_m I_m}{2}$

$100 = \dfrac{250 I_m}{2}$

$I_m = 0.8$ A

II. $I_m = \dfrac{V_m}{R}$

$R = 312.5\Omega$

III. $I = 0.8\sin\omega t$

The resistance is independent of frequency, so the variation of frequency will not affect the resistance of the resistor.

Pure Inductive Circuit:

In this circuit, an alternating voltage is applied across a pure inductor (L) is shown in Fig. 2.13.

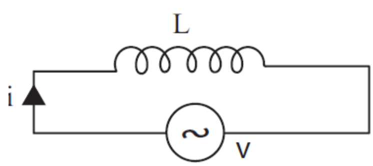

Figure 2.13

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

We know that the self induced emf always opposes the applied voltage.

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int v\,dt = \frac{1}{L}\int V_m \sin\omega t\,dt$$

$$= \frac{V_m}{\omega L}(-\cos\omega t) = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\left[\because I_m = \frac{V_m}{\omega L}\right]$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Phasor representation:

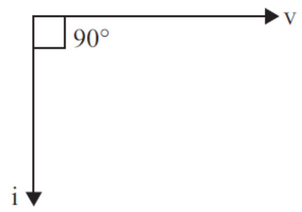

Figure 2.14

Comparing equations, the applied voltage and the resulting current are 90^0 out-of phase. Therefore in a purely inductive circuit there is a phase difference of 90^0 ie., phase angle is 90^0 ($\Phi = 90^0$). Clearly, the current **lags** behind the applied voltage.

Waveform representation:

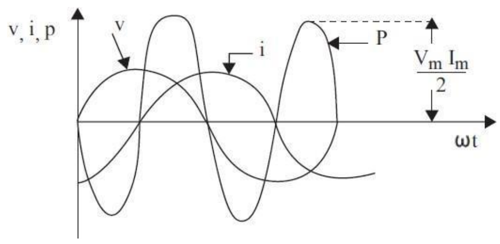

Figure 2.15

The waveform for applied voltage and the resulting current and the power were shown in Fig.2.15. The current waveform is lagging behind the voltage waveform by 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

$Z = X_L$ [Impedance is equal to inductive reactance]

Power:

(i) Instantaneous power:

$$p = v\,i = V_m \sin\omega t \; I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t\,(-\cos \omega t)$$

$$= -V_m I_m \sin\omega t \cos \omega t = -V_m I_m \sin\theta \cos \theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = -\frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta \, d\theta$$

$$= -\frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} \, d\theta \qquad [\because \sin 2\theta = 2\sin\theta \cos\theta]$$

$$= -\frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}\left[\cos 2\pi - \cos 0\right]$$

$$= \frac{V_m I_m}{4\pi}\left[1-1\right] = 0$$

Thus, a pure inductor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. It is seen that power wave is a sine wave of frequency double that of the voltage and current waves. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:

In a pure inductor the phase angle between the current and the voltage is 90^0 (lags).

$$\Phi = 90^0; \cos \Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero lagging.

Problems:

A coil of wire which may be considered as a pure inductance of 0.225H connected to a 120V, 50Hz source. Calculate (i) Inductive reactance (ii) Current (iii) Maximum power delivered to the inductor (iv) Average power and (v) write the equations of the voltage and current.

Solution:

Given: L = 0.225 H

$V_{RMS} = V = 120$ V

f = 50Hz

I. Inductive reactance, XL = 2πfL =2π x 50 x 0.225 = 70.68Ω

II. Instantaneous current, i =$-I_m$ cosωt

$\square I_m = \frac{V_m}{\omega L}$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$, calculate I_m and V_m

$V_m = V_{RMS}\sqrt{2} = 169.71V$

$I_m = \frac{V_m}{\omega L} = \frac{169.71}{70.68} = 2.4A$

Maximum power, $P_m = \frac{V_m I_m}{2} = 203.74$ W

III. Average power, P=0

IV. Instantaneous voltage, v = Vm sinωt = 169.71 sin 344t volts

Instantaneous current, i = -2.4 cosωt A

A pure inductance, L = 0.01H takes a current, 10 cos 1500t. Calculate (i) inductive reactance, (ii) the equation of voltage across it and (iii) at what frequency will the inductive reactance be equal to 40Ω.

Solution:

Given: L = 0.01 H

I = 10cos1500t

$I_m = 10A$

ω = 1500 rad/sec

I. Inductive reactance, X_L = ωL = 1500 x 0.01 = 15Ω

II. The voltage across the inductor, e= $L\frac{di}{dt}$

$$= 0.01 \frac{d(10\cos 1500t)}{dt}$$

III. $X_L = 40\Omega$; $2\pi fL = 40$

Find (i) frequency (ii) Maximum values of voltage and current (iii) RMS value of voltage and current (iv) Average values of both (v) Draw the phasor diagram (vi) circuit element and its values

Solution:

Given: $V_m = 200V$

$I_m = 20A$

$\omega = 314$ rad/sec

I. $\omega = 2\pi f$

$f = 50Hz$

II. $V_m = 200V$ and $I_m = 20A$

III. $V_{RMS} = \frac{V_m}{\sqrt{2}} = 141.42V$

$I_{RMS} = \frac{I_m}{\sqrt{2}} = 14.142A$

IV. For a sinusoidal wave, Average value of current,

$I_{av} = \frac{2I_m}{\pi} = 12.732A$

Average value of voltage,

$V_{av} = \frac{2V_m}{\pi} = 127.32A$

V. Phasor diagram

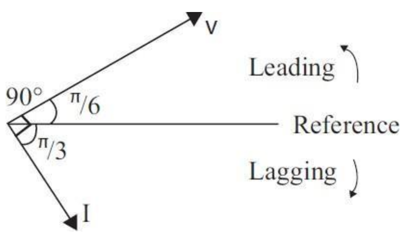

Figure 2.16

VI. From the phasor diagram, it is clear that I lags V by some angle(90^0). ~~So th~~e circuit is purely inductive.

Pure Capacitive Circuit:

In this circuit, an alternating voltage is applied across a pure capacitor(C) is shown in Fig.2.17

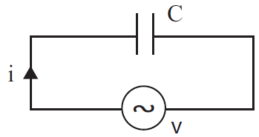

Figure 2.17

Let the instantaneous voltage applied across the inductance (L) be

$$v = V_m \sin\omega t$$

Let at any instant i be the current and Q be the charge on the plates.

So, charge on capacitor, Q = C.v

$$= C.\, V_m \sin\omega t$$

$$\text{Current, } i = \frac{dQ}{dt}$$

$$i = \frac{d}{dt}\left(CV_m \sin\omega t\right) = \omega CV_m \cos\omega t$$

$$= \omega CV_m \sin\left(\omega t + \pi\right)$$

$m \quad \left(\qquad \overline{2} \right)$

$[\because I_m = \omega C V_m]$

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

From the above equations , we find that there is a phase difference of 90^0 between the voltage and current in a pure capacitor.

Phasor representation:

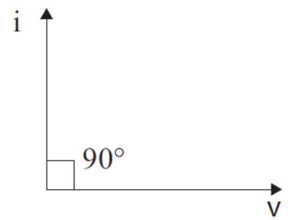

Figure 2.18

In the phasor representation, the current leads the voltage by an angle of 90^0.

Waveform representation:

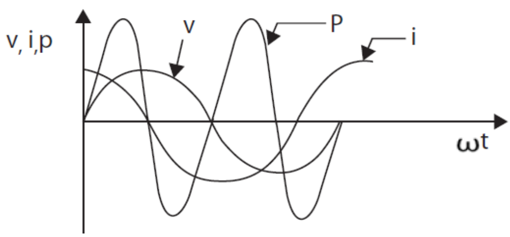

Figure 2.19

The current waveform is ahead of the voltage waveform by an angle of 90^0.

Impedance (Z):

$$Z = \frac{V_m}{I_m}$$

$$= \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

$Z = X_C$ [Impedance is equal to capacitive reactance]

Power:

(i)Instantaneous power:

$$p = v\,i$$

$$= V_m \sin\omega t \; I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin\omega t \,(\cos \omega t)$$

$$= V_m I_m \sin\theta \cos\theta$$

(ii) Average power:

Since the waveform in Fig. is symmetrical, the average power is calculated for one cycle.

$$P = \frac{1}{\pi}\int_0^{\pi} V_m I_m \sin\theta \cos\theta \, d\theta$$

$$= \frac{V_m I_m}{\pi}\int_0^{\pi} \frac{\sin 2\theta}{2} d\theta$$

$$[\because \sin 2\theta = 2\sin\theta \cos\theta]$$

$$= \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\theta}{2}\right]_0^{\pi} = \frac{V_m I_m}{4\pi}[-\cos 2\pi + \cos 0]$$

$$= \frac{V_m I_m}{4\pi}[-1+1] = 0$$

Thus, a pure capacitor does not consume any real power. It is also clear from Fig. that the average demand of power from the supply for a complete cycle is zero. Again, it is seen that power wave is a sine wave of frequency double that of the voltage and current. The maximum value of instantaneous power is $\left(\frac{V_m I_m}{2}\right)$.

Power Factor:
In a pure capacitor, the phase angle between the current and the voltage is 90^0 (leads).

$$\Phi = 90^0;\ \cos\Phi = \cos 90^0 = 0$$

Thus the power factor of a pure inductive circuit is zero leading.

Problems:

A 135μF capacitor has a 150V, 50Hz supply. Calculate (i) capacitive reactance (ii) equation of the current (iii) Instantaneous power (iv) Average power (v) RMS current (vi) Maximum power delivered to the capacitor.

Solution:
Given: $V_{RMS} = V = 150V$

$C = 135\mu F$

$f = 50Hz$

I. $X_C = \frac{1}{\omega C} = 23.58\Omega$

II. $i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$ $\square I_m = \omega C V_m$ and $V_{RMS} = \frac{V_m}{\sqrt{2}}$

$V_m = 150X\sqrt{2} = 212.13V$

$I_m = 314X135X10^{-6}X212.13 = 8.99A$

$i = 8.99\sin\left(314t + \frac{\pi}{2}\right)A$

III. $p = V_m I_m \sin\omega t\,(\cos\omega t) = 212.13X8.99\ \sin 314t.\cos 314t$

$= 66642.6\ \sin 314t.\cos 314t = 66642.6\ \frac{\sin 628t}{2}$

$[\square \sin 2\theta = 2\sin\theta\ \cos\theta]$

$= 33321.3\ \sin 628t\ W$

IV. Average power, P = 0

V. $I_{RMS} = \frac{I_m}{\sqrt{2}} = 6.36A$

A voltage of 100V is applied to a capacitor of 12μF. The current is 0.5 A. What must be the frequency of supply

Solution:

Given: $V_{RMS} = V = 100V$

$C = 12\mu F$

$I = 0.5A$

I. Find V_m and I_m

$$V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$V_m = 100X\sqrt{2} = 141.42V$

$$I_{RMS} = \frac{I_m}{\sqrt{2}}$$

$I_m = 0.5X\sqrt{2} = 0.707A$

II. $I_m = \omega C V_m = 2\pi f C V_m$

f = 66.3Hz

RL Series Circuit

Let us consider a circuit is which a pure resistance R and a purly inductive coil of inductance L are connected in series as shown in diagram.

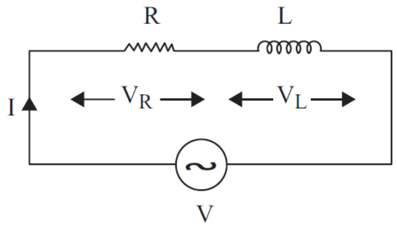

Figure 2.20

Let $V = V_m \sin \omega t$ be the applied voltage.

i = Circuit current at any constant.

I = Effective Value of Circuit Current.

V_R= Potential difference across inductor.

V_L= Potential difference across inductor.

F= Frequency of applied voltage.

The same current I flows through R and L hence I is taken as reference vector.

Voltage across resistor V_R= IR in phase with I
Voltage with inductor V_L= IX_L leading I by 90°

The phasor diagram of RL series circuit is shown below.

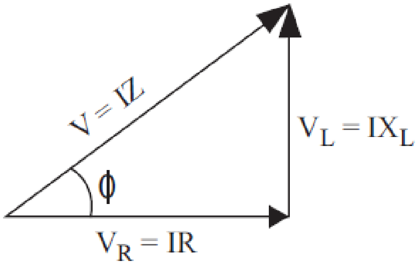

Figure 2.21

At any constant, applied voltage

$$V = V_R + V_L$$
$$V = IR + jIX_L$$
$$V = I\,(R + jx_L)$$
$$\frac{V}{I} = R + jx_L$$
$$= z \text{ impedance of circuit}$$
$$Z = R + j\,x_L$$
$$|z| = \sqrt{R^2 + X_L^2}$$

From phasor disgram,

$$\tan\phi = \frac{x_L}{R}$$
$$\phi = \tan^{-1}\left(\frac{x_L}{R}\right)$$

ϕ is called the phasor angle and it is the angle between V and I, its value lies between 0 to 90^0.

So impedence $Z = R + jX_L$
$$= |Z| < \phi$$

The current and voltage waveform of series RL Circuit is shown below.

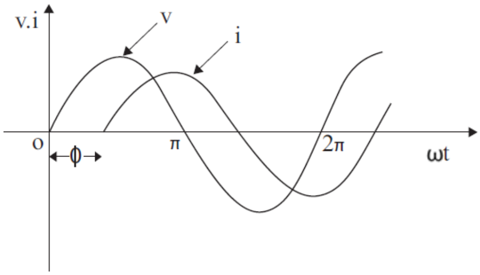

Figure 2.22

$V = V_m \sin\omega t$

$I = I_m \sin(\omega t - \phi)$

The current I lags behind the applied voltage V by an angle ϕ.

From phasor diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual Power P = VI cos ϕ – Current component is phase with voltage

Reactive or Quadrature Power

Q = VI sinϕ – Current component is quadrature with voltage

Complex or Apparent Power

S = VI – Product of voltage and current

S = P + jQ

Problem

A series RL Circuit has

$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$ and $V(t) = 20 \sin\left(314t + \frac{5\pi}{3}\right)$

Determine (a) the impedence of the circuit
(b) the values of R_1L and power factor
(c) average power of the circuit

Solution:

$i(t) = 5 \sin(314t + \frac{2\pi}{3})$

$V(t) = 20 \sin(314t + \frac{5\pi}{3})$

Phase angle of current $\theta_i = \frac{2\pi}{3} = \frac{2 \times 180}{3} = 120°$

Phase angle of voltage $\theta_v = \frac{5\pi}{3} = \frac{5 \times 180}{} = 150°$

phase angle between voltage and current $\theta = \theta_v \sim \theta_i$

$$= 150 - 120$$

$$\theta = 30^0$$

Power factor = $\cos\theta$

$$= \cos 30$$

$$= 0.866 \text{ (lagging)}$$

Impedence of the circuit $Z = \frac{V_m}{I_m}$

$$= \frac{20}{5}$$

$$Z = 4\Omega$$

(i) But $\cos\phi = \frac{R}{Z}$

$$0.866 = \frac{R}{4}$$

$$\therefore R = 4 \times 0.866$$

$$R = 3.46\Omega$$

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$X_L = \sqrt{Z^2 + R^2}$$

$$= \sqrt{(4)^2 - (3.46)^2}$$

$$X_L = 2\Omega$$

$$\omega L = 2\Omega$$

$$L = \frac{2}{\omega}$$

$$= \frac{2}{314}$$

$$L = 6.37 \times 10^{-3} \text{ H}$$

(ii) Average power = VI $\cos\phi$

$$= \frac{20}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} (0.866)$$

$= 43.3$ watts

A 10Ω resistor and a 20 mH inductor are connected is series across a 250V, 60 Hz supply. Find the impedence of the circuit, Voltage across the resistor, voltage across the inductor, apparent power, active power and reactive power.

Solution:

$R = 10\Omega$

$L = 20\text{ mH} = 20\times10^{-3}H$

$X_L = 2\pi fL$

$= 2\pi\times60\times20\times10^{-3}$

$X_L = 7.54\Omega$

(i) $|Z| = \sqrt{R^2 + (X_L)^2} = \sqrt{(10)^2 + (7.54)^2} = 12.5\Omega$

(ii) $I = \frac{V}{Z} = \frac{250}{12.5} = 20\,amps$

$V_R = IR = 20\times10 = 200$ volts

(iii) $V_L = I\,X_L = 20\times7.54 = 150.8$ volts

(iv) Apparent power $S = VI$

$= 250\times20$

$S = 5000VA$

$$\cos\phi = \frac{R}{Z} = \frac{10}{12.5} = 0.8 \text{ (lagging)}$$

Active power = VI cos ϕ

$= 250\times20\times0.8$

$P = 4000$ Watts

$$\sin\phi = \sqrt{1 - \cos^2\Phi} = \sqrt{1-(0.8)^2} = 0.6$$

Reactive Power Q = VI sin ϕ

$= 250\times20\times0.6$

$Q = 3000$ KVAR

2.18) Two impedances (5+j7)Ω and (10-j7)Ω are connected in series across a 200V supply. Calculate the current, power factor and power.

Solution:

$Z_1 = 5 + j7$

$Z_2 = 10 - j7$

$V = 200$ volts

$Z_{Total} = Z_1 + Z_2$

$= 5 + j7 + 10 - j7$

$Z_{Total} = 15 < 0^{\circ}$

$\phi = 0$ ∴

Taking V as referenve,

$$V = 200 < 0° \text{ Volts}$$

(i) $I = \dfrac{V}{Z} = \dfrac{200\angle 0°}{15\angle 0°} = 13.33\angle 0°\ amps$

(ii) $\phi = 0$

PF = $\cos\phi = \cos 0 = 1$

(iii) Power = $VI\cos\phi$

$= 200 \times 13.33 \times 1$

Power = 2666 watts

RC Series Circuit

Let us consider the circuit shown in diagram in which a pure resistance R and a pure capacitance C are connected in series.

Figure 3.24

Let

$V = V_m \sin\omega t$ be the applied voltage.

I = Circuit current of any instant

I = Effective value of circuit current

V_R = Potential Difference across Resistor

V_c = Potential Difference across Capacitor

f = Frequency of applied voltage

The same Current I flows through R and C

Voltage across R = V_R = IR in phase with I

Voltage across C = V_c = IX_c lagging I by 90^0

Applied voltage $V = IR - jIX_c$

$= I(R - jx_c)$

$$\frac{V}{I} = R - jX_c = Z$$

Z – Impedence of circuit

$$|Z| = \sqrt{R^2 + X_c^2}$$

Phasor Diagram of RC series circuit is,

Figure 3.25

From Triangle

$$\tan\phi = \frac{X_c}{R} = \frac{1/\omega c}{R} = \frac{1}{\omega c R}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega c R}\right)$$

ϕ is called Phase angle and it is angle between V and I. Its value lies between 0 and -90^o.

The current and voltage waveform of series RC Circuit is,

Figure 3.26

$V = V_m \sin \omega t$

$I = I_m \sin(\omega t - \phi)$

The current I leads the applied voltage V by an angle ϕ.

From Phasor Diagram,

Power factor $\cos\phi = \frac{R}{Z}$

Actual or real power $P = VI \cos\phi$

Reactive or Quardrature power $Q = VI \sin\phi$

Complex or Apparent Power $S = P + jQ$

PROBLEMS

3.20 A capacitor having a capacitarce of 10 μF is connected in series with a non-inductive resistor of 120Ω across 100V, 50HZ calculate the current, power and the Phase Difference between current and supply voltage.

(Non-inductive Resistor means a Pureresistor)

Solution:

C = 10 μF

R = 120Ω

V = 100V

F = 50Hz

$$X_c = \frac{1}{2\pi fc} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}}$$

$= 318\Omega$

$$|Z| = \sqrt{R^2 + X_c^2}$$

$= 340\ \Omega$

(a) $|I| = \frac{|V|}{|Z|}$

$= \frac{100}{340}$

$= 0.294\ amps$

(b) $PhaseDifference\ \phi = \tan^{-1}\left(\frac{X_c}{R}\right)$

$$= \tan^{-1}\left(\frac{318}{120}\right)$$

$$\phi = 69.3° \ (Leading)$$

$$\cos\phi = \cos(69.3)°$$

$$= 0.353 \ (Leading)$$

$$Power = |V||I|\cos\phi$$

$$= 100 \times 0.294 \times 0.353$$

$$= 10.38 \ Watts$$

The Resistor R in series with capacitance C is connected to a 50HZ, 240V supply. Find the value of C so that R absorbs 300 watts at 100 volts. Find also the maximum charge and the maximum stored energy in capacitance.

Solution:

V = 240 volt
F = 50Hz

Power absorbed by R = 300 watts
Voltage across R = 100 volts

$$|V|^2 = |V_R|^2 + |V_C|^2$$

$$|V_C| = \sqrt{|V|^2 - |V_R|^2}$$

$$= \sqrt{(240)^2 - (100)^2}$$

$$|V_C| = 218.2 \ volts$$

For Resistor, Power absorbed = 300 volts

$$|I|^2 R = |V_R||I| = 300$$

$$|I| = \frac{300}{|V_R|} = \frac{300}{100} = 3 amps$$

$$|X_C| = \frac{V_C}{|I|} \ (Apply \ ohm's \ law \ for \ C)$$

$$= \frac{218.2}{3} = 72.73\Omega$$

$$\frac{1}{2\pi fc} = 72.73 \qquad C =$$

$$\frac{1}{2\pi \times 50 \times 72.73}$$

$$= 43.77 \times 10^{-6} F$$

$$C = 43.77 \mu F$$

Maximum charge = Q_m = C × maximum V_c
Maximum stared energy = 1/2 (C × maximum V_c^2)
Maximum V_c = $\sqrt{2}$ × Rms value of V_c
= $\sqrt{2}$ × 218.2 = 308.6 volts

Now

Maximum charge = Q_m = 43.77 x 10^{-6} x 308.6
= 0.0135 Coulomb
Maximum energy stored
= ½ (43.77 x 10^{-6}) $(308.6)^2$
= 2.08 joules.

A Capacitor and Resistor are connected in series to an A.C. supply of 60 volts, 50HZ. The current is 2A and the power dissipated in the Resistor is 80 watts. Calculate (a) the impedance (b) Resistance (c) capacitance (d) Power factor.

Solution

$|V| = 60 volts$

$F = 50\ Hz$

$|I| = 2\ amps$

Power Dissipated = P = 80 watts

(a) $|Z| = \frac{|V|}{|I|} = \frac{60}{2} = 30\Omega$

(b) $As\ P = I^2 R$

$R = \frac{P}{I^2} = \frac{80}{4}$

$= 20\Omega$

(c) $Since, |Z|^2 = R^2 + X_c^2$

$X_c = \sqrt{(z)^2 - R^2}$

$= \sqrt{30^2 - 20^2} = 22.36\Omega$

$\frac{1}{2\pi fc} = 22.36$

$c = \frac{1}{2\pi f (22.36)}$

$= \frac{1}{2\pi \times 50 \times 22.36}$

$= 142 \times 10^{-6}$ F

C = 142 μF

(or) Power factor = $\cos\phi = \frac{R}{|Z|}$

$= \ = \frac{20}{30}$

= 0.67(Leading)

It is capacitive circuit.

A metal filament lamp, Rated at 750 watts, 100V is to be connected in series with a capacitor across a 230V, 60Hz supply. Calculate (i) The capacitance required (ii) The power factor

Solution

Rating of the metal filament W =750watts

$V_R = 100$ volts

$W = I^2R = V_RI$

$$I = \frac{W}{V_R} = \frac{750}{100} = 7.5\ amps$$

It is like RC Series Circuit

So

$$V^2 = V_R^2 + V_C^2$$

$$V_C = \sqrt{V^2 - V_R^2}$$

$$= \sqrt{(230)^2 - (100)^2}$$

$$= 207 volts$$

Applying Ohm's Law for C

$$|X_C| = \frac{|V_C|}{|I|} = \frac{207}{7.5}$$

$$= 27.6\Omega$$

$$\frac{1}{2\pi fc} = 27.6$$

$$c = \frac{1}{2\pi \times f \times 27.6} = \frac{1}{2\pi \times 60 \times 27.6}$$

$$= 96.19\ \mu F$$

$$Power\ factor = \cos\phi = \frac{R}{|Z|}$$

$$|Z| = \frac{|V|}{|I|} = \frac{230}{7.5} = 30.7\Omega$$

$$R = \frac{W}{I^2} = \frac{750}{(7.5)^2}$$

$$= 13.33\Omega$$

$$Powerfactor = \cos\phi = \frac{R}{Z}$$

$$\cos\phi = \frac{13.33}{30.7}$$

Solution

= 0.434(Leading)

RLC series circuit

Let v = RMS value of the voltage applied to series combination
I = RMS value of the current flowing
V_R = voltage across R
V_L = voltage across L
V_C = voltage across C

Figure 3.28

A circuit consisting of pure R, pure L and pure C connected in series is known as RLC series circuit.

Phasor diagram

Take I as reference
$V_R = I \times R$
$V_L = I \times X_L$
$V_C = I \times X_C$

Assume $X_L > X_C$

Then $V_L > V_C$

Figure 3.29

The above figure shows the phasor diagram for the combined circuit.
From the voltage triangle

$$|V|^2 = |V_R|^2 + (|V_L| - |V_C|)^2$$

$$= |IR|^2 + (|IX_L| - |IX_C|)^2$$

$$= |I|^2 [R^2 + (X_L - X_C)^2]$$

$$|V| = |I| \sqrt{R^2 + (X_L - X_C)^2}$$

$$|Z| = \frac{|V|}{|I|}$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \quad \because X = (X_L - X_C)$$

$$= \sqrt{R^2 + X^2}$$

Three cases of Z

Case 1 If $X_L > X_C$

The circuit behaves like RL circuit. Current lags behind voltage. So power factor is lagging.

Case 2 If $X_L < X_C$

The circuit behaves like RC circuit current leads applied voltage power factor is leading.

Case 3 When $X_L = X_C$, the circuit behaves like pure resistance.

Current is in phase with the applied voltage power factor is unity.

Impedance triangle

Figure 3.30

For $X_L > X_C$ For $X_L > X_C$.

1. If applied voltage
   $V = V_m \sin \omega t$ and ϕ is phase angle then 'i' is given by
   1) $i = I_m \sin(\omega t - \theta)$, for $X_L < X_C$
   2) $i = I_m \sin(\omega t + \theta)$, for $X_L > X_C$

3) $i = I_m \sin \omega t$ for $X_L = X_C$

2. Impedance for RLC series circuit in complex form (or) rectangular form is given by

$$Z = R + j (X_L - X_C)$$

Problems

In a RLC series circuit, the applied voltage is 5V. Drops across the resistance and inductance are 3V and 1V respectively. Calculate the voltage across the capacitor. Draw the phaser diagram.

$V = 5V$ $\quad V_R = 3V$ $\quad V_L = 1V$

$V^2 = V_R^2 + (V_L - V_C)^2$

$(V_L - V_C)^2 = V^2 - V_R^2$

$= 25 - 9 = 16$

$V_L - V_C = \pm 4$

$V_C = V_L \pm 4 = 1+4$

$V_C = 5V$

A coil of resistance 10Ω and in inductance of 0.1H is connected in series with a capacitance of 150μF across a 200v, 50HZ supply. Calculate

a) the inductive reactance of the coil.
b) the capacitive reactance
c) the reactance
d) current
e) power factor

$R = 10\Omega$

$L = 0.1$ H

$C = 150\ \mu F \quad = 150 \times 10^{-6}$ F

$V = 200V \quad f = 50$ Hz

a) $X_L = 2\pi f L = 2\pi (50)\ 0.1$

$= 31.4\ \Omega$

b) $X_C = \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi\ (50)(150 \times 10^{-6})}$

$= 21.2\Omega$

c) the reactance $X = X_L - X_C$

$= 31.4 - 21.2$

$= 10.2\ \Omega$ (Inductive)

d) $|Z| = \sqrt{R^2 + X^2}$

$= \sqrt{10^2 + (10.2)^2}$

$= 14.28\Omega (Inductive)$

e) $P.F = \cos\phi = \frac{R}{|Z|} = \frac{100}{14.28}$

= 0.7 (lagging) (I lags behind V)

Parallel AC circuit

When the impedance and connected in parallel and the combination is excited by AC source it is called parallel AC circuit.

Consider the parallel circuit shown in figure.

$$X_{C1} = \frac{1}{2\pi f c_1} = \frac{1}{\omega c_1}$$

$$X_{C2} = 2\pi f L_2 = \omega L_2$$

Impedance $|Z_1| = \sqrt{R_1^2 + X_{C1}^2}$

$$\phi_1 = \tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$|Z_2| = \sqrt{R_2^2 + X_{L2}^2}$$

$$\phi_2 = \tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

Conductance = g

Susceptance = b

Admittance = y

Branch 1

Conductance $g_1 = \frac{R_1}{|Z_1|^2}$

$$b_1 = \frac{X_{C1}}{|Z_1|^2} \ (positive)$$

$$|Y_1| = \sqrt{g_1^2 + b_1^2}$$

Branch 2

$$g_2 = \frac{R_2}{|Z_2|^2}$$

$$b_2 = \frac{X_{C2}}{|Z_2|^2} \ \text{(Negative)}$$

$|Y_2| = \sqrt{g_2^2 + b_2^2}$

Total conductance $G = g_1 + g_2$

Total Suceptance $B = b_1 - b_2$

Total admittance $|Y| = \sqrt{G^2 + B^2}$

Branch current $|I_1| = |V||Y_1|$

$|I_2| = |V||Y_2|$

$|I| = |V||Y|$

Phase angle = $\tan^{-1}\left(\frac{B}{G}\right)$ lag if B-negative

Power factor $\cos\phi = \frac{|G|}{|Y|}$

Problems:

Two impedances of parallel circuit can be represented by (20 + j15) and (10 – j60) Ω. If the supply frequency is 50 Hz, find the resistance, inductance or capacitance of each circuit.

$Z_1 = 20 + j15\ \Omega$

$Z_2 = 10 - j60\ \Omega$

$F = 50$ Hz

$Z_1 = R_1 + jX_L$

$Z_2 = R_2 - jX_C$

J term positive for in inductive

J term negative for capacitive.

For circuit 1, $R_1 = 20\Omega$

$X_1 = X_L = 2\pi fL = 2\pi(50)(L)$

$X_L = 15$

$2\pi(50)L = 15$

$$L = \frac{15}{2\pi(50)}$$

L = 48 mH

For circuit 2

$Z_2 = 10 - j60$

$R_2 = 10$

$X_2 = X_C = 60\ \Omega$

ie, $\frac{1}{2\pi fC} = 60$

C = $\frac{1}{}$

$2\pi\ (50)60$

$C = 53\ \mu F.$

2.3.27 Two circuits, the impedances of which are $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (6 - j8)\ \Omega$ are connected in parallel. If the total current supplied is 15A. What is the power taken by each branch.

$$Z1 = (10 + j15)\Omega = 18.03\angle 56.3$$

$$Z\ 2 = (6 - j8)\Omega = 10\angle - 53.13$$

$$I = 15\ A$$

$$I_1 = I\frac{Z_2}{Z_1 + Z_2} \quad \text{(Current divider rule)}$$

$$= \frac{15\angle 0^0 \times 10\angle - 53.13^0}{16 + j7}$$

$(Z_1 + Z_2 = 10 + j15 + 6 - j8)$

$$I_1 = \frac{150\angle -53.13^0}{17.46\angle 23.63}$$

$$I_1 = 8.6\angle - 76.76\ A$$

By KCL $I_2 = I - I_1$

$$= 15\angle 0 - 8.6\angle - 76.76$$

$$= 15 - (1.97 - j8.37)$$

$$= 15.5 - 32.7A$$

Power taken by branch 1

$= $ power dissipated in resistance of branch 1

$= |I_1|^2 R_1 = (8.6)^2 \times 10$

$= 739.6$ watts

Power taken by branch 2

$= |I_2|^2 R_2$

$= (15.5)^2 \times 6$

$= 1442$ watts

3.28 A 100Ω resistance and 0.6H inductance are connected in parallel across a 230v 50 Hz supply. Find the line current, impedance, power dissipated and parameter of the equivalent series circuit.

$Z_1 = R = 100\Omega$

$Z_2 = j\,X_L = j2\pi fL$

$= j\,(2\pi\times 50\times 0.6)$

$= j\,188.5\Omega$

$= 188.5 \angle 90$

$Z_T = Z_1 * Z_2$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{100\angle 0 \times 188.5\angle 90}{100 + j188.5}$$

$$= \frac{18850\angle 90}{213.4\angle 62}$$

$= 88.33 \angle 28$

$= 78 + j41.46 \Rightarrow R + jX_L$

Total impedance $|Z_T| = 88.33\Omega$

$R = 78\Omega$, $X_L = 41.46\Omega$

$X_L = 2\pi f L_{eq}$

$41.46 = 2\pi \times 50 \times L_{eq}$

$$Leq = \frac{41.46}{2\pi \times 50}$$

Leq = 132 mH

$= 30 - j40 + 24 + j32$

$= 54 - j8$

$= 54.6 \angle -8.43$ A

Comparing 'V' and 'I_T' current I_T lag voltage 'V'

$\therefore \phi = 8.43°$ lag

Power factor $= \cos\phi = \cos 8.43$

$= 0.99$ lag

True Power = $W = V\,I\cos\phi$

$= 200 \times 54.6 \times \cos 8.43$

$= 10802$ watts

$= 10.802$ KW

Apparent Power $= |V|\,I$

$= 200 \times 54.6$

$= 10920$ VA $= 10.920$ KVA

$= 3.663 \angle 8.43$

$= 3.623 + j0.54$

$= R + j X_L$

$R = 3.623\Omega \qquad X_L = 0.54\ \Omega$

(or)

$Z_{Total} = Z1 * Z2$

$$= \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(2.4 + j3.2)(3 - j4)}{2.4 + j3.2 + 3 - j4}$$

$$= \frac{4\angle 53.13 \times 5\angle -53.13}{5.46\angle -8.43}$$

$$= \frac{20\angle 0^0}{5.46\angle -8.43}$$

$= 3.663 \angle 8.43$

$= 3.623 + j\,0.54\ \Omega$

Reactive Power $= |V|\,I\sin\phi$

$= 200 \times 54.6 \times \sin 8.43$

$= 1601$ VAR

$= 1.601$ KVAR

Let Z_{total} = Total impedance

$$Z_{Total} = \frac{V}{I_{total}} = \frac{200\angle 0^0}{54.6\angle -8.43}$$

THREE PHASE A.C. CIRCUITS

Three Phase Connection

We have seen above only about single phase systems. Generally generation transmission and distribution of electrical energy are of three phase in nature. Three phase system is a very common poly phase system. It could be viewed combination of three single phase system with a phase difference of 120° between every pair. Generation, transmission and distribution of three phase power is cheaper. Three phase system is more efficient compared to single phase system. Uniform torque production occurs in three phase system where as pulsating torque is produced in the case of single phase system. Because of these advantages the overall generation, transmission and distribution of electrical power is usually of three phase.

There are two possible connections in 3-phase system. One is star connection and the other one is delta or mesh connection. Each type of connection is governed by characteristics equations relating the currents and the voltages.

Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called star point or the neutral point. The free ends of the three phase coils will be operating at specific potential with respect to the zero potential of star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system and three phase three wire system.

Analysis

Let us analyze the relationship between currents and voltages. In a three phase circuit, the voltage across the individual coil is known as phase voltage and the voltage between two lines is called line voltage. Similarly the current flowing through the coil is called phase current and the current flowing through the line is called line current.

Notations Defined

E_R , E_Y , E_B	: Phase voltages of R, Y and B phases.
I_R , I_Y, I_B	: Phase currents
V_{RY}, V_{YB}, V_{BR}	: Line voltages
I_{L1}, I_{L2}, I_{L3}	: Line currents

Figure 3.32

A balanced system is one in which the currents in all phases are equal in magnitude and are displaced from one another by equal angles. Also the voltages in all the phases are equal in magnitude and are displaced from one another by equal angles. Thus,

$E_R=E_Y=E_B=E_P \qquad V_{RY}=V_{YB}=V_{BR}=V_L$

$I_R=I_Y=I_B=I_P \qquad I_{L1}=I_{L2}=I_{L3}=I_L$

Figure 3.33

Current Relationship:

Apply Kirchhoff's current law at nodes R_1, Y_1, B_1 We get

$I_R= I_{LI}$; $I_Y = I_{L1}$; $I_B=I_{L3}$

This means that in a balanced star connected system, phase current equals the line current

$$I_P=I_L$$

Phase current = Line current

Voltage relationship:

Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R; V_{Ry} and E_y.

$$\vec{E_R}-\vec{E_Y}=\vec{V_{RY}}$$

Using law of parallelogram

$$V_{RY} = V_{RY} = \sqrt{E_R^2 + E_R^2 + 2E_R E_Y \cos 60}$$

$$= \sqrt{E_P^2 + E_P^2 + 2E_P E_P \cos 60} = E_P\sqrt{3}$$

Similarly,

$$\vec{E}_Y - \vec{E}_B = \vec{V}_{YB} \text{ and } \vec{E}_B - \vec{E}_R = \vec{V}_{BR}$$

$$V_{RY} = E_P\sqrt{3} \text{ and } V_{BR} = E_P\sqrt{3}$$

Hence $V_L = \sqrt{3}\, E_P$

Line Voltage $= \sqrt{3}$ phase voltage

Power relationship:

Let cosϕ be the power factor of the system.

Power consumed in one phase=$E_P I_P \cos\phi$

Power consumed in three phase $= 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi \text{ watts}$$

Reactive power in one phase $= E_P I_P \sin\phi$

Total Reactive power $= 3E_P I_P \sin\phi$

Apparent Power=$3E_P I_P$

Delta Connection:

The dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Figure 3.33

Let us analyze the relationship between currents and voltages. The system is balanced one. Notation used in the star connection are used here.

E_R, E_Y, E_B : Phase voltages of R, Y and B phases.
I_R, I_Y, I_B : Phase currents
V_{RY}, V_{YB}, V_{BR}: Line voltages
I_{L1}, I_{L2}, I_{L3} : Line currents

Voltage relationship:
Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R,V_{RY}

We Have $E_R=V_{RY}$
Similarly $E_Y = V_{YB}$ *and* $E_B = V_{BR}$

Thus $E_P=V_L$
Phase voltage= line voltage

Current Relationship:
Apply Kirchhoff's current law at node A (i.e.) R_1 , B_2 We get

$$\vec{I}_R - \vec{I}_B = \vec{I}_{1,1}$$

Referring to the phasor diagram and applying the law of parallelogram,
We get

$$I_{L1} = \sqrt{I_R^2 + I_Y^2 + 2 I_R I_Y \cos 60}$$
$$= \sqrt{I_P^2 + I_P^2 + 2 I_P I_P \cos 60}$$

Similarly,

$$\vec{I}_Y - \vec{I}_R = \vec{I}_{1,2} \text{ and } \vec{I}_B - \vec{I}_Y = \vec{I}_{1,3}$$

Hence $I_{L2} = I_P \sqrt{3}$ and $I_{L3} = I_P \sqrt{3}$

Thus Line current $= \sqrt{3}$ Phase current

$$I_L = I_P \sqrt{3}$$

Power relationship:
Let $\cos\phi$ be the power factor of the system.

Power consumed in one phase = $E_P l_P \cos\phi$

Power consumed in three phase = $3V_L \left(\frac{I_L}{\sqrt{3}}\right) \cos\phi$

$$= \sqrt{3} V_L I_L \cos\phi \text{ watts}$$

Reactive power in one phase = $E_p I_p \sin\phi$

$$\text{Total Reactive power} = 3E_p I_p \cos\phi$$

$$= \sqrt{3} V_L I_L \sin\phi \text{ VAR}$$

Apparent power per phase = $E_p I_p$

$$\text{Total Apparent Power} = 3E_p I_p = \sqrt{3} V_L I_L \text{ volt}$$

MEASUREMENT OF POWER IN THREE PHASE CIRCUITS:

A three phase circuit supplied from a balanced three phase voltage may have balanced load or unbalanced load. The load in general can be identified as a complex impedance. Hence the circuit will be unbalanced when the load impedance in all the phase are not of same value. As a result, the current flowing in the lines will have unequal values. These line currents will have equal values when the load connected to the three phases have equal values. The two cases mentioned above can exist when the load is connected in star or delta. The three phase power can be measured by using three watt maters in each phases. The algebraic sum of the reading gives the total three phase power consumed. However three phase power can also measured using two watt meter.

Case I Star Connected load

In this section we analyse the measurement of three phase power using two wattmeter, when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Diagram 4

Figure 3.35

For Wattmeter 1

$$Current\ measured = \vec{I}_{L1} = \vec{I}_R$$

$$Voltage\ measured = \vec{V}_{RY}$$

$$Phase angle between them = 30 + \phi$$

$$Power\ measured = P1 = V_{RY} I_R \cos(30 + \phi)$$

For Wattmeter 2

$$Current\ measured = \vec{I}_{L3} = \vec{I}_B$$

$$Voltage\ measured = \vec{V}_{BY}$$

$$Phase angle between them = 30 - \phi$$

$$Power\ measured = P1 = V_{BY} I_B \cos(30 - \phi)$$

$$= V_L I_L \cos(30 - \phi)$$

$$Now, \quad P1 + P2 = V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi + \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3}\, V_L I_L \cos\phi = Total\ power\ in\ a three\ phase circuit$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$
$$= V_L I_L \times 2 \times \sin 30 \sin\phi$$
$$= V_L I_L \sin\phi$$

$$\frac{P2 - P1}{P2 + P1} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]$$

$$\tan\phi = \sqrt{3}\ (P2 - P1/P2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Thus, two wattmeters connected appropriately in a three phase circuit can measure the total power consumed in the circuit.

Case II Delta Connected load

In this section we analyse the measurement of three phase power using two wattmeter, power when the load is star connected. The following assumption made:

(I) The three phase supply to which the load in connected is balanced.
(II) The phase sequence is R, Y, B.
(III) The load is balanced.
(IV) The load is R-L in nature.

Figure 3.36

For Wattmeter 1

$$\textit{Current measured} = \vec{I}_{1,1} = \vec{I}_R - \vec{I}_B$$

$$\textit{Voltage measured} = \vec{V}_{RY} = \vec{E}_R$$

$$\textit{Phase angle between them} = 30 + \phi$$

$$\textit{Power measured} = P1 = V_{RY} I_{L1} \cos(30 + \phi)$$

$$= V_L I_L \cos(30 + \phi)$$

For Wattmeter 2

$$\text{Current measured} = \vec{I}_{1,3} = \vec{I}_B - \vec{I}_Y$$

$$\text{Voltage measured} = \vec{V}_{BY} = -\vec{E}$$

$$\text{Phase angle between them} = 30 - \phi$$

$$\text{Power measured} = P1 = V_{BY} I_{1,3} \cos(30 - \phi)$$

$$= V_L I_L \cos(30 - \phi)$$

$$\text{Now,} \quad P1 + P2 = V_L I_L \cos(30 + \phi) + V_L I_L \cos(30 - \phi)$$

$$= V_L I_L [\cos 30 \cos\phi - \sin 30 \sin\phi + \cos 30 \cos\phi - \sin 30 \sin\phi]$$

$$= V_L I_L \times 2 \times \frac{\sqrt{3}}{2} \cos\phi$$

$$= \sqrt{3} V_L I_L \cos\phi = \text{Total power in a three phase circuit}$$

$$P2 - P1 = V_L I_L [\cos(30 - \phi) - \cos(30 + \phi)]$$

$$= V_L I_L \times 2 \times \sin 30 \sin\phi$$

$$= V_L I_L \sin\phi$$

$$\tan\phi = \sqrt{3}(P2 - P1/P2 + P1)$$

$$\text{Power factor} = \cos\left\{\tan^{-1}\sqrt{3}\left[\frac{P2 - P1}{P2 + P1}\right]\right\}$$

Problems 3.30

Three similar coils of Resistance of 10Ω and inductance 0.15 Henry are connected in star across a 3Φ, 440V, 50Hz supply. Find the line and phase values of current. Also find the above values when they are connected in Delta.

Solution:

Given Data

$$V_L = 440V,\ R_{ph} = 10\Omega,\ L_{ph} = 0.15H,\ f = 50Hz$$

$$X_{L\,ph} = 2\pi f L_{ph} = 2 \times \pi \times 50 \times 0.15 = 47.12\ \Omega$$

$$|Z_{ph}| = \sqrt{R_{ph}^2 + X_{L\,ph}^2} = \sqrt{10^2 + (47.12)^2}$$

$$= 48.17\Omega$$

In star Connection

$$I_L = I_{ph} \qquad V_L = \sqrt{3} V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 230.95 \text{ Volt}$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{230.95}{48.17} = 4.794\ A$$

$$I_L = I_{ph} = 4.794\ A$$

Active power = $3 V_{ph} I_{ph} \cos \Phi$

$$\cos\Phi = \frac{R_{ph}}{Z_{ph}} = 0.2075$$

$$\text{Activepower} = 3*230.95*\ 4.794*0.2075$$

$$= 689.54W$$

$$\text{Reactive power} = 3V_{ph} I_{ph} \sin \Phi$$

$$\sin \Phi = \sqrt{1 - \cos^2 \Phi} = 0.9782$$

$$\text{Reactive power} = 3*\ 230.95*\ 4.794 *\ 0.9782$$

$$= 3249.23VAR$$

$$\text{Apparent power} = 3V_{ph} I_{ph} = 3*\ 230.95*\ 4.794$$

$$= 3321.52V$$

If it is Delta connected coils, then

$$VL = V_{ph}\ \&\ IL = \sqrt{3}\ I_{ph}$$

$$VL = V_{ph} = 440V$$

$$I_{ph} = \frac{V_{ph}}{Z_{ph}} \frac{440}{48.17} = 9.134\ A$$

$$IL = \sqrt{3}\ I_{ph} = \sqrt{3} *9.134 = 15.82\ A$$

$$\text{Active power} = 3\ V_{ph} I_{ph} \cos \Phi$$

$$= 3*\ 440 *9.134 *\ 0.2075$$

$$= 2501.80\ watt$$

$$\text{Reacive power} = 3\,V_{ph} I_{ph} \sin\Phi$$

$$= 3 * 440 * 9.134 * 0.9782$$

$$\text{Apparent power} = 3V_{ph} I_{ph} = 3 * 440 * 9.134$$

$$= 12056.88\ VA$$

Problem 3.31

Two wattmeters connected to measure the 3Φ power indicate 1000 watts and 500 watts respectively. Calculate the power factor of the ckt.

Solution:

Given data

$$p_1 = 500 \text{ watts},\ p_2 = 1000 \text{ watts},$$

$$p_1 + p_2 = 1000 + 500 = 1500\ watts$$

$$p_2 - p_1 = 1000 - 500 = 500\ watts$$

$$p_1 = VL\,IL \cos(30 + \Phi)$$

$$p_2 = VL\,IL \cos(30 - \Phi)$$

$$p_1 + p_2 = \sqrt{3}\,VL\,IL \cos\Phi$$

$$p_2 - p_1 = \sqrt{3} * \frac{(p_2 - p_1)}{(p_1 + p_2)} = \frac{\sqrt{3} * 500}{1500}$$

$$= 0.5773$$

$$\Phi = 29.99°$$

Power factor cos Φ = 0.866

Problem 3.32

A balanced star connected load of (3+j4) Ω impedance is connected to 400V, three phase supply. What is the real power consumed by the load?

Solution:

Given data

$$V_L = 400\ volt$$

$$\text{Impedence / phase} = Z_{ph} = 3 + j4 = 5\angle 53°$$

In starconnection

$$I_L = I_{ph}\ \&\ V_L = \sqrt{3}\,V_{ph}$$

$$V_{ph} = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231 volt$$

$$\text{Current in each } I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{231}{5\angle 53°}$$

$$= 46.02\angle -53^\circ\ A$$

$$\text{Line current } I_L = 46.02\ A$$

$$\text{Total power consumed in the load} = \sqrt{3} V_L I_L \cos\Phi$$

$$= \sqrt{3} * 400 * 46.02 * \cos(-53^\circ)$$

$$= 19188\ watt$$

PART A – QUESTIONS

1. Define Form factor and Peak factor
2. What is meant by average value?
3. Give the relation between line voltage and phase voltage, line current and phase current for star and delta connection.
4. What are the advantages of polyphase system ?
5. Define power factor?
6. What is phase sequence?
7. Define inductance and write its unit.
8. What is meant by balanced system?
9. Write down the expression for power factor in two wattmeter method.

PART B – QUESTIONS

1. Explain with neat figures the power measurement in three phase circuits using two-wattmeter method.
2. A given load consisting of a resistor R & a capacitor C, takes a power of 4800W from 200V, 60HZ supply mains, Given that the voltage drop across the resistor is 120V, Calculate the (a) impedance (b) current (c) power factor (d) resistance (e) capacitance. Write down the equations for the current and voltage.
3. A coil of 10 ohms and inductance of 0.1H in series with a 150μF capacitor across 200V,250HZ supply. Calculate (i) inductive reactance, capacitive reactance and impedance of the circuit (ii) current (iii)power factor(iv)voltage across the coil and capacitor respectively.
4. An impedance z_1= (2.4+j3.2) ohms is in parallel with another impedance z_2= (3-j4) ohms. The combination is given a supply of 200 V. Calculate (i) total impedance (ii) individual & total currents (iii) power factor (iv) power in the circuit.
5. A balanced three phase load consists of 6 ohms resistor & 8 ohms reactor (inductive) in each phase. The supply is 230V, 3 phases, 50HZ. Find (a) phase current (b) line current (c) total power. Assume the load to be connected in star & delta.
6. A 3phase, 4 wire 208 V, ABC system supplies a star connected load in which $Z_A=10\angle 0$, $Z_B=15\angle 30$, $Z_C=10\angle -30$. Find the line currents,

the neutral current and the load power.

7. A coil having R = 10Ω and L = 0.2H is connected to a 100V, 50 Hz supply. Calculate (i) the impedance of the coil (ii) the current (iii) the phase difference between the current and voltage and (iv) the power.
8. Three similar coils of resistance of 10Ω and inductance 0.15H are connected in star across a 3 phase 440V, 50 Hz supply. Find the line and phase values of current. Also find the above values when they are connected in delta.
9. Each phase of a delta connected load comprises a resistor of Ohm and a capacitor of μF in series. Calculate the line current for a 3 - ϕ voltages of 400V at 50 Hz. Also evaluate the power factor and the total 3 - ϕ power absorbed by the load.

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SCHOOL OF ELECTRICAL AND ELECTRONICS

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

UNIT 3- INTRODUCTION T**O MACHINES**

SEEA1101- BASIC ELECTRICAL AND ELECTRONICS ENGINEERING

III. INTRODUCTION TO MACHINES

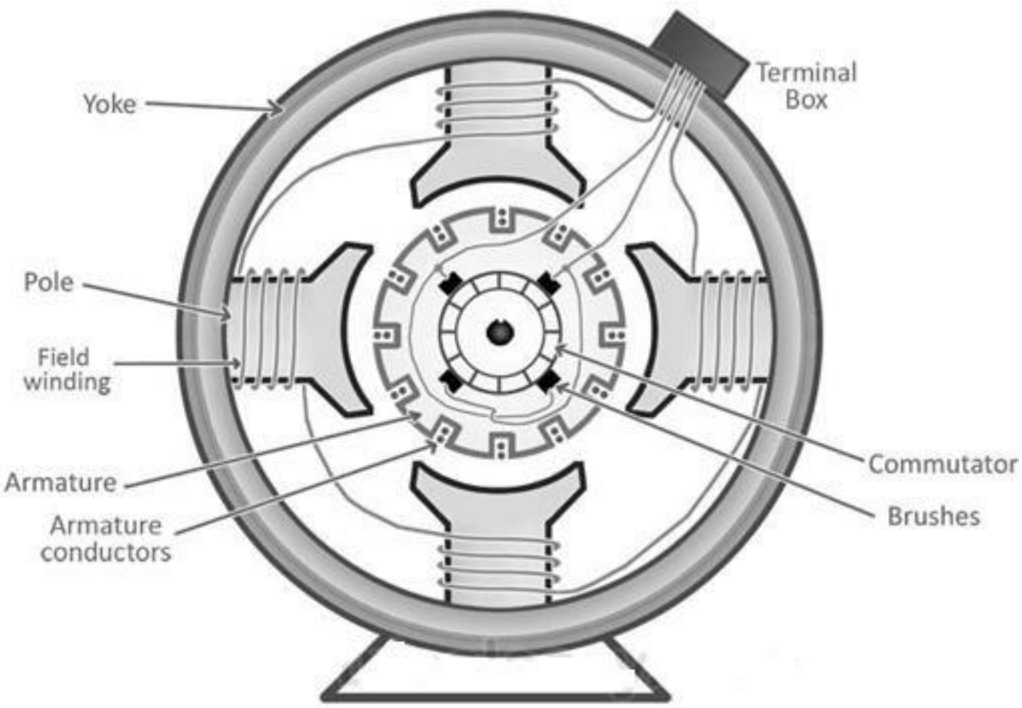
Construction and Principle of Operation of DC Generators - DC Motors - Single Phase Transformer - Single Phase Induction Motors - Stepper Motor**.**

3.1 DC Generator

A dc generator is an electrical machine which converts mechanical energy into direct current electricity. This energy conversion is based on the principle of production of dynamically induced emf. This article outlines basic construction and working of a DC generator.

3.2 Construction of A DC Machine:

A DC generator can be used as a DC motor without any constructional changes and vice versa is also possible. Thus, a DC generator or a DC motor can be broadly termed as a DC machine. These basic constructional details are also valid for the construction of a DC motor. Hence, let's call this point as construction of a DC machine instead of just 'construction of a dc generator'.



The above figure shows the constructional details of a simple 4-pole DC machine. A DC machine consists two basic parts; stator and rotor. Basic constructional parts of a DC machine are described below.

Yoke: The outer frame of a dc machine is called as yoke. It is made up of cast iron or steel. It not only provides mechanical strength to the whole assembly but also carries the magnetic flux produced by the field winding.

Poles and pole shoes: Poles are joined to the yoke with the help of bolts or welding. They carry field winding and pole shoes are fastened to them. Pole shoes serve two purposes; (i) they support field coils and (ii) spread out the flux in air gap uniformly.

Field winding: They are usually made of copper. Field coils are former wound and placed on each pole and are connected in series. They are wound in such a way that, when energized, they form alternate North and South poles.

Armature core (rotor)

Armature core: Armature core is the rotor of the machine. It is cylindrical in shape with slots to carry armature winding. The armature is built up of thin laminated circular steel disks for reducing eddy current losses. It may be provided with air ducts for the axial air flow for cooling purposes. Armature is keyed to the shaft.

Armature winding: It is usually a former wound copper coil which rests in armature slots. The armature conductors are insulated from each other and also from the armature core. Armature winding can be wound by one of the two methods; lap winding or wave winding. Double layer lap or wave windings are generally used. A double layer winding means that each armature slot will carry two different coils.

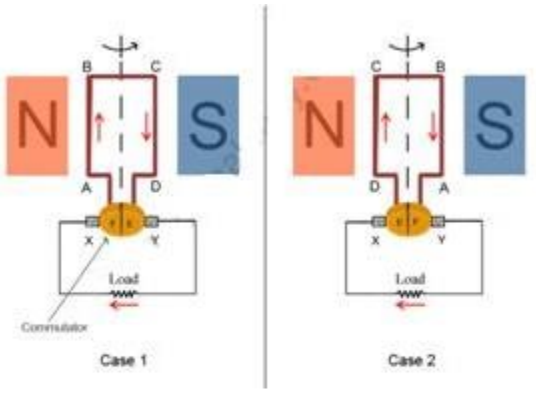
Commutator and brushes: Physical connection to the armature winding is made through a commutator-brush arrangement. The function of a commutator, in a dc generator, is to collect the current generated in armature conductors. Whereas, in case of a dc motor, commutator helps in providing current to the armature conductors. A commutator consists of a set of copper segments which are insulated from each other. The number of segments is equal to the number of armature coils. Each segment is connected to an armature coil and the commutator is keyed to the shaft. Brushes are usually made from carbon or graphite. They rest on commutator segments and slide on the segments when the commutator rotates keeping the physical contact to collect or supply the current.

Commutator

3.3 Working Principle of A DC Generator

According to Faraday's laws of electromagnetic induction, whenever a conductor is placed in a varying magnetic field (OR a conductor is moved in a magnetic field), an emf (electromotive force) gets induced in the conductor. The magnitude of induced emf can be calculated from the emf equation of dc generator. If the conductor is provided with the closed path, the induced current will circulate within the path. In a DC generator, field coils produce an electromagnetic field and the armature conductors are rotated into the field. Thus, an electromagnetically induced emf is generated in the armature conductors. The direction of induced current is given by Fleming's right hand rule.

Need of a Split ring commutator :



According to Fleming's right hand rule, the direction of induced current changes whenever the direction of motion of the conductor changes. Let's consider an armature rotating clockwise and a conductor at the left is moving upward. When the armature completes a half rotation, the direction of motion of that particular conductor will be reversed to downward. Hence, the direction of current in every armature conductor will be alternating. If you look at the above figure, you will know how the direction of the induced current is alternating in an armature conductor. But with a split ring commutator, connections of the armature conductors also gets reversed when the current reversal occurs. And therefore, we get unidirectional current at the terminals.

3.4 EMF equation of DC Generator

We know that the working principle of dc generator, that when conductors begin to cut the magnetic lines of force and therefore, the e.m.f. induces in the conductors according to 'Faraday's Law of Electromagnetic Induction'. The value of induced e.m.f. depends upon the lengths of the conductor, the magnetic field strength, and the speed at which the coil rotates. Let us see the equation for induced e.m.f.

Let,

φ = Flux per pole in Weber.

Z = Total number of armature conductors.

N = Armature rotation in revolution per minute (r.p.m).

P = Number of poles.

A = Number of parallel paths in armature.

E = e.m.f induced in any parallel path or generated e.m.f.

According to Faraday's law of Electromagnetic induction.

Average e.m.f generated per conductors,

$$= \frac{d\phi}{dt} = \frac{flux\ cut}{time\ taken}\ volt$$

Flux cut per Conductors in one revolution,

$$d\phi = \phi P \quad wb$$

Number of revolutions per minute,

$$= N / 60$$

Time taken for one revolution,

$$dt = 60 / N\ sec$$

E.m.f generated per conductor,

$$= \frac{d\phi}{dt} = \frac{\phi P}{60 / N} \times \frac{\phi PN}{60}\ volt$$

Therefore, the total emf E generated between the terminals if given as, E = Average e.m.f generated per conductor * Number of conductor in each parallel path

$$= \frac{\phi PN}{60} \times \frac{Z}{A} \; volt$$

$$E = \frac{\phi PN}{60} \times \frac{Z}{A} \; volt$$

3.5 Types of DC Generators

The DC generator converts mechanical power into electrical power. The magnetic flux in a DC machine is produced by the field coils carry current. The circulating current in the field windings produces a magnetic flux, and the phenomenon is known as Excitation.

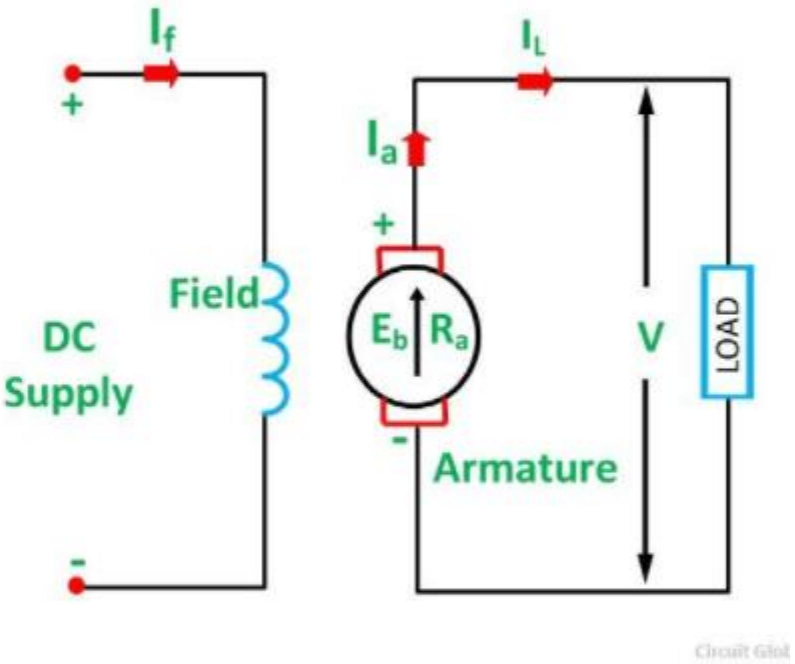
DC Generator is classified according to the methods of their field excitation.

- By excitation, the DC Generators are classified as Separately excited DC Generators and Self-excited DC Generators. There is also Permanent magnet type DC generators.
- The self-excited DC Generators are further classified as Shunt wound DC generators; Series wound DC generators and Compound wound DC generators.
- The Compound Wound DC generators are further divided as long shunt-wound DC generators, and short shunt-wound DC generators.
- The field pole of the DC generator is stationary, and the armature conductor rotates. The voltage generated in the armature conductor is of alternating nature, and this voltage is converted into the direct voltage at the brushes with the help of the commutator.

3.5.1 Separately Excited DC Generator

A DC generator whose field winding or coil is energised by a separate or external DC source is called a separately excited DC Generator. The flux produced by the poles depends upon the field current with the unsaturated region of magnetic material of the poles. i.e. flux is directly proportional to the field current. But in the saturated region, the flux remains constant.

The figure of self-excited DC Generator is shown below:



Separately Excited DC Generator

Here,

$I_a = I_L$ where I_a is the armature current and I_L is the line current.

Terminal voltage is given as:

$$V = E_g - I_a R_a \quad \ldots\ldots(1)$$

If the contact brush drop is known, then the equation (1) is written as:

$$V = E_g - I_a R_a - 2v_b \quad \ldots\ldots(2)$$

The power developed is given by the equation shown below:

$$\text{Power developed} = E_g I_a \quad \ldots\ldots\ldots(3)$$

$$\text{Power output} = VI_L = VI_a \quad \ldots\ldots\ldots(4)$$

3.5.2 Self Excited DC Generator

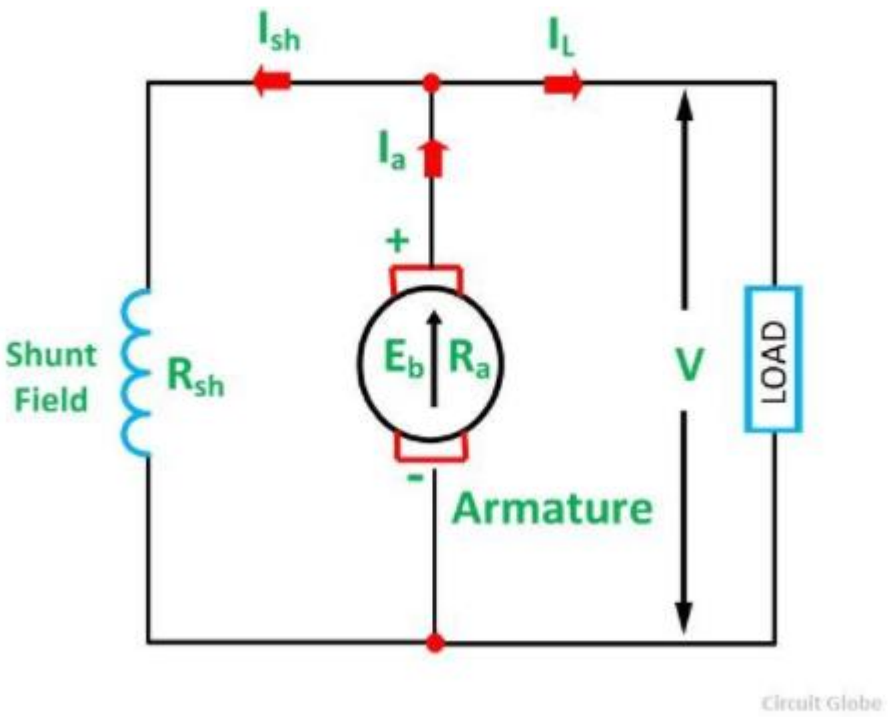
Self-excited DC Generator is a device, in which the current to the field winding is supplied by the generator itself. In self-excited DC generator, the field coils may be connected in parallel with the armature in the series, or it may be connected partly in series and partly in parallel with the armature windings.

The self-excited DC Generator is further classified as

a) Shunt Wound Generator

In a shunt-wound generator, the field winding is connected across the armature winding forming a parallel or shunt circuit. Therefore, the full terminal voltage is applied across it. A very small field current Ish, flows through it because this winding has many turns of fine wire having very high resistance Rsh of the order of 100 ohms.

The connection diagram of shunt-wound generator is shown below:



Shunt Wound DC Generator

The shunt field current is given as:

$$I_{sh} = \frac{V}{R_{sh}}$$

Where R_{sh} is the shunt field winding resistance.

The current field I_{sh} is practically constant at all loads. Therefore, the DC shunt machine is considered to be a constant flux machine.

Armature current is given as:

$$I_a = I_L + I_{sh}$$

Terminal voltage is given by the equation shown below:

$$V = E_g - I_a R_a$$

If the brush contact drop is included, the equation of the terminal voltage becomes

$$V = E_g - I_a R_a - 2v_b$$

$$\text{Power developed} = E_g I_a$$

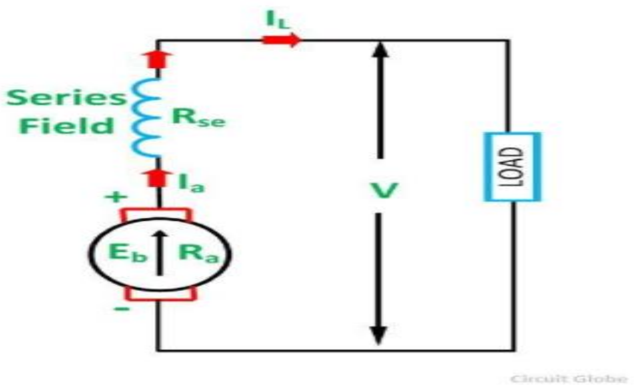
$$\text{Power output} = VI_L$$

b) Series Wound Generator

A series-wound generator the field coils are connected in series with the armature winding. The series field winding carries the armature current.

The series field winding consists of a few turns of wire of thick wire of larger cross-sectional area and having low resistance usually of the order of less than 1 ohm because the armature current has a very large value.

Its convectional diagram is shown below:



Series Wound DC Generator

Series field current is given as:

$$I_{se} = I_L = I_a$$

R_{se} is known as the series field winding resistance.

Terminal voltage is given as:

$$V = E_g - I_aR_a - I_{se}R_{se}$$

$$V = E_g - I_a(R_a + R_{se})$$

If the brush contact drop is included, the terminal voltage equation is written as:

$$V = E_g - I_a(R_a + R_{se}) - 2V_b$$

$$\text{Power developed} = E_gI_a$$

$$\text{Power output} = VI_L = VI_a$$

The flux developed by the series field winding is directly proportional to the current flowing through it. But it is only true before magnetic saturation after the saturation flux becomes constant even if the current flowing through it is increased.

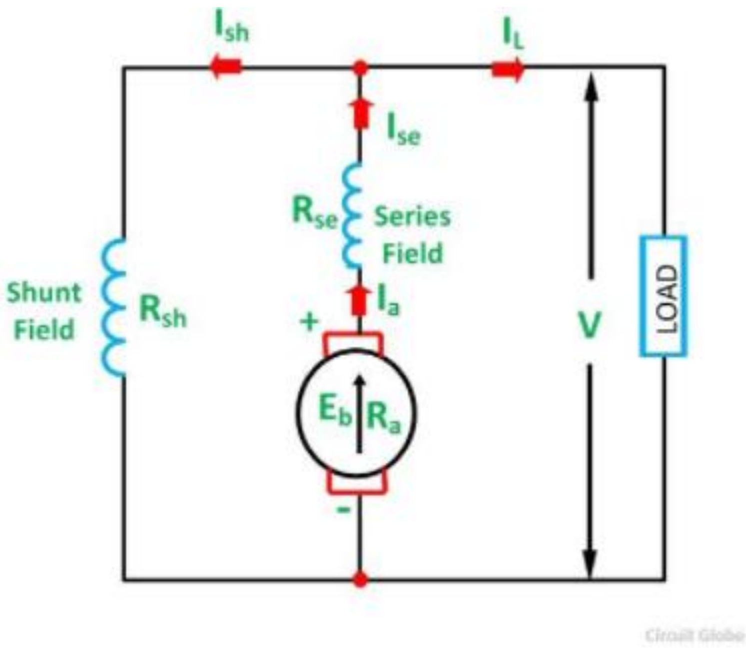
c) Compound Wound Generator

In a compound-wound generator, there are two field windings. One is connected in series, and another is connected in parallel with the armature windings. There are two types of compound-wound generator.

(i) Long shunt compound-wound generator
(ii) Short shunt compound-wound generator

(i)Long Shunt Compound Wound Generator

In a long shunt-wound generator, the shunt field winding is parallel with both armature and series field winding. The connection diagram of the long shunt-wound generator is shown below:



Long Shunt Compound Wound Generator

The shunt field current is given as:

$$I_{sh} = \frac{V}{R_{sh}}$$

Series field current is given as:

$$I_{se} = I_a = I_L + I_{sh}$$

Terminal voltage is given as:

$$V = E_g - I_a R_a - I_{se} R_{se} = E_g - I_a (R_a + R_{se})$$

If the brush contact drop is included, the terminal voltage equation is written as:

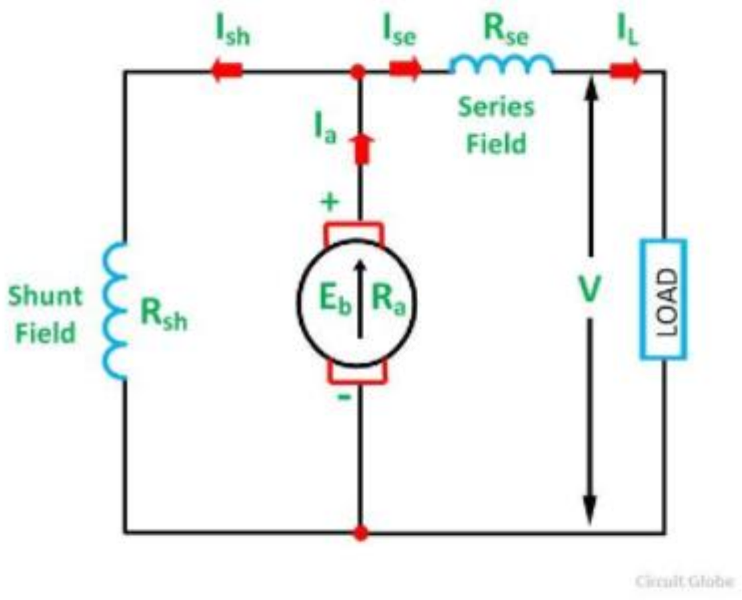
$$V = E_g - I_a (R_a + R_{se}) - 2V_b$$

$$\text{Power developed} = E_g I_a$$

$$\text{Power output} = VI_L$$

(ii)Short Shunt Compound Wound Generator

In a Short Shunt Compound Wound Generator, the shunt field winding is connected in parallel with the armature winding only. The connection diagram of a short shunt-wound generator is shown below.



Short Shunt Compound Wound Generator

Series field current is given as:

$$I_{se} = I_L$$

The shunt field current is given as:

$$I_{sh} = \frac{V + I_L R_{se}}{R_{sh}} = \frac{E_g - I_a R_a}{R_{sh}}$$

$$I_a = I_L + I_{sh}$$

Terminal voltage is given as:

$$V = E_g - I_a R_a - I_L R_{se}$$

If the brush contact drop is included, the terminal voltage equation is written as:

$$V = E_g - I_a R_a - I_L R_{se} - 2V_b$$

$$\text{Power developed} = E_g I_a$$

$$\text{Power output} = VI_L$$

In this type of DC generator, the field is produced by the shunt as well as series winding. The shunt field is stronger than the series field. If the magnetic flux produced by the series winding assists the flux produced by the shunt field winding, the generator is said to be **Cumulatively Compound Wound** generator.

If the series field flux opposes the shunt field flux, the generator is said to be **Differentially Compounded.**

3.6 Working Principle of A DC Motor

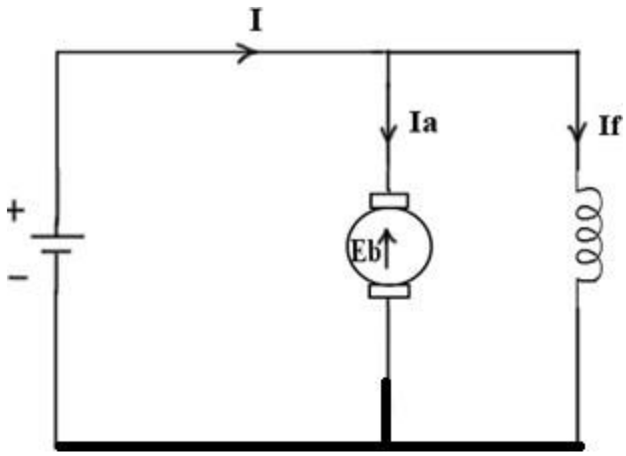
A motor is an electrical machine which converts electrical energy into mechanical energy. The principle of working of a DC motor is that "whenever a current carrying conductor is placed in a magnetic field, it experiences a mechanical force". The direction of this force is given by Fleming's left hand rule and it's magnitude is given by F = BIL. Where, B = magnetic flux density, I = current and L = length of the conductor within the magnetic field.

Fleming's left hand rule: If we stretch the first finger, second finger and thumb of our left hand to be perpendicular to each other AND direction of magnetic field is represented by the first finger, direction of the current is represented by second finger then the thumb represents the direction of the force experienced by the current carrying conductor.

3.6.1 Back EMF

According to fundamental laws of nature, no energy conversion is possible until there is something to oppose the conversion. In case of generators this opposition is provided by magnetic drag, but in case of dc motors there is back emf.

When the armature of the motor is rotating, the conductors are also cutting the magnetic flux lines and hence according to the Faraday's law of electromagnetic induction, an emf induces in the armature conductors. The direction of this induced emf is such that it opposes the armature current (Ia). The circuit diagram below illustrates the direction of the back emf and armature current. Magnitude of Back emf can be given by the emf equation of DC generator.



3.6.2 Significance of Back Emf:

Magnitude of back emf is directly proportional to speed of the motor. Consider the load on a dc motor is suddenly reduced. In this case, required torque will be small as compared to the current torque. Speed of the motor will start increasing due to the excess torque. Hence, being proportional to the speed, magnitude of the back emf will also increase. With increasing back emf armature current will start decreasing. Torque being proportional to the armature current, it will also decrease until it becomes sufficient for the load. Thus, speed of the motor will regulate.

On the other hand, if a dc motor is suddenly loaded, the load will cause decrease in the speed. Due to decrease in speed, back emf will also decrease allowing more armature current. Increased armature current will increase the torque to satisfy the load requirement. Hence, presence of the back emf makes a dc motor 'self-regulating'.

3.6.3 Types of DC Motor

A Direct Current Motor, DC is named according to the connection of the field winding with the armature. Mainly there are two types of DC Motors. One is Separately Excited DC Motor and other is Self-excited DC Motor.

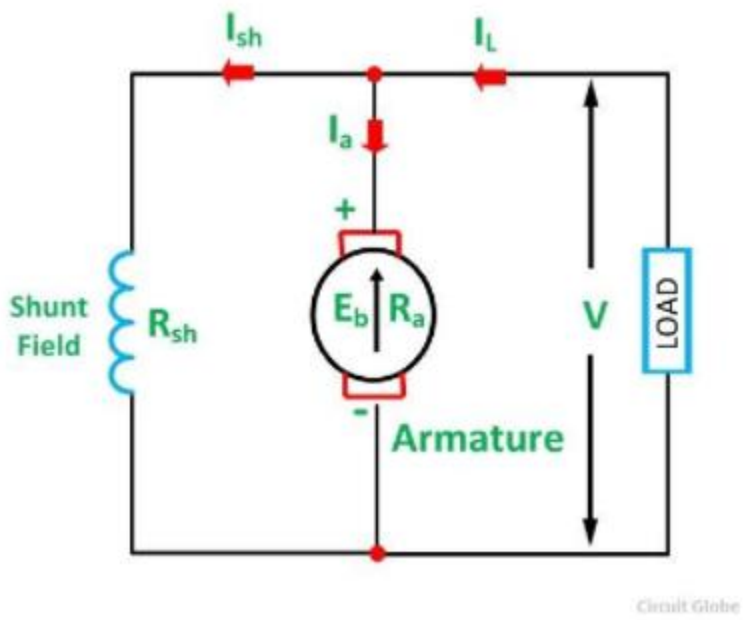
1. DC Shunt Motor

2. DC Seires Motor

3. DC Compound Motor

1. DC Shunt Motor

This is the most common types of DC Motor. Here the field winding is connected in parallel with the armature as shown in the figure below:



Shunt Wound DC Motor

The current, voltage and power equations for a shunt motor are written as follows.

By applying KCL at junction A in the above figure.

The sum of the incoming currents at A = Sum of the outgoing currents at A.

$$I = I_a + I_{sh} \ldots\ldots\ldots..(1)$$

Where,

I is the input line current
Ia is the armature current
Ish is the shunt field current

Equation (1) is the current equation.

The voltage equations are written by using Kirchhoff's voltage law (KVL) for the field winding circuit.

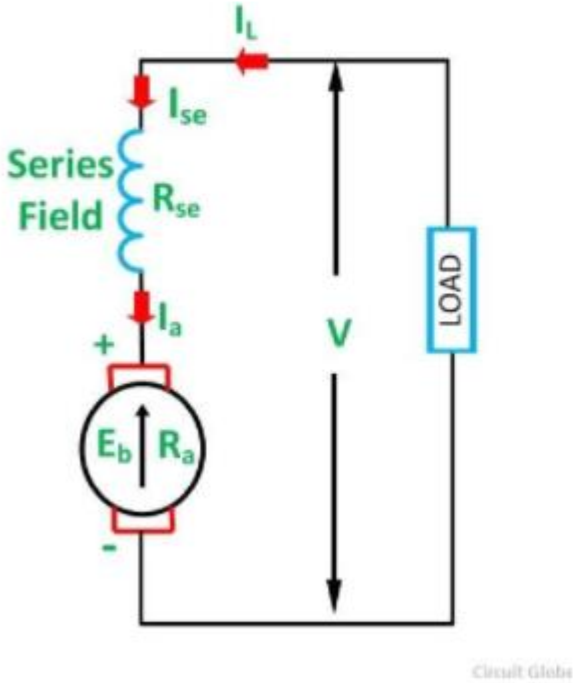
$$V = I_{sh}R_{sh} \ \ldots\ldots..(2)$$

For armature winding circuit the equation will be given as:

$$V = E + I_aR_a \ \ldots\ldots\ldots(3)$$

2. DC Seires Motor

In the series motor, the field winding is connected in series with the armature winding. The connection diagram is shown below:



Series Wound Motor

By applying the KCL in the above figure:

$$I = I_{se} = I_a$$

Where,

Ise is the series field current

The voltage equation can be obtained by applying KVL in the above figure.

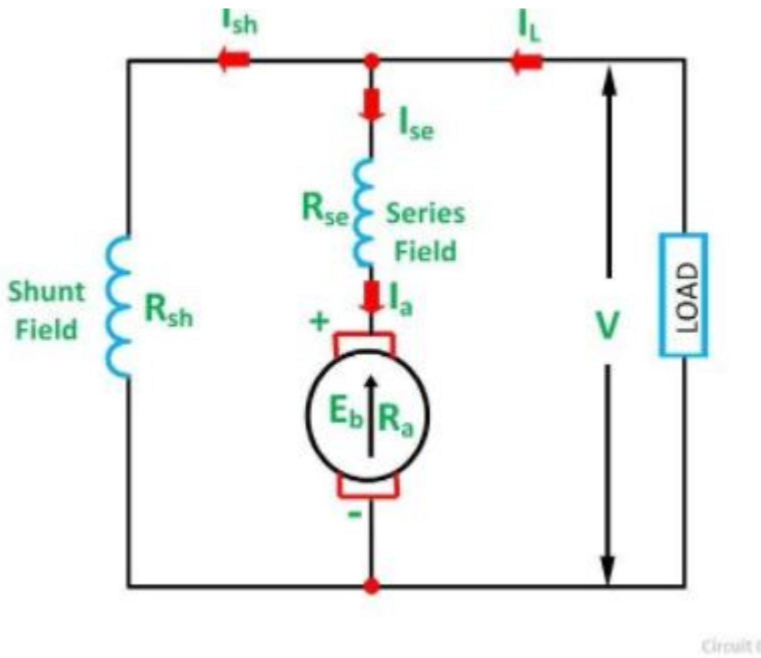
$$V = E + I\,(R_a + R_{se}) \ldots \ldots \ldots (8)$$

The power equation is obtained by multiplying equation (8) by I we get

$$VI = EI + I^2\,(R_a + R_{se}) \ldots \ldots \ldots (9)$$

3. DC Compound Motor

A DC Motor having both shunt and series field windings is called a Compound Motor. The connection diagram of the compound motor is shown below:



Compound Motor

The compound motor is further subdivided as **Cumulative Compound** Motor and **Differential Compound** Motor. In a cumulative compound motor the flux produced by both the windings is in the same direction, i.e.

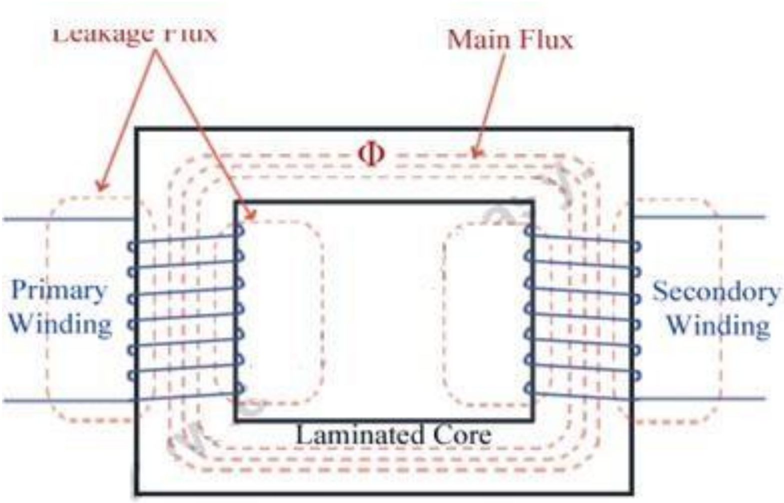
$$\varphi_r = \varphi_{sh} + \varphi_{se}$$

In differential compound motor, the flux produced by the series field windings is opposite to the flux produced by the shunt field winding, i.e.

$$\varphi_r = \varphi_{sh} - \varphi_{se}$$

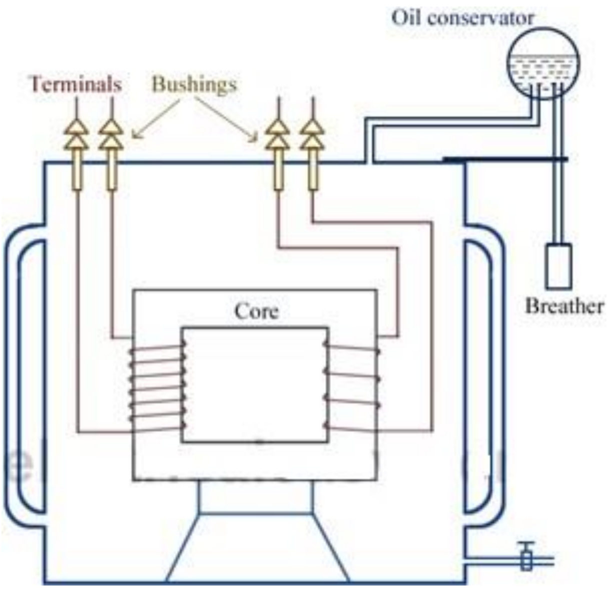
3.7 Electrical Transformer - Basic Construction, Working and Types

Electrical transformer is a static electrical machine which transforms electrical power from one circuit to another circuit, without changing the frequency. Transformer can increase or decrease the voltage with corresponding decrease or increase in current. Working Principle of Transformer



The basic principle behind working of a transformer is the phenomenon of mutual induction between two windings linked by common magnetic flux. The figure at right shows the simplest form of a transformer. Basically a transformer consists of two inductive coils; primary winding and secondary winding. The coils are electrically separated but magnetically linked to each other. When, primary winding is connected to a source of alternating voltage, alternating magnetic. The core provides magnetic path for the flux, to get linked with the secondary winding. Most of the flux gets linked with the secondary winding which is called as 'useful flux' or main 'flux', and the flux which does not get linked with secondary winding is called as 'leakage flux'. As the flux produced is alternating (the direction of it is continuously changing), EMF gets induced in the secondary winding according to Faraday's law of electromagnetic induction. This emf is called 'mutually induced emf', and the frequency of mutually induced emf is same as that of supplied emf. If the secondary winding is closed circuit, then mutually induced current flows through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit.

3.7.1 Basic Construction of Transformer



Basically a transformer consists of two inductive windings and a laminated steel core. The coils are insulated from each other as well as from the steel core. A transformer may also consist of a container for winding and core assembly (called as tank), suitable bushings to take our the terminals, oil conservator to provide oil in the transformer tank for cooling purposes etc. The figure at left illustrates the basic construction of a transformer.

In all types of transformers, core is constructed by assembling (stacking) laminated sheets of steel, with minimum air-gap between them (to achieve continuous magnetic path). The steel used is having high silicon content and sometimes heat treated, to provide high permeability and low hysteresis loss. Laminated sheets of steel are used to reduce eddy current loss. The sheets are cut in the shape as E,I and L. To avoid high reluctance at joints, laminations are stacked by alternating the sides of joint. That is, if joints of first sheet assembly are at front face, the joints of following assemble are kept at back face.

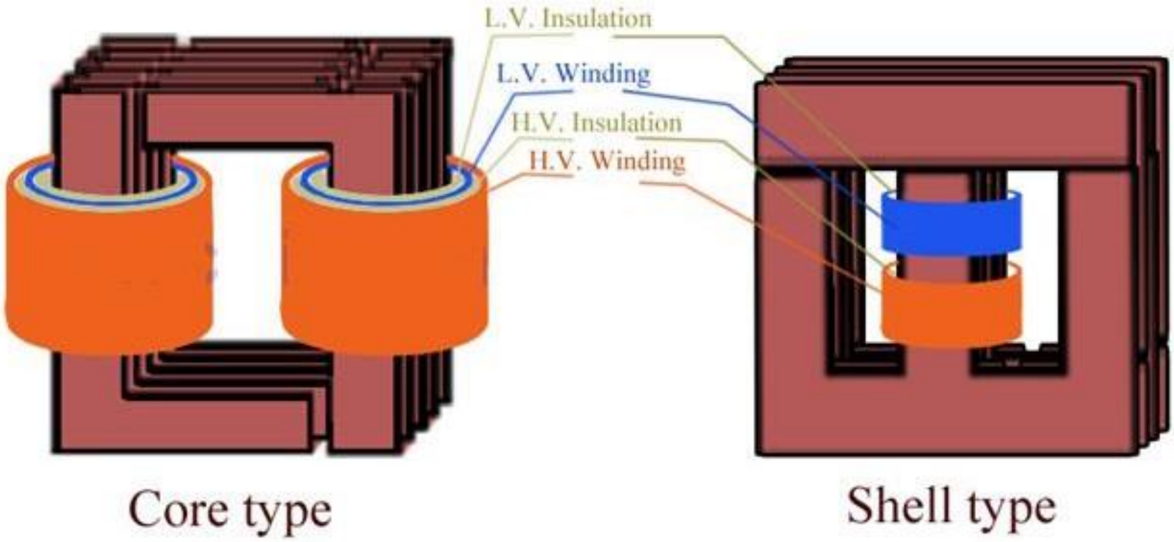
3.7.2 Types of Transformers

Transformers can be classified on different basis, like types of construction, types of cooling etc.

(A) On the basis of construction, transformers can be classified into two types as;

(i) Core type transformer and

(ii) Shell type transformer, which are described below.



(I) **Core Type Transformer**

In core type transformer, windings are cylindrical former wound, mounted on the core limbs as shown in the figure above. The cylindrical coils have different layers and each layer is insulated from each other. Materials like paper, cloth or mica can be used for insulation. Low voltage windings are placed nearer to the core, as they are easier to insulate.

(Ii) **Shell Type Transformer**

The coils are former wound and mounted in layers stacked with insulation between them. A shell type transformer may have simple rectangular form (as shown in above fig

(A) On the basis of their purpose
Step up transformer: Voltage increases (with subsequent decrease in current) at secondary. Step down transformer: Voltage decreases (with subsequent increase in current) at secondary.

(B) **On the basis of type of supply Single phase transformer**
Three phase transformer

(C) On the basis of their use
Power transformer: Used in transmission network, high rating

Distribution transformer: Used in distribution network, comparatively lower rating than that of power transformers.

Instrument transformer: Used in relay and protection purpose in different instruments in industries

Current transformer (CT)

Potential transformer (PT)

(E) **On the basis of cooling employed Oil-filled self cooled type**

Oil-filled water cooled type Air blast type (air cooled)

3.7.3 Comparison between core type and shell type transformer

Basis for Comparison	Core Type Transformer	Shell Type Transformer
Definition	The winding surround the core.	The core surround the winding.
Lamination Shape	The lamination is cut in the form of the L strips.	Lamination are cut in the form of the long strips of E and L.
Cross Section	Cross-section may be square, cruciform and three stepped	The cross section is rectangular in shape.
Copper Require	More	Less
Other Name	Concentric Winding or Cylindrical Winding.	Sandwich or Disc Winding
Limb	Two	Three
Insulation	More	Less
Flux	The flux is equally distributed on the side limbs of the core.	Central limb carry the whole flux and side limbs carries the half of the flux.
Winding	The primary and secondary winding are placed on the side limbs.	Primary and secondary windings are placed on the central limb
Magnetic Circuit	Two	One

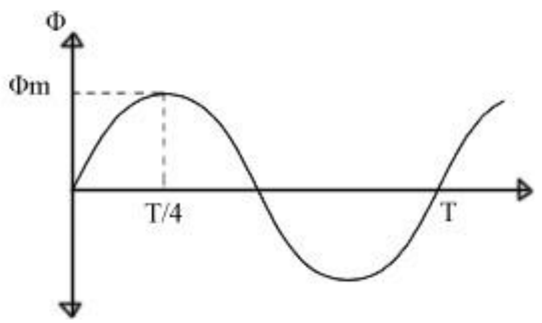
Basis for Comparison	Core Type Transformer	Shell Type Transformer
Losses	More	Less
Maintenance	Easy	Difficult
Mechanical Strength	Low	High
Output	Less	High
Natural Cooling	Does not Exist	Exist

3.7.4 EMF Equation of the Transformer

Let,

N1 = Number of turns in primary winding N2 = Number of turns in secondary winding

Φm = Maximum flux in the core (in



As, shown in the fig., the flux rises sinusoidally to its maximum value Φm from 0. It reaches to the maximum value in one quarter of the cycle i.e in T/4 sec (where, T is time period of the sin wave of the supply = 1/f).

Therefore,

average rate of change of flux = Φm /(T/4) = Φm/(1/4f) Therefore,

average rate of change of flux = 4f Φm............... (Wb/s).

Now,

Induced emf per turn = rate of change of flux per turn

Therefore, average emf per turn = 4f Φm............. (Volts).

Now, we know, Form factor = RMS value / average value

Therefore, RMS value of emf per turn = Form factor X average emf

per turn. As, the flux Φ varies sinusoidally, form factor of a sine

wave is 1.11 Therefore, RMS value of emf per turn = 1.11 x 4f Φm =

4.44f Φm.

RMS value of induced emf in whole primary winding (E1) = RMS value of emf per turn X Number of turns in primary winding

E1 = 4.44f N1 Φm....................................... eq 1

Similarly, RMS induced emf in secondary winding (E2) can be

given as E2 = 4.44f N2 Φm......................... eq 2

from the above equations 1 and 2,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\,\Phi m$$

This is called the emf equation of transformer, which shows, emf / number of turns is same for both primary and secondary winding.

For an ideal transformer on no load, E1 = V1 and E2 = V2 . where, V1 = supply voltage of primary winding

V2 = terminal voltage of secondary winding

Voltage Transformation Ratio (K) As derived above,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = K$$

Where, K = constant

This constant K is known as voltage transformation ratio.

If N2 > N1, i.e. K > 1, then the transformer is called step-up transformer.

If N2 < N1, i.e. K < 1, then the transformer is called step-down transformer.

3.8 Working Principle of a Single Phase Induction Motor

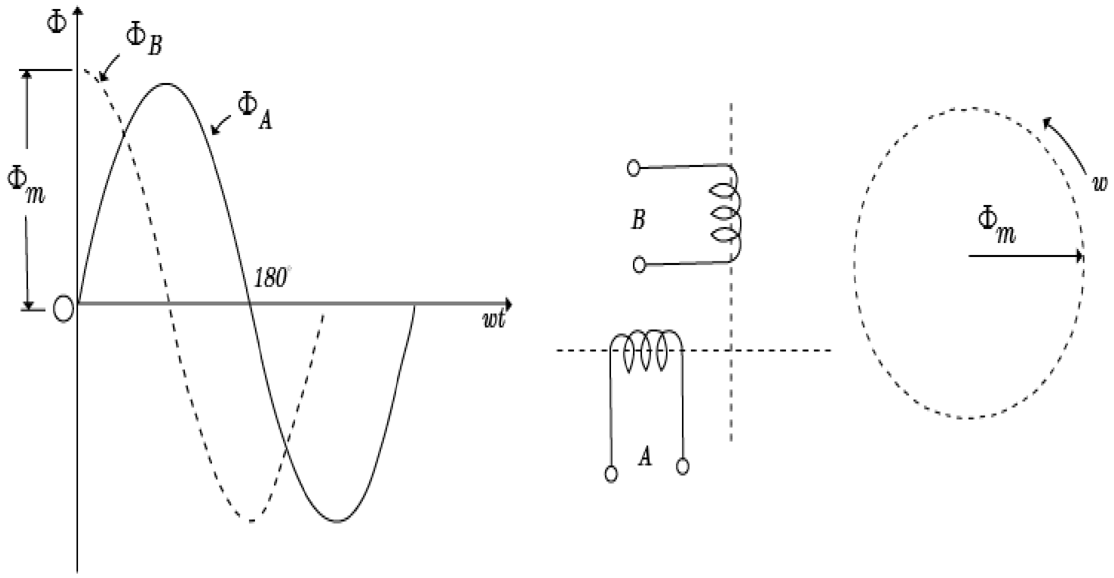
Production of Rotating Field

Consider two winding 'A' and 'B' so displaced that they produce magnetic field 90° apart in space. The resultant of these two fields is a rotating magnetic field of constant magnitude ϕm. Non-Uniform magnetic field produces a non-uniform torque which makes the operation of the motor noisy, affect starting torque.

Consider two winding 'A' and 'B' so displaced that they produce magnetic field 90° apart in space. The resultant of these two fields is a rotating magnetic field of constant magnitudeϕ_m. Non-Uniform magnetic field produces a non-uniform torque which makes the operation of the motor noisy, affect starting torque.

$$\varphi_A = \varphi_m \sin \omega t$$

$$\varphi_B = \varphi_m \sin(\omega t + 90°)$$



Starting Principle

A single phase induction motor consists of a single phase winding on the stator and a cage winding on the rotor. When a 1 phase supply is connected to the stator winding, a pulsating magnetic field is produced. In the pulsating field, the rotor does not rotate due to inertia. Therefore a single phase induction motor is not self-starting and requires some particular starting means. Two theories have been suggested to find the performance of a single phase induction motor.

3.8.2 Types of Single Phase Induction Motors

As mentioned above that, due to the rotating magnetic field of the stator, the induction motor becomes self starting. There are many methods of making a single phase induction motor as self starting one.

Based on the starting method, single phase induction motors are basically classified into the following types.

Split-phase motor

Capacitor start motor

Capacitor start capacitor run motor

Shaded pole motor

a) Split-phase motor

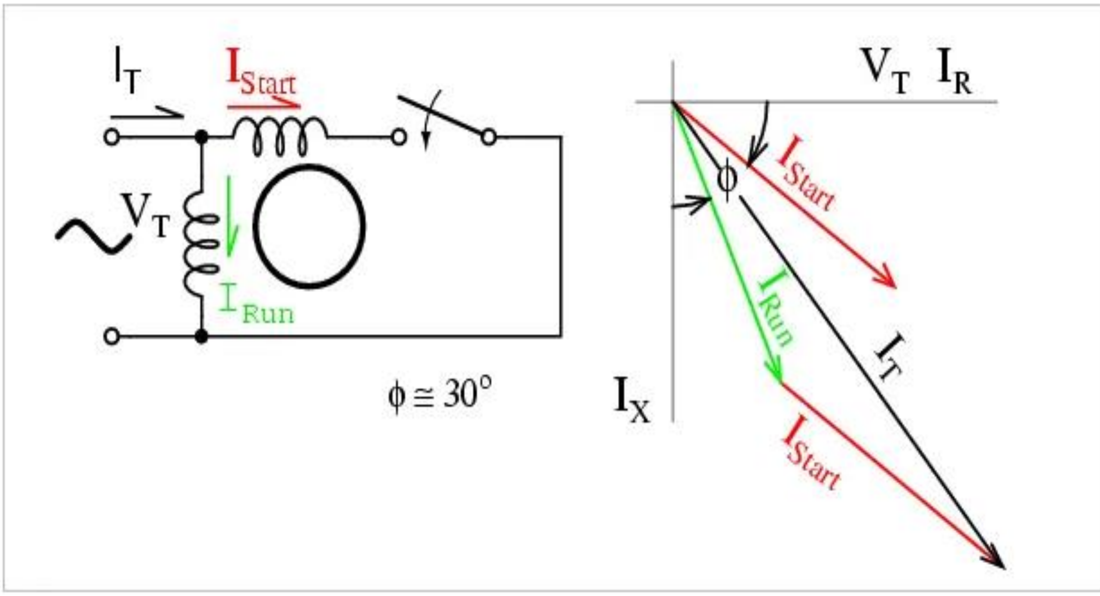
In addition to the main winding or running winding, a single-phase induction motor's stator carries another winding called auxiliary winding or starting winding. A centrifugal switch is connected in series with auxiliary winding.

This switch aims to disconnect the auxiliary winding from the main circuit when the motor attains a speed up to 75 to 80% of the synchronous speed.

We know that the running winding is inductive in nature. We aim to create the phase difference between the two winding, and this is possible if the starting winding carries high resistance.

In the figure below, the variables represent:

- I_{run} is the current flowing through the main or running winding,
- I_{start} is the current flowing in starting winding,
- V_T is the supply voltage



For a highly resistive winding, the current is almost in phase with the voltage, and for a highly inductive winding, the current lag behind the voltage by a large angle.

The starting winding is highly resistive so, the current flowing in the starting winding lags behind the applied voltage by a very small angle and the running winding is highly inductive in nature so, the current flowing in running winding lags behind applied voltage by a large angle.

The resultant of these two current is IT—the resultant of these two current produce rotating magnetic field which rotates in one direction.

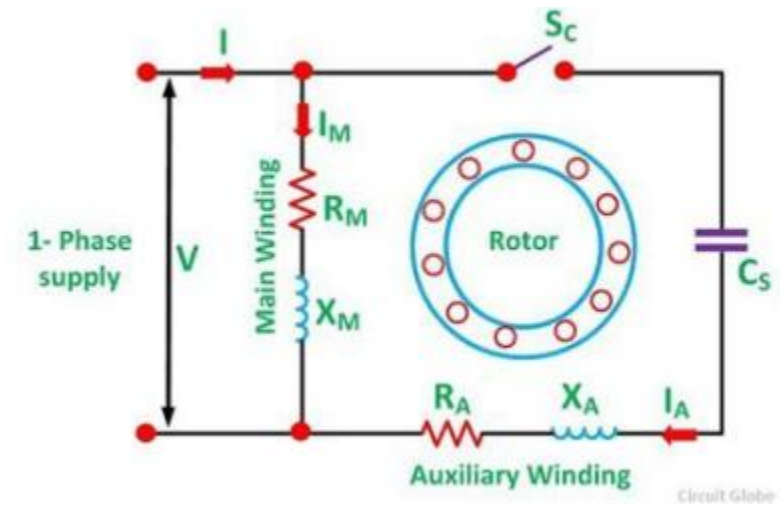
b) Capacitor start motor

This motor is similar to the split phase motor, but in addition a capacitor is connected in series to auxiliary winding. This is a modified version of split phase motor.

Since the capacitor draws a leading current, the use of a capacitor increases the phase angle between the two currents (main and auxiliary) and hence the starting torque. This is the main reason for using a capacitor in single phase induction motors.

Here the capacitor is of dry-type electrolytic one which is designed only for alternating current use. Due to the inexpensive type of capacitors, these motors become more popular in wide applications.

These capacitors are designed for definite duty cycle, but not for continuous use. The schematic diagram of capacitor start motor is shown in figure below.



The operation of this motor is similar to the split phase motor where the starting torque is provided by additional winding.

Once the speed is picked up, the additional winding along with capacitor is removed from the circuit with the help of centrifugal switch. But, the difference is that the torque produced by this motor is higher than split phase motor due to the use of capacitor.

Due to the presence of a capacitor, the current through auxiliary winding will leads the applied voltage by some angle which is more than that of split case type.

Thus, the phase difference between main and auxiliary currents is increased and thereby starting torque.

The performance of this motor is identical to the split phase motor when it runs near full load RPM. Due to the capacitor, the inrush currents are reduced in this motor.

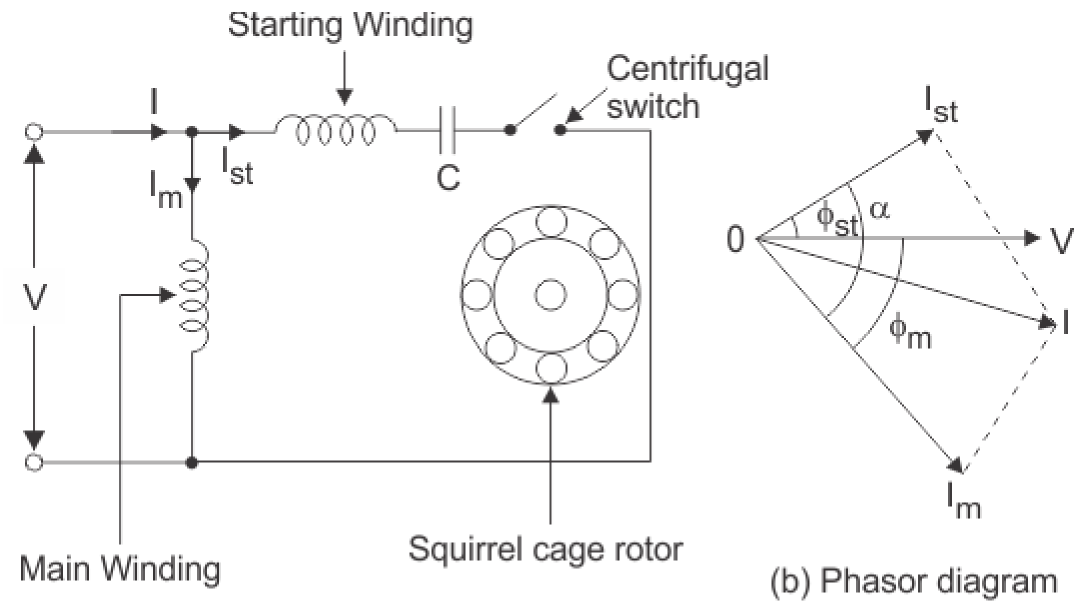
These motors have very high starting torque up to 300% full load torque. However the power factor is low at rated load and rated speed.

Owing to the high starting torque, these motors are used in domestic as well as industrial applications such as water pumps, grinders, lathe machines, compressors, drilling machines, etc.

c) Capacitor start capacitor run motor

The working principle of capacitor-start inductor motors is almost the same as capacitor-start capacitor-run induction motors.

We already know that a single-phase induction motor is not self-starting because the magnetic field produced is not a rotating type. To produce a rotating magnetic field, there must be some phase difference.



(a) Schematic representaion

(b) Phasor diagram

In the case of a split-phase induction motor, we use resistance for creating phase difference, but here we use a capacitor for this purpose. We are familiar with the fact that the current flowing through the capacitor leads to the voltage.

So, in capacitor start inductor motor and capacitor start capacitor run induction motor, we are using two winding, the main winding, and the starting winding.

With starting winding, we connect a capacitor, so the current flowing in the capacitor, i.e., Ist leads the applied voltage by some angle, φst.

The running winding is inductive in nature so, the current flowing in running winding lags behind applied voltage by an angle, φm.

Now there occur large phase angle differences between these two currents, which produce a resultant current. This will produce a rotating magnetic field since the torque produced by these motors depends upon the phase angle difference, which is almost 90o.

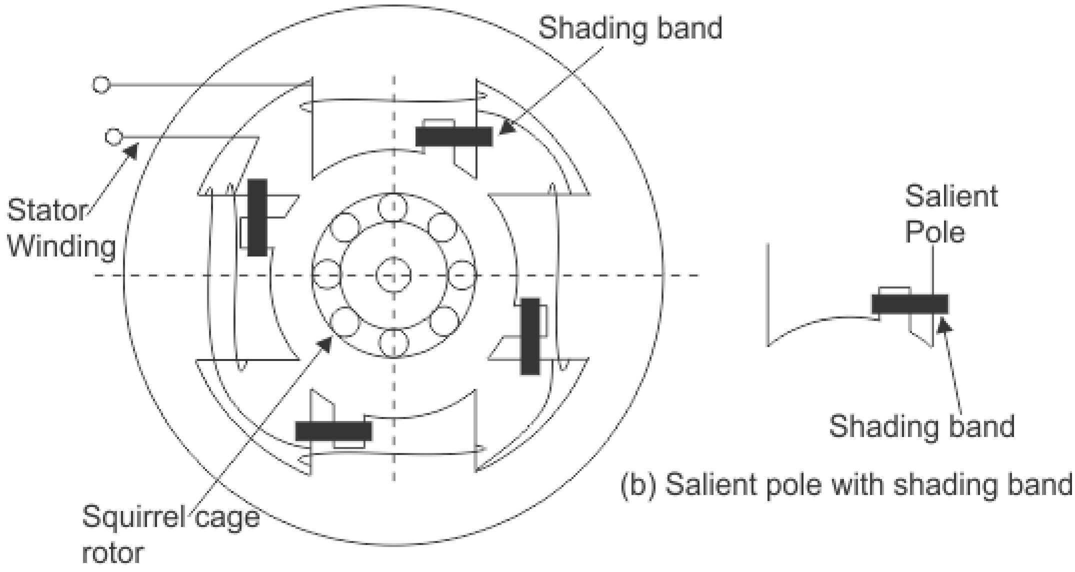
So, these motors produce very high starting torque. In the case of capacitor start induction motor, the centrifugal switch is provided to disconnect the starting winding when the motor attains a speed up to 75 to 80% of the synchronous speed but in the case of capacitor start capacitors run induction motor.

There is no centrifugal switch so, the capacitor remains in the circuit and improves the power factor and the running conditions of the single-phase induction motor.

d) Shaded pole motor

The stator of the shaded pole single-phase induction motor has salient or projected poles. These poles are shaded by a copper band or ring, which is inductive in nature.

The poles are divided into two unequal halves. The smaller portion carries the copper band and is called the shaded portion of the pole.



(b) Salient pole with shading band

(a) 4-pole shaded pole construction

ACTION: When a single-phase supply is given to a shaded pole induction motor's stator, an alternating flux is produced.

This change of flux induces emf in the shaded coil. Since this shaded portion is short-circuited, the current is produced in it in such a direction to oppose the main flux.

The flux in the shaded pole lags behind the flux in the unshaded pole. The phase difference between these two fluxes produces resultant rotating flux.

3.9 Stepper Motor

A Stepper Motor or a step motor is a brushless, synchronous motor which divides a full rotation into a number of steps. Unlike a brushless DC motor which rotates continuously when a fixed DC voltage is applied to it, a step motor rotates in discrete step angles. The Stepper Motors therefore are manufactured with steps per revolution of 12, 24, 72, 144, 180, and 200, resulting in stepping angles of 30, 15, 5, 2.5, 2, and 1.8 degrees per step. The stepper motor can be controlled with or without feedback.

Stepper motors work on the principle of electromagnetism. There is a soft iron or magnetic rotor shaft surrounded by the electromagnetic stators. The rotor and stator have poles which may be teethed or not depending upon the type of stepper. When the stators are energized the rotor moves to align itself along with the stator (in case of a permanent magnet type stepper) or moves to have a minimum gap with the stator (in case of a variable reluctance stepper). This way the stators are energized in a sequence to rotate the stepper motor. Get more information about working of stepper motors through interesting images at the stepper motor Insight.

3.9.1 Types of Stepper Motor

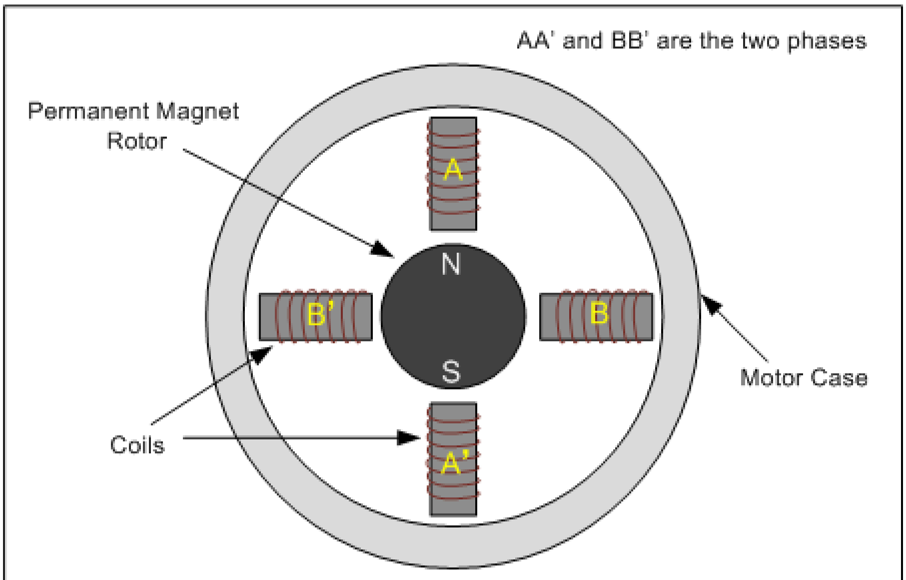
By construction the step motors come into three broad classes:

1. Permanent Magnet Stepper
2. Variable Reluctance Stepper
3. Hybrid Step Motor

These three types have been explained in detail in the following sections.

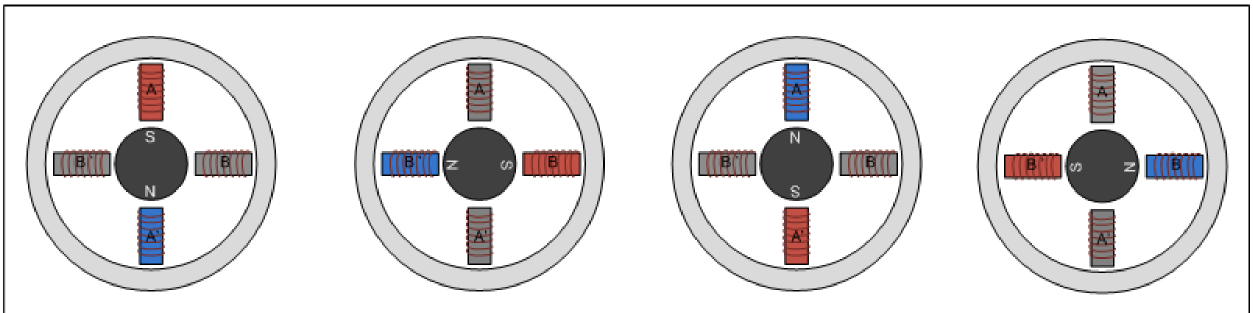
1. Permanent Magnet Stepper :

The rotor and stator poles of a permanent magnet stepper are not teethed. Instead the rotor have alternative north and south poles parallel to the axis of the rotor shaft.



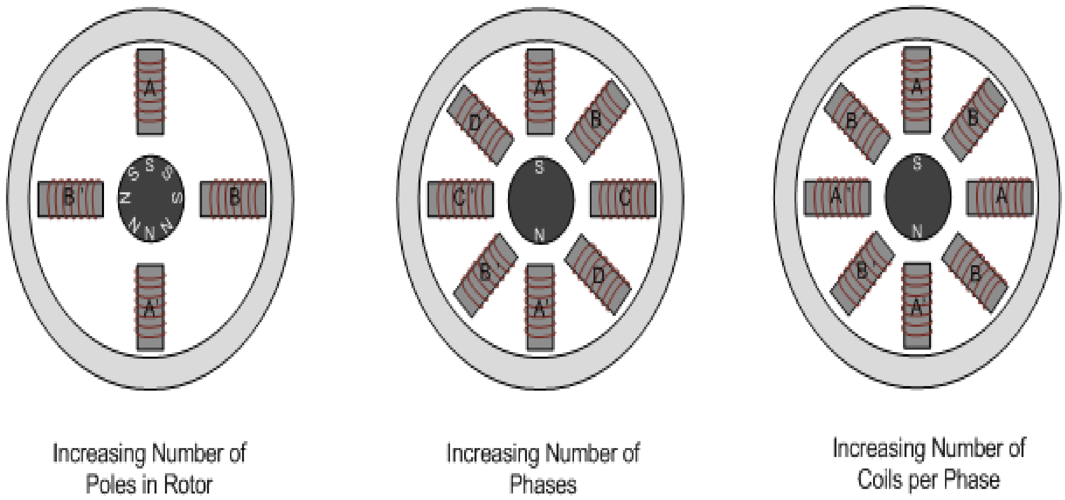
Two-Phase Permanent Magnet type Stepper

When a stator is energized, it develops electromagnetic poles. The magnetic rotor aligns along the magnetic field of the stator. The other stator is then energized in the sequence so that the rotor moves and aligns itself to the new magnetic field. This way energizing the stators in a fixed sequence rotates the stepper motor by fixed angles.



Working of a Permanent Magnet Stepper Motor

The resolution of a permanent magnet stepper can be increased by increasing number of poles in the rotor or increasing the number of phases.

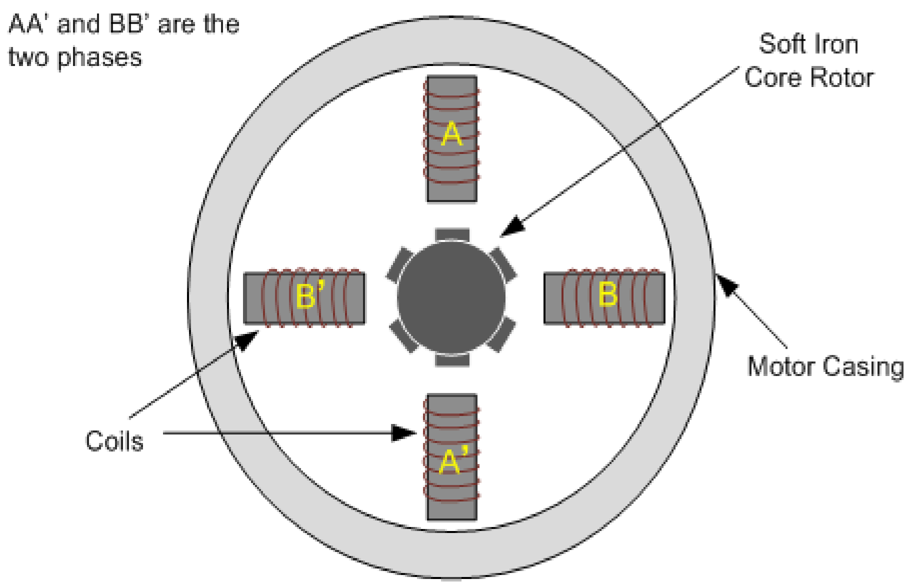


Methods to increase resolution of a Permanent Magnet Stepper

2. **Variable reluctance stepper**

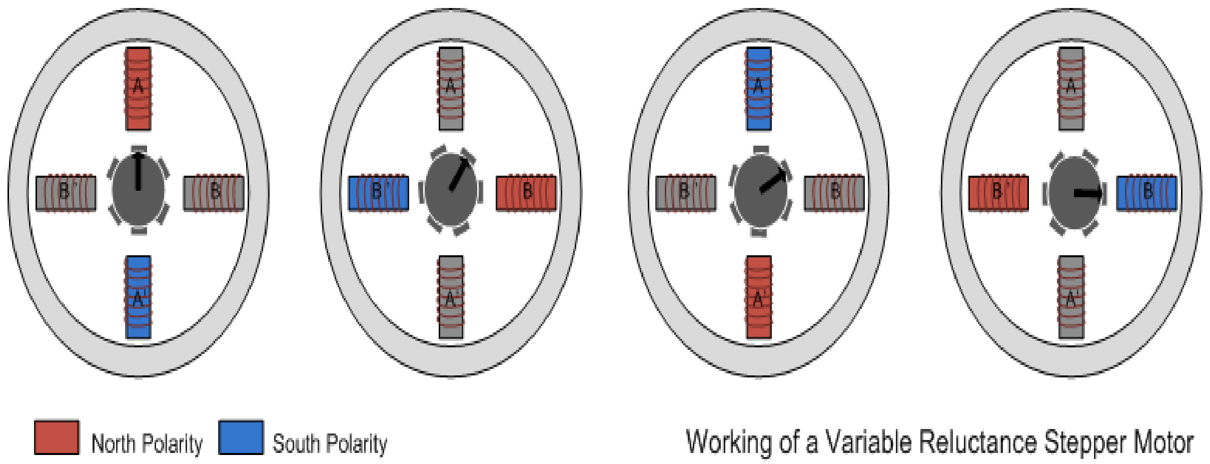
The variable reluctance stepper has a toothed non-magnetic soft iron rotor. When the stator coil is energized the rotor moves to have a minimum gap between the stator and its teeth.

Two-Phase Variable Reluctance Stepper Motor



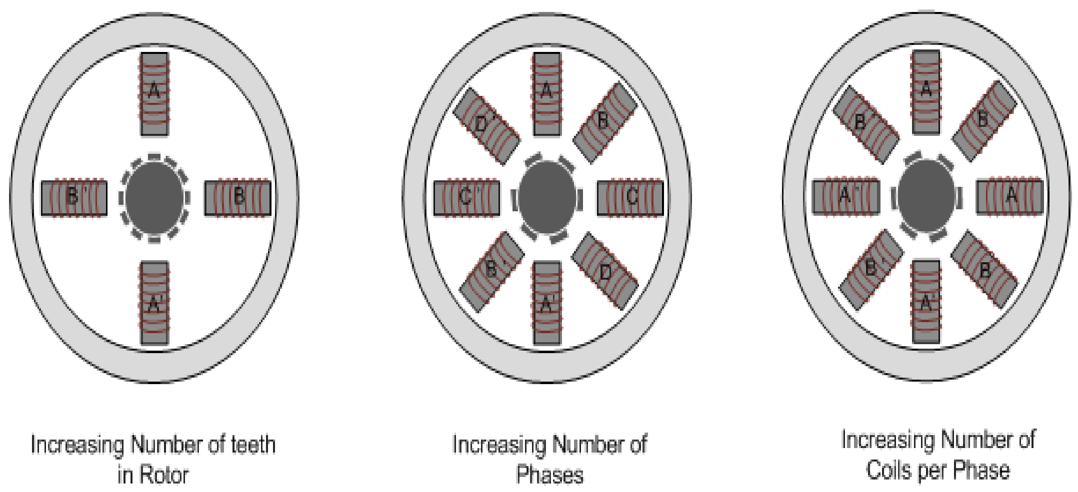
Notice that the teeth of the Rotor are so designed that when they are aligned to one phase, they get misaligned to the other

The teeth of the rotor are designed so that when they are aligned with one stator they get misaligned with the next stator. Now when the next stator is energized, the rotor moves to align its teeth with the next stator. This way energizing stators in a fixed sequence completes the rotation of the step motor.



Working of a Variable Reluctance Stepper Motor

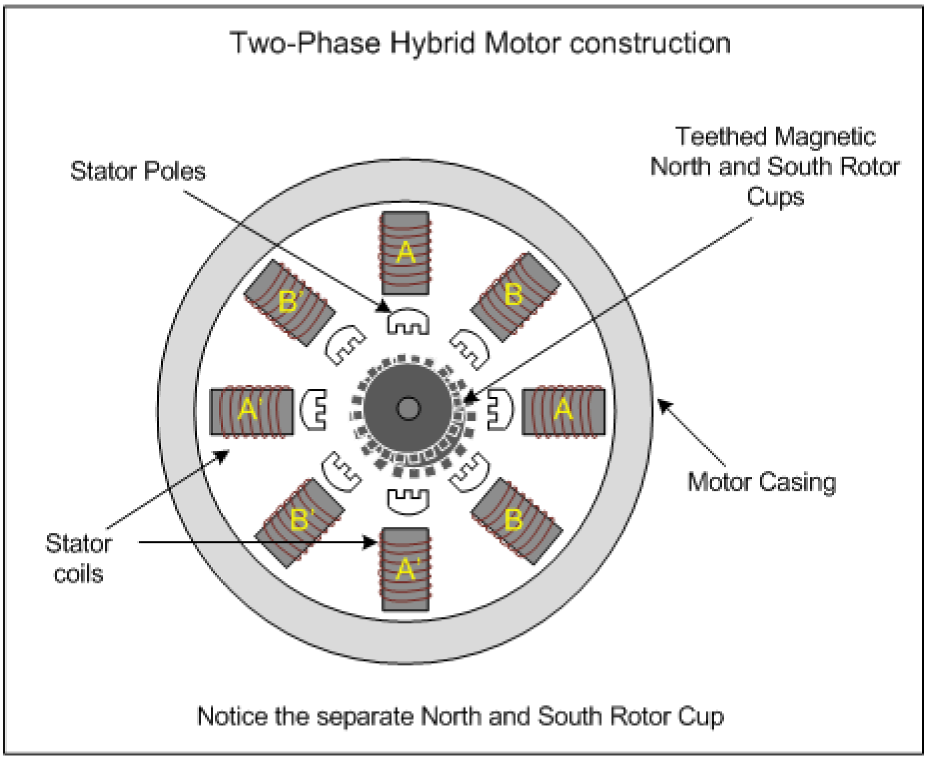
The resolution of a variable reluctance stepper can be increased by increasing the number of teeth in the rotor and by increasing the number of phases.



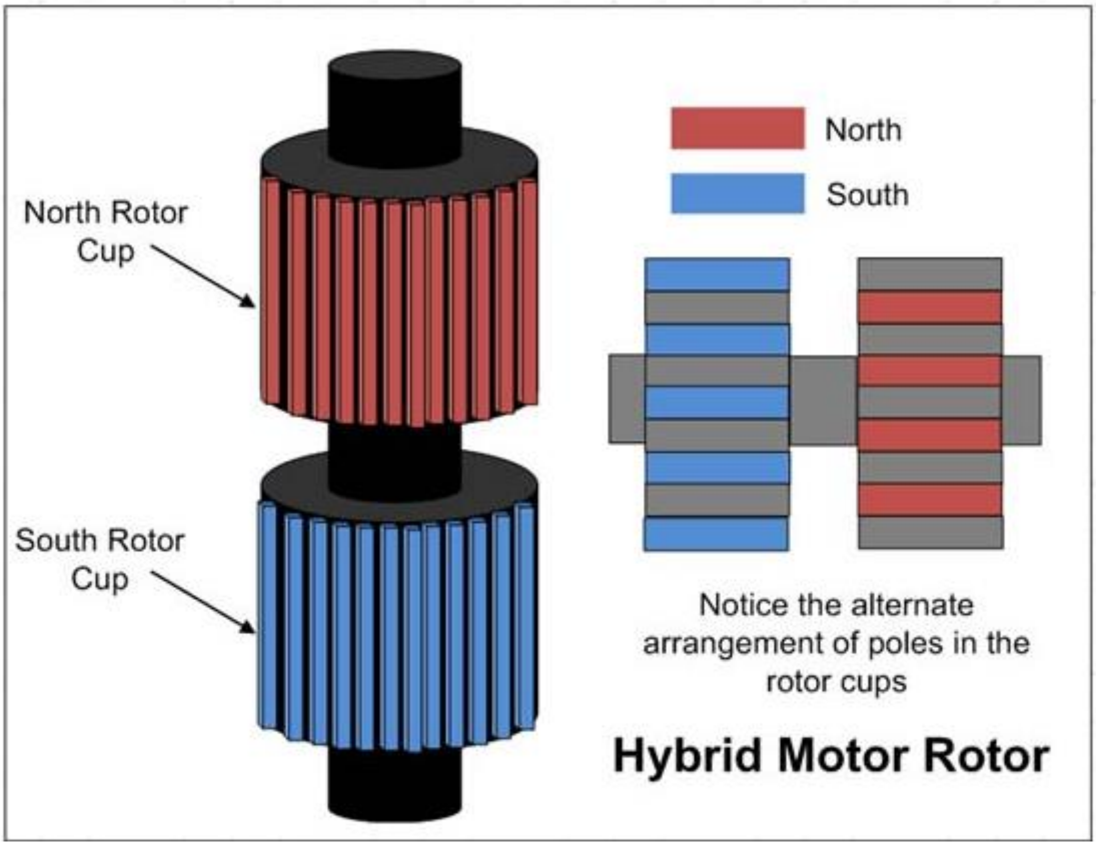
Methods to increase resolution of a Variable Reluctance Stepper

3. Hybrid stepper :

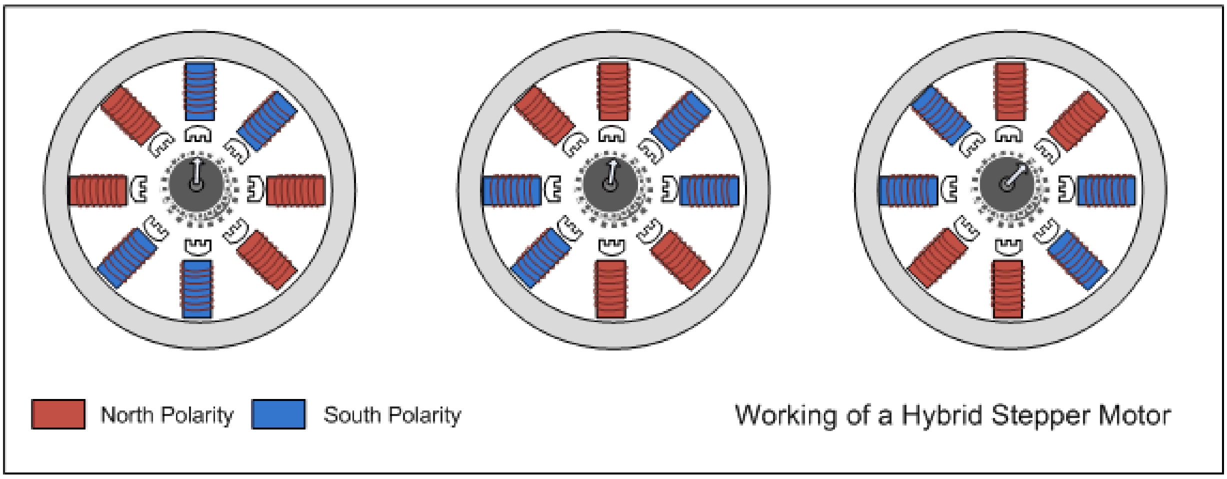
A hybrid stepper is a combination of both permanent magnet and the variable reluctance. It has a magnetic teethed rotor which better guides magnetic flux to preferred location in the air gap.



The magnetic rotor has two cups. One for north poles and second for the south poles. The rotor cups are designed so that that the north and south poles arrange in alternative manner. Check out the insight of a Hybrid Stepper Motor.



The Hybrid motor rotates on same principle of energizing the stator coils in a sequence.



3.9.2 Applications of Stepper Motors

Stepper motors are diverse in their uses, but some of the most common include:

3D printing equipment

Textile machines

Printing presses

Gaming machines

Medical imaging machinery

Small robotics

CNC milling machines

Welding equipment

Questions

S.NO	QUESTION
1	List the applications of DC Shunt generators.
2	What is the function of a commutator in a dc machine?
3	List the applications of DC series motors.
4	What are the types of transformer?
5	Write the EMF equation of a transformer.
6	Write down the EMF equation of DC generator
7	State the significance of back EMF.
8	What are the different types of DC motor?
9	List the applications of stepper motor
10	List the applications of single phase induction motors,

S.NO	QUESTION
1	Describe the constructional features of a dc machine with neat sketch.
2	Explain the working principle of DC Generator and derive its emf equation.
3	Draw and explain the connection diagram of DC series, shunt and compound motor.
4	With neat sketch explain the constructional details of a single phase transformer.
5	Explain with neat sketch the types of transformer.
6	Explain the working principle of a DC motor
7	Explain the significance of back emf.
8	Explain the Principle and operation of single phase induction motor.
9	Explain with neat sketch about a) Split phase induction motor, b) capacitor start induction motor, c) capacitor start- capacitor run induction motor, d) shaded pole induction motor.
10	Explain with neat sketch about a) permanent magnet stepper motor b) variable reluctance stepper motor c) hybrid stepper motor

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UNIT – IV
SEMICONDUCTOR DEVICES

UNIT IV SEMICONDUCTOR DEVICES

VI Characteristics of PN-junction diodes and Zener diodes, BJT and its configurations –input/ output Characteristics, Junction Field Effect Transistor –Drain and Transfer Characteristics, MOSFET –Depletion type and Enhancement type, Uni Junction Transistors –Silicon Controlled Rectifiers.

CLASSIFICATION OF MATERIALS :

Materials can be classified based on its conductivity property as:

Conductor: A conductor is a material which allows free flow of charge when a voltage is applied across its terminals. i.e. it has very high conductivity. Eg: Copper, Aluminum, Silver, Gold.
Semiconductor: A semiconductor is a material that has its conductivity somewhere between the insulator and conductor. The resistivity level is in the range of 10 and 104 Ω- cm. Eg: Silicon and Germanium. Both have 4 valance electrons. Electronic devices like PNdiode, Zener diode Bipolar Junction Transistor are made using these semiconductors.
Insulator: An insulator is a material that offers a very low level (or negligible) of conductivity when voltage is applied. Eg: Paper, Mica, glass, quartz.

Classification of semiconductors

- Intrinsic semiconductor
- Extrinsic semiconductor

Intrinsic semiconductor
They are semi-conducting materials which are pure and no impurity atoms are added to it. Eg: Germanium and Silicon.

Properties:

- Number of electrons is equal to the number of holes. I.e., ne=nh.
- Electrical conductivity is low.
- Electrical conductivity of intrinsic semiconductors depends on their temperatures.
-

Extrinsic semiconductors
Intrinsic semiconductor has very limited applications as they conduct very small amounts of current at room temperature. The current conduction capability of intrinsic semiconductor can be increased significantly by adding a small amounts impurity to the intrinsic semiconductor. By adding impurities it becomes impure or extrinsic semiconductor. This process of adding impurities is called as doping. The amount of impurity added is 1 part in 106 atoms

Properties:

- The number of electrons is not equal to the number of holes.
- The electrical conductivity is high.
- The electrical conductivity depends on the temperature and the amount of impurity added in them. They are further subdivided as

- P type semiconductor
- N type semiconductor

P type semiconductor:

When a intrinsic semiconductor is added with Trivalent impurity it becomes a P- Type semiconductor. Examples of trivalent impurities are Boron, Gallium, indium etc.

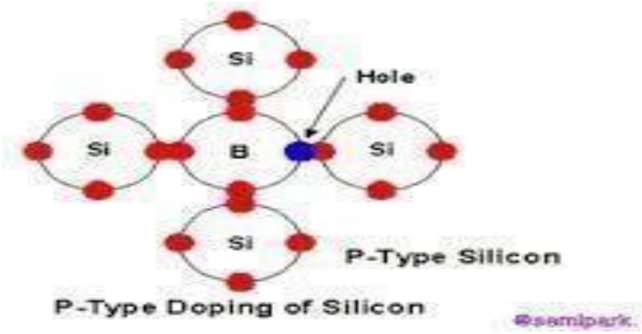


Fig. 4.1 P type Semiconductor

The crystal structure of P type semiconductor is shown in the Fig. 4.1. The three valance electrons of the impurity (boron).

forms three covalent bonds with the neighboring atoms and a vacancy exists in the fourthbond giving rise to the holes. The hole is ready to accept an electron from the neighboring atoms. Each trivalent atom contributes to one hole generation and thus introduces a largeno. of holes in the valance band. At the same time the no. electrons are decreased comparedto those available in intrinsic semiconductor because of increased recombination due to creation of additional holes. Thus in P type semiconductor,

- Holes are majority carriers and electrons are minority carriers.
- The semiconductor is rich in holes.N

type semiconductor:

If the added impurity is a pentavalent atom then the resultant semiconductor is called N-type semiconductor. Examples of pentavalent impurities are Phosphorus, Arsenic, Bismuth, Antimony etc.
A pentavalent impurity has five valance electrons. Fig 4.2 shows the crystal structure of N type semiconductor material where four out of five valance electrons of the impurity atom(antimony) forms covalent bond with the four intrinsic semiconductor atoms. The fifth electron is loosely bound to the impurity atom. This loosely bound electron can be easily excited from the valance band to the conduction band by the application of electric field or increasing the thermal energy. The energy required to detach the fifth electron form the impurity atom is very small of the order of 0.01ev for Ge and 0.05 eV for Si. Thus in a N type semiconductor

- Electrons are majority carriers and holes are minority carriers.
- The semiconductor is rich in electrons.

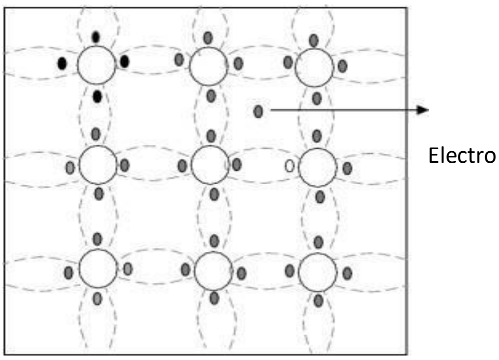


Fig. 4.2 N type Semiconductor

4.1.PN JUNCTION THEORY

When P and N type semiconductors are fused together, we obtain PN junction. When firstjoined together, very large density gradient exists between both sides of the PN junction.Therefore at the junction there is a tendency of free electrons from N side to diffuse overto the P side and the holes to the N side. This process is called diffusion. Hence some of the free electrons from the N side begin to migrate across this newly formed junction to fill up the holes in the P-type material.

As the free electrons move across the junction from N type to P type, they leave behind positively charge (donor ions) on the negative side and hence a positive charge is built onthe N-side of the junction. Similarly, the holes from the P side migrate across the junctionin the opposite direction into the N region where there are large numbers of free electrons.As a result, the charge density of the P-type along the junction is filled with negatively charged acceptor ions, and hence a negative charge is built on the P-side of the junction. The width of these layers depends on how heavily each side is doped with acceptor density and donor density respectively.

The electrostatic field across the junction caused by the positively charged N-Type regiontends to drive the holes away from the junction and negatively charged P type regions tend to drive the electrons away from the junction. Thus near the junction, a region depleted of mobile charge carriers is formed. This is called depletion layer, space region, and transition region. The depletion region is of the order of 0.5µm thick. There are no mobile carriers in this narrow depletion region. Hence no current flows across the junction and the system is in equilibrium.

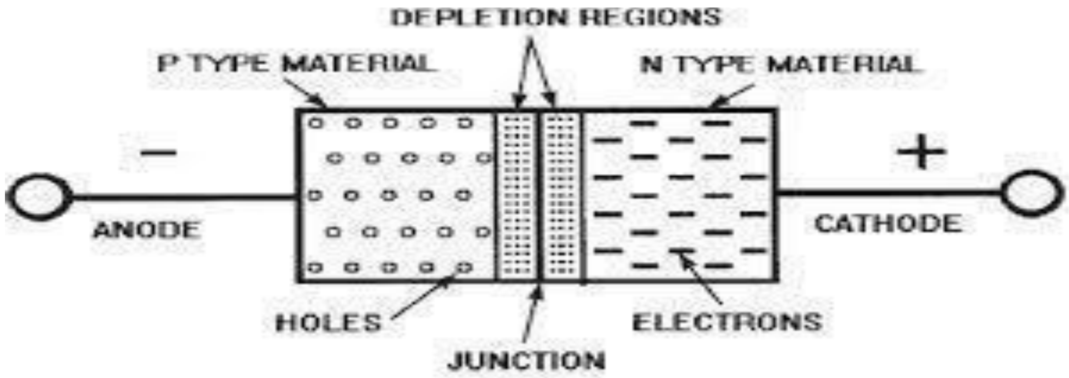


Fig. 4.3 PN junction

FORWARD BIASED OPERATION

When external voltage is applied then the potential difference is altered between the P and N regions. Positive terminal of the source is connected to the P side and the negative terminal is connected to N side then the PN junction diode is said to be connected in forward bias condition. This lowers the potential across the junction. The majority charge carriers in N and P regions are attracted towards the PN junction and the width of the depletion layer decreases with diffusion of the majority

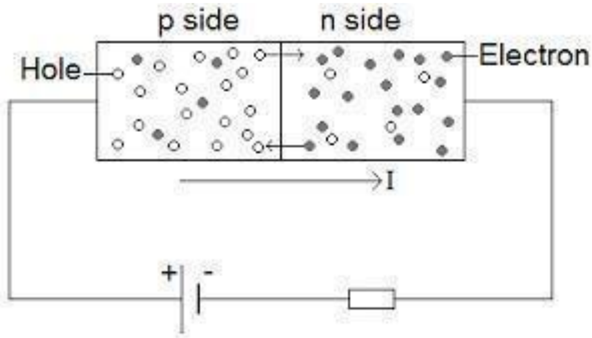


Fig. 4.4 Forward bias PN junction diode

charge carriers. The external biasing causes a departure from the state of equilibrium and also in the depletion layer. With the increase in forward bias greater than the built in potential, at a particular value the depletion region becomes very much thinner so that a large number of majority charge carriers can cross the PN junction and conducts an electric current. The current flowing up to built in potential is called as ZERO current orKNEE current.

Reverse Bias Operation

Positive terminal of the source is connected to the N side and the negative terminal is connected to P side. Here majority charge carriers are attracted away from the depletion layer by their respective battery terminals connected to PN junction. Positive terminal attracts the electrons away from the junction in N side and negative terminal attracts the holes away from the junction in P side. As a result of it, the width of the potential barrier increases that impedes the flow of majority carriers in N side and P side. The width of the free space charge layer increases, thereby electric field at the PN junction increases and the PN junction diode acts as a resistor. The current that flows in a PN junction diode is the small leakage current, due to minority carriers generated at the depletion layer or minority carriers which drift across the PN junction. The growth in the width of the depletion layer presents a high impedance path which acts as an insulator.

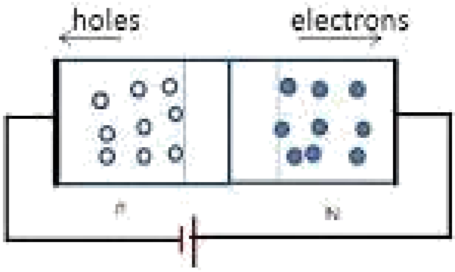


Fig. 4.5 Reverse bias PN junction diode

.**VI characteristics of PN Diode**

The VI characteristics of PN junction diode in forward bias are non linear, that is, not a straight line. This nonlinear characteristic illustrates that during the operation of the PN junction, the resistance is not constant. The slope of the PN junction diode in forward bias shows the resistance is very low. When forward bias is applied to the diode if this external voltage becomes greater than the value of the potential barrier, approx. 0.7 volts for silicon and 0.3 volts for germanium, then it causes a low impedance path and permits to conduct a large amount of current. Thus the current starts to flow above the knee point with a small amount of external potential.

In reverse bias condition, the P-type of the PN junction is connected to the negative terminal and N-type is connected to the positive terminal of the external voltage. This results in increased potential barrier at the junction. Hence, the junction resistance becomes very high and as a result practically no current flows through the circuit. However, a very small current of the order of µA, flows through the circuit in practice. This is knows as reverse saturation current and it is due to the minority carriers in the junction.

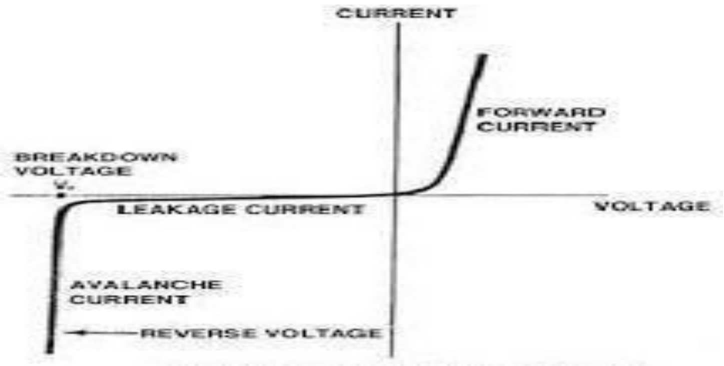


Fig. 4.6 VI characteristics of PN Diode

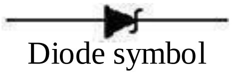
Applications of PN junction Diode:

The P-N junction diode has many applications.

- P-N junction diode in reverse biased configuration is sensitive to light from a range between 400nm to 1000nm, which includes VISIBLE light. Therefore, it can be usedas a photodiode.
- It can also be used as a solar cell.
- P-N junction forward bias condition is used in all LED lighting applications.
- The voltage across the P-N junction biased is used to create Temperature Sensors, and Reference voltages.
- It is used in many circuits‟ rectifiers, varactor for voltage controlled oscillators.

4.2.ZENER DIODE:

A zener diode is a special type of device designed to operate in the zener breakdown region which is heavily doped than the normal PN junction diode. Hence, it has very thin depletion region. Therefore, Zener diode allow more electric current than the normal PN junction diodes under forward bias like a normal diode but also allows electric current in the reverse direction if the applied reverse voltage is greater than the zener voltage. Thus they are always connected in reverse direction because it is specifically designed to work in reverse direction. The breakdown voltage of a Zener diode is carefully set by controlling the doping level during manufacture. The name Zener diode was named afterthe American physicist Clarance Melvin Zener who discovered the zener effect.



Diode symbol

Breakdown in Zener diode

There are two types of reverse breakdown regions in a Zener diode: Avalanche breakdown and Zener breakdown.

Avalanche breakdown

The avalanche breakdown occurs at high reverse voltage. When high reverse voltage is applied to the diode, the free electrons gains large amount of energy and accelerated to greater velocities. The free electrons moving at high speed will collides with the atoms and knock off more electrons. These electrons are again accelerated and collide with other atoms. Because of this continuous collision with the atoms, a large number of free electrons are generated. This cumulative process is referred to as avalanche multiplication which results in the flow of large reverse current and this breakdown of the diode is called avalanche breakdown. Avalanche breakdown occurs in zener diodes with zener voltage greater than 6V.

Zener breakdown:

The zener breakdown occurs in heavily doped diodes because of their narrow depletion region. When reverse biased voltage applied to the diode is increased, the narrow depletion region generates strong electric field. When it reaches close to zener voltage, the electric field in the depletion region is strong enough to pull electrons from their valence band. The valence electrons which gains sufficient energy from the strong electric field of depletion region will breaks bonding with the parent atom. The valance electrons which break bonding with parent atom will become free electrons. This free electrons results in large electric current, a small increase in voltage will rapidly increases the electric current. This breakdown is referred to as Zener breakdown.

Note:

- Zener breakdown occurs at low reverse voltage whereas avalanche breakdown occurs at high reverse voltage.
- Zener breakdown occurs in Zener diodes because they have very thin depletion region.
- Breakdown region is the normal operating region for a zener diode.
- Zener breakdown occurs in Zener diodes with Zener voltage less than 6V.

ZENER DIODE CHARACTERISTICS:

When a Zener diode is biased in the forward direction it behaves just like a normal PN junction diode.

Under reverse-biased condition, the reverse voltage is applied. As the reverse bias voltage is increased, breakdown of the junction occurs. The breakdown voltage depends upon the amount of doping. If the diode is heavily doped, depletion layer will be thin and consequently, breakdown occurs at lower reverse voltage and further, the breakdown voltage is sharp. A lightly doped diode has a higher breakdown voltage. Thus breakdown voltage can be selected with the amount of doping .This breakdown voltage point is called the "Zener voltage or breakdown voltage " and a large amount of current flows through the Zener diodes. This Zener breakdown voltage on the I-V curve is almost a vertical straight line.

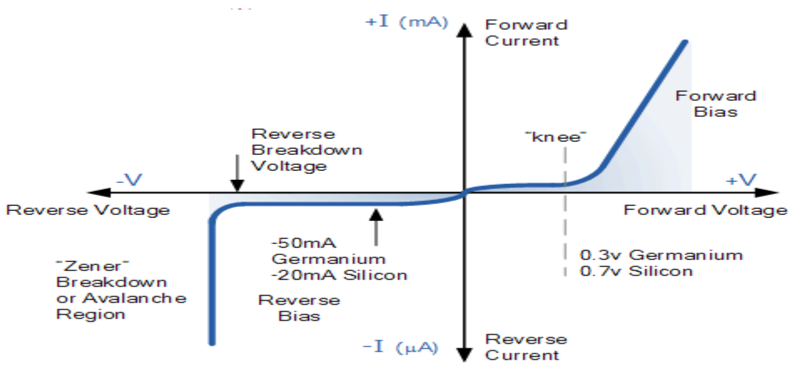


Fig. 4.7 VI characteristics of Zener diode ZENER DIODE AS A VOLTAGE

REGULATOR

From the Zener Characteristics shown, under reverse bias condition, the voltage across the diode remains constant although the current through the diode increases as shown. Thus the voltage across the zener diode serves as a reference voltage. Hence the diode canbe used as a voltage regulator.

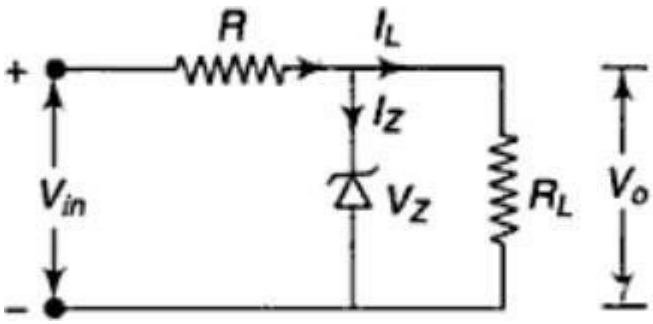


Fig. 4.8 Voltage regulator

It is required to provide constant voltage across load resistance RL , whereas the input voltage may be varying over a range. As shown, Zener diode is reverse biased and as longas the input voltage does not fall below Zener breakdown voltage, the voltage across the diode will be constant and hence the load voltage will also be constant.

4.3 BIPOLAR JUNCTION TRANSISTOR INTRODUCTION

The transistor was developed by Dr. Shockley along with Bell Laboratories team in 1951. It is a three terminal device whose output current, voltage and power are controlled by its input current. In communication systems it is the primary component in the amplifier. The important property of the transistor is that it can raise the strength of a weak signal. Thisproperty is called amplification. Transistors are used in digital computers, satellites, mobile phones and other communication systems, control systems etc., A transistor consists of two P-N junction. The junction are formed by sand witching either p-type or n-type semiconductor layers between a pair of opposite types which is shown below

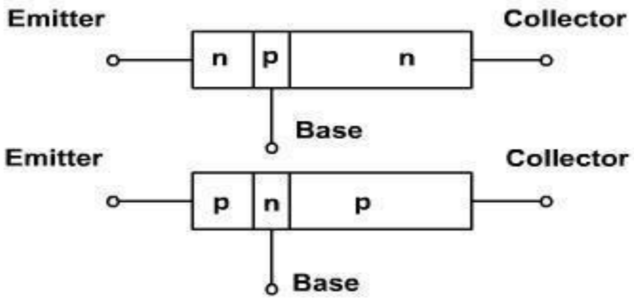


Fig. 4.9 Transistor

TRANSISTOR CONSTRUCTION:

A transistor has three regions known as emitter, base and collector.
Emitter:

- It is a region situated in one side of a transistor, which supplies charge carriers (ie., electrons and holes) to the other two regions
- Emitter is heavily doped region

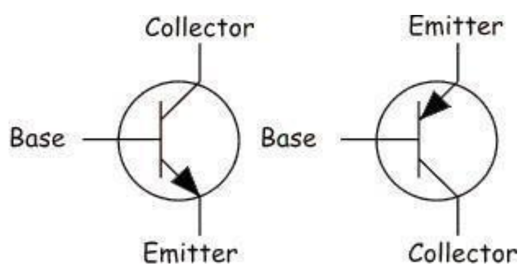
Base:

- It is the middle region that forms two P-N junction in thetransistor
- The base of the transistor is thin as compared to the emitter and is a lightly doped region

Collector:

- It is a region situated in the other side of a transistor (ie., side opposite to the emitter) which collects the charge carriers.
- The collector of the transistor is always larger than the emitter and base of atransistor
- The doping level of the collector is intermediate between the heavy doping of emitter and the light doping of the base.

TRANSISTOR SYMBOLS



NPN Transistor PNP transistor

Fig. 4.10 Transistor

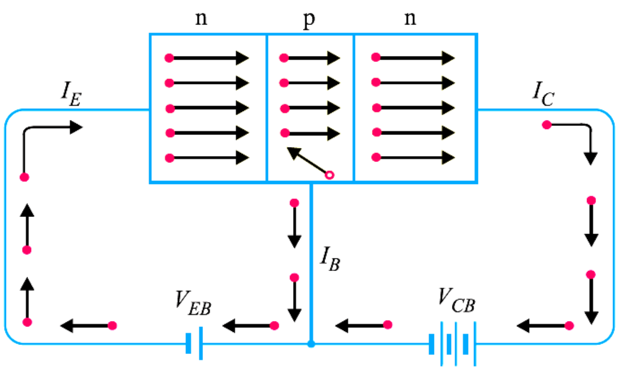
- The transistor symbol carries an arrow head in the emitter pointing from the P-region towards the N- region

- The arrow head indicates the direction of a conventional current flow in atransistor.
- The direction of arrow heads at the emitter in NPN and PNP transistor is oppositeto each other.
- The PNP transistor is a complement of the NPN transistor.
- In NPN transistor the majority carriers are free electrons, while in PNP transistorthese are the holes.

UNBIASED TRANSISTORS

A transistor with three terminals (Emitter, Base, Collector) left open is called an unbiased transistor or an open – circuited transistor. The diffusion of free electrons acrossthe junction produces two depletion layers. The barrier potential of three layers isapproximately 0.7v for silicon transistor and 0.3v for germanium transistor. Since the regions have different doping levels therefore the layers do not have the same width. The emitter base depletion layer penetrates slightly into the emitter as it is a heavily doped region where as it penetrates deeply into the base as it is a lightly doped region. Similarly the collector- base depletion layer penetrates more into the base region and less into the collector region. The emitter- base depletion layer width is smaller than that of collector base depletion layer. The unbiased transistor is never used in actual practice. Because of this we went for transistor biasing.

OPERATION OF NPN TRANSISTOR



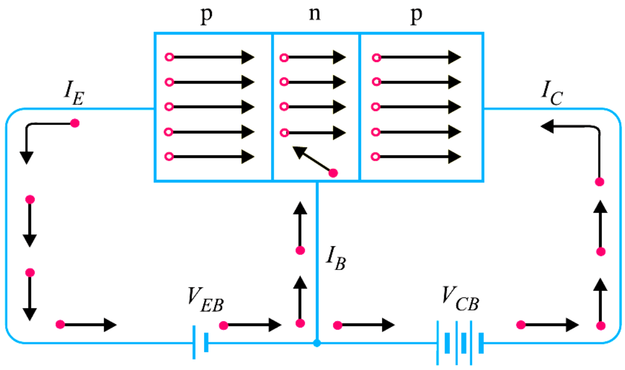
Basic connection of *npn* transistor

Fig. 4.11 NPN Transistor

- The NPN transistor is biased in forward active mode ie., emitter – base of transistor is forward biased and collector base junction is reverse biased
- The emitter – base junction is forward biased only if V is greater than barrier potential which is 0.7v for silicon and 0.3v for germanium transistor
- The forward bias on the emitter- base junction causes the free electrons in the N – type emitter to flow towards the base region. This constitutes the emitter current (IE). Direction of conventional current is opposite to the flow of electrons.
- Electrons after reaching the base region tend to combine with the holes.
- If these free electrons combine with holes in the base, they constitute base current (IB).
- Most of the free electrons do not combine with the holes in the base.
- This is because of the fact that the base and the width is made extremely small and electrons do not get sufficient holes for recombination.

- Thus most of the electrons will diffuse to the collector region and constitutes collector current (IC). This collector current is also called injected current, becauseof this current is produced due to electrons injected from the emitter region
- There is another component of collector current due to the thermal generated carriers.
- This is called as reverse saturation current and is quite small.

OPERATION OF PNP TRANSISTOR



Basic connection of *pnp* transistor

Fig. 4.12 PNP Transistor

- Operation of a PNP transistor is similar to npn transistor
- The current within the PNP transistor is due to the movement of holes where as, inan NPN transistor it is due to the movement of free electrons
- In PNP transistor, its emitter – base junction is forward biased and collector base junction is reverse biased.
- The forward bias on the emitter – base junction causes the holes in the emitter region to flow towards the base region
- This constitutes the emitter current (IE).
- The holes after reaching the base region combine with the electrons in the base and constitute base current (IB).
- Most of the holes do not combine with the electrons in the base region
- This is due to the fact that base width is made extremely small, and holes does notget sufficient electrons for recombination.
- Thus most of the holes diffuse to the collector region and constitutes collector current (IC).
- This current is called injected current, because it is produced due to the holes injected from the emitter region
- There is small component of collector current due to the thermally generated carriers
- This is also called as reverse saturation current.

TRANSISTOR CONFIGURATIONS

- A transistor is a three terminal device, but we require four terminals (two forinput and two for output) for connecting it in a circuit.
- Hence one of the terminal is made common to the input and output circuits.
- The common terminal is grounded.

- There are three types of configuration for the operation of a transistor.

Common base configuration

- This is also called grounded base configuration
- In this configuration emitter is the input terminal, collector is the output terminal and base is the common terminal

Common emitter configuration(CE)

- This is also called grounded emitter configuration
- In this configuration base is the input terminal, collector is the output terminal and emitter is the common terminal

Common collector configuration(CC)

- This is also called grounded collector configuration
- In this configuration, base is the input terminal, emitter is the output terminal and collector is the common terminal.

COMMON BASE CONFIGURATION (CB)

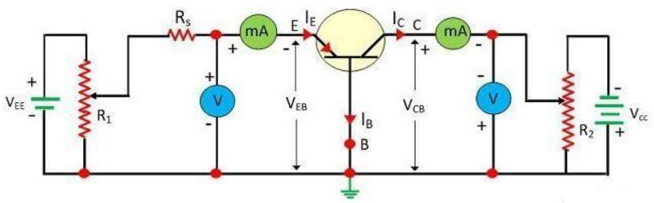


Fig. 4.13 CB Configuration

- The input is connected between emitter and base and output is connected across collector and base
- The emitter – base junction is forward biased and collector – base junction is reverse biased.
- The emitter current, flows in the input circuit and the collector currentflows in the output circuit.
- The ratio of the collector current to the emitter current is called current amplification factor.

CHARACTERISTICS OF CB CONFIGURATION

- The performance of transistors determined from their characteristic curvesthat relate different d.c currents and voltages of a transistor
- Such curves are known as static characteristics curves There are two important characteristics of a transistor

➢ Input characteristics
➢ Output characteristics

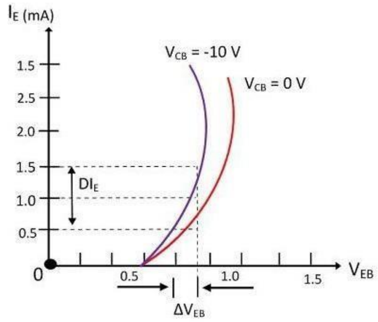


Fig. 4.14 Characteristics of CB Configuration

INPUT CHARACTERISTICS

The curve drawn between emitter current and emitter – base voltage for a given value ofcollector – base voltage is known as input Characteristic curves.

The following points are taken into consideration from the characteristic curve.

- For a specific value of VCB, the curve is a diode characteristic in the forward region. The PN emitter junction is forward biased.
- When the value of the voltage base current increases the value of emitter current increases slightly. The junction behaves like a better diode. The emitter and collector current is independent of the collector base voltage VCB.
- The emitter current IE increases with the small increase in emitter-base voltage VEB. It shows that input resistance is small.

Input Resistance

The ratio of change in emitter-base voltage to the resulting change in emitter current at constant collector base voltage VCB is known as input resistance. The input resistance is expressed by the formula. $r_i = \frac{\Delta V_{EB}}{\Delta I_E}$

Base width modulation (or) Early effect

- In a transistor, since the emitter – base junction is forward biased there is noeffect on the width of the depletion region.
- However, since collector – base junction is reverse biased, as the reverse bias voltage across the collector – base junction increases the width of thedepletion region also increases.
- Since the base is lightly doped the depletion region penetrates deeper into the base region.
- This reduces the effective width of the base region.
- This variation or modulation of the effective base width by the collector – base voltage is known as base width modulation or early effect.

The decrease in base width by the collector voltage has the following three effects base region. Hence current gain increases with increase in collector – base voltage.

- The concentration gradient of minority carriers within the base increases. This increases the emitter current.
- For extremely collector voltage, the effective base width may be reduced to zero, resulting in voltage breakdown of a transistor. This phenomenon is known as punch through.

Output characteristics

The curve drawn between collector current and collector – base voltage, for a given valueof emitter current is known as output characteristics.

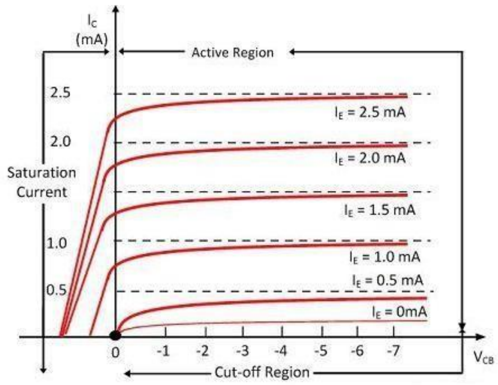


Fig. 4.15 Characteristics of CB Configuration

- The active region of the collector-base junction is reverse biased, the collector current IC is almost equal to the emitter current IE. The transistor is always operated in this region.
- The curve of the active regions is almost flat. The large charges in VCB produce only a tiny change in IC. The circuit has very high output resistancero.
- When VCB is positive, the collector-base junction is forward bias and the collector current decrease suddenly. This is the saturation state in which the collector current does not depend on the emitter current.
- When the emitter current is zero, the collector current is not zero. The current which flows through the circuit is the reverse leakage current, i.e., ICBO. The current is temperature depends and its value range from 0.1 to 1.0 µA for silicon transistor and 2 to 5 µA for germanium transistor.

Output Resistance

$$r_0 = \frac{\Delta V_{CB}}{\Delta I_C}$$

The ratio of change in collector-base voltage to the change in collector current atconstant emitter current IE is known as output resistance.

COMMON – EMITTER CONFIGURATION

- The input is connected between base and emitter, while output is connected between collector and emitter
- Emitter is common to both input and output circuits.
- The bias voltage applied are Vce and Vbe.
- The emitter-base junction is forward biased and collector-emitter junction is reverse biased.
- The base current Ib flows in the input circuit and collector current Ic flows sin the output circuit.

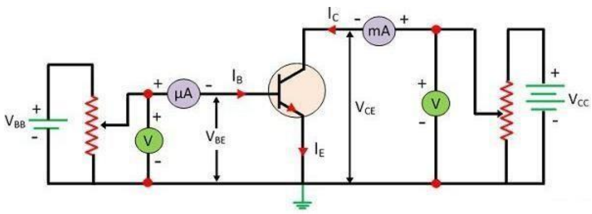


Fig. 4.16 CE Configuration

INPUT CHARACTERISTICS

- The curve plotted between base current IB and the base-emitter voltage VEB is called Input characteristics curve.
- For drawing the input characteristic the reading of base currents is taken throughthe ammeter on emitter voltage VBE at constant collector-emitter current.
- The curve for different value of collector-base current is shown in the figure below.

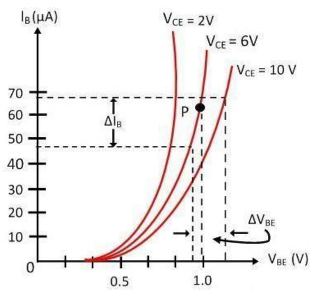


Fig. 4.17 Characteristics of CE Configuration

- The curve for common Emitter configuration is similar to a forward diode characteristic.
- The base current IB increases with the increases in the emitter-base voltage VBE. Thus the input resistance of the CE configuration is comparatively higher that of CB configuration.
- The effect of CE does not cause large deviation on the curves, and hence the effect of a change in VCE on the input characteristic is ignored.

Input Resistance:

The ratio of change in base-emitter voltage VBE to the change in base current ΔIB at constant collector-emitter voltage VCE is known as input resistance, i.e.,

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} \ at\ constant\ V_{CE}$$

OUTPUT CHARACTERISTIC

In CE configuration the curve draws between collector current IC and collector-emitter voltage VCE at a constant base current IB is called output characteristic. The characteristic curve for the typical NPN transistor in CE configuration is shown in the figure below.

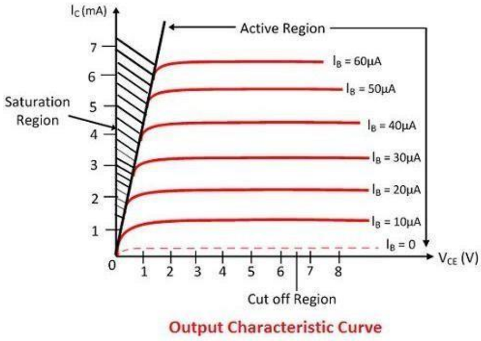


Fig. 4.18 Characteristics of CE Configuration

- In the active region, the collector current increases slightly as collector-emitter VCE current increases. The slope of the curve is quite more than the output characteristic of CB configuration. The output resistance of the common base connection is more than that of CE connection.
- The value of the collector current IC increases with the increase in VCE at constant voltage IB, the value β of also increases.
- When the VCE falls, the IC also decreases rapidly. The collector-base junction of the transistor always in forward bias and work saturate. In the saturation region, the collector current becomes independent and free from the input current IB
- In the active region IC = βIB, a small current IC is not zero, and it is equal to reverse leakage current ICEO.

Output Resistance:

The ratio of the variation in collector-emitter voltage to the collector-emitter current is known at collector currents at a constant base current IB is called output resistance ro.

$$r_o = \frac{\Delta V_{CE}}{\Delta I_C} \ at\ constant\ I_B$$

The value of output resistance of CE configuration is more than that of CB.

COMMON - COLLECTOR CONFIGURATION

CC configuration. In CC configuration, the input circuit is connected between emitter and base and the output is taken from the collector and emitter. The collector is common to both the input and output circuit and hence the name common collector connection or common collector configuration.

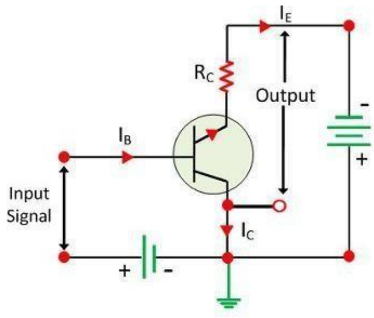


Fig. 4.19 CB Configuration

INPUT CHARACTERISTICS

The input characteristic of the common collector configuration is drawn between collector base voltage VCE and base current IB at constant emitter current voltage VCE. The value of the output voltage VCE changes with respect to the input voltage VBC and IB With the help of these values, input characteristic curve is drawn. The input characteristic curve is shown below.

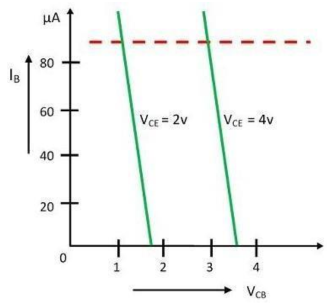


Fig. 4.20 Input Characteristics of CC Configuration

OUTPUT CHARACTERISTICS

The output characteristic of the common emitter circuit is drawn between the emitter- collector voltage VEC and output current IE at constant input current IB. If the input current IB is zero, then the collector current also becomes zero, and no current flows through the transistor.

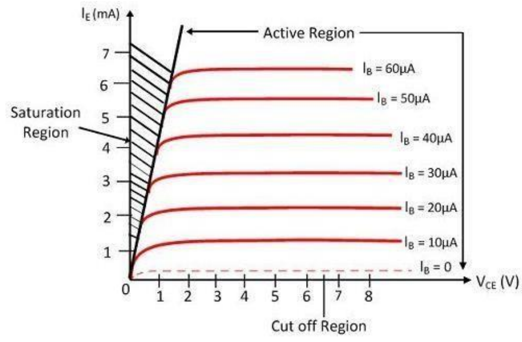


Fig. 4.21 Output Characteristics of CC Configuration

The transistor operates in active region when the base current increases and reaches to saturation region. The graph is plotted by keeping the base current IB constant and varying the emitter-collector voltage VCE, the values of output current IE are noticed withrespect to VCE. By using the VCE and IE at constant IB the output characteristic curve isdrawn.

4.4 JUNCTION FIELD EFFECT TRANSISTOR (JFET)

JFET is a unipolar-transistor, which acts as a voltage controlled current device andis a device in which current at two electrodes is controlled by the action of an electric fieldat a PN junction.

A JFET, or junction field-effect transistor, or JUGFET, is a FET in which the gate is created by reverse-biased junction (as opposed to the MOSFET which creates a junction via a field generated by conductive gate, separated from the gate region by a thin insulator).

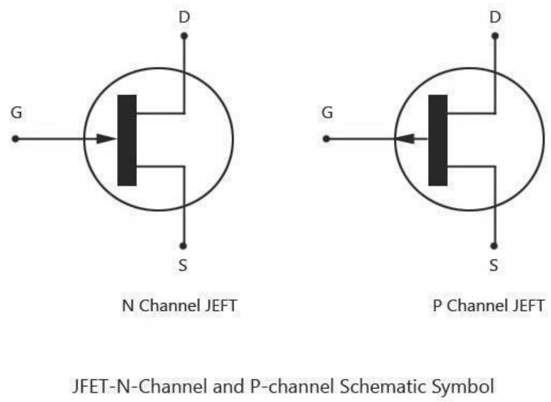


Fig. 4.22 JFET

CONSTRUCTION

N-Channel JFET

The figure shows construction and symbol of N-channel JFET. A small bar ofextrinsic semiconductor material, N type is taken and its two ends, two ohmic contacts are made which is the drain and source terminals of FET.

Heavily doped electrodes of P type material form PN junctions on each side of the bar. The thin region between the two P gates is called the channel. Since this channel is in the N type bar, the FET is known as N-channel JFET.

The electrons enter the channel through the terminal called source and leave through the terminal called drain. The terminals taken out from heavily doped electronics of P type material are called gates. These electrodes are connected together and only one terminal is taken out, which is called gate, as shown below.

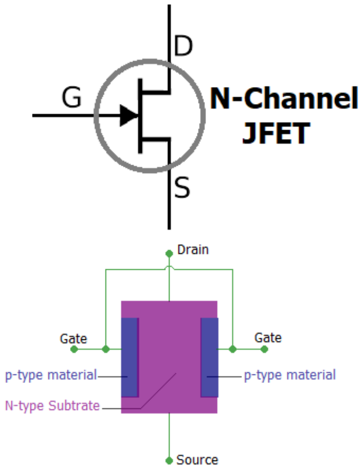


Fig. 4.23 N-Channel JFET

P-Channel JFET

The device could be made of P type bar with two N type gates as shown in the figure below. This will be P- channel JFET. The principle of working of N-channel JFET

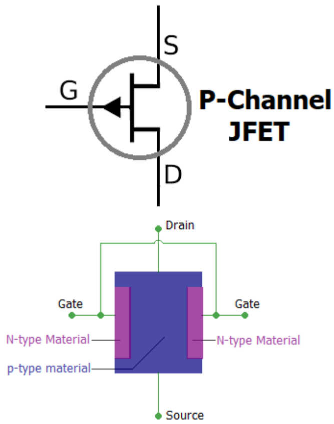


Fig. 4.24 P-Channel JFET

and P-channel JFET are similar. The only difference being that in N-channel JFET the current is carried by electrons while in P-channel JFET, it is carried by holes.

WORKING OF JFET

One best example to understand the working of a JFET is to imagine the garden hose pipe. Suppose a garden hose is providing a water flow through it. If we squeeze the hose the waterflow will be less and at a certain point if we squeeze it completely there will be zero water flow. JFET works exactly in that way. If we interchange the hose with a JFET and the waterflow with a current and then construct the current-carrying channel, we could control the current flow.

When there is no voltage across gate and source, the channel becomes a smooth path which is wide open for electrons to flow. But the reverse thing happens when a voltage is applied between gate and source in reverse polarity, which makes the P-N junction, reversed biased and makes the channel narrower by increasing the depletion layer and could put the JFET in cut-off or pinch off region.

In the below image we can see the saturation mode and pinch off mode and we will be

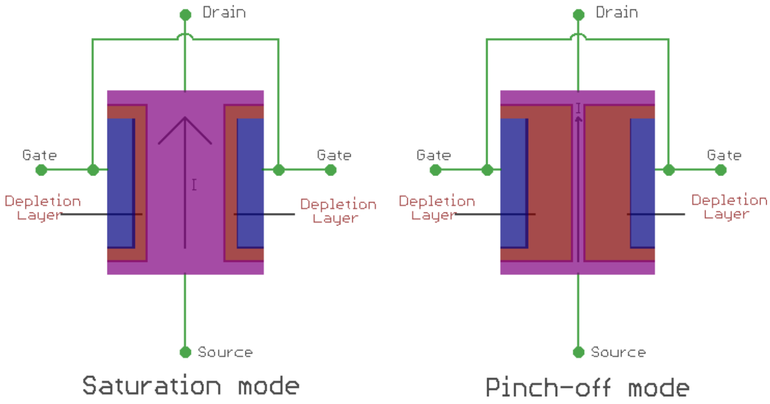


Fig. 4.25 Operation of JFET

able to understand the depletion layer became wider and the current flow becomes less.If we want to switch off a JFET we need to provide a negative gate to source voltage denotedas VGS for an N-type JFET. For a P-type JFET, we need to provide positive VGS.

JFET only works in the depletion mode.

CHARACTERISTICS OF JFETS

There are two types of static characteristics viz

1. Output or drain characteristics and
2. Transfer characteristic.

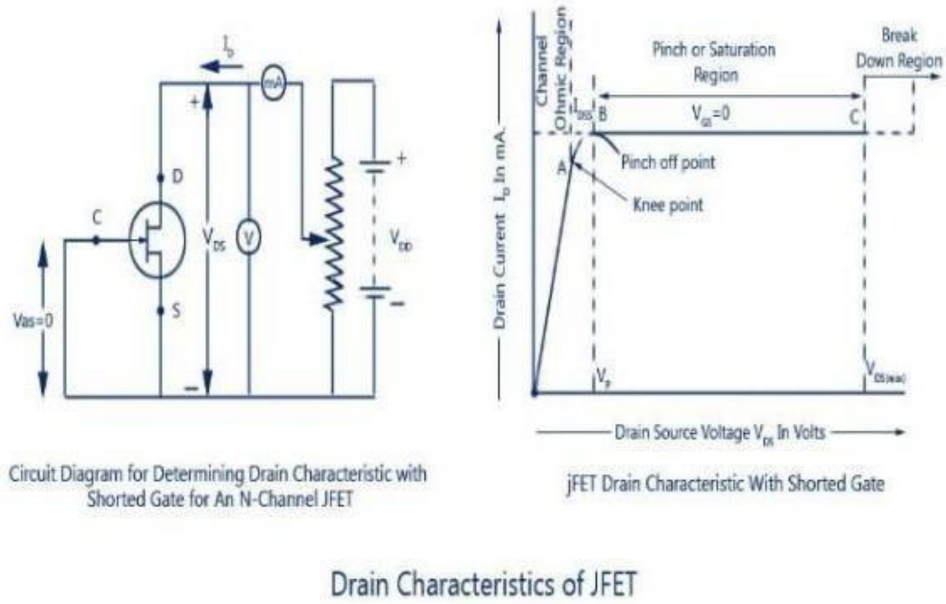


Fig. 4.26 Drain Characteristics of JFET

OUTPUT OR DRAIN CHARACTERISTICS.

- The curve drawn between drain current Ip and drain-source voltage VDS with gate-to source voltage VGS as the parameter is called the drain or output characteristic. This characteristic is analogous to collector characteristic of a BJT:

TRANSFER CHARACTERISTICS OF JFET

- The transfer characteristic for a JFET can be determined experimentally, keeping drain-source voltage, VDS constant and determining drain current, ID for various values of gate-source voltage, VGS.
- The circuit diagram is shown in fig. The curve is plotted between gate-source voltage, VGS and drain current, ID , as illustrated in fig. It is similar to the transconductance characteristics of a vaccum tube or a transistor.
- It is observed that
  1. Drain current decreases with the increase in negative gate-source bias
  2. Drain current, ID = IDSS when VGS = 0
  3. Drain current, ID = 0 when VGS = VD

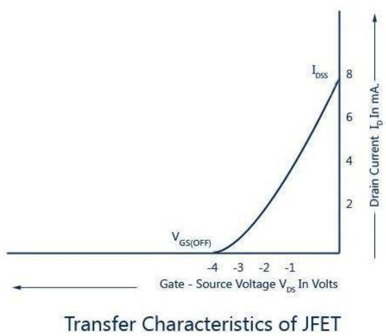


Fig. 4.27 Transfer Characteristics of JFET

The pinch off voltage is the value of VDS at which the drain current reaches its constantsaturation value. Any further increase in VDS does not have any effect on the value of ID.

4.5 METAL OXIDE SEMICONDUCTOR FIELD EFFECT TRANSISTOR (MOSFET)

MOSFET stands for metal oxide semiconductor field effect transistor which is widely used for switching and amplifying electronic signals in the electronic devices. It is capable of voltage gain and signal power gain. The MOSFET is a core of integrated circuit and it can be designed and fabricated in a single chip because of these very small sizes.

The MOSFET is a four terminal device with source(S), gate (G), drain (D) and body **(B)** or substrate terminals. The body of the MOSFET is frequently connected to the source terminal so making it a three terminal device like field effect transistor. The MOSFET is very far the most common transistor and can be used in both analog and digital circuits.

The drain and source terminals are connected to the heavily doped regions. The gate terminal is connected top on the oxide layer.The metal of the gate terminal and the semiconductor acts the parallel and the oxide layer acts as insulator of the state MOS capacitor. Between the drain and source terminal inversion layer is formed and due to theflow of carriers in it, the current flows in MOSFET the inversion layer is properties are controlled by gate voltage. Thus it is a voltage controlled device.

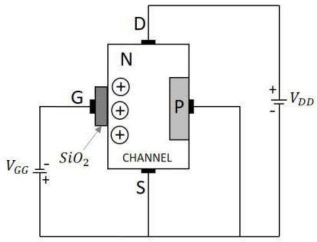


Fig. 4.28 MOSFET

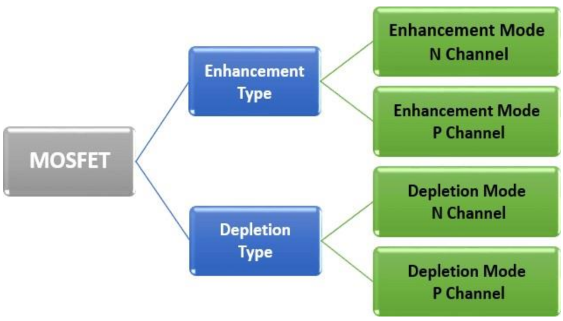


Fig. 4.29 Types of MOSFET

TYPES OF MOSFET

- Depletion type MOSFET
  - Depletion mode – Negative Gate - Source Voltage (VGS) is applied
  - Enhancement mode – Positive Gate - Source Voltage (VGS) isapplied.
- Enhancement type MOSFET

Depletion Type MOSFET:

When there is zero voltage on the gate terminal, the channel shows its maximum conductance. As the voltage on the gate is negative or positive, then decreases the channelconductivity.

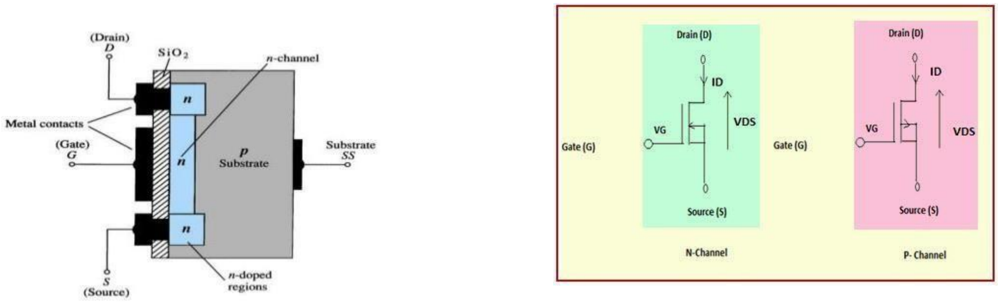


Fig. 4.30 Construction of Depletion Type MOSFET & Symbols

- The Drain (D) and source (S) leads connect to the n – doped regions
- The n doped regions are connected by an N – Channel
- This N-Channel is connected to the Gate (G) through a thin insulating layer of SiO2.
- The n-doped material lies on a p-doped substrate that may have an additional terminal connection called SS.

Enhancement type MOSFET:

When there is no voltage on the gate terminal the device does not conduct. More voltage applied on the gate terminal, the device has good conductivity.

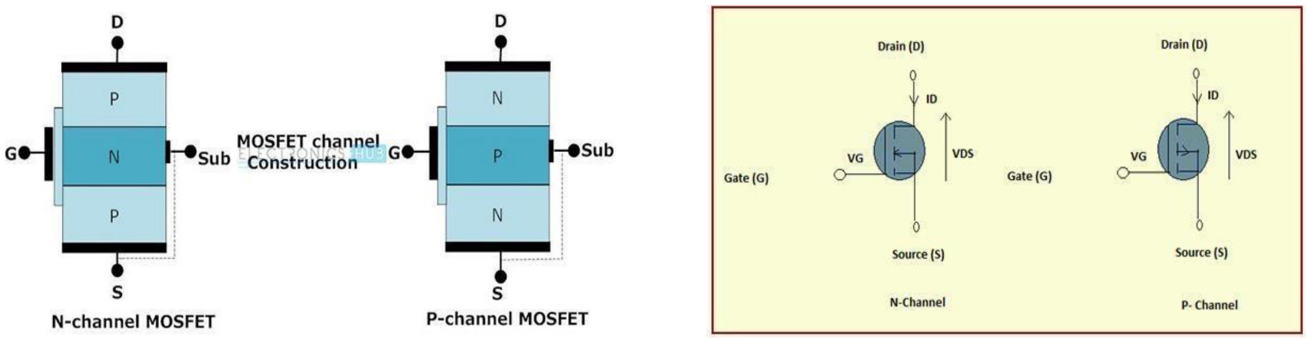


Fig. 4.31Construction of Enhancement Type of MOSFET & Symbols

- The Enhancement type MOSFET is equivalent to "Normally Open" switch and these types of transistors require gate-source voltage to switch ON the device.
- The broken line is connected between the source and drain which represents the enhancement type. In enhancement type MOSFETs the conductivity increases by increasing the oxide layer which adds the carriers to the channel.
- Generally, this oxide layer is called as „Inversion layer". The channel is formed between the drain and source in theopposite type to the substrate, such as N-channel is made with a P-type substrate and P-channel is made with an N-type substrate. The conductivity of the channel due to electrons or holes depends on N-type or P- type channel respectively.

WORKING OF DEPLETION TYPE MOSFET:

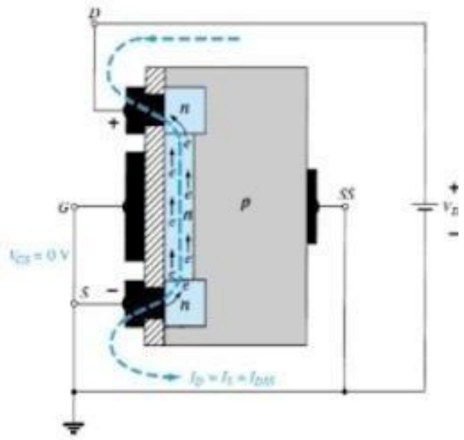


Fig. 4.32 Construction of Enhancement Type of MOSFET & Symbols

Working of Depletion Type MOSFET

The gate-to-source voltage is set to zero volts by the direct connection from one terminal to the other and a voltage VDS is applied across the drain to source terminals. The result is an attraction for the positive potential at the drain by the free electrons of the n-channel and a current similar to that established through the channel of the JFET. In fact, the resulting current with VGS = 0 V continues to be labeled IDSS, as shown in the characteristics of depletion type MOSFET in the below figure.

VGS has been set at a negative voltage such as 1 V. The negative potential at the gate will tend to pressure electrons toward the p-type substrate (like charges repel) and attract holes from the p-type substrate (opposite charges attract) as shown in the above figure

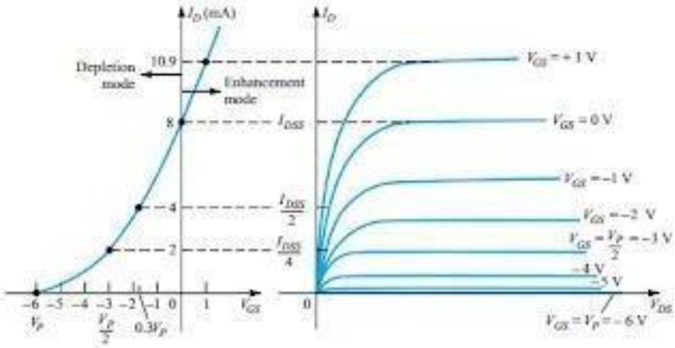


Fig. 4.33 Drain and Transfer Characteristics of Depletion type MOSFET

Depending on the magnitude of the negative bias established by VGS, a level of recombination between electrons and holes will occur that will reduce the number of free electrons in the n-channel available for conduction. The more negative the bias, the higher the rate of recombination. The resulting level of drain current is therefore reduced with increasing negative bias for VGS as shown in the figure below for VGS = - 1 V, - 2 V, and so on, to the pinch-off level of 6 V. The resulting levels of drain current and the plotting of the transfer curve proceeds exactly as described for the JFET.

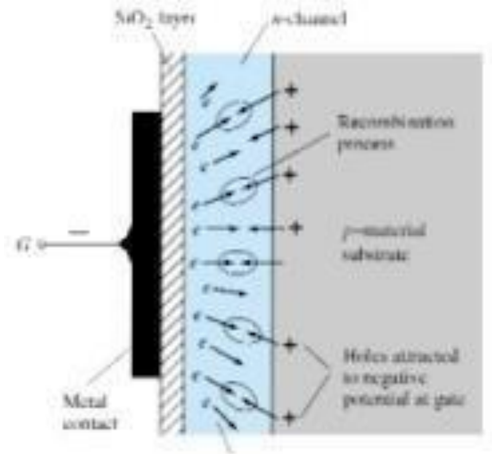


Fig. 4.34 Depletion type MOSFET

For positive values of VGS, the positive gate will draw additional electrons (free carriers) from the p-type substrate due to the reverse leakage current and establish new carriers through the collisions resulting between accelerating particles. As the gate to source voltage continues to increase in the positive direction, characteristics of depletion type MOSFET reveals that the drain current will increase at a rapid rate for the reasons listed above.

The vertical spacing between the VGS = 0 V and VGS = 1 V curves in the characteristic curve is a clear indication of how much the current has increased for the 1-V change in VGS. Due to the rapid rise, the user must be aware of the maximum drain current rating since it could be exceeded with a positive gate voltage. That is, for the device of figure showing characteristics of depletion type MOSFET, the application of a voltage VGS = 4 V would result in a drain current of

mA, which could possibly exceed the maximum rating (current or power) for the device.

As revealed above, the application of a positive gate-to-source voltage has “enhanced” the level of free carriers in the channel compared to that encountered with VGS = 0 V. For this reason the region of positive gate voltages on the drain or transfer characteristics is often referred to as the enhancement region, with the region between cutoff and the saturation level of IDSS referred to as the depletion region.

WORKING OF ENHANCEMENT TYPE MOSFET(EMOSFET):

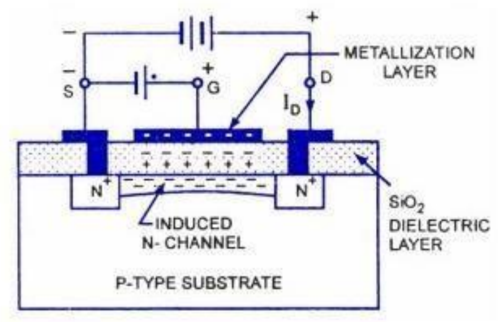


Fig. 4.35 Operation of Enhancement type MOSFET

Working of an EMOSFET

As its name indicates, this MOSFET operates only in the *enhancement mode* and has no depletion mode. It operates with large positive gate voltage only. It does not conduct when the gate-source voltage VGS = 0. This is the reason that it is called normally-off MOSFET. In these MOSFETâ€™s drain current ID flows only when VGS exceeds VGST [gate-to- source threshold voltage].

When drain is applied with positive voltage with respect to source and no potential is applied to the gate two N-regions and one P-substrate from two P-N junctions connected back to back with a resistance of the P-substrate. So a very small drain current that is, reverse leakage current flows. If the P-type substrate is now connected to the source terminal, there is zero voltage across the source substrate junction, and the drain- substrate junction remains reverse biased.

When the gate is made positive with respect to the source and the substrate, negative (i.e. minority) charge carriers within the substrate are attracted to the positive gate and accumulate close to the-surface of the substrate. As the gate voltage is increased, more and more electrons accumulate under the gate. Since these electrons can not flow across the insulated layer of silicon dioxide to the gate, so they accumulate at the surface of the substrate just below the gate. These accumulated minority charge carriers N -type channel stretching from drain to source. When this occurs, a channel is induced by forming what is termed an *inversion layer* (N-type). Now a drain current start flowing. The strength of the drain current depends upon the channel resistance which, in turn, depends upon the number of charge carriers attracted to the positive gate. Thus drain current is controlled by the gate potential.

Since the conductivity of the channel is enhanced by the positive bias on the gate so this device is also called the *enhancement MOSFET* or E- MOSFET.

The minimum value of gate-to-source voltage VGS that is required to form the inversion layer (N-type) is termed the *gate-to- source threshold voltage* VGST. For VGS below VGST,the drain current ID = 0. But for VGS exceeding VGST an N-type inversion layer connectsthe source to drain and the drain current ID is large. Depending upon the device being used, VGSTmay vary from less than 1 V to more than 5 V.
JFETs and DE-MOSFETs are classified as the depletion-mode devices because their conductivity depends on the action of depletion layers. E-MOSFET is classified as an enhancement-mode device because its conductivity depends on the action of the inversion layer. Depletion-mode devices are normally ON when the gate-source voltage VGS = 0, whereas the enhancement-mode devices are normally OFF when VGS = 0.

CHARACTERISTICS OF AN EMOSFET.

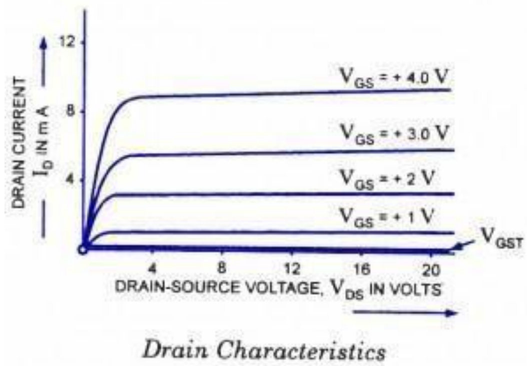


Fig. 4.36 Characteristics of Enhancement type MOSFETDrain

Characteristics-EMOSFET

Drain characteristics of an N-channel E-MOSFET are shown in the above figure. The lowest curve is the VGST curve. When VGS is lesser than VGST, ID is approximately zero.When VGS is greater than VGST, the device turns- on and the drain current ID is controlled by the gate voltage. The characteristic curves have almost vertical and almost horizontal parts. The almost vertical components of the curves correspond to the ohmic region, and the horizontal components correspond to the constant current region. Thus E-MOSFET can be operated in either of these regions i.e. it can be used as a variable- voltage resistor (WR) or as a constant current source.

EMOSFET-Transfer Characteristics

The current IDSS at VGS <=0 is very small, being of the order of a few nano-amperes. When the VGS is made positive, the drain current ID increases slowly at first, and then much more rapidly with an increase in VGS. The equation for the transfer characteristic does not obey equation. However it does follow a similar "square law type" of relationship. The equation for the transfer characteristic of E-MOSFETs is given as: $ID=K(VGS\text{-}VGST)^2$

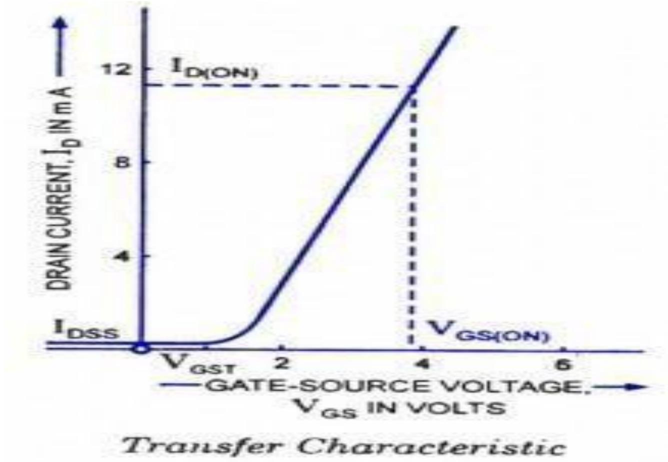


Fig. 4.37 Transfer Characteristics of Enhancement type MOSFET

Handling Precautions for MOSFET

The MOSFET has the drawback of being very susceptible to overload voltage and may require special handling during installation. The MOSFET gets damaged easily if it is not properly handled. A very thin layer of SiO2, between the gate and channel is damaged due to high voltage and even by static electricity. The static electricity may result from the sliding of a device in a plastic bag. If a person picks up the transistor by its case and brushes the gate against some grounded objects, a large electrostatic discharge may result. In a relatively dry atmosphere, a static potential of 300V is not uncommon on a person who has high resistance soles on his footwear.

MOSFETs are protected by a shorting ring that is wrapped around all four terminals during shipping and must remain in place until after the devices soldered in position. prior to soldering ,the technician should use a shorting strap to discharge his static electricity and make sure that the tip of the soldering iron is grounded. Once in circuit, there are usually low resistances present to prevent any excessive accumulation of electro static charge .However, the MOSFET should never be inserted into or removed from a circuit with the power ON.JFET is not subject to these restrictions, and even some MOSFETs have a built in gate protection known as "integral gate protection", a system built into the device to get around the problem of high voltage on the gate causing a puncturing of the oxide layer. The manner in which this is done is shown in the cross sectional view of Fig.7.11.The symbol clearly shows that between each and the source is placed a back-to- back (or front-to-front) pair of diodes, which are built right into P type substrate.

FET as Voltage-Variable Resistor

FET is operated in the constant-current portion of its output characteristics for the linear applications. In the region before pinch-off, where VDS is small, the drain to sourceresistance rd can be controlled by the bias voltage VGS. The FET is useful as a voltage variable resistor (VVR) or voltage dependent resistor (VDR).

In JFET, the drain to source conductance gd =ID/VDS for small values of VDS, whichmay also be expressed as gd=g do $[1-(VGS/VP)^{1/2}]$

Where, gdo is the value of drain conductance when the bias voltage VGS is zero. The variation of the rd with VGS can be closely approximated by the empirical expression,

rd =ro/(1-KVGS)

Where ro=drain resistance at zero gate bias, and K=a constant, dependent upon FET type.

Comparison of MOSFET and JFET

1. In enhancement and depletion types of MOSFET, the transverse electric field induced across an insulating layer deposited on the semiconductor material controls the conductivity of the channel. In the JFET the transverse electric field across the reverse biased PN junction controls the conductivity of the channel.
2. The gate leakage current in a MOSFET is of the order of 10^{-12}A.Hence the input resistance of a MOSFET is very high in the order of 10^{10} to 10^{15} ohm. The gate leakage current of a JFET is of the order of 10^{-9}A and its input resistance is of theorder of 10^{8} ohm.
3. The output characteristics of the JFET are flatter than those of the MOSFET and hence, the drain resistance of a JFET(0.1 to 1Mohm) is much higher than that of aMOSFET(1 to 50 K ohm)
4. JFETs are operated only in the depletion mode. The depletion type MOSFET may be operated in both depletion and enhancement mode.
5. Comparing to JFET, MOSFETs are easier to fabricate.
6. MOSFET is very susceptible to overload voltage and needs special handling during installation. It gets damaged easily if it is not properly handled.
7. MOSFET has zero offset voltage. As it is a symmetrical device, the source and draincan be interchanged. These two properties are very useful in analog signal switching.
8. Special digital CMOS circuits are available which involves near –zero power dissipation and very low voltage and current requirements. This makes them most suitable for portable systems.

Comparison of JFET and BJT

1. FET operations depend only on the flow of majority carrier-holes for P-channel FETs and electrons for N-channel FETs. Therefore, they are called Unipolar devices. Bipolar transistor (BJT) operation depends on both minority and majoritycurrent carrier.
2. As FET has no junctions and the conduction is through an N-type or P-type semiconductor material, FET is less noisy than BJT.

3. As the input circuit of FET is reverse biased, FET exhibits as much higher input impedance (in the order of 100MOHM) and lower output impedance and there willbe a high degree of isolation between input and output. So, FET can act as excellent buffer amplifier but the BJT has low input impedance because its input circuit is forward biased.
4. FET is a voltage control device, i.e. voltage at the input terminal controls the output current, whereas BJT is a current control device, i.e. the input current controls theoutput current.
5. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
6. The performance of BJT is degraded by neutron radiations because of reduction in minority carrier life time, whereas FET can tolerate a much higher level of radiation since they do not rely on minority carrier for their operation.
7. The performance of FET is relatively unaffected by ambient temperature changes. As it has a negative temperature coefficient at high current levels, it prevents the FET from thermal break down. The BJT has a positive temperature coefficient at high current levels which leads to thermal break down.
8. Since FET does not suffer from minority carrier storage effects, it has a higher switching speeds and cut off frequencies.BJT suffers a minority carrier storage effects and therefore has lower switching speed and cut off frequencies.
9. FET amplifiers have low gain bandwidth product due to the junction capacitive effects and produce more signal distortion except for small signal operation.
10. BJT are cheaper to produce than FETs.

4.6 Silicon Controlled Rectifier (SCR)

A silicon controlled rectifier is a semiconductor device that acts as a true electronic switch. It can change alternating current into direct current and at the same time can control the amount of power fed to the load. Thus *SCR* combines the features of a rectifierand a transistor.

Constructional details.

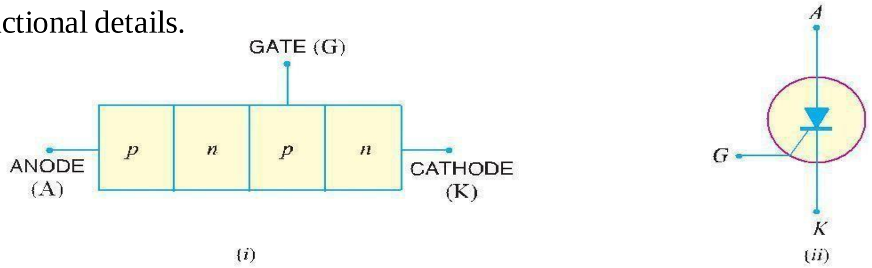


Fig. 4.38 SCR

When a PN junction is added to a junction transistor, the resulting three pn junction device is called a silicon controlled rectifier. Fig. I shows its construction. It is clear that it is essentially an ordinary rectifier (pn) and a junction transistor (npn) combined in one unit to form pnpn device. Three terminals are taken; one from the outer p-type material called anode A, second from the outer n-type material called cathode K and the third from the base of transistor section and is called gate

G. In the normal operating conditions of SCR, anode is held at high positive potential with respect to cathode and gate at small positive potential with respect to cathode. Fig. (ii) shows the symbol of SCR. The silicon controlled rectifier is a solid state equivalent of thyratron. The gate, anode and cathode of SCR correspond to the grid, plate and cathode of thyratron. For this reason, SCR is sometimes called thyristor.

Working of SCR

In a silicon controlled rectifier, load is connected in series with anode. The anode is alwayskept at positive potential *w.r.t.*cathode. The working of *SCR* can be studied under the following two heads:
When gate is open. The below diagram shows that the SCR circuit with gate open i.e. no voltage applied to the gate. Under this condition, junction J2 is reverse biased while junctions J1 and J3 are forward biased. Hence, the situation in the junctions J1 and J3 is just as in a npn transistor with base open. Consequently, no current flows through the load RL and the SCR. However, if the applied voltage is gradually increased, a stage is reachedwhen reverse biased junction J2 breaks down. The SCR now conducts heavily and is saidto be in the ON state. The applied voltage at which SCR conducts heavily without gate voltage is called Break over voltage.

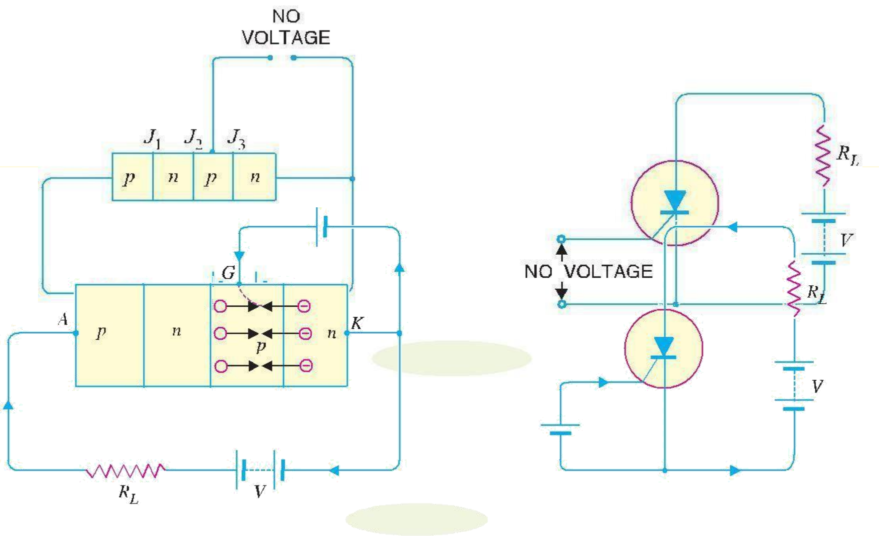


Fig. 4.39 Operation of SCR

(i) When gate is positive w.r.t. cathode.

The SCR can be made to conduct heavily at smaller applied voltage by applying a small positive potential to the gate as shown in Now junction J3 is forward biased and junction J2 is reverse biased. The electrons from n-type material start moving across junction J3 towards left whereas holes from p-type towards the right. Consequently, the electrons from junction J3 are attracted across junction J2 and gate current starts flowing. As soon as the gate current flows, anode current increases. The increased anode current in turn makes more electrons available at junction J2. This process continues and in an extremely small time, junction J2 breaks down and the SCR starts conducting heavily.

Once SCR starts conducting, the gate (the reason for this name is obvious) loses all control. Even if gate voltage is removed, the anode current does not decrease at all. The only way to stop conduction (i.e. bring SCR in off condition) is to reduce the applied voltage to zero.

The whole applied voltage V appears as reverse bias across junction J2 as junctions J1 andJ3 are forward biased. Because J1 and J3 are forward biased and J2 has broken down.

Conclusion. The following conclusions are drawn from the working of SCR : An SCR hastwo states i.e. either it does not conduct or it conducts heavily. There is no state in between. Therefore, SCR behaves like a switch.There are two ways to turn on the SCR. The first method is to keep the gate open and make the supply voltage equal to the breakover voltage. The second method is to operate SCR with supply voltage less than breakover voltage and then turn it on by means of a small voltage (typically 1.5 V, 30 mA) applied tothe gate.Applying small positive voltage to the gate is the normal way to close an SCR because the breakover voltage is usually much greater than supply voltage.To open the SCR (i.e. to make it non- conducting), reduce the supply voltage to zero.

VI CHARACTERISTICS OF SCR:

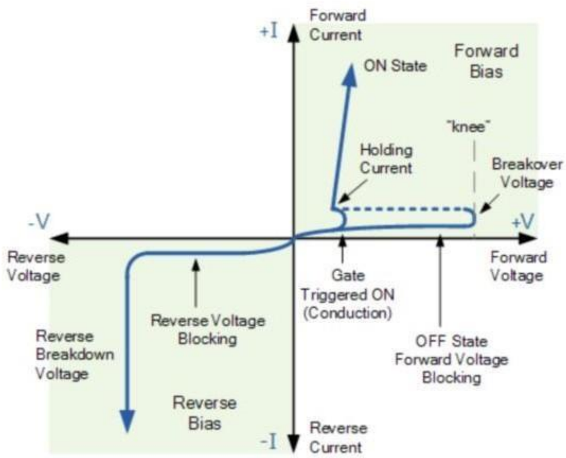


Fig. 4.40 V-I characteristics of SCR

The V-I characteristics of the SCR reveal that the SCR can be operated in three modes. Forward blocking mode (off state) Forward conduction mode (on state) Reverse blockingmode (off state) Forward blocking mode

In this mode of operation, the anode is given a positive potential while the cathode is given a negative voltage, keeping the gate at zero potential i.e. disconnected. In this case junction J1 and J3 are forward biased while J2 is reversed biased due to which only a small leakage current exists from the anode to the cathode until the applied voltage reaches its breakover value, at which J2 undergoes avalanche breakdown and at this breakover voltage it starts conducting, but below breakover voltage it offers very high resistance to the current and is said to be in the off state.

Forward conduction mode

SCR can be brought from blocking mode to conduction mode in two ways: either by increasing the voltage across anode to cathode beyond break over voltage or by applying of positive pulse at gate. Once it starts conducting, no more gate voltage is required to maintain it in the on state. There are two ways to turn it off: 1. Reduce the current through it below a minimum value called the holding current and 2. With the Gate turned off, short out the Anode and Cathode momentarily with a push- button switch or transistor across the junction.

Reverse blocking mode

In this mode SCR is reversed biased , ie when anode is negative compared to cathode. The characteristic of this region are similar to those of an ordinary PN junction diode. in this region ,junctionJ1and J3 are reversed biased whereas j2 is farward biased

.the device behaves as if two diodes are connected in series with a reverse voltage applied to them. A small leakage current of the order of mill amperes or micro amperes flow in the device. This reverse blocking mode is called the OFF state of the thyristor .when the reverse voltage of the SCR increases to a large extent breakdown occurs and the current in the device increases rapidly. Thus when the SCR is biased in this region the power dissipated is very high, if the power dissipated is more than the rated value of the SCR , the SCR is permanently damaged .thus in the reverse bias condition the voltage should never cross the breakdown voltage.

Characteristics of SCR

It is the curve between anode-cathode voltage (V) and anode current (I)of an SCR at constant gate current

Forward characteristics.

When anode is positive w.r.t. cathode, the curve between V and I is called the forward characteristic. In the above fig, OABC is the forward characteristic of SCR at IG = 0.If the supply voltage is increased from zero, a point is reached (point A) when the SCR starts conducting. Under this condition, the voltage across SCR suddenly drops as shown by dotted curve AB and most of supply voltage appears across the load resistance RL. If proper gate current is made to flow,SCR can close at much smaller supply voltage.

Reverse characteristics.

When anode is negative w.r.t. cathode, the curve between V and I is known as reverse characteristic. The reverse voltage does come across SCR when it is operated witha.c. supply. If the reverse voltage is gradually increased, at first the anode current remains small (i.e.leakage current) and at some reverse voltage, avalanche breakdown occurs and the SCR starts conducting heavily in the reverse direction as shown by the curve DE. This maximum reverse voltage at which SCR starts conducting heavily is known as reverse breakdown voltage.

EQUIVALENT CIRCUIT OF SCR

The *SCR* shown in Fig. 20.4 (*i*) can be visualised as separated into two transistors as shown in

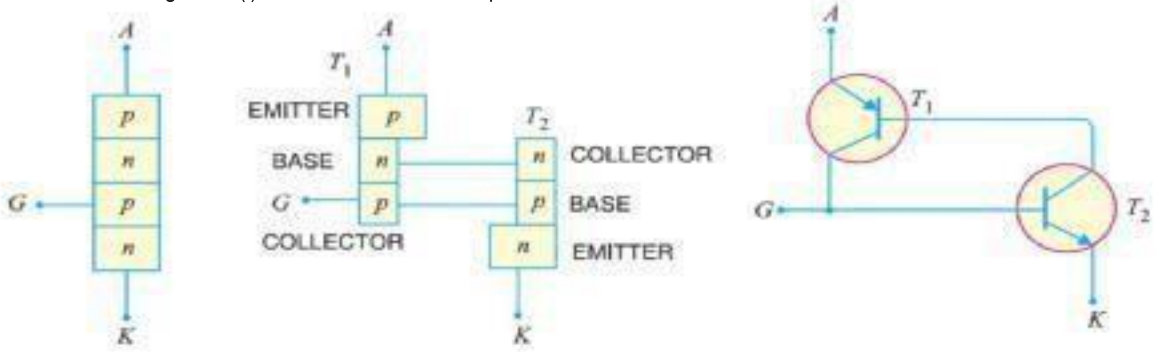


Fig. 4.41 Equivalent circuit of SCR

Thus, the equivalent circuit of *SCR* is composed of *pnp* transistor and *npn* transistor connected as shown in the above fig. It is clear that collector of each transistor is coupledto the base of the other, thereby making a positive feedback loop. The working of *SCR* canbe easily explained from its equivalent circuit. The above fig shows the equivalent circuit of *SCR* with supply voltage *V* and load resistance *RL*. Assume the supply voltage *V* is less than break over voltage as is usually the case.

With gate open (*i.e.* switch *S* open), there is no base current in transistor *T*2. Therefore, no current flows in the collector of *T*2 and hence that of *T*1. Under such conditions, the

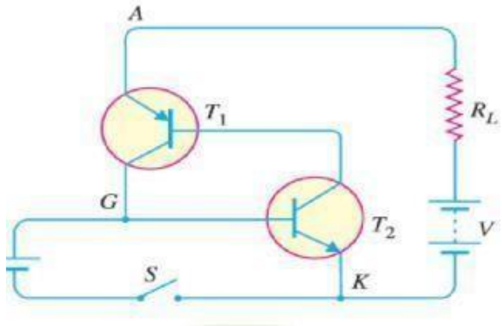


Fig. 4.42 Equivalent circuit of SCR – Two Transistor model

SCR is open. However, if switch *S* is closed, a small gate current will flow through the baseof *T*2 which means its collector current will increase.

The collector current of *T*2 is the base current of *T*1.Therefore, collector current of *T*1 increases. But collector current of *T*1 is the base current of *T*2. This action is accumulativesince an increase of current in one transistor causes an increase of current in the other transistor. As a result of this action, both transistors are driven to saturation, and heavy current flows through the load *RL*. Under such conditions, the *SCR* closes.

APPLICATIONS

SCRs are used in many areas of electronics where they find uses in a variety of different applications. Some of the more common applications for them are outlined below:

- AC power control (including lights, motors,etc).
- Overvoltage protection crowbar for power supplies.
- AC power switching.
- Control elements in phase angle triggered controllers.
- Within photographic flash lights where they act as the switch to discharge a stored voltage through the flash lamp, and then cut it off at the required time.

Thyristors are able to switch high voltages and withstand reverse voltages making them ideal for switching applications, especially within AC scenarios.

4.7 UNI JUNCTION TRANSISTOR (UJT)

A unijunction transistor (abbreviated as UJT) is a three-terminal semiconductor switching device. This device has a unique characteristic that when it is triggered, the emitter current increases regeneratively until it is limited by emitter power supply. Due to this characteristic, the unijunction transistor can be employed in a variety of applications e.g., switching, pulse generator, saw-tooth generator etc

CONSTRUCTION.

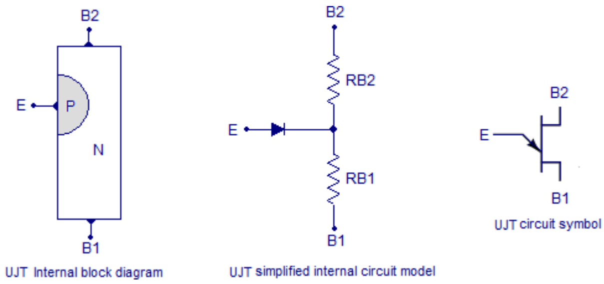


Fig. 4.43 UJT

The above Fig shows the basic structure of a unijunction transistor. It consists of an n-type silicon bar with an electrical connection on each end. The lead to these connections arecalled base leads base-one B1 and base two B2. Part way along the bar between the twobases, nearer to B2 than B1, a pn junction is formed between a p-type emitter and the bar. The lead to this junction is called the emitter lead E. Fig shows the symbol of unijunction transistor. Note that emitter is shown closer to B2 than B1.
i)Since the device has one pn junction and three leads, it is commonly called a unijunction transistor (uni means single).

With only one pn-junction, the device is really a form of diode. Because the two base terminals are taken from one section of the diode, this device is also called double-based diode.

The emitter is heavily doped having many holes. The n region, however, is lightly doped. For this reason, the resistance between the base terminals is very high (5 to 10 kΩ) when emitter lead is open.

WORKING PRINCIPLE OF UJT

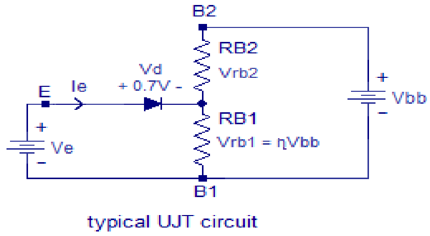


Fig. 4.44 Operation of UJT

The above fig shows the basic circuit operation of a unijunction transistor. The device has normally B2 positive w.r.t. B1.If voltage VBB is applied between B2 and B1 with emitter open, a voltage gradient is established along the n-type bar. Since the emitter is located nearer to B2, more than half of VBB appears between the emitter and B1. The voltage V1 between emitter and B1 establishes a reverse bias on the pn junction and the emitter current is cut off. Of course, a small leakage current flows from B2 to emitter due to minority carriers.

If a positive voltage is applied at the emitter, the pn junction will remain reverse biased so long as the input voltage is less than V1. If the input voltage to the emitter exceeds V1, the pn junction becomes forward biased. Under these conditions, holes are injected from p- type material into the n-type bar. These holes are repelled by positive B2 terminal and they are attracted towards B1 terminal of the bar. This accumulation of holes in the emitter to B1 region results in the decrease of resistance in this section of the bar. The result is that internal voltage drop from emitter to B1 is decreased and hence the emitter current IE increases. As more holes are injected, a condition of saturation will eventually be reached. At this point, the emitter current is limited by emitter power supply only. The device is now in the ON state.

If a negative pulse is applied to the emitter, the pn junction is reverse biased and the emitter current is cut off. The device is then said to be in the OFF state.

Intrinsic standoff ratio:

For ease of understanding, the internal model of the UJT is used in the circuit. B2 terminalof the UJT is made positive with respect to B1 terminal using the voltage source Vbb. Emitter terminal E of the UJT is forward biased using the voltage source Ve. Current starts flowing into the emitter only when the bias voltage Ve has exceeded the forward drop of the internal diode (Vd) plus the voltage drop across RB1 (Vrb1). This condition can be expressed using the following equation.

$Ve = Vd + Vrb1$

$Vrb1 = Vbb^* (RB1/(RB1+RB2))$

Considering the intrinsic stand off ratio η = RB1/(RB1+RB2), the equation becomes

$Ve = Vd + \eta \cdot Vbb$

A typical silicon diode has a forward voltage drop of 0.7V. When this factor is considered, the equation can be re written as

$Ve = 0.7V + \eta \cdot Vbb$

CHARACTERISTICS OF UJT

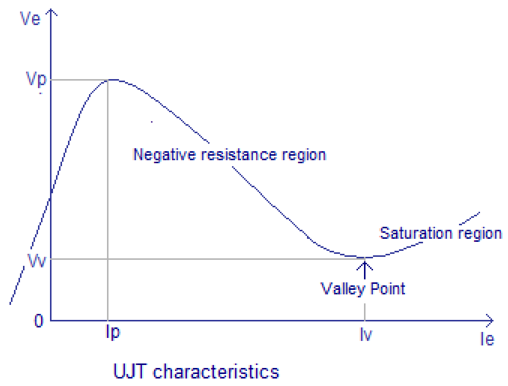


Fig. 4.45 Characteristics of UJT

The above Fig. shows the curve between emitter voltage (VE) and emitter current (IE) ofa UJT at a given voltage VBB between the bases. This is known as the emitter characteristicof UJT. The following points may be noted from the characteristics :
(I)Initially, in the cut-off region, as VE increases from zero, slight leakage current flows from terminal B2 to the emitter. This current is due to the minority carriers in the reversebiased diode.
Above a certain value of VE, forward IE begins to flow, increasing until the peak voltage VP and current IP are reached at point P.
After the peak point P, an attempt to increase VE is followed by a sudden increase in emitter current IE with a corresponding decrease in VE. This is a negative resistance portion of the curve because with increase in IE, VE decreases. The device, therefore, has a negative resistance region which is stable enough to be used with a great deal of reliability in many areas e.g., trigger circuits, sawtooth generators, timing circuits.

ADVANTAGES OF UJT

The UJT was introduced in 1948 but did not become commercially available until 1952.Since then, the device has achieved great popularity due to the following reasons :

- It is a low cost device.
- It has excellent characteristics.
- It is a low-power absorbing device under normal operatingconditions.

APPLICATIONS OF UJT

Due to above reasons, this device is being used in a variety of applications. A few include oscillators, trigger circuits, saw- tooth generators, bistable network etc.
The UJT is very popular today mainly due to its high switching speed.

A few selected applications of the UJT are as follows:
It is used to trigger SCRs and TRIACs It is used in non-sinusoidal oscillators

It is used in phase control and timing circuits It is used in saw tooth generatorsIt is used in oscillator circuit design.

QUESTIONS

PART A

1. What is a semiconductor?
2. Mention the application of PN junction diode.
3. What is a Zener diode?
4. List the three configuration of transistor.
5. Define α and β.
6. What is a FET?
7. List two applications of JFET and SCR.
8. What is a SCR? Give two applications of it.

PART B

1. Explain the V-I characteristics of PN junction diode.
2. Draw the circuit symbol for MOSFET and explain its characteristics.
3. Explain the application of the following devices,
   (a) UJT
   (b) FET
4. Explain the working principle and V-I characteristics of Zener diode.
5. What is a SCR? Explain its characteristics.
6. Draw the circuit diagram of a transistor. Explain in detail the transistor configuration.

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UNIT – 5 DIGITAL ELECTRONICS

UNIT V- DIGITAL ELECTRONICS

Number systems – Binary arithmetic - Boolean algebra, laws & theorems – Boolean Functions - Simplification of Boolean functions - Logic gates - Implementation of Boolean expressions using logic gate - Standard forms of Boolean expression.

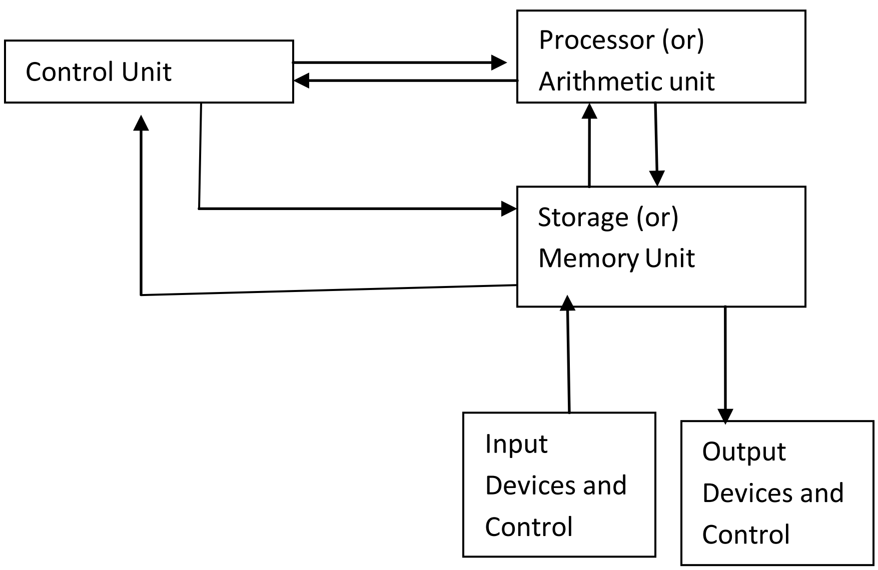
Number System

A number system relates quantities and symbols. In digital system how information is represented is key and there are different radices, i.e. number bases, which a numbering system can use.

Digital computer

Any class of devices capable of solving problems by processing information in discrete form. It operates on data, including letters and symbols, which are expressed in binary form i.e using only two digits 0 and 1.

The block diagram of digital computer is given below:



The memory unit stores programs as well as input, output and intermediate data. The processor unit performs arithmetic and other data processing tasks as specified by the program.The control unit supervises the flow of information between various units. The program and data prepared by the user are transferred into the memory unit by means of an input device such as punch card reader (or) tele typewriter. An output device, such as printer, receives the result of the computations and the printed results are presented to the user.

Number Representation:

It can have different base values like: binary (base-2), octal (base-8), decimal (base 10) and hexadecimal (base 16),here the base number represents the number of digits used in that numbering system. As an example, in decimal numbering system the digits used are: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Therefore the digits for binary are: 0 and 1, the digits for octal are: 0, 1, 2, 3, 4, 5, 6 and 7. For the hexadecimal numbering system, base 16, the digits are: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

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2. Binary numbers

Numbers that contain only two digit 0 and 1 are called Binary Numbers. Each 0 or 1 is called a Bit, from binary digit. A binary number of 4 bits is called a Nibble. A binary number of 8 bits is called a Byte. A binary number of 16 bits is called a Word on some systems, on others a 32-bit number is called a Word while a 16-bit number is called a Halfword.

Using 2 bit 0 and 1 to form

a binary number of 1 bit, numbers are 0 and 1

a binary number of 2 bit, numbers are 00, 01, 10, 11

a binary number of 3 bit, such numbers are 000, 001, 010, 011, 100, 101, 110, 111

a binary number of 4 bit, such numbers are 0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100,1101,1110,1111

Therefore , using n bits there are 2n binary numbers of n bits

Each digit in a binary number has a value or weight. The LSB has a value of 1. The second from the right has a value of 2, the next 4 , etc.,

16	8	4	2	1
2^4	2^3	2^2	2^1	2^0

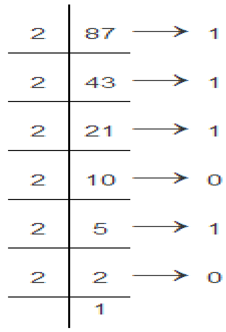
The binary equivalent for some decimal numbers are given below.

Decimal	0	1	2	3	4	5	6	7	8	9	10	11
Binary	0	1	10	11	100	101	110	111	1000	1001	1010	1011

3. Number Base Conversions

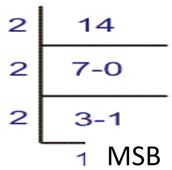
3.1 Conversion of decimal number to any number system
Step 1 convert the integer part by doing successive division using the radix of asked number systems.
Step 2 convert the fractional part by doing successive multiplication using radix of asked number system

3.2 Conversion of decimal to binary number system
The radix of asked number system is 2
Convert 87_{10} to $(\ )_2$



$(1010111)_2$

Convert $(14.625)_{10}$ decimal number to binary number



$(1110)_2$

<u>1st Multiplication Iteration</u>
Multiply 0.625 by 2
0.625 x 2 = 1.25(Product) Fractional part=0.25 Carry=1 **(MSB)**

<u>2nd Multiplication Iteration</u>
Multiply 0.25 by 2
0.25 x 2 = 0.50(Product) Fractional part = 0.50 Carry = 0

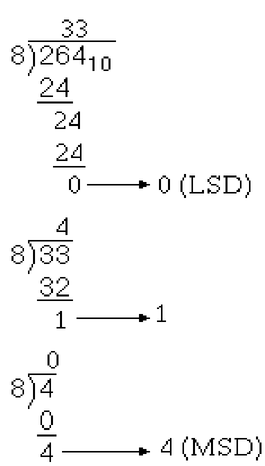
<u>3rd Multiplication Iteration</u>
Multiply 0.50 by 2
0.50 x 2 = 1.00(Product) Fractional part = 1.00 Carry = 1 **(LSB)**
$(101)_2$

The binary number of $(16.625)_{10}$ is $(1110.101)_2$

<u>3.3 Conversion of decimal to octal number system</u>

The radix of asked number system is 8

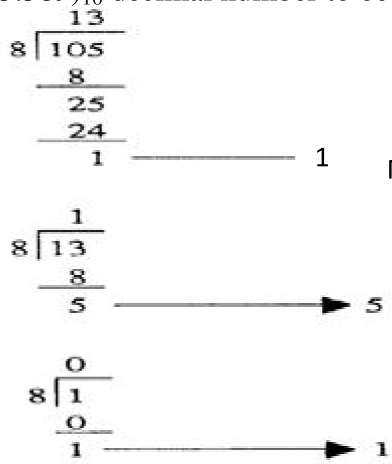
Convert $(264)_{10}$ decimal number to octal number



$(410)_8$

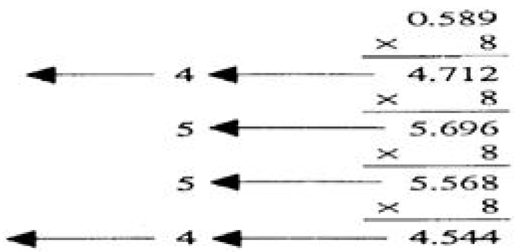
The octal number of $(264)_{10}$ is $(410)_8$

Convert $(105.589)_{10}$ decimal number to octal number



1 MSB

LSB (151)



MSB

LSB

(0.4554)

The octal number of $(105.589)_{10}$ is $(151.4554)_8$

3.4 Conversion of decimal to Hexadecimal number system

The radix of asked number system is 16

Convert $(1693)_{10}$ decimal number to Hexadecimal number

1693/16 = 105	Reminder	(13) D	(LSB)
105/16 = 6	Reminder	9	
6/16 = 0	Reminder	6	(MSB)

(1693)10 (69D)16

Convert $(1693.0628)_{10}$ decimal fraction to hexadecimal fraction $(?)_{16}$

1693/16 = 105	Reminder	(13) D	(LSB)
105/16 = 6	Reminder	9	
6/16 = 0	Reminder	6	(MSB)

(69D)

Multiply 0.0628 by 16

0.0628 x 16 = 1.0048(Product) Fractional part=0.0048 Carry=1 (MSB)

Multiply 0.0048 by 16

0.0048 x 16 = 0.0768(Product) Fractional part = 0.0768 Carry = 0

Multiply 0.0768 by 16

0.0768 x 16 = 1.2288(Product) Fractional part = 0.2288 Carry = 1

Multiply 0.2288 by 16

0.2288 x 16 = 3.6608(Product) Fractional part = 0.6608 Carry = 3 (LSB)

(.1013)

$(1693.0628)_{10} = (69D.1013)_{16}$

3.5 Conversion of any number system to decimal number system

In general the numbers can be represented as

$N= A_{n-1}r_{n-1} + = A_{n-2}r_{n-2} +........+ A_1 r^1 + A_0 r^0 + A_{-1} r^{-1} + A_{-2} r^{-2} +.........$

Where n= number in decimal
A= digit
r= radix of number system
n= The number of digits in the integer portion of number
m= the number of digits in the fractional portion of number

3.6 Conversion of binary to decimal number system

Convert $(101.101)_2= (?)_{10}$

101.101

$= 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 . \ 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$

$= 1 \times 4 + 0 \times 2 + 1 \times 1 . \ 1 \times (1 / 2) + 0 \times (1 / 4) + 1 \times (1 / 8)$

$= 4 + 0 + 1 . (1 / 2) + 0 + (1 / 8)$

= 5 + 0.5 + 0.125
= 5 . 625

Therefore $(1 0 1 . 1 0 1)_2 = (5.625)_{10}$

3.7 Conversion of octal to decimal number system

Convert $(128)_8= (?)_{10}$

$123_8 = 1*8^2 + 2*8^1 + 3*8^0 = 64 + 16 + 3 = 73$

the decimal equivalent of the number 123_8 is 73_{10}

Convert $(2 1. 2 1)_8= (?)_{10}$

2 1. 2 1

$= 2 \times 8^1 + 1 \times 8^0. \ 2 \times 8^{-1} + 1 \times 8^{-2}$

$= 2 \times 8 + 1 \times 1. \ 2 \times (1 / 8) + 1 \times (1 / 64)$

= 16 + 1 . (0. 2 5) + (0. 0 1 5 6 2 5)
= 17 + 0. 265625
= 17. 265625

Therefore $(2 1. 2 1)_8 = (1 7. 2 6 5 6 2 5)_{10}$

3.8 Conversion of hexadecimal to decimal number system

Convert $(E F. B 1)_{16}= (?)_{10}$

$= E \times 16^1 + F \times 16^0. \ B \times 16^{-1} + 1 \times 16^{-2}$

= 14 x 16 + 15 x 1 . 11 x (1 / 16) + 1 x (1 / 256)
= 224 + 15 + (0. 6 8 7 5) + (0. 0 0 3 9 0 6 2 5)
= 239 + 0. 6914
= 239. 691406

Therefore $(E F. B 1)_{16} = (2 3 9. 6 9 1 4 0 6)_{10}$

Convert $(0.9D9)16= (?)_{10}$

= 0 x 160. 9 x 16-1 + D x 16-2 + 9 x 16-3
= 0 x 1. 9 x (1 / 16) + 13 x (1 / 256) + 9 x (1 / 4096)
= 0 . (0. 5625) + (0. 050781) + (0. 0021972)
= 0. (0. 6154782)

= 0. 6154782

3.9 <u>Conversion of binary to octal number system</u>

Convert $(101101001)_2$ to $(\ )_8$

Divide the binary into group of three digits from LSB we will find the following pattern 101|101|001 Now writing the equivalent decimal number of each group we get 5 | 5 | 1 So the equivalent octal number is 551_8

Convert 11001100.101 to $(\ )_8$

011|001|100. |101|

3 1 4 . 5

So the equivalent octal number is 314.5

3.10 Conversion of binary to hexadecimal number system

Convert 111100010 to $(\ )_{16}$

Divide the binary into group of four digits from LSB

0001|1110|0010

Now writing the equivalent hexadecimal number of each group

1|E|2

So the equivalent Hexa decimal number is $1E2_{16}$

Convert 11000011001.101 to $(\ )_{16}$

0110|0001|1001|.1010|

6 1 9 . A

So the equivalent Hexa decimal number is $619.A_{16}$

3.11 Conversion of octal number system to hexa decimal number system

Convert $(25)_8$ to $(\ )_{16}$

First convert octal to binary

The binary equivalent of 25 is 010101

Divide the binary into group of four digits from LSB

0001|0101

1 5

So the equivalent Hexa decimal number is 15_{16}

3.12 Conversion of hexa decimal number system to octal number system

Convert $(1A.2B)_{16}$ to $(\ )_8$

First convert hexadecimal to binary

The binary equivalent of 1A.2B is 00011010.00101011

Divide the binary into group of Three digits

011|010|.|001|010|110

3 2 . 1 2 6

so the equivalent octal number is 32.126_8

4. COMPLEMENTS

In digital computers to simplify the subtraction operation and for logical manipulation complements are used . There are two types of complements for each radix system the radix complement and diminished radix complement. The first is referred to as the r's complement and the second as the (r-1)'s complement.

r's Complement

Given a positive number N in base r with an integer part of n digits, the r's complement of N is defined as r^n-N if $N \neq 0$ and 0 if N=0

(r-1)'s Complement

Given a positive number N in base r with an integer part of n digits and a fraction part of m digits, the (r-1)'s complement of N is defined as $r^n - r^{-m} - N$

Subtraction with r's complement

- The direct method of subtraction uses the borrow concept
- When subtraction is implemented by means of digital components, this method is found to be less efficient. So, instead the following procedure can be followed.

The subtraction of two positive numbers (M-N), both of base r, may be done as follows.

(1) Add the minuend M to the r's complement of the subtrahend N.
(2) Inspect the result obtained in step 1 for an end carry.
   - If an end-carry occurs, discard it.
   - If an end-carry does not occurs, take the r's complement of the number obtained in step 1 and place a negative sign in front.

Subtraction with (r-1)'s Complement

- The procedure for subtraction with (r-1)'s complement is same as r's complement except for end-around carry.
- The subtraction of M-N, both positive numbers in base r, may be calculated in the following manner.
  1. Add the minuend M to the (r-1)'s complement of the subtrahend N.
  2. Inspect the result obtained in step 1 for an end carry.
     - If an end-carry occurs, add 1 to the least significant digit (end-around carry)
     - If an end-carry does not occur, take the (r-1)'s complement of the number obtained in step 1 and place a negative sign in front.

It is classified into four types they are 1's complement , 2's complement , 9's complement and 10's complement.

4.1 <u>1's complement representation:</u> The 1's complement of a binary number is the number that results when we change all 1's to zeros and the zeros to ones.

2's complement representation:

The 2's complement is the binary number that results when we add 1 to the 1's complement.

Problems related to 1's complement and 2's complement :

1. *Express the following numbers in sign magnitude 1's and 2's complement :*
i) −56   ii)107

Solution : i) − 56

56	=	0 1 1 1 0 0 0	
− 56	=	1 0 0 0 1 1 1	1's Complement
		+ 1	
	=	1 0 0 1 0 0 0	2's Complement

ii) 107

107	=	0 1 1 0 1 0 1 1	
−107	=	1 0 0 1 0 1 0 0	1's Complement
		+ 1	
	=	1 0 0 1 0 1 0 1	2's Complement.

2. *Find 2's complement of* $(1\ 0\ 0\ 1)_2$

Solution :

1 0 0 1	number
0 1 1 0	1's complement
+ 1	
0 1 1 1	2's complement

3. *Find 2's complement of* $(1\ 0\ 1\ 0\ \ 0\ 0\ 1\ 1)_2$

Solution :

1 0 1 0 0 0 1 1	number
0 1 0 1 1 1 0 0	1's complement
+ 1	
0 1 0 1 1 1 0 1	2's complement

4.2 1's complement subtraction

Subtraction of binary numbers can be accomplished by the direct method by using the 1's complement method, which allows to perform subtraction using only addition . for subtraction of two numbers we have two cases.

1. Subtraction of smaller number from larger number and
2. Subtraction of larger number from smaller number.

1's complement Subtraction of smaller number from larger number

Method:

1. Determine the 1's complement of the smaller number.
2. Add the 1's complement to the larger number.
3. Remove the carry and add it to the result.
   This is called end -around carry.

4. *Subtract* 101011_2 *from* 111001_2 *using the 1's complement method.*

Solution :

```
    1 1 1 0 0 1
+   0 1 0 1 0 0   1's complement of 1 0 1 0 1 1
 ---------------
 ① 0 0 1 1 0 1
 └──────→ +   1   Add end-around carry
 ---------------
    0 0 1 1 1 0   Final answer
```

1's complement Subtraction of larger number from smaller number
Method:

1. Determine the 1's complement of the larger number.
2. Add the 1's complement to the smaller number.
3. Answer is in 1's complement form. To get the answer in true form take the 1's complement and assign negative sign to the answer.

5. *Subtract* 111001_2 *from* 101011_2 *using the 1's complement method.*

Solution :

```
   1 0 1 0 1 1
 + 0 0 0 1 1 0     1's complement of 1 1 1 0 0 1
 -------------
   1 1 0 0 0 1     Answer in 1's complement form
 - 0 0 1 1 1 0     Answer in true form
```

Advantages of 1's complement subtraction :

1. The 1's complement subtraction can be accomplished with an binary adder. Therefore , this method is useful in arithmetic logic circuits.
2. The 1's complement of a number is easily obtained by inverting each bit in the number.

4.3 2's complement Subtraction:

Like 1's complement subtraction, in 2's complement subtraction, the subtraction is accomplished by only addition.

2's complement Subtraction of smaller number from larger number

Method

1. Determine the 2's complement of the smaller number.
2. Add the 2's complement to the larger number.
3. Discard the carry.

6. *Subtract* 101011_2 *from* 111001_2 *using the 2's complement method.*

Solution :

```
   1 1 1 0 0 1
 + 0 1 0 1 0 1     2's complement of 1 0 1 0 1 1
 -------------
   0 0 1 1 1 0
   0 0 1 1 1 0     Final answer
```

2's complement Subtraction of larger number from smaller number
Method:

1. Determine the 2's complement of the larger number.
2. Add the 2's complement to the smaller number.

3. Answer is in 2's complement form. To get the answer in true form take the 2's complement and assign negative sign to the answer.

7. *Subtract 111001_2 from 101011_2 using 2's complement method.*

Solution :

	1 0 1 0 1 1	
+	0 0 0 1 1 1	2's complement of 1 1 1 0 0 1
	1 1 0 0 1 0	Answer in 2's complement form
–	0 0 1 1 1 0	Answer in true form

4.4 9's complement and 10's complement

Before knowing about 9's complement and 10's complement we should know why they are used and why their concept came into existence. Addition of signed BCD numbers can be performed by using 9's and 10's complement. The complements are used to make the arithmetic operations in digital system easier. Various topics and related problems we going to see here are

1. 9s complement
2. 10s complement
3. 9s complement subtraction
4. 10s complement subtraction

Now first of all let us know what 9's complement is and how it is done. To obtain the 9,s complement of any number we have to subtract the number with (10n - 1) where n = number of digits in the number, or in a simpler manner we have to divide each digit of the given decimal number with 9. The table 1. will explain the 9's complement more easily.

Table 1. 9's complement equivalent for decimalo numbers

Decimal digit	9s complement
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

Now coming to 10's complement, it is relatively easy to find out the 10's complement after finding out the 9,s complement of that number. We have to add 1 with the 9,s complement of any number to obtain the desired 10's complement of that number. Or if we want to find out the 10's complement directly, we can do it by following the formula, (10^n - number), where n = number of digits in the number. An example is given below to illustrate the concept of obtaining 10’s complement

A decimal number 456, find 9's complement and 10’s complement of this number

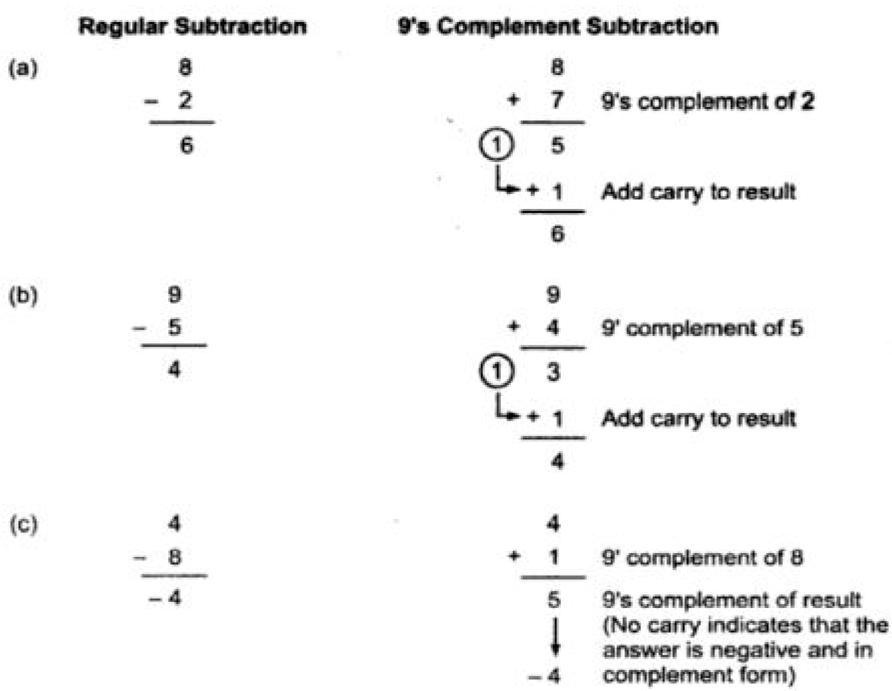
```
    999
(-) 456
-------
    543
```

10's complement of that no. is

```
 543
(+)1
----
 544
```

In 9’s complement subtraction when 9’s complement of smaller number number is added to the larger number carry is generated. It is necessary to add this carry to the result. (this is called an end around carry).when larger number is subtracted from the smaller number, there is no carry, and the result is in 9’s compliment form and negative. This is explained with following examples.

Subtraction using 9’s complements:



Steps for 9’s complement BCD subtraction

1. Find the 9’s complement of a negative number.
2. Add two numbers using BCD addition
3. If carry is generated add carry to the result otherwise find the 9’s complement of the result.

9. *Perform each of the following decimal subtractions in 8-4-2-1 BCD using 9's complement method.* *a)* 79 − 26 *b)* 89 − 54

Solution :

```
a)    79       0 1 1 1      1 0 0 1
    - 26     + 0 1 1 1      0 0 1 1      73 – 9's complement for BCD 26
    ----       -------      -------
      53       1 1 1 0      1 1 0 0
                            0 1 1 0      1100 > 9 so add 6
             -------------------------
               1 1 1 0 1    0 0 1 0
                   + 1 <--'              Propagate carry
             -------------------------
             1 1 1 1        0 0 1 0
           + 0 1 1 0                     Add 6
             -------------------------
           1 0 1 0 1        0 0 1 0
           L---------->   +       1      End around carry
           ---------------------------
             0 1 0 1        0 0 1 1      BCD for 53

b)    89     1 0 0 0        1 0 0 1
    - 54     0 1 0 0        0 1 0 1      (45) 9's complement of 54 BCD
    ----     -------        -------
      35     1 1 0 0        1 1 1 0
                          + 0 1 1 0      1110 > 9 so add 6
             -------------------------
             1 1 0 0        1 0 1 0 0
                 + 1 <--------'          Propagate carry
             -------        ---------
             1 1 0 1          0 1 0 0
             0 1 1 0                     Add 6
             -------          -------
           1 0 0 1 1          0 1 0 0
           L---------->   +         1    End around carry
             -------          -------
             0 0 1 1          0 1 0 1    BCD for 35
```

Subtraction using 10's complements:

The 10's complement of the decimal is equal to 9's complement plus 1. The 10's complement can be used to perform subtraction by adding the minuend to the 10's complement of the subtrahend and dropping the carry. This is explained with following examples.

```
      Regular Subtraction      10's Complement Subtraction
(a)          8                        8
           - 2                      + 8   10's complement of 2
           ---                      ---
             6                      ~~1~~6   Drop carry

(b)          9                        9
           - 5                      + 5   10's complement of 5
           ---                      ---
             4                      ~~1~~4   Drop carry

(c)          4                        4
           - 8                      + 2   10's complement of 8
           ---                      ---
            -4                        6   10's complement of result
                                      |   (No carry indicates that the
                                      v   answer is negative and in
                                     -4   the 10's complement form)
```

Steps for 10's complement BCD subtraction

1. Find the 10's complement of a negative number.
2. Add two numbers using BCD addition
3. If carry is not generated find the 10's complement of the result.

5.SIGNED NUMBERS

- Digital systems like computer, must be able to handle both positive and negative numbers.
- A signed binary number consists of both sign and magnitude information.
- The sign indicates whether a number is positive or negative.

5.1 Representation

There are three forms in which the signed integer (whole numbers) can be represented. They include,

1. Sign – Magnitude Form – Rarely used
2. 1's Complement Form
3. 2's Complement Form – Mostly used

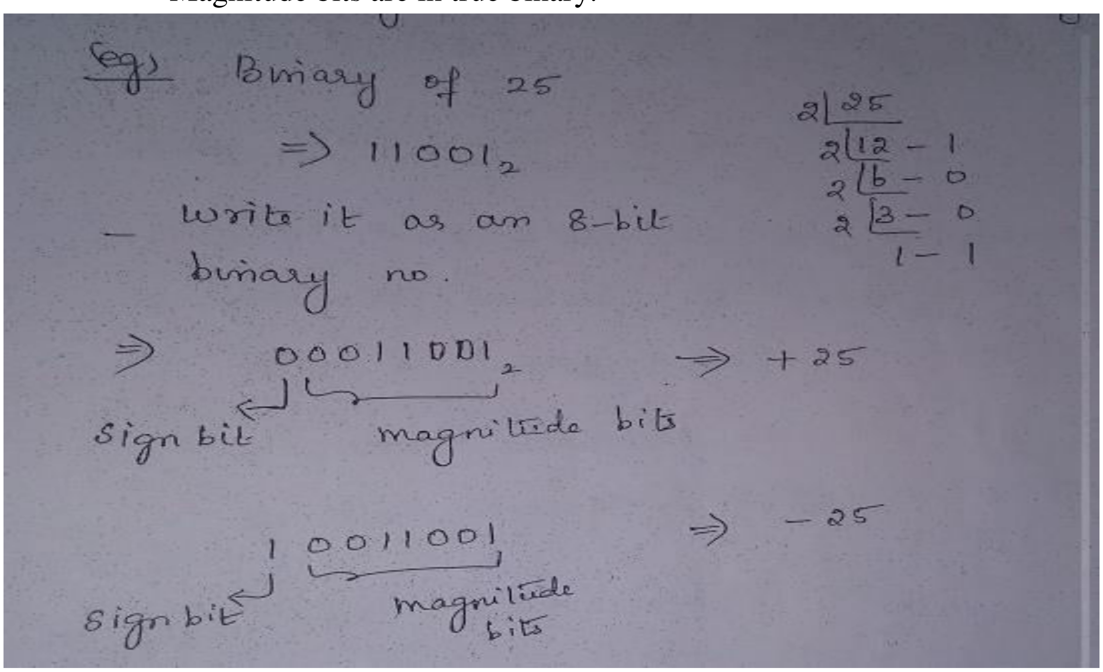
Note:

Sign bit – leftmost bit in a signed binary numbers

- 0 for positive, 1 for negative

5.11 Sign Magnitude Form

- Here, leftmost bit is the sign bit.
- Remaining bits are magnitude bits.
- Magnitude bits are in true binary.



5.12 1's Complement Form

- In this Form, positive numbers are represented the same way as positive sign-magnitude numbers.

- Negative numbers, are the 1's complement of the corresponding positive numbers.

(eg)

+25 is represented as,

00011001 → same as sign-magnitude form

-25 is represented as,

11100110 → 1's complement of +25

5.13 2's Complement Form

- Positive numbers in 2's complement form are represented as same as in sign-magnitude and 1's Complement Form.
- Negative numbers are the 2's complement of the corresponding positive numbers

(eg)

+25 is represented as,

00011001 → same as sign-magnitude form

-25 is represented as,

```
        11100110 +
               1
--------------------------------
        11100111₂  → 2's complement of +25
-------------------------------
```

Decimal value of Signed Numbers

(1) Sign Magnitude

- Decimal values of positive and negative numbers in this form are determined by summing the weights in all the magnitude – bit positions.
- The sign is determined by examining the sign bit.

(eg) 1. Determine the decimal value of this signed binary number expressed in sign – magnitude. 10010101

Soln:

- The seven magnitude bits and their powers of 2 weights are as follows.

  1 0010101

  ↓ $2^6 2^5 2^4 2^3 2^2 2^1 2^0$

  Sign bit

- Summing weights where there are 1's.

  → 16+4+1 = 21

- Since, the sign bit is 1, the decimal number is -21

(2) 1's Complement

- Decimal values of positive numbers in this form are determined by summing the weights in all bit postions.
- Decimal values of negative numbers are determined by assigning a negative value to the weight of the sign bit, summing all the weights where there are 1's and adding 1 to the result.

(eg) Determine the decimal value of the signed binary number expressed in 1's complement 11101000

Soln:

- The bits and their powers- of- two weights are as follows.

Note: for sign bit, it is -2^7 (or) -128

1 1 1 0 1 0 0 0

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

- Summing the weights where there are 1's

-128+64+32+8 = -24 (if +ve, write this as the result)

- Since, it is a negative number, add 1 to the result

-24+1 = -23

(3) 2's Complement

- Decimal values of positive and neagative numbers in this form are determined by summing the weights in all bit positions.
- The weight of the sign – bit in a negative number is given a negative value.

(eg): Determine the decimal values of the signed binary numbers expressed in 2's complement from 10101010

Soln:

- The bits and their corresponding powers of -2 weights are as follows

1 0 1 0 1 0 1 0

-2^7 2^6 2^5 2^4 2^3 2^2 2^1 2^0

- Summing weights where there are 1's

-128+32+8+2 = -86

Range of signed integer numbers that can be Represented

- Since 8-bit (1byte) grouping is common in most computers, the illustrations are all 8-bits. With 8-bits, we can represent 256 different numbers.
- With 16-bits (2 bytes), we can represent 65,536 different numbers.
- With 32-bits (4 bytes), we can represent 4.295×10^9 different numbers.

The formula for finding the number of different combinations of n-bits is,

Total combinations = 2^n

Range of values for n-bit numbers is,

$-(2^{n-1})$ to $+(2^{n-1} - 1)$

So, for 8 bits the range is,

-128 to +127

For 16 bits the range is,

-32768 to +32767 etc

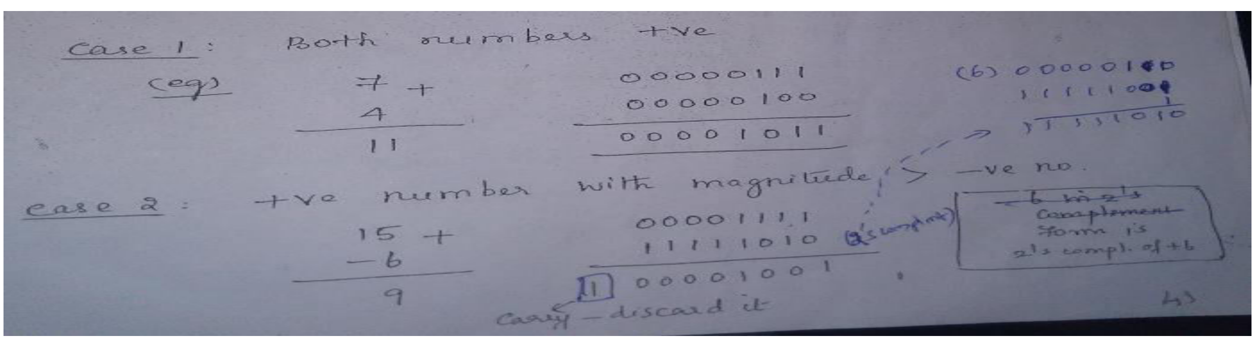
5.2 Arithmetic operations with Signed Numbers

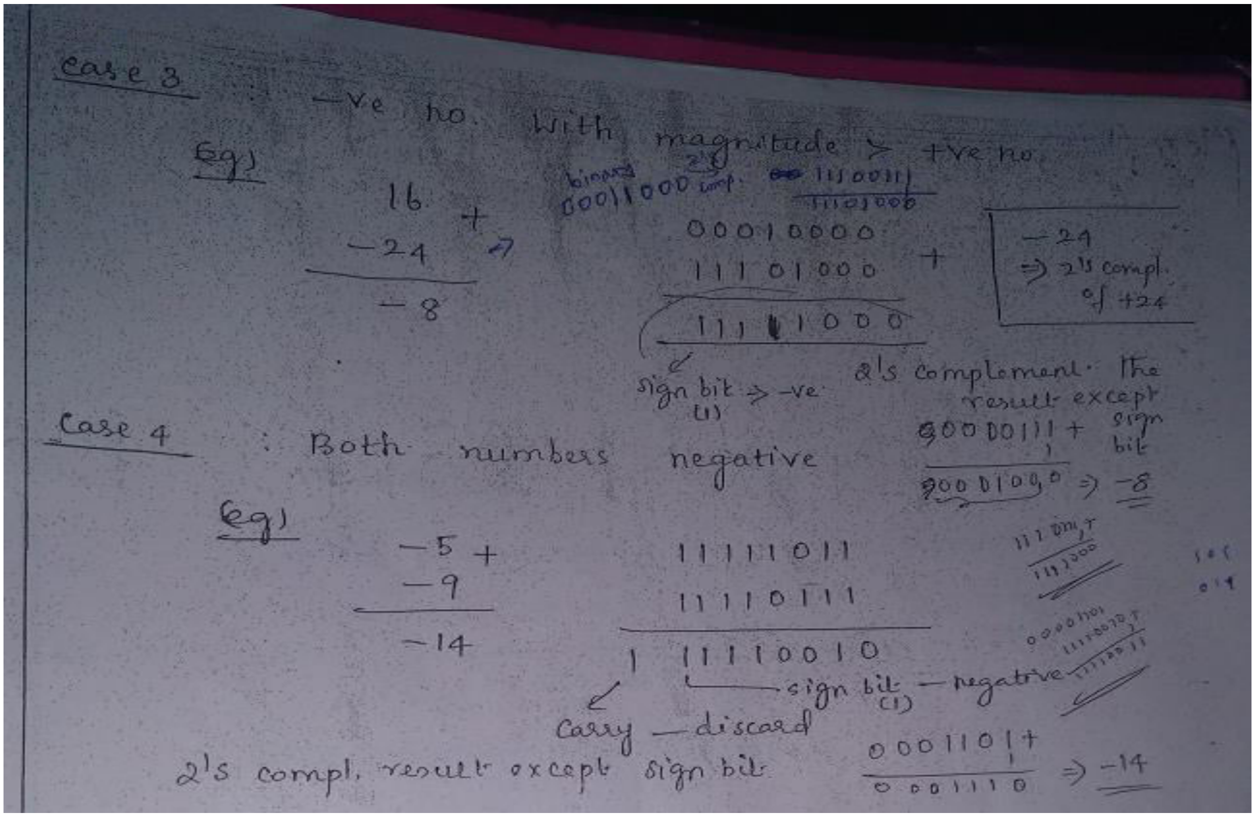
- Here, we use 2's complement representation

Addition

- The two numbers in an addition are the addend and the augend
- The result is sum.
- There are four cases that can occur when two signed binary numbers are added.
  - (1) Both numbers positive.
  - (2) Positive number with magnitude larger than negative number.
  - (3) Negative number with magnitude larger than positive number
  - (4) Both numbers negative.

Case 1: Both numbers +ve





Subtraction

- It is a special case of addition.
- The two numbers in subtraction are subtrahend and minuend.
- The result is the difference.
- To subtract +6 from +9, it is also equivalent to add -6 to +9.
- So, to subtract two signed numbers, take the 2's complement of the subtrahend and add. Discard any final carry bit.

6. BINARY ARITHMETIC

6.1 BINARY ADDITION

The binary addition table is as follows:

A+B	SUM	CARRY
0+0	0	0
0+1	1	0
1+0	1	0
1+1	0	1

Illustration 1:

Add $(1010)_2$ and $(0011)_2$

1010 (Augend)
0011 (Addend)

1101 (sum)

The addition manipulated above as follows.

Step 1: The least significant bits are added, i.e. 0+1 =1 with a carry of 0

Step 2: The carry in the previous is added to the next higher significant bits, i.e. 0+1+1= 0 with a carry 1.
Step 3: The carry in the previous is added to the next higher significant bits, i.e. 1+0+0 =1 with a carry 0.
Step 4: The preceding carry is added to the most significant bit i.e. 0+1+0 = 1 with a carry 0.
Thus the sum is 1101.

6.2 BINARY SUBTRACTION

The binary subtraction table is as follows:

A-B	DIFFERENCE	BORROW
0-0	0	0
0-1	1	1
1-0	1	0
1-1	0	0

Illustration 1:

Subtract $(0101)_2$ from $(1011)_2$

1011 (Minuend)
0101 (Subtrahend)

0110 (Difference)

The steps are described below

Step1: the LSB in the first column are 1 and 1. Hence, the difference is 1 - 1 = 0
Step2: The column, the subtraction is performed as 1 – 0 = 1
Step3: In the third column, the difference is given by 0 – 1 =1
Step 4: In the fourth column (MSB), the difference is given by 0 – 0 = 0 since 1 is borrowed for third column.

6.3 BINARY MULTIPLICATION

The binary multiplication table is as follows:

A *B	PRODUCT
0 * 0	0
0 * 1	0
1 * 0	0
1 * 1	1

- Binary multiplication uses add and shift process
- Binary multiplication is similar to decimal multiplication.

Illustration 1:

Multiplicand * Multiplier
10110.1x01001.1

101101
101101
000000
000000
101101
000000

Partial Product

011010101.11 **(Final product)**

The steps are described below

Step 1: The LSB of the multiplier is taken. If multiplier bit is 1, the multiplicand is copied as such and if the multiplier bit is 0 zero is placed in all the bit positions.
Step 2: The next higher significant bit of the multiplier is taken and, the partial product is written with the shift to the left, as in step 1.
Step 3: step 2 is repeated for all other higher significant bits.
Step 4: The partial product terms are added which gives the actual product of multiplier and the multiplicand.

6.4 BINARY DIVISION:

The binary division table is as follows:

A÷B	Result
0÷0	Not allowed
0÷1	0
1÷0	Not allowed
1÷1	1

- Binary division uses subtract and shift process
- Binary division is similar to decimal division.
- Division by 0 is meaningless.

Illustration 1:

Dividend ÷ Divisor

11011.1÷ 101

```
               101.1          (QUOTIENT)
DIVISOR 101  √11011.1          (DIVIDEND)
              101
             --------------
              111
              101
             --------------
              101
              101
             --------------
                0
             --------------
```

7.BINARY CODES

Binary codes are codes which are represented in binary system with modification from the original one. The group of symbols is called as a code. The digital data is represented, stored and transmitted as group of binary bits. This group is also called as binary code. The binary code is represented by the number as well as alphanumeric letter.

Advantages of Binary Code

Following is the list of advantages that binary code offers.

1. Binary codes are suitable for the computer applications.
2. Binary codes are suitable for the digital communications.
3. Binary codes make the analysis and designing of digital circuits if we use the binary codes.
4. Since only 0 and 1 are being used, implementation becomes easy.

7.1 Classification of binary codes:The codes are broadly categorized into following four categories.

- Weighted Codes
- Non-Weighted Codes
- Binary Coded Decimal Code
- Alphanumeric Codes

- Error Codes

7.1.1 Weighted codes: Weighted binary codes are those binary codes which obey the positional weight principle. Each position of the number represents a specific weight

Decimal	8421	5421	2421	5211
0	0000	0000	0000	0000
1	0001	0001	0001	0001
2	0010	0010	0010	0011
3	0011	0011	0011	0101
4	0100	0100	0100	0111
5	0101	1000	1011	1000
6	0110	1001	1100	1010
7	0111	1010	1101	1100
8	1000	1011	1110	1110
9	1001	1100	1111	1111

For example, in 8421BCD code, 1001 the weights of 1, 0, 0, 1 (from left to right) are 8, 4, 2 and 1 respectively. The codes 8421BCD, 2421BCD, 5211BCD are all weighted codes.

7.1.2 Non-weighted codes: The non-weighted codes are not positionally weighted. In other words, each digit position within the number is not assigned a fixed value (or weight).

Examples are

- Excess-3
- Gray code

DECIMAL	EXCESS - 3	GRAY CODE
0	0011	0000
1	0100	0001
2	0101	0011

6.1.3 EXCESS – 3 CODES:-

- This is another form of BCD code, in which each decimal digit is coded into a 4-bit binary code.
- The code for each decimal digit is obtained by adding decimal 3 to the natural BCD code of the digit.

GRAY CONVERSION:-

- Record the mostsignificant bit add the binary MSB to the next significant bit of the Gray code.
- Record the result, ignoring carrier continue the process, until the LSB is reached.

REFLECTIVE CODES: A code is reflective when the code is self-complementing. In otherwords, when the code for 9 is the complement the code for 0, 8 for 1, 7 for 2, 6 for 3 and 5 for 4. 2421BCD, 5421BCD and Excess-3 code are reflective codes.

SEQUENTIAL CODES: In sequential codes, each succeeding 'code is one binary number greater than its preceding code. This property helps in manipulation of data. 8421 BCD and Excess-3 are sequential codes.

ALPHANUMERIC CODES: Codes used to represent numbers, alphabetic characters, symbols and various instructions necessary for conveying intelligible information. ASCII, EBCDIC, UNICODE are the most-commonly used alphanumeric codes.

8.Decimal code

Binary codes for decimal digits require a minimum of four bits. Numerous different codes can be obtained by arranging four or more bits in ten distinct possible combinations. A few possibilities are tabulated.

DECIMAL DIGIT	8421	84-2-1	7421	5421	2421	BIQUINARY
	8 4 2 1	8 4 -2 -1	7 4 2 1	5 4 2 1	2 4 2 1	5 0 4 3 2 1 0
0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 1 0 0 0 0 1
1	0 0 0 1	0 1 1 1	0 0 0 1	0 0 0 1	0 0 0 1	0 1 0 0 0 1 0
2	0 0 1 0	0 1 1 0	0 0 1 0	0 0 1 0	0 0 1 0	0 1 0 0 1 0 0
3	0 0 1 1	0 1 0 1	0 0 1 1	0 0 1 1	0 0 1 1	0 1 0 1 0 0 0
4	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 0	0 1 0 0	0 1 1 0 0 0 0
5	0 1 0 1	1 0 1 1	0 1 0 1	0 1 0 1	1 0 1 1	1 0 0 0 0 0 1
6	0 1 1 0	1 0 1 0	0 1 1 0	0 1 1 0	1 1 0 0	1 0 0 0 0 1 0
7	0 1 1 1	1 0 0 1	1 0 0 0	0 1 1 1	1 1 0 1	1 0 0 0 1 0 0
8	1 0 0 0	1 0 0 0	1 0 0 1	1 0 1 1	1 1 1 0	1 0 0 1 0 0 0
9	1 0 0 1	1 1 1 1	1 0 1 0	1 1 0 0	1 1 1 1	1 0 1 0 0 0 0

9.Error detection code

In data transmission, Interference and physical defects in the communication medium can cause random bit errors. As the signal is transmitted through a media, the signal gets corrupted because of noise and distortion. Therefore the media is not reliable. To achieve a reliable communication through this unreliable media, there is need for detecting the error in the signal so that suitable mechanism can be devised to take corrective actions.

Error coding is a method of detecting and correcting these errors to ensure information is transferred intact from its source to its destination

The errors can be divided into two types:

• Single-bit Error: only one bit of given data unit (such as a byte, character, or data unit) is changed from 1 to 0 or from 0 to 1.

• Burst Error: two or more bits in the data unit have changed from 0 to 1 or vice-versa. (Here doesn't necessary means that error occurs in consecutive bits)

Error Detecting Codes:

Basic approach used for error detection is the use of redundancy, where additional bits are added to facilitate detection and correction of errors.

Popular techniques are:

• Simple Parity check
• Two-dimensional Parity check
• Checksum
• Cyclic redundancy check

Detecting Errors using simple parity check

Suppose we are transmitting 7-bit ASCII characters. A parity bit is added to each character to make it 8 bits. Parity can detect all single-bit errors

–If even parity is used and a single bit changes, it will change the parity to odd, which will be detected at the receiver end

–The receiver end can detect the error, but cannot correct it because it does not know which bit is erroneous

Parity can also detect some multiple-bit errors

Table 1 shows the four bit data word and its corresponding code words

Decimal value	Data block	Parity bit	Code word
0	0000	0	00000
1	0001	1	00011
2	0010	1	00101

3	0011	0	00110
4	0100	1	01001
5	0101	0	01010
6	0110	0	01100
7	0111	1	01111
8	1000	1	10001
9	1001	0	10010
10	1010	0	10100
11	1011	1	10111
12	1100	0	11000
13	1101	1	11011
14	1110	1	11101
15	1111	0	11110

10.Gray Code- Reflection and Self Complementary codes

- Gray Code is a non-weighted code which belongs to a class of codes called minimum change codes.
- Gray Code is an alternative binary representation, devised such that, between any two adjacent numbers, ***only one bit*** changes at a time.

Binary	Dec	Gray
00000	0	00000
00001	1	00001
00010	2	00011
00011	3	00010
00100	4	00110
00101	5	00111
00110	6	00101
00111	7	00100
01000	8	01100
01001	9	01101
01010	10	01111
01011	11	01110
01100	12	01010
01101	13	01011
01110	14	01001
01111	15	01000

- To the left we see three columns of data. These are representations of the same numbers 0-15 in different ways.
    - In the middle is the decimal value.
    - On the left is positional notation binary
    - On the right is Gray code.
- You will notice that, on the right, each adjacent row is different from it's neighbours by no more than one bit.
- The term Gray code is often used to refer to a "reflected" code, or more specifically still, the binary reflected Gray code.

10.1 Self-complementary Code

• A code is said to be self-complementary if the code for 9's complement of N i.e. 9-N can be obtained by interchanging all 0s and 1s.
• Decimal 9 is the complement of code for 0, 8 for 1, 7 for 2 and so on.
• For a code to be self complementing, the sum of all its weights must be 9. digit.8421 and 5421 codes are not self complementing codes whereas 5211,2421,3321, 4321 are self complementing.
• In general, a code is self-complementary if we produce a code by taking the first complement of the digit which is same as 9's complement of the number.

10.2 Reflective code

- Imaged about the centre entries with one bit changed
- Example ï 9ís complement of a reflected BCD code word is formed by changing only one of its bits
- In the Gray code example shown below, the MSB bit alone is changing and the remaining bits is reflected mirror image about the centre. For clarity, the MSB is removed.
- Gray code Reflected property of Gray code

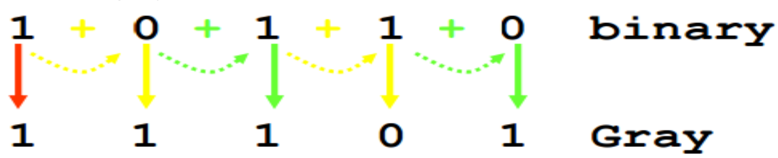
```
0000
0001
0011
0010
0110
0111
0101
0100
1100
1101
1111
1110
1010
1011
1001
1000
```

```
x000
x001
x011
x010
x110
x111
x101
x100
-------- mirror
x100
x101
x111
x110
x010
x011
x001
x000
```

Binary-to-Gray code conversion

- The MSB in the Gray code is the same as corresponding MSB in the binary number.
- Going from left to right, add each adjacent pair of binary code bits to get the next Gray code bit.
- Discard carries.

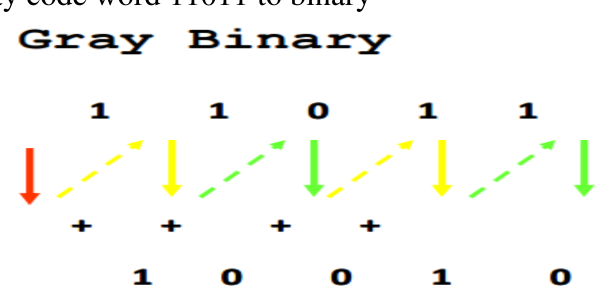
Problem: Convert 10110 to gray code



Gray-to-Binary Conversion

- The MSB in the binary code is the same as the corresponding bit in the Gray code.
- Add each binary code bit generated to the Gray code bit in the next adjacent position.
- Discard carries.

Problem: Convert the Gray code word 11011 to binary



11. Binary-Coded Decimal Code

Although the binary number system is the most natural system for a computer because it is readily represented in today's electronic technology, most people are more accustomed to the decimal system. One way to resolve this difference is to convert decimal numbers to binary, perform all arithmetic calculations in binary, and then convert the binary results back to decimal. This method requires that we store decimal numbers in the computer so that they can be converted to binary. Since the computer can accept only binary values, we must represent the decimal digits by means of a code that contains 1's and 0's. It is also possible to perform the arithmetic operations directly on decimal numbers when they are stored in the computer in coded form.

A binary code will have some unassigned bit combinations if the number of elements in the set is not a multiple power of 2. The 10 decimal digits form such a set. A binary code that distinguishes among 10 elements must contain at least four bits, but 6 out of the 16 possible combinations remain unassigned. Different binary codes can be obtained by arranging four bits into 10 distinct combinations. This scheme is called ***binary-coded decimal*** and is commonly referred to as **BCD**.

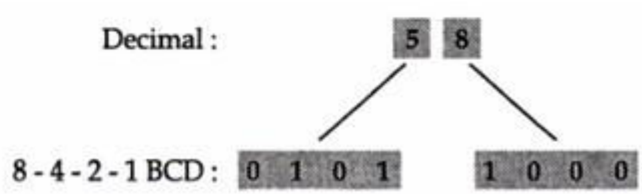
A number with *k* decimal digits will require 4*k* bits in BCD. Decimal 396 is represented in BCD with 12 bits as 0011 1001 0110, with each group of 4 bits representing one decimal digit. A decimal number in BCD is the same as its equivalent binary number only when the number is between 0 and 9. A BCD number greater than 10 looks different from its equivalent binary number, even though both contain 1's and 0's. Note that the BCD code is not self-complementing. Moreover, the binary combinations 1010 through 1111 are not used and have no meaning in BCD. Consider decimal 185 and its corresponding value in BCD and binary:

$(185)_{10} = (0001\ 1000\ 0101)_{BCD} = (10111001)_2$

Decimal	BCD Code			
Digit	8	4	2	1
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

Table 1

In multi digit BCD coding

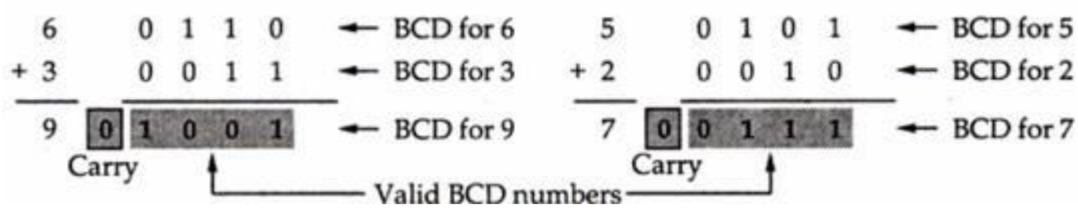


11.1 BCD addition:

The addition of two BCD numbers can be best understood by considering the three cases that occur when two BCD digits are added.

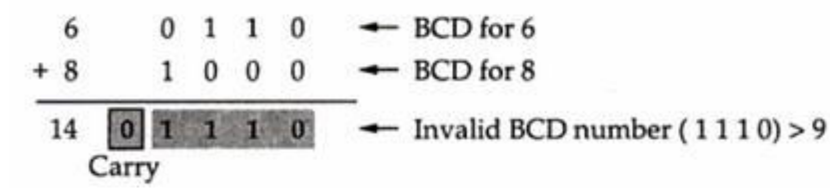
Sum equals 9 or less with carry 0

Let us consider additions of 3 and 6 in BCD.

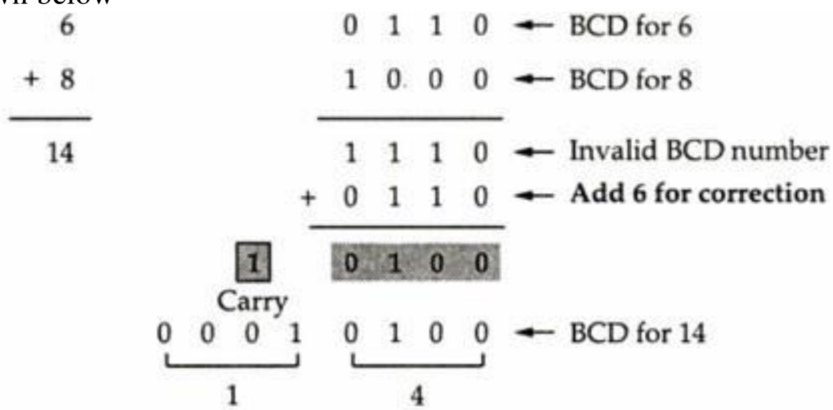


Sum greater than 9 with carry 0

Let us consider addition of 6 and 8 in BCD

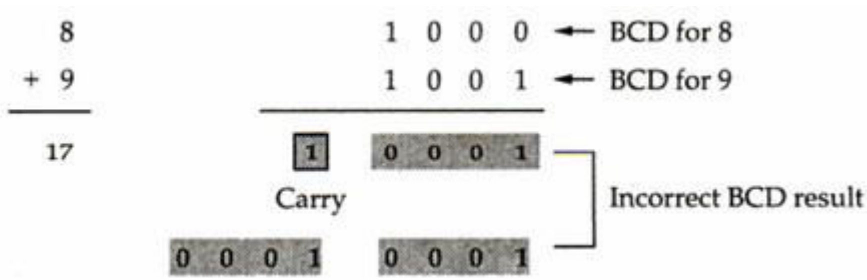


The sum 1110 is an invalid BCD number. This has occurred because the sum of the two digits exceeds 9. Whenever this occurs this occurs the sum has to be corrected by the addition of six (1110) in the invalid BCD number, as shown below

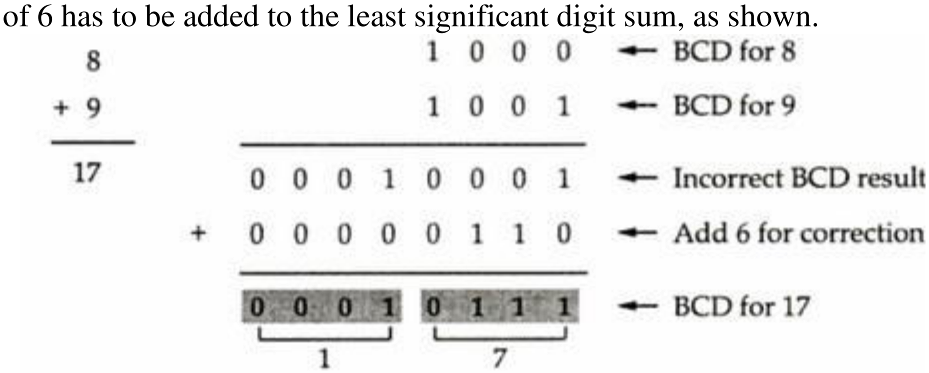


Sum equals 9 or less with carry 1

Let us consider addition of 8 and 9 in BCD



In this case, result (001 0001) is valid BCD number, but it is incorrect. To get the correct BCD result correction factor of 6 has to be added to the least significant digit sum, as shown.



BCD addition procedure

1. Add two BCD numbers using ordinary binary addition.
2. If four bit sum is equal to or less than 9, no correction is needed. The sum is in proper BCD form.

3. If the four bit sum is greater than 9 or if a carry is generated from the four-bit sum, the sum is invalid.
4. To correct the invalid sum, add 0110_2 to the four-bit sum. If a carry results from this addition, add it to the next higher-order BCD digit.

Example 1 : *Perform the code conversion :*

$(137)_{10}$ = (?) NBCD

Solution : NBCD = 8421 BCD

$\therefore$ $(137)_{10}$ = $(0001\ \ 0011\ \ 0111)_{NBCD}$

Example 2 : *Perform each of the following decimal additions in 8-4-2-1 BCD.*

a) 24 + 18 *b)* 48 + 58

Solution :

```
a)   24      0 0 1 0      0 1 0 0
   + 18      0 0 0 1      1 0 0 0
   ----     ---------    ---------
     42      0 0 1 1      1 1 0 0      Invalid BCD number 1 1 0 0 > 9
                      +   0 1 1 0      Add 6 for correction
            ----------------------
             0 0 1 1    1 0 0 1 0
        +           1 <-'              Propagate carry to next higher digit
        --------------------------
             0 1 0 0      0 0 1 0      BCD for 42
               4            2

b)   48    0 1 0 0      1 0 0 0
   + 58  + 0 1 0 1      1 0 0 0
   ----   ---------------------
    106    1 0 0 1    1 0 0 0 0
                  1 <-' 0 1 1 0      Propagate carry and
                                     Add 6 for correction
          ---------------------
           1 0 1 0      0 1 1 0      1 0 1 0 > 9 so add 6 for correction
           0 1 1 0
          ---------------------
       1   0 0 0 0      0 1 1 0      Corrected sum
0 0 0 1    0 0 0 0      0 1 1 0
   1          0            6
```

12.Alphanumeric codes

Alphanumeric codes are sometimes called character codes due to their certain properties. Now these codes are basically binary codes. We can write alphanumeric data, including data, letters of the alphabet, numbers, mathematical symbols and punctuation marks by this code which can be easily understandable and can be processed by the computers. Input output devices such as keyboards, monitors, mouse can be interfaced using these codes. 12-bit Hollerith code is the better known and perhaps the first effective code in the days of evolving computers in early days. During this period punch cards were used as the inputting and outputting data. But nowadays these codes are termed obsolete as many other modern codes have evolved. The most common **alphanumeric codes** used these days are **ASCII code**, **EBCDIC code** and Unicode.

12.1 ASCII Character Code

Many applications of digital computers require the handling not only of numbers, but also of other characters or symbols, such as the letters of the alphabet. For instance, consider a high-tech company with thousands of employees. To represent the names and other pertinent information, it is necessary to formulate a binary code for the letters of the alphabet. In addition, the same binary code must represent numerals and special characters (such as $). An alphanumeric character set is a set of elements that includes the 10 decimal digits, the 26 letters of the alphabet, and a number of special characters. Such a set contains between 36 and 64 elements if only capital letters are included, or between 64 and 128 elements if both uppercase and lowercase letters are included. In the first case, we need a binary code of six bits, and in the second, we need a binary code of seven bits. The standard binary code for the alphanumeric characters is the **American Standard Code for Information Interchange (ASCII)**, which uses seven bits to code 128 characters, as shown in Table below. The seven bits of the code are designated by $b1$ through $b7$, with $b7$ the most significant bit. The letter *A*, for example, is represented in ASCII as 1000001 (column 100, row 0001). The ASCII code also contains 94 graphic characters that can be printed and 34 nonprinting characters used for various control functions.

The graphic characters consist of the 26 uppercase letters (A through Z), the 26 lowercase letters (a through z), the 10 numerals (0 through 9), and 32 special printable characters, such as %, *, and $.characters. Format effectors are characters that control the layout of printing. They include the familiar word processor and typewriter controls such as backspace (BS), horizontal tabulation (HT), and carriage return (CR). Information separators are used to separate the data into divisions such as paragraphs and pages. They include characters such as record separator (RS) and file separator (FS). The communication-control characters are useful during the transmission of text between remote devices so that it can be distinguished from other messages using the same communication channel before it and after it. Examples of communication-control characters are STX (start of text) and ETX (end of text), which are used to frame a text message transmitted through a communication channel.

ASCII is a seven-bit code, but most computers manipulate an eight-bit quantity as a single unit called a *byte*. Therefore, ASCII characters most often are stored one per byte. The extra bit is sometimes used for other purposes, depending on the application.

For example, some printers recognize eight-bit ASCII characters with the most significant bit set to 0. An additional 128 eight-bit characters with the most significant bit set to 1 are used for other symbols, such as the Greek alphabet or italic type font.

DEC	OCT	HEX	BIN	Symbol	Description
0	000	00	00000000	NUL	Null char
1	001	01	00000001	SOH	Start of Heading
2	002	02	00000010	STX	Start of Text
3	003	03	00000011	ETX	End of Text
4	004	04	00000100	EOT	End of Transmission
5	005	05	00000101	ENQ	Enquiry
6	006	06	00000110	ACK	Acknowledgment
7	007	07	00000111	BEL	Bell
8	010	08	00001000	BS	Back Space
9	011	09	00001001	HT	Horizontal Tab
10	012	0A	00001010	LF	Line Feed
11	013	0B	00001011	VT	Vertical Tab
12	014	0C	00001100	FF	Form Feed
13	015	0D	00001101	CR	Carriage Return
14	016	0E	00001110	SO	Shift Out / X-On
15	017	0F	00001111	SI	Shift In / X-O

12.2 EBCDIC

The EBCDIC stands for Extended Binary Coded Decimal Interchange Code. IBM invented this code to extend the Binary Coded Decimal which existed at that time. All the IBM computers and peripherals use this code. It is an 8 bit code and therefore can accommodate 256 characters. Below is given some characters of **EBCDIC code** to get familiar with it.

Char	EBCDIC	HEX	Char	EBCDIC	HEX	Char	EBCDIC	HEX
A	1100 0001	C1	P	1101 0111	D7	4	1111 0100	F4
B	1100 0010	C2	Q	1101 1000	D8	5	1111 0101	F5
C	1100 0011	C3	R	1101 1001	D9	6	1111 0110	F6
D	1100 0100	C4	S	1110 0010	E2	7	1111 0111	F7
E	1100 0101	C5	T	1110 0011	E3	8	1111 1000	F8
F	1100 0110	C6	U	1110 0100	E4	9	1111 1001	F9
G	1100 0111	C7	V	1110 0101	E5	blank	...	...
H	1100 1000	C8	W	1110 0110	E6	.	...	...
I	1100 1001	C9	X	1110 0111	E7	(	...	...
J	1101 0001	D1	Y	1110 1000	E8	+	...	...
K	1101 0010	D2	Z	1110 1001	E9	$	...	...
L	1101 0011	D3	0	1111 0000	F0	*	...	...
M	1101 0100	D4	1	1111 0001	F1	)	...	...
N	1101 0101	D5	2	1111 0010	F2	-	...	...
O	1101 0110	D6	3	1111 0011	F3	/		

13. HAMMING CODE-ERROR DETECTION AND CORRECTION

Hamming code is a set of error-correction code s that can be used to detect and correct bit errors that can occur when computer data is moved or stored.

13.1 Error Detecting Codes

Basic approach used for error detection is the use of redundancy, where additional bits are added to facilitate detection and correction of errors. Popular techniques are: • Simple Parity check • Two-dimensional Parity check • Checksum • Cyclic redundancy check

Simple Parity Checking or One-dimension Parity Check The most common and least expensive mechanism for error- detection is the simple parity check. In this technique, a redundant bit called parity bit, is appended to every data unit so that the number of 1s in the unit (including the parity becomes even). Blocks of data from the source are subjected to a check bit or Parity bit generator form, where a parity of 1 is added to the block if it contains an odd number of 1's (ON bits) and 0 is added if it contains an even number of 1's. At the receiving end the parity bit is computed from the received data bits and compared with the received parity bit, as shown in Fig 1. This scheme makes the total number of 1's even, that is why it is called even parity checking. Considering a 4-bit word, different combinations of the data words and the corresponding code words are given in Table 1. Note that for the sake of simplicity, we are discussing here the even-parity checking, where the number of 1's should be an even number. It is also possible to use odd-parity checking, where the number of 1's should be odd.

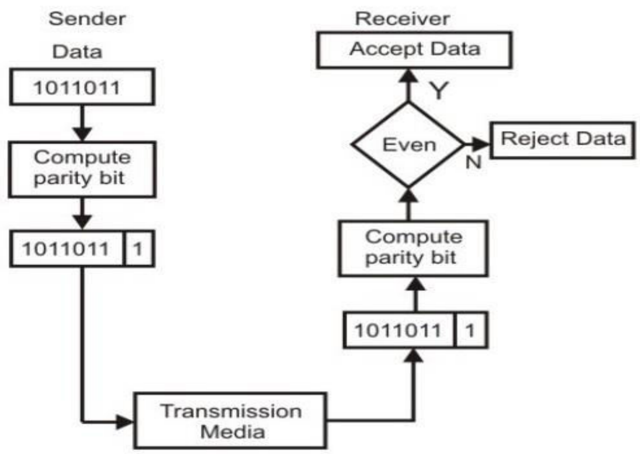


Fig 1) Even parity checking scheme

Decimal value	Data Block	Parity bit	Code word
0	0000	0	00000
1	0001	1	00011
2	0010	1	00101
3	0011	0	00110
4	0100	1	01001
5	0101	0	01010
6	0110	0	01100
7	0111	1	01111
8	1000	1	10001
9	1001	0	10010
10	1010	0	10100
11	1011	1	10111
12	1100	0	11000
13	1101	1	11011
14	1110	1	11101
15	1111	0	11110

Table 1:Possible 4 bit data words and corresponding code words

Two-dimension Parity Check

Performance can be improved by using two-dimensional parity check, which organizes the block of bits in the form of a table. Parity check bits are calculated for each row, which is equivalent to a simple parity check bit. Parity check bits are also calculated for all columns then both are sent along with the data. At the receiving end these are compared with the parity bitcalculated on the received data. This is illustrated in Fig. 2. Performance Two- Dimension Parity Checking increases the likelihood of detecting burst errors. As we have shown in Fig. 2, that a 2-D Parity check of n bits can detect a burst error of n bits. A burst error of more than n bits is also detected by 2-D Parity check with a highprobability. There is, however, one pattern of error that remains elusive. If two bits in one data unit are damaged and two bits in exactly same position in another data unit are also damaged, the 2-D Parity check checker will not detect an error. For example, if two data units: 11001100 and 10101100. If first and second from last bits in each of them is changed, making the data units as 01001110 and 00101110, the error cannot be detected by 2-D Parity check.

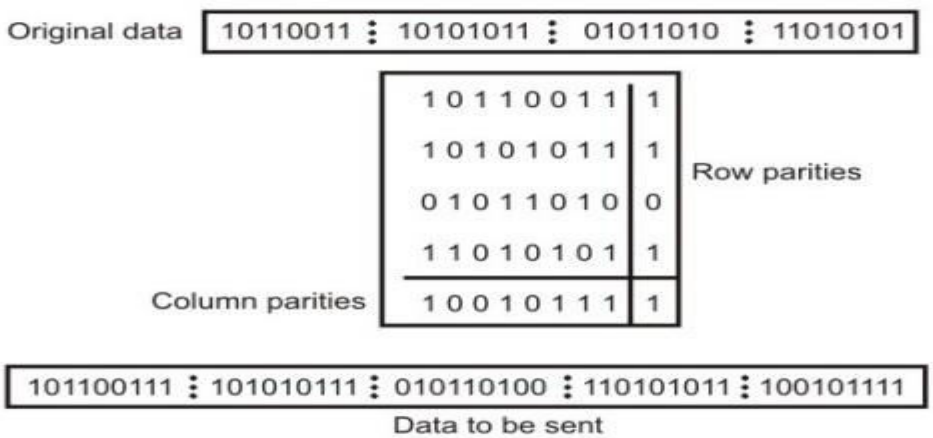


Fig 2) Two dimension parity checking

Example of Hamming Code Generation

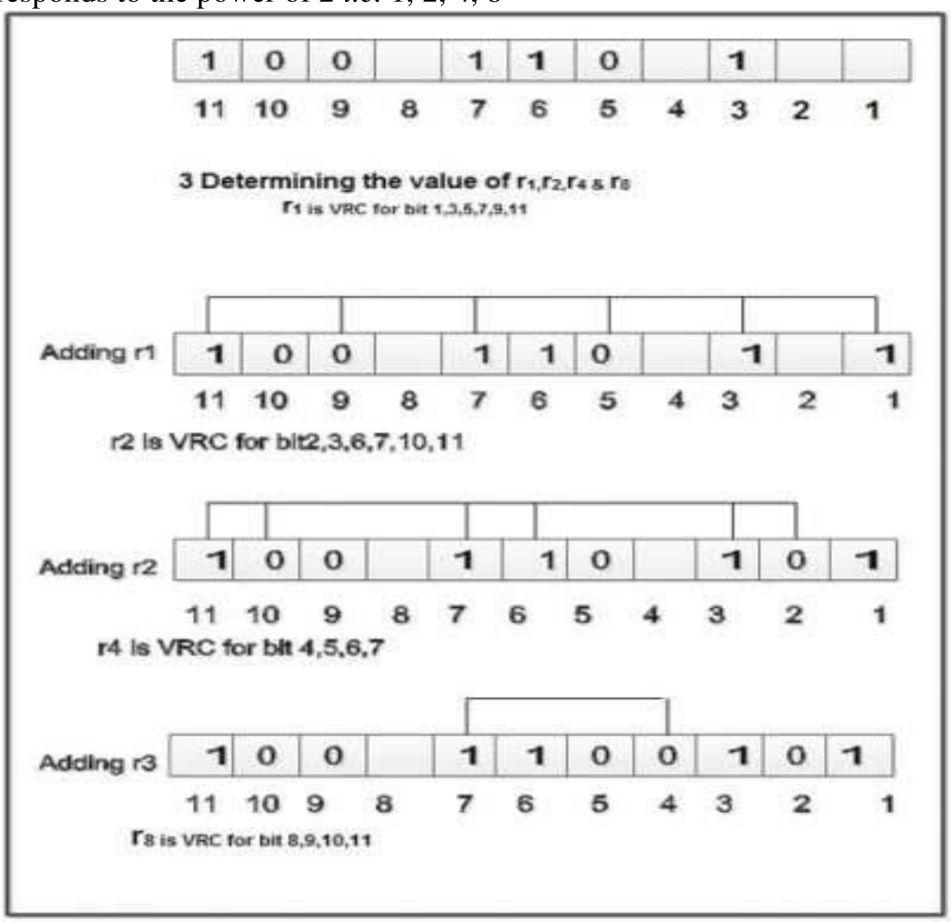
Suppose a binary data 1001101 is to be transmitted. To implement hamming code for this, following steps are used:

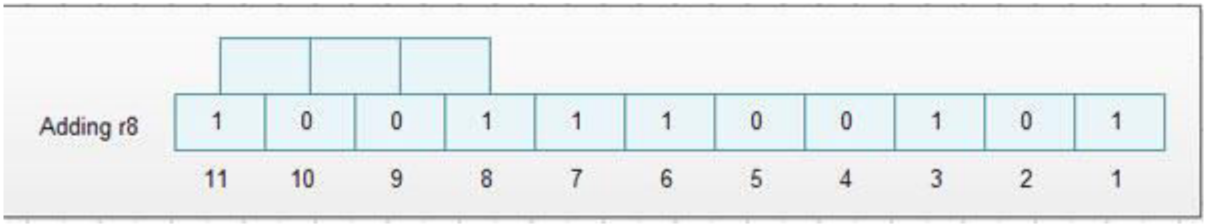
1. Calculating the number of redundancy bits required. Since number of data bits is 7, the value of r is calculated as

$$2^r \geq m + r + 1$$
$$2^4 \geq 7 + 4 + 1$$

Therefore no. of redundancy bits = 4

2. Determining the positions of various data bits and redundancy bits. The various r bits are placed at the position that corresponds to the power of 2 *i.e.* 1, 2, 4, 8

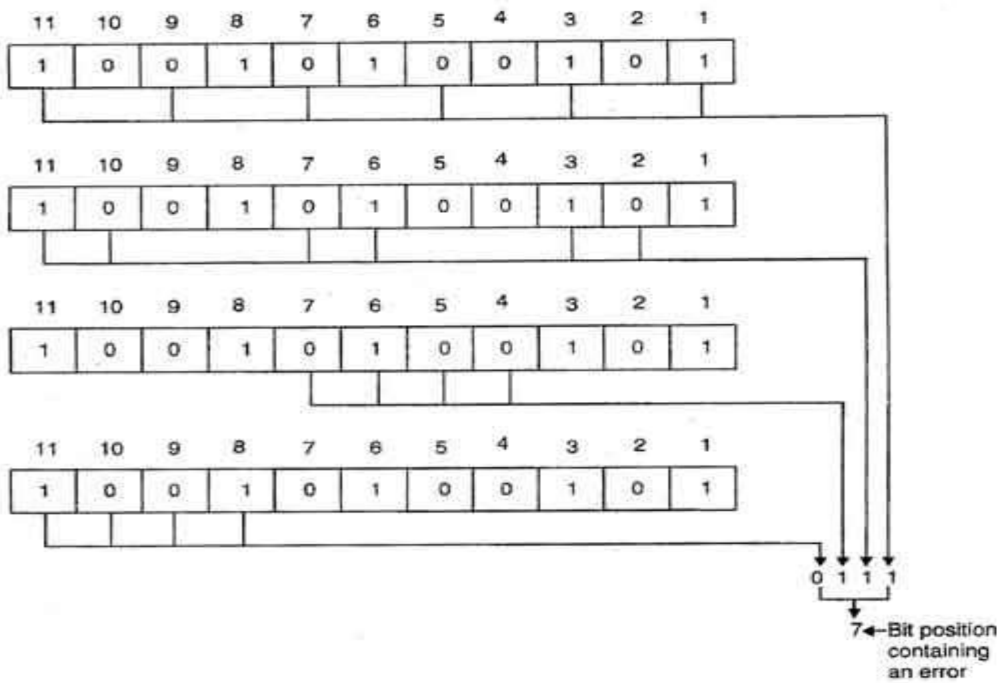




4. Thus data 1 0 0 1 1 1 0 0 1 0 1 with be transmitted.

13.1 Error Detection & Correction

Considering a case of above discussed example, if bit number 7 has been changed from 1 to 0.The data will be erroneous.



Data sent: 1 0 0 1 1 1 0 0 1 0 1

Data received: 1 00 1 0 1 00 1 0 1 (seventh bit changed)

The receive takes the transmission and recalculates four new VRCs using the same set of bits used by sender plus the relevant parity (r) bit for each set as shown in fig.

Then it assembles the new parity values into a binary number in order of r position (r_8, r_4, r_2, r_1).

In this example, this step gives us the binary number 0111. This corresponds to decimal 7. Therefore bit number 7 contains an error. To correct this error, bit 7 is reversed from 0 to 1.

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UNIT II-BOOLEAN ALGEBRA AND LOGIC GATES

Axiomatic definitions of Boolean Algebra - Basic Theorems and Properties of Boolean Algebra - Boolean Functions- Canonical and Standard forms - Digital Logic Gates- Simplification of Boolean Expressions, The map method- SOP and POS - NAND and NOR implementation - Don’t Cares - The Tabulation Method - Determination and Selection of Prime Implicants.

2.1 Axiomatic Definition of Boolean algebra

1. Closure
   a. Closure with respect to (wrt) OR (+)
   b. Closure with respect to AND (·)
2. Identity
   a. Identity element wrt to OR : 0
   b. Identity element wrt to AND : 1
3. Commutative Property
   a. Commutative Property wrt to OR : $x + y = y + x$
   b. Commutative Property wrt to AND : $x \cdot y = y \cdot x$
4. Distributive Property
   $X \cdot (y + z) = (x \cdot y) + (x \cdot z)$
   $x + (y \cdot z) = (x + y)(x + z)$
5. Existence of Complement
   $x + x' = 1$
   $x \cdot x' = 0$

Precedence:

(1) Parentheses (2) NOT (3) AND (4) OR

2. 2Basic Theorems and Properties of Boolean algebra

Operations with 0 and 1:

- $X + 0 = X$
- $X \bullet 1 = X$
- $X + 1 = 1$
- $X \bullet 0 = 0$

Idempotent laws

- $X + X = X$
- $X \bullet X = X$

Involution law:

- $(X')' = X$

Laws of complementarity:

- $X + X' = 1$
- $X \bullet X' = 0$

Commutative laws:

- $X + Y = Y + X$
- $X \bullet Y = Y \bullet X$

Associative laws:

- $(X + Y) + Z = X + (Y + Z) = X + Y + Z$
- $(XY)Z = X(YZ) = XYZ$

Distributive laws:

- $X(Y + Z) = XY + XZ$
- $X + YZ = (X + Y) (X + Z)$

Simplification theorems:

- $X Y + X Y' = X$
- $(X + Y) (X + Y') = X$
- $X + XY = X$
- $X (X + Y) = X$
- $(X + Y') Y = XY$
- $XY' + Y = X + Y$

DeMorgan’s laws:

There are two “de Morgan´s” rules or theorems,

- Two separate terms NOR´ed together is the same as the two terms inverted (Complement) and AND´ed for example, $(X+Y)' = X'.Y'$.
- Two separate terms NAND´ed together is the same as the two terms inverted (Complement) and OR´ed for example, $(X.Y)' = X' + Y'$.

Duality:

“Every algebraic expression deducible from the postulates of Boolean Algebra remains valid if the operations and identity elements are interchanged.”

- $(X + Y + Z + \dots)^D = X Y Z \dots$
- $(X Y Z \dots)^D = X + Y + Z + \dots$
- $[f (X1, X2, \dots XN, 0, 1, +, \bullet)]^D = f (X1, X2, \dots XN, 1, 0, \bullet, +)$

3. Boolean Functions

A simple 2-input AND, OR and NOT Gates can be represented by 16 possible functions as shown in the following table.

3.1 Laws of Boolean Algebra

Function		Description	Expression
1.	NULL	0	
2.	IDENTITY	1	
3.	Input	A	A
4.	Input	B	B
5.	NOT	A	A’
6.	NOT	B	B’
7.	A AND B (AND)	A . B	
8.	A AND NOT B	A . B’	
9.	NOT A AND B	A’ . B	
10.	NOT A AND NOT B (NAND)		A’ . B’
11.	A OR B (OR)	A + B	
12.	A OR NOT B	A + B’	
13.	NOT A OR B	A’ + B	
14.	NOT OR (NOR)	(A + B)’	
15.	Exclusive-OR	A.B’ + A’.B	
16.	Exclusive-NOR	A’.B’ + A.B	

Example

Using the above laws, simplify the following expression: (A + B)(A + C)

Q =	(A + B).(A + C)	
	A.A + A.C + A.B + B.C	– Distributive law
	A + A.C + A.B + B.C	– Idempotent AND law (A.A = A)
	A(1 + C) + A.B + B.C	– Distributive law
	A.1 + A.B + B.C	– Identity OR law (1 + C = 1)
	A(1 + B) + B.C	– Distributive law
	A.1 + B.C	– Identity OR law (1 + B = 1)
Q =	A + (B.C)	– Identity AND law (A.1 = A)

Then the expression: (A + B)(A + C) can be simplified to A + (B.C) as in the Distributive law.

4. Canonical and Standard Forms

Logical functions are generally expressed in terms of different combinations of logical variables with their true forms as well as the complement forms. Binary logic values obtained by the logical functions and logic variables are in binary form. An arbitrary logic function can be expressed in the following forms.

(i) Sum of the Products (SOP)
(ii) Product of the Sums (POS)

Product Term:

In Boolean algebra, the logical product of several variables on which a function depends is considered to be a product term. In other words, the AND function is referred to as a product term or standard product. The variables in a product term can be either in true form or in complemented form. For example, ABC′ is a product term.

Sum Term:

An OR function is referred to as a sum term. The logical sum of several variables on which a function depends is considered to be a sum term. Variables in a sum term can also be either in true form or in complemented form. For example, A + B + C′ is a sum term.

Sum of Products (SOP):

The logical sum of two or more logical product terms is referred to as a sum of products expression. It is basically an OR operation on AND operated variables. For example, Y = AB + BC + AC or Y = A′B + BC + AC′ are sum of products expressions.

Product of Sums (POS):

Similarly, the logical product of two or more logical sum terms is called a product of sums expression. It is an AND operation on OR operated variables. For example, Y = (A + B + C)(A + B′ + C)(A + B + C′) or Y = (A + B + C)(A′ + B′ + C′) are product of sums expressions.

Standard form:

The standard form of the Boolean function is when it is expressed in sum of the products or product of the sums fashion. The examples stated above, like Y =AB + BC + AC or Y = (A + B + C)(A + B′ + C)(A + B + C′) are the standard forms. However, Boolean functions are also sometimes expressed in nonstandard forms like F = (AB + CD)(A′B′ + C′D′), which is neither a sum of products form nor a product of sums form. However, the same expression can be converted to a standard form with help of various Boolean properties, as:

F = (AB + CD)(A′B′ + C′D′) = A′B′CD + ABC′D′

4.1 Minterm

A product term containing all n variables of the function in either true or complemented form is called the minterm. Each minterm is obtained by an AND operation of the variables in their true form or complemented form. For a two-variable function, four different combinations are possible, such as, A′B′, A′B, AB′, and AB. These product terms are called the fundamental products or standard products or minterms. In the minterm, a variable will possess the value 1 if it is in true or uncomplemented form, whereas, it contains the value 0 if it is in complemented form. For three variables function, eight minterms are possible as listed in the following table

A	B	C	Minterm
0	0	0	A’B’C’
0	0	1	A’B’C

0	1	0	A’BC’
0	1	1	A’BC
1	0	0	AB’C’
1	0	1	AB’C
1	1	0	ABC’
1	1	1	ABC

So, if the number of variables is n, then the possible number of minterms is 2n. The main property of a minterm is that it possesses the value of 1 for only one combination of n input variables and the rest of the 2n – 1 combinations have the logic value of 0. This means, for the above three variables example, if A = 0, B = 1, C = 1 i.e., for input combination of 011, there is only one combination A′BC that has the value 1, the rest of the seven combinations have the value 0.

Canonical Sum of Product Expression:

When a Boolean function is expressed as the logical sum of all the minterms from the rows of a truth table, for which the value of the function is 1, it is referred to as the canonical sum of product expression. The same can be expressed in a compact form by listing the corresponding decimal-equivalent codes of the minterms containing a function value of 1.

For example, if the canonical sum of product form of a three-variable logic function F has the minterms A′BC, AB′C, and ABC′, this can be expressed as the sum of the decimal codes corresponding to these minterms as below.

F (A,B,C) = (3,5,6)

= m3 + m5 + m6

= A′BC + AB′C + ABC′

where Σ (3,5,6) represents the summation of minterms corresponding to decimal codes 3, 5, and 6. The canonical sum of products form of a logic function can be obtained by using the following procedure:

1. Check each term in the given logic function. Retain if it is a minterm, continue to examine the next term in the same manner.

2. Examine for the variables that are missing in each product which is not a minterm. If the missing variable in the minterm is X, multiply that minterm with (X+X′).

2. Multiply all the products and discard the redundant terms.

4.2 Maxterm

A sum term containing all n variables of the function in either true or complemented form is called the maxterm. Each maxterm is obtained by an OR operation of the variables in their true form or complemented form. Four different combinations are possible for a two-variable function, such as, A′ + B′, A′ + B, A + B′, and A + B. These sum terms are called the standard sums or maxterms. Note that, in the maxterm, a variable will possess the value 0, if it is in true or uncomplemented form, whereas, it contains the value 1, if it is in complemented form. Like minterms, for a three-variable function, eight maxterms are also possible as listed in the following table

A	B	C	Maxterm
0	0	0	A+B+C
0	0	1	A+B+C’
0	1	0	A+B’+C
0	1	1	A+B’+C’
1	0	0	A’+B+C
1	0	1	A’+B+C’
1	1	0	A’+B’+C
1	1	1	A’+B’+C’

So, if the number of variables is n, then the possible number of maxterms is 2n. The main property of a maxterm is that it possesses the value of 0 for only one combination of n input variables and the rest of the 2n –1 combinations have the logic value of 1. This means, for the above three variables example, if A = 1, B = 1, C = 0 i.e., for input combination of 110, there is only one combination A′ + B′ + C that has the value 0, the rest of the seven combinations have the value 1.

Canonical Product of Sum Expression:

When a Boolean function is expressed as the logical product of all the maxterms from the rows of a truth table, for which the value of the function is 0, it is referred to as the canonical product of sum expression. The same can be expressed in a compact form by listing the corresponding decimal equivalent codes of the maxterms containing a function value of 0. For example, if the canonical product of sums form of a three-variable logic function F has the maxterms A + B + C, A + B′ + C, and A′ + B + C′, this can be expressed as the product of the decimal codes corresponding to thesemaxterms as below,

F (A,B,C) = Π (0,2,5)

= M0 M2 M5

= (A + B + C) (A + B′ + C) (A′ + B + C′)

where Π (0,2,5) represents the product of maxterms corresponding to decimal codes 0, 2, and 5. The canonical product of sums form of a logic function can be obtained by using the following procedure.

1. Check each term in the given logic function. Retain it if it is a maxterm, continue to examine the next term in the same manner.

2. Examine for the variables that are missing in each sum term that is not a maxterm. If the missing variable in the maxterm is X, add that maxterm with (X.X′).

3. Expand the expression using the properties and postulates as described earlier and discard the redundant terms. Some examples are given here to explain the above procedure.

5. **Boolean Function**

Boolean algebra deals with binary variables and logic operation. A **Boolean Function** is described by an algebraic expression called **Boolean expression** which consists of binary variables, the constants 0 and 1, and the logic operation symbols. Consider the following example

$$\underbrace{F(A, B, C, D)}_{\text{Boolean Function}} = \underbrace{A + \overline{BC} + ADC}_{\text{Boolean Expression}} \qquad \text{Equation No. 1}$$

5.1 Truth Table Formation

A truth table represents a table having all combinations of inputs and their corresponding result.

It is possible to convert the switching equation into a truth table. For example, consider the following switching equation.

$$F(A, B, C) = A + BC$$

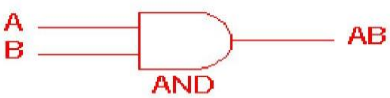
The output will be high (1) if A = 1 or BC = 1 or both are 1. The truth table for this equation is shown by Table (a). The number of rows in the truth table is 2^n where n is the number of input variables (n=3 for the given equation). Hence there are 2^3 = 8 possible input combination of inputs.

Inputs			Output
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

6. DIGITAL LOGIC GATES

A large number of electronic circuits (in computers, control units, and so on) are made up of logic gates.Digital systems are said to be constructed by using logic gates. These process signals which represent true or false. The basic gates are the AND, OR, NOT gates. The most common symbols used to represent logic gates are shown below.

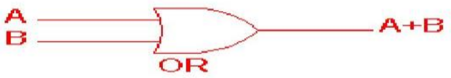
AND gate:



2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high. A dot (.) is used to show the AND operation i.e. A.B. Bear in mind that this dot is sometimes omitted i.e. AB.

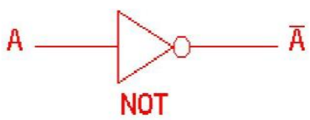
OR gate:



2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

The OR gate is an electronic circuit that gives a high output (1) if **one or more** of its inputs are high. A plus (+) is used to show the OR operation.

NOT gate:



NOT gate	
A	$\bar{A}$
0	1
1	0

7. Simplification of Boolean Expressions

Minimization of Boolean functions is an approach where a given Boolean expression can be transformed from one form to another equivalent form by applying Boolean Theorems. By minimizing the expressions the individual components used in electrical circuits can be minimized or reduced. This allows designers to make use of fewer components, thus reducing the cost of a particular system. It should be noted that there are no fixed rules that can be used to minimize a given expression. It is left to an individual's ability to apply Boolean Theorems in order to minimize a function.

Examples:

Example 1:

Using Boolean algebra techniques, simplify the expression X . Y + X (Y + Z) + Y (Y + Z)

Solution:

Given: X . Y + X (Y + Z) + Y (Y + Z).
Applying distributive property, we get

X . Y + X (Y + Z) + Y (Y + Z) = X . Y + X . Y + X . Z + Y . Y + Y . Z

We know B . B = B
= X . Y + X . Y + X . Z + Y + Y . Z

We know A . B + A . B = A . B
= X . Y + X . Z + Y + Y . Z
= X . Y + X . Z + Y [We know (B + BC = B)]
= Y + XZ

Example 2:

Using Boolean algebra techniques, simplify this expression: AB + A(B + C) + B(B + C)

Solution

Apply the distributive law to the second and third terms in the expression, as follows:

AB + A(B + C) + B(B + C)=AB + AB + AC + BB + BC =AB + AB + AC + B + BC
[BB = B] =AB + AC + B + BC [AB + AB = AB] =AB + AC + B [B
+ BC = B] =B+AC
[AB + B = B]

Example 3:

Using Boolean algebra techniques, simplify this expression A.B' + A.B + B.C

Solution

A.B' + A.B + B.C= A. (B' + B) + B.C
= A.1 + B.C
= A + B.C

Example 4:

Using Boolean algebra techniques, simplify this expression A'.B.C + A.B'.C + A.B.C' + A.B.C

Solution:

A'.B.C + A.B'.C + A.B.C' + A.B.C = A'.B.C + A.B'.C + A.B.C' + A.B.C + A.B.C + A.B.C
= (A'.B.C + A.B.C) + (A.B'.C + A.B.C) + (A.B.C' + A.B.C)
= (A' + A). B.C + (B' + B). C.A + (C' + C). A.B
= B.C + C.A + A.B

7.1 STANDARD FORMS OF BOOLEAN EXPRESSIONS

All Boolean expressions, regardless of their form, can be converted into either of two standard forms: the sum-of-products form or the product-of-sums form.

Standardization makes the evaluation, simplification, and implementation of Boolean expressions much more systematic and easier.

7.1.1 The Sum-of-Products (SOP) Form

When two or more product terms are summed by Boolean addition, the resulting expression is a sum-of-products (SOP). Some examples are:

AB + ABC
ABC + C'DE + B'CD'
AB + BCD + AC

Also, an SOP expression can contain a single-variable term, as in

A + ABC' + BCD'.

In an SOP expression a single over bar cannot extend over more than one variable.

Example

Convert each of the following Boolean expressions to SOP form:
(a) AB + B(CD + EF)
(b) (A + B)(B + C + D)
(c) [(A + B)′ + C+′

The Standard SOP Form

So far, you have seen SOP expressions in which some of the product terms do not contain all of the variables in the domain of the expression.

For example, the expression A′BC′ + AB′D + ABC′D′ has a domain made up of the variables A, B, C. and D. However, notice that the complete set of variables in the domain is not represented in the first two terms of the expression; that is, D or D′ is missing from the first term and C or C′ is missing from the second term.

A standard SOP expression is one in which all the variables in the domain appear in each product term in the expression. For example, A′BCD′ + ABC′D+ AB′CD are a standard SOP expression.

Converting Product Terms to Standard SOP:

Each product term in an SOP expression that does not contain all the variables in the domain can be expanded to standard SOP to include all variables in the domain and their complements. As stated in the following steps, a nonstandard SOP expression is converted into standard form using Boolean algebra rule → (A + A′ = 1) i.e., A variable added to its complement equals 1.

Step 1: Multiply each nonstandard product term by a term made up of the sum of a missing variable and its complement. This results in two product terms. As you know, you can multiply anything by 1 without changing its Value.

Step 2: Repeat Step 1 until all resulting product terms contain all variables in the domain in either complemented or uncomplemented form. In converting a product term to standard form, the number of product terms is doubled for each missing variable.

Example
Convert the following Boolean expression into standard SOP form: AB′C + A′B′ + ABC′D

Solution

The domain of this SOP expression A, B, C, D. Take one term at a time.

The first term, ABC, is missing variable D or D′, so multiply the first term by(D + D) as follows: AB′C =
AB′C(D + D′) = AB′CD + AB′CD′
In this case, two standard product terms are the result.

The second term, A′B′; is missing variables C or C′ and D or D′, so first multiply the second term by C + C′ as follows:
A′B′ = A′B′(C + C′) = A′B′C + A′B′C′

The two resulting terms are missing variable D or D′, so multiply both terms by (D + D) as follows
A′B′C(D + D′) + A′B′C′(D + D′)= A′B′CD + A′B′CD′ + A′B′C′D + A′B′C′D′

In this case, four standard product terms are the result.

The third term, ABC′D, is already in standard form. The complete standard SOP form of the original expression is as follows:
AB′C + A′B′ + ABC′D = AB′CD + AB′CD ‘+ A′ B′CD + A′B′CD′ + A′B′C′D +A′B′C′D′ + ABC′D

7.1.2 The Product-of-Sums (POS) Form

A sum term was defined before as a term consisting of the sum (Boolean addition) of literals (variables or their complements). When two or more sum terms are multiplied, the resulting expression is a product-of-sums (POS). Some examples are

(A' + B)(A + B' + C)
(A + B' + C')(C + D' + E)(B + C + D) (A +
B')(A + B' + C)(A + C)

A POS expression can contain a single-variable term, as inA(A + B + C)(B + C + D).

In a POS expression, a single over bar cannot extend over more than one variable; however, more than one variable in a term can have an over-bar. For example, a POS expression can have the term A' + B' + C' but not [A + B + C+'.

Implementation of a POS Expression simply requires ANDing the outputs of two or more OR gates. A sum term is produced by an OR operation and the product of two or more sum terms is produced by an AND operation.

The Standard POS Form

So far, you have seen POS expressions in which some of the sum terms do not contain all of the variables in the domain of the expression.

For example, the expression(A' + B + C) (A + B + D') (A + B' + C' + D)has a domain made up of the variables A, B, C, and D. Notice that the complete set of variables in the domain is not represented in e first two terms of the expression; that is, D or D' is missing from the first term and C or C' is missing from the second term.

A standard POS expression is one in which all the variables in the domainappear in each sum term in the expression. For example,(A' + B' + C + D)(A + B' + C + D)(A + B + C + D)is a standard POS expression.

Converting a Sum Term to Standard POS

Each sum term in a POS expression that does not contain all the variables in the domain can be expanded to standard form to include all variables in the domain and their complements. As stated in the following steps, a Nonstandard POS expression is converted into standard form using Boolean algebra rule → (A'. A = 0) i.e., A variable multiplied to its complement equals 0.

Step 1. Add to each nonstandard product term a term made up of the product of the missing variable and its complement. This results in two sum terms. As you know, you can add 0 to anything without changing its value.

Step 2. Apply rule A + BC = (A + B)(A + C).

Step 3. Repeat Step 1 until all resulting sum terms contain all variables in the domain in either complemented or non-complemented form.

Example

Convert the following Boolean expression into standard POS form:(A' + B + C)(B' + C + D')(A + B' + C' + D)

Solution

The domain of this POS expression is A, B, C, D. Take one term at a time.

The first term, A + B + C, is missing variable D or D', so add D'D and apply rule as follows: A' + B +
C = A' + B + C + D'D
= (A' + B + C + D')(A' + B + C + D)

The second term, B' + C + D', is missing variable A or A', so add A'A andapply rule as follows: B' + C +
D' = B' + C + D' + A'A

= (A’ + B’ + C + D’)(A + B’ + C + D’)

The third term, A + B’ + C’ + D, is already in standard form. The standardPOS form of the original expression is as follows:

(A’ + B + C)(B’ + C + D’)(A + B’ + C’ + D) = (A’ + B + C + D’)(A’ + B + C +D) (A’ + B’ + C + D’)(A + B’ + C + D’) (A + B’ + C’ + D)

7.2 CANONICAL FORMS OF BOOLEAN EXPRESSIONS

With one variable x & x.

With two variables x y, x y, x y and x y.

With three variables x’ y’ z’, x’ y’ z, x’ y z’, x’ y z, x y’ z’, x y’ z, x y z’ & x y z.

These eight AND terms are called Minterms.

X	Y	Z	MINTERM	DESIGNATION
0	0	0	X’Y’Z’	m0
0	0	1	X’Y’Z	m1
0	1	0	X’YZ’	m2
0	1	1	X’YZ	m3
1	0	0	XY’Z’	m4
1	0	1	XY’Z	m5
1	1	0	XYZ’	m6
1	1	1	XYZ	m7

Maxterm is the complement of its corresponding minterm and vice versa

X	Y	Z	MAXTERMS	DESIGNATION
0	0	0	X+Y+Z	M0
0	0	1	X+Y+Z’	M1
0	1	0	X+Y’+Z	M2
0	1	1	X+Y’+Z’	M3
1	0	0	X’+Y+Z	M4
1	0	1	X’+Y+Z’	M5
1	1	0	X’+Y’+Z	M6
1	1	1	X’+Y’+Z’	M7

For example the function F (for minterms)

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

F = x’ y’ z + x y’ z’ + x y z F = m1 + m4 + m7

Any Boolean function can be expressed as a sum of minterms (sum of products **SOP**) or product of maxterms (product of sums **POS**).

For example the function F (for maxterms)

F’ = x’ y’ z’ + x’ y z’ + x’ y z + x y’ z + x y z’

The complement of F’ = (F’)’= F

F = (x + y + z) (x + y' + z) (x + y' + z') (x' + y + z') (x' + y' + z)
F = M0M2M3M5M6

Example 1
Express the Boolean function F = A + B'C in a sum of minterms (SOP).
Solution
The term A is missing two variables because the domain of F is (A, B, C)
A = A(B + B') = AB + AB' because B + B' = 1
BC missing A, so
B'C(A + A') = ABC + A'B'C
AB(C + C') = ABC + ABC'
AB'(C + C') = AB'C + AB'C'
F = ABC + ABC' + AB'C + AB'C' + ABC + A'B'C
Because A + A = A
F = ABC + ABC' + AB'C + AB'C' + A'B'C
F = m7 + m6 + m5 + m4 + m1
In short notation
F(A, B, C) = Σ(1, 4, 5, 6, 7)
F' (A, B, C) = Σ(0, 2, 3)

- ➔ **The complement of a function expressed as the sum of minterms equalto the sum of minterms missing from the original function.**

Truth table for F = A + B'C

	A	B	C	B'	B'C	F
0	0	0	0	1	0	0
1	0	0	1	1	1	1
2	0	1	0	0	0	0
3	0	1	1	0	0	0
4	1	0	0	1	0	1
5	1	0	1	1	1	1
6	1	1	0	0	0	1
7	1	1	1	0	0	1

Example 2
Express F = xy + x'z in a product of maxterms form.

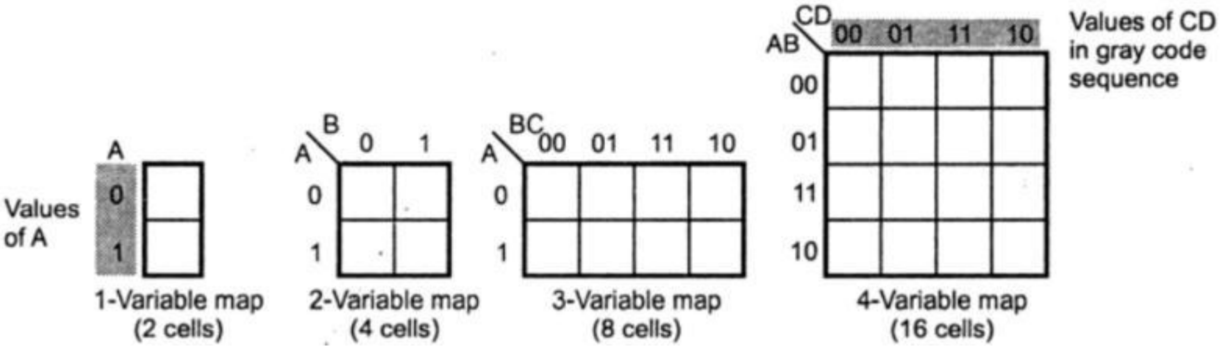
Solution
F = xy + x'z = (xy + x')(xy + z) = (x + x')(y + x')(x + z)(y + z) remember x + x' = 1
F = (y + x')(x + z)(y + z)
F = (x' + y + zz')(x + yy' + z)(xx' + y + z)
F = (x '+ y + z)(x' + y + z')(x + y + z)(x + y' + z)(x + y +z)(x' + y + z) F = (x' + y + z)(x' + y + z')(x + y + z)(x + y' + z)
F = M4 M5M0M2 F(x, y, z) = Π(0, 2, 4, 5)
F(x, y, z) = Π(1, 3, 6, 7)
The complement of a function expressed as the product of maxterms equal to the product of maxterms missing from the original function.
To convert from one canonical form to another, interchange the symbols Σ,Π and list those numbers missing from the original form.

F = M4 M5M0M2 = m1 + m3 + m6 + m7
F(x, y, z) = Π(0, 2, 4, 5) = Σ(1, 3, 6, 7)

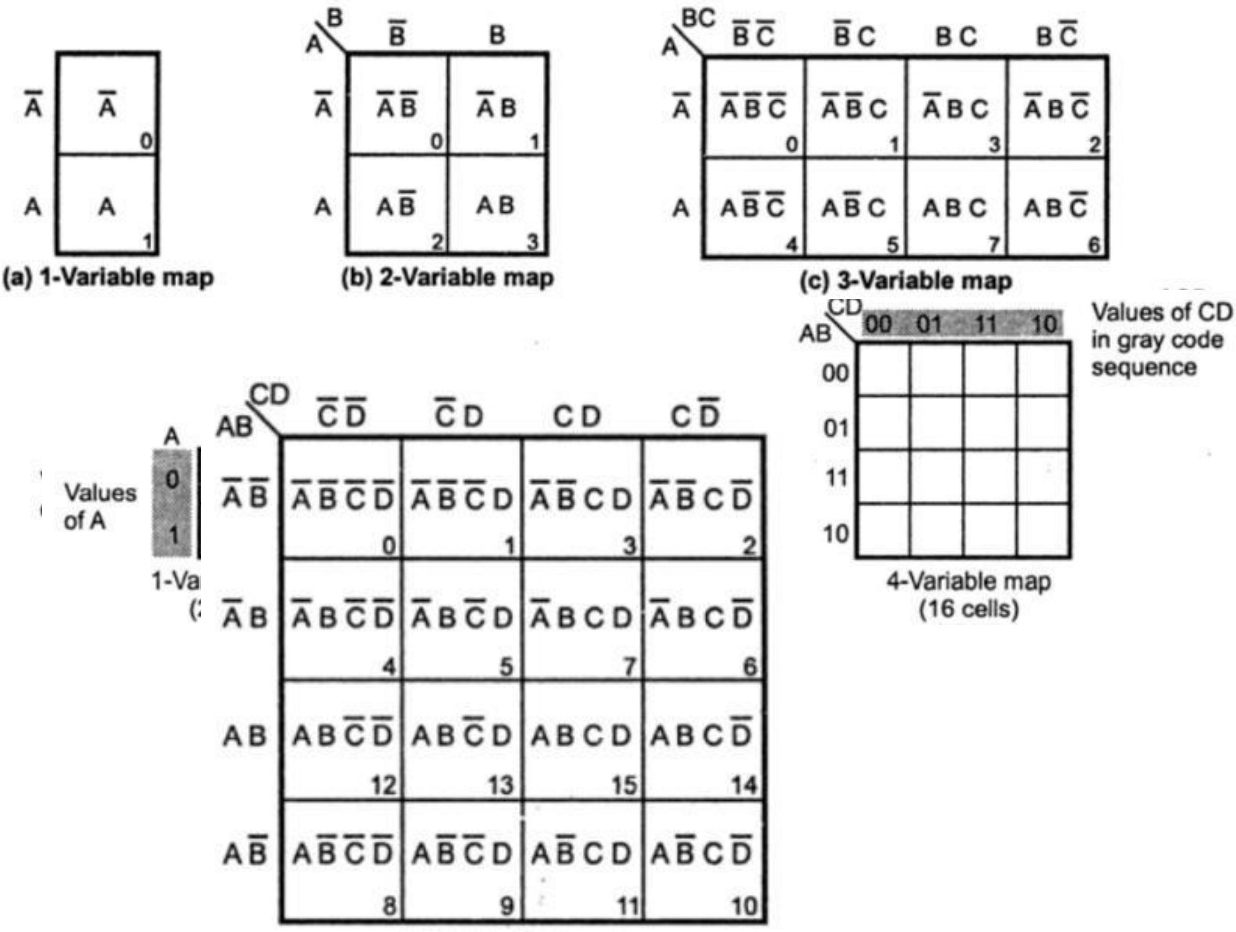
8. Karnaugh Map

Karnaugh map method gives us a systematic approach for simplifying a Boolean expression. Karnaugh map method was first proposed by Veitch and modified by Karnaugh, hence it is known as Karnaugh Map or K-map.

K-map contains boxes called cells. Each of the cell represents one of the 2^n possible products that can be formed from n variables. A two variable mzp contains 2^2 =4 cells, a three variable contains 2^3 =8 cells and four variable contains 2^4 =16 cells. The following figure shows the outline of 1, 2, 3 and 4 variable maps.

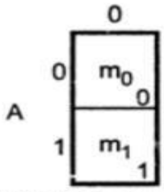


The product term(minterm) assigned to the cells of K-map by labelling each row and column is shown in 1, 2, 3 and 4 variable map and the product term(minterm) corresponding to each cell is shown in the below figure (a),(b),(c) and (d).

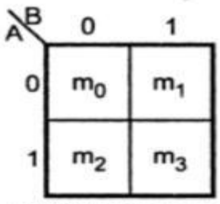


(a) 1-Variable map (b) 2-Variable map (c) 3-Variable map (d) 4-Variable map

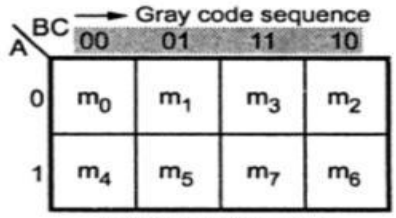
The labelling of the rows and columns of a 1, 2, 3 and 4 variable K-map using Gray code and the product terms(minterm) corresponding to each cell is shown in the figure(a) (b) (c) and (d).



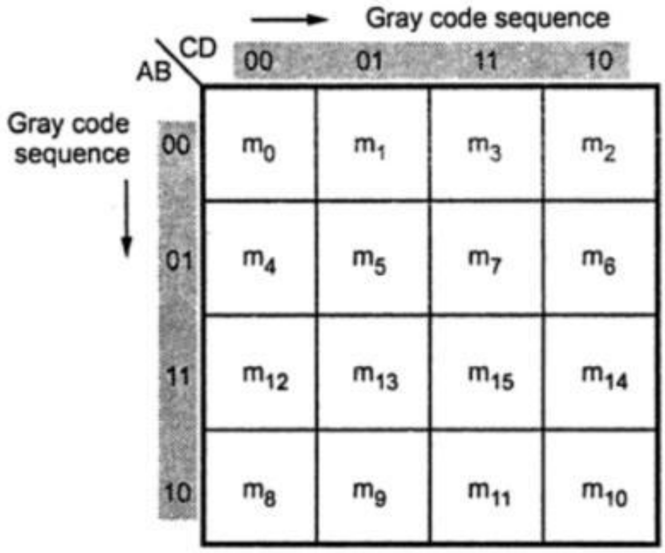
(a) 1-Variable map



(b) 2-Variable map



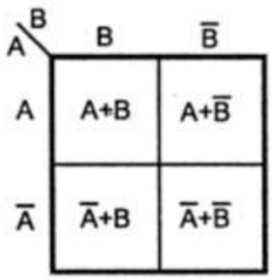
(c) 3-Variable map



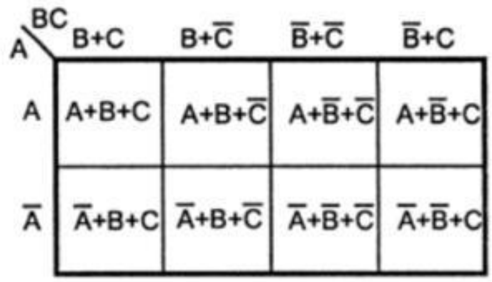
(d) 4-Variable map

The sum term(maxterm) assigned to the cells of K-map by labelling each row and column is shown in 1, 2, 3 and 4 variable map and the sum term(maxterm) corresponding to each cell is shown in the below figure (a),(b),(c) and (d).

(a) 1-Variable map



(b) 2-Variable map



(c) 3-Variable map

AB \ CD	C+D	$C+\overline{D}$	$\overline{C}+\overline{D}$	$\overline{C}+D$
A+B	A+B+C+D	$A+B+C+\overline{D}$	$A+B+\overline{C}+\overline{D}$	$A+B+\overline{C}+D$
$A+\overline{B}$	$A+\overline{B}+C+D$	$A+\overline{B}+C+\overline{D}$	$A+\overline{B}+\overline{C}+\overline{D}$	$A+\overline{B}+\overline{C}+D$
$\overline{A}+\overline{B}$	$\overline{A}+\overline{B}+C+D$	$\overline{A}+\overline{B}+C+\overline{D}$	$\overline{A}+\overline{B}+\overline{C}+\overline{D}$	$\overline{A}+\overline{B}+\overline{C}+D$
$\overline{A}+B$	$\overline{A}+B+C+D$	$\overline{A}+B+C+\overline{D}$	$\overline{A}+B+\overline{C}+\overline{D}$	$\overline{A}+B+\overline{C}+D$

(d) 4-Variable map

The labelling of the rows and columns of a 1, 2, 3 and 4 variable K-map using Gray code and the sum terms(maxterm) corresponding to each cell is shown in the figure(a) (b) (c) and (d)

	0
0	M_0
1	M_1

(a) 1-Variable map

A \ B	0	1
0	M_0	M_1
1	M_2	M_3

(b) 2-Variable map

→ Gray code sequence

A \ BC	00	01	11	10
0	M_0	M_1	M_3	M_2
1	M_4	M_5	M_7	M_6

(c) 3-Variable map

→ Gray code sequence

Gray code sequence ↓

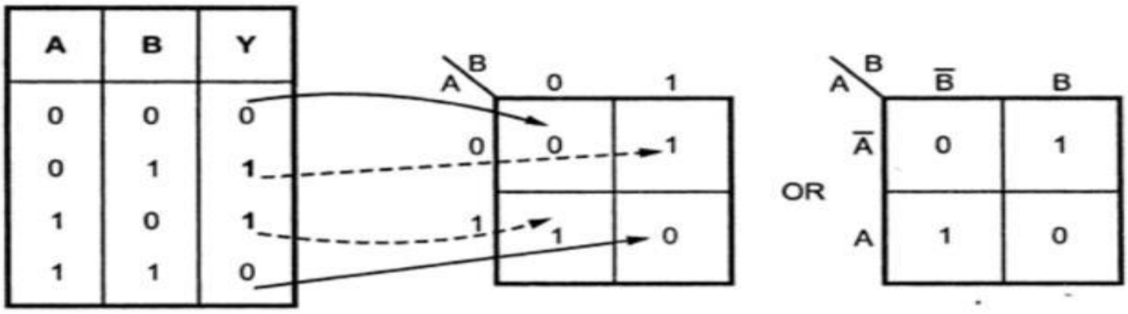
AB \ CD	00	01	11	10
00	M_0	M_1	M_3	M_2
01	M_4	M_5	M_7	M_6
11	M_{12}	M_{13}	M_{15}	M_{14}
10	M_8	M_9	M_{11}	M_{10}

(d) 4-Variable map

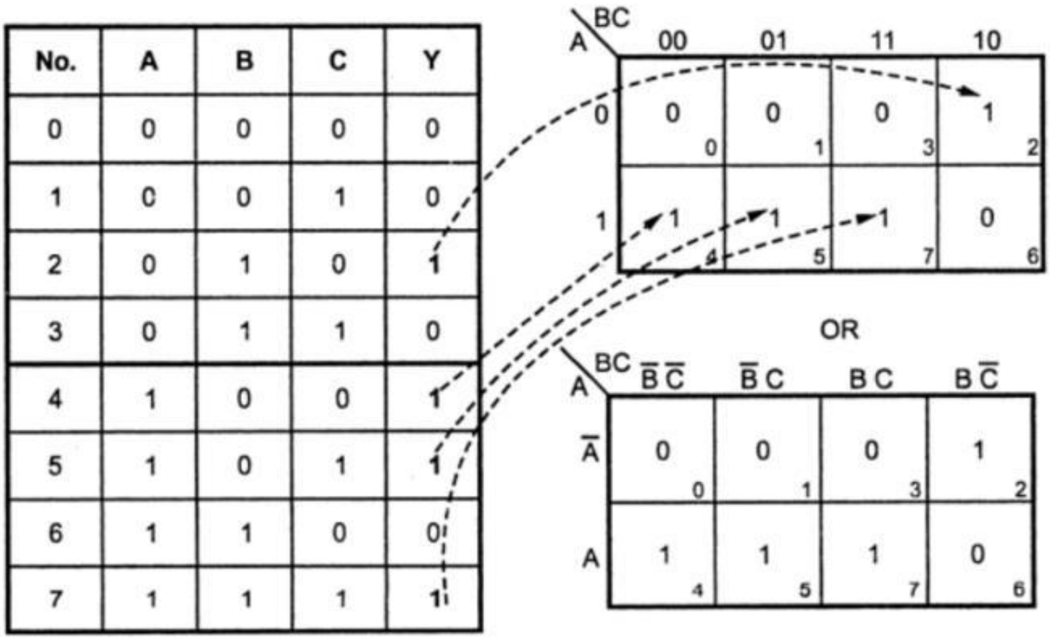
8.1 Plotting a Karnaugh Map

Representation of truth table on K-map

The representation of a two variable truth table on a Karnaugh map is shown below.

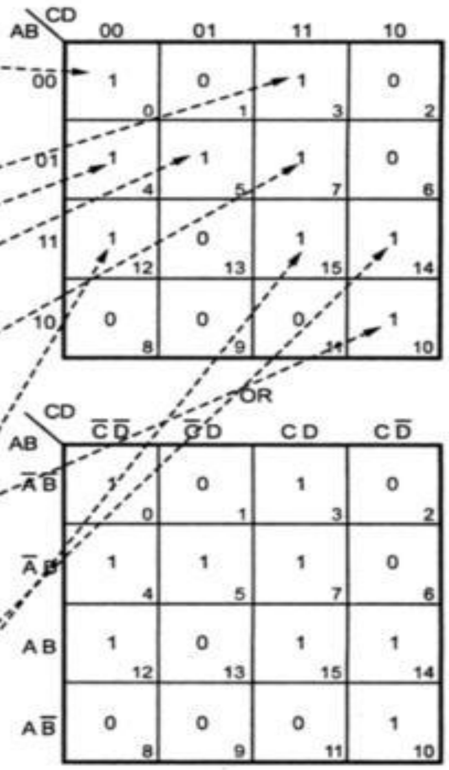


The representation of a three variable truth table on a Karnaugh map is shown below



The representation of a four variable truth table on a Karnaugh map is shown below

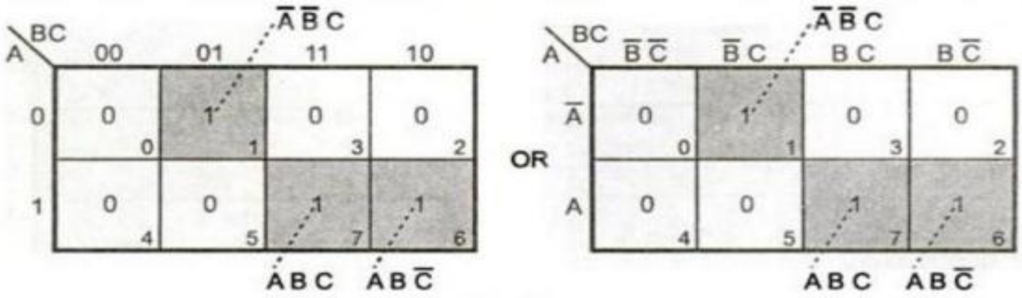
No.	A	B	C	D	Y
0	0	0	0	0	1
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	0
14	1	1	1	0	1
15	1	1	1	1	1



Representation standard SOP on K-map

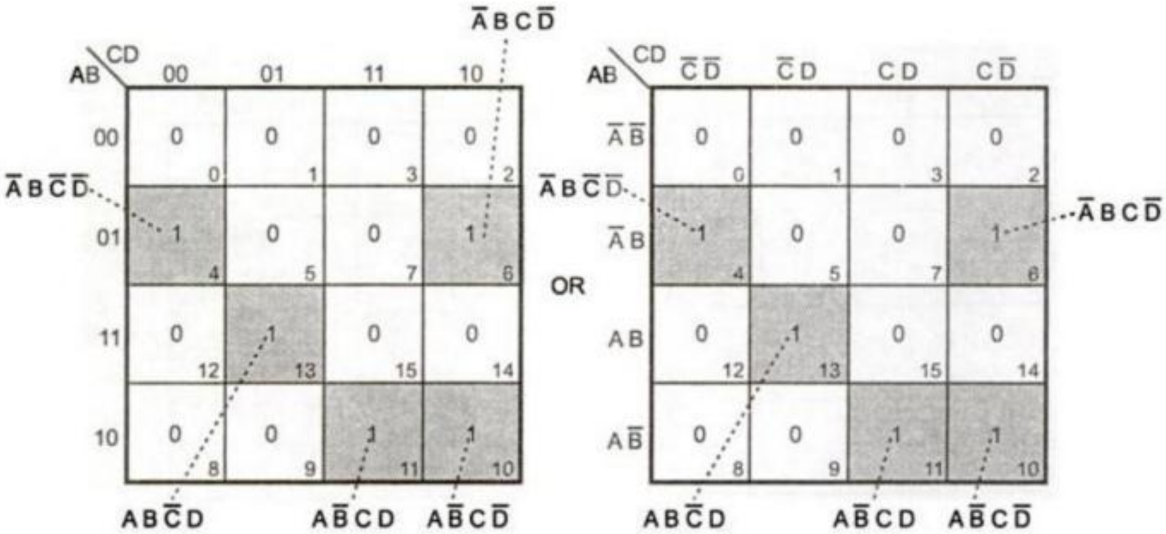
Example 1:

Plot Boolean expression $Y=ABC' + ABC + A'B'C$ on the Karnaugh map



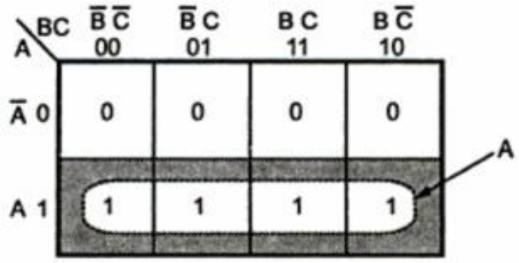
Example 2:

Plot Boolean expression $Y=A'BC'D' + AB'CD + A'BCD' + AB'CD' + ABC'D$ on the karnaugh map.

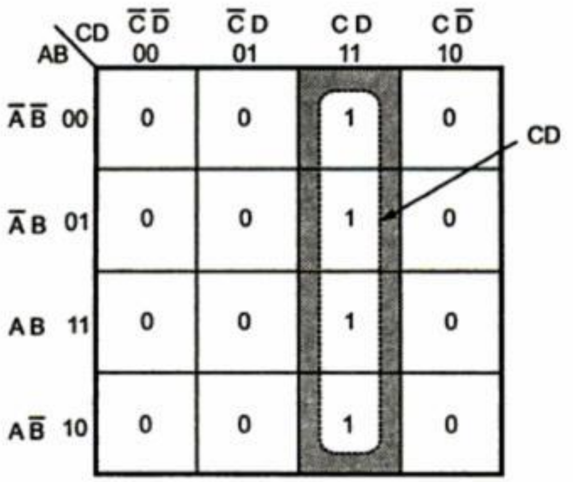


Grouping Cells for Simplification

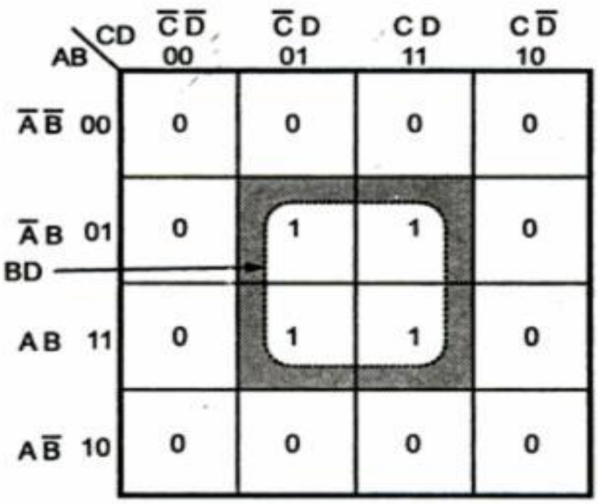
1. Grouping Two adjacent Pairs & Grouping Four adjacent ones (Quad)



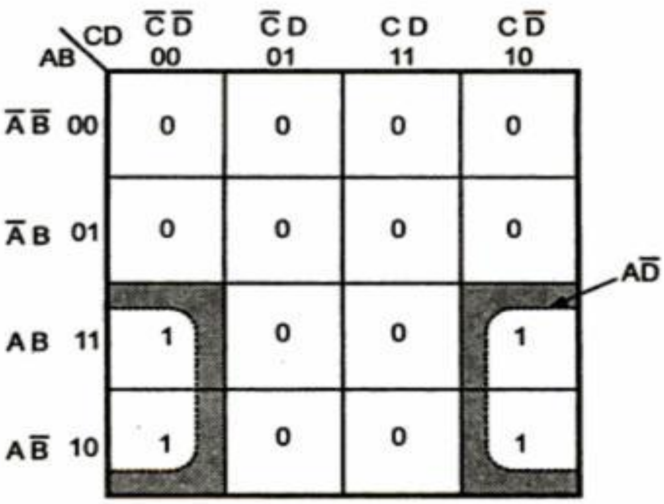
(a) Y = A



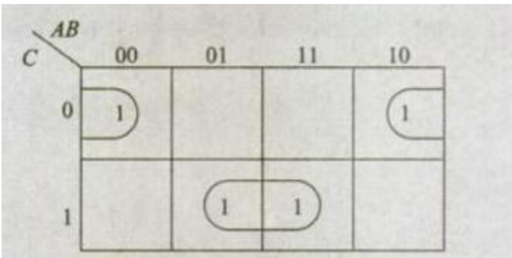
(b) Y = CD

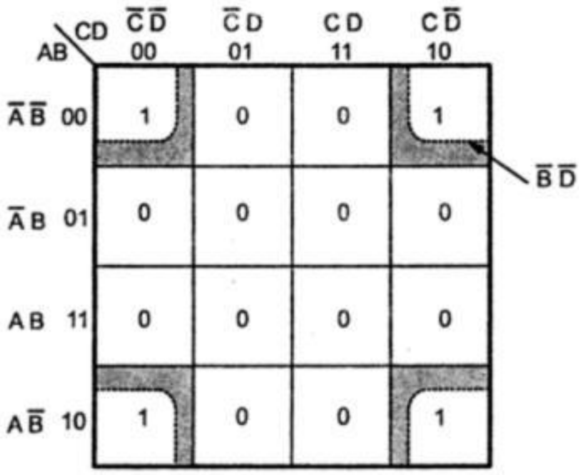


(c) Y = BD

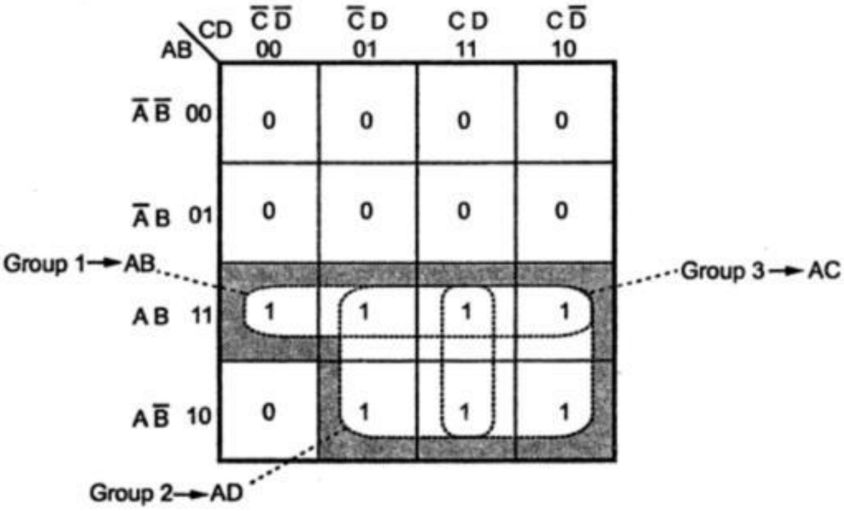


(d) Y = A$\bar{D}$



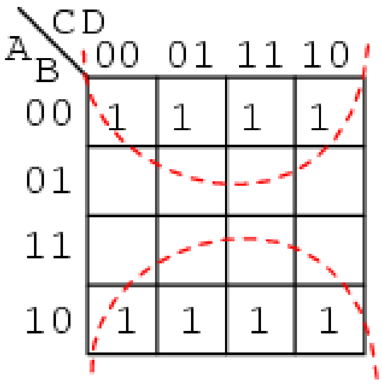


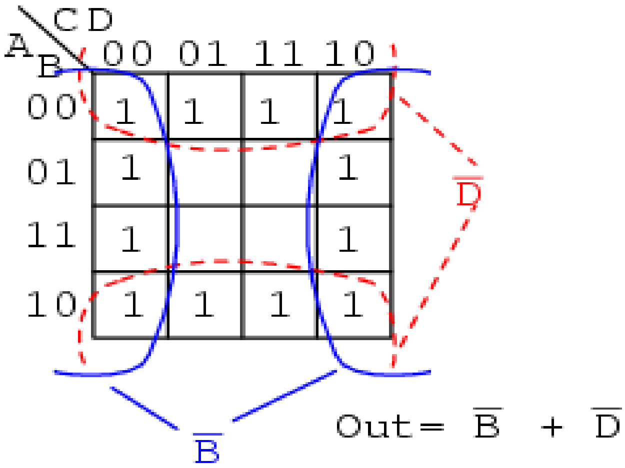
(e) $Y = \bar{B}\bar{D}$



(f) Y = AB + AD + AC

2. Grouping Eight adjacent ones (Octet)

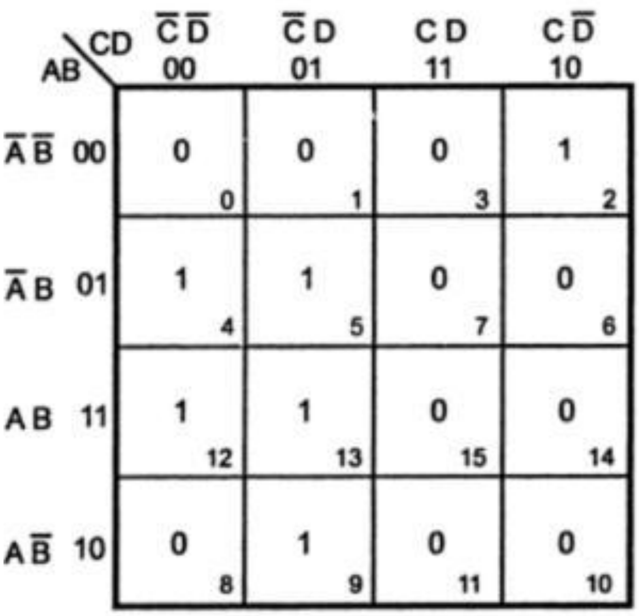


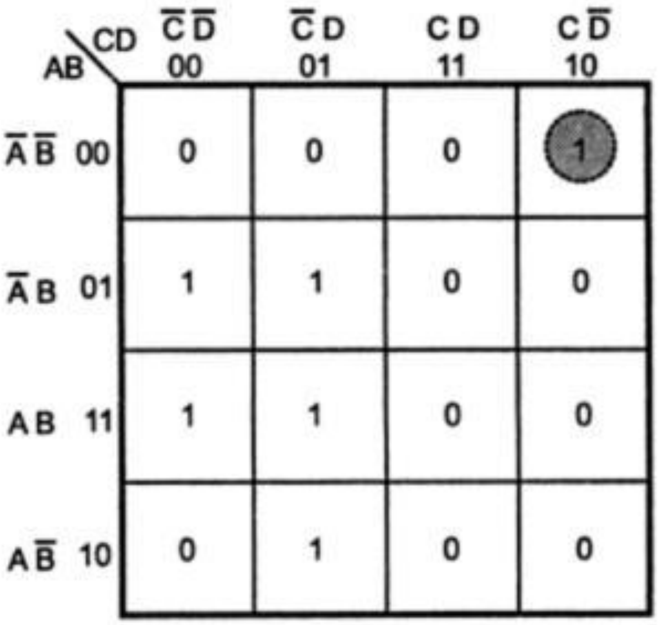


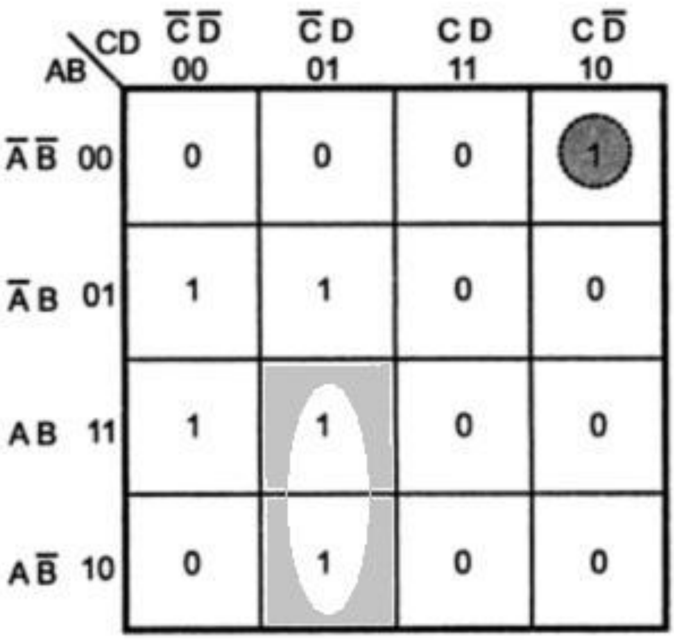
Simplification of Sum of Products Expression (SOP)

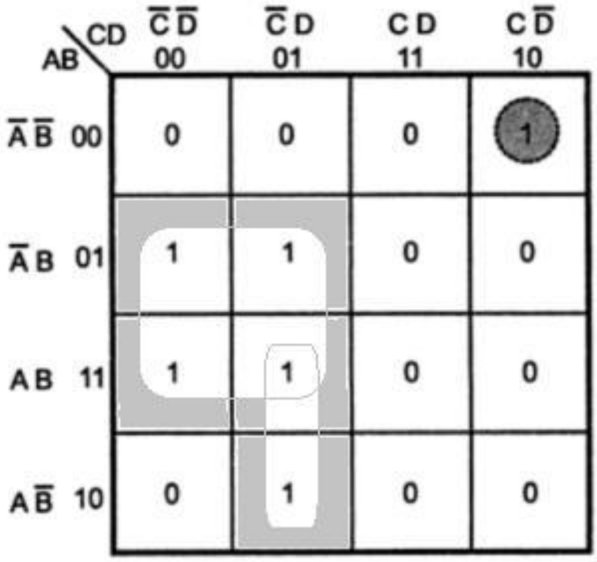
Example 1:

Minimize the Boolean expression $Y = A'BC'D' + A'BC'D + ABC'D' + ABC'D + AB'C'D + A'B'CD'$ on Karnaugh map




AB
CD
$\bar{C}\bar{D}$ 00
$\bar{C}D$ 01
CD 11
$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00
$\bar{A}B$ 01
AB 11
$A\bar{B}$ 10
0 0 0 1
1 1 0 0
1 1 0 0
0 1 0 0


AB
CD
$\bar{C}\bar{D}$ 00
$\bar{C}D$ 01
CD 11
$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00
$\bar{A}B$ 01
AB 11
$A\bar{B}$ 10
0 0 0 1
1 1 0 0
1 1 0 0
0 1 0 0

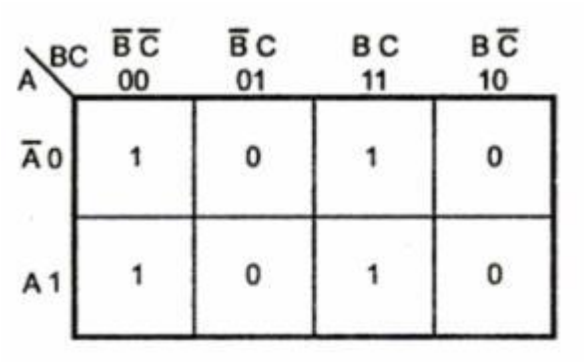


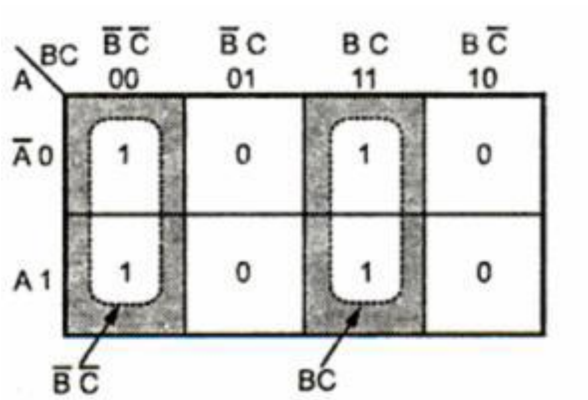
$Y = A'B'CD' + AC'D + BC'$

<u>Example 2:</u>

Simplify the logic function specified by the truth table using Karnaugh map method. Y is the output variable and A,B,C are the input variable

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1





$$Y=B'C'+BC$$

9. DON'T CARE CONDITIONS

- ***An output condition that can be regarded as either high or low***

The logical sum of the minterms associated with a Boolean function specifies the conditions under which the function is equal to 1. The function is equal to 0 for the rest of the minterms. This pair of conditions assumes that all the combinations of the values for the variables of the function are valid. In practice, in some applications the function is not specified for certain combinations of the variables. As an example, the four-bit binary code for the decimal digits has six combinations that are not used and consequently are considered to be unspecified. Functions that have unspecified outputs for some input combinations are called incompletely specified functions. In most applications, we simply don't care what value is assumed by the function for the unspecified minterms. For this reason, it is customary to call the unspecified minterm of a function don't care conditions. These don't care conditions can be used on a map to provide further simplification of the Boolean expression.

A don't care minterm is a combination of variables whose logical value is not specified. Such a minterm cannot be marked with a 1 in the map, because it would require that the function always be a 1 for such a combination. Likewise putting a 0 on the square requires the function to be 0. To distinguish don't care condition from 1's or the 0's an X is used. Thus an X inside a square in the map indicates that we don't care whether the value of 0 or 1 is assigned to F for the particular minterm.

In choosing the adjacent squares to simplify the function in a map the don't care minterms may be assumed to be either 0 0r 1. When simplifying the function, we can choose to include each don't care minterm with either the 1's or the 0's depending on which combination gives the simplest expression.

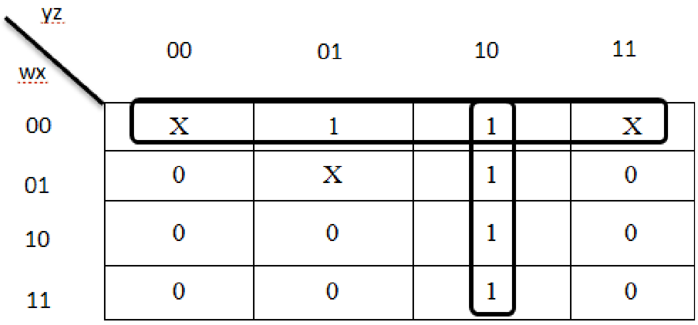
Example Problem:

Simplify the Boolean function F(w,x,y,z) = ∑(1,3,7,11,15) which has the don't care conditions d(w,x,y,z) = ∑(0,2,5).

Solution

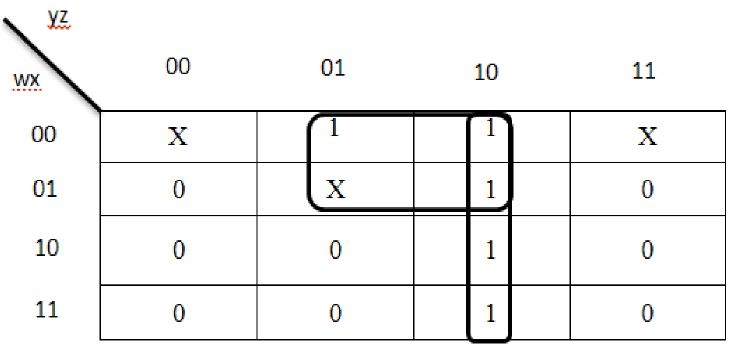
The minterms of F are the variable combinations that make the function equal to 1. The minterms of "d" are don't care minterms that may be assigned either 0 or 1. The map simplification is shown in fig. the minterms of F are marked by 1's. Those of d are marked by X's and remaining squares are filled with 0's.

To get simplified expression in sum-of- product form we must include all five 1's in the map but we may



In the part of the diagram, don't care minterm 0 and 2 is included the units 1's and the simplified function is now

F = yz+w'x'



In the second don't care minterm 5 is included with the 1's , and the simplified function is now

F = yz + w'z

<u>9.1 NAND AND NOR IMPLIMENTATION</u>

Digital circuits are frequently constructed with NAND and NOR gates rather than with AND and OR gates. NAND and NOR gates are easier to fabricate. So rules and procedures have been developed for the conversion from Boolean functions given in terms of AND, OR and NOT into equivalent NAND and NOR logic diagrams.

Two level NAND- NAND implementation

To facilitate the conversion to NAND logic, it is convenient to define an alternative graphic symbol for the gate. The alternate representation of NAND gate is shown in fig. according to De Morgan's theorem

Steps to be followed

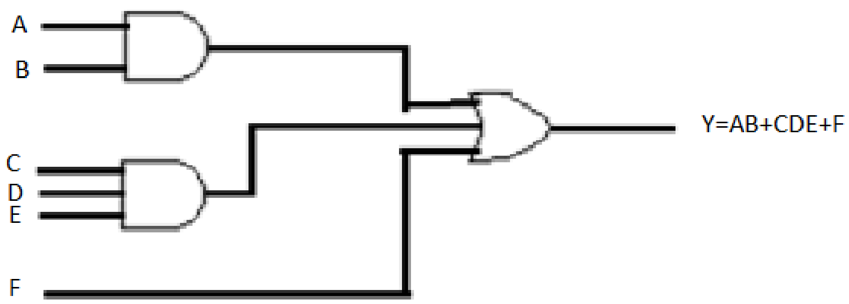
1. Simplify the given logic expression and convert it in the SOP form
2. Draw the logic circuit using AND,OR and NOT gate
3. Replace every AND gate by a NAND gate, Every OR gate by a bubbled OR gate and NOT gate by a NAND inverter.
4. Replace bubbled-OR gate by NAND gate.

Example Problem:

Implement the following Boolean equation using only NAND gatesY=AB+CDE+F

Solution

Step 1: realization using basic gates

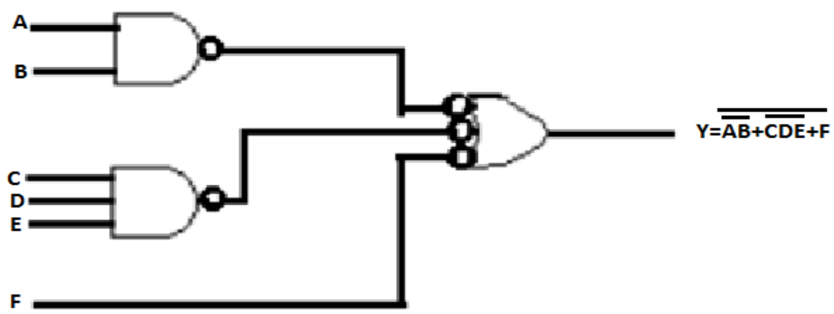


Step 2: replace

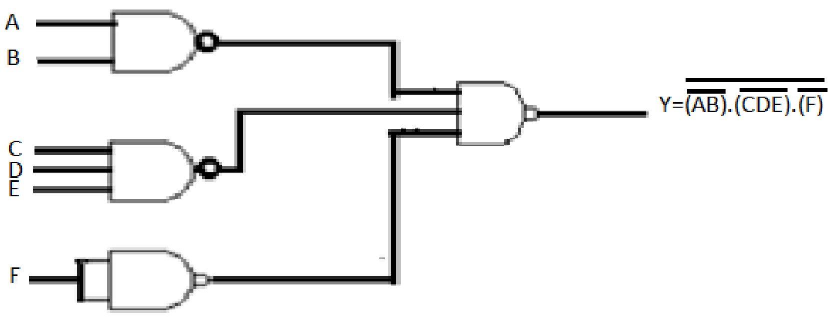
AND →NAND

OR →bubbled – OR

NOT →NAND inverter



Step 3: draw the logic circuit using only NAND gates



9.2 Multilevel NAND circuits

The standard form of expressing Boolean function results in a two-level implementation. If has digital system three or more levels then the most common procedure in the design of multilevel circuits is to express the Boolean function in terms of AND, OR and compliments operations.

The general procedure for converting multilevel AND – OR logic diagram into an all NAND logic diagram is as follows

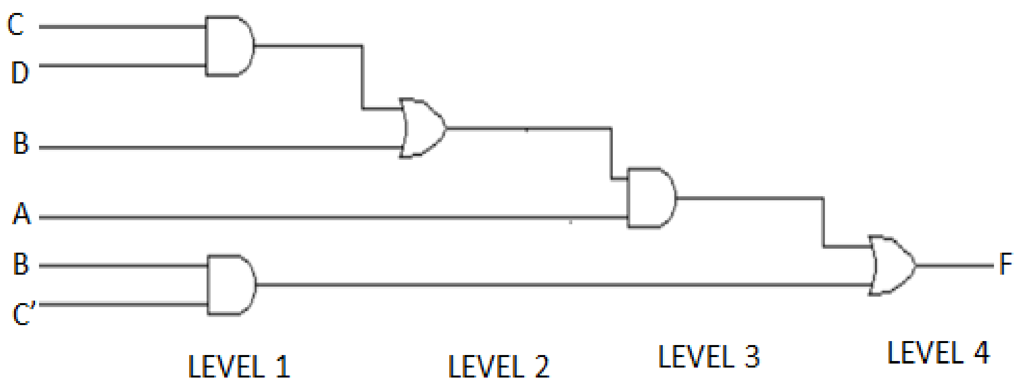
1. Convert all AND gates to NAND gates with AND – invert graphic symbols
2. Convert all OR gates to NAND gates with invert –OR graphic symbol.
3. Check all the bubbles in the diagram. For every bubble that is not compensated by other small circle along the same line insert an inverter or compliment the input literal.

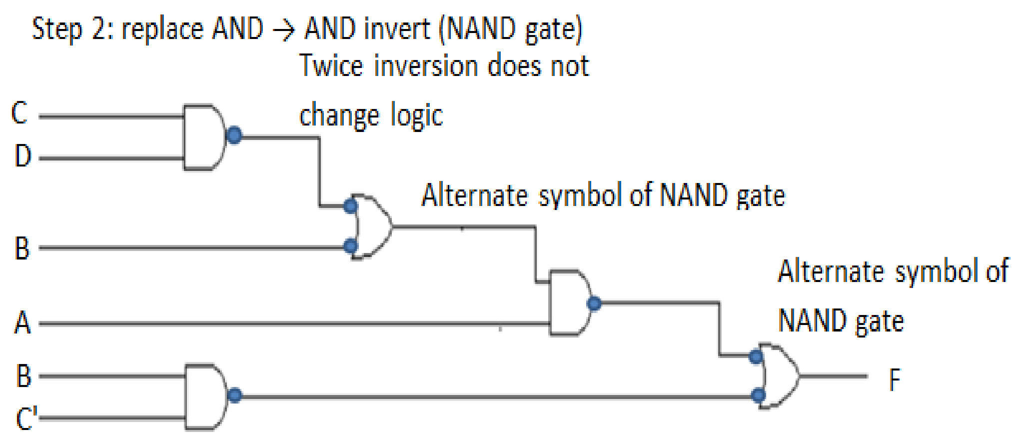
Example Problem:

Implement the following Boolean expression using NAND gates onlyF=A(CD+B)+BC

Solution:

Step 1: Draw logic diagram using AND,OR and NOT gate as shown in the fig.





9.3 NOR IMPLEMENTATION

The NOR operation is the dual of the AND operation. Therefore all procedures and rules for NOR logic are the dual for the corresponding procedures and rules developed for NAND logic. The NOR gate is another universal gate that can be used to implement any Boolean function. The alternative representation of NOR gate according to demorgan's theorem is shown below.

Steps to be followed

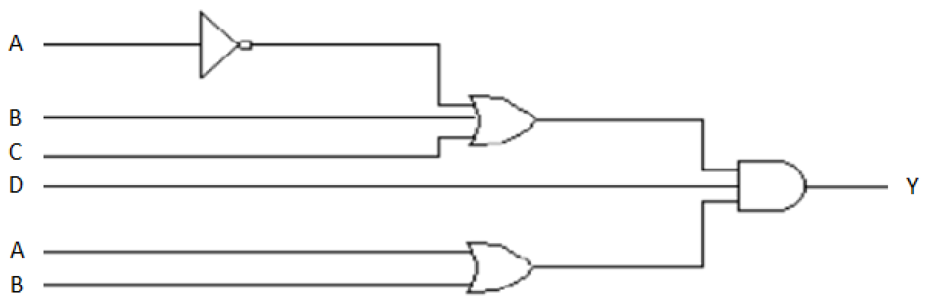
1. Simply the given logic expression and convert it into product of sum (POS) form.
2. Draw the AND – OR-NOT realization.
3. Replace every OR gate by NOR, every AND gate by a bubbled AND gate and ever inerter by a NOR inverter.
4. Draw the final circuit using only the NOR gates.

Example Problem:

Implement the following function by using NOR gates Y=(A'+B+C)(A+B)D

Solution:

Step 1: Implement the given Boolean function by usingAND, ORandNOTgate as shown below.

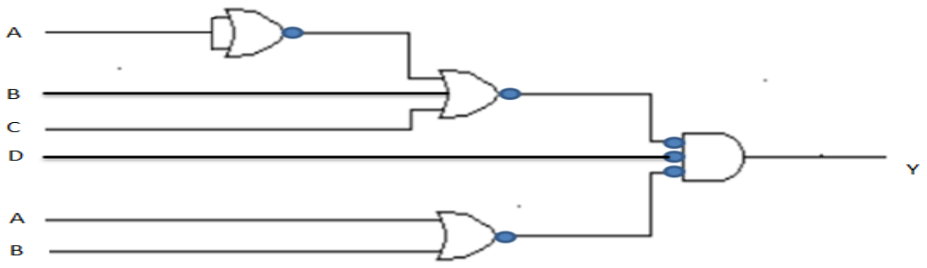


Step 2:

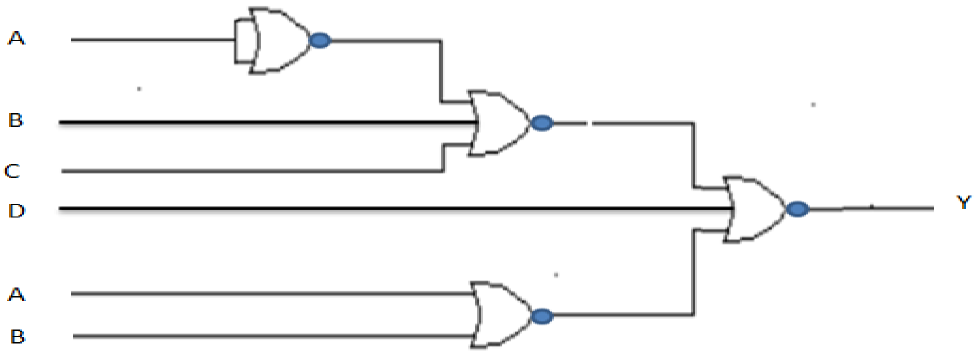
Replace OR → NOR

AND → invert AND

NOT → NOR invert



Step 3: Replace invert AND gate by NOR gate shown in fig.



9.4 MULTILEVEL NOR IMPLEMENTATION

The procedure for converting a multilevel AND-OR diagram to an all NOR diagram is similar to multilevel NAND implements. The following steps are followed for multilevel-NOR implementation

Step 1.impliment the logic function using AND, OR and NOT gate.

Step 2.convert all AND gates to NOR gates with invert-AND graphic symbol.

Step 3.convert all OR gates with OR invert graphic symbols.

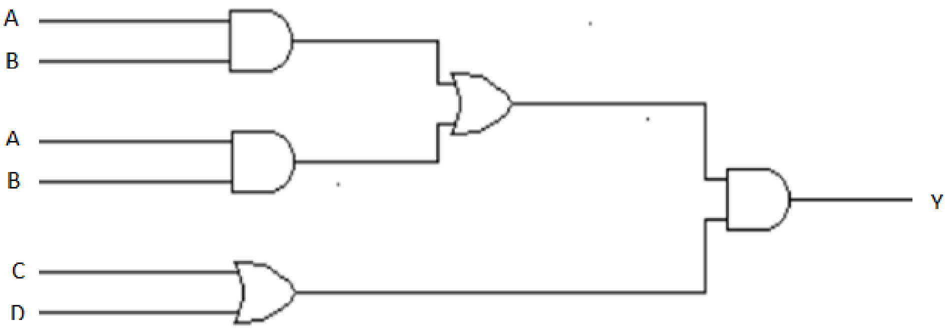
Step 4.Check all the bubbles in the diagram. For every bubble that is not compensated by another small circle along the same line, insert an inverter or compliment the input literal.

Example Problem:

Implement the following Boolean function using NOR gatesY=(AB'+A'B)(C+D')

Solution

Step 1: Implement the Boolean function using AND,OR and NOT gate as shown in fig.

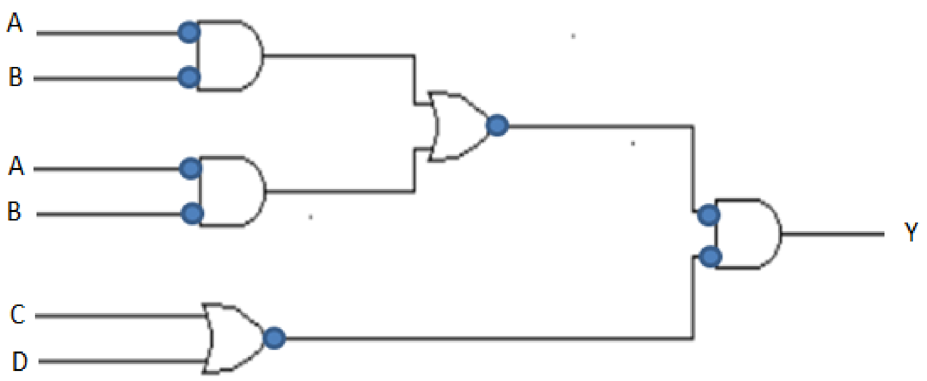


Step 2:

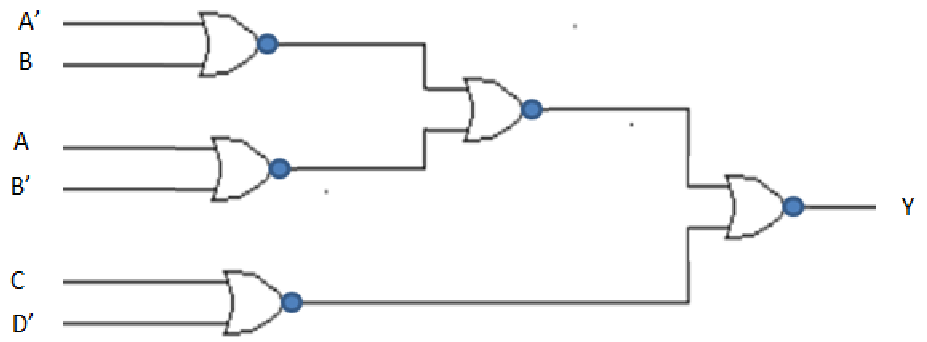
Replace

AND → invert-AND symbol

OR → NOR gate



Step 3: Check each line has even number of bubbles. If any line does not have even number of bubbles the insert bubble (i.e. input **A, B',A', B**has odd number of bubbles. Therefore apply the inverted inputs to make even numbers of bubbles)



10. QUINE-MC CLUSKEY (OR) TABULATION METHOD
11.

Definition: It is used to simplify the Boolean expression for more variables.

The map methodf implication is convenient method as long as the numbers of variables do not exceed fivevariables. If the number of variable increases, it is difficult to make the simplification of expression. F the number of variables increases it is difficult to make the simplification of expression. To avoid this complex and to meet this need W.V. Quine and E.J. McCluskey developed an exact tabulation method to simplify the Boolean expression. This method is called as tabulation method or Quine McCluskey method.

The summary steps are as follows to simplify the Boolean expression.

Step 1. List all minterms in the binary form.

Step 2. Separate the number of groups according to the number of 1's.

Step 3. Compare each binary number with every group in the adjacent next highest category group and they differ only one bit position. Put check mark if comparison is possible (-) and copy remaining term in the next column. Put (√) mark for every comparison.The essential prime implicants are identified if they have no tick mark.

Step 4. Apply the same process described in step 3 for the resultant column and continue the cycles until a single pass through cycle yieldsfurther elimination of literals.

Step 5. From prime implicant chart

(a) The prime implicants should be represented in rowsand each minterm of the function in a column.
(b) Crosses (X) should be placed in each row to show the composition of minterm that makes the prime implicants.
(c) A completed prime implicants table should be inspected for columns containing only a single cross in their columns are called essential prime implicants.

Step 6. Getting the simplified expression after the above step

Example Problem: Simplify the Boolean function by using tabulation method.

F(a,b,c,d)=∑m(0,1,2,5,6,7,8,9,10,14)

Solution

Group	Column I		Column II		Column III
	abcd		abcd		abcd
Group 0	0	0000√	0,1	000√	0,1,8,9-00-
Group 1 Number of 1's one	1	0001√	0,2	00-0√	0,2,8,10-0-0
	2	0010√	0,8	-000√	0,8,1,-00-
	8	1000√	1,5	0-01√	0,8,2,10-0-0
Group 2 Number of 1's two	5	0101√	1,9	-001√	2,6,10,14-10
	6	0110√	2,6	0-10√	2,10,6,14-10
	9	1001√	2,10	-0-10√	
	10	1010√	8,9	100√	
Group 3 Number of 1's three	7	0111√	8,10	10-0√	
	14	1110√	5,7	01-1	
			6,7	011-	
			6,14	=110√	
			10,14	1-10√	

Table: Prime implicant table

Prime implicants	minterms										
		0	1	2	5	6	7	8	9	10	14
1,5	d'c'd		X		X						
5,7	a'bd				X		X				
6,7	a'bc					X	X				
0,1,8,9*	b'c'	X	X					X	X		
0,2,8,10	b'd'	X		X				X		X	
2,10,6,14*	cd'			X		X				X	X
									√		√

Note that the cells (5,7), (1,5), (6,7),(0,1,8,9),(0,2,8,10) and (2,10,6,14)are prime implicants. The prime implicants table can be plotted as shown in table above. All the unticked terms in the above simplification are given as prime implicant of this Boolean expression, these prime implicants chart is shown in table. In the chart all the specified implicants form columns a cross is put in the row of eachprime implicant under the columns of the implicants which it covers.

A tick mark is placed against every essential prime implicant (which column contains a singlecross(X). the sum of essential prime implicantsF = b’c’ + cd’

The prime implicants which covers the minterms 0,1,8,9 and 2,10,6,14 therefore in order to coverthe remaining minterms, the reduced prime implicants chart is formed as follows.

To cover the minterms the prime implicants (6,7) and (0,2,8,10) can be selected in additionto theessential prime implicants for obtaining the minimal Boolean expression is given

$$F = b'c'+cd'+a'bc+b'd'$$

Reduced prime implicant table

Prime implicants	Minterms										
		0	1	2	5	6	7	8	9	10	14
1,5	a’c’d		X		X						
5,7	a’bd				X		X				
6,7	a’bc					X	X				
0,2,8,10*	b’d’	X		X				X		X	
		√		√		√		√		√	

QUESTIONS

Part :A

1. Empirical formula to find the number of parity bits in the Hamming code generation.
2. Point out the gray code for given Binary Numbers i. $(10101101)_2$ ii. $(1010111000)_2$
3. Perform the operation 9 – 3 using 2’s complement.
4. Convert the given non-canonical SOP into canonical form. F = AB’+AC+B’C
5. Brief the redundant literal rule.

Part :B

1. Assess the following conversions
   a. $(12AEF)_{16}$ to $(?)_{10}$ and $(?)_2$
   b. $(11011.0101)_2$ to $(?)_8$ and $(?)_{16}$
   c. $(358.255)_{10}$ to $(?)_8$ and $(?)_{16}$.
2. Suppose that an incoming message 11110101111 is received. Detect the error bit and Correct it with even parity scheme at the receiver end.
3. Point out the Canonical form for the given SOP and POS functions.
   a. F(A,B,C,D)=AB + A’BD + A’CD’
   b. F(A,B,C)=(A + B’) (A + C’)
   c. F(A,B,C)=A (A + C’) (A + B)

4. Reduce the given functions using Boolean algebra techniques.
   a. AB + A(B + C) + B(B + C)
   b. A'B + B (A + C)' + A (B + C)'
   c. A'BC + AB'C' + A'B'C' + AB'C + ABC.

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